1. Number

YOUR NOTES
⬇

CONTENTS

1.1 Numbers & Accuracy
- 1.1.1 Multiplication (non-Calc)
- 1.1.2 Division (non-Calc)
- 1.1.3 Prime Factors
- 1.1.4 HCFs & LCMs
- 1.1.5 Rounding & Estimation

1.2 Sets & Venn Diagrams
- 1.2.1 Set Notation & Venn Diagrams

1.3 Conversion
- 1.3.1 Recurring Decimals

1.4 Order by Size
- 1.4.1 Order by Size

1.5 Standard Form
- 1.5.1 Standard Form – Basics
- 1.5.2 Standard Form – Harder

1.6 Working with Fractions
- 1.6.1 Mixed Numbers & Top Heavy Fractions
- 1.6.2 Adding & Subtracting Fractions
- 1.6.3 Multiplying & Dividing Fractions

1.7 Bounds
- 1.7.1 Bounds & Error Intervals – Basics
- 1.7.2 Calculations using Bounds

1.8 Ratios
- 1.8.1 Ratios
- 1.8.2 Speed, Density & Pressure
- 1.8.3 Speed, Density & Pressure – Harder

1.9 Percentages
- 1.9.1 Basic Percentages
- 1.9.2 Percentage Increases & Decreases
- 1.9.3 Reverse Percentages

1.10 Using a Calculator
- 1.10.1 Using a Calculator

1.11 Time

© copyright Save My Exams

1. Number

YOUR NOTES
⬇

1.11.1 Time
1.12 Currency
1.12.1 Exchange Rates
1.13 Exponential Growth & Decay
1.13.1 Compound Interest

1.1 NUMBERS & ACCURACY

1.1.1 MULTIPLICATION (NON-CALC)

(Non-calculator) multiplication – why so many methods?

- Different methods work for different people, and some are better **depending** on the **size of number** you are dealing with
- We recommend the following **3 methods** depending on the size of number you are dealing with
  (If in doubt all methods will work for all numbers!)

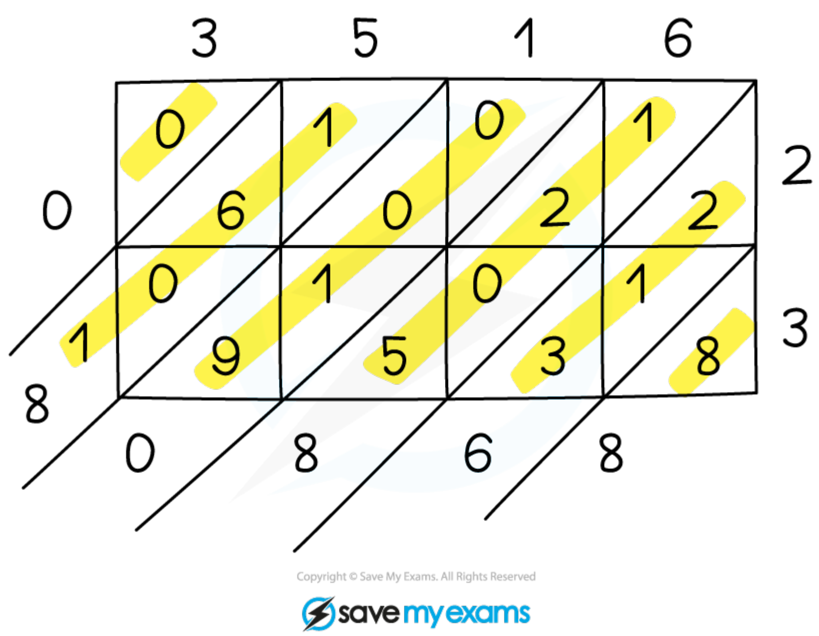
1. Number

YOUR NOTES

1. Lattice method

(Best for numbers with two or more digits)

- This method allows you to work with digits
- So in the number 3 516 you would only need to work with the digits 3, 5, 1 and 6
- So if you can multiply up to 9×9 you can't go wrong!



So, 3516 × 23 = 80 868

1. Number

YOUR NOTES

2. Partition method

(Best when one number has just one digit)

- This method keeps the value of the larger number intact
- So with 3 516 you would use 3000, 500, 10 and 6
- This method is **not suitable** for two larger numbers as you can end up with a lot of zero digits that are hard to keep track of

	3000	500	10	6
7	21 000	3 500	70	42

Copyright © Save My Exams. All Rights Reserved
save my exams

$$\begin{array}{r} 21\,000 \\ 3\,500 \\ 70 \\ +\quad 42 \\ \hline 24612 \end{array}$$

Copyright © Save My Exams. All Rights Reserved
save my exams

So, 3516 × 7 = 24 612

1. Number

YOUR NOTES
⬇

3. Repeated addition method

(Best for smaller, simpler cases)

- You may have seen this called 'chunking'
- It is a way of building up to the answer using simple multiplication facts that can be worked out easily

eg. 13×23
$1 \times 23 = 23$
$2 \times 23 = 46$
$4 \times 23 = 92$
$8 \times 23 = 184$

So, $13 \times 23 = 1 \times 23 + 4 \times 23 + 8 \times 23 = 23 + 92 + 184 = 299$

Decimals

- These 3 methods can easily be adapted for use with decimal numbers
- You ignore the decimal point whilst multiplying but put it back in the correct place in order to reach a final answer

eg. 1.3×2.3
Ignoring the decimals this is 13×23, which from above is 299
There are two decimal places in total in the question, so there will be two decimal places in the answer

So, $1.3 \times 2.3 = 2.99$

Exam Tip

If you do forget your times tables then in the exam write a list out of the table you need as you do a question.
So for example, if you need to multiply by 8, and you've forgotten your 8 times tables, write it down: 8, 16, 24, 32, 40, 48, etc. as far as you need to.

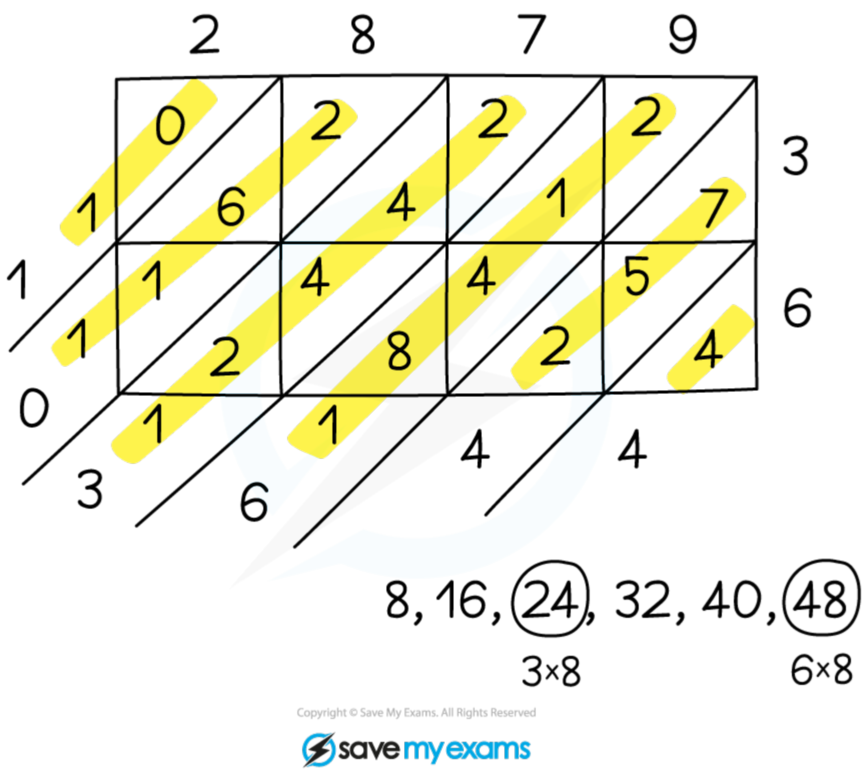
1. Number

YOUR NOTES

Worked Example

1. Multiply 2879 by 36

- As you have a 4-digit number multiplied by a 2-digit number then the lattice method (1) is the best choice
- Start with a 4×2 grid....



- Notice the use of listing the 8 times table at the bottom to help with any you may have forgotten
  2879 × 36 = 103 644
- Note that the method would still work if you had set it up as a 2×4 grid

1. Number

YOUR NOTES
⬇

2. Pencils are sold in boxes. Each box costs £1.25 and each box contains 15 pencils.

Tyler buys 35 boxes of pencils.

(a) Work out how many pencils Tyler has in total.

(b) Work out the total cost for all the boxes Tyler buys.

(a)

This is a roundabout way of asking you to work out 15×35
As this is a simpl-ish case (3) you should use the repeated addition method

$1 \times 35 = 35$
$2 \times 35 = 70$
$4 \times 35 = 140$
$8 \times 35 = 280$
$16 \times 35 = 560$

It doesn't matter if you go past 15 …

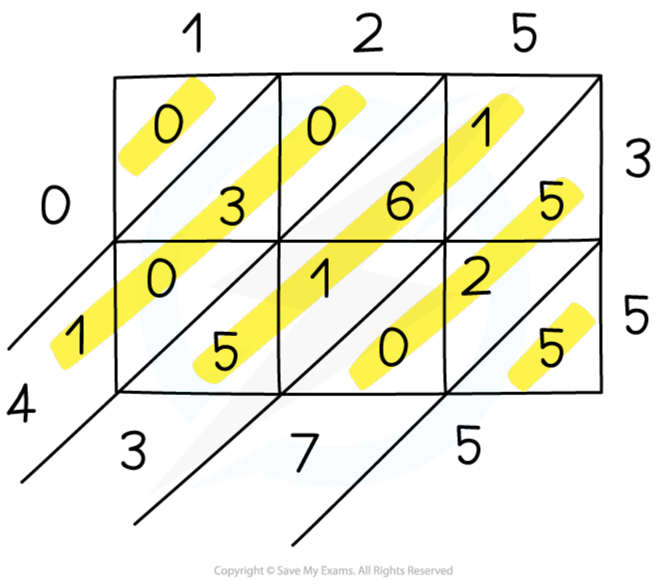
$15 \times 35 = 16 \times 35 - 1 \times 35 = 560 - 35$
$15 \times 35 = 525$

1. Number

YOUR NOTES

(b)

This question is 1.25×35 so involves decimals (4)
Ignoring the decimals it becomes 125×35 and so the lattice method is best



125 × 35 = 4375

Now count the decimal places from the question and put the decimal point back in the correct place

£1.25 × 35 = £43.75

Exam Tip

Okay, getting the highlighter out during an exam may be a touch excessive! But do use your grid/diagram to help you answer the question – the highlighter in the example above makes it clear which digits to add up at each stage. You can do this in pen or pencil but do make sure you can still read the digits underneath as it is all part of your method/working.

1. Number

YOUR NOTES

1.1.2 DIVISION (NON-CALC)

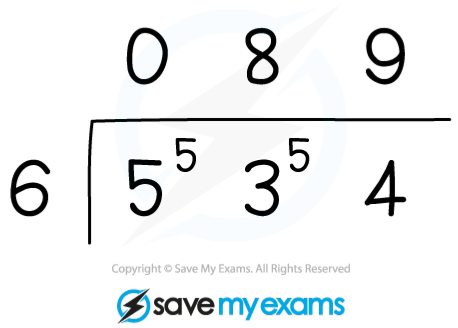
(Non-calculator) division – more methods

- Most students will have seen short division (bus stop method) and long division and there is often confusion between the two
- Fortunately, you only need one – so use short division
- While short division is best when dividing by a single digit, for bigger numbers you need a different approach
- You can use other areas of maths that you know to help – eg. cancelling fractions, “shortcuts” for dividing by 2 and 10, and the repeated addition (“chunking”) method covered in (Non-Calculator) Multiplication

1. Short division (bus stop method)

- Apart from where you can use shortcuts such as dividing by 2 or by 5, this method is best used when dividing by a single digit

eg. 534 ÷ 6



So, 534 ÷ 6 = 89

1. Number

YOUR NOTES

2. Factoring & cancelling

- This involves treating division as you would if you were asked to cancel fractions
- You can use the fact that with division, most non-calculator questions will have only number answers
- The only thing to be aware of is that this might not be the case if you've been asked to write a fraction as a mixed number (but if you are asked to do that it should be obvious from the question)

eg. $1008 \div 28$

$1008 \div 28 = 504 \div 14 = 252 \div 7 = 36$

- You may have spotted the first two values (1008 and 28) are both divisible by 4 which is fine but if not, divide top and bottom by any number you can
- To do the last part ($252 \div 7$) you can use the short division method above

3. Intelligent repeated addition

- This is virtually identical to the version for multiplication – the process stops when the number dividing into is reachedeg $1674 \div 27$This is the same as saying $? \times 27 = 1674$So we can build up in "chunks" of 27 until we get to 16741 $\times 27 = 27$
$10 \times 27 = 270$
$20 \times 27 = 540$
$40 \times 27 = 1080$
$60 \times 27 = 1620$ … by using the last two results added together.Now you are close we can add on 27 one at a time again.
$61 \times 27 = 1647$
$62 \times 27 = 1674$
So $1674 \div 27 = 62$

4. Dividing by 10, 100, 1000, … (Powers of 10)

- This is a case of moving digits (or decimal points) or knocking off zeros
eg. $380 \div 10 = 38$
$45 \div 100 = 0.45$

5. Dividing by 2, 4, 8, 16, 32, … (Powers of 2)

- This time it is a matter of repeatedly halving
eg. $280 \div 8 = 140 \div 4 = 70 \div 2 = 35$
$1504 \div 32 = 752 \div 16 = 376 \div 8 = 188 \div 4 = 94 \div 2 = 47$

1. Number

YOUR NOTES
⬇

Exam Tip

On the non-calculator paper, division is very likely to have a whole number (exact) answer. So if, when using the repeated addition method, you do not reach this figure then it is likely you've made an error in your calculations somewhere.

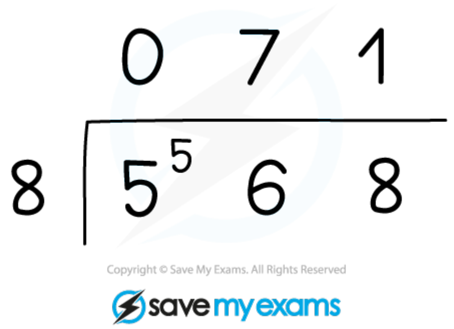
1. Number

YOUR NOTES
⬇

Worked Example

1. After a fundraising event, the organiser wishes to split the £568 raised between 8 charities. How much will each charity get?

- This is division by a single digit so short division would be an appropriate method
- If you spot it though, 8 is also a power of 2 so you could just halve three times
- Method 1 – short division:



568 ÷ 8 = 71

- Method 2 – powers of 2:

568 ÷ 2 = 284
284 ÷ 2 = 142
142 ÷ 2 = 71
568 ÷ 8 = 71

568 ÷ 8 = 71

You know to halve three times since
2 × 2 × 2 = 8

1. Number

YOUR NOTES

2. A robot packs tins of soup into boxes. Each box holds 24 cans of soup.
The robot has 1824 cans of soup to pack into boxes.
How many boxes will be produced by the robot?

$1824 \div 24$

Both numbers are large so intelligent repeated addition is the best approach

$1 \times 24 = 24$
$10 \times 24 = 240$
$20 \times 24 = 480$
$40 \times 24 = 960$
$80 \times 24 = 1920$ … going too far doesn't matter as you can subtract …
$79 \times 24 = 1896$
$78 \times 24 = 1872$
$77 \times 24 = 1848$
$76 \times 24 = 1824$

- Although this may at first look like a trial and improvement method it is important to show logic throughout as you build the number up – that's why we call it INTELLIGENT repeated addition here at SME!

$1824 \div 24 = 76$

The robot will produce 76 boxes of cans of soup

1. Number

YOUR NOTES
⬇

1.1.3 PRIME FACTORS

What are prime factors?

- Factors are things that are multiplied together
- Prime numbers are numbers which can only be divided by themselves and 1
- The prime factors of a number are therefore all the prime numbers which multiply to give that number
- You should remember the first few prime numbers:
  2, 3, 5, 7, 11, 13, 17, 19, …

How to find prime factors

- Use a FACTOR TREE to find prime factors
- Write the prime factors IN ASCENDING ORDER with × between
- Write with POWERS if asked

Language

This is one of those topics where questions can use different phrases that all mean the same thing …

- Express … as the product of prime factors
- Find the prime factor decomposition of …
- Find the prime factorisation of …

© copyright Save My Exams

1. Number

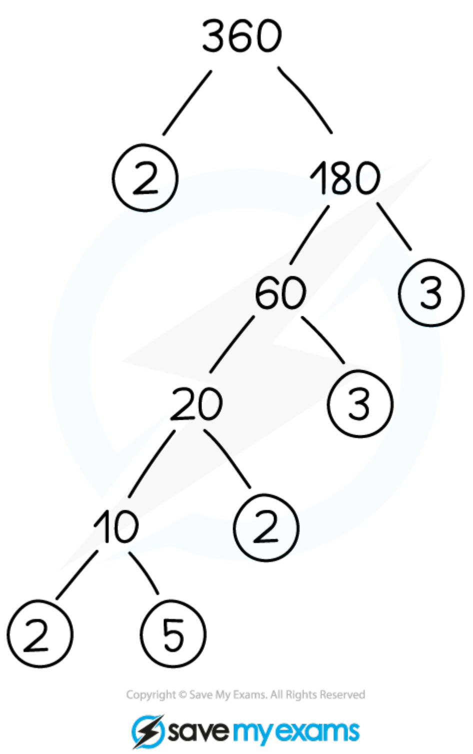
YOUR NOTES

Worked Example

1. *Find the prime factors of* 360.

Give your answer in the form $2^p \times 3^q \times 5^r$ *where* p, q *and* r *are integers to be found.*

1 − *For each number find any two numbers which are factors (not 1 ...) and write those as the next pair of numbers in the tree. If a number is prime, put a circle round it. When all the end numbers are circled you're done!*



1. Number

YOUR NOTES ⬇

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$ $2 \;-\;$ *Be careful to get them all, especially those repeated ones*

$360 = 2^3 \times 3^2 \times 5^1$ $3 \;-\;$ *Write using powers as the question asks for this*

The 1 as a power of 5 isn't really necessary but the question asked for it...

(*So* $p = 3,\ q = 2,\ r = 1$)

1. Number

YOUR NOTES
⬇

1.1.4 HCFS & LCMS

What are HCFs & LCMs?

- HCF is Highest Common Factor
- This is the biggest number which is a factor of (divides into) two numbers
- LCM is Lowest Common Multiple
- This is the smallest number which two numbers divide into (are factors of)
- You should remember the first few Prime Numbers:
  2, 3, 5, 7, 11, 13, 17, 19, …

How to find HCFs & LCMs

1. Find PRIME FACTORS
2. Create a VENN DIAGRAM
3. ∩ is HIGHEST COMMON FACTOR (Overlap)
4. ∪ is LOWEST COMMON MULTIPLE (Union)

© copyright Save My Exams

1. Number

YOUR NOTES

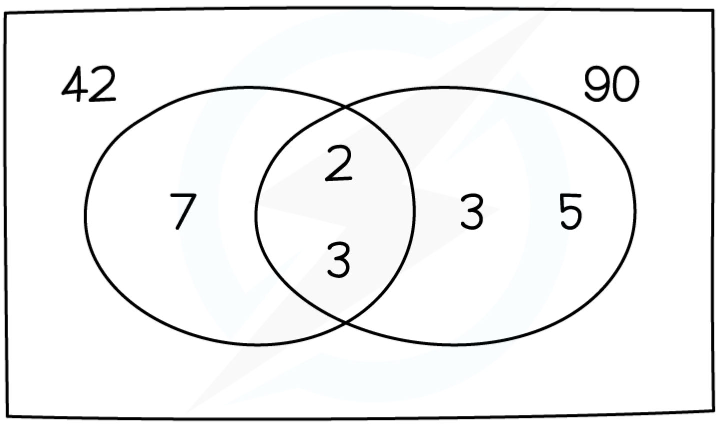
Worked Example

1. *Find the HCF and LCM of 42 and 90.*

$42 = 2 \times 3 \times 7$

$90 = 2 \times 3 \times 3 \times 5$

1 – *See the separate notes for finding Prime Factors*



Copyright © Save My Exams. All Rights Reserved

2 – *Box is not essential for our purposes here*

$HCF = 2 \times 3$

$HCF = 6$

3 – $\cap$, *intersection/overlap*

$LCM = 7 \times 2 \times 3 \times 3 \times 5$

$LCM = 630$

4 – $\cup$, *union/'or'*

1. Number

YOUR NOTES
⬇

1.1.5 ROUNDING & ESTIMATION

Why use estimation?

- We **estimate** to find approximations for difficult sums
- Or to check our answers are about the right size (right order of magnitude)

How to estimate

- We **round** numbers to something sensible before **calculating**
- GENERAL RULE:
  Round numbers to 1 significant figure
    - 7.8 → 8
    - 18 → 20
    - 3.65 × 10-4 → 4 × 10-4
    - 1080 → 1000
- EXCEPTIONS:
  It can be more sensible (or easier) to round to something convenient
    - 16.2 → 15
    - 9.1 → 10
    - 1180 → 1200

  It wouldn't usually make sense to round a number to zero
- FRACTIONS get bigger when the top is bigger and/or the bottom is smaller and vice versa

© copyright Save My Exams

1. Number

YOUR NOTES
⬇

Worked Example

1. *Calculate an estimate for* $\frac{15.9 \times 3.87}{18.7}$.

$3.87 \rightarrow 4$ 1 − *Round both to 1 significant figure*

$18.7 \rightarrow 20$

$15.9 \rightarrow 15$ 2 − *An exception as 15.9 is quite a long way from*

$\frac{15.9 \times 3.87}{18.7} \approx \frac{15 \times 4}{20}$ *Now use the rounded figures to find an approximation*

$\approx \frac{60}{20}$

≈ 3

2. (*a*) *Use your calculator to work out Calculate an estimate for* 108.6×27.3.

(*b*) *Show an estimate to verify your answer to* (*a*) *is of the right order of magnitude.*

(*a*) 2964.78   *Write down all the digits from your calculator display*

(*b*) $108.6 \rightarrow 100$ *Round both numbers*

$27.3 \rightarrow 30$

$108.6 \times 27.3 \approx 100 \times 30$ *Use the rounded figures to find an approximation*

≈ 3000 *Check both answers are the same order of magnitude*

1. Number

YOUR NOTES
⬇

1.2 SETS & VENN DIAGRAMS

1.2.1 SET NOTATION & VENN DIAGRAMS

What do I need to know?

- You'll be drawing **Venn diagrams** so make sure you are familiar with those first
- **Notation**

ξ is the **universal** set (the set of **everything**)a ∈ B means a is an **element** of B (a is in the set B)

A ∩ B means the **intersection** of A and B (the **overlap** of A **and** B)

A ∪ B means the **union** of A and B (**everything** in A **or** B **or** both)

A' is "**not** A" (everything **outside** A)

Sets can be written as a list of **elements** (**members**) or described in words – either way curly brackets are used:

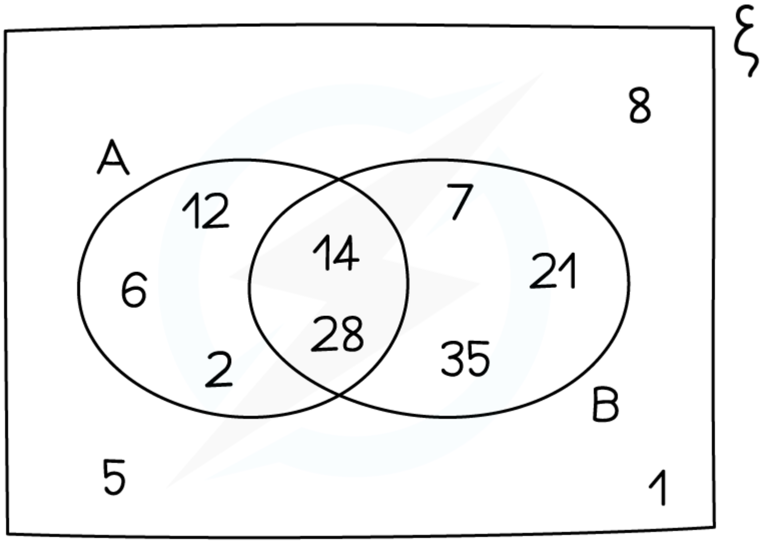
- A={3, 6, 9, 12, 15, 18}
- A={Multiples of 3 less than 20}

Worked Example

1. Use the Venn diagram below to answer the following questions.

1. Number

YOUR NOTES



Copyright © Save My Exams. All Rights Reserved

(a) Write down all the members of the set A,

(b) Describe in words the members of the set B,

(c) Write down all the members of the set $A' \cap B$,

(d) Suggest, with justification, where the element 49 could go within the Venn diagram,

(e) If one of the numbers in the Venn diagram is chosen at random, find the probability that the number is in the set $A \cup B$.

1. Number

YOUR NOTES
⬇

(a)

2, 6, 12, 14, 28

They don't have to be in order but make sure you get them all!

(b)

Multiples of 7 less than 40

(c)

$A' \cap B = \{7, 21, 35\}$

Draw a quick sketch if and shade the area(s) required if unsure

(d)

49 could go in set B as these are multiples of 7.

There are alternative answers – especially if there is a limit to how high the multiples of 7 in set B can be (see answer (b)!)

(e)

Number of elements in the set $A \cup B$ is 8

Number of elements in total 11

P (randomly chosen number is in set $A \cup B$) = $\frac{8}{11}$

1. Number

YOUR NOTES
⬇

1.3 CONVERSION

1.3.1 RECURRING DECIMALS

What are recurring decimals?

- A rational number is any number that can be written as an integer (whole number) divided by another integer
- When you write a **rational number as a decimal** you either get a decimal that stops (eg ¼ = 0.25) or one that recurs (eg ⅓ = 0.333333...)
- The recurring part can be written with a dot (or dots) over it instead as in the example below

What do we do with recurring decimals?

- Normally, you will be asked to write a recurring decimal as a fraction in its lowest terms
- To do this:
  Write out a few decimal places... ...and then:

1. Write as **f = ...**
2. **Multiply by 10** repeatedly until two lines have the same decimal part
3. **Subtract** those two lines
4. **DIVIDE to get f** = ... (and cancel if necessary to get fraction in lowest terms)

eg Write $0.\dot{3}\dot{7}$ *as a fraction in its lowest terms.*

Write out a few decimal places :

$$0.\dot{3}\dot{7} = 0.3737373737\ldots$$

1. $f = 0.3737373737\ldots$

2. $100f = 37.37373737\ldots$

3. *$f = \ldots$ and $100f = \ldots$ have the same decimal part so we subtract those :*

$$100f - f = 37.37373737\ldots - 0.3737373737..$$

$$99f = 37$$

4. $\div 99: \quad f = \frac{37}{99}$

(No cancelling is necessary in this case as it is already in its lowest terms)

1. Number

YOUR NOTES
⬇

Worked Example

Write $0.4\dot{2}\dot{7}$ *as a fraction in its lowest terms.*

Write out a few decimal places :

$0.4\dot{2}\dot{7} = 0.42727272727\ldots$

1. $f == 0.42727272727\ldots$

2. $10f = 4.2727272727\ldots$

$100f = 42.727272727\ldots$

$1000f = 427.27272727\ldots$

3. *Here* $10f = \ldots$ *and* $1000f = \ldots$ *have the same decimal part so we subtract those :*

$1000f - 10f = 427.27272727\ldots - 4.2727272727..$

$990f = 423$

4. $\div 990 :\quad f = \frac{423}{990}$

Both 423 *and* 990 *are multiples of* 9, *so cancel to get the fraction into its lowest terms :*

$f = \frac{423}{990} = \frac{423 \div 9}{990 \div 9} = \frac{47}{110}$

1. Number

YOUR NOTES
⬇

1.4 ORDER BY SIZE

1.4.1 ORDER BY SIZE

Putting numbers into order – isn't this easy?

- Yes! Until the numbers start including a mixture of **fractions**, **decimals** and/or **percentages**
- There could also be numbers written as **powers/indices** and (square/cube) **roots** included too
- Then there is the use of mathematical inequality symbols too:
  - ≠ **not equal** to
  - **< less than**
  - **≤ less than or equal to**
  - **> greater than**
  - **≥ greater than or equal to**

1. Number

YOUR NOTES

NUMBER FORMATS

FRACTIONS: e.g. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5},$

DECIMALS: e.g. $0.2, 0.03, 0.\dot{4}$

RECURRING
$0.\dot{4} = 0.444444...$

PERCENTAGES: e.g. 50%, 25%, 10%, 0.2%

POWERS/INDICES: e.g. $2^4, 3^2, 4^{-1}, 9^{\frac{1}{2}}$

POWER OR INDEX

ROOTS: e.g. $\sqrt{49}, \sqrt{36}, \sqrt[3]{8}, \sqrt[3]{64}$

SQUARE ROOT SYMBOL, CAN ALSO BE $\sqrt[2]{49}$

CUBE ROOT SYMBOL

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES ⬇

- To arrange numbers in order they all need to be written in the same **format**
- Usually, **decimals** are the easiest to compare
- Before continuing make sure you can convert:
  - **fractions to decimals**
  - **percentages to decimals**

CHANGE TO DECIMALS

FRACTIONS TO DECIMALS

- MANY SHOULD BE FAMILIAR:

$\frac{1}{2} = 0.5$ $\frac{1}{5} = 0.2$ $\frac{1}{10} = 0.1$

$\frac{1}{4} = 0.25$ $\frac{3}{4} = 0.75$

$\frac{1}{3} = 0.\dot{3}$ $\frac{2}{3} = 0.\dot{6}$

- IF UNFAMILIAR, DIVIDE NUMERATOR BY DENOMINATOR:

$\frac{5}{8}$ $8\,\overline{)\,5.{}^{5}0\,{}^{2}0\,{}^{4}0}$ with answer 0.625

ADD ZEROES AS NEEDED
USE SHORT DIVISION

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES

PERCENTAGES TO DECIMALS

- AGAIN, MANY SHOULD BE FAMILIAR

$50\% = 0.5$	$10\% = 0.1$
$25\% = 0.25$	$20\% = 0.2$
$75\% = 0.75$	$1\% = 0.01$

- IF UNFAMILIAR, DIVIDE BY 100 – MOVE DIGITS TWO PLACES RIGHT

$37\% = 37 \div 100 = 0.37$

$9\% = 9 \div 100 = 0.09$

BE CAREFUL WITH PERCENTAGES UNDER 10%! 0.9 IS 90%

$12.3\% = 12.3 \div 100 = 0.123$

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES ⬇

- Also make sure you can evaluate ...
  - **powers/indices** (eg $\mathbf{4^2 = 4 \times 4 = 16}$)
  - common **square roots** and **cube roots** (eg $\mathbf{\sqrt{4} = 2}$)

EVALUATING POWERS/INDICES

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$5^3 = 5 \times 5 \times 5 = 125$$

$$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

NEGATIVE POWERS MEAN "1 OVER"

$$9^{\frac{1}{2}} = \sqrt{9} = 3$$

FRACTIONAL POWERS MEAN ROOTS – SEE BELOW

EVALUATING SQUARE & CUBE ROOTS

$$\sqrt{25} = 5$$

$$5 \times 5 = 25$$

$\sqrt{25}$ IS LIKE ASKING "WHICH NUMBER MULTIPLIED BY ITSELF IS 25"?

$$\sqrt[3]{27} = 3$$

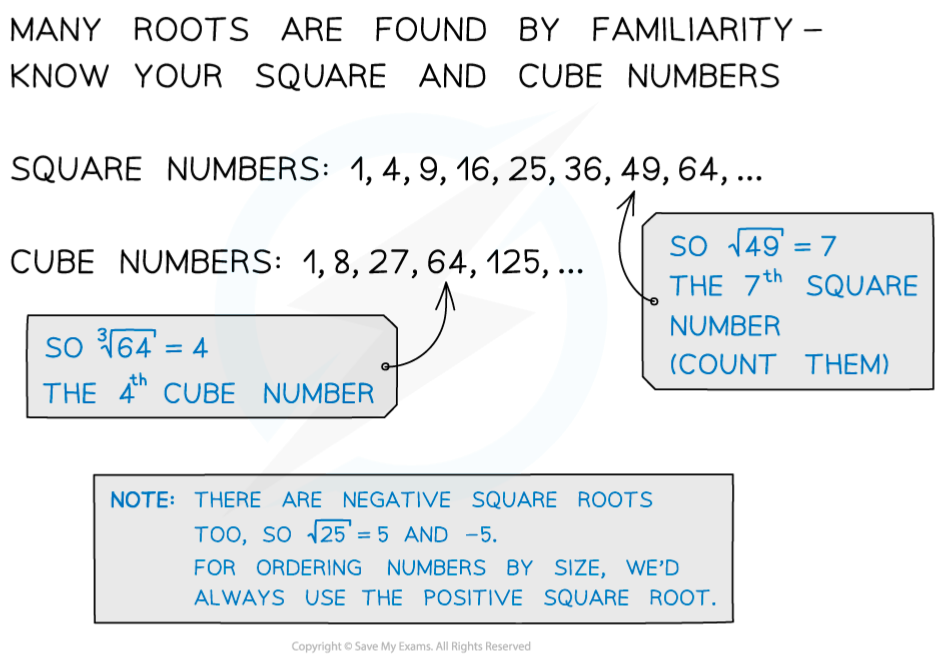
$$3 \times 3 \times 3 = 27$$

$\sqrt[3]{27}$ IS LIKE ASKING "WHICH NUMBER MULTIPLIED BY ITSELF, AND AGAIN, IS 27"?

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES ⬇



- For more awkward cases you may need to use a calculator

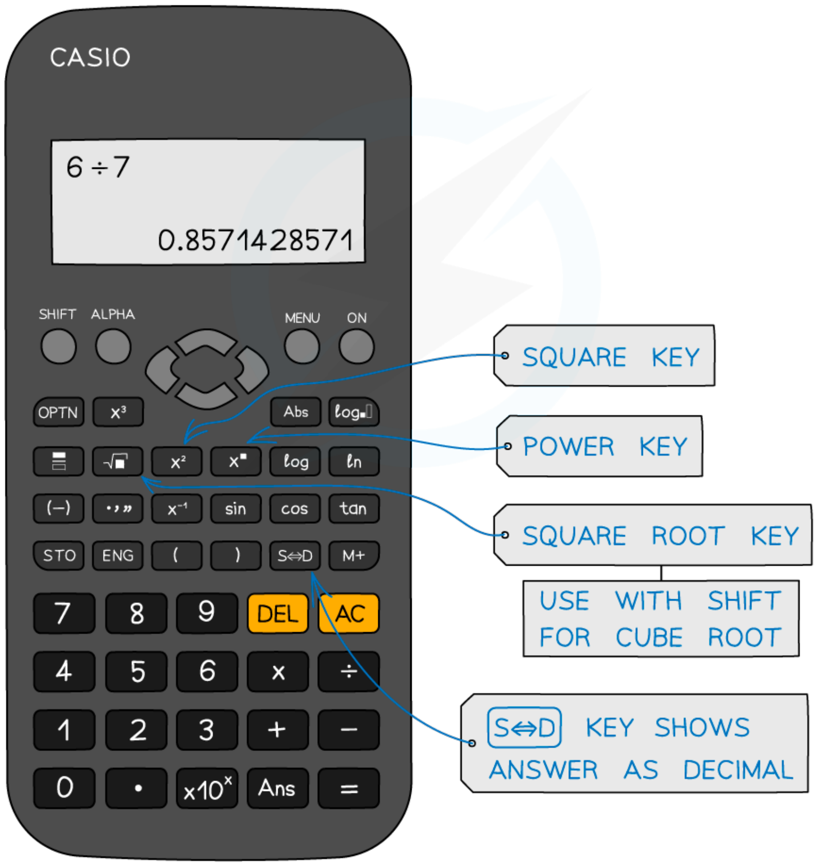
1. Number

YOUR NOTES
⬇

USING A CALCULATOR

... IF ALLOWED!

e.g. $\frac{6}{7}$ S⇔D BUTTON MAY BE NEEDED



Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES
⬇

How do I put decimals in order?

- The easiest way is to compare the numbers digit by digit
  - Write the numbers one underneath the other, in neat, lined up columns
  - Reveal each number one digit at a time, from the left (ie highest place value first)
  - Make a note of the order as you go

e.g. WRITE THE FOLLOWING IN ORDER OF SIZE, STARTING WITH THE SMALLEST. (DO NOT USE A CALCULATOR)

$0.31 \quad \frac{2}{3} \quad \sqrt{0.09} \quad 32\% \quad 3^{-1}$

STEP 1: CONVERT TO DECIMALS...

0.31 — ALREADY A DECIMAL

$\frac{2}{3} = 0.666\,666\ldots \quad (0.\dot{6})$ — RECOGNISE

$\sqrt{0.09} = 0.3$ — RECOGNISE FROM $\sqrt{9} = 3$

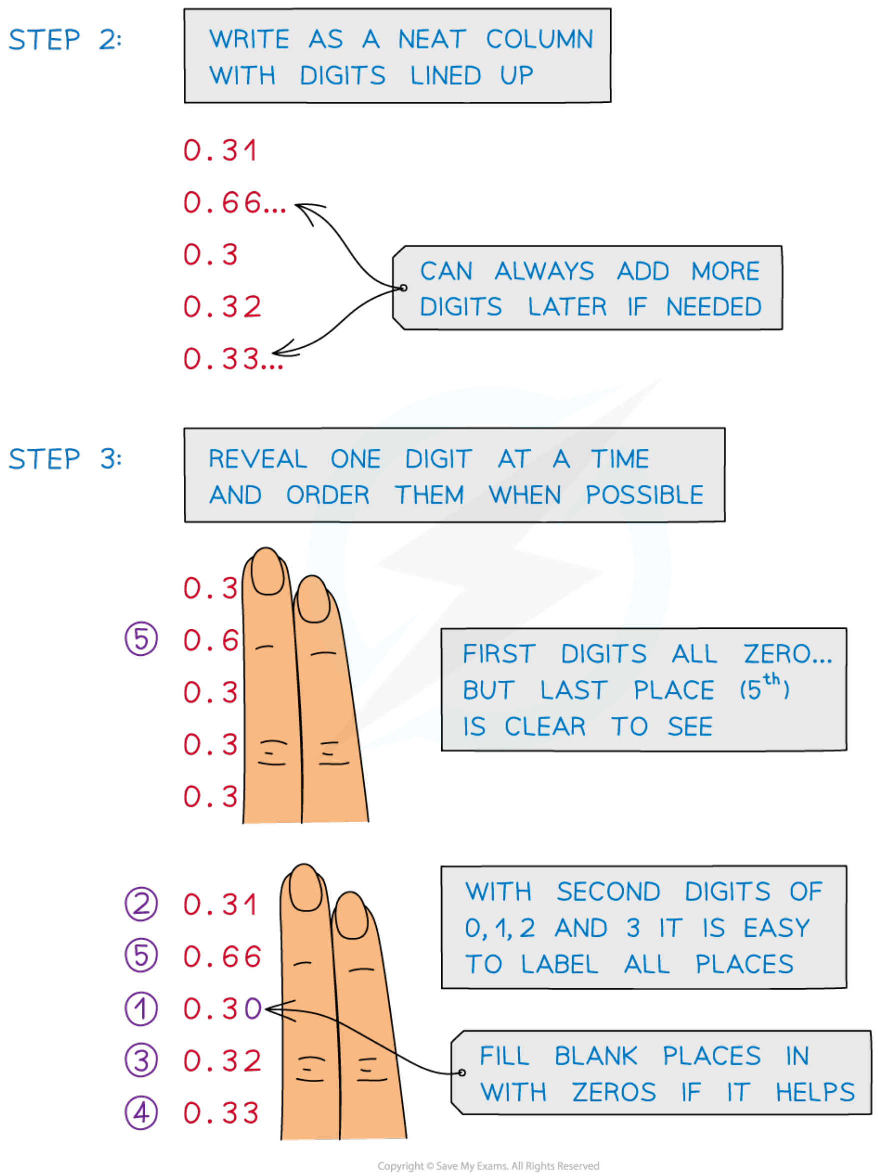
$32\% = 0.32$ — $32 \div 100$

$3^{-1} = \frac{1}{3} = 0.333\,333\ldots \quad (0.\dot{3})$ — NEGATIVE POWER MEANS "1 OVER"

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES



1. Number

YOUR NOTES

STEP 4: WRITE DECIMALS IN ORDER

① ② ③ ④ ⑤

0.3, 0.31, 0.32, 0.33…, 0.66…

STEP 5: WRITE FINAL ANSWER USING NUMBERS IN THEIR ORIGINAL FORMAT

$\sqrt{0.09}$ 0.31 32% 3^{-1} $\frac{2}{3}$

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES ⬇

Summary

- STEP 1 Convert all values to decimals
- STEP 2 Write as a neat column with digits lined up
- STEP 3 Reveal one digit at a time and order them when possible
- STEP 4 Write decimals in order
- STEP 5 Write final answer using numbers in their original format

Exam Tip

Questions may use the words **ascending** or **descending** to describe the order.

Ascending means "**upwards**" – so **increasing** in size, ie. start with the smallest, finish with the largest. **Descending** means "**downwards**" – so **decreasing** in size,

ie. start with the largest, finish with the smallest.

Worked Example

1. Number

YOUR NOTES
⬇

?

(a) Insert the correct symbol, either $=$, $\leq$, $\geq$, $<$ or $>$ between each pair of numbers.

(i) -3 5

(ii) 4.2 4.06

(iii) $\frac{4}{5}$ 0.6

(iv) 75% $\frac{3}{4}$

(b) Write the following in order of size, starting with the smallest.

2^{-1} 90% $\frac{4}{5}$ 0.501 $\sqrt{0.36}$

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES
⬇

a) i) $-3 < 5$

USE A NUMBER LINE IF UNSURE WITH NEGATIVES

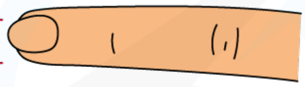
-3 -2 0 2 4 5

ii) $4.2 > 4.06$

PLACE VALUE IMPORTANT WITH DECIMALS

USE "REVEAL" METHOD:

4

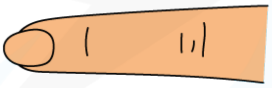


4

... THEN ...

CAN SEE THIS WILL BE BIGGER

4 . 2



4 . 0

iii) $\frac{4}{5} > 0.6$ $\frac{3}{5} = 0.6$

RECOGNISE $\frac{1}{5} = 0.2$

SO $\frac{4}{5} = 0.2 \times 4 = 0.8$

iv) $75\% = \frac{3}{4}$ $75\% = \frac{75}{100} = \frac{15}{20} = \frac{3}{4}$

YOU SHOULD RECOGNISE THESE ARE EQUIVALENT

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES

b) STEP 1: $\boxed{\text{CONVERT TO DECIMALS}}$

$2^{-1} = \frac{1}{2^1} = \frac{1}{2} = 0.5$ NEGATIVE POWER MEANS "1 OVER"

$90\% = 90 \div 100 = 0.9$ $\% \xrightarrow[\div 100]{}$ DECIMAL

$\frac{4}{5} = \frac{1}{5} \times 4 = 0.2 \times 4 = 0.8$ RECOGNISE $\frac{1}{5} = 0.2$

0.501 ALREADY A DECIMAL!

$\sqrt{0.36} = 0.6$ $\sqrt{36} = 6$

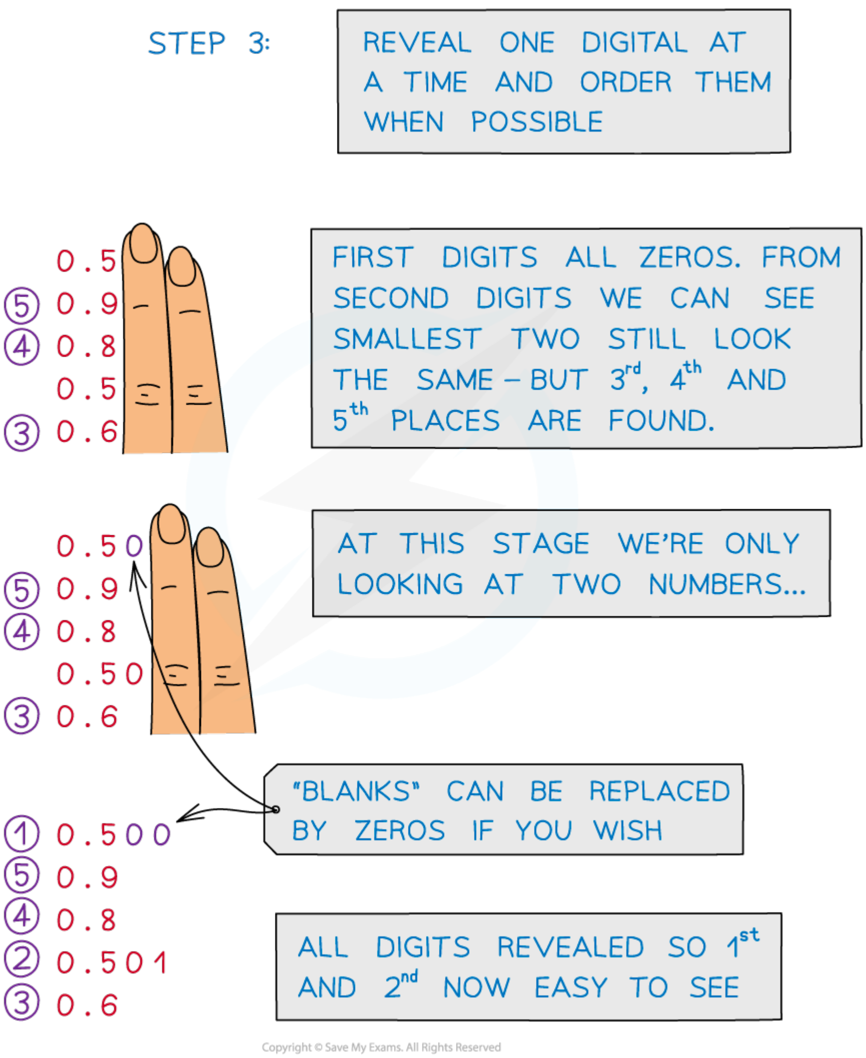
STEP 2: $\boxed{\text{WRITE AS NEAT COLUMN WITH DIGITS LINED UP}}$

0 . 5
0 . 9
0 . 8
0 . 5 0 1
0 . 6

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES
⬇



1. Number

YOUR NOTES

STEP 4: WRITE DECIMALS IN ORDER

① 0.5, ② 0.501, ③ 0.6, ④ 0.8, ⑤ 0.9

STEP 5: WRITE FINAL ANSWER USING NUMBERS IN THEIR ORIGINAL FORMAT

2^{-1}, 0.501, $\sqrt{0.36}$, $\frac{4}{5}$, 90%

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES
⬇

1.5 STANDARD FORM

1.5.1 STANDARD FORM - BASICS

What is standard form?

- Standard Form (sometimes called Standard Index Form) is a way of writing very big and very small numbers using powers of 10

Why do we use standard form?

- Writing big (and small) numbers in Standard Form allows us to:
  - write them more neatly
  - compare them more easily
  - and it makes things easier when doing calculations

How do we use standard form?

- Using Standard Form numbers are always written in the form:

  $\mathbf{a \times 10^n}$
- The rules:
  - $\mathbf{1 \le a < 10}$ so there is one non-zero digit before the decimal point
  - $\mathbf{n > 0}$ for LARGE numbers – how many times a is multiplied by 10
  - $\mathbf{n < 0}$ for SMALL numbers – how many times a is divided by 10
  - **Do calculations on a calculator** (if allowed)
    Otherwise follow normal rules (including indices) but adjust answer to fit Standard Form (move decimal point and change n

1. Number

YOUR NOTES

Worked Example

1. *Without using a calculator, multiply* 5×10^{18} *by* 7×10^{-4}.

   Give your answer in standard form.

$$5 \times 10^{18} \times 7 \times 10^{-4} = 5 \times 7 \times 10^{18} \times 10^{-4}$$ *Separate into numbers and powers of 10*

$$= 35 \times 10^{18+(-4)}$$ *Use Laws of Indices on the powers of 10*

$$= 35 \times 10^{14}$$

$$= 3.5 \times 10 \times 10^{14}$$ *Write in standard form (this isn't as 35 > 10)*

$$= 3.5 \times 10^{15}$$

2. *Use your calculator to find* $\frac{1.275 \times 10^{6}}{3.4 \times 10^{-2}}$.

   Write your answer in the form $A \times 10^{n}$, *where* $1 \leq A < 10$ *and n is an integer.*

$$\frac{1.275 \times 10^{6}}{3.4 \times 10^{-2}} = 37\ 500\ 000$$ *Your calculator will not necessarily give the answer in standard form. Copy the digits, especially those zeros, carefully!*

$$= 3.75 \times 10^{7}$$

1. Number

YOUR NOTES
⬇

1.5.2 STANDARD FORM - HARDER

Standard form – harder questions

- Make sure you are familiar with:
  Roots & Indices – Basics
  Standard Form – Basics
- Harder problems often combine algebra and laws of indices with numbers written in standard form
- Other areas of mathematics may be used and questions are often in context
- Units may be muddled up to make things trickier

1. Number

YOUR NOTES

Worked Example

1. *Given that* $x = 25 \times 10^{4n}$ *write* $x^{\frac{3}{2}}$ *in standard form.*

$$x^{\frac{3}{2}} = \left(25 \times 10^{4n}\right)^{\frac{3}{2}}$$

$$= 25^{\frac{3}{2}} \times \left(10^{4n}\right)^{\frac{3}{2}}$$ *Apply the power to both factors in the bracket*

$$= 25^{\frac{3}{2}} \times 10^{4n \times \frac{3}{2}}$$ *Use the Laws of Indices on the power of 10*

$$= 125 \times 10^{6n}$$ *Simplify*

$$= 1.25 \times 10^{2} \times 10^{6n}$$

Write in Standard Form (this isn't as 125 > 10)

$$= 1.25 \times 10^{6n+2}$$

2. *The diameter of a hydrogen atom is* 1.06×10^{-10} *m.*

The nucleus of a hydrogen atom has diameter 2.40×10^{-13} *cm.*

The diameter of a hydrogen atom is k times the size of the nucleus of a hydrogen atom.

Find the value of k correct to three significant figures.

First spot that we have a mixture of m and cm!

$$2.40 \times 10^{-13} \ cm = 2.40 \times 10^{-13} \div 100 \ m$$

Change one (doesn't matter which) so units match

$$= 2.40 \times 10^{-15} \ m$$

$$k = 1.06 \times 10^{-10} \div 2.40 \times 10^{-15} = 44\ 166.6666\ ...$$

To find k divide the (diameter of) the atom by the nucleus

$k = 44\ 200$ *Final answer for k rounded to 3 significant figures*

1. Number

YOUR NOTES

1.6 WORKING WITH FRACTIONS

1.6.1 MIXED NUMBERS & TOP HEAVY FRACTIONS

What are mixed numbers & top heavy fractions?

- A mixed number has a whole number (integer) part and a fraction
  eg. 3 3/4 means "three and three quarters"
- A top heavy fraction – also called an **improper fraction** – is one with the top (numerator) bigger than the bottom (denominator)
  eg. 15/4 means "fifteen quarters"

Turning mixed numbers into top heavy fractions

1. **Multiply** the big number by the bottom (denominator)
2. **Add** that to the top (numerator)
3. Write as **top heavy** fraction

Turning top heavy fractions into mixed numbers

- **Divide** the top by the bottom (to get a whole number and a remainder)
- The **whole number** is the big number
- The **remainder** goes over the bottom

Worked Example

1. *Write* $3\frac{3}{4}$ *as a top heavy fraction.*

$3 \times 4 = 12$	1 − *multiply the big number by the bottom*
$12 + 3 = 15$	2 − *add to the top*
$3\frac{3}{4} = \frac{15}{4}$	3 − *your final answer should be top heavy*

2. *Write* $\frac{17}{5}$ *as a mixed number.*

$17 \div 5 = 3$ *remainder* 2	4 − *divide the top by the bottom*
$\frac{17}{5} = 3\frac{2}{5}$	5 − *final answer is a mixed number*

1. Number

YOUR NOTES
⬇

1.6.2 ADDING & SUBTRACTING FRACTIONS

Dealing with mixed numbers

- Always turn Mixed Numbers into Top Heavy Fractions before doing calculations

Adding & Subtracting

- Adding and subtracting are treated in exactly the same way:

1. Find the **lowest** common bottom (denominator)
2. Write fractions with the **new** bottoms
3. **Multiply** tops by same as bottoms
4. Write as a **single** fraction (take care if subtracting)
5. **Simplify** the top
6. Turn Top Heavy Fractions back into Mixed Numbers (if necessary)

Worked Example

1. *Work out* $3\frac{3}{4} + \frac{3}{8}$, *giving your answer as a mixed number.*

$3\frac{3}{4} = \frac{3\times4+3}{4} = \frac{15}{4}$ — *First turn the mixed number* $3\frac{3}{4}$ *into a top heavy fraction*

1 – *Spot that the LOWEST common denominator is* 8

$\frac{15}{4} + \frac{3}{8} = \frac{15\times2}{8} + \frac{3}{8}$ — 2, 3 – *Note in this case* $\frac{3}{8}$ *remains unchanged*

$= \frac{15\times2+3}{8}$ — 4

$= \frac{33}{8}$ — 5 – *Be careful, this is NOT your final answer!*

$3\frac{3}{4} + \frac{3}{8} = 4\frac{1}{8}$ — 6 – *In this case the answer has to be a mixed number*

1. Number

YOUR NOTES
⬇

1.6.3 MULTIPLYING & DIVIDING FRACTIONS

Dealing with mixed numbers

- Always turn Mixed Numbers into Top Heavy Fractions before doing calculations

Dividing fractions

- Never try to divide fractions
- Instead “flip’n’times”
- So “**÷a/b**” becomes “**×b/a**”
- And follow the rules for multiplying…

Multiplying fractions

- **Simplify** by factorising and cancelling (ignore the × between the fractions)
- Multiply the **tops**
- Multiply the **bottoms**
- **Simplify** by factorising and cancelling (if you missed something earlier)
- Turn Top Heavy Fractions back into Mixed Numbers (if necessary)

Worked Example

1. *Divide* $3\frac{1}{4}$ *by* $\frac{3}{8}$, *giving your answer as a mixed number.*

$3\frac{1}{4} = \frac{3\times4+1}{4} = \frac{13}{4}$ — *First turn the mixed number* $3\frac{1}{4}$ *into a top heavy fraction*

$\frac{13}{4} \div \frac{3}{8} = \frac{13}{4} \times \frac{8}{3}$ — *A division question – so "flip 'n' times"*

Now follow the rules for multiplying

$\frac{13}{4} \times \frac{8}{3} = \frac{13}{4} \times \frac{4\times2}{3}$ — 1. *It is not essential to write* 8 *as* 4×2 *but you should spot it*

$= \frac{13}{1} \times \frac{2}{3}$ — *Cancel the* 4*'s*

$= \frac{26}{3}$ — 2, 3

No need to factorise and cancel again (all was done in stage 1)

$3\frac{1}{4} \div \frac{3}{8} = 8\frac{2}{3}$ — *In this case the answer has to be a mixed number*

© copyright Save My Exams

1. Number

YOUR NOTES
⬇

1.7 BOUNDS

1.7.1 BOUNDS & ERROR INTERVALS - BASICS

What are bounds?

- Bounds are the smallest – the **Lower Bound (LB)** – and largest – the **Upper Bound (UB)** – numbers that a **rounded** number can lie between

How do we find bounds?

- The basic rule is "Half Up, Half Down"
- More formally:
  - UPPER BOUND – add on half the degree of accuracy
  - LOWER BOUND – take off half the degree of accuracy
  - ERROR INTERVAL: LB ≤ **x** < UB
- Note:
  It is very tempting to think that the Upper Bound should end in a 9, or 99, etc. but if you look at the Error Interval – LB ≤ x < UB – it does NOT INCLUDE the Upper Bound so all is well

Worked Example

1. *The length of a road*, l, *is given as* $l = 3.6$ *km*, *correct to* 1 *decimal place*.
   Find the Lower and Upper Bounds for l.

$UB \text{ for } l = 3.6 + 0.05$

$= 3.65$

1 – *The degree of accuracy is* 1 *decimal place* (*or* 0.1) – *half is* 0.05.

$LB \text{ for } l = 3.6 - 0.05 = 3.55$

$= 3.55$

2 – *As for the upper bound half the degree of accuracy is* 0.05.

3 – *This question doesn't require it but if the error interval had been asked for the final answer would be* $3.55 \le l < 3.65$.

1. Number

YOUR NOTES
⬇

1.7.2 CALCULATIONS USING BOUNDS

What are bounds?

- Bounds are the smallest – the **Lower Bound (LB)** – and largest – the **Upper Bound (UB)** – numbers that a **rounded** number can lie between

How do we find bounds?

- The basic rule is "Half Up, Half Down"
- More formally:
  - UPPER BOUND – add on half the degree of accuracy
  - LOWER BOUND – take off half the degree of accuracy
  - ERROR INTERVAL: LB ≤ **x** < UB

Calculations using bounds

- Find bounds before calculating and then:
  - For FRACTIONS/DIVISION:
    UB = UB ÷ LB and
    LB = LB ÷ UB
  - Otherwise:
    UB = UB × UB etc.

Worked Example

1. Number

YOUR NOTES

1. *A room measures* $4m$ *by* $7m$, *where each measurement is made to the nearest metre.*
Find Upper and Lower Bounds for the area of the room.

$3.5 \le 4 < 4.5$	1, 2 − *First find the bounds for each dimension*
$6.5 \le 7 < 7.5$	*It is not essential to write these as an error intervals,*
	stating $LB = 3.5$, $UB = 4.5$, *etc is fine*
$Area\ LB = 3.5 \times 6.5$	*We do not have fractions so* 5 − $LB = LB \times LB$
$Area\ LB = 22.75\ m^2$	
$Area\ UB = 4.5 \times 7.5$	5 − $UB = UB \times UB$
$Area\ UB = 33.75\ m^2$	

2. *David is trying to work out how many slabs he needs to buy in order to lay a garden path.*
Slabs are 50 *cm long, measured to the nearest* 10 *cm.*
The length of the path is 6 *m, measured to the nearest* 10 *cm.*
Find the maximum number of slabs David will need to buy.

$45 \le 50 < 55$	1, 2 − *First find the bounds for each dimension*
$0.45 \le 0.5 < 0.55$	*Change the units if necessary but do as a separate step*
$5.95 \le 6 < 6.05$	
$6.05 \div 0.45 = 13.44...$	*The maximum number of slabs will be the Upper Bound*
	4 − *This is a division calculation so use* $UB = UB \div LB$
Max no. of slabs = 14	*The context of the question means it is sensible for the final*
	answer to be a whole number − *but more than* 13.44...

1. Number

YOUR NOTES
⬇

1.8 RATIOS

1.8.1 RATIOS

What is a ratio?

- A **ratio** is a way of comparing one part of a whole to another
- A **ratio** can also be expressed as a **fraction** (of the whole)
- We often use a ratio (instead of a fraction) when we are trying to show how things are shared out or in any situation where we might use **scale factors**

How to work with ratios

1. Put what you know in **RATIO** form (use more than one line if necessary)
2. Add "extra bits" (eg Total, Difference, Sum) if you think they might be useful
3. Use **SCALE FACTORS** to complete lines
4. Pick out the ANSWER!

Exam Tip

One less obvious place to use Ratios is when dealing with Currency Conversion problems – see Exchange Rates.

1. Number

YOUR NOTES ⬇

Worked Example

1. *Jemima is 5 years old, Karl is 8 and Lonnie is 11. They are sharing some money in the ratio of their ages. If Karl gets £5.60, what is the total amount of money they are sharing?*

$Total\ of\ ages = 5 + 8 + 11 = 24$

	J	:	K	:	L	:	Total
Age	5	:	8	:	11	:	24
Money	j	:	5.60	:	l	:	t

1, 2 − *Put the given information into ratio form − and in this case it may be useful to look at a total column (since a total amount is what we've been asked to find)*
Note the use of (useful) letters to mark unknown values in the table − we may use none, some or all of them later

$Scale\ Factor = \frac{5.6}{8} = 0.7$

3 − *To find t (and j and l too!) we need to compare the age row with the money row.*

$\frac{t}{24} = 0.7$

The same scale factor applies to the Total column

$t = 0.7 \times 24$

$t = £16.80$

Remember your final answer is money in this case

1. Number

YOUR NOTES

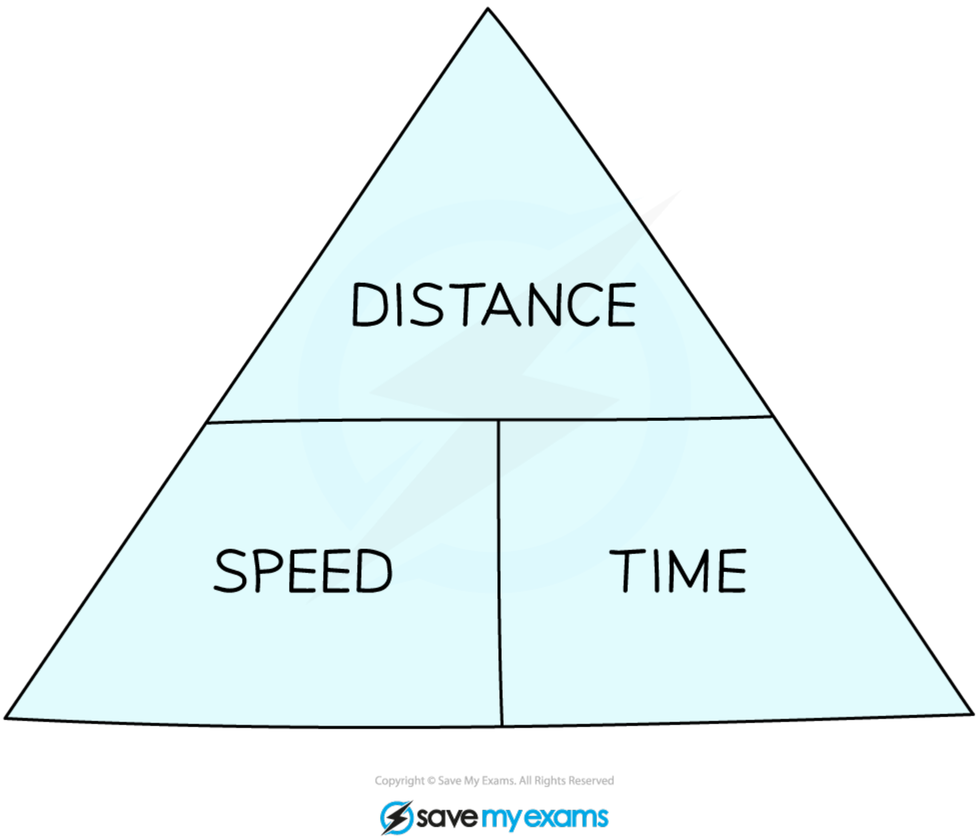
1.8.2 SPEED, DENSITY & PRESSURE

What connects speed, density & pressure?

- Speed, sensity and pressure are all examples of variables which are calculated by dividing one thing by another:
  Speed = Distance ÷ Time
  Density = Mass ÷ Volume
  Pressure = Force ÷ Area
- In that respect they can all be treated in the same way

Doing speed, density & pressure questions

1. Use **UNITS** in Q (or other info) to write down a formula
2. Create BLUE TRIANGLE:
e.g. for Speed, Distance and Time



1. Number

YOUR NOTES

3. For each part of the Q write down
what you know
what you want to know
4. Use Blue Triangle to **REARRANGE** formula (if necessary)
5. **SUBSTITUTE** numbers and SOLVE

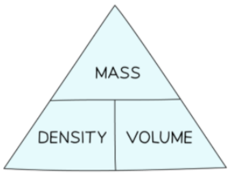
Worked Example

1. *The density of pure gold is* $19.3\ g/cm^3$.

What is the volume of a gold bar which has a mass of $454\ g$?

$Density = \frac{Mass}{Volume}$ 1 − *The units of density are given as* g/cm^3 *which is* $Mass \div Volume$

Note you may see the units of density written as $g\ cm^{-3}$



2 − *Create your blue triangle*

$Density = 19.3\ g/cm^3,\ Mass = 454\ g$ 3 − *Write down what you know ...*

To find : Volume ... *and what you are trying to find*

$Volume = \frac{Mass}{Density}$ 4 − *Covering up Volume in the blue triangle*

leaves "Mass over Density"

$Volume = \frac{454}{19.3}$ 5 − *Substitute numbers and solve*

$Volume = 23.5\ cm^3$ 1 *decimal place is sensible to round to given*

19.3 *was used in the question.*

1. Number

YOUR NOTES
⬇

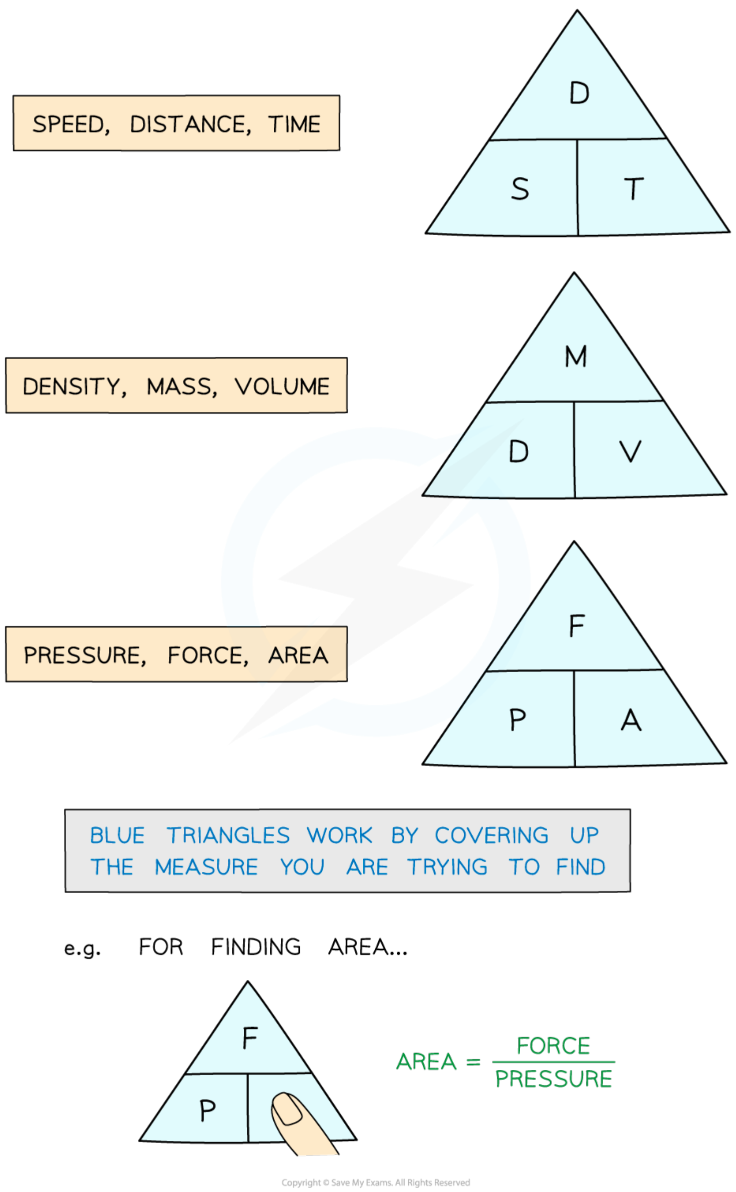
1.8.3 SPEED, DENSITY & PRESSURE - HARDER

What are speed, density and pressure?

- **Speed**, **density** and **pressure** are compound measures – they are made from other measures
  - **Speed** is related to the measures **distance** and **time**
  - **Density** is related to **mass** and **volume**
  - **Pressure** is related to **force** and **area**
- The relationship between each of these sets of measures follows the same pattern – what we refer to as "blue triangles"

1. Number

YOUR NOTES



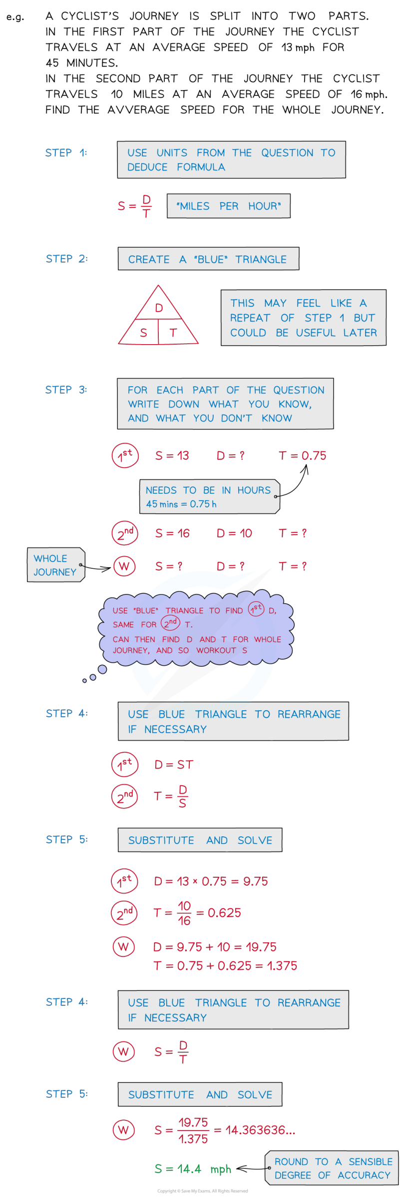
- If you do not remember the blue triangles – do not worry, these can often be deduced from information given in the question – see the examples below
- It is important you understand and can do the basic questions with speed, density and pressure

1. Number

YOUR NOTES
⬇

Speed, distance and time harder problems

- **Speed** is commonly measured in **metres per second** (**m/s**) or **miles per hour** (**mph**)
  - There are other possibilities such as **kilometres per hour** (**kmph**)
  - The units indicate **speed** is **distance per time**
    ie **speed = distance ÷ time**
- “**Speed**” (in this formula) means “**average speed**”
- In harder problems there are often two journeys – or two parts to one longer journey

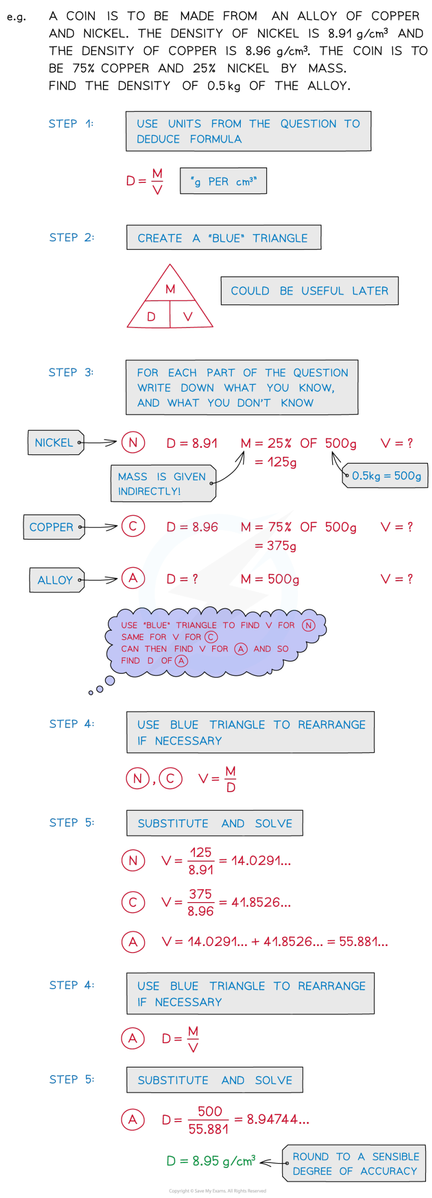


1. Number

YOUR NOTES
⬇

Density, mass and volume harder problems

- Density is usually measured in **grams per cubic centimetre** (**g/cm^3**) or **kilograms per cubic metre** (**kg/m^3**)
    - The units indicate that **density** is **mass per volume** ie **density = mass ÷ volume**
- In harder problems there are often two metals (alloys), liquids or gases that have been combined rather than working with a single substance



1. Number

YOUR NOTES

Pressure, force and area harder problems

- Pressure is usually measured in Newtons per square metre (**N/m²**)

  The units of pressure are often called **Pascals** (**Pa**) rather than **N/m²**

  - The units indicate that **pressure** is **force per area**
    ie **pressure = force ÷ area**

- Remember that **weight** is a **force;** it is different to **mass**

Exam Tip

Do look out for a mixture of units:

• **Time** can be given as **minutes** but common phrases like "half an **hour**" (ie 30 minutes) could also be used in the same question.

• Any mixed units should be those in common use and easy to convert

g to **kg** (and vice versa)

m to **km** (and vice versa)

1. Number

YOUR NOTES ⬇

Worked Example

Nichrome is a metal alloy, made from nickel and chromium, one use of which is for the heating element in toasters.

The density of nickel is 8.91 g/cm^3 and the density of chromium is 7.19 g/cm^3.

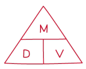
To create nichrome, nickel and chromium are mixed in the ratio 4:1.

Find the density of 2 kg of nichrome.

STEP 1: USE UNITS FROM THE QUESTION TO DEDUCE FORMULA

$D = \frac{M}{V}$ g/cm^3 IS MASS PER VOLUME

STEP 2: CREATE A 'BLUE' TRIANGLE



COULD BE USEFUL LATER

STEP 3: FOR EACH PART OF THE QUESTION WRITE DOWN WHAT YOU KNOW, AND WHAT YOU DON'T KNOW

NICKEL N: D = 8.91 M = ? V = ?
CHROMIUM C: D = 7.19 M = ? V = ?
NICHROME Ni: D = ? M = 2000 V = ?

UNITS! 2kg = 2000g

FIND THE MASS OF N & C BY USING THE GIVEN RATIO

N : C : Ni
4 : 1 : 5
1600 : 400 : 2000

STEP 4: USE BLUE TRIANGLE TO REARRANGE IF NECESSARY

$V = \frac{M}{D}$

STEP 5: SUBSTITUTE AND SOLVE

N: $V = \frac{1600}{8.91} = 179.5735...$

C: $V = \frac{400}{7.19} = 55.6328...$

DON'T ROUND TOO EARLY – USE CALCULATOR MEMORY

Ni: $V = 179.5735... + 55.6328...$
$V = 235.2063...$

STEP 4: $D = \frac{M}{V}$

STEP 5: $D = \frac{2000}{235.2063...} = 8.5031...$

DENSITY OF 2kg OF NICHROME IS 8.50 g/cm^3 TO 2 DECIMAL PLACES

QUESTION GIVES OTHER DENSITIES TO 2dp SO IT MAKES SENSE TO DO THE SAME

1. Number

YOUR NOTES

ALTERNATIVE SOLUTION

THE BLUE TRIANGLE FORMULA WORKS ON TWO ASSUMPTIONS:

- NO MASS OR VOLUME IS LOST WHEN THE TWO METALS ARE COMBINED
- DENSITY IS CONSTANT, NO MATTER HOW MUCH OF THE SUBSTANCE THERE IS

THE SECOND BULLET POINT MEANS THE 2kg IS NOT NEEDED – JUST USE THE RATIO AS MASSES

STEP 3:

N : $D = 8.91$ $M = 4$ $V = ?$

C : $D = 7.19$ $M = 1$ $V = ?$

Ni : $D = ?$ $M = 5$ $V = ?$

STEP 5:

N : $V = \frac{4}{8.91} = 0.44893...$

C : $V = \frac{1}{7.19} = 0.13908...$

Ni : $V = 0.44893... + 0.13908...$

$V = 0.58801...$

STEP 4: $D = \frac{M}{V}$

STEP 5: $D = \frac{5}{0.58801...}$

$D = 8.50$ g/cm^3 (2 dp)

IF YOU UNDERSTOOD THIS ALTERNATIVE SOLUTION, SEE IF YOU CAN APPLY SIMILAR SKILLS TO THE DENSITY PROBLEM IN THE REVISION NOTES ABOVE

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES
⬇

1.9 PERCENTAGES

1.9.1 BASIC PERCENTAGES

What is a percentage?

- “Per-cent” simply means “ ÷ 100” (or “out of 100”)
- You can think of a percentage as a standardised way of expressing a fraction – by always expressing is “out of 100”
- That means it is a useful way of comparing fractions.
- Eg. ½ = 50% (50% means 50 ÷ 100)
  ⅖ = 40%
  ¾ = 75%

Things to remember:

- Decimal equivalent = percentage ÷ 100
- Percentage = decimal equivalent × 100
- To find “a percentage of A”: multiply by the decimal equivalent
- To find “A as a percentage of B”: do A ÷ B to get decimal equivalent

Exam Tip

You can always use the decimal equivalent instead of doing a more traditional percentage calculation:
For example, to find 35% of 80 you can just do 80 × 0.35 = 28
(rather than doing the more complicated calulation 80 × 35 ÷ 100)

1. Number

YOUR NOTES

Worked Example

$Jamal\ earns\ £1200\ for\ a\ job\ he\ does\ and\ pays\ his\ agent\ £150\ in\ commission.$

$Express\ his\ agent's\ commission\ as\ a\ percentage\ of\ Jamal's\ earnings.$

1. $Commission \div Earnings = 150 \div 1200$

   $= 0.125$

2. $You\ should\ recognise\ this\ as\ the\ decimal\ equivalent\ of\ 12.5\%$

   $(or\ do\ 0.125 \times 100 = 12.5\%\ if\ not\ so\ confident)$

1. Number

YOUR NOTES
⬇

1.9.2 PERCENTAGE INCREASES & DECREASES

How to increase or decrease by a percentage

- Identify “before” & “after” quantities
- Find percentage of “before” that we want:
  - Increase – add percentage to 100
  - Decrease – subtract percentage from 100
- Write down a statement connecting “before” and “after”:
  - “after is a percentage of before”
- Write down the statement as an equation using decimal equivalent
  - remember “is” means “=”
- Substitute and solve

Worked Example

Jennie earns £1200 *per week in her job.*

She is to receive a 5% *pay rise.*

Find her new weekly pay.

We can call the "before" Old Pay and the "after" New Pay

1. *We want* $100 + 5 = 105\%$

2. *New Pay is* 105% *of Old Pay*

The decimal equivalent of 105% *is* 1.05

3. $\text{New Pay} = 1.05 \times \text{Old Pay}$

4. $\text{New Pay} = 1.05 \times 1200$

$\text{New Pay} = £1260 \text{ per week}$

© copyright Save My Exams

1. Number

YOUR NOTES
⬇

1.9.3 REVERSE PERCENTAGES

What is a reverse percentage?

- A reverse percentage question is one where we are given **the value after a percentage increase or decrease** and asked to find the value before the change

How to do reverse percentage questions

- You should do these in exactly the same way as percentage increase & decrease questions!
- Identify “before” & “after” quantities
- FIND percentage we want:
  - Increase – ADD percentage to 100
  - Decrease – SUBTRACT from 100
- Write down a STATEMENT connecting “before” and “after”:
  - “after is a percentage of before”
- Write down the statement as an EQUATION using decimal equivalent
  - remember “is” means “=”
- SUBSTITUTE and SOLVE

Worked Example

Jennie now earns £31500 per year in her job.

She has recently had a 5% pay rise.

Find her annual pay before the pay rise.

We can call the "before" Old Pay and the "after" New Pay

1. *We want* $100 + 5 = 105\%$

2. *New Pay is 105% of Old Pay*

The decimal equivalent of 105% is 1.05

3. $New\ Pay = 1.05 \times Old\ Pay$

4. $31500 = 1.05 \times Old\ Pay$

Divide by 1.05 : $Old\ Pay = 31500 \div 1.05$

$Old\ Pay = £3000\ per\ year$

© copyright Save My Exams

1. Number

YOUR NOTES
⬇

1.10 USING A CALCULATOR

1.10.1 USING A CALCULATOR

Why the fuss about using a calculator?

- GCSE Mathematics goes beyond using the basic features of a calculator and explores many of the special functions of a scientific calculator
- It is important to get to know your calculator, the earlier you get one and learn about the scientific functions the better you will be at using them
- It's not just maths that uses these, some of the scientific functions can be used in science exams too

What do I need to know?

- The notes below apply to most if not all scientific calculators but the images are based on the Casio fx-83GTX
- The Casio fx-85GTX is the same model but also has solar power. Both are labelled "Classwiz" too but be careful here at there is a more advanced "Classwiz" calculator that is used at A level (fx-991EX)

1. Number

YOUR NOTES

The Casio fx-83GTX Classwiz

- Be aware if you have an old or very basic scientific calculator that they may work backwards
- For example, if you wanted to find sin (57) you would type 57 then press the sin button
- Modern calculators tend to work in the order in which we write things

1. Mode/setup

- Make sure you know how to change the mode of your calculator, especially if someone else has used it
- The "Angle Unit" needs to be degrees – normally indicated by a "D" symbol across the top of the display
- Make sure you can switch between "exact" answers (fractions, surds, in terms of π, etc) and "approximate" answers (decimals)
- Most calculators default to "Math" mode with the word Math written across the top of the display or using a symbol
- When in "Math" mode you can switch whatever is on the answer line between exact and decimals by pressing the "S-D" button

1. Number

YOUR NOTES

2. Templates

- These are largely the shortcut buttons – the fraction button, the square, cube and power buttons, square roots

Calculator shortcut buttons

3. Trigonometry (sin/cos/tan)

- Remember to use SHIFT (sometimes called 2nd or INV button) when finding angles
- When using these buttons you will find that before you type the angle the calculator automatically gives you an open bracket "(". You should get into the habit of making sure you use a closed bracket ")" after typing the angle in
- This is very important if there is something else to type in that comes after sin/cos/tan

4. Standard Form and π

- Find the $\times 10^x$ button and know how to use it
- Modern calculators display standard form in the way it is written
- Older models may use a small capital letter "$_E$" in place of $\times 10^x$ on the display line
- π is often near or under SHIFT with the standard form button

1. Number

YOUR NOTES

5. Memory

- The **ANS** (answer) button is very useful – especially when working with decimals in the middle of solutions that you should avoid rounding until your final answer
- ANS recalls the last answer the calculator calculated

6.Table

- If your calculator has a **table** function or mode, use it
- This can be extremely useful in those "complete the table of values and draw the graph" type questions

7. Brackets and negative numbers

- Use as you would in written mathematics
- Remember to use the (-) button for a negative number, not the subtract button

8. Judgement and special features

- The rule of thumb is to use your calculator to do one calculation at a time
- However, you can also make a judgement call on this as to how many marks are available in the question and whether a question asks you to "write down all the digits on your calculator display"
- You are better off writing too much down than not enough!

9. Practise!

- This is a long list but we will finish by going back to the start – there is nothing better you can do than getting a calculator early and learning how to use it by practising the varying types of questions you are likely to come across

© copyright Save My Exams

1. Number

YOUR NOTES
⬇

Exam Tip

Always put negative numbers in brackets. For a quick example, try using your calculator to work out -3^2 and then $(-3)^2$.

In working out always write down more digits than the final answer requires and don't round them (write something like 9.3564… using the three dots shows you haven't rounded). Use the ANS button when you next need that number on your calculator.

Worked Example

3. Complete the table of values for $y = x^3 - 6x + 1$

x	-3	-2	-1	0	1	2	3
y		5					10

$(-3)^3 - 6 \times (-3) + 1 = -8$ Use brackets around negatives and (-) key

$(-1)^3 - 6 \times (-1) + 1 = 6$ Use arrow keys and change "3"s to "1"s

$0^3 - 6 \times 0 + 1 = 1$

$1^3 - 6 \times 1 + 1 = -4$ You can use the TABLE mode/feature

$2^3 - 6 \times 2 + 1 = -3$ of your calculator if it has one

x	-3	-2	-1	0	1	2	3
y	-8	5	6	1	-4	-3	10

4. Solve the quadratic equation $2x^2 + 6x + 3 = 0$, giving your answers in the form $\frac{a \pm b\sqrt{3}}{2}$

$\text{"}a\text{"} = 2, \ \text{"}b\text{"} = 6, \ \text{"}c\text{"} = 3$ a, b, (c) from quadratic formula have nothing to do with the a and b mentioned in question

$\text{"}b^2 - 4ac\text{"} = 6^2 4 \times 2 \times 3 = 12$ Find the discriminant first (the bit under square root)

$x = \frac{-6 \pm \sqrt{12}}{2 \times 2}$ Now put into full quadratic formula

$x = \frac{-3 \pm \sqrt{3}}{2}$ … and your calculator will simplify for you

Note: You'll have to choose "+" or "-" when You type it into your calculator.

1. Number

YOUR NOTES

1. Use your calculator to work out

$$\frac{\sqrt{4.69}}{0.34^3 + sin\,(45°)}$$

Give your answer as a decimal.

Write down all the figures on your calculator display.

$\sqrt{4.69} = 2.16564 \ldots$ To show your working write down the top

$0.34^3 + sin(45) = 0.746410 \ldots$ and bottom separately …

2.901406085 … but you can type it all in one go for the final answer using the fraction button, etc

2. $a^5 = \frac{p+q}{p^2q}$

Find the value of a when $p = 1.2 \times 10^{-4}$ and $q = 7.83 \times 10^5$

Give your answer to 3 decimal places.

$p + q = 783000.0001$ Show each stage as working

$p^2q = 0.0112752$ Use brackets when it gets long or awkward

$a^5 = 69444444.46$ Write down all digits at these stages

$\boldsymbol{a = 37.01071 \ldots = 37.011}$ Write more digits than you need, then round

1. Number

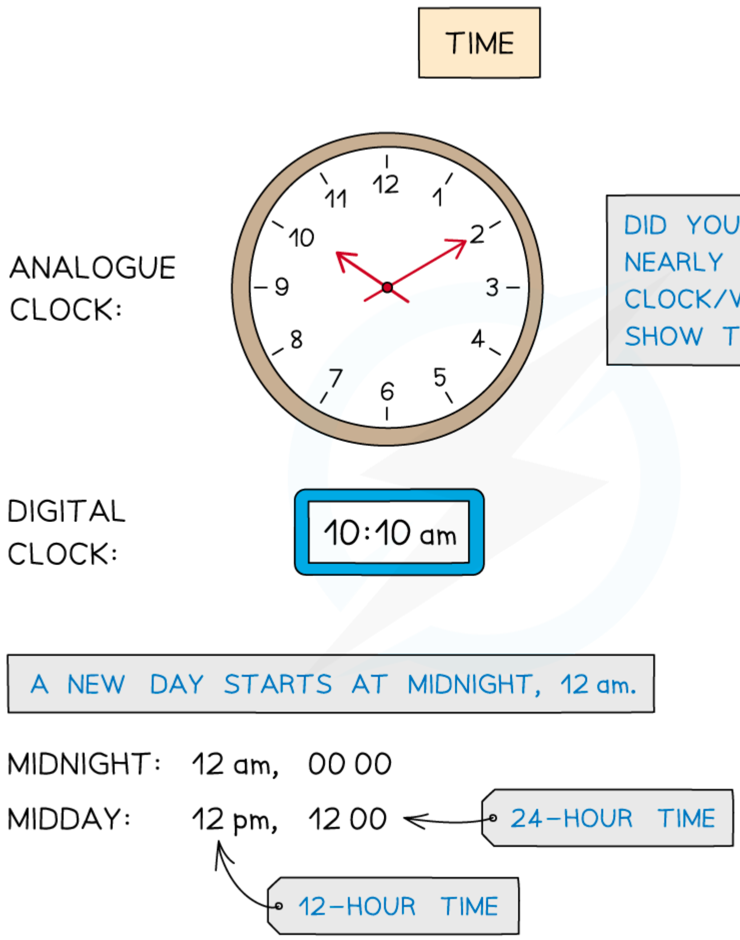
YOUR NOTES
⬇

1.11 TIME

1.11.1 TIME

What do I need to know about time for IGCSE?

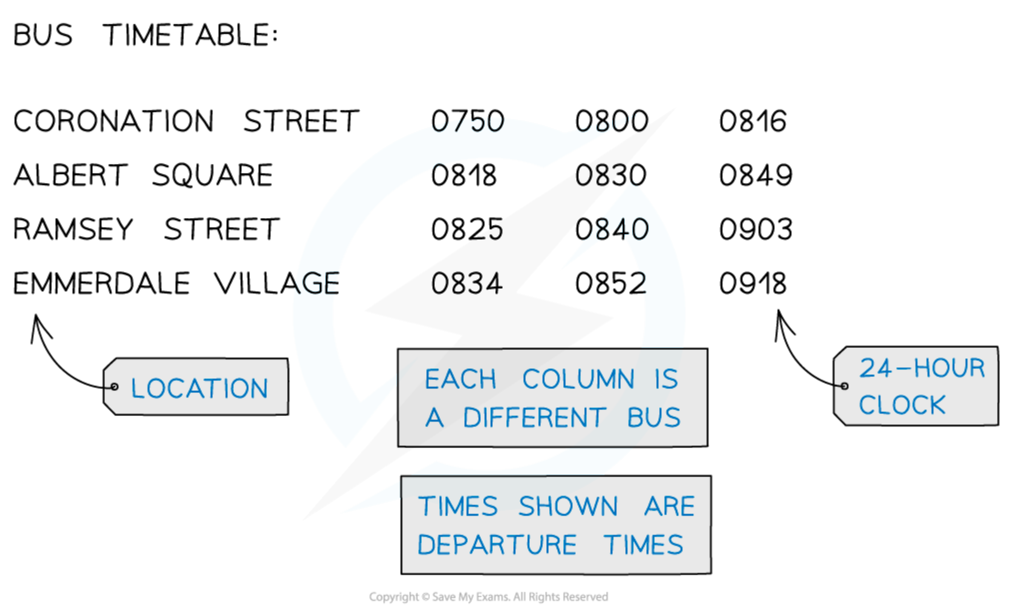
- Both 12-hour and 24-hour times could be used
- In the 12-hour clock system ...
  - AM is between midnight (12am) and midday (12pm)
  - PM is between midday (12pm) and midnight (12am)
- Times may have to be read from both analogue and digital clocks
- Times may have to be read from timetables



Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES
⬇


BUS TIMETABLE:

CORONATION STREET	0750	0800	0816
ALBERT SQUARE	0818	0830	0849
RAMSEY STREET	0825	0840	0903
EMMERDALE VILLAGE	0834	0852	0918

1. Number

YOUR NOTES

- Time does not work like the rest of the number system (based on 10s, 100s, etc) so calculations can get awkward
    - 60 seconds in a minute
    - 24 hours in a day
    - ... and many more !

TIME CONVERSIONS

THERE ARE...

... 60 SECONDS IN A MINUTE
... 60 MINUTES IN AN HOUR
... 24 HOURS IN A DAY
... 7 DAYS IN A WEEK
... 365 DAYS IN A YEAR
(366 IN A LEAP YEAR)
... 52 WEEKS IN A YEAR
(PLUS ONE DAY)
... 12 MONTHS IN A YEAR
... 1000 YEARS IN A MILLENIUM

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES

DO LEARN THESE – AND HOW MANY DAYS ARE IN EACH MONTH...

THIRTY DAYS HATH SEPTEMBER, APRIL, JUNE AND NOVEMBER,
ALL THE REST HAVE THIRTY-ONE.
EXCEPTING FEBRUARY ALONE.
AND THAT HAS TWENTY-EIGHT DAYS CLEAR,
AND TWENTY-NINE IN EACH LEAP YEAR.

JANUARY	31 DAYS
FEBRUARY	28 (29 IN A LEAP YEAR)
MARCH	31
APRIL	30
MAY	31
JUNE	30
JULY	31
AUGUST	31
SEPTEMBER	30
OCTOBER	31
NOVEMBER	30
DECEMBER	31

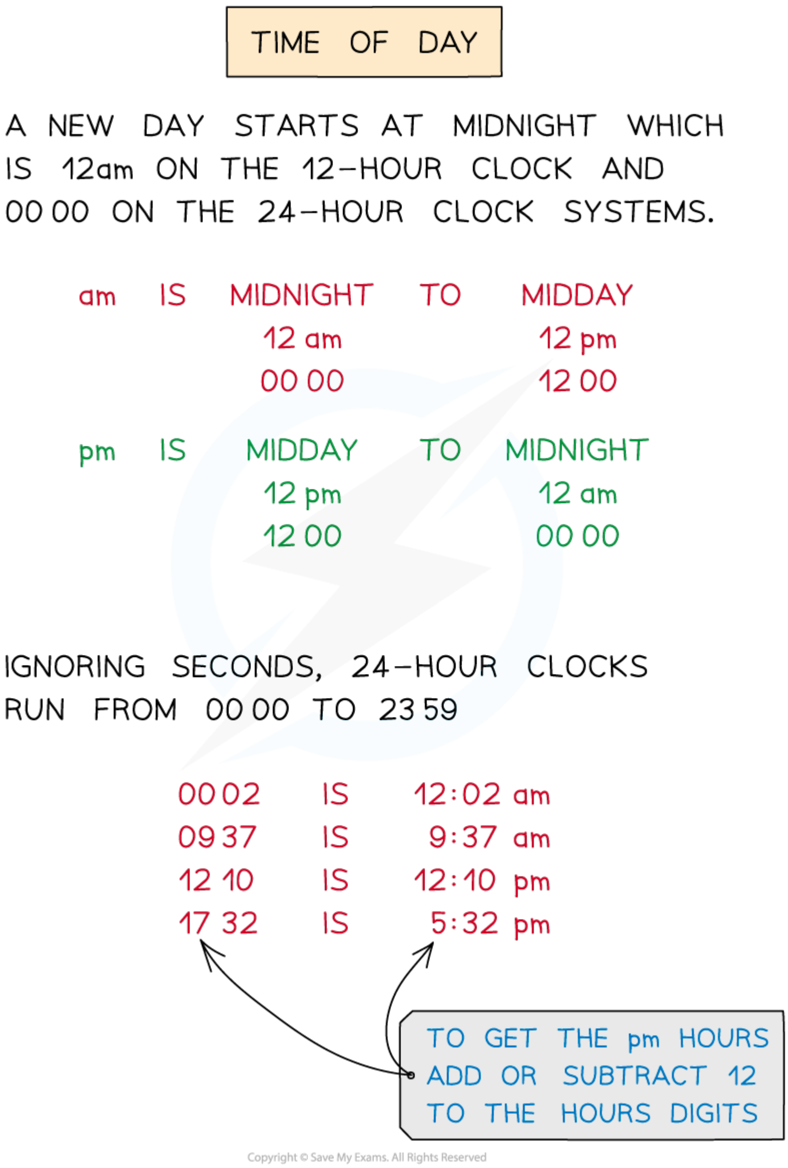
Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES
⬇

How do I read a clock?

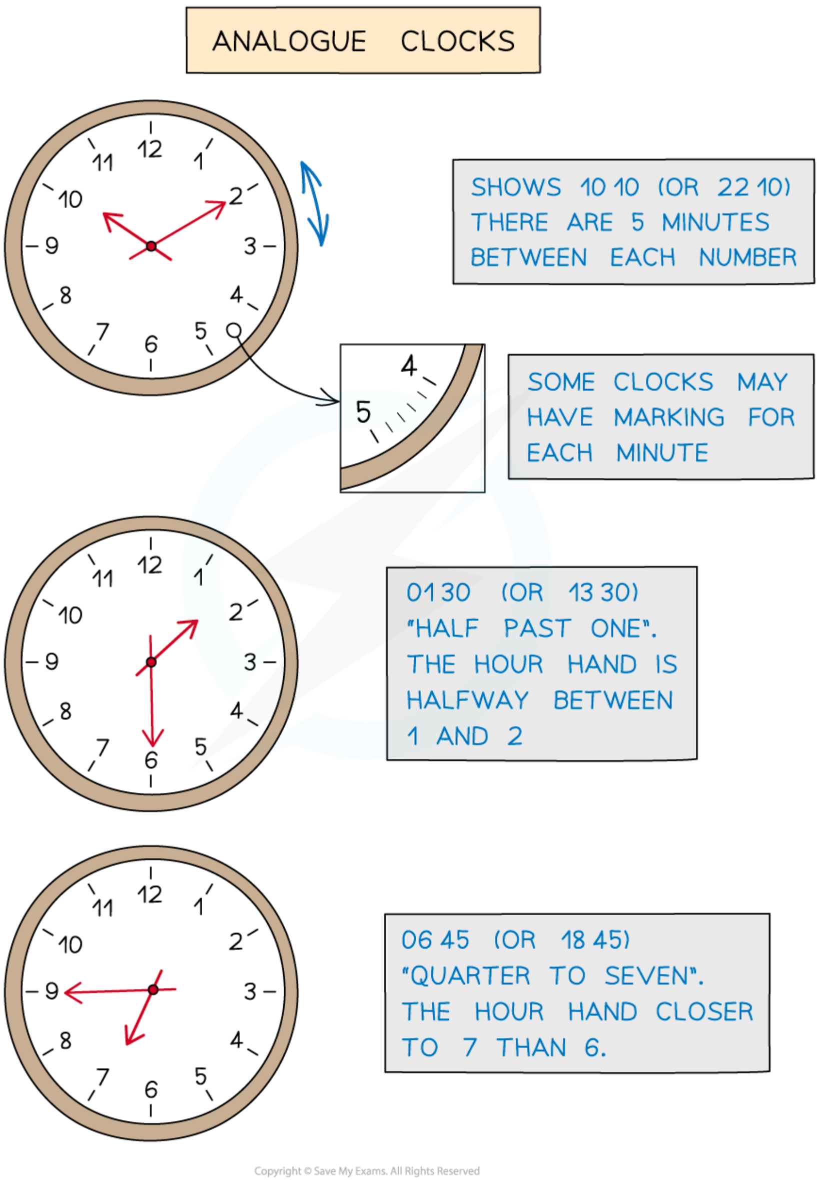
- A 12-hour clock goes round once for am and once for pm
  - am is midnight (12am) to midday (12pm)
  - pm is midday (12pm) to midnight (12am)
- A 24-hour clock uses four digits – two for the hour, two for the minutes
  - 1134 is 11.34am
  - The day starts at midnight which is 0000
  - 1pm is 1300, 2pm is 1400, … 10pm is 2200, 11pm is 2300



1. Number

YOUR NOTES

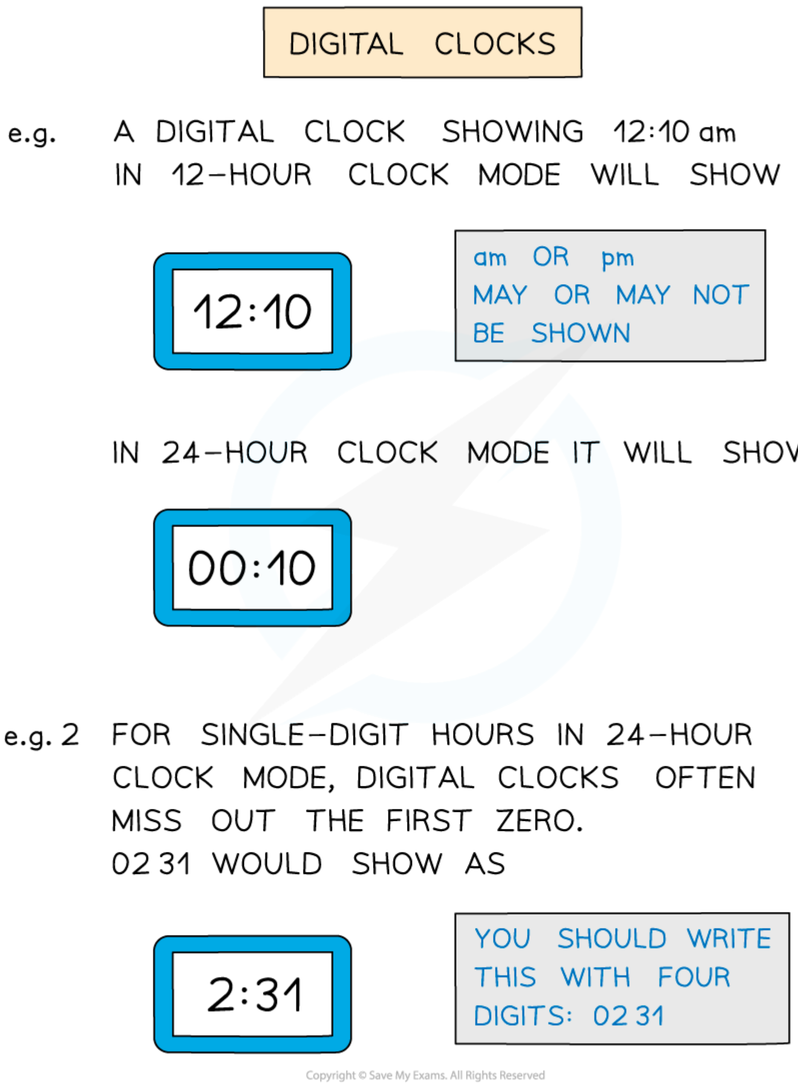
- Analogue clocks work in 12-hour time
- On the minute hand each number is worth five minutes
  - Some clocks will have markings for individual minutes
- The hour hand is always moving
  - At "half past" the hour hand should be halfway between two numbers



1. Number

YOUR NOTES
⬇

- Digital clocks can use either 24 hour time or 12-hour time
  - A ":" is often displayed between the hours and minutes
    eg 1245 would be displayed as 12:45
  - am or pm does not need to be specified with 24-hour time, it may or may not be shown on a 12-hour time
  - For single-digit hours, clocks often miss out the first zero
    eg 09:23 would be displayed as 9:23
- Timetables (for a bus or train for example) use the 24-hour time
  - Times are listed as four digits without the ":"

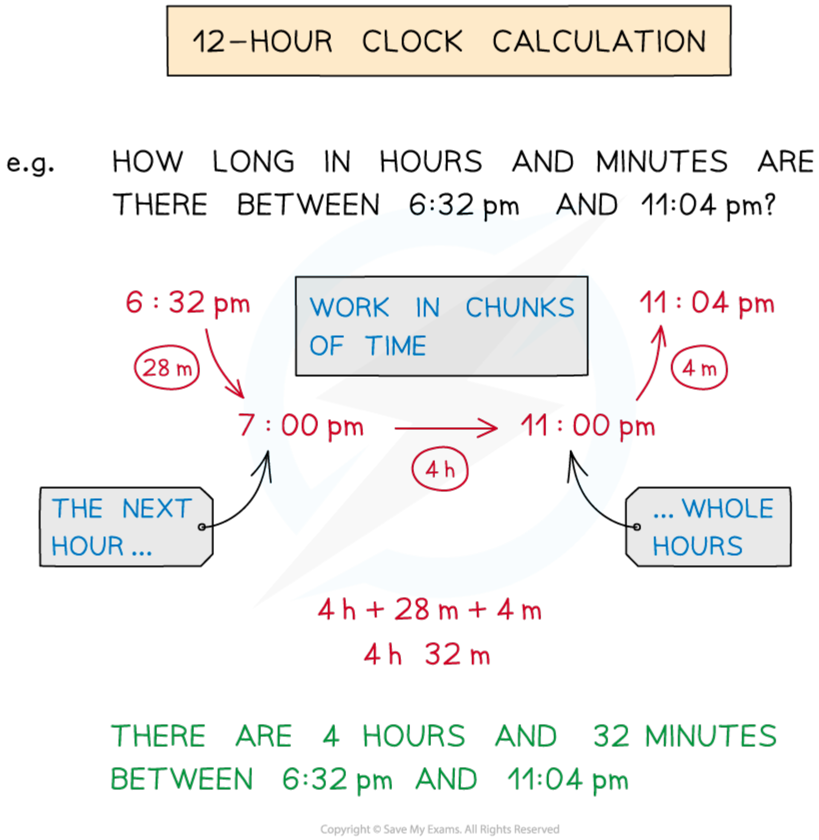


1. Number

YOUR NOTES
⬇

How do I calculate with time in terms of the 12-hour clock?

- Work in chunks of time
  - eg calculate the minutes until the next hour, then whole hours, then minutes until a final time
- Ensure you know when the 12-hour clock switches from am to pm
  - Remember midday is 12pm and midnight is 12am

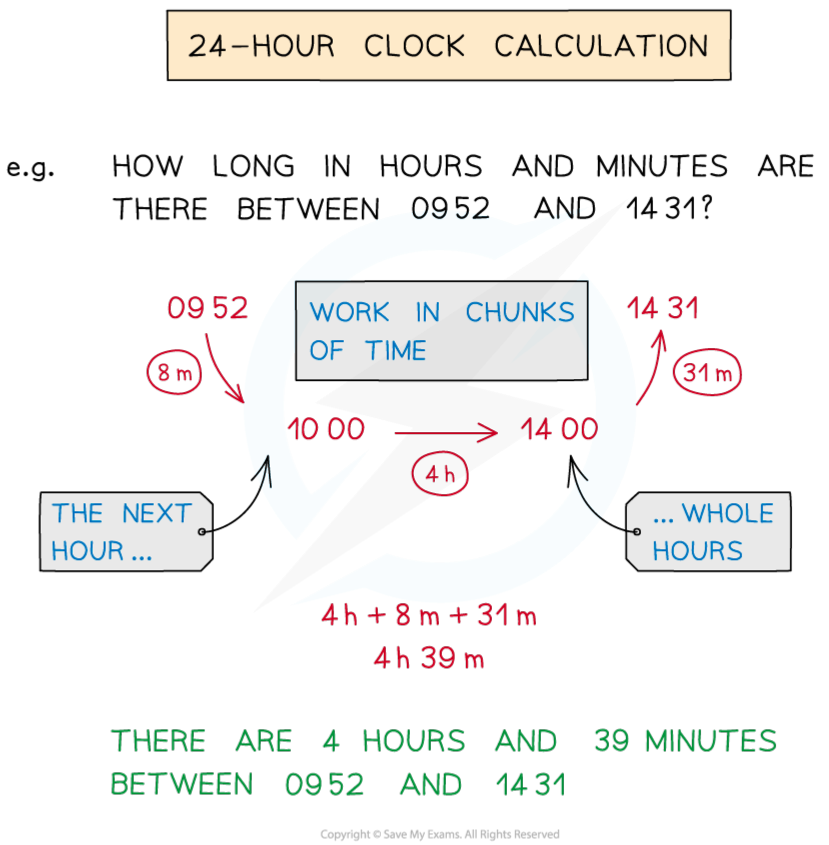


1. Number

YOUR NOTES

How do I calculate with time in terms of the 24-hour clock?

- Work in chunks of time just like the 12 hour clock calculations
  - eg. calculate the minutes until the next hour, then whole hours, then minutes until a final time
- If the hour is greater than 12, subtract 12 from it to find the 12-hour PM hour



How do I use bus and train timetables?

- These tend to use the 24-hour clock system
- Each column represents a different bus/train – these are often called "services"
  - eg "The 0810 service from London King's Cross"
- The time in each cell usually indicate departure times (when the bus/train leaves that stop/station)
  - The last location on the list usually shows the arrival time

BUS/TRAIN TIMETABLES

e.g.

Coronation Street	0750	0800	0816	0835	0855	0925	1000
Albert Square	0818	0830	0849	0915	0927	0955	1027
Ramsey Street	0825	0840	0903	0926	0937	1003	1034
Emmerdale Village	0834	0852	0918	0941	0949	1013	1043

HOW LONG IS THE JOURNEY...

i) ...FROM CORONATION STREET TO ALBERT SQUARE ON THE 08 35 BUS?

CS, 08 35 → 09 00 → AS, 09 15

25 m   15 m

25 m + 15 m

40 MINUTES

ii) ...FROM RAMSEY STREET TO EMMERDALE VILLAGE ON THE 10 34 BUS?

RS, 10 34 → EV, 10 43

9 m

9 MINUTES

Copyright © Save My Exams. All Rights Reserved

iii) ...FROM ALBERT SQUARE TO EMMERDALE VILLAGE ON THE 08 49 BUS?

AS, 08 49 → 09 00 → EV, 09 18

11m   18 m

11 m + 18 m

29 MINUTES

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES

Exam Tip

Even when allowed, put that calculator away for time-based questions, they are pretty useless for these calculations!

Worked Example

(a) A film starts at 1535 and lasts for 1 hour 45 minutes.
Find the time at which the film ends.

(b) It is due to start raining at 0235 and not stop until 0814.
Find the time (in hours and minutes) it is expected to rain for.

(c) A train journey started at 11.23am and finished at 2.27pm.
How long, in hours and minutes, was the train journey.

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES

(d) The table below shows part of a bus timetable

Coronation Street	0750	0800	0816
Albert Square	0818	0830	0849
Ramsey Street	0825	0840	0903
Emmerdale Village	0834	0852	0918

(i) How long should the journey between Coronation Street and Ramsey Street take on the 0800 bus?

(ii) What is the latest bus that can be taken from Albert Square if there is the need to arrive in Emmerdale Village by 0900?

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES
⬇

a) 15 35 + 1h 45 m

↓

16 35 + 45 m

↓

17 00 + 20 m

↓

17 20

WORK IN "CHUNKS" OF TIME

ADD THE HOUR ON, EASY!

GO TO THE NEXT HOUR: 60 MINUTES IN AN HOUR SO 25 OF 45 NEEDED. LEAVING ANOTHER 20 MINUTES

THE FILM ENDS AT 1720

THE QUESTION DIDN'T SAY WHETHER TO USE 12- OR 24-HOUR CLOCK SO EITHER FINE (5:20 pm) BUT IT'S USUALLY EASIEST TO STICK TO WHAT THE QUESTION DOES.

b) 02 35 $\xrightarrow{25\text{ m}}$ 03 00 $\xrightarrow{5\text{ h}}$ 08 00 $\xrightarrow{14\text{ m}}$ 08 14

NEXT HOUR

WHOLE HOURS

5 h + 25 m + 14 m

5 h 39 m

IT IS EXPECTED TO RAIN FOR 5 h 39 m.

IT'S FINE TO USE ABBREVIATIONS LIKE h AND m

Copyright © Save My Exams. All Rights Reserved

c) 11 : 23 am — 37 m → 12 : 00 pm — 2 h → 2 : 00 pm — 27 m → 2 : 27 pm

CAREFUL WORKING ACROSS AN am/pm CHANGE

2 h + 37 m + 27 m

2 h 64 m

3 h 4 m

SPOT THAT THIS IS OVER 60 SO THERE'S AN EXTRA HOUR

THE TRAIN JOURNEY WAS 3h 4m.

Copyright © Save My Exams. All Rights Reserved

d) i) THE 08 00 BUS IS THE MIDDLE COLUMN

CORONATION STREET – 08 00
RAMSEY STREET – 08 40

40 MINUTES

YOU COULD HIGHLIGHT THESE ON THE TIMETABLE IN THE QUESTION

ii) EMMERDALE VILLAGE 08 34, 08 52, 09 18

LATEST BUS

TO ARRIVE BEFORE 09 00

IT WILL BE THE SECOND BUS WHICH LEAVES ALBERT SQUARE AT 08 30.

THE 08 30 BUS IS THE LATEST BUS THAT CAN BE TAKEN FROM ALBERT SQUARE TO ARRIVE IN EMMERDALE VILLAGE BY 09 00.

Copyright © Save My Exams. All Rights Reserved

1. Number

YOUR NOTES
⬇

1.12 CURRENCY

1.12.1 EXCHANGE RATES

Simplifying exchange rate questions

Use ratios!

1. Put exchange rates in **ratio** form (use more than one line if necessary)
2. Add lines for prices/costs
3. Use **scale factors** to complete lines
4. Pick out the **answer!**

It can be that simple!

Worked Example

1. €1 (*Euro*) *is worth* \$21.48 (*Mexican Peso*).
   ฿1 (*Bitcoin*) *is worth* €6882.55 (*Euro*).
   A vintage car costs \$1 000 000 (*Mexican Peso*).
   What is the cost of the car in Bitcoins?

$1\ -\ \textit{Using ratios}$

$2\ -\ \textit{Add a line for each rate/cost}$

$\textit{Use unknowns } (x,\ b) \textit{ for values to find}$

1. Number

YOUR NOTES

	€ EURO	:	\$ MP	:	฿ BITCOIN
RATE €/\$	1	:	21.48	:	
RATE ฿/€	6882.55	:	x	:	1
RATE \$/฿		:	1 000 000	:	b

Copyright © Save My Exams. All Rights Reserved

$x = 21.48 \times 6882.55$

$x = \$147\ 837.174$

3 – Looking at the Euro column, the Scale Factor is 6882.55

$Scale\ Factor = 1\ 000\ 000 \div 147837.174$

3 – Looking at the MP column, we can find the Scale Factor to convert between the rate for ฿/€ and cost.

$= 6.764$

$Cost = 1 \times 6.764 =$ ฿6.764 (*Bitcoins*)

4 – We can now pick out the answer in Bitcoins

1. Number

YOUR NOTES
⬇

1.13 EXPONENTIAL GROWTH & DECAY

1.13.1 COMPOUND INTEREST

What is compound interest?

- Compound interest is where interest is paid on the interest from the year (or whatever time frame is being used) before as well as on the original amount
- This is different from **simple interest** where interest is only paid on the original amount

How do you work with compound interest?

- For COMPOUND changes (can be a decrease as well as an increase):
  - Keep multiplying by the decimal equivalent of the percentage you want
- Otherwise do the same as normal:
- Identify "before" & "after" quantities
- FIND percentage we want:
  - Increase – ADD percentage to 100
  - Decrease – SUBTRACT from 100
- Write down a STATEMENT connecting "before" and "after":
  - "after is a percentage of before"
- Write down the statement as an EQUATION using decimal equivalent
  - remember "is" means "="
- SUBSTITUTE and SOLVE

Exam Tip

This method works for any Compound Change – increase or decrease.

Remembering "× m × m × m = × m^3" can make life a lot quicker:

It is usually much easier to multiply by decimal equivalent raised to a power than to multiply by the decimal equivalent several times in a row.

1. Number

YOUR NOTES

Worked Example

Jasmina invests £1200 in a Savings Account which pays Compound Interest at the rate of 2% per year for 7 years.

To the nearest pound, what is her investment worth at the end of the 7 years?

We can call the "before" Investment and the "after" Final Value

1. *We want* $100 + 2 = 102\%$ *EACH YEAR*

   The decimal equivalent of 102% is 1.02 and we want to apply it 7 times so our multiplier is 1.02^7

2. *Final Value is 102% (applied for 7 years) of Investment*

3. $\text{Final Value} = 1.02^7 \times \text{Investment}$

4. $\text{Final Value} = 1.02^7 \times 1200 = 1378.4228...$

   $\text{Final Value} = £1378$ *(to the nearest £)*