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EXERCISE 3

$R_{school} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ $R_{stay} = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$

STEP 1

1) FIND r_π for sunny

for Sunny:

$r_\pi = 0.5\,(5) + 0.5\,(-5) = 2.5 + (-2.5) = 0$

2) For cloudy:

$r_\pi = 0.5\,(3) + 0.5\,(1) = 1.5 + 0.5 = 2$

3) MATRIX: $r_\pi \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

STEP 2

ROW 1
SUNNY: • $P_\pi\,(1,1) = 0.5\,(0.8) + 0.5\,(0.9) = 0.4 + 0.45 = 0.85$

• $P_\pi\,(1,2) = 0.5\,(0.2) + 0.5\,(0.1) = 0.1 + 0.05 = 0.15$

ROW 2
CLOUDY: • $P_\pi\,(2,1) = 0.5\,(0.4) + 0.5\,(0.3) = 0.2 + 0.15 = 0.35$

• $P_\pi\,(2,2) = 0.5\,(0.6) + 0.5\,(0.7) = 0.3 + 0.35 = 0.65$

4) MATRIX: $P_\pi \begin{bmatrix} 0.85 & 0.15 \\ 0.35 & 0.65 \end{bmatrix}$

STEP 3 (SUNNY)

$v_1 = 0 + 0.9\,(0.85 v_1 + 0.15 v_2) = 0.765 v_1 + 0.135 v_2$

$v_1 - 0.765 v_1 - 0.135 v_2 = 0$ | $0.235 v_1 - 0.135 v_2 = 0$

STEP 3 (CLOUDY)

$v_2 = 2 + 0.9\,(0.35 v_1 + 0.65 v_2)$

$v_2 = 2 + 0.315 v_1 + 0.585 v_2$ | $v_2 - 0.315 v_1 - 0.585 v_2 = 2$

$-0.315 v_1 + 0.415 v_2 = 2$

STEP 4 (CLOUDY)

$0.235 v_1 - 0.135 v_2 = 0$

$0.235 v_1 = 0 + 0.135 v_2$

$\frac{0.235 v_1}{0.235} = \frac{0 + 0.135 v_2}{0.235}$

$v_1 = \frac{0 + 0.135 v_2}{0.235}$

$-0.315 v_1 + 0.415 v_2 = 2$

$-0.315 \left(\frac{0 + 0.135 v_2}{0.235} \right) + 0.415 v_2 = 2$

$\left(-0.315 \times \frac{0}{0.235} \right) \left(-0.315 \times \frac{0.135}{0.235} v_2 \right) + 0.415 v_2 = 2$

$(-0.315 \times 0)(-0.315 \times 0.574 v_2) + 0.415 v_2 = 2$

$0 - 0.181 v_2 + 0.415 v_2 = 2$

$0 + 0.234 v_2 = 2$

$0.234 v_2 = 2 + 0$

$V_\pi\,(\text{CLOUDY}) = \frac{2+0}{0.234} = \frac{2}{0.234} = 8.547$

STEP 4 (SUNNY)

$V_1 = \frac{0 + 0.135 v_2}{0.235}$

$V_1 = \frac{0 + 0.135(8.347)}{0.235} = \frac{0 + 1.154}{0.235} = \frac{1.154}{0.235}$

V.R (SUNNY) = 4.911 "

STEP 5

FOR SUNNY:

$V_* (\text{sunny}) = 5 + 0.4(0.8 v_1 + 0.2 v_2)$

$V_1 = 5 + 0.72 v_1 + 0.18 v_2$

$V_1 - 0.72 v_1 - 0.18 v_2 = 5$ $\quad 0.28 v_1 - 0.18 v_2 = 5$ "

FOR CLOUDY:

$V_* (\text{cloudy}) = 3 + 0.9(0.4 v_1 + 0.6 v_2)$

$V_2 = 3 + 0.36 v_1 + 0.54 v_2$

$v_2 - 0.36 v_1 - 0.54 v_2 = 3$ $\quad -0.36 v_1 + 0.46 v_2 = 3$ "

STEP 6

FROM SUNNY:

$0.28 v_1 = 5 + 0.18 v_2$

$V_1 = \frac{5 + 0.18 v_2}{0.28}$

FOR CLOUDY:

$-0.36 v_1 + 0.46 v_2 = 3$

$-0.36 \left(\frac{5 + 0.18 v_2}{0.28}\right) + 0.46 v_2 = 3$

STEP 6 (CLOUDY)

$-0.36 \left(\frac{5 + 0.18 v_2}{0.28}\right) + 0.46 v_2 = 3$

$\left(-0.36 \times \frac{5}{0.28}\right) \left(-0.36 \times \frac{0.18 v_2}{0.28}\right) + 0.46 v_2 = 3$

$-6.429 - 0.231 v_2 + 0.46 v_2 = 3$

$-6.429 + 0.229 v_2 = 3$

$0.229 v_2 = 3 + 6.429$

$0.229 v_2 = 9.429$

$V_* (\text{CLOUDY}) = \frac{9.429}{0.229} = 41.175$ "

STEP 6 (SUNNY)

$V_1 = \frac{5 + 0.18 v_2}{0.28} \rightarrow V_1 = \frac{5 + 0.18(41.175)}{0.28} \rightarrow V_1 = \frac{5 + 7.412}{0.28} = \frac{12.412}{0.28}$ $\quad V_* (\text{SUNNY}) = 44.329$ "

STEP 7

$q(1, \text{School}) = 5 + 0.9(0.8 v_1 + 0.2 v_2) = 44.33$

$q(1, \text{Home}) = -5 + 0.9(0.9 v_1 + 0.1 v_2) = 34.61$

$q(2, \text{School}) = 3 + 0.9(0.4 v_1 + 0.6 v_2) = 41.18$

$q(2, \text{Home}) = 1 + 0.9(0.3 v_1 + 0.72 v_2) = 38.91$