Vectors
Difficulty: Medium

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 1

Time allowed: **82 minutes**

Score: **/71**

Percentage: **/100**

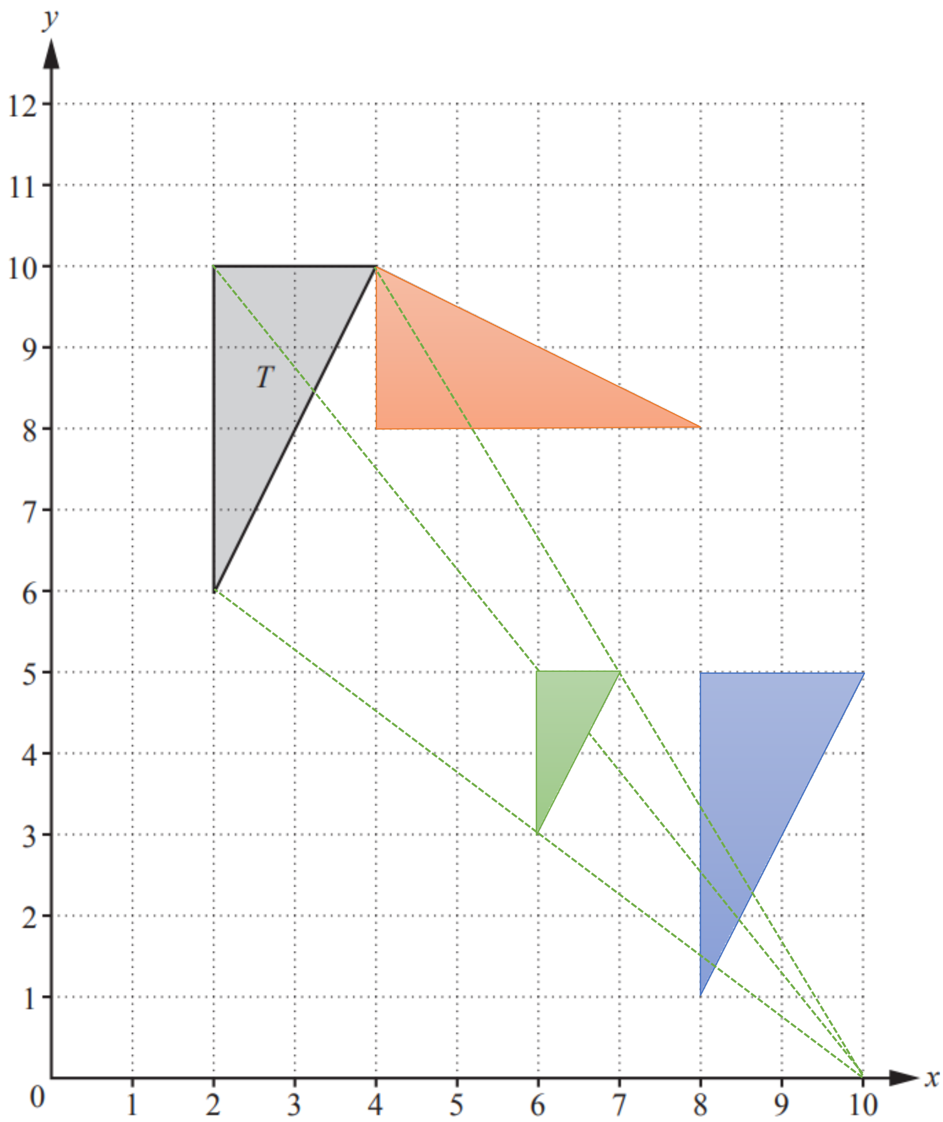
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) ~~Assembled by AS~~

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



On the grid, draw the image of $\begin{pmatrix} 6 \\ -5 \end{pmatrix}$,

(i) triangle T after translation by the vector [2]

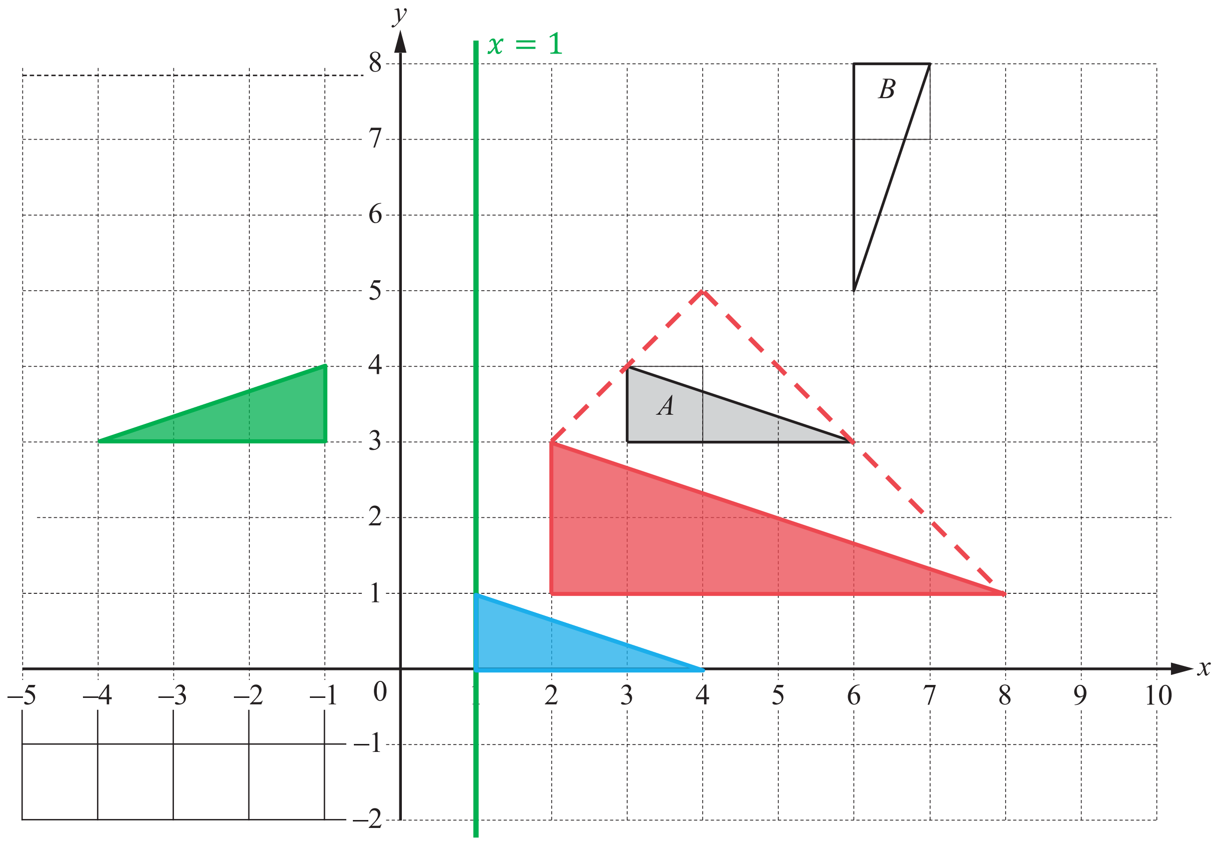
The blue triangle above

(ii) triangle T after rotation through 90° anticlockwise with centre (4, 10), [2]

The orange triangle above

(iii) triangle T after enlargement with scale factor $\frac{1}{2}$, centre (10, 0). [2]

The green triangle above



(a) Draw the image when triangle A is reflected in the line $x = 1$. [2]

Green triangle

(b) Draw the image when triangle A is translated by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. [2]

Move 2 to the left and 3 down – Blue Triangle

(c) Draw the image when triangle A is enlarged by scale factor 2 with centre (4, 5). [2]

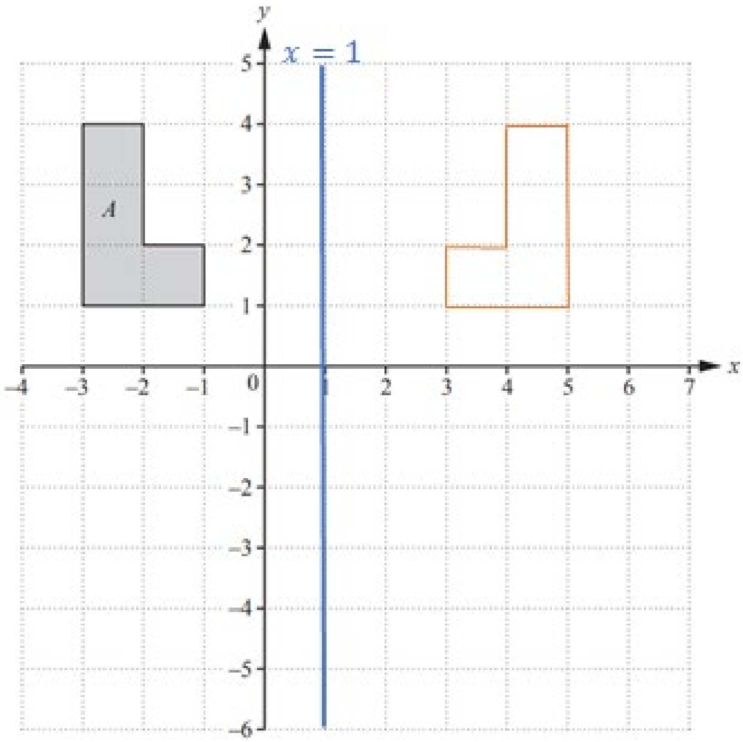
Centre of enlargement and construction lines marked – Red triangle

(d) Describe fully the **single** transformation that maps triangle A onto triangle B. [3]

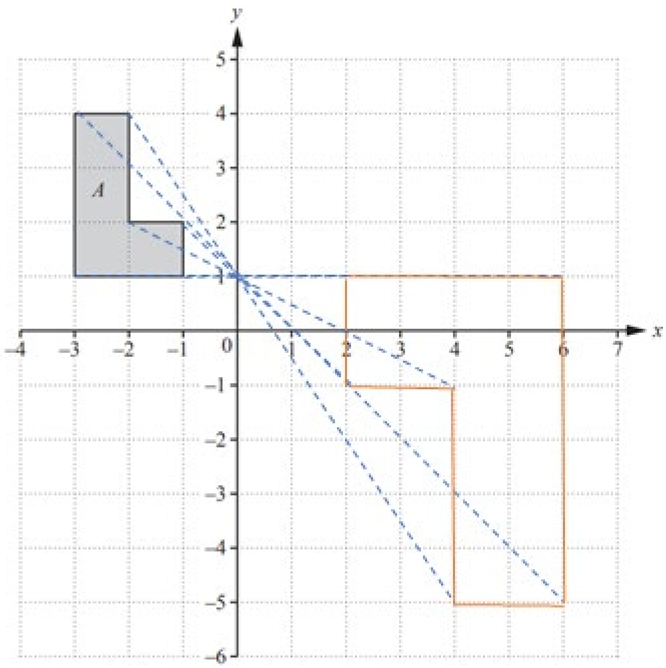
Rotation 90° anticlockwise about $(7, 4)$

(a) On the grid, draw the image of

(i) shape A after a reflection in the line $x = 1$, [2]



(ii) shape A after an enlargement with scale factor –2, centre (0, 1), [2]



(iii) shape A after the transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [3]

We are transforming a set of coordinates $\begin{pmatrix} x \\ y \end{pmatrix}$, which become

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$$

Applying this to each vertex of the shape we get the new vertices

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix} \to \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

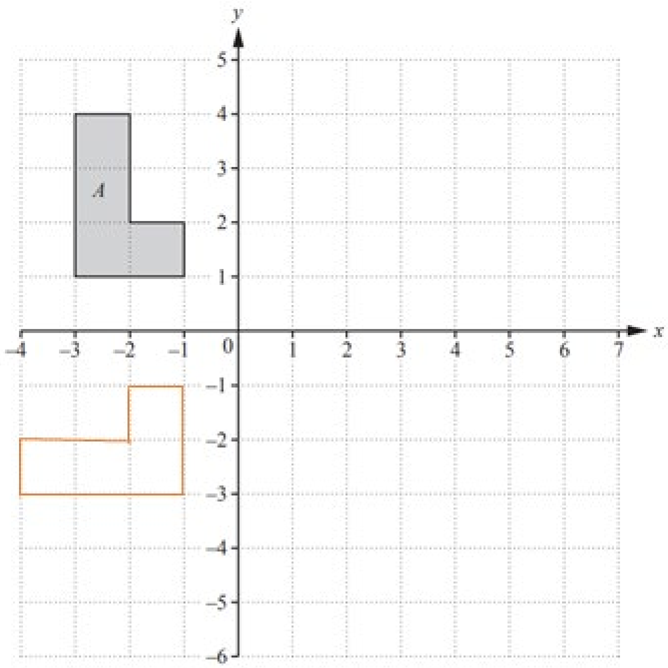
$$\begin{pmatrix} -1 \\ 2 \end{pmatrix} \to \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 2 \end{pmatrix} \to \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} \to \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix} \to \begin{pmatrix} -4 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 1 \end{pmatrix} \to \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

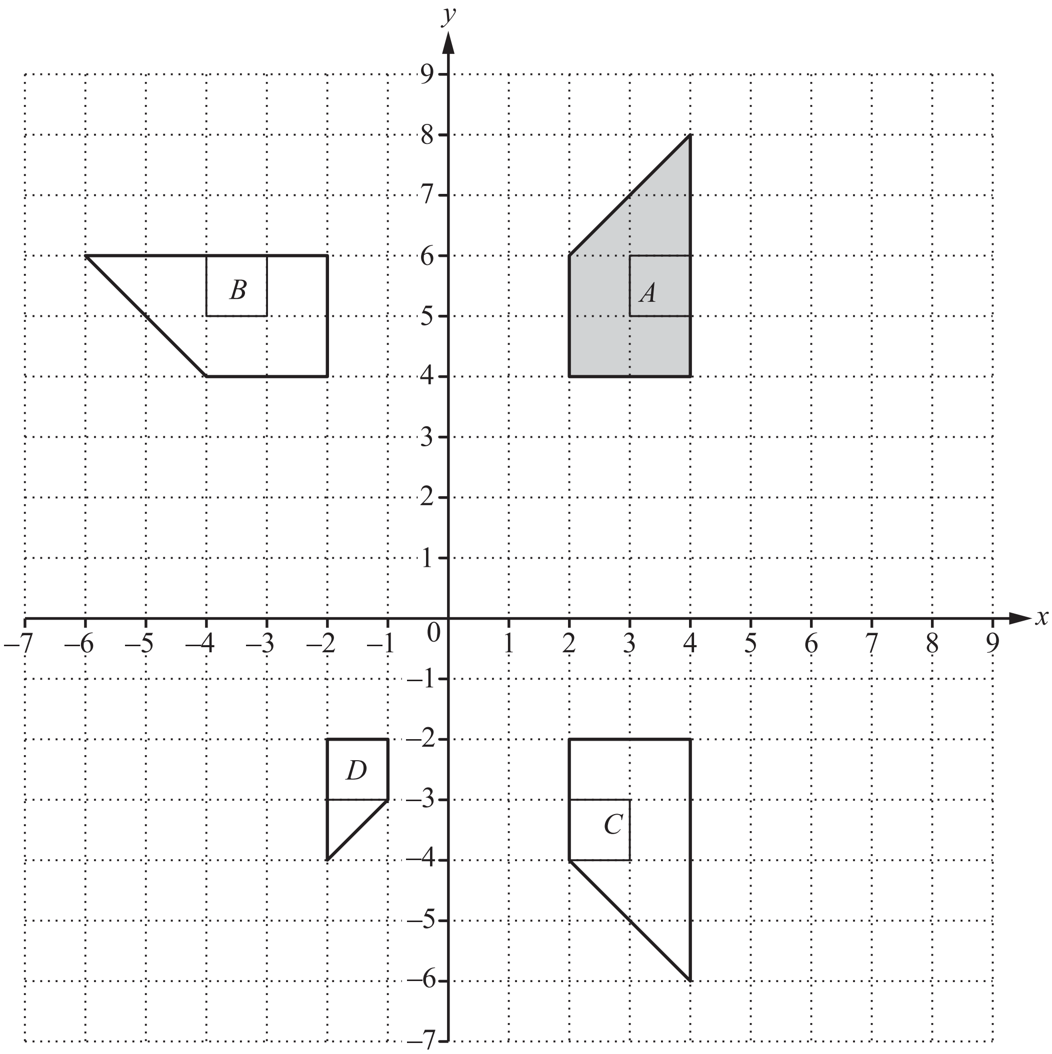


(b) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$. [3]

With this transformation the set of points $\begin{pmatrix} x \\ y \end{pmatrix}$ become

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x \\ 3y \end{pmatrix} = 3\begin{pmatrix} x \\ y \end{pmatrix}$$

This is an **enlargement of scale factor 3 with centre at the origin**.

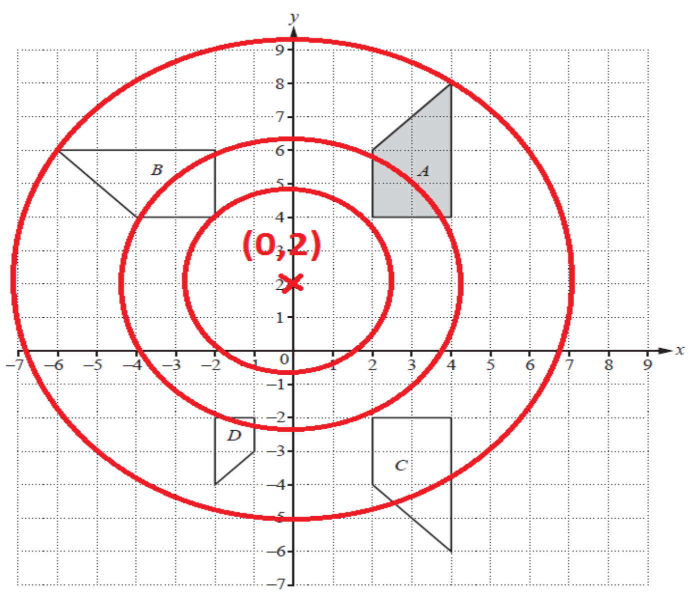


(a) Describe fully the **single** transformation that maps

(i) shape A onto shape B,

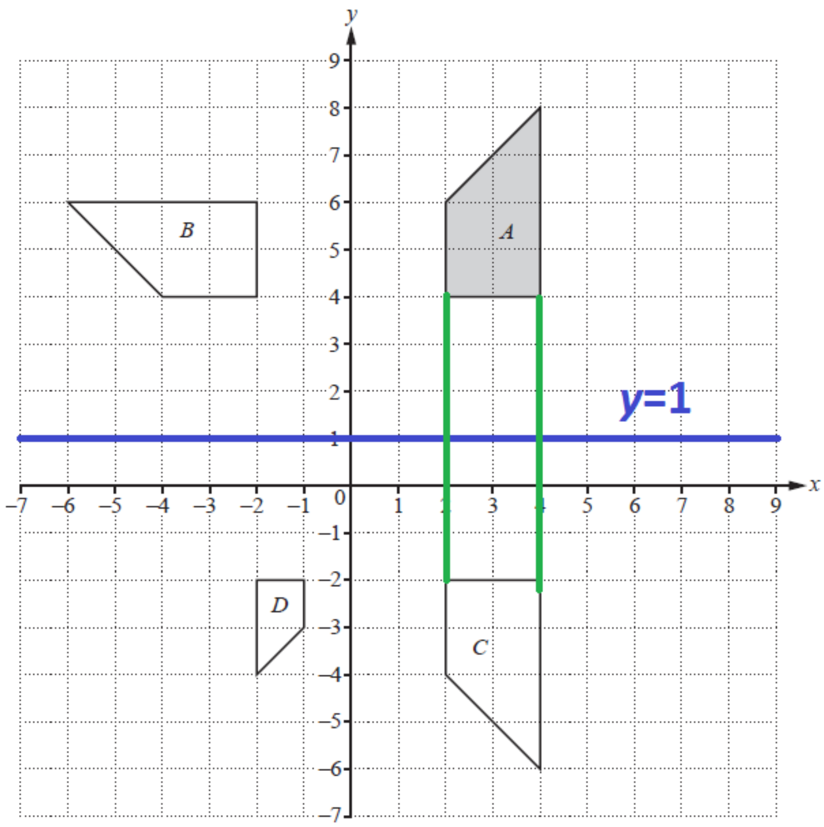
[3]

By drawing circles connecting the corresponding vertices of the shapes, we can see that the transformation is a **rotation by 90° anticlockwise around the point (0,2)** (the centre of the circles)



(ii) shape A onto shape C, [2]

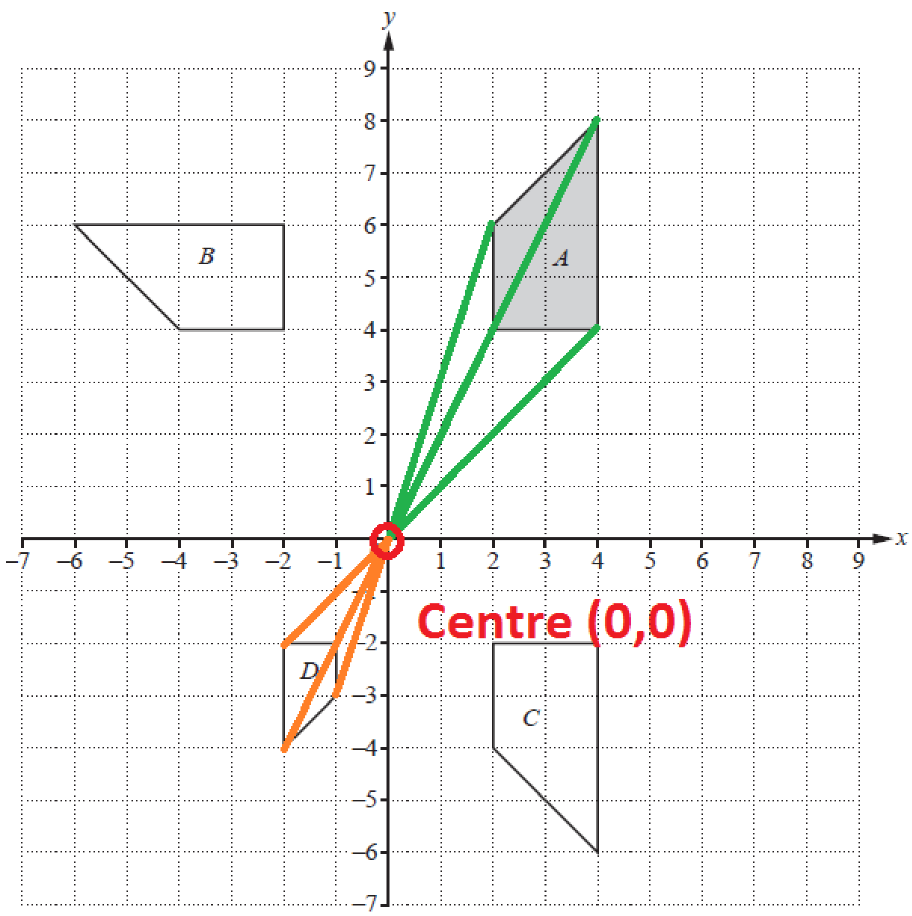
This transformation is a reflection in y=1 as the shapes are symmetric by this line.



(iii) shape *A* onto shape *D*. [3]

When we join the corresponding vertices of shapes A and D, the lines cross at point (0,0).

The distance from (0,0) to a vertex of shape A is twice as long as the distance from (0,0) to a corresponding vertex of shape D. This suggests that the scale factor of the enlargement is -1/2 (minus sign as the lines point in the opposite direction from (0,0)).



The transformation is **an enlargement with centre (0,0) and the scale factor -1/2**.

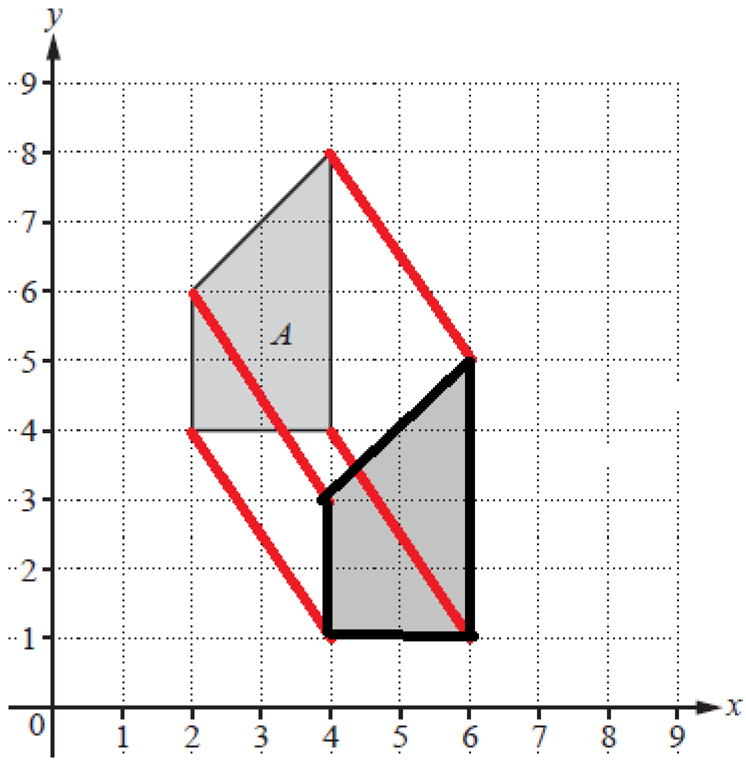
(b) Find the 2 × 2 matrix that represents the transformation in **part (a)(iii)**. [2]

We know that the centre is (0,0) and the factor is -1/2, therefore we simply multiply a identity matrix with our scale factor.

$$-\frac{1}{2}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

(c) On the grid, draw the image of shape A after a translation by the vector $\begin{pmatrix}2\\-3\end{pmatrix}$. [2]

This transformation represents a translation by 2 units in the positive *x* direction and by 3 units in the negative *y* direction.



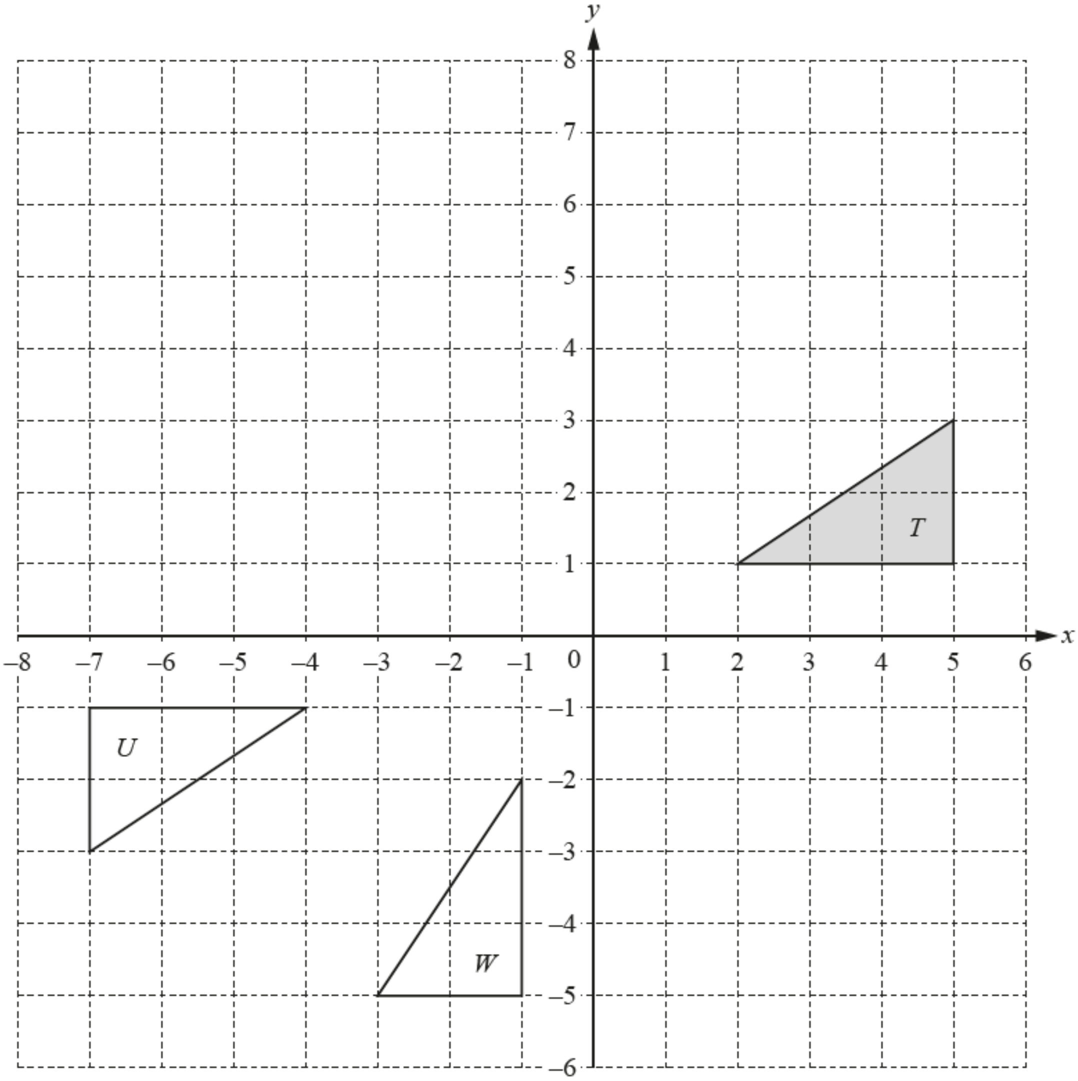
The new shape has vertices **(4,1), (6,1), (6,5) and (4,3).**

(d) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$. [2]

This matrix switches the coordinates (*x* into *y* and vice versa).

$$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}y\\x\end{pmatrix}$$

Therefore it is a **reflection by *y*=*x*.**



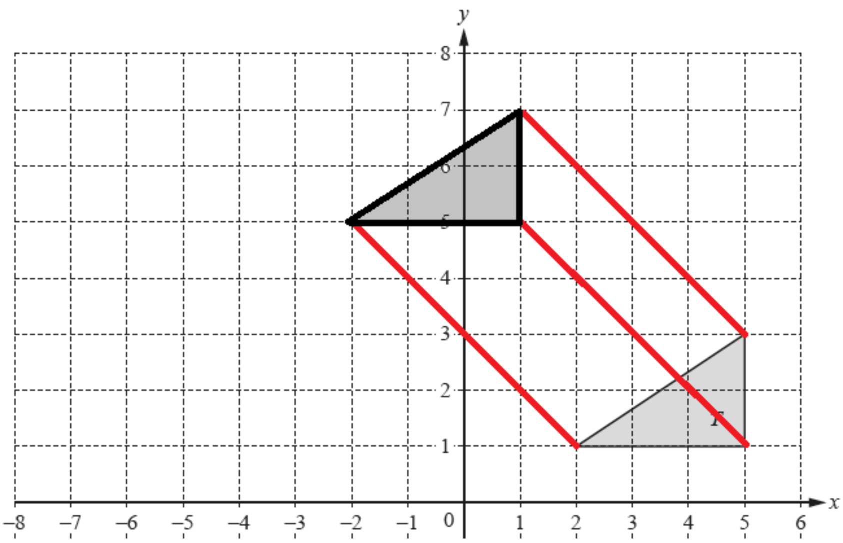
(a) On the grid, draw the image of

(i) triangle T after a translation by the vector $\begin{pmatrix} -4 \\ 4 \end{pmatrix}$, [2]

This transformation represents a shift by 4 units in the negative x direction and by 4 units in the positive y direction.

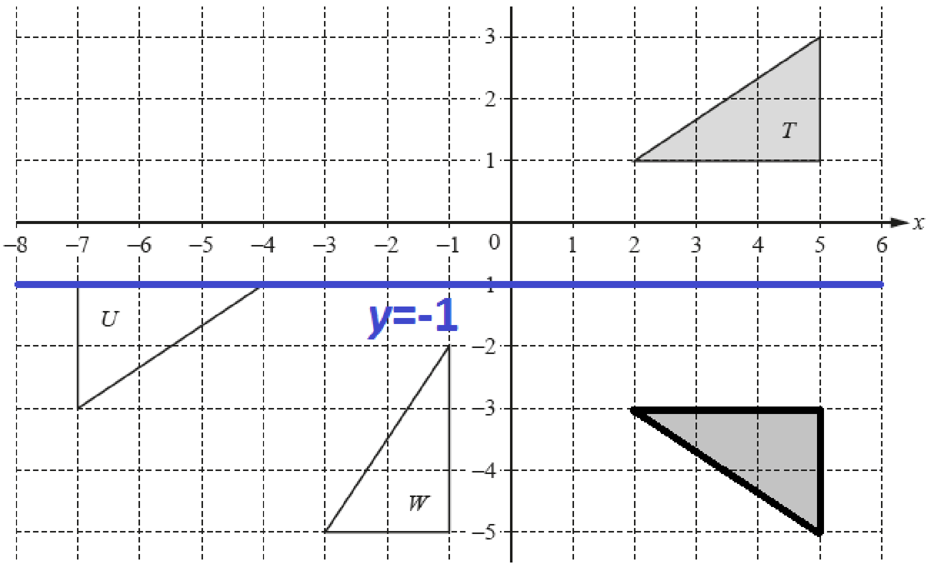
The vertices of the new triangle are:

(-2,5), (1,5) and (1,7).



(ii) triangle T after a reflection in the line $y = -1$. [2]

To draw a reflection in y=-1, the distances of corresponding vertices of the object and image triangles to the mirror line must be the same. The triangles must by symmetrical about this line.



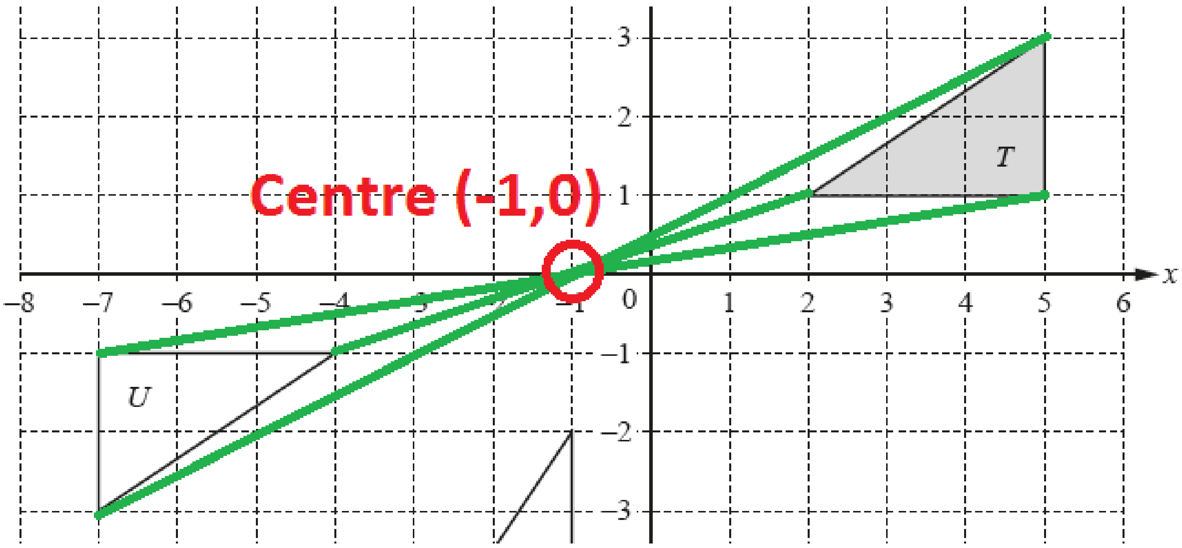
The vertices of the new triangle are: **(2,-3), (5,-3) and (5,-5).**

(b) Describe fully the **single** transformation that maps triangle T onto triangle U. [3]

When we join the corresponding vertices of triangles T and U, the lines cross at point (-1,0).

The distance from (-1,0) to a vertex of triangle T is the same as the distance from (-1,0) to a corresponding vertex of the triangle U.

This suggests that the scale factor of the enlargement is -1 (minus sign as the lines point in the opposite direction to (-1,0)).



The transformation is **an enlargement with centre (-1,0) and the scale factor -1**.

(Alternatively, this transformation can be interpreted as a rotation by 180° with a centre (-1,0).

(c) (i) Describe fully the **single** transformation that maps triangle T onto triangle W. [2]

By connecting corresponding vertices, we can see that the green lines are parallel.

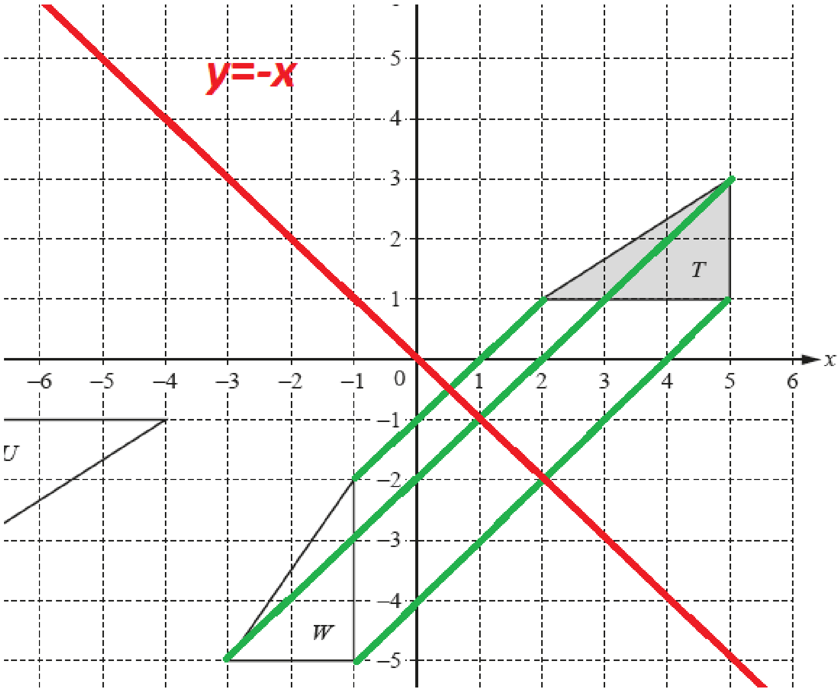
This transformation, however, is clearly not a simple rotation (shape is rotated), therefore

it must be a reflection.

All green lines have a common bisector $y=-x$.

Therefore this transformation is a **reflection in line**

***y=-x*.**



(ii) Find the 2 × 2 matrix that represents the transformation in **part (c)(i)**. [2]

We are looking for a matrix, which switches coordinates and changes their sign as well.

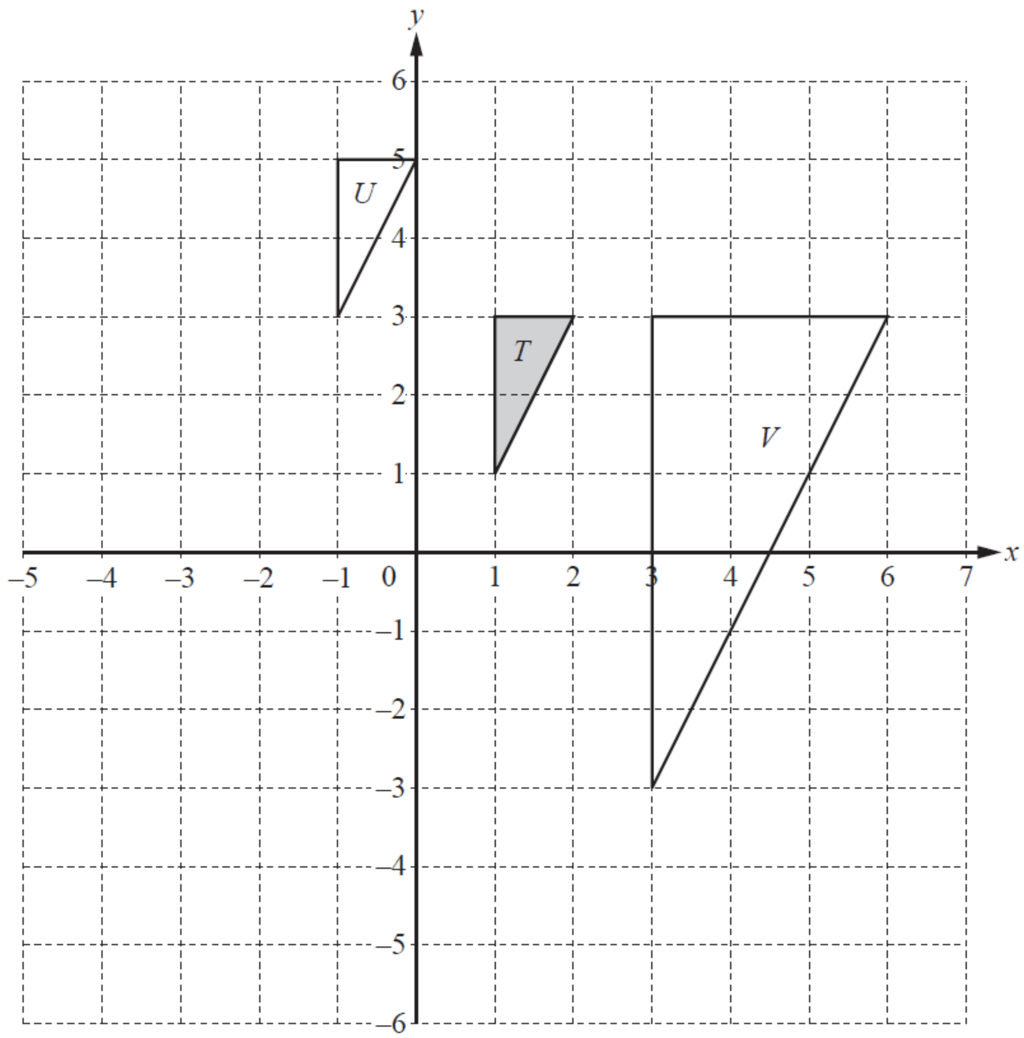
The matrix that swaps coordinates is:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

The change of sign can be easily added by multiplying the matrix by factor (-1).

Therefore the matrix corresponding to the transformation in c)i) is:

$$\begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$$

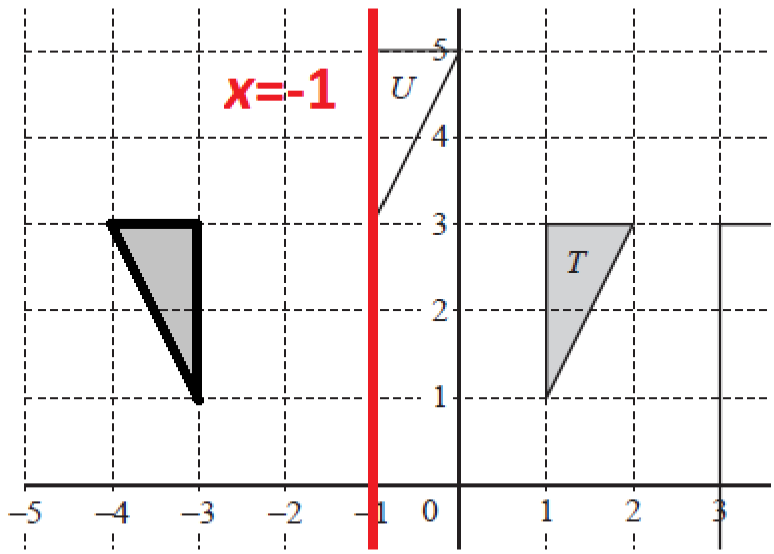


(a) On the grid, draw the image of

(i) triangle T after a reflection in the line $x = -1$, [2]

To draw a reflection in the line x=-1, the distances of corresponding vertices of the object and image triangles from the mirror line must be the same. The triangles must by symmetrical about this line.

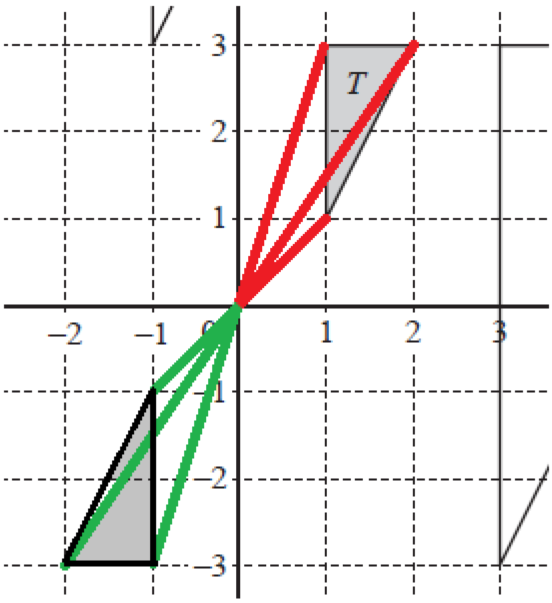
The vertices of the new triangle are **(-3,3), (-3,1), (-4,-3)**



(ii) triangle T after a rotation through 180° about $(0, 0)$. [2]

A rotation through 180° about (0,0) is equivalent to an enlargement of scale factor -1 with the centre (0,0).

- To perform this enlargement, draw a lines connecting vertices to the point (0,0).
- Extend those lines one more time (to twice their original length).
- Mark the vertices of the new triangle at the end of the lines.
- Join the vertices to form a new triangle.

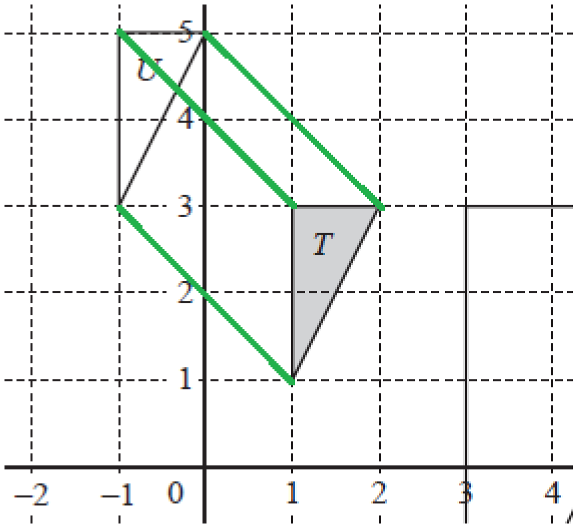


The vertices of the new triangle are **(-1,-1), (-2,-3), (-1,-3)**

(b) Describe fully the **single** transformation that maps

(i) triangle T onto triangle U, [2]

By drawing lines connecting corresponding vertices, we can see that all lines are parallel.



This suggests that the transformation is a translation.

The lines from T to U go 2 units in negative x direction and 2 units in positive y direction.

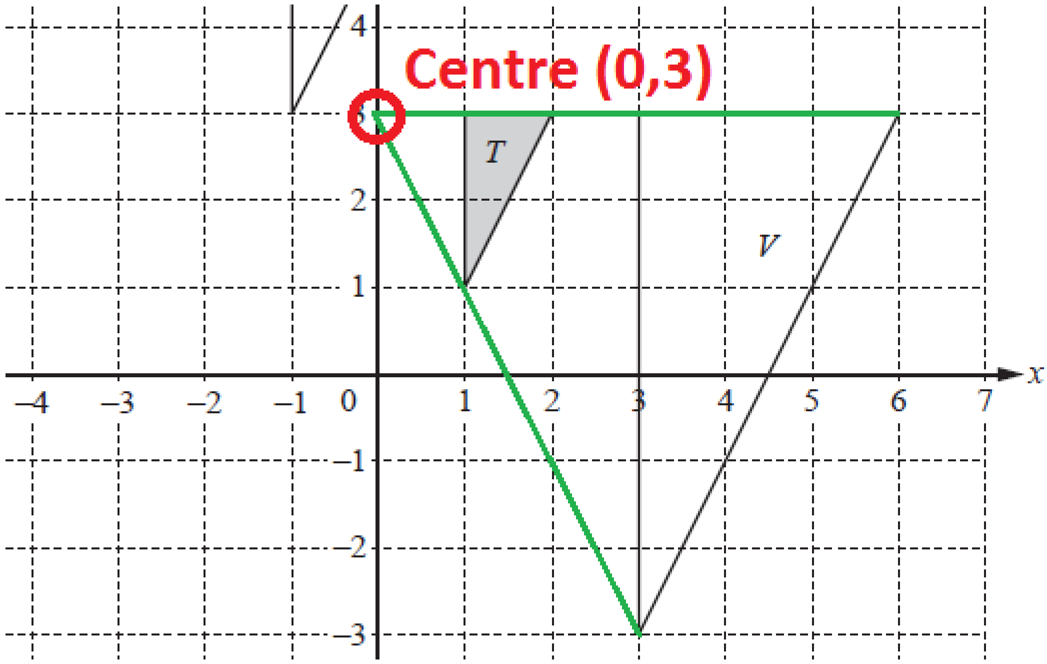
Therefore it is a **translation by vector** $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

(ii) triangle T onto triangle V. [3]

When we join the corresponding vertices of triangles T and V, the lines cross at point (0,3).

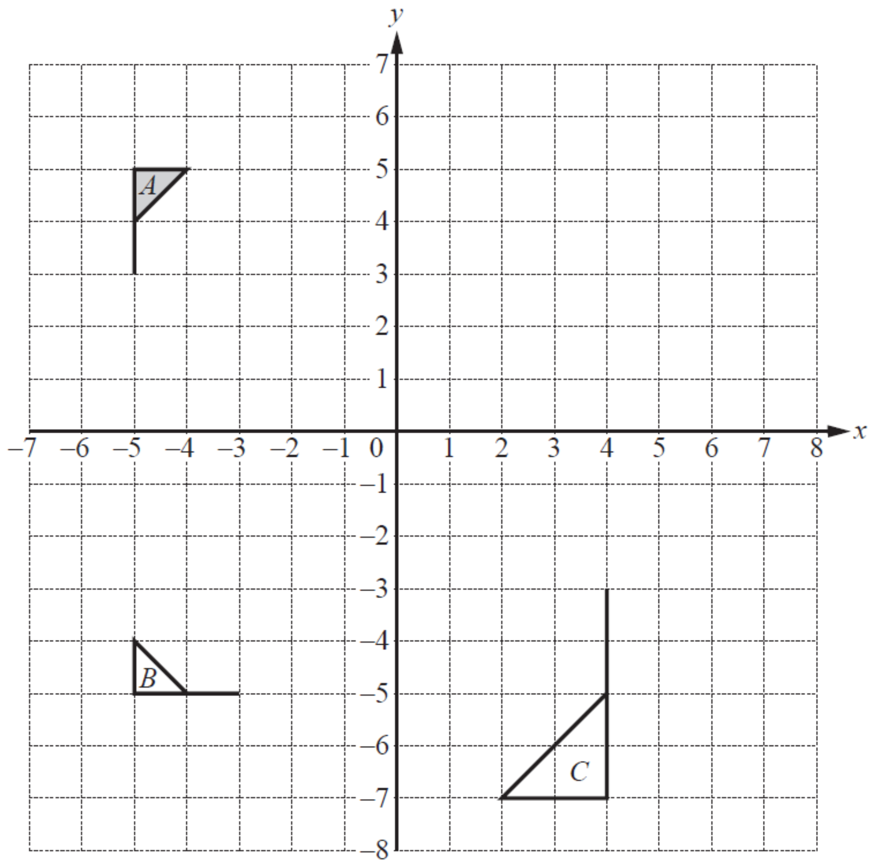
The distance from (0,3) to a vertex of triangle V is three times longer than the distance from (0,3) to a corresponding vertex of the triangle T.

This suggests that the scale factor of the enlargement is 3.



The transformation is **an enlargement with centre (0,3) and the scale factor 3**.

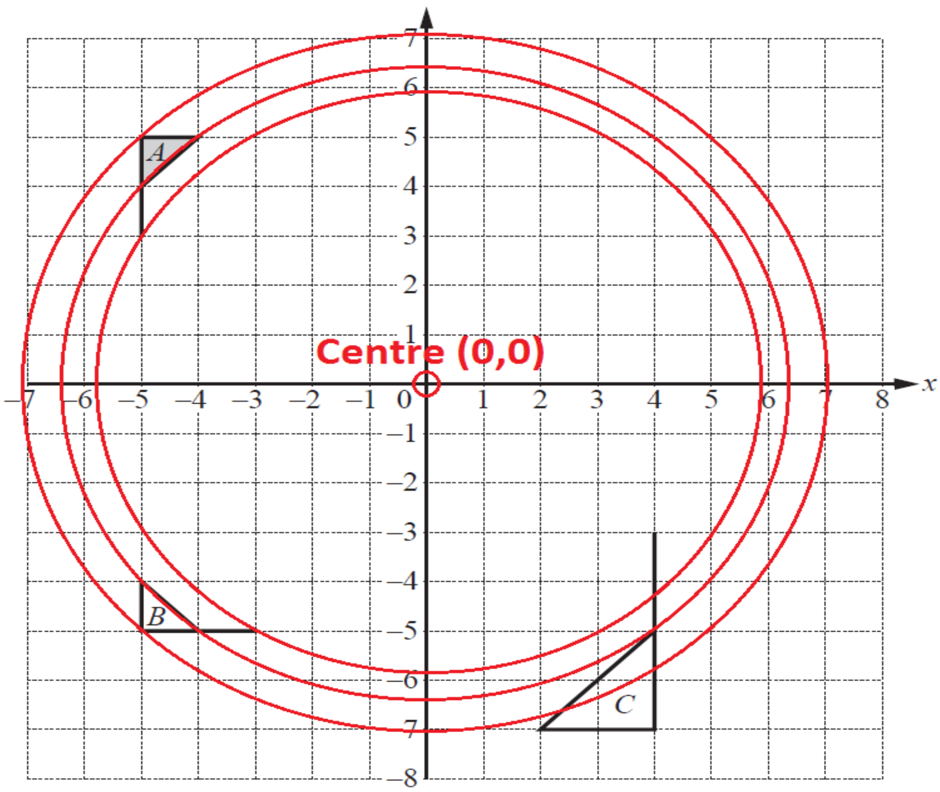
Question 7



(a) Describe fully the **single** transformation that maps

(i) flag A onto flag B, [3]

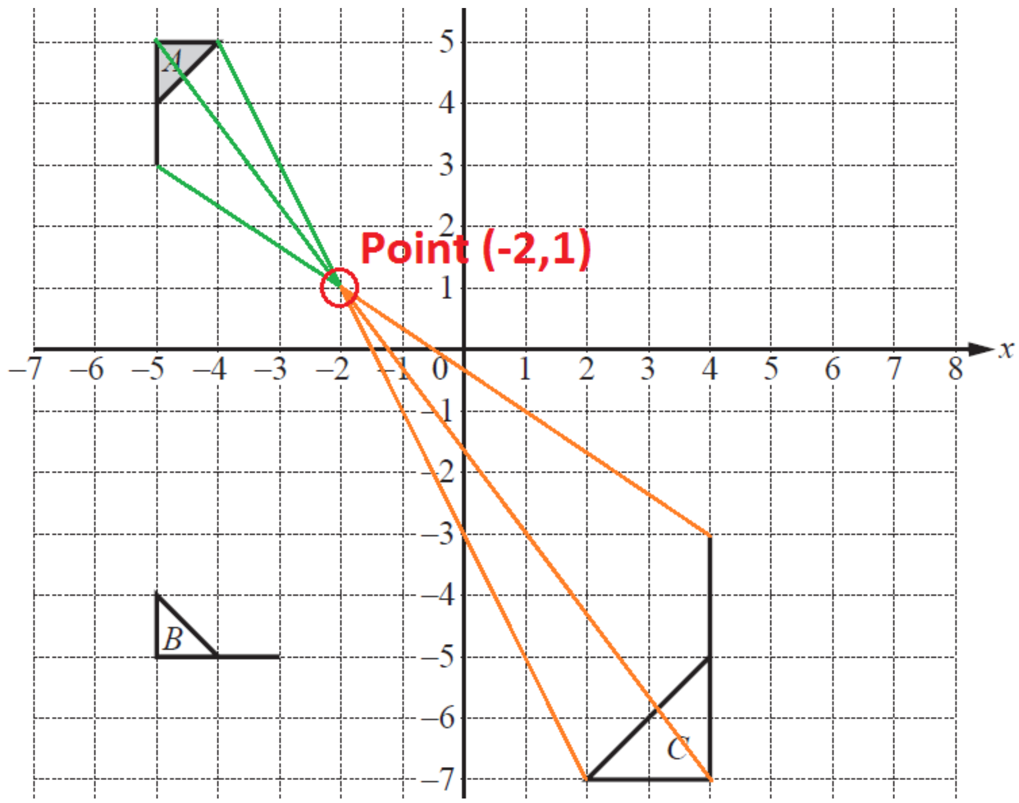
By drawing circles connecting the corresponding vertices of the flags, we can see that the transformation is a **rotation by 90° anticlockwise around the point (0,0)** (the centre of the circles).



(ii) flag A onto flag C. [3]

When we join the corresponding vertices of flags A and C, the lines cross at point (-2,1).

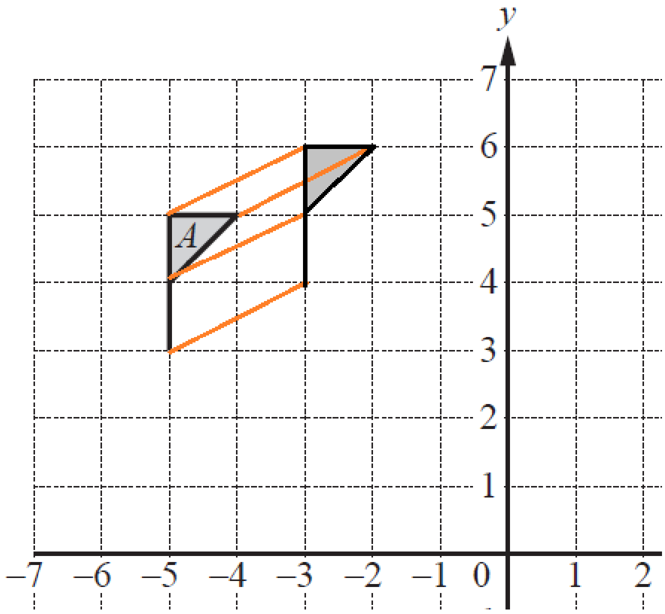
The distance from (-2,1) to a vertex of flag C is twice as long as the distance from (-2,1) to a corresponding vertex of flag A. This suggests that the scale factor of the enlargement is -2 (minus sign as the lines point in the opposite direction from (-2,1)).



The transformation is **an enlargement with centre (-2,1) and the scale factor -2**.

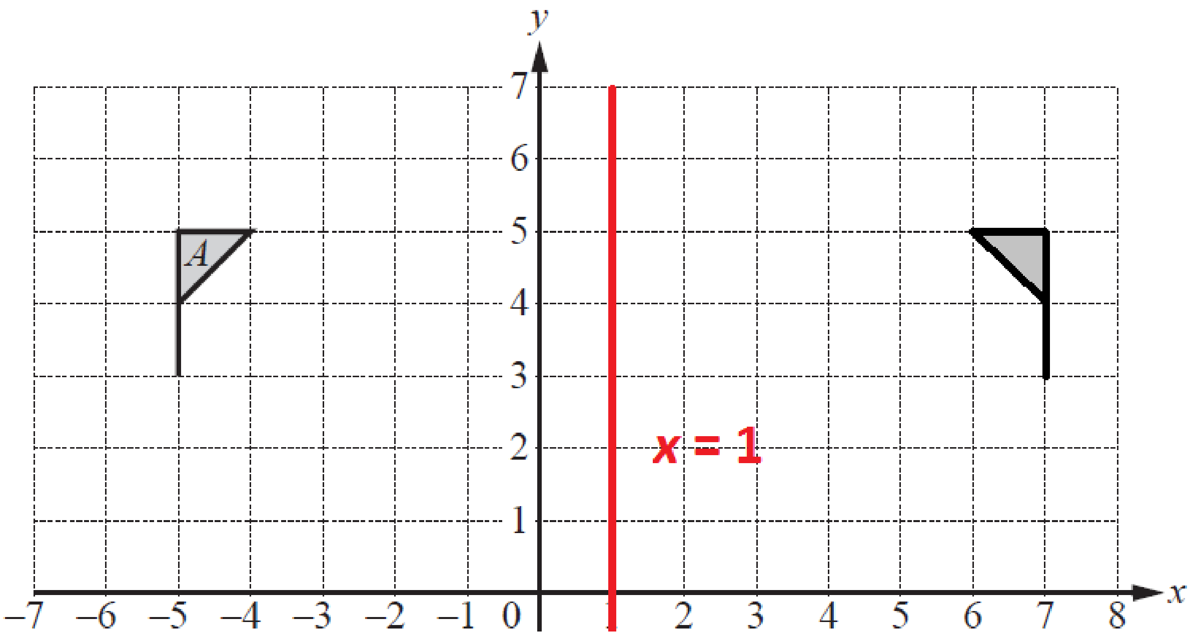
(b) Draw the image of flag A after a translation by the vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. [2]

This is a simple translation of each vertex by 2 units in the positive x-direction and 1 unit in the positive y-direction.



The vertices of the new flag are **(-3,4), (-3, 5), (-3,6) and (-2,6)**.

(c) Draw the image of flag A after a reflection in the line $x = 1$. [2]



The new flag has vertices **(7,3), (7,4), (7,5) and (6,5)**.

(d) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. [2]

The transformation is represented by the matrix:

x unchanged $\quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad$ flips the sign of y

This transformation is **a reflection in the *x*-axis** as it flips the sign of y-coordinate, but leaves the x-coordinate unchanged.

$$\begin{pmatrix} x \\ y \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} x \times 1 + y \times 0 \\ x \times 0 + y \times (-1) \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

Vectors
Difficulty: Medium

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 2

Time allowed: **109 minutes**

Score: **/95**

Percentage: **/100**

Grade Boundaries:

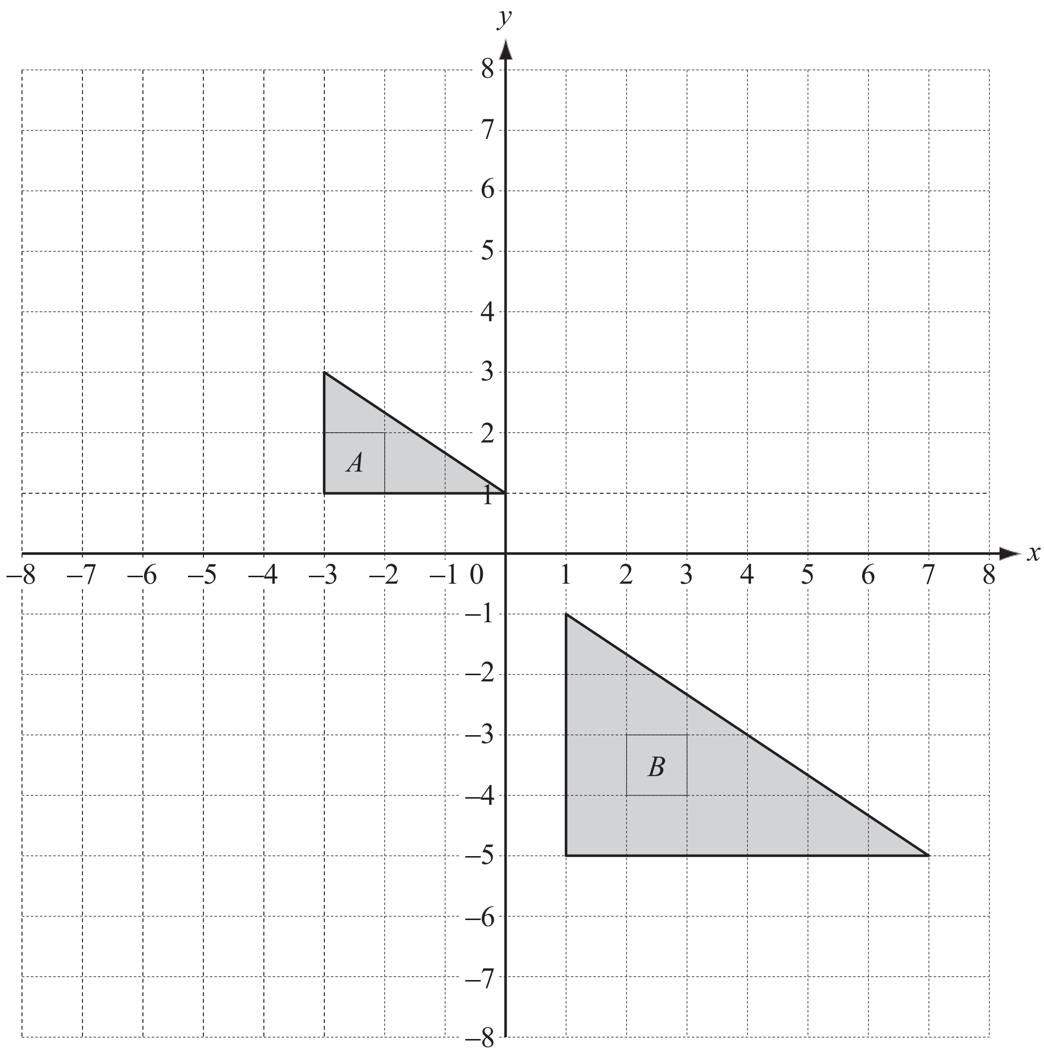
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Question 1

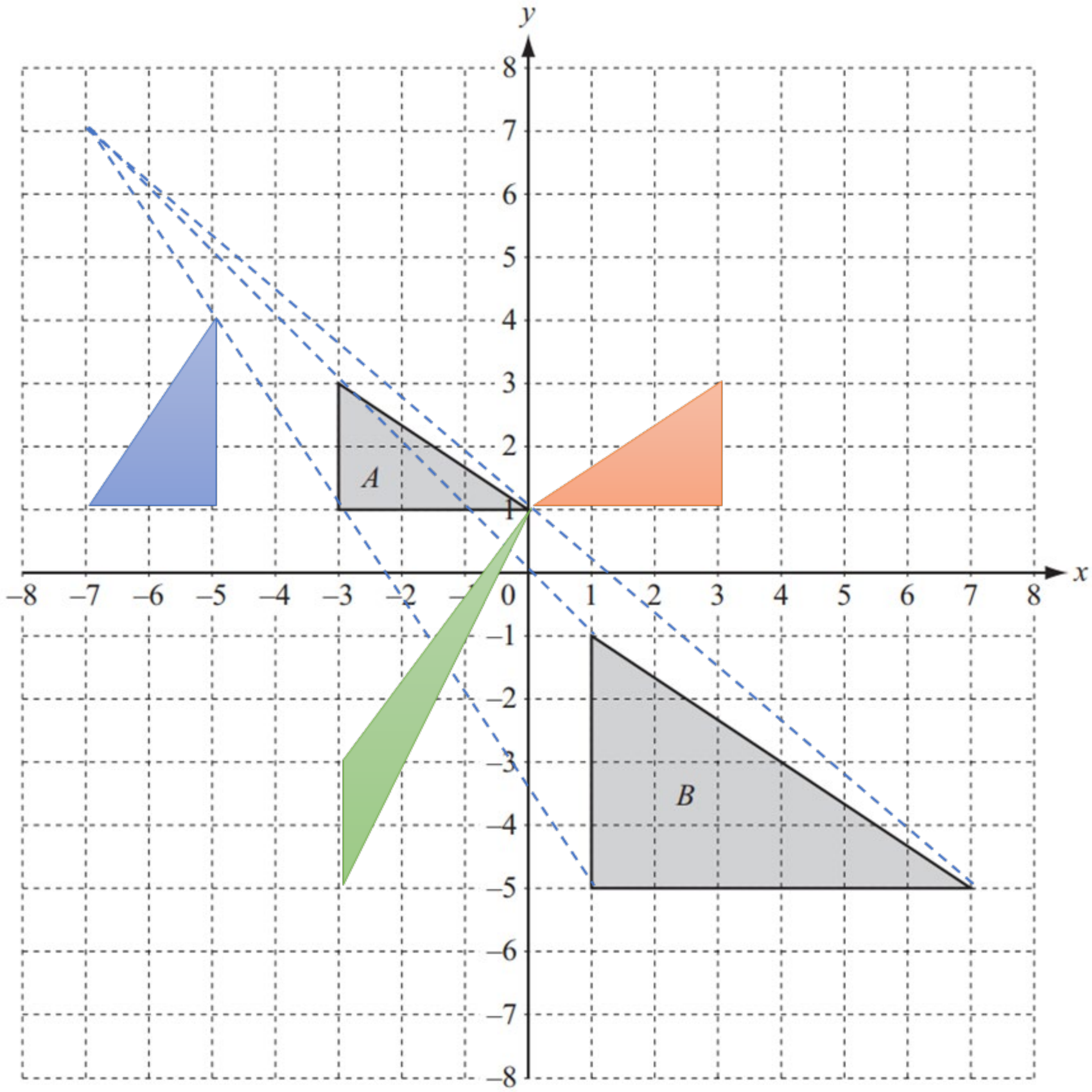


(a) Draw the image when triangle A is reflected in the line $x = 0$. [1]

Orange triangle below

(b) Draw the image when triangle A is rotated through 90° anticlockwise about $(-4, 0)$. [2]

Blue triangle below



(c) (i) Describe fully the **single** transformation that maps triangle A onto triangle B. [3]

Enlargement, scale factor 2, centre (-7, 7)

(ii) Complete the followingstatement. [2]

$$Area\ of\ triangle\ A : Area\ of\ triangle\ B = \mathbf{1 : 4}$$

(d) Write down the matrix that represents a stretch, factor 4 with the y-axis invariant. [2]

$$\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$$

(e) (i) On the grid, draw the image of triangle A after the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. [3]

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x + y \end{pmatrix}$$

Green triangle drawn above.

(ii) Describe fully this **single** transformation. [3]

Shear with y-axis invariant of scale factor 2.

(iii) Find the inverse of the matrix $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. [2]

Firstly, find the determinant (**Δ**):

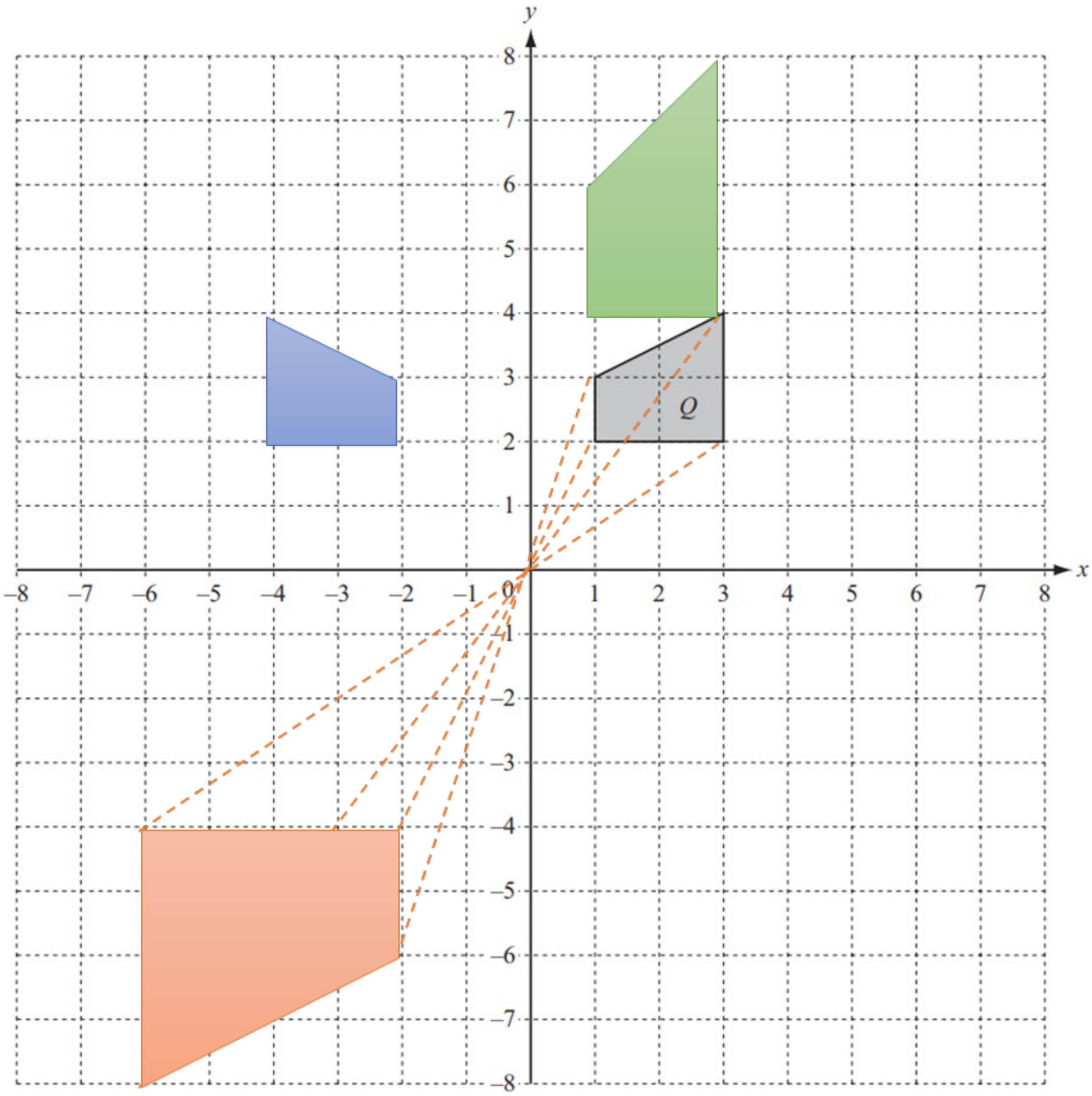
$$\Delta = 1 \times 1 - 2 \times 0$$

$$= 1$$

Now, swap the elements on the leading diagonal and change the sign of elements on the antidiagonal to get the inverse as:

$$\rightarrow M^{-1} = \frac{1}{\Delta}\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{-2} & \mathbf{1} \end{pmatrix}$$



(a) Draw the reflection of shape Q in the line $x = -1$. [2]

The blue shape on the graph above.

(b) (i) Draw the enlargement of shape Q, centre (0, 0), scale factor -2 . [2]

The orange shape above (construction lines in dotted orange).

(ii) Find the 2×2 matrix that represents an enlargement, centre (0, 0), scale factor -2 . [2]

We require that

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -2x \\ -2y \end{pmatrix}$$

This is done by the matrix

$$\begin{pmatrix} \mathbf{-2} & \mathbf{0} \\ \mathbf{0} & \mathbf{-2} \end{pmatrix}$$

(c) **(i)** Draw the stretch of shape Q, factor 2, x-axis invariant. [2]

The green shape on the graph above.

(ii) Find the 2×2 matrix that represents a stretch, factor 2, x-axis invariant. [2]

We require that

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ 2y \end{pmatrix}$$

This is done by the matrix

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} \end{pmatrix}$$

(iii) Find the inverse of the matrix in **part (c)(ii)**. [2]

First find the determinant (**Δ)**

$$\Delta = 1 \times 2 - 0 \times 0$$

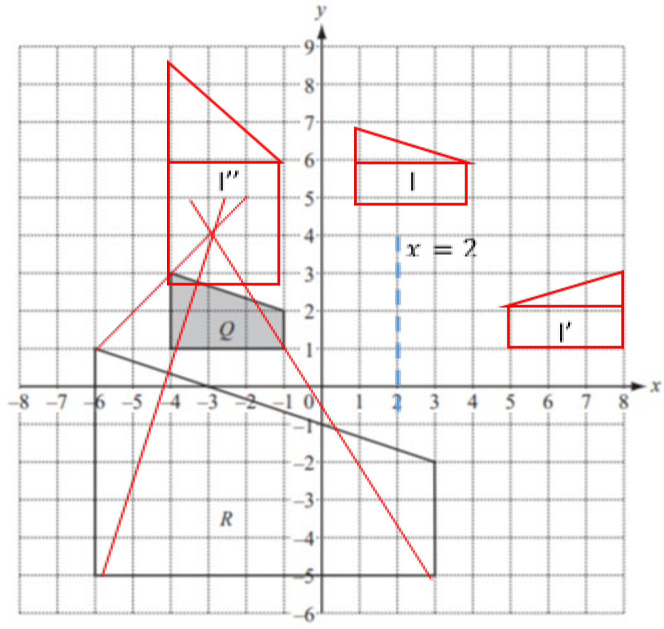
$$= 2$$

Now swap the elements on the leading diagonal and multiply the antidiagonal by -1. Then divide the resulting matrix by the determinant.

$$\boldsymbol{M^{-1} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}}$$

(iv) Describe fully the **single** transformation represented by the matrix in **part(c)(iii)**. [3]

This is a **stretch, factor ½, x-axis invariant.**



(a) Describe fully the **single** transformation that maps shape Q onto shape R. [3]

By counting the size of the edges, we can see R is three times larger than Q. To find the focus we draw lines from each of R's vertices through Q's corresponding vertices. The point of intersection is the focus of enlargement as shown on the graph.

From the graph, there has been an enlargement of scale factor 3 about the point (-3, 4)

(b) **(i)** Draw the image when shape Q is translated by the vector $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$. [2]

A translation along the vector $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ means the shape is moved 5 in the positive x direction and 4 in the positive y direction.

Image shown as I.

(ii) Draw the image when shape Q is reflected in the line $x = 2$. [2]

To reflect, draw the line x = 2 and ensure all corresponding points, on Q and the Image, are the same distance either side of the line of reflection.

Image shown as I’

(iii) Draw the image when shape Q is stretched, factor 3, x-axis invariant. [2]

To stretch, x axis invariant, all points of the image should be three times further away from the x axis then that of Q.

Image shown as I’’

(iv) Find the 2 × 2 matrix that represents a stretch of factor 3, x-axis invariant. [2]

Here you want only the y coordinates to be changed by a factor of 3, so when you multiply each coordinate by the matrix, the x coordinate remains unchanged, but the y coordinate is multiplied by 3, hence:

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{3} \end{pmatrix}$$

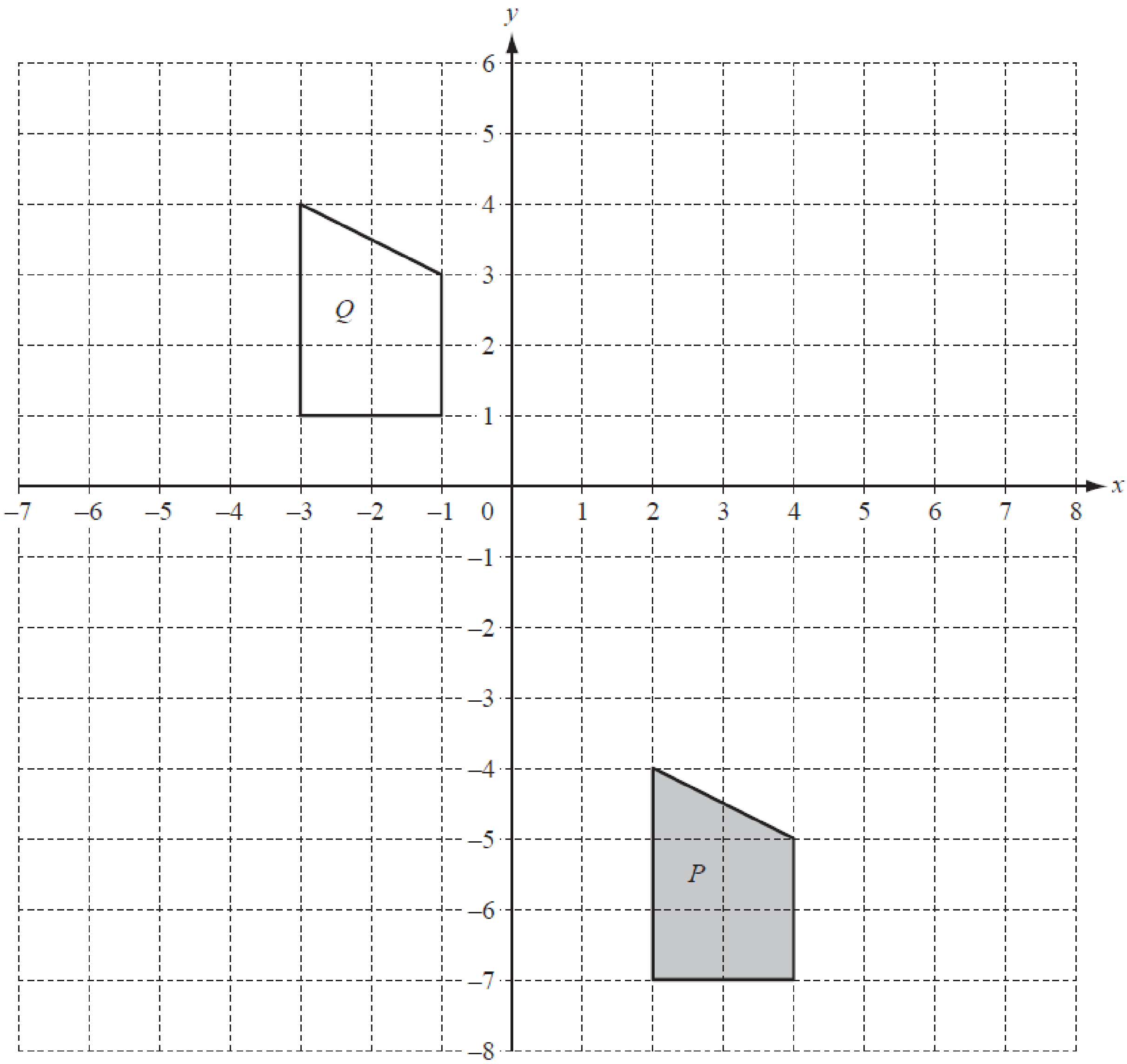
(c) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. [2]

To do this, you can simply apply that matrix to each of the vertices of Q, find the image and hence identify the transformation.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

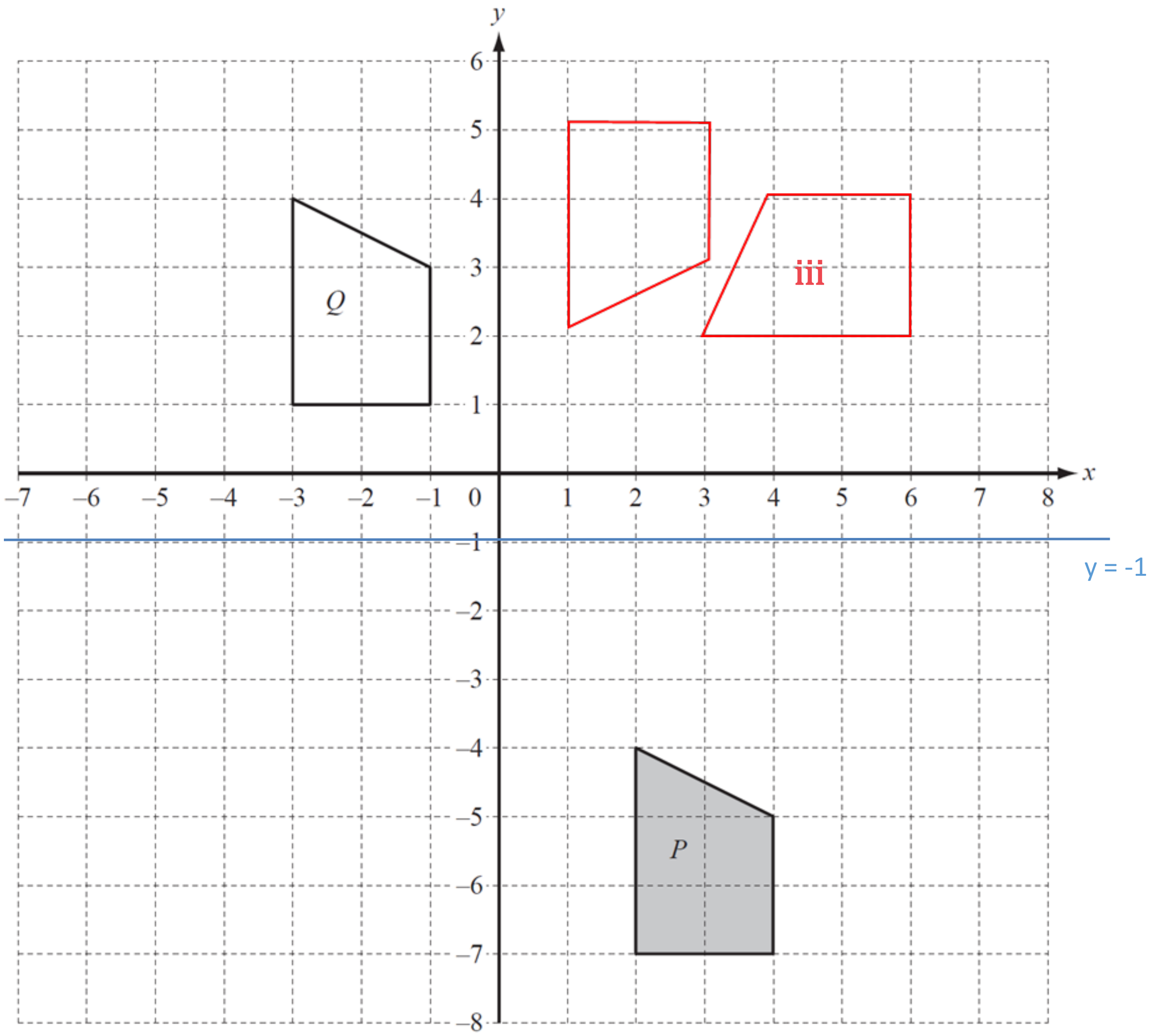
You can start to see that this matrix swaps the x and y coordinate, by drawing the image you can quickly see that this is **a reflection in $y = x$.**



(i) Describe fully the **single** transformation which maps shape P onto shape Q. [2]

Translation, $\begin{pmatrix} -5 \\ 8 \end{pmatrix}$

(ii) On the grid above, draw the image of shape P after reflection in the line $y = -1$. [2]

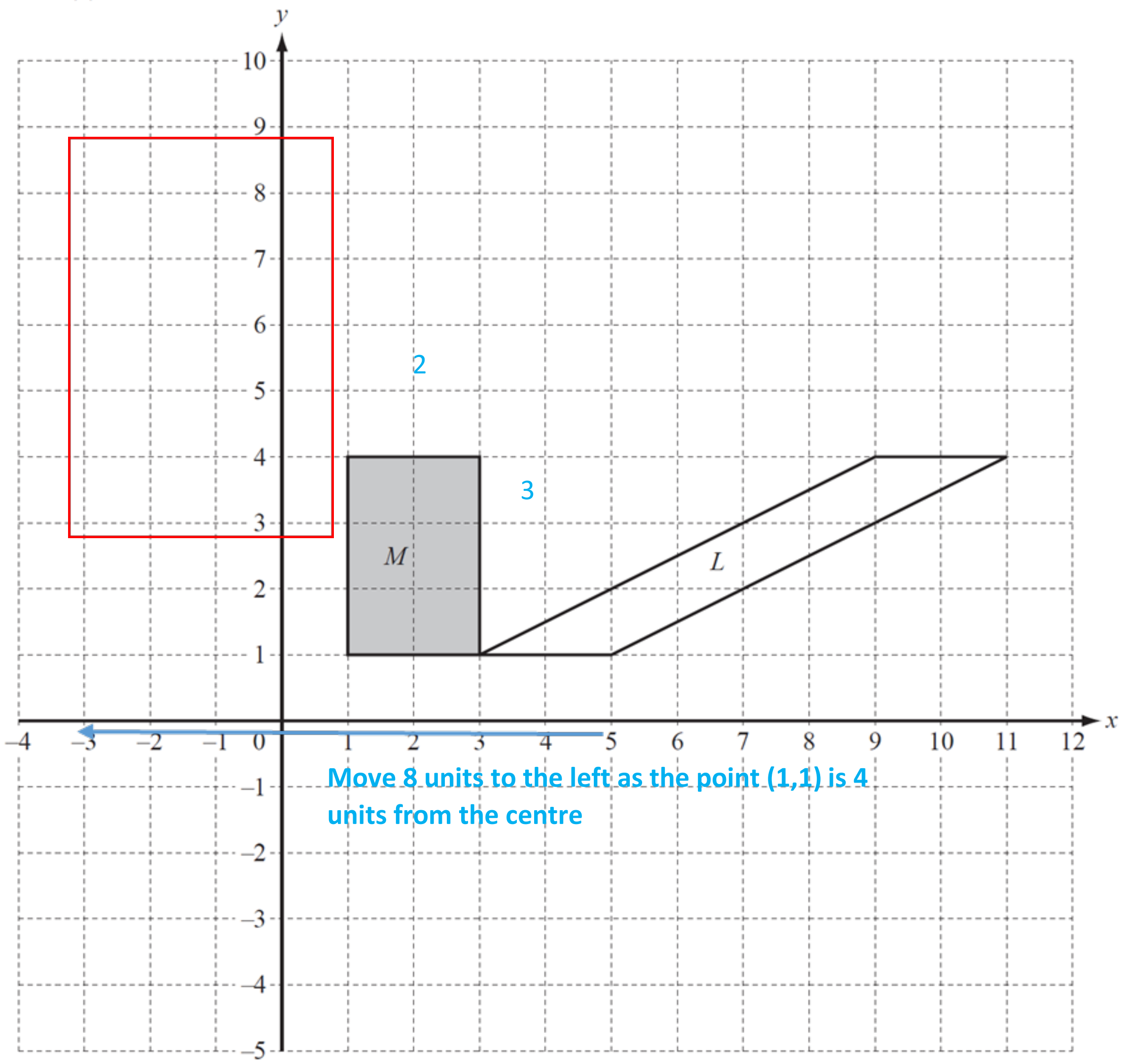


(iii) On the grid above, draw the image of shape P under the transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [3]

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 2 & 4 & 4 \\ -4 & -7 & -7 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 7 & 7 & 5 \\ 2 & 2 & 4 & 4 \end{pmatrix}$$

(b)



(i) Describe fully the **single** transformation which maps shape M onto shape L. [3]

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 3 & 1 \\ 1 & 1 & 4 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 5 & 11 & 9 \\ 1 & 1 & 4 & 4 \end{pmatrix}$$

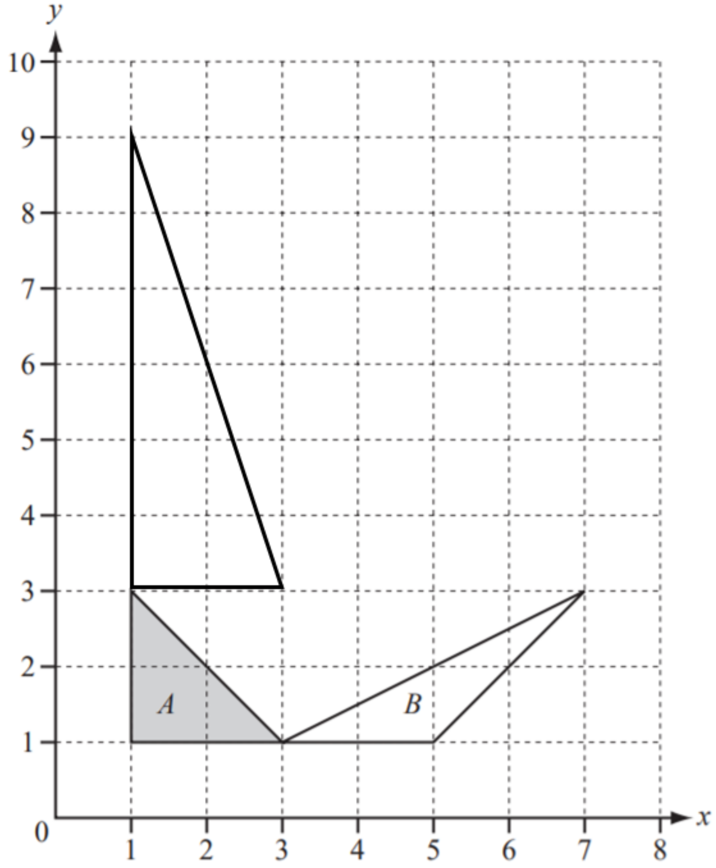
We see above that when there is a shear of 2 units, the resultant

transformation is figure L.

Therefore,

Shear with x-axis invariant by 2 units

(ii) On the grid above, draw the image of shape M after enlargement by scale factor 2, centre (5, 0). [2]



(a) (i) Draw the image of shape A after a stretch, factor 3, x-axis invariant. [2]

The shape will only stretch along the y axis as it is x-axis invariant. Each y value will triple as the factor is 3 but the x values stay the same.

(ii) Write down the matrix representing a stretch, factor 3, x-axis invariant. [2]

A stretch by factor k along the y-axis is given by the matrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$$

Therefore, for factor 3:

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{3} \end{pmatrix}$$

(b) (i) Describe fully the **single** transformation which maps shape A onto shape B. [3]

The area of A and B are the same therefore this transformation must be a shear.

The displacement from C to C' is 6 units and the distance from the x-axis to C is 3. Therefore, the shear factor is: $\frac{6}{3} = 2$

The same can be proved with point D. The displacement from D to D' is 2 units and the distance from the x-axis is 1 therefore the factor is $\frac{2}{1} = 2$

Shear, x-axis invariant, factor 2

(ii) Write down the matrix representing the transformation which maps shape A onto shape B. [2]

A shear of factor k parallel to the x-axis is given by the matrix:

$$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$$

Therefore, for factor 2:

$$\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

(a) Calculate the magnitude of the vector $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$. [2]

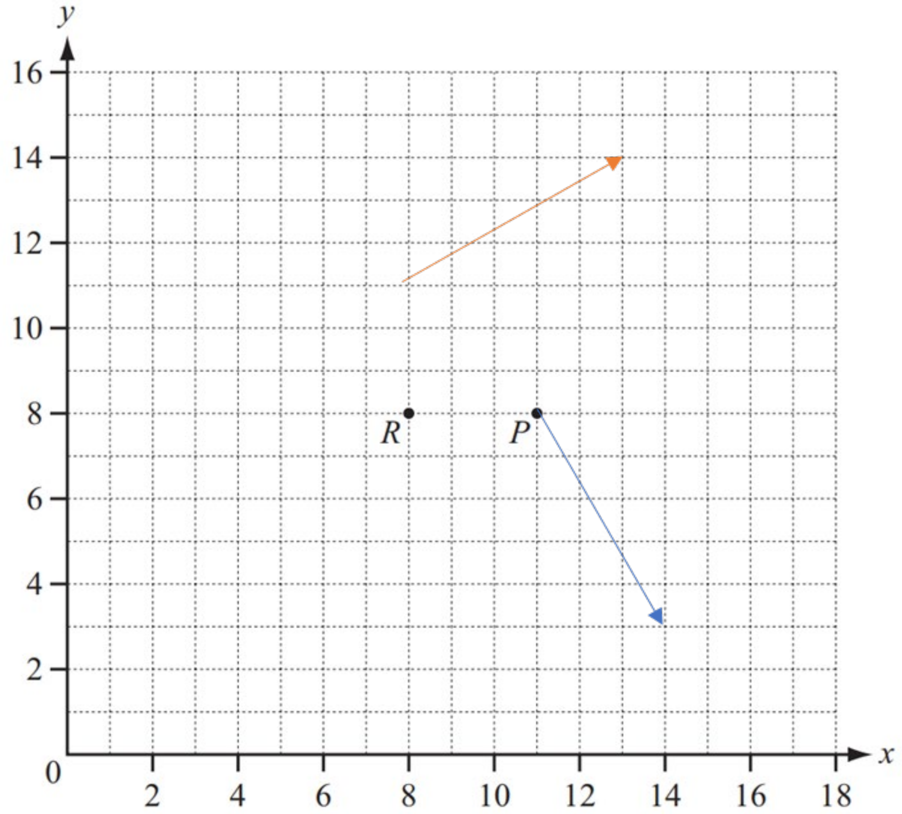
$|v| = \sqrt{3^2 + (-5)^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

$= \mathbf{5.83}$

(b)



(i) The points P and R are marked on the grid above.

$\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$. Draw the vector $\overrightarrow{PQ}$ on the grid above. [1]

The blue arrow on the diagram below.

(ii) Draw the image of vector $\overrightarrow{PQ}$ after rotation by 90° anticlockwise about R. [2]

The orange arrow on the diagram below.

(c) $\overrightarrow{DE} = 2\mathbf{a} + \mathbf{b}$ and $\overrightarrow{DC} = 3\mathbf{b} - \mathbf{a}$.

Find $\overrightarrow{CE}$ in terms of **a** and **b**. Write your answer in its simplest form. [2]

$$\overrightarrow{CE} = -\overrightarrow{DC} + \overrightarrow{DE}$$

$$= -3\vec{b} + \vec{a} + 2\vec{a} + \vec{b}$$

$$= \mathbf{3\vec{a} - 2\vec{b}}$$

(d) $\overrightarrow{OT} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\overrightarrow{OV} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

Write $\overrightarrow{TV}$ as a column vector. [2]

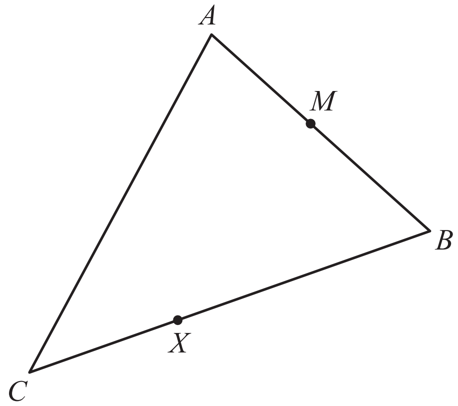
$$\overrightarrow{TV} = -\overrightarrow{OT} + \overrightarrow{OV}$$

$$= -\begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 5+2 \\ -5-1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{7} \\ \mathbf{-6} \end{pmatrix}$$

(e)



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$\overrightarrow{AB} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$.

(i) Find $\overrightarrow{CB}$ in terms of **b** and **c**. [1]

$\overrightarrow{CB} = -\overrightarrow{AC} + \overrightarrow{AB}$

$= \vec{b} - \vec{c}$

(ii) X divides CB in the ratio $1:3$.
M is the midpoint of AB.

Find $\overrightarrow{MX}$ in terms of **b** and **c**.
Show all your working and write your answer in its simplest form. [4]

$\overrightarrow{MX} = \overrightarrow{MB} + \overrightarrow{BX}$

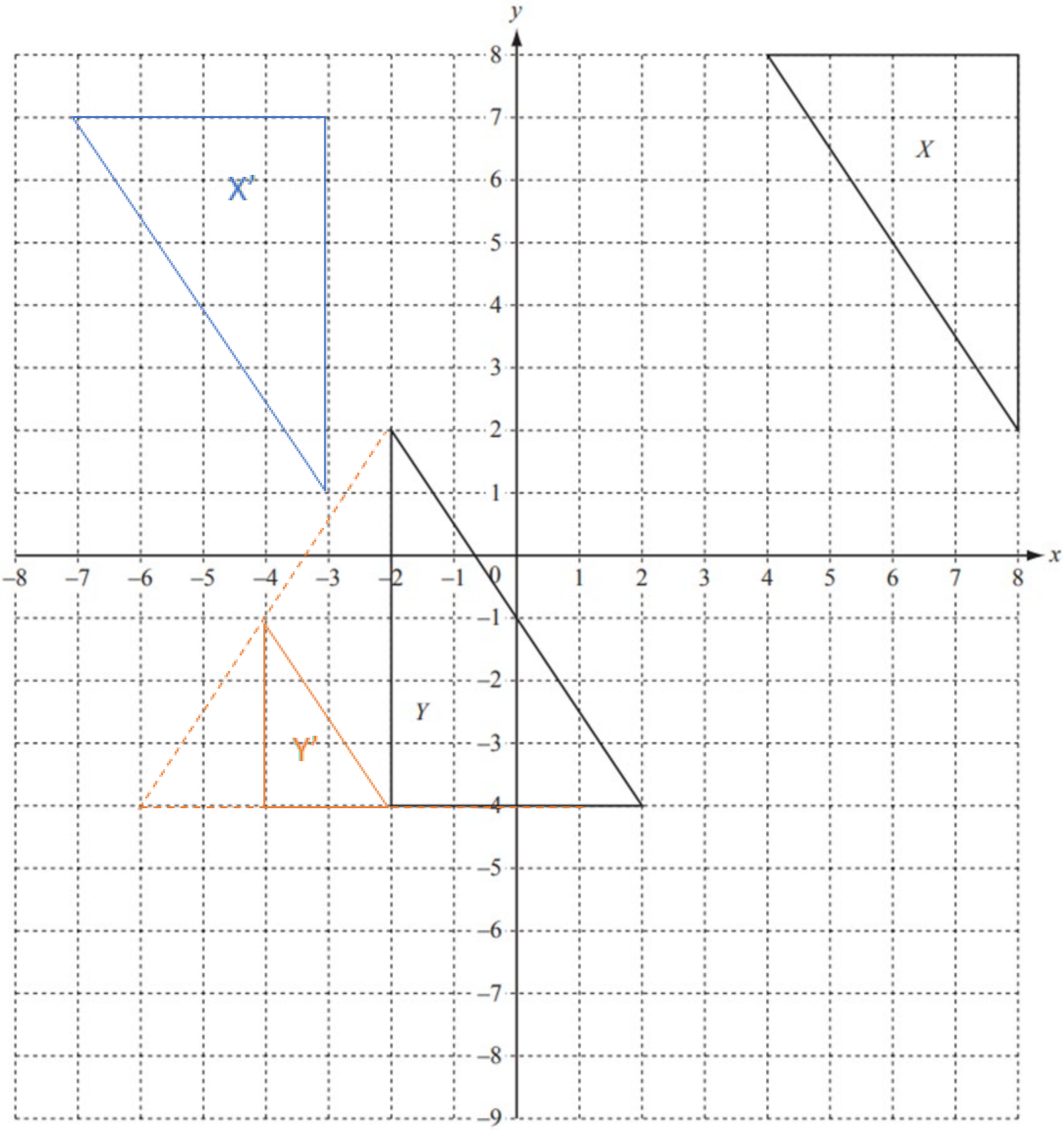
$= \frac{1}{2}\overrightarrow{AB} + \frac{3}{4}\overrightarrow{BC}$

$= \frac{1}{2}\vec{b} - \frac{3}{4}\overrightarrow{CB}$

$= \frac{1}{2}\vec{b} - \frac{3}{4}(\vec{b} - \vec{c})$

$= \frac{3}{4}\vec{c} - \frac{1}{4}\vec{b}$

(a)



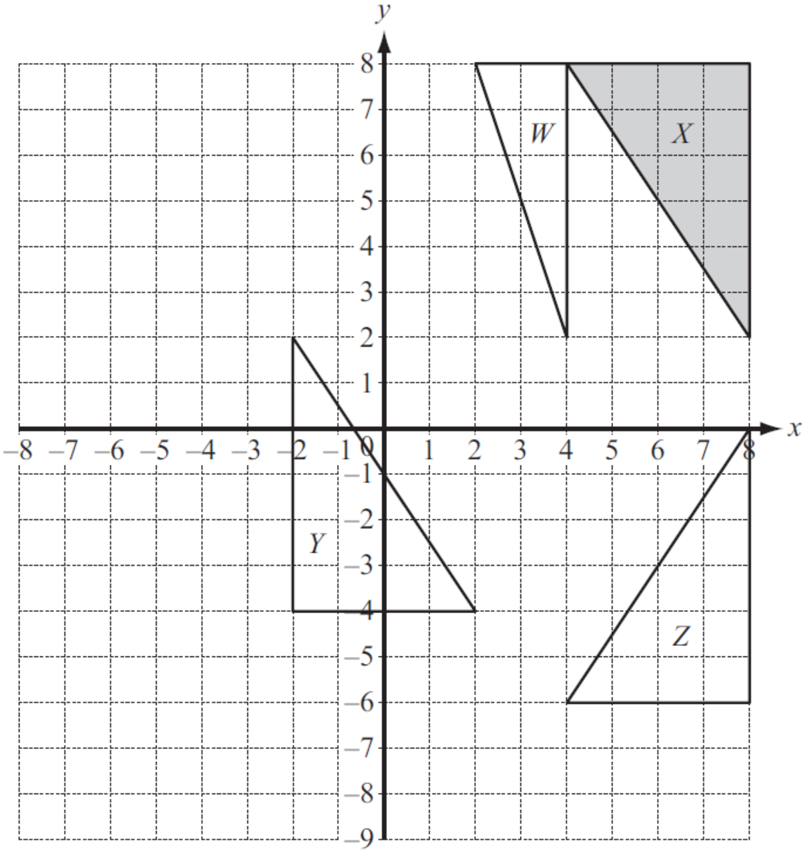
(i) Draw the translation of triangle X by the vector $\begin{pmatrix} -11 \\ -1 \end{pmatrix}$. [2]

Blue triangle labelled X'. Translated 11 in the negative x-direction and 1 in the negative y-direction.

(ii) Draw the enlargement of triangle Y with centre $(-6, -4)$ and scale factor $\frac{1}{2}$. [2]

Orange triangle labelled Y'.

(b)



Describe fully the **single** transformation that maps

(i) triangle X onto triangle Z, [2]

Reflection in the line $y = 1$.

(ii) triangle X onto triangle Y, [3]

Enlargement, centre (3, 2), scale factor -1

or Rotation of 180 degrees about (3,2).

(iii) triangle X onto triangle W. [3]

Stretch, factor 1/2, invariant line y-axis.

(c) Find the matrix that represents the transformation in part (b)(iii). [2]

We need

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2}x \\ y \end{pmatrix}$$

The matrix must therefore be

$$\begin{pmatrix} \mathbf{0.5} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Vectors
Difficulty: Medium

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 3

Time allowed: **110 minutes**

Score: **/96**

Percentage: **/100**

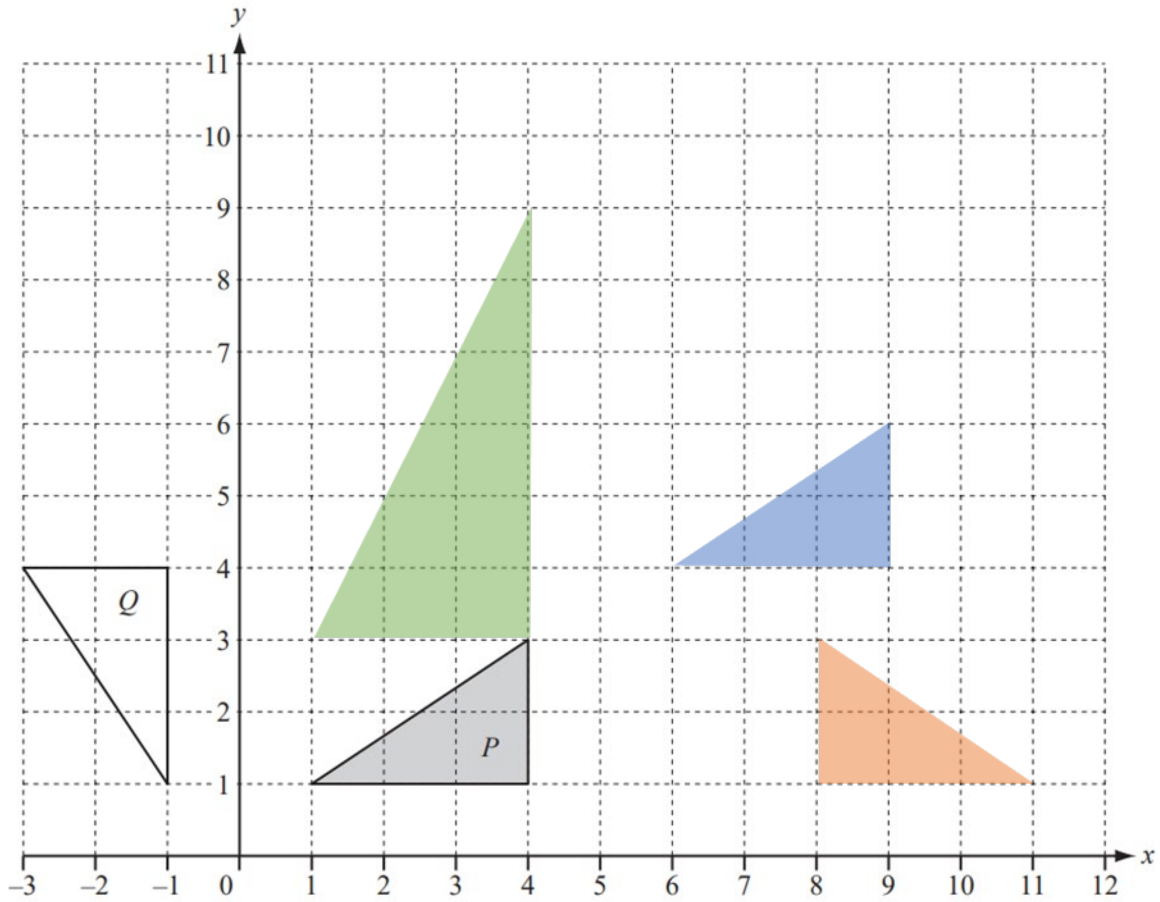
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



(a) Draw the translation of triangle P by $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$. [2]

The blue triangle drawn below.

(b) Draw the reflection of triangle P in the line $x = 6$. [2]

The orange triangle drawn below.

(c) (i) Describe fully the **single** transformation that maps triangle P onto triangle Q. [3]

Rotation of 90˚, anti-clockwise, about the origin

(ii) Find the 2 by 2 matrix which represents the transformation in **part(c)(i).** [2]

We require

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x \end{pmatrix}$$

This is done with the matrix

$$\begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$$

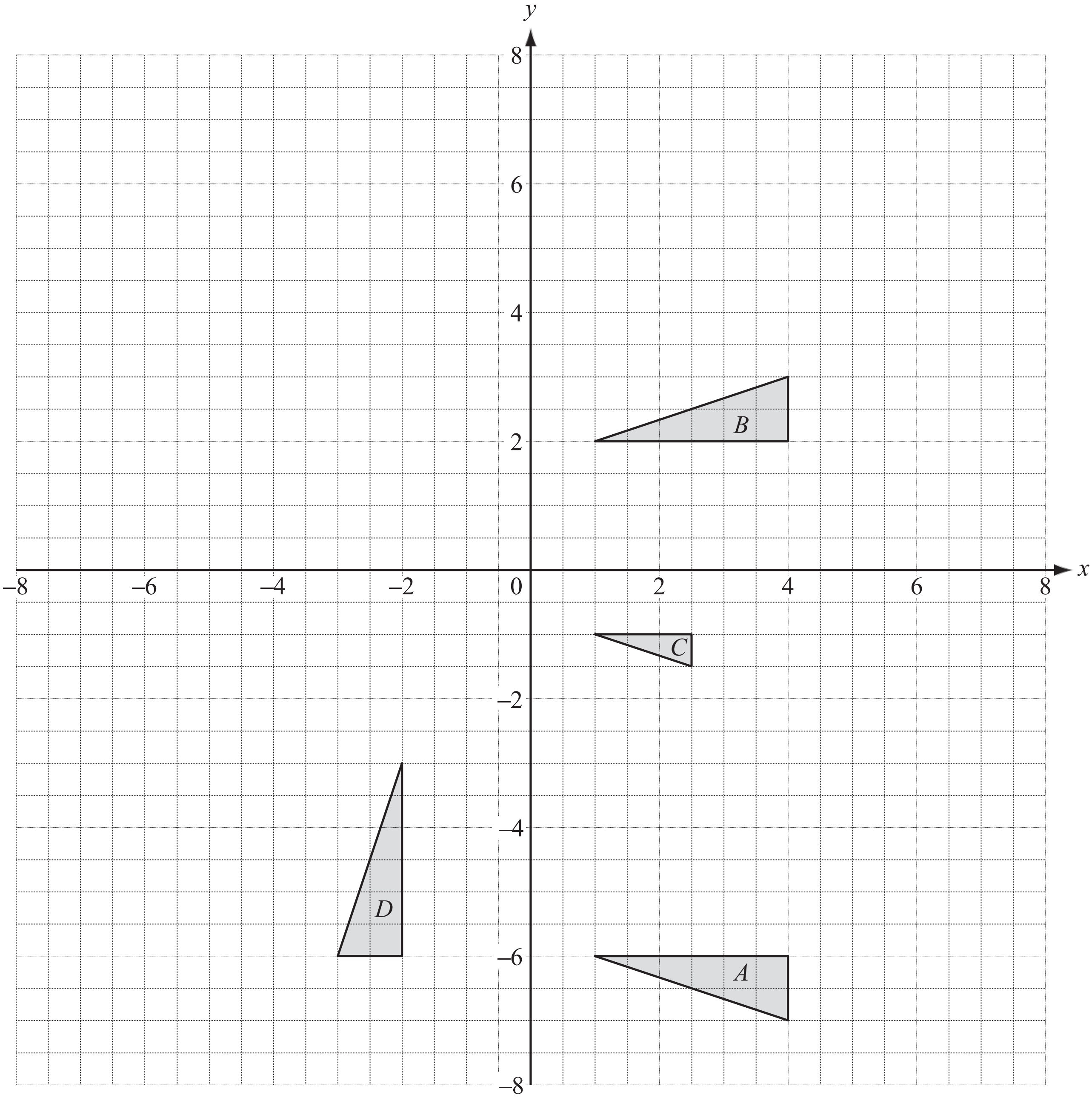
(d) (i) Draw the stretch of triangle P with scale factor 3 and the x-axis as the invariant line. [2]

The green triangle drawn above

(ii) Find the 2 by 2 matrix which represents a stretch, scale factor 3 and x-axis invariant. [2]

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{3} \end{pmatrix}$$

Question 2



(a) Describe fully the **single** transformation which maps

(i) triangle A onto triangle B, [2]

The 2 triangles are mirror images of each other, the size being the same. Therefore, the single transformation is a reflection.

Every point in the image will be the same distance from the reflection line as the original shape. The line that is halfway between the 2 figures is y = -2. This line is represented on the graph below. The lines joining each point in the object to the corresponding point in the image are perpendicular on the reflection line.

(ii) triangle A onto triangle C, [3]

The size of the triangle is changed between the object A and the image C, therefore, the single transformation is an enlargement. The image is smaller, so the enlargement factor is smaller than 1.

Each side of the image will be the scale factor times the corresponding side in the initial image.

In our case, the sides of triangle C are half the size of the side in triangle A, therefore, the scale factor is 0.5.

When enlarged, the distance from the centre to each corner will be multiplied by the scale factor to obtain the image, in this case, 0.5.

The distance from the centre of enlargement to each of the vertices of C needs to be half the distance from the centre of enlargement to the corresponding vertex of A.

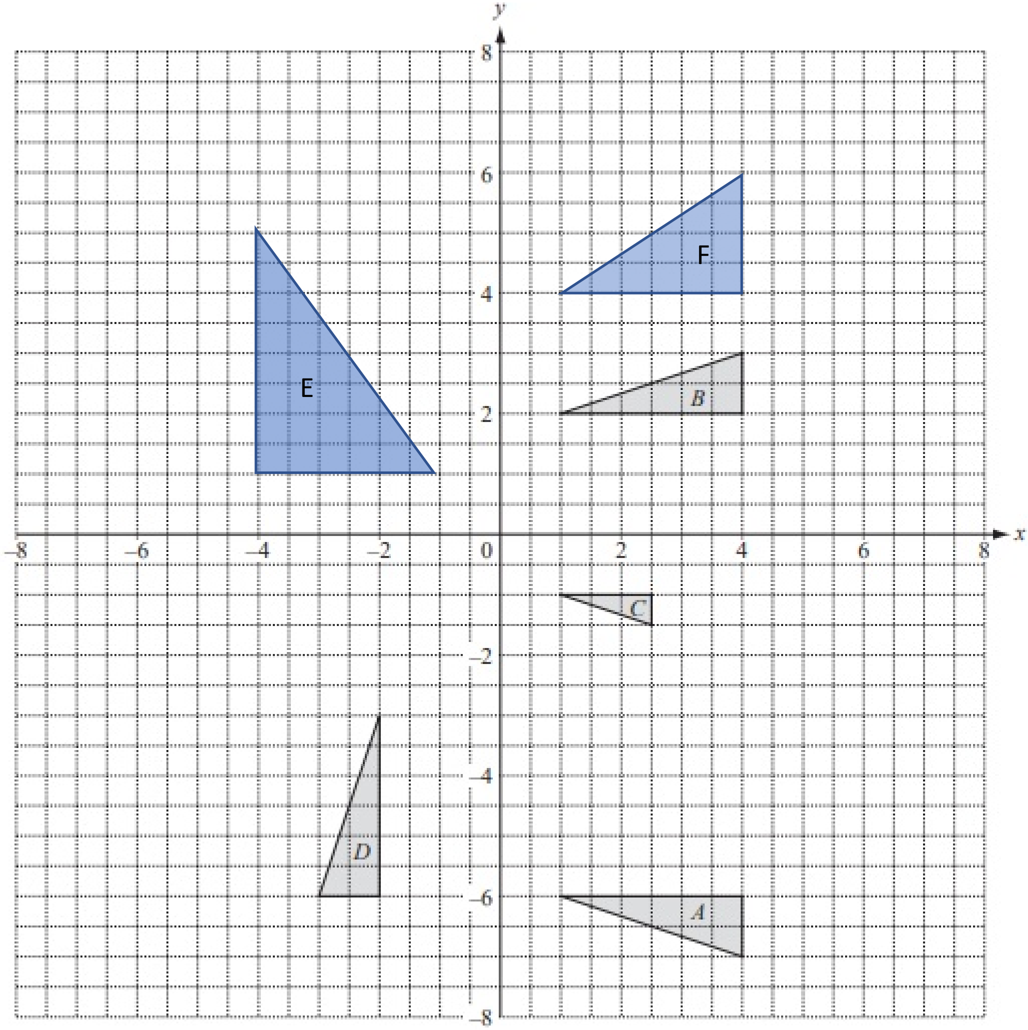
The centre of enlargement will be the point of coordinates (1, 4)

(iii) triangle A onto triangle D. [3]

The shape is rotated to the right (clockwise) by 90° around the centre point of coordinates (1, -3).

(b) Draw the image of

(i) triangle B after a translation of $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$, [2]



We put the coordinates of each of the vertices in ordered pairs as columns in the matrix.

For example, for the triangle with the coordinates (1, 2); (4, 2); (4, 3), the matrix will be:

$$\begin{pmatrix} 1 & 4 & 4 \\ 2 & 2 & 3 \end{pmatrix}$$

In a column matrix, the top number represents movements either to the left or to the right while the bottom number represents movements either up or down. A positive number represents a movement either to the right or up while a negative number represents a movement to the left or down.

We obtain the coordinates of the image by adding up the corresponding elements from the tringle matrix and translation matrix. Each column is summed up one by one.

The image matrix will be:

$$\begin{pmatrix} -4 & -1 & -1 \\ 4 & 4 & 5 \end{pmatrix}$$

The triangle with these coordinates is represented above as triangle E.

(ii) triangle B after a transformation by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. [3]

We transform it by multiplying the matrix by the given matrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 4 & 4 \\ 2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 1 & 4 \\ 4 & 4 & 6 \end{pmatrix}$$

We multiply the 2 matrices by multiplying the elements from each row of the first matrix by the corresponding elements from each column in the second matrix.

The image will therefore have the coordinates: (4, 4); (1, 4); (4, 6)

We represent this image on the grid above as image F.

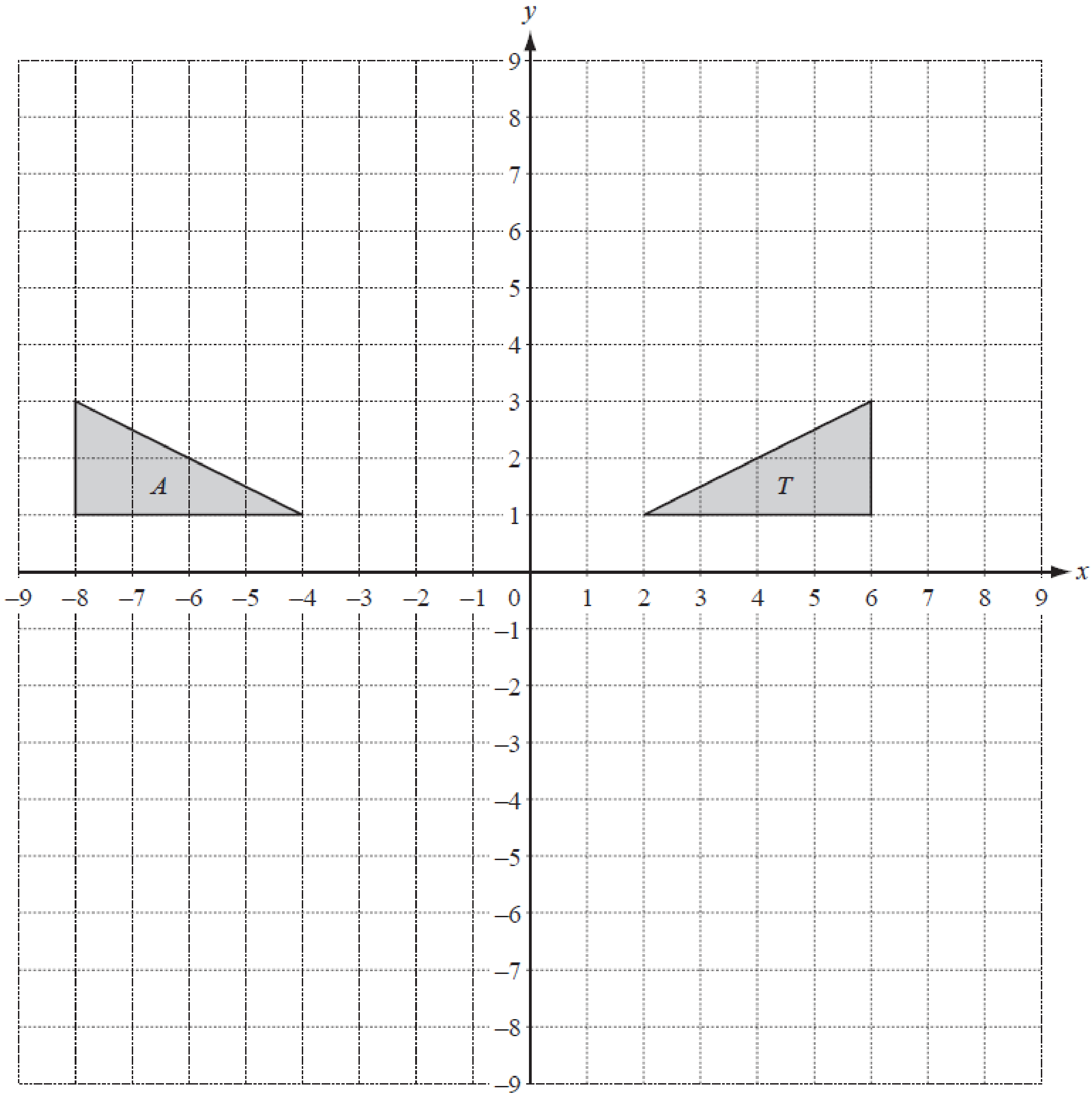
(c) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. [3]

The single transformation mapped by the matrix is a stretch.

$\begin{pmatrix} 1 & 0 \\ 0 & m \end{pmatrix}$ represents a stretch parallel to the y-axis (vertical) by the stretch factor m.

In our case, m = 2.

This linear transformation will enlarge the distances in a direction, in this case vertical, but will not affect the distances in the other direction, horizontal, keeping the x-axis invariant.



Triangles T and A are drawn on the grid above.

(a) Describe fully the single transformation that maps triangle T onto triangle A. [2]

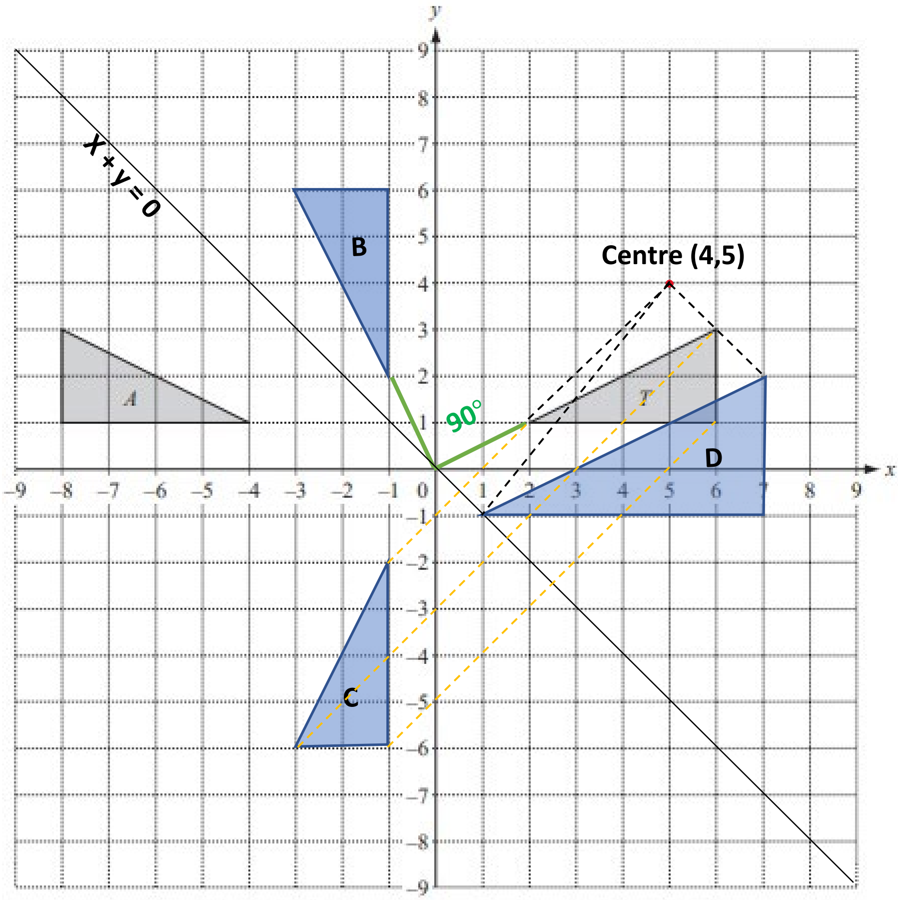
The triangle is reflected across a mirror line to create an image. Every point in the original shape is the same distance from the mirror line as every point in the image.

(b) (i) Draw the image of triangle T after a rotation of 90° anticlockwise about the point $(0,0)$.

Label the image B. [2]

A rotation will turn the shape around a fixed point, the centre of rotation, creating the image. The centre of rotation will be the origin, (0, 0), in this case.

The rotated image will have each corner (from image B) at the same distance from the centre of rotation as the original shape, image T. The rotation takes place anticlockwise, to the left relative to the centre of rotation and by 90°.



(ii) Draw the image of triangle T after a reflection in the line $x + y = 0$.

Label the image C. [2]

We first need to draw the line x + y = 0, equivalent to the line y = -x. This can be seen in black on the graph above.

Every point in the image will be the same distance from the reflection line as the original shape.

The lines joining each point in the object to the corresponding point in the image are perpendicular on the reflection line.

(iii) Draw the image of triangle T after an enlargement with centre (4, 5) and scale factor 1.5.

Label the image D. [2]

The centre of enlargement will be the point of coordinates (4, 5)

The enlargement will change the size of the initial shape. When enlarged, the distance from the centre to each corner will be multiplied by the scale factor to obtain the image. In this case, the scale factor is 1.5.

Each side of the image will be 1.5 x the corresponding side in the initial image.

(c) (i) Triangle T has its vertices at co-ordinates (2, 1), (6, 1) and (6, 3).

Transform triangle T by the matrix $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

Draw this image on the grid and label it E. [3]

We put the coordinates of each of the vertices in ordered pairs as columns in the matrix.

For example, for the triangle with the coordinates (2, 1); (6, 1); (6, 3), the matrix will be:

$$\begin{pmatrix} 2 & 6 & 6 \\ 1 & 1 & 3 \end{pmatrix}$$

We transform it by multiplying this matrix by the given matrix:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 6 & 6 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 6 \\ 3 & 7 & 9 \end{pmatrix}$$

We multiply the 2 matrices by multiplying the elements from each row of the first matrix by the corresponding elements from each column in the second matrix.

The image will therefore have the coordinates: (2, 3); (6, 7); (6, 9)

We represent this image on the grid above.

(ii) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. [3]

$\begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix}$ represents a vertical shear matrix with m – the shear factor.

In our case, the shear factor is m = 1.

Vertical shear leaves lines parallel to the y-axis invariant but tilts any other line about the point where they intersect the y-axis. Horizontal lines get tilted to become lines with the slope equal to m, where m is the shear factor.

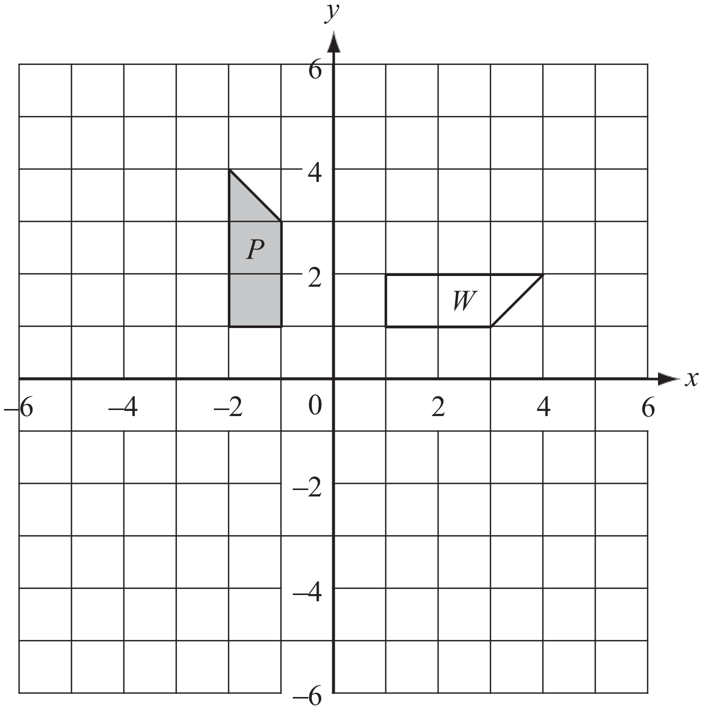
(d) Write down the matrix that transforms triangle *B* onto triangle *T*. [2]

The triangle T has the matrix: $\begin{pmatrix} 2 & 6 & 6 \\ 1 & 1 & 3 \end{pmatrix}$

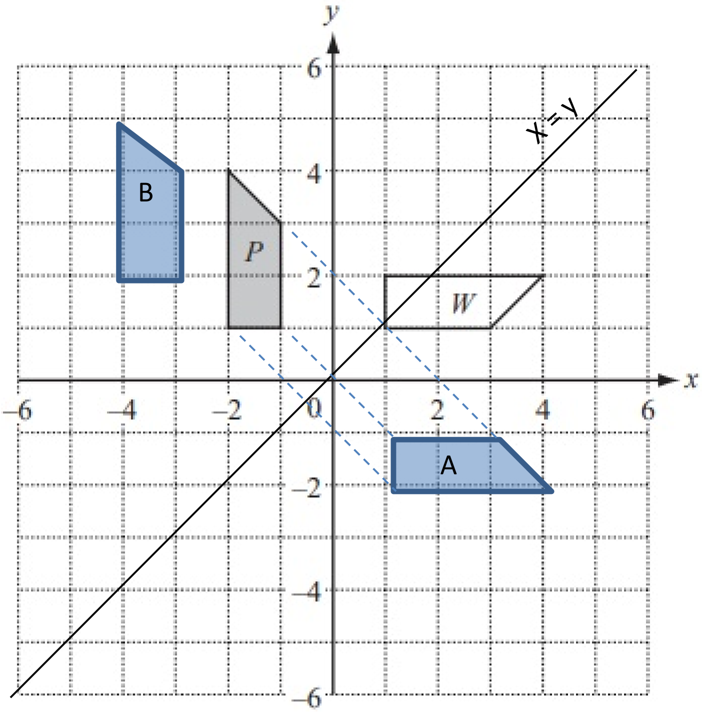
And triangle B has the matrix: $\begin{pmatrix} -1 & -1 & -3 \\ 2 & 6 & 6 \end{pmatrix}$

The latter needs to be multiplied by a matrix, obtaining the matrix of triangle T.

This matrix is: $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$



(a) Draw the reflection of shape P in the line $y = x$. [2]



We first need to draw the line y = x. This can be seen in black on the graph above.

Every point in the image will be the same distance from the reflection line as the original shape.

The lines joining each point in the object to the corresponding point in the image are perpendicular on the reflection line. The shape is labelled as A on the grid above.

(b) Draw the translation of shape P by the vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$. [2]

We put the coordinates of each of the vertices in ordered pairs as columns in the matrix.

For example, for the shape P with the coordinates (-1, 1); (-2, 1); (3, -1) and (4, -2) the matrix will be:

$$\begin{pmatrix} -1 & -2 & -1 & -2 \\ 1 & 1 & 3 & 4 \end{pmatrix}$$

In a column matrix, the top number represents movements either to the left or to the right while the bottom number represents movements either up or down. A positive number represents a movement either to the right or up while a negative number represents a movement to the left or down.

We obtain the coordinates of the image by adding up the corresponding elements from the tringle matrix and translation matrix. Each column is summed up one by one.

The image matrix will be:

$$\begin{pmatrix} -3 & -4 & -3 & -4 \\ 2 & 2 & 4 & 5 \end{pmatrix}$$

The shape with these coordinates is represented above as B.

(c) (i) Describe fully the **single** transformation that maps shape P onto shape W. [3]

A rotation will turn the shape around a fixed point, the centre of rotation, creating the image. The centre of rotation will be the origin, (0, 0), in this case.

The rotated image will have each corner (from P) at the same distance from the centre of rotation as the original shape, image W. The rotation takes place clockwise, to the right relative to the centre of rotation and by 90°.

(ii) Find the 2 by 2 matrix which represents this transformation. [2]

The matrix which represents this transformation is:

$$\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$$

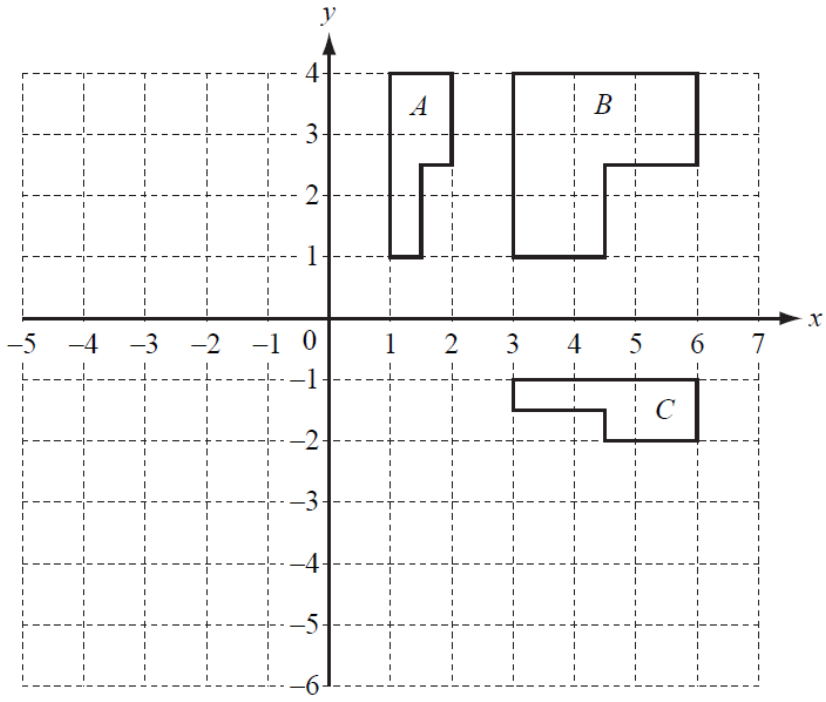
(d) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. [3]

The single transformation mapped by the matrix is a stretch.

$\begin{pmatrix} 1 & 0 \\ 0 & m \end{pmatrix}$ represents a stretch parallel to the y-axis (vertical) by the stretch factor m.

In our case, m = 2.

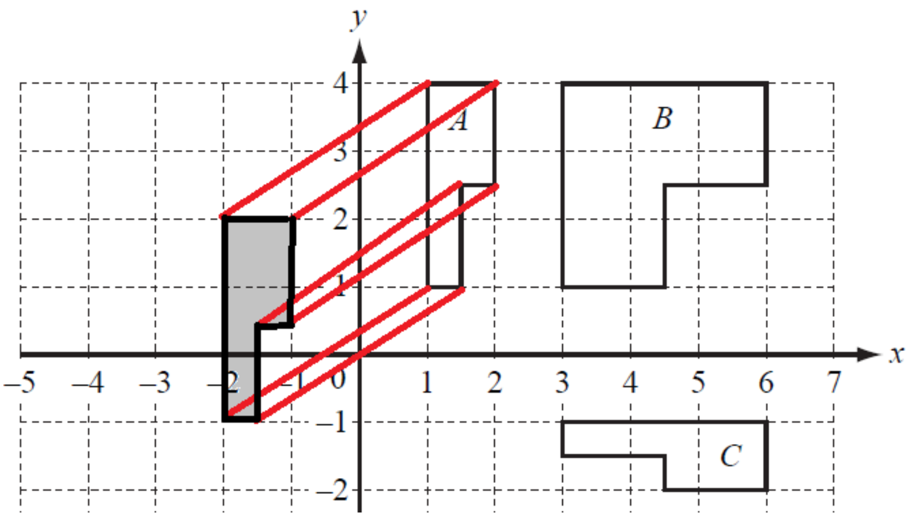
This linear transformation will enlarge the distances in a direction, in this case vertical, but will not affect the distances in the other direction, horizontal, keeping the x-axis invariant.



(a) On the grid above, draw the image of

(i) shape A after translation by the vector $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$, [2]

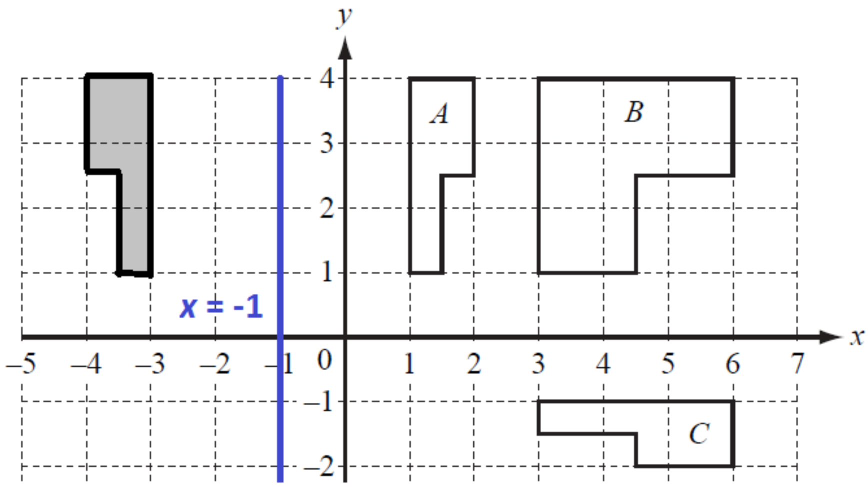
This transformation represents a shift by 3 units in the negative *x* direction and by 2 units in the negative*y* direction. (both term in the translation vectors are negative)



(ii) shape A after reflection in the line $x = -1$. [2]

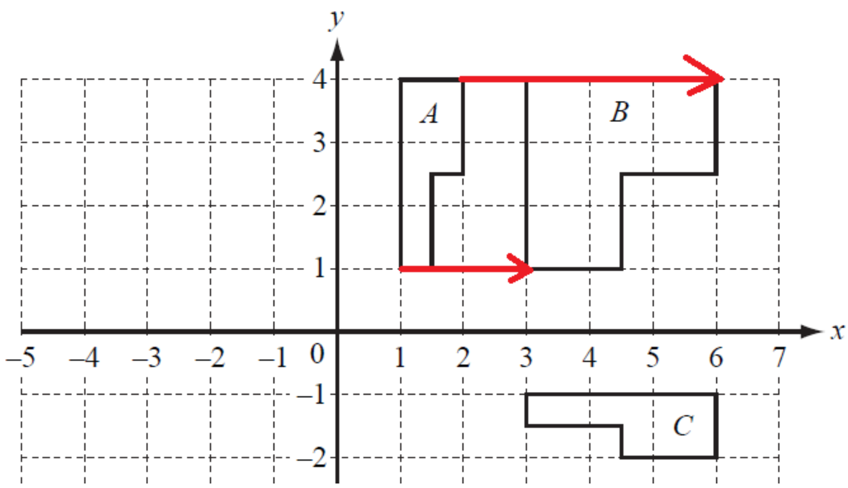
To draw a reflection in the line x = -1, the distances of corresponding vertices of the original and mirror triangles must be the same.

The shape must by symmetric by this line.



(b) Describe fully the **single** transformation which maps [3]

(i) shape A onto shape B,



The arrows show translation of points:

$(2,4)\ to\ (6,4)$

$(1,1)\ to\ (3,1)$

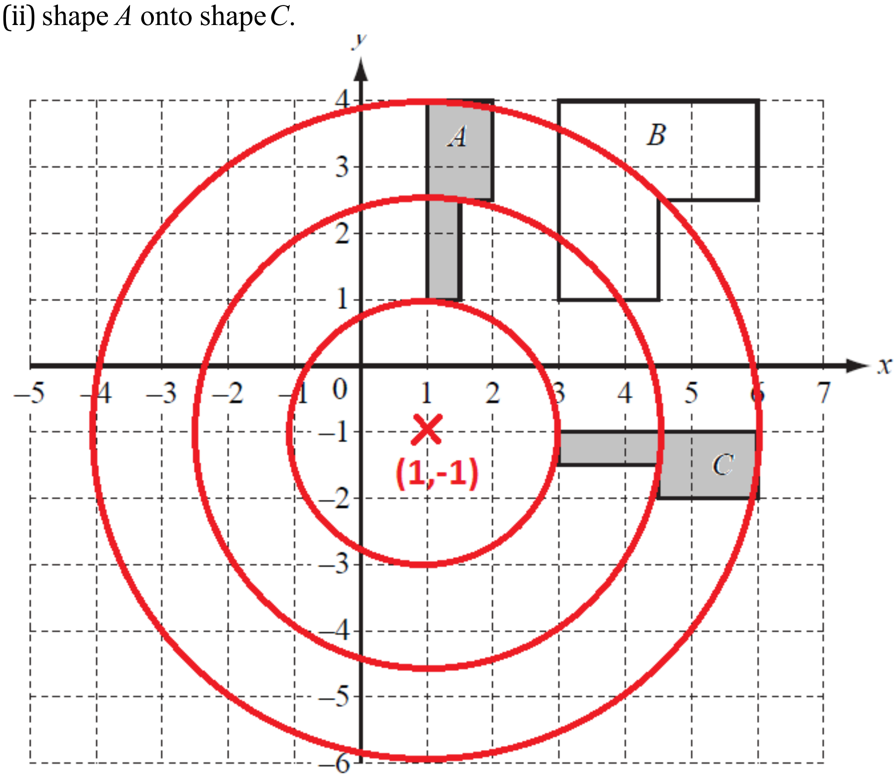
We can see that the *x* coordinate is multiplied by the factor 3.

The *y* coordinate on the other hand is unchanged.

Therefore this operations is:

Stretching by factor 3 with the *y*-axis invariant.

(ii) shape A onto shape C. [3]



By drawing circles connecting the corresponding vertices of the shapes, we can see that the transformation is:

a **rotation by 90° clockwise around the point (1,-1)**(the centre of the circles)

(c) Find the matrix representing the transformation which maps shape A onto shape B. [2]

From part b)i) we know that the x coordinate of every point is multiplied by factor 3.

The y coordinate is unchanged (multiplied by factor 1)

Hence we can construct a matrix for this transformation.

- Diagonal terms: 3 and 1 corresponding to the factors
- Only simple stretching so no non-diagonal terms involved

Matrix of transformation A to B:

$$\begin{pmatrix} \mathbf{3} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

(d) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. [3]

A general matrix for rotation about the origin looks like

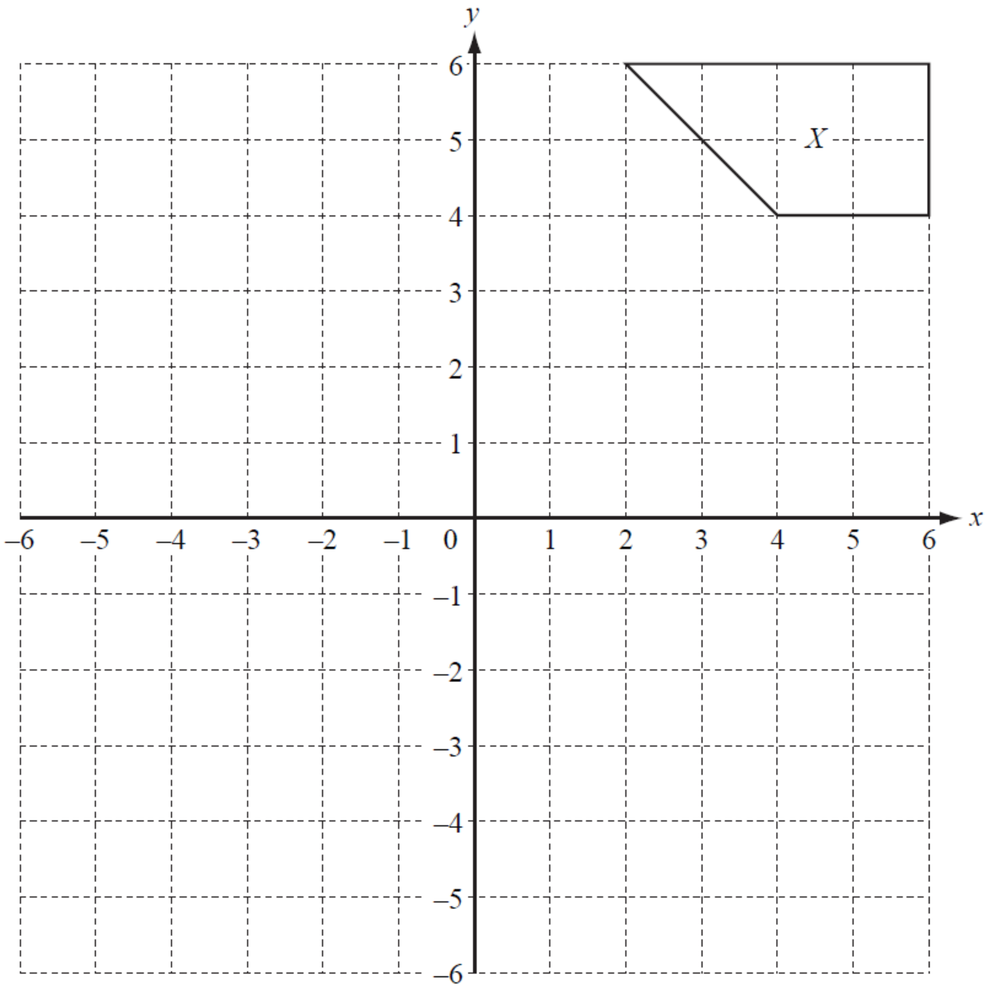
$$\begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$$

where x is an angle of anticlockwise rotation.

We can get the given matrix by setting x=180°.

Therefore the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is:

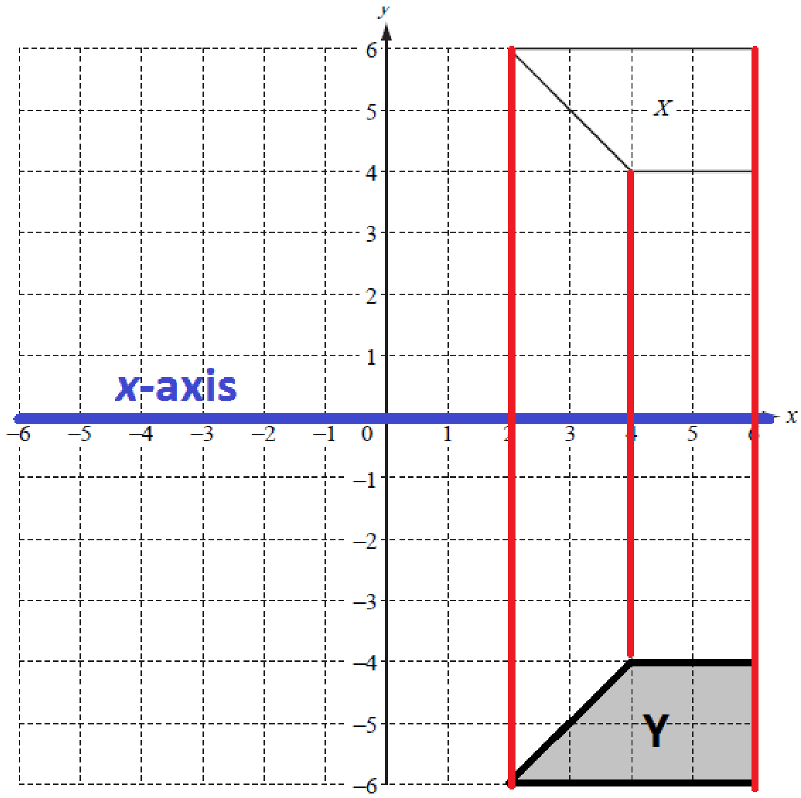
Rotation by 180° about the origin (0,0).



(a) (i) Draw the reflection of shape X in the x-axis. Label the image Y. [2]

To draw a reflection in the *x*-axis line (*y=0*), the distances of corresponding vertices of the original and mirror triangles must be the same.

The shapes must be symmetric by this axis (line).



The coordinated of the shape Y:

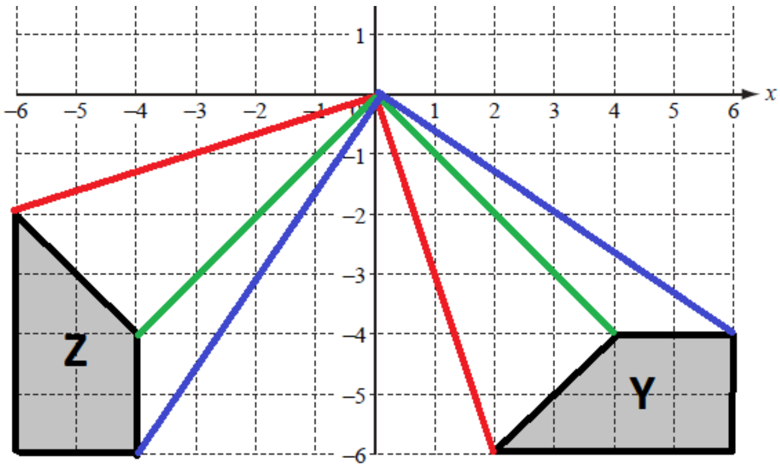
$(4, -4)$; $(6, -4)$; $(6, -6)$; $(2, -6)$;

(ii) Draw the rotation of **shape *Y***, 90° clockwise about (0, 0). Label the image *Z*. [2]

Remember that we are rotating Y. Rotation by 90° can be done by swapping the *x* and *y* coordinates of the points and then changing the sign of the new *x* coordinate.The coordinated of the shape Z:

$(-4, -4)$; $(-4, -6)$; $(-6, -6)$; $(-6, -2)$;

On the diagram below, we see that the lines of the same colour are directed at 90° with respect to each other, they all emerge from the centre of the rotation at (0,0).

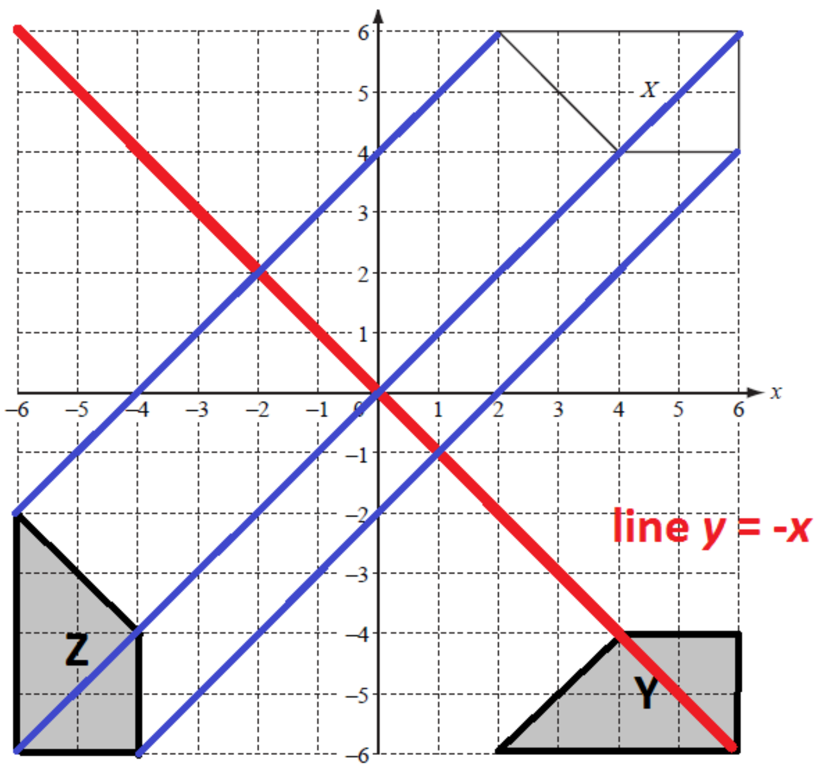


(iii) Describe fully the **single** transformation that maps shape Z onto shape X. [2]

The transformation that maps shape Z into shape X is:

a reflection in the line *y* = -*x*.

The shapes are symmetric by this line.



(b) (i) Draw the enlargement of shape X, centre (0, 0), scale factor $\frac{1}{2}$. [2]

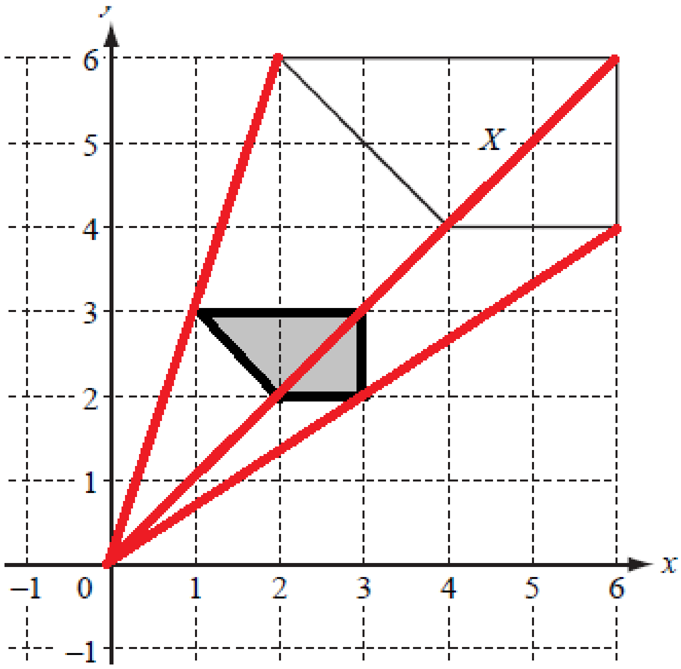
To apply enlargement with centre (0,0) with factor ½. All *x* and *y* coordinates will be multiplied by the factor ½ .

The coordinates of the original shape X were

$$(4,4);\ (4,6);\ (2,6)\ and\ (6,6)$$

Therefore the coordinates of the new shape will be:

$$\mathbf{(2,2);\ (3,2);\ (1,3)\ and\ (3,3)}$$



(ii) Find the matrix which represents an enlargement, centre (0, 0), scale factor $\frac{1}{2}$. [2]

The transformation multiplies both the *x* and *y* coordinate by a factor 1/2, hence both diagonal terms will equal to this factor.

As this is a pure enlargement with centre at (0,0), the off-diagonal coordinates are 0.

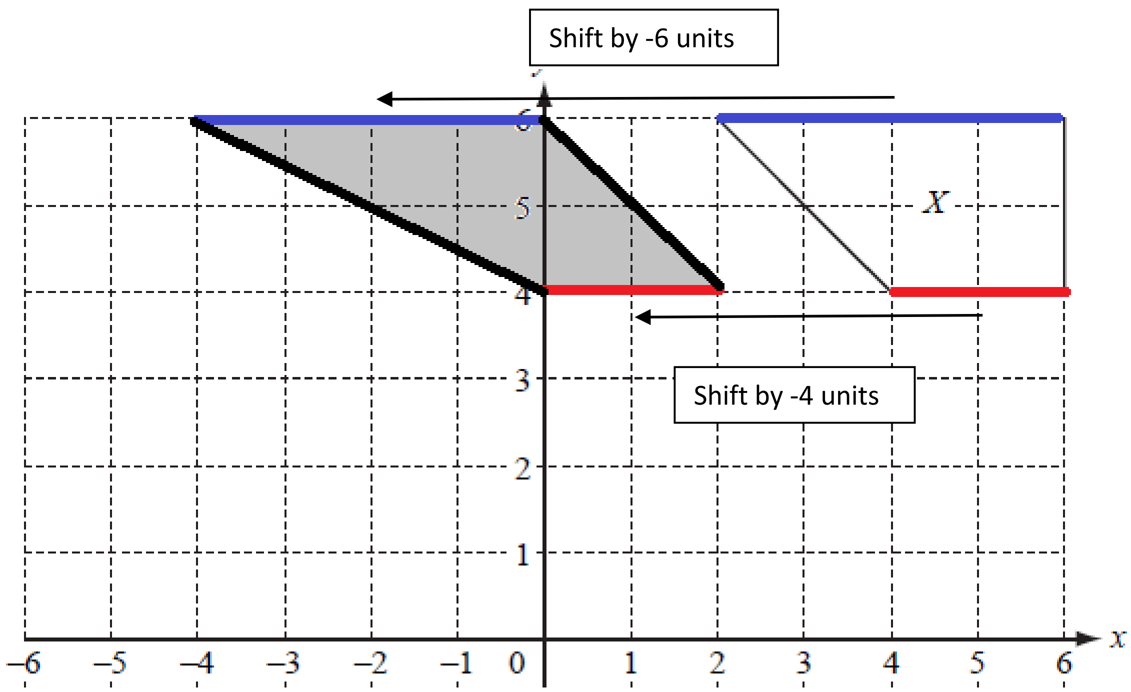
This matrix representing an enlargement, centre (0,0) with scale factor ½ is

$$\begin{pmatrix} \mathbf{0.5} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.5} \end{pmatrix}$$

(c) (i) Draw the shear of **shape *X*** with the x-axis invariant and shear factor -1. [2]

The bottom face (red) is at $y=4$ so when shear with factor -1 is applied, it should be shifted by -4 units in the x-direction. The shift is the y value of the face times the factor.

The top face (blue) is shifted at $y=6$. Using a similar method, this face will be shifted by 6 units in the negative x-direction.



Therefore the coordinates of the new shape are:

$$(0,4);\ (2,4);\ (0,6) \, and \, (-4,6)$$

(ii) Find the matrix which represents a shear with the x-axis invariant and shear factor -1. [2]

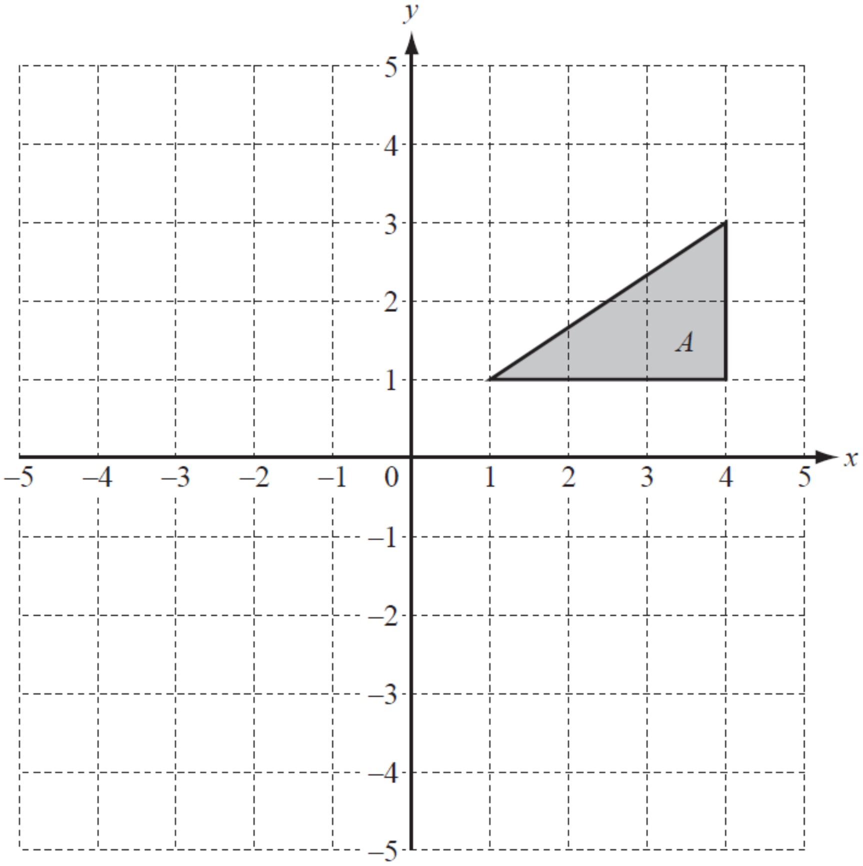
Shear transformation matrix with the x-axis invariant has a general form:

$$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$$

where k is the shear factor. In our case this factor is (-1) so the matrix becomes:

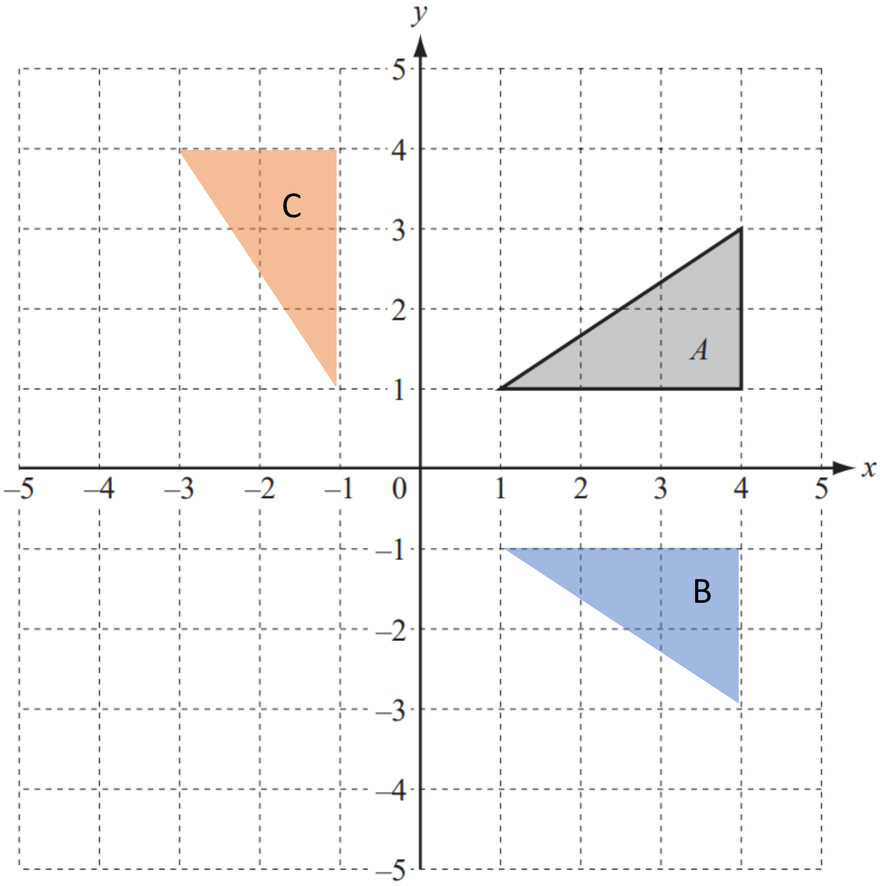
$$\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

(a)



(i) Draw the image when triangle A is reflected in the line $y = 0$. Label the image B. [2]

(ii) Draw the image when triangle A is rotated through 90 anticlockwise about the origin. Label the image C. [2]



(iii) Describe fully the **single** transformation which maps triangle B onto triangle C. [2]

Reflection through the line $y = x$.

(b) Rotation through $90°$ anticlockwise about the origin is represented by the matrix $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

(i) Find $\mathbf{M}^{-1}$, the inverse of matrix $\mathbf{M}$. [2]

The determinant (Δ) of M is

$$\Delta = 0 \times 0 - -1 \times 1$$

$$= 1$$

Then swap the elements on the leading diagonal and

multiply the anti-diagonal by -1

$$M^{-1} = \frac{1}{\Delta}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$$

(ii) Describe fully the **single** transformation represented by the matrix $\mathbf{M}^{-1}$. [2]

It is the inverse of M, hence it represents a

rotation though 90˚ clockwise about the origin.

Vectors
Difficulty: Medium

Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 4

Time allowed: **92 minutes**

Score: **/80**

Percentage: **/100**

Grade Boundaries:

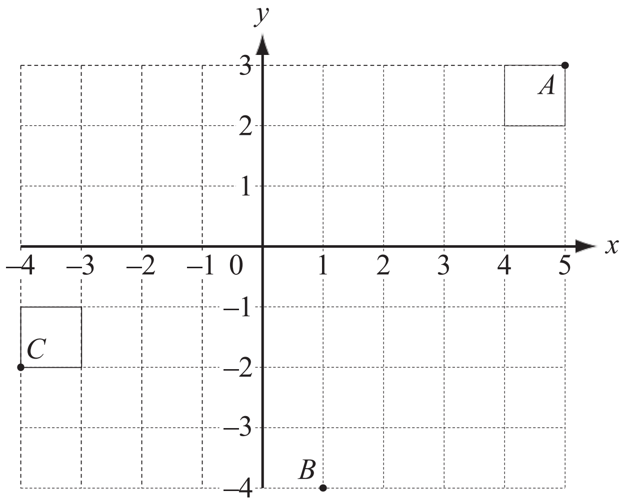
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

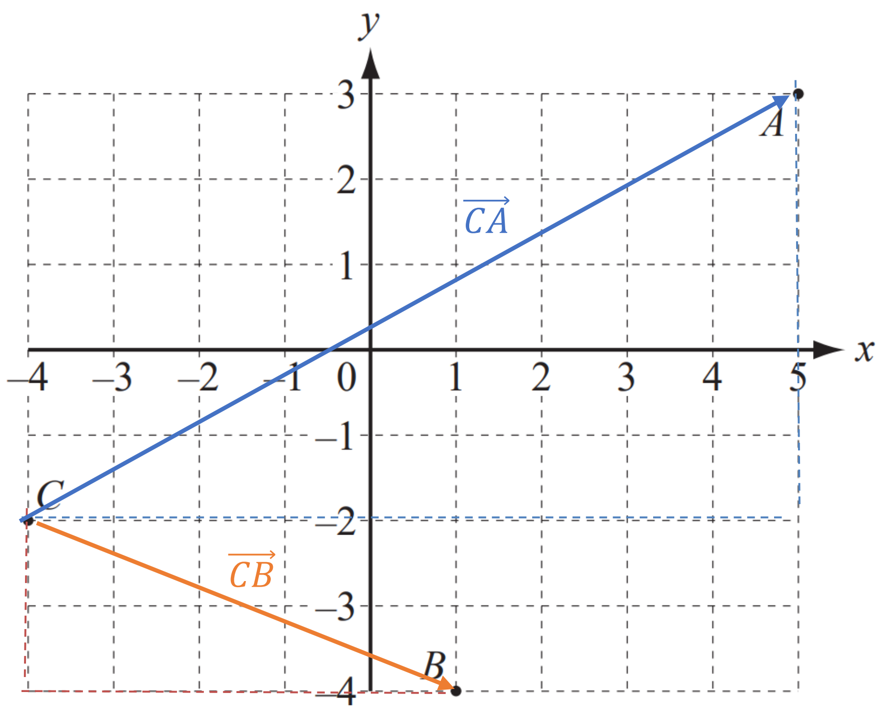
CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a)



The points A (5, 3), B (1, –4) and C (–4, –2) are shown in the diagram.



(i) Write $\overrightarrow{CA}$ as a column vector. [1]

$$\overrightarrow{CA} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$$

(ii) Find $\overrightarrow{CA} - \overrightarrow{CB}$ as a single column vector. [2]

$$\overrightarrow{CB} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$\rightarrow \overrightarrow{CA} - \overrightarrow{CB} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{4} \\ \mathbf{7} \end{pmatrix}$$

(iii) Complete the following statement. [1]

$$\boldsymbol{\overrightarrow{CA} - \overrightarrow{CB} = \overrightarrow{BA}}$$

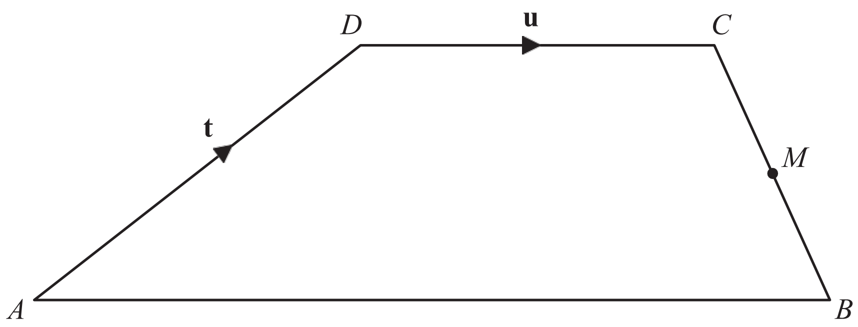
(iv) Calculate $|\overrightarrow{CA}|$. [2]

$$|\overrightarrow{CA}| = \sqrt{9^2 + 5^2}$$

$$= \sqrt{106}$$

$$= \mathbf{10.3}$$

(b)



NOT TO SCALE

$ABCD$ is a trapezium with DC parallel to AB and $DC = \frac{1}{2}AB$.

M is the midpoint of BC.

$\overrightarrow{AD} = \mathbf{t}$ and $\overrightarrow{DC} = \mathbf{u}$.

Find the following vectors in terms of t and / or u.

Give each answer in its simplest form.

(i) $\overrightarrow{AB}$ [1]

$\boldsymbol{\overrightarrow{AB} = 2\vec{u}}$

(ii) $\overrightarrow{BM}$ [2]

$\overrightarrow{BM} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CM}$

$= -2\vec{u} + \vec{t} + \vec{u} + \frac{1}{2}\overrightarrow{CB}$

$= \vec{t} - \vec{u} - \overrightarrow{BM}$

$\rightarrow 2\overrightarrow{BM} = \vec{t} - \vec{u}$

$\rightarrow \boldsymbol{\overrightarrow{BM} = \frac{1}{2}(\vec{t} - \vec{u})}$

(iii) $\overrightarrow{AM}$ [2]

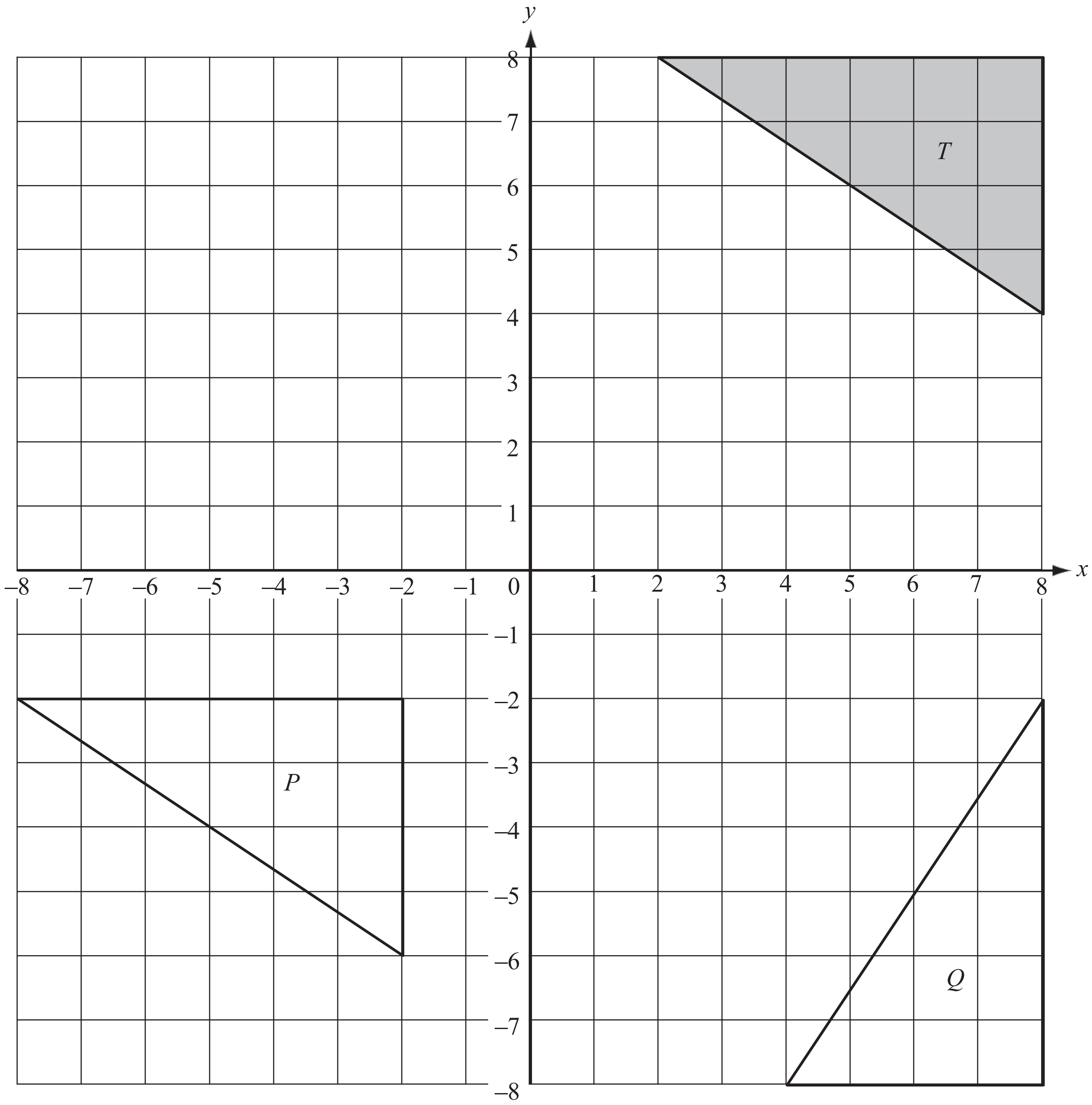
$$\overrightarrow{AM} = \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CM}$$

$$= \vec{t} + \vec{u} + \frac{1}{2}\overrightarrow{CB}$$

$$= \vec{t} + \vec{u} - \overrightarrow{BM}$$

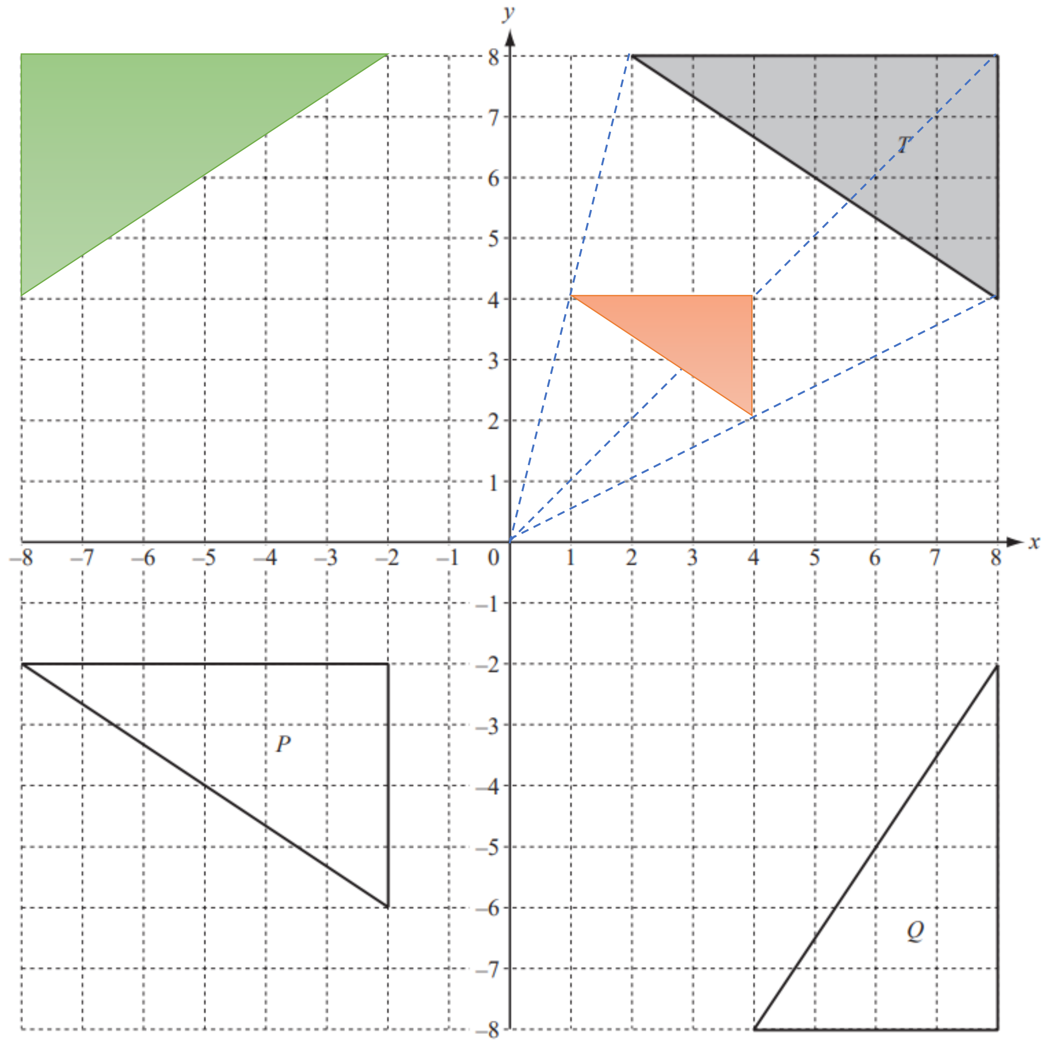
$$= \vec{t} + \vec{u} - \frac{1}{2}(\vec{t} - \vec{u})$$

$$= \mathbf{\frac{1}{2}\vec{t} + \frac{3}{2}\vec{u}}$$



(a) On the grid, draw the enlargement of the triangle T, centre (0, 0), scale factor $\frac{1}{2}$. [2]

The orange triangle, drawn below (blue lines are construction lines).



(b) The matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a transformation.

(i) Calculate the matrix product $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 8 & 8 & 2 \\ 4 & 8 & 8 \end{pmatrix}$. [2]

$$\begin{pmatrix} -8+0 & -8+0 & -2+0 \\ 0+4 & 0+8 & 0+8 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-8} & \mathbf{-8} & \mathbf{-2} \\ \mathbf{4} & \mathbf{8} & \mathbf{8} \end{pmatrix}$$

(ii) On the grid, draw the image of the triangle T under this transformation. [2]

The green triangle, drawn above.

(iii) Describe fully this **single** transformation. [2]

Reflection in the y-axis (x=0).

(c) Describe fully the **single** transformation which maps

(i) triangle T onto triangle P, [2]

Translation of 10 in the negative x-direction and 10 in the negative y-direction = $\begin{pmatrix} -10 \\ -10 \end{pmatrix}$.

(ii) triangle T onto triangle Q. [3]

Rotation of 90˚ clockwise, centre (0, 0).

(d) Find the 2 by 2 matrix which represents the transformation in **part (c)(ii).** [2]

We require

$$\begin{pmatrix} 8 & 8 & 2 \\ 4 & 8 & 8 \end{pmatrix} \to \begin{pmatrix} 4 & 8 & 8 \\ -8 & -8 & -2 \end{pmatrix}$$

i.e.

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y \\ -x \end{pmatrix}$$

The matrix that does this is

$$\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$$

(a) $\mathbf{p} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

(i) Find, as a single column vector, $\mathbf{p} + 2\mathbf{q}$. [2]

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 + 12 \\ 2 + 6 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{15} \\ \mathbf{8} \end{pmatrix}$$

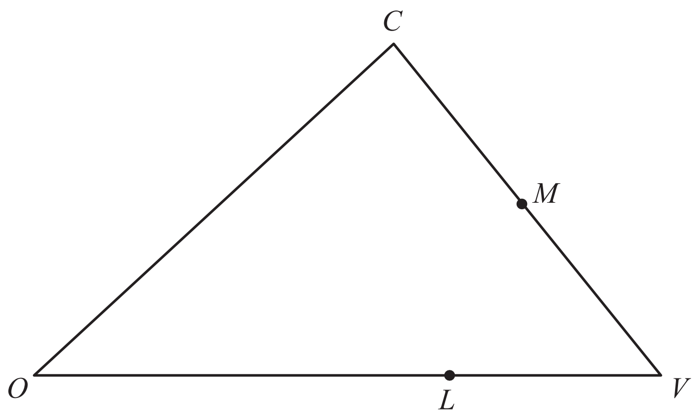
(ii) Calculate the value of $|\,\mathbf{p} + 2\mathbf{q}\,|$. [2]

$$\sqrt{15^2 + 8^2}$$

$$= \sqrt{289}$$

$$= \mathbf{17}$$

(b)



NOT TO SCALE

In the diagram, $CM = MV$ and $OL = 2LV$.
O is the origin. $\overrightarrow{OC}$ = c and $\overrightarrow{OV}$ = v.

Find, in terms of c and v, in their simplest forms

(i) $\overrightarrow{CM}$, [2]

$$\overrightarrow{CM} = \frac{1}{2}\overrightarrow{CV}$$

$$= \frac{1}{2}\left(\overrightarrow{CO} + \overrightarrow{OV}\right)$$

$$= \frac{\mathbf{1}}{\mathbf{2}}(-\vec{\boldsymbol{c}} + \vec{\boldsymbol{v}})$$

(ii) the position vector of M, [2]

$$\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CM}$$

$$= \vec{c} + \frac{1}{2}(-\vec{c} + \vec{v})$$

$$= \frac{\mathbf{1}}{\mathbf{2}}(\vec{\boldsymbol{c}} + \vec{\boldsymbol{v}})$$

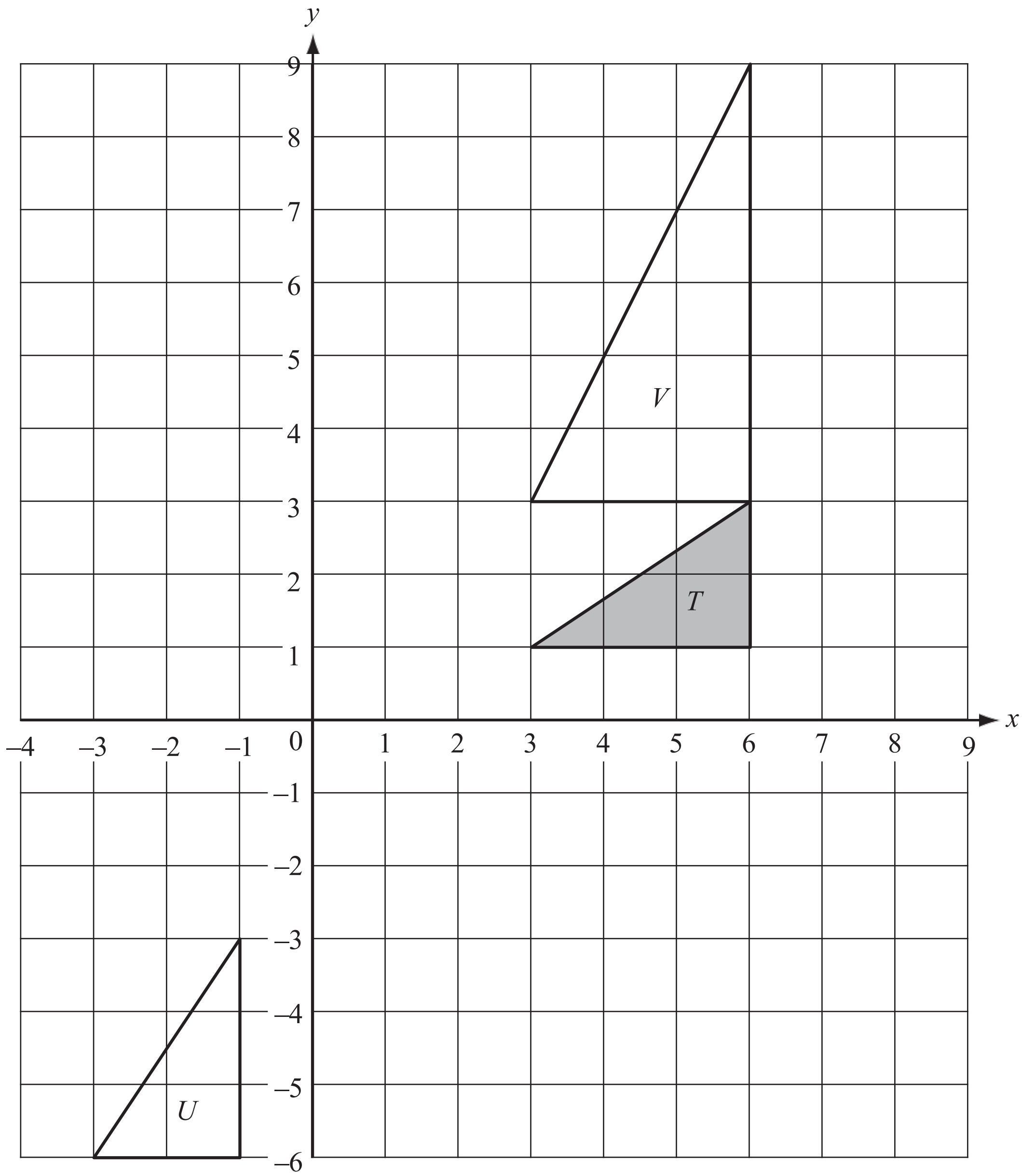
(iii) $\overrightarrow{ML}$. [2]

$$\overrightarrow{ML} = \overrightarrow{MC} + \overrightarrow{CO} + \overrightarrow{OL}$$

$$= -\overrightarrow{CM} - \vec{c} + \frac{2}{3}\vec{v}$$

$$= \frac{1}{2}(\vec{c} - \vec{v}) - \vec{c} + \frac{2}{3}\vec{v}$$

$$= -\frac{\mathbf{1}}{\mathbf{2}}\vec{\boldsymbol{c}} + \frac{\mathbf{1}}{\mathbf{6}}\vec{\boldsymbol{v}}$$



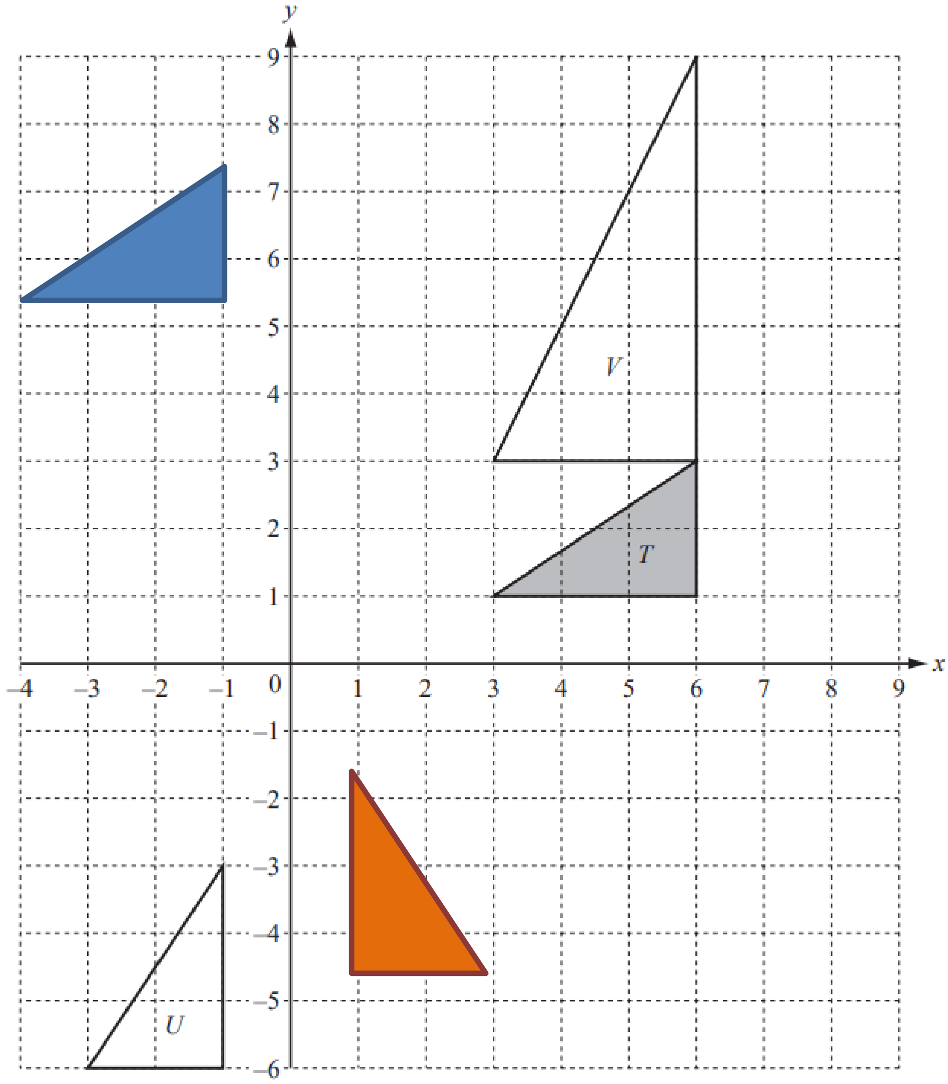
(a) On the grid, draw

(i) the translation of triangle T by the vector $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$, [2]

The blue triangle drawn below

(ii) the rotation of triangle T about (0, 0), through 90° clockwise. [2]

The orange triangle drawn below



(b) Describe fully the **single** transformation that maps

(i) triangle T onto triangle U, [2]

Reflection through the line $y = -x$

(ii) triangle T onto triangle V. [3]

Stretch of factor 3 in the y-direction (x-axis invariant).

(c) Find the 2 by 2 matrix which represents the transformation that maps

(i) triangle T onto triangle U, [2]

We require

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ -x \end{pmatrix}$$

So, we need the matrix

$$\begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$$

(ii) triangle T onto triangle V, [2]

We require

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ 3y \end{pmatrix}$$

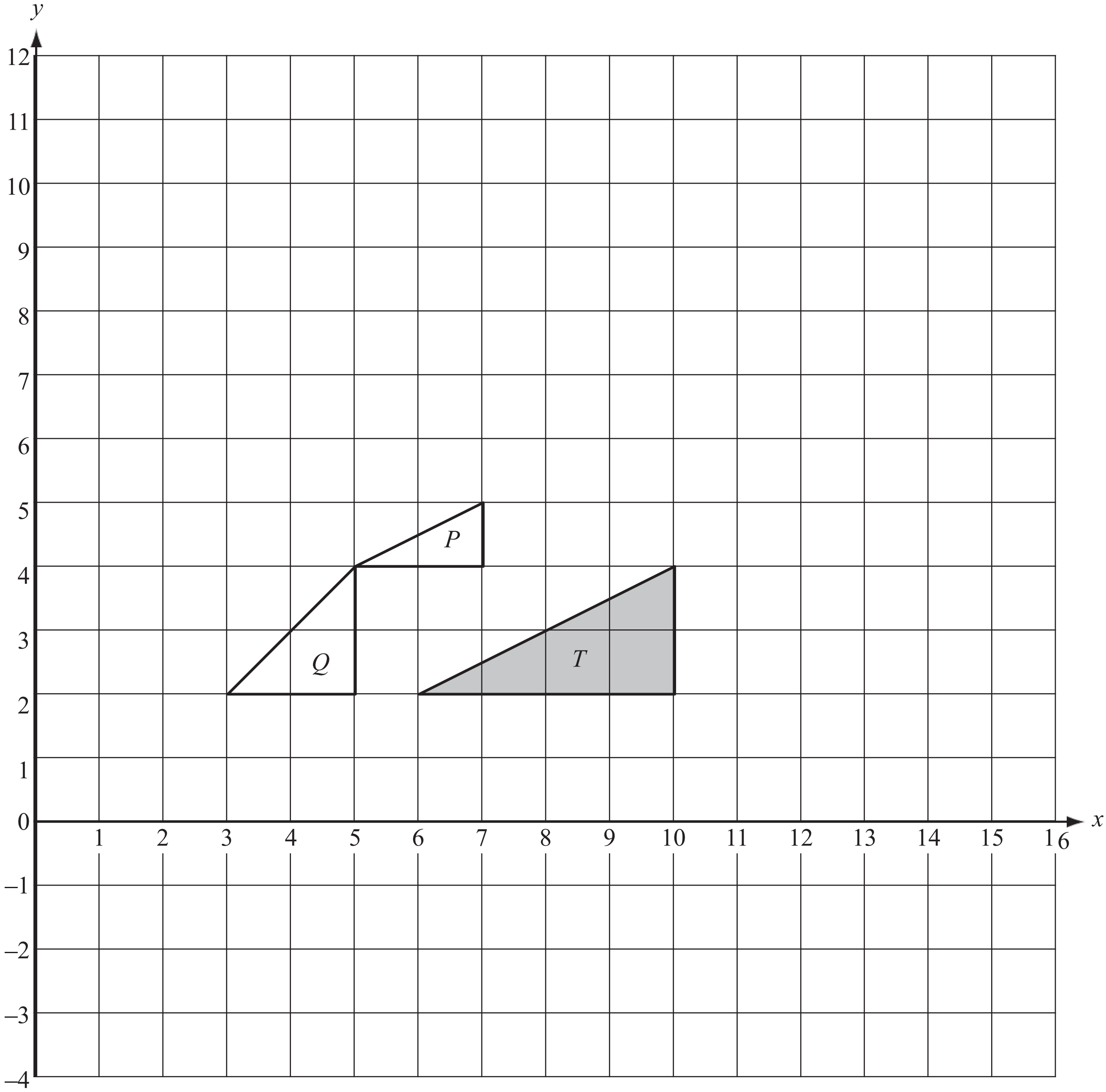
So, we need the matrix

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{3} \end{pmatrix}$$

(iii) triangle V onto triangle T. [1]

Need the inverse of the previous matrix, which is just

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{\frac{1}{3}} \end{pmatrix}$$

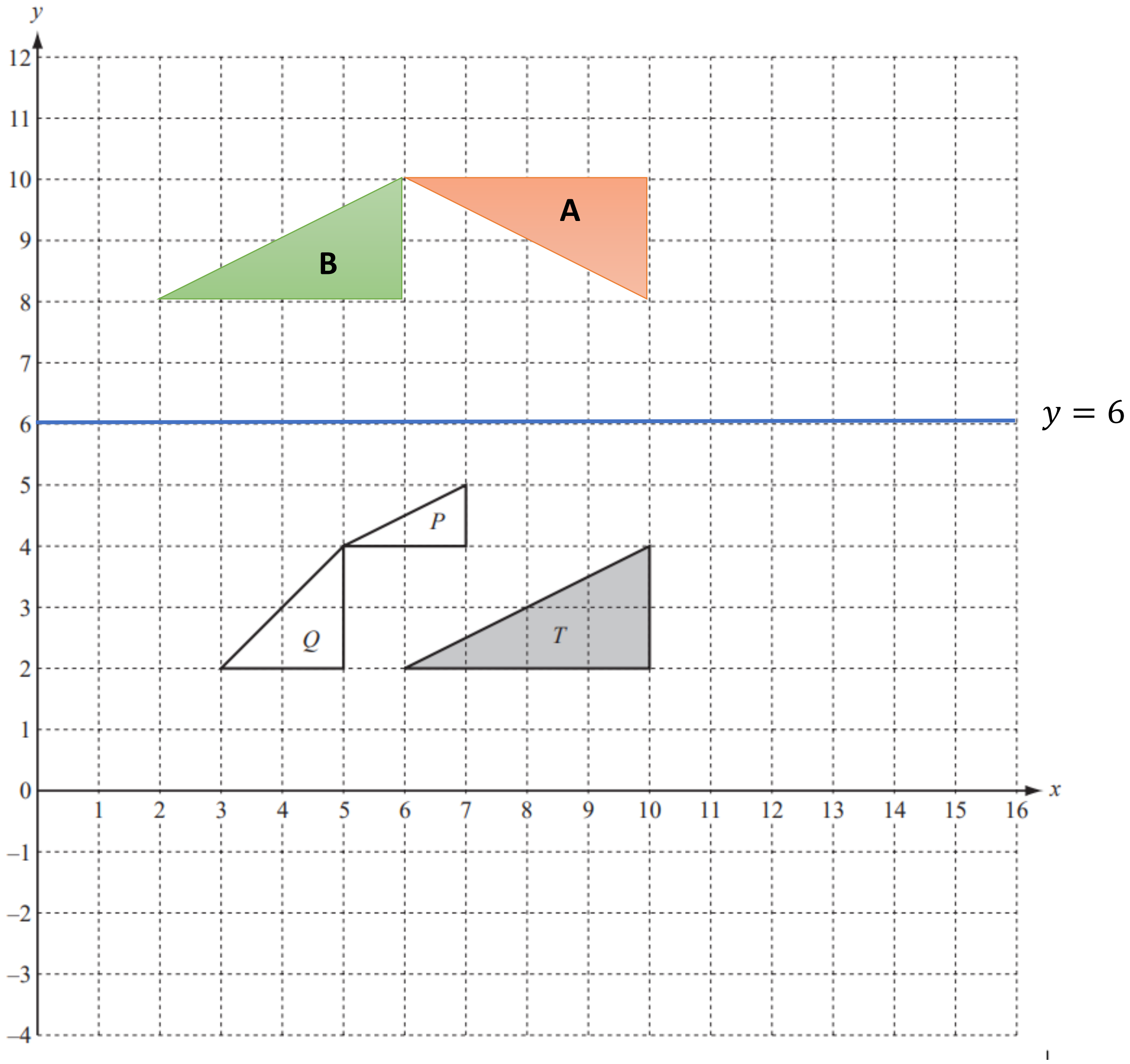


(a) Draw the reflection of triangle T in the line $y = 6$.

Label the image A. [2]

(b) Draw the translation of triangle T by the vector $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$.

Label the image B. [2]



(c) Describe fully the **single** transformation which maps triangle B onto triangle T. [2]

Translation by vector $\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

(d) (i) Describe fully the **single** transformation which maps triangle T onto triangle P. [3]

Enlargement of scale factor 0.5 with centre (4, 6).

(ii) Complete the following statement. [1]

Area of triangle P = 0.25 × Area of triangle T

(e) (i) Describe fully the **single** transformation which maps triangle T onto triangle Q. [3]

Stretch, y-axis invariant, of factor 0.5

(ii) Find the 2 by 2 matrix which represents the transformation mapping triangle T onto triangle Q.

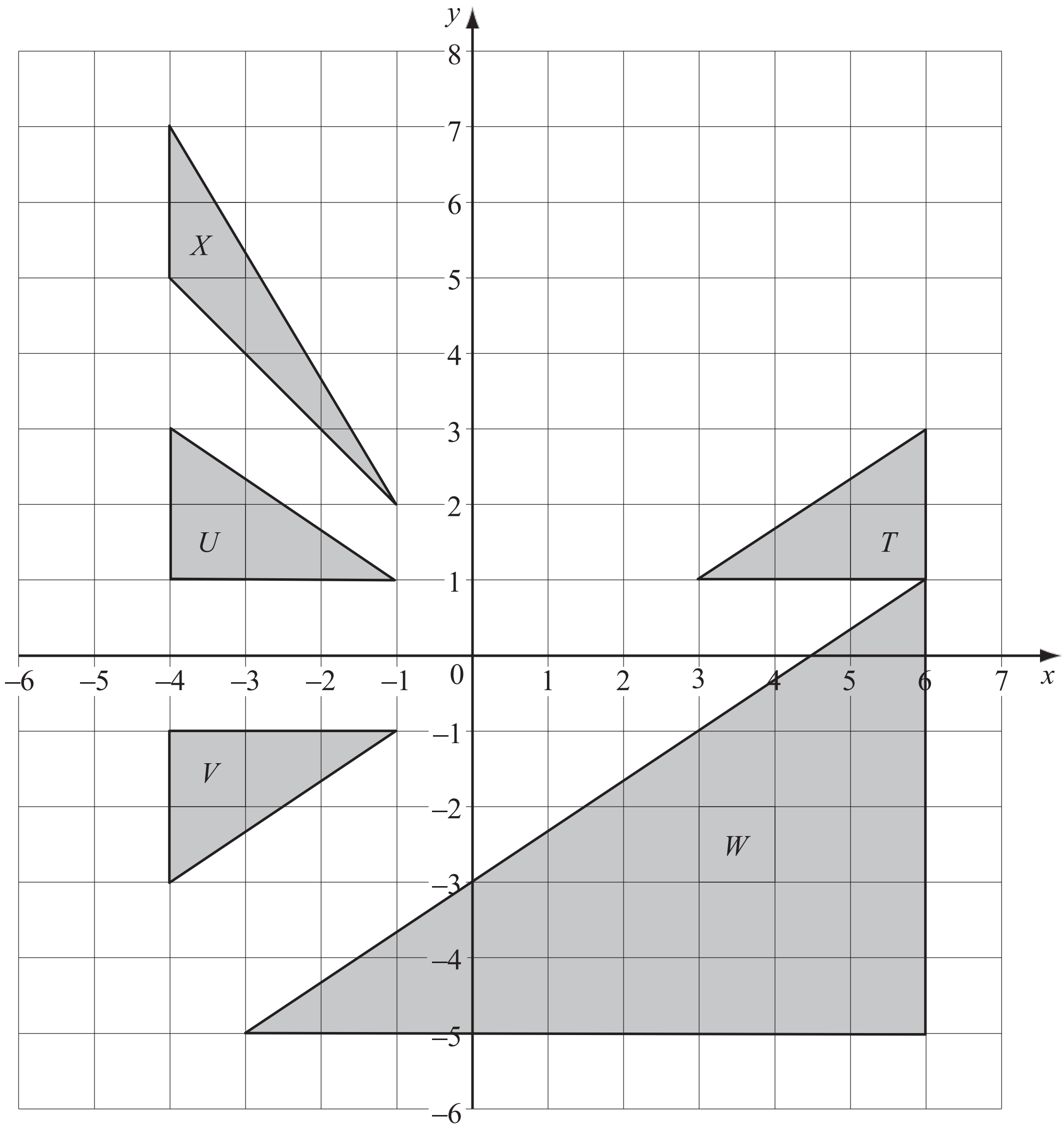
[2]

We require that

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} \frac{1}{2}x \\ y \end{pmatrix}$$

This is achieved with the matrix

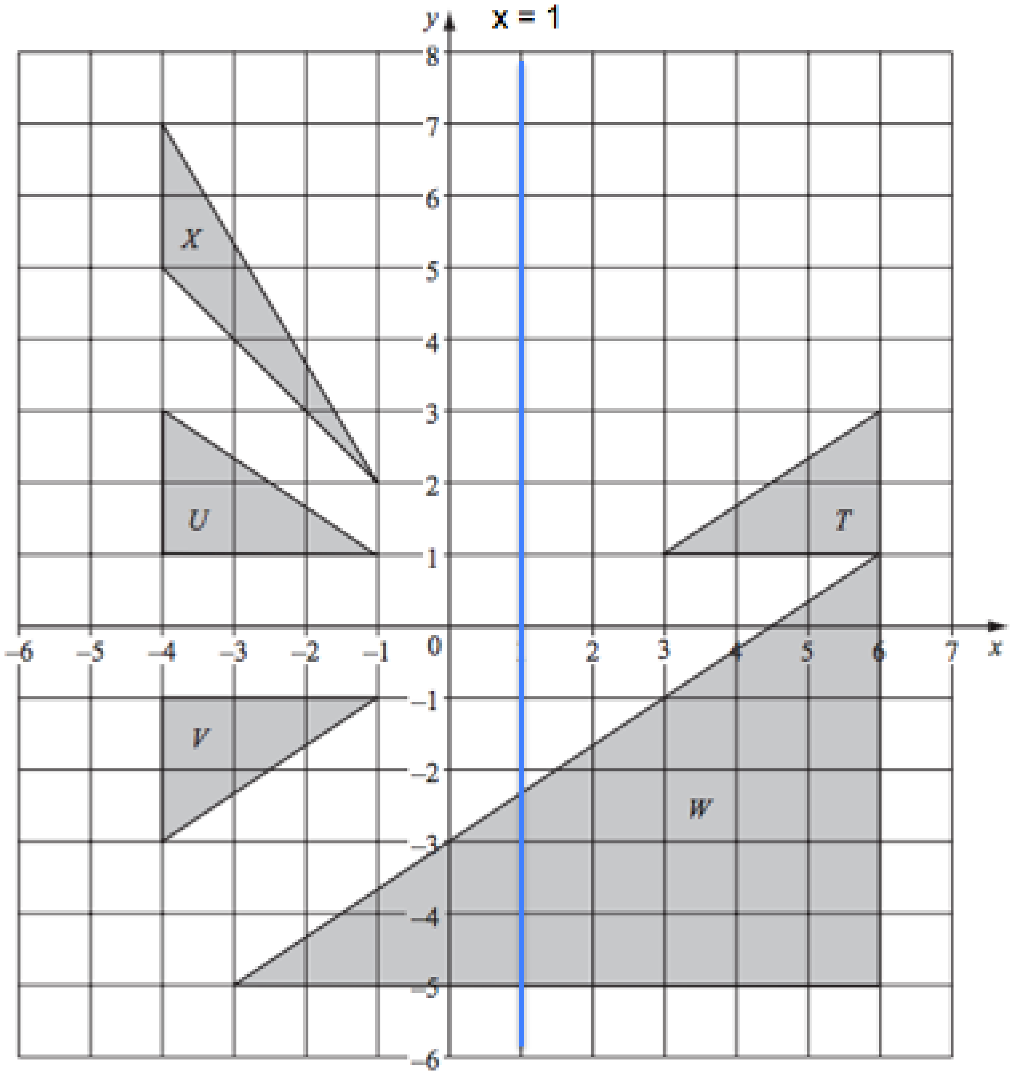
$$\begin{pmatrix} \mathbf{0.5} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$


y
8
7
6
5
4
3
2
1
0
–1
–2
–3
–4
–5
–6
x
–6
–5
–4
–3
–2
–1
1
2
3
4
5
6
7
X
U
V
T
W

(a) Describe fully the **single** transformation which maps

(i) triangle *T* onto triangle *U*, [2]

Reflection in x = 1



(ii) triangle *T* onto triangle *V*, [3]

Triangles T and V are equal distances from the point 1,0

Rotate 180 degrees around point (1,0)

(iii) triangle T onto triangle W, [3]

Enlargement

The length of the opposite side of triangle T = 2, length of opposite side of triangle W = 6

6/2 = 3 (scale factor)

To find centre, draw lines through corresponding corners in each shape, where lines cross is the centre of transformation, which in this case is **(6,4).**

(iv) triangle U onto triangle X. [3]

Shear

Draw lines through hypotenuses and adjacent sides on both triangles and mark where the corresponding lines meet. Both points are on the y axis meaning the y axis is the invariant line.

Shear factor = distance of point displacement from transformation/distance from invariant line to point

e.g. point -4,3 on triangle U is displaced to point -4,7 on triangle X after transformation. The distance from point -4,3 to the y axis is -4

Shear factor = (7 – 3)/ -4

Shear scale factor = -1

(b) Find the matrix representing the transformation which maps

(i) triangle U onto triangle V, [2]

reflection in x axis, so y coordinate changes sign

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix}$$

(ii) triangle U onto triangle X. [2]

Replace y coordinate with shear scale factor

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{1} \end{pmatrix}$$

Vectors
Difficulty: Medium

Model Answers 5

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 5

Time allowed: **85 minutes**

Score: **/74**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

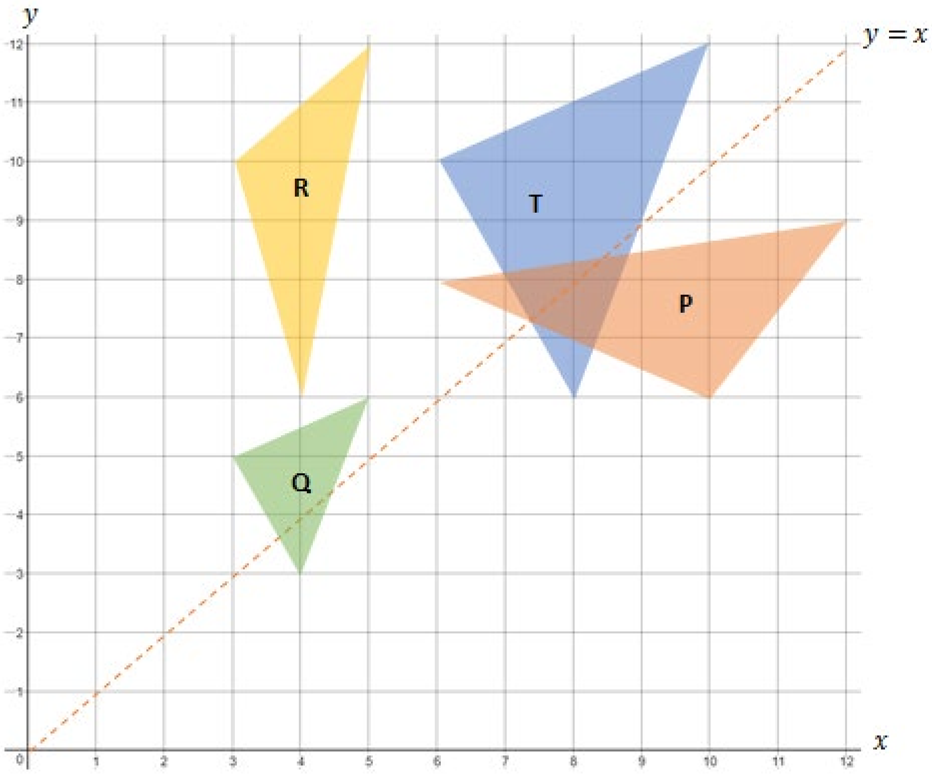
A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Answer the whole of this question on a sheet of graph paper.

(a) Draw x and y axes from 0 to 12 using a scale of 1 cm to 1 unit on each axis. [1]



(b) Draw and label triangle T with vertices (8, 6), (6, 10) and (10, 12). [1]

Blue triangle above.

(c) Triangle T is reflected in the line $y = x$.

(i) Draw the image of triangle T. Label this image P. [2]

Orange triangle above.

(ii) Write down the matrix which represents this reflection. [2]

We require

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y \\ x \end{pmatrix}$$

This is done by the matrix

$$\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$$

(d) A transformation is represented by the matrix $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

(i) Draw the image of triangle T under this transformation. Label this image Q. [2]

Green triangle above

(ii) Describe fully this single transformation. [3]

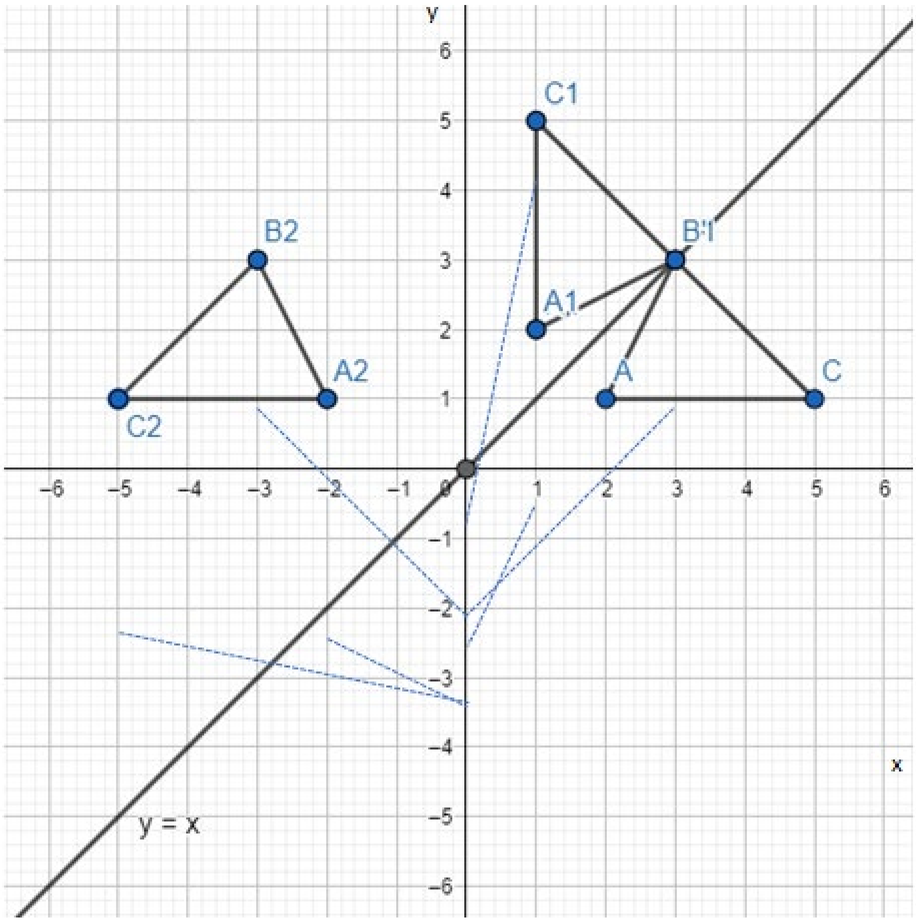
Enlargement, scale factor ½, centre (0, 0)

(e) Triangle T is stretched with the y-axis invariant and a stretch factor of $\frac{1}{2}$.

Draw the image of triangle T. Label this image R. [2]

Yellow triangle above.

(a) Draw and label x and y axes from -6 to 6, using a scale of 1 cm to 1 unit. [1]



(b) Draw triangle ABC with A (2,1), B (3,3) and C (5,1). [1]

(c) Draw the reflection of triangle ABC in the line $y = x$. Label this $A_1B_1C_1$. [2]

We first draw the line of equation y = x.

A reflection will have each point of the object and the corresponding point in the image equidistant form the line y = x. The distance from a point in the object to its corresponding point in the image will also be perpendicular on the line y = x.

In this case, B(3, 3) is a point on the line y = x, therefore, B = B1.

(d) Rotate **triangle** $A_1B_1C_1$ about (0,0) through 90° anti-clockwise. Label this $A_2B_2C_2$. [2]

Anti-clockwise represents a rotation towards left.

There is a 90° angle between the distance of one point from the centre of rotation and the distance of the corresponding point in the image from the centre of rotation.

(e) Describe fully the single transformation which maps triangle ABC onto triangle $A_2B_2C_2$. [2]

By looking at the image above we can see that the image A2B2C2 is the reflection of triangle ABC across the y-axis. Each point in the triangle ABC and its corresponding point in the image A2B2C2 are equidistant from the y-axis and the distance from one point to its corresponding image is perpendicular on the y-axis.

(f) A transformation is represented by the matrix $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$.

(i) Draw the image of triangle ABC under this transformation. Label this $A_3B_3C_3$. [3]

We represent the triangle ABC through the matrix:

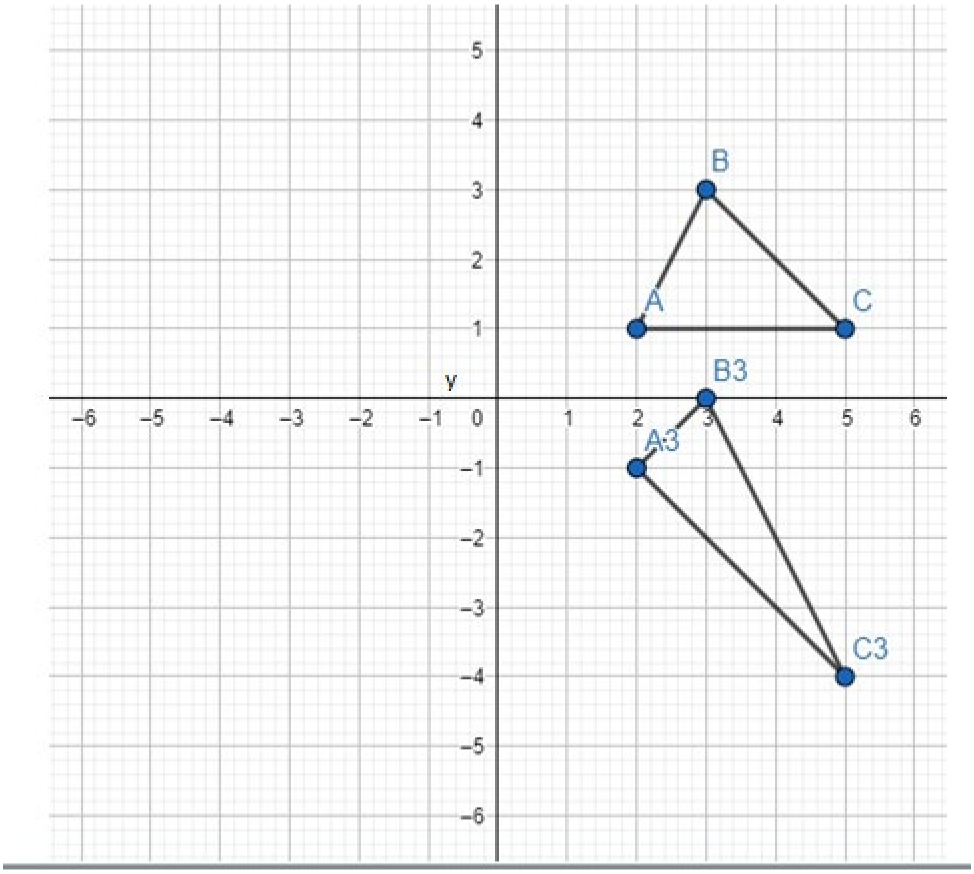
$$\begin{pmatrix} 2 & 3 & 5 \\ 1 & 3 & 1 \end{pmatrix}$$

We multiply this matrix by the transformation matrix to obtain the coordinates of the image.

$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 & 5 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1\,x\,2+0\,x\,1 & 1\,x\,3+0\,x\,3 & 1\,x\,5+0\,x\,1 \\ -1\,x\,2+1\,x\,1 & -1\,x\,3+1\,x\,3 & -1\,x\,5+1\,x\,1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 & 5 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 5 \\ -1 & 0 & -4 \end{pmatrix}$$

The image has the coordinates: A3(2, -1) B3 (3, 0), C3 (5, -4).



(ii) Describe fully the single transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$. [2]

The single transformation described by the matrix is a shear transformation. The transformation is a vertical shear, y-variant. The horizontal lines become tilted by the shear angle to become lines with slope m, where m is the shear factor.

(iii) Find the matrix which represents the transformation that maps triangle $A_3B_3C_3$ onto triangle ABC. [2]

The matrix is: $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} \end{pmatrix}$ since this matrix multiplied by the A3B3C3 triangle matrix will give the matrix containing the coordinates of triangle ABC.

Transformation T is translation by the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Transformation M is reflection in the line $y = x$.

(a) The point A has co-ordinates (2, 1).

Find the co-ordinates of

(i) T(A), [1]

Point A of coordinates (2, 1) can be written as: $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

We sum up this vector with the translation column vector to obtain the coordinates of the image.

$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

T(A) = (5, 3)

(ii) MT(A). [2]

The reflection in the line y = x of the point of coordinates (5, 3) gives the image of coordinates **(3, 5).**

(b) Find the 2 by 2 matrix **M**, which represents the transformation M. [2]

The reflection matrix in the line of equation y = x is: $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$

(c) Show that, for any value of k, the point Q $(k-2, k-3)$ maps onto a point on the line $y = x$ following the transformation TM(Q). [3]

Point Q of coordinates (k – 2 , k – 3) can be written as: $\begin{pmatrix} k-2 \\ k-3 \end{pmatrix}$

To reflect this point in the line y = x we multiply its matrix by the reflection matrix from b).

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} k-2 \\ k-3 \end{pmatrix} = \begin{pmatrix} 0\,x\,(k-2) + 1\,x\,(k-3) \\ 1\,x\,(k-2) + 0\,x\,(k-3) \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} k-2 \\ k-3 \end{pmatrix} = \begin{pmatrix} k-3 \\ k-2 \end{pmatrix}$$

We sum up this vector with the translation column vector to obtain the coordinates of the image.

$$\begin{pmatrix} k-3 \\ k-2 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} k \\ k \end{pmatrix}$$

TM(Q) = (k, k)

y = x = k so this point is on the line y = x.

(d) Find $\mathbf{M}^{-1}$, the inverse of the matrix $\mathbf{M}$. [2]

The inverse of the M matrix is M = $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$

(e) **N** is the matrix such that $N + \begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 0 & 0 \end{pmatrix}$.

(i) Write down the matrix **N.** [2]

$N + \begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 0 & 0 \end{pmatrix}$

We represent the matrix with the unknowns: $N = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

a + 0 = 0 -> a = 0

b + 3 = 4 -> b = 1

c + 1 = 0 -> c = -1

d + 0 = 0 -> d = 0.

$N = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

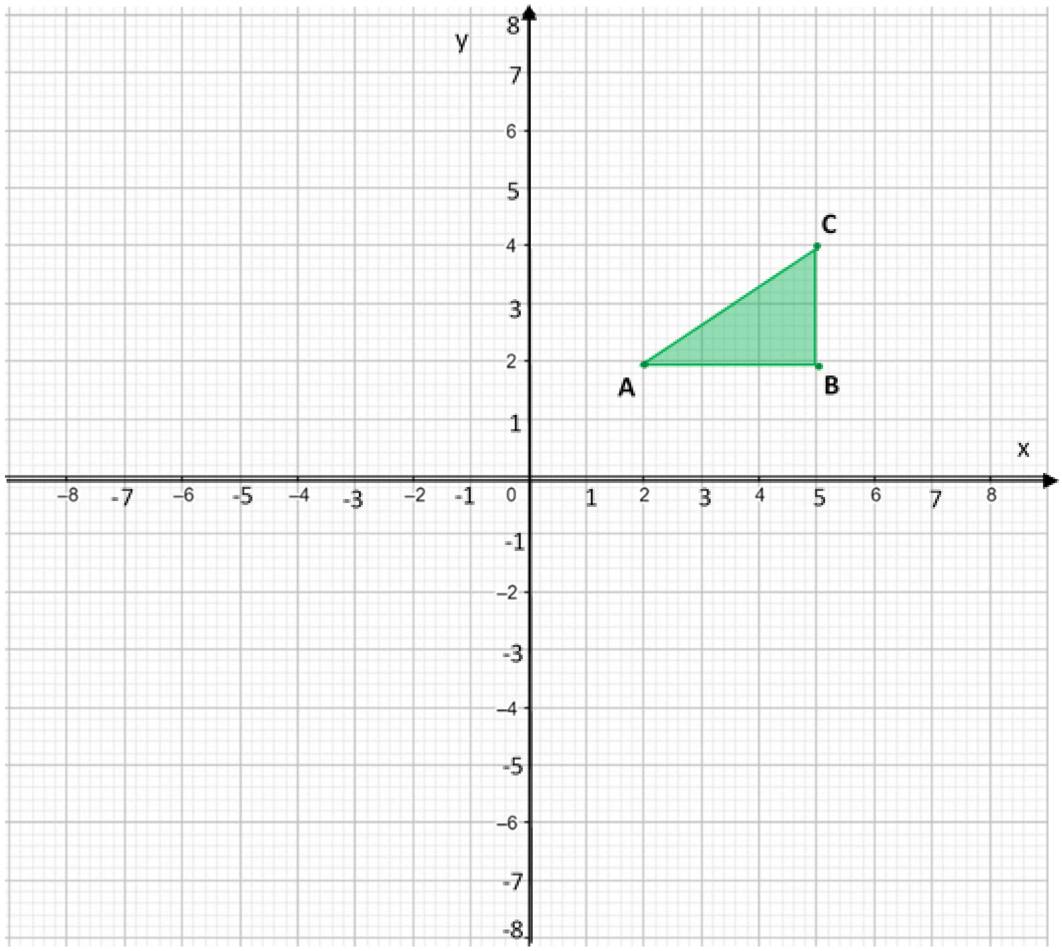
(ii) Describe completely the **single** transformation represented by N. [3]

N represents a 90° clockwise rotation around the centre (0, 0).

Answer the whole of this question on one sheet of graph paper.

(a) Draw and label x and y axes from –8 to +8, using a scale of 1 cm to 1 unit on each axis. [1]

(b) Draw and label triangle ABC with A (2, 2), B (5, 2) and C (5, 4). [1]



(c) On your grid:

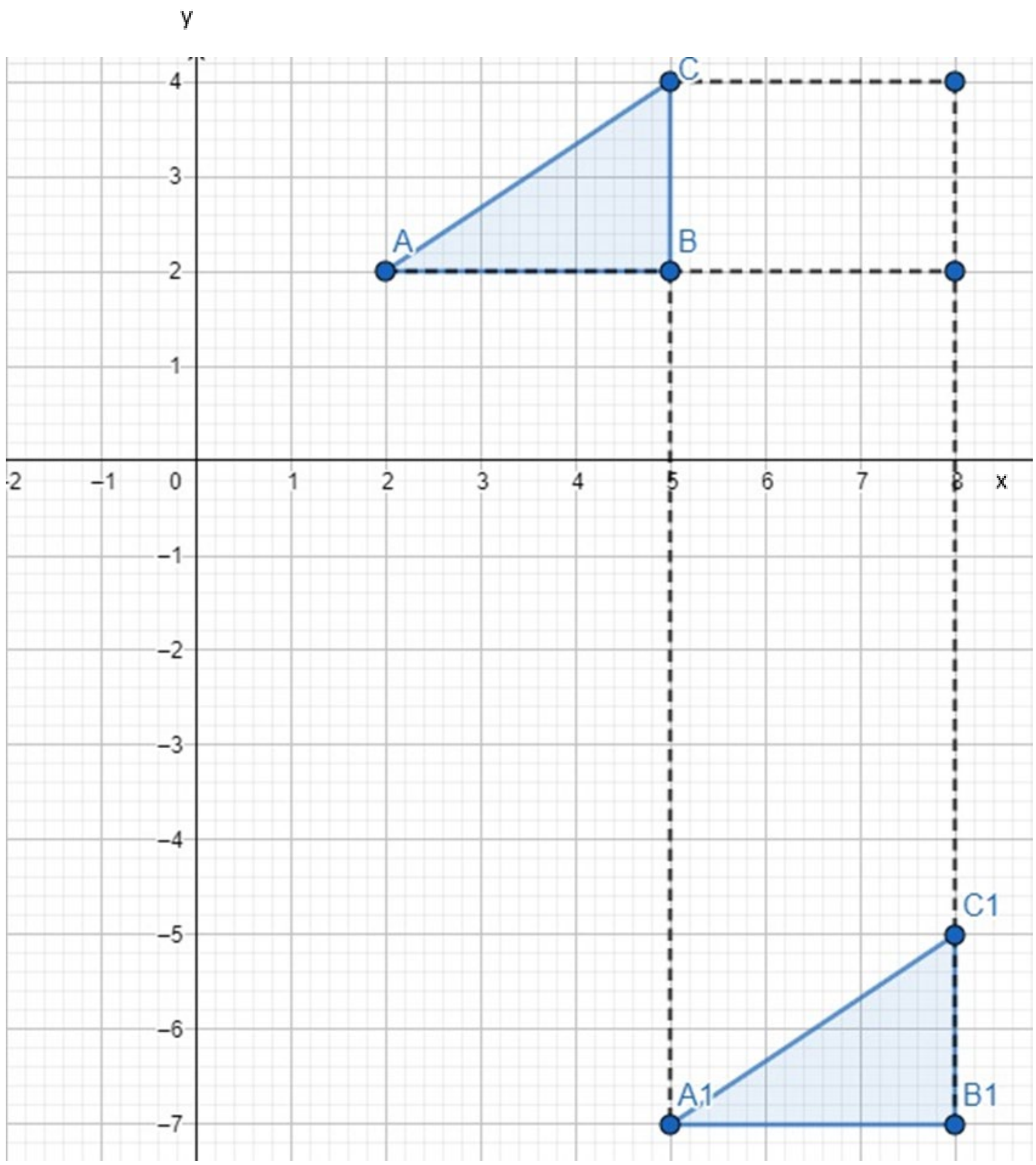
(i) translate **triangle** $\boldsymbol{ABC}$ by the vector $\begin{pmatrix} 3 \\ -9 \end{pmatrix}$ and label this image $A_1B_1C_1$; [2]

Every point in the triangle ABC is translated the same distance and in the same direction onto triangle A1B1C1.

We use positive numbers to describe a translation up or the right and negative numbers to describe a translation down or to the left.

In a column vector, the upper value represents a movement to the left or right while the lower value represents a movement either up or down.

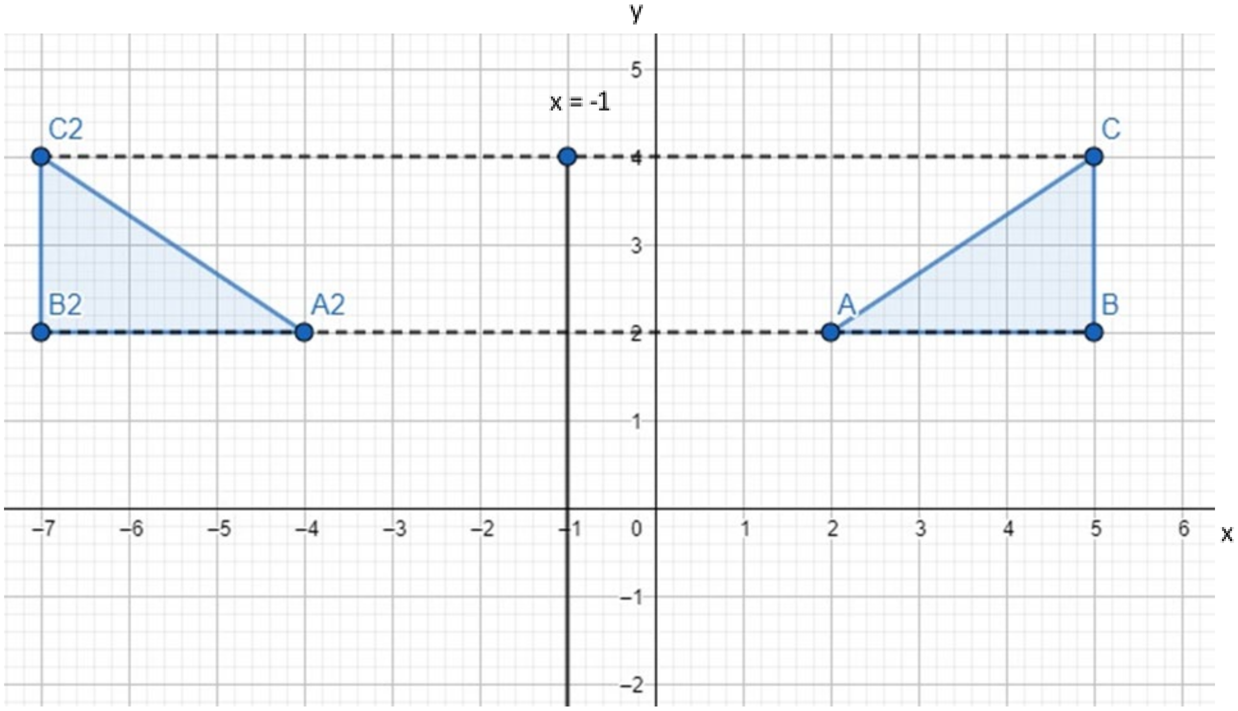
In our case, triangle ABC is translated by 3 units to the right and by nine units down.



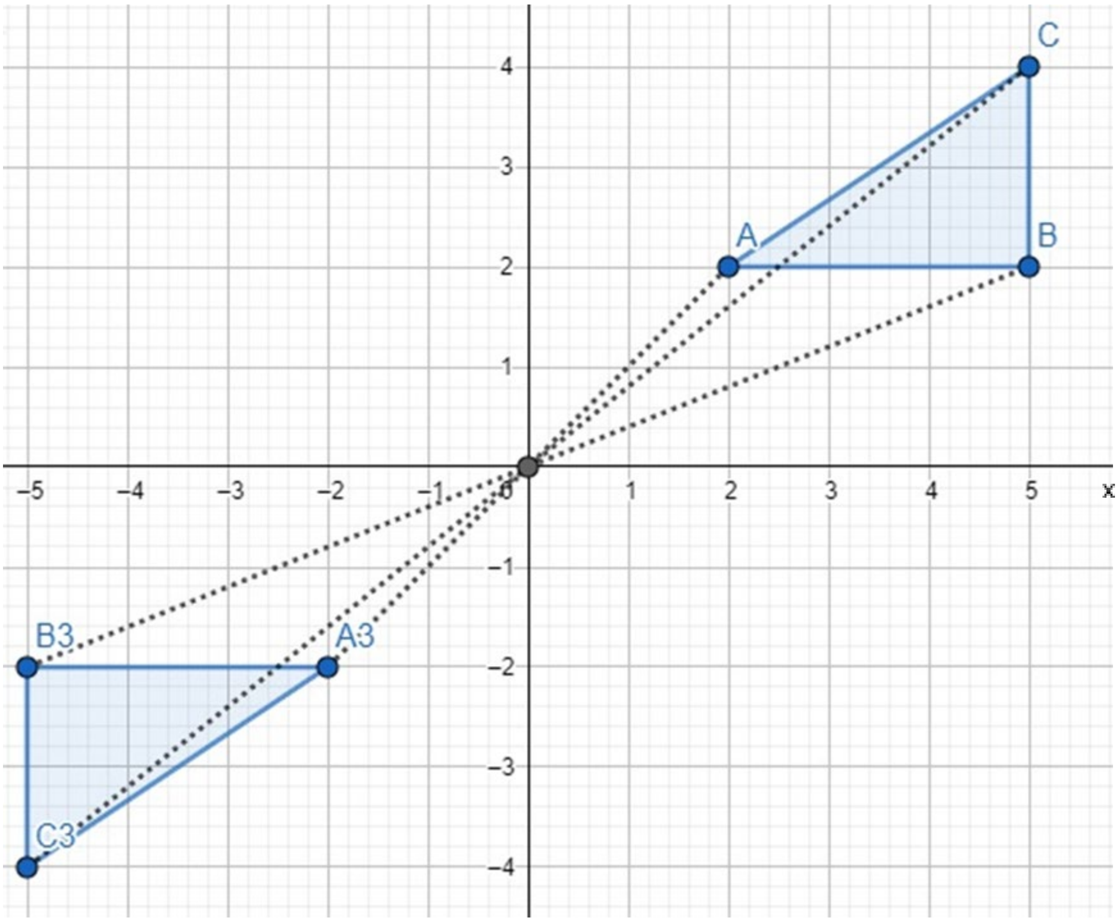
(ii) reflect **triangle** $\boldsymbol{ABC}$ in the line $x = -1$ and label this image $A_2B_2C_2$; [2]

In our case, we define the reflection across the mirror line of equation x = -1

Every point in triangle ABC is at the same distance from the mirror line as every corresponding point in triangle A2B2C2. Also, these distances are perpendicular on the mirror line.



(iii) rotate **triangle *ABC*** by 180° about (0, 0) and label this image $A_3B_3C_3$. [2]



A rotation turns the shape around a fixed point, in our case around the origin (0, 0).

The rotation is clockwise and by 180. Therefore, all the points in triangle ABC are in a straight line with the corresponding point in the image.

(d) A stretch is represented by the matrix $\begin{pmatrix} 1.5 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Draw the image of **triangle *ABC*** under this transformation. Label this image $A_4B_4C_4$. [3]

A stretch is a transformation which enlarges all sides in the same direction by a constant factor.

A matrix in the form:

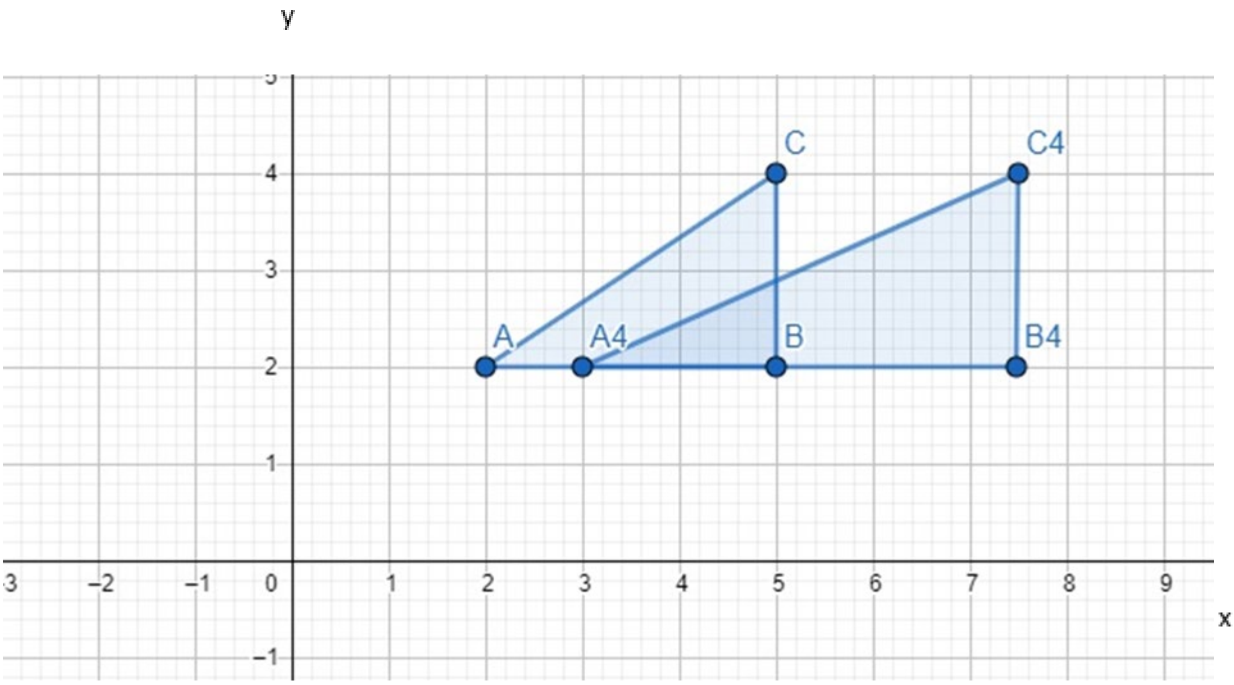
$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ represents a stretch along the x-axis with the constant factor k.

A matrix in the form:

$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$ represents a stretch along the y-axis with the constant factor k.

In our case, the matrix means that the x coordinate of each point will be equal to the x coordinate of the corresponding point in the image multiplied by 1.5.

The y coordinates of each point remain the same.



(ii) Work out the inverse of the matrix $\begin{pmatrix} 1.5 & 0 \\ 0 & 1 \end{pmatrix}$. [2]

To work out the inverse of a 2x2 matrix we use:

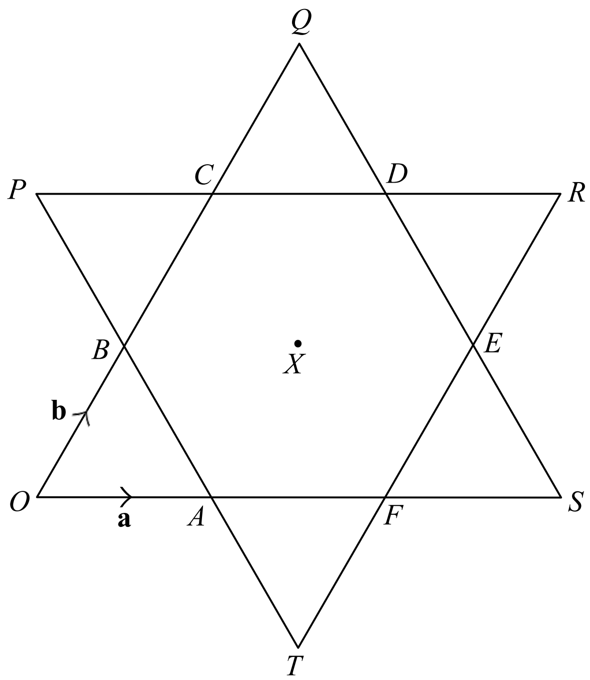
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

In our case:

$$\begin{pmatrix} 1.5 & 0 \\ 0 & 1 \end{pmatrix}^{-1} = \frac{1}{1.5}\begin{pmatrix} 1 & 0 \\ 0 & 1.5 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 1 & 0 \\ 0 & 1.5 \end{pmatrix}$$

(iii) Describe **fully** the single transformation represented by this inverse. [3]

Using the inverse matrix, we can obtain a stretch of triangle ABC since y will be invariant in this case and the constant factor is $\frac{2}{3}$.



A star is made up of a regular hexagon, centre X, surrounded by 6 equilateral triangles.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Write the following vectors in terms of **a** and/or **b**, giving your answers in their simplest form.

(i) $\overrightarrow{OS}$, [1]

The regular hexagon has all 6 sides equal.

The triangles are equilateral, therefore, having all 3 sides equal as well.

We can deduce that:

$\overrightarrow{OA} = \overrightarrow{AF} = \overrightarrow{FS} = \text{a}$

$\boldsymbol{\overrightarrow{OS}} = \mathbf{3a}$

(ii) $\overrightarrow{AB}$, [1]

$\overrightarrow{OA} - \overrightarrow{OB} = \overrightarrow{BA}$

$\overrightarrow{BA} = -\overrightarrow{AB}$

$\boldsymbol{\overrightarrow{AB}} = \boldsymbol{\overrightarrow{OB}} - \boldsymbol{\overrightarrow{OA}} = \mathbf{b - a}$

(iii) $\overrightarrow{CD}$, [1]

Similar to i), $\overrightarrow{OA} = \overrightarrow{AF} = \overrightarrow{CD}$ = a

$\overrightarrow{CD}$ = a

(iv) $\overrightarrow{OR}$, [2]

$\overrightarrow{OR} = \overrightarrow{OF} + \overrightarrow{OC}$

$\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{AF}$

$\overrightarrow{OA} = \overrightarrow{AF}$ = a

$\overrightarrow{OF}$ = 2a

$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$

$\overrightarrow{OB} = \overrightarrow{BC}$ = b

$\overrightarrow{OC}$ = 2b

$\overrightarrow{OR} = \overrightarrow{OF} + \overrightarrow{OC}$

$\overrightarrow{OR}$ = 2a + 2b

(v) $\overrightarrow{CF}$. [2]

$\overrightarrow{CF} = \overrightarrow{OF} - \overrightarrow{OC}$

$\overrightarrow{CF}$ = 2a – 2b

(b) When $|\mathbf{a}| = 5$, write down the value of

(i) $|\mathbf{b}|$, [1]

The triangles are equilateral, so:

$|\boldsymbol{a}| = |\boldsymbol{b}| = 5$

(ii) $|\mathbf{a} - \mathbf{b}|$. [1]

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \text{b} - \text{a}$

Therefore:

$\overrightarrow{BA} = \text{a} - \text{b}$

$|\boldsymbol{a} - \boldsymbol{b}| = |\overrightarrow{BA}| = 5$

(c) Describe fully a single transformation which maps

(i) triangle OBA onto triangle OQS, [2]

The size of the triangle is changed between the object OBA and the image OQS, therefore, the single transformation is an enlargement. The image is bigger, so the enlargement factor is greater than 1.

Each side of the image will be the scale factor times the corresponding side in the initial image.

In our case, the sides of triangle OQS are 3 times the size of the side in triangle OBA, therefore, the scale factor is 3.

When enlarged, the distance from the centre to each corner will be multiplied by the scale factor to obtain the image, in this case, 3.

The distance from the centre of enlargement to each of the vertices of OQS needs to be 3 times the distance from the centre of enlargement to the corresponding vertex of OBA.

The centre of enlargement will be the point of coordinates (0, 0)

(ii) triangle OBA onto triangle RDE, with O mapped onto R and B mapped onto D. [2]

The 2 triangles are mirror images of each other, therefore, the single transformation is a reflection.

The distance from each vertex of the object to the mirror line needs to equal to the distance from each corresponding vertex of the image to the mirror line. All these distances are also perpendicular on the mirror line.

In our case, this line is CF.

(d) (i) How many lines of symmetry does the star have? [1]

The star has 6 lines of symmetry.

(ii) When triangle OQS is rotated clockwise about X, it lies on triangle PRT, with O on P.
Write down the angle of rotation. [1]

The triangle is rotated by 1 sixth of the total angle around angle X, 360°.

360°/6

= 60°

Vectors
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **69 minutes**

Score: **/60**

Percentage: **/100**

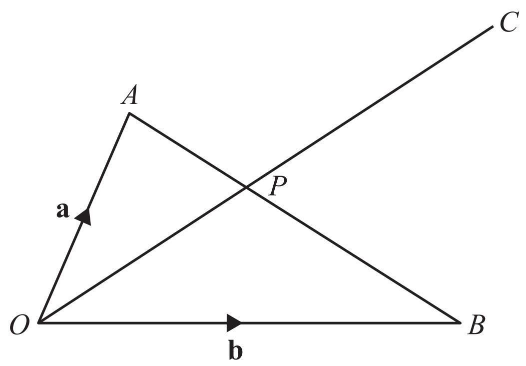
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



NOT TO SCALE

In the diagram, O is the origin and P lies on AB such that $AP : PB = 3 : 4$.
$\overrightarrow{OA} =$ **a** and $\overrightarrow{OB} =$ **b**.

(i) Find $\overrightarrow{OP}$, in terms of **a** and **b**, in its simplest form. [3]

$$\overrightarrow{AB} = \vec{b} - \vec{a}$$

$$\overrightarrow{OP} = \vec{a} + \frac{3}{7}\overrightarrow{AB}$$

$$= \frac{4}{7}\vec{a} + \frac{3}{7}\vec{b}$$

(ii) The line OP is extended to C such that $\overrightarrow{OC} = mOP$ and $BC = k$**a**.

Find the value of m and the value of k. [2]

$$\overrightarrow{BC} = -\vec{b} + \overrightarrow{OC}$$

$$\to k\vec{a} = -\vec{b} + m\left(\frac{4}{7}\vec{a} + \frac{3}{7}\vec{b}\right)$$

Equating coefficients of $\vec{b}$

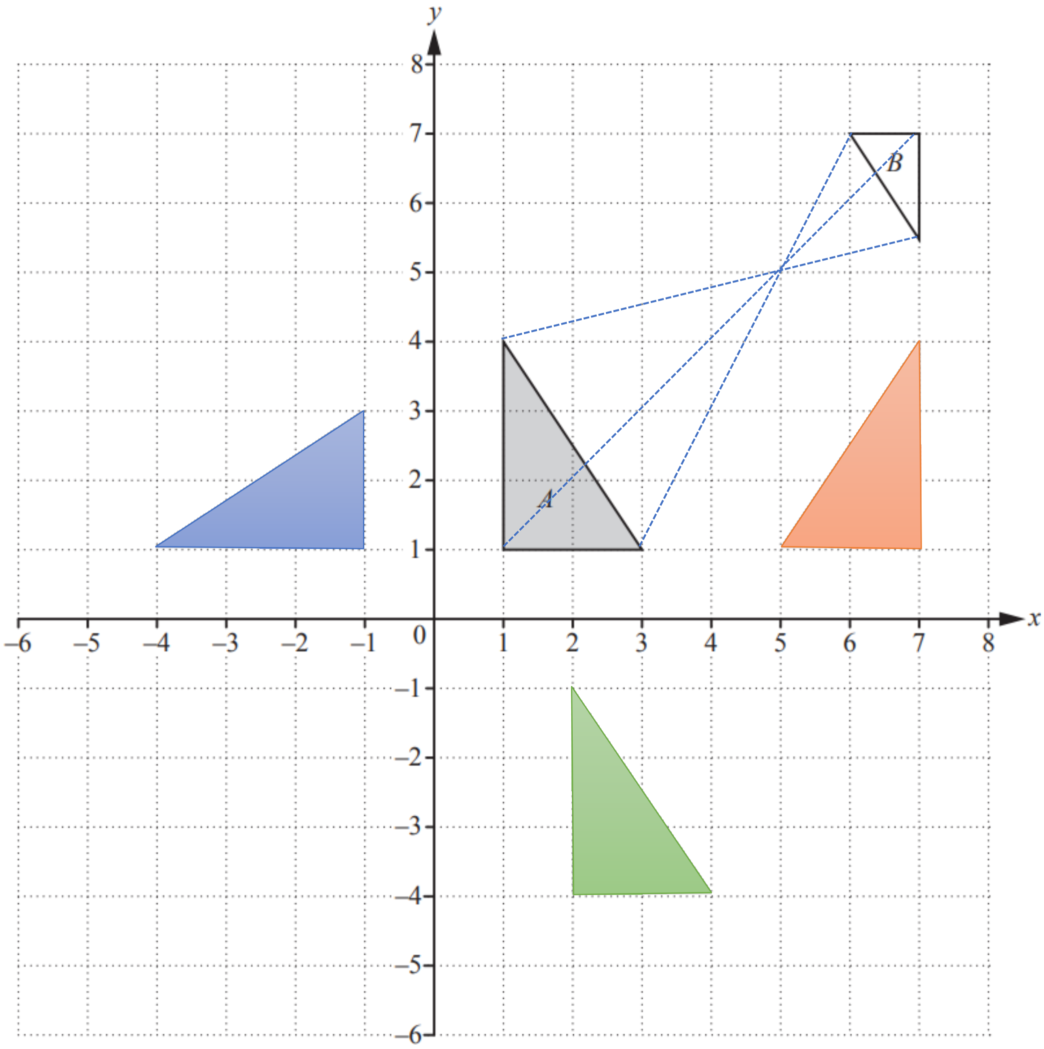
$$\frac{3}{7}m - 1 = 0$$

$$\to m = \frac{7}{3}$$

Equating coefficients of $\vec{a}$

$$\frac{4}{7}m = k$$

$$\to k = \frac{4}{3}$$



(a) (i) Draw the image of triangle A after reflection in the line $x = 4$. [2]

Orange triangle

(ii) Draw the image of triangle A after rotation of 90° anticlockwise about (0, 0). [2]

Blue triangle

(iii) Draw the image of triangle A after translation by the vector $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$. [2]

Green triangle

(b) Describe fully the **single** transformation that maps triangle A onto triangle B. [3]

Construction lines are drawn in dotted blue

Enlargement, scale factor -0.5, centre (5, 5)

(a) Describe fully the **single** transformation that maps shape A onto

(i) shape B,

[2]

Translation 3 units in the x-direction and -13 units in the y-direction $= \begin{pmatrix} 8 \\ -13 \end{pmatrix}$.

(ii) shape C.

[3]

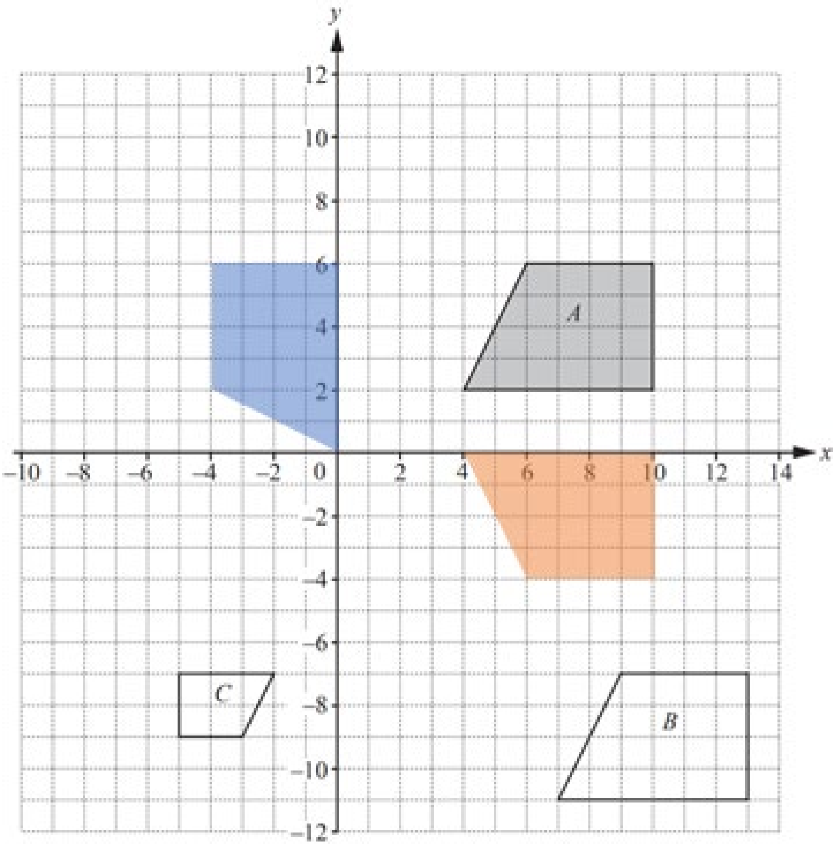
Enlargement, scale factor -1/2, centre (0, -4)

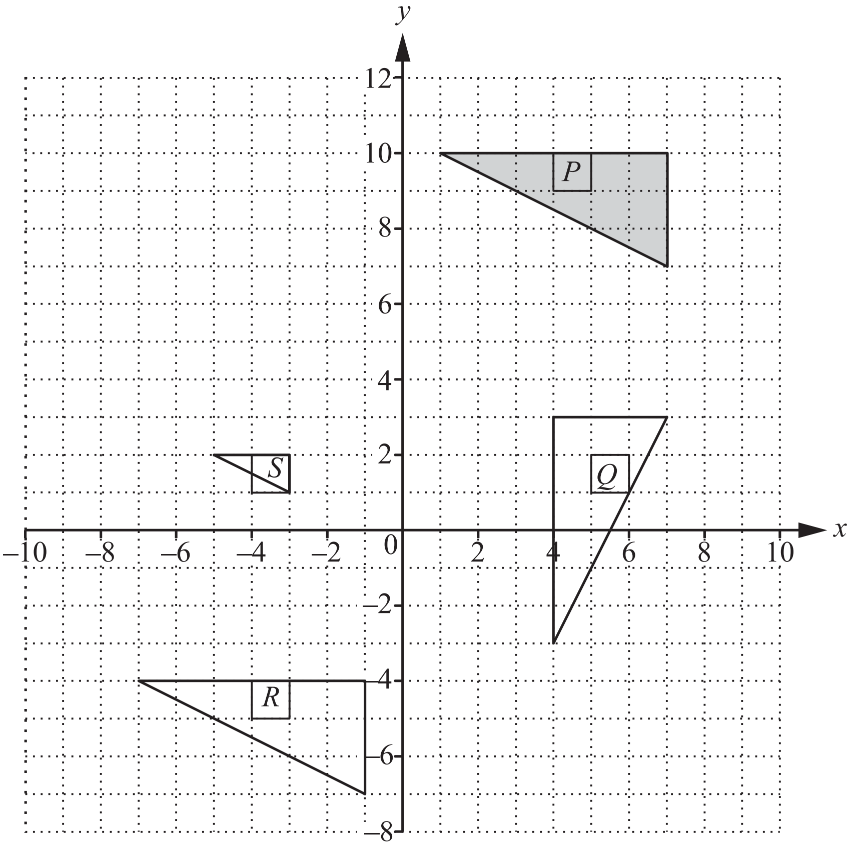
(b) Draw the image of shape A after rotation through 90° anticlockwise about the point (3, - 1). [2]

Drawn in blue below.

(c) Draw the image of shape A after reflection in $y = 1$. [2]

Drawn in orange below

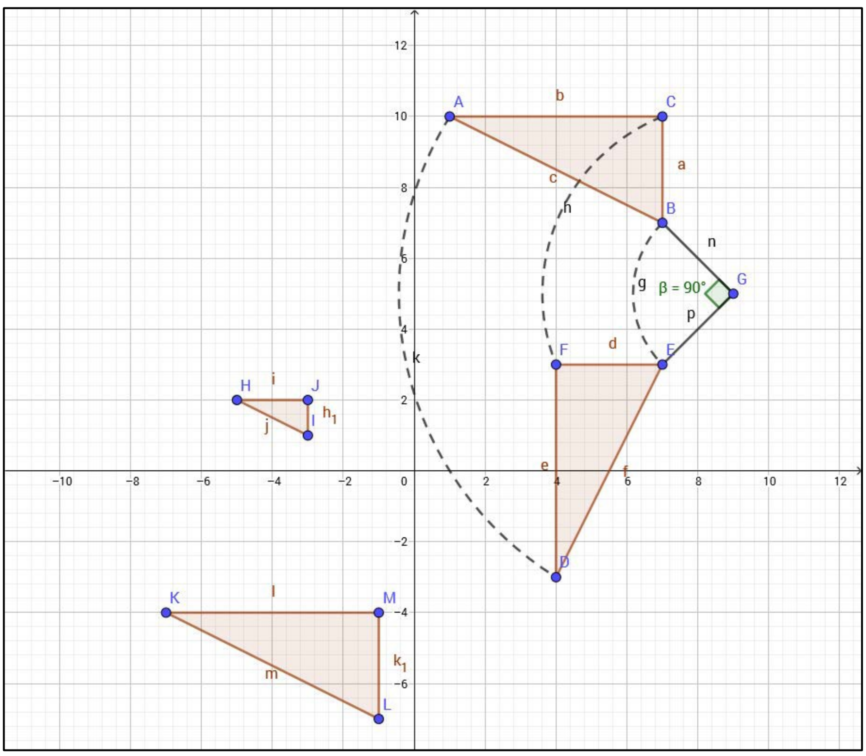




(a) Describe fully the **single** transformation that maps

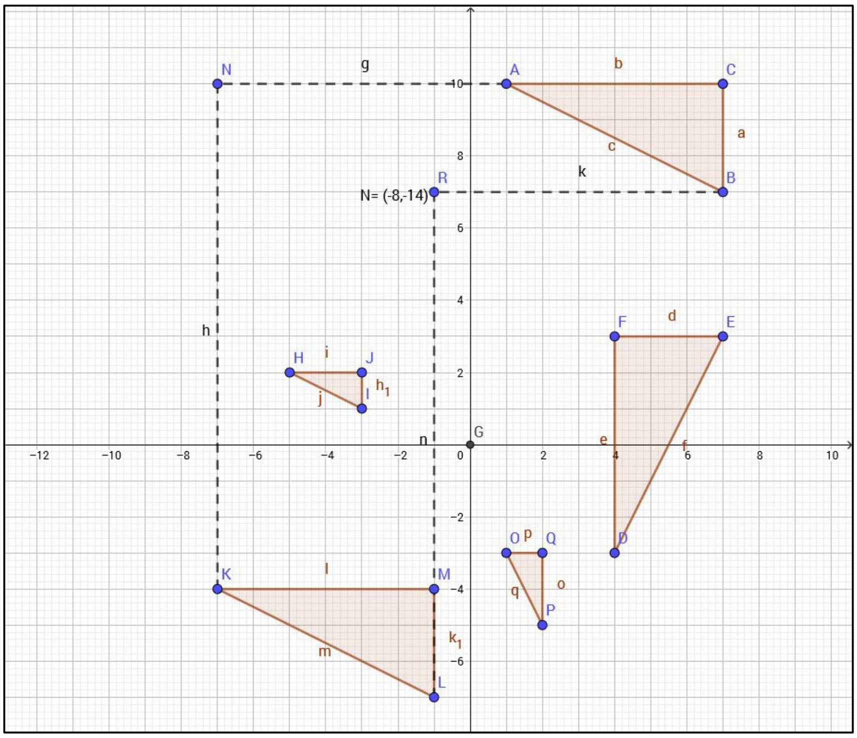
(i) shape P onto shape Q, [3]

We can see by inspection that P to Q is a *90 degree anticlockwise rotation around (9,5)*



(ii) shape P onto shape R, [2]

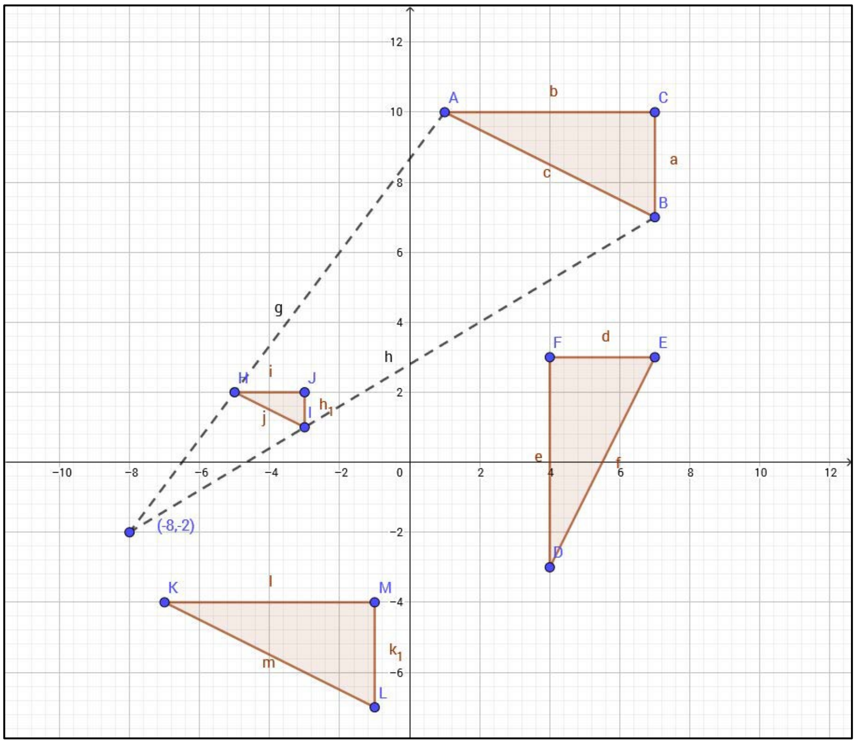
We can see by inspection that P to R is a *translation of (-8,-14)*



(iii) shape P onto shape S. [3]

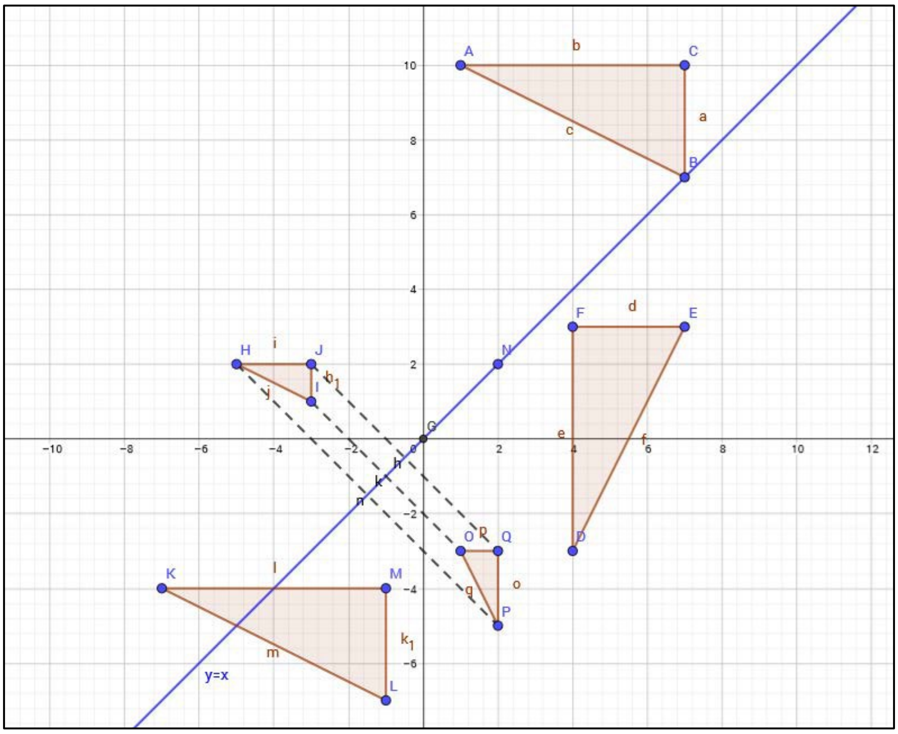
By drawing lines connecting the points of the triangle, we can see on the diagram below

that P to S is an *enlargement, scale factor 1/3, centre (-8,-2)*



(b) Draw the reflection of **shape S** in the line $y = x$. [2]

To reflect the shapes in the line $y = x$ it will help to draw the line first. Then it will be useful to draw lines from each corner of the triangle to the line, meeting the line at a right angle, and then extending the same distance beyond the line. If we do this for all three points, we can then connect them up to make the reflected triangle.



(a) $\mathbf{m} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\mathbf{n} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

(i) Work out $2\mathbf{m} - 3\mathbf{n}$. [2]

$$2\boldsymbol{m} - 3\boldsymbol{n} = 2 \times \begin{pmatrix} 3 \\ 2 \end{pmatrix} - 3 \times \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \times 3 - 3 \times (-2) \\ 2 \times 2 - 3 \times 3 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{12} \\ \mathbf{-5} \end{pmatrix}$$

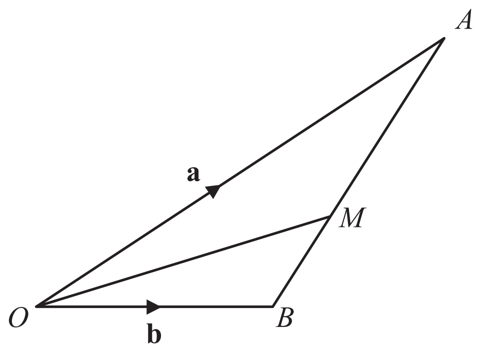
(ii) Calculate $|2\mathbf{m} - 3\mathbf{n}|$. [2]

The Modulus (length) of a Vector is found using Pythagoras' Theorem:

$$|2\boldsymbol{m} - 3\boldsymbol{n}| = \sqrt{12^2 + (-5)^2}$$

$$= \mathbf{13}$$

(b) (i)



NOT TO SCALE

In the diagram, O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
The point M lies on AB such that $AM : MB = 3 : 2$.

Find, in terms of **a** and **b**, in its simplest form

(a) $\overrightarrow{AB}$, [1]

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} - \boldsymbol{a}$$

(b) $\overrightarrow{AM}$, [1]

Since M splits AB in the ratio 3 : 2

$$\overrightarrow{AM} = \frac{3}{5} \times \overrightarrow{AB}$$

$$\overrightarrow{\boldsymbol{AM}} = \frac{\mathbf{3}}{\mathbf{5}} \times (\boldsymbol{b} - \boldsymbol{a})$$

(c) the position vector of M. [2]

The position vector of a point is the vector from O to that point.

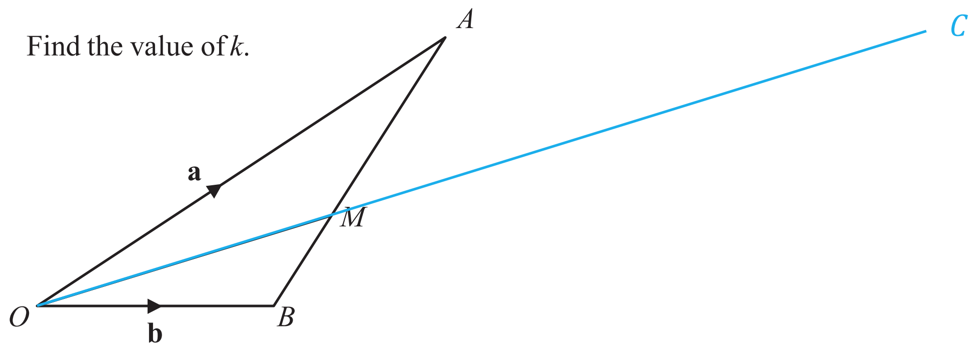
$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

$$= \boldsymbol{a} + \frac{3}{5} \times (\boldsymbol{b} - \boldsymbol{a})$$

$$\overrightarrow{\boldsymbol{OM}} = \frac{\mathbf{2}}{\mathbf{5}}\boldsymbol{a} + \frac{\mathbf{3}}{\mathbf{5}}\boldsymbol{b}$$

(ii) OM is extended to the point C.
The position vector of C is $\mathbf{a} + k\mathbf{b}$.

Find the value of k. [1]

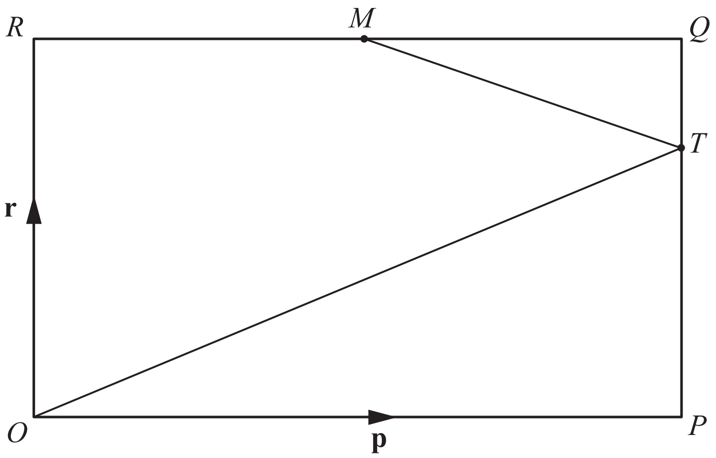


Since OC is parallel to OM, $\overrightarrow{OC}$ is a multiple of $\overrightarrow{OM}$.

And since the coefficient of $\boldsymbol{a}$ in $\overrightarrow{OC}$ is 1, that multiple must be $\frac{5}{2}$.

So, applying that multiple to the coefficient of $\boldsymbol{b}$:

$$k = \frac{5}{2} \times \frac{3}{5} = \frac{\mathbf{3}}{\mathbf{2}}$$



NOT TO SCALE

$OPQR$ is a rectangle and O is the origin.
M is the midpoint of RQ and $PT : TQ = 2 : 1$.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.

(a) Find, in terms of **p** and/or **r**, in its simplest form

(i) $\overrightarrow{MQ}$, [1]

$$\overrightarrow{MQ} = \frac{1}{2}\boldsymbol{p}$$

(ii) $\overrightarrow{MT}$, [1]

$$\overrightarrow{MT} = \frac{1}{2}\boldsymbol{p} - \frac{1}{3}\boldsymbol{r}$$

(iii) $\overrightarrow{OT}$. [1]

$$\overrightarrow{OT} = \boldsymbol{p} + \frac{2}{3}\boldsymbol{r}$$

(b) RQ and OT are extended to meet at U.

Find the position vector of U in terms of **p** and **r**.
Give your answer in its simplest form. [2]

$$\overrightarrow{OU} = \frac{3}{2}\overrightarrow{OT}$$

$$= \boldsymbol{r} + \frac{3}{2}\boldsymbol{p}$$

(c) $\overrightarrow{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$ and $|\overrightarrow{MT}| = \sqrt{180}$.

Find the positive value of k. [3]

$$\sqrt{(2k)^2 + (-k)^2} = \sqrt{180}$$

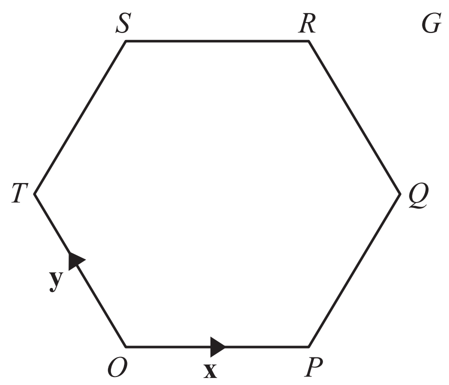
$$(2k)^2 + (-k)^2 = 180$$

$$4k^2 + k^2 = 180$$

$$5k^2 = 180$$

$$k^2 = 36$$

$$k = 6$$



NOT TO SCALE

O is the origin and $OPQRST$ is a regular hexagon.

$\overrightarrow{OP} = \mathbf{x}$ and $\overrightarrow{OT} = \mathbf{y}$.

(a) Write down, in terms of **x** and/or **y**, in its simplest form,

(i) $\overrightarrow{QR}$, [1]

The shape OPQRST is a regular hexagon; hence vector QR is identical with vector OT.

$\overrightarrow{QR} = \overrightarrow{OT}.$

We are given that:

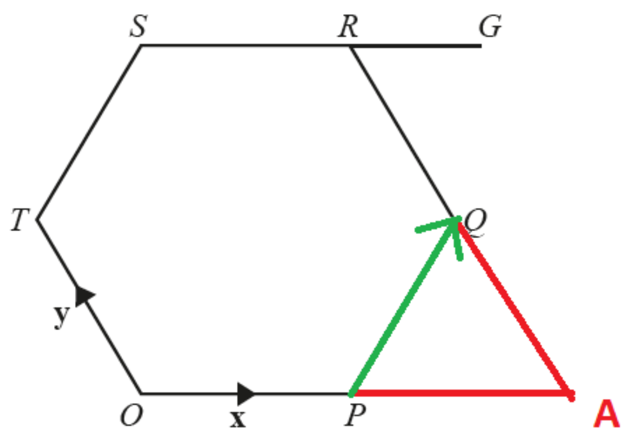
$\overrightarrow{OT} = \boldsymbol{y}.$

Therefore

$\overrightarrow{\boldsymbol{QR}} = \boldsymbol{y}.$

(ii) $\overrightarrow{PQ}$, [1]

Add point A such that PQA form an equilateral triangle.



The vector PQ can be split into two vectors.

$$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$$

Points O, P and A lie on the same line. We conclude that

$\overrightarrow{PA} = \overrightarrow{OP} = x$.

Vector AQ is identical with vectors OT and QR.

$\overrightarrow{AQ} = \overrightarrow{OT = y}$.

Therefore

$$\boldsymbol{\overrightarrow{PQ} = x + y}$$

(iii) the position vector of S. [2]

The position vector of S is the vector OS. This vector can be split into two vectors.

$$\overrightarrow{OS} = \overrightarrow{OT} + \overrightarrow{TS}$$

Again, from the argument that the shape is a hexagon, we see that the vector TS is identical with PQ and we already know vector PQ from the previous part.

$$\overrightarrow{TS} = \overrightarrow{PQ} = x + y$$

Therefore

$$\overrightarrow{OS} = y + (x + y)$$

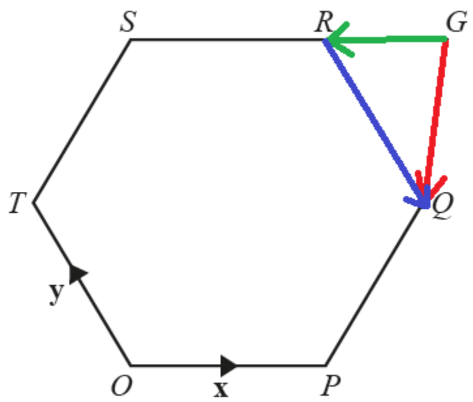
$$\boldsymbol{\overrightarrow{OS} = x + 2y}$$

(b) The line SR is extended to G so that $SR : RG = 2 : 1$.

Find $\overrightarrow{GQ}$, in terms of **x** and **y**, in its simplest form. [2]

Vector GQ can be split into two vectors.

$$\overrightarrow{GQ} = \overrightarrow{GR} + \overrightarrow{RQ}$$



We are given that SR : RG = 2 : 1. From this information we conclude:

$$\overrightarrow{RG} = \frac{1}{2}\overrightarrow{SR}$$

The vector SR is identical to vector OP.

$$\overrightarrow{RG} = \frac{1}{2}\overrightarrow{OP}$$

$$\overrightarrow{RG} = \frac{1}{2}x$$

We are already established that:

$$\overrightarrow{QR} = y$$

The problem is these are not quite the vectors we want. It is important to remember that for vectors:

$\overrightarrow{AB} = -\overrightarrow{BA}$.

Therefore $\overrightarrow{GQ} = -(\overrightarrow{RG} + \overrightarrow{QR})$

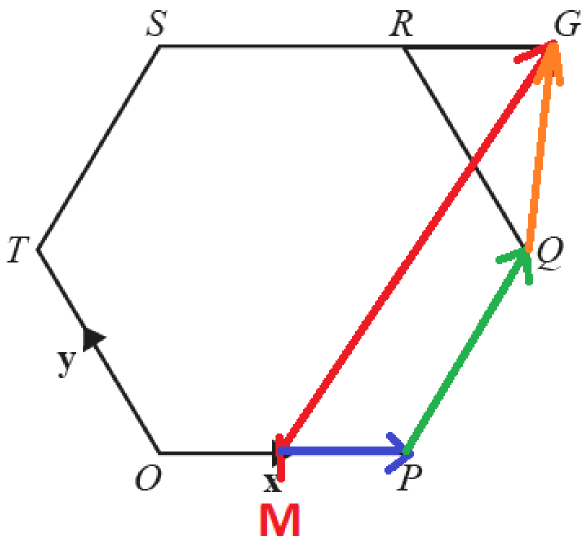
$$\boldsymbol{\overrightarrow{GQ} = -(\frac{1}{2}x + y)}$$

(c) *M* is the midpoint of *OP*.

(i) Find $\overrightarrow{MG}$, in terms of **x** and **y**, in its simplest form. [2]

The vector MG can be split into multiple vectors.

$$\overrightarrow{MG} = \overrightarrow{MP} + \overrightarrow{PQ} + \overrightarrow{QG}$$



M is the midpoint of OP.

$$\overrightarrow{MP} = \frac{1}{2}\overrightarrow{OP} = \frac{1}{2}x$$

We already know vectors PQ and GQ. It is important to remember that for vectors:

$\overrightarrow{QG} = -\overrightarrow{GQ}$.

Therefore

$$\overrightarrow{MG} = \frac{1}{2}x + (x + y) + (\frac{1}{2}x + y)$$

$$\boldsymbol{\overrightarrow{MG} = 2x + 2y}$$

(ii) H is a point on TQ such that $TH : HQ = 3 : 1$.

Use vectors to show that H lies on MG. [2]

In order to prove that H lines on MG, we show that the vector MG is a multiple of vector MH.

The vector MG can be split into multiple vectors.

$$\overrightarrow{MH} = \overrightarrow{MO} + \overrightarrow{OT} + \overrightarrow{TH}$$

M is the midpoint of OP.

$$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OP} = \frac{1}{2}x$$

We are given that TH : HQ = 3 : 1. From this information we conclude:

$$\overrightarrow{TH} = \frac{3}{4}\overrightarrow{TQ}$$

The shape is a hexagon so we can conclude that

$$\overrightarrow{TQ} = 2\overrightarrow{OP} = 2x$$

Now we know all the necessary vectors.

$$\overrightarrow{MH} = -\overrightarrow{OM} + \overrightarrow{OT} + \frac{3}{2}\overrightarrow{OP}$$

$$\overrightarrow{MH} = -\frac{1}{2}x + y + \frac{3}{2}(x)$$

$$\overrightarrow{MH} = x + y$$

Hence we see that

$$\boldsymbol{\overrightarrow{MG} = 2\overrightarrow{MH}}$$

Proving that H lies on MG.

Vectors
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **74 minutes**

Score: **/64**

Percentage: **/100**

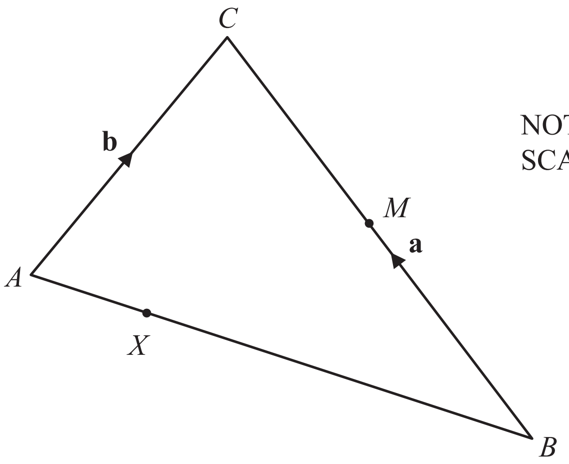
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



$\overrightarrow{BC} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$.

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**. [1]

The vector AB can be split into two vectors.

$$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$$

It is important to remember that for vectors: $\overrightarrow{BC} = -\overrightarrow{CB}$.

Using this fact:

$$\overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BC}$$

$$\boldsymbol{\overrightarrow{AB} = b - a}$$

(b) M is the midpoint of BC.
X divides AB in the ratio $1:4$.

Find $\overrightarrow{XM}$ in terms of **a** and **b**.
Show all your working and write your answer in its simplest form. [4]

The vector XM can be split into more vectors.

$$\overrightarrow{XM} = \overrightarrow{XA} + \overrightarrow{AC} + \overrightarrow{CM}$$

Since M is the midpoint of MC, =1/2 BC.

This means that $\overrightarrow{MC} = \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}a$

As X divides AB into the ratio 1:4, we know that AX= 1/5 AB

$$\overrightarrow{AX} = \frac{1}{5}\overrightarrow{AB} = \frac{1}{5}(b - a)$$

$$\overrightarrow{XM} = \overrightarrow{XA} + \overrightarrow{AC} + \overrightarrow{CM}$$

Using the vector property mentioned in part a):

$$\overrightarrow{XM} = -\overrightarrow{AX} + \overrightarrow{AC} - \overrightarrow{MC}$$

$$\overrightarrow{XM} = -\frac{1}{5}(b - a) + b - \frac{1}{2}a$$

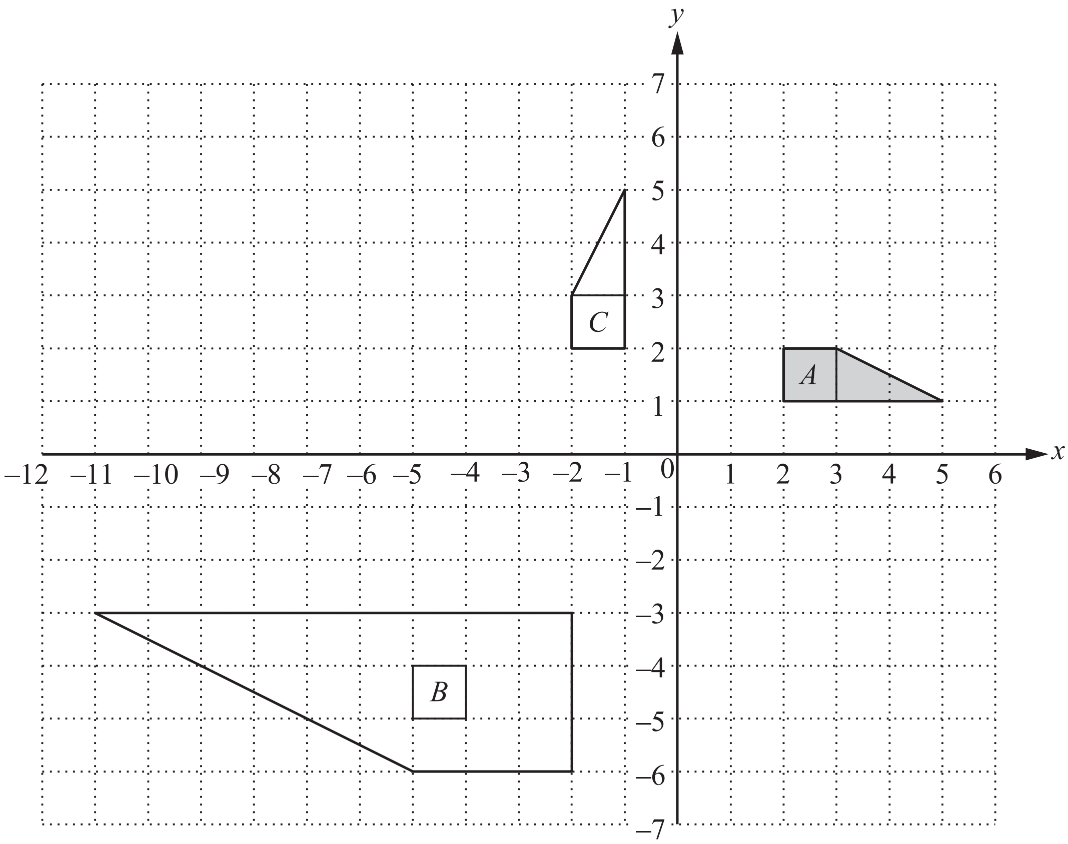
Multiply out the bracket and add corresponding vectors.

$$\overrightarrow{XM} = -\frac{1}{5}b + \frac{1}{5}a + b - \frac{1}{2}a$$

$$\overrightarrow{XM} = \frac{\mathbf{4}}{\mathbf{5}}\boldsymbol{b} - \frac{\mathbf{3}}{\mathbf{10}}\boldsymbol{a}$$

Or

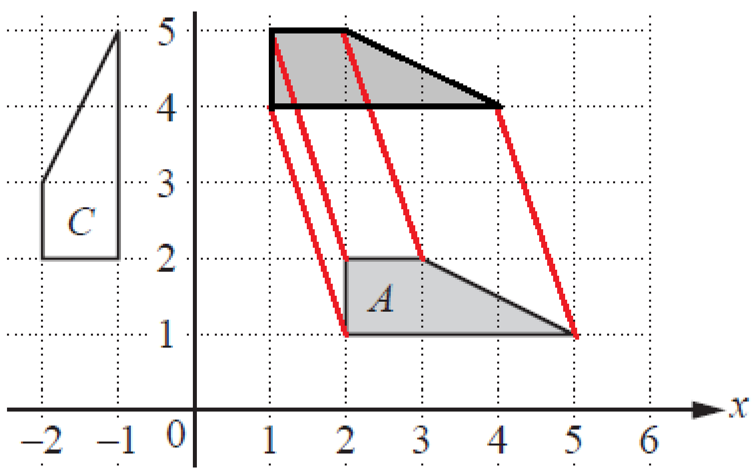
$$\overrightarrow{XM} = \frac{\mathbf{1}}{\mathbf{10}}(\mathbf{8}\boldsymbol{b} - \mathbf{3}\boldsymbol{a})$$



(a) Draw the image of

(i) shape A after a translation by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$, [2]

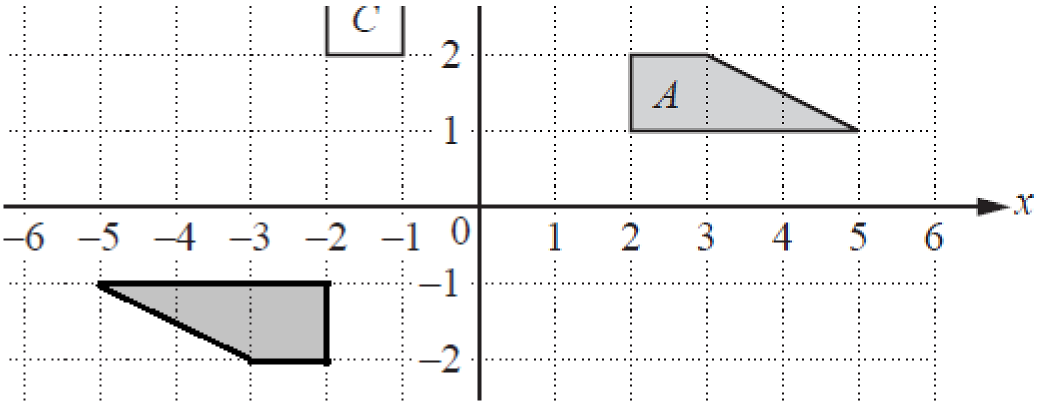
This transformation represents a shift by 1 unit in the negative *x* direction and by 3 units in the positive *y* direction.



The vertices of the new shape are: **(1,4), (1,5), (2,5) and (4,4).**

(ii) shape A after a rotation through 180° about the point (0, 0), [2]

We rotate the shape by 180°. This is essentially the same as reflecting the shape in line *y*=-*x*.



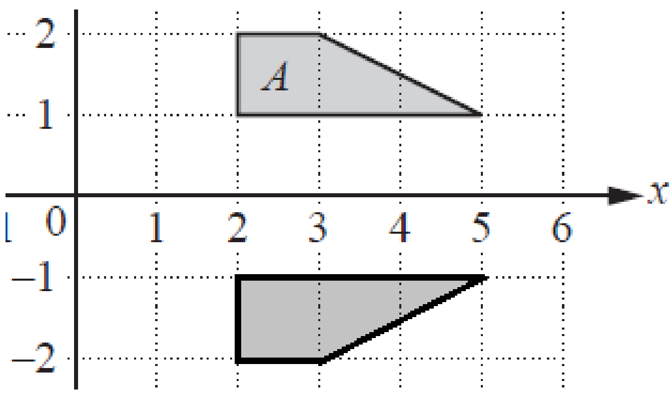
The vertices of the rotated shape are: **(-2,-1), (-5,-1), (-2,-2) and (-3,-2).**

(iii) shape A after the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. [3]

This matrix transformation represents a reflection in the *x*-axis.

The *x* coordinate does not change, but the *y* coordinate flips sign.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$



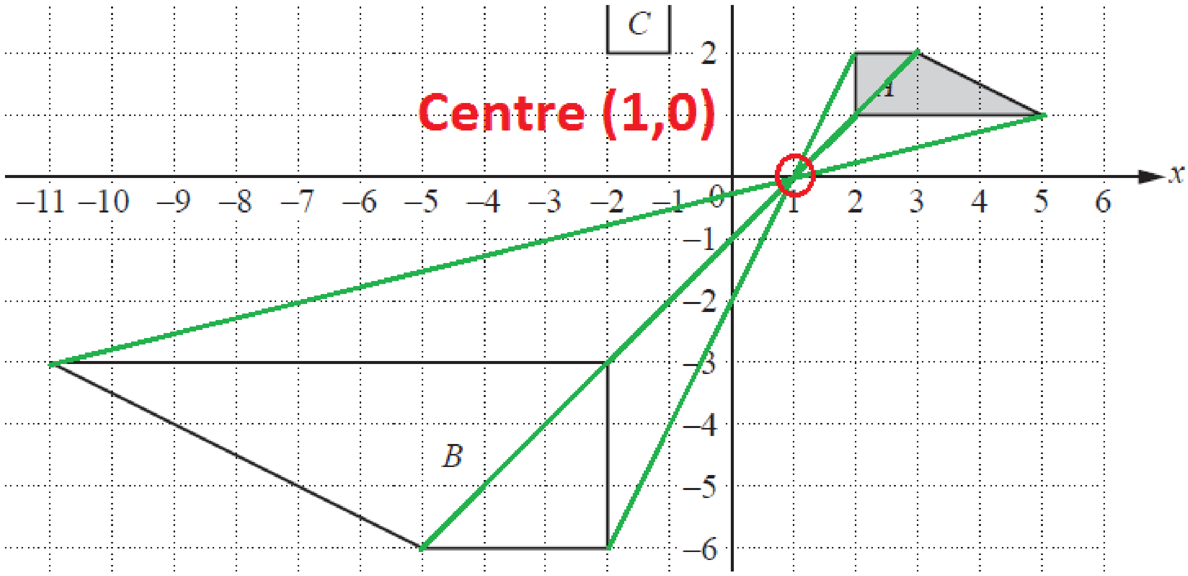
The vertices of the new shape are: **(2,-1), (5,-1), (2,-2) and (3,-2).**

(b) Describe fully the **single** transformation that maps shape A onto shape B. [3]

When we join the corresponding vertices of shapes A and B, the lines cross at point (1,0).

The distance from (1,0) to a vertex of shape B is three times as long as the distance from (1,0) to a corresponding vertex of shape A.

This suggests that the scale factor of the enlargement is -3(minus sign as the lines point in the opposite direction from (1,0)).



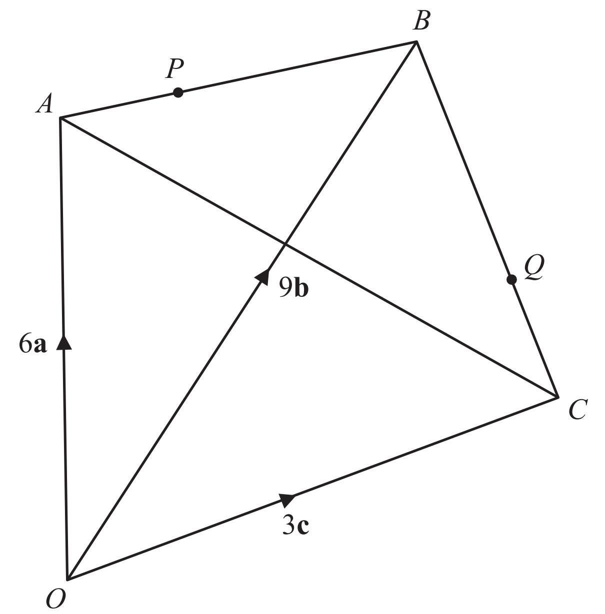
The transformation is **an enlargement with centre (1,0) and the scale factor -3**.

(c) Find the matrix which represents the transformation that maps shape A onto shape C. [2]

The transformation that maps shape A onto shape C is a rotation by 90° in anticlockwise direction.

A general matrix for rotation looks like $\begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$ where x is an angle of anticlockwise rotation.

This matrix becomes $\begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$ for x= 90°.



In the diagram, O is the origin and $\overrightarrow{OA} = 6\mathbf{a}$, $\overrightarrow{OB} = 9\mathbf{b}$ and $\overrightarrow{OC} = 3\mathbf{c}$.
The point P lies on AB such that $\overrightarrow{AP} = 3\mathbf{b} - 2\mathbf{a}$.
The point Q lies on BC such that $\overrightarrow{BQ} = 2\mathbf{c} - 6\mathbf{b}$.

(a) Find, in terms of **b** and **c**, the position vector of Q.
Give your answer in its simplest form. [2]

$$\overrightarrow{OQ} = \overrightarrow{OB} + \overrightarrow{BQ}$$

$$= 9\vec{b} + \left(2\vec{c} - 6\vec{b}\right)$$

$$= \mathbf{3\vec{b} + 2\vec{c}}$$

(b) Find, in terms of **a** and **c**, in its simplest form

(i) $\overrightarrow{AC}$, [1]

$$\overrightarrow{AC} = -\overrightarrow{OA} + \overrightarrow{OC}$$

$$= \mathbf{3\vec{c} - 6\vec{a}}$$

(ii) $\overrightarrow{PQ}$. [2]

$$\overrightarrow{PQ} = -\overrightarrow{AP} - \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{BQ}$$

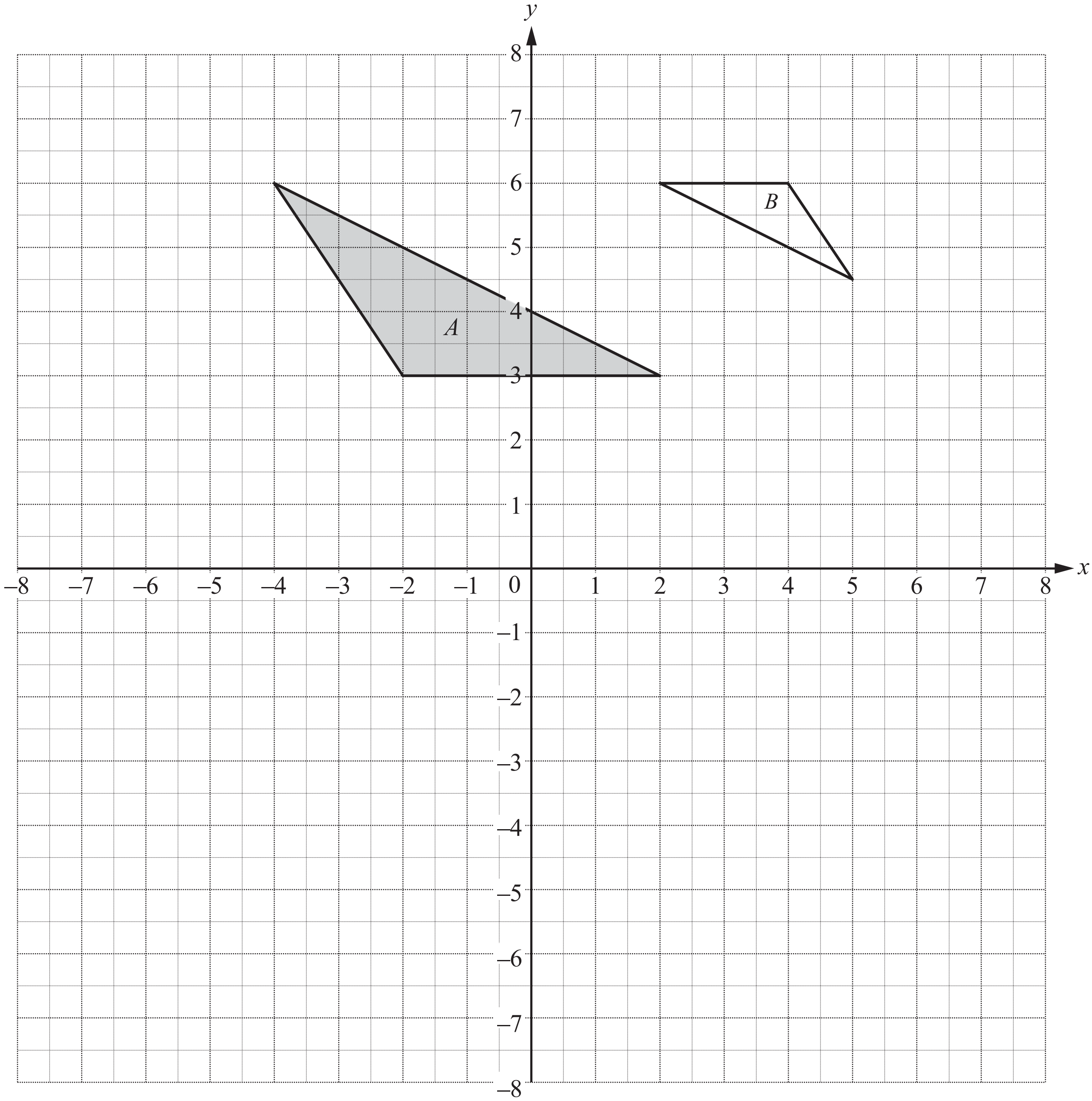
$$= -(3\vec{b} - 2\vec{a}) - 6\vec{a} + 9\vec{b} + 2\vec{c} - 6\vec{b}$$

$$= \boldsymbol{-4\vec{a} + 2\vec{c}}$$

(c) Explain what your answers in **part (b)** tell you about PQ and AC. [2]

$$\overrightarrow{PQ} = \frac{3}{2}\overrightarrow{AC}$$

These tell us that PQ and AC are **parallel**.



(a) Describe fully the **single** transformation that maps triangle A onto triangle B. [3]

Enlargement, scale factor $-\frac{1}{2}$, centre (2, 5)

(b) On the grid, draw the image of

(i) triangle A after a reflection in the line $x = -3$, [2]

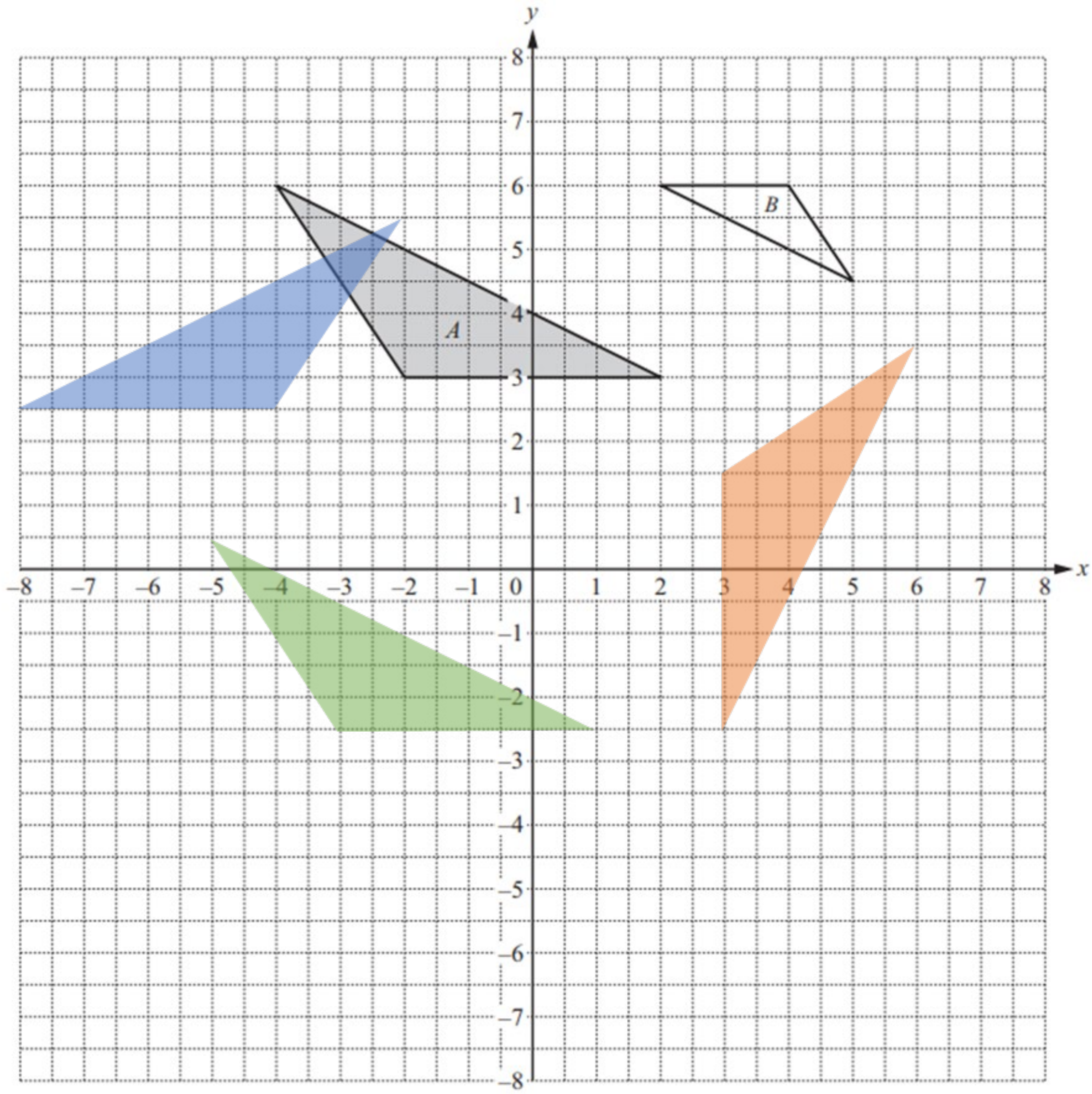
The blue triangle on the diagram below.

(ii) triangle A after a rotation about the origin through 270° anticlockwise, [2]

The orange triangle on the diagram below.

(iii) triangle A after a translation by the vector $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$. [2]

The green triangle on the diagram below.



(c) **M** is the matrix that represents the transformation in **part (b)(ii)**.

(i) Find **M**. [2]

We require:

$$M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix}$$

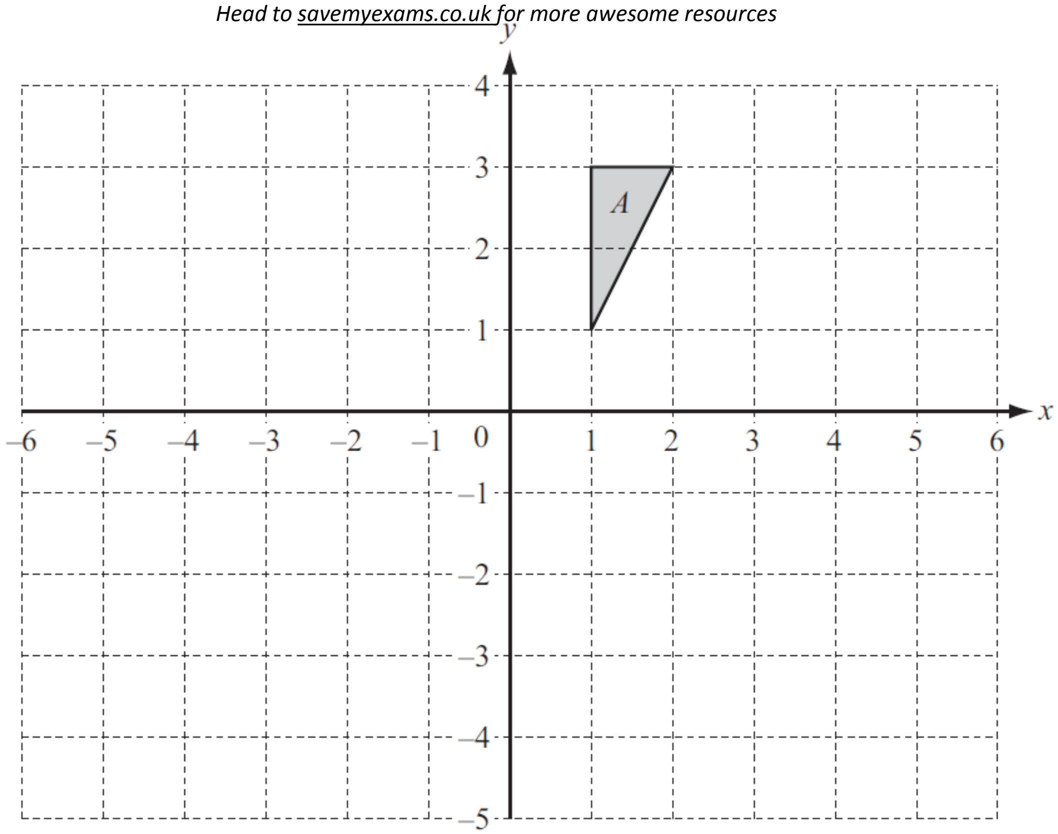
This is done by:

$$\boldsymbol{M} = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$$

(ii) Describe fully the **single** transformation represented by $\mathbf{M}^{-1}$, the inverse of **M**. [2]

M is rotation of 90˚ clockwise about the origin, therefore its inverse is:

$\boldsymbol{M}^{-1}$ is rotation of 90˚ anti-clockwise about the origin.



(a) On the grid,

(i) draw the image of shape A after a translation by the vector $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$, [2]

The blue triangle below.

(ii) draw the image of shape A after a rotation through 90° clockwise about the origin. [2]

The orange triangle below.

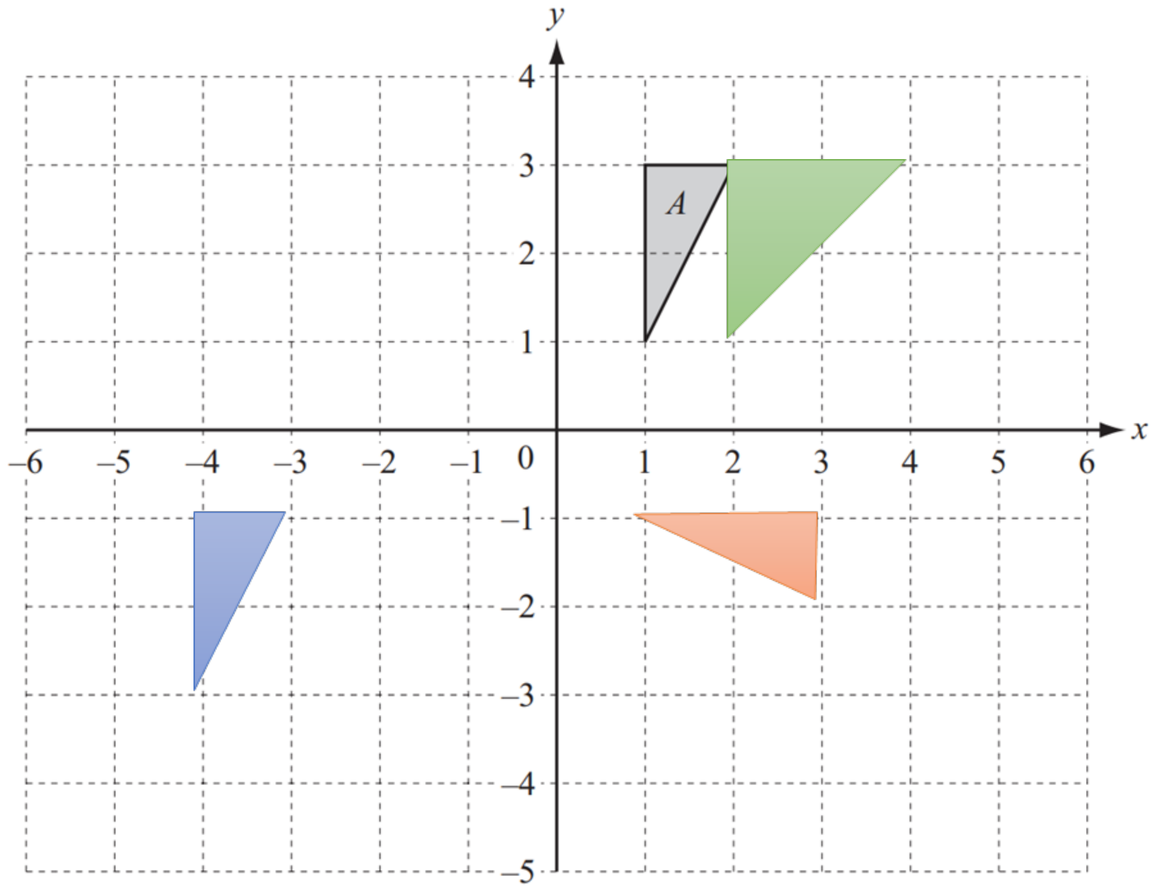
(b) (i) On the grid, draw the image of shape A after the transformation represented by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ y \end{pmatrix}$$ [3]

The green triangle below.

(ii) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. [3]

A **stretch of scale factor 2 in the x-direction (y-axis invariant).**


y
x
4
3
2
1
0
–1
–2
–3
–4
–5
–6
–5
–4
–3
–2
–1
1
2
3
4
5
6
A

(a) $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

(i) P is the point (–2, 3).

Work out the co-ordinates of Q. [1]

$$\begin{pmatrix} x_Q \\ y_Q \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$\rightarrow x_Q + 2 = -3$$

$$\rightarrow x_Q = -5$$

and

$$y_Q - 3 = 4$$

$$\rightarrow y_Q = 7$$

Hence

$$\boldsymbol{Q = (-5, 7)}$$

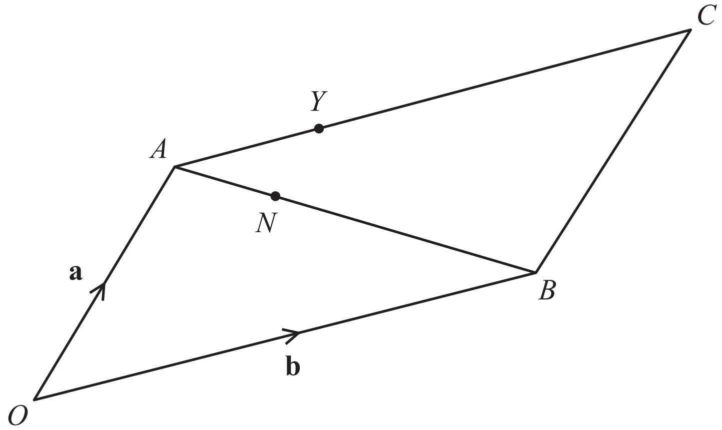
(ii) Work out $|\overrightarrow{PQ}|$, the magnitude of $\overrightarrow{PQ}$. [2]

$$\sqrt{3^2 + 4^2}$$

$$= \sqrt{25}$$

$$= \mathbf{5}$$

(b)



$OACB$ is a parallelogram.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
$AN : NB = 2 : 3$ and $AY = \frac{2}{5}AC$.

(i) Write each of the following in terms of **a** and/or **b**.
Give your answers in their simplest form.

(a) $\overrightarrow{ON}$ [2]

$$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$$

$$= \vec{a} + \frac{2}{5}\overrightarrow{AB}$$

$$= \vec{a} + \frac{2}{5}(\vec{b} - \vec{a})$$

$$= \frac{3}{5}\vec{a} + \frac{2}{5}\vec{b}$$

(b) $\overrightarrow{NY}$ [2]

$$\overrightarrow{NY} = \overrightarrow{NA} + \overrightarrow{AY}$$

$$= -\tfrac{2}{5}(\vec{b} - \vec{a}) + \tfrac{2}{5}\overrightarrow{AC}$$

$$= -\tfrac{2}{5}(\vec{b} - \vec{a}) + \tfrac{2}{5}\vec{b}$$

$$= \tfrac{2}{5}\vec{a}$$

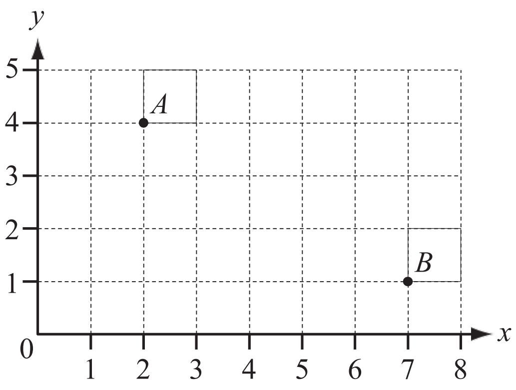
(ii) Write down two conclusions you can make about the line segments NY and BC. [2]

$$\overrightarrow{NY} = \frac{2}{5}\overrightarrow{BC}$$

and

$\overrightarrow{NY}$ and $\overrightarrow{BC}$ are parallel

(a)



(i) Write down the position vector of A. [1]

$$\begin{pmatrix} \mathbf{2} \\ \mathbf{4} \end{pmatrix}$$

(ii) Find $|\overrightarrow{AB}|$, the magnitude of $\overrightarrow{AB}$. [2]

$$\overrightarrow{AB} = \vec{b} - \vec{a}$$

$$= \begin{pmatrix} 7 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

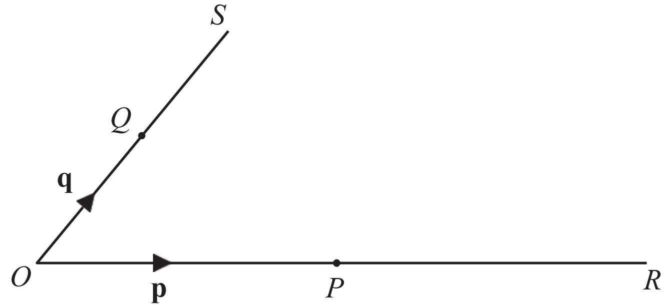
Hence

$$|\overrightarrow{AB}| = \sqrt{5^2 + 3^2}$$

$$= \sqrt{34}$$

$$= \mathbf{5.83}$$

(b)



NOT TO SCALE

O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.
OP is extended to R so that $OP = PR$.
OQ is extended to S so that $OQ = QS$.

(i) Write down $\overrightarrow{RQ}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. [1]

$$\overrightarrow{RQ} = -2\vec{p} + \vec{q}$$

(ii) PS and RQ intersect at M and $RM = 2MQ$.

Use vectors to find the ratio $PM : PS$, showing all your working. [4]

First find PS

$$\overrightarrow{PS} = 2\vec{q} - \vec{p}$$

Then find PM (through R, using RQ from part i)

$$\overrightarrow{PM} = \vec{p} + \tfrac{2}{3}\overrightarrow{RQ} = \vec{p} + \tfrac{2}{3}(-2\vec{p} + \vec{q})$$

Expand and simplify comparing to PS

$$\overrightarrow{PM} = \tfrac{1}{3}(2\vec{q} - \vec{p})$$

So ratio PM:PS

$$\tfrac{1}{3}(2\vec{q} - \vec{p}) \quad : \quad 2\vec{q} - \vec{p}$$

1:3

Vectors
Difficulty: Hard

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 3

Time allowed: **106 minutes**

Score: **/92**

Percentage: **/100**

Grade Boundaries:

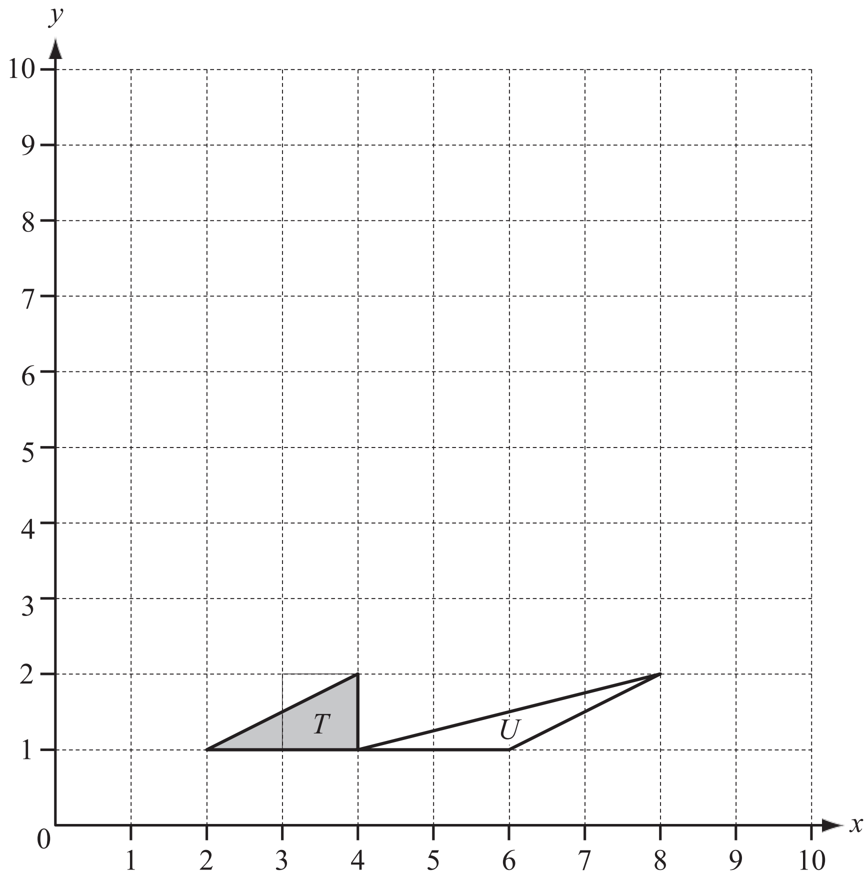
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

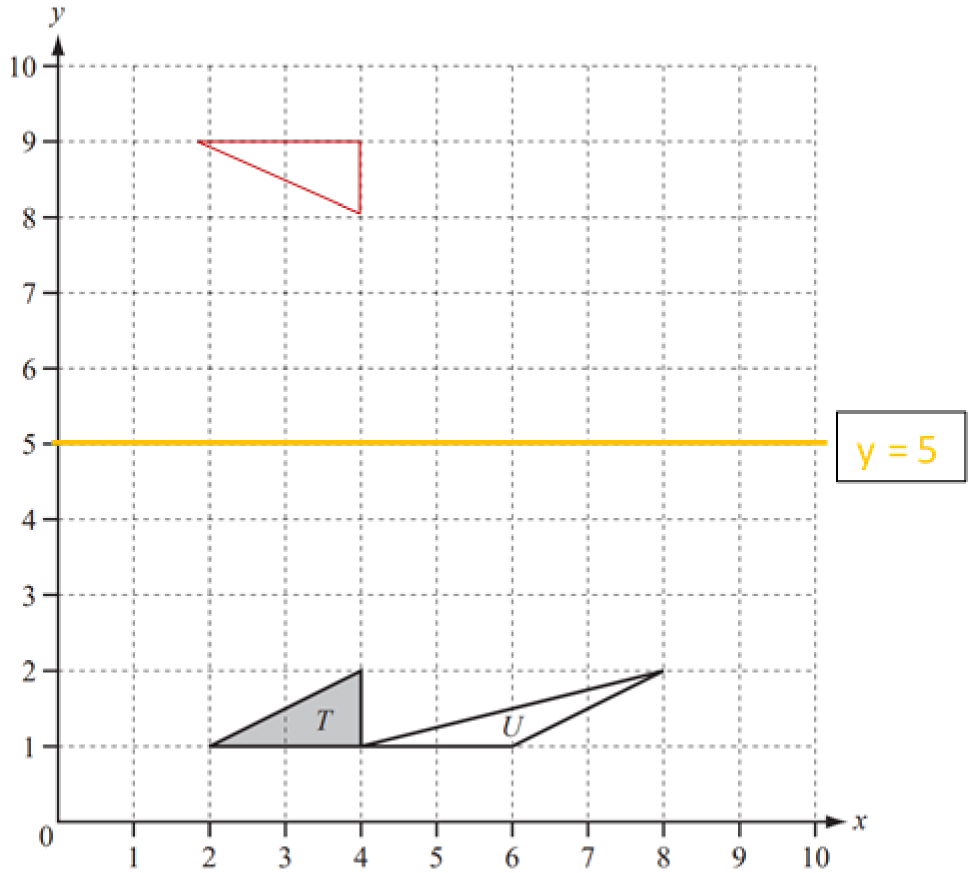
CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

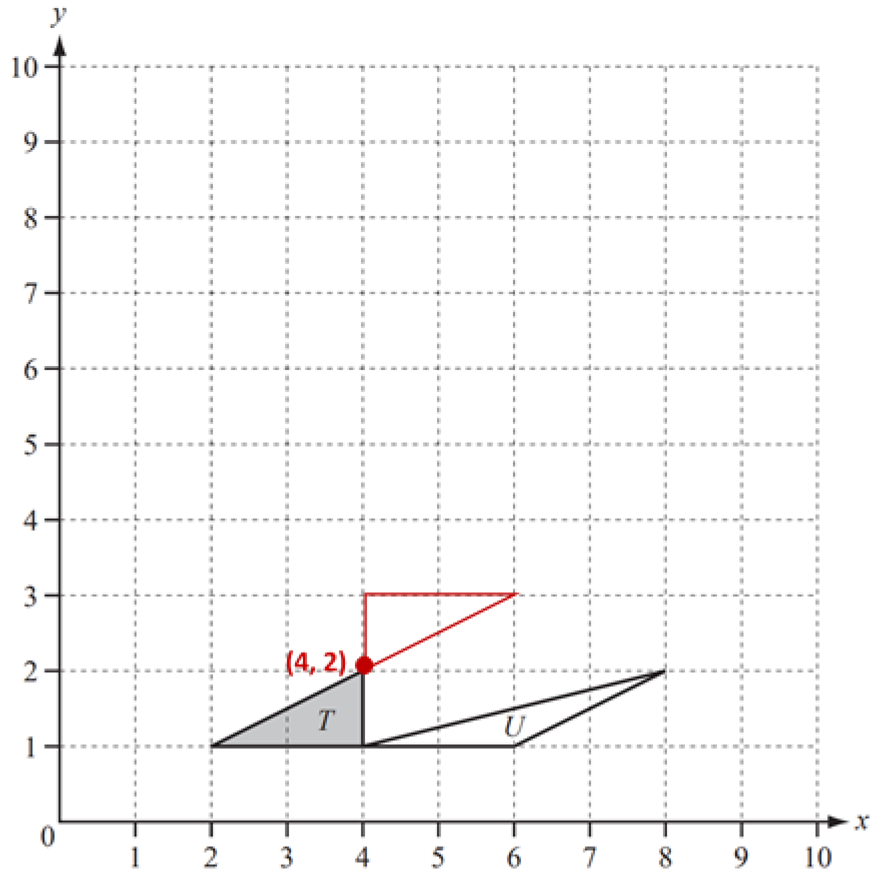
(a)



(i) Draw the reflection of triangle T in the line $y = 5$. [2]



(ii) Draw the rotation of triangle T about the point (4, 2) through 180°. [2]



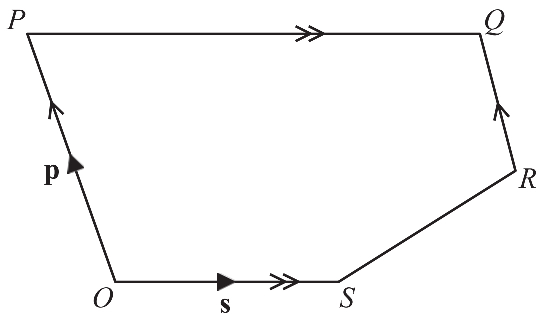
(iii) Describe fully the **single** transformation that maps triangle T onto triangle U. [3]

Shear, x-axis invariant, factor 2

(iv) Find the 2 × 2 matrix which represents the transformation in **part (a)(iii)**. [2]

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

(b)



NOT TO SCALE

In the pentagon $OPQRS$, OP is parallel to RQ and OS is parallel to PQ.
$PQ = 2OS$ and $OP = 2RQ$.
O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OS} = \mathbf{s}$.

Find, in terms of **p** and **s**, in their simplest form,

(i) the position vector of Q, [2]

$$\overrightarrow{PQ} = 2\overrightarrow{OS}$$

$$= 2\boldsymbol{s}$$

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$$

$$= \boldsymbol{p} + 2\boldsymbol{s}$$

(ii) $\overrightarrow{SR}$. [2]

$$\overrightarrow{OS} + \overrightarrow{SR} + \overrightarrow{RQ} = \overrightarrow{OQ}$$

Since OP = 2RQ,

$$\overrightarrow{RQ} = \frac{1}{2}\boldsymbol{p}$$

Rearranging,

$$\overrightarrow{SR} = \overrightarrow{OQ} - \overrightarrow{OS} - \overrightarrow{RQ}$$

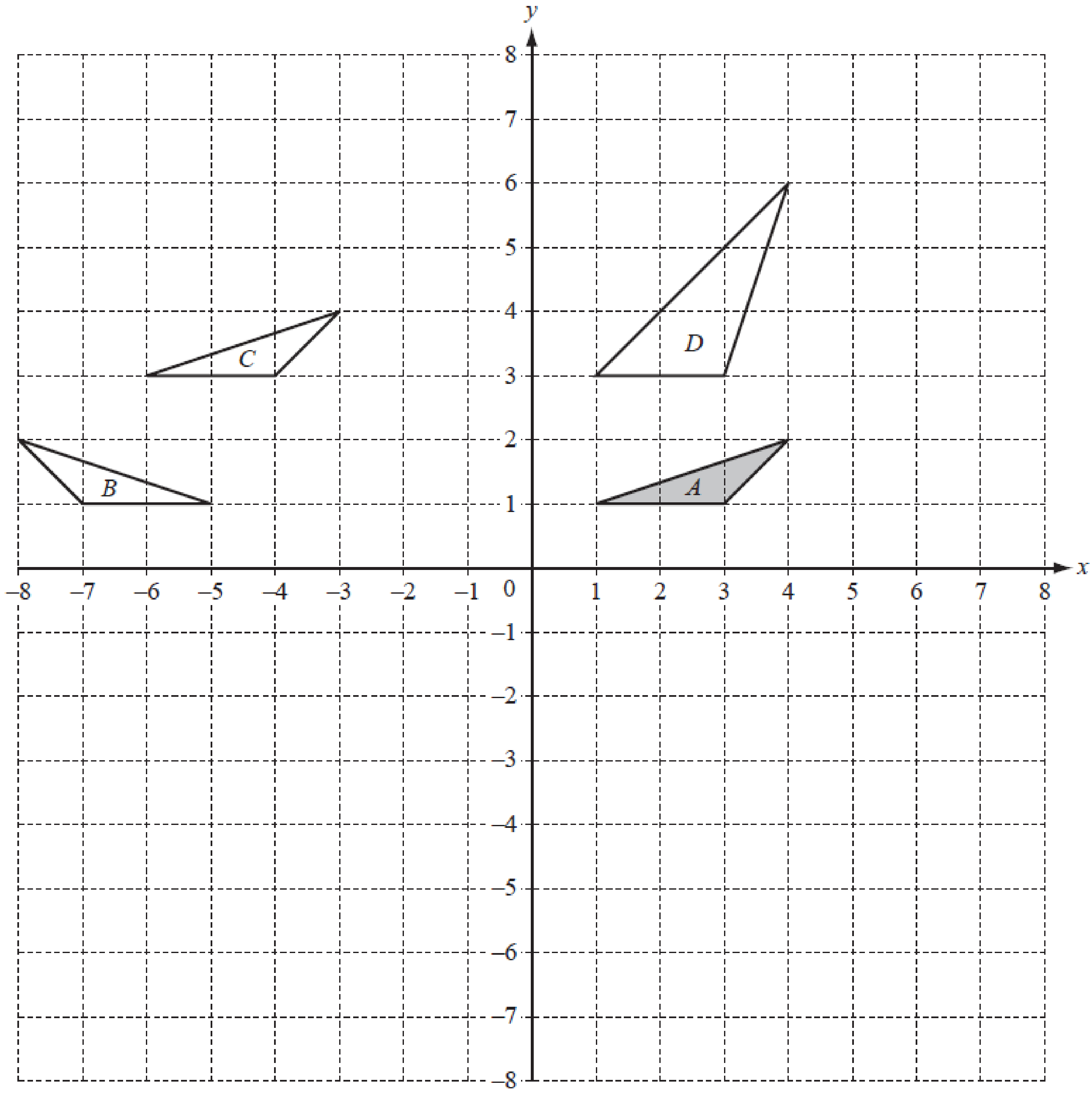
$$= (\boldsymbol{p} + 2\boldsymbol{s}) - \boldsymbol{s} - \frac{1}{2}\boldsymbol{p}$$

$$= \frac{1}{2}\boldsymbol{p} + \boldsymbol{s}$$

(c) Explain what your answers in **part (b)** tell you about the lines *OQ* and *SR*. [1]

Note here that SR is half of OQ.

Hence the lines are **parallel** and **OQ = 2SR.**



(a) Describe fully the **single** transformation that maps triangle *A* onto

(i) triangle *B*, [2]

Reflection about the line x = -2

(ii) triangle C, [2]

To get A to map onto C, we have to move A 7 units to the left and 2 units up.

Therefore the transformation is a **translation** by:

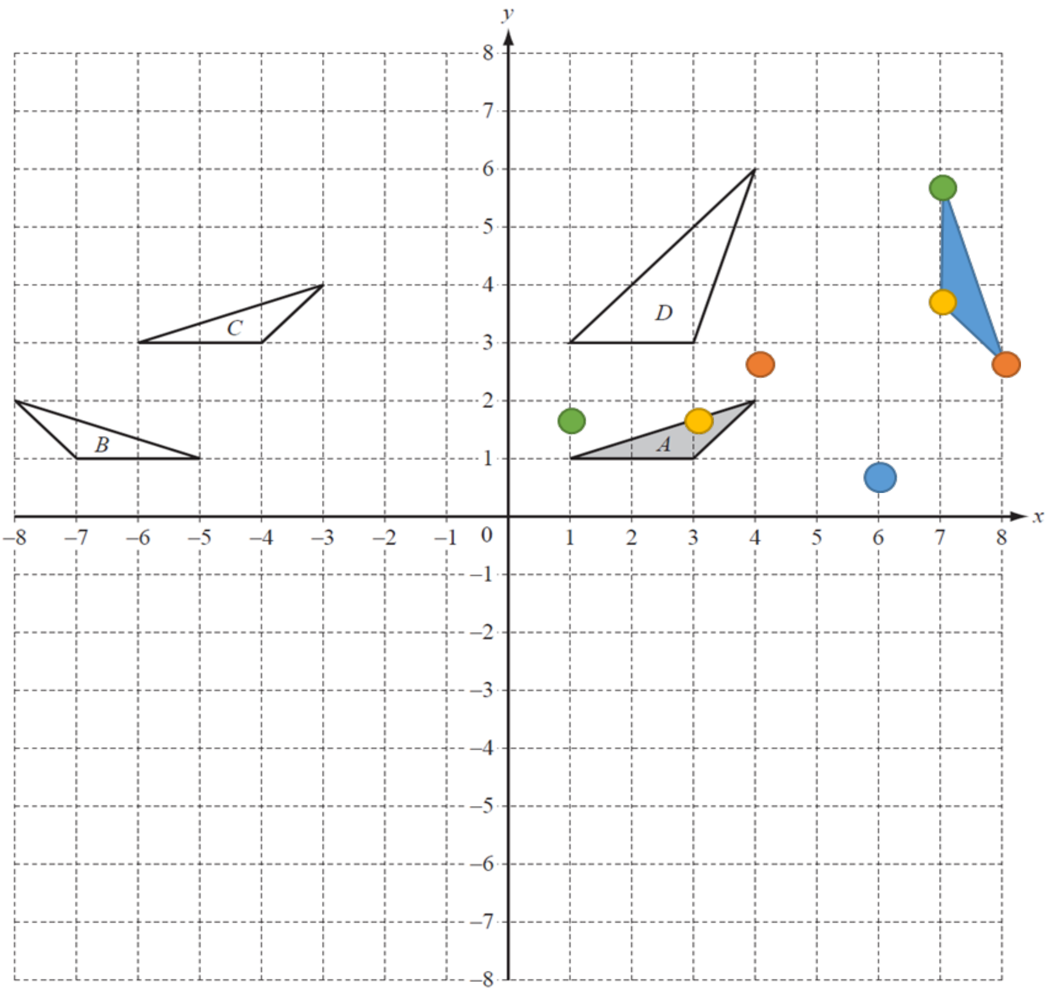
$$\begin{pmatrix} -7 \\ 2 \end{pmatrix}$$

(iii) triangle D. [3]

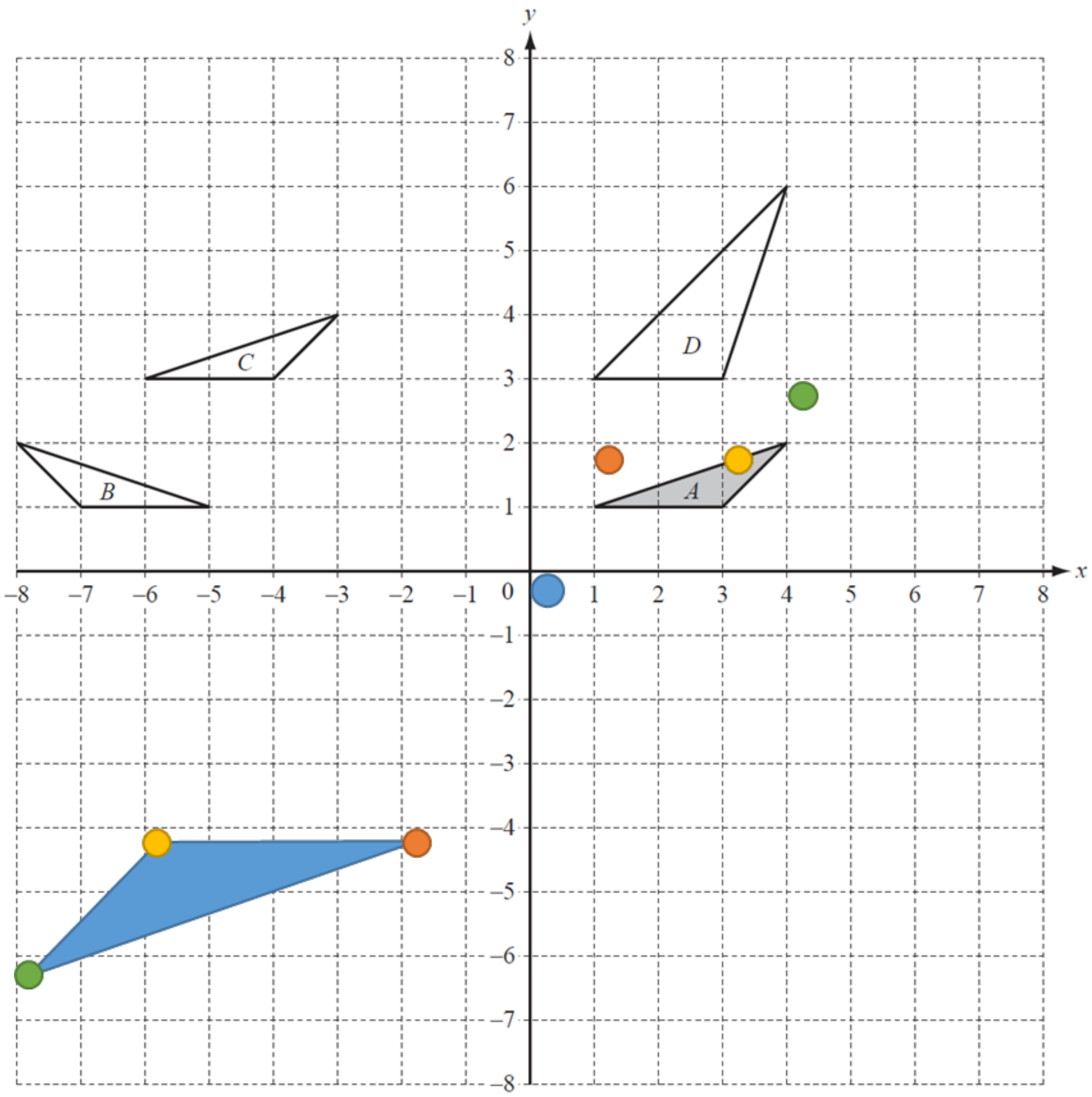
This is **a stretch transformation invariant on the x-axis by factor of 3.**

(b) On the grid, draw

(i) the rotation of triangle A about (6, 0) through 90° clockwise, [2]



(ii) the enlargement of triangle A by scale factor -2 with centre $(0, -1)$, [2]

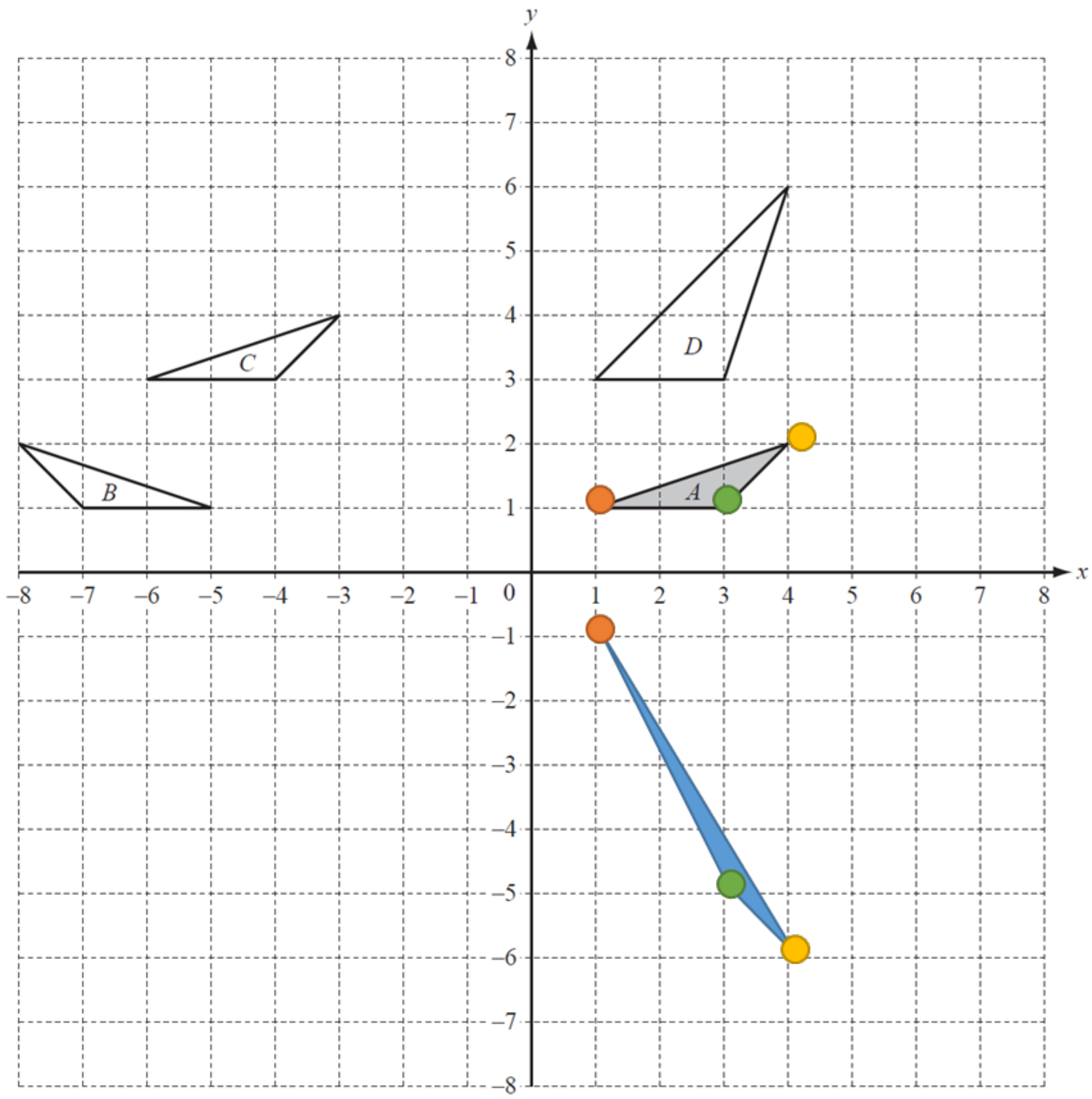


Orange: 2 left and 4 down from centre

Yellow: 6 left and 4 down from centre

Green: 8 left and 6 down from centre

(iii) the shear of triangle A by shear factor -2 with the y-axis invariant. [2]



Keep y-axis invariant,

Orange: Multiply 1 with -2 gives -2. Shear downwards by 2 units.

Green: Multiply 3 with -2 gives -6. Shear downwards by 6 units.

Yellow: Multiply 4 with -2 gives -8. Shear downwards by 8 units.

(c) Find the matrix that represents the transformation in **part (b)(iii)**. [2]

The matrix is given by:

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{-2} & \mathbf{1} \end{pmatrix}$$

-2 is in the bottom left as in this case the y-axis is invariant.

(a) The co-ordinates of P are $(-4, -4)$ and the co-ordinates of Q are $(8, 14)$.

(i) Find the gradient of the line PQ. [2]

$$Gradient = \frac{14-(-4)}{8-(-4)}$$

$$= \frac{18}{12}$$

$$= \mathbf{1.5}$$

(ii) Find the equation of the line PQ. [2]

$$The\ equation\ of\ a\ straight\ line: y = mx + c$$

$$y = 1.5x + c$$

Substitute any point, in this case let's choose point Q (8, 14):

$$14 = 1.5(8) + c$$

$$c = 2$$

$$\boldsymbol{y = 1.5x + 2}$$

(iii) Write $\overrightarrow{PQ}$ as a column vector. [1]

$$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 8+4 \\ 14+4 \end{pmatrix} = \begin{pmatrix} 12 \\ 18 \end{pmatrix}$$

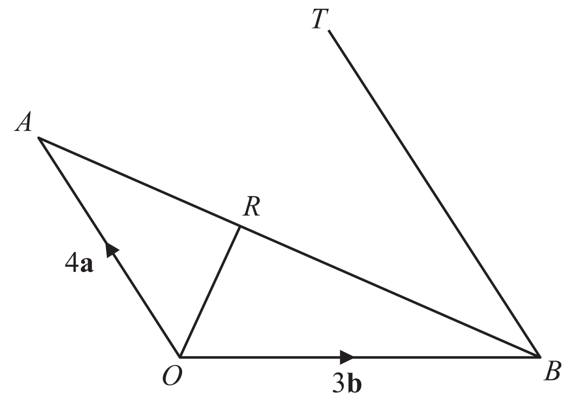
(iv) Find the magnitude of $\overrightarrow{PQ}$. [2]

$$\overrightarrow{PQ} = \begin{pmatrix} 12 \\ 18 \end{pmatrix}$$

$$Magnitude = \sqrt{12^2 + 18^2}$$

$$= \mathbf{21.6}$$

(b)



In the diagram, $\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OB} = 3\mathbf{b}$.

R lies on AB such that $\overrightarrow{OR} = \frac{1}{5}(12\mathbf{a} + 6\mathbf{b})$.

T is the point such that $\overrightarrow{BT} = \frac{3}{2}\overrightarrow{OA}$.

(i) Find the following in terms of **a** and **b**, giving each answer in its simplest form.

(a) $\overrightarrow{AB}$ [1]

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

$$4\boldsymbol{a} + \overrightarrow{AB} = 3\boldsymbol{b}$$

$$\overrightarrow{AB} = 3\boldsymbol{b} - 4\boldsymbol{a}$$

(b) $\overrightarrow{AR}$ [2]

$$\overrightarrow{OR} = \frac{1}{5}(12\boldsymbol{a} + 6\boldsymbol{b})$$

$$\overrightarrow{OA} + \overrightarrow{AR} = \overrightarrow{OR}$$

$$4\boldsymbol{a} + \overrightarrow{AR} = \frac{1}{5}(12\boldsymbol{a} + 6\boldsymbol{b})$$

$$\overrightarrow{AR} = \frac{12}{5}\boldsymbol{a} + \frac{6}{5}\boldsymbol{b} - 4\boldsymbol{a}$$

$$= \frac{6}{5}\boldsymbol{b} - \frac{8}{5}\boldsymbol{a}$$

$$= \frac{1}{5}(6\boldsymbol{b} - 8\boldsymbol{a})$$

(c) $\overrightarrow{OT}$ [1]

$$\overrightarrow{BT} = \frac{3}{2}\overrightarrow{OA} = \frac{3}{2}(4\boldsymbol{a})$$

$$= 6\boldsymbol{a}$$

$$\overrightarrow{OB} + \overrightarrow{BT} = \overrightarrow{OT}$$

$$\overrightarrow{\boldsymbol{OT}} = \boldsymbol{6a + 3b}$$

(ii) Complete the following statement. [1]

The points O, R and T are in a straight line because **OR is parallel to OT.**

(iii) Triangle OAR and triangle TBR are similar.

Find the value of $\dfrac{\text{area of triangle } TBR}{\text{area of triangle } OAR}$. [2]

$$\overrightarrow{BT} = \frac{3}{2}\overrightarrow{OA}$$

Since the area of a triangle has units of the square of

length, and the triangles are similar,

The ratio is:

$$\frac{Area\ of\ triangle\ TBR}{Area\ of\ triangle\ OAR} = \left(\frac{3}{2}\right)^2$$

$$= \frac{\boldsymbol{9}}{\boldsymbol{4}}$$

(a) $\mathbf{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} -10 \\ 21 \end{pmatrix}$

(i) Find $2\mathbf{a} + \mathbf{b}$. [1]

$$2a + b$$

$$= 2\begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$= \begin{pmatrix} -4 + 2 \\ 6 + (-7) \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-2} \\ \mathbf{-1} \end{pmatrix}$$

(ii) Find $|\mathbf{b}|$. [2]

$$|b| = \sqrt{2^2 + (-7)^2}$$

$$= \sqrt{4 + 49}$$

$$= \pm\sqrt{\mathbf{53}}$$

(iii) $m\mathbf{a} + n\mathbf{b} = \mathbf{c}$

Find the values of m and n.
Show all your working. [6]

$$m\begin{pmatrix}-2\\3\end{pmatrix} + n\begin{pmatrix}2\\-7\end{pmatrix} = \begin{pmatrix}-10\\21\end{pmatrix}$$

Split into 2 simultaneous equations

$-2m + 2n = -10$ (1)

$3m - 7n = 21$ (2)

Multiply (1) by 3 and (2) by 2 to get

$-6m + 6n = -30$ (3)

$6m - 14n = 42$ (4)

Add (3) and (4) together

$$-8n = 12$$

$$\rightarrow \boldsymbol{n = -1.5}$$

Sub this into (1)

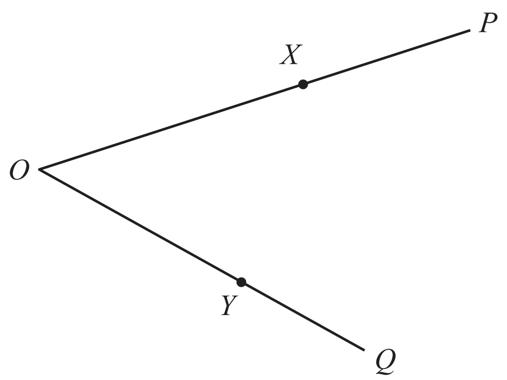
$$-2m + 2(-1.5) = -10$$

$$\rightarrow -2m - 3 = -10$$

$$\rightarrow -2m = -7$$

$$\rightarrow \boldsymbol{m = 3.5}$$

(b)



In the diagram, $OX : XP = 3 : 2$ and $OY : YQ = 3 : 2$.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.

(i) Write $\overrightarrow{PQ}$ in terms of **p** and **q**. [1]

$$\overrightarrow{PQ} = -\vec{p} + \vec{q}$$

(ii) Write $\overrightarrow{XY}$ in terms of **p** and **q**. [1]

We know that

$$\overrightarrow{OX} = \frac{3}{5}\overrightarrow{OP} = \frac{3}{5}\vec{p}$$

and

$$\overrightarrow{OY} = \frac{3}{5}\overrightarrow{OQ} = \frac{3}{5}\vec{q}$$

Hence

$$\overrightarrow{XY} = -\frac{3}{5}\vec{p} + \frac{3}{5}\vec{q}$$

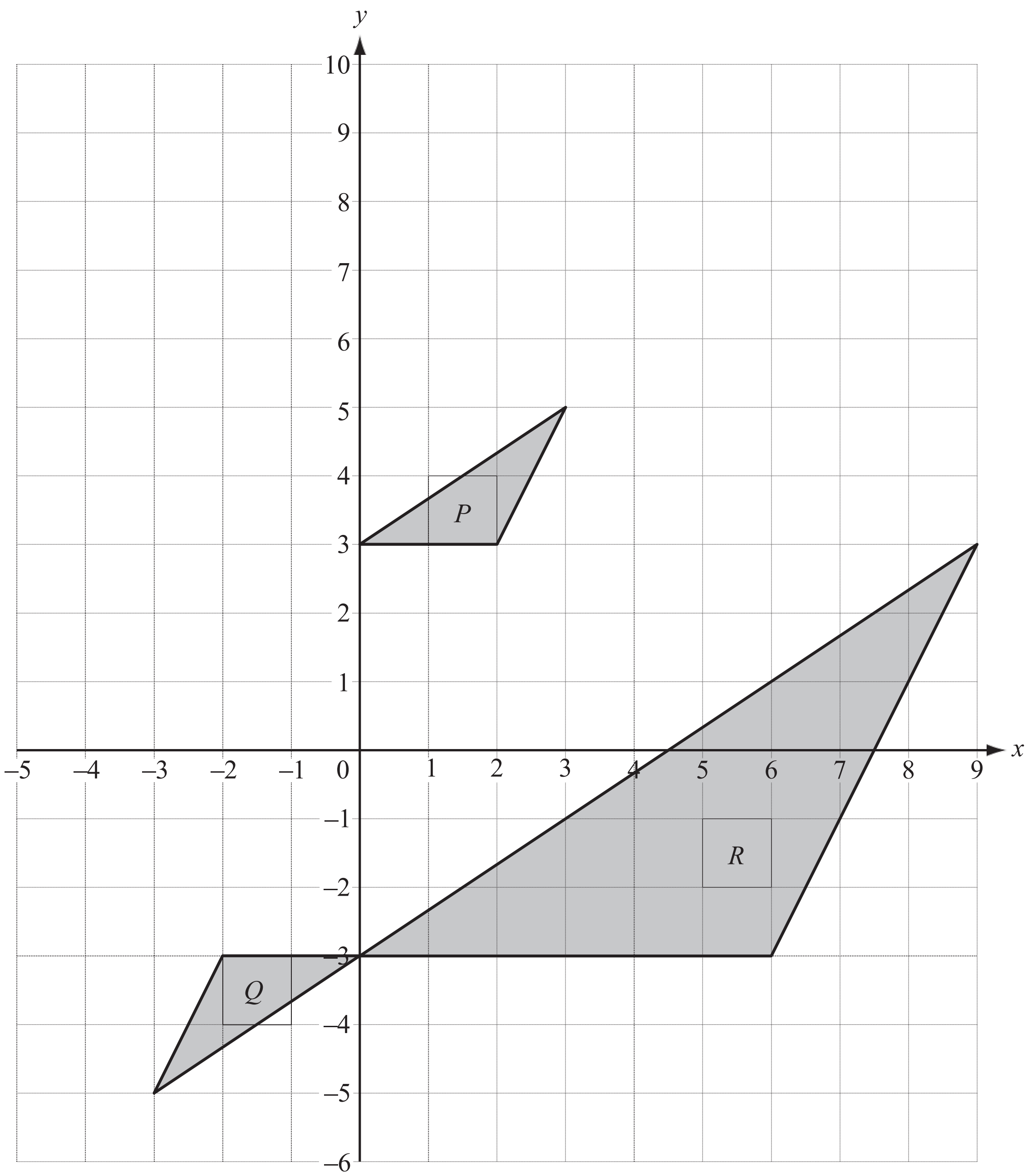
(iii) Complete the following sentences. [3]

The lines XY and PQ are **parallel**. *One is a scalar multiple of the other.*

The triangles OXY and OPQ are **similar**.

The ratio of the area of triangle OXY to the area of OPQ is $3^2 : 5^2$

$$= 9 : 25$$



(a) Describe fully

(i) the **single** transformation which maps triangle P onto triangle Q, [3]

Rotation, about the origin, of 180°

(ii) the **single** transformation which maps triangle Q onto triangle R, [3]

Enlargement, scale factor -3, centre $(0, -3)$

(iii) the **single** transformation which maps triangle R onto triangle P. [3]

Enlargement, scale factor 1/3, centre $(0, 6)$

(b) On the grid, draw the image of

(i) **triangle *P*** after translation by $\begin{pmatrix} -4 \\ -5 \end{pmatrix}$, [2]

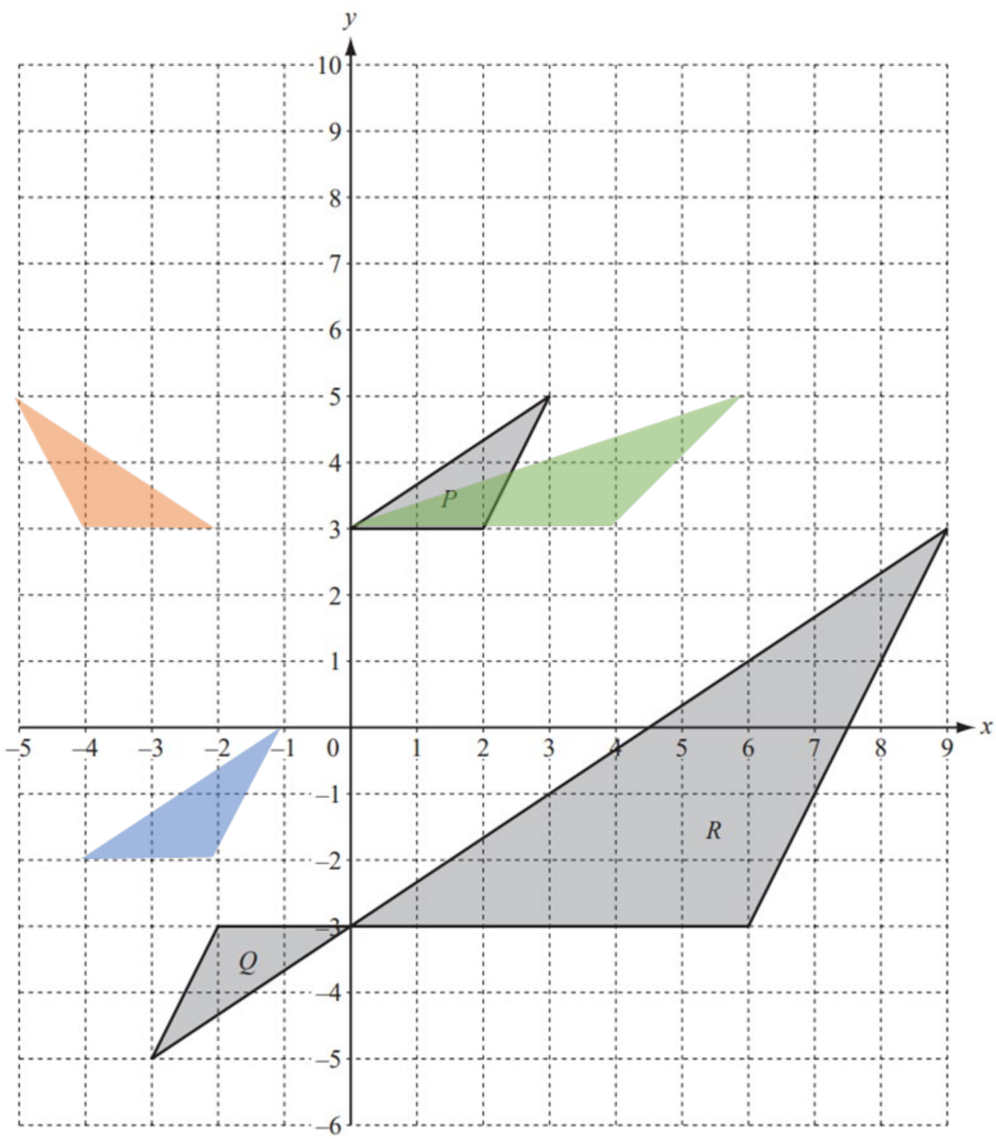
The blue triangle on the graph below.

(ii) **triangle *P*** after reflection in the line $x = -1$. [2]

The orange triangle on the graph below.

(c) (i) On the grid, draw the image of **triangle *P*** after a stretch, scale factor 2 and the y-axis as the invariant line. [2]

The green triangle on the graph below



(ii) Find the matrix which represents this stretch. [2]

We require that

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2x \\ y \end{pmatrix}$$

This is done with the matrix

$$\begin{pmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

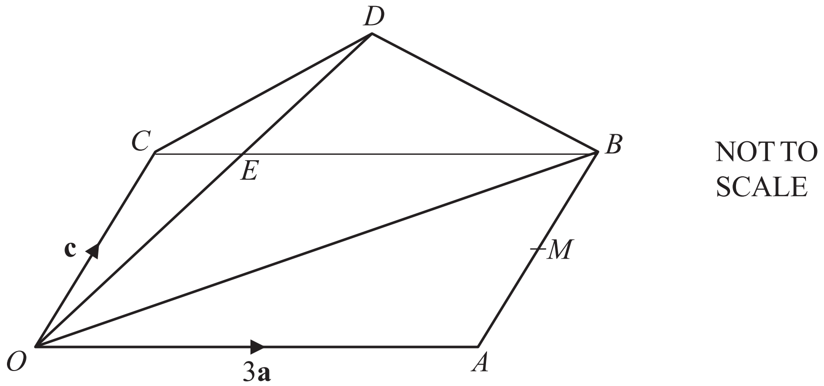
(a) P is the point (2, 5) and $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

Write down the co-ordinates of Q. [1]

$Q = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{5} \\ \mathbf{3} \end{pmatrix}$

(b)



O is the origin and $OABC$ is a parallelogram.
M is the midpoint of AB.

$\overrightarrow{OC} = \mathbf{c}$, $\overrightarrow{OA} = 3\mathbf{a}$ and $CE = \frac{1}{3}CB$.

OED is a straight line with $OE : ED = 2 : 1$.

Find in terms of **a** and **c**, in their simplest forms

(i) $\overrightarrow{OB}$, [1]

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \mathbf{3\vec{a} + \vec{c}}$

(ii) the position vector of M, [2]

$\vec{m} = \overrightarrow{OM}$

$= \mathbf{3\vec{a} + \frac{1}{2}\vec{c}}$

(iii) $\overrightarrow{OE}$, [1]

$$\overrightarrow{OE} = \vec{c} + \frac{1}{3}\overrightarrow{CB}$$

$$= \vec{\boldsymbol{c}} + \vec{\boldsymbol{a}}$$

(iv) $\overrightarrow{CD}$. [2]

$$\overrightarrow{CD} = -\vec{c} + \overrightarrow{OD}$$

$$\overrightarrow{OD} = \frac{3}{2}\overrightarrow{OE}$$

$$= \frac{3}{2}(\vec{c} + \vec{a})$$

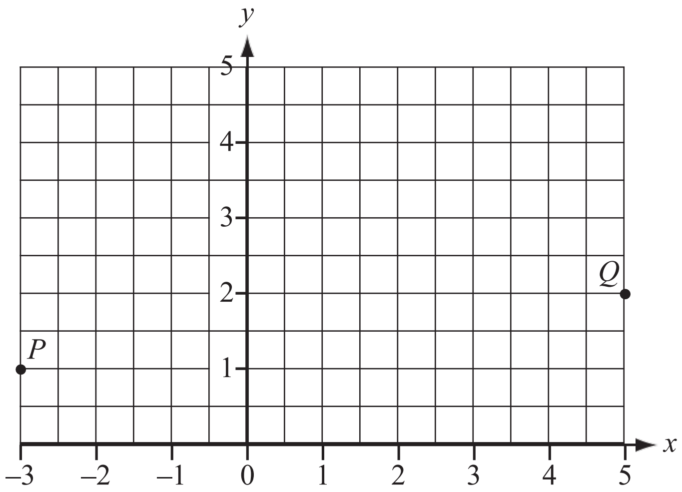
$$\rightarrow \overrightarrow{\boldsymbol{CD}} = \frac{\mathbf{1}}{\mathbf{2}}(\vec{\boldsymbol{c}} + \mathbf{3}\vec{\boldsymbol{a}})$$

(c) Write down two facts about the lines CD and OB. [2]

$\overrightarrow{CD}$ and $\overrightarrow{OB}$ are **parallel** and

$$\overrightarrow{\boldsymbol{CD}} = \frac{\mathbf{1}}{\mathbf{2}}\overrightarrow{\boldsymbol{OB}}$$

(a)



The points P and Q have co-ordinates (–3, 1) and (5, 2).

(i) Write $\overrightarrow{PQ}$ as a column vector. [1]

The column vector has the top number equivalent to the change in x of the line PQ and the bottom value equal to the change in y of the line PQ.

We can work out these values using the coordinates of the point P and Q.

Change in y = 2 – 1 = 1

Change in x = 5 – (-3) = 8

The column vector: $\overrightarrow{PQ} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$

(ii) $\overrightarrow{QR} = 2\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Mark the point R on the grid. [1]

$\overrightarrow{QR} = 2\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

Similar to point i), we can calculate each of the values as:

$-2 = x_R - x_Q$

$2 = y_R - y_Q$

Q(5, 2)

-2 = x_R – 5

x_R = 3

2 = y_R - 2

y_R = 4

R(3, 4)

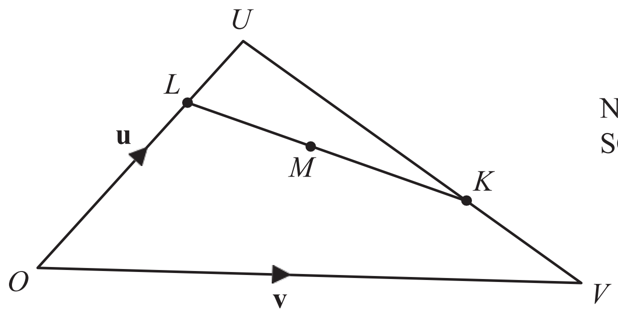
(iii) Write down the position vector of the point P. [1]

The position vector represents the coordinates of the point.

Point P has the coordinates x = -3 and y = 1

P = $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

(b)



NOT TO SCALE

In the diagram, $\overrightarrow{OU} = \mathbf{u}$ and $\overrightarrow{OV} = \mathbf{v}$.

K is on UV so that $\overrightarrow{UK} = \frac{2}{3}\overrightarrow{UV}$ and L is on OU so that $\overrightarrow{OL} = \frac{3}{4}\overrightarrow{OU}$.

M is the midpoint of KL.

Find the following in terms of $\mathbf{u}$ and $\mathbf{v}$, giving your answers in their simplest form.

(i) $\overrightarrow{LK}$ [4]

By looking at the figure, we can see that:

$\overrightarrow{LK} = \overrightarrow{LU} + \overrightarrow{UK}$

We know that $\overrightarrow{OL} = \frac{3}{4}\overrightarrow{OU}$,

Therefore:

$\overrightarrow{LU} = \frac{1}{4}\overrightarrow{OU} = \frac{1}{4}$ u

We know that $\overrightarrow{UK} = \frac{2}{3}\overrightarrow{UV}$,

$\overrightarrow{UV}$ = u + v

Therefore:

$\overrightarrow{UK} = \frac{2}{3}$ (v - u)

We obtain:

$\overrightarrow{LK} = \overrightarrow{LU} + \overrightarrow{UK}$

$\overrightarrow{LK} = \frac{1}{4}$ u + $\frac{2}{3}$ v - $\frac{2}{3}$ u

$\boldsymbol{\overrightarrow{LK} = \frac{-5}{12}}$ **u** $\boldsymbol{+ \frac{2}{3}}$ **v**

(ii) $\overrightarrow{OM}$ [2]

$\overrightarrow{OM} = \overrightarrow{OL} + \overrightarrow{LM}$

We know that $\overrightarrow{OL} = \frac{3}{4}\overrightarrow{OU}$, Therefore:

$\overrightarrow{OL} = \frac{3}{4}\overrightarrow{OU} = \frac{3}{4}$ u

M is the mid-point of KL, therefore:

$\overrightarrow{LM} = \frac{1}{2}\overrightarrow{LK}$

From b) i), we know that $\overrightarrow{LK} = \frac{-5}{12}$ u + $\frac{2}{3}$ v

We substitute $\overrightarrow{LK}$ in: $\overrightarrow{LM} = \frac{1}{2}\overrightarrow{LK}$

$\overrightarrow{LM} = \frac{1}{2}(\frac{-5}{12}$ u + $\frac{2}{3}$ v)

$\overrightarrow{LM} = \frac{-5}{24}$ u + $\frac{1}{3}$ v

$\overrightarrow{OM} = \overrightarrow{OL} + \overrightarrow{LM}$

$\overrightarrow{OM} = \frac{3}{4}$ u + $\frac{-5}{24}$ u + $\frac{1}{3}$ v

$\overrightarrow{OM} = \frac{13}{24}$ u + $\frac{1}{3}$ v

Vectors
Difficulty: Hard

Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 4

Time allowed: **87 minutes**

Score: **/76**

Percentage: **/100**

Grade Boundaries:

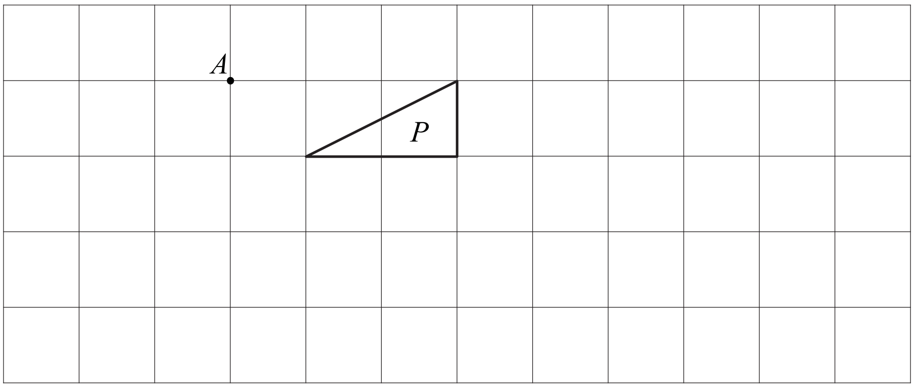
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

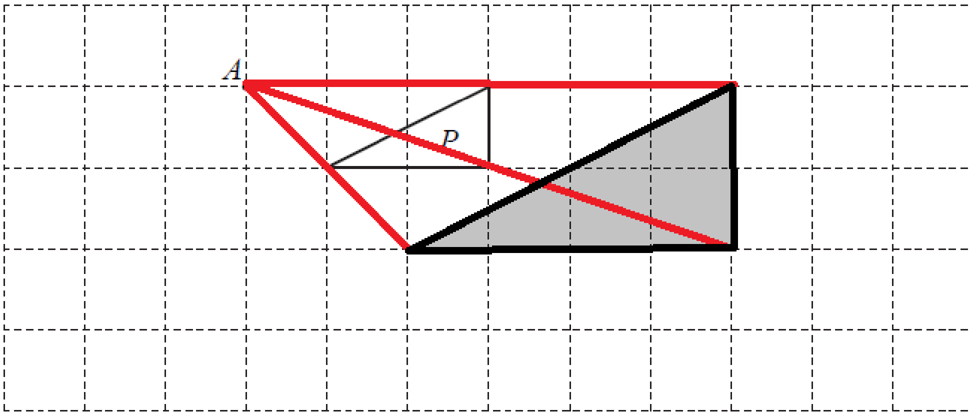
9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a)

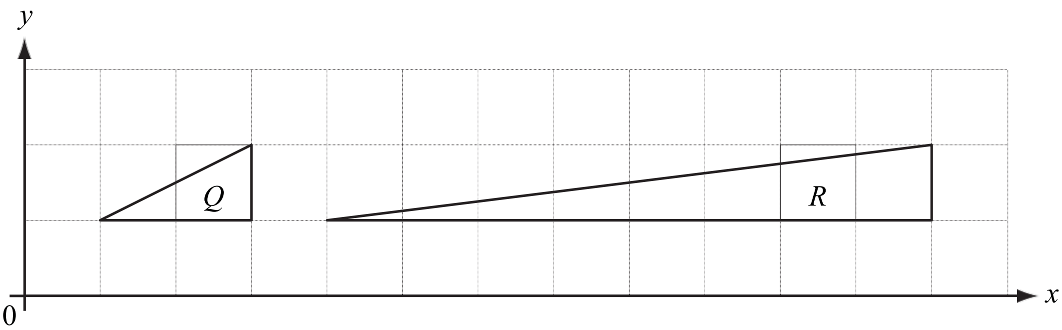


Draw the enlargement of triangle P with centre A and scale factor 2. [2]

To draw the enlargement, draw line from A in the direction of vertices of the triangle P, but then double the distance to get the scale factor of 2.



(b)



(i) Describe fully the **single** transformation which maps shape Q onto shape R. [3]

The arrows show translation of points:

$$(3,2)\ to\ (12,2)$$

$$(1,1)\ to\ (341)$$

$(3,1)\ to\ (12,1)$(blue arrow for clarity)

We can see that the *x* coordinate is multiplied by the factor 4.

The *y* coordinate on the other hand is unchanged.

Therefore this operations is:

Stretching by factor 4 with the *y*-axis invariant.

(ii) Find the matrix which represents this transformation. [2]

From part b)i) we know that the *x* coordinate of every point is multiplied by factor 4.

The *y* coordinate is unchanged (multiplied by factor 1)

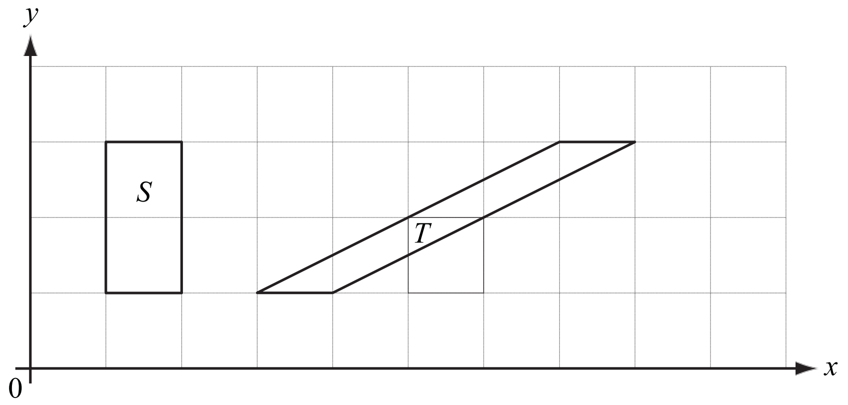
Hence we can construct a matrix for this transformation.

- Diagonal terms: 4 and 1 corresponding to the factors
- Only simple stretching so no non-diagonal terms involved

Matrix of transformation of Q to R:

$$\begin{pmatrix} \mathbf{4} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

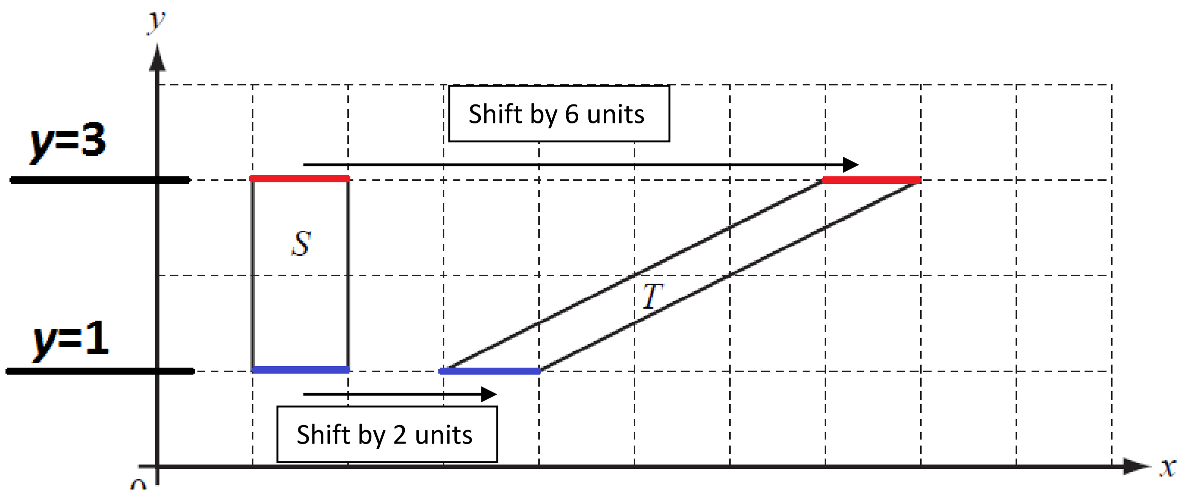
(c)



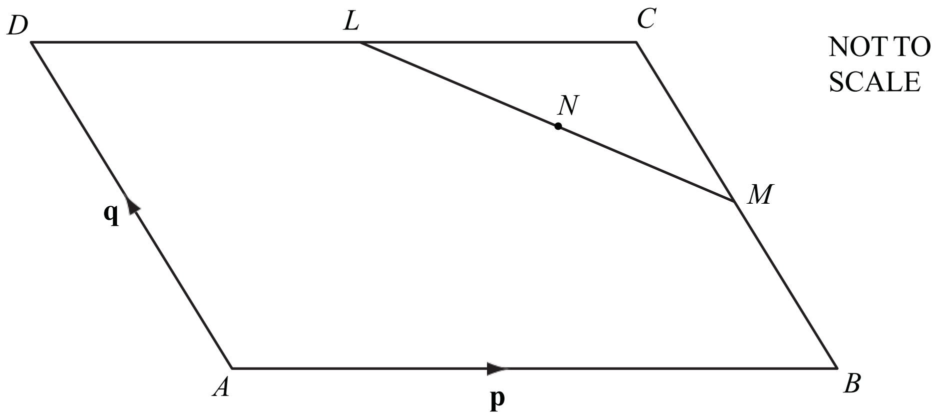
Describe fully the **single** transformation which maps shape *S* onto shape *T*. [3]

The bottom face is shifted by 2 units at *y*=1 and the top face is shifted by 6 units at *y*=3. As the ratio of the shift to the *y* value is fixed (equal to 2), we conclude that is operation is:

Shear only by factor 2 with the *x*-axis invariant.



(a)



ABCD is a parallelogram.
L is the midpoint of *DC*, *M* is the midpoint of *BC* and *N* is the midpoint of *LM*.
$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{q}$.

(i) Find the following in terms of **p** and **q**, in their simplest form. [1]

(a) $\overrightarrow{AC}$

The vector AC can be split into two vectors.

$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$$

Since ABCD is a parallelogram, we have:

$$\overrightarrow{AB} = \overrightarrow{DC} \quad and \quad \overrightarrow{AD} = \overrightarrow{BC}$$

Hence we can write the vector AC as:

$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{AB}$$

$$\boldsymbol{\overrightarrow{AC} = q + p}$$

(b) $\overrightarrow{LM}$ [2]

Similarly, the vector LM can be split into two vectors.

$$\overrightarrow{LM} = \overrightarrow{LC} + \overrightarrow{CM}$$

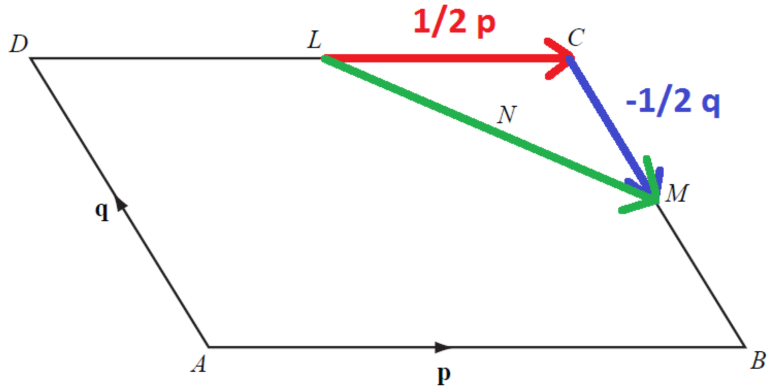
It is important to remember that for vectors:

$\overrightarrow{CM} = -\overrightarrow{MC}$.

We know that L and M are midpoints of DC and BC repsectivelly.

$$\overrightarrow{MC} = \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD} = \frac{1}{2}\boldsymbol{q}$$

$$\overrightarrow{LC} = \frac{1}{2}\overrightarrow{DC} = \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}\boldsymbol{p}$$



Putting all this together:

$$\overrightarrow{LM} = \overrightarrow{LC} - \overrightarrow{MC}$$

$$\overrightarrow{LM} = \frac{1}{2}\boldsymbol{p} - \frac{1}{2}\boldsymbol{q}$$

(c) $\overrightarrow{AN}$ [2]

N is the midpoint of LM, therefore:

$$\overrightarrow{LN} = \frac{1}{2}\overrightarrow{LM} = \frac{1}{2}(\frac{1}{2}p - \frac{1}{2}q)$$

$$\overrightarrow{LN} = \frac{1}{4}p - \frac{1}{4}q$$

The vector AN can be split into two vectors.

$$\overrightarrow{AN} = \overrightarrow{AL} + \overrightarrow{LN}$$

Vector AL can be split further into some vectors that we already know

$$\overrightarrow{AL} = \overrightarrow{AC} - \overrightarrow{LC}$$

Putting this together and substituting from previous parts

$$\overrightarrow{AN} = \overrightarrow{AC} - \overrightarrow{LC} + \overrightarrow{LN}$$

$$\overrightarrow{AN} = q + p - \frac{1}{2}p + \frac{1}{4}p - \frac{1}{4}q$$

By combining the vectors, find the vector AN in its simplest form.

$$\boldsymbol{\overrightarrow{AN} = \frac{3}{4}q + \frac{3}{4}p}$$

(ii) Explain why your answer for $\overrightarrow{AN}$ shows that the point N lies on the line AC. [1]

From part a)i)a), we have:

$$\overrightarrow{AC} = q + p$$

and then from a)i)c):

$$\overrightarrow{AN} = \frac{3}{4}q + \frac{3}{4}p$$

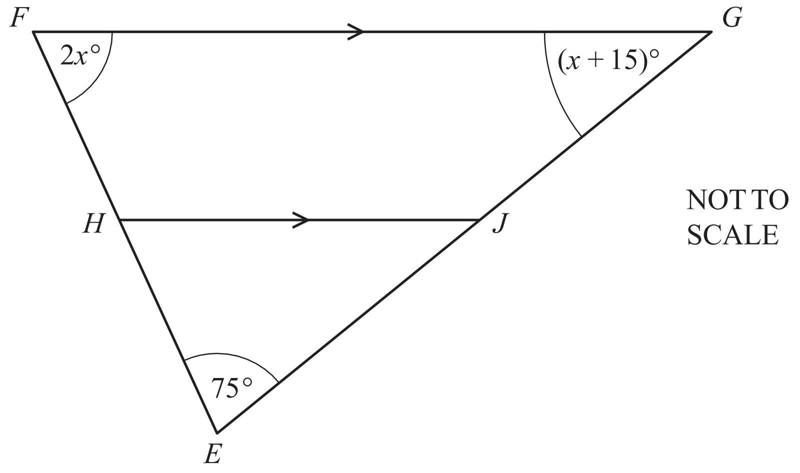
Hence we can write:

$$\overrightarrow{AN} = \frac{3}{4}\overrightarrow{AC}$$

Proving that:

The point N lies on the line AC (more precisely ¾ from A towards C)

(b)



EFG is a triangle.
HJ is parallel to FG.
Angle $FEG = 75°$.
Angle $EFG = 2x°$ and angle $FGE = (x + 15)°$.

(i) Find the value of x. [2]

The sum of all three internal angles of a triangle EFG must sum up to 180°.

$$180° = \measuredangle FEG + \measuredangle EGF + \measuredangle GFE$$

$$180° = 75° + (x + 15)° + 2x°$$

Subtract 90° from both sides.

$$90° = 3x°$$

Divide both sides by 3 to get the value of *x*.

$$\boldsymbol{x = 30}$$

(ii) Find angle HJG. [1]

The angles HJE and HJG are supplementary angles, they must sum up to 180°.

$$180° = \measuredangle HJE + \measuredangle HJG$$

As HJ is parallel to FG, the angles HJE and FGE are corresponding angles of the same size:

$$\measuredangle HJE = \measuredangle FGE$$

$$\measuredangle HJE = x + 15$$

Using the previous part of the question:

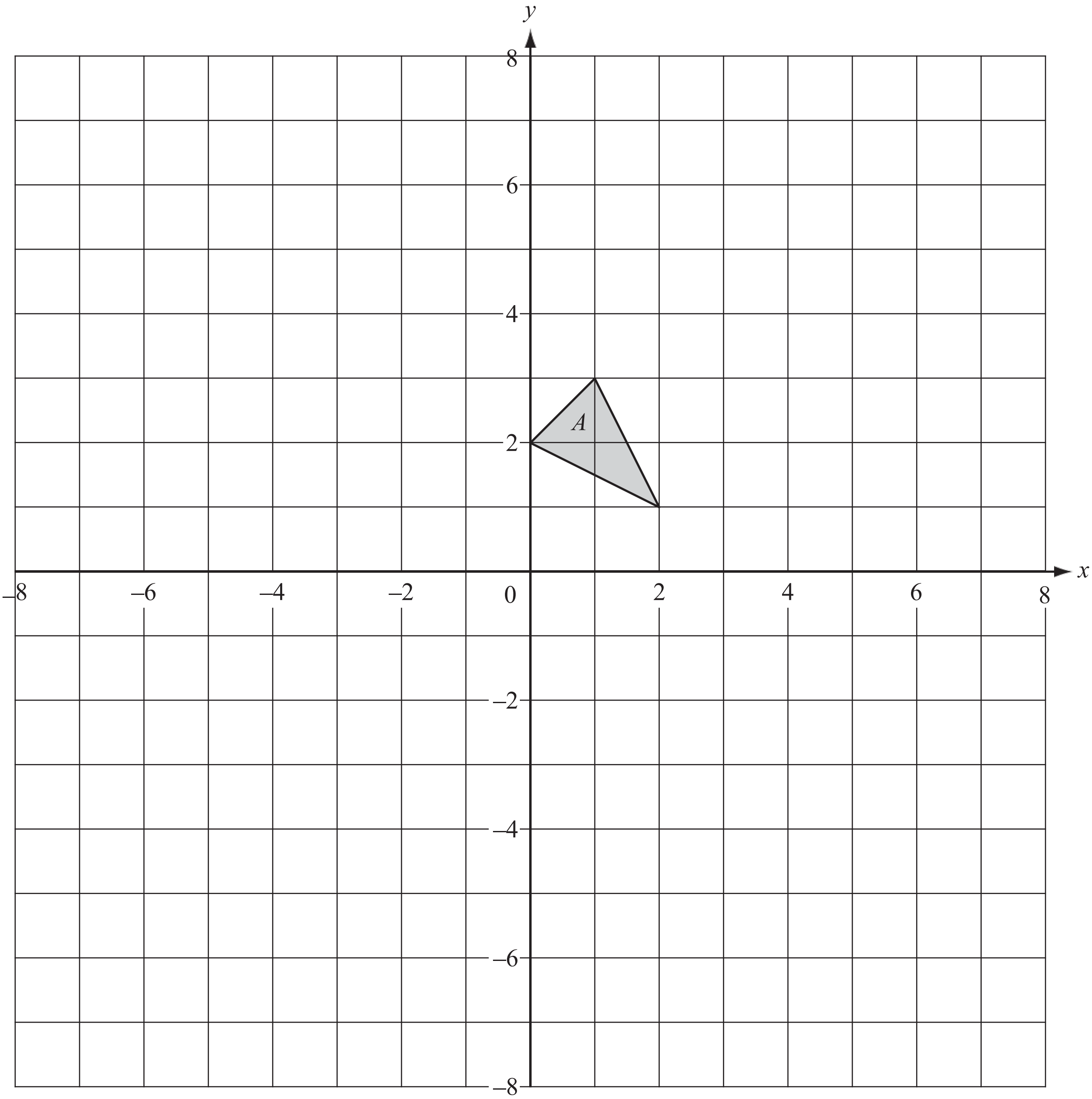
$$\measuredangle HJE = 45°$$

Substituting this result into the original equation, we can find the angle HJG.

$$180° = 45° + \measuredangle HJG$$

$$\boldsymbol{HJG = 135°}$$

(a)



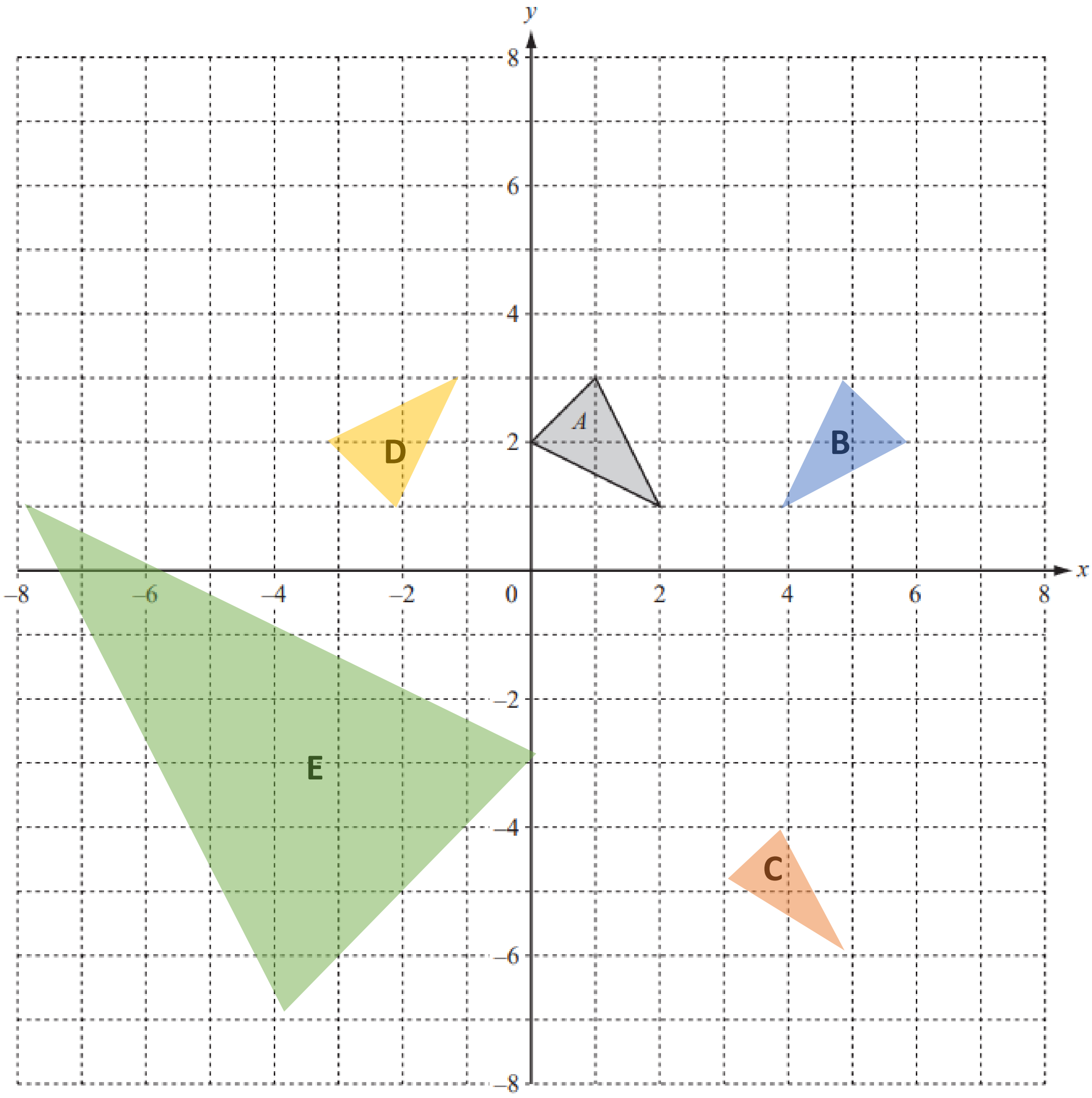
Draw the images of the following transformations on the grid above.

(i) Translation of triangle A by the vector $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$. Label the image B. [2]

(ii) Reflection of triangle A in the line $x = 3$. Label the image C. [2]

(iii) Rotation of triangle A through 90° anticlockwise around the point (0, 0).
Label the image D. [2]

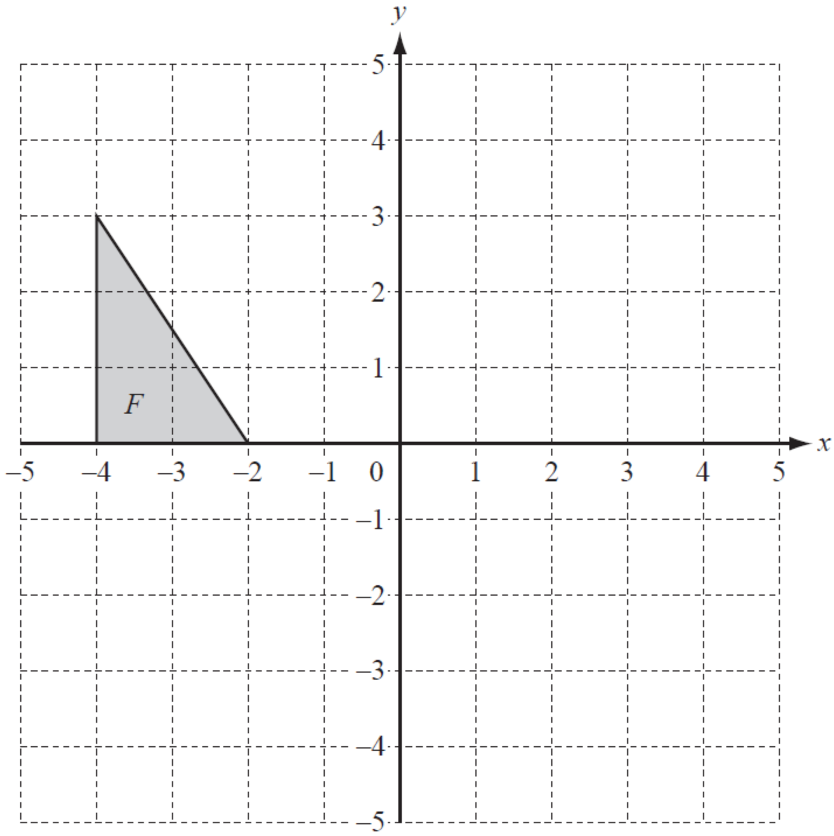
(iv) Enlargement of triangle A by scale factor –4, with centre (0, 1).
Label the image E. [2]


y
x
8
6
4
2
0
–2
–4
–6
–8
–8
–6
–4
–2
2
4
6
8
A
B
C
D
E

(b) The area of triangle E is $k \times$ area of triangle A. [1]
Write down the value of k.

16

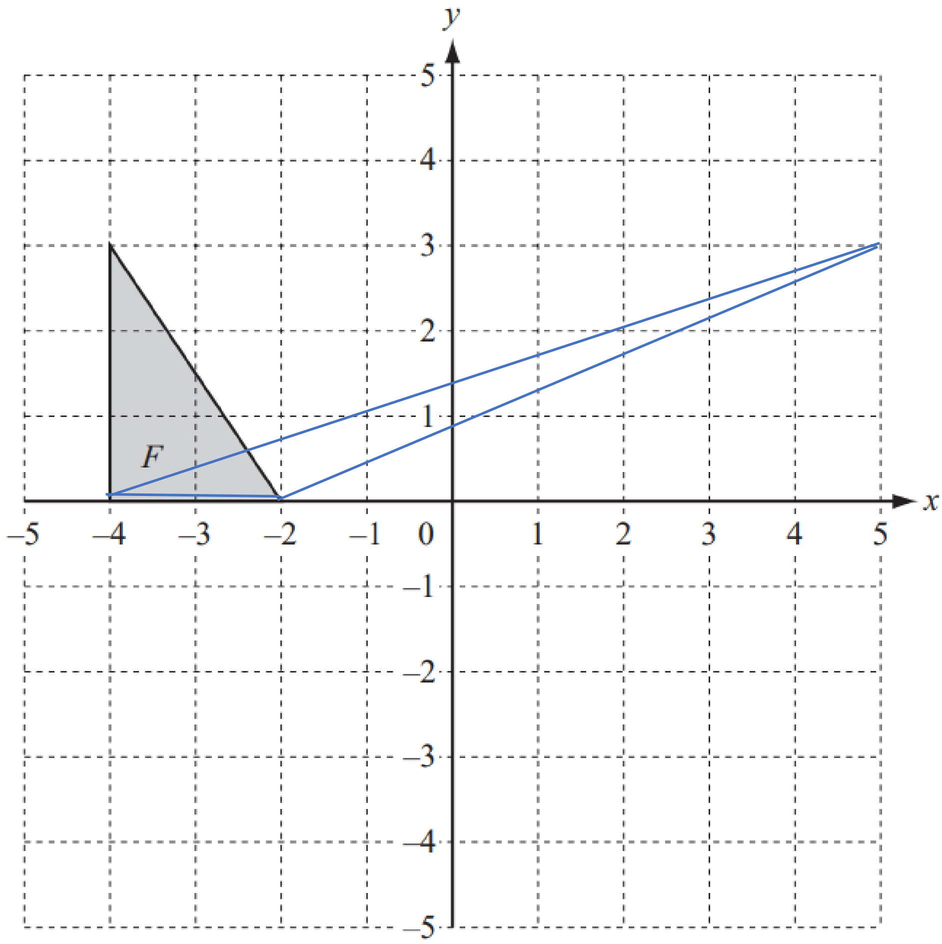
(c)



(i) Draw the image of triangle F under the transformation represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. [3]

We have

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 3y \\ y \end{pmatrix}$$



(ii) Describe fully this single transformation. [3]

Shear with x-axis invariant and scale factor 3

(iii) Find $\mathbf{M}^{-1}$, the inverse of the matrix $\mathbf{M}$. [2]

Determinant (Δ) of M

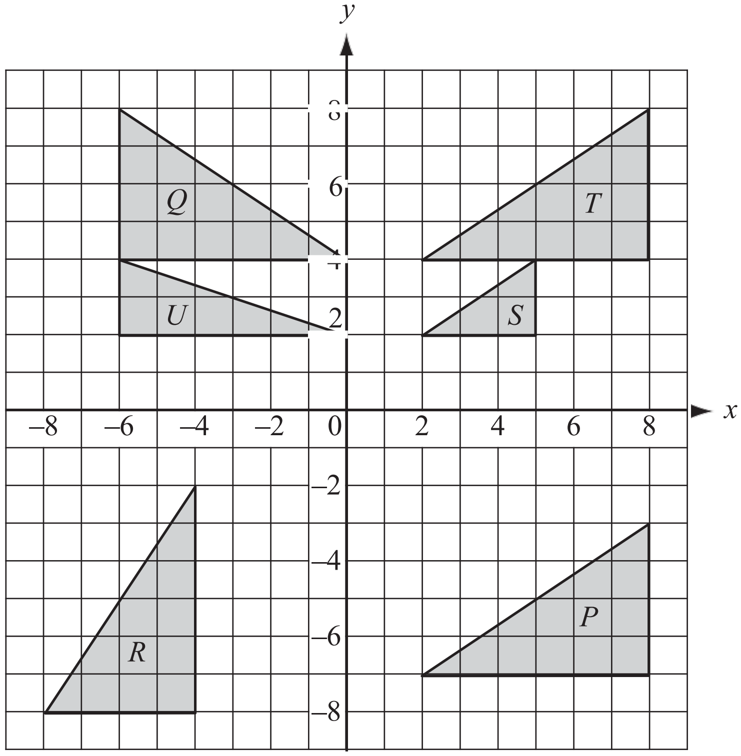
$$\Delta = 1 \times 1 - 0 \times 3$$

$$= 1$$

Now swap the elements of the leading diagonal and multiply the anti-diagonal by -1

$$M^{-1} = \frac{1}{\Delta}\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{1} & \mathbf{-3} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$



The diagram shows triangles P, Q, R, S, T and U.

(a) Describe fully the **single** transformation which maps triangle

(i) T onto P, [2]

Translation, 11 units, in the $-y$ direction = translation with vector $\begin{pmatrix} 0 \\ -11 \end{pmatrix}$.

(ii) Q onto T, [2]

Reflection in the line $x = 1$

(iii) T onto R, [2]

Reflection in the line $y = -x$

(iv) T onto S, [3]

Enlargement, scale factor 0.5, centre (2, 0)

(v) U onto Q. [3]

Stretch, scale factor 2, parallel to the y-axis (x-axis invariant).

(b) Find the 2 by 2 matrix representing the transformation which maps triangle

(i) T onto R, [2]

We require

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ -x \end{pmatrix}$$

This is done by

$$\begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$$

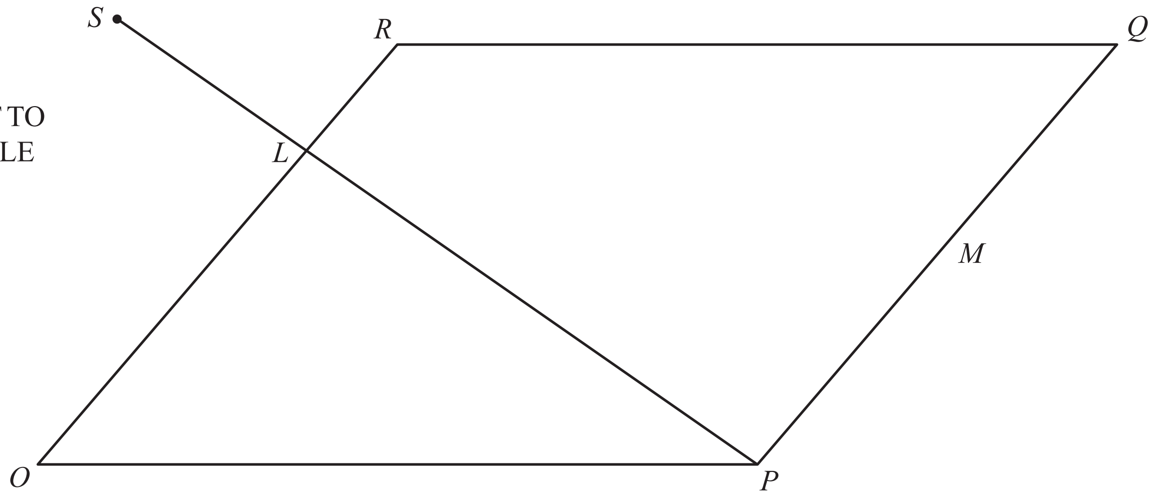
(ii) U onto Q. [2]

We require

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ 2y \end{pmatrix}$$

This is done by

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} \end{pmatrix}$$



$OPQR$ is a parallelogram.
O is the origin.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.
M is the mid-point of PQ and L is on OR such that $OL : LR = 2 : 1$.
The line PL is extended to the point S.

(a) Find, in terms of $\mathbf{p}$ and $\mathbf{r}$, in their simplest forms,

(i) $\overrightarrow{OQ}$, [1]

$$\overrightarrow{OQ} = \vec{p} + \vec{r}$$

(ii) $\overrightarrow{PR}$, [1]

$$\overrightarrow{PR} = -\vec{p} + \vec{r}$$

(iii) $\overrightarrow{PL}$, [1]

$$\overrightarrow{PL} = -\vec{p} + \frac{2}{3}\vec{r}$$

(iv) the position vector of M. [1]

$$\overrightarrow{OM} = \vec{p} + \frac{1}{2}\vec{r}$$

(b) PLS is a straight line and $PS = \frac{3}{2}PL$.

Find, in terms of **p** and/or **r**, in their simplest forms,

(i) $\overrightarrow{PS}$, [1]

$$\overrightarrow{PS} = \frac{3}{2}\overrightarrow{PL}$$

$$= \frac{3}{2}\left(-\vec{p} + \frac{2}{3}\vec{r}\right)$$

$$= -\frac{3}{2}\vec{p} + \vec{r}$$

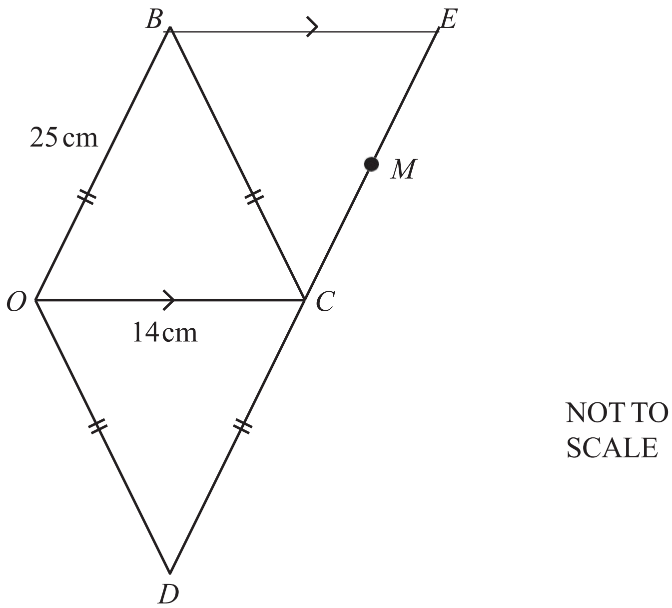
(ii) $\overrightarrow{QS}$. [2]

$$\overrightarrow{QS} = -\vec{r} + \left(-\frac{3}{2}\vec{p} + \vec{r}\right)$$

$$= -\frac{3}{2}\vec{p}$$

(c) What can you say about the points Q, R and S? [1]

QRS form a straight line (colinear)



OBCD is a rhombus with sides of 25 cm. The length of the diagonal *OC* is 14 cm.

(a) Show, **by calculation**, that the length of the diagonal *BD* is 48 cm. [3]

BD is the diagonal in the OBCD rhombus.

BD is therefore perpendicular on OC and passes through the middle of the line OC.

In the right-angled triangle formed with BD perpendicular on OC and OB as hypothenuse,

we can use Pythagoras' Theorem:

$(BD/2)^2 + (OC/2)^2 = OB^2$

$OC/2 = 7$ cm

$(BD/2)^2 + 7^2 = 25^2$

$BD/2 = 24$

BD = 48 cm

(b) Calculate, correct to the nearest degree,

(i) angle *BCD*, [2]

OB = BC in triangle OBC, therefore, angle BCO = angle BOC.

Sin BOC = $\frac{\mathrm{BD}/2}{\mathrm{BO}}$

Sin BOC = $\frac{24}{25}$

Angle BOC = angle BCO = 73.73°

Angle BCD = 2 x angle BCO

Angle BCD = 2 x 73.73°

Angle BCD = 147°

(ii) angle *OBC*. [1]

Angle OBC = 180° - 2 x angle BCO

Angle OBC = 180° - 147°

Angle OBC = 33°

(c) $\overrightarrow{DB} = 2\mathbf{p}$ and $\overrightarrow{OC} = 2\mathbf{q}$.
Find, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(i) $\overrightarrow{OB}$, [1]

$\overrightarrow{OB} = \frac{1}{2}\overrightarrow{DB} + \frac{1}{2}\overrightarrow{OC}$

$\overrightarrow{OB}$ = **p + q**

(ii) $\overrightarrow{OD}$. [1]

$\overrightarrow{OD} = \frac{1}{2}\overrightarrow{BD} + \frac{1}{2}\overrightarrow{OC}$

$\overrightarrow{BD} = -\overrightarrow{DB} = -2\mathrm{p}$

$\overrightarrow{OD}$ = **-p + q**

(d) BE is parallel to OC and DCE is a straight line.
Find, in its simplest form, $\overrightarrow{OE}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. [2]

$\overrightarrow{OE} = \overrightarrow{CE} + \overrightarrow{OC}$

$\overrightarrow{CE} = \overrightarrow{OB} = \mathrm{p} + \mathrm{q}$

$\overrightarrow{OE} = \mathrm{p} + \mathrm{q} + 2\mathrm{q}$

$\overrightarrow{OE}$ = **p + 3q**

(e) M is the mid-point of CE.

Find, in its simplest form, $\overrightarrow{OM}$ in terms of **p** and **q**. [2]

$\overrightarrow{OM} = \overrightarrow{CM} + \overrightarrow{OC}$

$\overrightarrow{CM} = \frac{1}{2}\overrightarrow{CE}$

$\overrightarrow{CM} = \frac{1}{2}(p + q)$

$\overrightarrow{OM} = \frac{1}{2}(p + q) + 2q$

$\boldsymbol{\overrightarrow{OM} = \frac{1}{2}p + \frac{5}{2}q}$

(f) O is the origin of a co-ordinate grid. OC lies along the x-axis and $\mathbf{q} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$.

($\overrightarrow{DB}$ is vertical and $|\overrightarrow{DB}| = 48$.)

Write down as column vectors

(i) **p**, [1]

$\overrightarrow{DB}$ = 2p

p = $|\overrightarrow{DB}|/2$

p is a vertical line of size 24.

As a column vector: $\begin{pmatrix} \mathbf{0} \\ \mathbf{24} \end{pmatrix}$

(ii) $\overrightarrow{BC}$. [2]

$\overrightarrow{BC} = \overrightarrow{OD}$ = -p + q

$\overrightarrow{BC} = \begin{pmatrix} 0 \\ -24 \end{pmatrix} + \begin{pmatrix} 7 \\ 0 \end{pmatrix}$

$\boldsymbol{\overrightarrow{BC} = \begin{pmatrix} 7 \\ -24 \end{pmatrix}}$

(g) Write down the value of $|\overrightarrow{DE}|$. [1]

$\overrightarrow{DE} = \overrightarrow{CE} + \overrightarrow{DC} = 2 \times \overrightarrow{DC}$

$|\overrightarrow{DC}| = 25$

$|\overrightarrow{DE}|$ **= 25 x 2 = 50**

Vectors
Difficulty: Hard

Model Answers 5

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 5

Time allowed: **90 minutes**

Score: **/78**

Percentage: **/100**

Grade Boundaries:

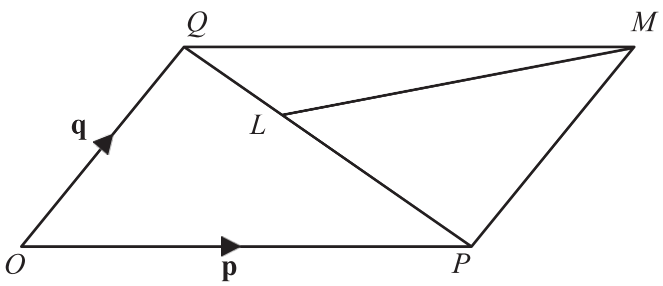
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a)



OPMQ is a parallelogram and *O* is the origin.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.
L is on *PQ* so that *PL* : *LQ* = 2:1.

Find the following vectors in terms of **p** and **q**. Write your answers in their simplest form.

(i) $\overrightarrow{PQ}$, [1]

$\overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$

(ii) $\overrightarrow{PL}$, [1]

$\overrightarrow{PL} = \frac{2}{3}\overrightarrow{PQ}$

$\overrightarrow{PL} = \frac{2}{3}(\mathrm{q} - \mathrm{p})$

$\overrightarrow{PL} = \frac{2}{3}\mathbf{q} - \frac{2}{3}\mathbf{p}$

(iii) $\overrightarrow{ML}$, [2]

$\overrightarrow{ML} = \overrightarrow{MP} + \overrightarrow{PL}$

$\overrightarrow{MP} = -\ \overrightarrow{OQ} = -\mathrm{q}$

$\overrightarrow{ML} = -\mathrm{q} + \frac{2}{3}(\mathrm{q} - \mathrm{p})$

$\overrightarrow{ML} = -\mathrm{q} + \frac{2}{3}\mathrm{q} - \frac{2}{3}\mathrm{p}$

$\overrightarrow{ML} = -\frac{1}{3}\mathbf{q} - \frac{2}{3}\mathbf{p}$

(iv) the position vector of L. [2]

The position vector of L is the vector $\overrightarrow{OL}$.

$\overrightarrow{OL} = \overrightarrow{OP} + \overrightarrow{PL}$

$\overrightarrow{OL} = \mathbf{p} + \frac{2}{3}\mathbf{q} - \frac{2}{3}\mathbf{p} = \frac{1}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}$

(b) R is the point (1,2). It is translated onto the point S by the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

(i) Write down the co-ordinates of S. [1]

$x_S = x_R + 3$ and $y_S = x_s - 4$

$x_S = 1 + 3 = 4$

$y_S = 2 - 4 = -2$

(ii) Write down the vector which translates S onto R. [1]

We represent the column vector which translates S onto R with the unknown $\begin{pmatrix} a \\ b \end{pmatrix}$.

$x_R = x_S + a$ and $y_R = y_s + b$

1 = 4 + a and 2 = -2 + b

a = -3 and b = 4

The column vector is: $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

(c) The matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents a **single** transformation.

(i) Describe fully this transformation. [3]

The matrix represents a clockwise 90° rotation around the centre (0, 0).

(ii) Find the co-ordinates of the image of the point (5, 3) after this transformation. [1]

The point of coordinates (5, 3) can be written as: $\binom{5}{3}$.

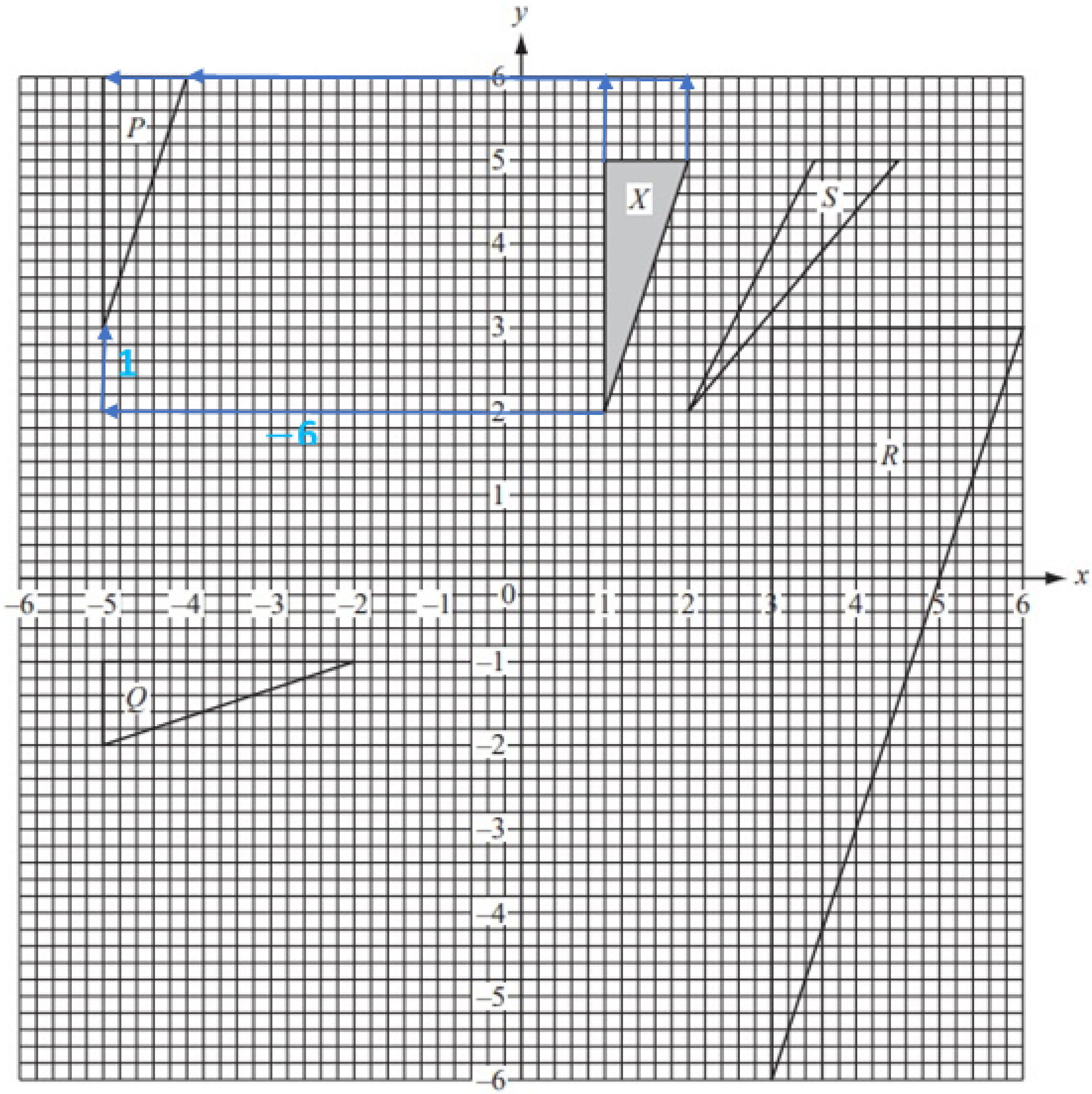
We multiply this column matrix by the rotation matrix to obtain the coordinates of the image.

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \times 5 + 1 \times 3 \\ -1 \times 5 + 0 \times 3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} \mathbf{3} \\ \mathbf{-5} \end{pmatrix}$$

(d) Find the matrix which represents a reflection in the line $y = x$. [2]

The reflection matrix across x = y is: $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$



(a) Describe fully the single transformation which maps

(i) triangle X onto triangle P, [2]

The transformation which maps triangle X onto triangle P is a translation, since the triangle X is moved up and to the left, but the shape remains the same. Every point in the triangle X is translated the same distance and in the same direction onto triangle P.

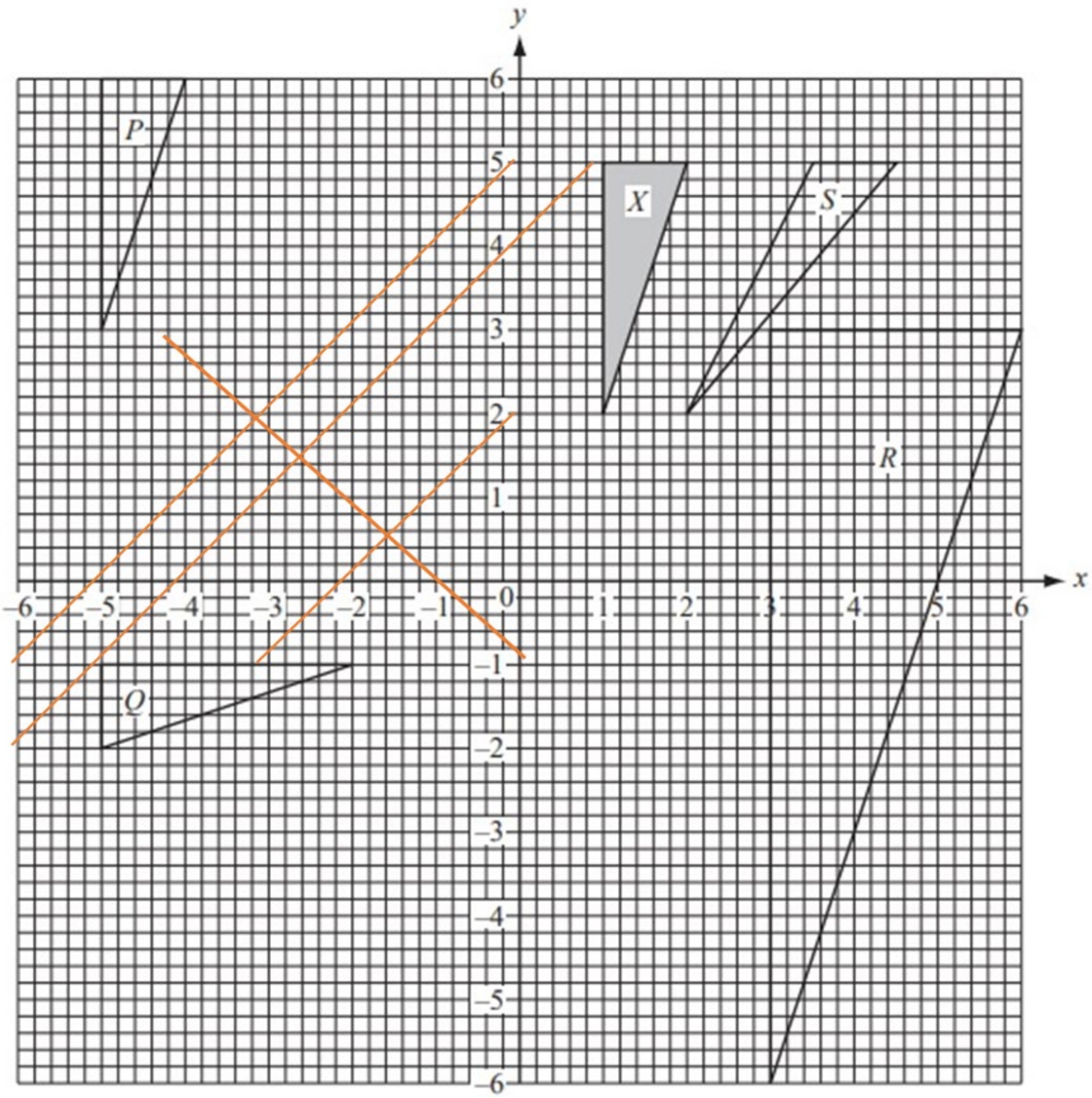
We use positive numbers to describe a translation up or the right and negative numbers to describe a translation down or to the left.

In a column vector, the upper value represents a movement to the left or right while the lower value represents a movement either up or down.

In our case, triangle X is translated by 6 squares to the left and by one square up.

The vector describing this translation is: $\begin{pmatrix} -6 \\ 1 \end{pmatrix}$

(ii) triangle X onto triangle Q, [2]



The transformation which maps triangle X onto triangle Q is a **reflection**. In our case, we define the reflection across the mirror line going through the origin and through the point of coordinates (-1, -1)

Every point in triangle X is at the same distance form the mirror line as every corresponding point in triangle P. Also, these distances are perpendicular on the mirror line.

To describe the reflection, we need to work out the equation of the mirror line.

The equation of a line takes up the form:

y = mx + n

where m is the gradient of the line and n is the y-intercept

In our case, since the line is passing through the origin, the y-intercept is n = 0.

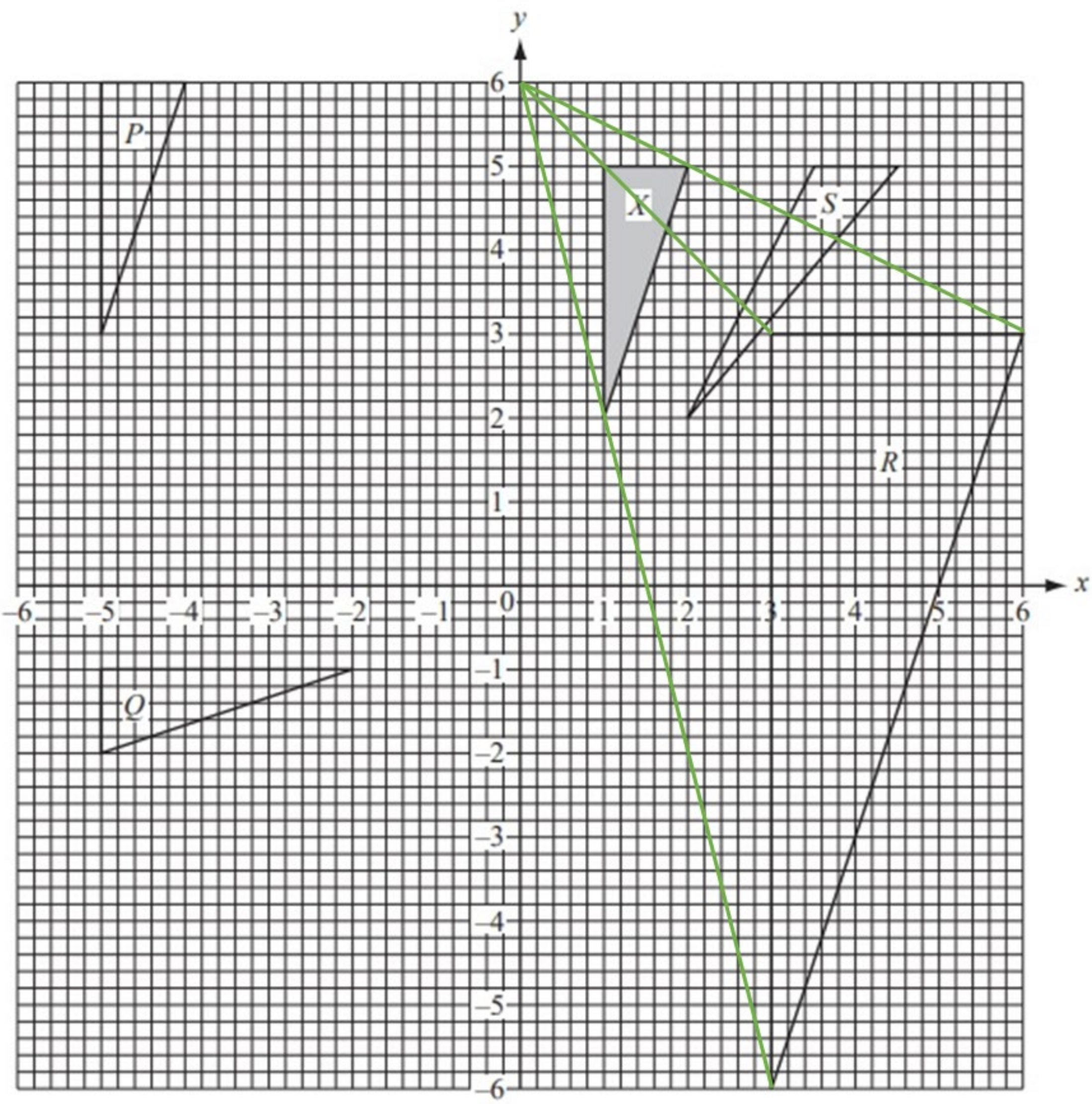
m = $\frac{\text{change in y}}{\text{change in x}}$

m = $\frac{1}{-1}$

m = -1

The equation of the line is y = -x.

(iii) triangle X onto triangle R, [3]



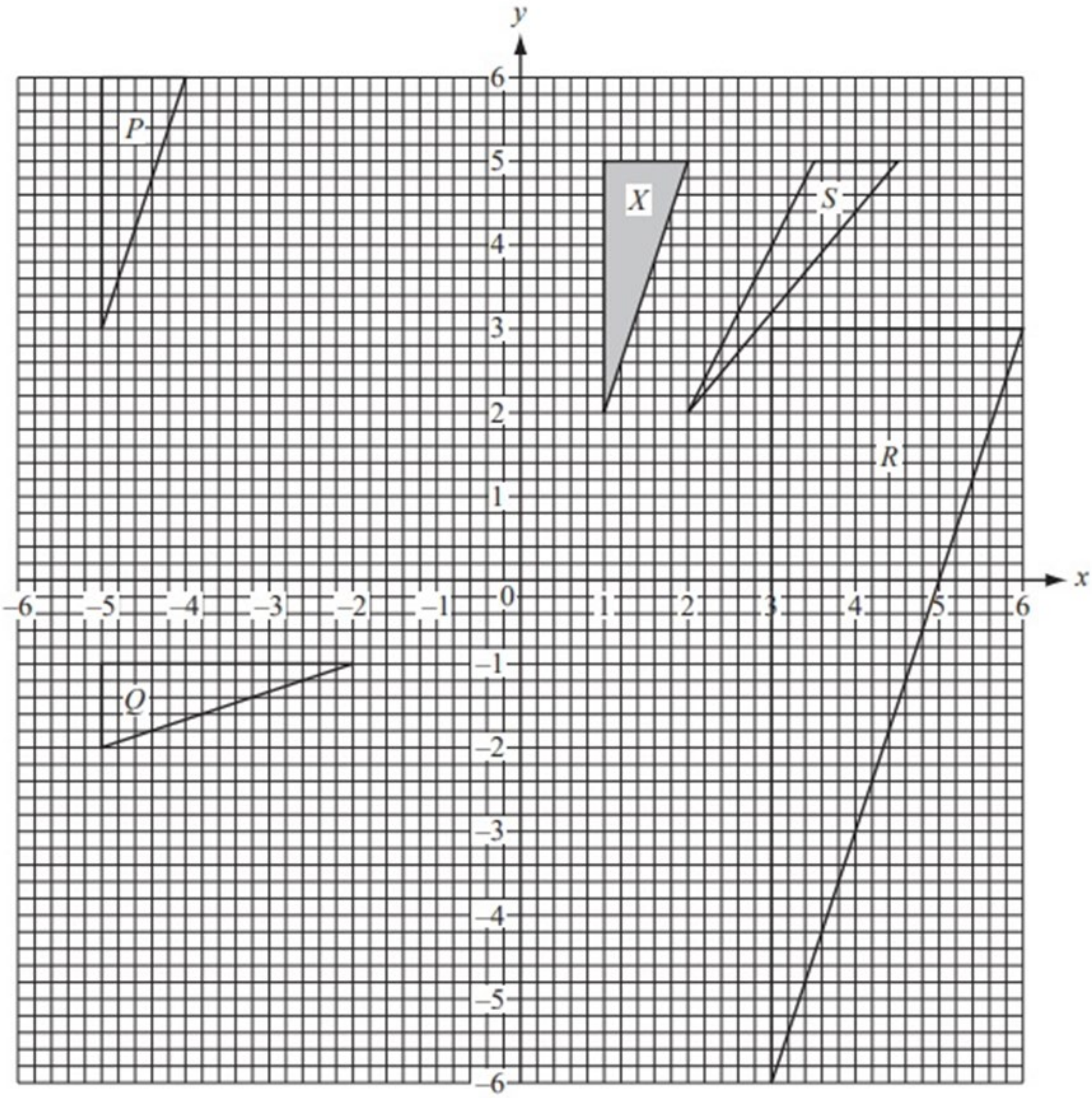
The transformation which maps triangle X onto triangle R is an **enlargement**, since the difference between the 2 triangles is the size of the shape. To describe an enlargement, we need to work out the scale factor and the centre of enlargement.

Each side of triangle R is 3 times the size of the corresponding side of triangle X, therefore the scale factor = 3.

The centre of enlargement is the point from which the distance to a point in triangle X is multiplied by the scale factor to obtain the size of the distance from the centre of enlargement to the corresponding point in triangle P.

In this case, the centre of enlargement has the coordinates (0, 6)

(iv) triangle X onto triangle S. [3]



The transformation which maps triangle X onto triangle S is a shear transformation. The y coordinates of triangle X are the same with the y coordinates of the corresponding points in triangle S, therefore, $y = 0$ is the invariant.

$x' = x + ky$

where x' is the x coordinate of the image

x is the x coordinate of the initial figure

k is the shear factor

and y is the y coordinate of the initial figure.

For one of the points in triangle X:

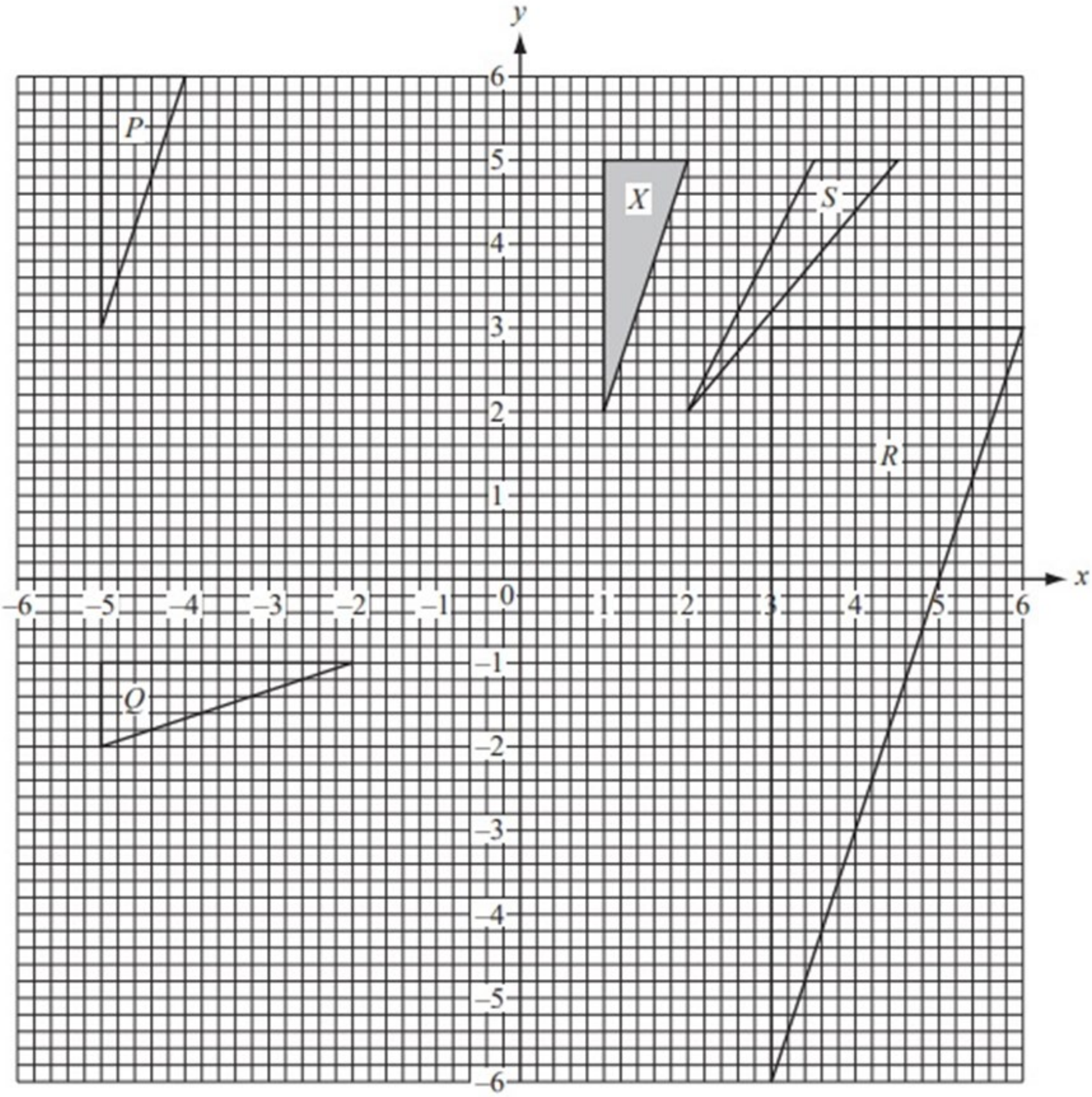
$4.5 = 2 + 5k$

$5k = 2.5$

$k = 0.5$

(b) Find the 2 by 2 matrix which represents the transformation that maps

(i) triangle X onto triangle Q, [2]



A 2 by 2 matrix which maps triangle X onto triangle Q is:

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

This is because we notice that the x-coordinates of each point in triangle X is equal to the x-coordinates of the corresponding point multiplied by -1.

(ii) triangle X onto triangle S. [2]

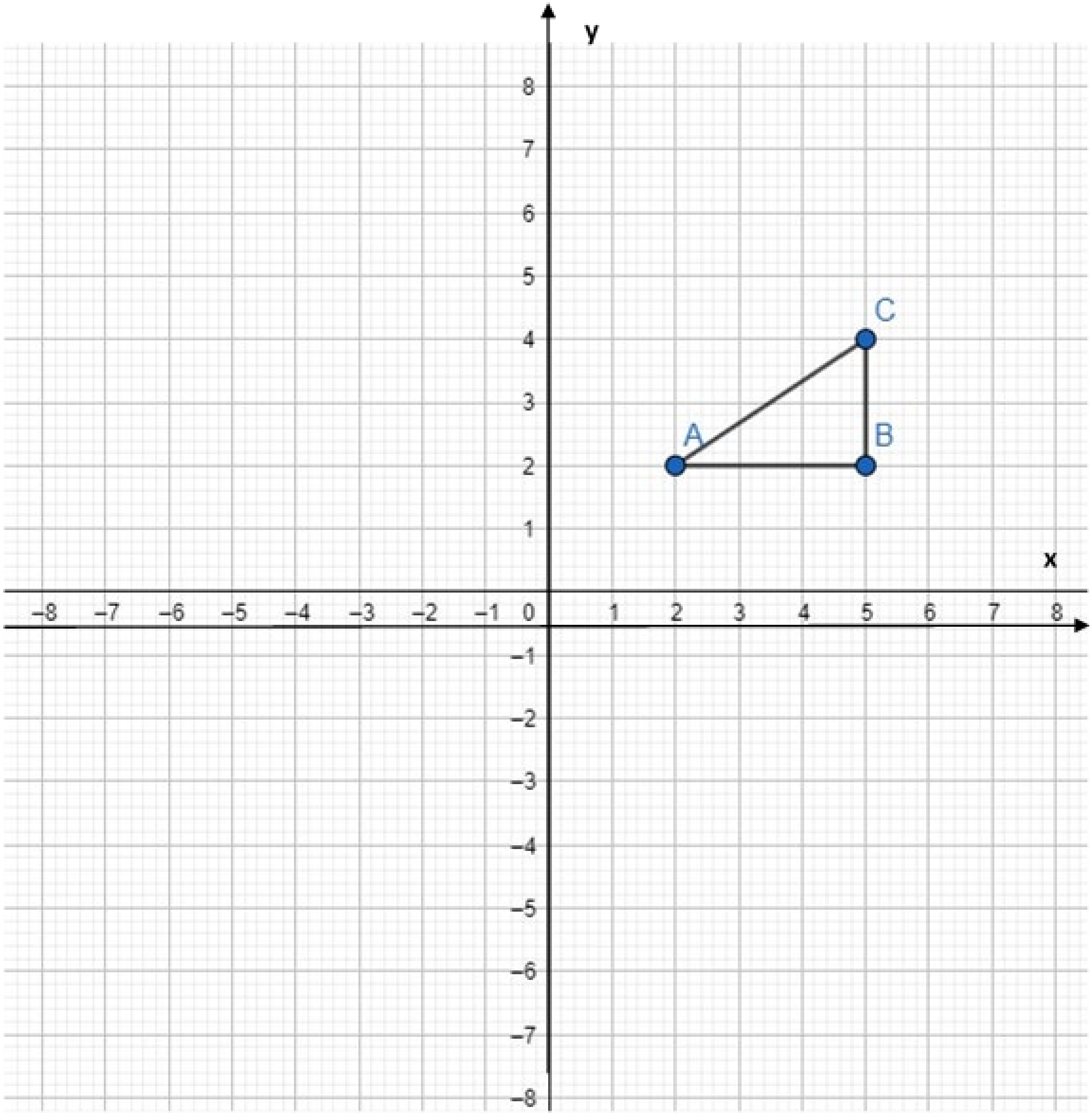
A 2 by 2 matrix which maps triangle X onto triangle Q is:

$$\begin{pmatrix} 1 & 0.5 \\ 0 & 1 \end{pmatrix}$$

This is because the transformation is a shear with constant factor 0.5 and the invariant y = 0.

Answer the whole of this question on a sheet of graph paper.

(a) Draw x- and y-axes from –8 to 8 using a scale of 1cm to 1 unit.
Draw triangle ABC with A (2, 2), B (5, 2) and C (5, 4). [2]



(b) Draw the image of triangle ABC under a translation of $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$.

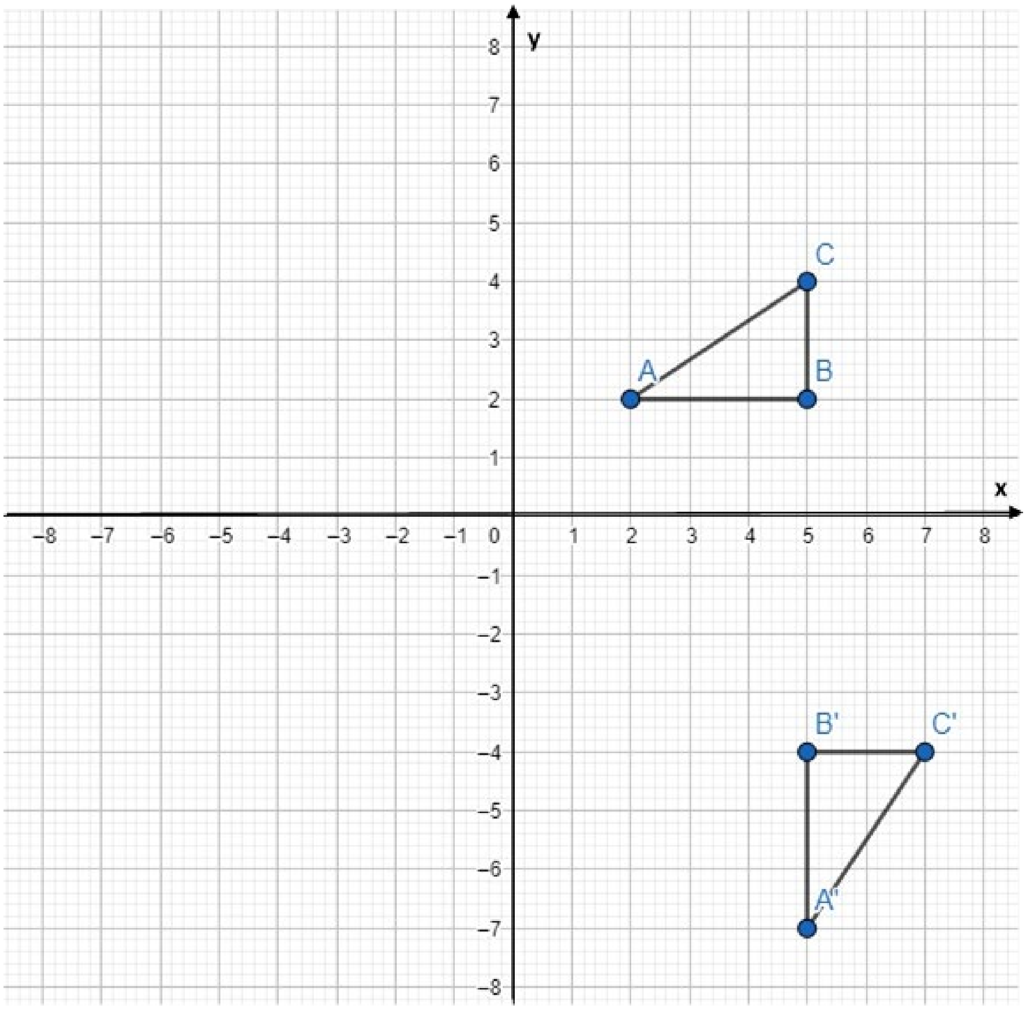
Label it $A_1B_1C_1$. [2]

A translation represents a transformation which moves the changes the position of the triangle but not the size or shape.

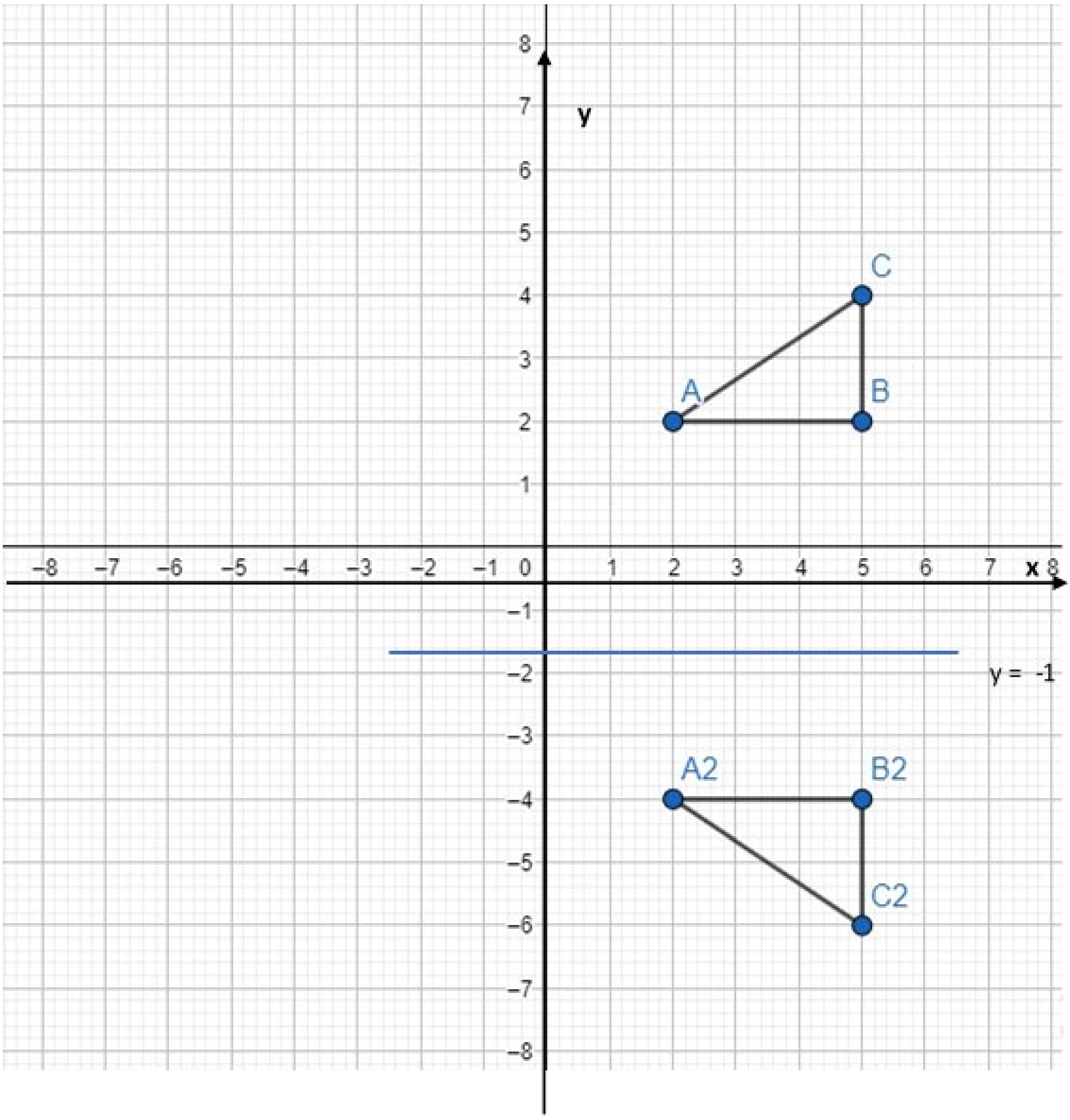
The upper number of the translation vector represents a movement either left or right and the lower number in the vector represents the movement either up or down. Positive values mean a movement t either up or right and negative values mean a movement either down or to the left.

In our case, the triangle will be moved 9 units to the left and 3 units up.

Taking this into account, the image A'B'C' is drawn on the grid below.



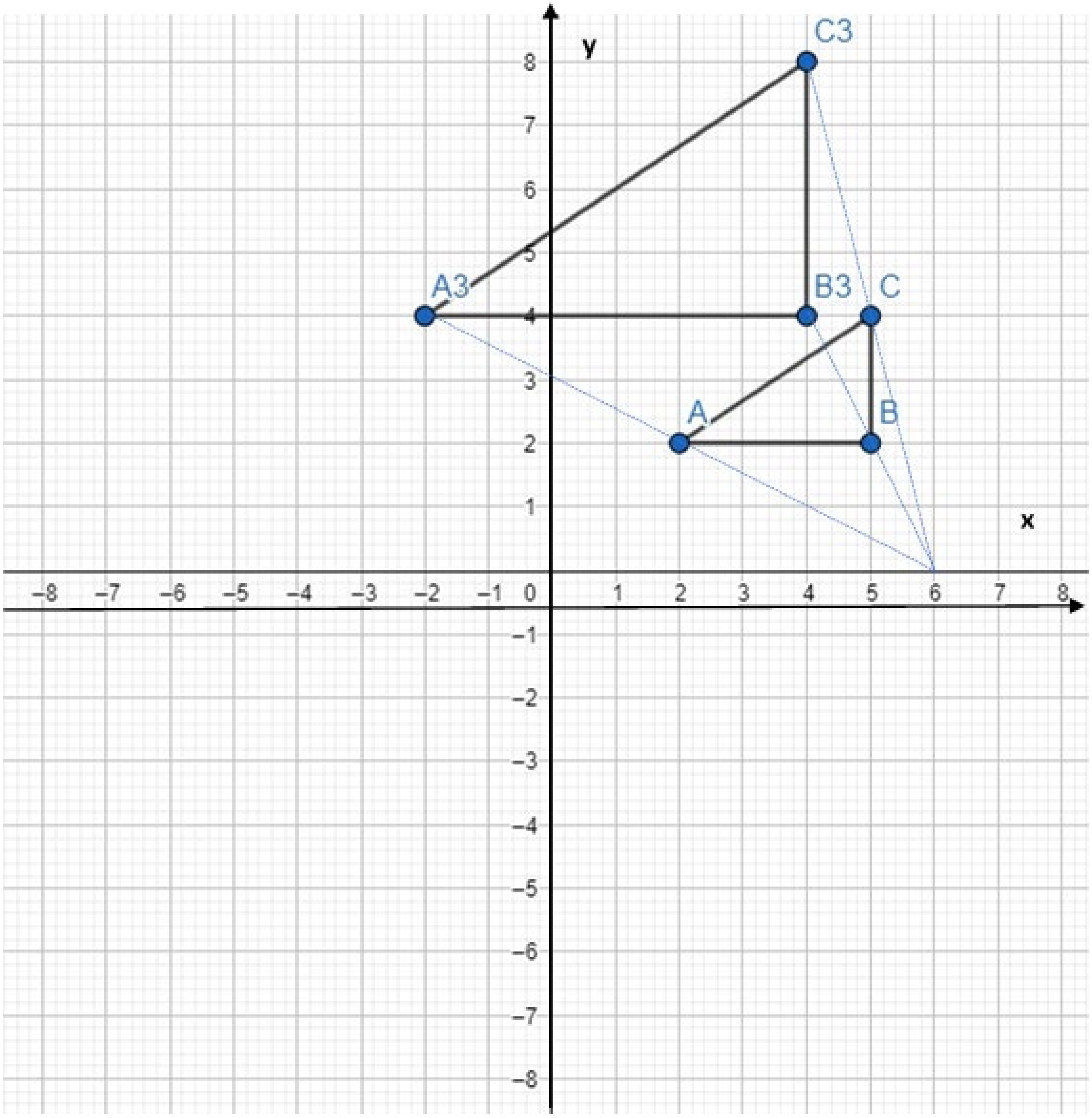
(c) Draw the image of triangle ABC under a reflection in the line $y = -1$. [2]
Label it $A_2B_2C_2$.



In our case, we define the reflection across the mirror line of equation y = -1

Every point in triangle ABC is at the same distance from the mirror line as every corresponding point in triangle A2B2C2. Also, these distances are perpendicular on the mirror line.

(d) Draw the image of triangle ABC under an enlargement, scale factor 2, centre (6,0). Label it $A_3B_3C_3$. [2]

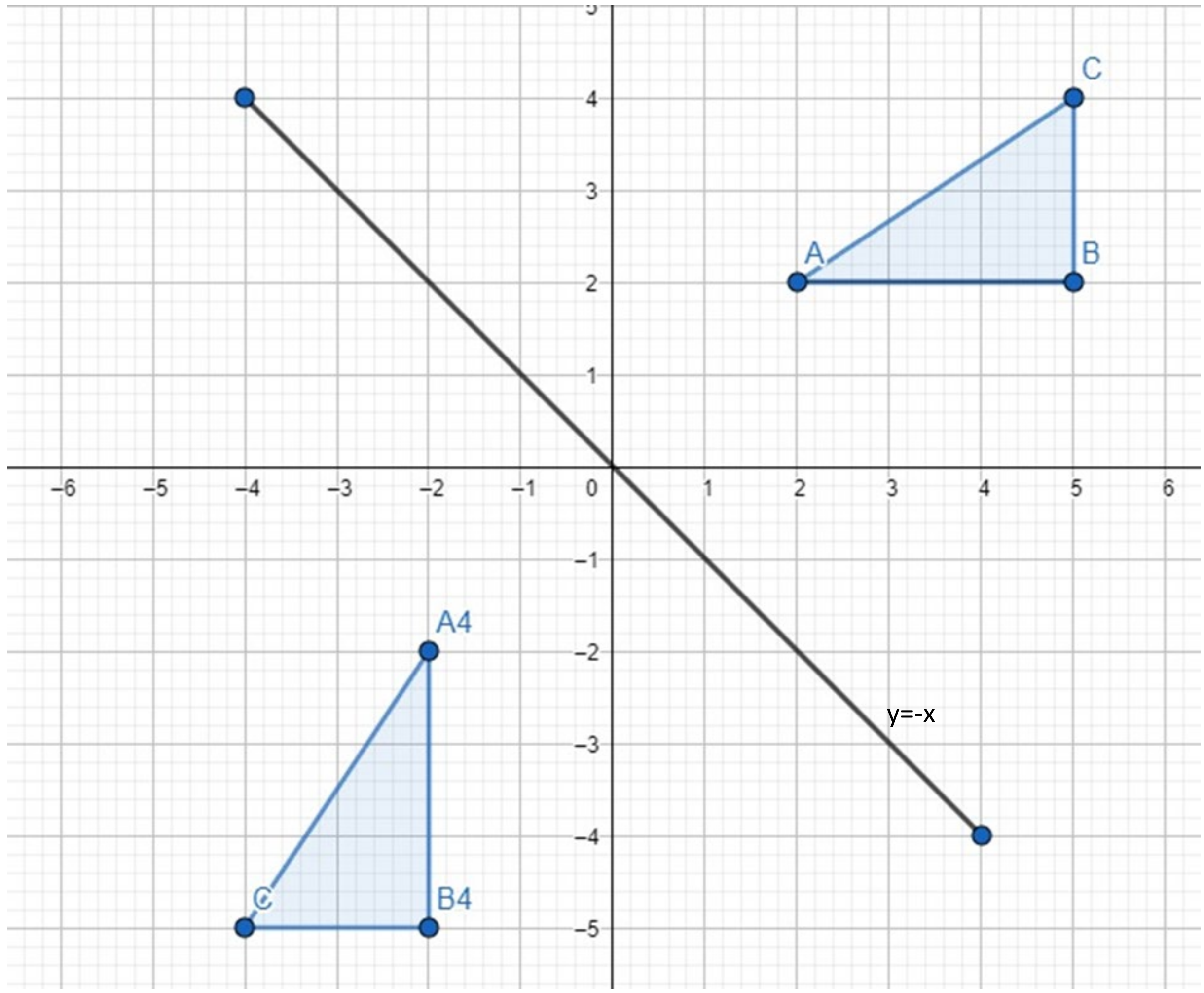


An enlargement by a scale factor of 2 means that every side of the image will be double the size of the corresponding size in the initial figure.

To enlarge a shape, we multiply the distances from the centre of enlargement to each point of the initial figure by the scale factor to obtain the distance from the centre to the corresponding point in the mirror image.

(e) The matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a transformation.

(i) Draw the image of triangle ABC under this transformation. Label it $A_4B_4C_4$. [2]

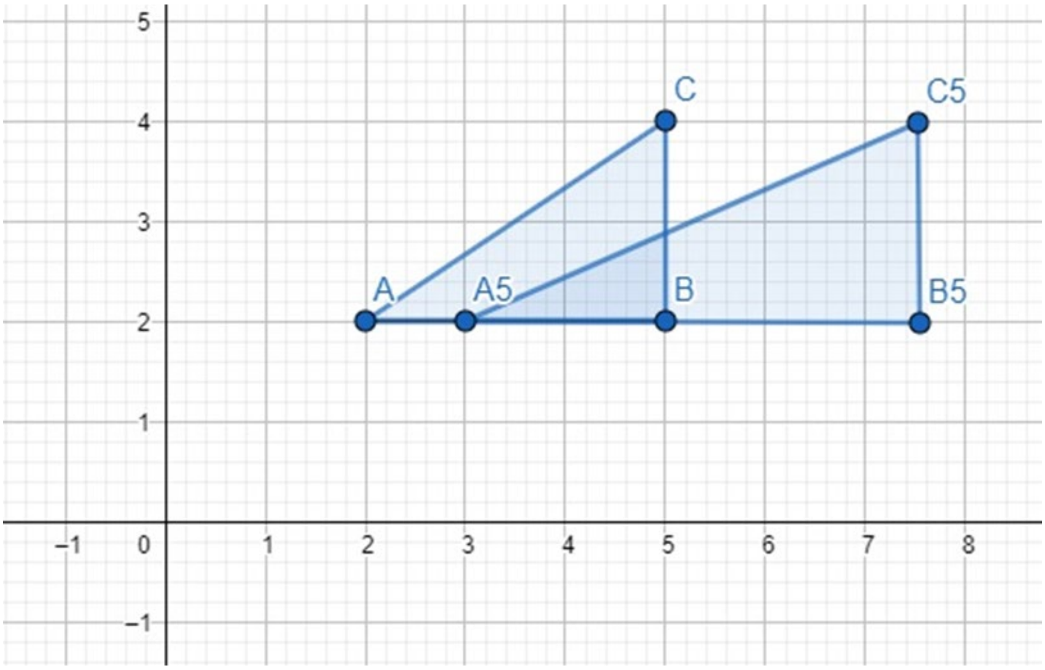


In our case, we define a reflection across the mirror line of equation y = -x

(ii) Describe fully this single transformation. [2]

Every point in triangle ABC is at the same distance from the mirror line as every corresponding point in triangle A2B2C2. Also, these distances are perpendicular on the mirror line.

(f) (i) Draw the image of triangle ABC under a stretch, factor 1.5, with the y-axis invariant. Label it $A_5B_5C_5$. [2]



A stretch is a transformation which enlarges all sides in the same direction by a constant factor.

In our case, the matrix means that the x coordinate of each point will be equal to the x coordinate of the corresponding point in the image multiplied by 1.5. The y coordinates of each point remain the same.

(ii) Find the 2 by 2 matrix which represents this transformation. [2]

A matrix in the form:

$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ represents a stretch along the x-axis with the constant factor k.

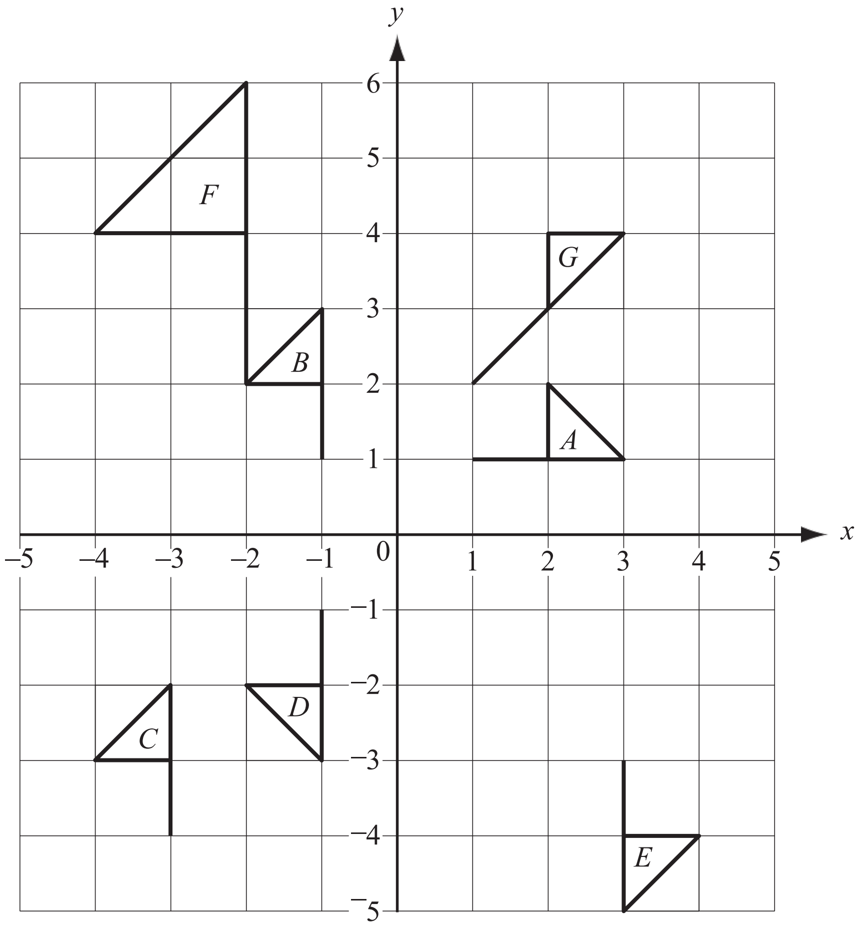
A matrix in the form:

$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$ represents a stretch along the y-axis with the constant factor k.

The constant factor here is 1.5 and the y = 0 is invariable.

The matrix is:

$$\begin{pmatrix} \mathbf{1.5} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$



(a) Describe fully the **single** transformation which maps

(i) shape A onto shape B, **Rotation 90° anticlockwise about $(0, 0)$** [2]

(ii) shape B onto shape C, **Translation by Vector $\begin{pmatrix} -2 \\ -6 \end{pmatrix}$** [2]

(iii) shape A onto shape D, **Reflection in the line $y = -x$** [2]

(iv) shape B onto shape E, **Rotation 180° about $(1, -1)$** [2]

(v) shape B onto shape F, **Enlargement, Scale Factor 2, Centre $(0, 0)$** [2]

(vi) shape A onto shape G. **Shear parallel to y-axis** [2]

(b) A transformation is represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

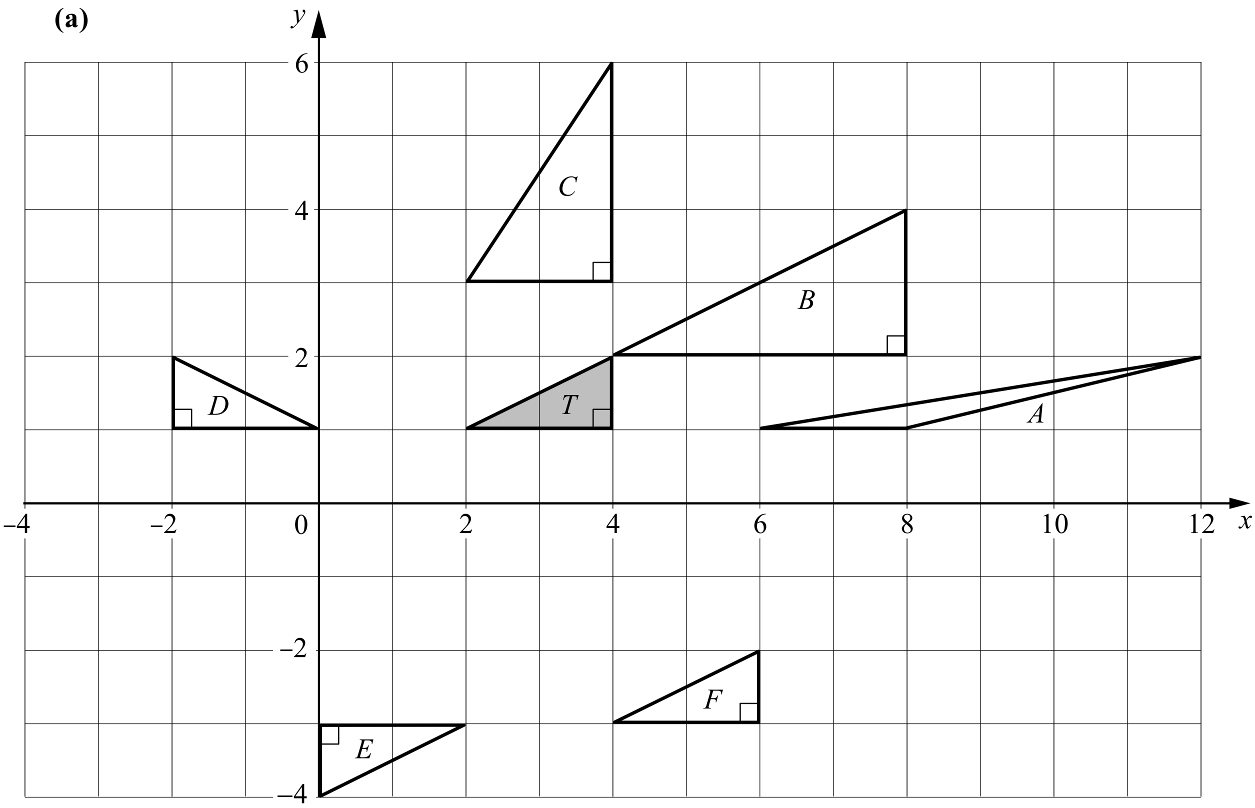
Which shape above is the image of shape A after this transformation? ***B*** [2]

(c) Find the 2 by 2 matrix representing the transformation which maps

(i) shape B onto shape D, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ [2]

(ii) shape A onto shape G. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ [2]

(a)



Use one of the letters A, B, C, D, E or F to answer the following questions.

(i) Which triangle is T mapped onto by a **translation**? Write down the translation vector. [2]

Triangle F is the one onto which T is mapped by translation since this single transformation does not change the size or the orientation of the object, but it changes its position.

The triangle has been moved 4 units down and 2 units to the right to obtain the image.

Therefore, the translation vector is: $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

(ii) Which triangle is T mapped onto by a **reflection**? Write down the equation of the mirror line. [2]

Triangle D is the one onto which T is mapped by reflection since this single transformation does not change the size of the object, but it represents its mirror image.

In a reflection, both the object and the image are at equal distances relative to the mirror line and the distances are perpendicular on the mirror line.

In our case, we observe that the mirror line is the line of equation

x = 1.

(iii) Which triangle is T mapped onto by a **rotation**? Write down the coordinates of the centre of rotation. [2]

Triangle E is the one onto which T is mapped by stretch with the x-axis invariant since this single transformation does not change the size of the object but changes its orientation. The object is rotated around the centre of rotation to obtain the image.

Each corner of the object needs to be at the same distance form the centre of rotation as its corresponding corner of the image.

In this case, the centre of rotation has the coordinates

(2, -1).

[2]

(iv) Which triangle is T mapped onto by a **stretch** with the x-axis invariant? Write down the scale factor of the stretch.

Triangle C is the one onto which T is mapped by stretch with the x-axis invariant since this single transformation changes the sizes of the sides only in one direction. Since the x-axis is invariant, all the x coordinates of each point remain the same, but the y coordinates change by multiplying them by a scale factor.

We notice that the side of the image is 3 times the corresponding side of the object on the y-axis, therefore, the

scale factor is 3.

(v) $\mathbf{M} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$ Which triangle is T mapped onto by $\mathbf{M}$?

Write down the name of this transformation. [2]

The matrix representing triangle T is:

$$\begin{pmatrix} 2 & 4 & 4 \\ 1 & 1 & 2 \end{pmatrix}$$

We multiply this matrix by M to work out the matrix representing the image after transformation.

$\begin{pmatrix}1 & 4\\0 & 1\end{pmatrix}\begin{pmatrix}2 & 4 & 4\\1 & 1 & 2\end{pmatrix} = \begin{pmatrix}2+4 & 4+6 & 4+8\\0+1 & 1+0 & 2+0\end{pmatrix}$

$\begin{pmatrix}1 & 4\\0 & 1\end{pmatrix}\begin{pmatrix}2 & 4 & 4\\1 & 1 & 2\end{pmatrix} = \begin{pmatrix}6 & 10 & 12\\1 & 1 & 2\end{pmatrix}$ – representing triangle A.

The triangle A is the one onto which T is mapped by

a shear, using the transformation matrix M.

(b) **P** = $\begin{pmatrix}1 & 3\\5 & 7\end{pmatrix}$, **Q** = $(-1 \quad -2)$, **R** = $(1 \quad 2 \quad 3)$, **S** = $\begin{pmatrix}-1\\2\\3\end{pmatrix}$.

Only some of the following matrix operations are possible with matrices **P**, **Q**, **R** and **S** above.

PQ, **QP**, **P** + **Q**, **PR**, **RS**

Write down and calculate each matrix operation that is possible. [6]

To multiply to matrices, the number of columns of the first matrix needs to be equal to the number of rows in the second matrix.

To add up two matrices, they need to be the same size.

For PQ, P has 2 columns and Q has one row, therefore, we cannot multiply them.

For QP, Q has 2 columns and P has 2 rows, therefore, they can be multiplied:

$(-1 \quad -2)\begin{pmatrix}1 & 3\\5 & 7\end{pmatrix} = ((-1)x1 + (-2)x5 \quad (-1)x3 + (-2)x7)$

$\mathbf{(-1 \quad -2)\begin{pmatrix}1 & 3\\5 & 7\end{pmatrix} = (-11 \quad -17)}$

For P + Q, P is a 2x2 matrix and Q is a 1x2 matrix, so we cannot add them up.

For PR, P has 2 columns and R has one row, therefore, we cannot multiply them.

For RS, R has 3 columns and S has 3 rows, therefore, we can multiply them:

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = (1\times(-1) + 2\times2 + 3\times3) = (12)$$