Specialist English: Assignment 4 (solutions)

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Here's my solutions; your solutions needn't be identical. To begin with, here's the snippet from the assignment:

> **Random permutation:** Random permutation function is well studied in Combinatorial mathematics and group theory. In Cauchy's two-line notation, there are the preimage elements in the first row, and the image elements of the preimage elements in the second row. The random permutation function can be expressed as:
>
> $$\pi\colon \begin{pmatrix} 1 & \cdots & n \\ p_1 & \cdots & p_n \end{pmatrix} \qquad (2)$$
>
> where $\pi(i) = p_i(i = 1, \cdots, n)$. We define π^{-1} as the inverse function of π. When the condition satisfies $\pi(i) = i$, the permutation is called identical permutation denoted by I. The description of random permutation generation is as follows:
>
> **Algorithm 1** Random Permutation generation
> 1: set $\pi = I$
> 2: **for** $i = n : 2$ **do**
> 3: select a random integer j where $1 \leq j \leq n$;
> 4: swap $\pi(i)$ and $\pi(j)$;
> 5: **end for**

First, the snippet should say "$1 \leq j \leq i$" and not "$1 \leq j \leq n$" in the algorithm—this was my mistake. (Sorry!)

It didn't seem to play a big role in assignment solutions though. Several students are familiar with this algorithm, and they auto-corrected the error. I think it affected one student's answer, so I was more lenient with marking them. It's called the Fisher-Yates Shuffle[1].

Question 1 Why is it improper to say "random permutation function" above?

There's two problems:

- A permutation is, by definition, a function. For example, a permutation of $\{1, \ldots, n\}$ is a function $\pi\colon \{1, \ldots, n\} \to \{1, \ldots, n\}$ which is a bijection. Thus, it's tautological to say "permutation function" (it's like saying "I have a hundred yuan in Chinese currency").
- Another problem is due to confusion with "function" in mathematics (as above) and "function" in computer science (e.g. a random permutation function `random_permutation(n)` might return a random permutation of $\{1, \ldots, n\}$).

  There's two reasons to naturally interpret this phrase as referring to the computer science definition: (a) "permutation function" is tautological in mathematics, and (b) "random permutation function" (instead of "random-permutation function") means a random "permutation function" (not a "random permutation" function). However, the authors talk about its mathematical definition, which is mismatched.

[1] https://en.wikipedia.org/wiki/Fisher-Yates_shuffle

Question 2 Which words are mistakenly capitalized?

There are two problems: the C in "...in Combinatorial mathematics" and the P in "Random Permutation generation" should be lowercase. (Alternatively, we could write "Random Permutation Generation" if we wanted to use title case in Algorithm 1.)

Question 3 What are the "preimage elements", the "image elements", and the "image elements of the preimage elements" of a permutation of $\{1, \ldots, n\}$? So what does the sentence beginning "In Cauchy's two-line notation ..." literally mean?

It seems this question was too hard—I tried to be lenient marking it. It was intended to be an example of where the mathematics and English clash. (I predicted it would be much easier than it actually turned out to be.)

I was expecting an answer along the lines of:

> For a permutation of $\{1, \ldots, n\}$, the "preimage elements", the "image elements", and the "image elements of the preimage elements" are all the same: the elements in $\{1, \ldots, n\}$. Therefore, the sentence beginning "In Cauchy's two-line notation ..." literally means:
>
> > In Cauchy's two-line notation, the elements in $\{1, \ldots, n\}$ are in the first row, and the elements in $\{1, \ldots, n\}$ are in the second row.
>
> While technically correct, it's uninformative: it fails to describe the order of elements in the two rows, the essential part of the notatation.

Another important point is that the authors mistakenly write "preimage elements" instead of "domain elements", and therefore it only happens to be technically correct by luck: "preimage elements" and "domain elements" are equal for permutations.

A lot of students surmised what the authors intended to write, but nobody recognized that what they (literally) wrote does not actually have that meaning. To explain the mathematics in more details:

A permutation is a function $\pi\colon \{1, \ldots, n\} \to \{1, \ldots, n\}$ which is a bijection. The following follow from the definitions:

- Its domain is $\{1, \ldots, n\}$.
- Its "preimage elements" would be interpreted as meaning the elements in the preimage of $\{1, \ldots, n\}$ under π. That is, the elements in

$$\begin{aligned}\pi^{-1}[\{1, \ldots, n\}] &= \{x \in \{1, \ldots, n\} \colon \pi(x) \in \{1, \ldots, n\}\} \\ &= \{1, \ldots, n\},\end{aligned}$$

  since π is a permutation of $\{1, \ldots, n\}$. So its "preimage elements" are the elements in $\{1, \ldots, n\}$.
- Its "image elements" would be interpreted as meaning the elements in the the image of $\{1, \ldots, n\}$ under π. That is, the elements in

$$\pi[\{1, \ldots, n\}] = \{1, \ldots, n\},$$

  since π is a permutation of $\{1, \ldots, n\}$. So its "image elements" are the elements in $\{1, \ldots, n\}$.
- The "image elements of the preimage elements" would be interpreted as meaning the elements in the image of the preimage of $\{1, \ldots, n\}$ under π. This is equal to $\pi[\pi^{-1}[\{1, \ldots, n\}]] = \{1, \ldots, n\}$ from above.

  So its "image elements of the preimage elements" are the elements in $\{1, \ldots, n\}$.

Question 4 Rewrite "... where $\pi(i) = p_i (i = 1, \cdots, n)$" to be more (humanly) readable and technically correct.

I was expecting the answer "... where $\pi(i) = p_i$ for all $i \in \{1, \ldots, n\}$." This corrected several bugs:

- By adding "for all", we clarify the relationship between the equation $\pi(i) = p_i$ and i.
- It separates two mathematical expressions with words.
- It fixes $i = 1, \cdots, n$, which is mathematically incorrect.
- It fixes the LaTeX in $1, \cdots, n$ (typeset `1,\cdots,n`), replacing it with $1, \ldots, n$ (typeset `1,\ldots,n`).

Question 5 How can we eliminate the notation p_i from the sentence beginning "The random permutation function ..."?

The notatation

$$\pi \colon \begin{pmatrix} 1 & \cdots & n \\ p_1 & \cdots & p_n \end{pmatrix}$$

can be rewritten with $\pi(i)$ in place of p_i, and we don't need the notatation p_i, thereby saving the reader's time because they don't need to learn what p_i means.

Question 6 Why is I a poor choice of notation for the identity permutation?

The paper is about matrices, and the notation I ordinarily refers to the identity matrix (not the identity permutation). It's also inconsistent to use a capital I when (a) other permutations are denoted using lowercase π, and (b) capital letters are used to denote matrices elsewhere in the paper.

Some students commented that it could be confused with l or 1; we avoid writing l and instead write ℓ (typeset `$\ell$`) for this reason. I don't think I and 1 are likely to be confused in they are properly formatted.

Some students commented that I is used for "current" in physics—it's an unlikely confusion, since the paper does not talk about current.

Question 7 Where do the authors actually define the term "random permutation"?

Despite the length of the snippet, they don't define "random permutation"! They describe two-line notation, and they give an algorithm for generating a random permutation, but they don't actually say what a random permutation actually is.

Actually, I feel it's better to *not* give a formal definition: the mathematical formalities are not needed in the paper, and the reader can probably understand "random permutation" without a definition.

(This should have been a big hint to answering Question 9.)

Question 8 For each sentence in the snippet and for Algorithm 1, identify how it helps the reader understand the rest of the paper (if at all).

- Random permutation function is well studied in Combinatorial mathematics and group theory.

  How does it helps the reader understand the rest of the paper? It doesn't—it's a tangential comment about how mathematicians study random permutations.

- In Cauchy's two-line notation, there are the preimage elements in the first row, and the image elements of the preimage elements in the second row.

  How does it helps the reader understand the rest of the paper? It introduces the term "Cauchy's two-row notation" and helps the reader understand the subsequent sentence which shows the notation (or, at least it attempts to). It does not help the reader otherwise.

- The random permutation function can be expressed as:

  $$\pi \colon \begin{pmatrix} 1 & \cdots & n \\ p_1 & \cdots & p_n \end{pmatrix} \tag{2}$$

  where $\pi(i) = p_i (i = 1, \cdots, n)$.

  How does it helps the reader understand the rest of the paper? This sentences shows Cauchy's two-row notation. It is not used in the paper, so it does not help the reader understand the rest of the paper. In fact, it wastes the reader's time (which is worse than writing nothing).

- We define π^{-1} as the inverse function of π.

  How does it helps the reader understand the rest of the paper? By not defining terms like "permutation" and "preimage", the reader is assumed to know what they mean. Such a reader is likely aware that π^{-1} denotes the inverse function of π. Moreover, if it were necessary to define the notation π^{-1}, the reader would not think to find it in a section entitled "Random permutation". Consequently, it does not help the reader to explain this notation.

  (Note: The notation is subsequently used in $\delta_{\pi_1^{-1}(i),j}$.)

- When the condition satisfies $\pi(i) = i$, the permutation is called identical permutation denoted by I.

  How does it helps the reader understand the rest of the paper? This (ungrammatically) introduces the identity permutation (incorrectly called the "identical permutation"), denoting it I. This notation is used in Algorithm 1, and nowhere else. Consequently, it does not help the reader understand the rest of the paper outside of Algorithm 1.

  (We could just replace I in Algorithm 1 with the words "the identity permutation", and get rid of the notation.)

- The description of random permutation generation is as follows: [Algorithm 1]

  How does it helps the reader understand the rest of the paper? Algorithm 1 is an algorithm for generating random permutations (not a "description"). Algorithm 1 is not referenced in the paper, other than by "as follows" from the preceding text. Consequently, Algorithm 1 (and the sentence it belongs to) does not help the reader understand the rest of the paper.

Question 9 Using at most one sentence, rewrite the snippet to give only the information identified in part 8. It needs to be grammatically correct and succinct.

This is what I would write:

I would just delete the whole snippet. There's virtually nothing useful for the reader who wants to understand *the paper*; if the reader wants to learn about *random permutations*, they should find it in a relevant reference, not some random paper on another topic. It's basically a complete waste of the reader's time. Also note that my proposed (empty) snippet does not contain any errors.

I did write "at most one sentence" on the assignment sheet, which means "one or fewer sentences" (but nobody got the hint).

Remember, to write well:

> ... you must have something to say ...
>
> — Halmos, *How to write mathematics*, 2009.

If we really want to write something:

In this paper, a *random permutation* is one chosen uniformly at random from the set of permutations of $\{1, \ldots, n\}$; these can be efficiently generated using the Fisher-Yates Shuffle.

This gives a bit more formality, and comes closer to defining a "random permutation". The reader can type "Fisher-Yates Shuffle" into a search engine if they want to know more.