7. Vectors & Transformations

YOUR NOTES
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7.1 VECTORS

7.1.1 VECTORS - BASICS

What are vectors?

- A **vector** is a type of number that has b**oth a size and a direction**
- At GCSE we only deal with two-dimensional vectors, although it is possible to have vectors with any number of dimensions

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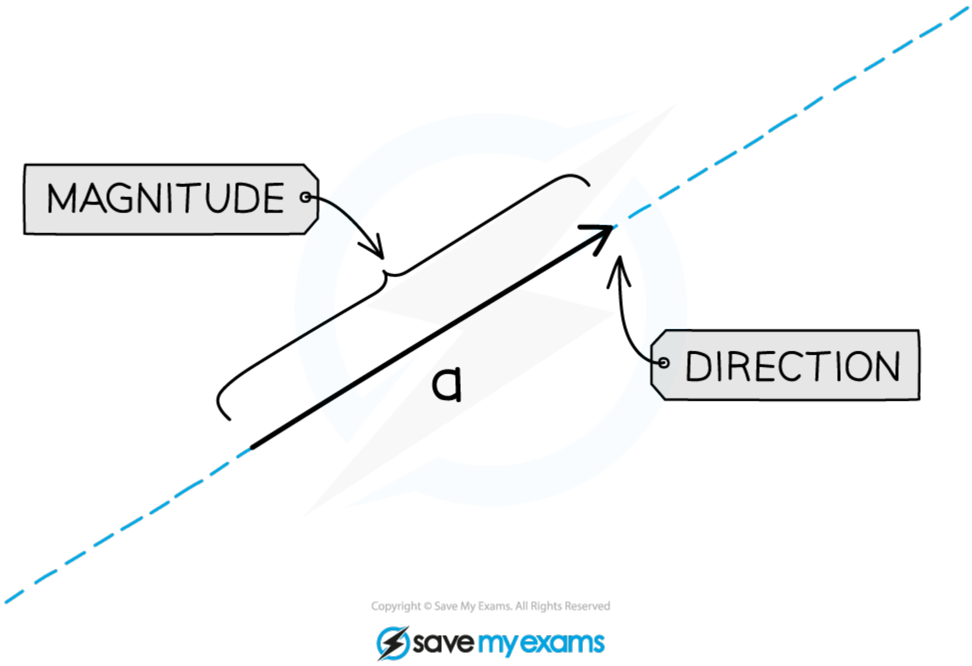
Representing vectors

- Vectors are represented as arrows, with the arrowhead indicating the **direction** of the vector, and the length of the arrow indicating the vector's **magnitude** (ie its size):



- In print vectors are usually represented by bold letters (as with vector a in the diagram above), although in handwritten workings underlined letters are normally used.

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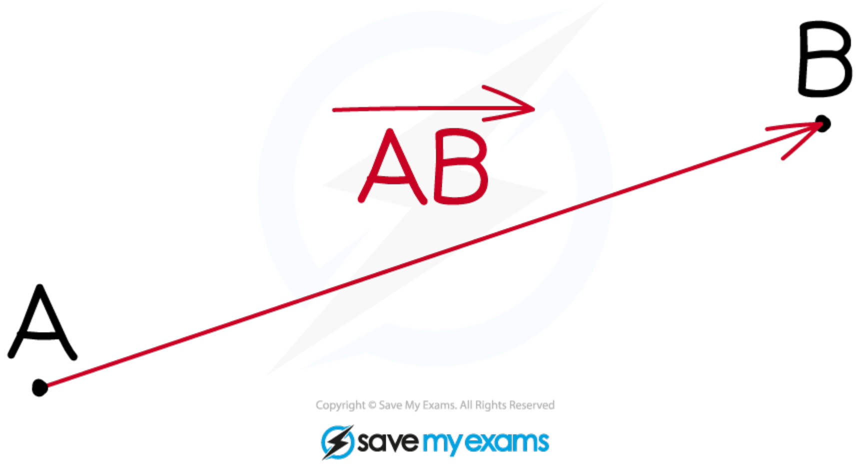
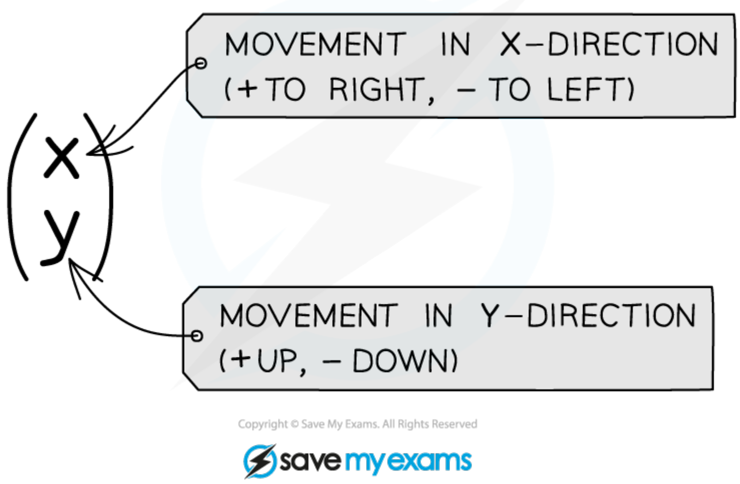
- Another way to indicate a vector is to write its starting and ending points with an arrow symbol over the top, as with the vector AB in the diagram below:



- Note that the order of the letters is important! Vector BA in the above diagram would point in the opposite direction (ie with its 'tail' at point B, and the arrowhead at point A).

Vectors and transformation geometry

In transformation geometry, translations are indicated in the form of a column vector:



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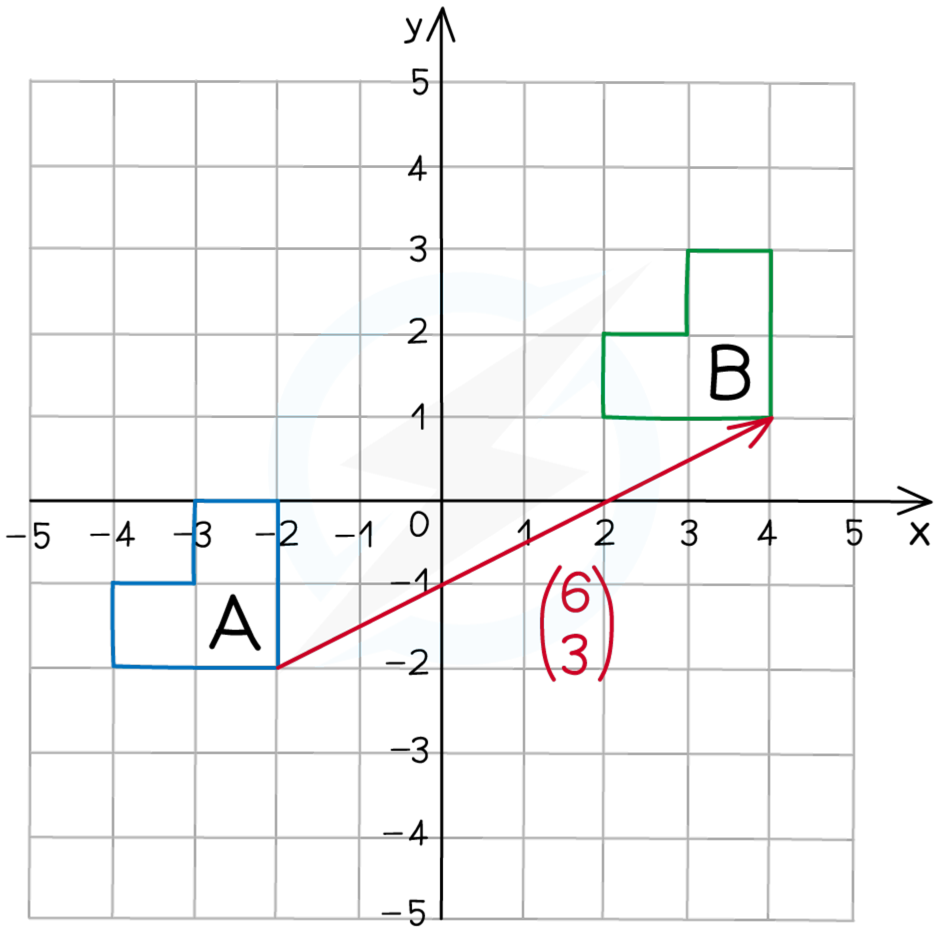
- In the following diagram, Shape A has been translated six squares to the right and 3 squares up to create Shape B
- This transformation is indicated by the **translation vector** (6 3) :



- Note: ‘Vector’ is a word from Latin that means ‘carrier’
- In this case, the vector ‘carries’ shape A to shape B, so that meaning makes perfect sense!

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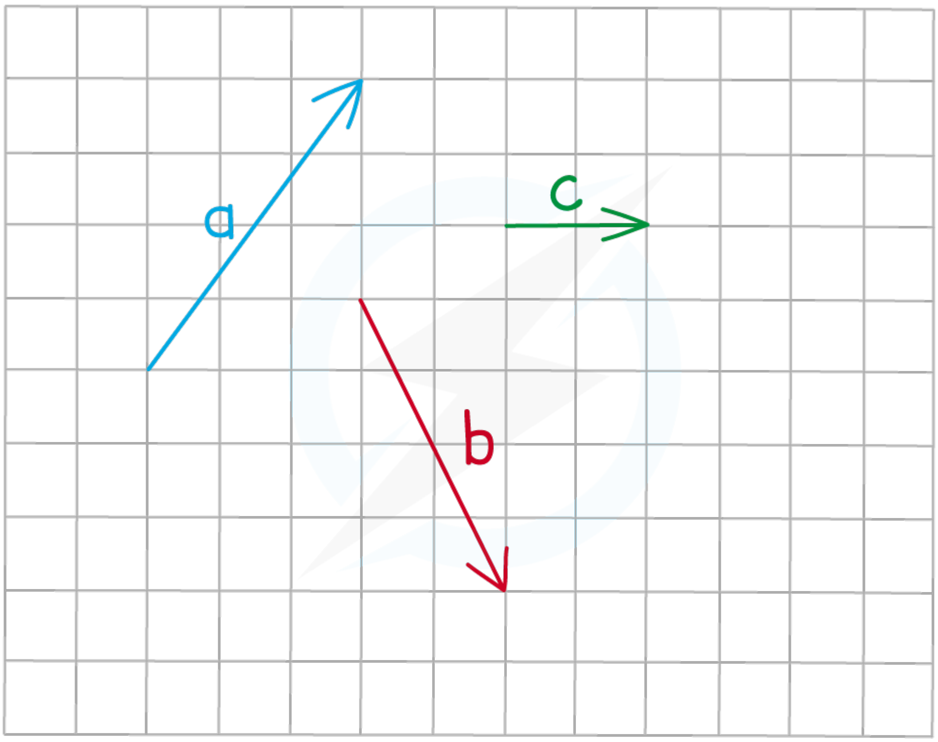
Vectors on a grid

- You also need to be able to work with vectors on their own, outside of the transformation geometry context
- When vectors are drawn on a grid (with or without x and y axes), the vectors can be represented in the same (x y) column vector form as above



a = (3 4) b = (2 - 4) c = (2 0)

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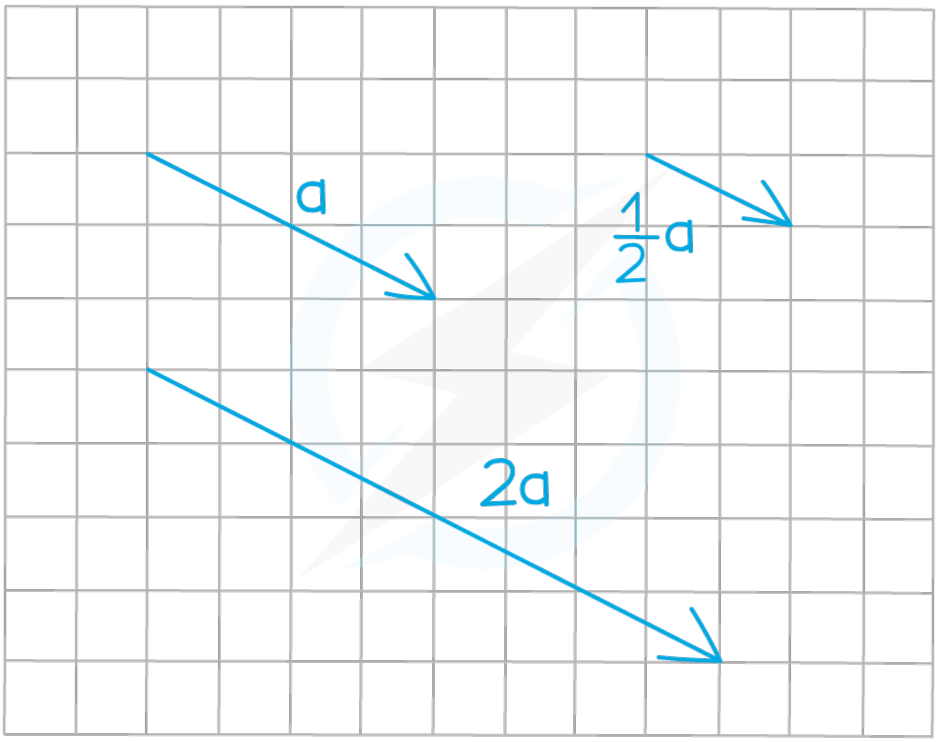
Multiplying a vector by a scalar

- A scalar is a number with a magnitude but no direction – ie the regular numbers you are used to using
- When a vector is multiplied by a scalar, the magnitude of the vector changes, but its direction stays the same
- If the vector is represented as a column vector, then each of the numbers in the column vector gets multiplied by the scalar



a = (4 - 2)

2a = 2 × (4 - 2) = (2 × 4 2 × (-2)) = (8 - 4)

½a= ½ × (4 - 2) = (½ × 4 ½ × (-2)) = (2 - 1)

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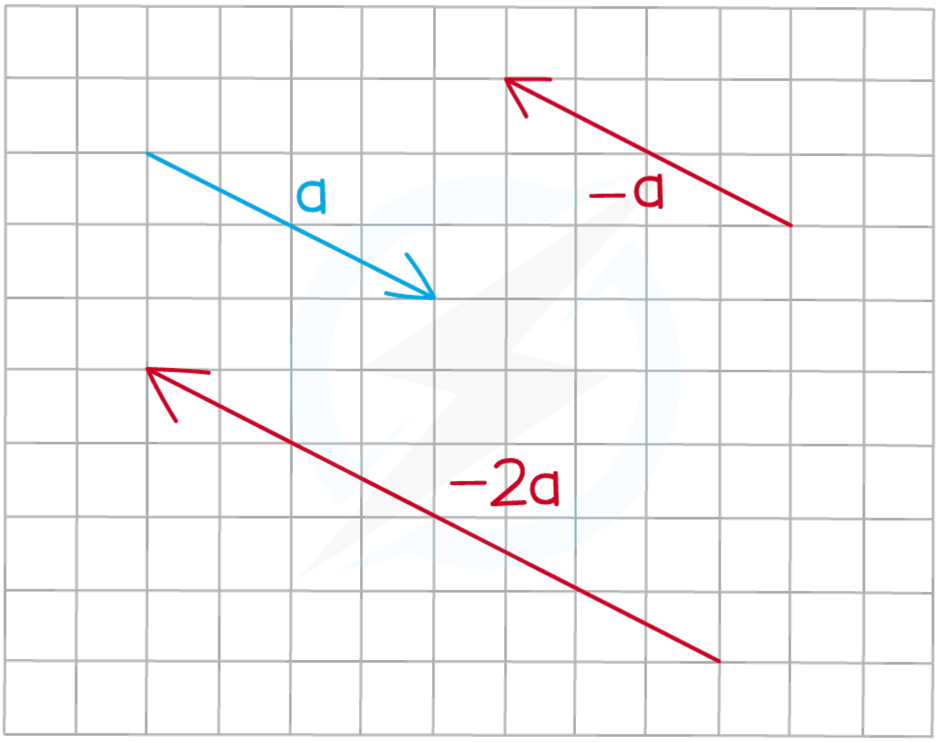
- Note that multiplying by a negative scalar also changes the direction of the vector:



a = (4 - 2)

-a = -1 × (4 - 2) = (-1 × 4 -1 × (-2)) = (-4 2)

-2a = -2 × (4 - 2) = (-2 × 4 - 2 × (-2)) = (-8 4)

- Note in particular that vector -a is the the same size as vector a, but points in the opposite direction!

7. Vectors & Transformations

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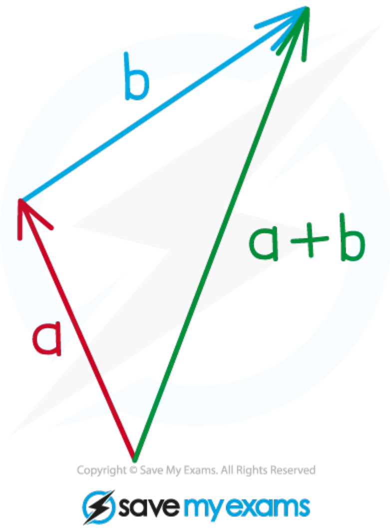
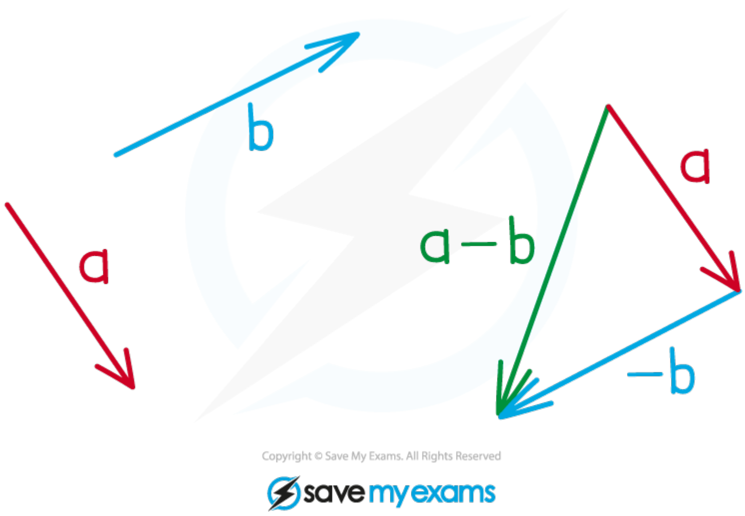
Adding and subtracting vectors

- Adding two vectors is defined geometrically, like this:



- Subtracting one vector from another is defined as addition of the negative of the subtracted vector



a - b = a + (-b)

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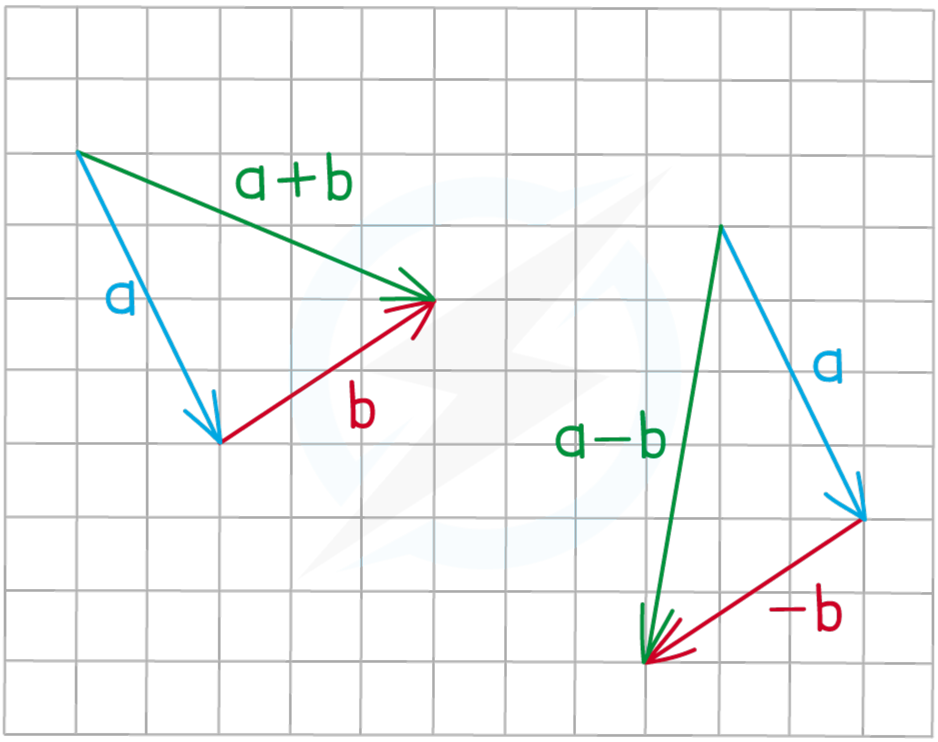
- When vectors are represented as **column vectors**, adding or subtracting is simply a matter of adding or subtracting the vectors' x and y coordinates
- For example:



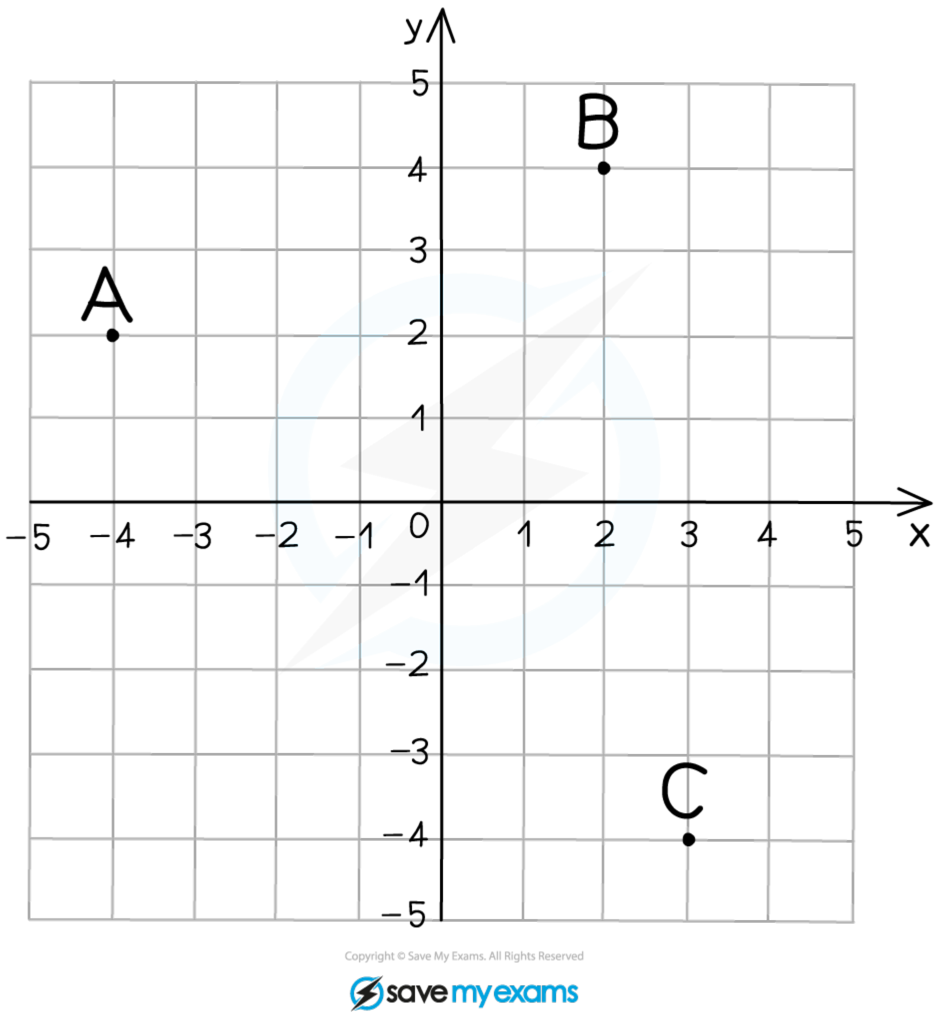
a = (2 - 4)   b = (3 2)

a + b = (2 - 4) + (3 2) = (2 + 3 -4 +2) = (5 - 2)

a - b = (2 - 4) - (3 2) = (2 - 3 -4 -2) = (-1 - 6)

Worked Example

The points A, B, and C are shown on the following coordinate grid:



a) Write the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{CB}$ as column vectors.

b) Using the column vectors from part (a), confirm that

$$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

a) Start by drawing the three vectors onto the grid:

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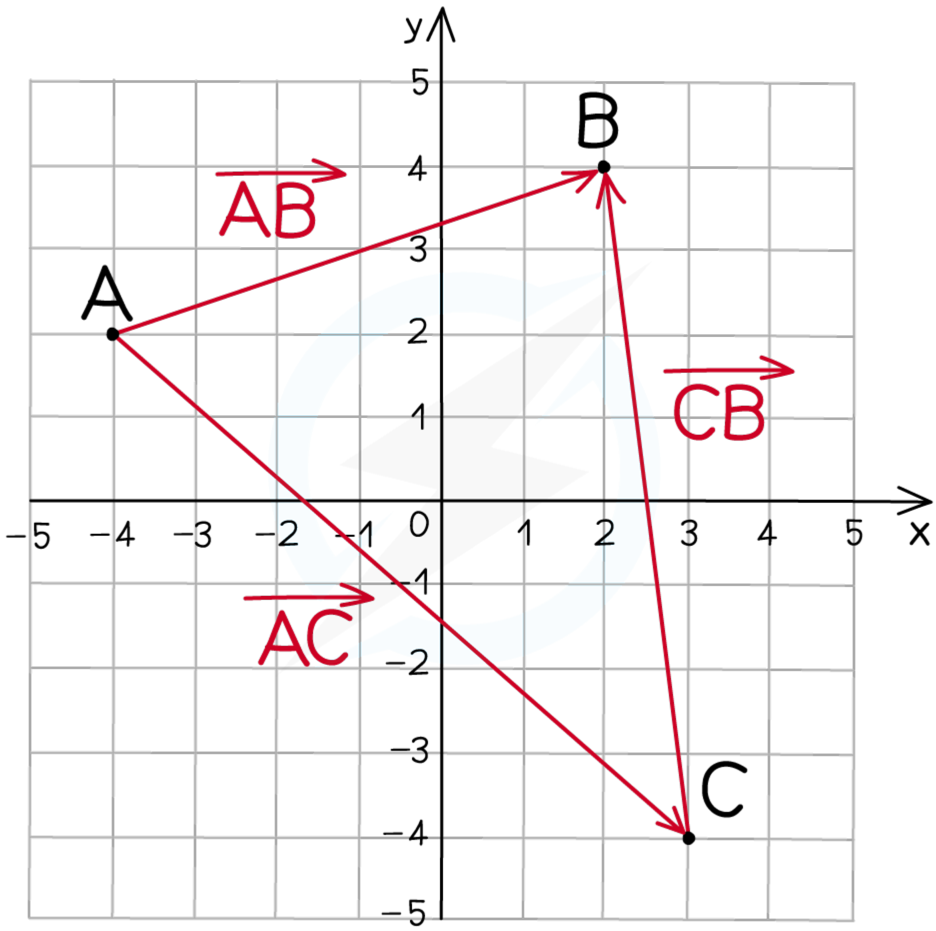

From A to B is ‘6 to the right and 2 up’, so:

$$\overrightarrow{AB} = (6\ 2)$$

From A to C is ‘7 to the right and 6 down’, so:

$$\overrightarrow{AC} = (7\ \ -6)$$

From C to B is ‘1 to the left and 8 up’, so:

$$\overrightarrow{CB} = (-1\ 8)$$

b) Now just perform subtraction on the column vectors from part (a):

$$\overrightarrow{AB} - \overrightarrow{AC} = (6\ 2) - (7\ \ -6) = (6-7\ \ 2-(-6)) = (-1\ 8) = \overrightarrow{CB}$$

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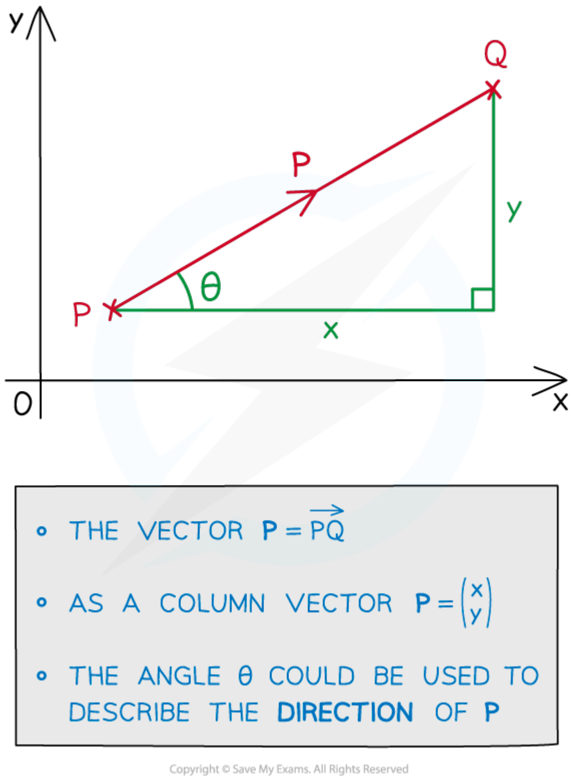
7.1.2 VECTORS - MODULUS

What is a vector?



- **Vectors** have various uses in mathematics
  - In mechanics vectors represent velocity, acceleration and forces
  - At IGCSE vectors are used in geometry – eg. translation
  - Ensure you are familiar with the Revision Notes **Vectors – Basics**
- Vectors have **magnitude** and **direction**
  - These notes look at finding the **magnitude**, or **modulus**, of a vector
  - Vectors are given in column vector form

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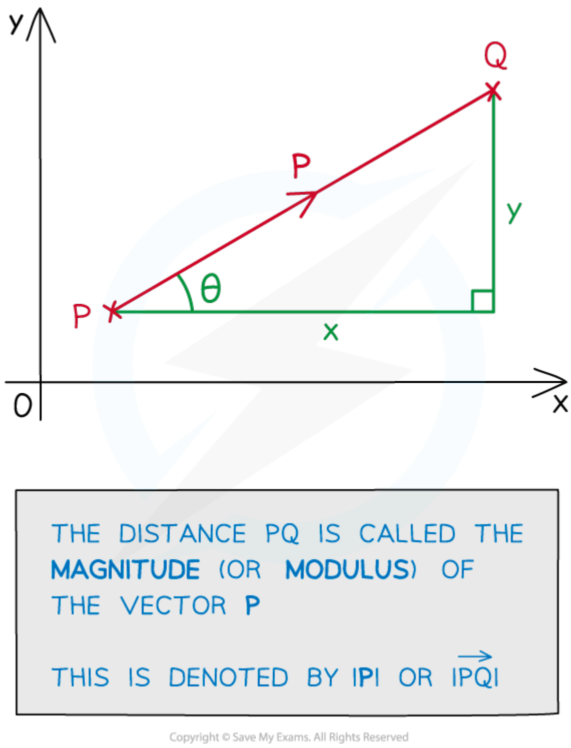
What is the magnitude or modulus of a vector?



- This depends on the use of the vector
  - For velocity, magnitude would be speed
  - For a force, magnitude would be the strength of the force (in Newtons)
- The words **magnitude** and **modulus** mean the same thing with vectors
- In geometry **magnitude** and **modulus** mean the **distance** of the vector
  - This is always a **positive** value
  - The direction of the vector is irrelevant
- Magnitude or modulus is indicated by vertical lines
  - |**a**| would mean the magnitude of vector **a**

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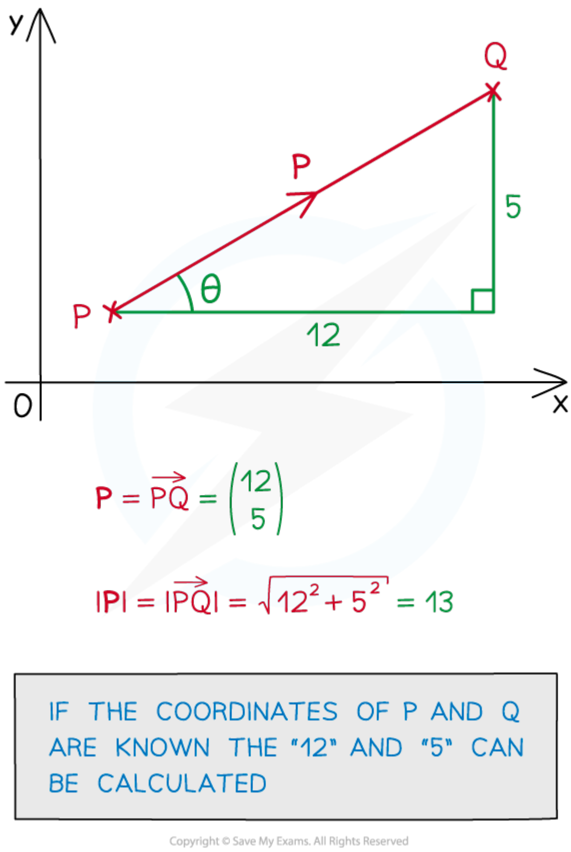
How do I find the magnitude or modulus of a vector

- Pythagoras' Theorem!



Exam Tip

Sketch a vector to help, it does not have to be to scale, then you can use this to form a right-angled triangle.

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Worked Example

(a) Write down the column vector, $\overrightarrow{AB}$, that would take you from the point A with coordiantes $(-3\ ,\ 5)$ to the point B with coordaintes $(7\ ,\ 1)$.

(b) Find the modulus of the vector $\overrightarrow{AB}$

(c) Briefly explain why $\left|\overrightarrow{BA}\right| = \left|\overrightarrow{AB}\right|$

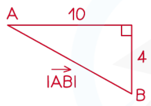
(d) Another vector, $\overrightarrow{CD}$, has three times the magnitude of vector $\overrightarrow{AB}$.

Write down the column vector $\overrightarrow{CD}$.

a) $\overrightarrow{AB} = \begin{pmatrix} 7-(-3) \\ 1-5 \end{pmatrix}$ "END" – "START"

$\overrightarrow{AB} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$

b) DRAW A DIAGRAM TO HELP



NO NEED TO WORRY ABOUT THE NEGATIVE

$|\overrightarrow{AB}| = \sqrt{10^2 + 4^2}$ APPLY "PYTHAGORAS" TO GET THE MODULUS

$= \sqrt{116}$

$|\overrightarrow{AB}| = 2\sqrt{29}$ (10.8 TO 3 SF)

c) $|\overrightarrow{BA}| = |\overrightarrow{AB}|$ SINCE BOTH VECTORS $\overrightarrow{AB}$ AND $\overrightarrow{BA}$ HAVE THE SAME DISTANCE

DIRECTION IS NOT INVOLVED IN MODULUS

d) $\overrightarrow{CD} = 3\overrightarrow{AB}$

$= 3\begin{pmatrix} 10 \\ -4 \end{pmatrix}$

$\overrightarrow{CD}$ IS THREE TIMES LONGER THAN $\overrightarrow{AB}$ SO IS LIKE APPLYING $\overrightarrow{AB}$ THREE TIMES

$\overrightarrow{CD} = \begin{pmatrix} 30 \\ -12 \end{pmatrix}$

7. Vectors & Transformations

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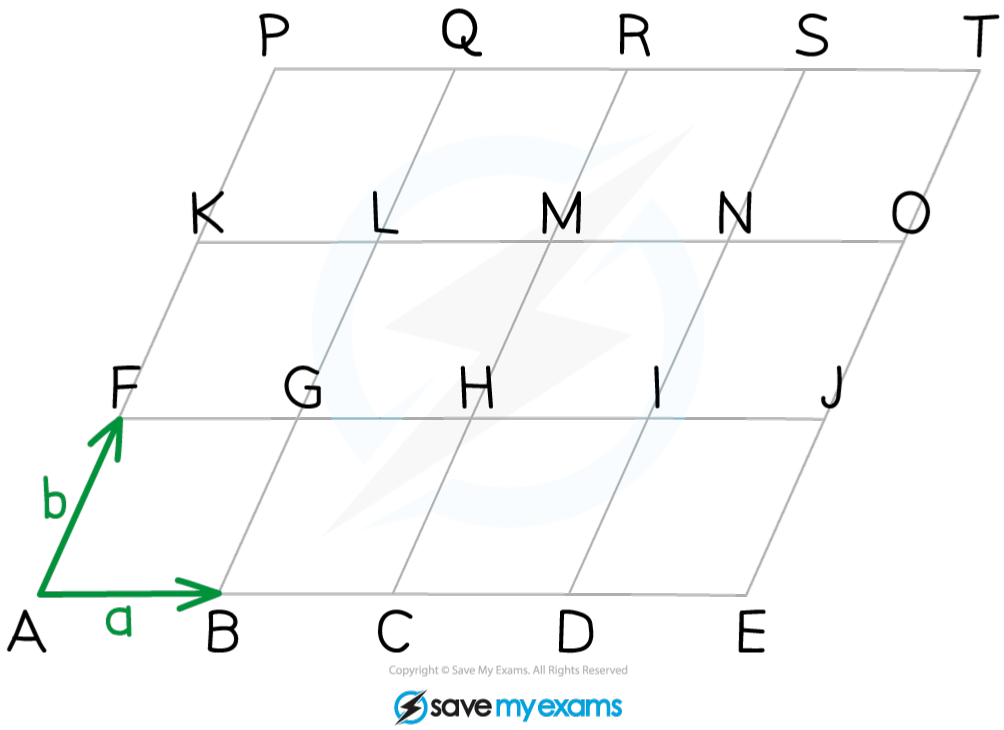
7.1.3 VECTORS - FINDING PATHS

Finding paths in vector diagrams

- It is important to be able to describe vectors by following paths through a geometric diagram
- The following grid is made up entirely of parallelograms, with the vectors and defined as marked in the diagram:



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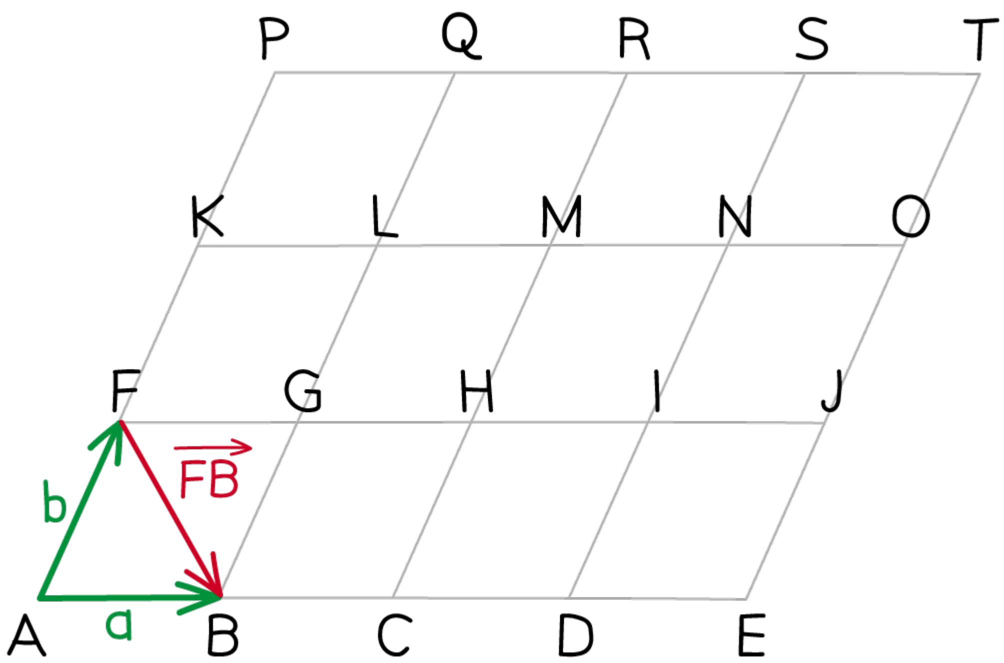
1. Note the difference between 'specific' and 'general' vectors

   The vector $\overrightarrow{AB}$ in the diagram is specific, and refers only to the vector starting at A and ending at B

   However the vector a is a general vector – *any* vector the same length as $\overrightarrow{AB}$ and pointing in the same direction is equal to a

   Similarly, any vector the same length as $\overrightarrow{AF}$ and pointing in the same direction is equal to b
2. Following a vector in the 'wrong' direction (ie from head to tail instead of from tail to head) makes a general vector negative. So in the diagram above $\overrightarrow{AB} = a$, but $\overrightarrow{BA} = -a$

   Similarly $\overrightarrow{AF} = b$, but $\overrightarrow{FA} = -b$
3. Note in particular the vector $\overrightarrow{FB}$:



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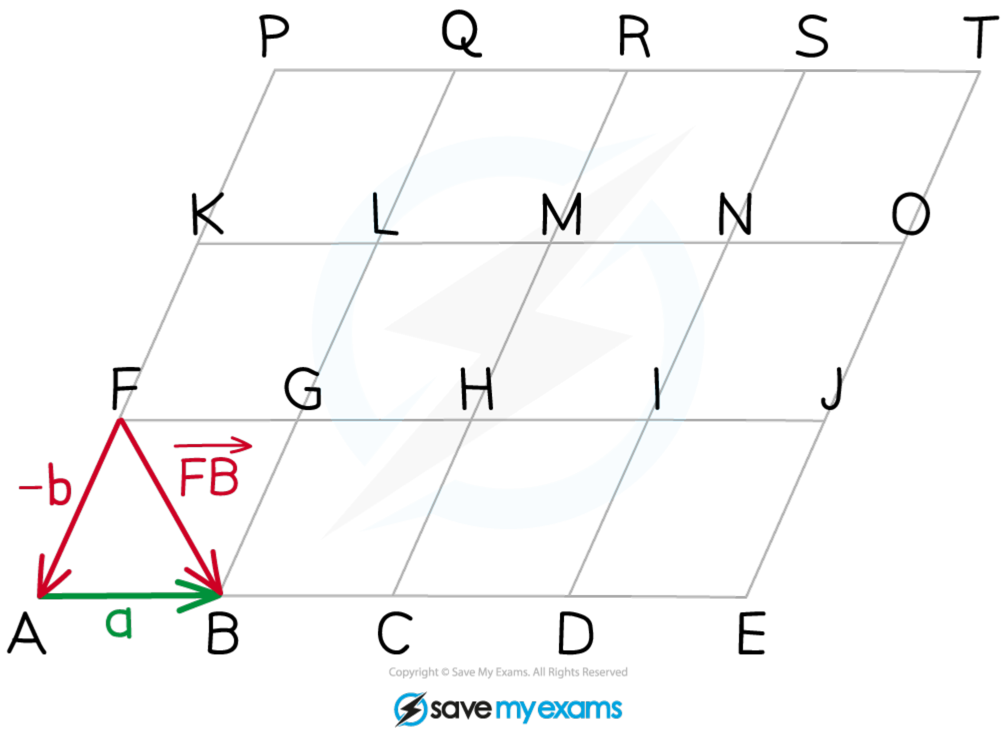
- Getting from point to point we have to go the ‘wrong way’ down and then the ‘right way’ along



- It follows that:

$\overrightarrow{FB} = -b + a = a - b$

and of course then:

$\overrightarrow{BF} = -\overrightarrow{FB} = -(a - b) = b - a$

- Keeping those things in mind, it is possible to describe any vector that goes from one point to another in the above diagram in terms of a and b

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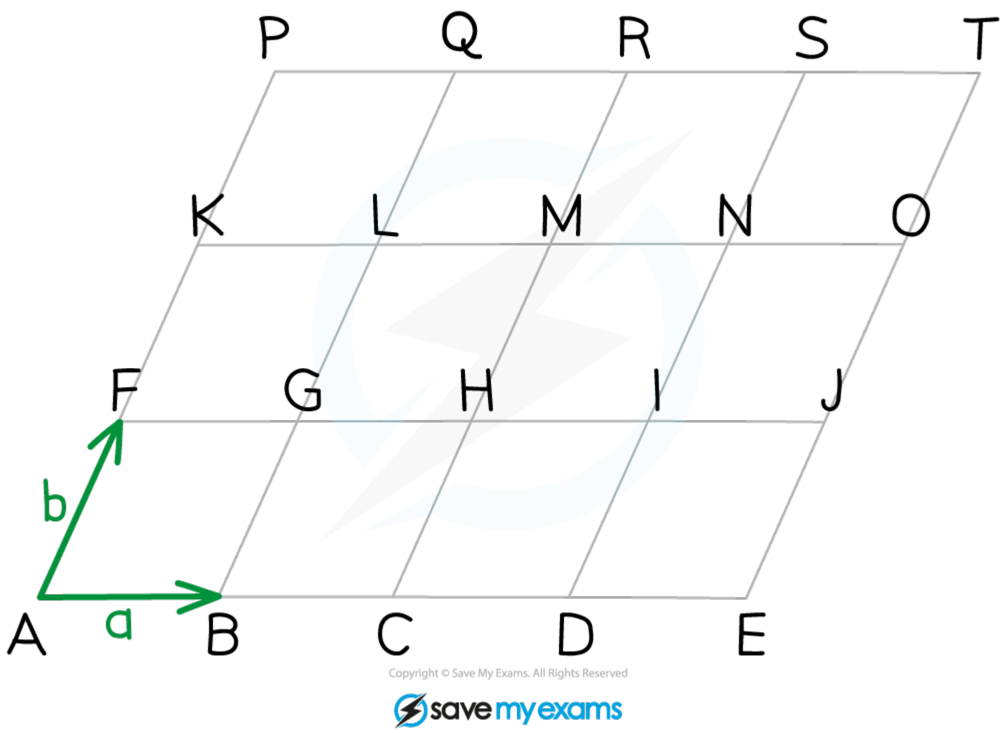
Exam Tip

Adding and subtracting vectors follows all the same rules as adding and subtracting letters like x and y in algebra (this includes collecting like terms). It doesn't matter exactly what path you follow through a diagram from starting point to ending point – as long as you add and subtract the general vectors correctly along the path you use, you will get the correct answer.

Worked Example

The following diagram consists of a grid of identical parallelograms. Vectors a and b are defined by $a = \overrightarrow{AB}$ and $b = \overrightarrow{AF}$.



7. Vectors & Transformations

YOUR NOTES

Write the following vectors in terms of a and b:

a) $\overrightarrow{AE}$

b) $\overrightarrow{GT}$

c) $\overrightarrow{EK}$

a) To get from A to E we need to follow vector a four times to the right. Ie

$$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$$
$$= a + a + a + a$$
$$\overrightarrow{\boldsymbol{AE}} = \boldsymbol{4a}$$

b) There are many ways to go from G to T. Here we go from G to Q (b twice), and then from Q to T (a three times):

$$\overrightarrow{\boldsymbol{GT}} = \overrightarrow{\boldsymbol{GL}} + \overrightarrow{\boldsymbol{LQ}} + \overrightarrow{\boldsymbol{QR}} + \overrightarrow{\boldsymbol{RS}} + \overrightarrow{\boldsymbol{ST}} = \boldsymbol{b} + \boldsymbol{b} + \boldsymbol{a} + \boldsymbol{a} + \boldsymbol{a}$$
$$\overrightarrow{\boldsymbol{GT}} = \boldsymbol{3a + 2b}$$

b) Here we go from E to O (b twice), and then from O to K (a 'the wrong way' four times):

$$\overrightarrow{\boldsymbol{EK}} = \overrightarrow{\boldsymbol{EJ}} + \overrightarrow{\boldsymbol{JO}} + \overrightarrow{\boldsymbol{ON}} + \overrightarrow{\boldsymbol{NM}} + \overrightarrow{\boldsymbol{ML}} + \overrightarrow{\boldsymbol{LK}} = \boldsymbol{b} + \boldsymbol{b} - \boldsymbol{a} - \boldsymbol{a} - \boldsymbol{a} - \boldsymbol{a}$$
$$\overrightarrow{\boldsymbol{EK}} = \boldsymbol{-4a + 2b}$$

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7.1.4 VECTORS - PROVING THINGS

What are vector proofs?

- In **vector proofs** we use vectors, along with a few key ideas, to prove that things are true in geometrical diagrams

Parallel vectors

- Two vectors are **parallel** if and only if one is a multiple of the other
- This tends to appear in vector proofs in the following ways:

1. If you find in your workings that one vector is a multiple of the other, then you know that the two vectors are parallel – you can then use that fact in the rest of the proof

2. If you need to *show* that two vectors are parallel, then all you need to do is show that one of the vectors multiplied by some number is equal to the other one

Eg If $\overrightarrow{AB} = a + 2b$ and $\overrightarrow{CD} = 3a + 6b$, then $\overrightarrow{CD} = 3(a + 2b) = 3\overrightarrow{AB}$

Therefore $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel

Points on a straight line

- Often you are asked to show in a vector proof that three points lie on a straight line (ie that they are **collinear**)
- This is generally done as follows:

3. To show that three points A, B, and C lie on a straight line, show that vectors connecting the three points are parallel
   For example, show that $\overrightarrow{AB}$ is a multiple of (and therefore parallel to) $\overrightarrow{AC}$, or that $\overrightarrow{AB}$ is a multiple of (and therefore parallel to) $\overrightarrow{BC}$

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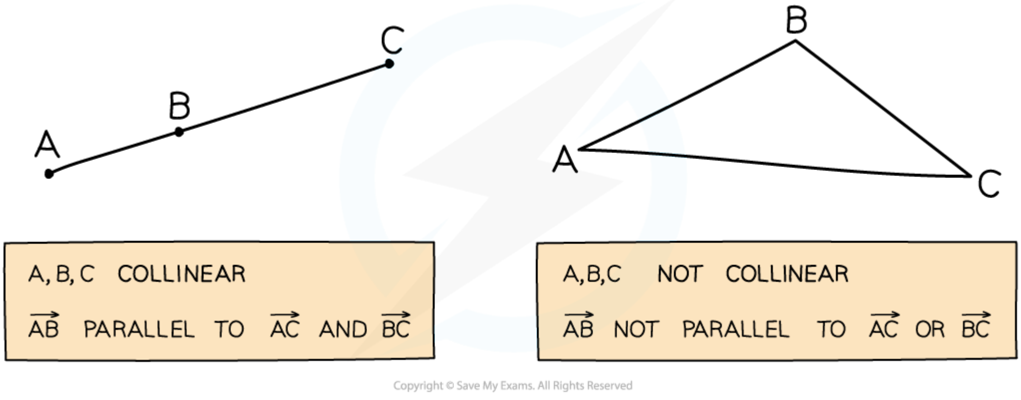


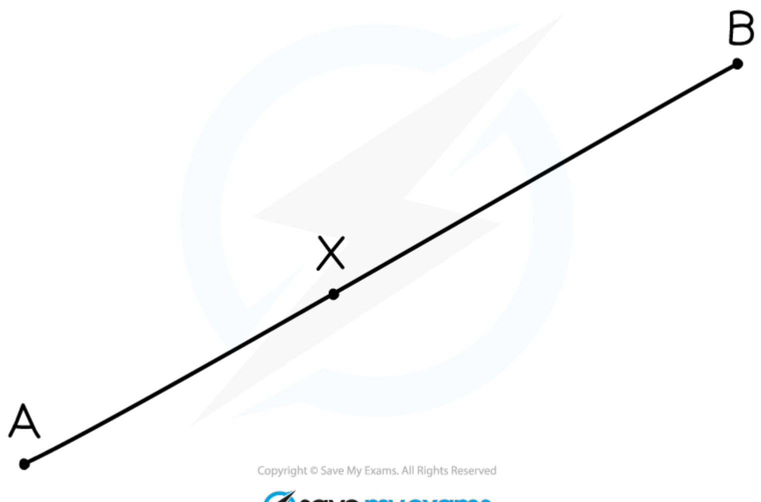
Vectors divided in ratios

- Be careful turning ratios into fractions in vector proofs!

4. If a point divides a line segment in the ratio p : q, then:

$$\overrightarrow{AX} = \frac{p}{p+q}\overrightarrow{AB} \quad and \quad \overrightarrow{XB} = \frac{q}{p+q}\overrightarrow{AB}$$

eg. In the following diagram, the point divides in the ratio 3: 5:



Therefore

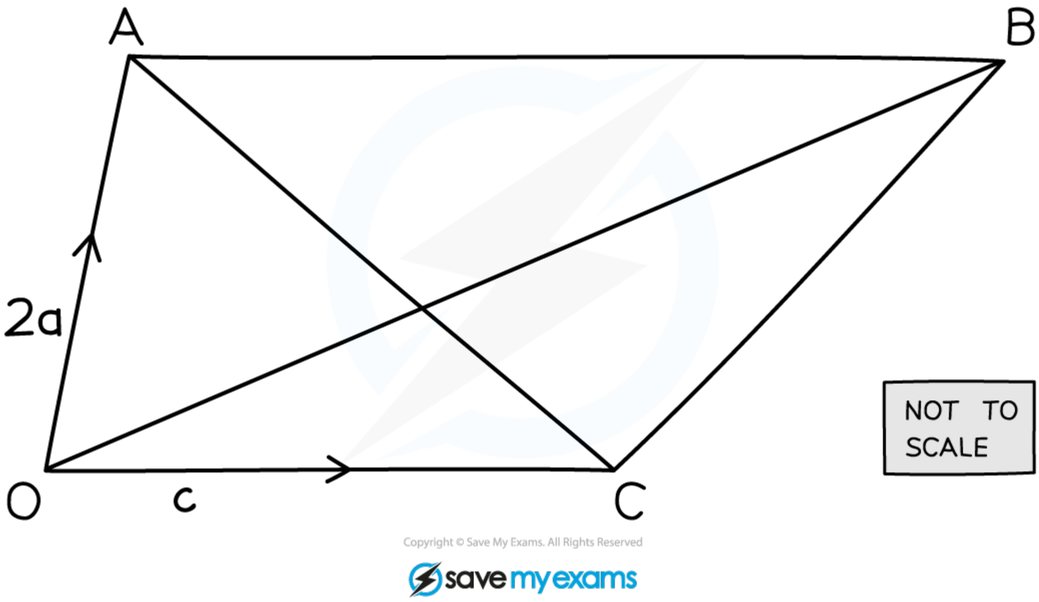
$$\overrightarrow{AX} = \frac{3}{8}\overrightarrow{AB} \quad and \quad \overrightarrow{XB} = \frac{5}{8}\overrightarrow{AB}$$

Worked Example

The diagram shows trapezium $OABC$.

$$\overrightarrow{OA} = 2a$$
$$\overrightarrow{OC} = c$$

AB is parallel to OC, with $\overrightarrow{AB} = 3\overrightarrow{OC}$



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a) Find expressions for vectors $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of a and b.

Point P lies on AC such that $AP: PC = 3:1$.

b) Find expressions for vectors $\overrightarrow{AP}$ and $\overrightarrow{OP}$ in terms of a and b.

c) Hence, prove that point P lies on line OB, and determine the ratio $\overrightarrow{OP}:\overrightarrow{PB}$

a) $\overrightarrow{AB} = 3\overrightarrow{OC}$ and $\overrightarrow{OC} = c$, so $\overrightarrow{AB} = 3c$.

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = 2a + 3c$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\overrightarrow{OA} + \overrightarrow{OC} = -2a + c$

4. $AP: PC = 3:1$ means that $\overrightarrow{AP} = \frac{3}{3+1}\overrightarrow{AC} = \frac{3}{4}\overrightarrow{AC}$

b) $\overrightarrow{AP} = \frac{3}{4}\overrightarrow{AC} = \frac{3}{4}(-2a + c)$

$\overrightarrow{AP} = -\frac{3}{2}a + \frac{3}{4}c$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 2a + \left(-\frac{3}{2}a + \frac{3}{4}c\right)$

$\overrightarrow{OP} = \frac{1}{2}a + \frac{3}{4}c$

3. To show that O, P, and B are colinear, note that $\overrightarrow{OP} = \frac{1}{2}a + \frac{3}{4}c = \frac{1}{4}(2a + 3c)$

c) $\overrightarrow{OP} = \frac{1}{4}(2a + 3c) = \frac{1}{4}\overrightarrow{OB}$

Therefore $\overrightarrow{OP}$ is parallel to $\overrightarrow{OB}$, so P must lie on the line OB.

4. If $\overrightarrow{OP} = \frac{1}{4}\overrightarrow{OB}$, then $\overrightarrow{PB} = \frac{3}{4}\overrightarrow{OB}$

$\overrightarrow{OP}:\overrightarrow{PB} = 1:3$

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7.2 TRANSFORMATIONS

7.2.1 TRANSFORMATIONS - ROTATION

What are transformations in maths?

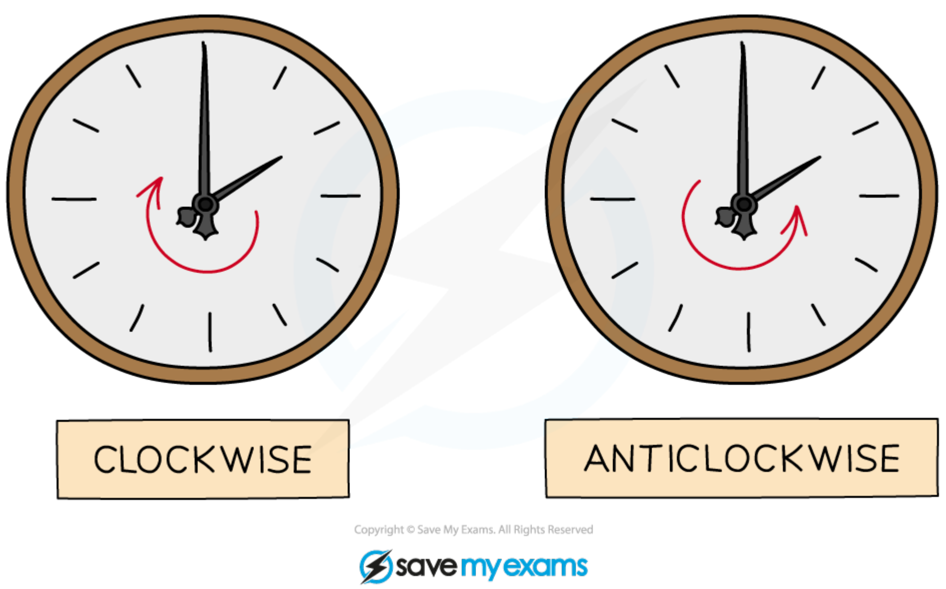
- There are 4 transformations in GCSE Maths – **rotation, reflection, translation and enlargement**
- All 4 change a shape in some way, useful in things like computer graphics.
- There is some language and notation often used in this topic – the original shape is called the **object** and the transformed shape is called the **image**
- Vertices on the object are labelled A, B, C, etc.
  And on the image they will be A', B', C' etc.
  If there is a second transformation then they will become A", B", C" etc.

Rotation – what do I need to know?

- You need to be able to perform a rotation (on a coordinate grid) as well as spotting and describing a rotation when presented with one
- Rotation has 3 features:

  1. Angle of rotation – how far we are going to rotate the shape by

  2. Direction – clockwise or anti-clockwise



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In the special case of 180° you will not need a direction

3. Centre of rotation – this is the point about which our rotation happens
It does NOT have to be a point that is on the shape nor in the middle of the shape; it can be anywhere

Exam Tip

Use tracing paper – it should be available although you will probably have to ask one of the invigilators for it. If you don’t want to do that during the exam, ask at the start – okay so a question may not turn up but if it does at least you’ve got it.
Most angles are “nice” – 90°, 180°, etc. Draw an arrow facing “up” on your tracing paper. Then as you rotate it, it’ll be really easy to see when you’ve turned 90° (arrow will be facing left or right), 180° (arrow facing down) and so on.

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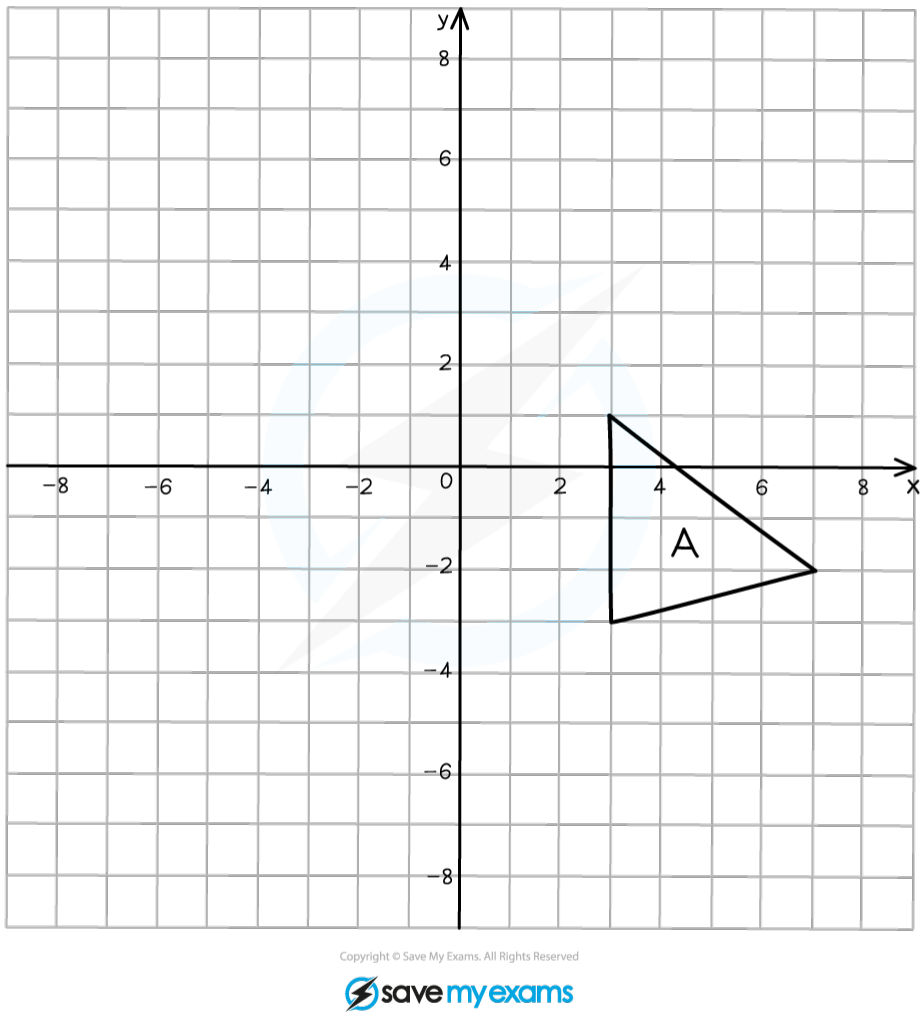
Worked Example

1. On the grid below rotate shape A by 90° anti-clockwise about the point (0, 2).

Label your answer A'.



This shape is easy to draw time and time again, even after a rotation or other transformation, so all you really need to do is rotate one point

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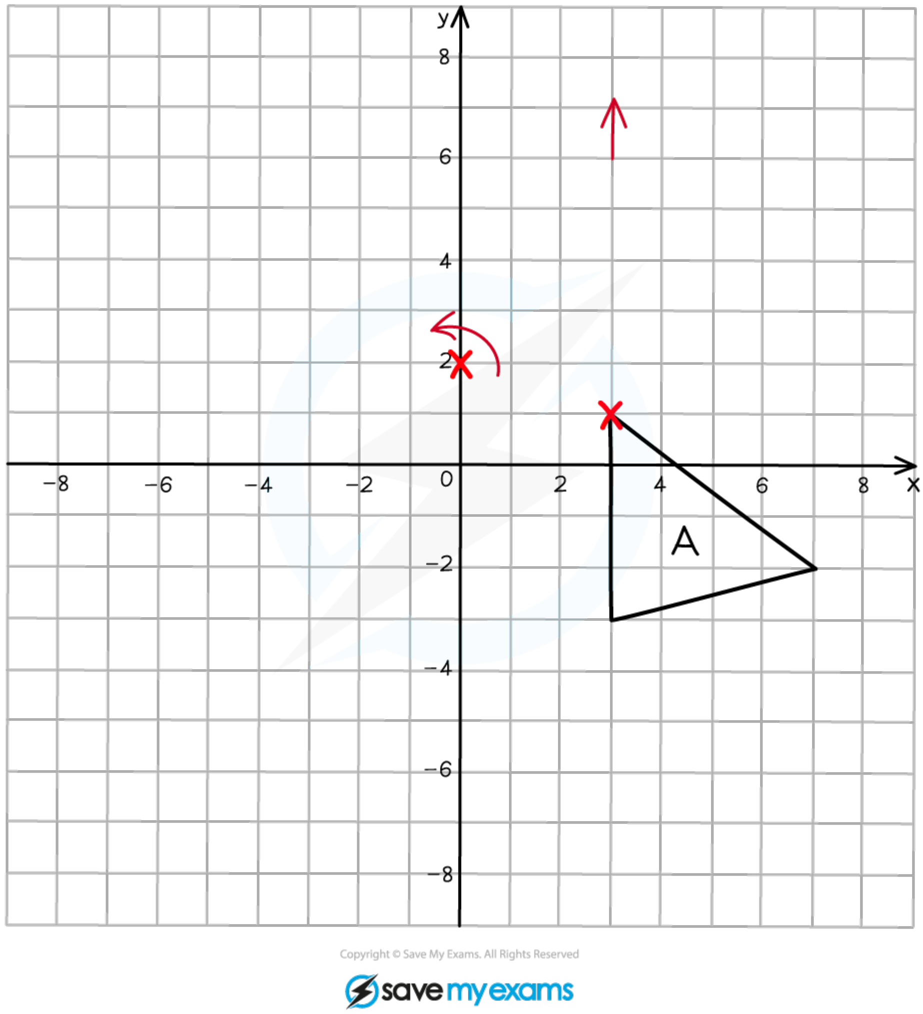

2, 3 - Using tracing paper mark one point on the shape (trace whole shape if you prefer, it is to only give you an idea of where shape ends up), mark the point of rotation, the direction and an arrow pointing up to make it easy to see when you've turned 90°

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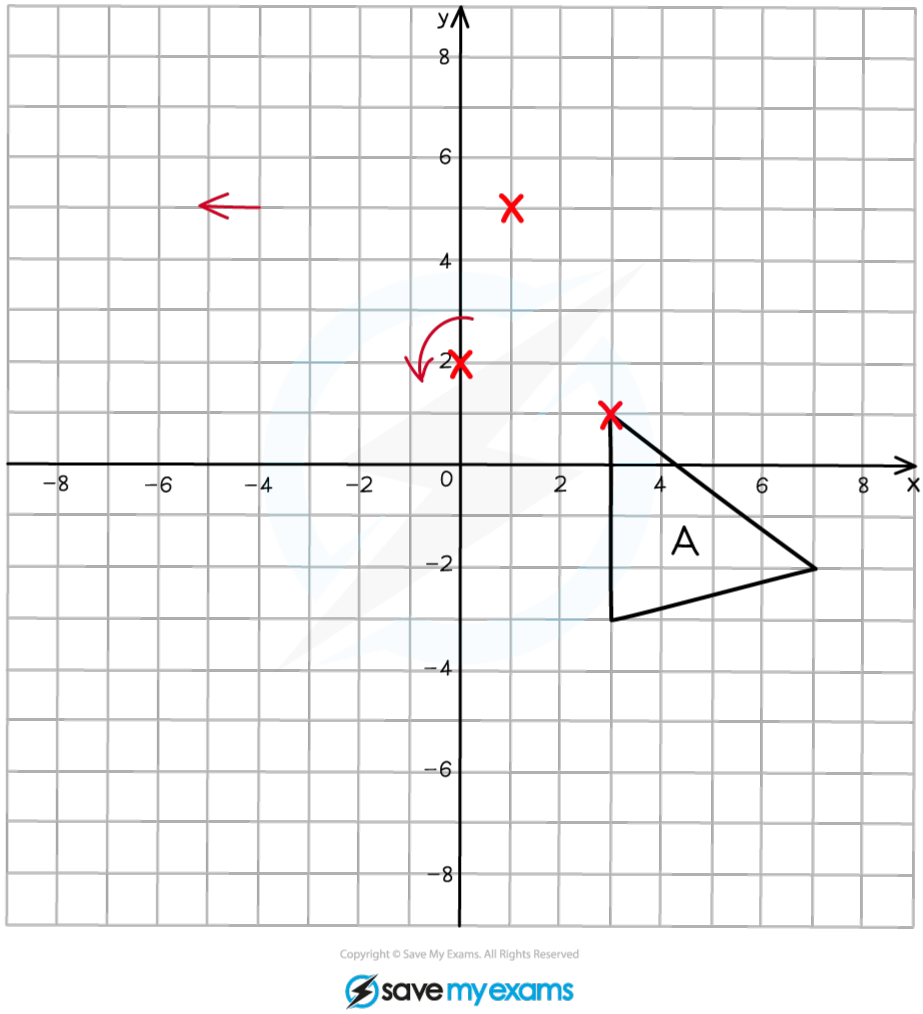

1 – Using the arrow from the top of the tracing paper it should be pointing to the left when we have turned 90° anti-clockwise

Make a mental note of the coordinates of your rotated point (1, 5)

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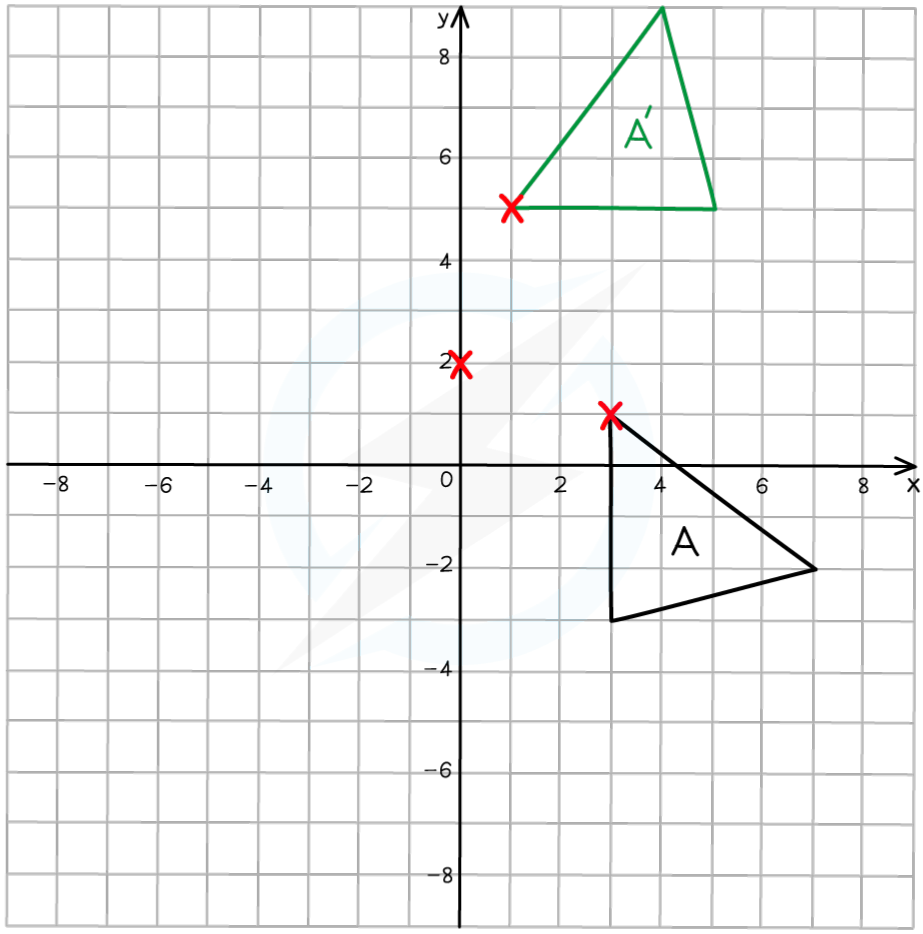


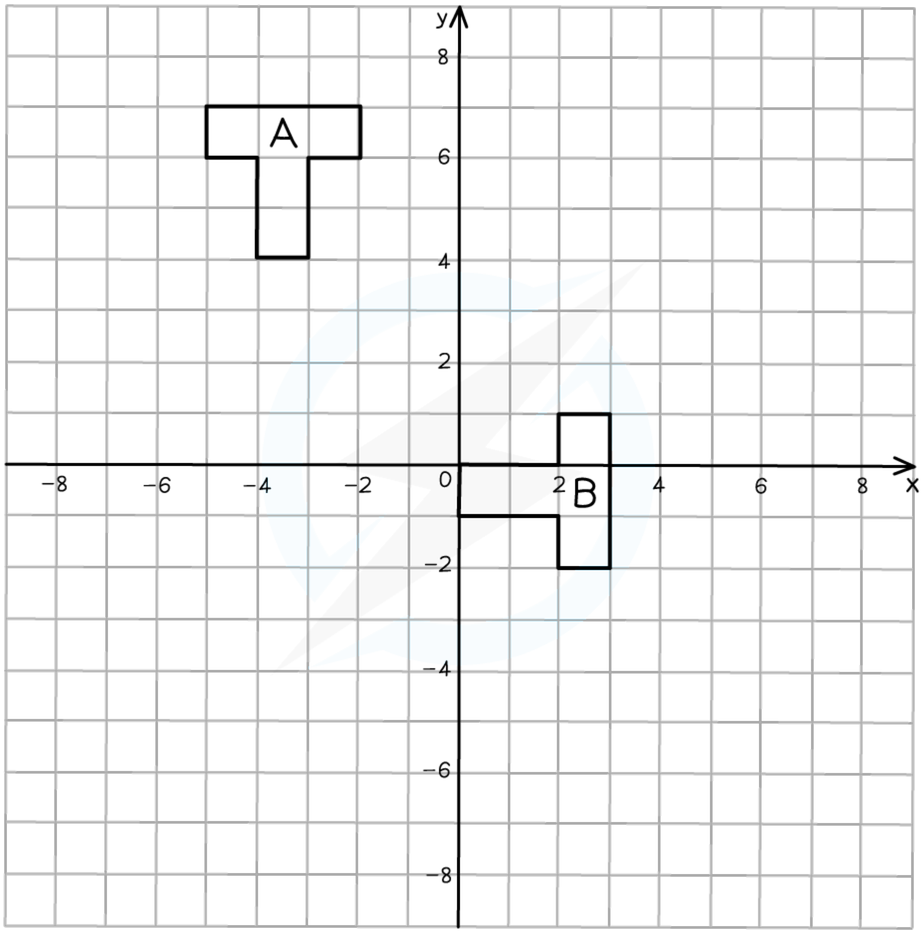
As the shape is easy to draw once you know where one point goes you can finish the question off

You can still use your tracing paper to check the other points or whole shape

2. Describe fully the single transformation that creates shape B from shape A.

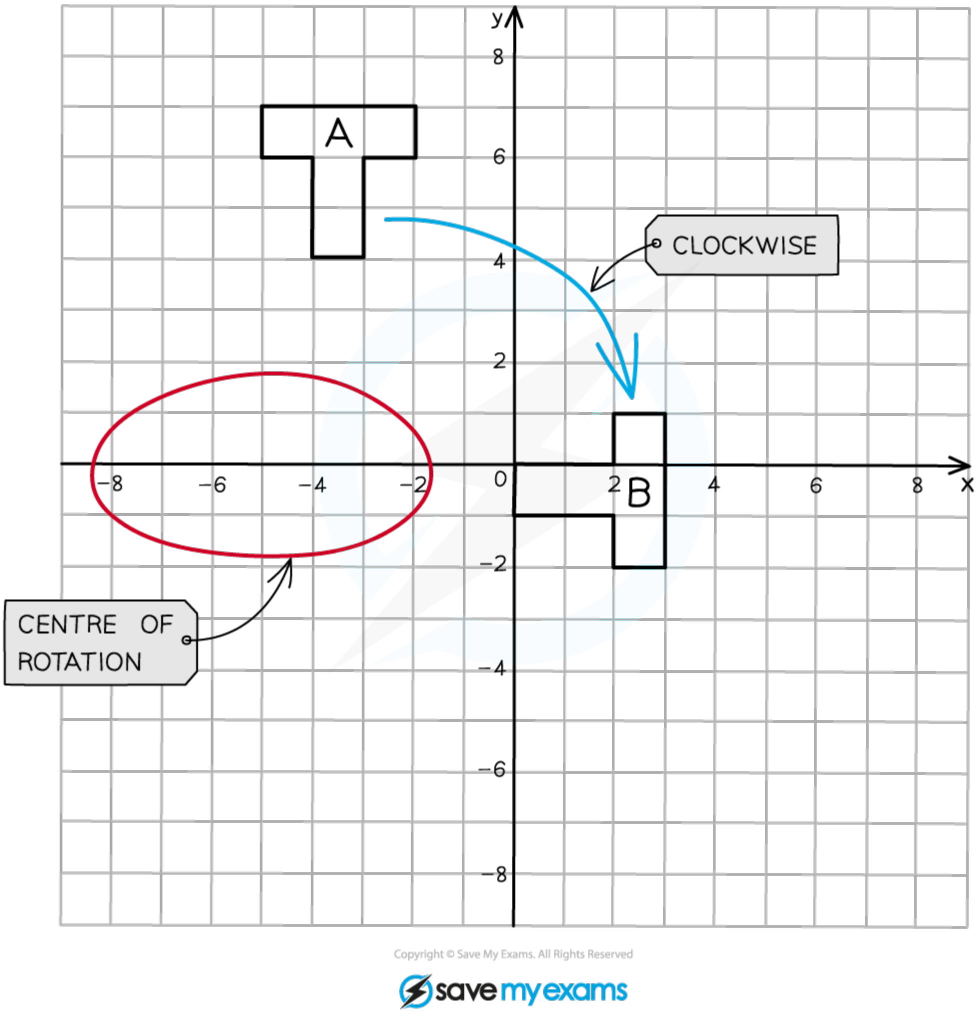
7. Vectors & Transformations

YOUR NOTES
⬇



7. Vectors & Transformations

YOUR NOTES

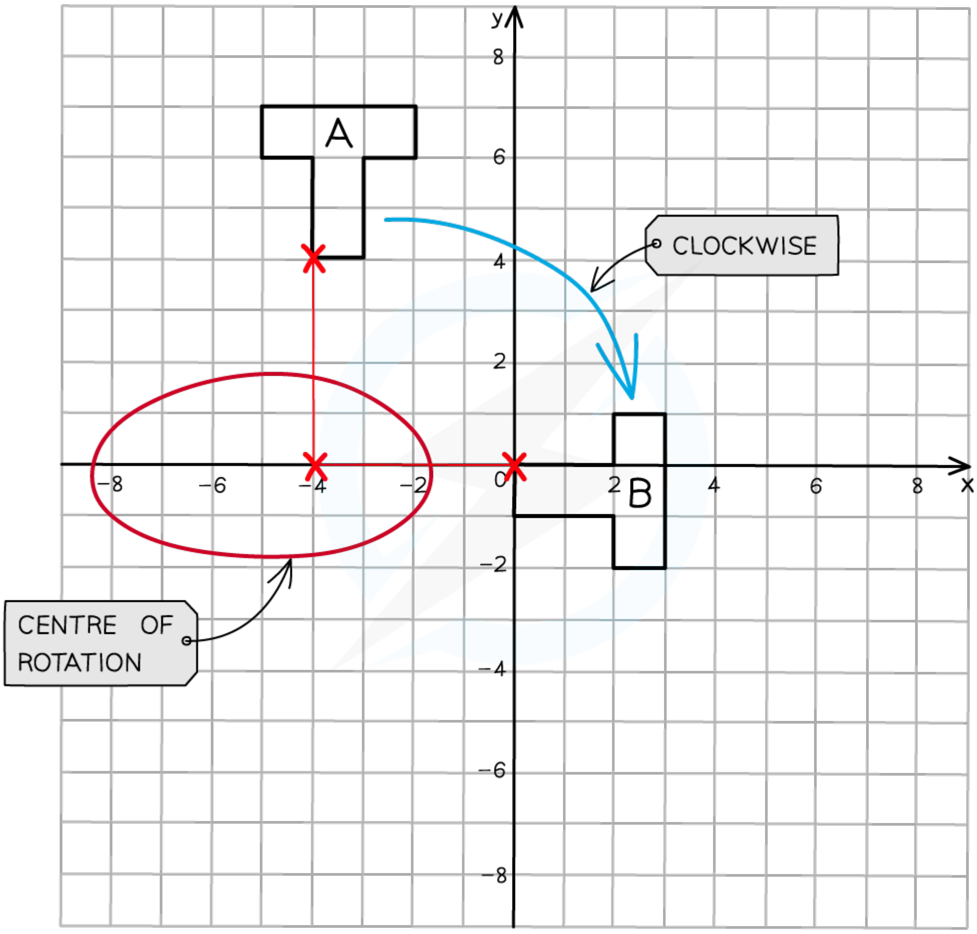


You should be able to see we have turned 90° clockwise (or 270° anticlockwise, both are correct)

The biggest problem is finding the centre of rotation but you should be able to quickly see roughly where this should be

7. Vectors & Transformations

YOUR NOTES
⬇



To find the exact coordinates of the centre of rotation you can either play around with tracing paper until you find it or you are looking for a point that is equal distance away from corresponding points

On the diagram we have the "bottom left" of the T-shape and can see the point (-4, 0) is 4 cm from both points – so this must be the centre of rotation

Rotation, 90° clockwise with centre of rotation (-4, 0)

Okay so this was always going to be a rotation given what these notes are about but make sure you always write the other three elements down – **angle**, **direction** and **centre** of **rotation**

7. Vectors & Transformations

YOUR NOTES

7.2.2 TRANSFORMATIONS - REFLECTION

Reflection – what do I need to know?

- You need to be able to perform a reflection (on a coordinate grid) as well as spotting and describing a reflection when presented with one
- Reflections only have one key thing to look for – the mirror line – but these will be described mathematically using the equations of straight lines making things a little more awkward

1. Vertical lines
These are in the form $x = k$, for some number k

2. Horizontal lines
These are in the form $y = k$, for some number k

3. Diagonal lines
Much harder to perform a reflection in these but lines are of the form

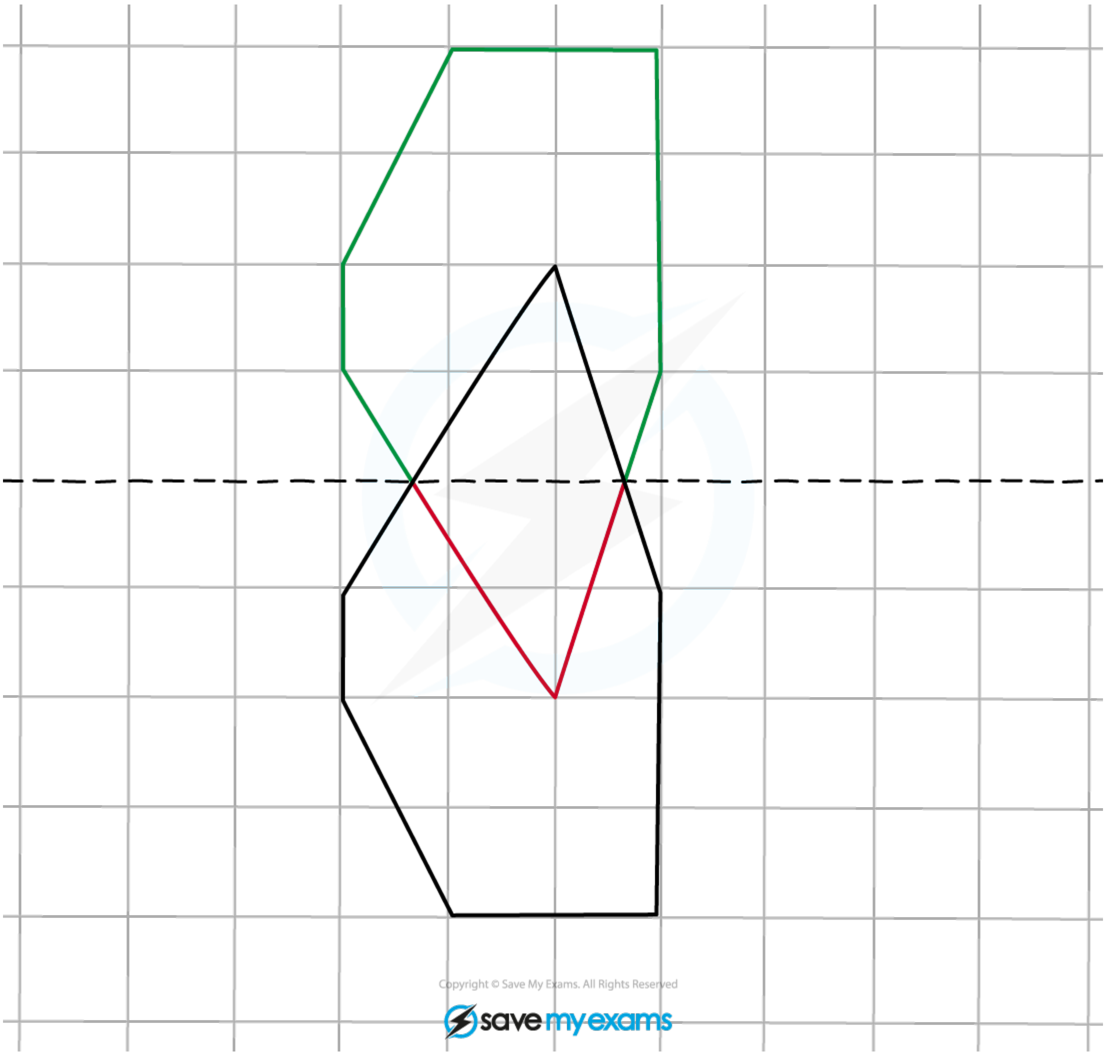
$\mathbf{y = mx + c}$ (see Straight Line Graphs)

4. Points on the mirror line
Do not move – they stay where they are

5. Double reflections
This is where the mirror line passes through the shape being reflected
Part of the shape gets reflected one way, the rest the other

7. Vectors & Transformations

YOUR NOTES
⬇



7. Vectors & Transformations

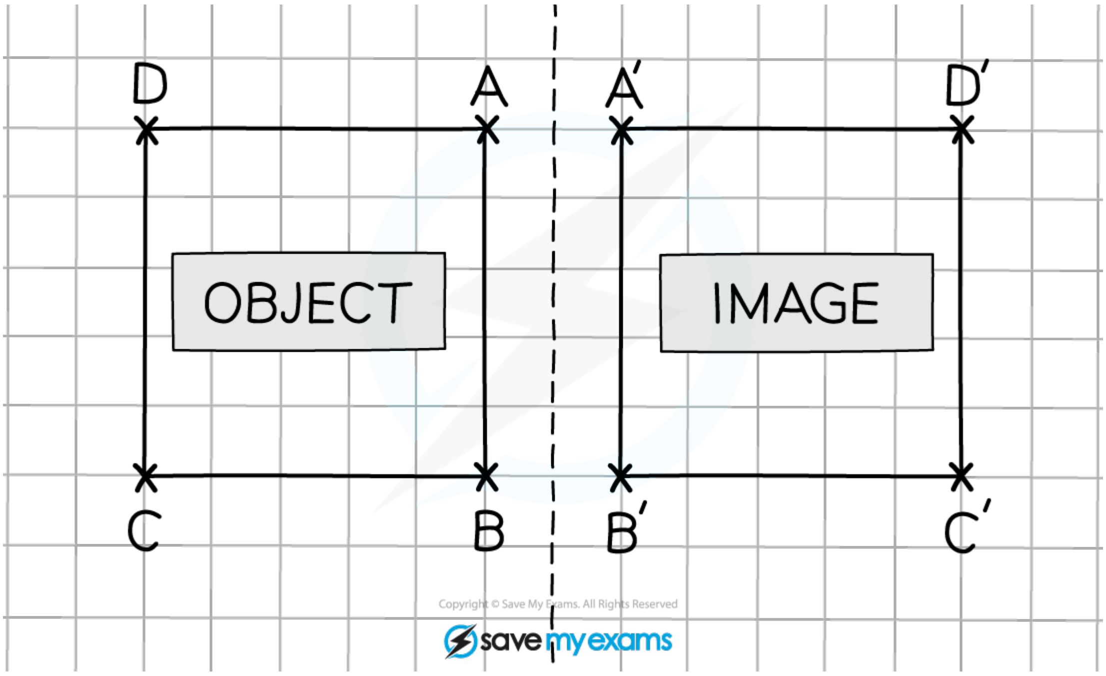
YOUR NOTES

6. Regular polygons

Squares and other regular polygons can look identical even after a reflection (and other transformations too) – there is no obvious sign the shape has been reflected – you may think a shape has been translated

The way to identify these is to look at one vertex (point) on the shape and its corresponding position

If it is a reflection it will be "back-to-front" on the other side



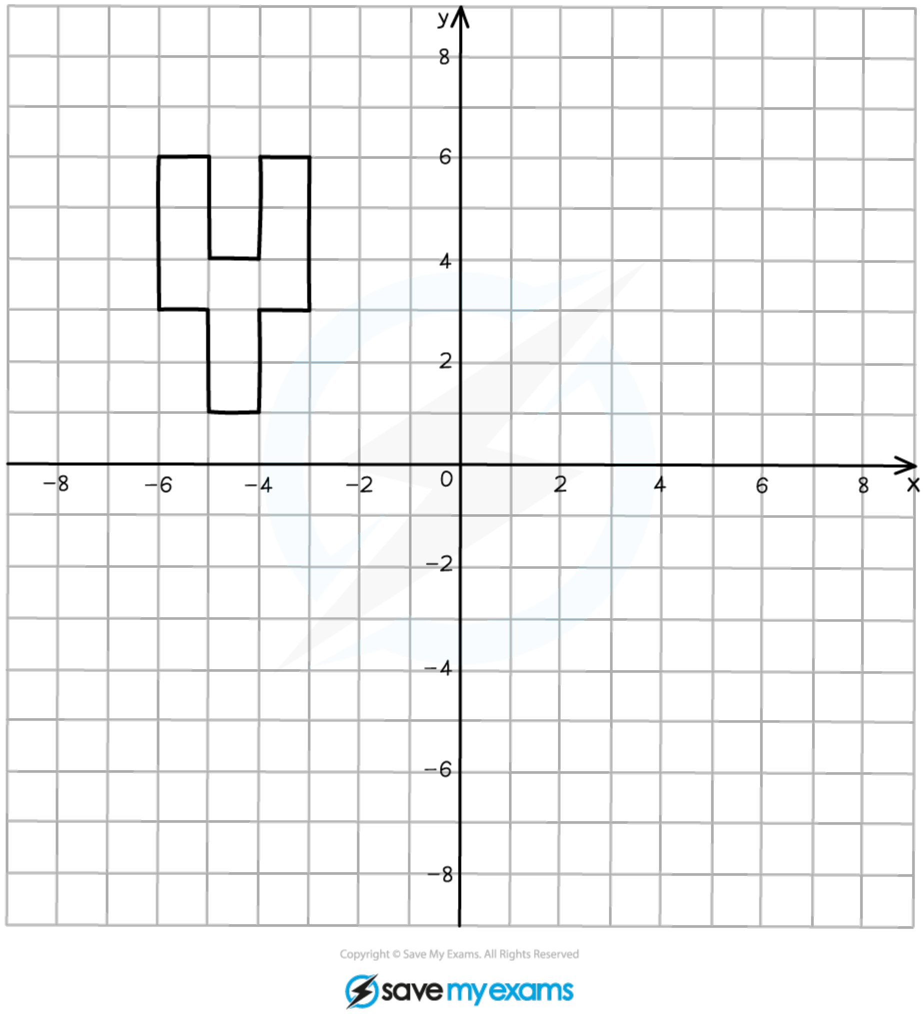
Worked Example

1. On the grid below reflect shape S in the line $y = x + 3$.

State the coordinates of all of the vertices of your reflected shape.

7. Vectors & Transformations

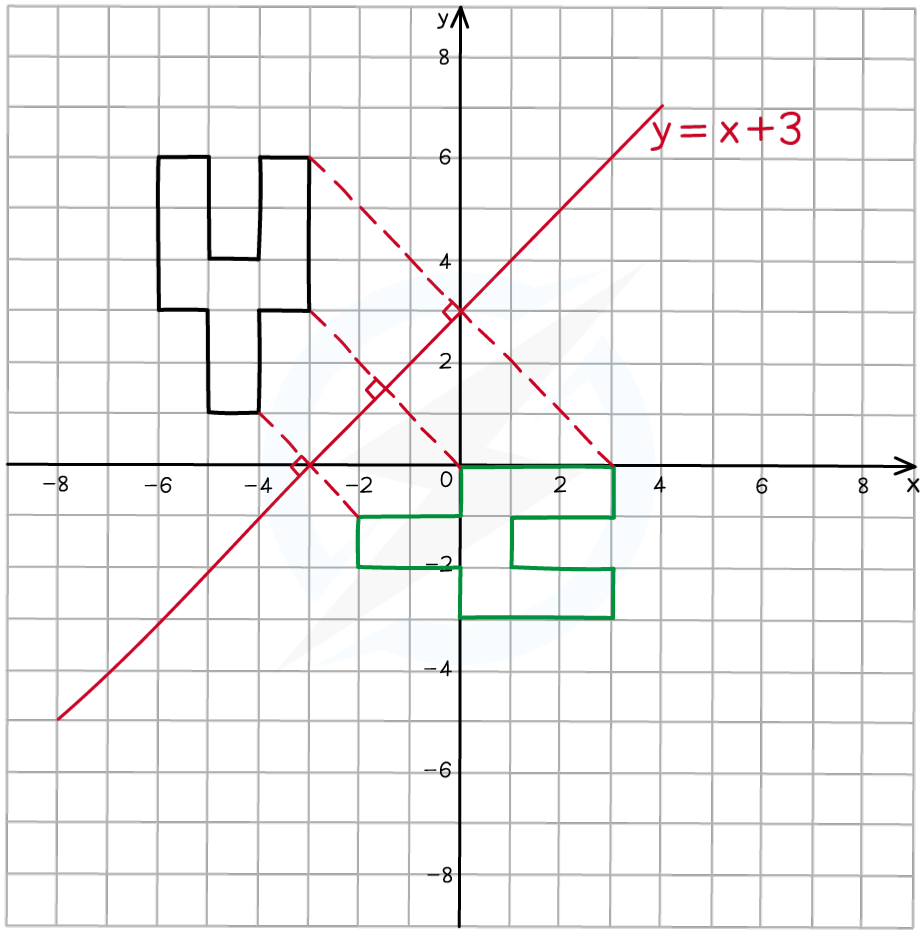
YOUR NOTES



3 - The first problem here is the mirror line but from your work with straight line graphs you know it has a gradient of 1 and a y-axis intercept of 3
Draw the line on the diagram, it doesn't have to be dotted!

7. Vectors & Transformations

YOUR NOTES
⬇



A slightly awkward shape and a diagonal line so move 2-3 vertices from the object before drawing your final answer

Remember reflections work "perpendicular to the mirror line"!

You need the same distance on either side of the mirror line so either measure with your ruler or count in "diagonal squares" – the bottom of the Y-shape is 1 "diagonal square" from the mirror line so we need 1 "diagonal square" on the other side to find where this point moves to.

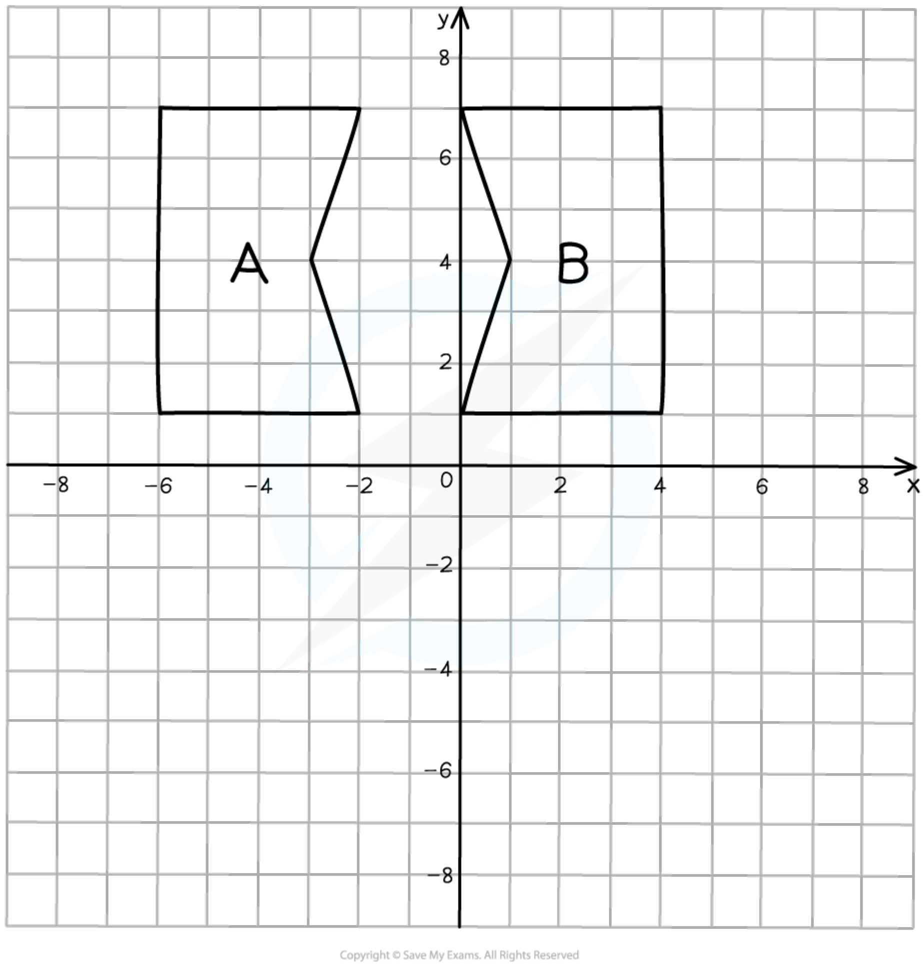
Vertices of reflected shape are: (0, 0), (3, 0), (3, -1), (1, -1), (1, -2), (3, -2), (3, -3), (0, -3), (-2, -2), (-2, -1) (0, -1)

Quite a lot to list so make sure you work logically and don't miss any out!

7. Vectors & Transformations

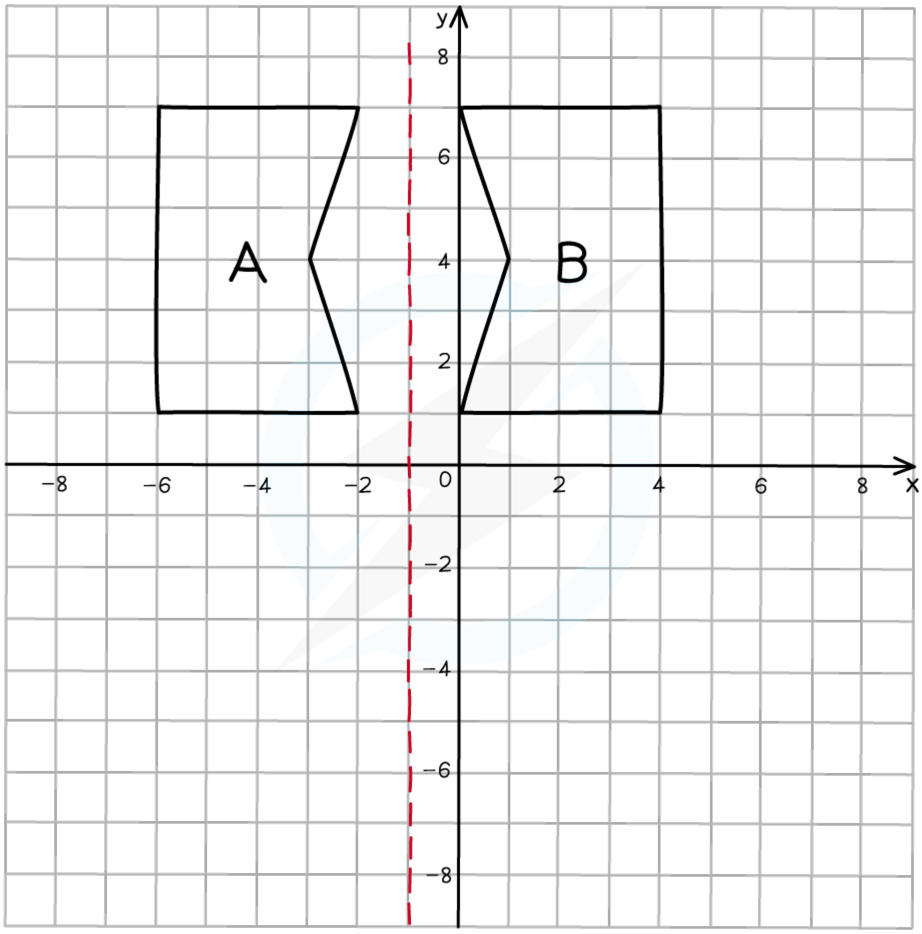
YOUR NOTES

2. Describe fully the single transformation that creates shape B from shape A.



7. Vectors & Transformations

YOUR NOTES



1 - You should be able to "see" this without much difficulty but it is still important to write the transformation in the correct language

Shape A has been reflected in the line $x = -1$ to create shape B

Okay so this was always going to be a reflection ...

Do be careful with mirror lines near the axes – it is very easy to miscount or think one of the axes is the mirror line when it isn't

7. Vectors & Transformations

YOUR NOTES

7.2.3 TRANSFORMATIONS - TRANSLATION

Translation – what do I need to know?

- You need to be able to perform a translation (on a coordinate grid) as well as spotting and describing a translation when presented with one
- Translations are **where an object has moved but remains the same way up**
- This movement is described by a **vector**
- You need to know how to write a translation using a vector (rather than words)

1. **Vectors**
    - Written in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where ...
        - x is the distance moved horizontally:
          Negative means move to the left
          Positive means move to the right
        - y is the distance moved vertically:
          Negative means move down
          Positive means move up
2. **Special cases**
    - In some cases where the vector is **small** enough the **image** can "overlap" the **object**
    - The vector is how the shape moves, not the size of the gap between the object and the image – watch out for this common error!

Exam Tip

As you are simply redrawing a given shape in a different place on the coordinate grid all you need to do is work out where one of the vertices of the shape translates to and draw the shape from there.

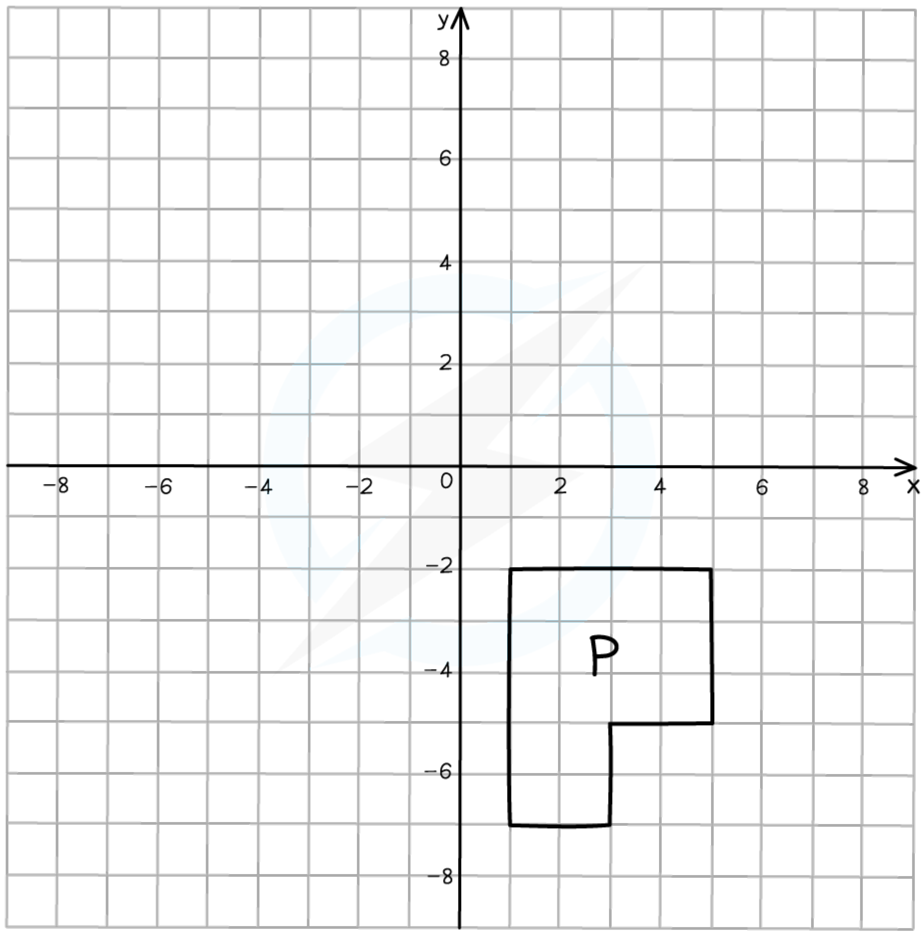
7. Vectors & Transformations

YOUR NOTES

Worked Example

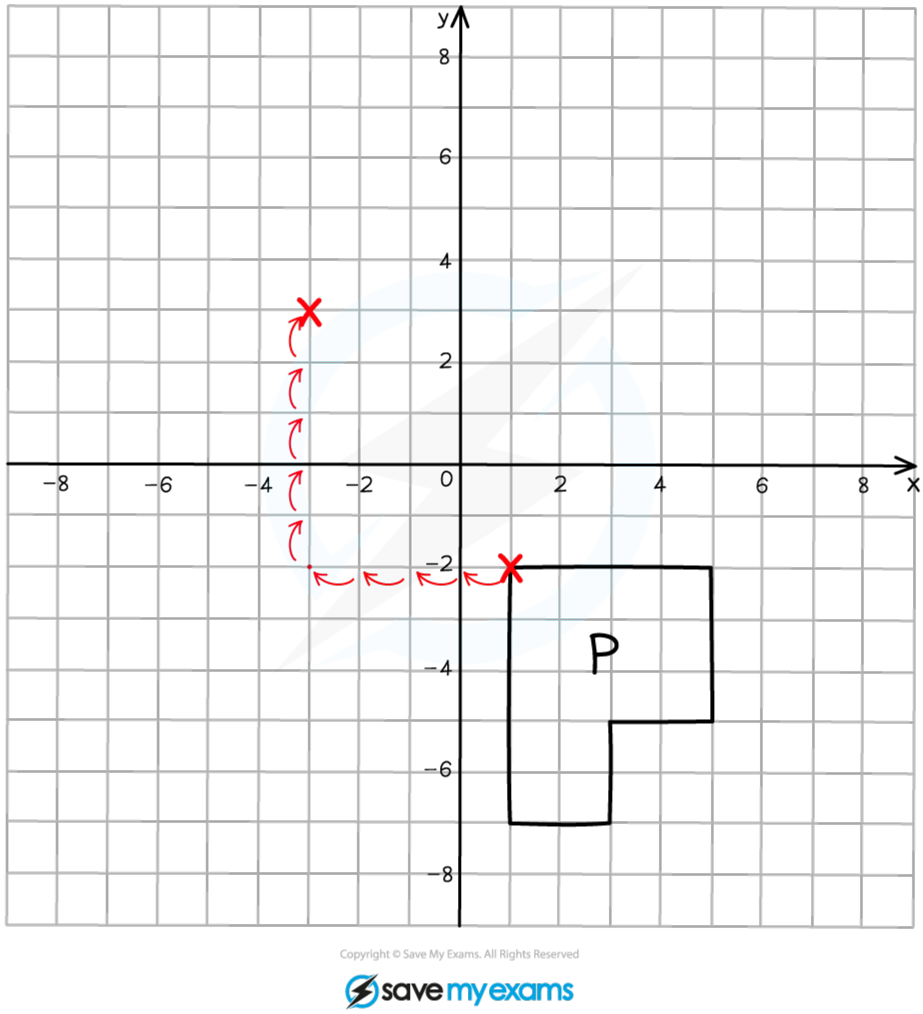
1. On the grid below translate shape P using the vector $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$.

Label your translated shape P'.



7. Vectors & Transformations

YOUR NOTES

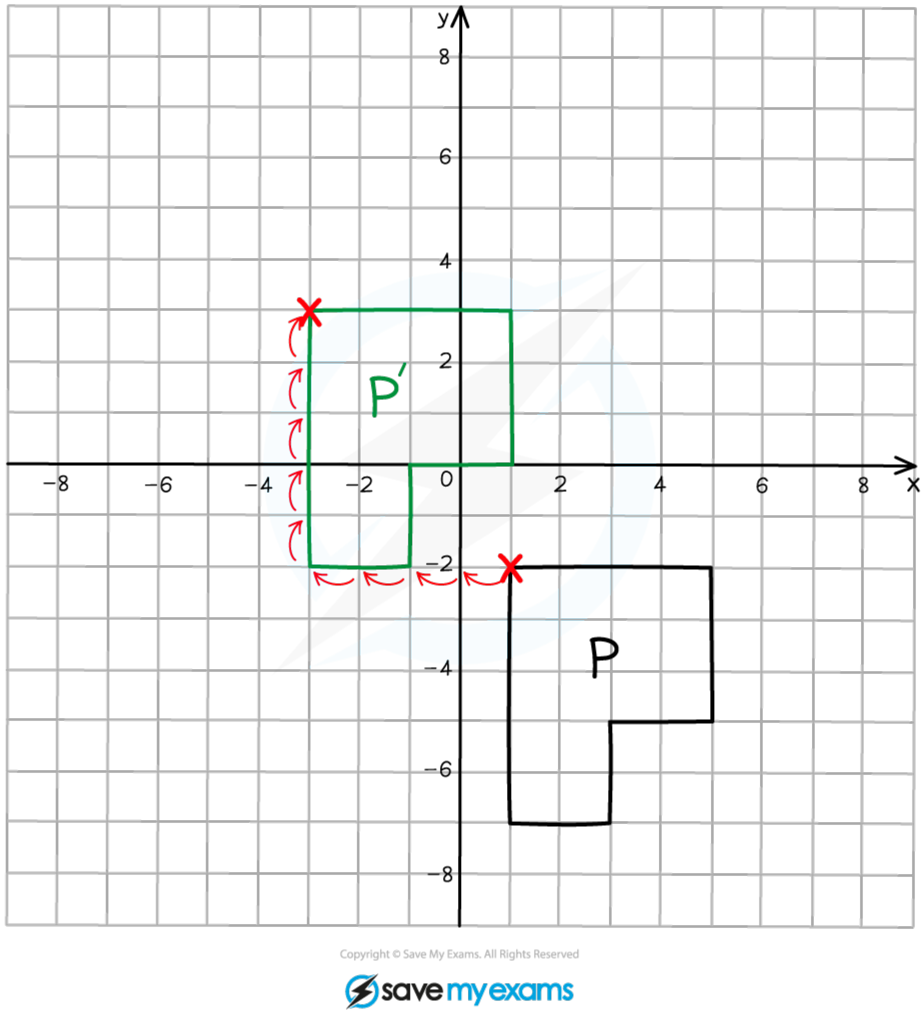


1 – The vector means “4 to the left” and “5 up”

You don’t have to draw any of the arrows but it is a good idea to mark the paper after counting across and again after counting up to make your understanding clear

7. Vectors & Transformations

YOUR NOTES

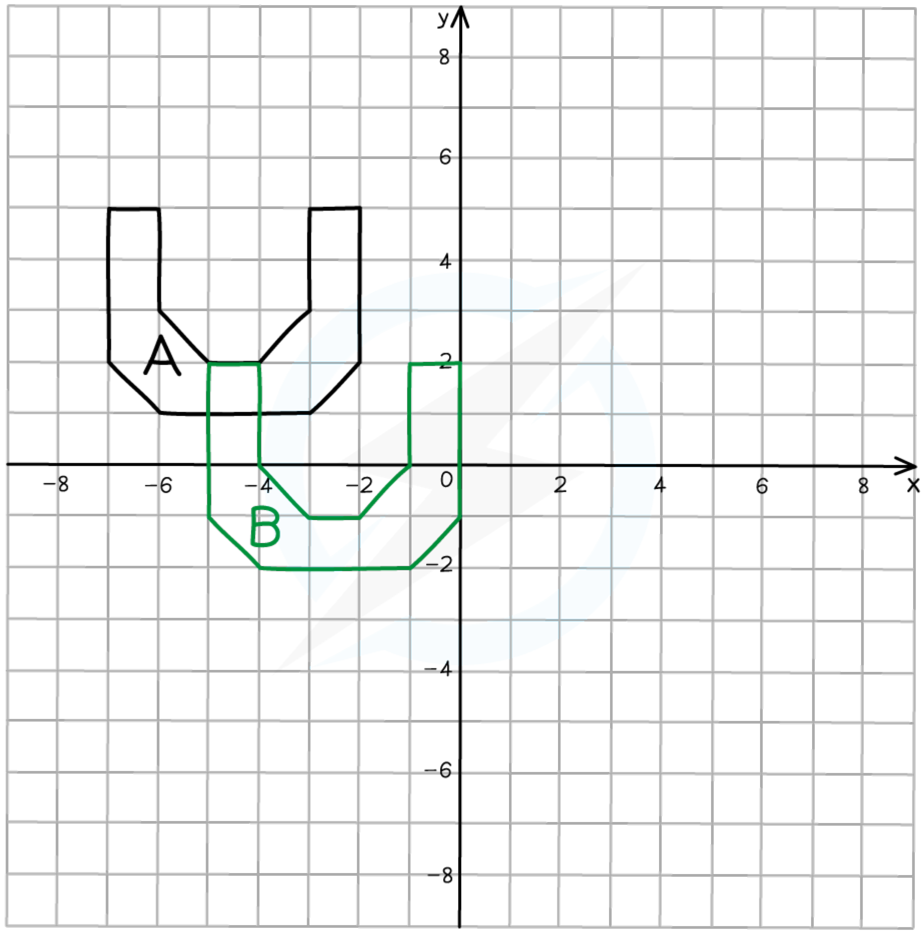


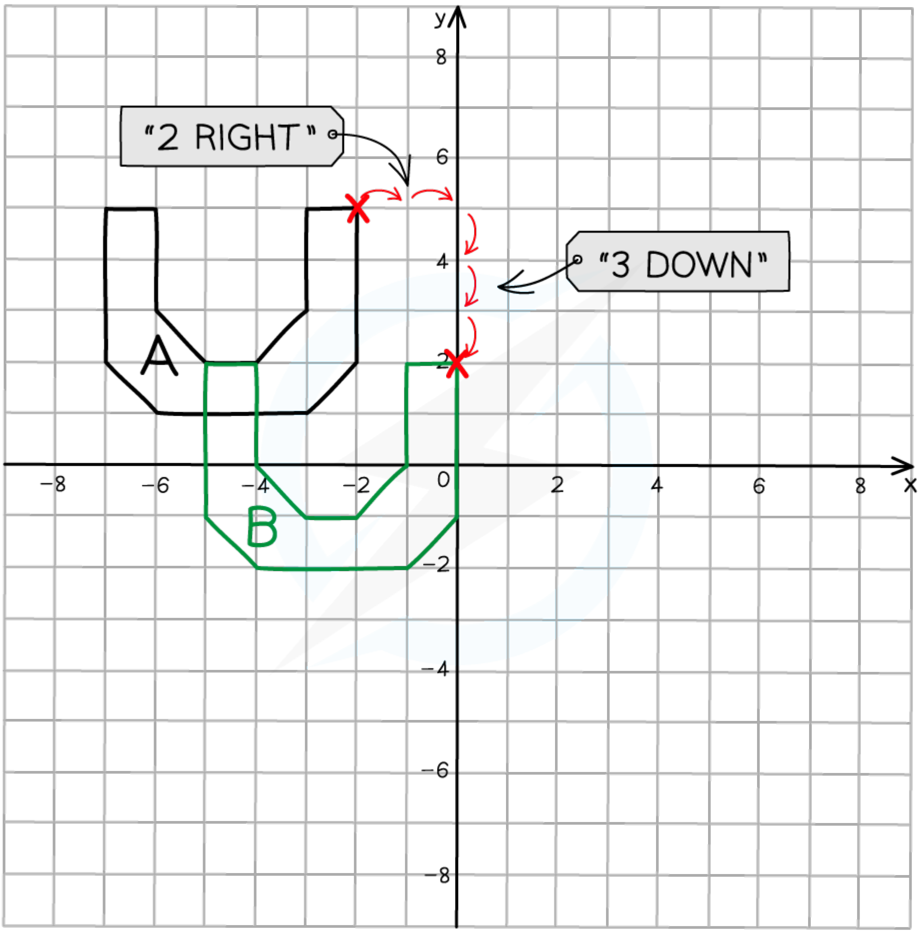
Picking on one vertex and following the shape round from there makes it really easy to get a translation in exactly the right place!

2. Describe fully the single transformation that creates shape B from shape A.

7. Vectors & Transformations

YOUR NOTES
⬇





1 – This is a case here where the image overlaps the object but you should still see the shape is the same way up and so has moved and it is a translation

Use a point that is well aware from any overlap area to avoid confusion but take care counting around the axes

Shape A has been translated using the vector $\binom{2}{-3}$ create shape B

Okay so this was always going to be a translation …

You do not need to write "2 right" and "3 down" but if it helps, do it!

7. Vectors & Transformations

YOUR NOTES
⬇

7.2.4 TRANSFORMATIONS - ENLARGEMENT

Enlargement – what do I need to know?

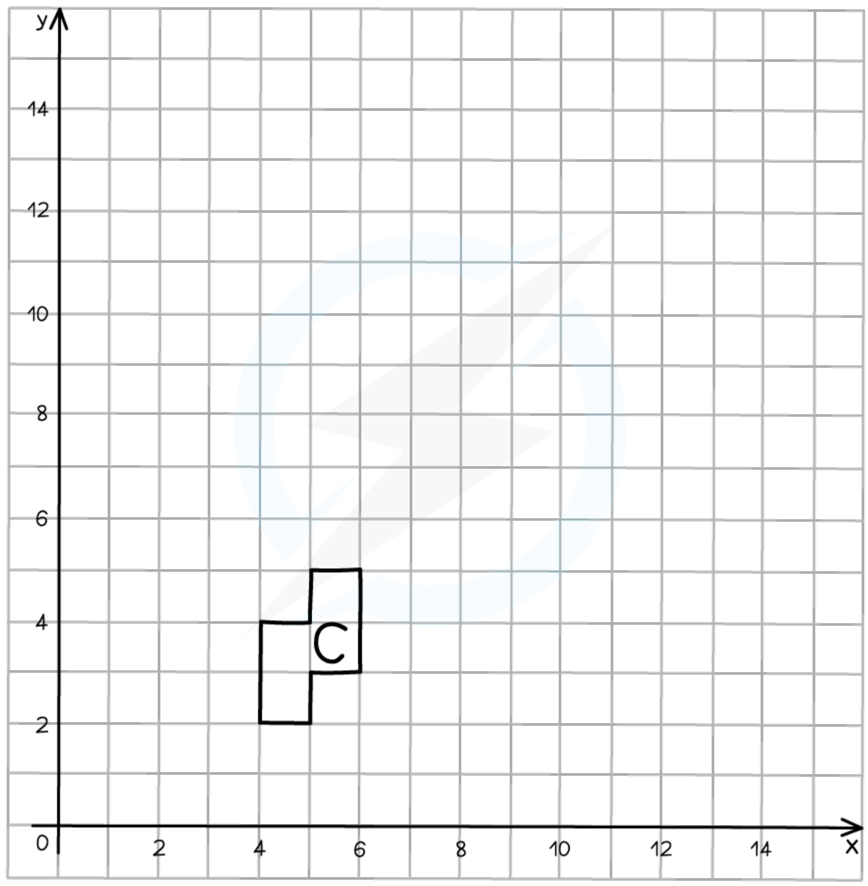
- You need to be able to perform an enlargement (on a coordinate grid) as well as spotting and describing an enlargement when presented with one.
- The key things with an enlargement are:
  - **Scale Factor**
    The scale factor is how many times every side of the image is bigger than the object
    However if the scale factor is a fraction the image will be smaller than the object
  - **Centre of Enlargement**
    This tells us where on the page the image is going to go but we have a bit of work to do first
    As with the other transformations in most cases it is easiest to move one vertex of the shape and draw the image from there

Worked Example

1. On the grid below enlarge shape C using scale factor 2 and centre of enlargement (2, 1).
   Label your translated shape C'.
   Without working out any areas explain why the area of C' is four times as large as the area of C.

7. Vectors & Transformations

YOUR NOTES



2 – Start by marking the centre of enlargement (CoE)

Pick a vertex from the image and draw a line through that and the CoE

7. Vectors & Transformations

YOUR NOTES
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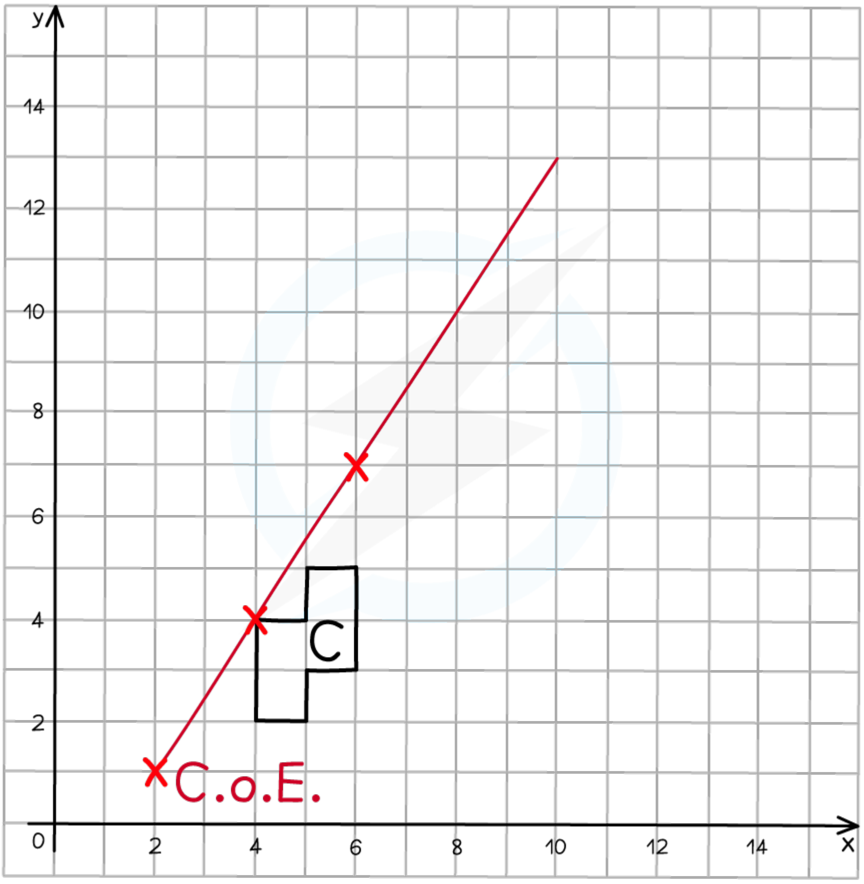
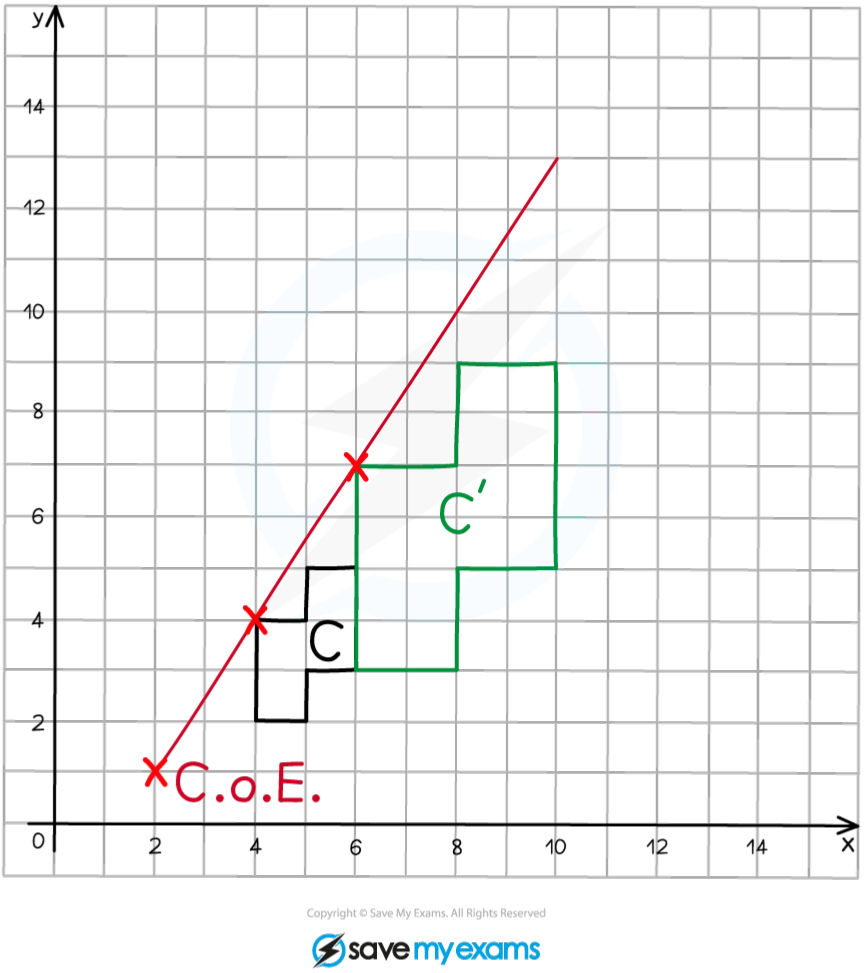


1 – As the scale factor is **2** the distance from the CoE to the new/image vertex will be **twice** the distance from the CoE to the original/object vertex

You can either measure this with a ruler or think of it as “2 across, 3 up” that needs to be done twice.

7. Vectors & Transformations

YOUR NOTES
⬇



As with the other transformations once we know where one vertex goes it is easy to complete the shape – but this time remember each distance on the shape will be multiplied by the scale factor.

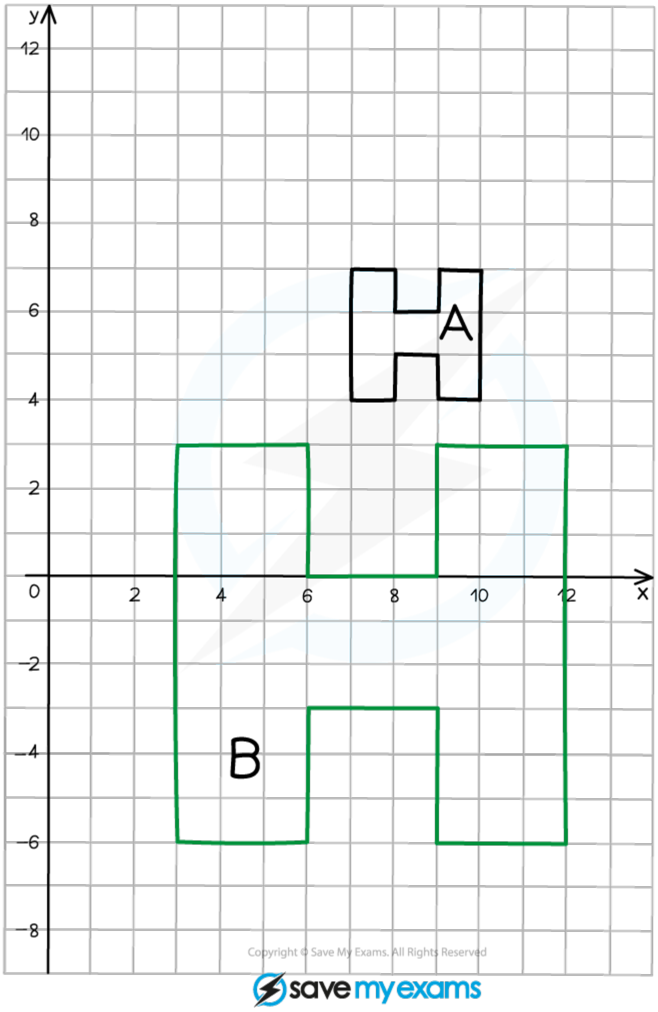
The area of C will need to be multiplied by the area scale factor, which is 2 x 2 = 4, in order to find the area of C'.

Hence C' has an area four times bigger than C.

2. Describe fully the single transformation that creates shape B from shape A.

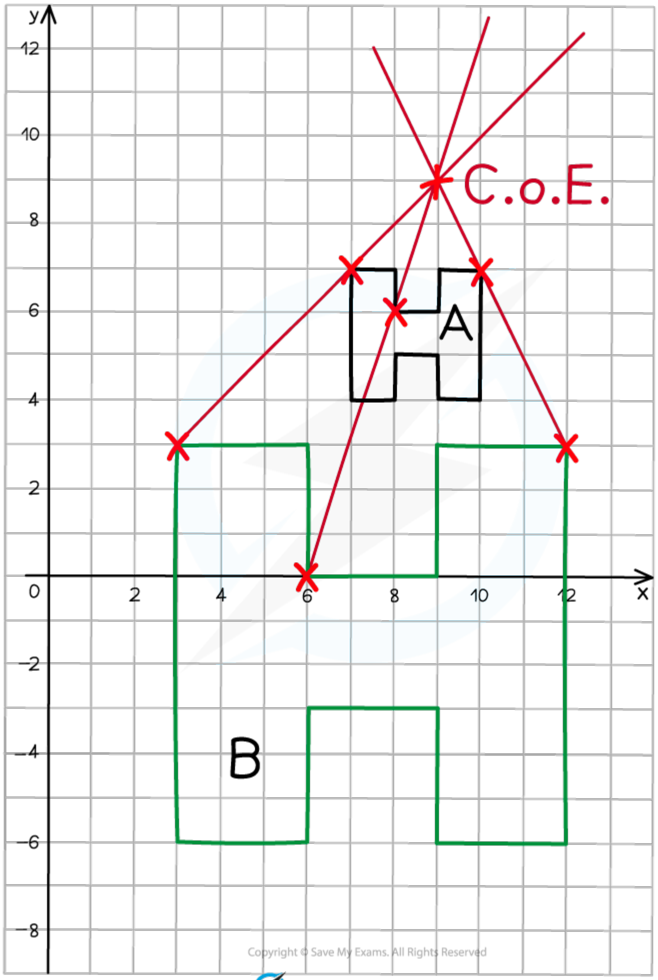
7. Vectors & Transformations

YOUR NOTES
⬇



7. Vectors & Transformations

YOUR NOTES ⬇



1 – You should be able to see the scale factor by comparing one of the distances on the two shapes – picking a distance of 1 is the easiest way to do this

2 - Starting from the **image**, draw lines ("rays of light") from corresponding vertices

You'll need to do at least two but doing a third acts as a check

Where these lines cross gives you the centre of enlargement

Shape A has been enlarged using a scale factor of 3 and centre of enlargement at (9, 9) to create shape B

Okay so this was always going to be an enlargement …

Do note that if A and B had swapped over the centre of enlargement would remain the same but the scale factor would've been $\frac{1}{3}$

7. Vectors & Transformations

YOUR NOTES
⬇

7.2.5 COMBINED TRANSFORMATIONS

Combined Transformations – what do I need to know?

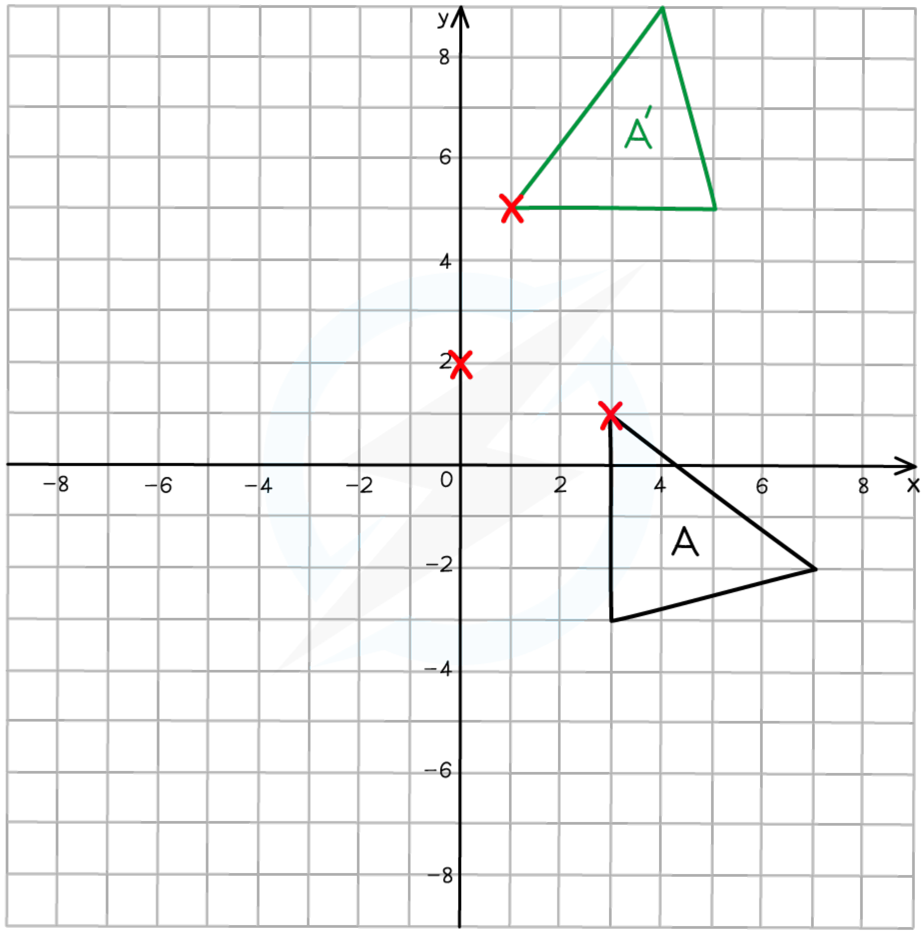
- Combined transformations are more than one transformation, one performed after the after
- It is often the case that 2 transformations can be equivalent to 1 alternative transformation and you will be expected to spot those
- Here's a reminder of the transformations:

1. Rotation

- Requires an angle, direction and centre of rotation
- It is usually easy to tell the angle from the orientation of the image
- Use some instinct and a bit of trial and error to find the centre of enlargement.

7. Vectors & Transformations

YOUR NOTES

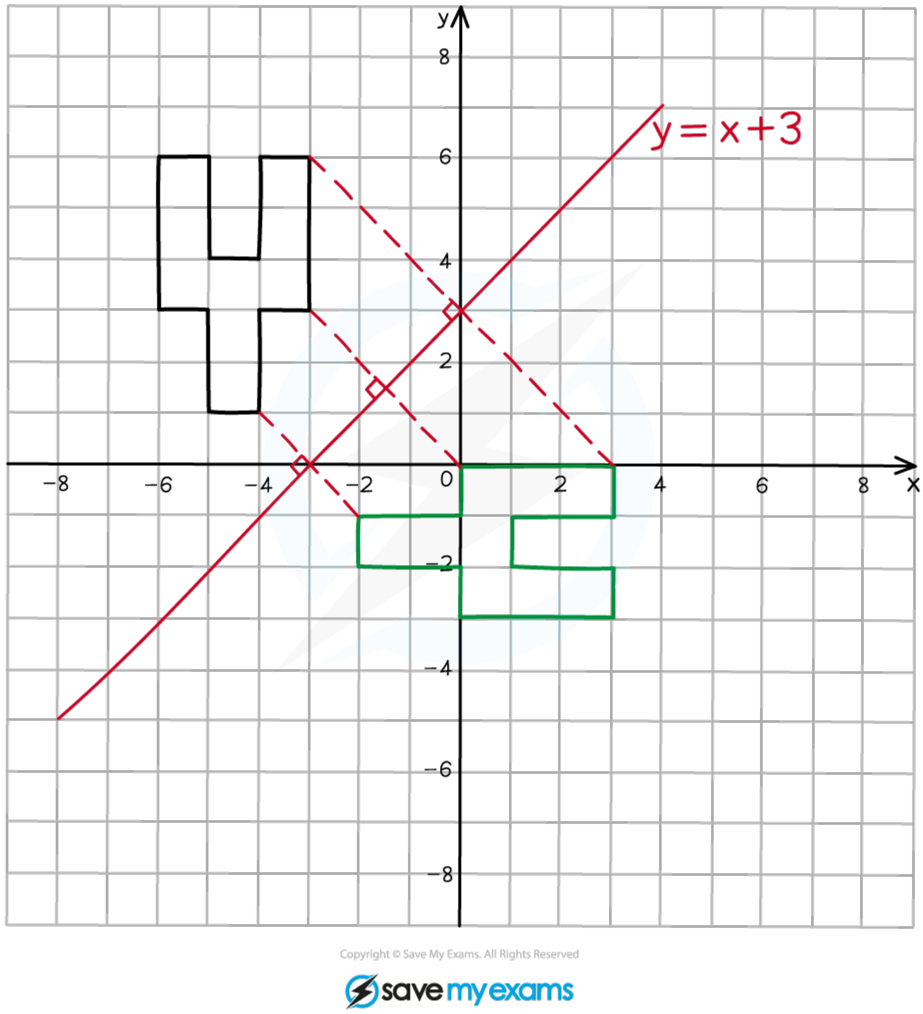


2. Reflection

- Mirror line – can be vertical (x = k), horizontal (y = k) or diagonal (y = mx + c)
- Points on the mirror line do not move
- Double reflections are possible if the mirror line passes through the object

7. Vectors & Transformations

YOUR NOTES



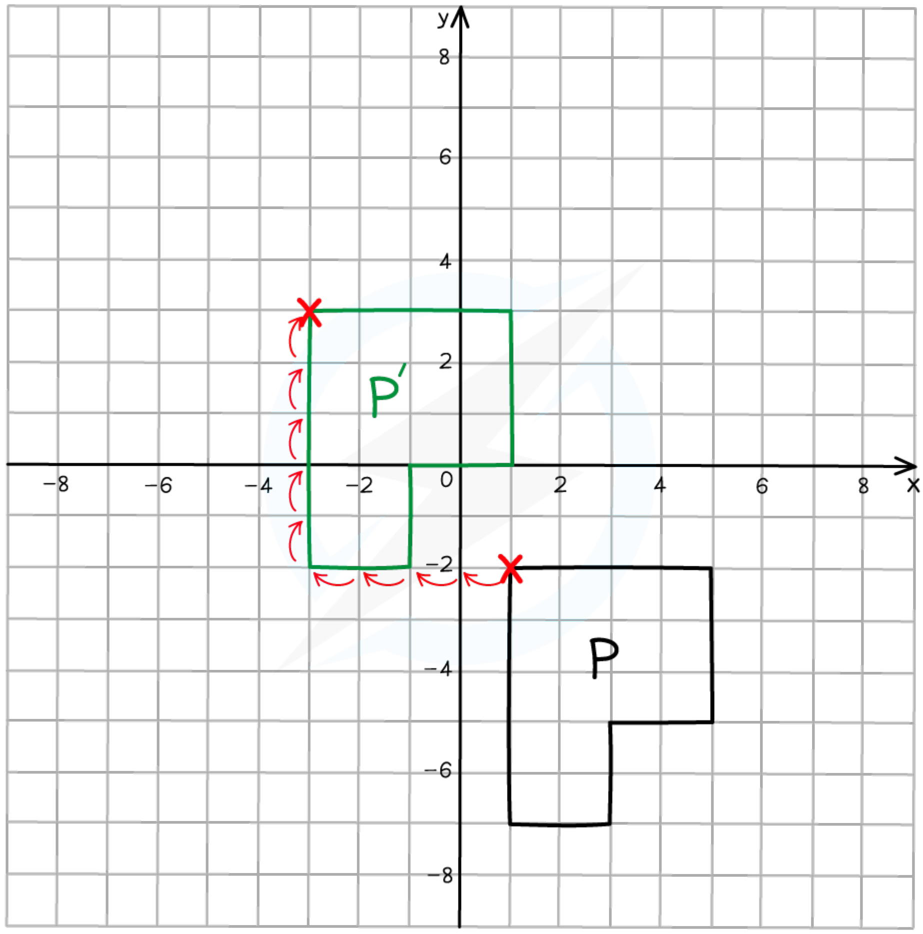
3. **Translation**

A vector in the form $\binom{x}{y}$ describes a vector.

A translation does not change which way up a shape is (and coordinates will correspond in the same order).

7. Vectors & Transformations

YOUR NOTES
⬇

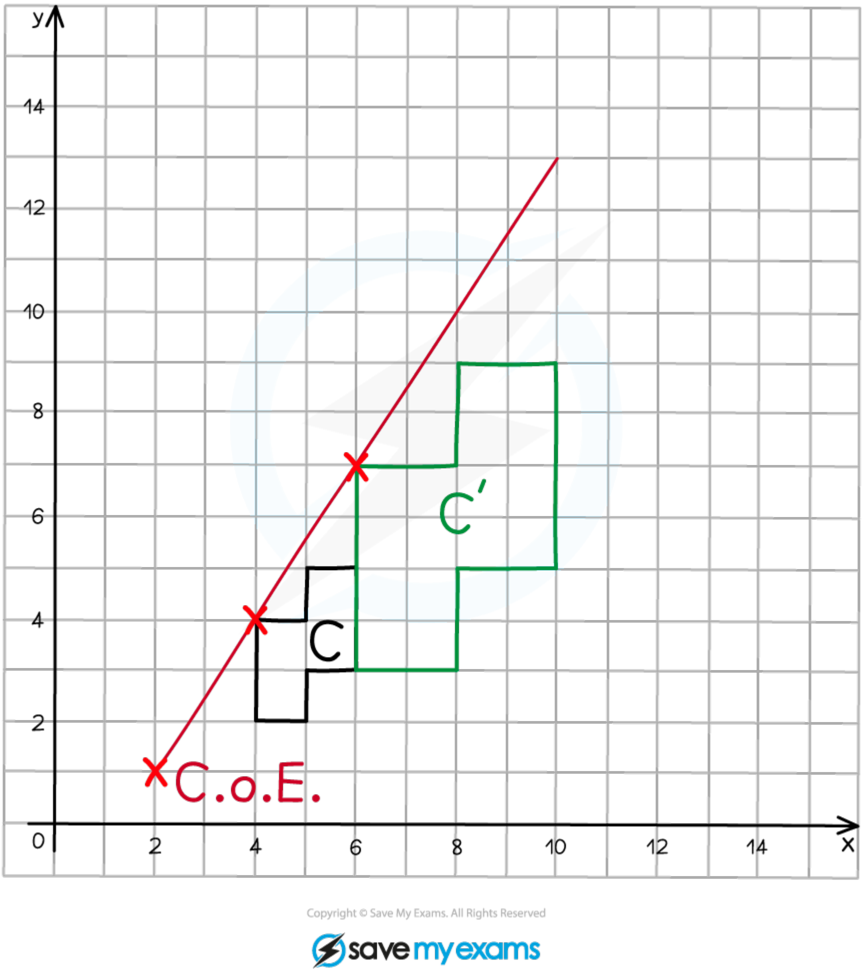


4. Enlargement

- A scale factor and centre of enlargement are needed for an enlargement
- Enlargements can make shapes smaller if scale factor is fractional
- Area scale factor is the scale factor squared
- Negative scale factors mean the shape is enlarged on the other side of the centre of enlargement

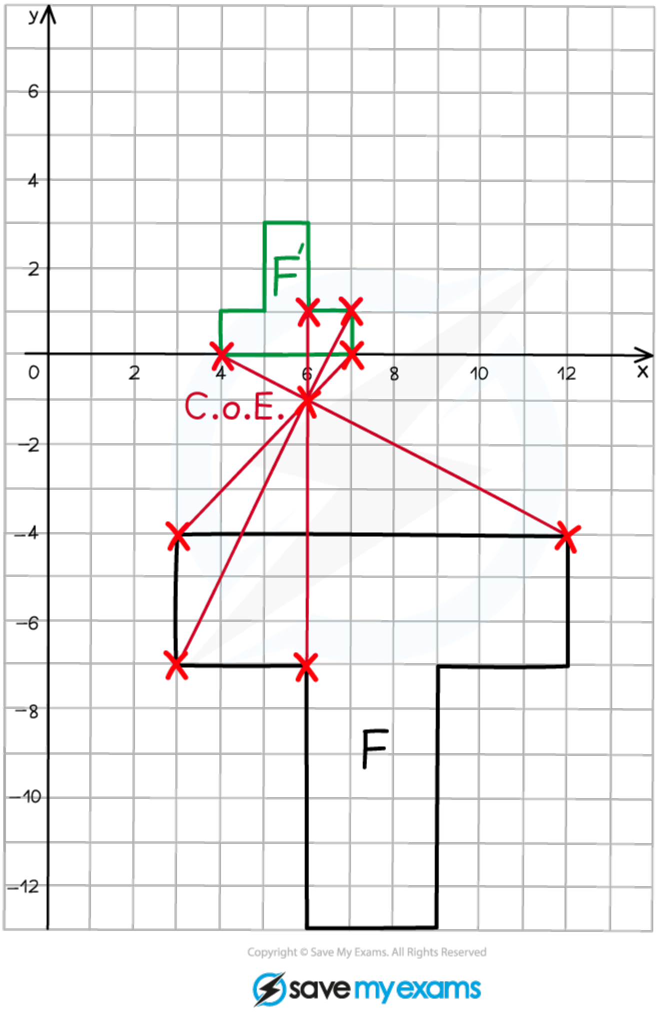
7. Vectors & Transformations

YOUR NOTES
⬇



7. Vectors & Transformations

YOUR NOTES



Worked Example

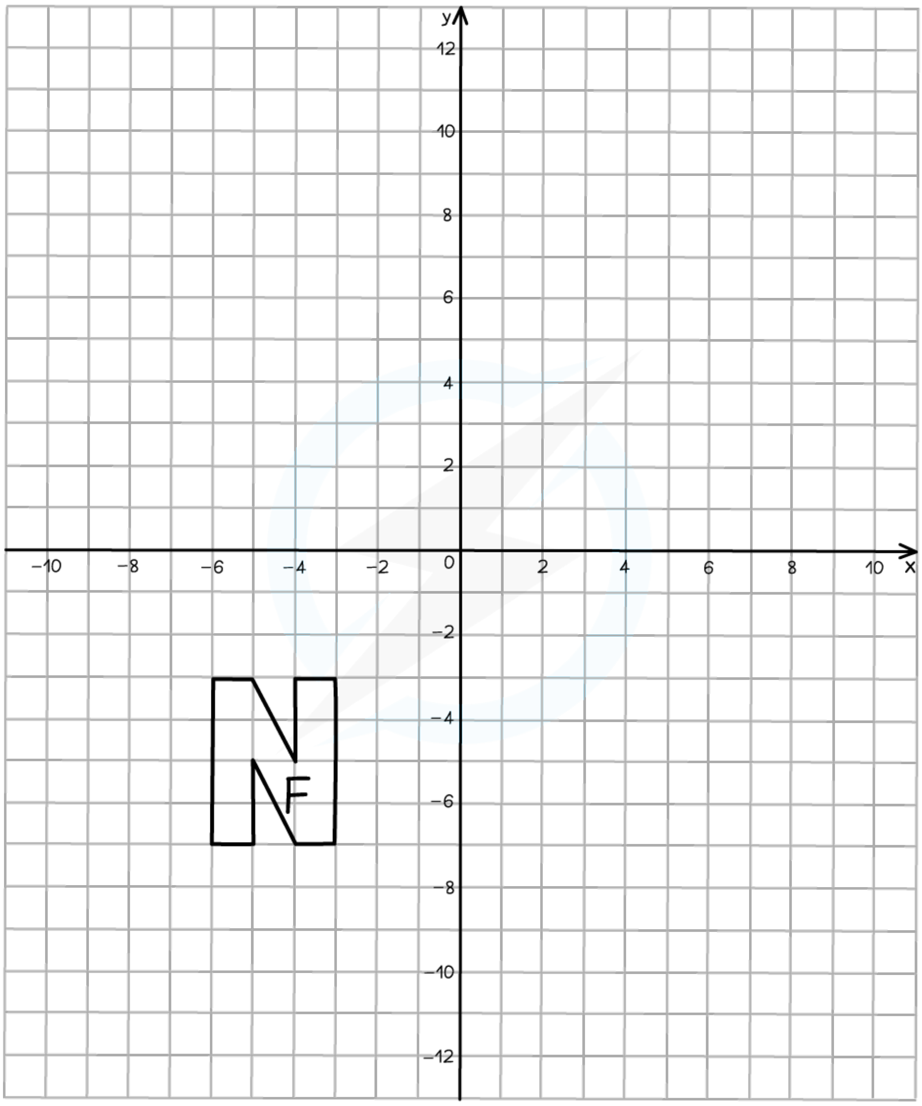
1. (a) On the grid below rotate shape F 180° using the origin as the centre of rotation.
   Label this shape F'.

   (b) Reflect shape F' in the line $y = 0$.
   Label this shape F''.

   (c) Fully describe the single transformation that would create shape F''
   from shape F.

7. Vectors & Transformations

YOUR NOTES



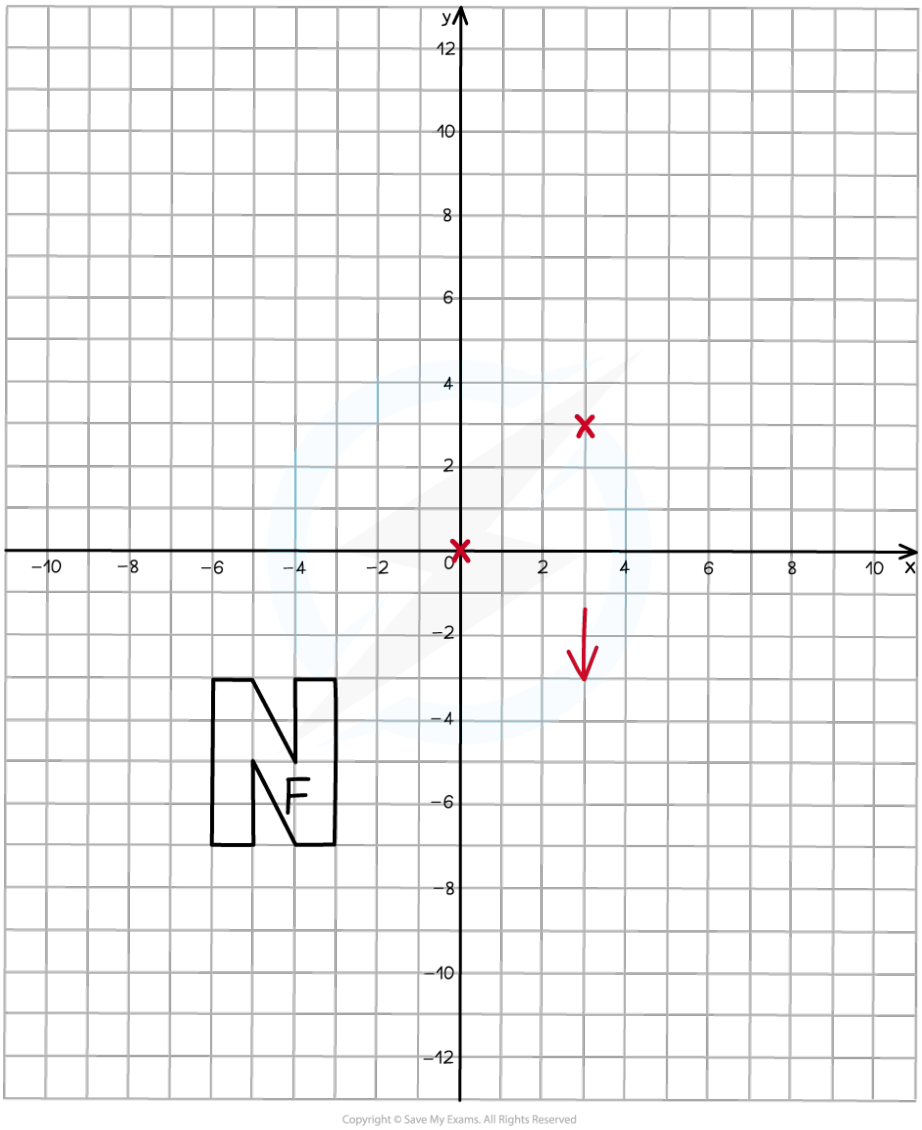
(a)

1 – Start with a rotation

The diagrams above and below show this being down using tracing paper

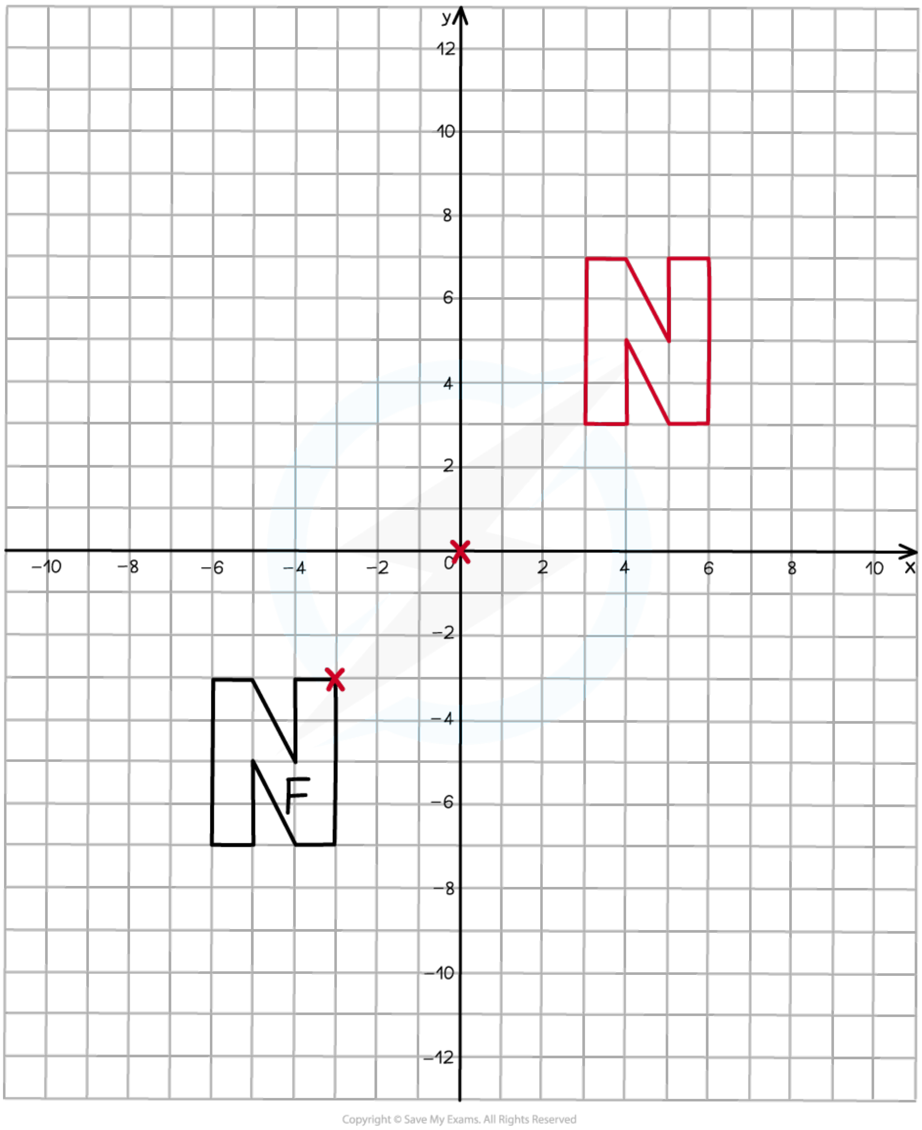
7. Vectors & Transformations

YOUR NOTES
⬇



7. Vectors & Transformations

YOUR NOTES



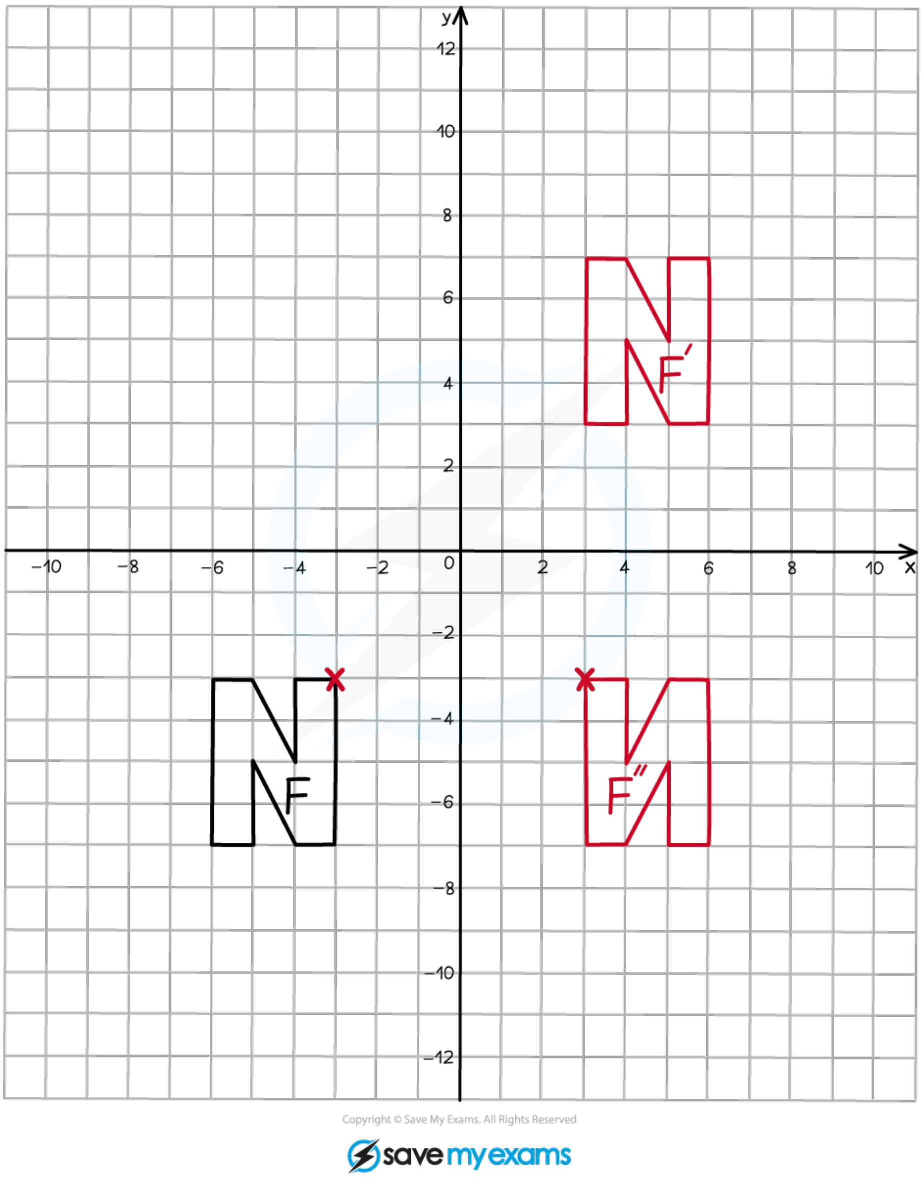
(b)

2 – Now we have a reflection to do

The line $y = 0$ is the same as the x-axis

7. Vectors & Transformations

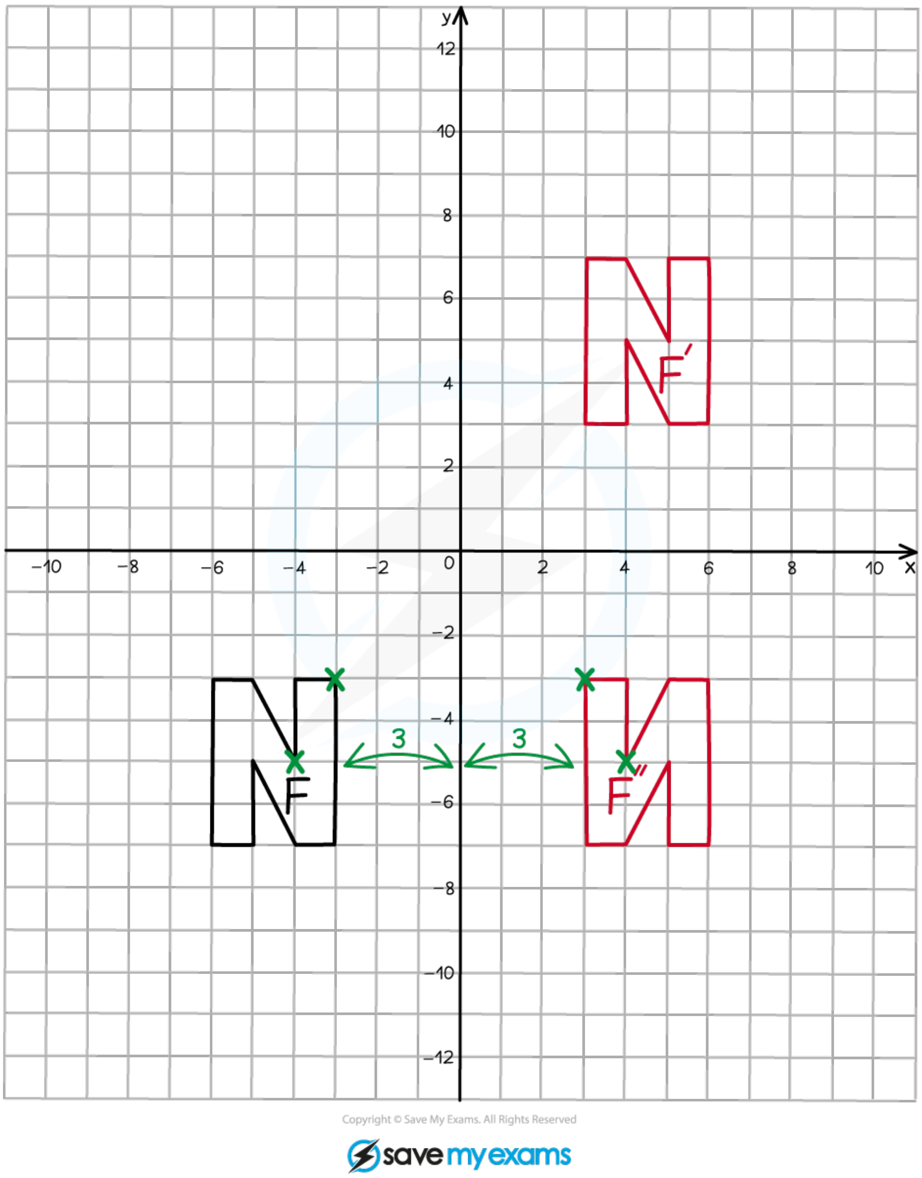
YOUR NOTES



(c)

7. Vectors & Transformations

YOUR NOTES



You should now be able to see how to get from F to F'' directly.

The single transformation from F to F'' is a reflection in the y-axis.

The y-axis is also the line x=0, either description is acceptable!

7. Vectors & Transformations

YOUR NOTES

7.2.6 TRANSFORMATIONS - ENLARGEMENT (NEGATIVE SCALE FACTOR)

Enlargement with negative scale factors – what do I need to know?

- There is only one key difference between enlargements when the scale factor is negative and normal enlargements
- You will still need to perform enlargements with negative scale factors. It is possible but unusual to be asked to identify one
- The key things with an enlargement are:

1. (Negative) Scale Factor

- This time it helps to think of the scale factor as how many times a vertex on the image is further away from the centre of enlargement than the corresponding vertex on the object
- Then, as it is negative, we measure that distance in the opposite direction from the centre of enlargement.

2. Centre of Enlargement

- This tells us where on the page the image is going to go but we have a bit of work to do first
- With negative scale factors drawing the shape from one vertex is not as straightforward
- So apply the enlargement to at least two vertices (helps if they are connected) and do more if necessary.

Exam Tip

Exam questions are quite keen on combining both negative and fractional scale factors!

Build your answer up following the rules and you will be fine!

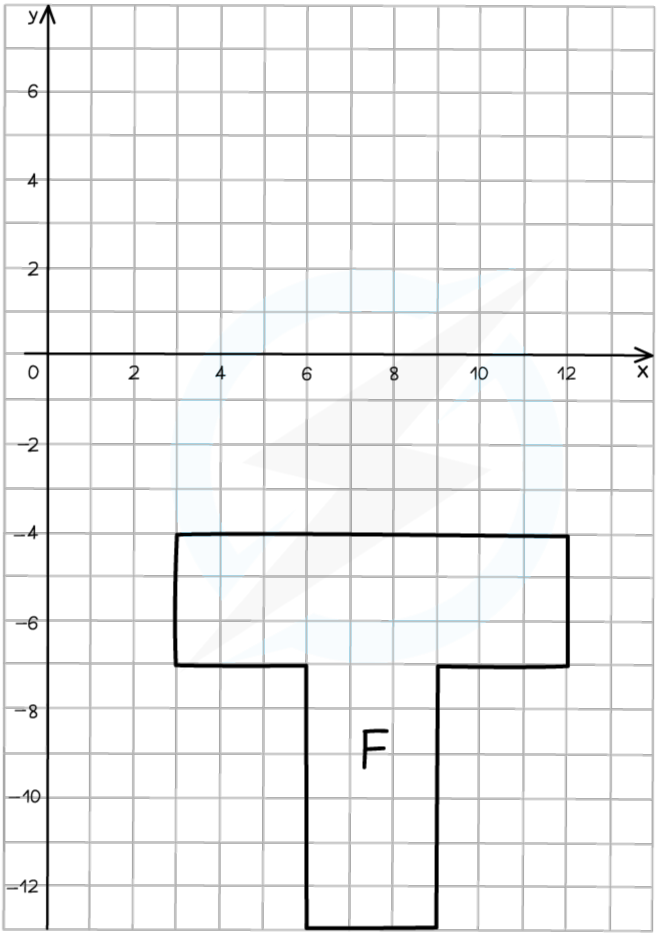
7. Vectors & Transformations

YOUR NOTES

Worked Example

1. On the grid below enlarge shape F using scale factor $-\frac{1}{3}$ and centre of enlargement (6, -1). Label this shape F'.

   If the area of F is 45 cm² write down the area of F'.



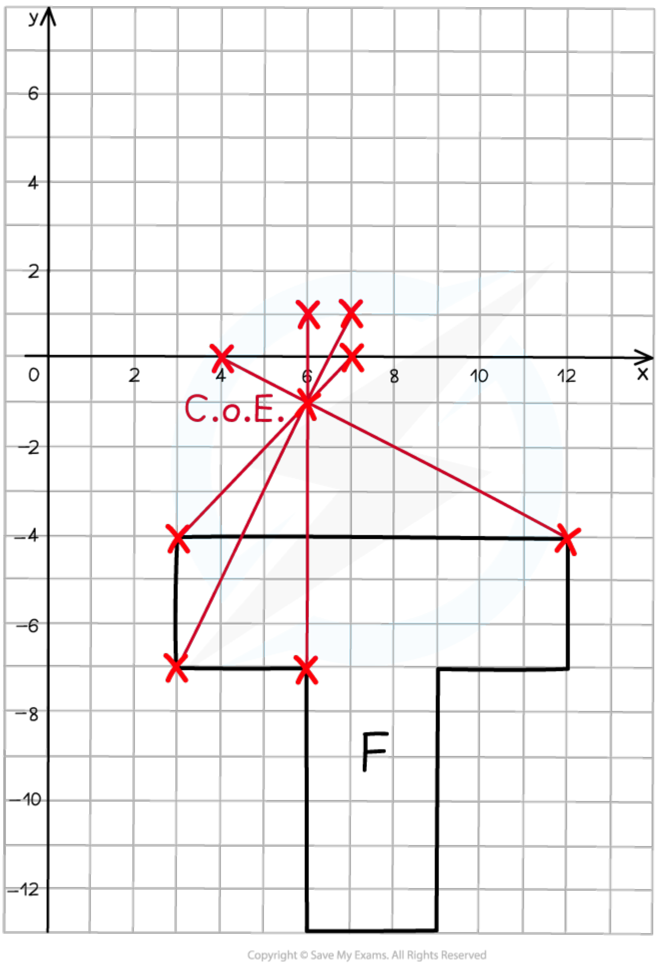
2 – Start by marking the centre of enlargement (CoE)

We have a negative and fractional scale factor

Pick 2-3 (working below has 4) vertices, drawing a line through that and the CoE

7. Vectors & Transformations

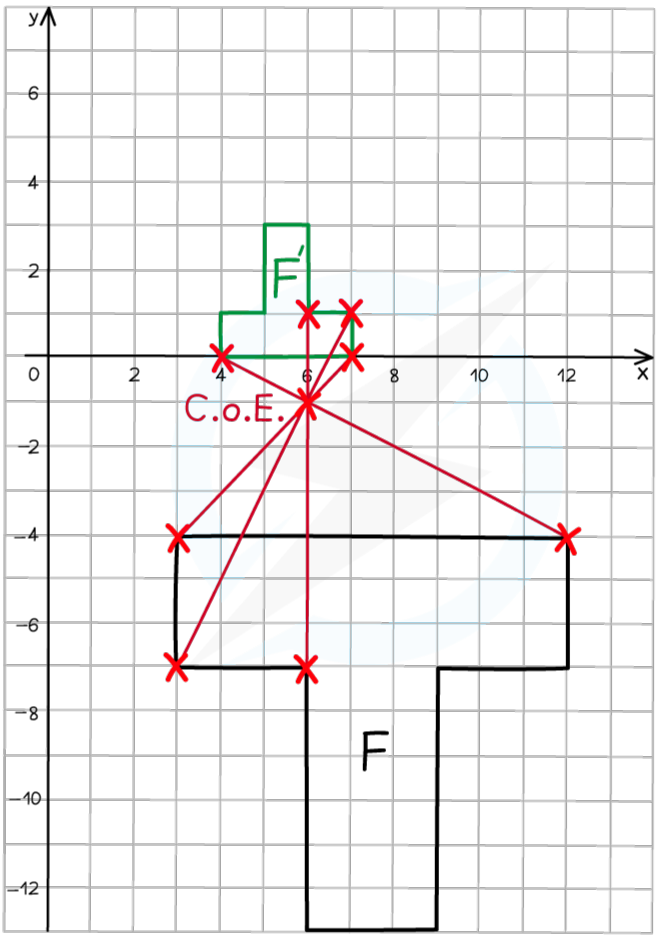
YOUR NOTES
⬇



1 – As the scale factor is $-\frac{1}{2}$ we should measure **half** the distance on the **other** side of these lines

7. Vectors & Transformations

YOUR NOTES



Unlike the other transformations it is not so easy to draw once we know where one vertex goes

Do as many as you feel you need to

(Notice the image is both "upside down" and vertices have "swapped sides")

The area scale factor will be $(\frac{1}{3})^2 = \frac{1}{9}$, so the area of F' will be 45 ÷ 9 = 5 cm²

The "-" doesn't matter here as anything to do with area will be positive

If the scale factor is -1 the image would be the same size as the object but would still be "upside down" with "swapped sides"