Started on	Wednesday, 20 December 2023, 9:00 AM
State	Finished
Completed on	Wednesday, 20 December 2023, 11:53 AM
Time taken	2 hours 53 mins

Question 1
Complete
Marked out of 3.00

If in a binary tree a node has two internal children, then its inorder predecessor internal node has

- a. depth 1
- b. no internal right child
- c. two internal children
- d. no internal left child

Question 2
Complete
Marked out of 3.00

The running time of RadixSort to sort n integers, each represented by D digits in radix base B is Θ (________)

- a. $D(B+n)$
- b. $B^D n$
- c. $n(B+D)$
- d. $B(D+n)$

Question 3
Complete
Marked out of 4.00

An undirected graph G is BIPARTITE if its vertices can be partitioned into two sets X and Y such that every edge in G has one end vertex in X and the other in Y. Based on this definition we can conclude that an undirected graph G is bipartite if and only if

a. G does not have any cycle of length 3 or 5 (length of a cycle is the number of edges on it)
b. G does not have any cycle of length 3 or more (length of a cycle is the number of edges on it)
c. in BFS(G) there is no edge between two vertices with the same BFS-depth
d. in DFS(G) there is no edge between two vertices with the same DFS-depth

Question 4
Complete
Marked out of 3.00

The expected memory space used in a randomized Skip List of n elements is Θ (__________)

a. n^2
b. n
c. $n \log n$
d. $\log n$

Question 5
Complete
Marked out of 3.00

In hashing with open addressing, if the hash table has size N and contains n elements, then the expected number of collisions for any randomly selected element is

a. less than $\frac{N}{N-n}$
b. less than $\frac{n}{2}$
c. less than $\frac{n}{N-1}$
d. less than $\frac{n}{N}$

Question 6

Complete

Marked out of 2.00

A simple undirected graph G is COMPLETE if every pair of distinct vertices in G are adjacent. Suppose G is such a graph with n vertices. Then

- a. In BFS(G) height of the BFS-tree is $O(1)$ and there are $O(n)$ non-tree edges
- b. In DFS(G) there are $(n-1)(n-2)/2$ back-edges,, and every node of the DFS-tree has at most one child
- c. In BFS(G) the root of the BFS-tree has $n-1$ children, and there are $\Theta(n^2)$ BFS back-edges
- d. In DFS(G) every node in the DFS-tree has at most $n-1$ children, there are $\Theta(n)$ DFS back-edges and $\Theta(n^2)$ cross-edges

Question 7

Complete

Marked out of 4.00

Suppose we apply Prim's Minimum Spanning Tree (MST) algorithm to the weighted graph G shown below starting from vertex a. Let T denote the resulting MST.

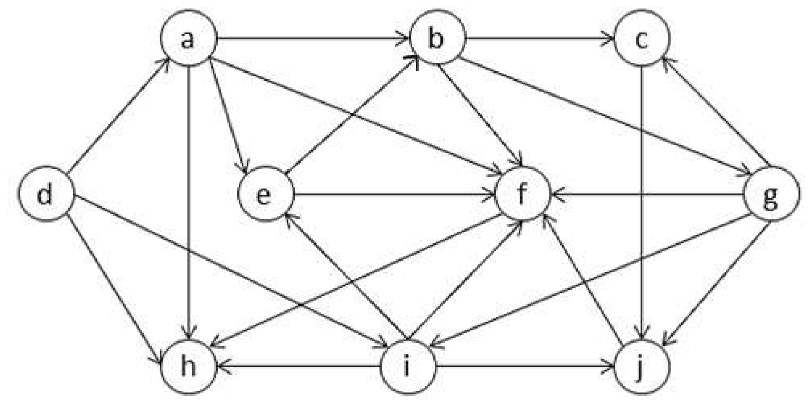
- a. There are 2 edges on the unique simple path in T between vertices f and c
- b. Edge (c, g) is added to T before edge (e, i)
- c. There are 6 edges on the unique simple path in T between vertices d and j
- d. Edge (g, h) is the 5th edge added to T

Question 8

Complete

Marked out of 4.00

Is the following digraph G a DAG? Answer: _________ , because _________



- a. Yes, because G is a simple digraph
- b. No, because DFS(G) has a back-edge
- c. No, because DFS(G) has at least one cross-edge or forward-edge
- d. Yes, because G admits a topological ordering

Question 9

Complete

Marked out of 5.00

We want to insert the keys 35, 55, 15, 25, in that order, into the Splay Tree T shown below. Which of the following is the resulting Splay Tree?

Which of the following is the resulting Splay Tree?

- a.
- b.
- c.

Question 10

Complete

Marked out of 4.00

Suppose T is an initially empty search tree and we insert the sequence of n keys $1, 2, 3, \ldots, n$, in increasing order, into T. Then the height of the resulting search tree T is

- a. $\Theta(n)$ if T is a Splay tree, but is $\Theta(\log n)$ if T is an AVL tree
- b. $\Theta(\log n)$ if T is a Splay tree, but is $\Theta(n)$ if T is a BST
- c. $\Theta(\log n)$, regardless of whether T is a Splay tree or an AVL tree
- d. $\Theta(n)$, regardless of whether T is a BST or a Splay tree or an AVL tree

Question 11

Complete

Marked out of 4.00

An array $A[0..n-1]$ of n arbitrary and unsorted real numbers is given. We want to determine whether more than a third of the array elements have the same value. Worst-case time complexity of the most efficient algorithm that solves this problem is $\Theta($ _________ $)$.

- a. \((n~+~\log n)\log n\)
- b. \(n/3~+~n\log n\)
- c. \(\log n ~+~ n~+~n\log(n/3)\)
- d. \({(~n\sqrt{n}~+~\log n}~)~/~(~{\sqrt{n/3~}~+~3\log n}~)\)

Question 12

Complete

Marked out of 4.00

Let G be a connected weighted graph. Suppose \(T_1\) is a Shortest Path Tree (SPT) of G rooted at some source vertex s, and \(T_2\) is a Minimum Spanning Tree (MST) of G. Now suppose we add a positive constant w to the weight of each edge of G and call the resulting weighted graph G'. Then, as a set of edges in G' ,

- a. \(T_1\) is an SPT of G', but \(T_2\) may not be an MST of G'
- b. \(T_1\) may not be an SPT of G', and \(T_2\) may not be an MST of G'
- c. \(T_1\) is an SPT of G', and \(T_2\) is an MST of G'
- d. \(T_1\) may not be an SPT of G', but \(T_2\) is an MST of G'

Question 13
Complete
Marked out of 3.00

Suppose \(A\) is an \(n\times n\) matrix of arbitrary positive integers. We want to find the largest integer, if any, that appears at least once in each row of \(A\). For example, the answer would be \(4\) for the following matrix: \(A ~~=~~ \left[\begin{matrix} 4 & 1 & 2 & 8 \\ 2 & 5 & 4 & 1 \\ 3 & 4 & 6 & 3 \\ 4 & 7 & 4 & 2 \end{matrix} \right] \) .

This problem can be solved most efficiently in \(\Theta\) (______________) time in the worst-case.

- a. \(n^3\)
- b. \(n\log n\)
- c. \(n\)
- d. \(n^2 \log n\)

Question 14
Complete
Marked out of 4.00

Consider the AVL tree T shown below where keys are shown inside the nodes, but the node heights are not shown. Now suppose we delete 51 according to the dictionary delete algorithm on AVL trees described in the course (i.e., lecture slide video recordings)

- a. Left subtree of the root remains unchanged, depth of 85 remains 3, and 93 becomes right child of 85
- b. 63 becomes the root, and 81 becomes right child of 79
- c. Left subtree of the root remains unchanged, and 71 becomes parent of 79
- d. 63 becomes the root, 71 and 81 become siblings, and 85 remains left child of 93

Question 15
Complete
Marked out of 4.00

Let T be an AVL tree with n elements. Direct each edge of T from parent to child to form a directed graph G with n vertices and n-1 directed edges. Then the transitive closure of G has \(\Theta\) (____________) edges.

- a. \(n^2\)
- b. \(n\log n\)
- c. \(n\)
- d. \(\log n\)

Question 16
Complete
Marked out of 4.00

The time complexity of the following recursive algorithm is Θ (____________)

```
public static double mystery( double x , int n) {
   if (n <= 0) return 4*x+1 ;
   for ( int i = 0 , double prod = 1.0 ; i < n ; i++ )  prod *= x + 5*i ;
   return mystery(prod+x , 3*n/4) + mystery(prod+x+2 , n/4) + x/2 - 3 ;
}
```

- a. 2^n
- b. $n\log n$
- c. $n\log^2 n$
- d. n^2

Question 17
Complete
Marked out of 3.00

Building a min-heap from an array of n items requires _________________ worst-case time.

- a. $\Theta (n\log n)$, since it requires that much time to HeapSort in the worst-case
- b. $\Omega(n\log n)$, based on the universal lower bound on comparison based sorting
- c. $\Theta(n)$, using upHeaps in preorder
- d. $\Theta(n)$, using downHeaps in postorder
- e. $\Theta(n)$, only if the array is already sorted

Question 18
Complete
Marked out of 3.00

The algorithm design technique used in Floyd-Warshall's algorithm is

- a. repeated doubling
- b. divide and conquer
- c. greedy method
- d. dynamic programming

Question 19

Complete

Marked out of 4.00

The maximum number of edges in any simple directed graph with n vertices that is NOT strongly connected is exactly (not just asymptotically) = ____________________ .

a. $(n-1)^2$

b. $n(n-1)/2$

c. $n(n-1)$

d. $(n/2)^2$

Question 20

Complete

Marked out of 2.00

The maximum degree of any vertex in a simple undirected graph with n vertices is

a. $n + 1$

b. $2n - 1$

c. $n - 1$

d. n

Question 21

Complete

Marked out of 2.00

The auxiliary data structure used in BFS(G) is

a. queue

b. stack

c. none of these

d. AVL tree (for efficiency)

Question 22

Complete

Marked out of 3.00

The time complexity of an efficient algorithm to generate the connected components of an undirected graph G with n vertices and m edges, represented by its adjacency list structure is Θ (________)

a. $n (n + m)$

b. $n + m$

c. $n \ast m$

d. n^2

Question 23
Complete
Marked out of 4.00

The link structure of a binary tree can be uniquely determined by its

- a. levelorder and inorder node sequences
- b. preorder and postorder node sequences
- c. inorder and preorder node sequences
- d. postorder and levelorder node sequences

Question 24
Complete
Marked out of 3.00

We have two divide-&-conquer algorithms, let's call them A and B, where A calls B.
Their time complexities, $T_A(n)$ and $T_B(n)$, are expressed by the following recurrence relations

$$T_A(n) ~=~ 4 T_A(n/2) ~~+~~T_B(n)~+~\Theta(n)$$
$$T_B(n) ~=~ 6 T_B(n/3) ~~+~~ \Theta(n^2\log n) .$$

Then

- a. $T_A(n) ~=~\Theta(n^2\log^3 n)$
- b. $T_B(n) ~=~\Theta(n^2\log^2 n)$
- c. $T_A(n) ~=~\Theta(n^2\log^2 n)$
- d. $T_A(n) ~=~\Theta(n^2\log n)$

Question 25
Complete
Marked out of 3.00

In an undirected graph with n vertices and m edges, the sum of the degrees of all n vertices is

- a. $2 m$
- b. $(m^2)/2$
- c. $2n$
- d. $(2n-1)/2$

Question 26
Complete
Marked out of 3.00

Suppose S is a sequence of n numbers, each equal to 0 or 1. The **worst-case** time complexity to sort S by MergeSort is $\Theta (T_1(n))$, and by randomized QuickSort with (L, E, G) tri-partitioning is $\Theta (T_2(n))$, where

- a. $T_1(n) = n\log n$, $T_2(n) = n^2$
- b. $T_1(n) = n\log n$, $T_2(n) = n$
- c. $T_1(n) = n$, $T_2(n) = n\log n$
- d. $T_1(n) = n$, $T_2(n) = n^2$

Question 27

Complete

Marked out of 4.00

Let G = (V, E) be a simple weighted digraph with n vertices and m edges, and non-negative edge weights. Let $s \in V$ be a selected source. We want to rank the vertices from 1 to n based on how close they are to the source vertex, where closeness of a vertex is measured in terms of its shortest path distance from the source. So, vertex s has rank 1. The problem is to find the rank k vertex for a given integer $k\in[1,n]$. This problem can be solved efficiently by

- a. doing a BFS and stopping at level k
- b. doing k Bellman-Ford iterations
- c. restricting Dijkstra's algorithm to k iterations of its main loop
- d. doing k iterations of Floyd-Warshall using the weigted adjacency matrix instead

Question 28

Complete

Marked out of 3.00

The fewest possible number of nodes in an AVL tree of height h is Θ (________)

- a. $h \log h$
- b. $2^{h/2}$
- c. $((1+\sqrt{5})/2)^h$
- d. 2^h

Question 29

Complete

Marked out of 2.00

Linear probing is equivalent to double hashing with secondary hash function $d(k) =$ ______________ .

- a. $(7 - (k \mod 7))$
- b. $(q - (k \mod q))$ where q is a prime number
- c. $(k \mod N)$ where N is the hash table size
- d. 1

Question 30

Complete

Marked out of 3.00

Consider the set of n distinct positive integers $A~=~\{1, 2, 3, \dots , n\}$. Give a permuted sequence S of A so that MergeSort(S) and HeapSort(S) would both require $\Theta(n\log n)$ time to sort S in increasing order, but InsertionSort(S) would only need $\Theta(n)$ time to sort S.

- a. $S~=~[1, 2, 3, ~n , n-1, n-2, n-3, \dots , 4]$ (first 3 in increasing order, last n-3 in decreasing order)
- b. $S~=$ odd integers in A in increasing order followed by even integers in A in increasing order
- c. $S~=~[n, n-1, n-2, ~1, 2, 3, \dots , n-3]$ (first 3 in decreasing order, last n-3 in increasing order)
- d. $S~=$ even integers in A in increasing order followed by odd integers in A in increasing order