Graphs
Difficulty: Medium

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 1

Time allowed: **128 minutes**

Score: **/111**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) Assembled by AS

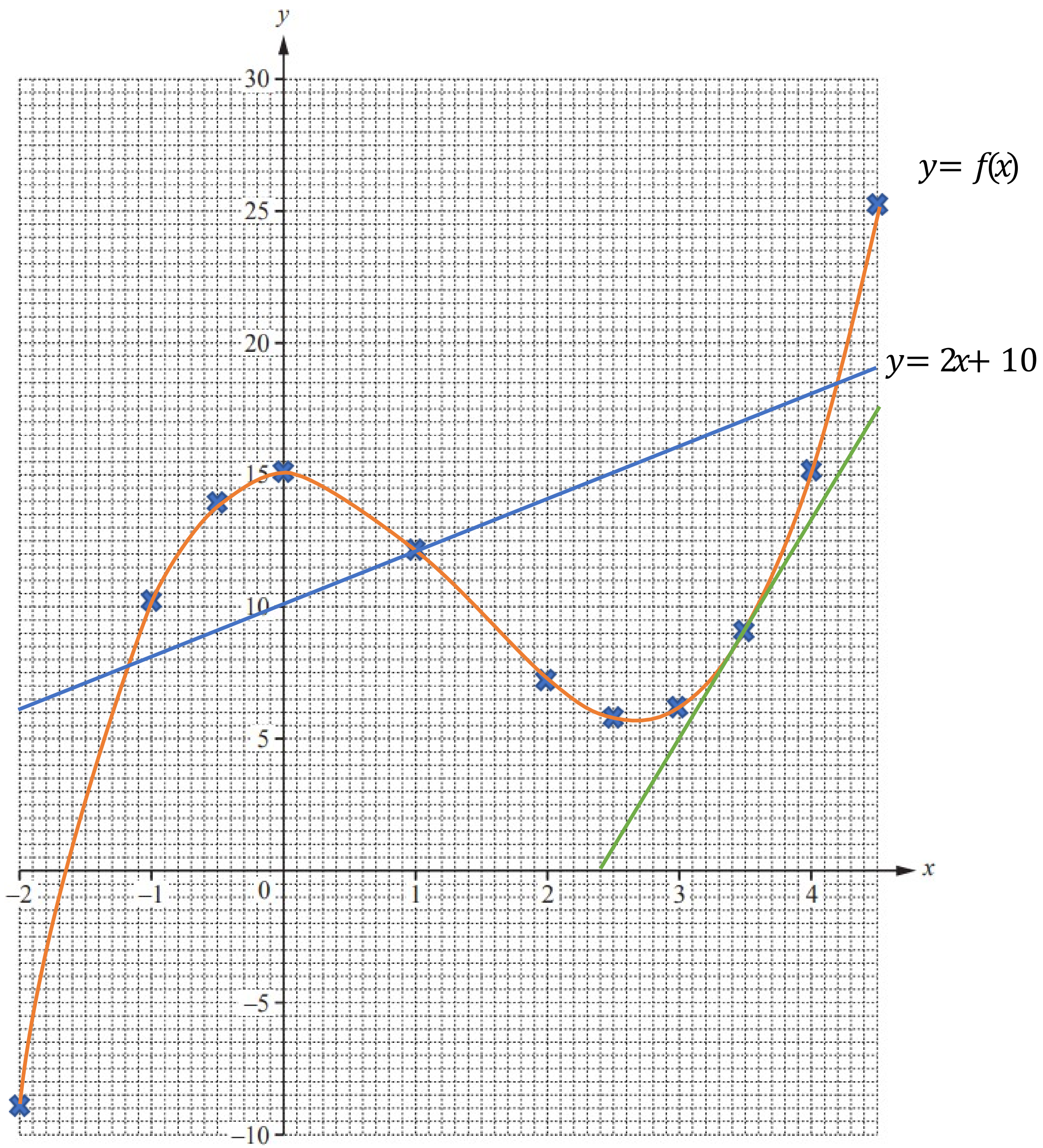
9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

$f(x) = x^3 - 4x^2 + 15$

(a) Complete the table of values for $y = f(x)$. [2]

x	–2	–1	–0.5	0	1	2	2.5	3	3.5	4	4.5
y	–9	**10**	13.9	15	12	**7**	5.6	6	8.9	15	25.1

(b) On the grid, draw the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 4.5$. [4]



(c) Use your graph to solve the equation f(x) = 0. [1]

$x = -1.65$

(d) By drawing a suitable tangent, estimate the gradient of the graph of y = f(x) when x = 3.5 . [3]

Tangent drawn in green on graph above.

Gradient is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{17.5 - 0}{4.5 - 2.4}$$

$$= \frac{17.5}{2.1}$$

$$= 8.3$$

(e) By drawing a suitable straight line on the grid, solve the equation $x^3 - 4x^2 - 2x + 5 = 0$. [4]

$$x^3 - 4x^2 - 2x + 5 = 0$$

$$\rightarrow x^3 - 4x^2 + 15 = 2x + 10$$

Line drawn in blue above

$x = -1.2, \quad x = 1, \quad x = 4.2$

Question 2

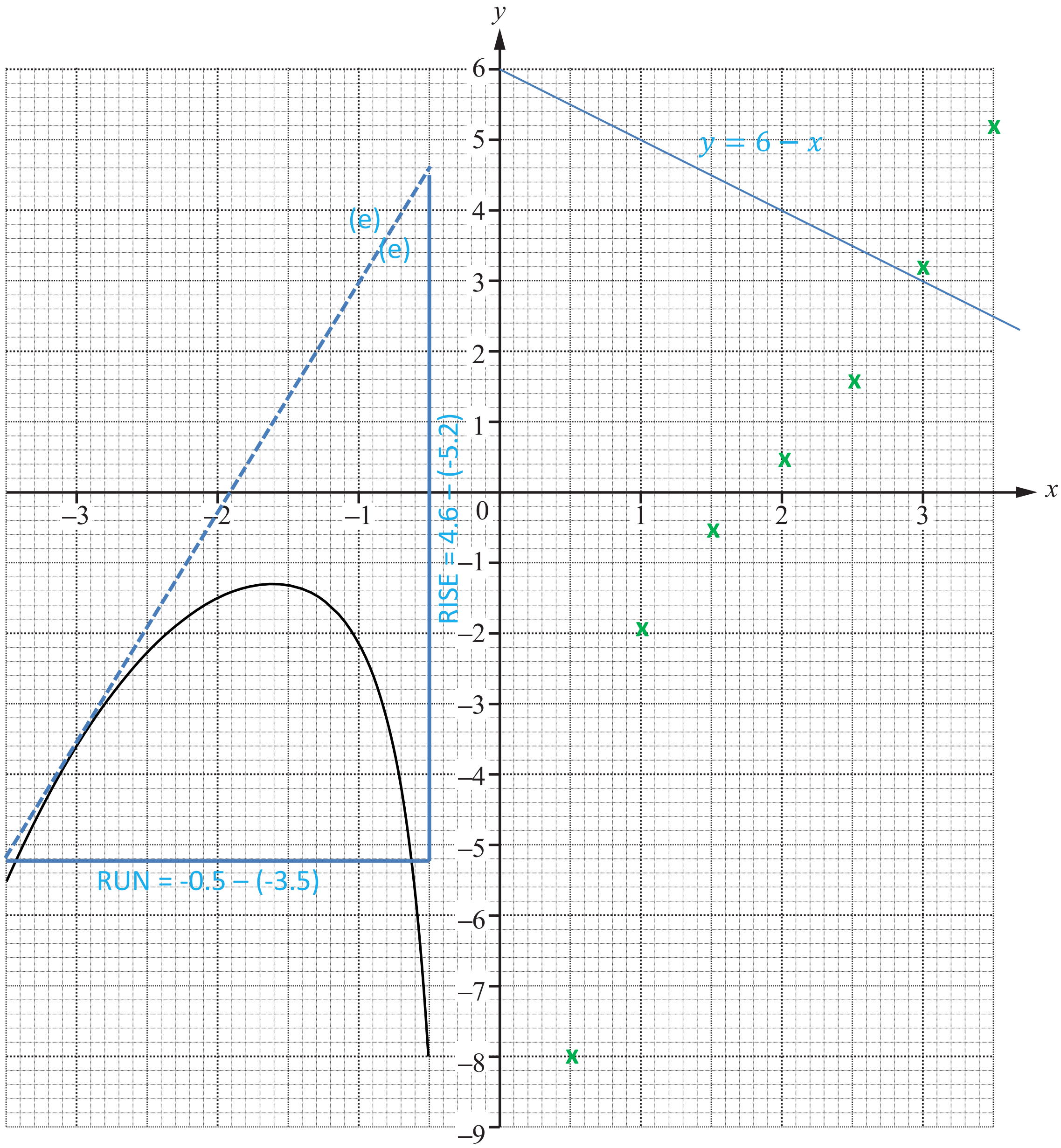
$$y = \frac{x^3}{8} - \frac{2}{x^2}\ ,\ x \neq 0$$

(a) Complete the table of values. Table function used on Calculator to get:

x	0.5	1	1.5	2	2.5	3	3.5
y	−8.0	−1.9	−0.5	0.5	1.6	**3.2**	**5.2**

[2]

(b)



The graph of $y = \frac{x^3}{8} - \frac{2}{x^2}$ for $-3.5 \leqslant x \leqslant -0.5$ has already been drawn.

On the grid, draw the graph of $y = \frac{x^3}{8} - \frac{2}{x^2}$ for $0.5 \leqslant x \leqslant 3.5$.

Points plotted – smooth (ish!) curve drawn.

[4]

(c) Use your graph to solve the equation $\frac{x^3}{8} - \frac{2}{x^2} = 0$. [1]

Curve crosses the x-axis at $\boldsymbol{x = 1.7}$

(d) $\frac{x^3}{8} - \frac{2}{x^2} = k$ and k is an integer.

Write down a value of k when the equation $\frac{x^3}{8} - \frac{2}{x^2} = k$ has

(i) one answer, [1]

Any value of k > -1.3 so that y = k only crosses curve once eg **k = 1**

(ii) three answers. [1]

Any value of k < -1.3 so that y = k crosses curve three times eg **k = -3**

(e) By drawing a suitable tangent, estimate the gradient of the curve where $x = -3$. [3]

Tangent drawn on graph. Use gradient$=\frac{\text{RISE}}{\text{RUN}}$

$$\text{radient} = \frac{4.6-(-5.2)}{-0.5-(-3.5)}$$

Gradient = 3.3

(f) (i) By drawing a suitable line on the grid, find x when $\frac{x^3}{8} - \frac{2}{x^2} = 6 - x$. [3]

Draw line $y = 6 - x$ on graph (y-intercept = 6, gradient = -1).

Find x-coordinate of point of intersection: $\boldsymbol{x = 2.9}$

(ii) The equation $\frac{x^3}{8} - \frac{2}{x^2} = 6 - x$ can be written as $x^5 + ax^3 + bx^2 + c = 0$.

Find the values of a, b and c. [4]

Multiply all terms by $8x^2$ to get rid of fractions:

$$\frac{x^3}{8} \times 8x^2 - \frac{2}{x^2} \times 8x^2 = 6 \times 8x^2 - x \times 8x^2$$

Simplify to get $x^5 - 16 = 48x^2 - 8x^3$

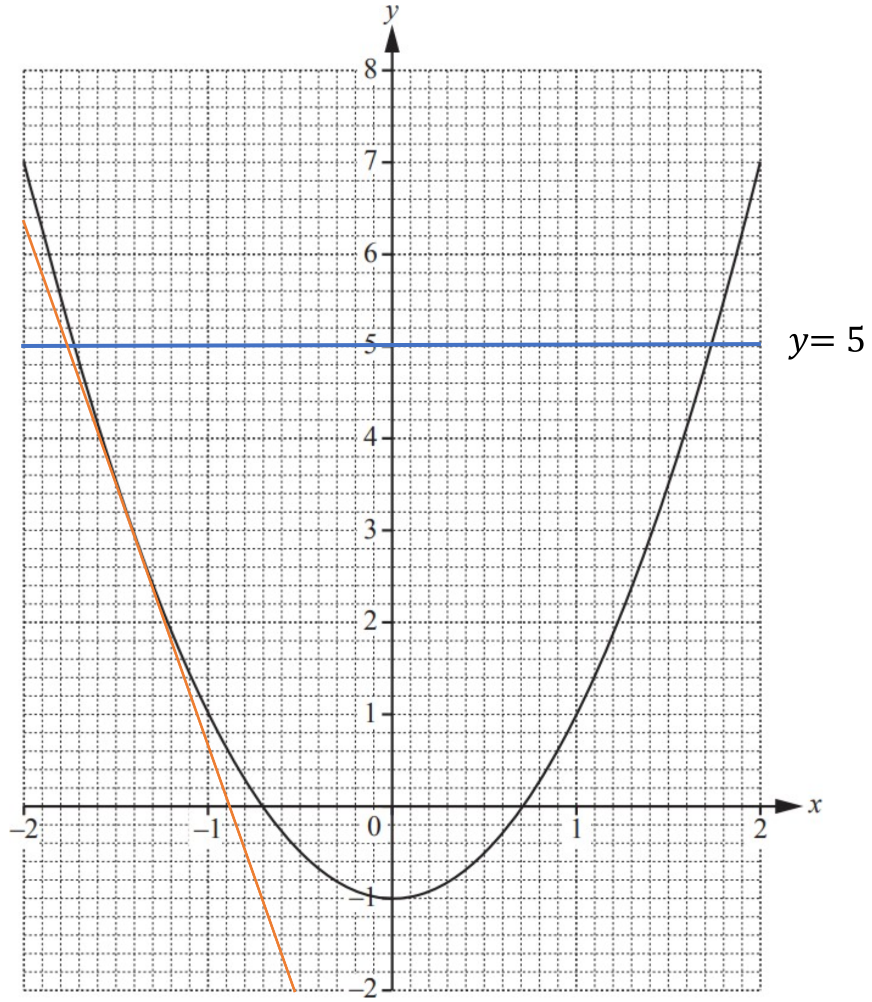
Add $8x^3$ and subtract $48x^2$ to/from both sides:

$$x^5 + 8x^3 - 48x^2 - 16 = 0$$

$$\boldsymbol{a = 8, b = -48 \text{ and } c = -16}$$

$f(x) = 2x^2 - 1$

The graph of $y = f(x)$, for $-2 \leqslant x \leqslant 2$, is drawn on the grid.



(a) Use the graph to solve the equation $f(x) = 5$.

[2]

Solutions to $f(x) = 5$ are where the line $y = 5$ and the curve $f(x)$ intersect.

$y = 5$ is drawn in blue on graph above and we can see intersections at

$$\boldsymbol{x = -1.7, \qquad x = 1.7}$$

(b) (i) Draw the tangent to the graph of $y = \mathrm{f}(x)$ at the point (-1.5, 3.5). [1]

Tangent at $(-1.5, 3.5)$ is drawn in orange on graph above.

(ii) Use your tangent to estimate the gradient of $y = \mathrm{f}(x)$ when $x = -1.5$. [2]

Gradient is calculated as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Picking two points on the orange line

$$m = \frac{3.5 - 6.4}{-1.5 - -2}$$

$$= -\frac{2.9}{0.5}$$

$$= \mathbf{-5.8}$$

(c) $\mathrm{g}(x) = 2^x$

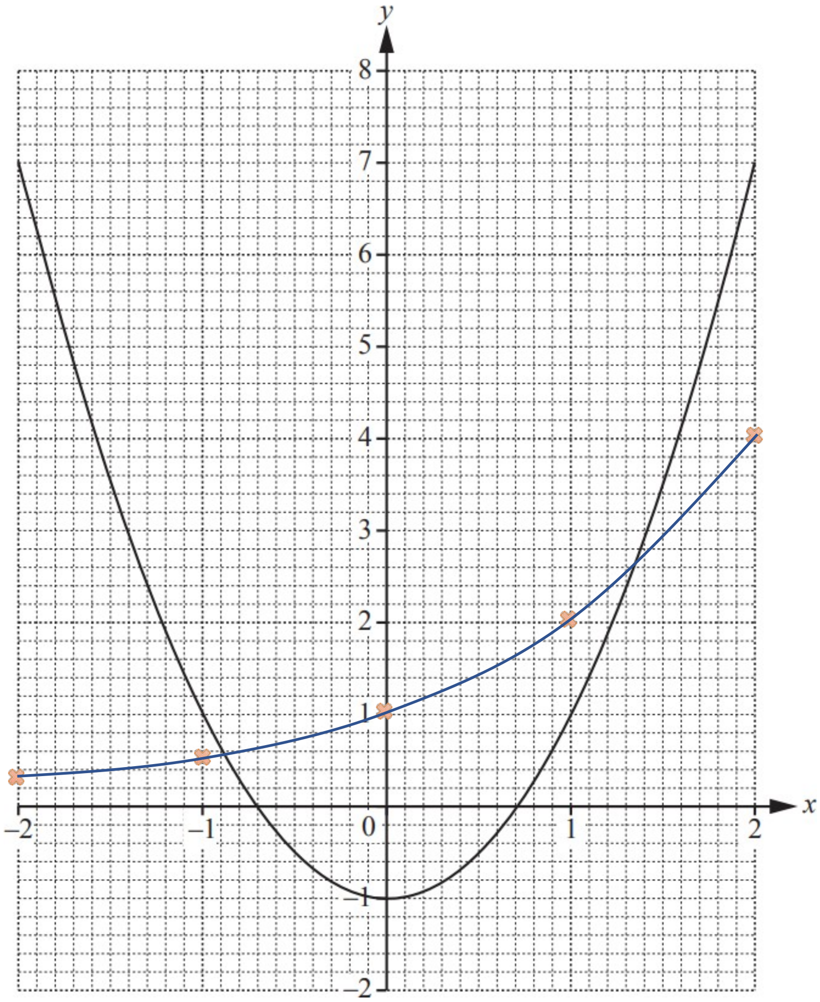
(i) Complete the table for $y = \mathrm{g}(x)$.

x	-2	-1	0	1	2
y	0.25	0.5	**1**	2	4

[1]

(ii) On the grid opposite, draw the graph of $y = g(x)$ for $-2 \leqslant x \leqslant 2$. [3]

Plotted in orange and drawn in blue on graph below.



(d) Use your graphs to solve

(i) the equation f (x) = g(x), [2]

$f(x) = g(x)$ is solved when the two curves intersect, which we can see happens at

$$x = -0.9, \qquad x = 1.4$$

(ii) the inequality f(x) < g(x). [1]

We can see that $f(x)$ is below $g(x)$ for

$$-0.9 < x < 1.4$$

(e) (i) Write down the three values.

g(- 3) =. g(- 5) =. g(- 10) = [1]

$$g(-3) = 0.125$$

$$g(-5) = 0.03125$$

$$g(-10) = 0.0009765625$$

(ii) Complete the statement.

As x decreases, g(x) approaches the value .... [1]

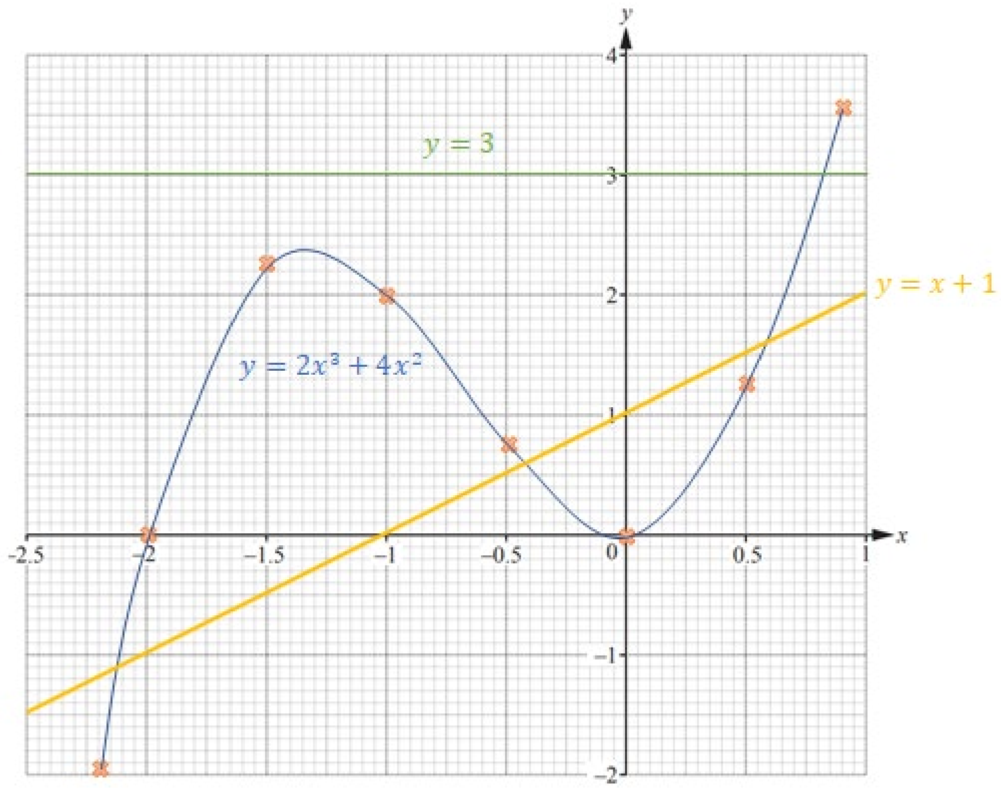
As x decreases, $g(x)$ approaches the value $\mathbf{0}$

The table shows some values for $y = 2x^3 + 4x^2$.

x	-2.2	-2	-1.5	-1	-0.5	-0	0.5	0.8
y	-1.94	**0**	**2.25**	**2**	0.75	0	**1.25**	3.58

(a) Complete the table. [4]

(b) Draw the graph of $y = 2x^3 + 4x^2$ for $-2.2 \leqslant x \leqslant 0.8$. [4]



(c) Find the number of solutions to the equation $2x^3 + 4x^2 = 3$. [1]

The line $y = 3$ is drawn on the graph above in green.

The solutions to

$2x^3 + 4x^2 = 3$

occur when the line and the curve intersect.

We can see that this happens once, hence **1 solution**.

(d) (i) The equation $2x^3 + 4x^2 - x = 1$ can be solved by drawing a straight line on the grid.

Write down the equation of this straight line. [1]

Rearranging this equation gives us

$$2x^3 + 4x^3 = 1 + x$$

Hence the line we draw is

$$\boldsymbol{y = 1 + x}$$

(ii) Use your graph to solve the equation $2x^3 + 4x^2 - x = 1$. [3]

The line is drawn on the graph above in yellow.

The intersections (and hence the solutions) are

$$\boldsymbol{x = -2.14, \qquad x = -0.42, \qquad x = 0.57}$$

(e) The tangent to the graph of $y = 2x^3 + 4x^2$ has a negative gradient when $x = k$.

Complete the inequality for k. [2]

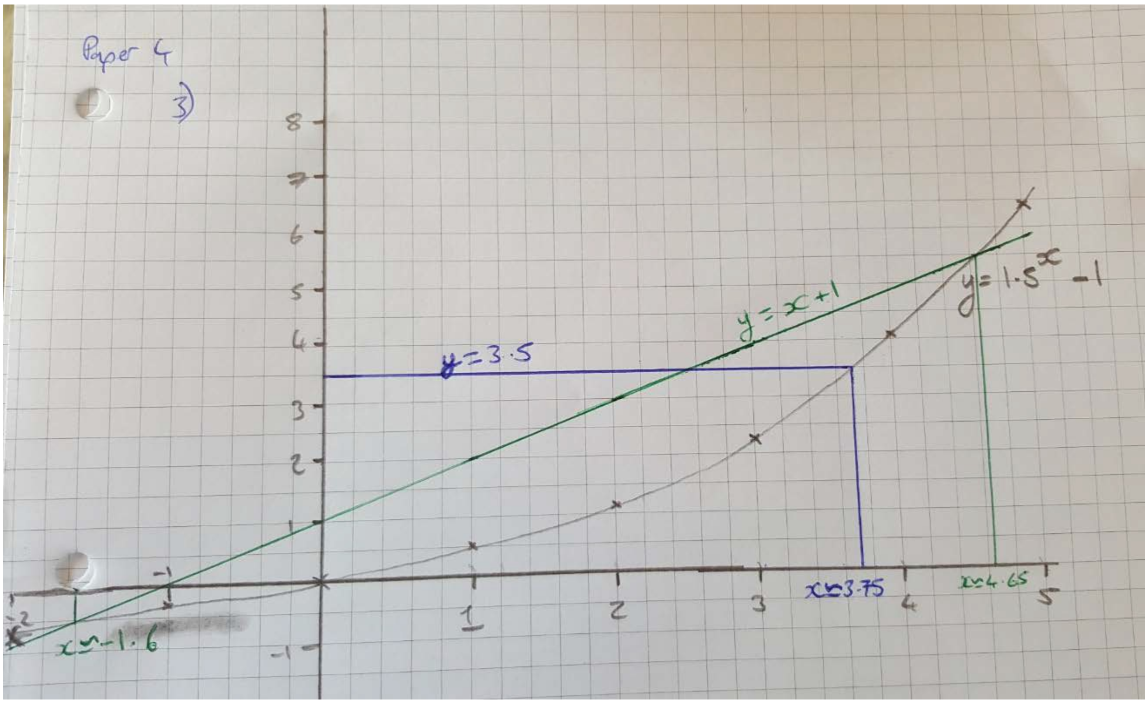
$$\boldsymbol{-1.35 < k < 0}$$

Between the maximum point and the minimum point, judged from own graph drawing.

The table shows some values for $y = 1.5^x - 1$.

x	–2	–1	0	1	2	3	4	5
y	–0.56	–0.33				2.38	4.06	6.59

An example of a hand drawn graph that would be acceptable for this question is shown here:



(a) Complete the table. [3]

We can put these x-values into our calculators to find the y-values.

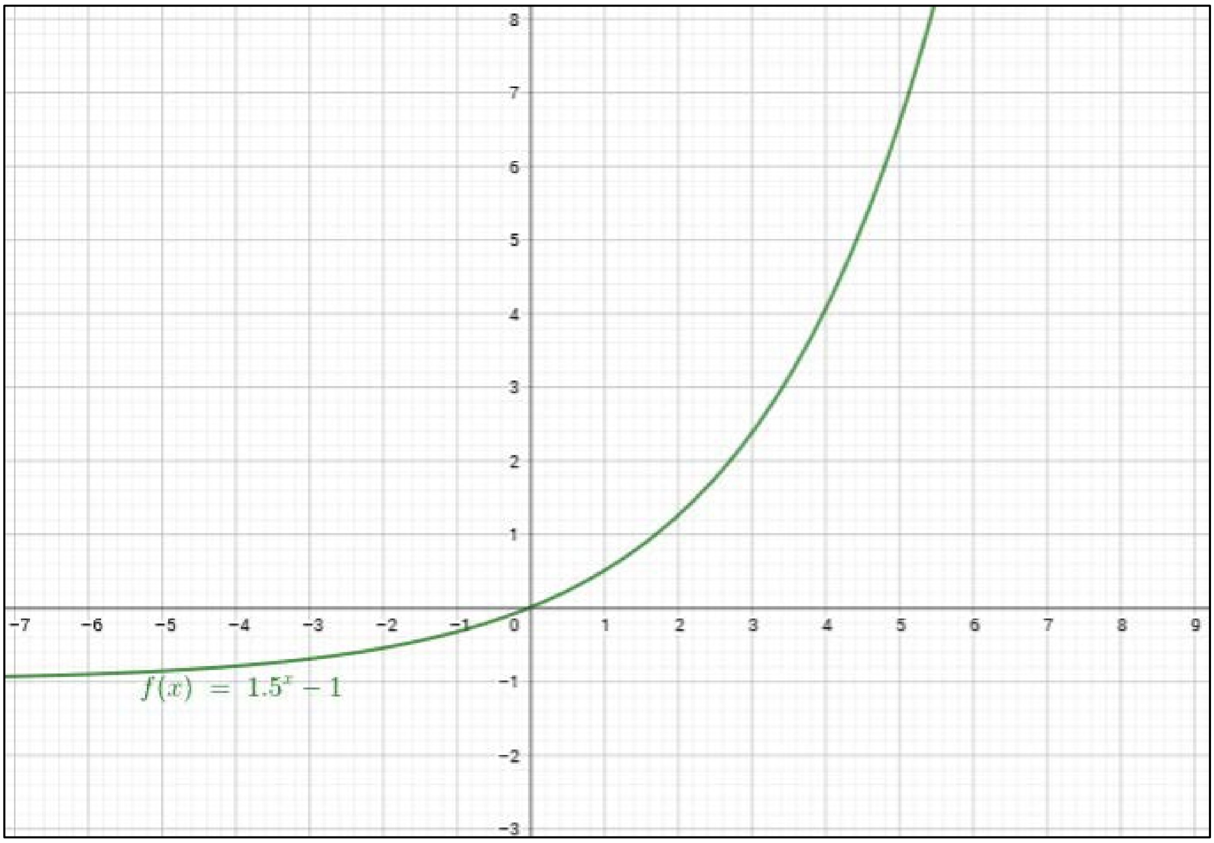
$$x = 0, y = 0$$

$$x = 1, y = 0.5$$

$$\boldsymbol{x = 2, y = 1.25}$$

(b) Draw the graph of $y = 1.5^x - 1$ for $-2 \leq x \leq 5$. [4]

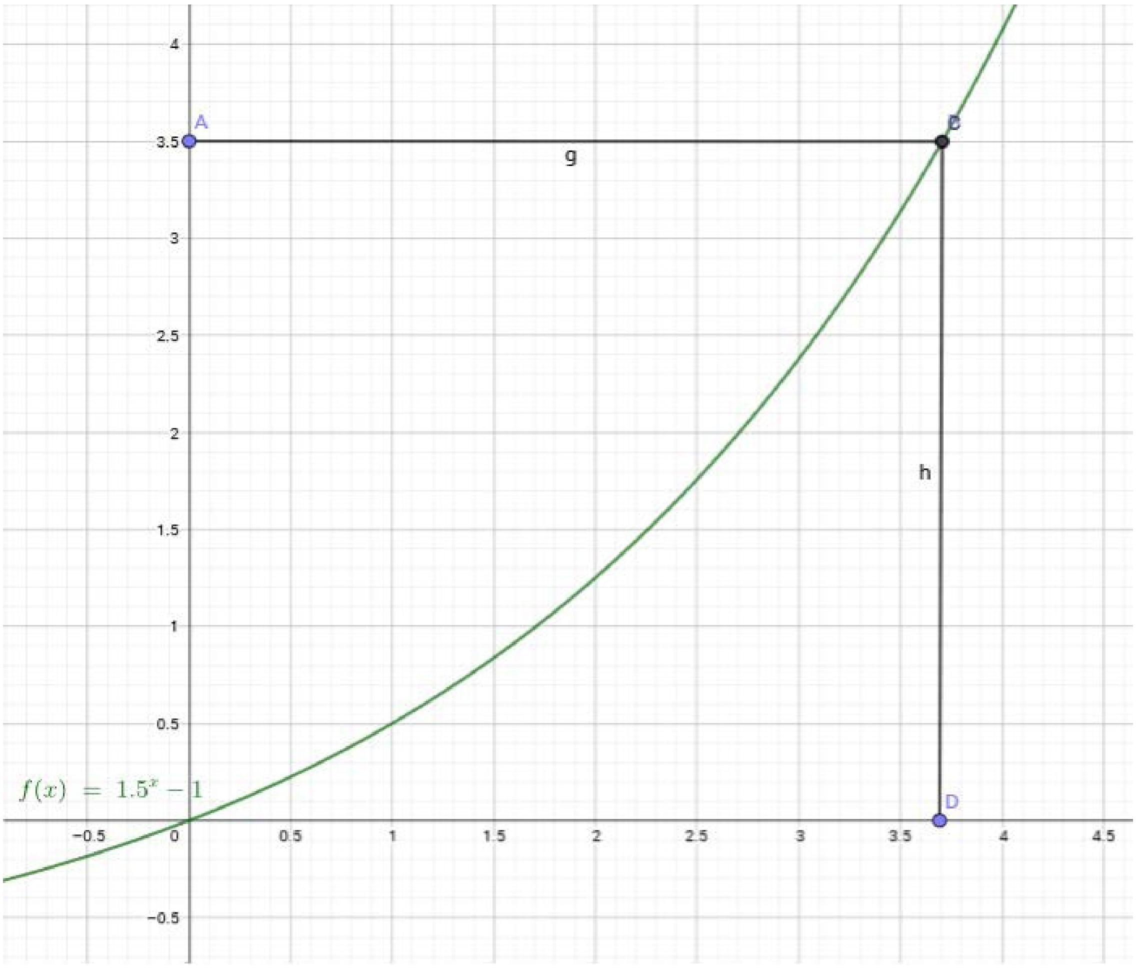
The graph of $y = 1.5^x - 1$ looks like this:



(c) Use your graph to solve the equation $1.5^x - 1 = 3.5$. [2]

We can solve this by drawing the line $y = 3.5$. Where these two lines intersect (at point A), we know that $3.5 = 1.5^x - 1$. We can then read down on to the x-axis to find what the x-coordinate is. In this case,

$$x \approx 3.7\ (1.d.p)$$



(d) By drawing a suitable straight line, solve the equation $1.5^x - x - 2 = 0$. [3]

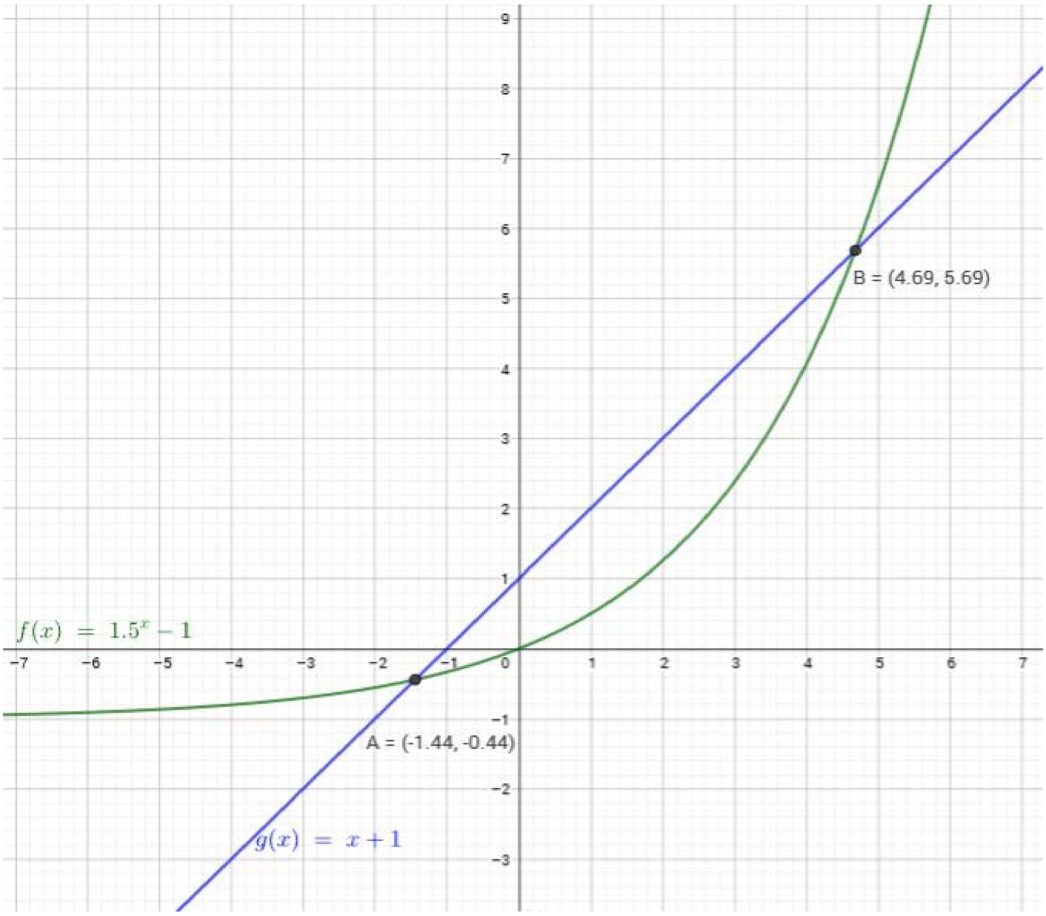
To do this, we need to rearrange the equation we are given to match what we have in part a) and part b). This looks like this

$$1.5^x - x - 2 = 0$$

$$1.5^x - 1 - x - 1 = 0$$

$$1.5^x - 1 = x + 1$$

So we could solve this by drawing the line $y = x + 1$.



Again, where these lines intersect we know the two values must be the same. Reading down to the x-axis gives us the x coordinate. This happens at:

$$x \approx 4.7\ (1.d.p)$$

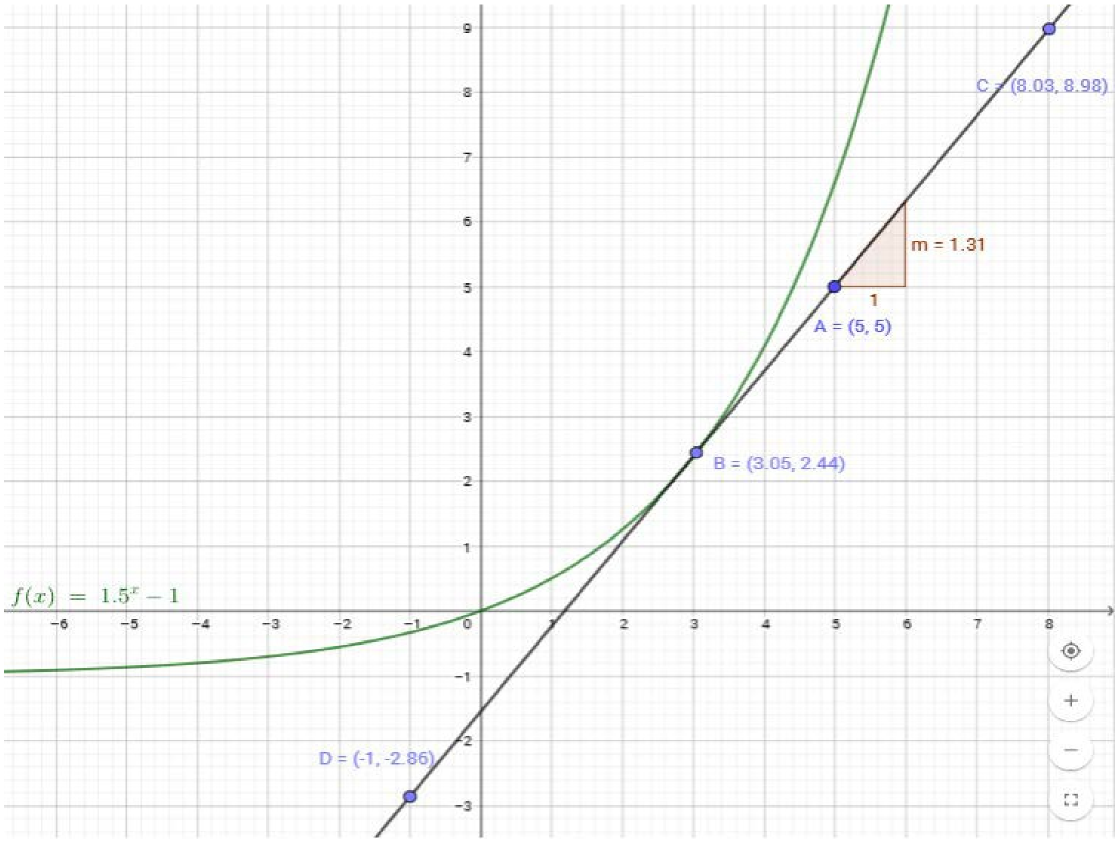
$$x \approx -1.4\ (1.d.p)$$

(e) (i) On the grid, plot the point A at (5, 5). [1]

(ii) Draw the tangent to the graph of $y = 1.5^x - 1$ that passes through the point A. [1]

(iii) Work out the gradient of this tangent. [2]

We start by plotting the point (5,5). Next we draw a line that goes through this point and only just touches the curve.



Then we find the gradient by taking two points that are far apart, which we approximate to 1 significant figure. In this case, we use C=(8,9) and D=(-1,-3).

$$Gradient = \frac{\Delta y}{\Delta x}$$

$$Gradient \approx \frac{9 - -3}{8 - -1}$$

$$Gradient \approx \frac{12}{9}$$

$$Gradient \approx 1.33333 \ldots$$

$$\boldsymbol{Gradient \approx 1.33\ (3.s.f)}$$

As we can see in the picture above (which was done with a computer), the real gradient is in fact 1.31, so we are quite close.

$y = 1 - \frac{2}{x^2}, x \neq 0$

(a) Complete the table.

x	–5	–4	–3	–2	–1	–0.5		0.5	1	2	3	4	5
y	**0.92**	0.88	0.78	**0.5**	**-1**	–7		–7	**-1**	**0.5**	0.78	0.88	**0.92**

[3]

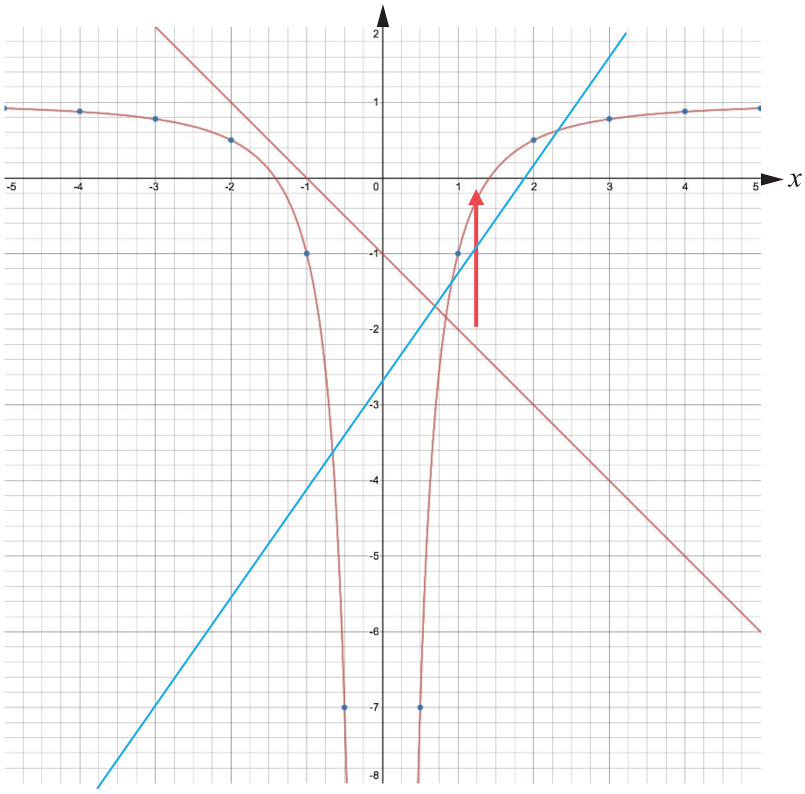
Use the table function on your calculator to recreate the table.

(Or plug the x values into the function...)

(b) On the grid, draw the graph of $y = 1 - \frac{2}{x^2}$ for $-5 \leq x \leq -0.5$ and $0.5 \leq x \leq 5$. [5]

Plot points and draw a smooth curve through them.

NB the curve does not cross the y-axis as it is not defined there when $x = 0$.



[2]

(c) (i) On the grid, draw the graph of $y = -x - 1$ for $-3 \leq x \leq 5$.

The equation of a straight line is $y = mx + c$ where m is the gradient and c is the y-intercept

So draw a line through -1 on the y-axis with a gradient of -1

(ii) Solve the equation $1-\frac{2}{x^2}=-x-1$. [1]

Find the x-coordinate of the point where the line crosses the curve.

See red dot and line on graph above.

$$x = 0.8$$

(iii) The equation $1-\frac{2}{x^2}=-x-1$ can be written in the form $x^3+px^2+q=0$.

Find the value of p and the value of q. [3]

$$1-\frac{2}{x^2}=-x-1$$

Multiply by x^2

$$x^2-2=-x^3-x^2$$

$$x^3+2x^2-2=0$$

$$p=2, q=-2$$

(d) The graph of $y=1-\frac{2}{x^2}$ cuts the positive x-axis at A.

B is the point $(0, -2)$.

(i) Write down the co-ordinates of A. [1]

Reading off the graph: $A(1.4, 0)$

(ii) On the grid, draw the straight line that passes through A and B. [1]

Blue line on graph drawn through A and B

(iii) Complete the statement.

The straight line that passes through A and B is a ......**Tangent**......

at the point ......A...... [2]

(a) Complete the table for $y = 3x + \frac{2}{x^2} + 1,\ x \neq 0$.

x	−3	−2	−1	−0.5	−0.3		0.3	0.5	1	2	3
y	−7.8	**−4.5**	0	7.5	22.3		24.1	**10.5**	6	7.5	10.2

Use the table function on your calculator to recreate the table.

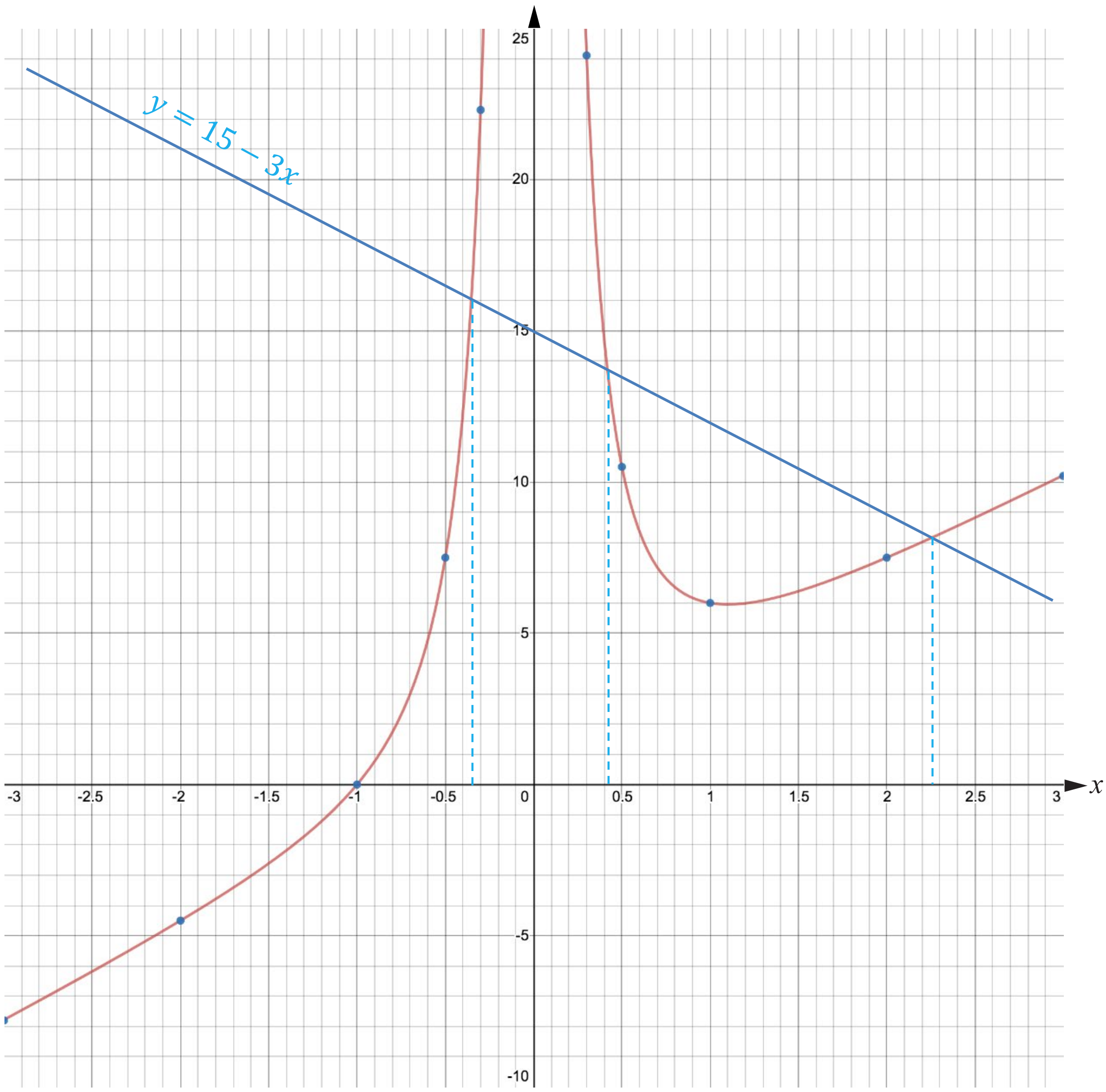
(Or plug the x values into the function…)

[2]

(b) On the grid, draw the graph of $y = 3x + \frac{2}{x^2} + 1$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$.

Plot points and draw a smooth curve through them.

NB the curve does not cross the y-axis as it is not defined there when $x = 0$.



[5]

(c) Write down the value of the largest integer, k, so that the equation $3x + \frac{2}{x^2} + 1 = k$ has exactly one solution.

[1]

$y = k$ is a horizontal line.

The largest integer value of k for which the line only crosses the curve once is

$$\boldsymbol{k = 5}$$

(d) (i) By drawing a suitable straight line on the grid, solve $3x + \frac{2}{x^2} + 1 = 15 - 3x$.

Draw the line $y = 15 - 3x$ (in blue on graph) and read of x-coordinates of points of intersection with the curve:

$x =$ $\boldsymbol{-0.34}$ or $x =$ $\boldsymbol{0.82}$ or $x =$ $\boldsymbol{2.37}$ [4]

(ii) The equation $3x + \frac{2}{x^2} + 1 = 15 - 3x$ can be written in the form $ax^3 + bx^2 + cx + 2 = 0$, where a, b and c are integers.

Find a, b and c. [3]

$$3x + \frac{2}{x^2} + 1 = 15 - 3x$$

Multiply by x^2:

$$3x^3 + 2 + x^2 = 15x^2 - 3x^3$$

Add $3x^3$ to and subtract $15x^2$ from both sides:

$$6x^3 - 14x^2 + 2 = 0$$

$$\boldsymbol{a = 6, b = -14 \text{ and } c = 0}$$

Graphs
Difficulty: Medium

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 2

Time allowed: **121 minutes**

Score: **/105**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a) Complete the table. [2]

x	-10	-8	-5	-2	-1.6		1.6	2	5	8	10
f(x)	-12	-10.5	-9	-12	-14.1		14.1	12	9	10.5	12

(b) On the grid, draw the graph of $y =$ f(x) for $-10 \leqslant x \leqslant -1.6$ and $1.6 \leqslant x \leqslant 10$. [5]

(c) Using your graph, solve the equation f(x) = 11. [2]

Trace across from y=11 and then down to the x-axis

$x = 2.3 \; or \; x = 8.7$

(d) k is a prime number and $f(x) = k$ has no solutions.

Find the possible values of k. [2]

Prime numbers between 0 and 9 **2, 3, 5, 7**

(e) The gradient of the graph of $y = f(x)$ at the point (2, 12) is -4.

Write down the co-ordinates of the other point on the graph of $y = f(x)$ where the gradient is -4. [1]

(-2, -12)

(f) (i) The equation $f(x) = x^2$ can be written as $x^3 + px^2 + q = 0$.

Show that $p = -1$ and $q = -20$. [2]

$$f(x) = x^2$$

$$\frac{20}{x} + x = x^2$$

$$x^2 = \frac{20}{x} + x$$

$$x^2 - \frac{20}{x} - x = 0$$

$$x^3 - 20 - x^2 = 0$$

$$\boldsymbol{x^3 - x^2 - 20 = 0}$$

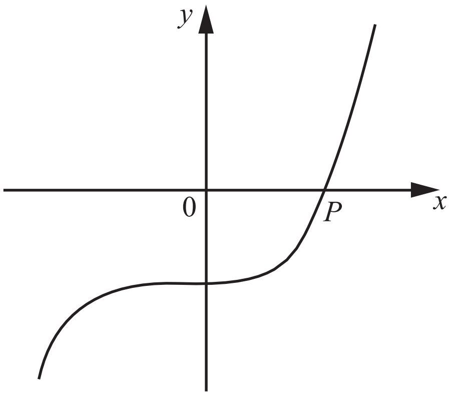
(ii) On the grid opposite, draw the graph of $y = x^2$ for $-4 \leqslant x \leqslant 4$. [2]

See diagram

(iii) Using your graphs, solve the equation $x^3 - x^2 - 20 = 0$. [1]

$\boldsymbol{x = 3.3}$ (See diagram. Intersection of curves)

(iv)



NOT TO SCALE

The diagram shows a **sketch** of the graph of $y = x^3 - x^2 - 20$.
P is the point $(n, 0)$.

Write down the value of n. [1]

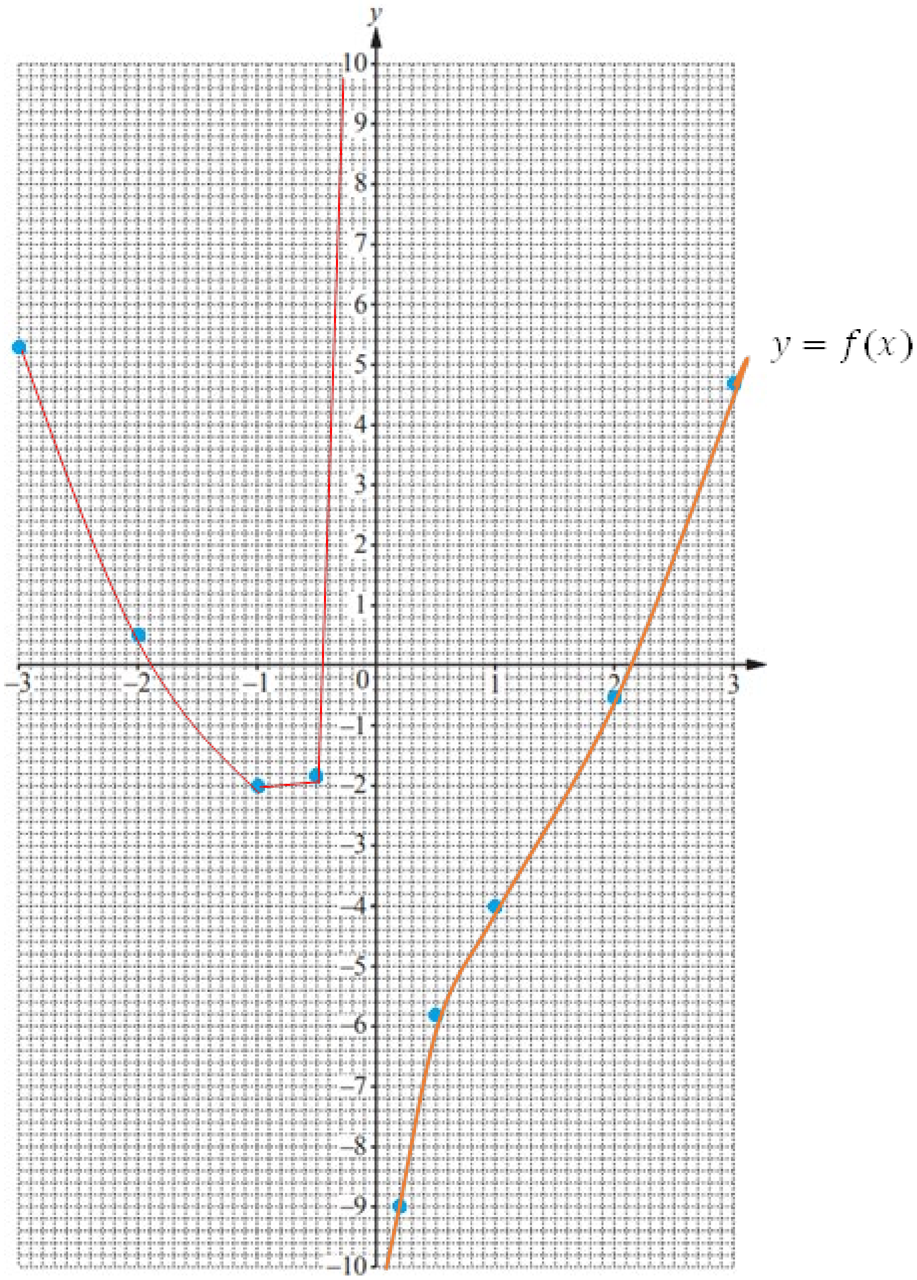
$n = 3.3$ (Use value from (iii))

$f(x) = x^2 - \frac{1}{x} - 4,\ x \neq 0$

(a) (i) Complete the table. [2]

x	–3	–2	–1	–0.5	–0.1		0.2	0.5	1	2	3
f(x)	5.3	0.5	**−2**	–1.8	6.0		–9.0	–5.8	–4	**−0.5**	4.7

(ii) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.1$ and $0.2 \leqslant x \leqslant 3$. [5]



(b) Use your graph to solve the equation f(x) = 0. [3]

From the graph we can see that $f(x) = 0$ for

$$\boldsymbol{x = -1.85, \quad x = -0.4, \quad x = 2.1}$$

(c) Find an integer k, for which f(x) = k has one solution. [1]

$$\boldsymbol{k = -8}$$

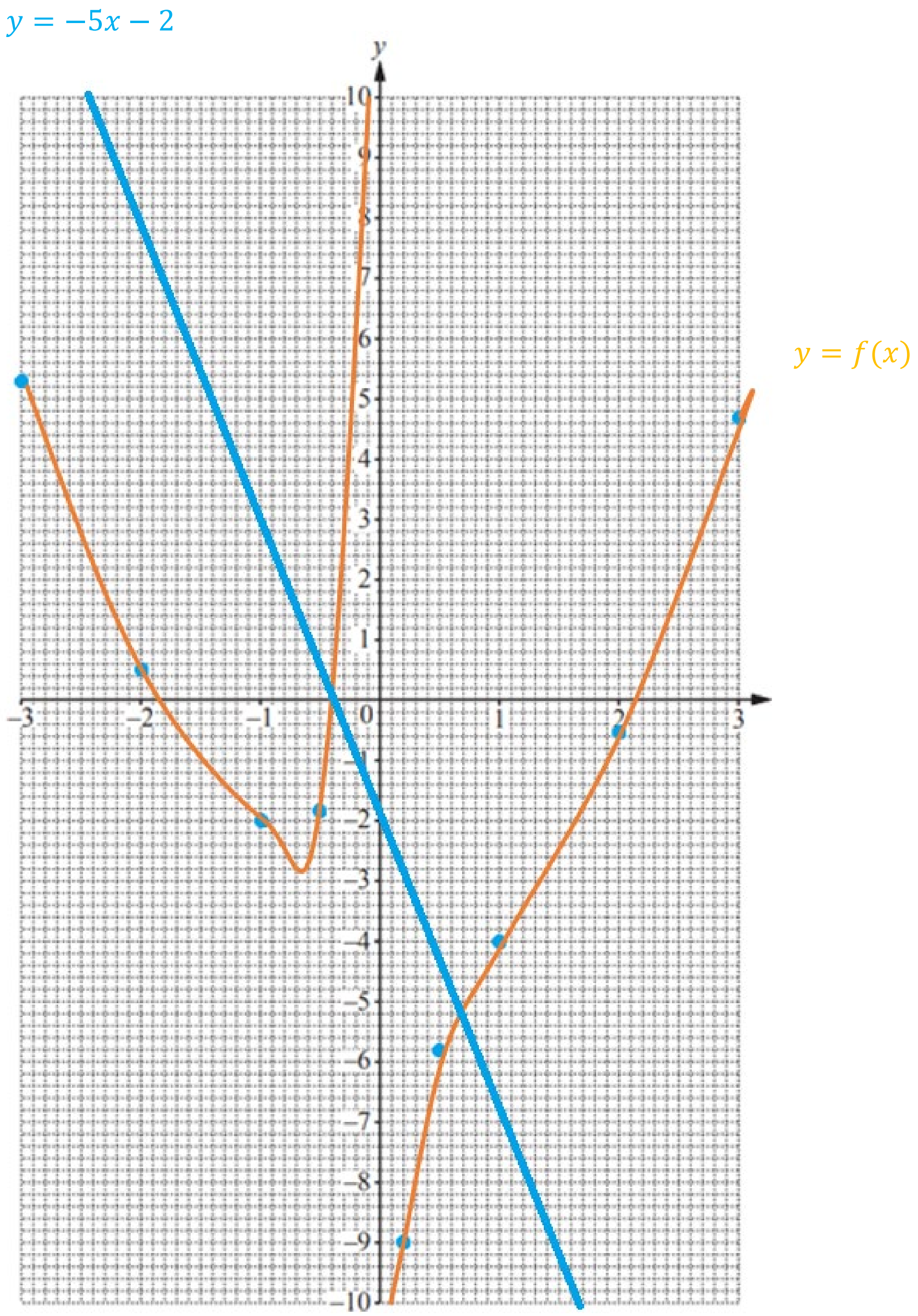
Note that any integer less than -3 would be correct here.

(d) (i) By drawing a suitable straight line, solve the equation $f(x) + 2 = -5x$. [4]

Rearranging gives us

$$f(x) = -5x - 2$$

So, if we draw $y = -5x - 2$ and see where the curve and this line intersect, we will have our solution.



The curve and the line intersect at

$$\boldsymbol{x = -0.4, \quad x = 0.7}$$

(ii) $f(x) + 2 = -5x$ can be written as $x^3 + ax^2 + bx - 1 = 0$.

Find the value of a and the value of b. [2]

Recall that

$$f(x) = x^2 - \frac{1}{x} - 4$$

We now have

$$f(x) + 2 = -5x$$

$$\rightarrow x^2 - \frac{1}{x} - 4 + 2 = -5x$$

$$\rightarrow x^2 - \frac{1}{x} - 2 = -5x$$

Multiply everything by x

$$x^3 - 1 - 2x = -5x^2$$

Now move everything to the left hand side (LHS)

$$x^3 + 5x^2 - 2x - 1 = 0$$

Comparing with the formula given we see that

$$\boldsymbol{a = 5, \qquad b = -2}$$

$$f(x) = x - \frac{1}{2x^2}, \quad x \neq 0$$

(a) Complete the table of values. [2]

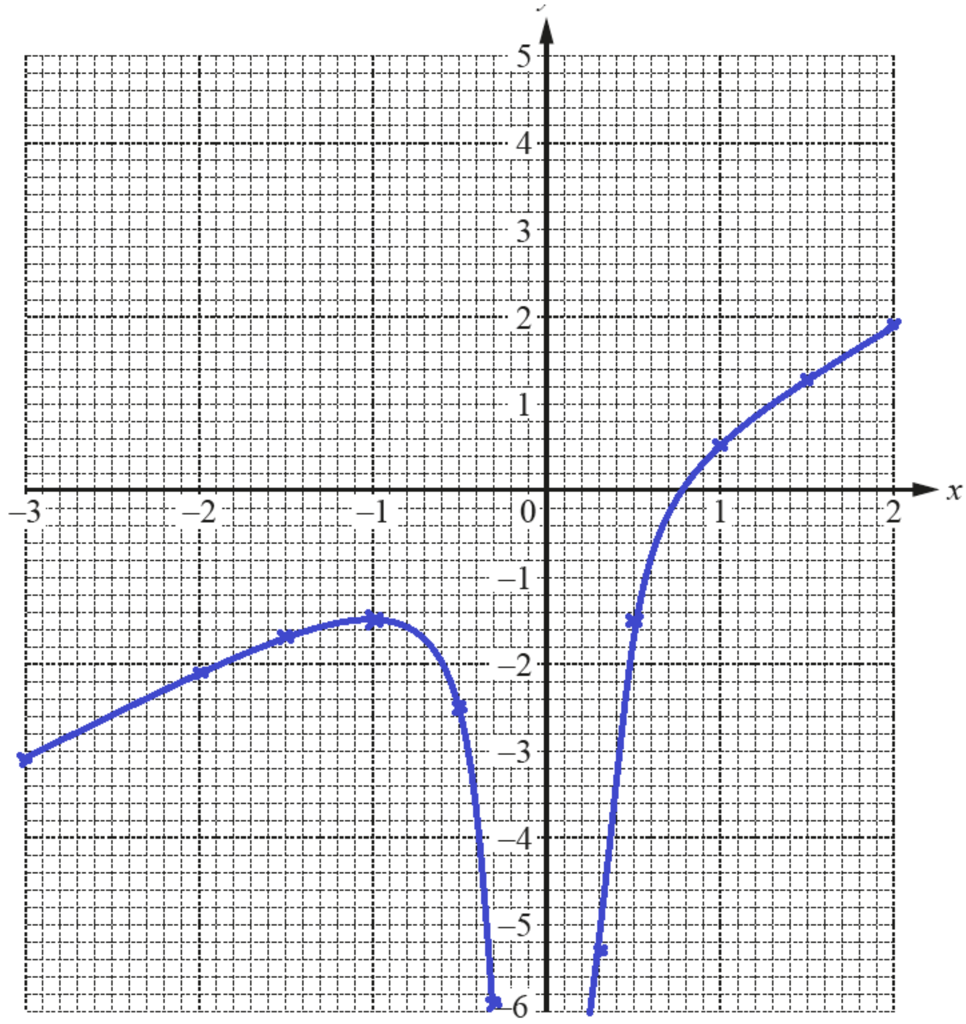
x	-3	-2	-1.5	-1	-0.5	-0.3		0.3	0.5	1	1.5	2
$f(x)$	-3.1	-2.1	-1.7		-2.5	-5.9		-5.3	-1.5		1.3	1.9

We use calculator to find the values of *f(x)* for *x*=-1 and *x*=1.

$$f(-1) = -1.5, \qquad f(1) = 0.5$$

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 2$. [5]

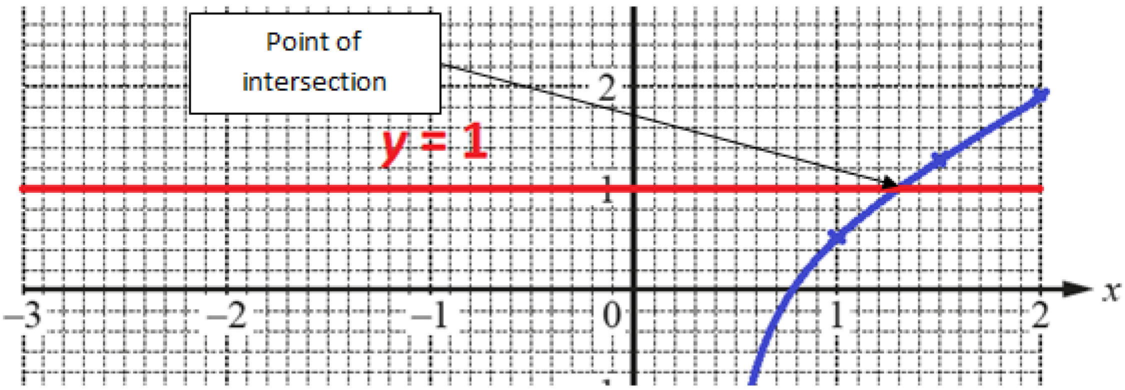
Plot the point from the table on the grid and then draw a smooth function connecting the points (blue curve).



(c) Use your graph to solve the equation f(x) = 1.

[1]

We plot the line y=1 and find the x-coordinate of the point of intersection.



From the graph, we can see that the x-coordinate of the point is ***x* = 1.3.**

(d) There is only one negative integer value, k, for which f(x) = k has only one solution for all real x.

Write down this value of k.

[1]

From the graph, we can clearly see that ***k*=-1**, since for -2 and any other negative integer, there are two solution to *f(x)=k*.

(e) The equation $2x - \frac{1}{2x^2} - 2 = 0$ can be solved using the graph of y = f(x) and a straight line graph.

(i) Find the equation of this straight line. [1]

Subtract (x-2) from both sides of the equation.

$$x - \frac{1}{2x^2} = 2 - x$$

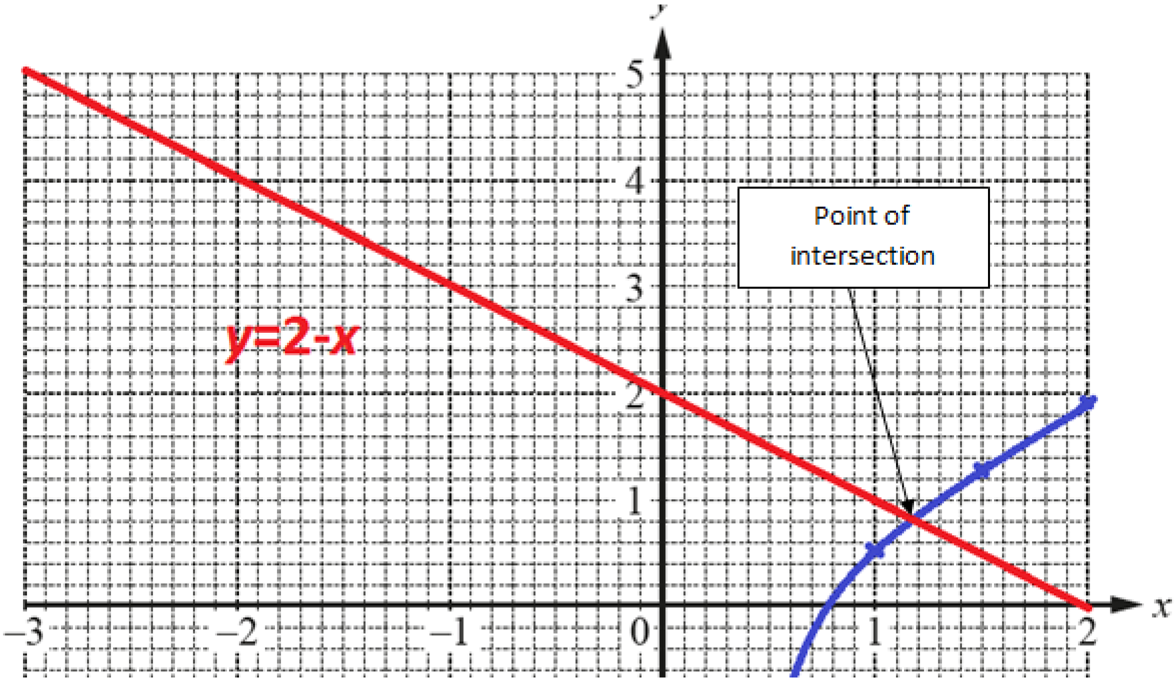
We can see that the right side of the equation is our original function.

Therefore the left hand side must be the straight line we are looking for.

$$\boldsymbol{y = 2 - x}$$

(ii) On the grid, draw this straight line and solve the equation $2x - \frac{1}{2x^2} - 2 = 0$ [3]

We plot a line $y = 2 - x$ and find the x-coordinate of the point of intersection with the original graph to solve $2x - \frac{1}{2x^2} - 2 = 0$



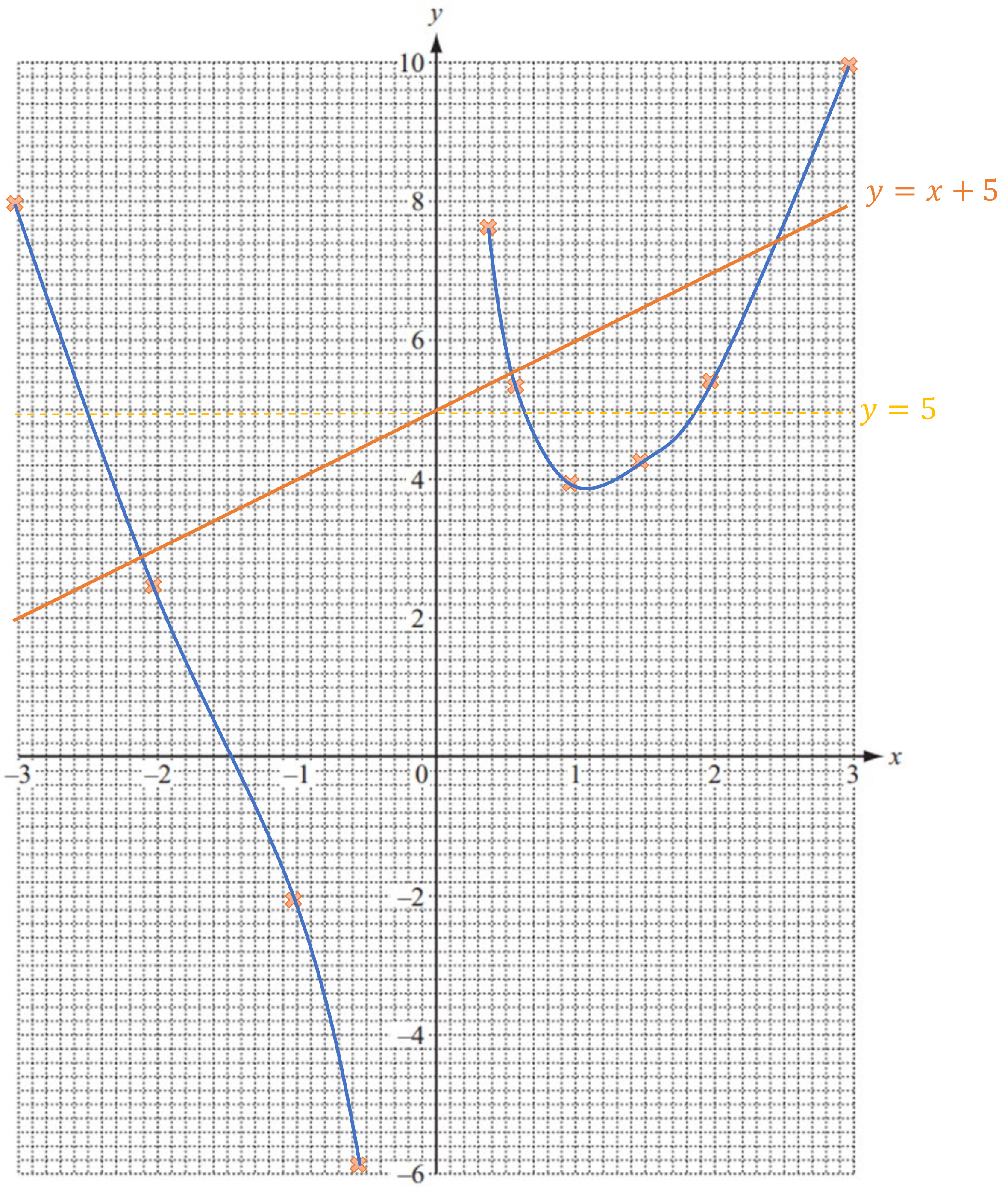
From the graph, we can see that the x-coordinate of the point, and hence the solution to the equation $2x - \frac{1}{2x^2} - 2 = 0$ is

$$\boldsymbol{x = 1.15}$$

(a) Complete the table of values for $y = x^2 + \frac{3}{x}$, $x \neq 0$. [2]

x	-3	-2	-1	-0.5	0.4	0.6	1	1.5	2	3
y	8	2.5	**-2**	-5.8	7.7	5.4	4	4.3	**5.5**	10

(b) Draw the graph of $y = x^2 + \frac{3}{x}$ for $-3 \leqslant x \leqslant -0.5$ and $0.4 \leqslant x \leqslant 3$. [5]



(c) Use your graph to solve the equation $x^2 + \frac{3}{x} = 5$. [3]

$x = -2.50,\ x = 0.65,\ x = 1.90$

(d) By drawing a suitable straight line, solve the equation $x^2 + \frac{3}{x} = x + 5$. [4]

Suitable line drawn in orange on graph above.

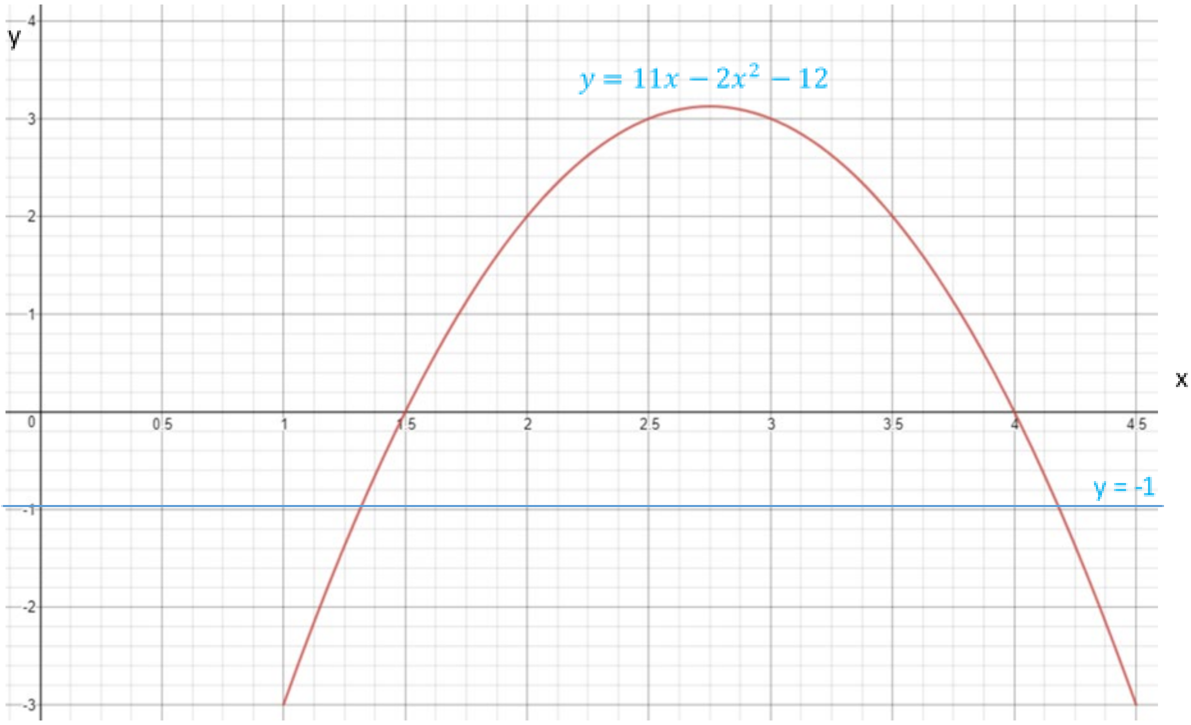
$x = -2.10, \quad x = 0.55, \quad x = 2.50$

The table shows some values for the function $y = 11x - 2x^2 - 12$ $1 \leqslant x \leqslant 4.5$. [3]

x	1	1.5	2	2.5	3	3.5	4	4.5
y	-3	**0**	2	3	3	**2**	**0**	**-3**

(a) Complete the table of values.

(b) On the grid below, draw the graph of $y = 11x - 2x^2 - 12$ for $1 \leqslant x \leqslant 4.5$. [4]



(c) By drawing a suitable line, use your graph to solve the equation $11x - 2x^2 = 11$. [2]

$$y = 11x - 2x^2 - 12$$

$$y + 12 = 11x - 2x^2$$

$$For, \quad 11x - 2x^2 = 11$$

$$y + 12 = 11$$

$$Therefore, \quad y = -1$$

Draw a line at where y = -1 and read off x values

x = 1.3 to 1.4 or

x = 4.1 to 4.2

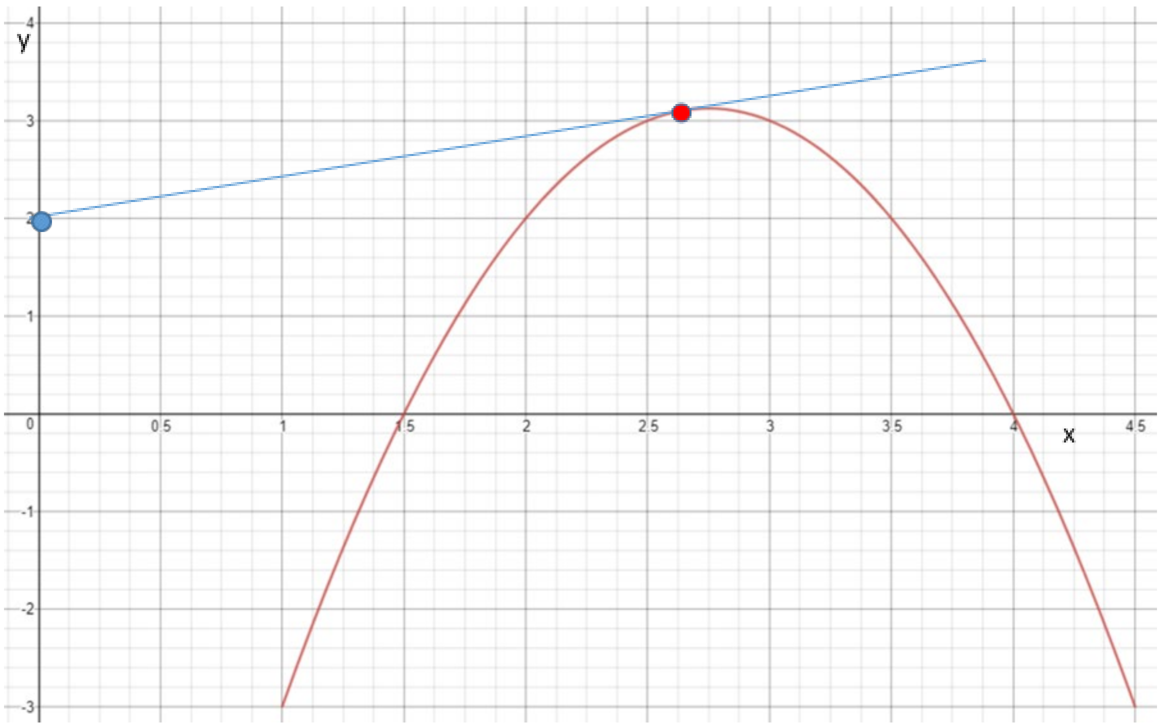
(d) The line $y = mx + 2$ is a tangent to the curve $y = 11x - 2x^2 - 12$ at the point P.

By drawing this tangent,

(i) find the co-ordinates of the point P, [2]

$$y = mx + 2$$

First, we have to correctly identify the y-intercept. From this equation, the y-intercept is 2. We then draw a tangential line towards the curve.



Based on where the tangent meets the line, the point P can be deduced.

Point P is (2.63, 3.1)

Any value within this range (2.5 to 2.75, 3 to 3.4) is acceptable.

(ii) work out the value of m. [2]

To work the value for m, identify 2 points on this tangential line and find the gradient.

To make things simple, since P has already been found, and we know the y-intercept, we already have 2 points:

Point P is (2.63, 3.1)

y-intercept is (0, 2)

$$Gradient = \frac{3.1 - 2}{2.63 - 0}$$

$$m = 0.42$$

(a) Complete this table of values for the function $f(x) = \frac{1}{x} - x^2,\ x \neq 0$. [3]

x	–3	–2	–1	–0.5	–0.2		0.2	0.5	1	2	3
f(x)	–9.33	–4.5	–2	–2.25	-5.04		4.96	1.75	0	–3.5	–8.67

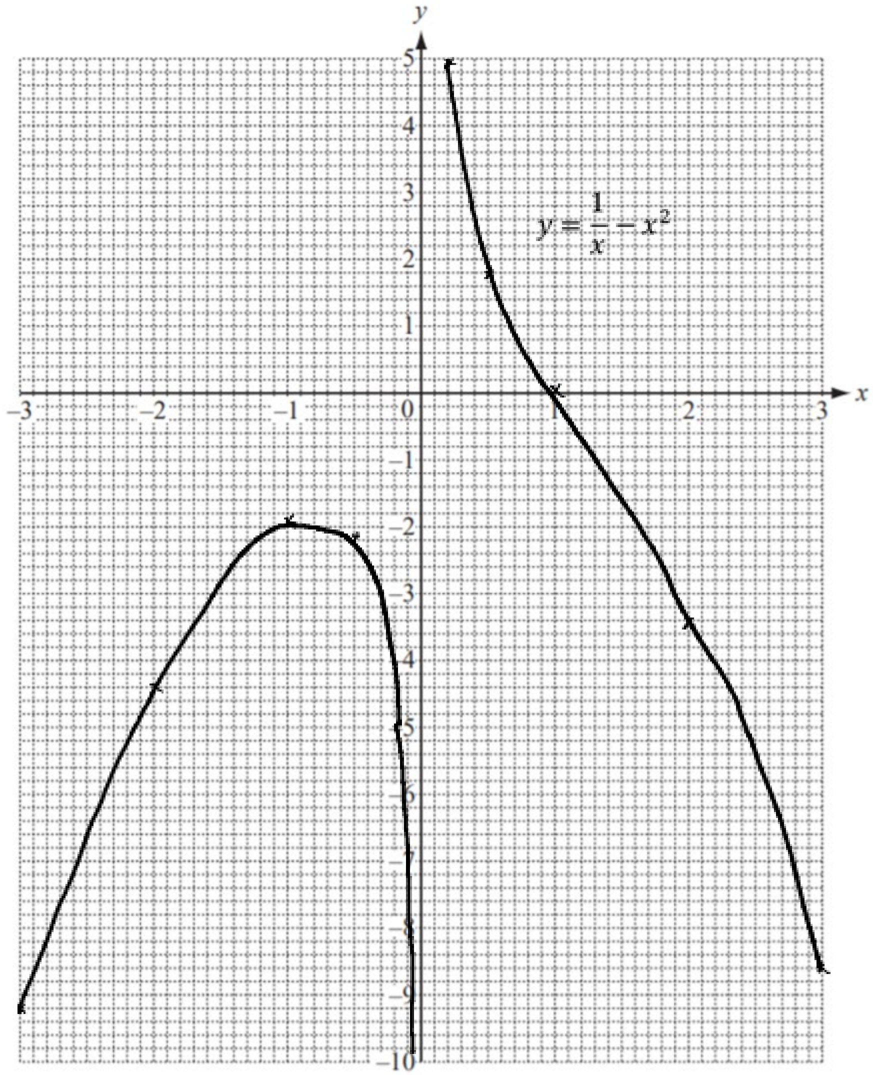
Use a calculator to fill in the gaps

$$f(-0.2) = \frac{1}{-0.2} - (-0.2)^2 = \mathbf{-5.04}$$

$$f(0.5) = \frac{1}{0.5} - 0.5^2 = \mathbf{1.75}$$

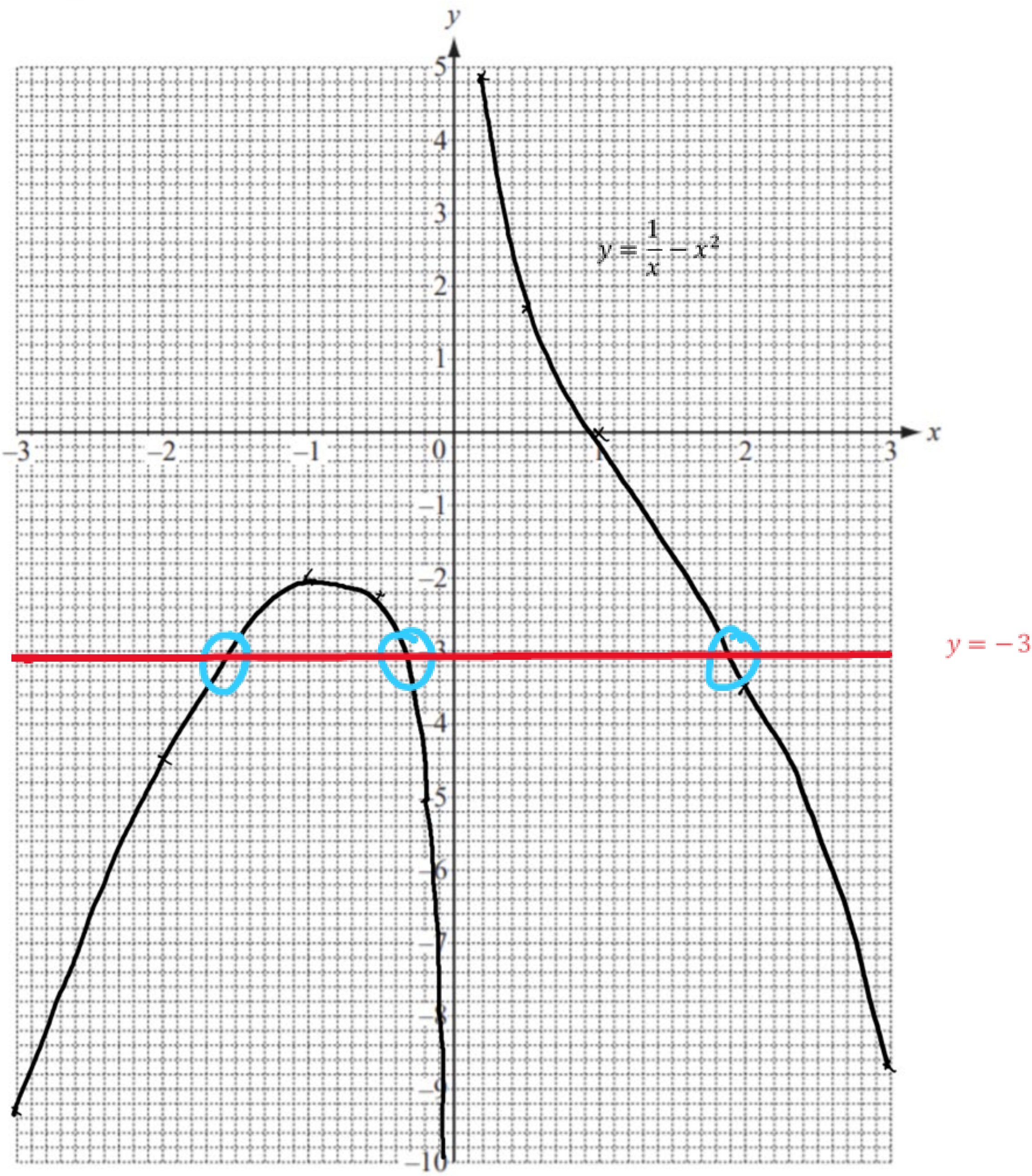
$$f(1) = \frac{1}{1} - 1^2 = \mathbf{0}$$

(b) Draw the graph of $f(x) = \frac{1}{x} - x^2$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$. [5]



(c) Use your graph to solve f(x) = –3. [3]

Draw the line $y = f(x) = -3$ on the diagram and find the values of x where the line

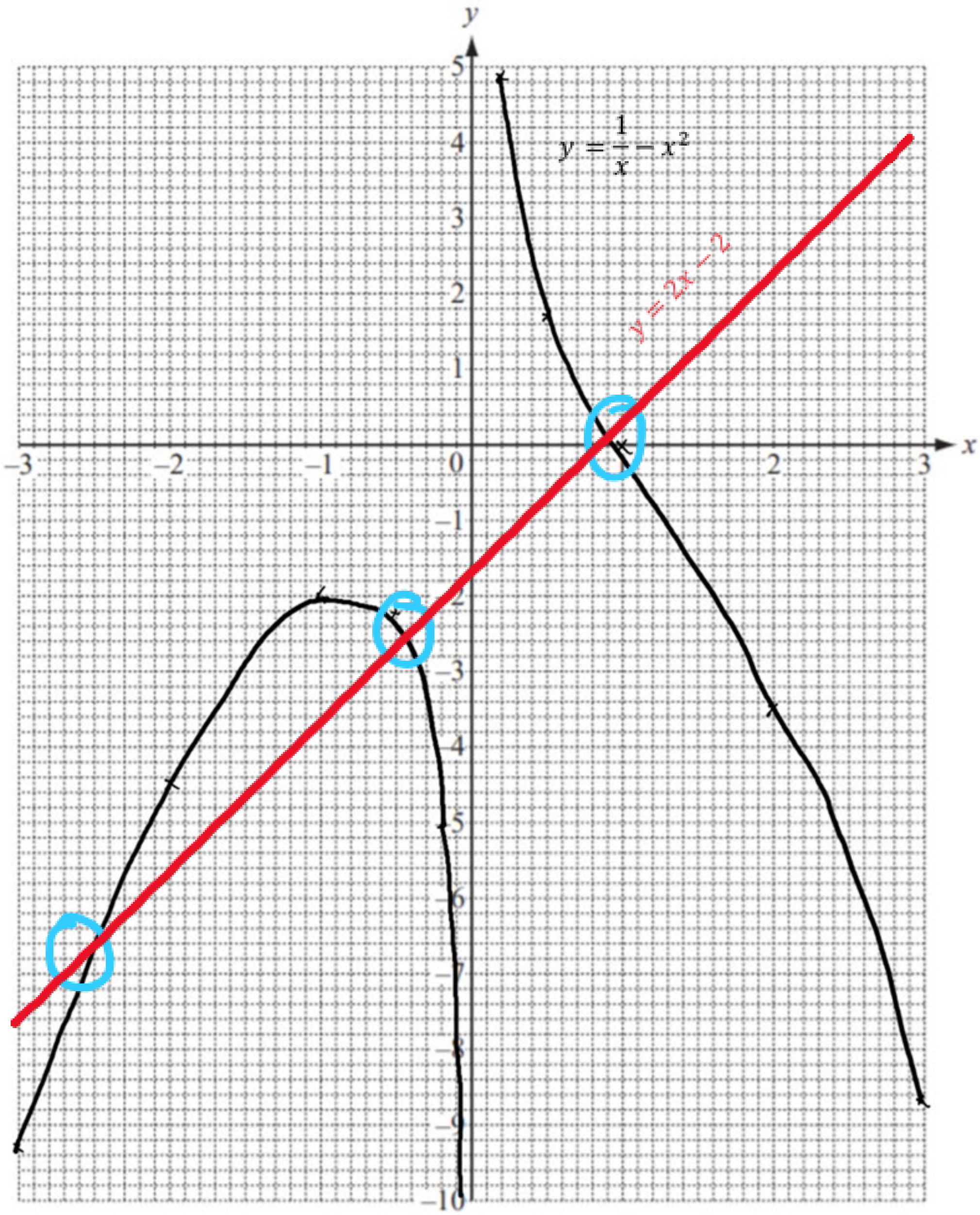


intersects the curve.

$x = -1.55, -0.3, 1.9$

(d) By drawing a suitable line on your graph, solve the equation f(x) = $2x - 2$. [3]

Draw the line $y = f(x) = 2x - 2$ on the diagram and find the values of x for where it intersects the curve

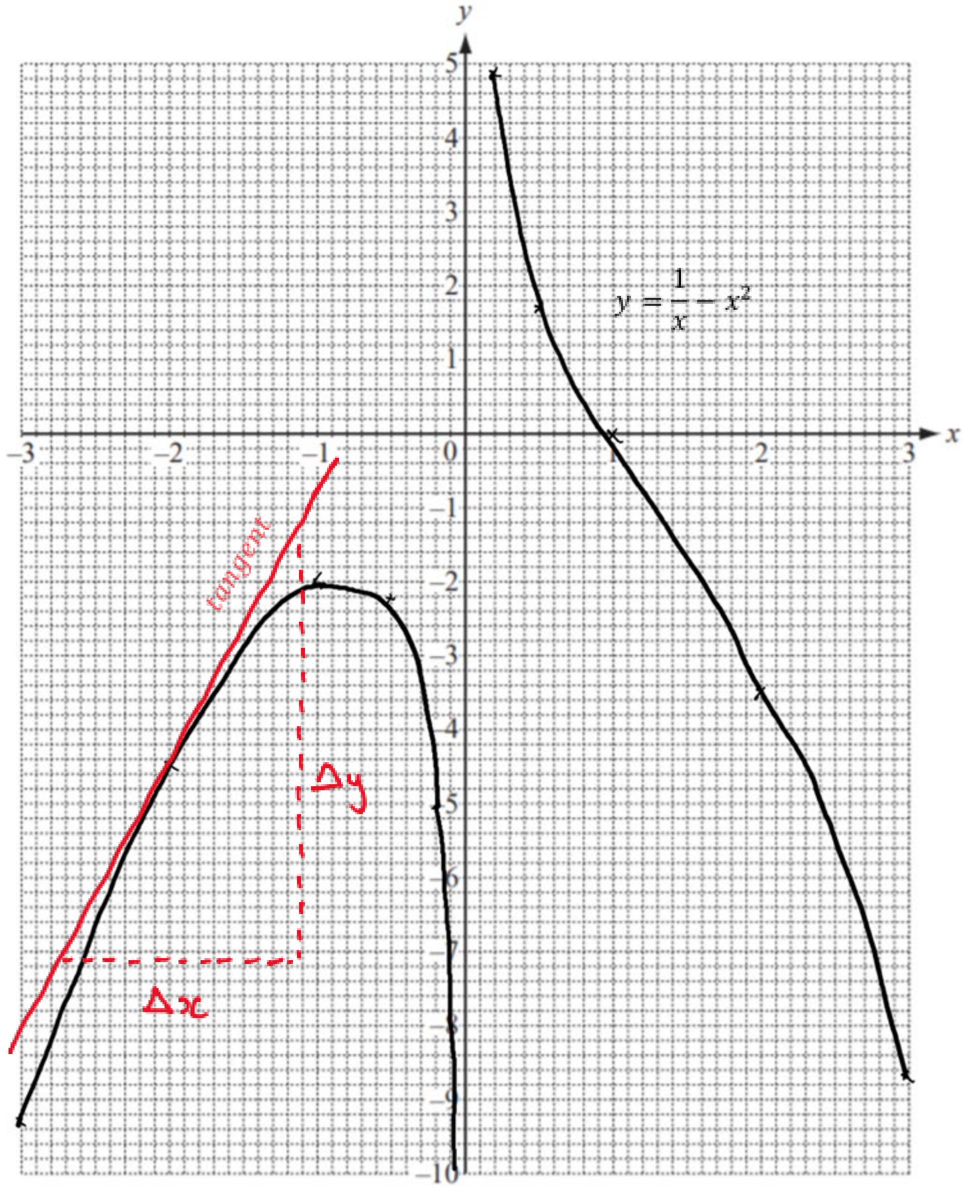


$x = -2.6, -0.4, 1$

(e) By drawing a suitable tangent, work out an estimate of the gradient of the curve at the point where $x = -2$.

You must show your working. [3]

Draw a tangent to the curve at $x = -2$ and use the equation $m = \frac{\Delta y}{\Delta x}$ to find the gradient



by using two points from the tangent.

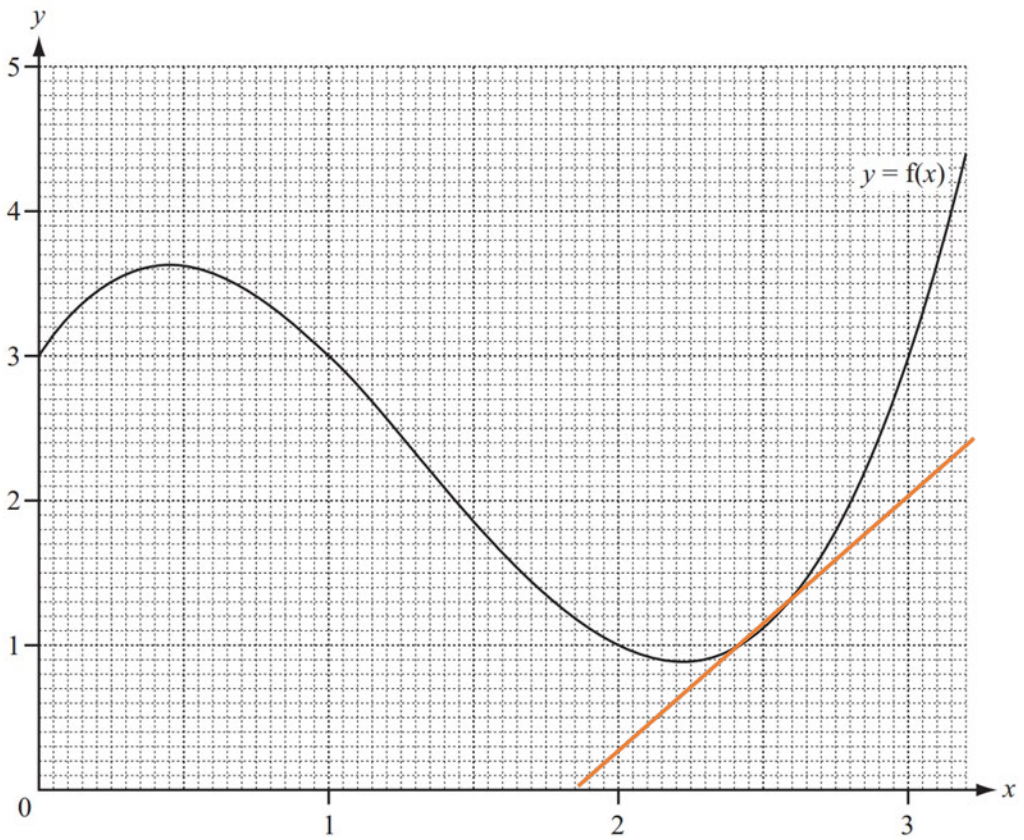
$Change\ in\ x\ (\Delta x) = -1.1 - -2.7 = 1.6$

$Change\ in\ y\ (\Delta y) = -1.1 - -7 = 5.9$

$$gradient = \frac{5.9}{1.6}$$

$$= \mathbf{3.69}$$

The graph of $y = f(x)$ is drawn on the grid for $0 \leqslant x \leqslant 3.2$.



(a) (i) Draw the tangent to the curve $y = f(x)$ at $x = 2.5$. [1]

The orange line on the diagram below.

(ii) Use your tangent to estimate the gradient of the curve at $x = 2.5$. [2]

We use the equation

$$\boldsymbol{m = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x}}$$

Where m is the gradient and $\Delta x, \Delta y$ represent the change in x and y respectively.

From the diagram we can see

$$\Delta y = 2.3$$

$$\Delta x = 1.3$$

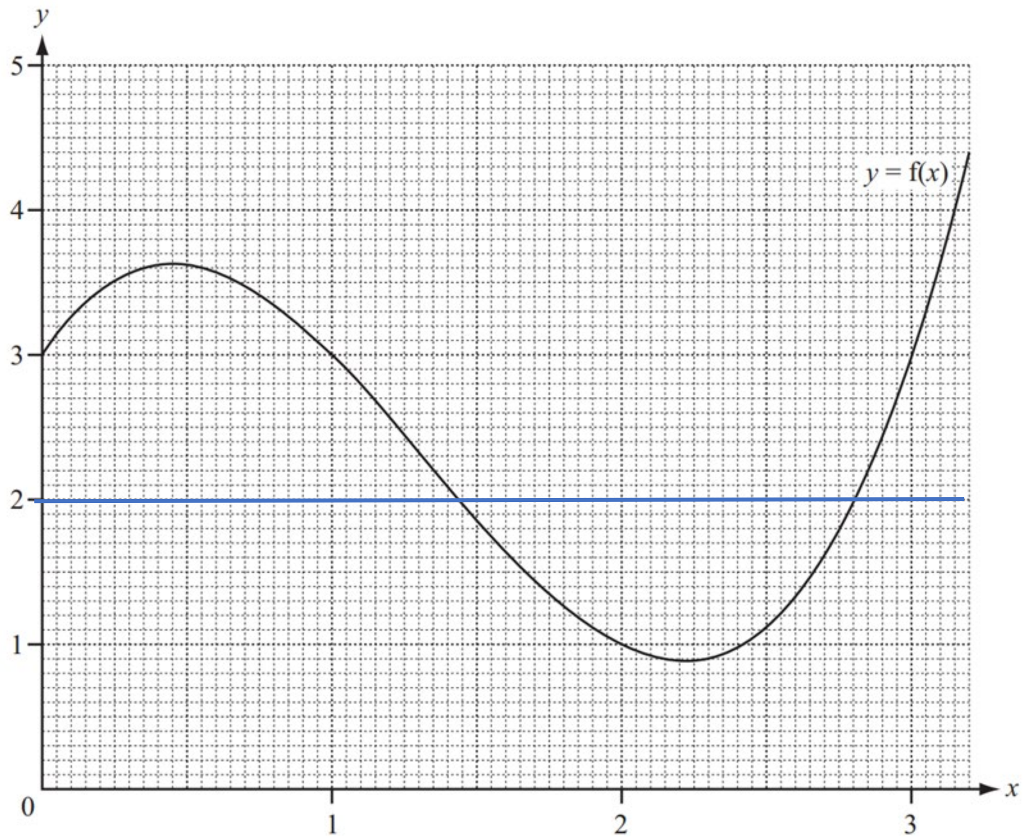
$$\rightarrow m = \frac{2.3}{1.3} = \mathbf{1.77}$$

(b) Use the graph to solve $f(x) = 2$, for $0 \leqslant x \leqslant 3.2$. [2]

The line $y = 2$ is drawn in blue on the diagram below.

Where this line and the curve $f(x)$ intersect are the solutions to

$$f(x) = 2$$



We can see that the solutions are

$$\boldsymbol{x = 1.45, x = 2.8}$$

(c) $$g(x) = \frac{x}{2} + \frac{2}{x^2} \quad x \neq 0.$$

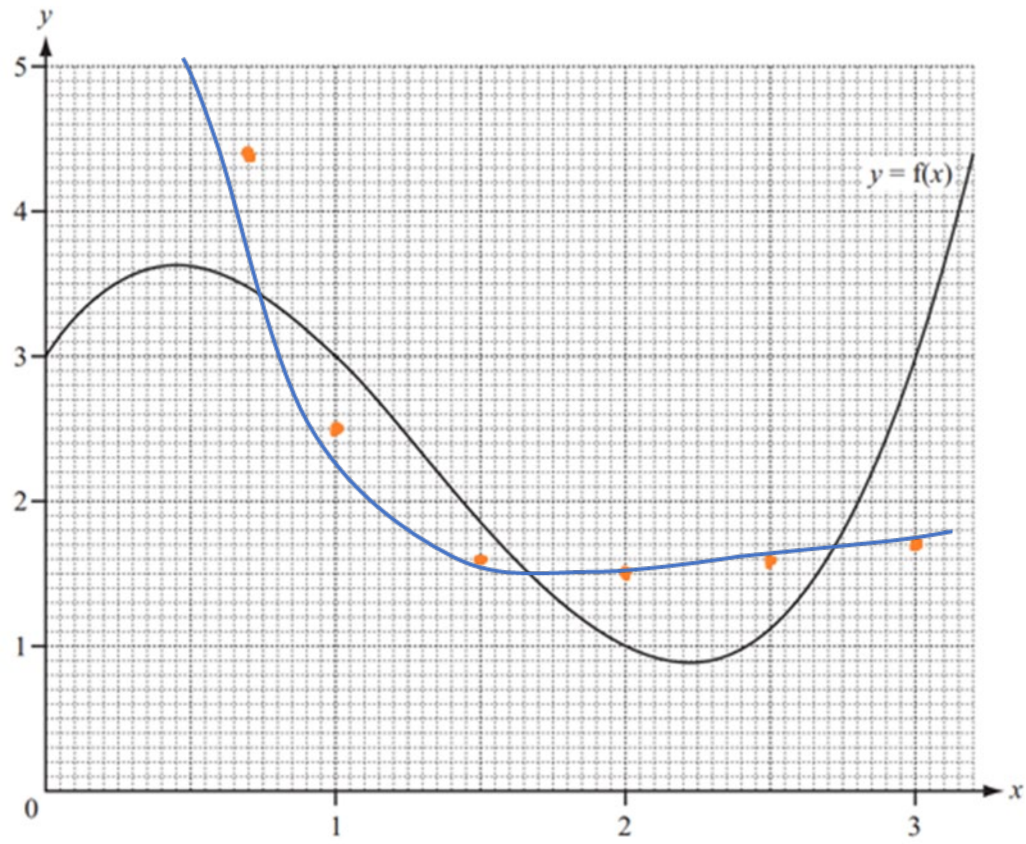
(i) Complete the table for values of g(x), correct to 1 decimal place.

x	0.7	1	1.5	2	2.5	3
g(x)	**4.4**	**2.5**	1.6	**1.5**	1.6	1.7

[2]

(ii) On the grid opposite, draw the graph of $y = g(x)$ for $0.7 \leqslant x \leqslant 3$. [3]

The points of $g(x)$ are plotted in orange and the curve is drawn in blue on the diagram below.



(iii) Solve $f(x) = g(x)$ for $0.7 \leqslant x \leqslant 3$. [3]

We can see that the curves intersect at

$$x = 0.85, x = 1.7, x = 2.7$$

Graphs
Difficulty: Medium

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 3

Time allowed: **122 minutes**

Score: **/106**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

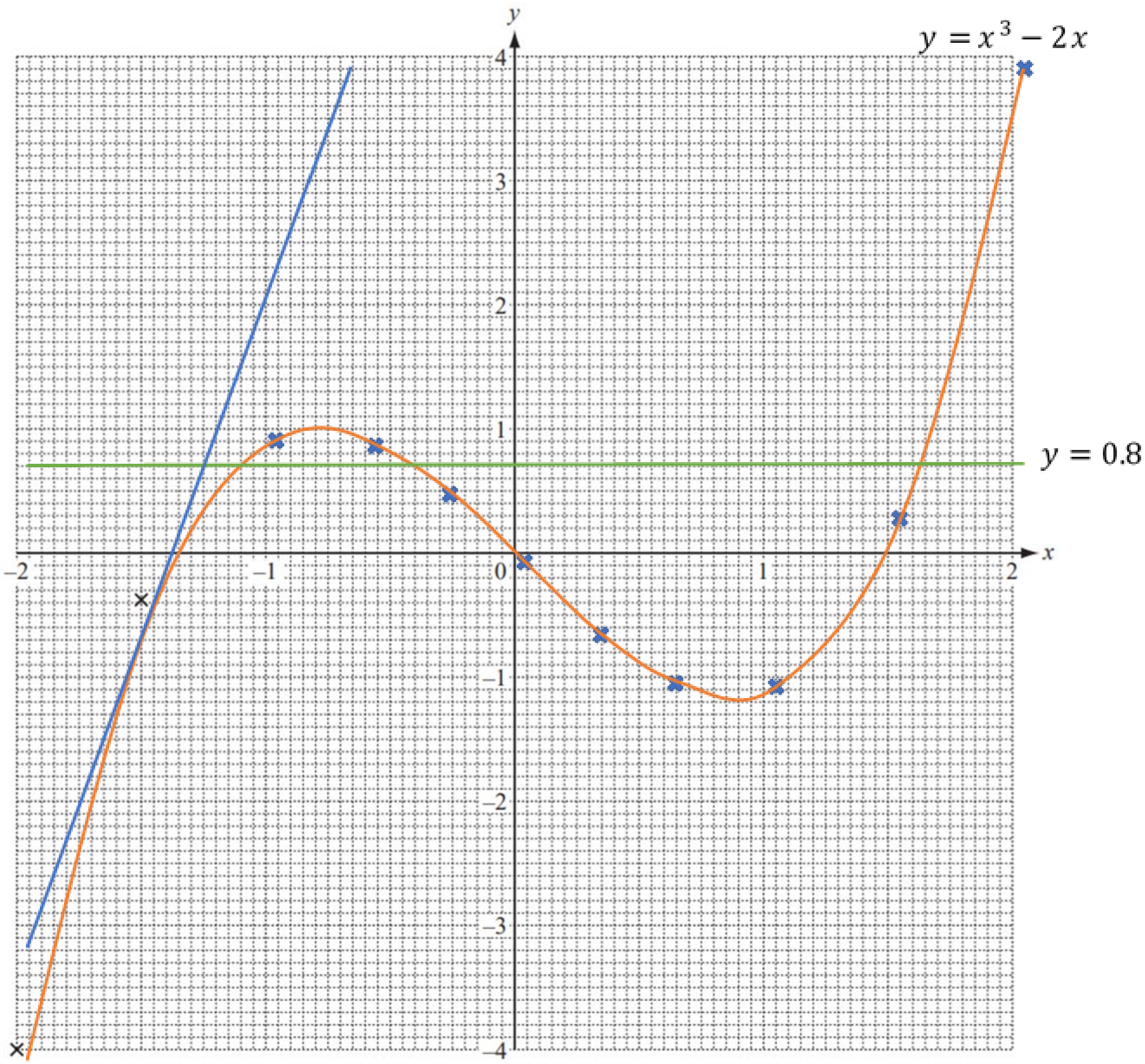
Question 1

The table shows some values for the equation $y = x^3 - 2x$ for $-2 \leqslant x \leqslant 2$.

x	-2	-1.5	-1	-0.6	-0.3	0	0.3	0.6	1	1.5	2
y	-4	-0.38	**1**	**0.98**	0.57	**0**	-0.57	**-0.98**	**-1**	0.38	4

(a) Complete the table of values. [3]

(b) On the grid below, draw the graph of $y = x^3 - 2x$ for $-2 \leqslant x \leqslant 2$. [4]
The first two points have been plotted for you.



(c) (i) On the grid, draw the line $y = 0.8$ for $-2 \leqslant x \leqslant 2$. [1]

The green line drawn above.

(ii) Use your graph to solve the equation $x^3 - 2x = 0.8$. [3]

The line and the curve intersect at (and hence solve the equation for)

$$\boldsymbol{x = -1.15, \quad -0.50, \quad 1.60}$$

(d) By drawing a suitable tangent, work out an estimate for the gradient of the graph of $y = x^3 - 2x$ where $x = -1.5$.

You must show your working. [3]

Tangent drawn in blue on graph above.

The gradient of the tangent is found using

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Hence, using the coordinates of its endpoints

$$m = \frac{4 - -3.1}{-0.7 - -2}$$

$$= \frac{7.1}{1.3}$$

$$= \boldsymbol{5.46}$$

(a) Complete the table for the function $f(x) = \frac{x^3}{2} - 3x - 1$. [3]

x	–3	–2	–1.5	–1	0	1	1.5	2	3	3.5
f(x)	–5.5		1.8	1.5		–3.5	–3.8	–3		9.9

We complete the table by substituting the values of x in the function

$f(x) = \frac{x^3}{2} - 3x - 1$

For x = -2

$f(x) = \frac{(-2)^3}{2} - 3 \times (-2) - 1$

f(x) = 1

For x = 0

$f(x) = \frac{0^3}{2} - 3 \times 0 - 1$

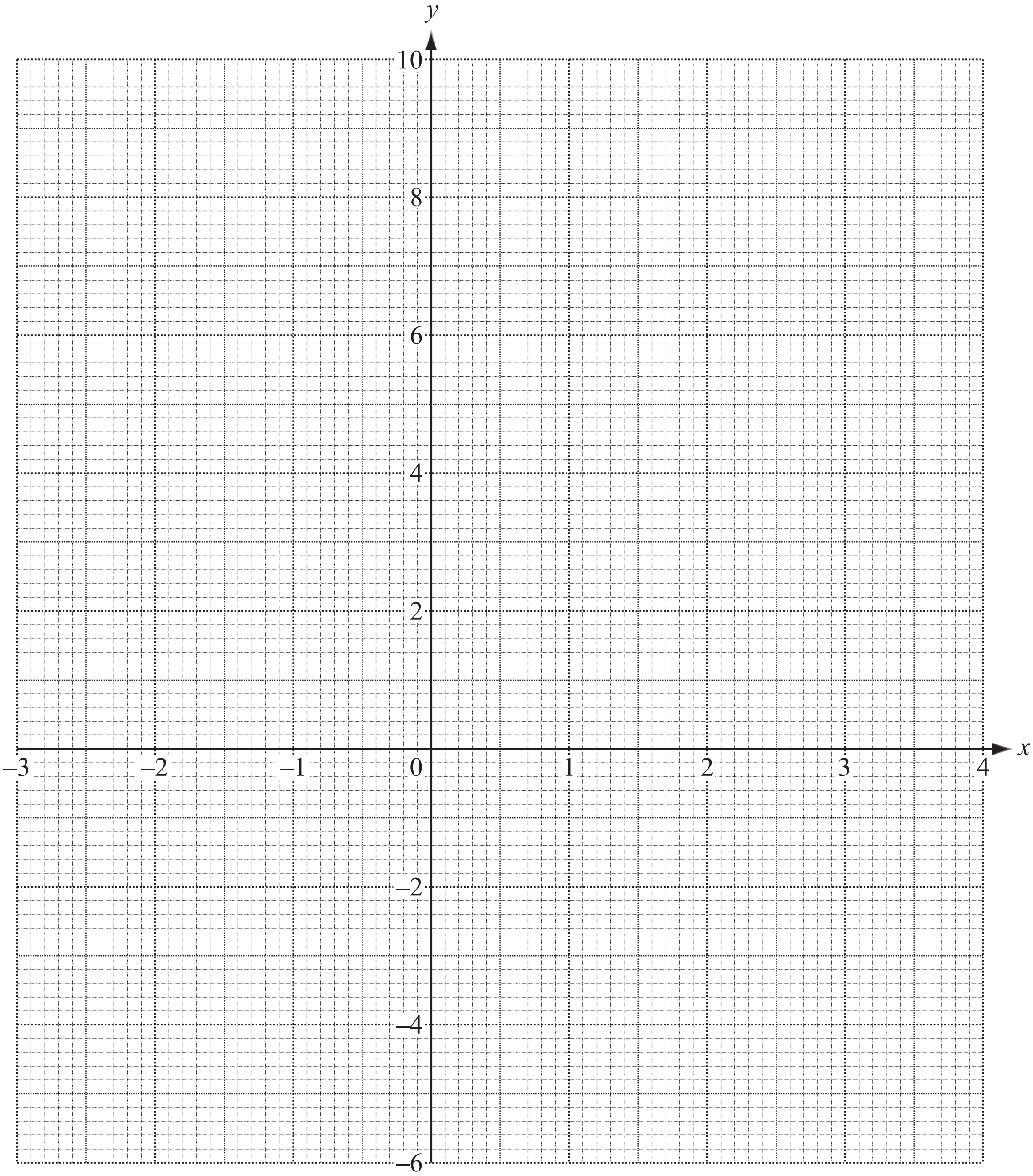
f(x) = -1

For x = 3

$f(x) = \frac{3^3}{2} - 3 \times 3 - 1$

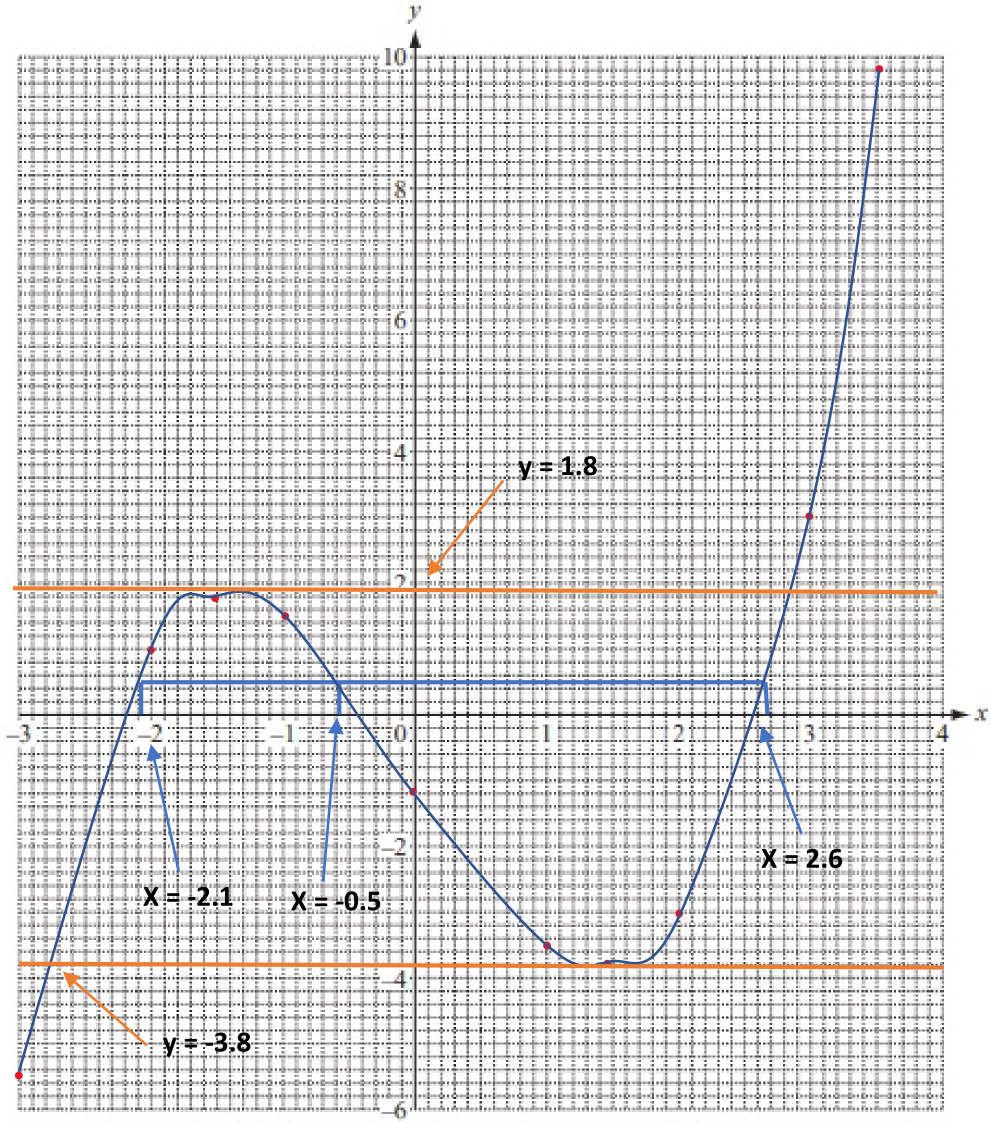
f(x) = 3.5

(b) On the grid draw the graph of $y = f(x)$ for $-3 \leq x \leq 3.5$ [4]



We plot the points (x, f(x)) for x = -3, -2, -1 0, 1, 2, 3

The corresponding f(x) for each x value can be seen in the table.



(c) Use your graph to

(i) solve f(x) = 0.5, [3]

f(x) = 0.5

To solve this equation using the graph we need to identify all the x coordinates for points on the graph with f(x) = y = 0.5

There are 3 such points, of coordinates (-2.1, 0.5), (-0.5, 0.5), (2.6, 0.5)

Therefore, the solutions of the equation are:

X = -2.1

X = -0.5

X = 2.6

(ii) find the inequalities for k, so that $f(x) = k$ has only 1 answer. [2]

f(x) = k

We need to select the y values for which the graph has only one corresponding x value.

By looking at the graph, we can see that these regions are for

f(x) = k < -3.8 and **f(x) = k > 1.8.**

(d) (i) On the same grid, draw the graph of $y = 3x - 2$ for $-1 \leq x \leq 3.5$ [3]

.

$-1 \leq x \leq 3.5$

For this interval, some of the values taken by are x = -1, 0, 1, 2, 3

The y values are worked out by substituting the x values above in the function y = 3x – 2

For x = -1:

y = 3 x (-1) – 2 = -5

For x = 0:

y = 3 x 0 – 2 = -2

For x = 1:

y = 3 x 1 – 2 = 1

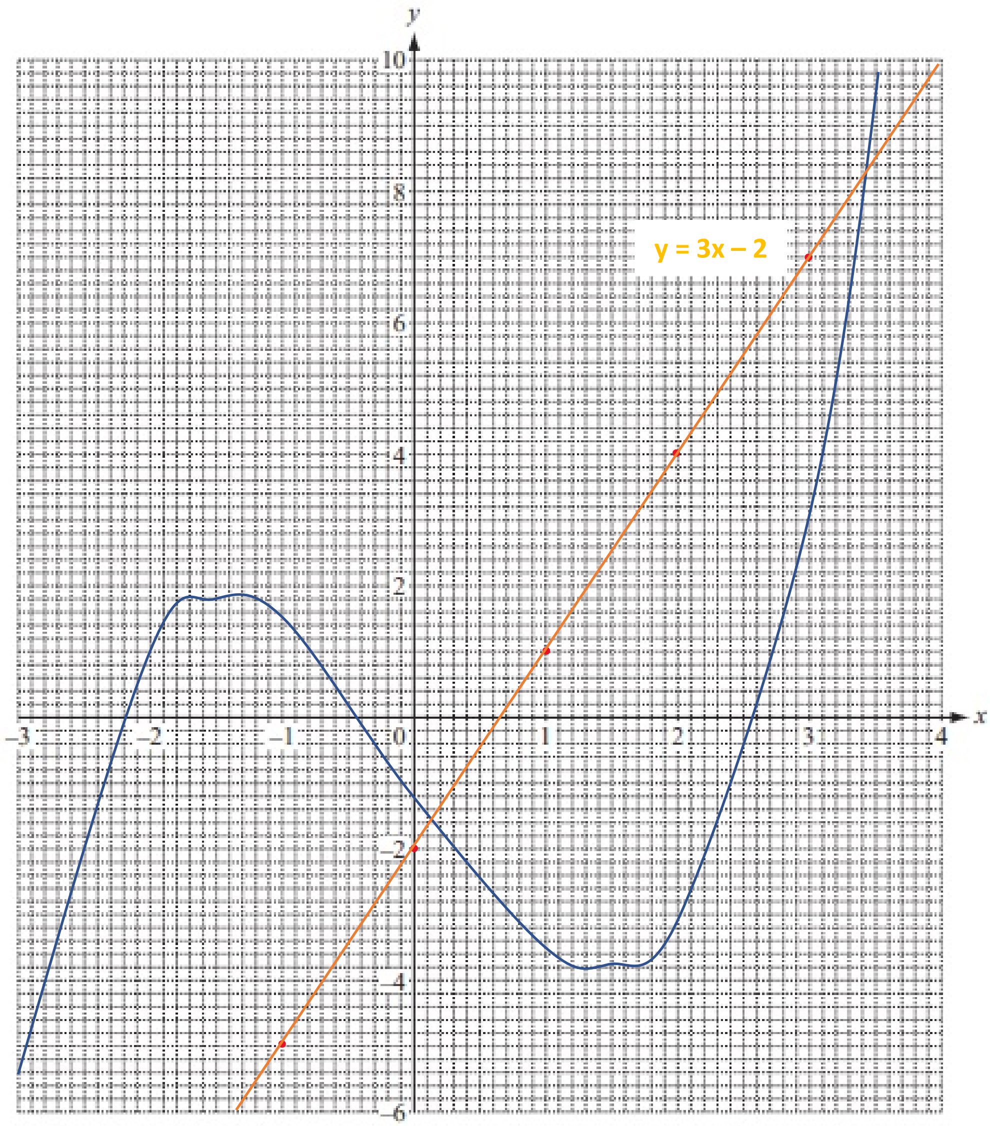
For x = 2:

y = 3 x 2 – 2 = 4

For x = 3:

Y = 3 x 3 – 2 = 7

We plot these values to obtain the graph below:



(ii) The equation $\frac{x^3}{2} - 3x - 1 = 3x - 2$ can be written in the form $x^3 + ax + b = 0$.

Find the values of a and b. [2]

We write the equation in the given form: $x^3 + ax + b = 0$ to work out a and b.

$\frac{x^3}{2} - 3x - 1 = 3x - 2$

We move all the terms on the same side and multiply each of them by 2 to have them all in the same form.

$X^3 - 6x - 2 - 6x + 4 = 0$

$X^3 - 12x + 2 = 0$

For the form $X^3 - 12x + 2 = X^3 + ax + b$

We can deduce that

a = -12 and b = 2

(iii) Use your graph to find the **positive** answers to $\frac{x^3}{2} - 3x - 1 = 3x - 2$ for $-3 \leq x \leq 3.5$. [2]

The solution of the equality: $\frac{x^3}{2} - 3x - 1 = 3x - 2$ is found at the intersection of the 2

graphs: $y = 3x - 2$ and $y = \frac{x^3}{2} - 3x - 1$

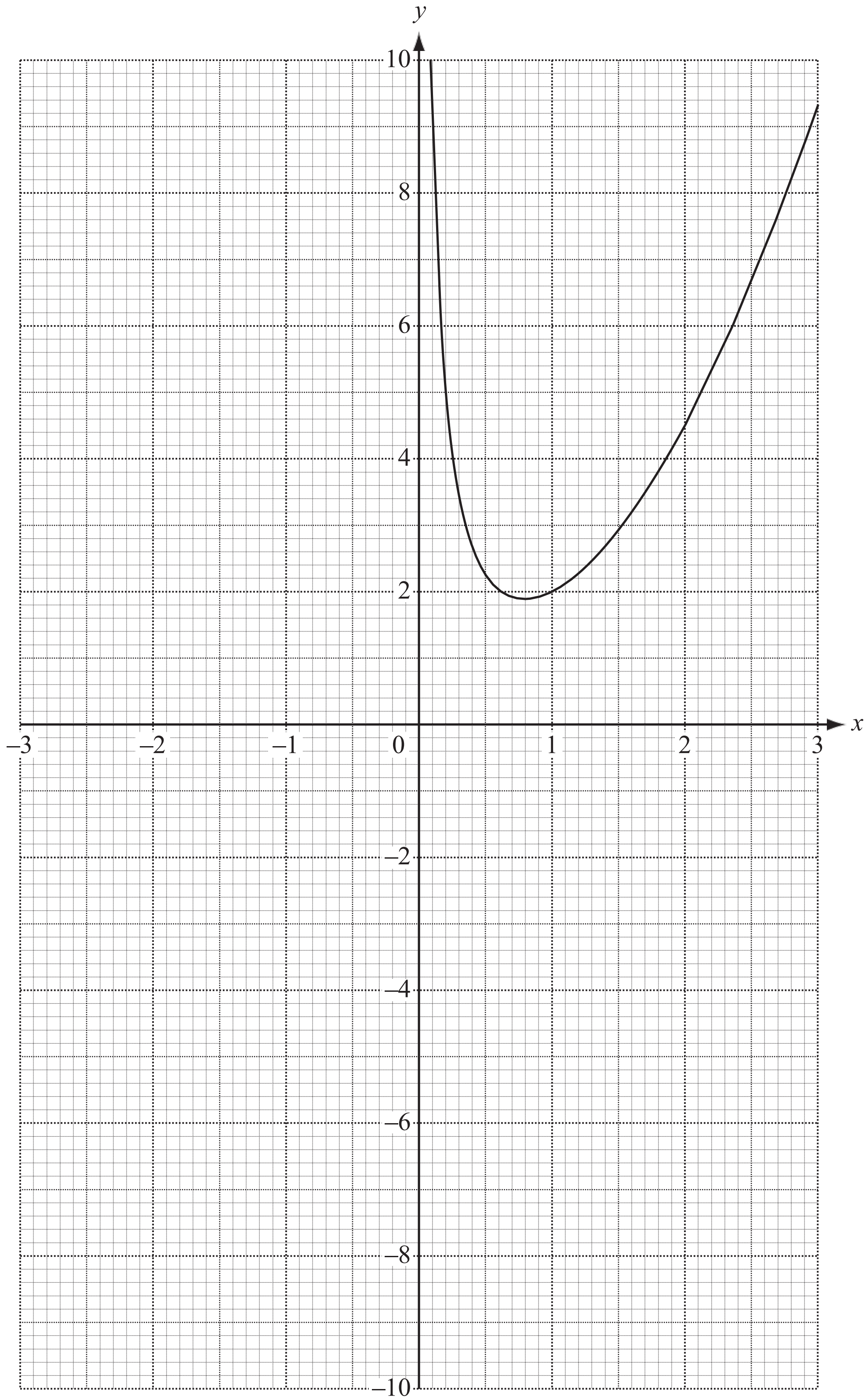
By looking at the 2 graphs plotted above, we can see that they intersect at 2 points:

(0.2, -1.6) and (3.3, 8.4)

Therefore, the equality has 2 positive solutions:

x = 0.2 and x = 3.3

The diagram shows the accurate graph of $y = f(x)$ where $f(x) = \frac{1}{x} + x^2$ for $0 < x \leqslant 3$.



(a) Complete the table for $f(x) = \frac{1}{x} + x^2$. [3]

x	-3	-2	-1	-0.5	-0.3	-0.1
$f(x)$		3.5	0	-1.8		

$f(x) = \frac{1}{x} + x^2$

We substitute the x values to work out the corresponding value of f(x).

For x = -3:

$f(-3) = \frac{1}{-3} + (-3)^2$

= **8.66**

For x = -0.3:

$f(-0.3) = \frac{1}{-0.3} + (-0.3)^2$

= **-3.24**

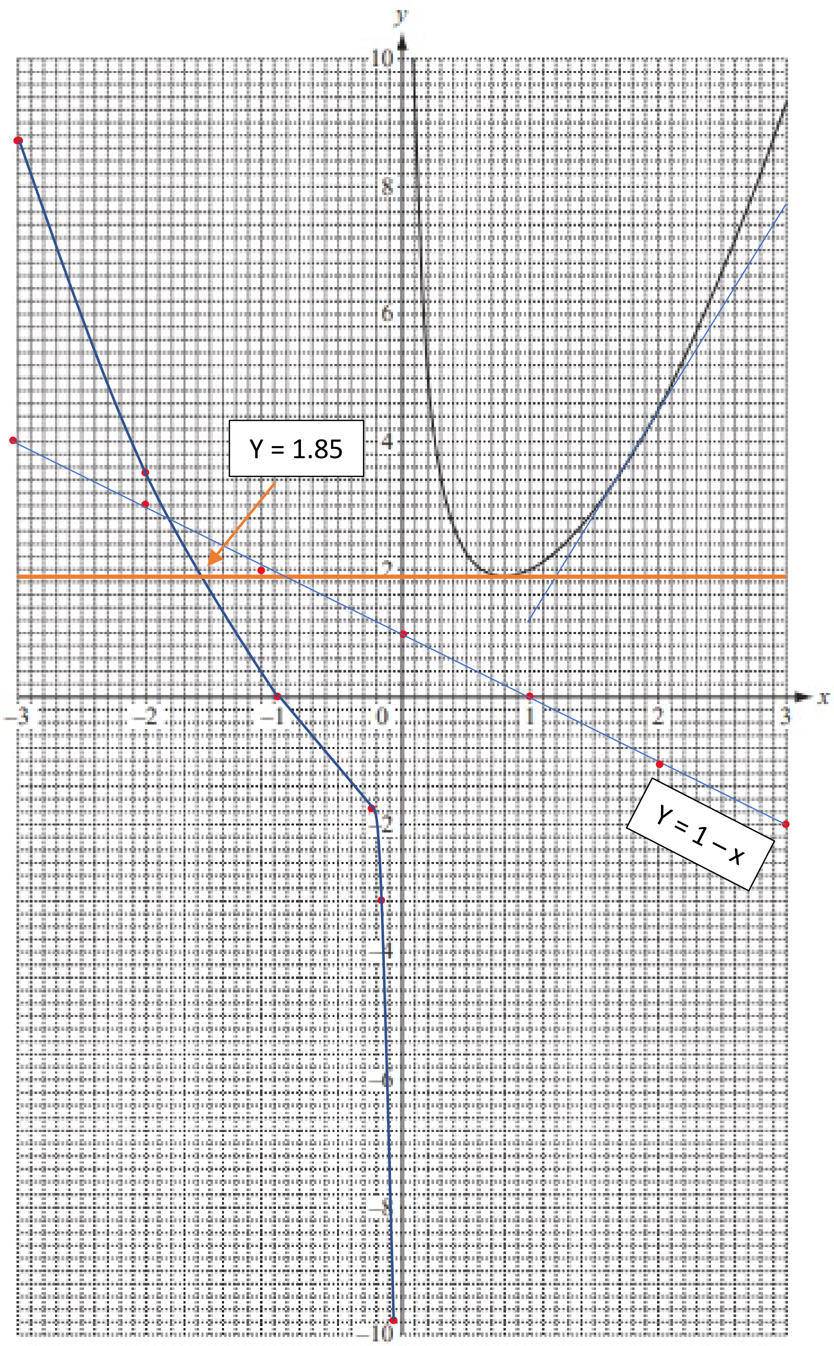
For x = -0.1:

$f(-0.1) = \frac{1}{-0.1} + (-0.1)^2$

= **-9.99**

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x < 0$. [3]

For the interval $-3 \leq x < 0$ we can select the values $x = -3, -2, -1, -0.5$to represent the graph.



(c) By drawing a tangent, work out an estimate of the gradient of the graph where $x = 2$. [3]

A tangent is a line which needs to pass through the point x = 2 and intersect the graph only at that point.

gradient = change in y/change in x

By looking at the graph above, we use the tangent to define the change in y as 6.8 units and the change in x as 2 units

Gradient = 6.8/2 = 3.4

(d) Write down the inequality satisfied by k when $f(x) = k$ has three answers. [1]

f(x) = k

We need to select the y values for which the graph has 3 simultaneous corresponding x values.

By looking at the graph, we can see that these regions are for

f(x) = k > 1.85.

(e) (i) Draw the line $y = 1 - x$ on the grid for $-3 \leqslant x \leqslant 3$. [2]

For the interval -3 ≤ x ≤ 3 we can select the values: x = -3, -2, -1, 0, 1, 2, 3 to represent the graph

The corresponding y values are:

For x = -3, y = 1 – (-3) = 4

For x = -2, y = 1 – (-2) = 3

For x = -1, y = 1 – (-1) = 2

For x = 0, y = 1 – 0 = 1

For x = 1, y = 1 – 1 = 0

For x = 2, y = 1 – 2 = -1

For x = 3, y = 1 – 3 = -2

We plot these values to obtain the line.

(ii) Use your graphs to solve the equation $1 - x = \frac{1}{x} + x^2$. [1]

The solutions will be at the points at which the 2 graphs intersect.

In this case, the intersection is at (-1.8, 2.8)

The solution is:

x = -1.8

(f) (i) Rearrange $x^3 - x^2 - 2x + 1 = 0$ into the form $\frac{1}{x} + x^2 = ax + b$, where a and b are integers. [2]

$x^3 - x^2 - 2x + 1 = 0$

We can divide each term by x to obtain x^2 and 1/x instead of x^3 and 1.

$x^2 - x - 2 + \frac{1}{x} = 0$

We move – x – 2 on the other side of the equation to get closer to the form required.

$x^2 + \frac{1}{x} = x + 2$

$x^2 + \frac{1}{x} = ax + b$

Therefore, we can deduce that a = 1 and b = 2 in this case.

(ii) Write down the equation of the line that could be drawn on the graph to solve $x^3 - x^2 - 2x + 1 = 0$. [1]

From f) i) we know that $x^3 - x^2 - 2x + 1 = 0$ can be written as: $x^2 + \frac{1}{x} = x + 2$

Therefore, the solutions of the equation can be worked out by looking at

the intersections between the graphs $f(x) = x^2 + \frac{1}{x}$ and the line $y = x + 2$

The equation of the line is y = x + 2

(a) Complete the table of values for $y = 2^x$. [2]

x	–2	–1	0	1	2	3
y	0.25		1	2		8

We substitute the values of x in the function $y = 2^x$ to obtain the corresponding y values.

For $x = -1$:

$y = 2^{-1}$

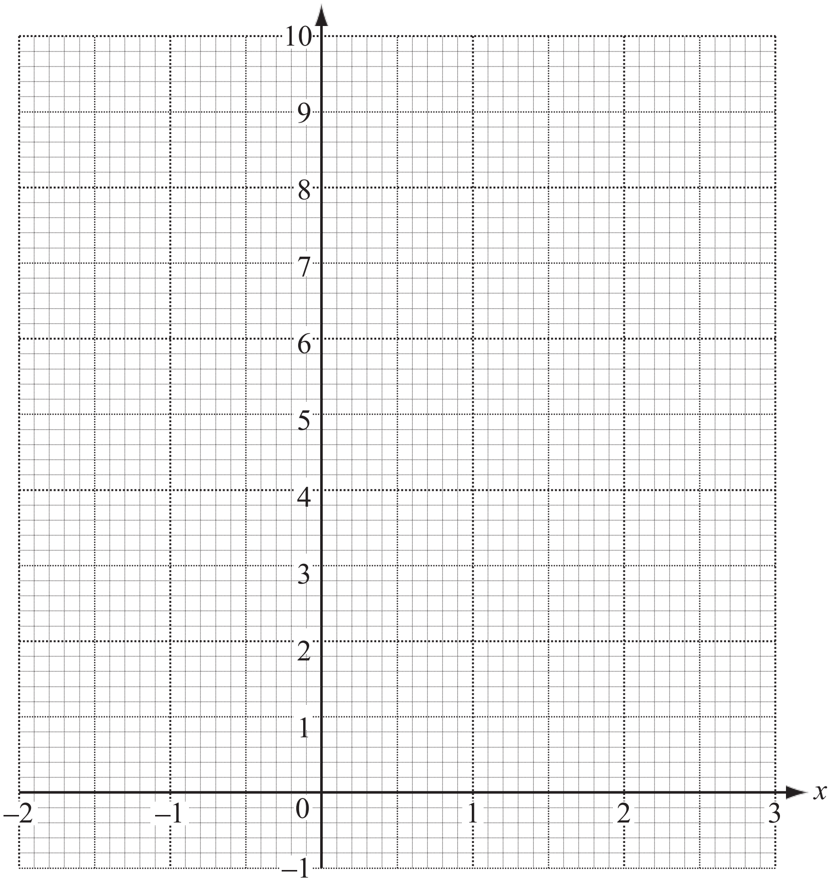
$y = 0.5$

For $x = 2$:

$y = 2^2$

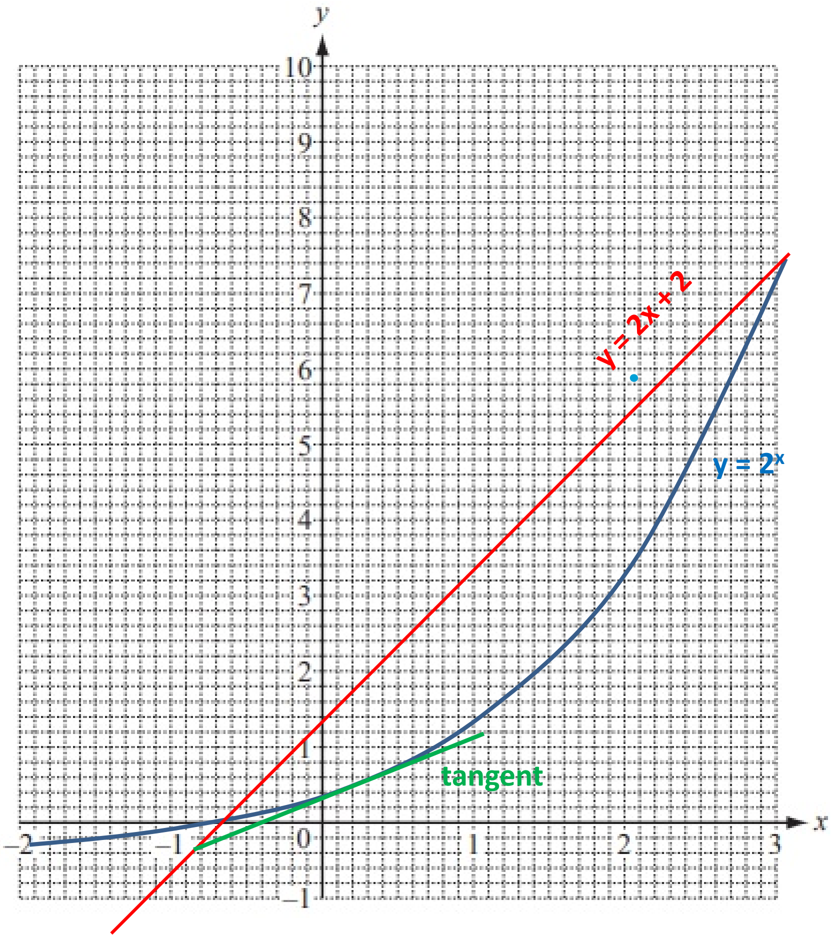
$y = 4$

(b) On the grid, draw the graph of $y = 2^x$ for $-2 \leqslant x \leqslant 3$. [3]



The corresponding x and y values are presented in the given table. We plot each point to obtain the function $y = 2^x$ for the given interval.

(c) (i) On the grid, draw the straight line which passes through the points (0, 2) and (3, 8). [1]



(ii) The equation of this line is $y = mx + 2$.

Show that the value of m is 2. [1]

y = mx + 2

We know that this line passes through the points of coordinates (0, 2) and (3, 8).

Therefore, we can substitute these pairs of values in the equation of the line.

We obtain:

2 = m x 0 + 2

And

8 = 3m + 2

From the second equation we can work out m:

3m = 6

m = 2

(iii) One answer to the equation $2^x = 2x + 2$ is $x = 3$.

Use your graph to find the other answer. [1]

The answers to the equation are found at the intersections between the 2 functions, $y = 2^x$ and $y = 2x + 2$.

The x coordinates of these intersections are solutions for both equations, giving the same y value.

Looking at the graph above, we can see that the intersections are at the points: x = 3, y = 8 and x = -0.8, y = 0.6.

The other solution is: x = -0.8

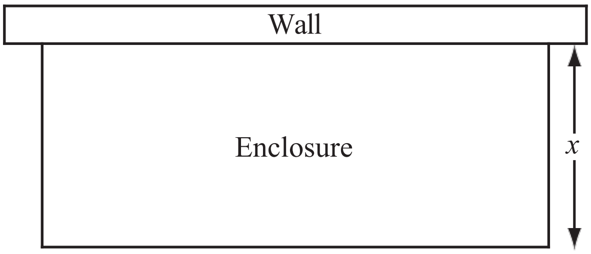
(d) Draw the tangent to the curve at the point where $x = 1$.

Use this tangent to calculate an estimate of the gradient of $y = 2^x$ when $x = 1$. [3]

gradient = $\frac{\text{change in x}}{\text{change in y}}$

gradient = $\frac{2}{1.5}$

gradient = 1.33



NOT TO SCALE

A farmer makes a rectangular enclosure for his animals.
He uses a wall for one side and a total of 72 metres of fencing for the other three sides.

The enclosure has width x metres and area A square metres.

(a) Show that $A = 72x - 2x^2$. [2]

The length of the longer side is

$$72 - 2x$$

The area is then

$$A = x \times (72 - 2x)$$

$$= \mathbf{72x - 2x^2}$$

(b) Factorise completely $72x - 2x^2$. [2]

$$\mathbf{2x(36 - x)}$$

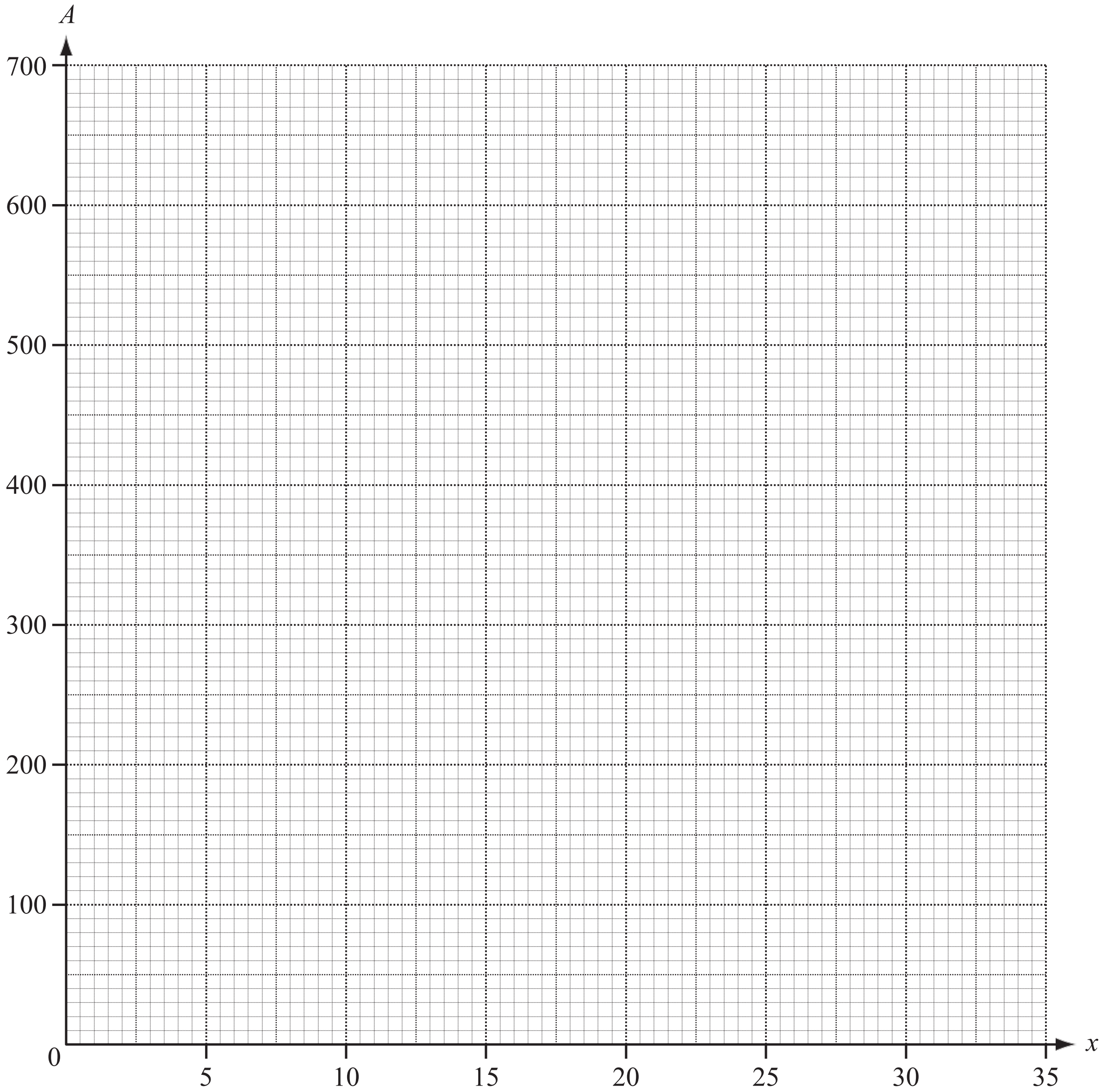
(c) Complete the table for $A = 72x - 2x^2$.

x	0	5	10	15	20	25	30	35
A	0	310	520			550	360	

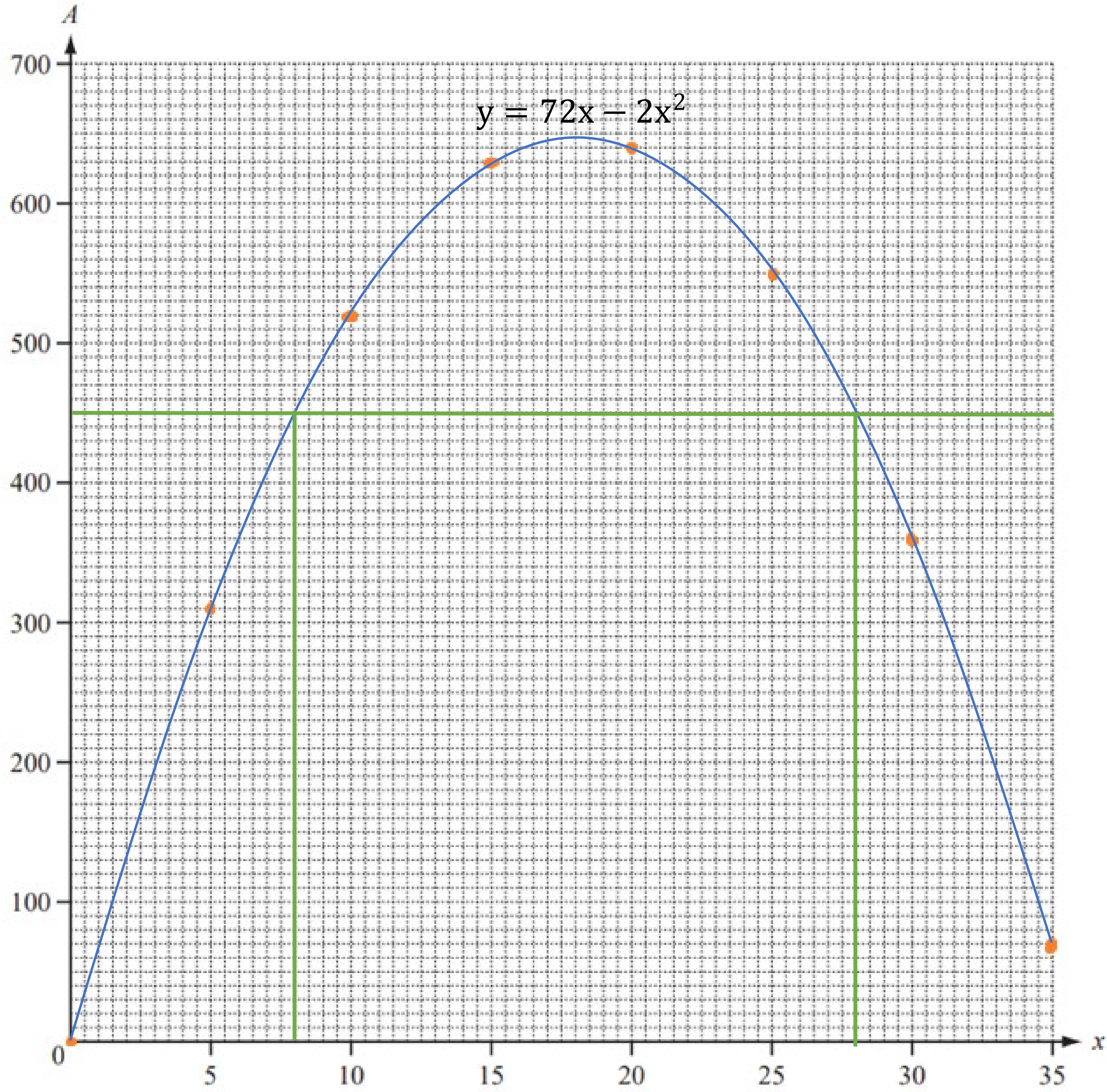
[3]

x	0	5	10	15	20	25	30	35
A	0	310	520	**630**	**640**	550	360	**70**

(d) Draw the graph of $A = 72x - 2x^2$ for $0 \leqslant x \leqslant 35$ on the grid opposite.



[4]



(e) Use your graph to find

(i) the values of x when $A = 450$, [2]

From our graph we can see that when $A = 450$ (green lines)

$$x = 8, x = 28$$

(ii) the maximum area of the enclosure. [1]

The maximum point of the graph (and hence the maximum area) is

$$A_M = 650$$

(f) Each animal must have at least $12\ m^2$ for grazing.

Calculate the greatest number of animals that the farmer can keep in an enclosure which has an area of $500\ m^2$. [2]

$$500 \div 12$$

$$= 41.67$$

$$= \mathbf{41}\ \boldsymbol{whole\ animals}$$

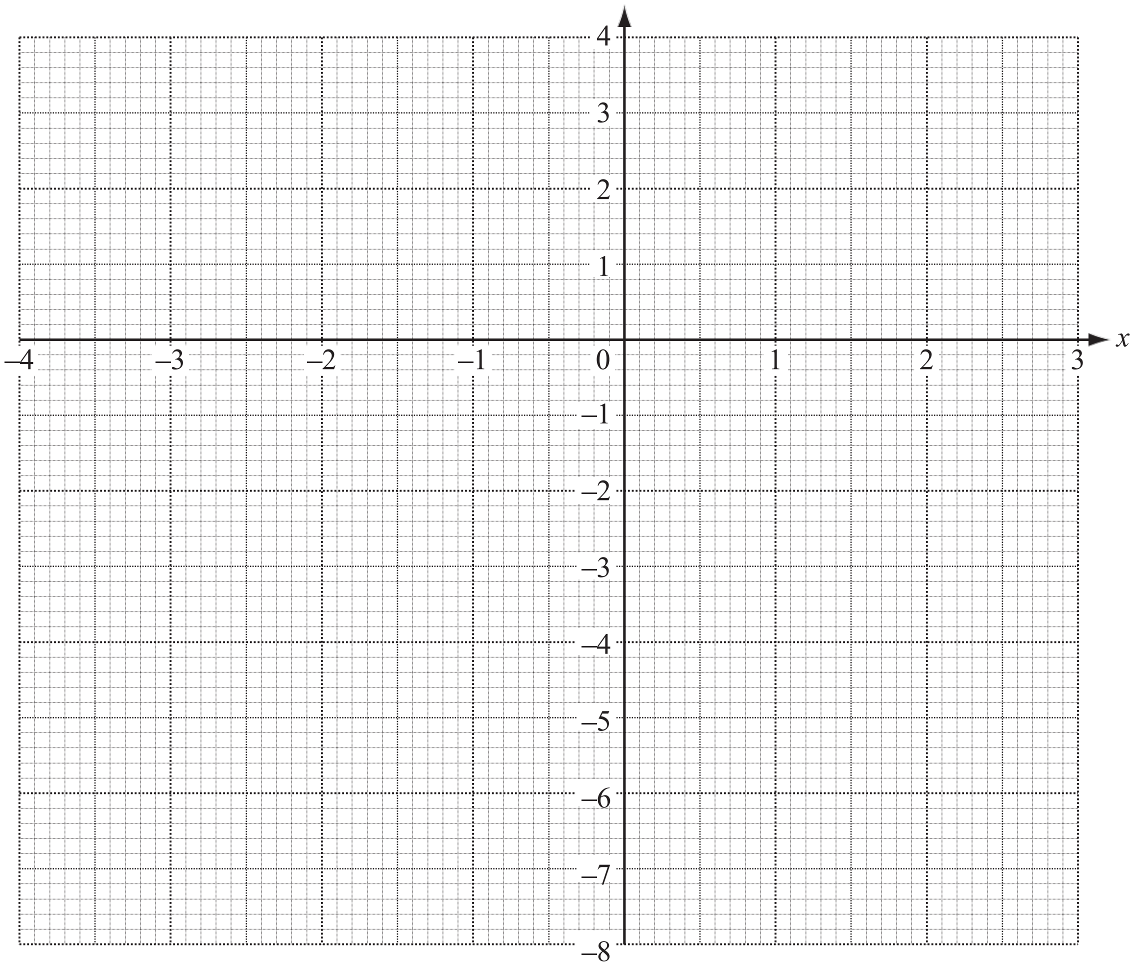
(a) Complete the table for the function $f(x) = \dfrac{x^3}{10} + 1$

x	-4	-3	-2	-1	0	1	2	3
$f(x)$		-1.7	0.2	0.9	1	1.1	1.8	

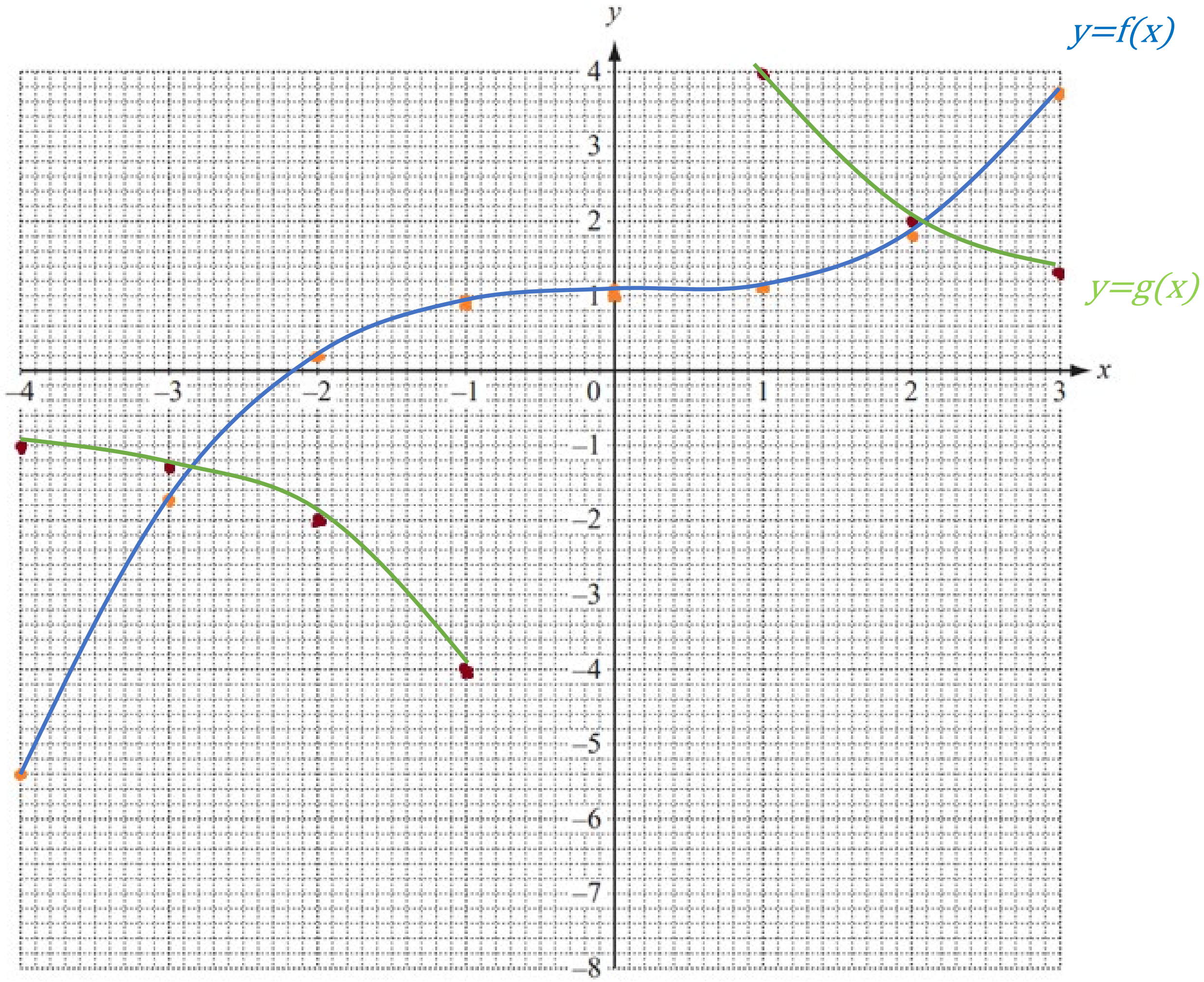
[2]

x	-4	-3	-2	-1	0	1	2	3
$f(x)$	**-5.4**	-1.7	0.2	0.9	1	1.1	1.8	**3.7**

(b) On the grid, draw the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 3$.



[4]



(c) Complete the table for the function $g(x) = \frac{4}{x},\ x \neq 0$.

x	–4	–3	–2	–1	1	2	3
g(x)	–1	–1.3				2	1.3

[2]

x	-4	-3	-2	-1	1	2	3
$g(x)$	-1	-1.3	-2	-4	4	2	1.3

(d) On the grid, draw the graph of $y = g(x)$ for $-4 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 3$. [3]

Drawn on the graph above in green

(e) (i) Use your graphs to solve the equation $\frac{x^3}{10} + 1 = \frac{4}{x}$. [2]

Need to solve

$$g(x) = f(x)$$

This is solved for where the curves on the graph above intersect.

We can read off

$$\boldsymbol{x = -2.85, x = 2.05}$$

(ii) The equation $\frac{x^3}{10} + 1 = \frac{4}{x}$ can be written as $x^4 + ax + b = 0$. [2]

Find the values of a and b.

Multiply through by x

$$\frac{x^4}{10} + x = 4$$

Multiply through by 10

$$x^4 + 10x = 40$$

Minus 40 from both sides

$$x^4 + 10x - 40 = 0$$

Hence

$$\boldsymbol{a = 10}$$

$$\boldsymbol{b = -40}$$

(a) $f(x) = 2^x$

Complete the table.

x	-2	-1	0	1	2	3	4
$y = f(x)$		0.5	1	2	4		

[3]

x	-2	-1	0	1	2	3	4
$y = f(x)$	**0.25**	0.5	1	2	4	**8**	**16**

(b) $g(x) = x(4 - x)$

Complete the table.

x	-1	0	1	2	3	4
$y = g(x)$		0	3		3	0

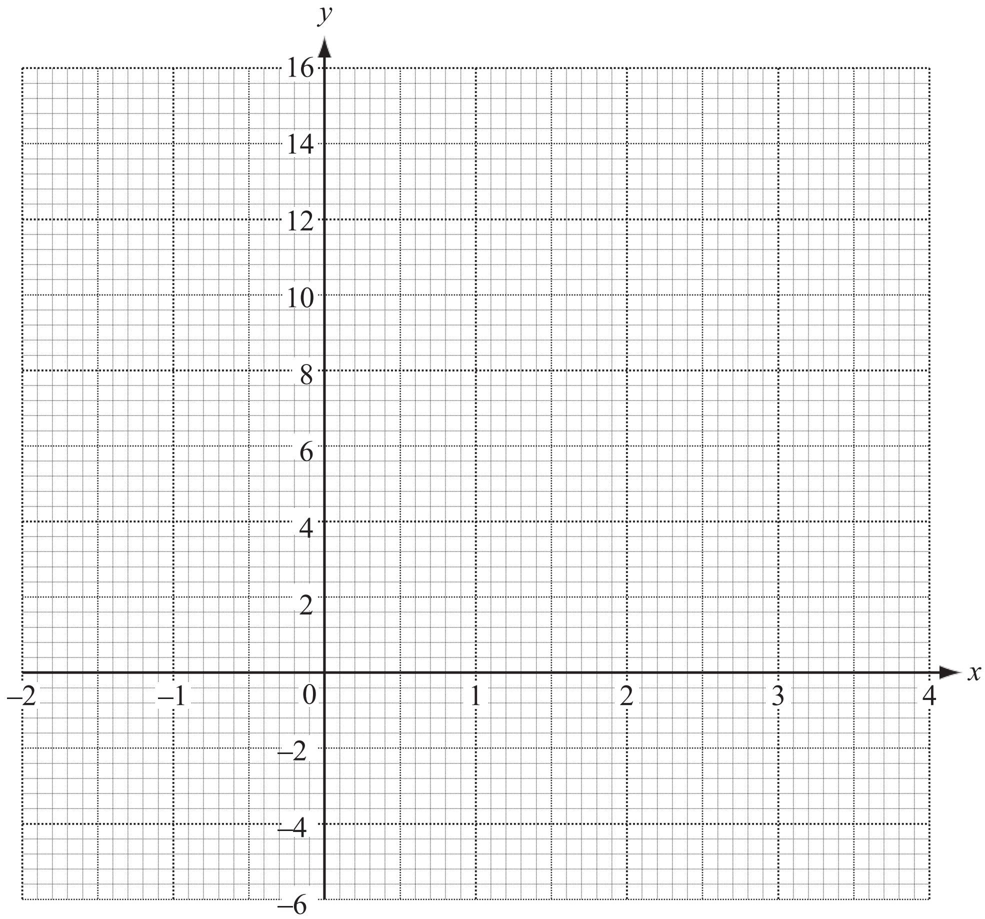
[2]

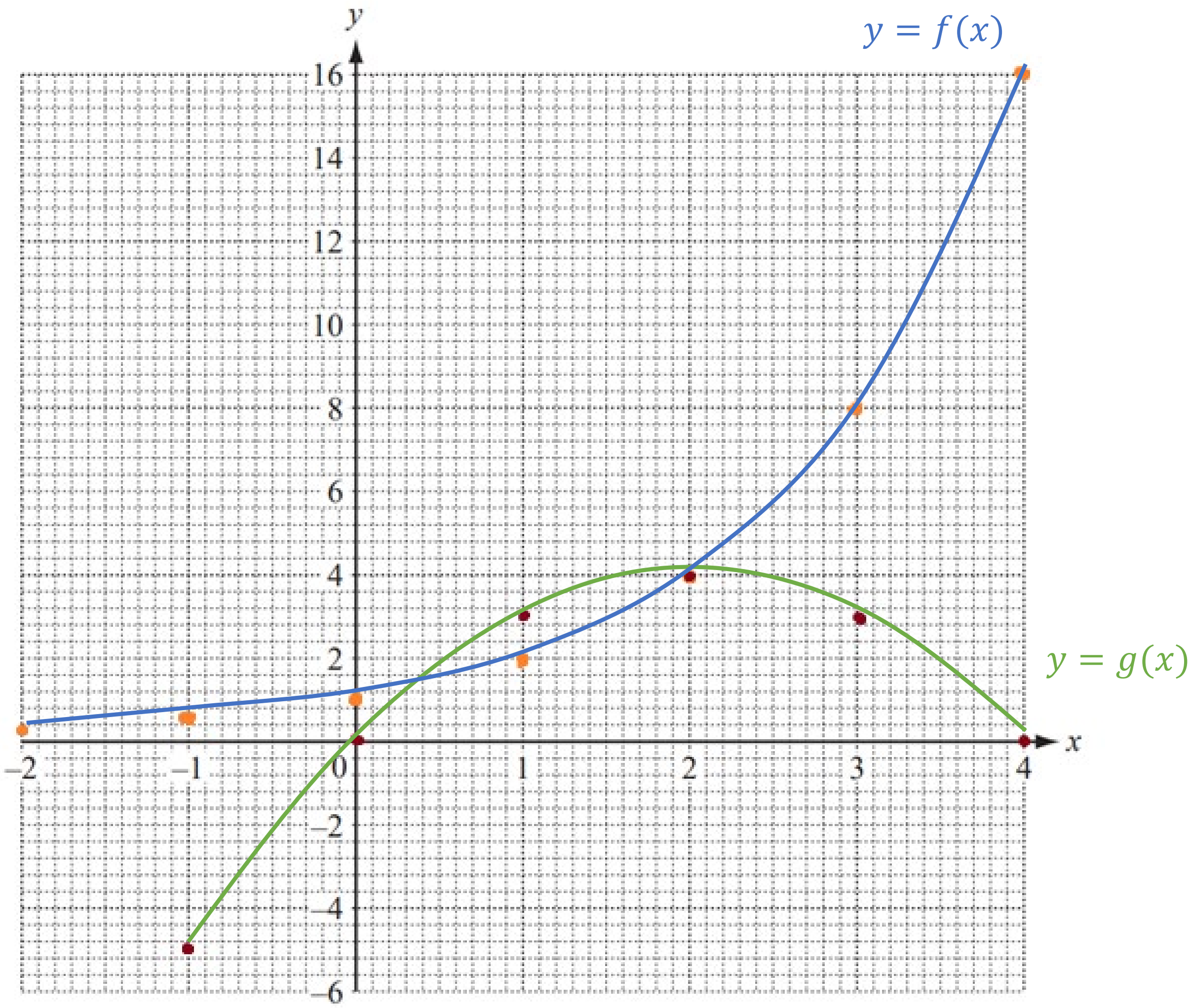
x	-1	0	1	2	3	4
$y = g(x)$	**-5**	0	3	**4**	3	0

(c) On the grid, draw the graphs of

(i) $y = f(x)$ for $-2 \leqslant x \leqslant 4$, [3]

(ii) $y = g(x)$ for $-1 \leqslant x \leqslant 4$. [3]





(d) Use your graphs to solve the following equations.

(i) $f(x) = 10$ [1]

$\mathbf{3.35}$

(ii) $f(x) = g(x)$ [2]

$\boldsymbol{x = 0.3,\ x = 2}$

(iii) $f^{-1}(x) = 1.7$ [1]

$\mathbf{3.2}$

Graphs
Difficulty: Medium

Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 4

Time allowed: **86 minutes**

Score: **/75**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

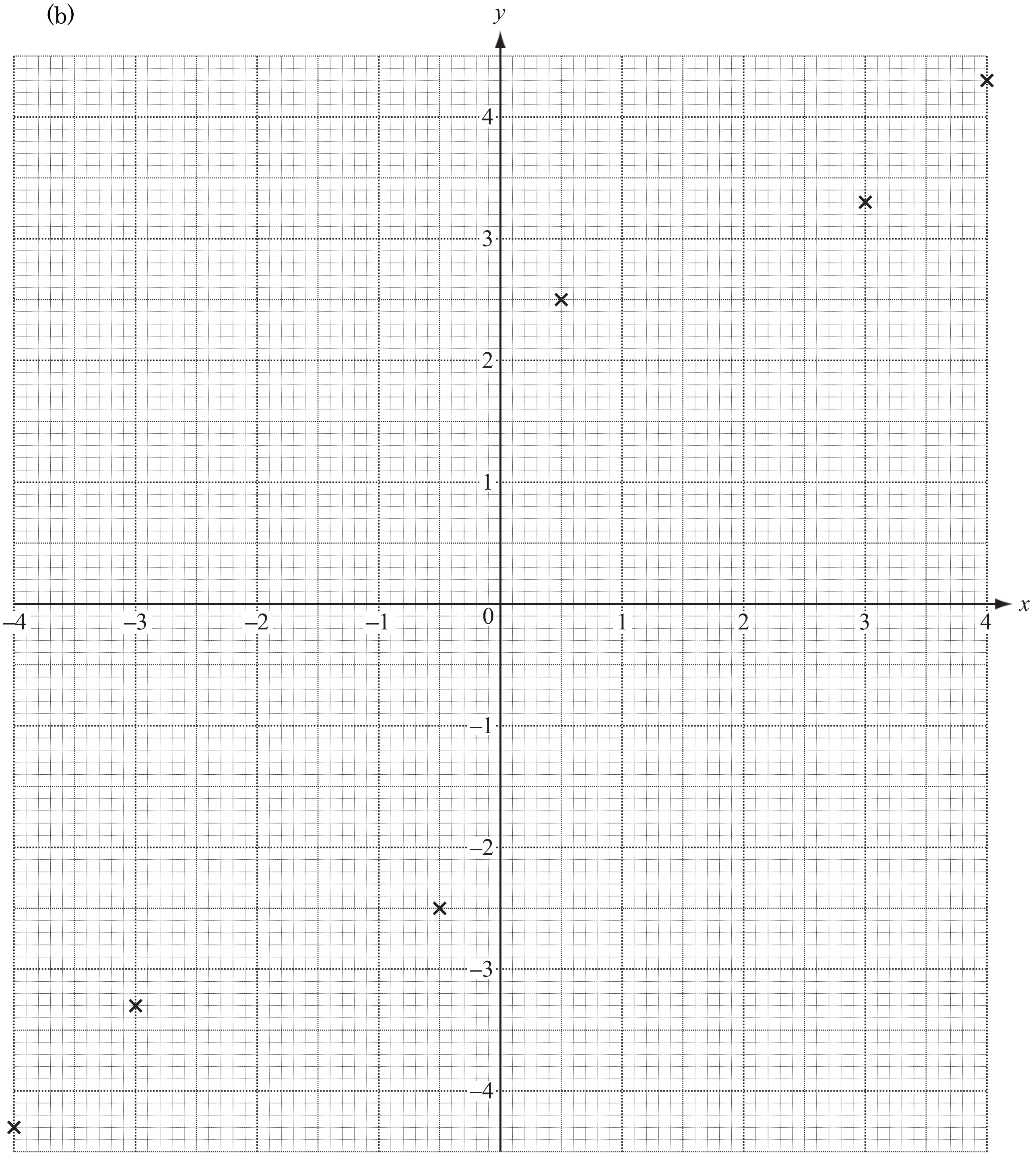
(a) Complete the table of values for $y = x + \frac{1}{x}$.

x	–4	–3	–2	–1	–0.5	0.5	1	2	3	4
y	–4.3	–3.3			–2.5	2.5			3.3	4.3

[2]

x	-4	-3	-2	-1	-0.5	0.5	1	2	3	4
y	-4.3	-3.3	**-2.5**	**-2**	-2.5	2.5	**2**	**2.5**	3.3	4.3

(b)



On the grid, draw the graph of $y = x + \frac{1}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

Six of the ten points have been plotted for you. [3]

(c) There are three integer values of k for which the equation $x + \frac{1}{x} = k$ has **no** solutions.

Write down these three values of k. [2]

$\boldsymbol{k = -1, \quad k = 0, \quad k = 1}$

(d) Write down the ranges of x for which the gradient of the graph of $y = x + \frac{1}{x}$ is positive. [2]

$\boldsymbol{x < -1}$

$\boldsymbol{x > 1}$

(e) To solve the equation $x + \frac{1}{x} = 2x + 1$, a straight line can be drawn on the grid.

(i) Draw this line on the grid for $-2.5 \leqslant x \leqslant 1.5$. [2]

Drawn in orange on the graph above

(ii) On the grid, show how you would find the solutions. [1]

Solutions are circled, and lines are drawn in green above.

(iii) Show how the equation $x + \frac{1}{x} = 2x + 1$ can be rearranged into the form $x^2 + bx + c = 0$ and find the values of b and c. [3]

Multiply through by x

$$x^2 + 1 = 2x^2 + x$$

Subtract x^2 from both sides

$$1 = x^2 + x$$

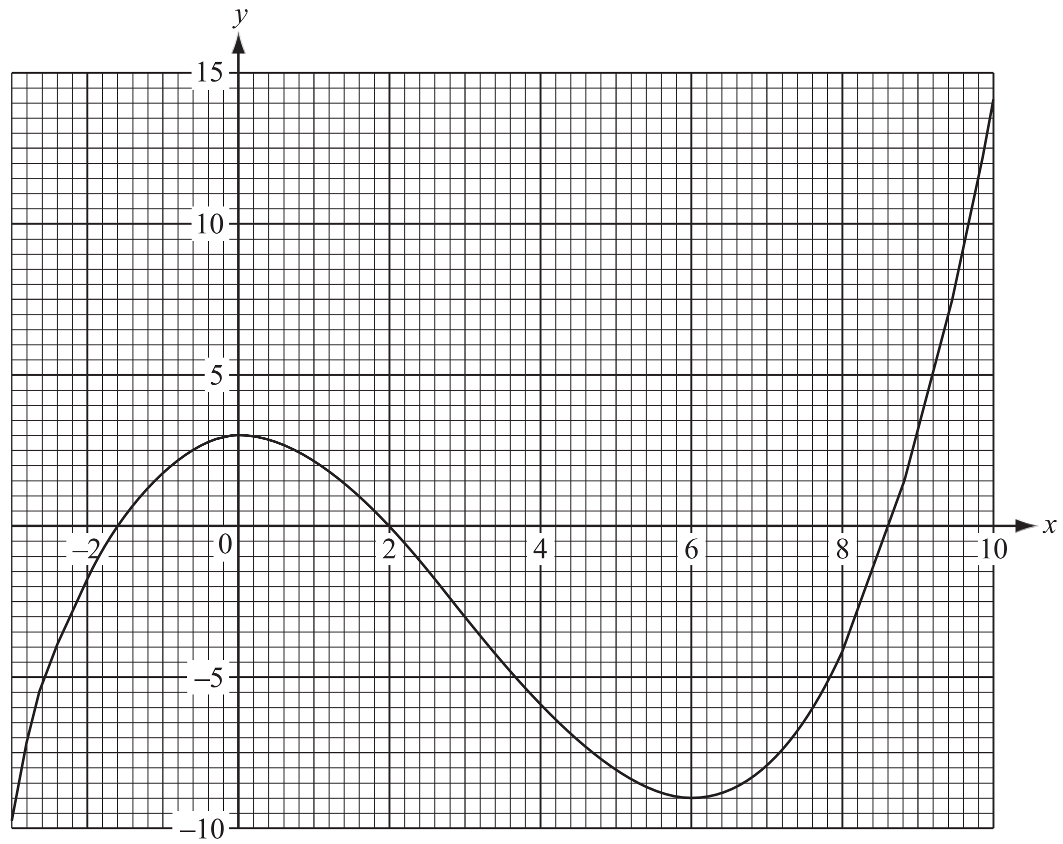
Subtract 1 from both sides

$$x^2 + x - 1 = 0$$

Hence

$$\boldsymbol{b = 1}$$

$$\boldsymbol{c = -1}$$



The diagram shows the accurate graph of $y = f(x)$.

(a) Use the graph to find

(i) f(0), [1]

We need to identify the y value corresponding to x = 0.

$f(x) = y$

For x = 0, **f(0) = 3**

(ii) f(8). [1]

For x = 8, **f(8) = -4**

(b) Use the graph to solve

(i) $f(x) = 0$, [2]

We need to find the x values for which f(x) = y = 0.

By looking at the graph, we see that for y = 0, **x takes 3 values: x = -1.6, x = 2 and x = 8.6**

(ii) $f(x) = 5$. [1]

f(x) = y = 5

We see that there is one corresponding x value, **x = 9.2.**

(c) k is an integer for which the equation $f(x) = k$ has exactly two solutions.

Use the graph to find the two values of k. [2]

We need to identify the y values for which the line of equation y = k intersects the graph only twice.

These intersections have the x values corresponding to y = k.

For **y = 3**, the corresponding x values are x = 0 and x = 9.

For **y = -9,** the corresponding x value are x = 6.1 and x = approximate -2.5

(d) Write down the range of values of x for which the graph of $y = f(x)$ has a negative gradient. [2]

gradient = $\frac{\text{change in y}}{\text{change in x}}$

The gradient will be negative if either the change in y or the change in x have negative values.

We notice that the change in y is negative for the range **0 < x < 6**, so the gradient is also negative for this range.

(e) The equation $f(x) + x - 1 = 0$ can be solved by drawing a line on the grid.

(i) Write down the equation of this line. [1]

$f(x) + x - 1 = 0$

$f(x) = y = 1 - x$

The equation of the line is $y = 1 - x$.

(ii) How many solutions are there for $f(x) + x - 1 = 0$? [1]

$f(x) = y = 1 - x$

By drawing the line of equation $y = 1 - x$ on the graph above, we can see it intersects the graph $f(x)$ in 3 different points, so there are 3 possible solutions.

Answer the whole of this question on a sheet of graph paper.

$$f(x) = 3x - \frac{1}{x^2} + 3, \quad x \neq 0.$$

(a) The table shows some values of f(x).

x	−3	−2.5	−2	−1.5	−1	−0.5	−0.4	−0.3	0.3	0.4	0.5	1	1.5	2	2.5	3
f(x)	p	−4.7	−3.3	−1.9	−1	−2.5	−4.5	−9.0	−7.2	−2.1	0.5	q	7.1	8.8	10.3	r

Find the values of p, q and r. [3]

For x = -3, f(-3) = p = 3 x (-3) - $\frac{1}{(-3)^2}$ +3

p = -6 - $\frac{1}{9}$ = **-6.1**

For x = 1, f(1) = q = 3 x 1 - $\frac{1}{1}$ +3

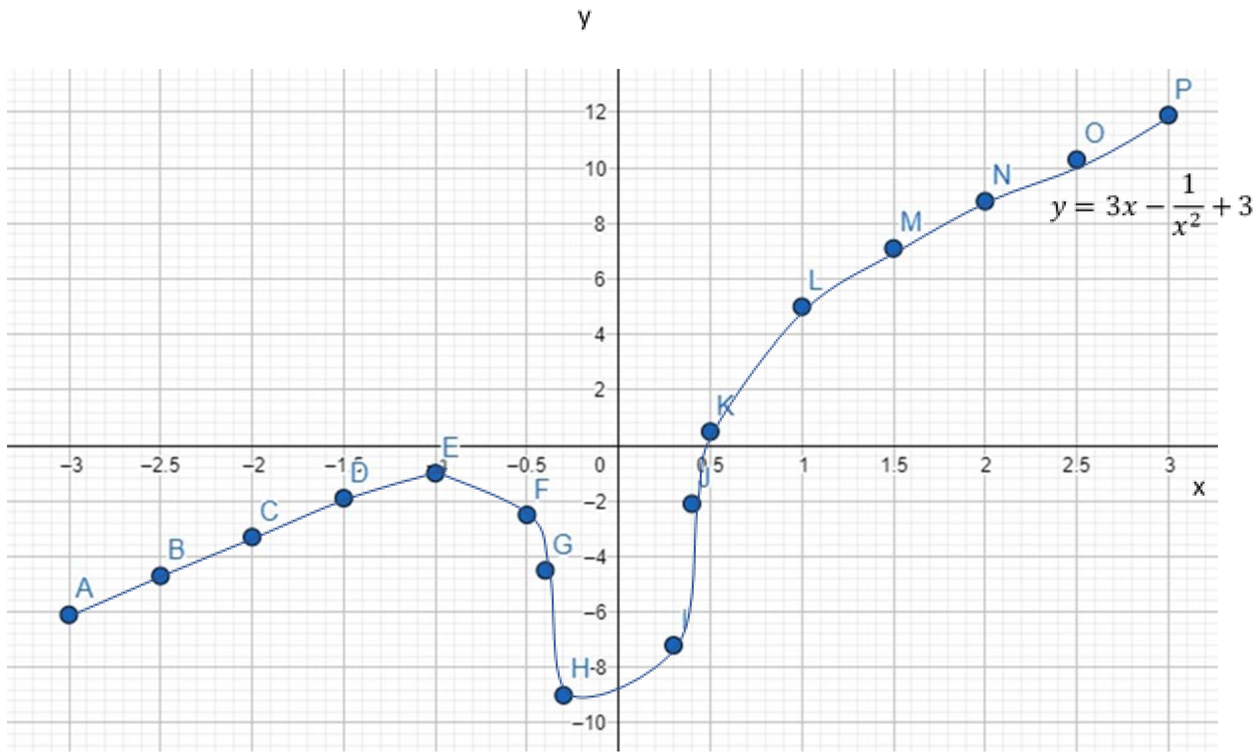
q = 5

For x = 3, f(3) = p = 3 x 3 - $\frac{1}{3^2}$ +3

r = 12 - $\frac{1}{9}$ = **11.9**

(b) Draw axes using a scale of 1 cm to represent 0.5 units for $-3 \ 2 \leqslant x \leqslant 3$ and 1 cm to represent units for $-10 \leqslant y \leqslant 12$. [1]

(c) On your grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$. [5]



(d) Use your graph to solve the equations

(i) $3x - \frac{1}{x^2} + 3 = 0$, [1]

f(x) = y = 0

The corresponding x value is **$0.45 \leq x \leq 0.5$**

(ii) $3x - \frac{1}{x^2} + 7 = 0.$ [3]

f(x) + 4 = y = 0

f(x) = y = - 4

There are 3 possible corresponding x values (because the line of equation y = -4 intersects the graph in 3 different points).

$-2.4 \leq x \leq -2.1$

$-0.5 \leq x \leq -0.4$

$0.3 \leq x \leq 0.4$

(e) $g(x) = 3x + 3$.
On the same grid, draw the graph of $y = g(x)$ for $-3 \leqslant x \leqslant 3$. [2]

g(x) = 3x + 3

For x = -3, g(x) = y = 3 x (-3) + 3 = **-6**

For x = -2, g(x) = y = 3 x (-2) + 3 = **-3**

For x = -1, g(x) = y = 3 x (-1) + 3 = **0**

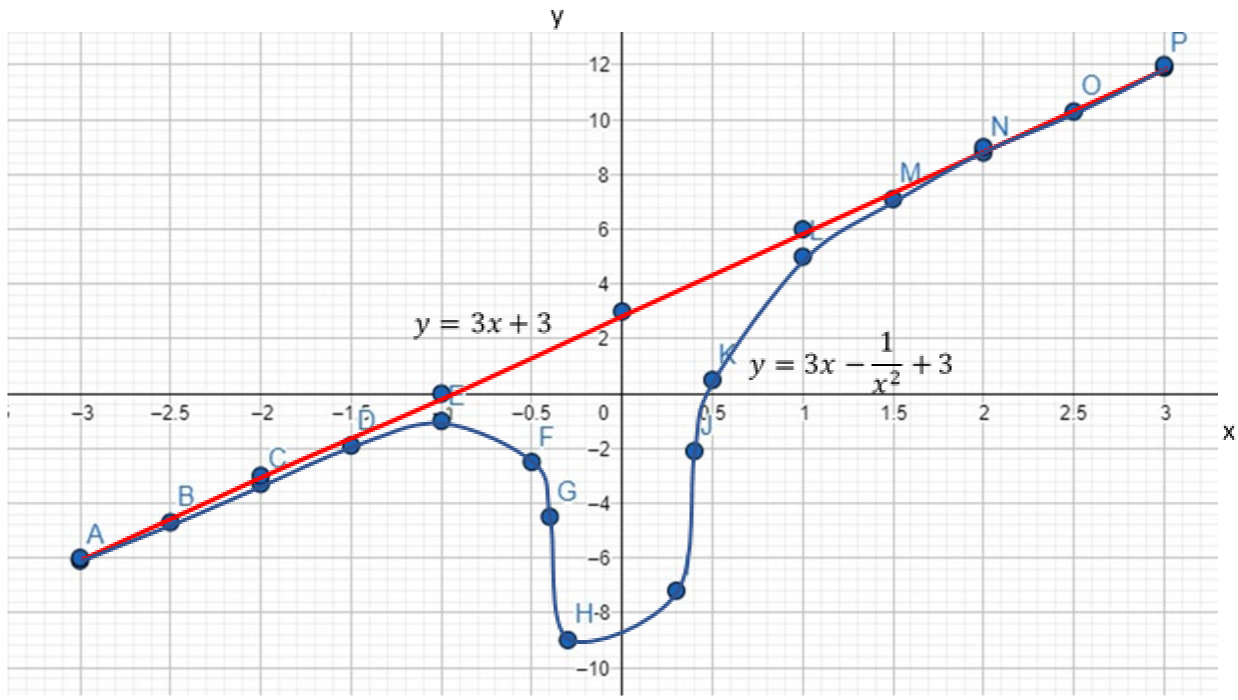
For x = 0, g(x) = y = 3 x 0 + 3 = **3**

For x = 1, g(x) = y = 3 x 1 + 3 = **6**

For x = 2, g(x) = y = 3 x 2 + 3 = **9**

For x = 3, g(x) = y = 3 x 3 + 3 = **12**

We plot these points on the graph above.



(f) (i) Describe briefly what happens to the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ for large positive or negative values of x. [1]

Around large positive or negative numbers the 2 graphs get close.

(ii) Estimate the gradient of $y = \mathrm{f}(x)$ when $x = 100$. [1]

The gradient will be similar to the one of the graph around x = 2.5 and x = 3,

therefore, **the gradient will be around 3.**

Answer the whole of this question on a sheet of graph paper.

(a) Find the values of k, m and n in each of the following equations, where $a > 0$.

(i) $a^0 = k$, [1]

Any number raised to be power of 0 equals 1.

k = 1

(ii) $a^m = \frac{1}{a}$, [1]

$\frac{1}{a} = a^{-1} = a^m$

m = -1

(iii) $a^n = \sqrt{a^3}$. [1]

$\sqrt{a^3} = a^{3/2} = a^n$

n = 3/2

(b) The table shows some values of the function $f(x) = 2^x$.

x	−2	−1	−0.5	0	0.5	1	1.5	2	3
$f(x)$	r	0.5	0.71	s	1.41	2	2.83	4	t

(i) Write down the values of r, s and t. [3]

$f(x) = 2^x$

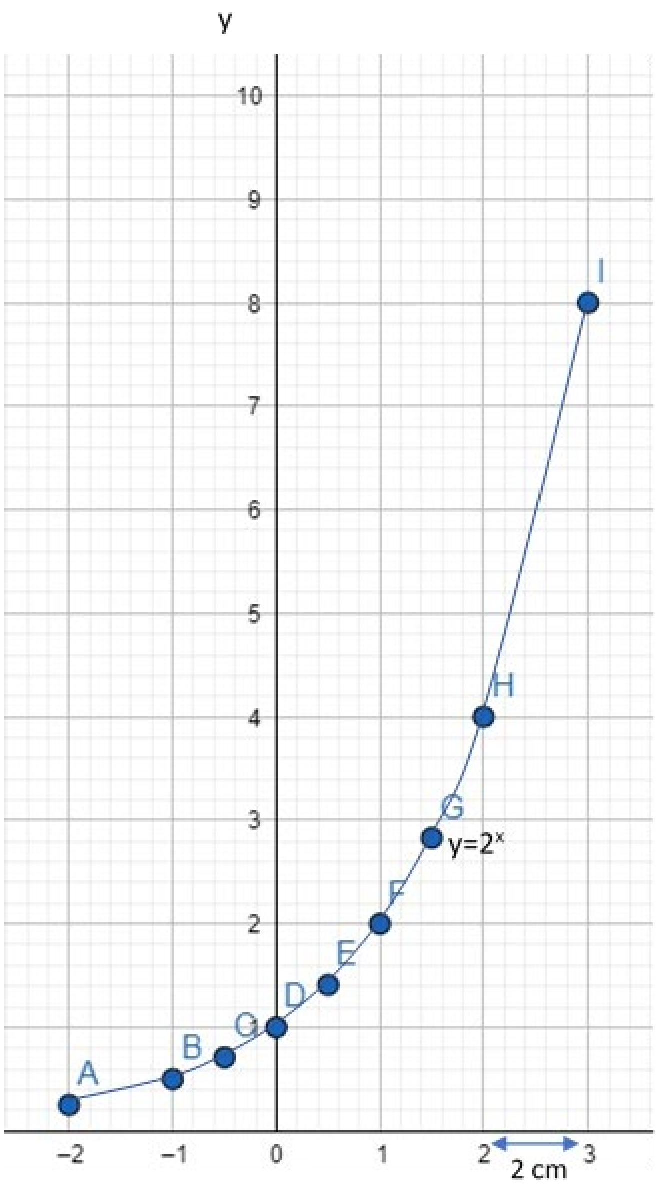
for $x = -2$, $f(-2) = 2^{-2}$ = **0.25 = r**

for $x = 0$, $f(0) = 2^0$ = **1 = s**

for $x = 3$, $f(3) = 2^3$ = **8 = t**

(ii) Using a scale of 2 cm to represent 1 unit on each axis, draw an x-axis from -2 to 3 and a y-axis from 0 to 10. [1]

(iii) On your grid, draw the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 3$. [4]



(c) The function g is given by g(x) = 6 – 2x.

(i) On the same grid as **part (b)**, draw the graph of y = g(x) for – 2 $\leqslant x \leqslant$ 3. [2]

g(x) = 6 – 2x

for x = -2, g(-2) = 6 – 2 x (-2) = **10**

for x = -1, g(-1) = 6 – 2 x (-1) = **8**

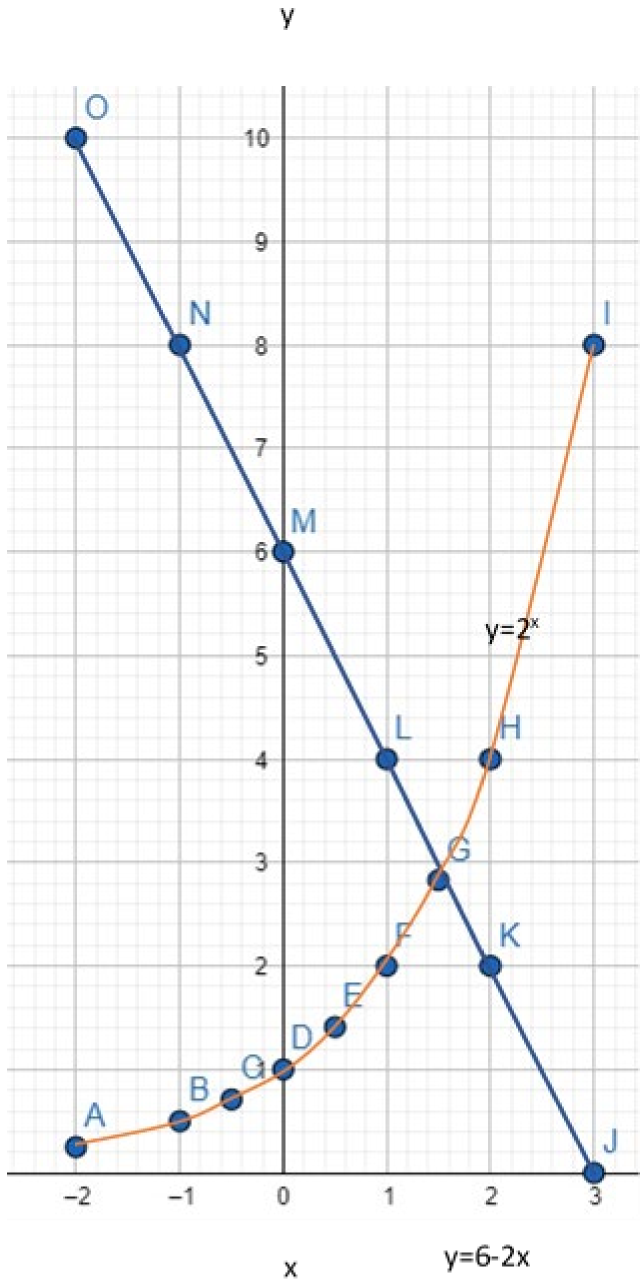
for x = 0, g(0) = 6 – 2 x 0 = **6**

for x = 1, g(1) = 6 – 2 x 1 = **4**

for x = 2, g(2) = 6 – 2 x 2 = **2**

for x = 3, g(3) = 6 – 2 x 3 = **0**

We plot the points on the graph above.



(ii) Use your graphs to solve the equation $2 = 6 - 2x$. [1]

The solution of the equation is the x coordinate of the

intersection of the 2 graphs, f(x) and g(x).

x = 1.52

(iii) Write down the value of x for which $2 \underset{x}{<} 6 - 2x$ for $x \in$ {positive integers}. [1]

x – positive integer

The solution is x = 1, since it is the only positive integer for which $2^x < 6 - 2x$.

Answer the whole of this question on a sheet of graph paper.

The table gives values of $f(x) = 2^x$, for $-2 \leqslant x \leqslant 4$.

x	-2	-1	0	1	2	3	4
f(x)	p	0.5	q	2	4	r	16

(a) Find the values of p, q and r. [3]

$f(x) = 2^x$

We substitute the values of x into the function $f(x) = 2^x$ to work out the value f(x).

For x = -2:

$p = f(-2) = 2^{-2}$

p = 0.25

For x = 0:

$q = f(0) = 2^0$

q = 1

For x = 3:

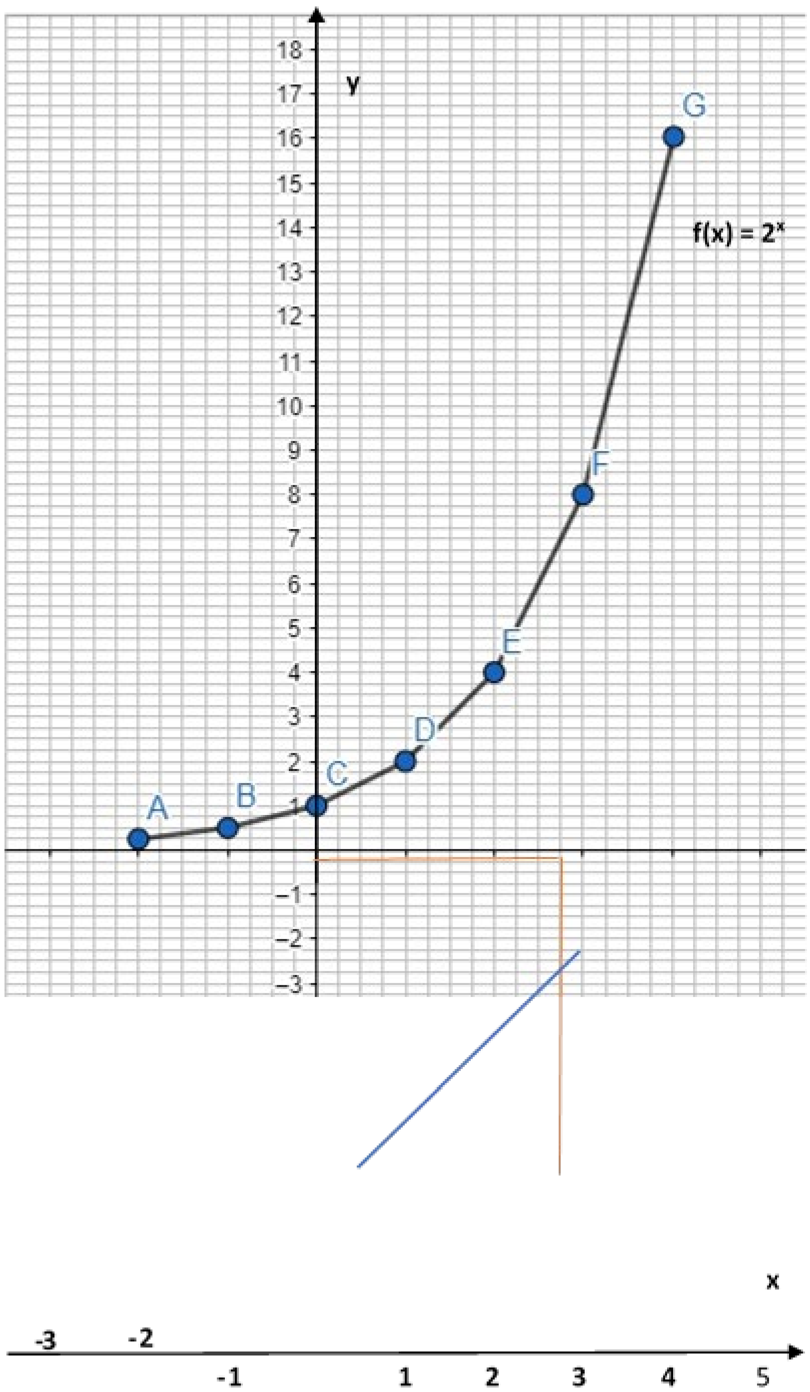
$r = f(3) = 2^3$

r = 8

(b) Using a scale of 2 cm to 1 unit on the x-axis and 1 cm to 1 unit on the y-axis, draw the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 4$. [5]

To work out the graph of the function, we plot all the points represented in the table.

The points will be: A (-2, 0.25); B (-1, 0.5); C (0, 1); D (1, 20); E (2, 4); F (3, 8); G (4, 16).



(c) Use your graph to solve the equation $2^x = 7$. [1]

On the graph above, for y = 7, x = 2.75.

(d) What value does f(x) approach as x decreases? [1]

Using the graph, we can see that as x decreases, f(x) approaches 0.

(e) By drawing a tangent, estimate the gradient of the graph of $y = f(x)$ when $x = 1.5$. [3]

We draw a tangent through the point x = 1.5.

The formula for working out the gradient, m, is:

$m = \frac{\text{change in y}}{\text{change in x}}$

By using the points D (1, 2); E (2, 4), which are also points on the tangent:

$m = \frac{2}{1}$

m = 2

(f) On the same grid draw the graph of $y = 2x + 1$ for $0 \leqslant x \leqslant 4$. [2]

y = 2x + 1

$0 \leq x \leq 4$

For x = 0:

y = 2 x 0 + 1

y = 1

For x = 1:

y = 2 x 1 + 1

y = 3

For x = 2:

y = 2 x 2 + 1

y = 5

For x = 3:

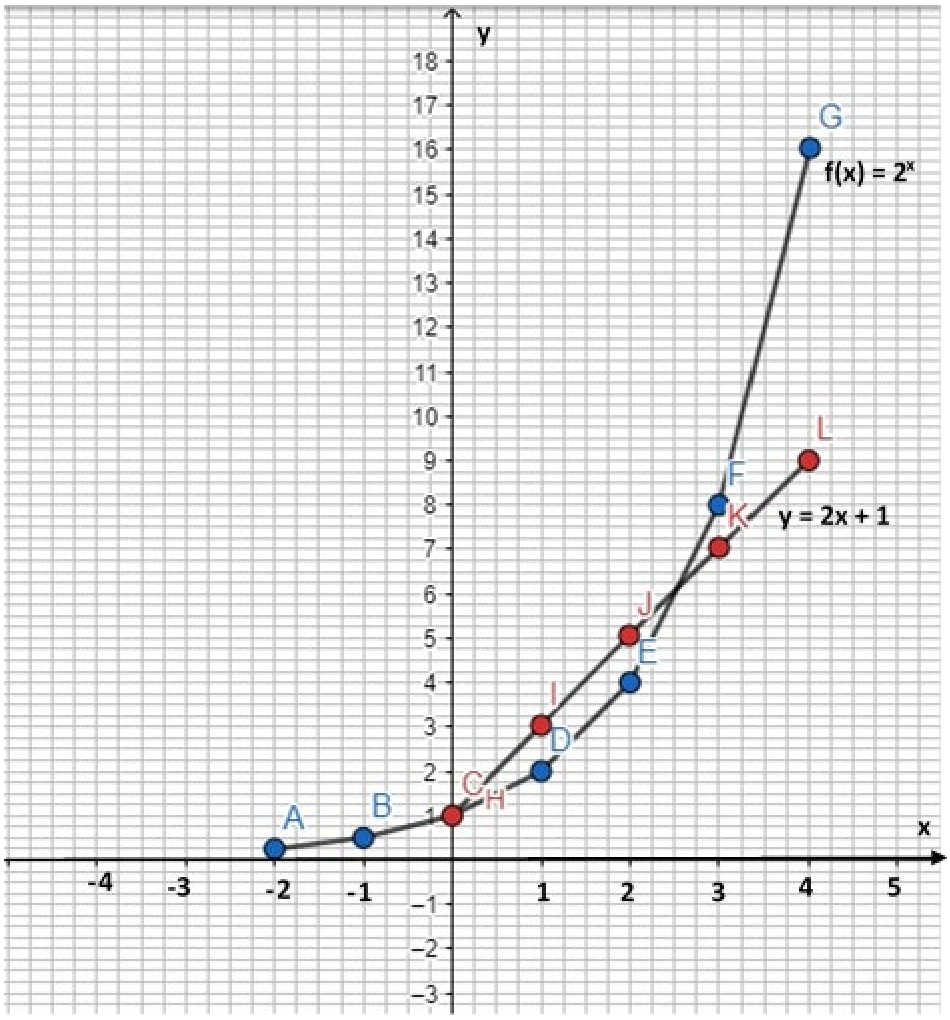
y = 2 x 3 + 1

y = 7

For x = 4:

y = 2 x 4 + 1

y = 9



The points we need to plot are: H (0, 1); I (1, 3); J (2, 5); K (3, 7); L (4, 9).

(g) Use your graph to find the non-integer solution of $2^x = 2x + 1$. [2]

The equality $2^x = 2x + 1$ has its solution at the intersection between the 2 graphs.

The 2 graphs intersect at 2 points:

One of the points, C (0, 1), has x = 0, an integer.

The second intersection is the point of coordinates (2.5, 6), has x = 2.5, a non-integer.

x = 2.5

Graphs
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **117 minutes**

Score: **/102**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a) Complete the table of values for $y = \frac{x^3}{3} - x^2 + 1$.

x	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5	3
y	–2.38	–0.33	0.71	**1**	0.79	0.33	–0.13	–0.33	–0.04	**1**

[2]

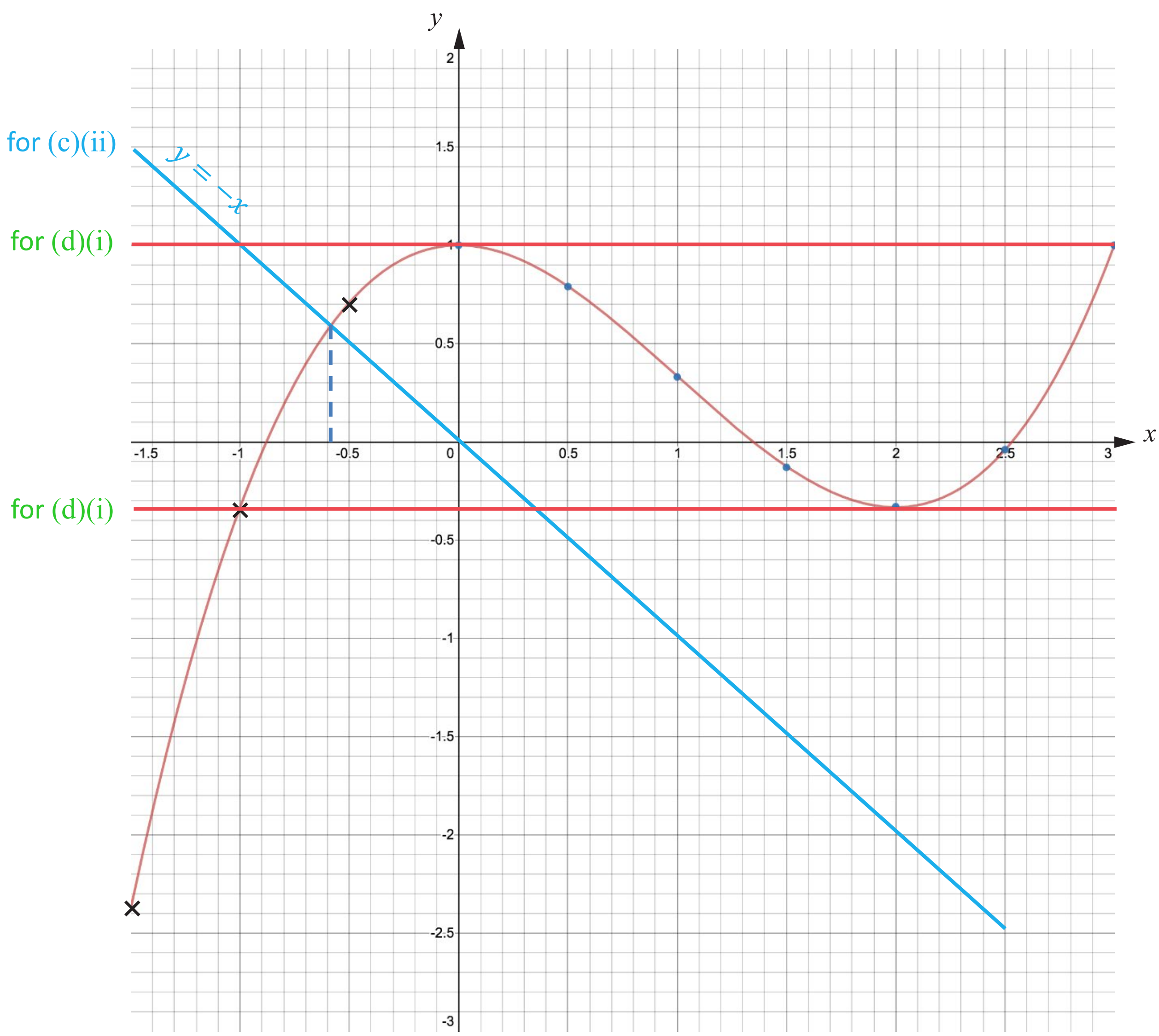
Use the table function on your calculator to recreate the table.

(Or plug the x values into the function…)

(b) Draw the graph of $y = \frac{x^3}{3} - x^2 + 1$ for $-1.5 \leq x \leq 3$.

The first 3 points have been plotted for you.

Plot points and draw a smooth curve through them.



[4]

(c) Using your graph, solve the equations.

(i) $\dfrac{x^3}{3} - x^2 + 1 = 0$ Simply look at the x values where the curve crosses the x-axis

$x =$ ....$\mathbf{-0.87}$.... or $x =$ ....$\mathbf{1.35}$.... or $x =$ ....$\mathbf{2.53}$....[3]

(ii) $\dfrac{x^3}{3} - x^2 + x + 1 = 0$ [2]

Make the left hand side of this equation look like the function drawn by

subtracting x from both sides:

$$\frac{x^3}{3} - x^2 + 1 = -x$$

Draw the right hand side, $y = -x$, (in blue on the graph) and read off the x-

coordinate of the point of intersection of the line and the curve.

$$\boldsymbol{x = -0.59}$$

(d) Two tangents to the graph of $y = \dfrac{x^3}{3} - x^2 + 1$ can be drawn parallel to the x-axis.

(i) Write down the equation of each of these tangents. [2]

These tangents are the horizontal lines drawn in red on the graph and have equations:

$$\boldsymbol{y = 1 \text{ and } y = -0.33}$$

(ii) For $0 \leq x \leq 3$, write down the smallest possible value of y. [1]

The minimum point (lowest point) of the graph for $0 \leq x \leq 3$ is the point the second

tangent was drawn though in (d)(i) so:

$$\boldsymbol{y = -0.33}$$

The table shows some values for $y = x^3 - 3x + 2$.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y		3.125		3.375	2		0		4

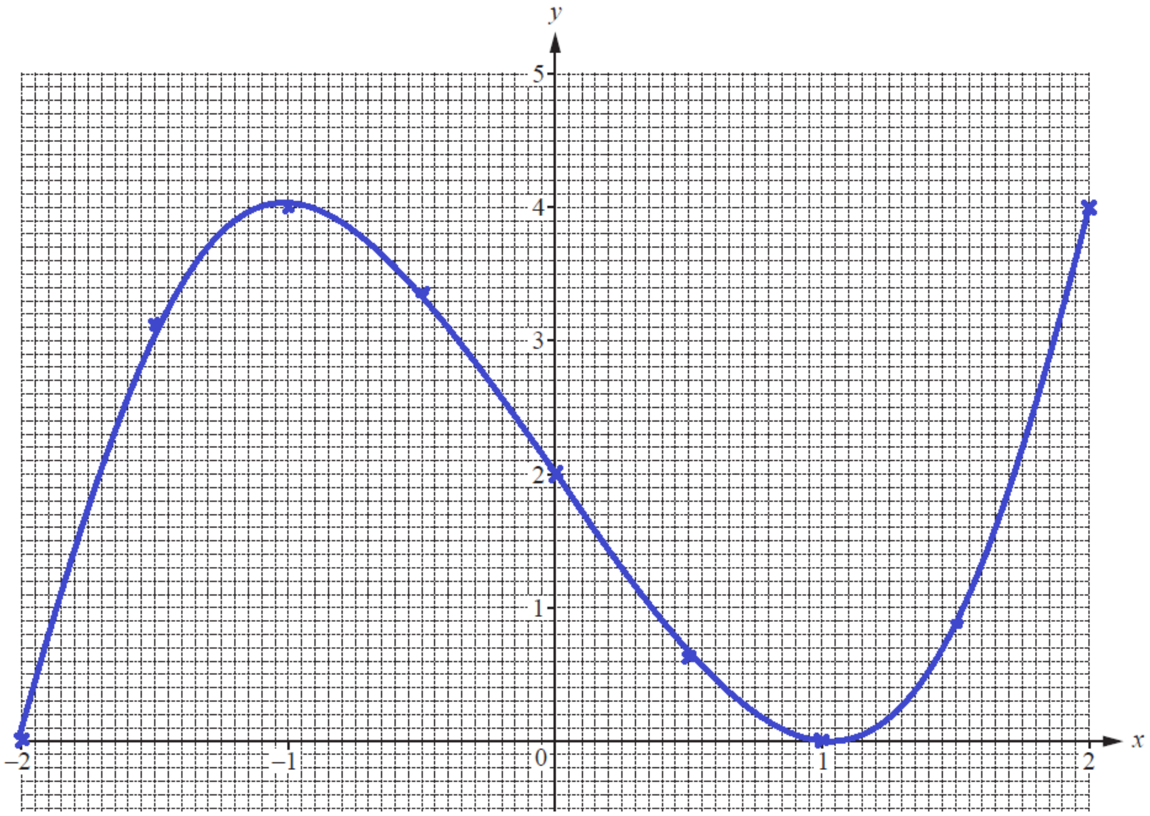
(a) Complete the table of values. [4]

We use calculator to find the values of y for x=-2, x=-1, x=0.5 and x=1.5.

$$y(-2) = 0, \quad y(-1) = 4, \quad y(0.5) = 0.625, \quad y(1.5) = 0.875$$

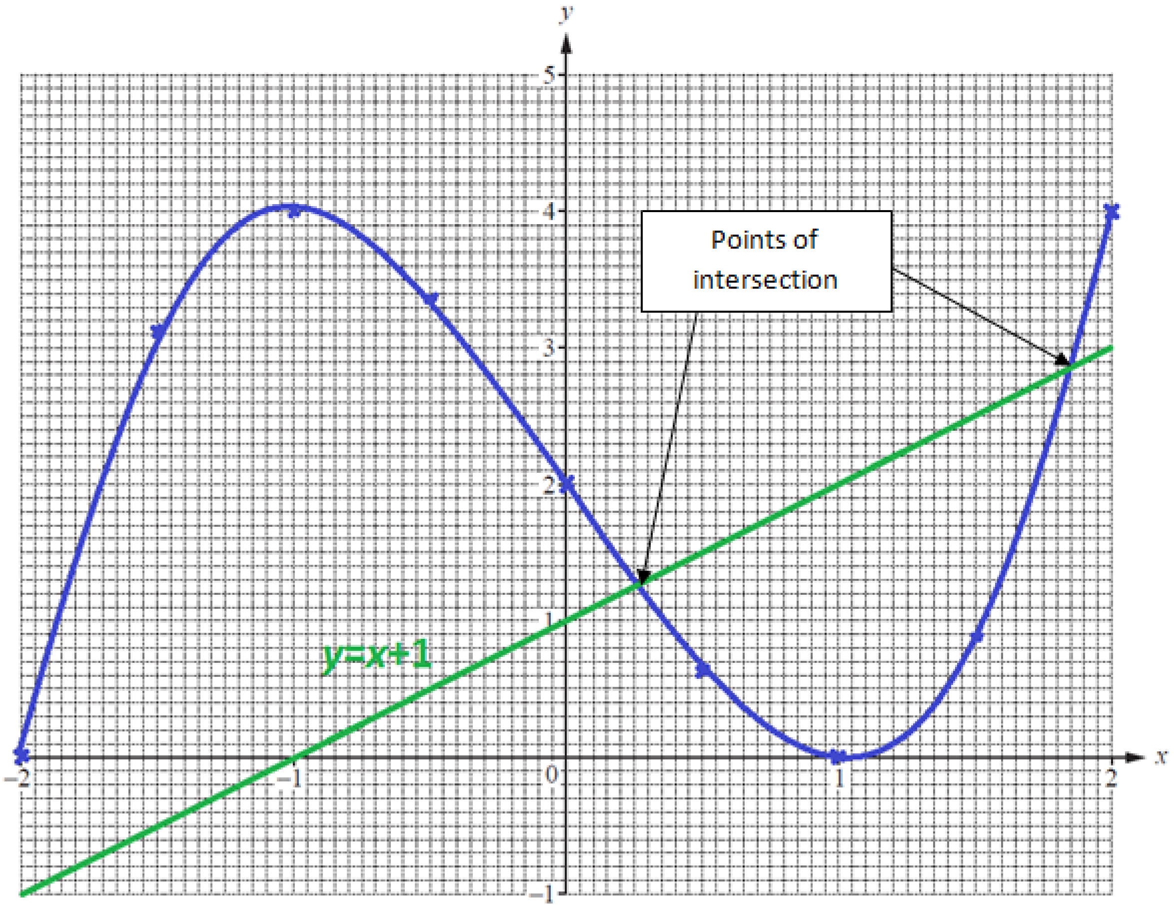
(b) On the grid, draw the graph of $y = x^3 - 3x + 2$ for $-2 \leqslant x \leqslant 2$. [4]

Plot the point from the table on the grid and then draw a smooth function connecting the points (blue curve).



(c) By drawing a suitable line, solve the equation $x^3 - 3x + 2 = x + 1$ for $-2 \leqslant x \leqslant 2$. [3]

The left hand side of the given equation is our original graph. We plot a line $y = x + 1$ and find the x-coordinate of the point of intersection with the original graph to solve $x^3 - 3x^2 + 2 = x + 1$



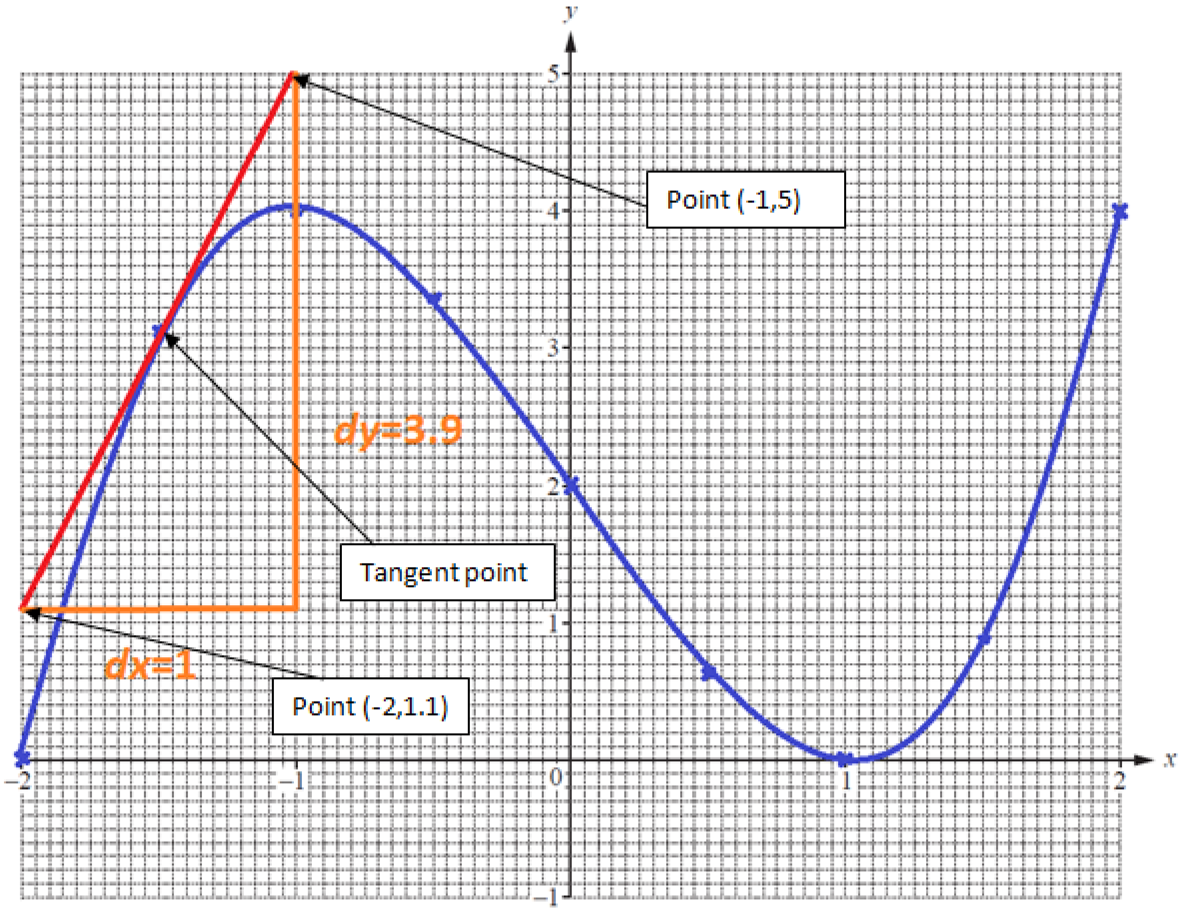
From the graph, we can see that the x-coordinates of the points, and hence the solutions to the equation $x^3 - 3x^2 + 2 = x + 1$ are

$$x = 0.25, \qquad x = 1.85$$

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = -1.5$.

[3]

We start by drawing a tangent line (red line) to the function at point *x*=-1.5 (which means that the line touches out function at this point).



The gradient is found as the change of y-coordinate over the change of x-coordinate between two points on the line. We pick two points (-2,1.1) and (-1,5).

$$\text{Gradient } m = \frac{dy}{dx} = \frac{5-1.1}{-1-(-2)} = \frac{3.9}{1}$$

$$= \mathbf{3.9}$$

$y = x^2 - 2x + \frac{12}{x}$, $x \neq 0$

(a) Complete the table of values.

x	−4	−3	−2	−1	−0.5		0.5	1	2	3	4
y	21	11		−9	−22.75		23.25	11	6		11

[2]

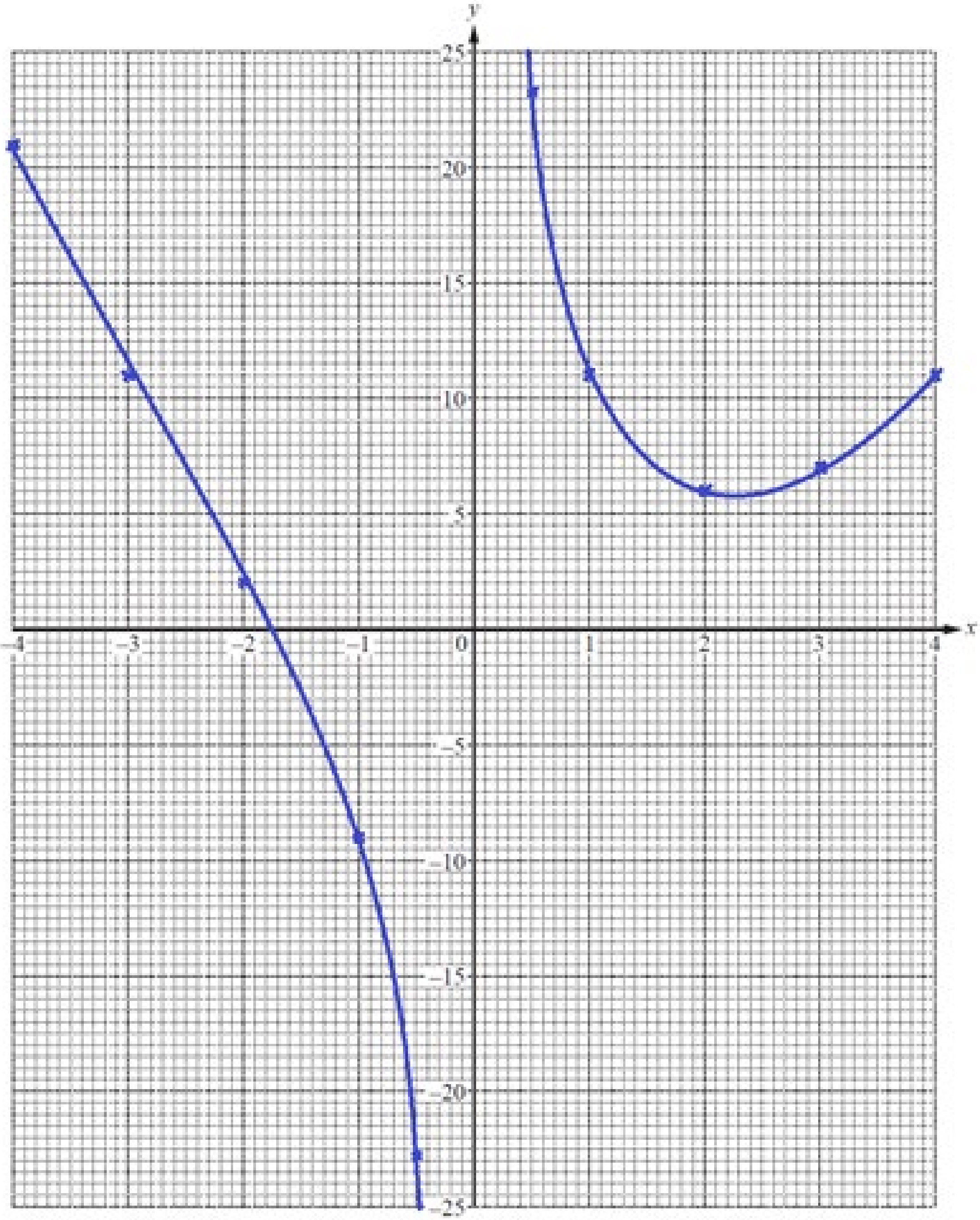
We use calculator to find the values of *y* for *x*=-2 and *x*=3.

$$y(-2) = 2, \qquad y(3) = 7$$

(b) On the grid, draw the graph of $y = x^2 - 2x + \frac{12}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

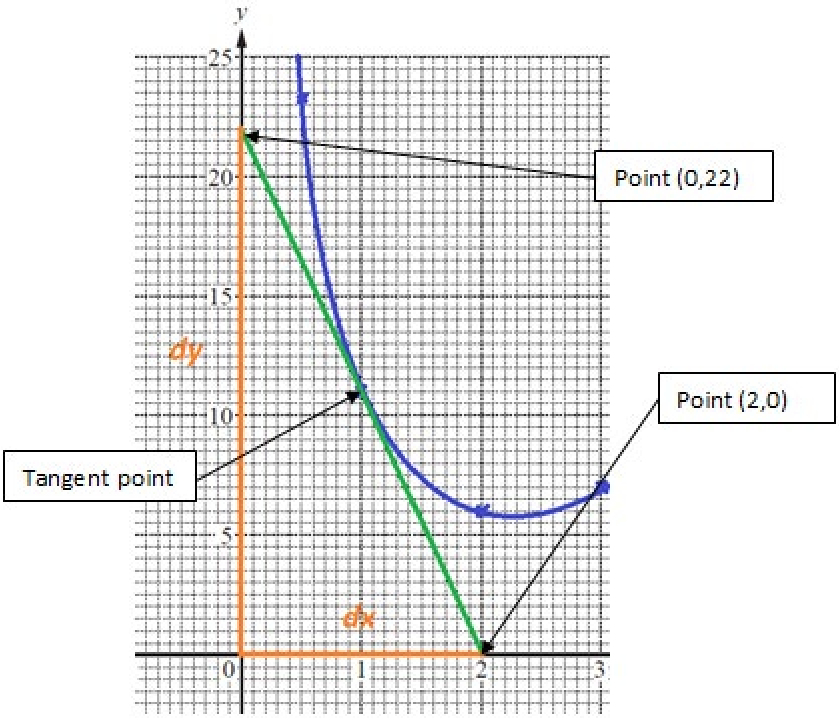
[5]

Plot the point from the table on the grid and then draw a smooth function connecting the points (blue curve).


y
25
20
15
10
5
0
−5
−10
−15
−20
−25
x
−4
−3
−2
−1
1
2
3
4

(c) By drawing a suitable tangent, find an estimate of the gradient of the graph at the point (1, 11). [3]

We start by drawing a tangent line (green line) to the function at point x=1 (which means that the line touches out function at this point).



The gradient is found as the change of y-coordinate (dy) over the change of x-coordinate (dx) between two points on the line. We pick two points (0,22) and (2,0).

$$\text{Gradient } m = \frac{dy}{dx} = \frac{0-22}{2-(0)} = \frac{-22}{2}$$

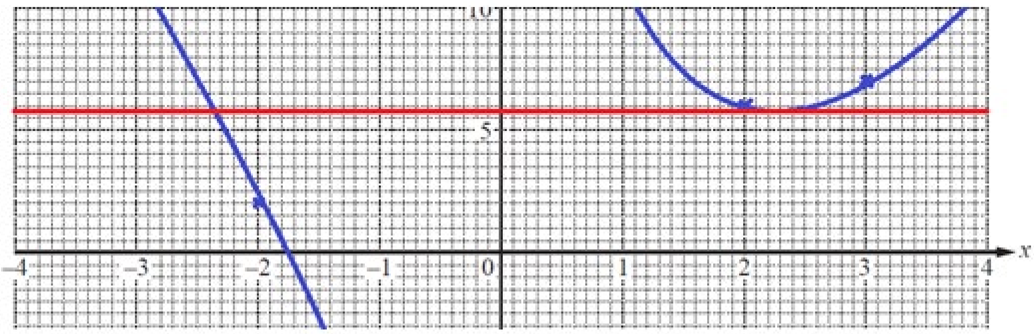
$$= \mathbf{-11}$$

(d) The equation $x^2 - 2x + \dfrac{12}{x} = k$ has exactly two distinct solutions.

Use the graph to find

(i) the value of k, [1]

The function has exactly 2 solutions at the turning point of the function. Above this functional value, there are three solutions and below there is only one solution.



Therefore $\boldsymbol{k}$**=5.75** (red line)

(ii) the solutions of $x^2 - 2x + \frac{12}{x} = k$. [2]

Find the x-coordinates of the intersections of the red line (y=k=5.75) and the blue graph.

$$\boldsymbol{x = -2.35, \qquad x = 2.3}$$

(e) The equation $x^3 + ax^2 + bx + c = 0$ can be solved by drawing the line $y = 3x + 1$ on the grid.

Find the value of a, the value of b and the value of c. [3]

Drawing the line y=3x+1 on the grid and finding the intersection with the original function is the same as equating the two functions.

$$x^2 - 2x + \frac{12}{x} = 3x + 1$$

Multiply both sides of the equation by x.

$$x^3 - 2x^2 + 12 = 3x^2 + x$$

Subtract $3x^2 + x$ from both sides.

$$x^3 - 5x^2 - x + 12 = 0$$

We have converted the equation into the form $x^3 + ax^2 + bx + c = 0$, where:

$$\boldsymbol{a = -5, \qquad b = -1, \qquad c = 12}$$

$$f(x) = \frac{8}{x^2} + \frac{x}{2}, \quad x \neq 0.$$

(a) Complete the table of values for f(x). [3]

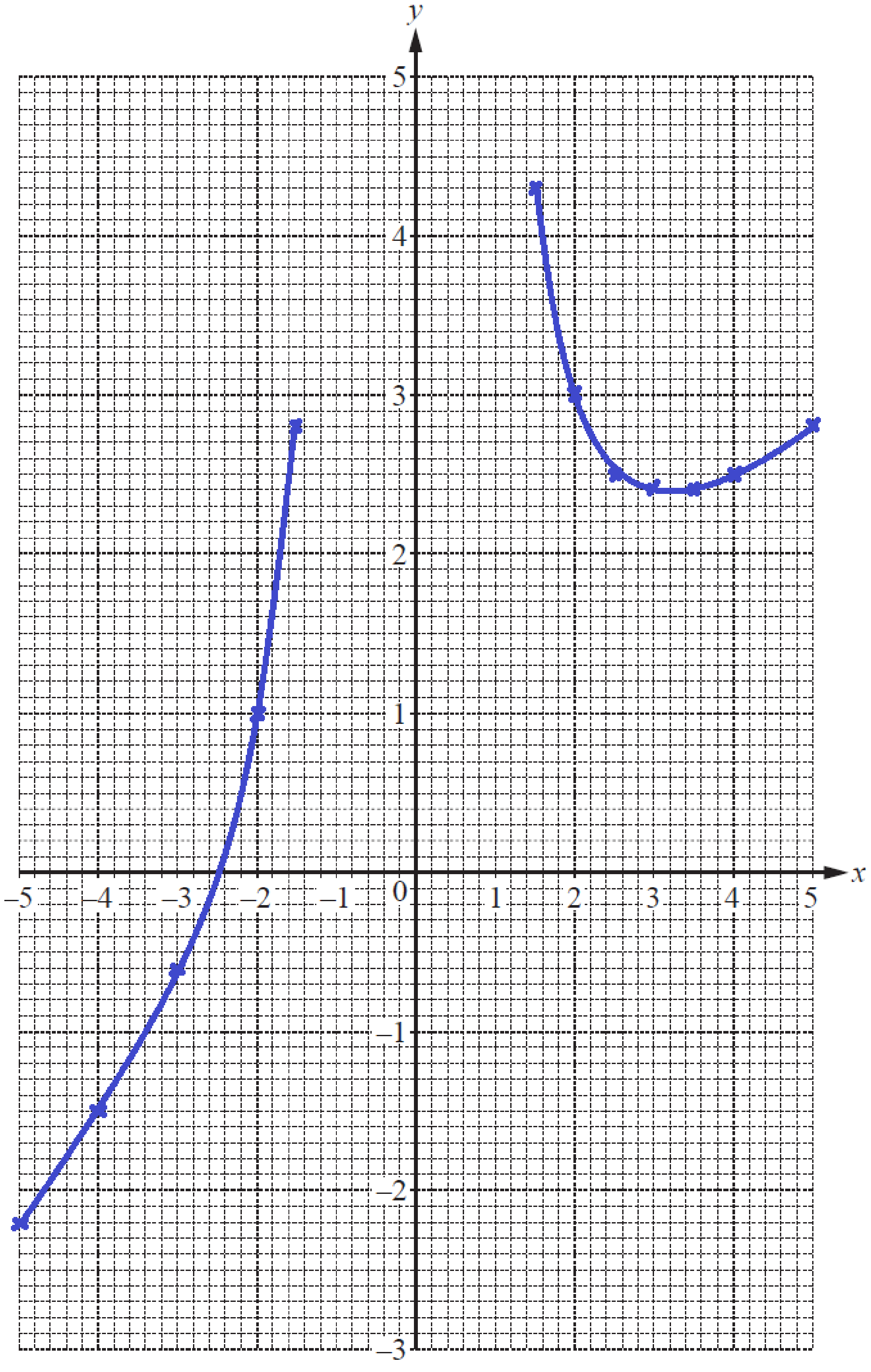
x	–5	–4	–3	–2	–1.5		1.5	2	2.5	3	3.5	4	5
f(x)	–2.2	–1.5	–0.6		2.8		4.3		2.5	2.4	2.4		2.8

We use a calculator to find the values of *f(x)* for *x*=-2, *x*=2 and *x*=3.

$$f(-2) = 1, \qquad f(2) = 3, \qquad f(3) = 2.5$$

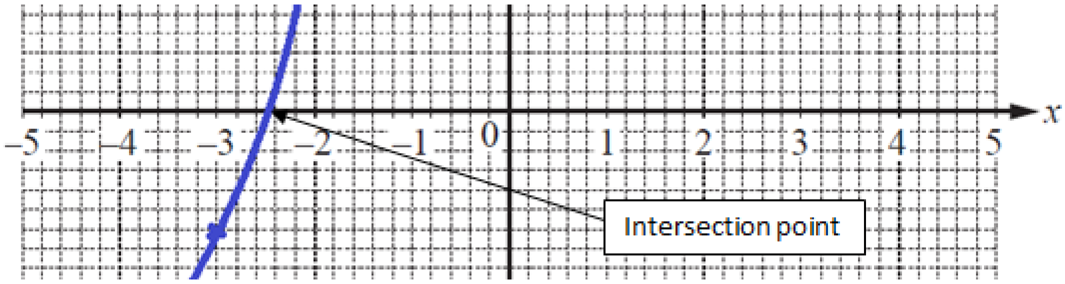
(b) On the grid, draw the graph of $y = f(x)$ for $-5 \leqslant x \leqslant -1.5$ and $1.5 \leqslant x \leqslant 5$. [5]

Plot the point from the table on the grid and then draw a smooth function connecting the points (blue curve).



(c) Solve f(x) = 0. [1]

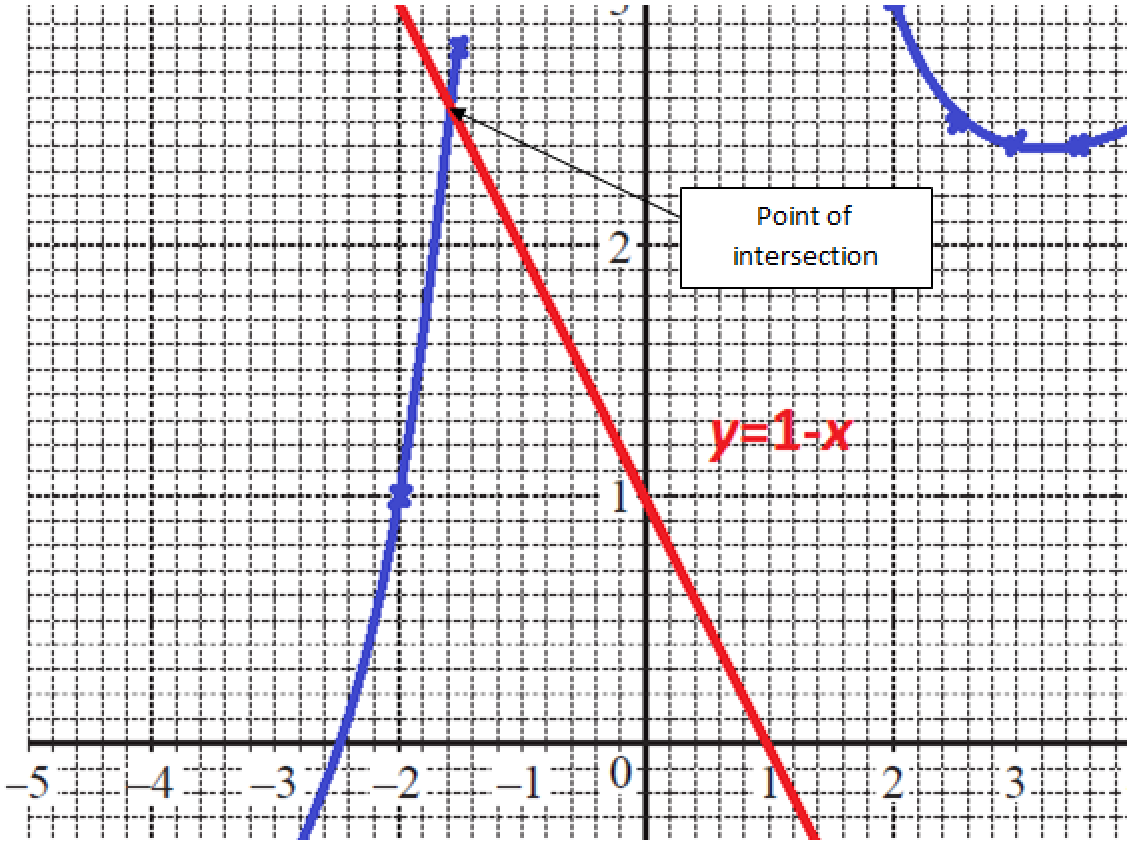
We are looking for the x-coordinate of the intersection point of the graph and the x-axis.



From the graph, we can see that the x-coordinate of the point is ***x* = -2.4.**

(d) By drawing a suitable line on the grid, solve the equation f(x) = 1 – x. [3]

We plot a line $y = 1 - x$ and find the x-coordinate of the point of intersection with the original graph to solve $f(x) = 1 - x$



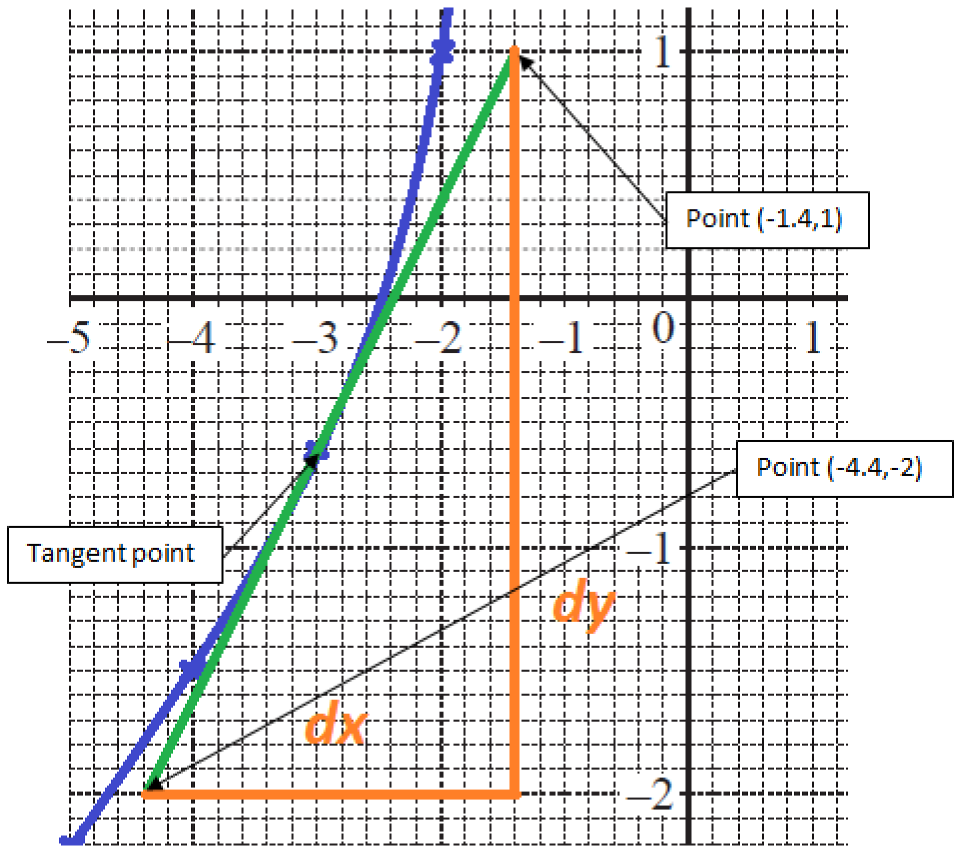
From the graph, we can see that the x-coordinate of the point, and hence the solution to the equation $f(x) = 1 - x$ is

$$x = -1.6$$

(e) By drawing a tangent at the point (–3, –0.6), estimate the gradient of the graph of $y = f(x)$ when $x = -3$.

[3]

We start by drawing a tangent line (green line) to the function at point x = -3 (which means that the line touches out function at this point).



The gradient is found as the change of y-coordinate (dy) over the change of x-coordinate (dx) between two points on the line. We pick two points (-4.4,-2) and (-1.4,1).

$$\text{Gradient } m = \frac{dy}{dx} = \frac{1-(-2)}{-1.4-(-4.4)} = \frac{3}{3}$$

$$= \mathbf{1}$$

The table shows some values of $y = x^3 + 3x^2 - 2$. [3]

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1
y	-2	1.13		1.38		-1.38		-1.13	

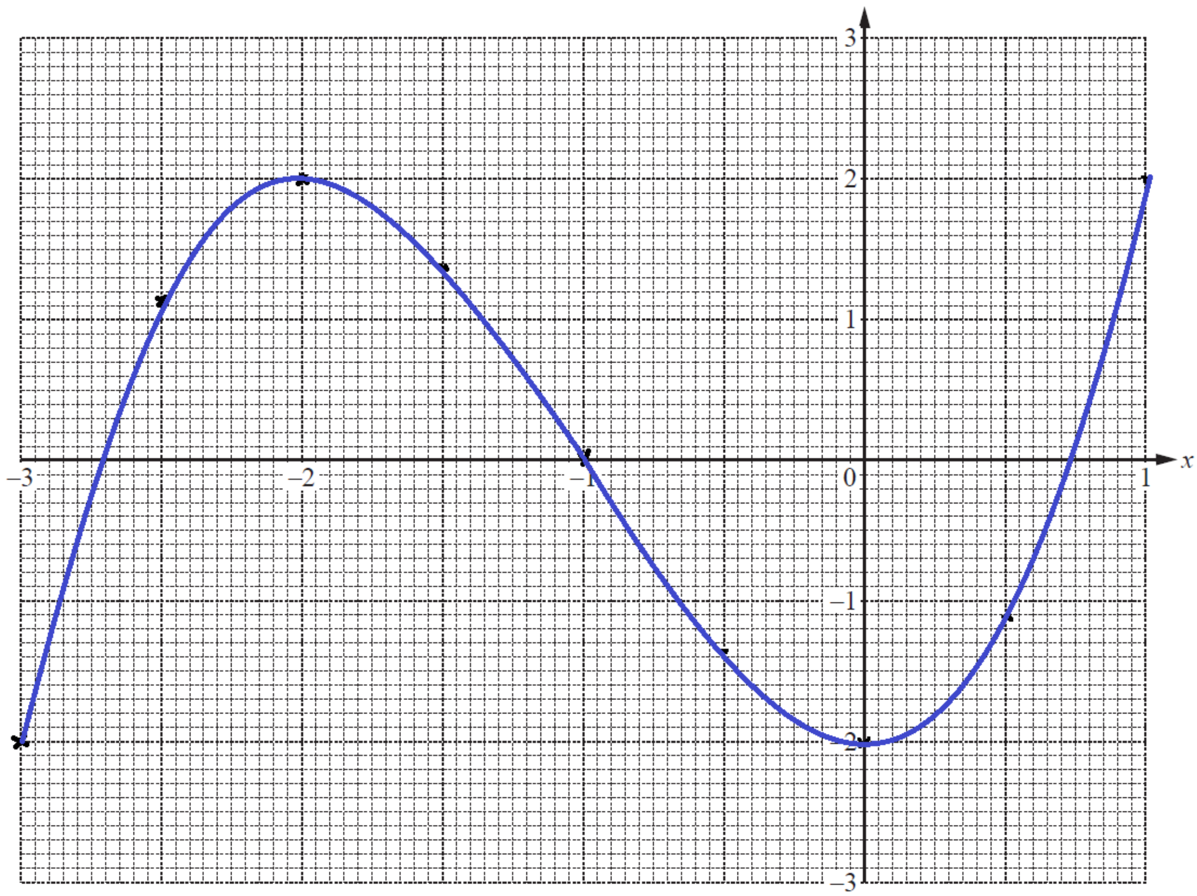
(a) Complete the table of values.

We use calculator to find the values of y for x=-2, x=-1, x=0 and x=1.

$y(-2)=2$, $y(-1) = 0$, $y(0) = -2$, $y(1) = 2$

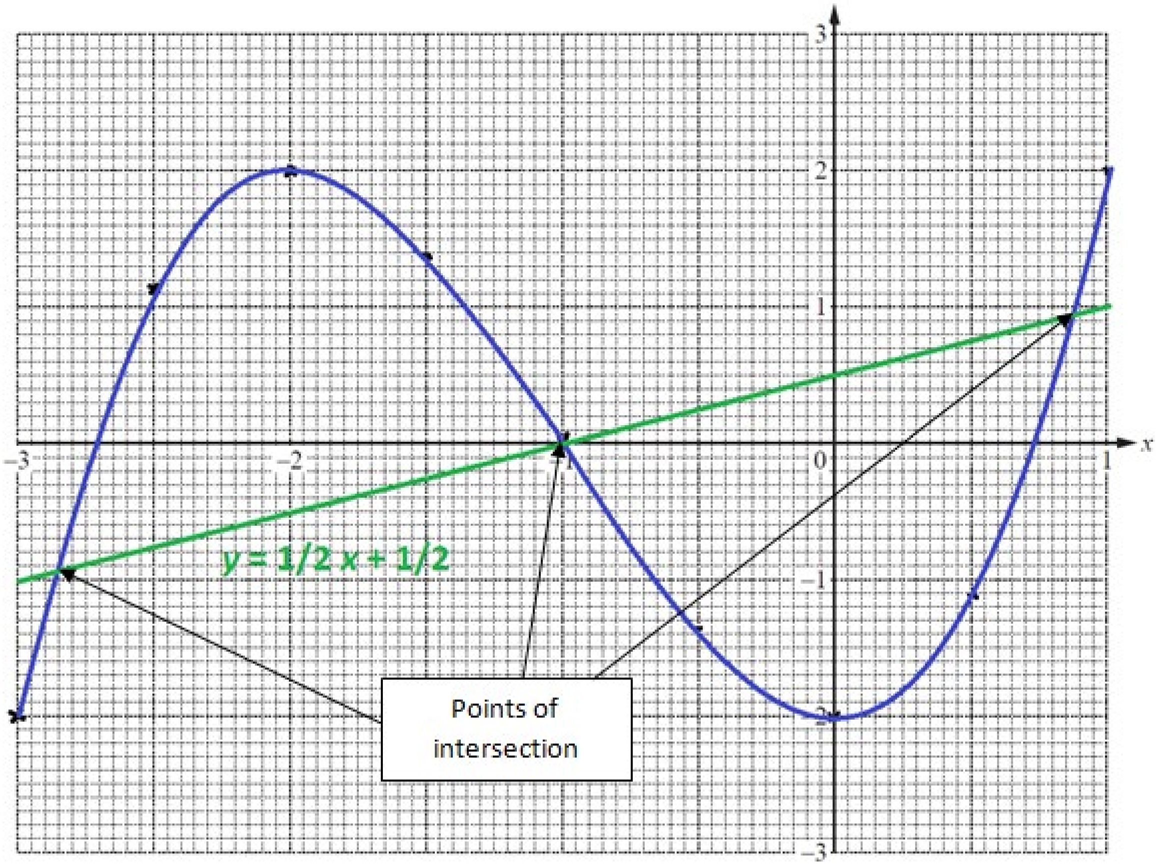
(b) On the grid, draw the graph of $y = x^3 + 3x^2 - 2$ for $-3 \leqslant x \leqslant 1$. [4]

Plot the point from the table on the grid and then draw a smooth function connecting the points (blue curve).



(c) By drawing a suitable line, solve the equation $x^3 + 3x^2 - 2 = \frac{1}{2}(x + 1)$. [4]

The left hand side of the given equation is our original graph. We plot a line $y = \frac{x}{2} + \frac{1}{2}$ and find the x-coordinate of the point of intersection with the original graph to solve $x^3 + 3x^2 - 2 = \frac{x}{2} + \frac{1}{2}$



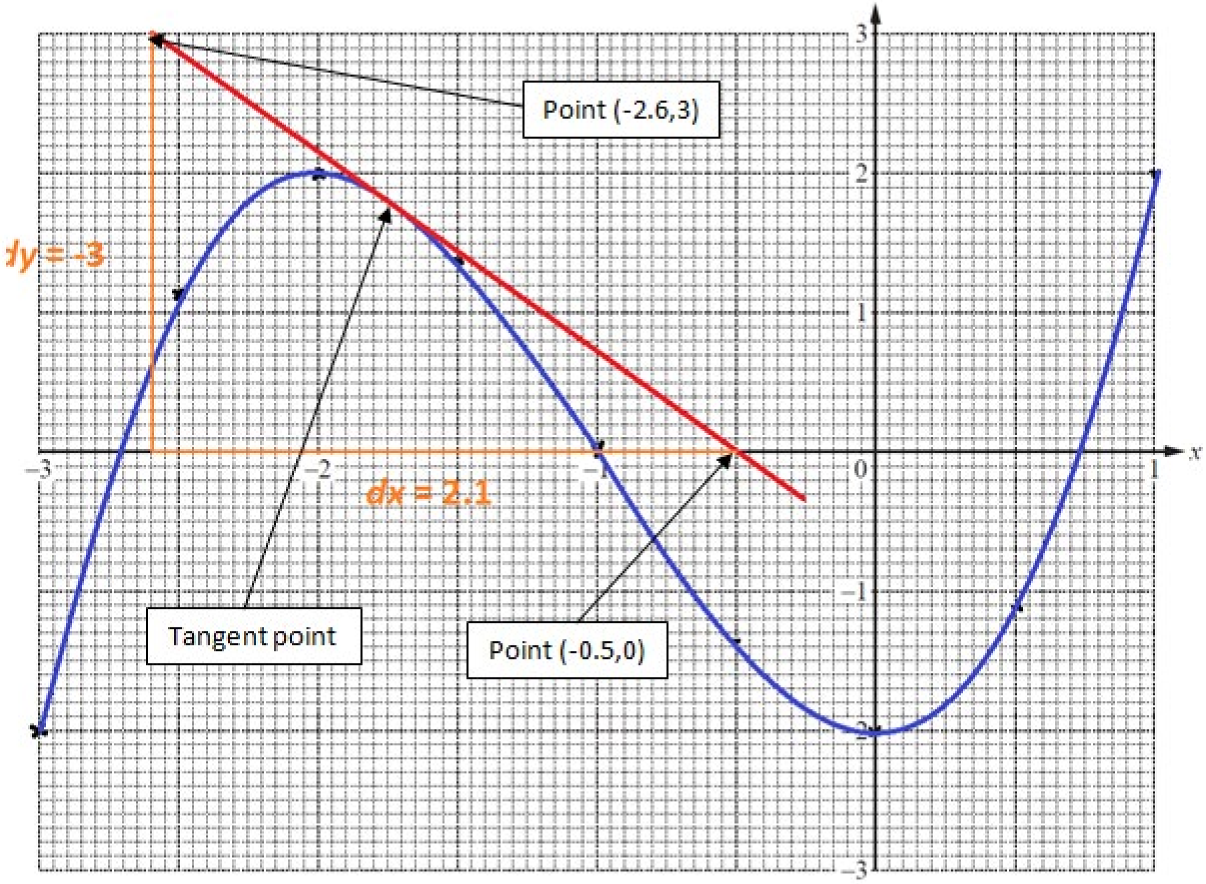
From the graph, we can see that the x-coordinates of the points, and hence the solutions to the equation $x^3 + 3x^2 - 2 = \frac{x}{2} + \frac{1}{2}$ are

$$x = -2.86, \quad x = -1, \quad x = 0.87$$

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = -1.75$.

[3]

We start by drawing a tangent line (red line) to the function at point *x*=-1.75 (which means that the line touches out function at this point).



The gradient is found as the change of y-coordinate over the change of x-coordinate between two points on the line. We pick two points (-2.6,3) and (-0.5,0).

$$\text{Gradient } m = \frac{dy}{dx} = \frac{0-3}{-0.5-(-2.6)} = \frac{-3}{2.1}$$

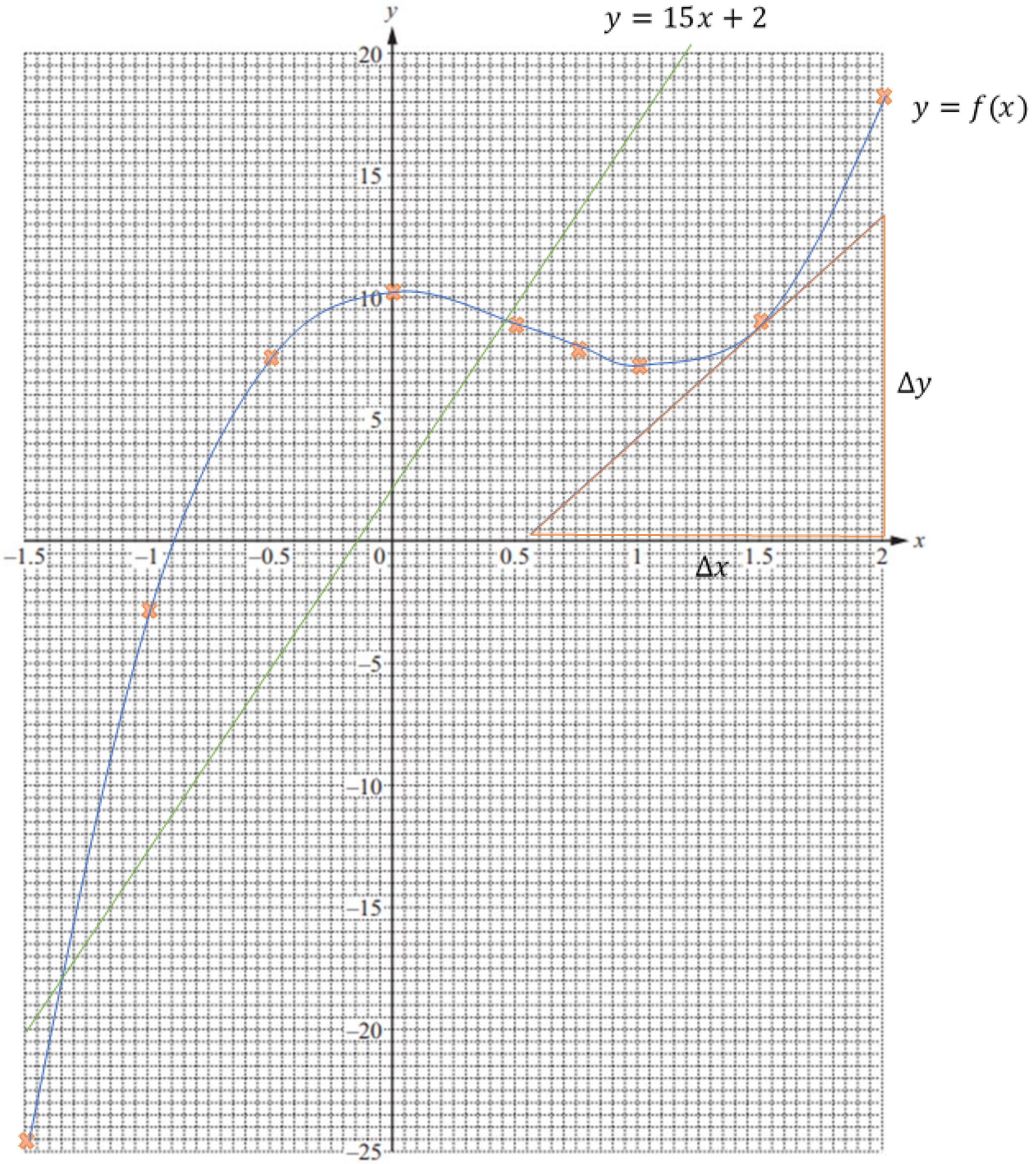
$$= -1.43.$$

$$f(x) = 5x^3 - 8x^2 + 10$$

(a) Complete the table of values. [3]

x	-1.5	-1	-0.5	0	0.5	0.75	1	1.5	2
$f(x)$	-24.9	**-3**	**7.375**	10	8.6	7.6	7	**8.875**	18

(b) Draw the graph of $y = f(x)$ for $-1.5 \leqslant x \leqslant 2$. [4]



(c) Use your graph to find an **integer** value of k so that $f(x) = k$ has

(i) exactly one solution, [1]

$\boldsymbol{k = 0}$

(ii) three solutions. [1]

$\boldsymbol{k} = 9$

(d) By drawing a suitable straight line on the graph, solve the equation $f(x) = 15x + 2$ for $-1.5 \leqslant x \leqslant 2$.

Suitable line drawn in green on graph above. [4]

Intersections, and hence solutions, at:

$$\boldsymbol{x = -1.35, \qquad x = 0.45}$$

(e) Draw a tangent to the graph of $y = f(x)$ at the point where $x = 1.5$.

Use your tangent to estimate the gradient of $y = f(x)$ when $x = 1.5$. [3]

Tangent drawn in orange above.

We have that the gradient is:

$$\boldsymbol{m = \frac{\Delta y}{\Delta x}}$$

$$= \frac{13 - 0}{2 - 0.55}$$

$$= \frac{13}{1.45}$$

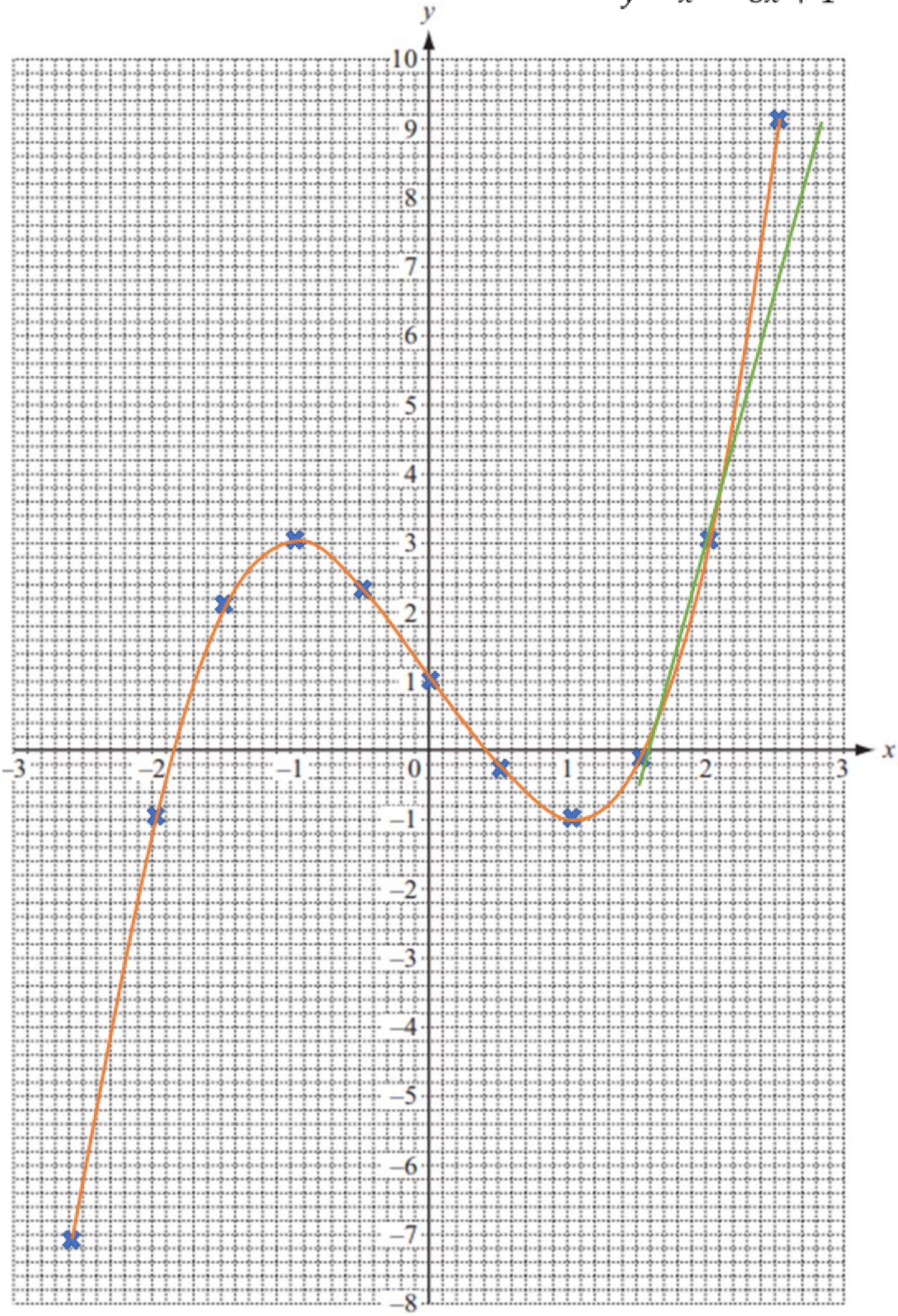
$$\boldsymbol{= 8.97}$$

(a) Complete the table of values for $y = x^3 - 3x + 1$. [2]

x	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
y	-7.125	-1	**2.125**	3	**2.375**	1	-0.375	-1	-0.125	3	9.125

(b) Draw the graph of $y = x^3 - 3x + 1$ for $-2.5 \leq x \leq 2.5$. [4]

$y = x^3 - 3x + 1$



(c) By drawing a suitable tangent, estimate the gradient of the curve at the point where $x = 2$. [3]

Suitable tangent drawn in green above.

Gradient is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(10 - 0)}{2.8 - 1.7}$$

$$= \frac{10}{1.1}$$

$$= 9.1$$

(d) Use your graph to solve the equation $x^3 - 3x + 1 = 1$. [2]

The graph is equal to 1 at:

$$x = -1.7, \qquad x = 0, \qquad x = 1.8$$

(e) Use your graph to complete the inequality in k for which the equation

$x^3 - 3x + 1 = k$ has three different solutions. [2]

$$-1 < k < 3$$

Graphs
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **135 minutes**

Score: **/117**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

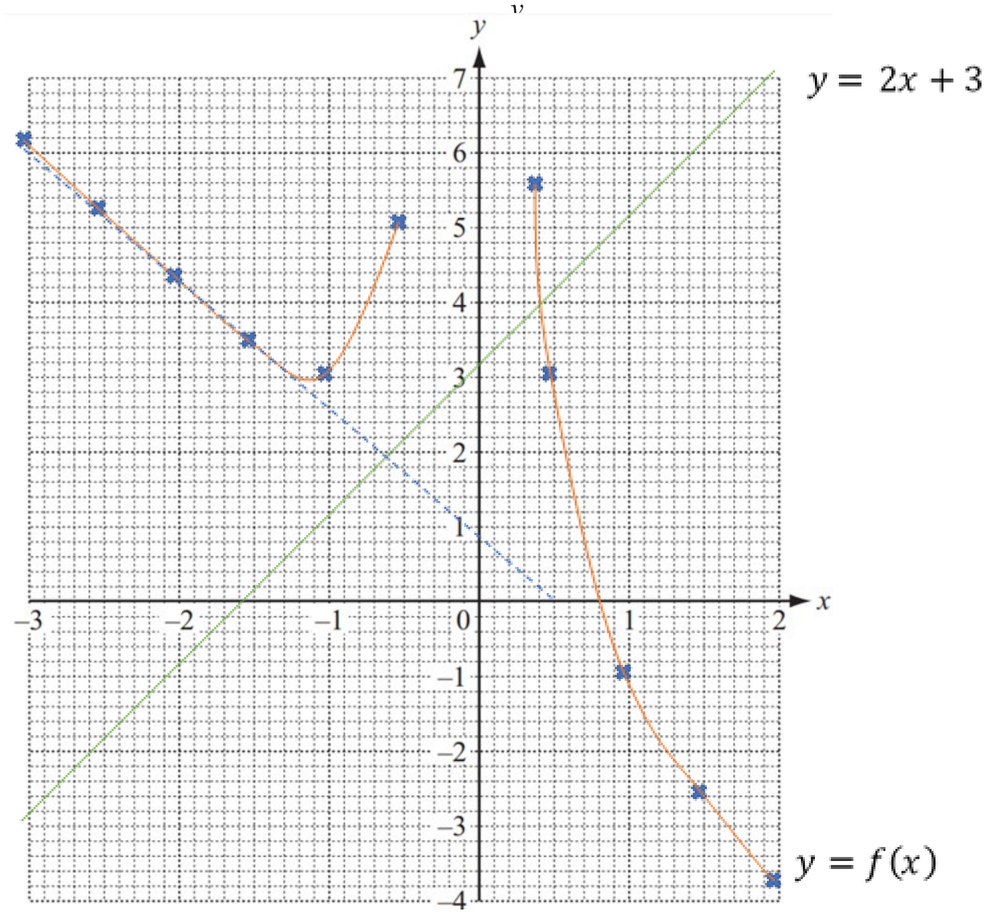
9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

$\mathrm{f}(x) = \dfrac{1}{x^2} - 2x\,,\ x \neq 0$

(a) Complete the table of values for $\mathrm{f}(x)$. [3]

x	-3	-2.5	-2	-1.5	-1	-0.5	0.4	0.5	1	1.5	2
$f(x)$	6.1	5.2	4.3	3.4	3	5	5.5	3	-1	-2.6	-3.8

(b) On the grid, draw the graph of $y = \mathrm{f}(x)$ for $-3 \leqslant x \leqslant -0.5$ and $0.4 \leqslant x \leqslant 2$. [5]



(c) Solve the equation $\mathrm{f}(x) = 2$. [1]

$x = 0.6$

(d) Solve the equation $f(x) = 2x + 3$. [3]

Straight line drawn on graph above in green.

Intersection, and hence solution, at

$$\boldsymbol{x = 0.4}$$

(e) (i) Draw the tangent to the graph of $y = f(x)$ at the point where $x = -1.5$. [1]

Tangent drawn in dotted blue on graph above.

(ii) Use the tangent to estimate the gradient of the graph of $y = f(x)$ where $x = -1.5$. [2]

Gradient is

$$\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$= \frac{0 - 6}{0.5 - -3}$$

$$= -\frac{6}{3.5}$$

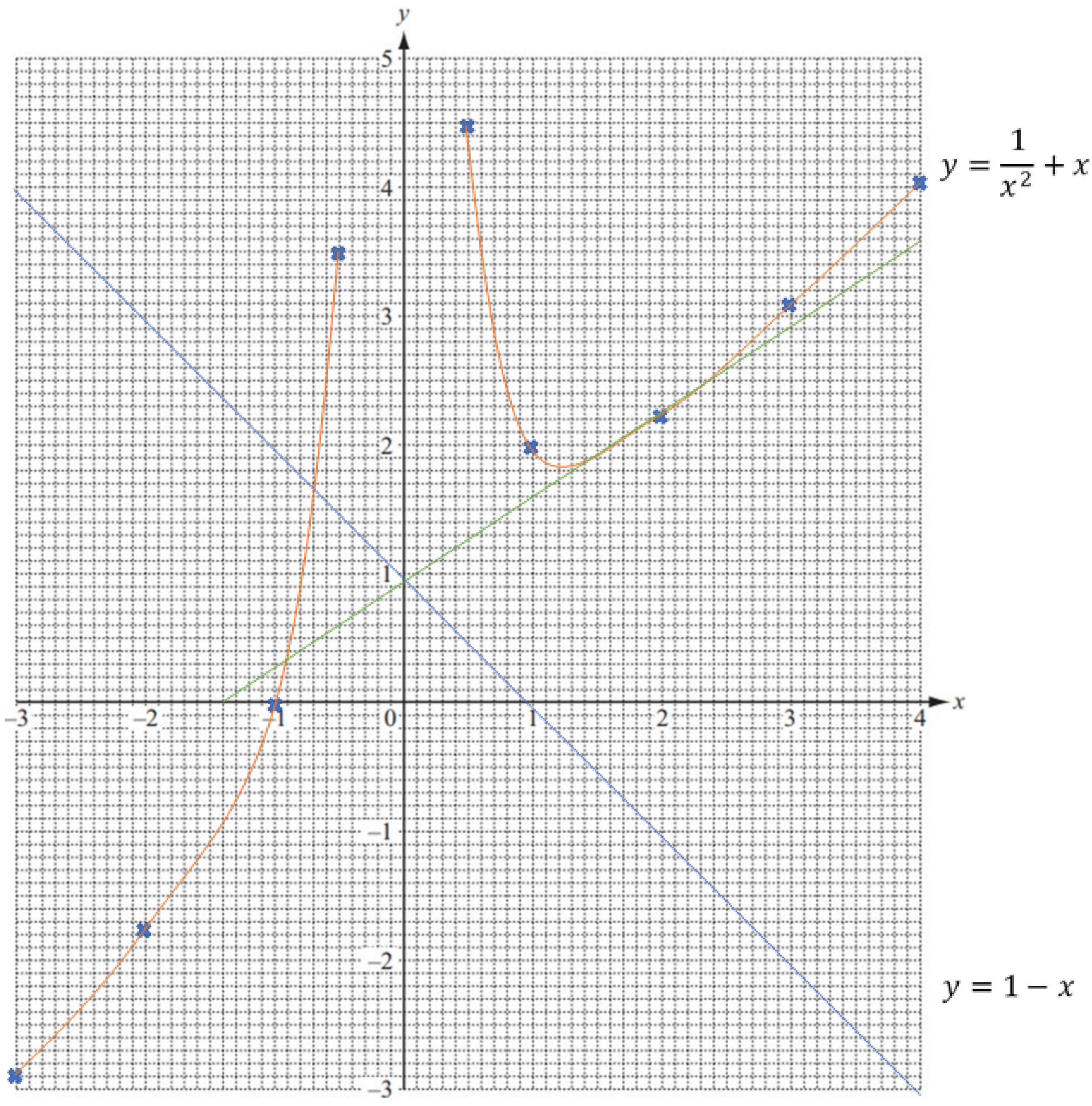
$$\boldsymbol{= -1.7}$$

The table shows some values for the function $y = \frac{1}{x^2} + x$, $x \neq 0$. [3]

(a) Complete the table of values.

x	-3	-2	-1	-0.5	0.5	1	2	3	4
y	-2.89	-1.75	**0**	3.5	**4.5**	2	2.25	**3.11**	4.06

(b) On the grid, draw the graph of $y = \frac{1}{x^2} + x$ for $-3 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$. [5]



(c) Use your graph to solve the equation $\frac{1}{x^2} + x - 3 = 0$. [3]

Need to solve

$$\frac{1}{x^2} + x = 3$$

i.e. where does our curve equal 3.

$$\boldsymbol{x = -0.55, \qquad x = 0.7, \qquad x = 2.9}$$

(d) Use your graph to solve the equation $\frac{1}{x^2} + x = 1 - x$. [3]

Where does the line $y = 1 - x$ (blue line above) intersect the curve?

$$\boldsymbol{x = -0.7}$$

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = 2$.

Tangent drawn in green above. [3]

Gradient is

$$\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$= \frac{3.6 - 0}{4 - -1.5}$$

$$= \frac{3.6}{5.5}$$

$$\boldsymbol{= 0.65}$$

(f) Using algebra, show that you can use the graph at $y = 0$ to find $\sqrt[3]{-1}$. [3]

$$\frac{1}{x^2} + x = 0$$

$$\rightarrow 1 + x^3 = 0$$

$$\rightarrow x^3 = -1$$

$$\rightarrow \boldsymbol{x = \sqrt[3]{-1}}$$

(a) Complete the table of values for $y = \frac{2}{x^2} - \frac{1}{x} - 3x$. [3]

x	-3	-2	-1	-0.5	-0.3		0.3	0.5	1	2	3
y	9.6	*7*	6	*11.5*	26.5		18.0	*4.5*	-2	-6	-9.1

Substitute different values for x into the equation to solve for the missing y values.

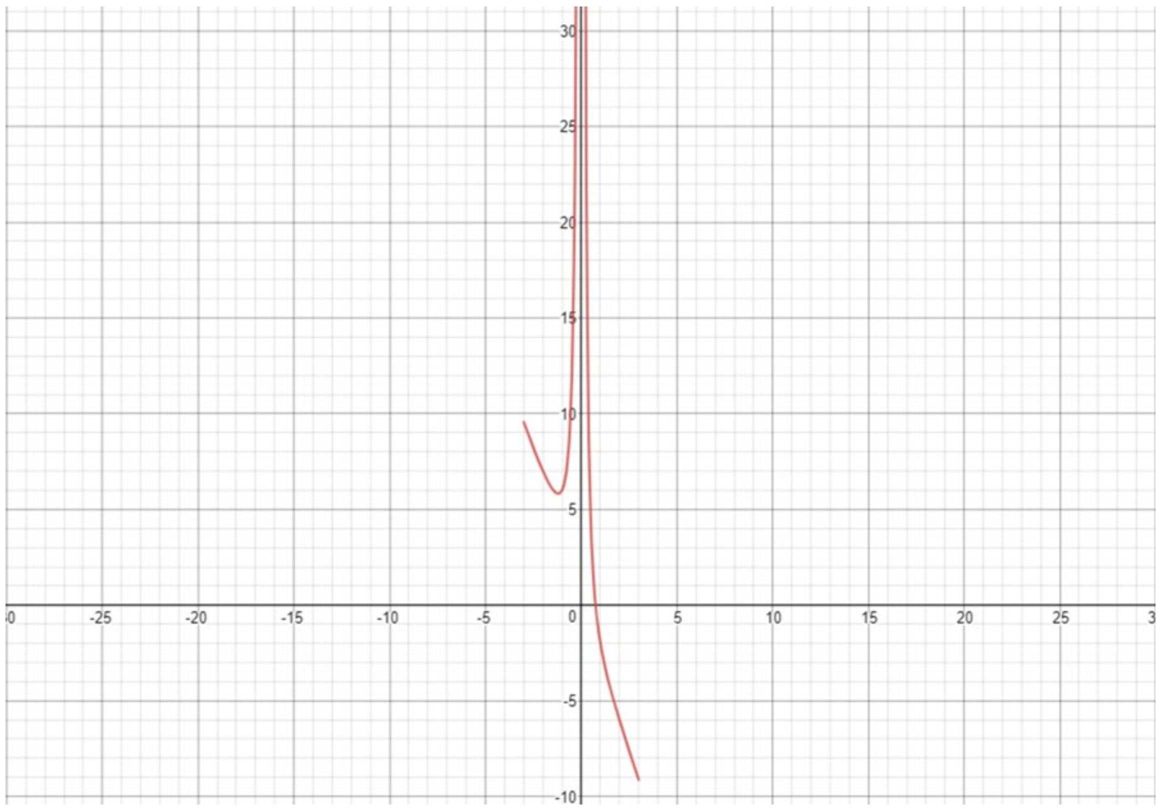
Example: x = -2,

$$y = \frac{2}{(-2)^2} - \frac{1}{-2} - 3(-2)$$

$$= \frac{2}{4} + \frac{1}{2} + 6$$

$$\mathbf{= 7}$$

(b) Draw the graph of $y = \frac{2}{x^2} - \frac{1}{x} - 3x$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$. [5]



(c) Use your graph to solve these equations.

(i) $\frac{2}{x^2} - \frac{1}{x} - 3x = 0$ [1]

y = 0,

Read from graph:

$$x = 0.7 \ to \ 0.8$$

(ii) $\frac{2}{x^2} - \frac{1}{x} - 3x - 7.5 = 0$ [3]

Rearrange to obtain:

$$7.5 = \frac{2}{(x)^2} - \frac{1}{x} - 3(x)$$

y = 7.5,

$$x = -2.3 \ to \ -2.2$$

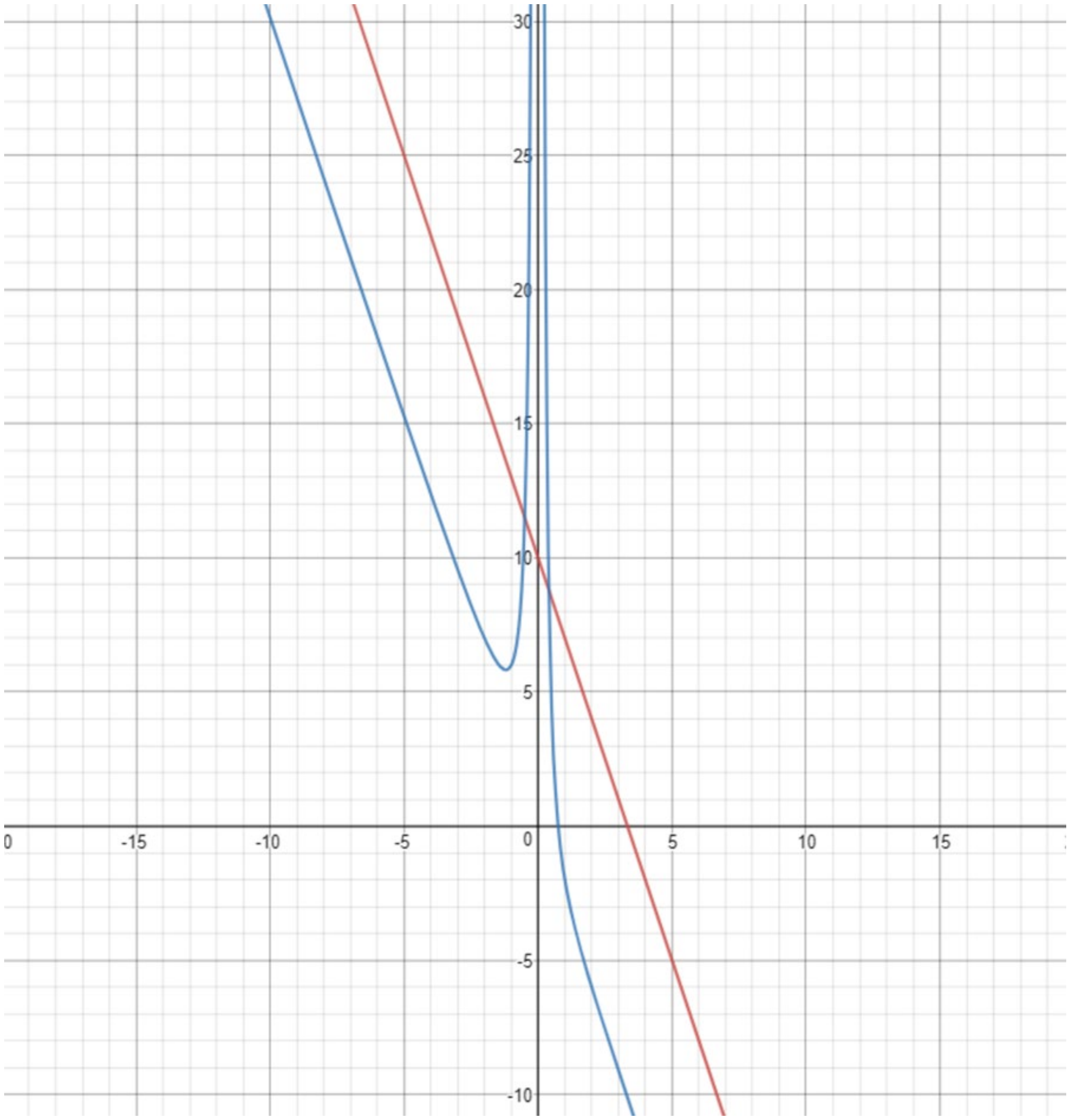
$$x = -0.8 \ to \ -0.6$$

$$x = 0.35 \ to \ 0.5$$

(d) (i) By drawing a suitable straight line on the graph, solve the equation $\frac{2}{x^2} - \frac{1}{x} - 3x = 10 - 3x$.

Plot y = 10 – 3x, use some values to create points: [4]

x	-3	-0.3	0.3	3
y	19	10.9	9.1	1



Read off the graph for the x values where the red line y = 10 – 3x intersects with the blue graph drawn earlier.

$$x = -0.55 \; to \; -0.45$$

$$x = 0.35 \; to \; 0.45$$

(ii) The equation $\frac{2}{x^2} - \frac{1}{x} - 3x = 10 - 3x$ can be written in the form $ax^2 + bx + c = 0$ where a, b and c are integers.

Find the values of a, b and c. [3]

$$10 - 3x = \frac{2}{(x)^2} - \frac{1}{x} - 3(x)$$

Multiply throughout by x^2:

$$10x^2 - 3x^3 = 2 - x - 3x^2$$

$$10x^2 = 2 - x$$

$$10x^2 + x - 2 = 0$$

Hence,

$$\boldsymbol{a = 10}$$

$$\boldsymbol{b = 1}$$

$$\boldsymbol{c = -2}$$

As the RHS is equal to zero, the signs can be flipped. Hence, also acceptable are:

$$\boldsymbol{a = -10}$$

$$\boldsymbol{b = -1}$$

$$\boldsymbol{c = 2}$$

$f(x) = 3 - x - x^2$ $\quad\quad$ $g(x) = 3^x$

(a) Complete the tables of values for f(x) and g(x). [3]

x	–1.5	–1	–0.5	0	0.5	1	1.5
f(x)	2.25	3	3.25		2.25	1	–0.75

x	–1.5	–1	–0.5	0	0.5	1	1.5
g(x)	0.19		0.58		1.73	3	5.20

$f(x) = 3 - x - x^2$

The unknown values for f(x) can be found by substituting

the corresponding x values into the equation.

x	-1.5	-1	-0.5	0	0.5	1	1.5
f(x)	2.25	3	3.25	**3**	2.25	1	-0.75

$g(x) = 3^x$

The unknown values for g(x) can be found by substituting

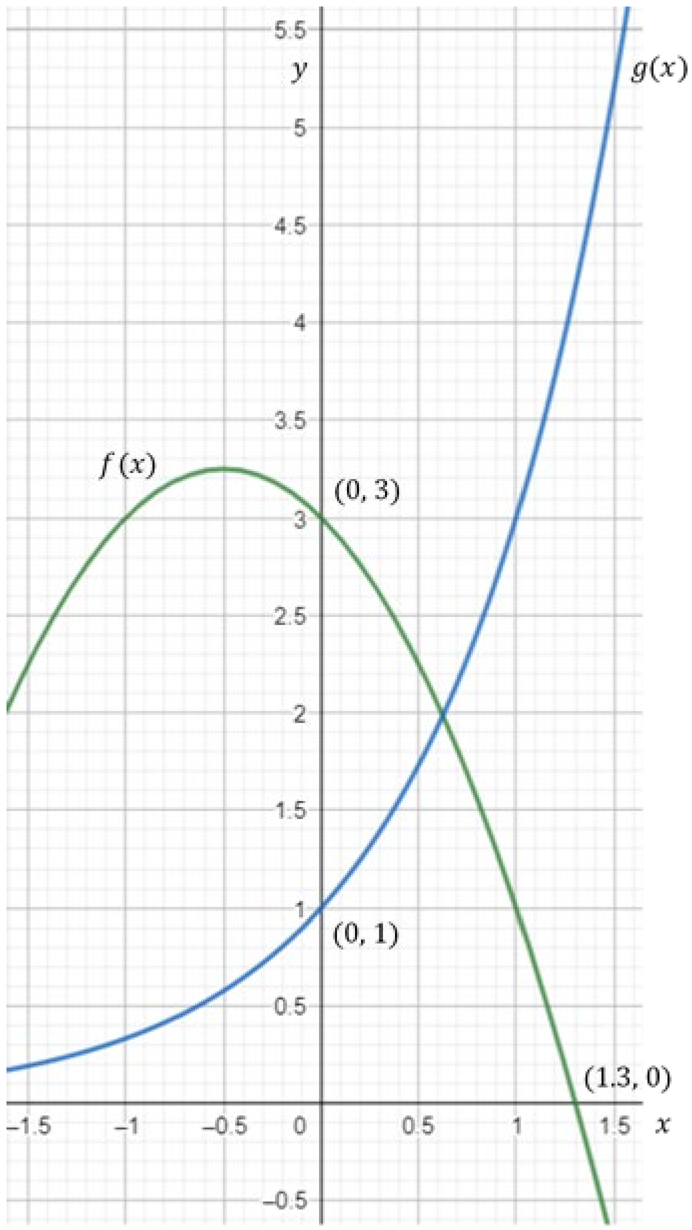
the corresponding x values into the equation.

x	-1.5	-1	-0.5	0	0.5	1	1.5
g(x)	0.19	**0.33**	0.58	**1**	1.73	3	5.20

(b) On the grid, draw the graphs of $y = f(x)$ and $y = g(x)$ for $-1.5 \leq x \leq 1.5$. [6]

Here we simply substiute the values from each table onto the graph and draw a line

the intercepting each group of points



(c) For $-1.5 \leq x \leq 1.5$, use your graphs to solve

(i) $f(x) = 0$, [1]

Here we find the x coordinate of f(x) where y is equal to 0

$\boldsymbol{x = 1.3}$

(ii) $g(x) = 4$, [1]

Here we find the x coordinate of g(x) where y is equal to 4

$x \approx 1.25$

(iii) $f(x) = g(x)$. [1]

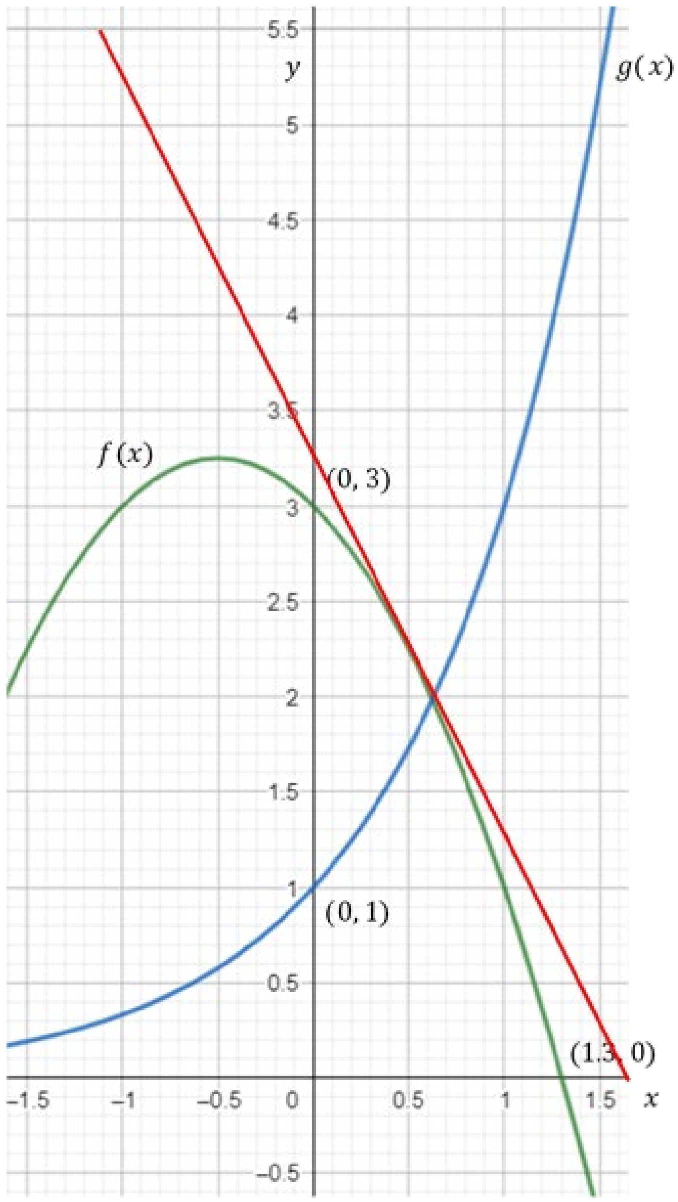
Here we find the x coordinate at the point of intersection between f(x) and g(x)

$x \approx 0.6$

(d) By drawing a suitable tangent, find an estimate of the gradient of the graph of $y = f(x)$ when $x = 0.5$.

[3]

Here we draw a line that touches f(x) only at the point where x = 0.5



To find the gradient of a straight line we use the following equation:

$$Gradient = \frac{change\ in\ y}{change\ in\ x}$$

To make this the best estimate, take the maximum and minimum possible values for x and y from the gradient.

$$= \frac{5.5 - 0.3}{-1.1 - 1.5}$$

$$= \frac{5.2}{-2.6}$$

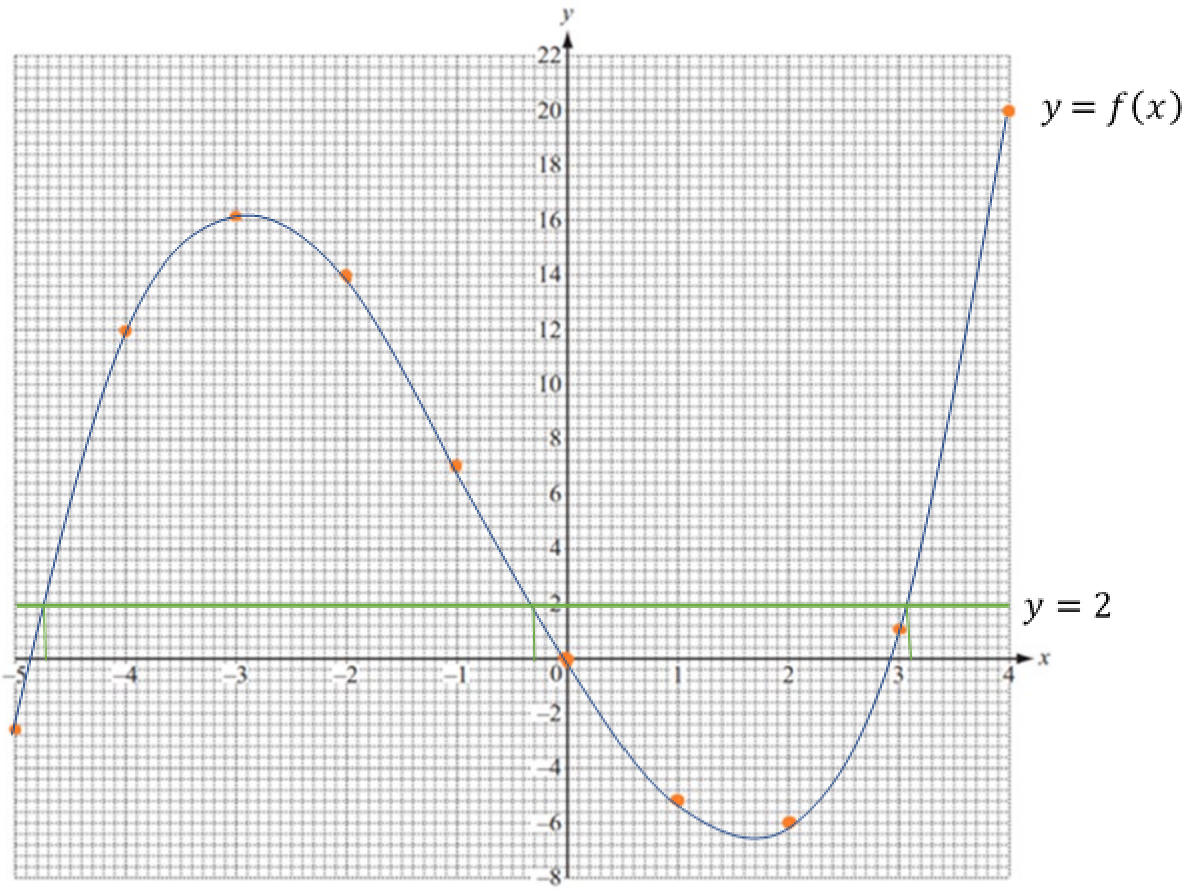
$$= -2$$

(a) (i) Complete the table of values for $y = \frac{1}{2}x^3 + x^2 - 7x$. [3]

x	–5	–4	–3	–2	–1	0	1	2	3	4
y	–2.5	12	16.5	**14**	7.5	0	**-5.5**	–6	1.5	**20**

(ii) On the grid, draw the graph of $y = \frac{1}{2}x^3 + x^2 - 7x$ for $-5 \leqslant x \leqslant 4$. [4]

Points plotted in orange, curve in blue (green line is for subsequent question).



(b) Use your graph to solve the equation $\frac{1}{2}x^3 + x^2 - 7x = 2$. [3]

On the graph above the line $y = 2$ is drawn in green.

The intersections, and hence the solutions, are

$$x = -4.7, \quad -0.3, \quad 3.1$$

(c) By drawing a suitable tangent, calculate an estimate of the gradient of the graph where $x = −4$. [3]

Straight line drawn at $x = -4$ in green below.

We can see that it passes through the points

$$(-5, 4)\ and\ (-3, 20)$$

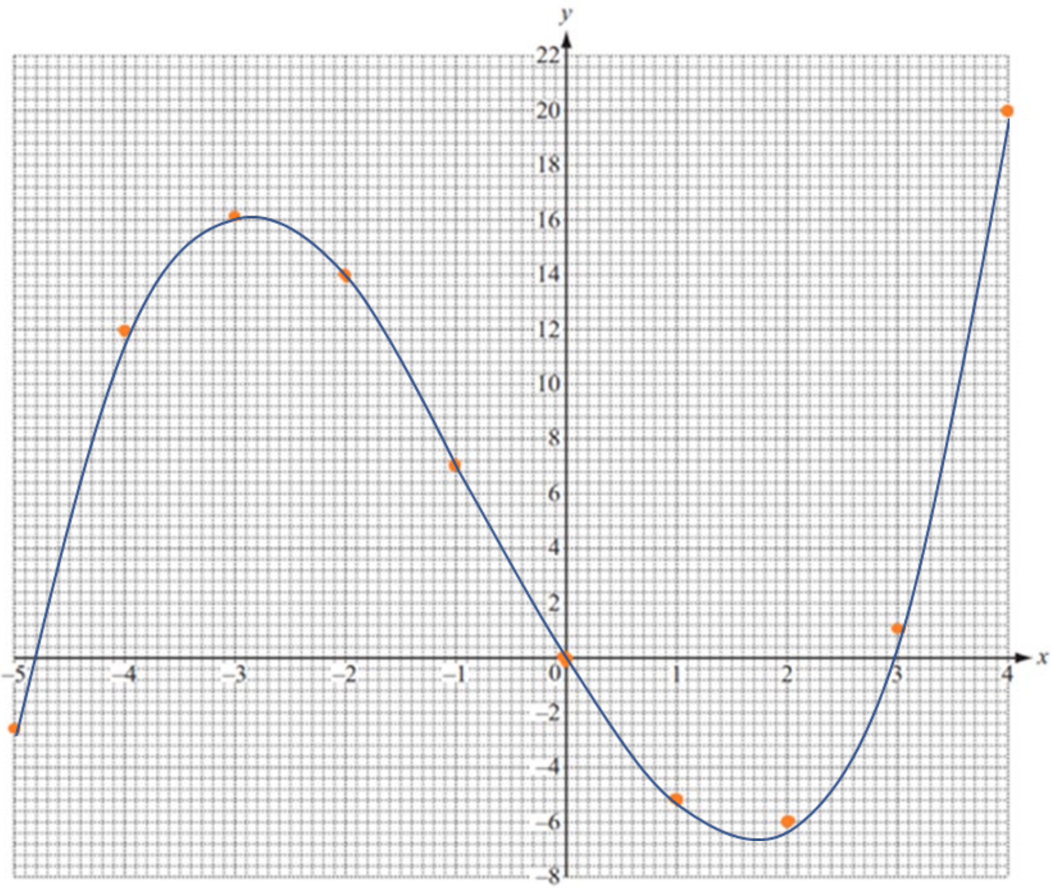
Using these, we calculate the gradient as

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{20 - 4}{-3 - -5}$$

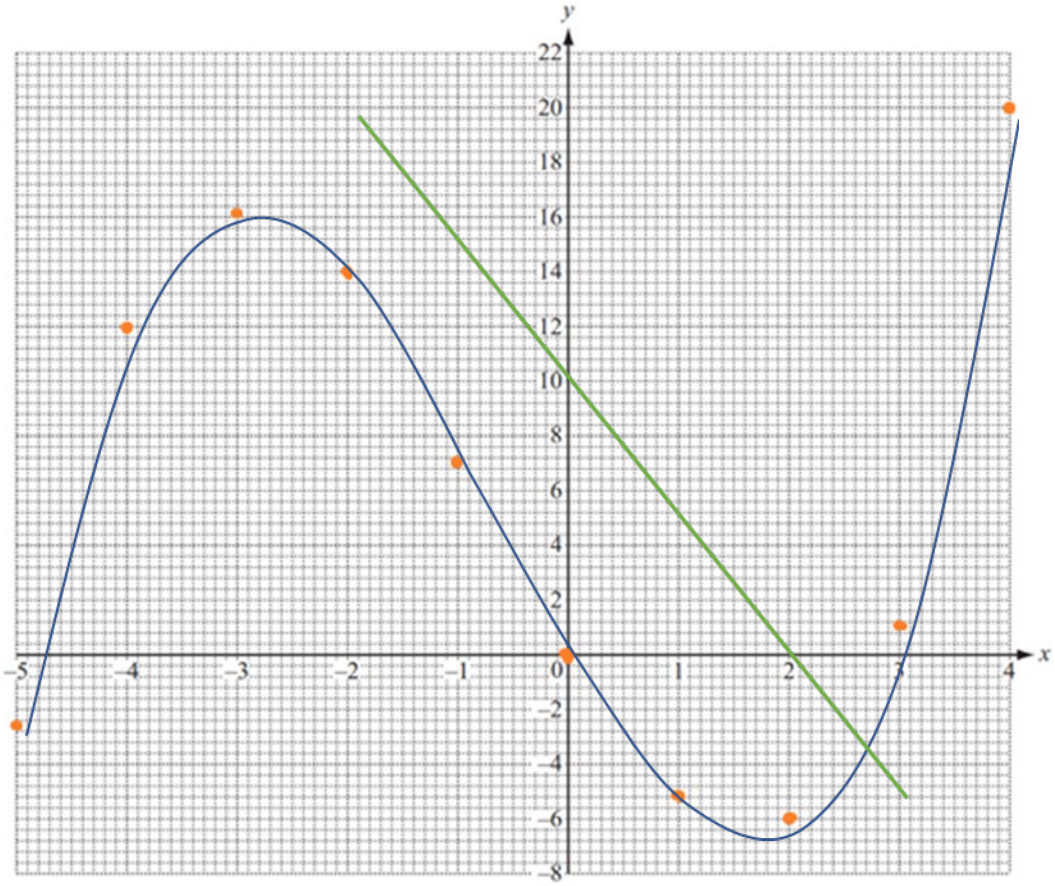
$$= \frac{16}{2}$$

$$= \mathbf{8}$$



(d) (i) On the grid draw the line $y = 10 - 5x$ for $-2 \leqslant x \leqslant 3$. [3]

Green line with gradient -5 and passing through the y-axis at 10 drawn below.



(ii) Use your graphs to solve the equation $\frac{1}{2}x^3 + x^2 - 7x = 10 - 5x$. [1]

The solution to

$$\frac{1}{2}x^3 + x^2 - 7x = 10 - 5x$$

is the x coordinate of the intersection of the curve and the straight line above.

This is

$$\boldsymbol{x = 2.6}$$

$f(x) = 2^x$

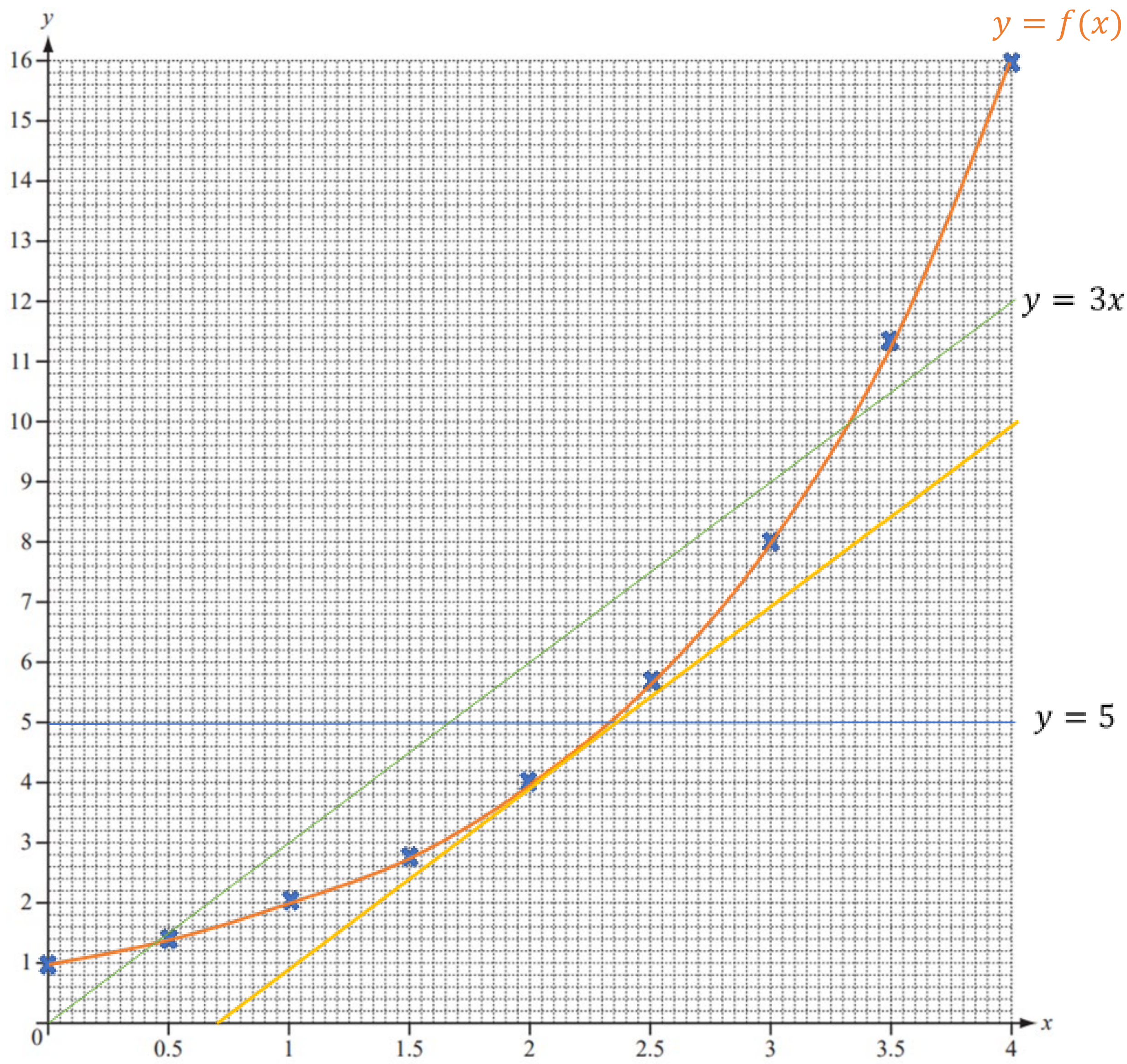
(a) Complete the table.

[3]

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$f(x)$	**1**	1.4	2	2.8	4	5.7	8	**11.3**	**16**

(b) Draw the graph of $y = f(x)$ for $0 \leqslant x \leqslant 4$.

[4]



[1]

(c) Use your graph to solve the equation $2^x = 5$.

This is solved for when the line $y = 5$ (blue line drawn above) and the curve $y = f(x)$ intersect. Using the graph above, we see that this is

$$\boldsymbol{x = 2.30}$$

(d) Draw a suitable straight line and use it to solve the equation $2^x = 3x$. [3]

Suitable line drawn in green on graph above ($y = 3x$).

They intersect at

$$\boldsymbol{x = 0.47, \qquad 3.30}$$

(e) Draw a suitable tangent and use it to find the co-ordinates of the point on the graph of $y = f(x)$ where the gradient of the graph is 3.

[3]

Draw a line parallel to the line $y = 3x$ (green line above) that is tangential to the curve.

This line is drawn in yellow on above graph and the coordinates where it cuts the curve are

$$\boldsymbol{(2.1, 4.2)}$$

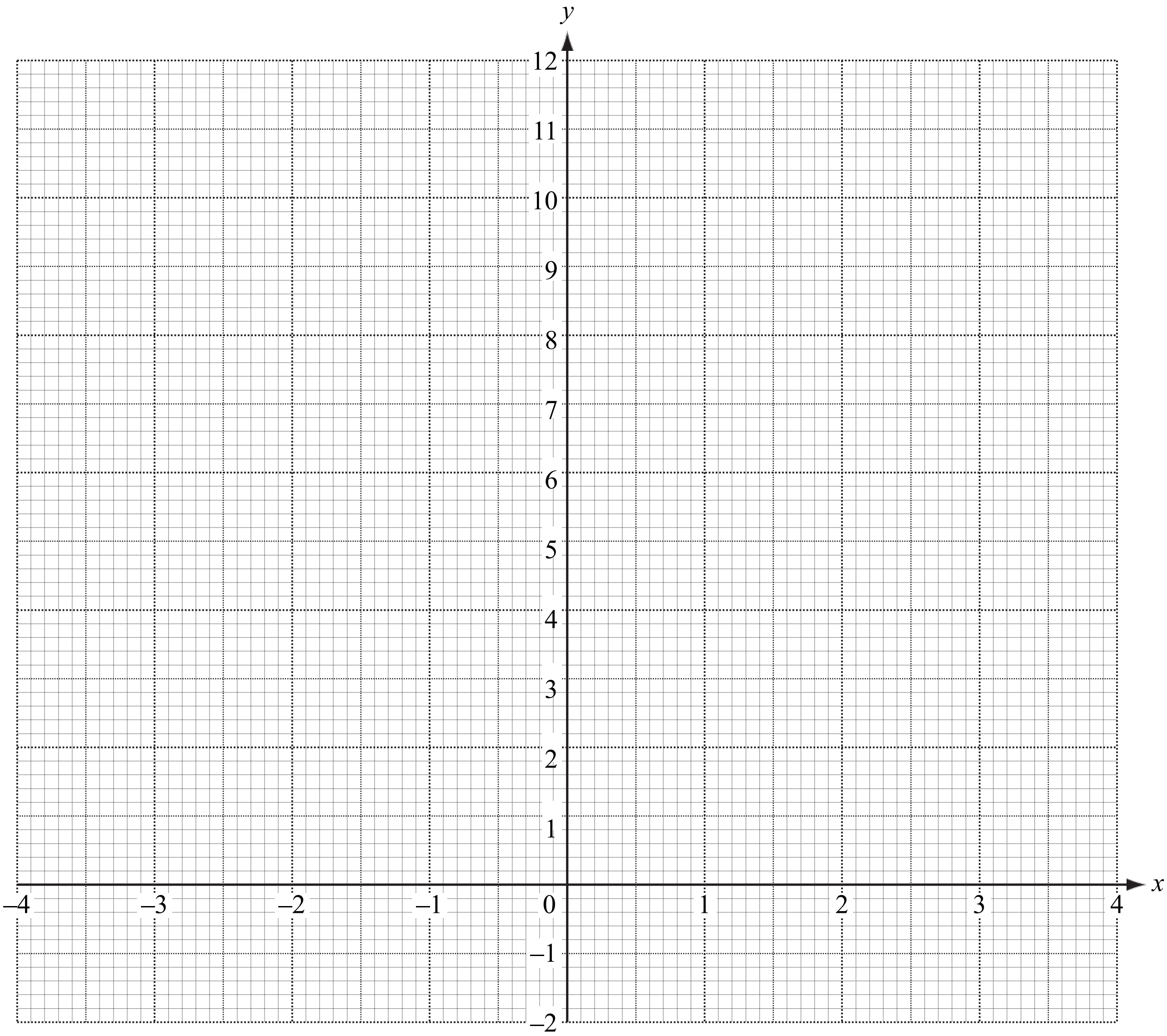
(a) Complete the table of values for the equation $y = \frac{4}{x^2}$, $x \neq 0$. [3]

x	- 4	- 3	- 2	- 1	- 0.6		0.6	1	2	3	4
y	0.25	0.44			11.11			4.00		0.44	

We use calculator to find the values of y for x=-2, x=-1, x= 0.6, x= 2 and x= 3.

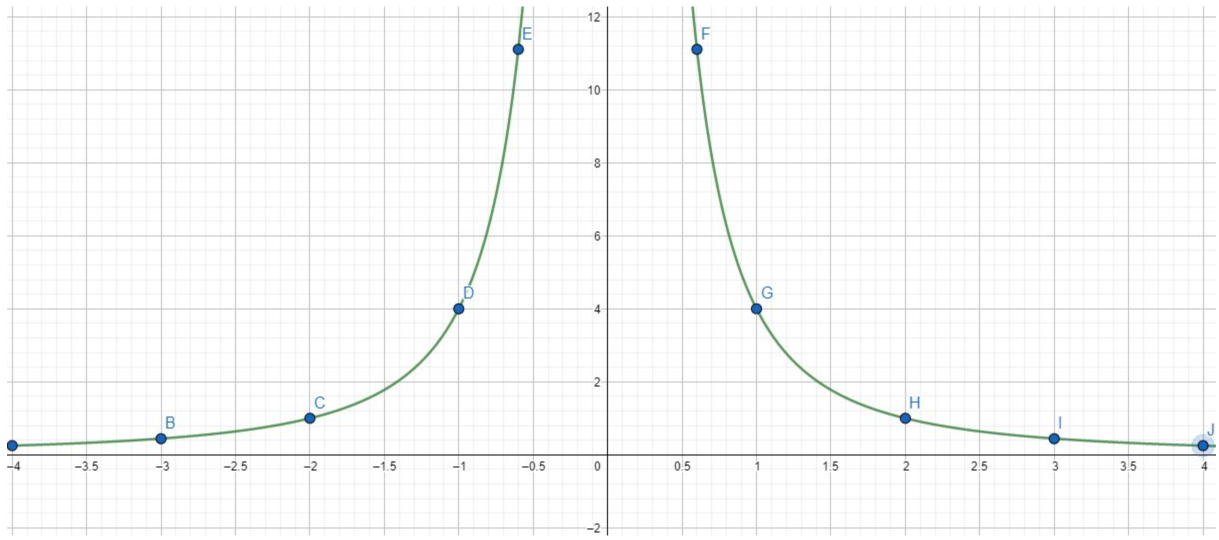
$$y(-2) = 1, \quad y(-1) = 4, \quad y(0.6) = 11.11, \quad y(2) = 1, \quad y(4) = 0.25$$

(b) On the grid, draw the graph of $y = \frac{4}{x^2}$ for $-4 \leqslant x \leqslant -0.6$ and $0.6 \leqslant x \leqslant 4$. [5]



Plot the point from the table on the grid and then draw a smooth function connecting the points.

There is an asymptote at x = 0 because we cannot divide by zero.



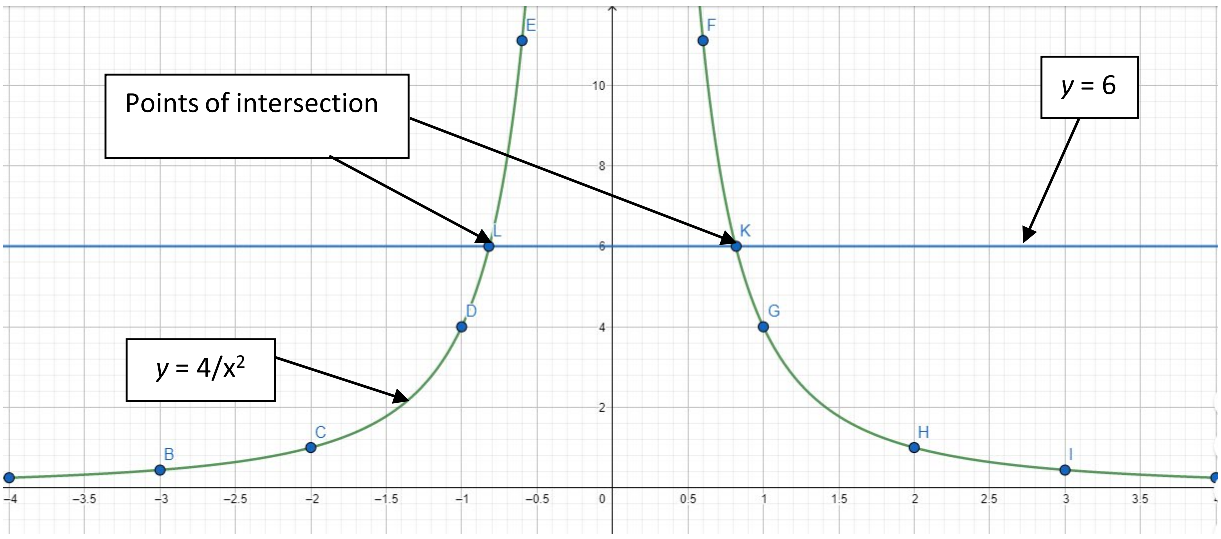
(c) Use your graph to solve the equation $\frac{4}{x^2} = 6$. [2]

The left hand side of the given equation:

$$\frac{4}{x^2} = 6$$

is our original graph and we want to find out when is the expression equal to 6.So we are looking for point, which lie on the intersection of graphs:

$$y = \frac{4}{x^2} \text{and} \quad y = 6$$



From the graph, we see that the points of intersection are approximately

$$(-0.82\,,6\,)\text{and}\quad(0.82\,,6)$$

By taking the *x* coordinates of these points, we have the two possible

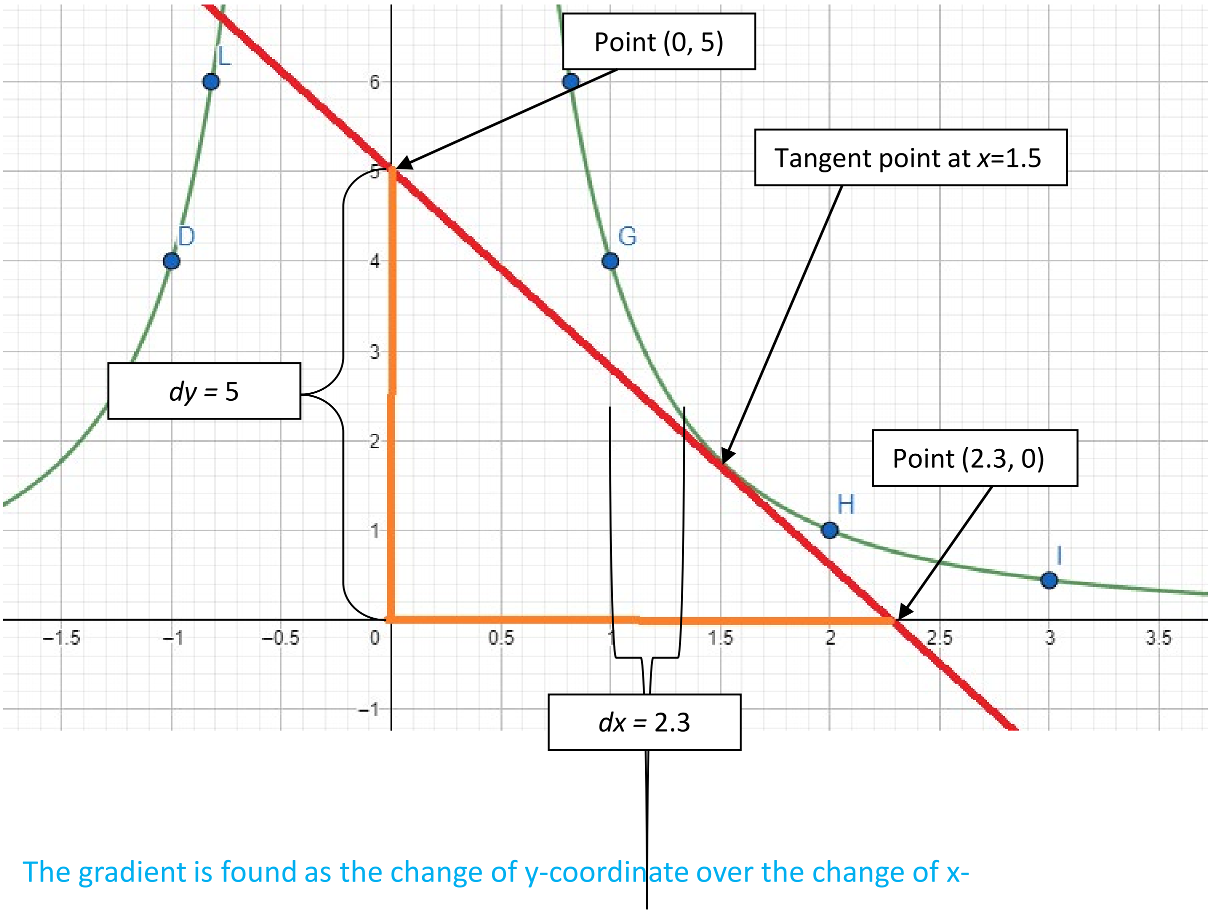
solutions of the equation.

$$\mathbf{-0.82\quad and\quad 0.82}$$

(d) By drawing a suitable tangent, estimate the gradient of the graph where $x = 1.5$. [3]

We start by drawing a tangent line (red line) to the function at point x=1.5

(which means that the line touches our function at this point).



The gradient is found as the change of y-coordinate over the change of x-coordinate between two points on the line.

We pick two points on the line:

$$(0\,,5)\ and\ (2.3\,,0)$$

Express the gradient as a ratio.

$$gradient\ m = \frac{dy}{dx}$$

$$m = \frac{5-0}{0-2.3} = \frac{5}{-2.3}$$

Calculate the gradient (correct to 3sf).

$$\boldsymbol{m = -2.17}$$

(e) (i) The equation $\frac{4}{x^2} - x + 2 = 0$ can be solved by finding the intersection of the graph of $y = \frac{4}{x^2}$ and a straight line.

Write down the equation of this straight line. [1]

We recognize that the left-most term is the term from our original equation. Hence by moving all other term to the right side, we will find the equation of the line.

Start with:

$$\frac{4}{x^2} - x + 2 = 0$$

Add *x - 2* to both sides of the equation.

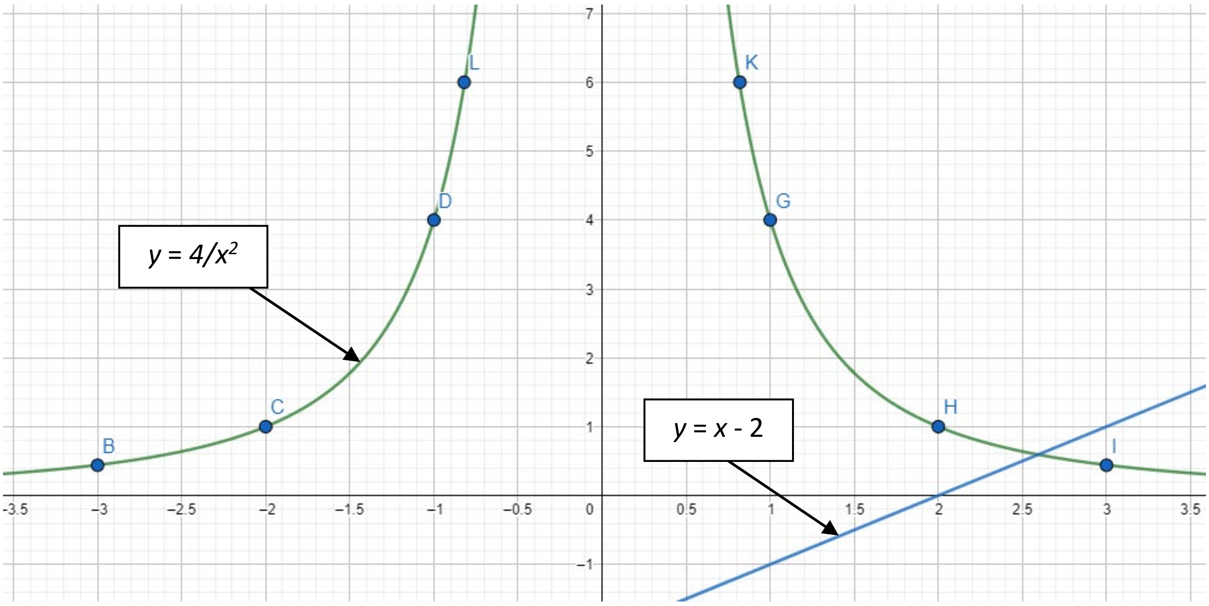
$$\frac{4}{x^2} = x - 2$$

From the right hand side we can see that the equation of the straight line we are looking for is:

$$\boldsymbol{y = x - 2}$$

(ii) On the grid, draw the straight line from your answer to part (e)(i). [2]

We plot the line on the same graph:



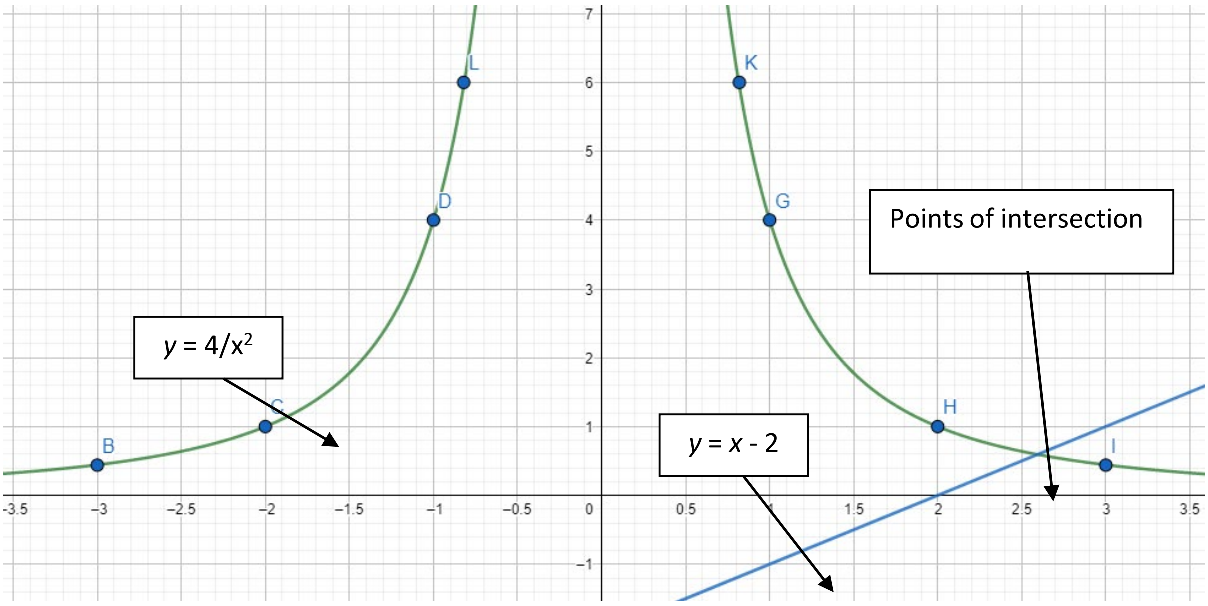
(iii) Use your graphs to solve the equation $\frac{4}{x^2} - x + 2 = 0$. [1]

The solution of the equation is the *x* coordinate of the intersection of the line:

$$y = x - 2$$

and the curve

$$y = \frac{4}{x^2}$$



The *x* coordinate of the intersection point is roughly:

$$\boldsymbol{x = 2.6}$$

Graphs
Difficulty: Hard

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 3

Time allowed: **132 minutes**

Score: **/115**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

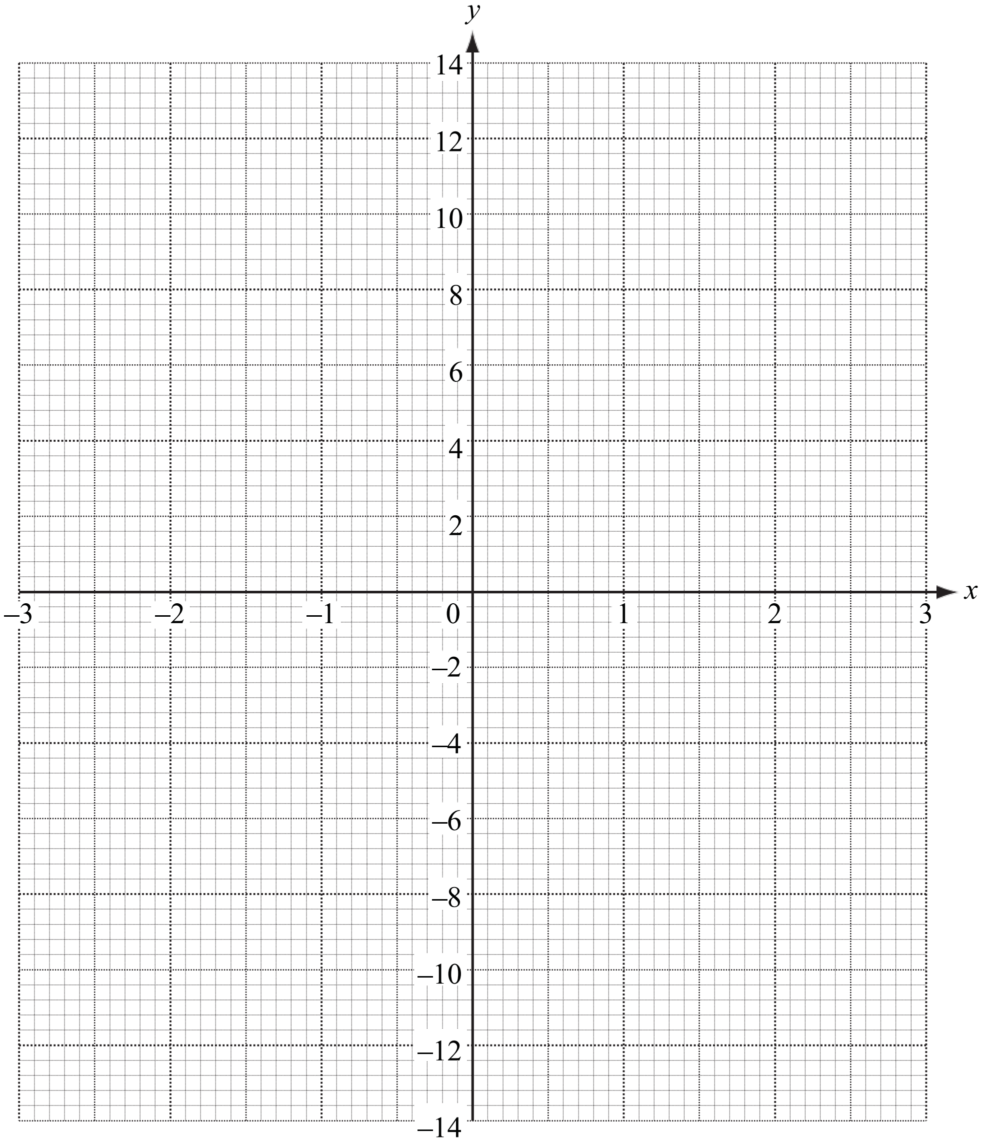
(a) Complete the table of values for the function $y = x^2 - \frac{3}{x}$, $x \neq 0$. [3]

x	- 3	- 2	- 1	- 0.5	- 0.25		0.25	0.5	1	2	3
y	10	5.5		6.3	12.1		- 11.9			2.5	8

We use calculator to find the values of *y* for *x*= -1, *x*= 0.5 and *x*= 1.

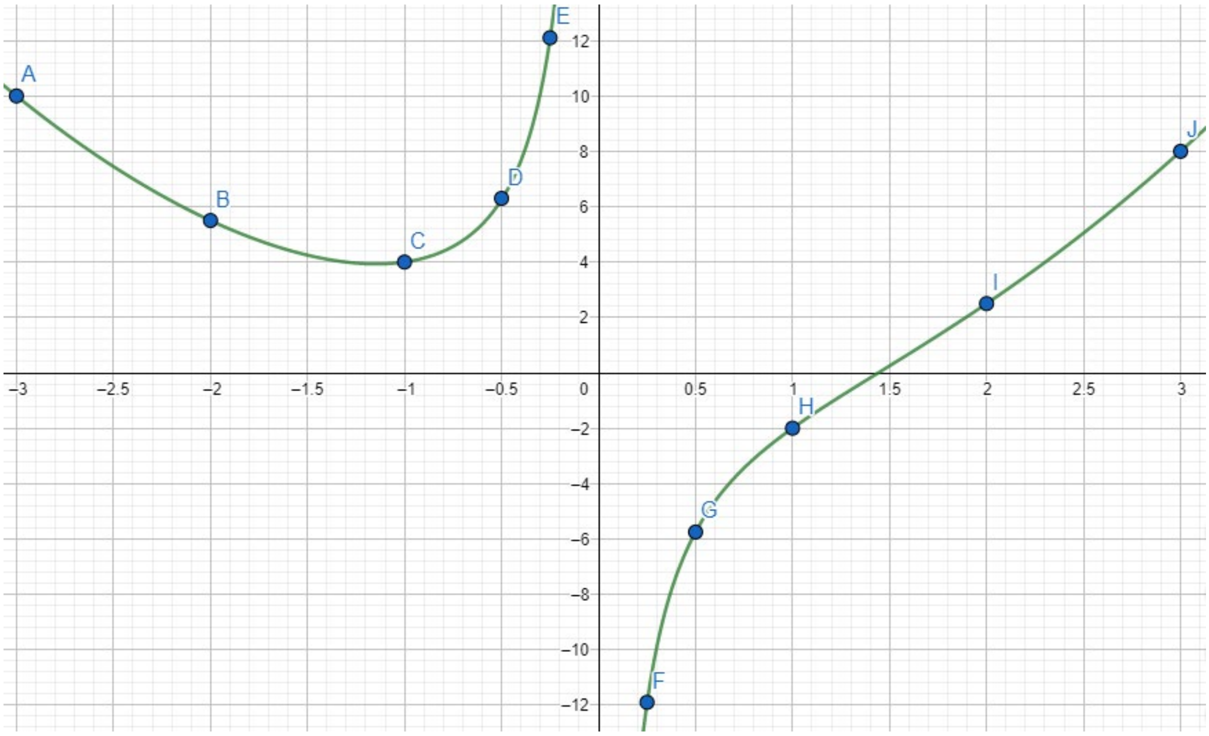
$$\boldsymbol{y(-1) = 4,\qquad y(0.5) = -5.75,\qquad y(1) = -2}$$

(b) Draw the graph of $y = x^2 - \frac{3}{x}$ for $-3 \leqslant x \leqslant -0.25$ and $0.25 \leqslant x \leqslant 3$. [5]



Plot the point from the table on the grid and then draw a smooth function connecting the points.

There is an asymptote at x = 0 because we cannot divide by zero.



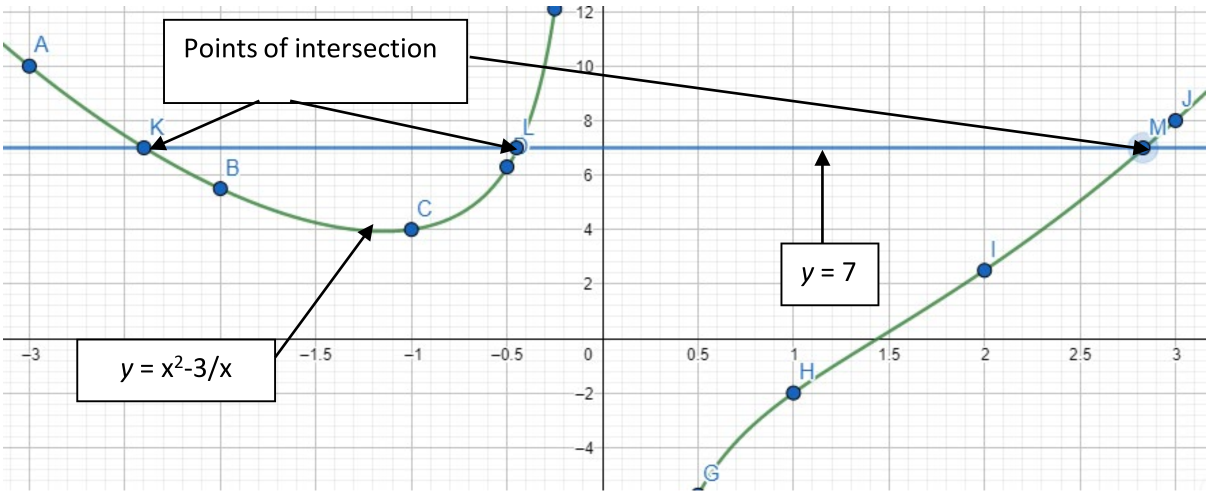
(c) Use your graph to solve $x^2 - \frac{3}{x} = 7$. [3]

The left hand side of the given equation:

$$x^2 - \frac{3}{x} = 7$$

is our original graph and we want to find out when is the expression equal to7.So we are looking for points, which lie on the intersection of graphs:

$$x^2 - \frac{3}{x} \text{and} \quad y = 7$$



From the graph, we see that the points of intersection are approximately

$$(-2.40\,,7\,)\,,(-0.45\,,7)\;\;\text{and}\;\;(2.83\,,7)$$

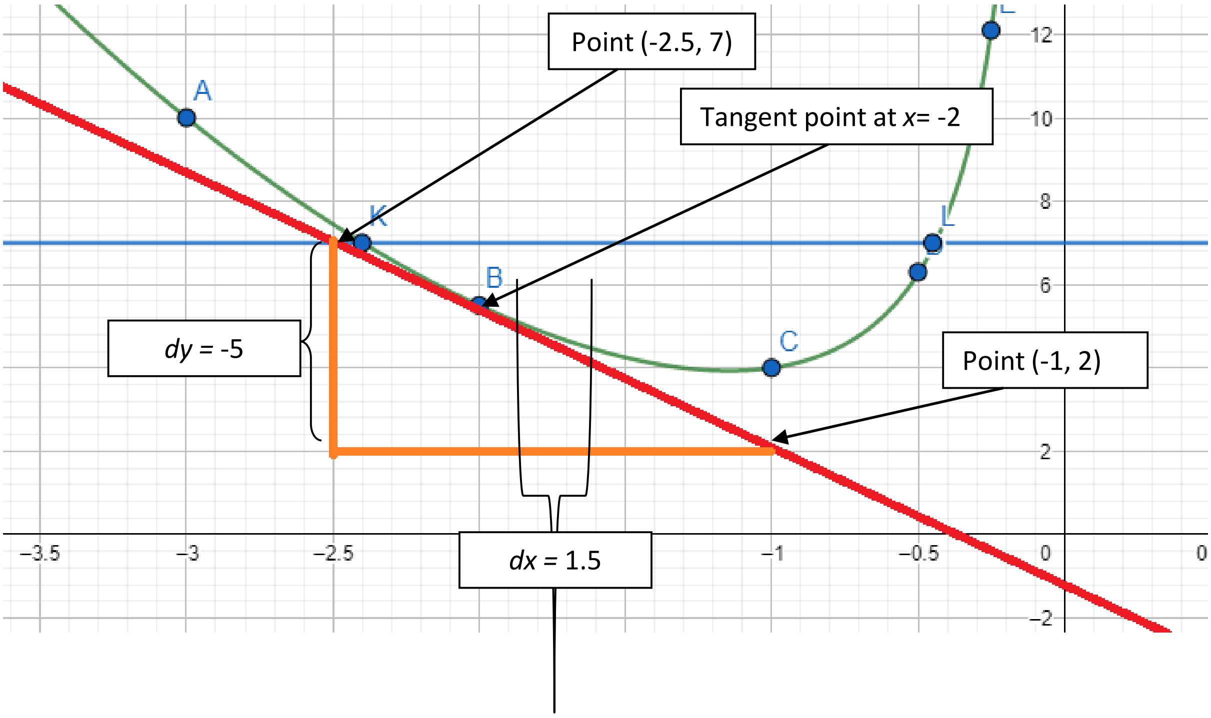
By taking the *x* coordinates of these points, we have the three possible solutions of the equation.

$$\mathbf{-2.40,\qquad -0.45\;\;and\;\;2.83}$$

(d) Draw the tangent to the curve where $x = -2$.
Use the tangent to calculate an estimate of the gradient of the curve where $x = -2$. [3]

We start by drawing a tangent line (red line) to the function at point *x*= -2

(which means that the line touches our function at this point).



The gradient is found as the change of y-coordinate over the change of x-coordinate between two points on the line.

We pick two points on the line:

$$(-2.5\,,7)\ and\ (-1\,,2)$$

Express the gradient as a ratio.

$$gradient\ m = \frac{dy}{dx}$$

$$m = \frac{2-7}{-1-(-2.5)} = \frac{-5}{1.5}$$

Calculate the gradient (correct to 3sf).

$$\boldsymbol{gradient\ m = -3.33}$$

Question 2

(a) Complete the table of values for the function f(x), where f(x) = $x^2 + \frac{1}{x^2}$, $x \neq 0$. [3]

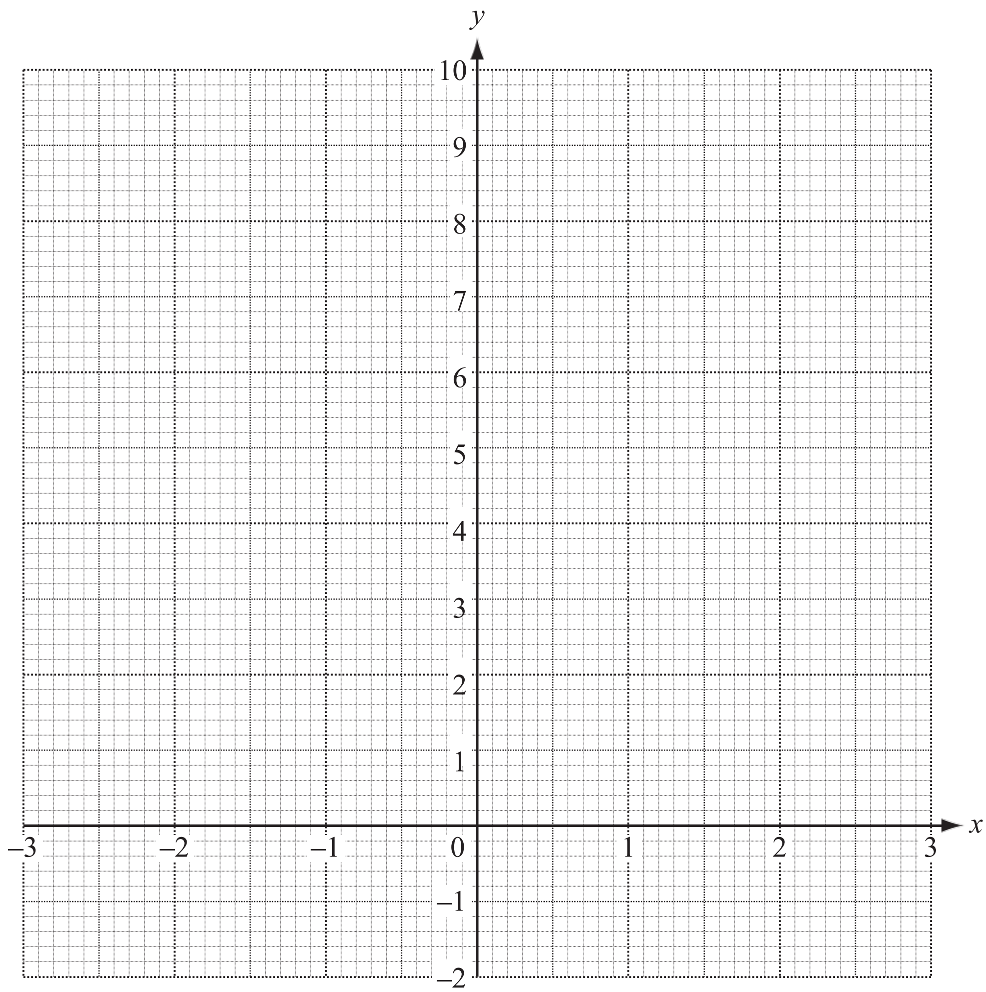
x	-3	-2.5	-2	-1.5	-1	-0.5		0.5	1	1.5	2	2.5	3
f(x)		6.41		2.69		4.25		4.25		2.69		6.41	

We use calculator to find the values of y for x= -3, x= -2, x= -1,x= 1, x = 2 and x = 3.

$$y(-3) = 9.11, \quad y(-2) = 4.25, \quad y(-1) = 2$$

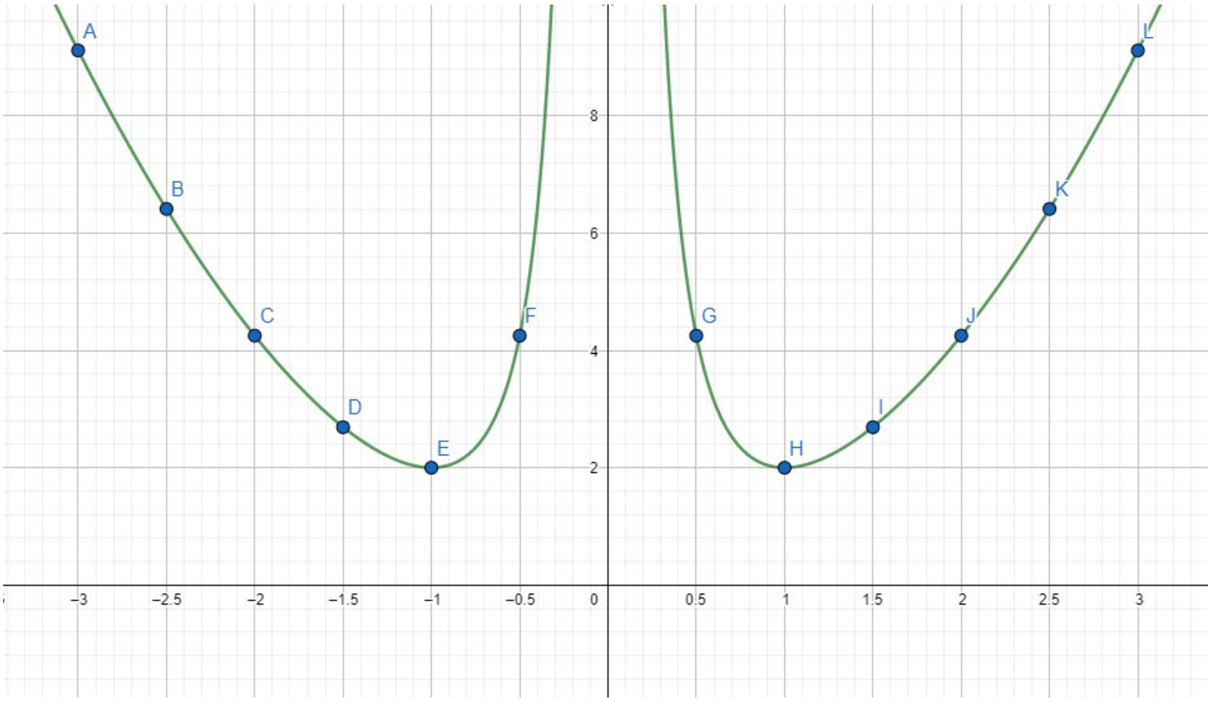
$$y(1) = 2, \quad y(2) = 4.25, \quad y(3) = 9.11$$

(b) On the grid, draw the graph of y = f(x) for $-3 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 3$. [5]



Plot the point from the table on the grid and then draw a smooth function connecting the points.

There is an asymptote at x = 0 because we cannot divide by zero.



(c) (i) Write down the equation of the line of symmetry of the graph. [1]

The function is even:

$$f(x) = f(-x)$$

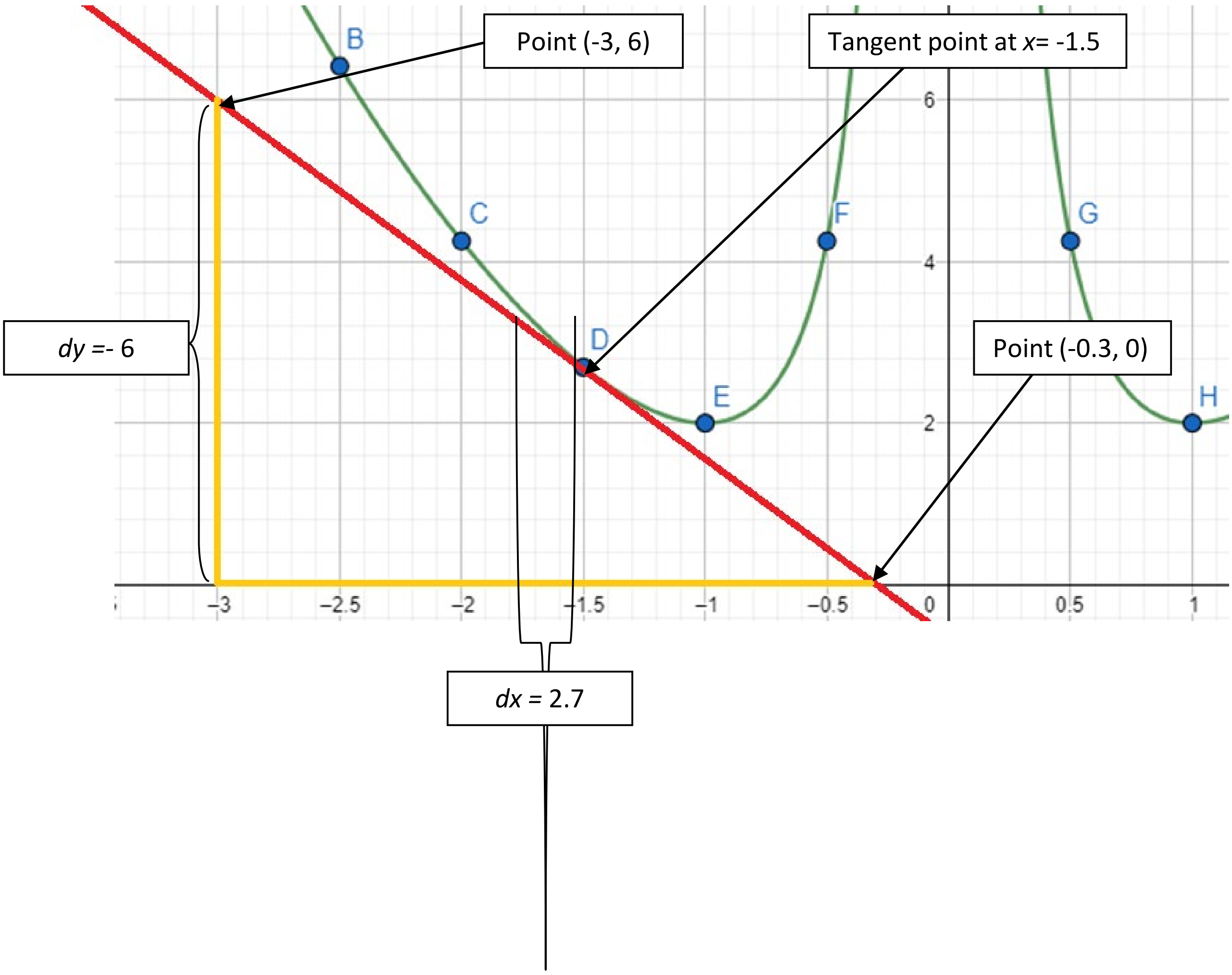
$$x^2 + \frac{1}{x^2} = (-x)^2 + \frac{1}{(-x)^2}$$

The image is mirrored in the y-axis (x=0). This is the line of symmetry:

$$\boldsymbol{x = 0}$$

(ii) Draw the tangent to the graph of $y = f(x)$ where $x = -1.5$.
Use the tangent to estimate the gradient of the graph of $y = f(x)$ where $x = -1.5$. [3]

We start by drawing a tangent line (red line) to the function at point *x*= -1.5 (which means that the line touches our function at this point).



The gradient is found as the change of y-coordinate over the change of x-coordinate between two points on the line.

We pick two points on the line:

$$(-3\,,6)\ and\ (-0.3\,,0)$$

Express the gradient as a ratio.

$$gradient\ m = \frac{dy}{dx}$$

$$m = \frac{0-6}{-0.3-(-3)} = \frac{-6}{2.7}$$

Calculate the gradient (correct to 3sf).

$$\boldsymbol{gradient\ m = -2.22}$$

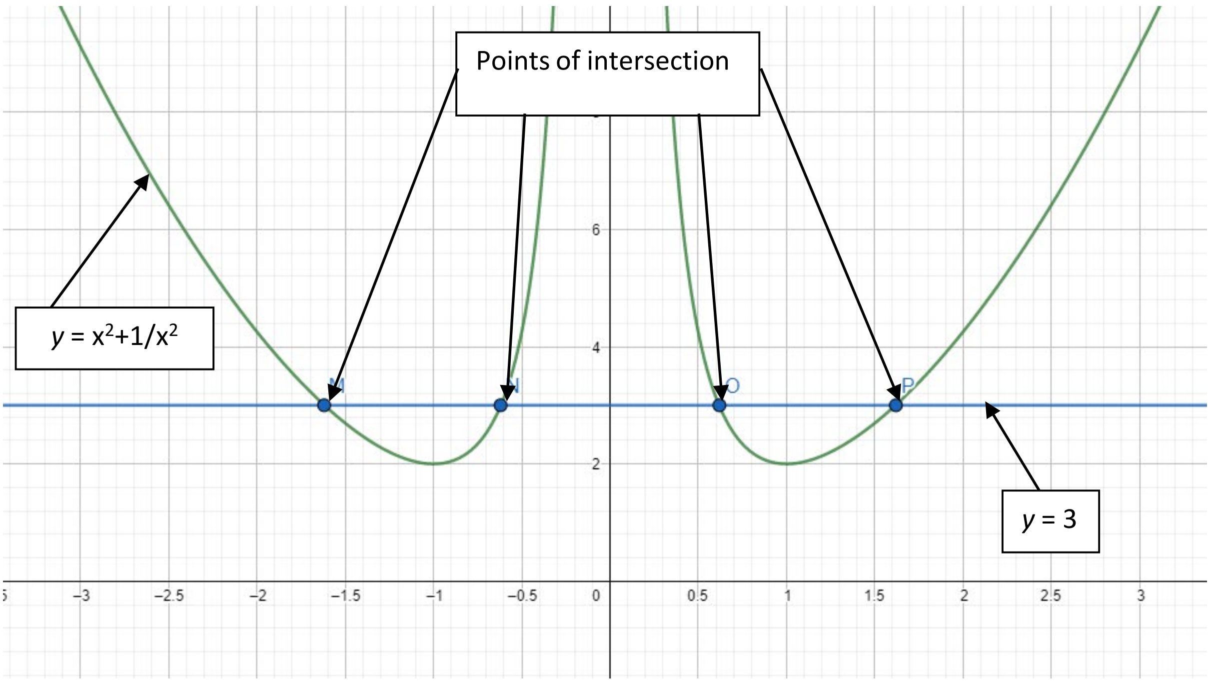
(iii) Use your graph to solve the equation $x^2 + \frac{1}{x^2} = 3$. [2]

The left hand side of the given equation:

$$x^2 + \frac{1}{x^2} = 3$$

is our original graph and we want to find out when is the expression equal to3.So we are looking for points, which lie on the intersection of graphs:

$$y = x^2 + \frac{1}{x^2} \text{and} \quad y = 3$$



From the graph, we see that the points of intersection are approximately

$$(-1.62\,,3\,),(-0.62\,,3\,)\,,(0.62\,,3)\;\text{ and }\;(-1.62\,,3)$$

By taking the *x* coordinates of these points, we have the four possible solutions of the equation.

$$\mathbf{-1.62,\quad -0.62,\quad 0.62\;\; and\;\; 1.62}$$

(iv) Draw a suitable line on the grid and use your graphs to solve the equation $x^2 + \frac{1}{x^2} = 2x$. [3]

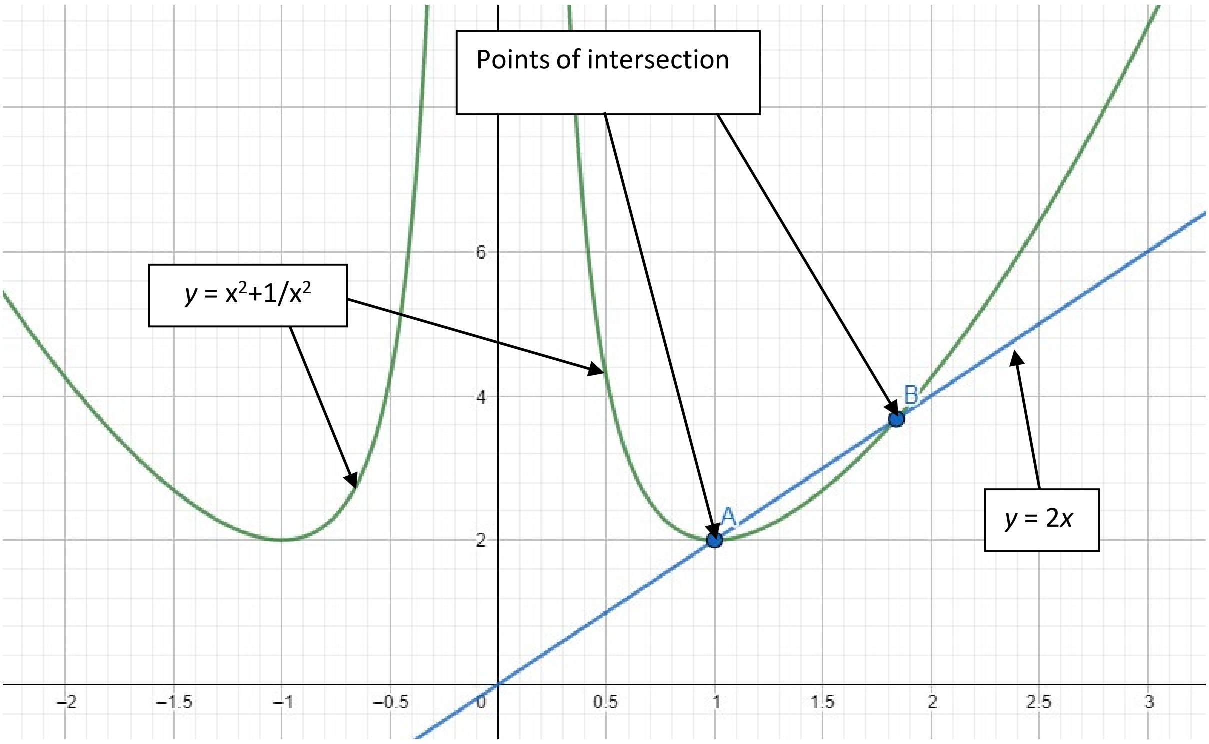
This equation is very similar to the one from part iii), however now we want for our function

$$y = x^2 + \frac{1}{x^2}$$

to be equal to:

$$y = 2x$$

Hence we plot this new function and look for points where the two functions equal (intersection points of the graphs).



From the graph, we see that the points of intersection are approximately

$$(1\ ,2\ )\text{and}(1.85\ ,3.60\ )$$

By taking the *x* coordinates of these points, we have the two possible solutions of the equation.

$$\mathbf{1 \quad and \quad 1.85}$$

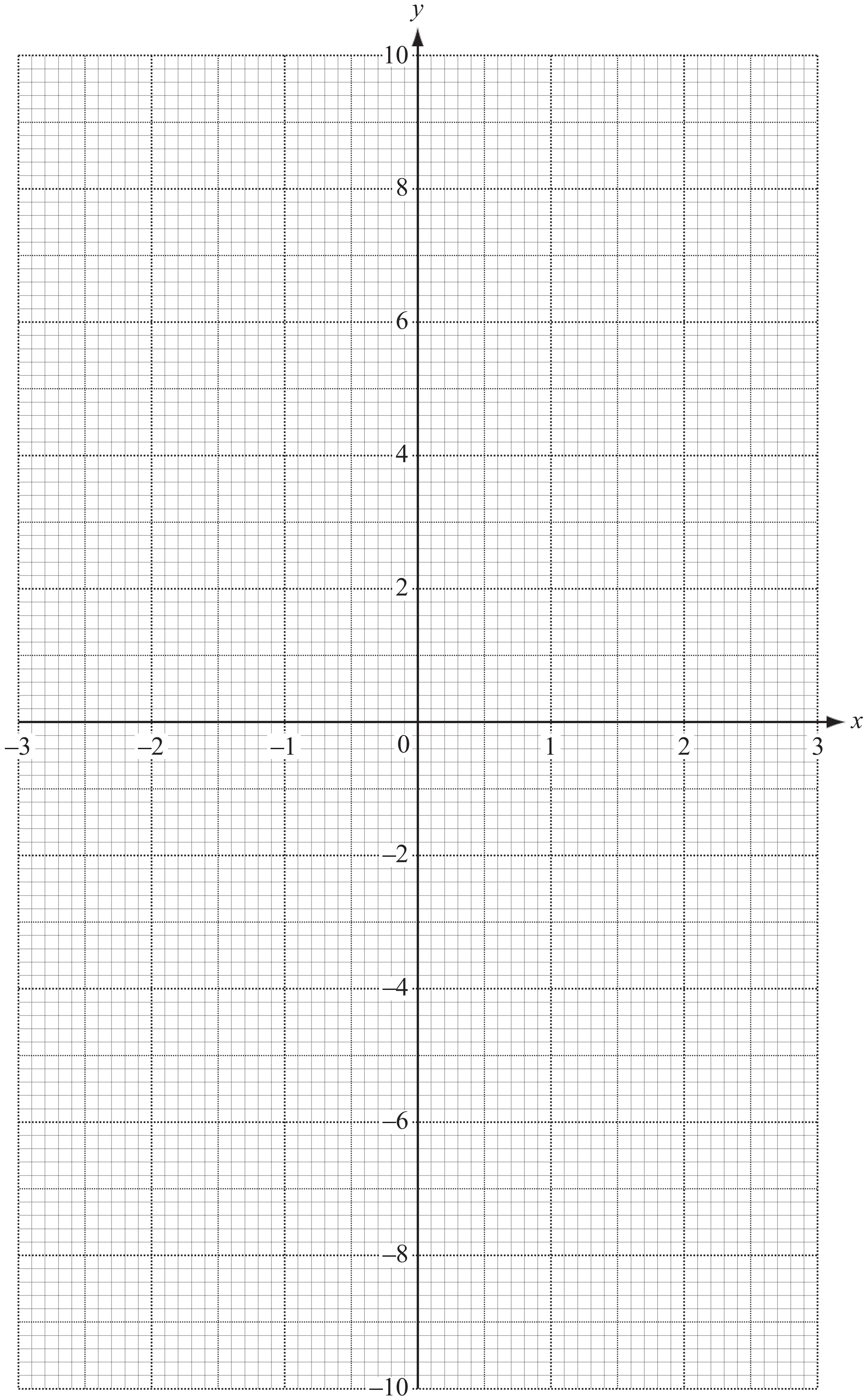
(a) Complete the table for the function $f(x) = \frac{2}{x} - x^2$.

x	–3	–2	–1	–0.5	–0.2	0.2	0.5	1	2	3
f(x)	–9.7	–5			–10.0	10.0	3.75	1		–8.3

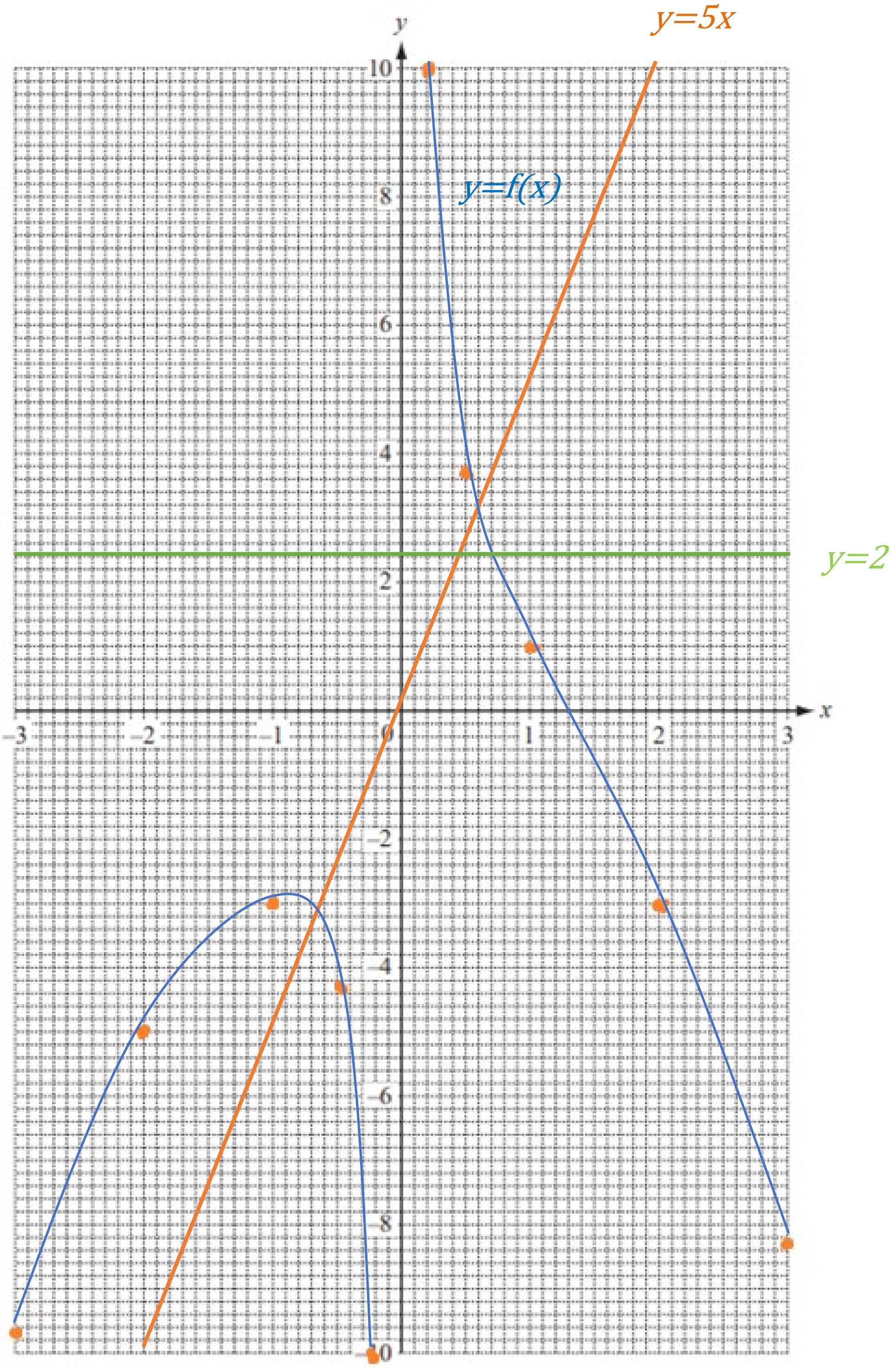
[3]

x	-3	-2	-1	-0.5	-0.2	0.2	0.5	1	2	3
$f(x)$	-9.7	-5	**-3**	**-4.25**	-10.0	10.0	3.75	1	**-3**	-8.3

(b) On the grid draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$.



[5]


y=5x
y
10
8
y=f(x)
6
4
2
y=2
-3
-2
-1
0
1
2
3
x
-2
-4
-6
-8
-10

(c) Use your graph to

(i) solve $f(x) = 2$, [1]

Where the green line ($y = 2$) intersects with the curve. We read it off as

$$\boldsymbol{x = 0.75}$$

(ii) find a value for k so that $f(x) = k$ has 3 solutions. [1]

We see that if we were to draw a line at $y = -4$ it would

intersect the curve at 3 points, so

$$\boldsymbol{k = 4}$$

(d) Draw a suitable line on the grid and use your graphs to solve the equation $\frac{2}{x} - x^2 = 5x$. [3]

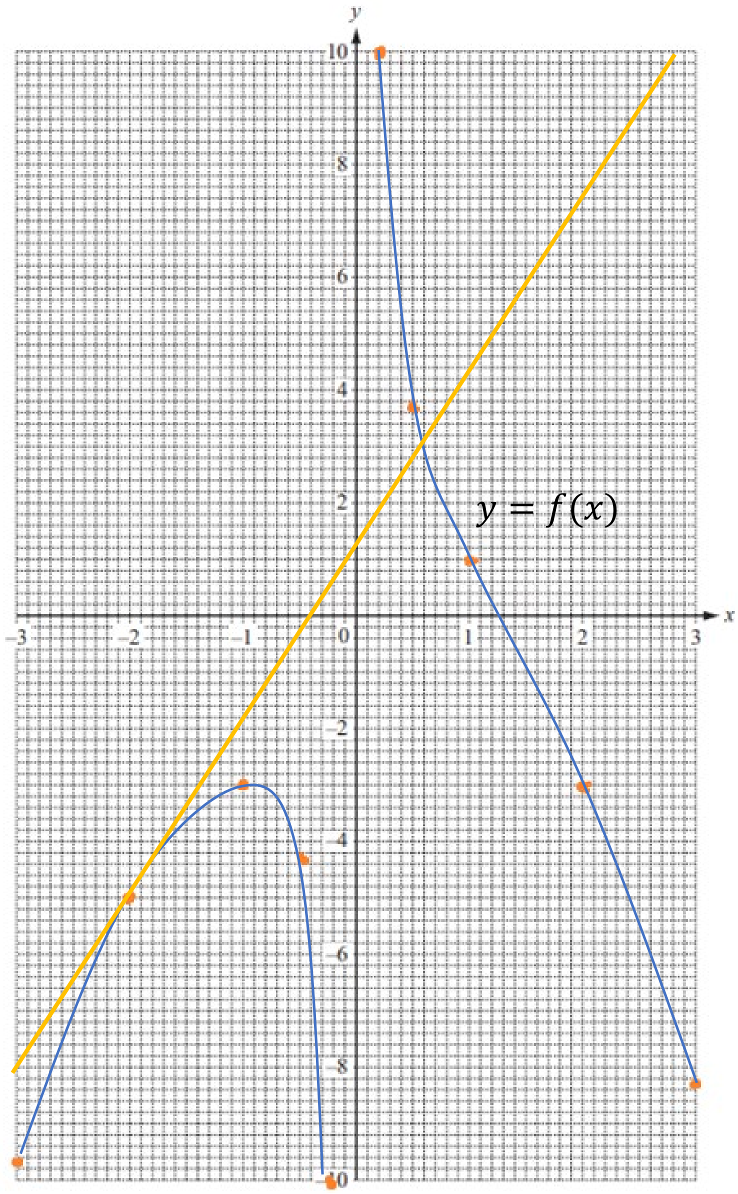
The line $y = 5x$ drawn on the graph in orange.

It intersects with our curve at

$$\boldsymbol{x = 0.6, x = -0.65}$$

(e) Draw the tangent to the graph of $y = f(x)$ at the point where $x = -2$.

Use it to calculate an estimate of the gradient of $y = f(x)$ when $x = -2$. [3]



Find the gradient as

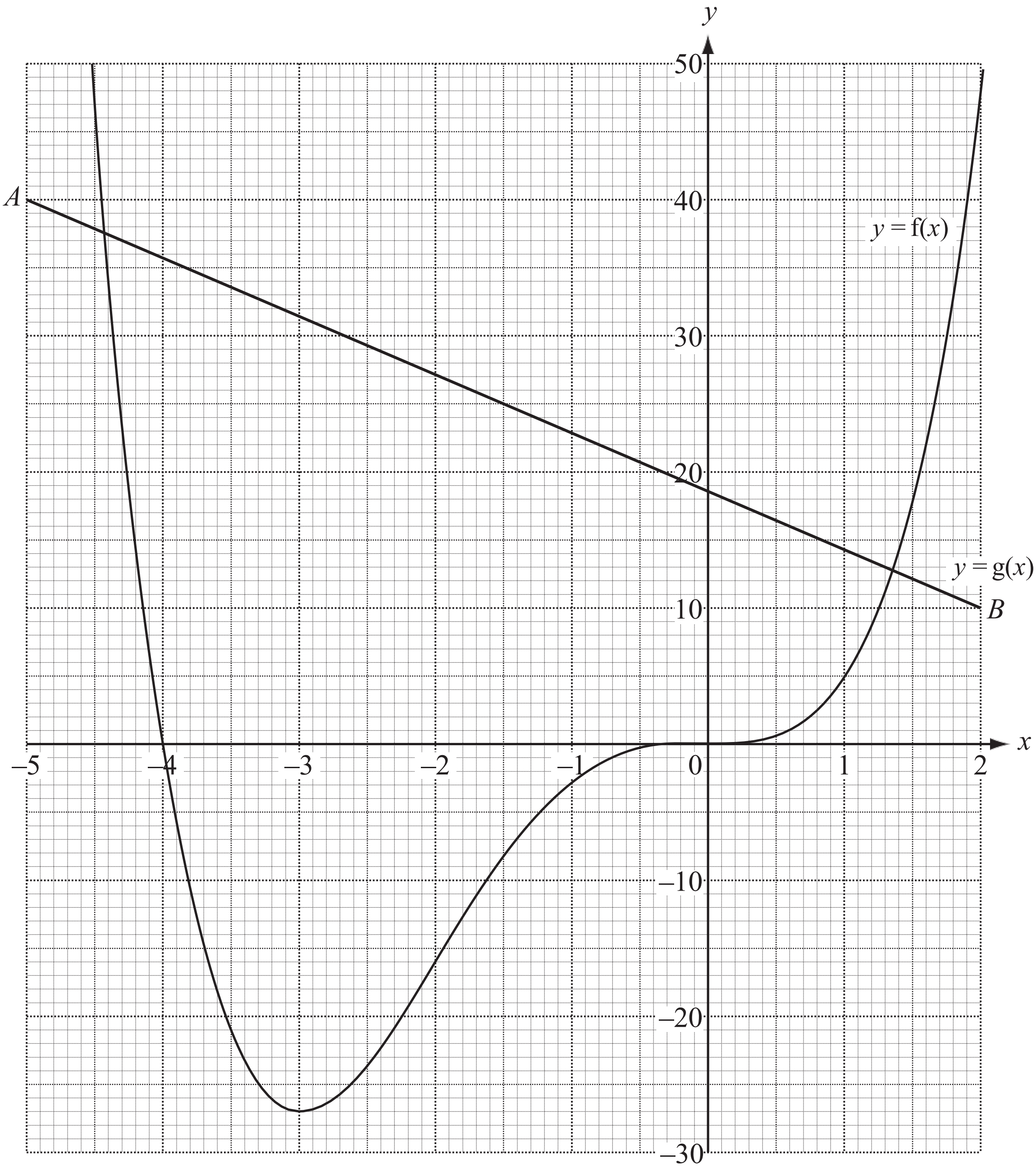
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Pick the points (0, 1) and (-2, -5)

$$m = \frac{1 - -5}{0 - -2}$$

$$= \frac{6}{2}$$

$$= \mathbf{3}$$



The graphs of $y = f(x)$ and $y = g(x)$ are shown above.

(a) Find the value of

(i) $f(-2)$, [1]

f(-2) = -16

(ii) $g(0)$. [1]

g(0) = 18

(b) Use the graphs to solve

(i) the equation $f(x) = 20$, [2]

x = 1.6 or **-4.4**

(ii) the equation $f(x) = g(x)$, [2]

Find where lines cross on graph

x = 1.3 or **x = -4.5**

(iii) the inequality $f(x) < g(x)$. [1]

-4.5 < x < 1.3

(c) Use the points A and B to find the gradient of $y = g(x)$ as an exact fraction. [2]

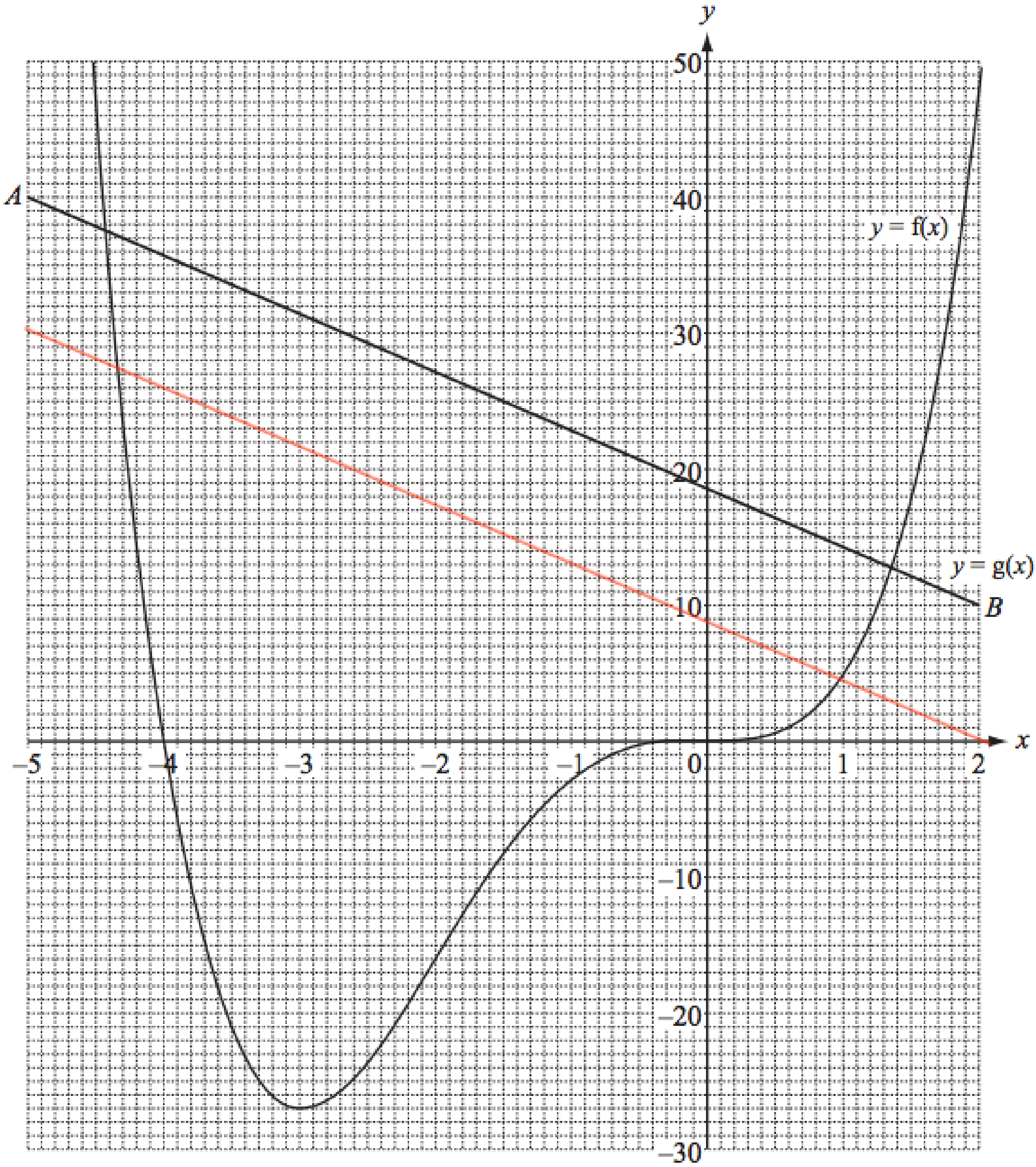
Gradient = change in y/change in x

change in y = 10 – 40 = -30

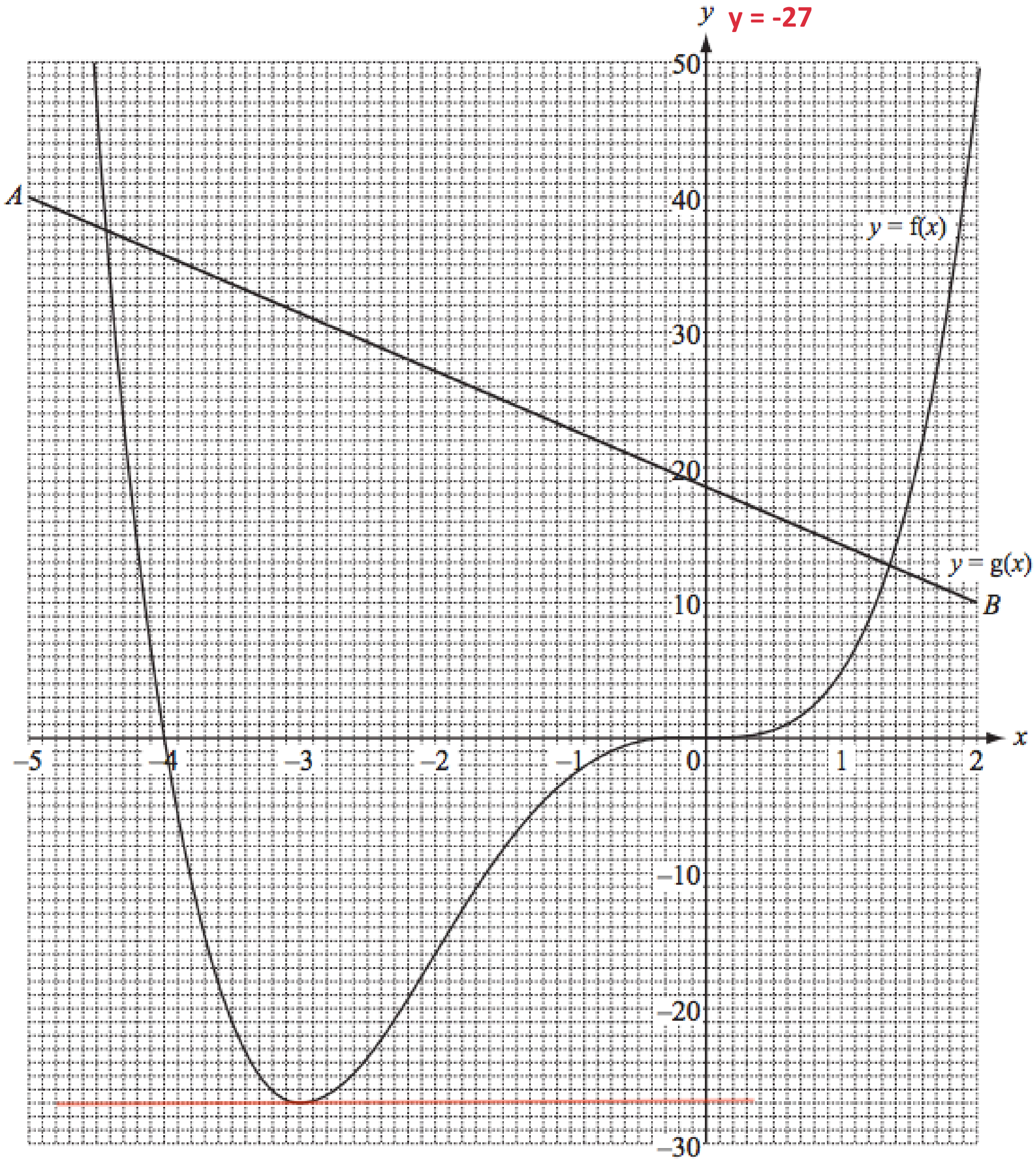
change in x = 2 – – 5 = 7

-30/7

(d) On the grid, draw the graph of $y = g(x) - 10$. [2]



(e) (i) Draw the tangent to the graph of $y = f(x)$ at $(-3, -27)$. [1]



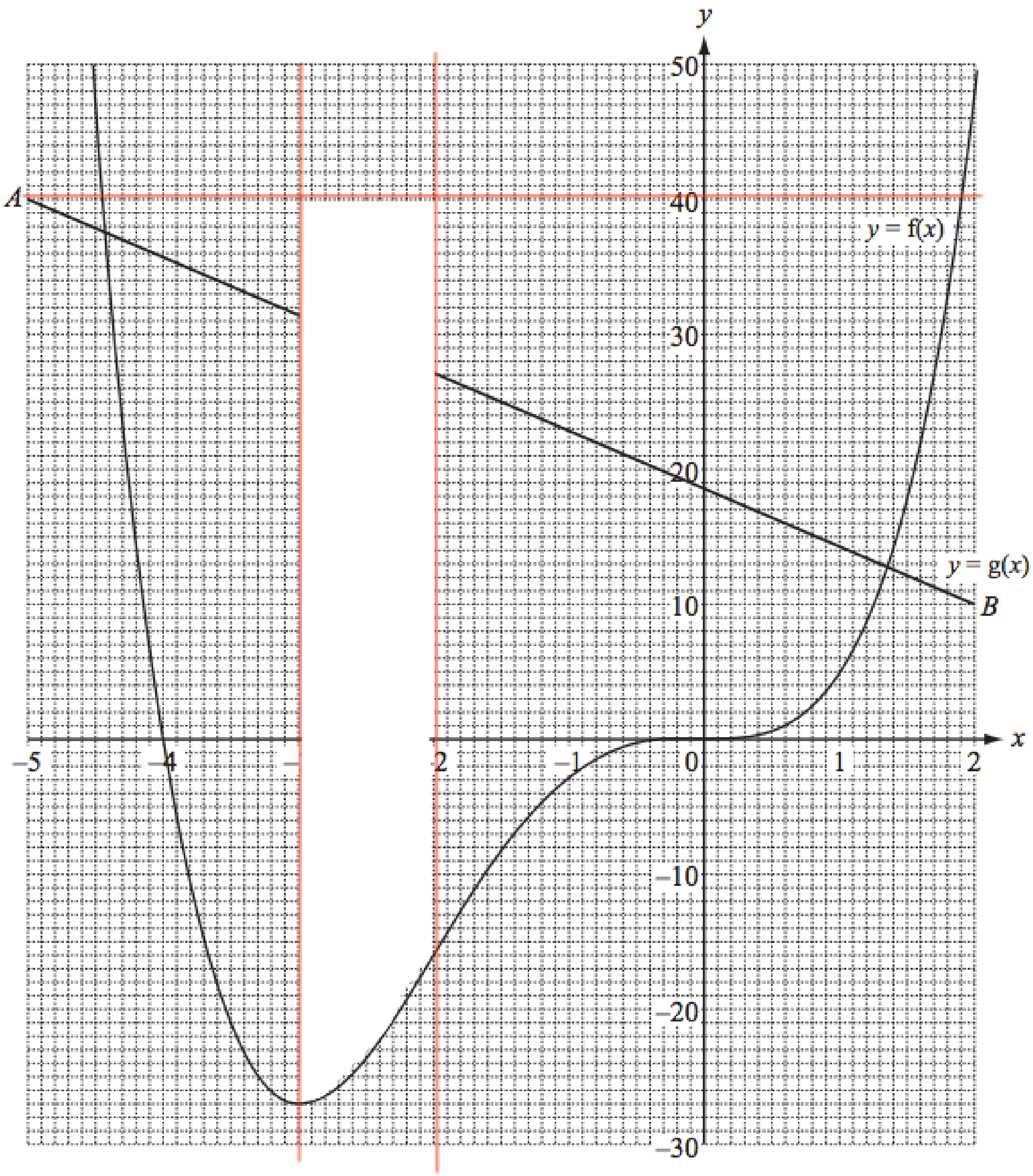
(ii) Write down the equation of this tangent. [1]

$y = -27$

(f) A region, R, contains points whose co-ordinates satisfy the inequalities

$-3 \leqslant x \leqslant -2, \qquad y \leqslant 40 \qquad$ and $\qquad y \geqslant \mathrm{g}(x).$

On the grid, draw suitable lines and label this region R. [2]



Question 5

(a) The table shows some values for the equation $y = \frac{x}{2} - \frac{2}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

x	–4	–3	–2	–1.5	–1	–0.5	0.5	1	1.5	2	3	4
y	–1.5	–0.83	0	0.58			–3.75		–0.58	0	0.83	1.5

(i) Write the missing values of y in the empty spaces. [3]

$y = \frac{x}{2} - \frac{2}{x}$

For x = -1:

$y = \frac{-1}{2} - \frac{2}{-1}$

$y = -0.5 - (-2)$

= 1.5

For x = -0.5:

$y = \frac{-0.5}{2} - \frac{2}{-0.5}$

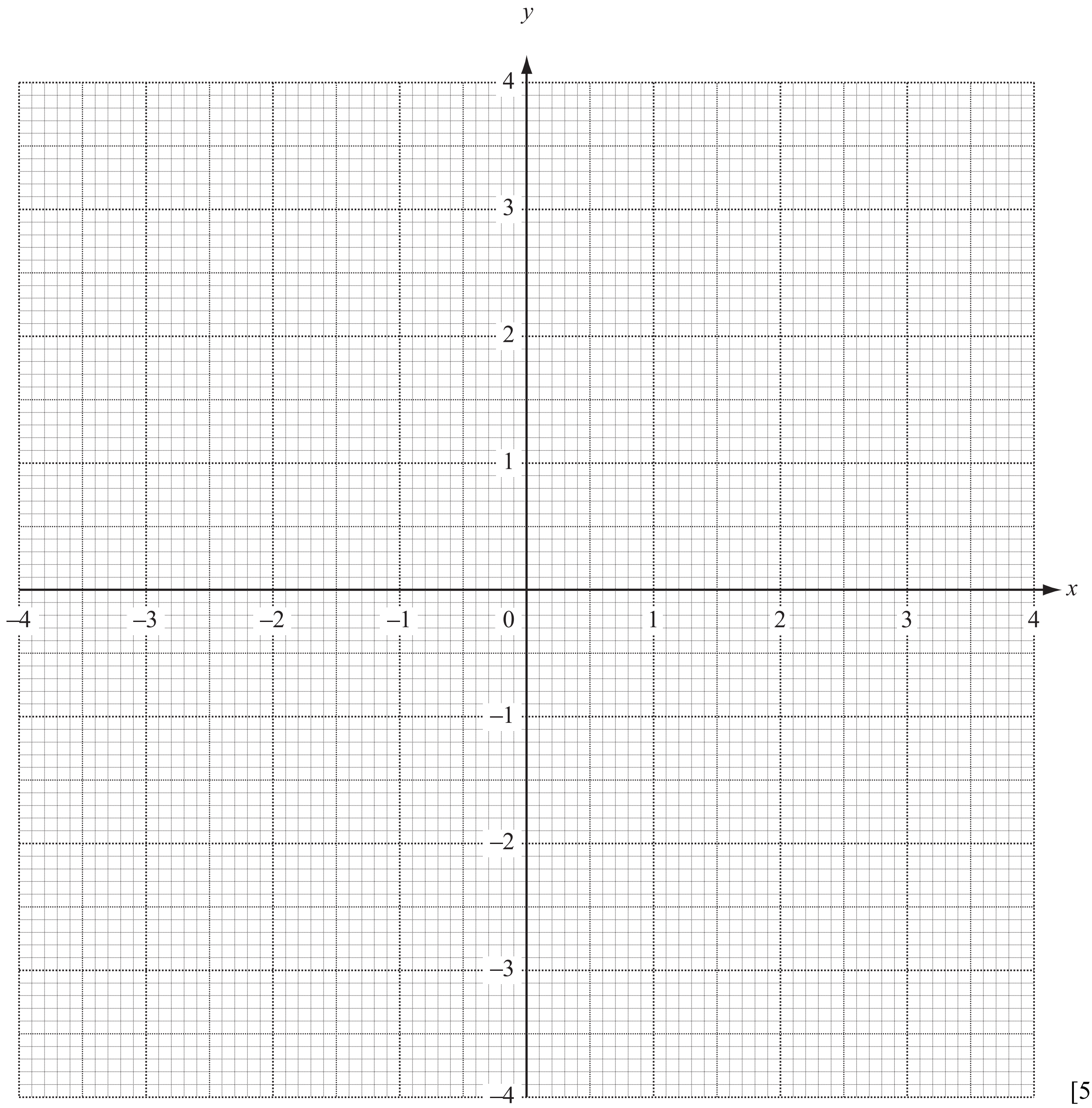
y = 3.75

For x = 1:

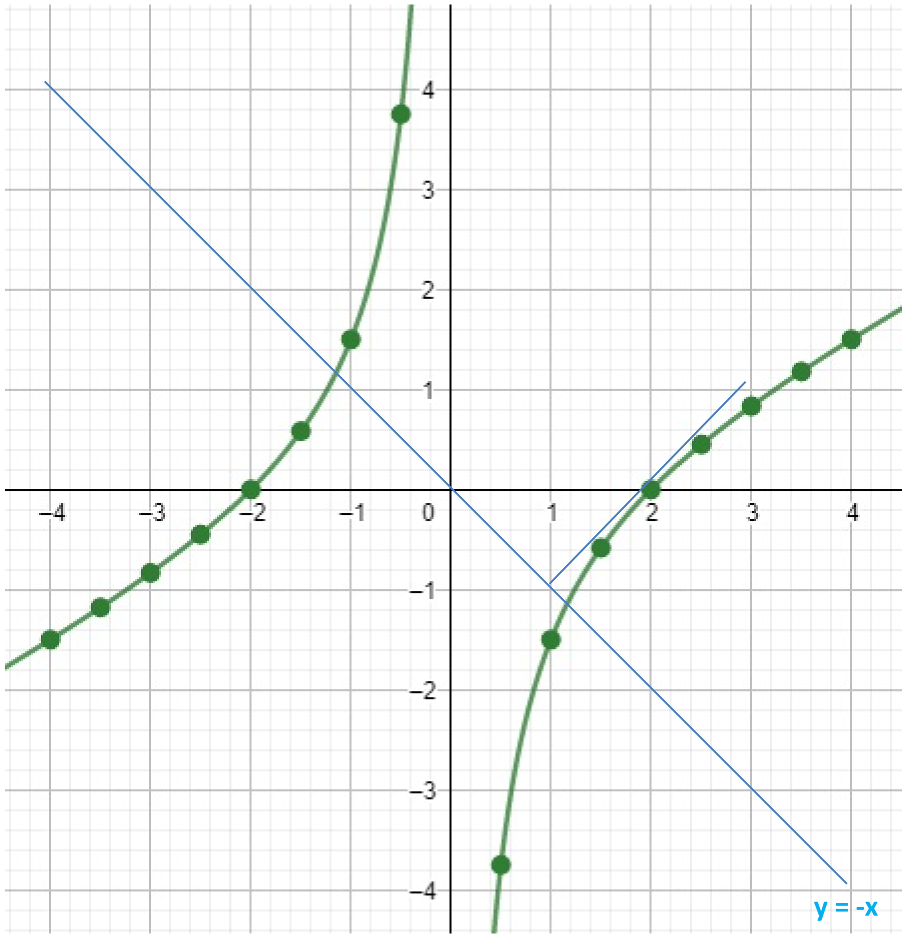
$y = \frac{1}{2} - \frac{2}{1}$

y = -1.5

(ii) On the grid, draw the graph of $y = \frac{x}{2} - \frac{2}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.



[5]



(b) Use your graph to solve the equation $\frac{x}{2} - \frac{2}{x} = 1$. [2]

Using the graph above, the x values corresponding to y = 1 are:

X = 3.2 and x = -1.2

(c) (i) By drawing a tangent, work out the gradient of the graph where $x = 2$. [3]

gradient = $\frac{\text{change in y}}{\text{change in x}}$

gradient = $\frac{2}{2}$

gradient = 1

(ii) Write down the gradient of the graph where $x = -2$. [1]

The 2 graphs are inverted mirror images of each other, therefore, the gradient

of the tangent in x = 2 will be equal to the gradient of the tangent in x = -2.

(d) (i) On the grid, draw the line $y = -x$ for $-4 \leqslant x \leqslant 4$. [1]

(ii) Use your graphs to solve the equation $\frac{x}{2} - \frac{2}{x} = -x$. [2]

The solutions of the equation y = -x are the x coordinates of the intersections

of the line of equation y = -x and the graph $y = \frac{x}{2} - \frac{2}{x}$.

Using the graph, the solutions are:

x = 1.2 and x = -1.2

(e) Write down the equation of a straight line which passes through the origin and does not intersect the graph of $y = \frac{x}{2} - \frac{2}{x}$. [2]

The equation passes through the origin, the point of coordinates (0, 0).

The equation of a line takes up the form:

y = mx + n

where m is the gradient and n is the y-intercept

In our case, we can substitute the values y = x = 0 in the equation above.

0 = 0 x m + n

n = 0

The line does not intersect the line of equation $y = \frac{x}{2} - \frac{2}{x}$.

For the line of equation $y = \frac{x}{2} - \frac{2}{x}$:

$y = \frac{x}{2} - \frac{2}{x}$.

$y = mx + n$

The gradient in this case is: $m = ½$

The gradient of the line needs to be greater than 1/2.

The equation of the line is:

$y = mx$

with $m \geq ½$

Answer the whole of this question on a sheet of graph paper.

The table shows some of the values of the function $f(x) = x^2 - \frac{1}{x}$, $x \neq 0$.

x	−3	−2	−1	−0.5	−0.2	0.2	0.5	1	2	3
y	9.3	4.5	2.0	2.3	p	−5.0	−1.8	q	3.5	r

(a) Find the values of p, q and r, correct to 1 decimal place. [3]

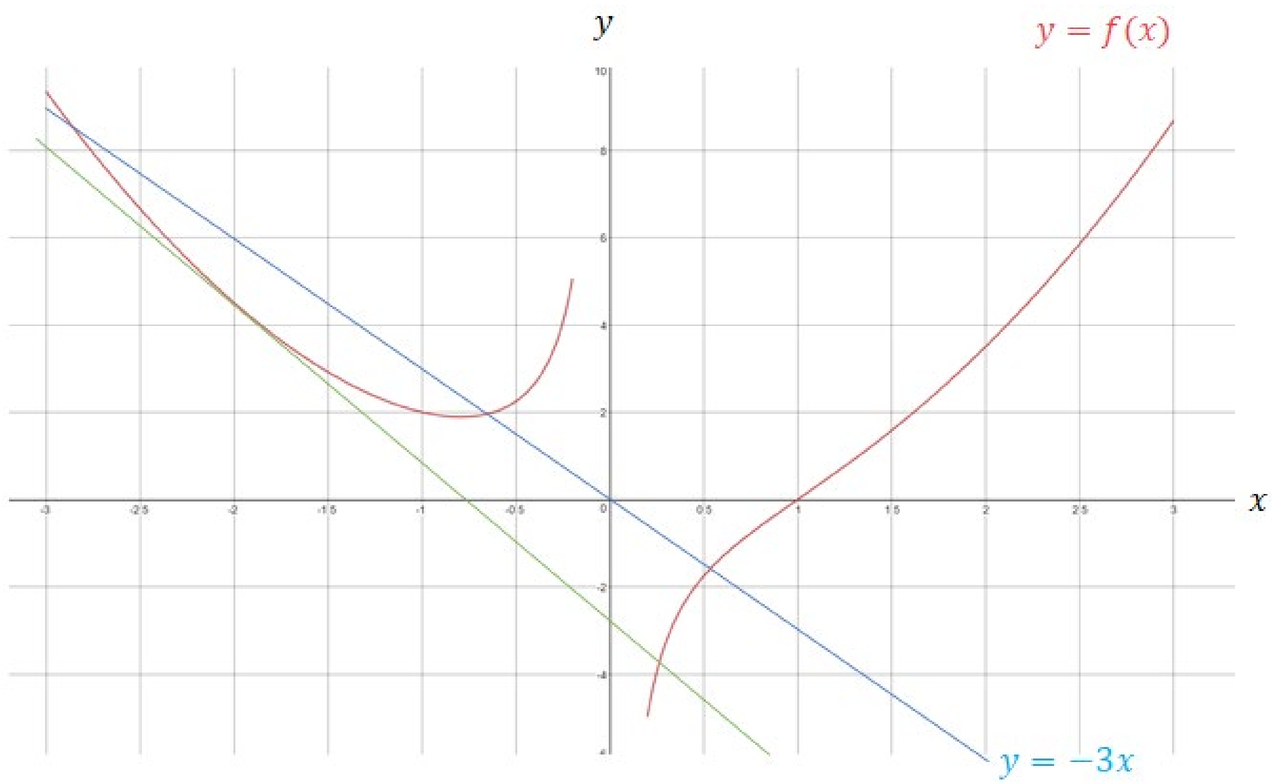
$p = 5.0$

$q = 0.0$

$r = 8.7$

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw an x-axis for $-3 \leqslant x \leqslant 3$ and a y-axis for $-6 \leqslant y \leqslant 10$.

Draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$. [6]



(c) (i) By drawing a suitable straight line, find the three values of x where f(x) = −3x. [3]

Line of $y = -3x$ drawn in blue above.

It intersects with the curve at

$$\boldsymbol{x = -2.75, \qquad x = -0.7, \qquad x = 0.55}$$

(ii) $x^2 - \frac{1}{x} = -3x$ can be written as $x^3 + ax^2 + b = 0$.

Find the values of a and b. [2]

Multiply through by x

$$x^3 - 1 = -3x^2$$

$$\rightarrow x^3 + 3x^2 - 1 = 0$$

$$\boldsymbol{a = 3}$$

$$\boldsymbol{b = -1}$$

(d) Draw a tangent to the graph of y = f(x) at the point where x = –2.

Use it to estimate the gradient of y = f(x) when x = –2. [3]

Tangent drawn in green above.

Now use

$$\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$\rightarrow m = \frac{-6 - 8}{0.8 - -3}$$

$$= -\frac{14}{3.8}$$

$$\boldsymbol{= -3.7}$$

Answer the whole of this question on a sheet of graph paper.
Use one side for your working and one side for your graphs.

Alaric invests \$100 at 4% per year compound interest.

(a) How many dollars will Alaric have after 2 years? [2]

100×1.04^2

$= \mathbf{108.16}$

(b) After x years, Alaric will have y dollars.
He knows a formula to calculate y.
The formula is $y = 100 \times 1.04^x$

x (Years)	0	10	20	30	40
y (Dollars)	100	p	219	q	480

Use this formula to calculate the values of p and q in the table. [2]

$p = 100 \times 1.04^{10}$

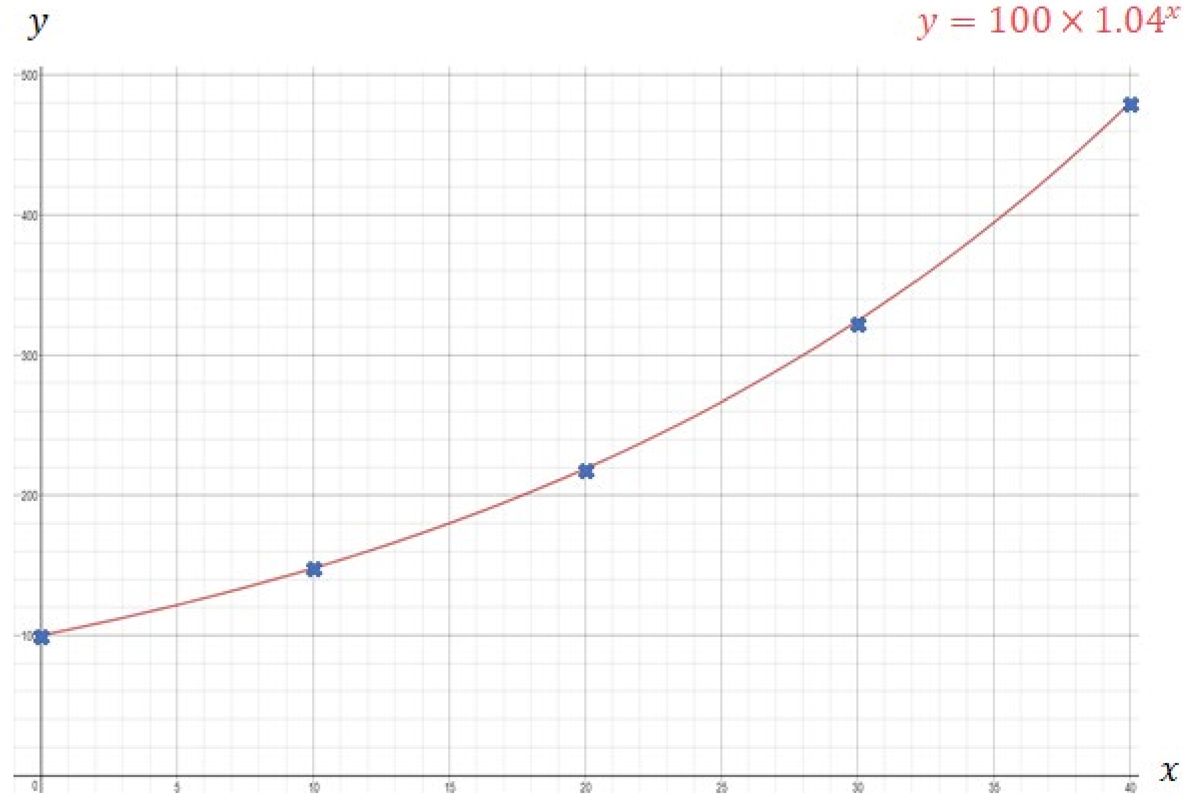
$= \mathbf{148}$

$q = 100 \times 1.04^{30}$

$= \mathbf{324}$

(c) Using a scale of 2 cm to represent 5 years on the x-axis and 2 cm to represent \$50 on the y-axis, draw an x-axis for $0 \leqslant x \leqslant 40$ and a y-axis for $0 \leqslant y \leqslant 500$.

Plot the five points in the table and draw a smooth curve through them. [5]



(d) Use your graph to estimate

(i) how many dollars Alaric will have after 25 years, [1]

Read off the graph at $x = 25$

$$y = 265$$

(ii) how many years, to the nearest year, it takes for Alaric to have \$200. [1]

Read off the graph at $y = 200$

$$x = 18$$

(e) Beatrice invests \$100 at 7% per year **simple interest**.

(i) Show that after 20 years Beatrice has \$240. [2]

$$100 + 100 \times 0.07 \times 20$$

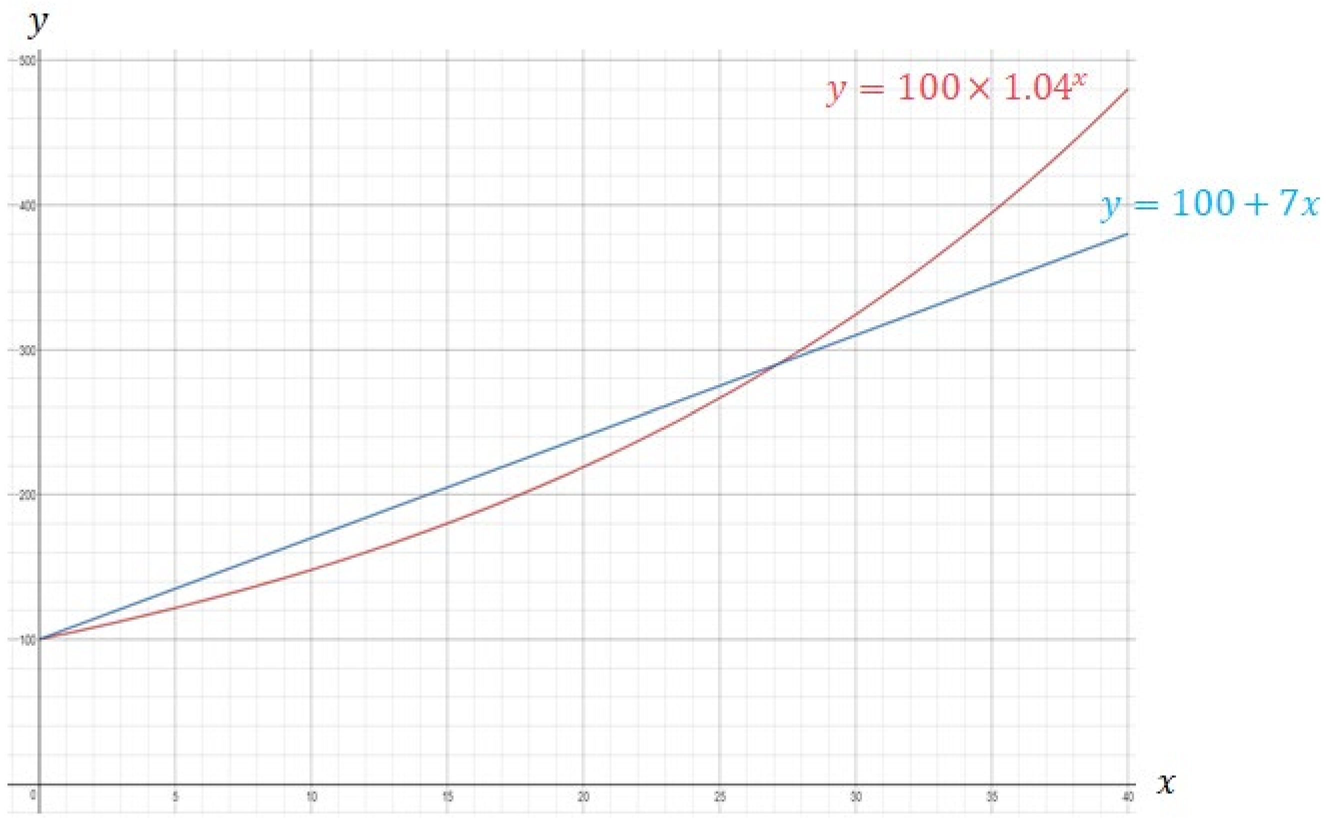
$$= \mathbf{240}$$

(ii) How many dollars will Beatrice have after 40 years? [1]

$$100 + 100 \times 0.07 \times 40$$

$$= 380$$

(iii) On the **same grid**, draw a graph to show how the \$100 which Beatrice invests will increase during the 40 years. [2]



(f) Alaric first has more than Beatrice after n years.
Use your graphs to find the value of n. [1]

Alaric (red curve) has more than Beatrice (blue line) at

$$\boldsymbol{x = 27}$$

Graphs
Difficulty: Hard

Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 4

Time allowed: **101 minutes**

Score: **/88**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Answer the whole of this question on one sheet of graph paper.

$$f(x) = 1 - \frac{1}{x^2}, \; x \neq 0.$$

(a)

x	-3	-2	-1	-0.5	-0.4	-0.3		0.3	0.4	0.5	1	2	3
f(x)	p	0.75	0	-3	-5.25	q		q	-5.25	-3	0	0.75	p

Find the values of p and q. [2]

f(x) = 1 - $\frac{1}{x^2}$

For x = -3:

f(x) = 1 - $\frac{1}{(-3)^2}$

f(x) = $\frac{1}{9}$

p = 0.88

For x = -0.3:

f(x) = 1 - $\frac{1}{x^2}$

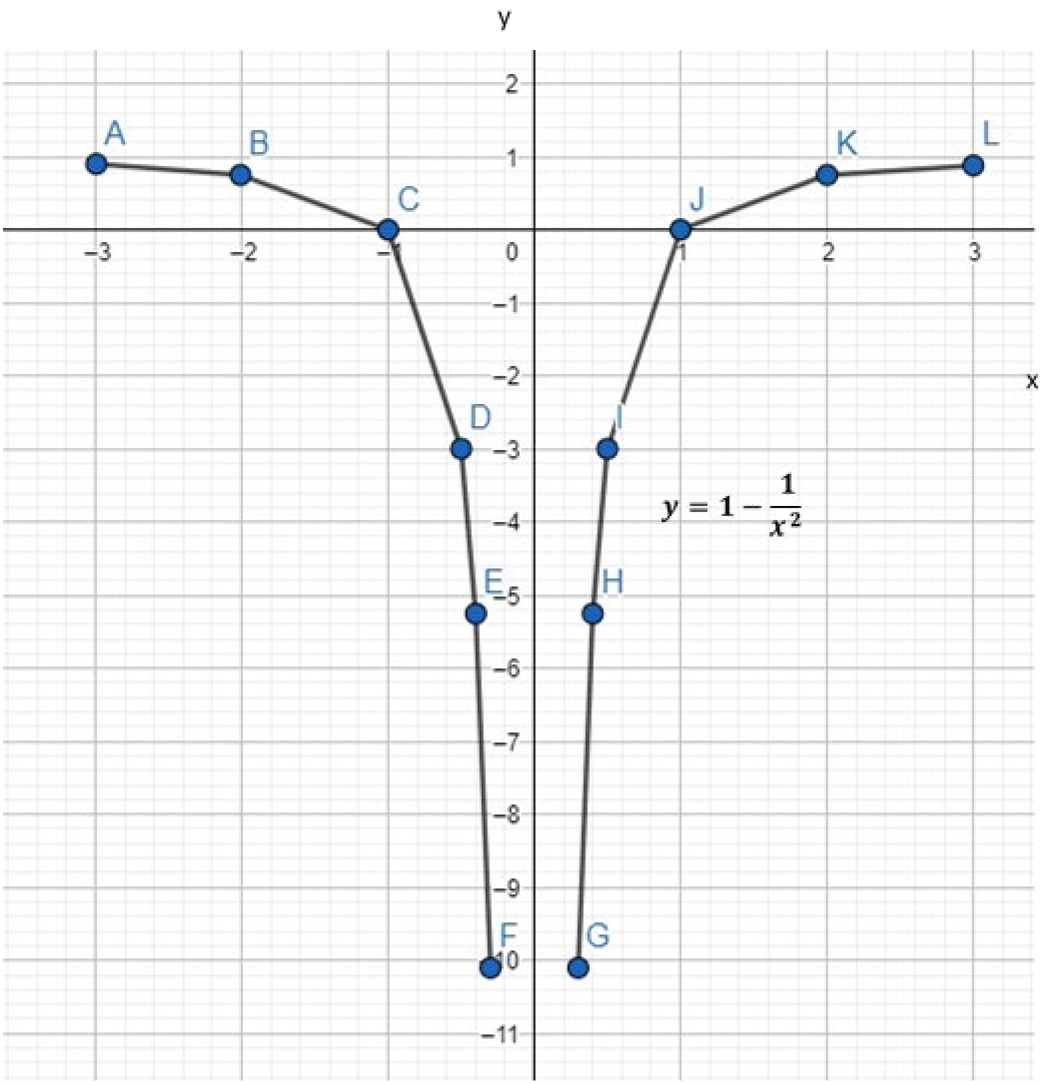
f(x) = $\frac{1}{(-0.3)^2}$

q = - 10.1

(b) (i) Draw an x-axis for $-3 \leqslant x \leqslant 3$ using 2 cm to represent 1 unit and a y-axis for $-11 \leqslant y \leqslant 2$ using 1 cm to represent 1 unit. [1]

(ii) Draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.3$ and for $0.3 \leqslant x \leqslant 3$. [5]

First we draw the correct scale and labelling axes. Then we need to plot all the points represented in the table



(c) Write down an integer k such that $f(x) = k$ has no solutions. [1]

f(x) = k

$1 - \frac{1}{x^2} = k$

$\frac{1}{x^2} = 1 - k$

For any $k \neq 1$, $1 - k \neq 0$.

For $\frac{1}{x^2} = 0$, x cannot have any values since the denominator of a fraction cannot be 0.

For $\frac{1}{x^2} < 0$, x cannot have any values since the fraction needs to have a negative denominator to take a negative value and x^2 cannot take negative values for any values of x.

So any integer (k) where $k \geq 1$ is correct.

(d) **On the same grid**, draw the graph of $y = 2x - 5$ for $-3 \leqslant x \leqslant 3$. [2]

y = 2x – 5

We need to work out the value of y for every x value in the interval $-3 \leq x \leq 3$.

For x = -3:

y = 2 x (-3) – 5

y = -11

For $x = -2$:

$y = 2 \times (-2) - 5$

$y = -9$

For $x = -1$:

$y = 2 \times (-1) - 5$

$y = -7$

For $x = 0$:

$y = 2 \times 0 - 5$

$y = -5$

For $x = 1$:

$y = 2 \times 1 - 5$

$y = -3$

For $x = 2$:

$y = 2 \times 2 - 5$

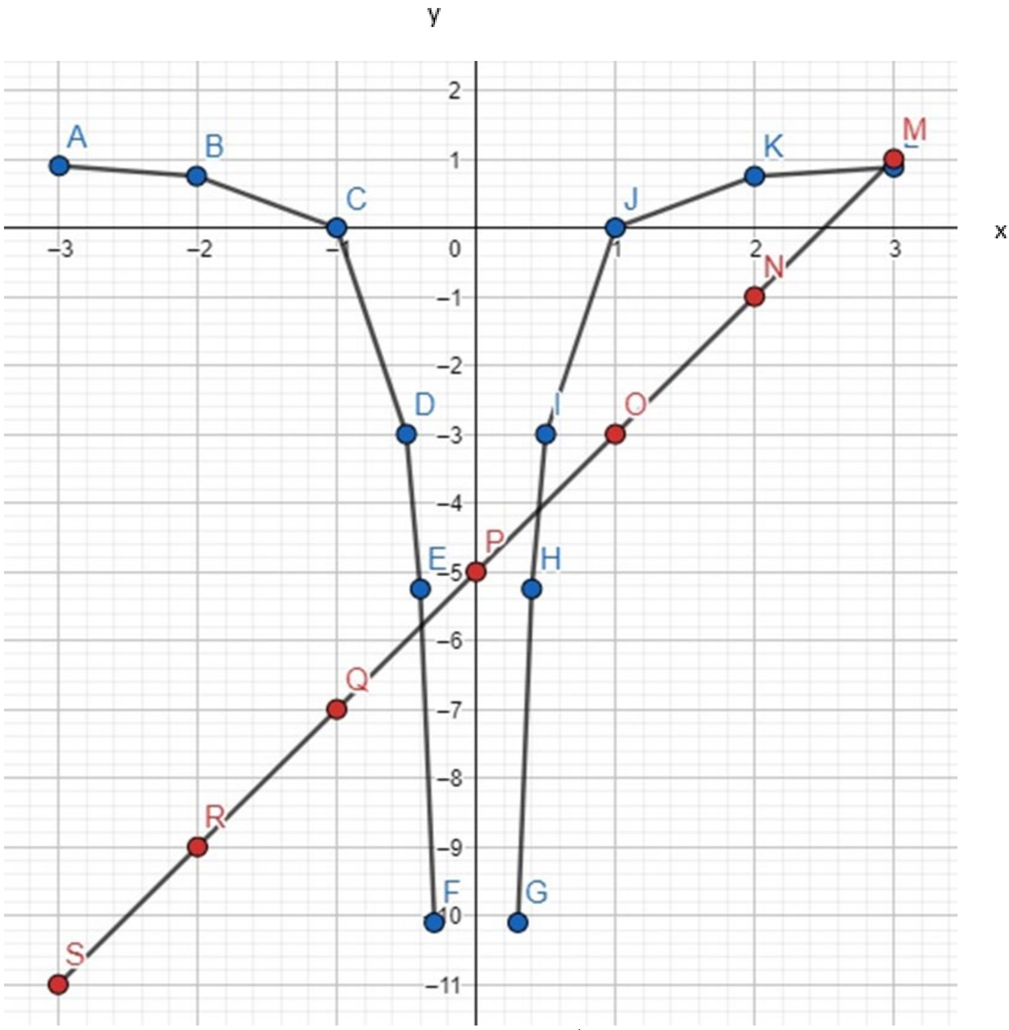
$y = -1$

For $x = 3$:

$y = 2 \times 3 - 5$

$y = 1$

We plot all the points described above:



(e) (i) Use your graphs to find solutions of the equation $1 - \frac{1}{x^2} = 2x - 5$ [3]

The solutions of the equation are the x coordinates of the points at which the 2 graphs intersect.

Using the graph above, we see that the 2 graphs intersect at 3 different points, therefore the equation has 3 different solutions.

The first point of intersection has the coordinates (2.9, 0.9), therefore the solution is **x = 2.9**

The second point of intersection has the coordinates (0.45, -4), therefore the solution is **x = 0.45**

The second point of intersection has the coordinates (-0.38, -5.77), therefore the solution is **x = -0.38**

(ii) Rearrange $1 - \frac{1}{x^2} = 2x - 5$ into the form $ax^3 + bx^2 + c = 0$, where a, b and c are integers. [2]

$1 - \frac{1}{x^2} = 2x - 5$

We multiply each term in the equality by x^2 to have all the terms in the same form.

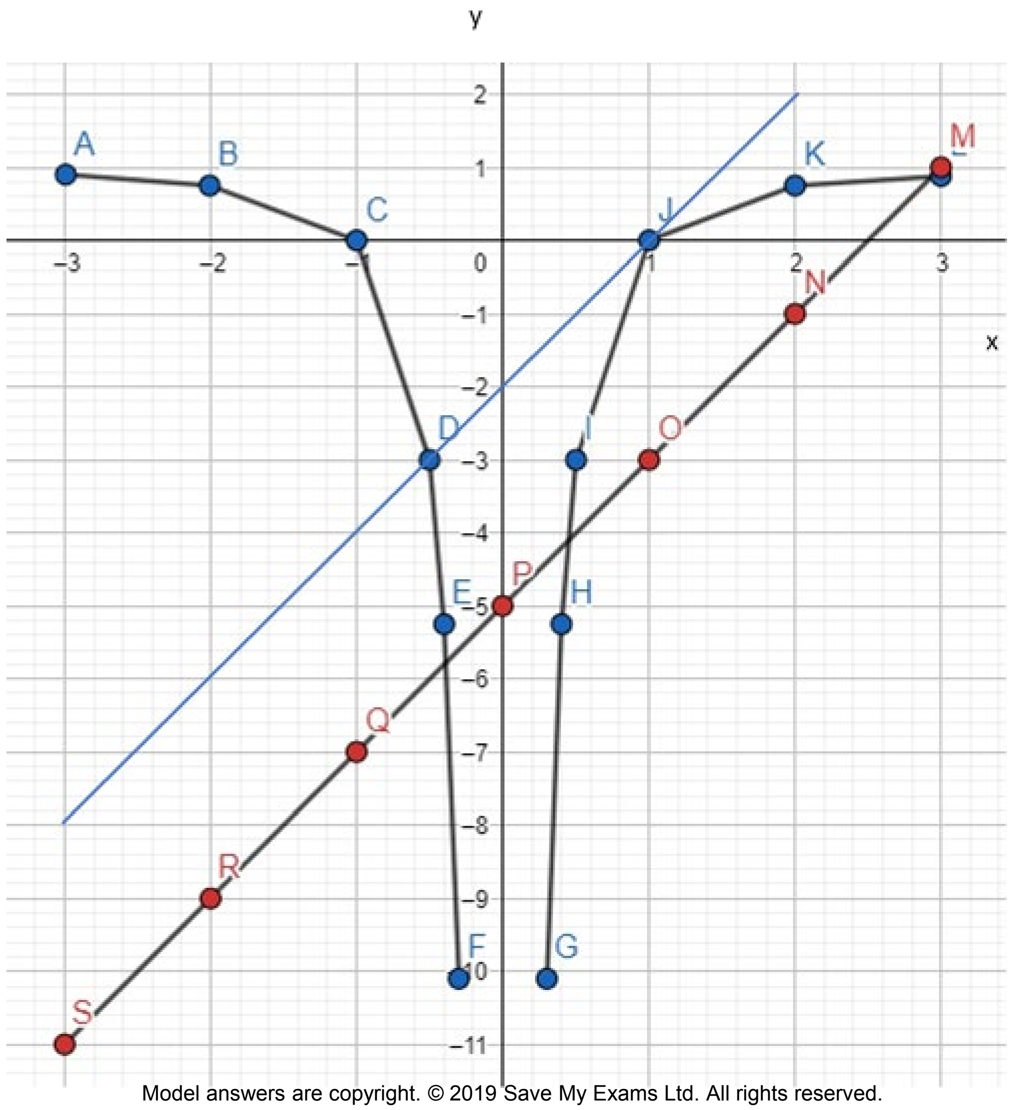
$x^2 - 1 = 2x^3 - 5x^2$

We move all the terms to one side to obtain an equation equal to 0.

$2x^3 - 6x^2 + 1 = 0$

a = 2, b = -6, c = 1

(f) (i) Draw a tangent to the graph of $y = f(x)$ which is parallel to the line $y = 2x - 5$. [1]



To have 2 parallel lines, they need to have the same gradient.

The equation of a line is in the form:

y = mx + n

where m is the gradient and n is the y-intercept

In our case, the line y = 2x – 5 has the gradient m = 2.

To draw a parallel tangent, the tangent needs to have the gradient equal to 2.

Gradient = $\frac{yB-yA}{xB-xA}$

Using this, we draw the tangent shown on the figure above.

(ii) Write down the equation of this tangent. [2]

From i), we know that the gradient of the tangent is m = 2.

Using the graph, we can see that the y-intercept is n = -2

Therefore, the equation of the line will be:

y = 2x - 2

Answer the whole of this question on a sheet of graph paper.

(a) $$f(x) = \frac{12}{x+1}$$

x	0	1	2	3	4	5	6	7	8	9	10	11
f(x)	p	6	4	3	2.4	2	1.71	q	1.33	r	1.09	1

(i) Calculate the values of p, q and r. [3]

$f(x) = \frac{12}{x+1}$

For x = 0:

$f(0) = \frac{12}{0+1}$

f(0) = 12

For x = 7:

$f(7) = \frac{12}{7+1}$

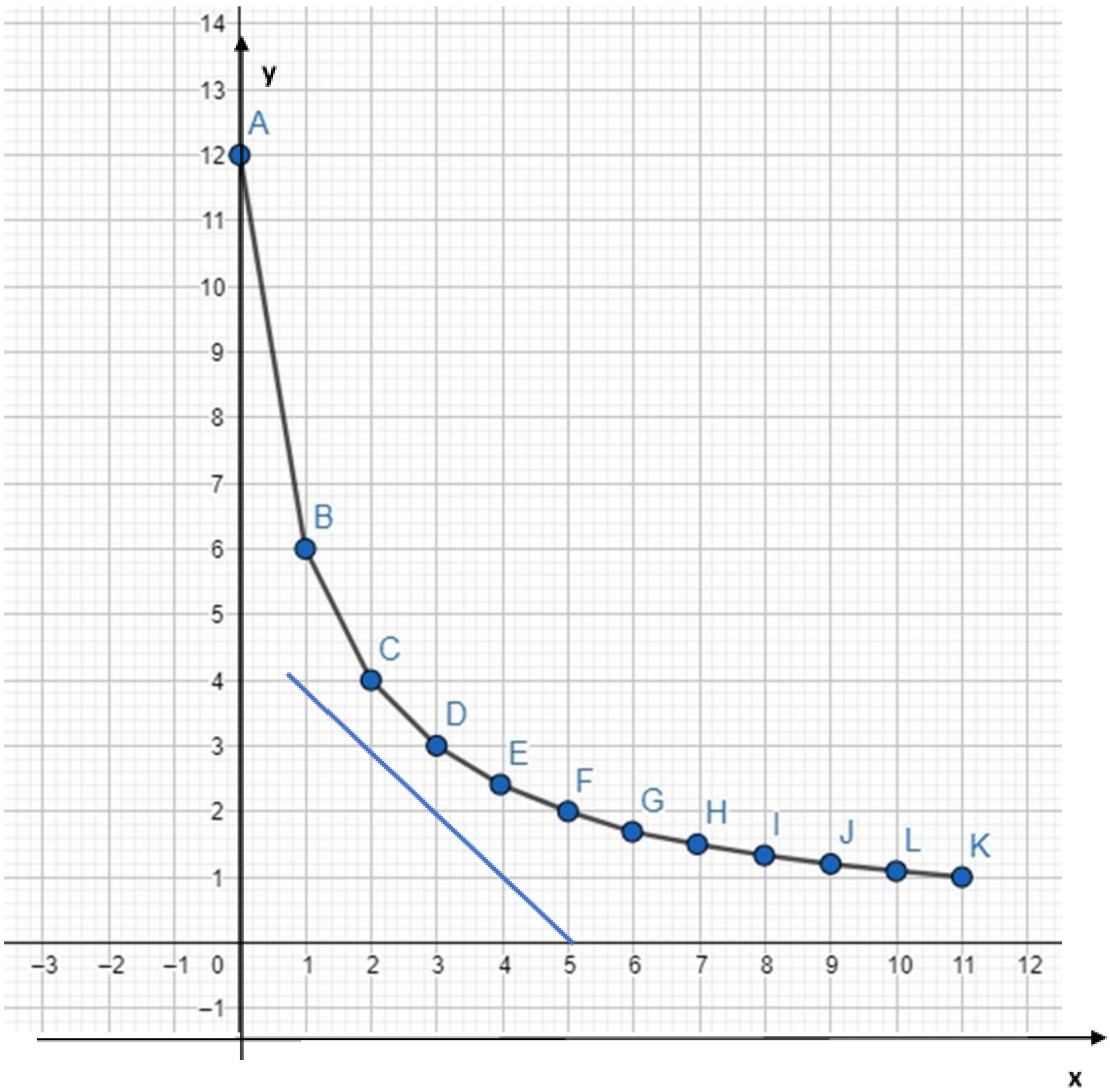
$f(7) = \frac{3}{2} = 1.5$

For x = 9:

$f(7) = \frac{12}{9+1}$

f(7) = 1.2

(ii) Draw the graph of $y = f(x)$ for $0 \leqslant x \leqslant 11$.
Use a scale of 1cm to 1 unit on each axis. [5]



(iii) By drawing a suitable line, find an estimate of the gradient of the graph at the point (3, 3). [3]

The tangent to the curve which passes through the point D (3, 3) can be used to calculate the gradient of the curve in that point.

The tangent passes also through the point C (4, 2).

We can work out the gradient, m, using:

$$m = \frac{\text{change in y}}{\text{change in x}}$$

m = -1

(b) On the same grid draw the graph of $y = 8 - x$ for $0 \leqslant x \leqslant 8$. [2]

We need to identify the value of y for each x from the interval.

For x = 0:

$y = 8 - 0$

$y = 8$

For x = 1

$y = 8 - 1$

$y = 7$

For x = 2:

$y = 8 - 2$

$y = 6$

For x = 3:

$y = 8 - 3$

$y = 5$

For x = 4:

$y = 8 - 4$

$y = 4$

For x = 5:

$y = 8 - 5$

y = 3

For x = 6:

y = 8 – 6

y = 2

For x = 7:

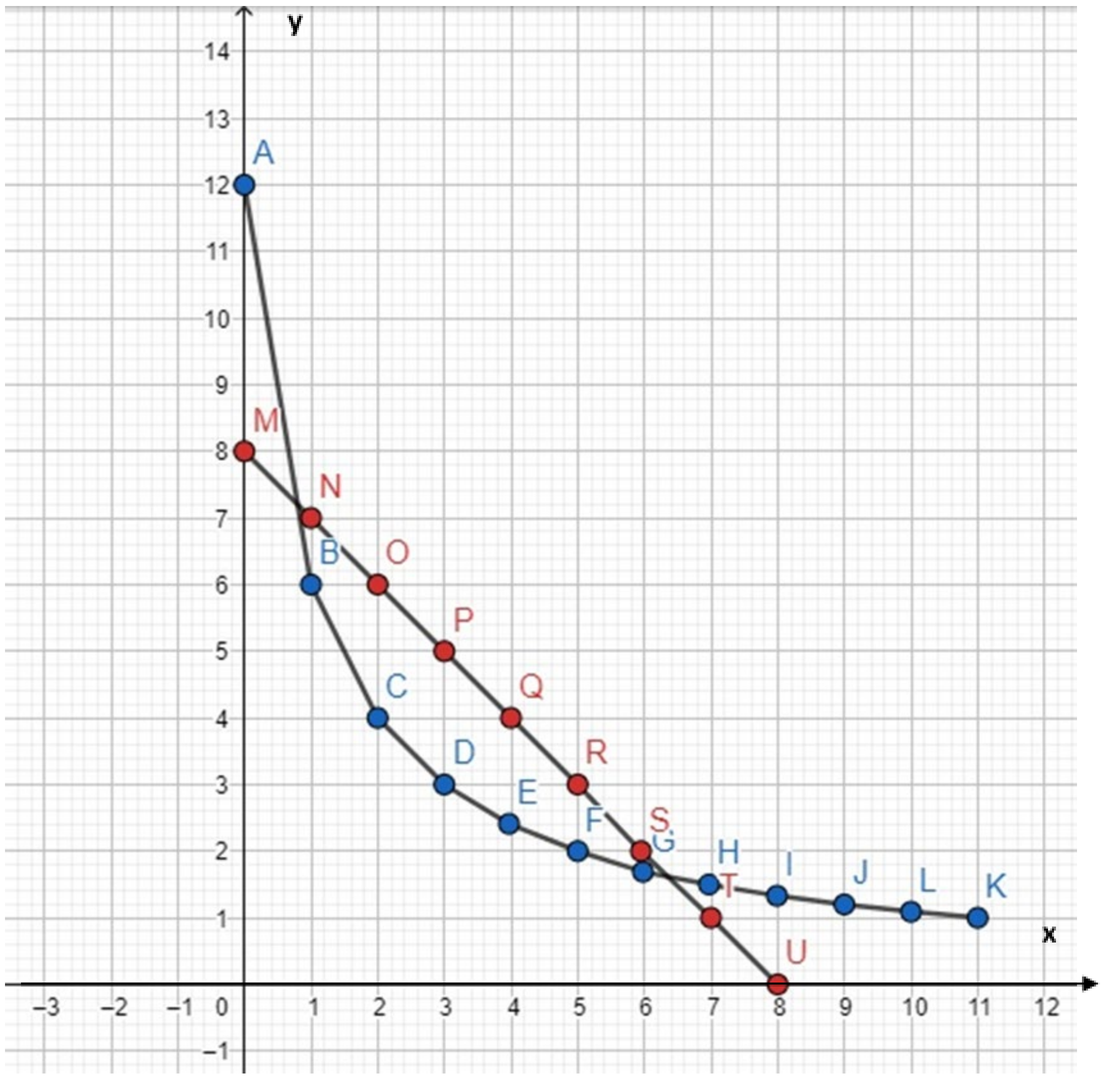
y = 8 – 7

y = 1

For x = 8:

y = 8 – 8

y = 0



We plot these points to obtain the graph of the function.

(c) (i) Show that the equation $f(x) = 8 - x$ simplifies to $x^2 - 7x + 4 = 0$. [2]

$\frac{12}{x+1} = 8 - x$

We multiply by (x + 1) to have all the numbers in the same form.

$12 = (8 - x)(x + 1)$

$12 = 8x - x^2 + 8 - x$

$12 = 7x - x^2 + 8$

$\mathbf{x^2 - 7x + 4 = 0}$

(ii) **Use your graph** to solve this equation, giving your answers correct to 1 decimal place. [2]

The solution of the equation is represented by the intersection of the 2 graphs.

The intersection is the point of coordinates x = 0.8 and y = 6.25.

Therefore, the solution is x = 0.8

(a) $f(x) = x^2 - x - 3$.

x	-3	-2	-1	0	1	2	3	4
$f(x)$	p	3	-1	-3	q	-1	3	r

Find the values of p, q and r. [3]

Use the table function on your calculator to recreate the table and read off p, q and r.

(Or plug the x values into the function…)

Either way $\boldsymbol{p = 9, q = -3}$ **and** $\boldsymbol{r = 9}$

(i) Draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 4$.
Use a scale of 1 cm to represent 1 unit on each axis. [4]

See next page – points plotted – smooth curve drawn – thinner line

(iii) By drawing a suitable line, estimate the gradient of the graph at the point where $x = -1$. [3]

Line drawn on next page in red

$$\text{RADIENT} = \frac{\text{RISE}}{\text{RUN}} = \frac{-9}{3} = \boldsymbol{-3}$$

(b) $g(x) = 6 - \dfrac{x^3}{3}$.

x	-2	-1	0	1	2	3
$g(x)$	8.67	u	v	5.67	3.33	-3

(i) Find the values of u and v. [2]

Use the table function on your calculator to recreate the table and read off u and v.

(Or plug the x values into the function…)

Either way $\boldsymbol{u = 6.33}$ **and** $\boldsymbol{v = 6}$

(ii) On the same grid as **part (a) (ii)** draw the graph of $y = \mathrm{g}(x)$ for $-2 \leqslant x \leqslant 3$. [4]

See next page – points plotted – smooth curve drawn – thicker line

(c) (i) Show that the equation $\mathrm{f}(x) = \mathrm{g}(x)$ simplifies to $x^3 + 3x^2 - 3x - 27 = 0$. [1]

$\mathrm{f}(x) = \mathrm{g}(x)$ $\quad x^2 - x - 3 = 6 - \dfrac{x^3}{3}$

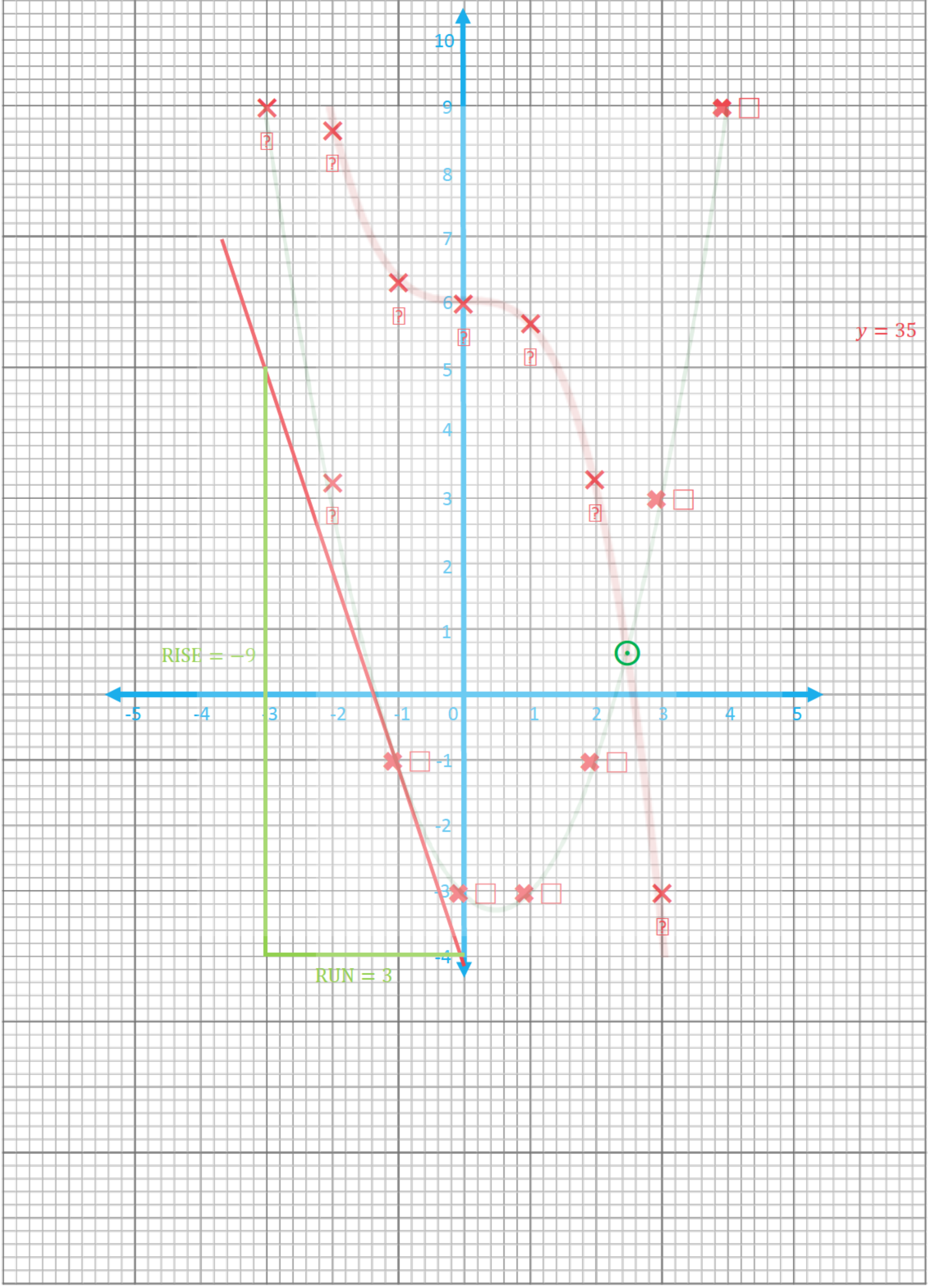
Multiply by 3 $\quad 3x^2 - 3x - 9 = 18 - x^3$

Add $x^3 - 18$ to both sides $\quad x^3 + 3x^2 - 3x - 27 = 0$

(ii) Use your graph to write down a solution of the equation $x^3 + 3x^2 - 3x - 27 = 0$. [1]

Write down x coordinate of the point of intersection (marked with $\odot$ on the graph)

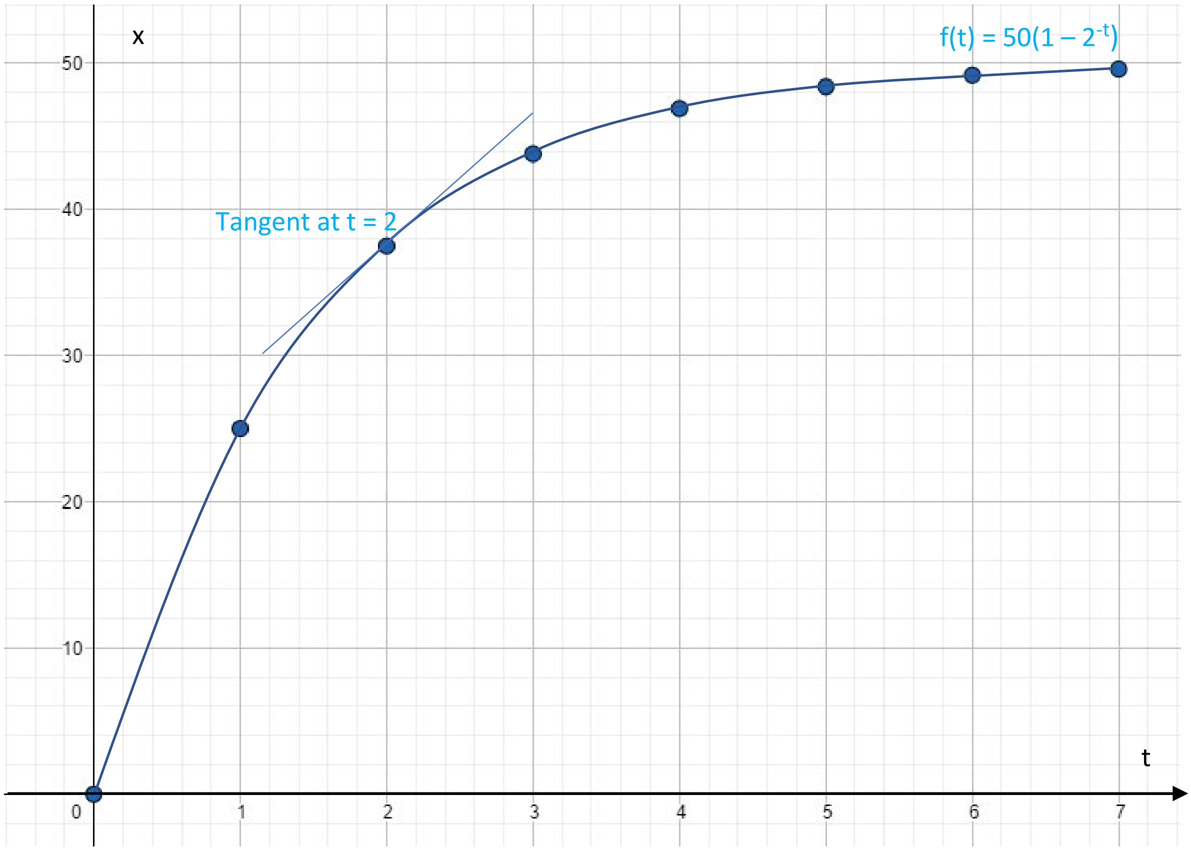
$\boldsymbol{x = 2.5}$


10
9
8
7
6
5
4
3
2
1
-5
-4
-3
-2
-1
0
1
2
3
4
5
-1
-2
-3
-4
y = 35
RISE = −9
RUN = 3

Answer the whole of this question on a sheet of graph paper.

t	0	1	2	3	4	5	6	7
f(t)	0	25	37.5	43.8	46.9	48.4	49.2	49.6

(a) Using a scale of 2 cm to represent 1 unit on the horizontal t-axis and 2 cm to represent 10 units on the y-axis, draw axes for $0 \le t \le 7$ and $0 \le y \le 60$.
Draw the graph of the curve $y = f(t)$ using the table of values above. [5]



(b) $f(t) = 50(1 - 2^{-t})$.

(i) Calculate the value of f(8) and the value of f(9). [2]

$f(8) = 50(1 - 2^{-8})$

f(8) = 49.8

$f(9) = 50(1 - 2^{-9})$

f(9) = 49.9

(ii) Estimate the value of f(t) when t is large. [1]

As t increases, 2^{-t} becomes closer to 0.

f(t) = 50(1 – approx. 0) = 50

Therefore, for a large t value, f(t) approaches 50.

(c) (i) Draw the tangent to $y = f(t)$ at $t = 2$ and use it to calculate an estimate of the gradient of the curve at this point. [3]

The tangent in point t = 2 is drawn on the diagram above.

Gradient = $\frac{\text{change in y}}{\text{change in x}}$

Gradient = $\frac{46.1-30}{3-1.2}$

Gradient = 8.94

(ii) The function f(t) represents the speed of a particle at time t. Write down what quantity the gradient gives. [1]

The gradient at time t gives the acceleration, with the unit m/s^2.

(d) (i) On the same grid, draw $y = g(t)$ where $g(t) = 6t + 10$, for $0 \le t \le 7$. [2]

g(t) = 6t + 10

For t = 0:

g(0) = 6 x 0 + 10 = 10

For t = 1:

g(1) = 6 x 1 + 10 = 16

For t =2:

g(2) = 6 x 2 + 10 = 22

For t = 3:

g(3) = 6 x 3 + 10 = 28

For t = 4:

g(4) = 6 x 4 + 10 = 34

For t = 5:

g(5) = 6 x 5+ 10 = 40

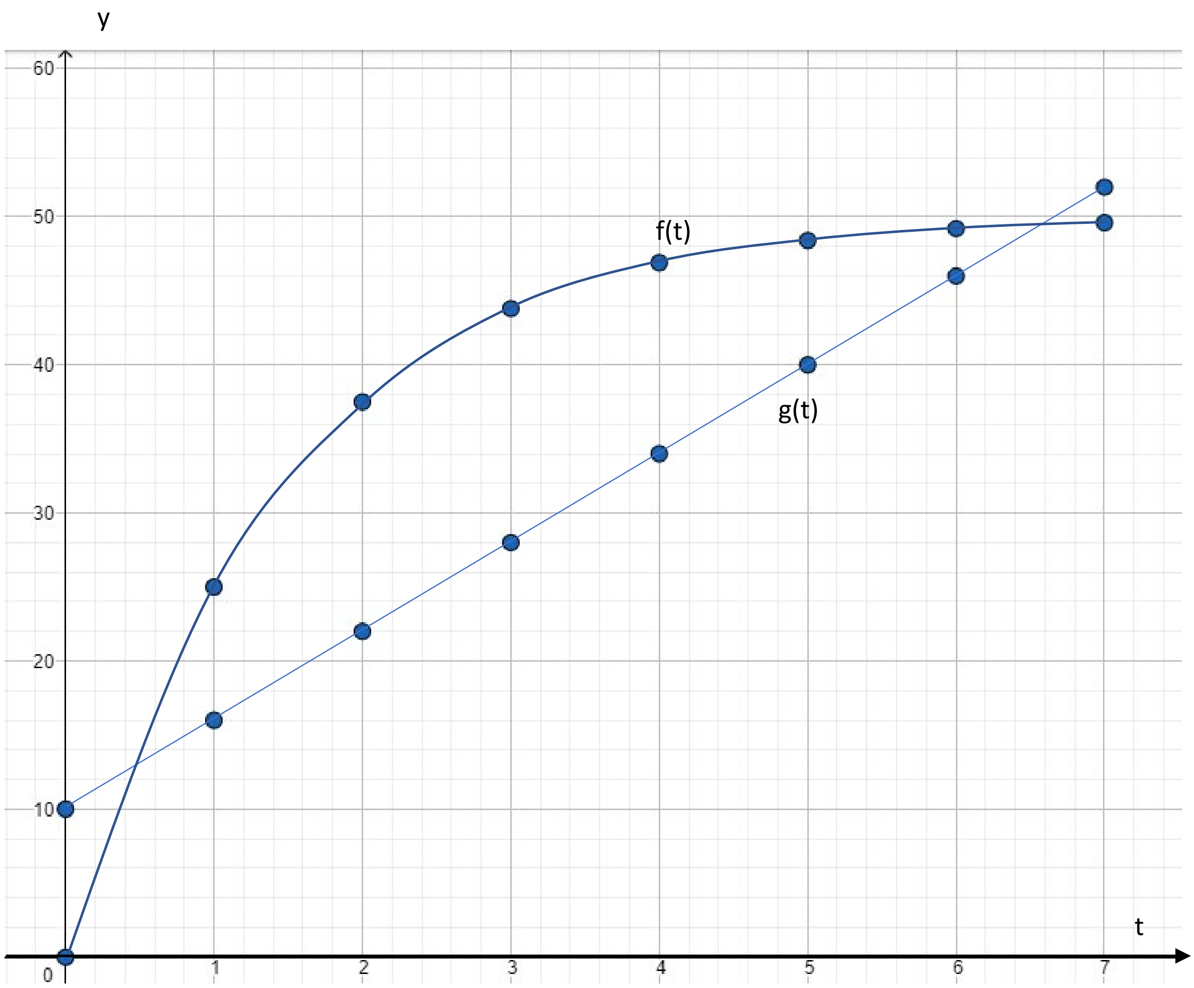
For t = 6:

g(6) = 6 x 6 + 10 = 46

For t = 7:

g(7) = 6 x 7 + 10 = 52

We plot these values to work out the graph of g(t).



(ii) Write down the range of values for t where $f(t) > g(t)$. [2]

By looking at the graph above, we see that the values for which f(t) > g(t)

represent the t values range

$13 \leq t \leq 48$ approximately.

(iii) The function $g(t)$ represents the speed of a second particle at time t.
State whether the first or second particle travels the greater distance for $0 \leq t \leq 7$. You **must** give a reason for your answer. [2]

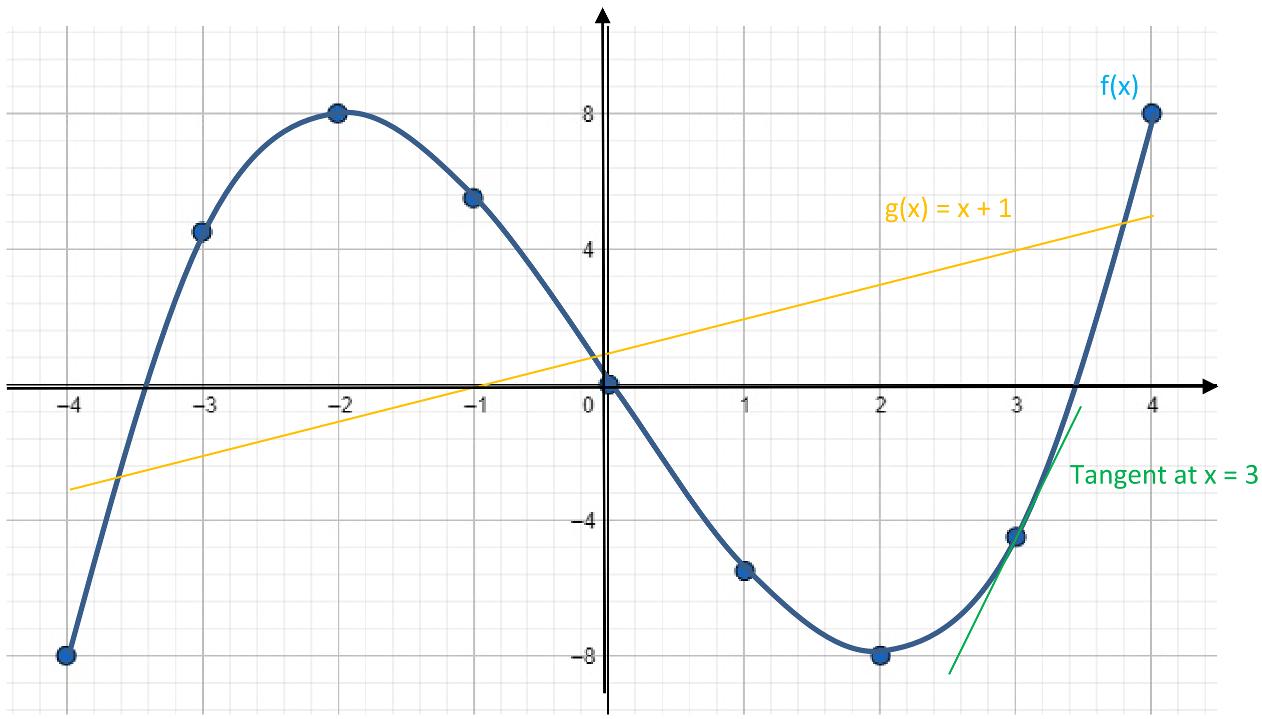
The distance in this case is represented by the area under the graph for each of the functions.

Therefore, the greatest distance is travelled by the first particle since the area under the curve is bigger than the area under the straight line.

Answer the whole of this question on a sheet of graph paper.

x	04	03	02	01	0	1	2	3	4
f(x)	08	4.5	8	5.5	0	05.5	08	04.5	8

(a) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 4 units on the y-axis, draw axes for $-4 \leq x \leq 4$ and $-8 \leq y \leq 8$.
Draw the curve y # f(x) using the table of values given above. [5]



x

(b) Use your graph to solve the equation f(x) = 0. [2]

For f(x) = y = 0 the corresponding x solution is x = 0, x = 3.4 and x = -3.4.

(c) On the same grid, draw y = g(x) for $-4 \leq x \leq 4$, where g(x) = x + 1. [2]

g(x) = x + 1

For x = -4: g(-4) = -3

For x = -3: g(-3) = -2

For x = -2: g(-2) = -1

For x = -1: g(-1) = 0

For x = 0: g(0) = 1

…

For x = 4: g(4) = 5

(d) Write down the value of [4]

(i) g(1),

g(1) = 1 + 1 = 2

(ii) fg(1),

fg(1) = f(g(1))

g(1) = 2

f(2) = fg(1)

By looking at the graph, the value corresponding to x = 2 for the function f(x) is

y = -8

(iii) $g^{01}(4)$,

g(x) = x + 1 = y

y – 1 = x

$g^{-1}(x) = x - 1$

$g^{-1}(4) = 4 - 1 = 3$

(iv) the **positive** solution of f(x) = g(x).

f(x) = g(x) has the solution as the x coordinates for the points in which

the graph for the 2 functions intersect.

The positive x value for one of these intersections is

x = 3.8

(e) Draw the tangent to y = f(x) at x = 3. Use it to calculate an estimate of the gradient of the curve at this point. [3]

The tangent to y = f(x) at x = 3 is drawn on the graph above.

Gradient = $\frac{\text{change in y}}{\text{change in x}}$

Gradient = $\frac{8}{1}$

Gradient = 8