

# Number

## Difficulty: Medium

### Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 1

**Time allowed:** 75 minutes

**Score:** /64

**Percentage:** /100

**Grade Boundaries:**

#### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

#### CIE IGCSE Maths (0980) *Assembled by NS*

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

$$\mathcal{E} = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$$

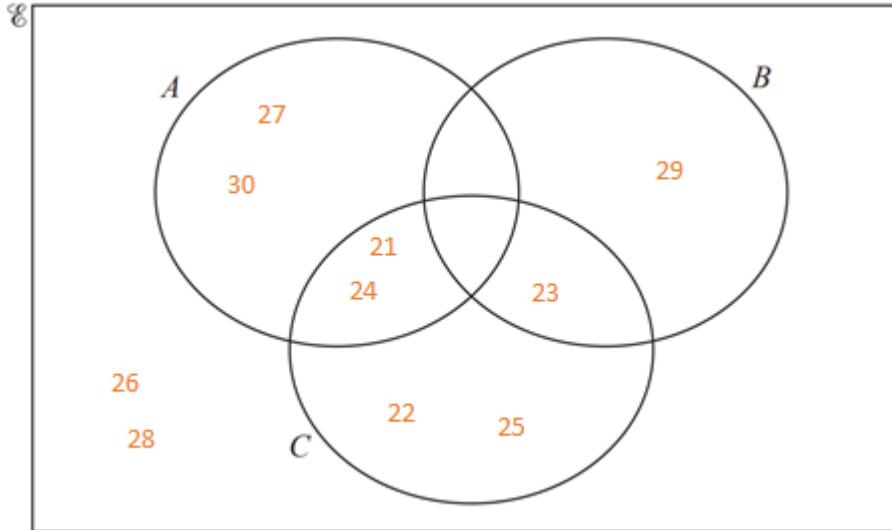
$$A = \{x : x \text{ is a multiple of } 3\}$$

$$B = \{x : x \text{ is prime}\}$$

$$C = \{x : x \geq 25\}$$

(a) Complete the Venn diagram.

[4]



(b) Use set notation to complete the statements.

(i)  $26 \dots \dots \dots B$   $\mathbf{26 \notin B}$  [1]

(ii)  $A \cap B = \dots \dots \dots$   $\mathbf{A \cap B = \emptyset}$  [1]

(c) List the elements of  $B \cup (C \cap A)$ . [2]

**21, 24, 23, 29**

(d) Find

(i)  $n(C)$ , [1]

**5**

(ii)  $n(B' \cup (B \cap C))$  [1]

**9**

(e)  $A \cap C$  is a subset of  $A \cup C$ .

Complete this statement using set notation. [1]

**$(A \cap C) \subset (A \cup C)$**

## Question 2

- (a) (i) Divide \$105 in the ratio 4 : 3.

[2]

When using ratios, create as many columns and rows as you

need and use Scale Factors to move between them:

$$\begin{array}{ccc} A & : & B & : \text{Total} \\ 4 & : & 3 & : & 7 \\ 4 \times 15 & : & 3 \times 15 & : & 105 \end{array}$$

Multiply by  $\frac{105}{7} = 15$

**60 : 45**

- (ii) Increase \$105 by 12%.

[2]

Write a statement down in the form: "After is a percentage of Before":

Answer is 112% of \$105

Translate this into an equation ("is" means "=" , "of" means "×"):

$$\text{Answer} = 1.12 \times 105$$

$$\text{Answer} = \$117.60$$

- (iii) In a sale the original price of a jacket is reduced by 16% to \$105.

Calculate the original price of the jacket.

[3]

Write a statement down in the form: "After is a percentage of Before":

Sale Price is 84% of Original Price

Translate this into an equation ("is" means "=" , "of" means "×"):

$$\text{Sale Price} = 0.84 \times \text{Original Price}$$

Put in the numbers:

$$105 = 0.84 \times \text{Original Price}$$

Solve (in this case by dividing by 0.84):

$$\begin{aligned} \text{Original Price} &= \frac{105}{0.84} \\ &= \$125 \end{aligned}$$

- (b) Jakob invests \$500 at a rate of 2% per year compound interest.  
 Claudia invests \$500 at a rate of 2.5% per year simple interest.

Calculate the difference between these two investments after 30 years.  
 Give your answer in dollars correct to the nearest cent.

[6]

For Compound Interest multiply by the scale factor to the power of the number of years:

$$\text{Jakob} = 500 \times 1.02^{30}$$

$$\text{Jakob} = \$905.68$$

For Simple Interest work out the interest for one year and multiply by the number of years:

$$\text{Claudia} = 500 + 500 \times 0.025 \times 30$$

$$\text{Claudia} = \$875$$

$$\text{Difference} = 905.68 - 875$$

$$\text{Difference} = \$30.68$$

- (c) Michel invests \$P at a rate of 3.8% per year compound interest.  
 After 30 years the value of this investment is \$1469.

Calculate the value of  $P$ .

[3]

Using the same principle as in (b) we can write

$$1469 = P \times 1.038^{30}$$

$$P = \frac{1469}{1.038^{30}}$$

$$P = \$479.85$$

- (d) The population of a city increases exponentially at a rate of  $x\%$  every 5 years.  
 In 1960 the population was 60100.  
 In 2015 the population was 120150.

Calculate the value of  $x$ .

[3]

Again the same principle, but spotting that we have 11 lots of 5 years from 1960 to 2015

we can write (using  $f$  as the scale factor):

$$120150 = 60100 \times f^{11}$$

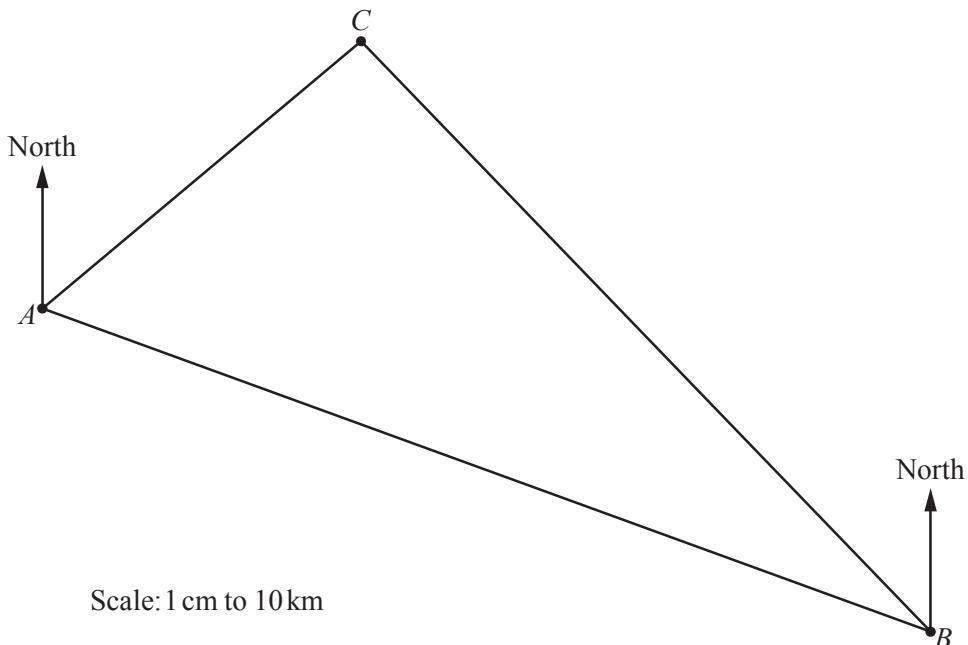
$$f = \sqrt[11]{\frac{120150}{60100}}$$

$$f = 1.065$$

1.065 is the scale factor for a 6.5% increase so  $x = 6.5$

### Question 3

The scale drawing shows the positions of three towns  $A$ ,  $B$  and  $C$  on a map.  
The scale of the map is 1 centimetre represents 10 kilometres.



- (a) Find the actual distance  $AB$ .

[1]

We measure the length between  $A$  and  $B$  to be approximately 12.5 cm.

As 1 cm represents 10 kilometres, we simply multiply by 10 to get the distance in kilometres.

$$AB = 12.5\text{cm} \times 10\text{km per cm}$$

$$= 125\text{km}$$

- (b) Measure the bearing of  $A$  from  $B$ .

[1]

We measure the angle  $CAB$  to be approximately  $70^\circ$ .

The bearing from  $A$  to  $B$  is  $360^\circ - 70^\circ$

= $290^\circ$ .

- (c) Write the scale 1 cm to 10 km in the form  $1:n$ .

[1]

One kilometre represents 1 000 metres and one metre represents 100 centimetres.

Multiply these factors to get a conversion between kilometres and centimetres.

$$10\text{ km} = 10 \times 1\,000 \times 100 = 1\,000\,000\text{ centimetres}$$

Therefore the ratio is  $1 : 1\,000\,000$ .

- (d) On the scale drawing, a lake inside the national park has area  $0.4 \text{ cm}^2$ .

Calculate the actual area of the lake.

[2]

One centimetre on the map represents 10 kilometres in reality, however this conversion is linear (one dimensional), therefore we must square both numbers to convert areas.

$$1 \text{ cm}^2 = 100 \text{ km}^2$$

Now we know that  $1 \text{ cm}^2$  on the map represents 100 square kilometres.

$$0.4 \text{ cm}^2 \times 100 \text{ km}^2 \text{ per } \text{cm}^2 = 40 \text{ km}^2$$

The actual area of the lake is **40 km<sup>2</sup>**.

## Question 4

- (a) Kolyan buys water for \$2.60 .  
He also buys biscuits.
- (i) The ratio cost of biscuits : cost of water = 3 : 2.

Find the cost of the biscuits.

[2]

The ratio of cost of biscuits and the cost of water is 3:2.

$$\frac{\text{cost of biscuits}}{\text{cost of water}} = \frac{3}{2}$$

Kolyan buys water for \$2.60

$$\frac{\text{cost of biscuits}}{\$2.60} = \frac{3}{2}$$

Multiply both sides by 2.60.

$$\text{cost of biscuits} = \frac{3}{2} \times \$2.60$$

$$\text{cost of biscuits} = \$3.90$$

- (ii) Kolyan has \$9 to spend.

Work out the total amount Kolyan spends on water and biscuits as a fraction of the \$9.  
Give your answer in its lowest terms.

[2]

To get the fraction of money Kolyan spends on water and biscuits, divide the amount he spends and the total amount of money he has.

$$\frac{\text{cost of biscuits} + \text{cost of water}}{\text{money he has}} = \frac{\$2.60 + \$3.90}{\$9} = \frac{\$6.50}{\$9}$$

Turn the fraction into its simplest form (multiply both top and bottom by 2)

$$\frac{6.50}{9}$$

$$= \frac{13}{18}$$

- (iii) The \$9 is 62.5% less than the amount Kolyan had to spend last week.

Calculate the amount Kolyan had to spend last week.

[3]

If \$9 is 62.5% less, then it represents 37.5% (=100%-62.5%) of the amount he spent last week.

Divide the amount he spent this week by 0.375 (=37.5%/100%) to get the amount he had to spend last week.

$$\begin{array}{r} \$9 \\ \hline 0.375 \\ = \$24 \end{array}$$

- (b) Priya buys a bicycle for \$250.

Each year the value of the bicycle decreases by 8% of its value at the beginning of that year.

Calculate the value of Priya's bicycle after 10 years.

Give your answer correct to the nearest dollar.

[3]

The decrease of value by 8% per year means that the value next year will be 0.92 (=1- 8%/100%) times the value in the previous year.

We want to find out the value after 10 years, so the initial value has to be multiplied by the factor 0.92 ten times.

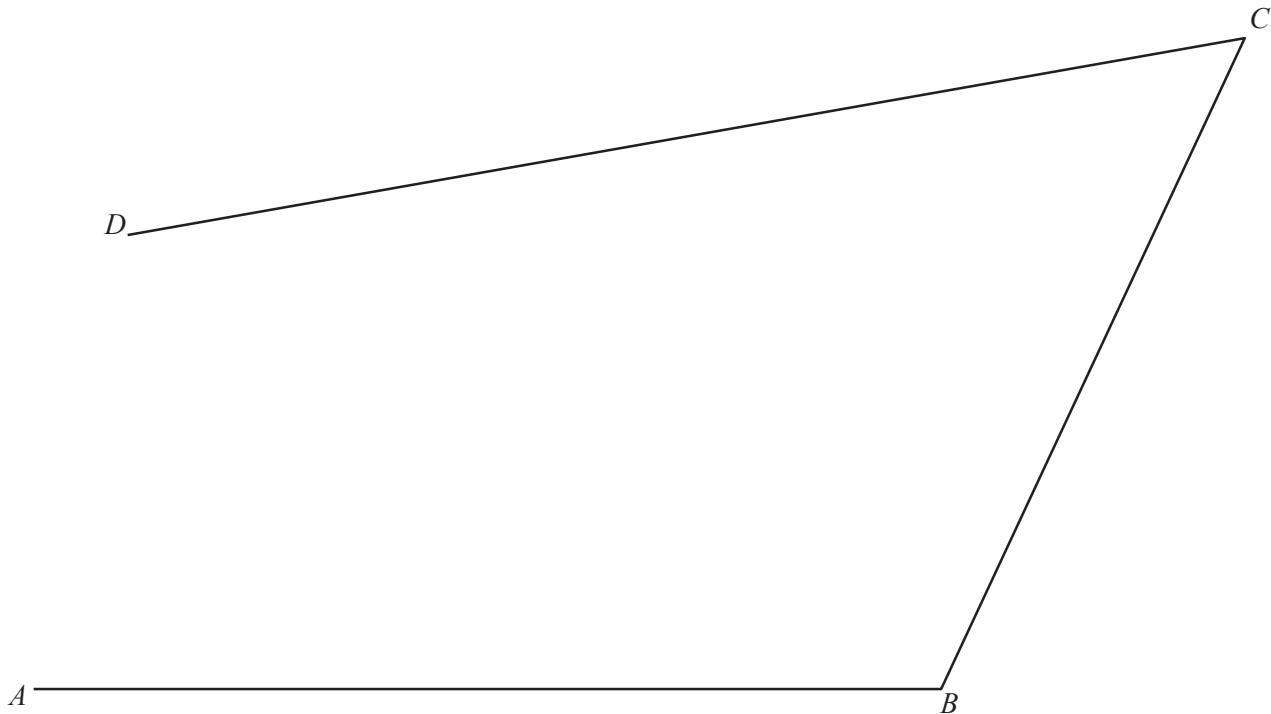
$$\$250 \times 0.92^{10} = \$108.60$$

Round your answer to the nearest dollar.

**The value of the bicycle after 10 years is \$109**

## Question 5

The diagram is a scale drawing of three straight roads,  $AB$ ,  $BC$  and  $CD$ .  
The scale is  $1 : 5000$ .



Scale  $1 : 5000$

Find the actual length of the road  $BC$ .  
Give your answer in metres.

[2]

We measure the length of  $BC$  to be roughly  $9.5\text{cm}$ .

To get the actual length of the road  $BC$ , we multiply this number by the scale factor (5000)

$$BC = 9.5\text{cm} \times 5000$$

$$BC = 47\,500\text{ cm}$$

Convert from centimetres into metres (divide by 100).

$$\mathbf{BC = 475m}$$

## Question 6

- (a) (i) Eduardo invests \$640 at a rate of 2% per year compound interest.

Show that, at the end of 6 years, Eduardo has \$721, correct to the nearest dollar.

[2]

The rate of 2% per year (compound interest) means that the amount of money next year will be  $1.02 (=1 + 2/100)$  times the previous amount.

We want to find out the amount after 6 years, so the initial amount needs to be multiplied by the factor 1.02 six times.

$$\$640 \times 1.02^6 = \$720.74$$

Round the number to the nearest dollar:

=**\$721**

- (ii) Manuela also invests \$640.  
At the end of 4 years, Manuela has \$721.

Find the yearly compound interest rate.

[4]

Using the same equation, we know that:

$$\$640 \times f^4 = \$721$$

Symbol  $f$  denotes the interest factor  $f = 1 + \frac{\text{interest}}{100}$

Divide both sides of the equation by 640.

$$f^4 = \frac{721}{640}$$

Apply fourth root to both sides.

$$f = \sqrt[4]{1.1266}$$

Use a calculator to work out the value  $f$ .

$$f = 1.0302$$

To get the interest, we use the formula:

$$f = 1 + \frac{\text{interest}}{100\%}$$

Subtract 1 from both sides and multiply by 100% to get the interest.

$$0.0302 = \frac{\text{interest}}{100\%}$$

$$\text{interest} = 3.02\%$$

- (b) Carlos buys a motor scooter for \$1200.  
Each year the value of the scooter decreases by 10% of its value at the beginning of that year.

Find the value of the scooter after 3 years.

[2]

The decrease of value by 10% per year means that the value next year will be 0.90

$(=1 - 10/100)$  times the value in the previous year.

We want to find out the value after 3 years, so the initial value has to be multiplied by the factor 0.90 three times.

$$\$1200 \times 0.90^3 = \$874.8$$

The value of the scooter after 3 years is \$874.8

## Question 7

(a)  $x$  is an integer.

$$\mathcal{E} = \{x: 1 \leq x \leq 10\}$$

$$A = \{x: x \text{ is a factor of } 12\}$$

$$B = \{x: x \text{ is an odd number}\}$$

$$C = \{x: x \text{ is a prime number}\}$$

- (i) Complete the Venn diagram to show this information.

[3]

We want to sort numbers 1 to 10 according to the following criteria using a Venn diagram.

A: a number is a factor of 12

B: a number is an odd number

C: a number is a prime number

First, we write down, which numbers belong to which categories:

A: 1, 2, 3, 4, 6

B: 1, 3, 5, 7, 9

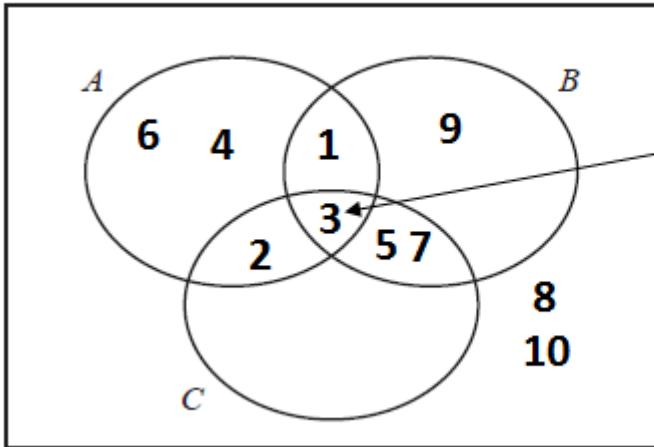
C: 2, 3, 5, 7

(Note that numbers 8 and 10 do not belong to any of the three categories.)

Second, we write the numbers into the Venn diagram based on their categories. If any number belongs to two or more categories, it must lie in the intersection of the corresponding circles.

If a number does not belong to any of the categories, the number lies outside the circles.

Q



For example, the number 3 belongs to all three categories, so it must lie in the intersection of all three circles

- (ii) Use set notation to complete each statement. [3]

$$6 \dots \dots \dots A$$

$$A \cap B \cap C = \dots \dots \dots$$

$$A \cap A' = \dots \dots \dots$$

From the Venn diagram, we can see that number 6 belongs to the group A. Symbol  $\in$  is used to state that a given element belongs to a given set.

$$6 \in A$$

$A \cap B \cap C$  means that we are looking for an element, which lies in the intersection of the three sets. The only number that satisfies all three conditions is the number 3.

$$A \cap B \cap C = \{3\}$$

$A \cap A'$  means that we are looking for an element, which belongs both to A and does not belong to A. This, however, does not make any sense, therefore there are no elements, which satisfy this condition.

$$A \cap A = \{ \}$$

(iii) Find  $n(B)$ .

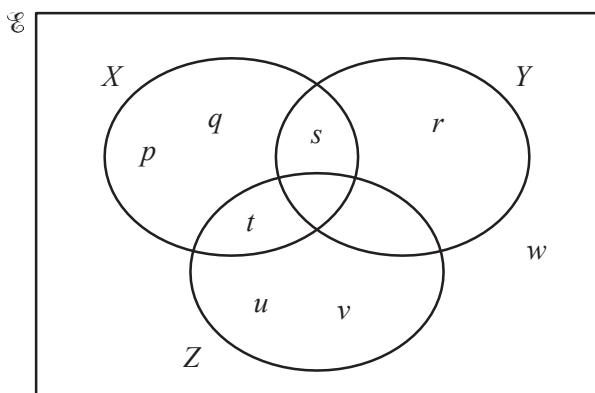
[1]

We are looking for the number of elements, which belong to the set B (odd numbers).

From part a) i) we know that only number 1, 3, 5, 7, 9 satisfy the condition B. Therefore there are 5 numbers that belong to this set.

$$n(B) = 5$$

(b)



(i) Use set notation to complete the statement.

$$\{u, v\} \dots Z$$

[1]

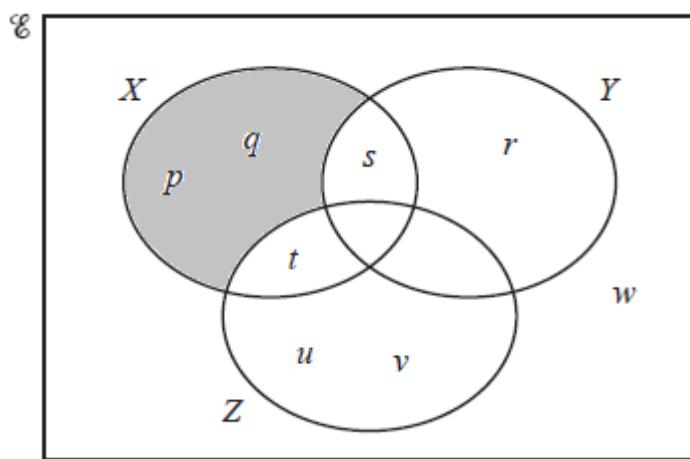
The set of elements  $u, v$  is a subset of Z. Symbol  $\subset$  is used to denote a subset.

$$\{u, v\} \subset Z$$

- (ii) Shade
- $X \cap (Z \cup Y)'$
- .

[1]

Region  $X \cap (Z \cup Y)'$  is a set of elements, which both belong to  $X$  and do not belong to  $(Z \cup Y)$  which is the union of  $Z$  and  $Y$ . Therefore such elements only belong to  $X$  and no other set.



# Number

## Difficulty: Medium

### Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 2

**Time allowed:** 93 minutes

**Score:** /81

**Percentage:** /100

**Grade Boundaries:**

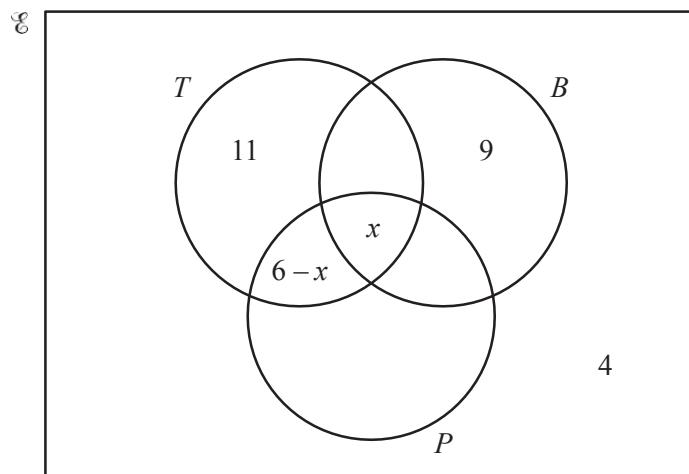
#### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

#### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1



In the Venn diagram,  $\mathcal{E} = \{\text{children in a nursery}\}$

$B = \{\text{children who received a book for their birthday}\}$

$T = \{\text{children who received a toy for their birthday}\}$

$P = \{\text{children who received a puzzle for their birthday}\}$

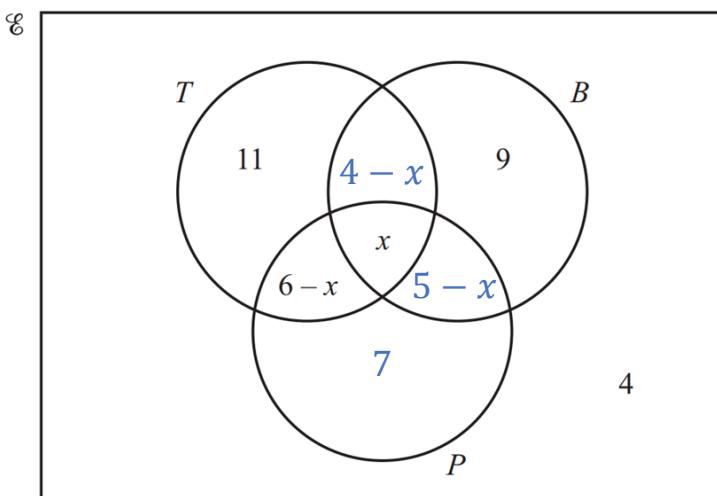
$x$  children received a book and a toy and a puzzle.

6 children received a toy and a puzzle.

- (a) 4 children received a book and a toy.  
5 children received a book and a puzzle.  
7 children received a puzzle but not a book and not a toy.

Complete the Venn diagram above.

[3]



(b) There are 40 children in the nursery.

Using the Venn diagram, write down and solve an equation in  $x$ . [3]

$$11 + 4 - x + 9 + 6 - x + x + 5 - x + 7 + 4 = 40$$

$$\rightarrow 46 - 2x = 40$$

Add  $2x$  and subtract 40 from both sides

$$2x = 6$$

Divide through by 2

$$x = 3$$

(c) Work out

(i) the probability that a child, chosen at random, received a book but not a toy and not a puzzle, [1]

$$\frac{9}{40}$$

(ii) the number of children who received a book and a puzzle but not a toy, [1]

$$5 - x$$

$$= 2$$

(iii)  $n(B)$ , [1]

$$9 + 2 + 1 + 3$$

$$= 15$$

(iv)  $n(B \cup P)$ ,

[1]

$$15 + 7 + 6 - x$$

$$= 15 + 7 + 3$$

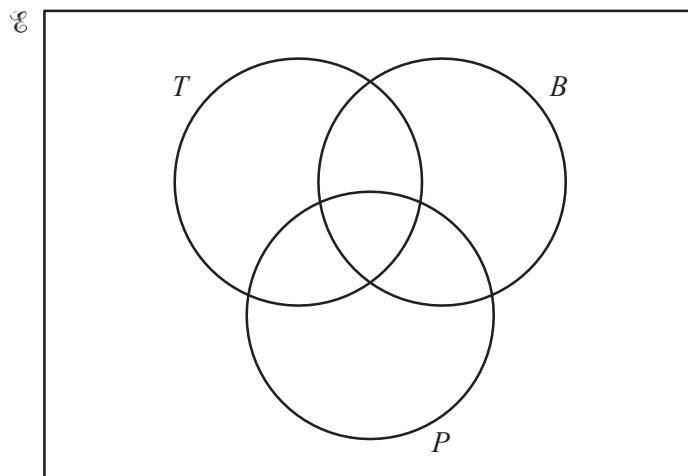
$$= 25$$

(v)  $n(B \cup T \cup P)'$ .

[1]

**4**

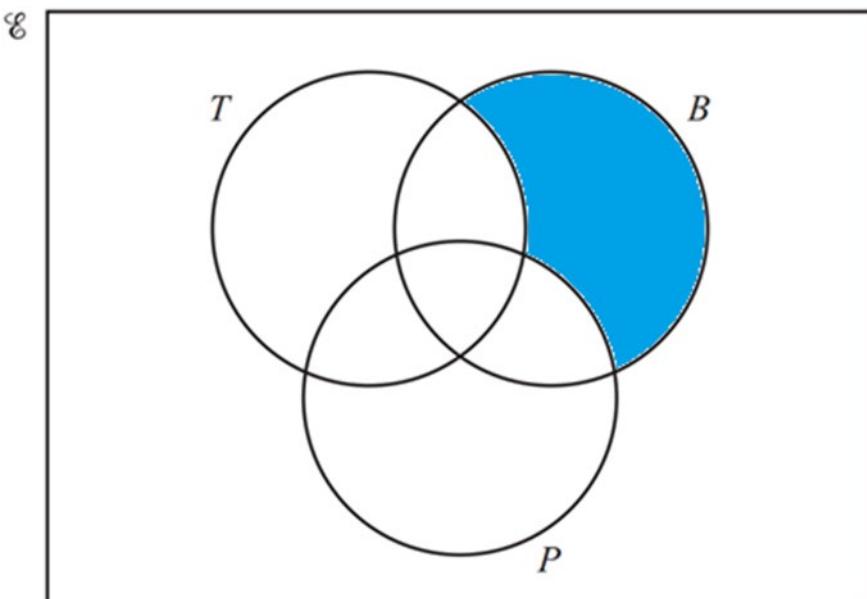
(d)



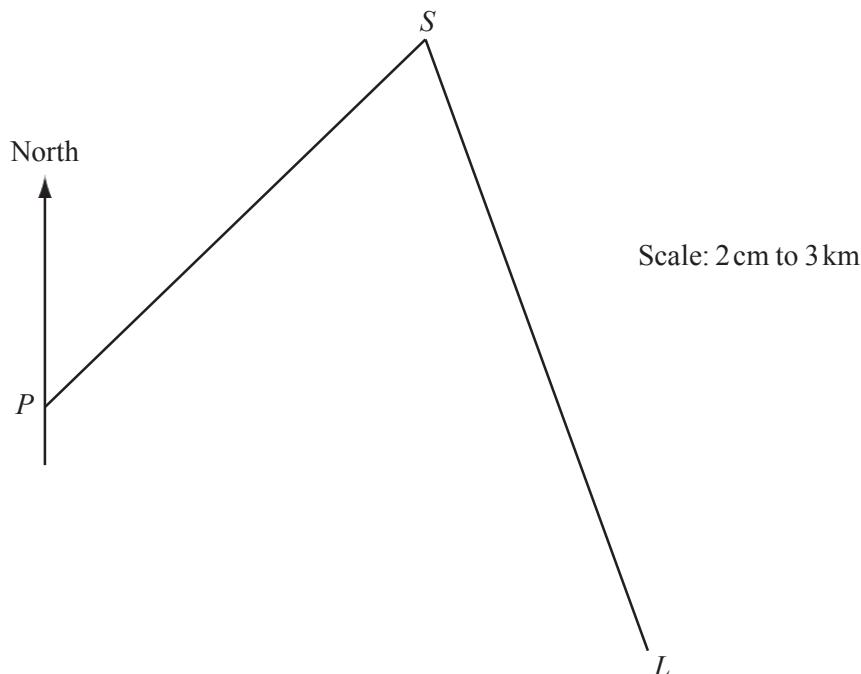
Shade the region  $B \cap (T \cup P)'$ .

[1]

E



## Question 2



In the scale drawing,  $P$  is a port,  $L$  is a lighthouse and  $S$  is a ship.  
The scale is 2 centimetres represents 3 kilometres.

(a) Measure the bearing of  $S$  from  $P$ .

[1]

**046**

(b) Find the actual distance of  $S$  from  $L$ .

[2]

Measured distance is: **8.6 cm**

Using the scale, we get the actual distance

$$8.6 \times \frac{3}{2}$$

$$= 12.9 \text{ km}$$

(c) The bearing of  $L$  from  $S$  is  $160^\circ$ .

[1]

Calculate the bearing of  $S$  from  $L$ .

$$360 - (180 - 160)$$

$$= 360 - 20$$

$$= 340$$

(d) Work out the scale of the map in the form  $1:n$ .

[2]

$$2 \text{ cm} : 3 \text{ km}$$

$$= 1 \text{ cm} : 1.5 \text{ km}$$

$$= 1 \text{ cm} : 1.5 \times 10^3 \text{ m}$$

$$= 1 \text{ cm} : 1.5 \times 10^5 \text{ cm}$$

$$= 1 : 150\,000$$

(e) The lighthouse stands on an island of area  $1.5 \text{ cm}^2$  on the scale drawing.

Work out the actual area of the island.

[2]

Need to square the length scale factor (found in part (d))

$$1.5 \times 150\,000^2$$

$$= 3.375 \times 10^{10}$$

This is the area in  $\text{cm}^2$ . Need to convert to  $\text{km}^2$ 

$$3.375 \times 10^{10} \div 100\,000^2$$

$$= 3.375 \text{ km}^2$$

### Question 3

Jane and Kate share \$240 in the ratio 5 : 7 .

- (a) Show that Kate receives \$140. [2]

$$240 \times \frac{7}{12}$$

$$= 140$$

- (b) Jane and Kate each spend \$20.

Find the new ratio Jane's remaining money : Kate's remaining money.  
Give your answer in its simplest form. [2]

Jane originally had

$$240 - 140$$

$$= 100$$

Hence, for the remaining money

*Jane : Kate*

$$= 80 : 120$$

$$= 2 : 3$$

- (c) Kate invests \$120 for 5 years at 4% per year simple interest.

Calculate the total amount Kate has after 5 years. [3]

$$4\% \text{ of } 120 = 0.04 \times 120$$

$$= 4.8$$

After 5 years this is

$$120 + 5 \times 4.8$$

$$= 144$$

(d) Jane invests \$80 for 3 years at 4% per year compound interest.

Calculate the total amount Jane has after 3 years.

[3]

Give your answer correct to the nearest cent.

$$80 \times 1.04^3$$

$$= 89.99$$

(e) An investment of \$200 for 2 years at 4% per year compound interest is the same as an investment of \$200 for 2 years at  $r\%$  per year simple interest.

Find the value of  $r$ .

[3]

The amount after 2 years is

$$200 \times 1.04^2$$

$$= 216.32$$

For simple interest we would have

$$2 \times \frac{r}{100} \times 200 = 16.32$$

$$\rightarrow 4r = 16.32$$

$$\rightarrow r = 4.08$$

## Question 4

In July, a supermarket sold 45 981 bottles of fruit juice.

- (a) The cost of a bottle of fruit juice was \$1.35 .

Calculate the amount received from the sale of the 45981 bottles.

Give your answer correct to the nearest hundred dollars.

[2]

$$45981 \times 1.35$$

$$= 62\,100$$

- (b) The number of bottles sold in July was 17% more than the number sold in January.

Calculate the number of bottles sold in January.

[3]

$$45981 \div 1.17$$

$$= 39\,300$$

- (c) There were 3 different flavours of fruit juice.

The number of bottles sold in each flavour was in the ratio apple: orange: cherry = 3 : 4 : 2.  
The total number of bottles sold was 45981.

Calculate the number of bottles of orange juice sold.

[2]

$$45\,981 \times \frac{4}{9}$$

$$= 20\,436$$

- (d) One bottle contains 1.5 litres of fruit juice.

Calculate the number of 330 ml glasses that can be filled completely from one bottle.

[3]

$$1500 \div 330$$

$$= 4.54$$

So, 4 whole glasses

- (e)  $\frac{5}{9}$  of the 45981 bottles are recycled.

Calculate the number of bottles that are recycled.

[2]

$$45\,981 \times \frac{5}{9}$$

$$= 25\,545$$

## Question 5

David sells fruit at the market.

- (a) In one week, David sells 120kg of tomatoes and 80kg of grapes.

(i) Write 80 kg as a fraction of the total mass of tomatoes and grapes.

Give your answer in its lowest terms.

[1]

The total mass of tomatoes and grapes is given by:

$$120\text{kg} + 80\text{kg} = 200\text{kg}$$

The fraction is then given by:

$$\frac{80\text{kg}}{200\text{kg}} = \frac{8}{20}$$

$$= \frac{2}{5}$$

- (ii) Write down the ratio mass of tomatoes:mass of grapes.

Give your answer in its simplest form.

[1]

The ratio of the mass of tomatoes to the mass of grapes is:

$$120:80$$

$$= 12:8$$

$$= 3:2$$

- (b) (i) One day he sells 28 kg of oranges at \$1.56 per kilogram.

He also sells 35 kg of apples.

The total he receives from selling the oranges and the apples is \$86.38 .

Calculate the price of 1 kilogram of apples.

[2]

Using a simple model:



- (ii) The price of 1 kilogram of oranges is \$1.56 .  
This is 20% more than the price two weeks ago.

Calculate the price two weeks ago.

[3]

\$1.56/kg is 120% of the price (includes 20% of the price

two weeks ago):

$$\frac{\$1.56/kg}{120\%} \times 100\%$$

$$= \$1.30/kg$$

- (c) On another day, David received a total of \$667 from all the fruit he sold.  
The cost of the fruit was \$314.20 .  
David worked for  $10\frac{1}{2}$  hours on this day.

Calculate David's rate of profit in dollars per hour.

[2]

Total profit for the day is given by:

$$\$667 - \$314.20 = \$352.80$$

But David worked for 10.5 hours, so his hourly profit will be:

$$\frac{\$352.80}{10.5} = \$33.60/h$$

## Question 6

- (a) One day, Maria took 27 minutes to walk 1.8 km to school.  
She left home at 0748.

- (i) Write down the time Maria arrived at school.

[1]

There are 60 minutes in an hour therefore:

$$\text{Arrival time} = 07\ 48 + 00\ 12 + 00\ 15$$

$$= 08\ 00 + 00\ 15$$

$$= \mathbf{08\ 15}$$

- (ii) Show that Maria's average walking speed was 4 km/h.

[2]

Since speed must be given as km/h, her travel time must  
be converted from minutes to hours.

$$\text{Time in hours} = \frac{27}{60}$$

$$= 0.45 \text{ hours}$$

Her average speed can be calculated by:

$$\text{Average speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{1.8}{0.45}$$

$$= \mathbf{4 \text{ km/h}}$$

(b) Another day, Maria cycled the 1.8 km to school at an average speed of 15 km/h.

(i) Calculate the percentage **increase** that 15 km/h is on Maria's walking speed of 4 km/h. [3]

The percentage increase in her speed can be calculated by:

$$\text{Percentage change} = \frac{\text{new speed} - \text{old speed}}{\text{old speed}} * 100$$

$$= \frac{15 - 4}{4} * 100$$

$$= 2.75 * 100$$

$$= 275\%$$

(ii) Calculate the percentage **decrease** that Maria's cycling time is on her walking time of 27 minutes.

[3]

First, Maria's cycling time must be calculated like so:

$$\text{Time in hours} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{1.8}{15}$$

$$= 0.12 \text{ hours}$$

Now we must find the percentage decrease in her cycling time using the

percentage change equation used before.

$$\text{Percentage change} = \frac{\text{new time} - \text{old time}}{\text{old time}} * 100$$

$$= \frac{0.12 - 0.45}{0.45} * 100$$

$$= -0.733 * 100$$

$$= -73.3\%$$

Since this is negative, it shows a percentage decrease.

$$\therefore \text{Percentage decrease} = 73.3\%$$

(iii) After school, Maria cycled to her friend's home.

This took 9 minutes, which was 36% of the time Maria takes to walk to her friend's home.

Calculate the time Maria takes to walk to her friend's home.

[2]

To find the time it takes Maria to walk, we can use the percentage equation.

$$\text{Percentage} = \frac{\text{time cycling}}{\text{time walking}} * 100$$

$$36 = \frac{9}{\text{time walking}} * 100$$

We can find the time to walk by dividing through by 36, and then multiplying

through by time walking, this gives:

$$\text{Time walking} = \frac{9}{36} * 100$$

$$\text{Time walking} = 25 \text{ minutes}$$

## Question 7

A tennis club has 560 members.

- (a) The ratio men : women : children = 5 : 6 : 3.

- (i) Show that the club has 240 women members.

[2]

Ratio is at 5:6:3, the total number of parts is:

$$\text{Total number of parts: } 5 + 6 + 3 = 14$$

Since there are a total of 560 members,

$$14 \text{ parts} \rightarrow 560 \text{ members}$$

$$6 \text{ parts (women)} \rightarrow \frac{560}{14} \times 6$$

$$= 240 \text{ members (shown)}$$

- (ii) How many members are children?

[1]

$$3 \text{ parts (children)} \rightarrow \frac{560}{14} \times 3$$

$$= 120 \text{ members were children}$$

- (b)  $\frac{5}{8}$  of the 240 women members play in a tournament.

[2]

How many women members do **not** play in the tournament?

Find the proportion of women who did not play in the tournament

$$\text{Women who did not play in the tournament} = 1 - \frac{5}{8}$$

$$= \frac{3}{8}$$

Multiply by total number of women which was found in (a) (i):

$$\frac{3}{8} \times 240$$

$$= 90 \text{ women}$$

(c) The annual membership fee in 2013 is \$198 for each adult and \$75 for each child.

(i) Calculate the total amount the 560 members pay in 2013.

[2]

Find the number of adults:

$$\text{Number of adult members} = 560 - 120$$

$$= 440 \text{ adult members}$$

Find the amount paid:

$$\text{Total amount} = (440 \times \$198) + (120 \times \$75)$$

$$= \$96120$$

(ii) The adult fee of \$198 in 2013 is 5.6% more than the fee in 2012.

Calculate the adult fee in 2012.

[3]

Take 2012 as the base 100%:

$$105.6\% \rightarrow \$198$$

$$100\% = \frac{198}{105.6} \times 100$$

$$= \$187.50$$

(d) The club buys 36 tennis balls for \$9.50 and sells them to members for \$0.75 each.

Calculate the percentage profit the club makes.

[3]

Calculate sales of tennis balls:

$$\text{Sales} = \$0.75 \times 36$$

$$= \$27$$

Since cost is \$9.50, we can calculate the profit:

$$\text{Profit} = \$27 - \$9.50$$

$$= \$17.50$$

$$\% \text{ Profit} = \frac{17.5}{9.5} \times 100\%$$

$$= 184\%$$

(e) A tennis court is a rectangle with length 23.7 m and width 10.9 m, each correct to 1 decimal place.

Calculate the upper and lower bounds of the perimeter of the court.

[3]

$$\text{Upper bound length} = 23.75m$$

$$\text{Lower bound length} = 23.65m$$

$$\text{Upper bound width} = 10.95m$$

$$\text{Lower bound width} = 10.85m$$

$$\text{Upper bound perimeter} = (2 \times 23.75) + (2 \times 10.95)$$

$$= 69.4m$$

$$\text{Lower bound perimeter} = (2 \times 23.65) + (2 \times 10.85)$$

$$= 69m$$

# Number

## Difficulty: Medium

### Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 3

**Time allowed:** **102 minutes**

**Score:** **/89**

**Percentage:** **/100**

**Grade Boundaries:**

#### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

#### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

- (a) Ali and Ben receive a sum of money.  
They share it in the ratio 5 : 1.  
Ali receives \$2345.

Calculate the total amount.

[2]

If Ali receives 5 times the amount that Ben does, then Ben receives:

$$\frac{\$2345}{5} = \$469$$

The total amount is therefore:

$$\$469 + \$2345$$

$$= \$2814$$

- (b) Ali uses 11% of his \$2345 to buy a television.

Calculate the cost of the television.

[2]

11% of 2345 can be calculated using your calculator as follows:

$$\frac{11\%}{100} \times \$2345$$

$$= \$257.95$$

- (c) A different television costs \$330.

- (i) Ben buys one in a sale when this cost is reduced by 15%.

How much does Ben pay?

[2]

If the television costs \$330 at 100% of its cost, we need to find the cost

when reduced by 15% which will be  $100\% - 15\% = 85\%$  of the cost

$$\frac{85\%}{100} \times \$330$$

$$= \$280.50$$

- (ii) \$330 is 12% less than the cost last year.

Calculate the cost last year.

[3]

If \$330 is 12% less this mean the cost last year was multiplied by

88% as  $100\% - 12\% = 88\%$

$$\text{old cost} \times 88\% = \text{new cost}$$

Therefore, to go backwards and find the old cost we must divide the new cost by 88%.

$$\frac{\$330}{0.88} = \$375$$

- (d) Ali invests \$1500 of his share in a bank account.  
The account pays compound interest at a rate of 2.3% per year.

Calculate the total amount in the account at the end of 3 years.

[3]

The formula for calculating compound interest in our case is as follows:

$$A = P(1 + r)^t$$

Where A is the amount at the end, P is the principal amount invested, r is the decimal interest rate, and t is the time in years

Therefore,

$$A = \$1500 \left(1 + \frac{2.3\%}{100}\right)^3$$

$$= \$1605.90$$

- (e) Ali also buys a computer for \$325.  
He later sells this computer for \$250.

Calculate Ali's percentage loss.

[3]

Percentage loss can be calculated as:

$$\frac{\text{original price} - \text{selling price}}{\text{original price}} \times 100\%$$

$$\frac{325 - 250}{325} \times 100\% = 23.1\%$$

$$= 23.1\%$$

## Question 2

Anna, Bobby and Carl receive a sum of money.  
They share it in the ratio 12:7:8.  
Anna receives \$504.

- (a) Calculate the **total** amount. [3]

We have that Anna receives

$$\frac{12}{12 + 7 + 8}$$

$$= \frac{12}{27}$$

of the total money.

Hence

$$\frac{12}{27} \times \text{total} = 504$$

$$\rightarrow \text{total} = \frac{27}{12} \times 504$$

$$= 1134$$

- (b) (i) Anna uses 7% of her \$504 to pay a bill.  
Calculate how much she has left.

[3]

$$504 - 504 \times 0.07$$

$$= 468.72$$

- (ii) She buys a coat in a sale for \$64.68.  
This was 23% less than the original price.  
Calculate the original price of the coat.

[3]

$$64.68 \div 0.77$$

$$= 84$$

- (c) Bobby uses \$250 of his share to open a bank account.

This account pays compound interest at a rate of 1.6% per year.

Calculate the amount in the bank account after 3 years.

Give your answer correct to 2 decimal places.

[3]

$$250 \times 1.016^3$$

$$= 262.19$$

- (d) Carl buys a computer for \$288 and sells it for \$324.

Calculate his percentage profit.

[3]

His profit is

$$324 - 288$$

$$= 36$$

As a percentage

$$\frac{36}{288} \times 100\%$$

$$= 12.5\%$$

### Question 3

$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$E = \{x : x \text{ is an even number}\}$$

$$F = \{2, 5, 7\}$$

$$G = \{x : x^2 - 13x + 36 = 0\}$$

(a) List the elements of set  $E$ .

[1]

$$\textcolor{red}{E = \{2, 4, 6, 8\}}$$

(b) Write down  $n(F)$ .

[1]

**3**

(c) (i) Factorise  $x^2 - 13x + 36$ .

[2]

$$(x - 9)(x - 4)$$

(ii) Using your answer to part (c)(i), solve  $x^2 - 13x + 36 = 0$  to find the two elements of  $G$ . [1]

$$x^2 - 13x + 36 = 0$$

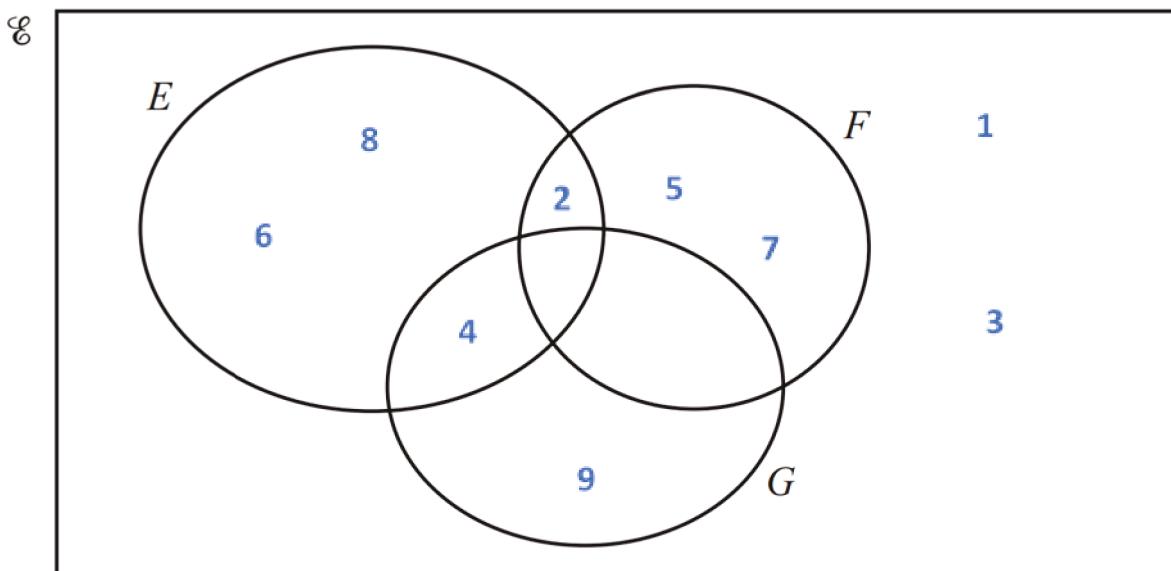
$$\rightarrow (x - 9)(x - 4) = 0$$

$$\rightarrow x = 9, \quad x = 4$$

Hence

$$\textcolor{red}{G = \{4, 9\}}$$

- (d) Write all the elements of  $\mathcal{E}$  in their correct place in the Venn diagram. [2]



- (e) Use set notation to complete the following statements.

(i)  $F \cap G = \dots$  [1]

$$\mathbf{F} \cap \mathbf{G} = \emptyset$$

Note that  $\emptyset$  denotes the empty set

$$\emptyset = \{ \}$$

(ii)  $7 \dots E$  [1]

$$\mathbf{7} \notin \mathbf{E}$$

(iii)  $n(E \dots F) = 6$  [1]

$$\mathbf{n}(\mathbf{E} \cup \mathbf{F}) = 6$$

## Question 4

- (a) In a sale, Jen buys a laptop for \$351.55.  
This price is 21% less than the price before the sale.

Calculate the price before the sale.

[3]

If  $x$  is the original price then we have

$$(1 - 0.21) \times x = 351.55$$

$$\rightarrow 0.79x = 351.55$$

$$\rightarrow x = 351.55 \div 0.79$$

$$= 445$$

- (b) Alex invests \$4000 at a rate of 8% per year simple interest for 2 years.  
Bob invests \$4000 at a rate of 7.5% per year compound interest for 2 years.

Who receives more interest and by how much?

[6]

Alex's interest is

$$4000 \times 0.08 \times 2$$

$$= 640$$

Bob's interest is

$$4000 \times 1.075^2 - 4000$$

$$= 622.5$$

Alex has more interest, and the difference is

$$640 - 622.5$$

$$= 17.5$$

## Question 5

Children go to camp on holiday.

- (a) Fatima buys bananas and apples for the camp.

- (i) Bananas cost \$0.85 per kilogram.

Fatima buys 20kg of bananas and receives a discount of 14%.

How much does she spend on bananas?

[3]

We first need to work out what she would normally pay for 20 kg and then apply the 14% discount on this amount.

Price: \$0.85/kg

For 20 kg, she would pay:

$$20 \text{ kg} \times \$0.85/\text{kg} = \$17$$

With the 14% discount:

$$\$17 - \frac{14}{100} \times \$17 = \$17 - \$2.38$$

$$\$17 - \frac{14}{100} \times \$17$$

$$= \$14.62$$

- (ii) Fatima spends \$16.40 on apples after a discount of 18%.

Calculate the original price of the apples.

[3]

We represent the original price with the unknown  $x$ .

The 18% discount has been reduced from the original price,  $x$

$$x - \frac{18}{100}x = \$16.40$$

$$82x = \$1640$$

$$x = \$20.$$

- (iii) The ratio number of bananas:number of apples = 4:5.

There are 108 bananas.

Calculate the number of apples.

[2]

A ratio represents the actual amount of money for each of the 2, divided by the highest common factor.

In this case, we represent the highest common factor with the unknown  $x$ .

The number of bananas =  $4x$

The number of apples =  $5x$

The number of bananas = 108

$$4x = 108$$

$$x = 27$$

We use this value to work out the number of apples.

The number of apples =  $5x$

The number of apples =  $5 \times 27$

**The number of apples = 135**

- (b) The cost to hire a tent consists of two parts.

$$\boxed{\$c} + \boxed{\$d \text{ per day}}$$

The total cost for 4 days is \$27.10 and for 7 days is \$34.30.

Write down two equations in  $c$  and  $d$  and solve them.

[4]

We can write the following equations as the amount  $c$  plus the amount  $d$  times the number of days, since  $d$  is a price/day.

For 4 days:

$$c + 4d = \$27.10$$

$$c + 7d = \$34.30$$

We subtract the simultaneous equations to work out  $d$ .

$$c + 4d - c - 7d = \$27.1 - \$34.3$$

$$-3d = -7.2$$

$$d = \$2.4$$

We substitute this value in one of the equations above to work out  $c$ .

$$c + 4 \times \$2.4 = \$27.10$$

$$c + \$9.6 = \$27.10$$

$$c = \$17.5$$

- (c) The children travel 270 km to the camp, leaving at 07 43 and arriving at 15 13. [3]

Calculate their average speed in km/h.

$$\text{Speed} = \text{distance}/\text{time}$$

$$\text{Distance} = 270 \text{ km}$$

$$\text{Time} = 7 \text{ hours and } 30 \text{ minutes} = 7.5 \text{ hours}$$

$$\text{Speed} = 270 \text{ km}/7.5 \text{ hours}$$

$$\text{Speed} = 36 \text{ km/h}$$

- (d) Two years ago \$540 was put in a savings account to pay for the holiday.

The account paid **compound** interest at a rate of 6% per year.

How much is in the account now?

[2]

We can use the compound interest formula:

$$5\% = \frac{6}{100} = 0.06$$

$$\$540 \times (1 + 0.06)^2 = \$540 \times 1.1236$$

$$\$540 \times (1.06)^2$$

$$= \$606.744$$

## Question 6

(a) Work out the following.

$$(i) \frac{1}{0.2^2} \quad [1]$$

Square the denominator:

$$\frac{1}{0.2^2} = \frac{1}{0.04}$$

Divide the numbers.

$$\frac{1}{0.2^2} = 25$$

$$(ii) \sqrt{5.1^2 + 4 \times 7.3^2} \quad [1]$$

Square the numbers with an exponent:

$$\sqrt{5.1^2 + 4 \times 7.3^2} = \sqrt{26.01 + 4 \times 53.29}$$

$$\sqrt{5.1^2 + 4 \times 7.3^2} = \sqrt{26.01 + 213.16}$$

Add the numbers together and apply square root.

$$\sqrt{5.1^2 + 4 \times 7.3^2} = \sqrt{239.17}$$

Get the answer correct to three significant figures:

$$\sqrt{5.1^2 + 4 \times 7.3^2} = 15.5$$

(iii)  $25^{\frac{1}{2}} \times 1000^{-\frac{2}{3}}$

[2]

Apply the exponents. (Remember that when an exponent is in a denominator, then it is the same as taking a root of a number. For example exponent of  $\frac{1}{2}$  is the same as square root.)

$$25^{\frac{1}{2}} \times 1000^{-\frac{2}{3}} = \sqrt{25} \times \sqrt[3]{1000}^{-2}$$

$$25^{\frac{1}{2}} \times 1000^{-\frac{2}{3}} = 5 \times 10^{-2}$$

$$25^{\frac{1}{2}} \times 1000^{-\frac{2}{3}} = 5 \times 100^{-1}$$

A negative exponent means that the number should be inverted (form  $1/x$  from  $x$ ).

$$25^{\frac{1}{2}} \times 1000^{-\frac{2}{3}} = 5 \times \frac{1}{100}$$

Multiply the numbers to get the final answer:

$$25^{\frac{1}{2}} \times 1000^{-\frac{2}{3}} = 0.05$$

- (b) Mia invests \$7500 at 3.5% per year **simple** interest.  
Calculate the total amount she has after 5 years.

[3]

We calculate 3.5% (simple interest) of \$550 as:

$$\$7500 \times 0.035 = \$262.50$$

Therefore Mia gets \$262.5 every year. In 5 years, this is:

$$\$262.50 \times 5 = \$1312.50$$

By adding this to the original investment, we have that Mia has

$$\$7\,500 + \$1\,312.50 = \$8\,812.50$$

- (c) Written as the product of prime factors  $48 = 2^4 \times 3$ .

- (i) Write 60 as the product of prime factors.

[2]

The simplest way to write a number as a product of prime numbers is to try to divide it by the small prime numbers (2,3,5...).

We can see that 60 is divisible by 2, so we will continue to divide it by 2 and then try 3 or higher primes once the remaining number is no longer divisible by 2.

$$60 \div 2 = 30$$

$$30 \div 2 = 15$$

15 is no longer divisible by 2, so we try higher primes.

$$15 \div 3 = 5$$

5 is already a prime, so we have fully factorized the number.

We have divided by 2 twice, by 3 once and got left with 5, therefore:

$$\mathbf{60 = 2^2 \times 3 \times 5}$$

- (ii) Work out the highest common factor (HCF) of 48 and 60.

[2]

We already have the prime factorization of 48:

$$48 = 2^4 \times 3$$

Then from the previous part:

$$60 = 2^2 \times 3 \times 5$$

From the factorization of 48 and 60, we see that both numbers contain number 2 at least twice and one factor of 3.

The highest common factor is the product of these common factors.

$$HCF(48,60) = 2^2 \times 3$$

$$\mathbf{HCF(48, 60) = 12}$$

- (iii) Work out the lowest common multiple (LCM) of 48 and 60. [2]

The simplest way to get the lowest common multiple, when we already know the highest common factor, is simply divide the product of the two numbers by their highest common factor.

$$LCM(48,60) = \frac{48 \times 60}{HCF(48,60)}$$

$$LCM(48,60) = \frac{2880}{12}$$

$$\mathbf{LCM(48, 60) = 240}$$

But we also provide an alternative method which can be used without knowing the highest common factor.

Both numbers contain number 2 at least twice and number 3 once (orange). The number 60 contains one extra factor 5 (green) and the number 48 contains number 2 two more times (blue). The LCM is therefore

$$LCM(48,60) = 2 \times 2 \times 3 \times 5 \times 2 \times 2$$

$$\mathbf{LCM(48, 60) = 240}$$

(This result agrees with the alternative method.)

## Question 7

Lucy works in a clothes shop.

- (a) In one week she earned \$277.20.

(i) She spent  $\frac{1}{8}$  of this on food.

Calculate how much she spent on food.

[1]

Multiply the amount Lucy has earned by  $\frac{1}{8}$  to get the amount of number she spent of food.

$$\$277.20 \times \frac{1}{8} = \$34.65$$

- (ii) She paid 15% of the \$277.20 in taxes.  
Calculate how much she paid in taxes.

[2]

The 15% can be represented as:

$$15\% = \frac{15}{100} = 0.15$$

Hence to get 15% of the \$227.20, multiply this about by 0.15. Lucy paid in taxes:

$$\$277.20 \times 0.15 = \$41.58$$

- (iii) The \$277.20 was 5% more than Lucy earned in the previous week.  
Calculate how much Lucy earned in the previous week.

[3]

The amount she earned in this week represents is 5% more than the amount she earned in the previous week. If the previous week represents 100%, then this week she earned 105% of that.

If \$227.20 is 105% then to find the 100%, we need to divide this amount by 1.05 (representing 105%).

$$\frac{\text{previous week}}{100\%} = \frac{\text{this week}}{105\%}$$

$$\text{previous week} = \frac{\$277.20}{1.05}$$

The amount Lucy earned in the previous week:

$$\text{previous week} = \$264$$

- (b) The shop sells clothes for men, women and children.
- (i) In one day Lucy sold clothes with a total value of \$2200 in the ratio

$$\text{men : women : children} = 2 : 5 : 4.$$

Calculate the value of the women's clothes she sold.

[2]

The ratio of clothes sold for men, women and children:

$$\text{men : women : children} = 2 : 5 : 4$$

When we sum all three ratio factor, we get:

$$2 + 5 + 4 = 11$$

Hence the proportion for any of the groups is the corresponding ratio factor to the total 11.

So for women, this proportion is:

$$\text{proportion of women clothes} = \frac{\text{ratio factor for women}}{\text{total}}$$

$$\text{proportion of women clothes} = \frac{5}{11}$$

Using this, we can calculate the value of the women clothes she sold:

$$\text{value of women clother sold}$$

$$= \text{total value} \times \text{proportion of women clothes}$$

$$\text{value of women clother sold} = \$2200 \times \frac{5}{11}$$

$$\text{value of women clother sold} = \$1000$$

- (ii) The \$2200 was  $\frac{44}{73}$  of the total value of the clothes sold in the shop on this day.

Calculate the total value of the clothes sold in the shop on this day.

[2]

If \$2200 represents  $44/73$  of the total value, then the total value can be

found by dividing the partial value of clothes sold by this proportion.

$$\text{total value} = \$2200 \div \frac{44}{73}$$

$$\text{total value} = \$3650$$

# Number

## Difficulty: Medium

### Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 4

Time allowed: 86 minutes

Score: /75

Percentage: /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

##### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

- (a) In 2008 the total number of tickets sold for an athletics meeting was 3136.  
The ratio child tickets sold : adult tickets sold = 17 : 32.
- (i) How many child tickets were sold? [2]

There were  $17 + 32 = 49$  total parts with 17 of those parts being children's tickets

$$3136 \times \frac{17}{49}$$
  
  
 $= 1088$

- (ii) Child tickets cost \$2 each and adult tickets cost \$4.50 each.

Show that the total amount received from the sale of the tickets in 2008 was \$11 392. [2]

Number of adult tickets is

$$3136 - 1088$$
  
  
 $= 2048$

Amount made was

$$1088 \times 2 + 2048 \times 4.5$$
  
  
 $= 11\,392$

- (b) In 2009 the amount received from the sale of tickets for the athletics meeting was \$12 748.

Calculate the percentage increase in the amount received from 2008 to 2009.

[3]

Percentage increase is

$$\frac{12746 - 11392}{11392} \times 100\%$$

$$= 11.9\%$$

- (c) In 2008 the amount of \$11 392 was 28% more than the amount received in 2007.

Calculate how much was received in 2007.

[3]

We have that

$$11392 = 1.28 \times n$$

Where n is the amount received in 2007

$$\rightarrow n = \frac{11392}{1.28}$$

$$= 8900$$

## Question 2

- (a) Hansi and Megan go on holiday.

The costs of their holidays are in the ratio Hansi : Megan = 7 : 4.

Hansi's holiday costs \$756.

Find the cost of Megan's holiday.

[2]

There are  $7 + 4 = 11$  parts and 7 of those parts are Hansi's.

We have that

$$\frac{7}{11}(756 + M) = 756$$

Where M is the cost of Megan's holiday.

Rearranging for M

$$M = \frac{11}{7}(756) - 756$$

$$= 432$$

- (b) In 2008, Hansi earned \$7800.

- (i) He earned 15% more in 2009.

Calculate how much he earned in 2009.

[2]

15% more means multiply by 1.15 (115%)

$$7800 \times 1.15$$

$$= 8970$$

- (ii) In 2010, he earns 10% more than in 2009.  
Calculate the percentage increase in his earnings from 2008 to 2010. [3]

Increase by another 10%

$$8970 \times 1.10$$

$$= 9867$$

The difference between this and 2008 is

$$9867 - 7800$$

$$= 2067$$

As a percentage this is

$$\frac{2067}{7800} \times 100\%$$

$$= 26.5\%$$

- (c) Megan earned \$9720 in 2009. This was 20% more than she earned in 2008. [3]  
How much did she earn in 2008?

We have that

$$9720 = 1.2 \times E_{2008}$$

$$\rightarrow E_{2008} = \frac{9720}{1.2}$$

$$= 8100$$

- (d) Hansi invested \$500 at a rate of 4% per year compound interest.  
Calculate the final amount he had after three years.

[3]

Increase of 4% per year means multiply by 1.04 each year.

After 3 years we have

$$500 \times 1.04^3$$

$$= 562.43$$

### Question 3

Thomas, Ursula and Vanessa share \$200 in the ratio

$$\text{Thomas : Ursula : Vanessa} = 3 : 2 : 5.$$

- (a) Show that Thomas receives \$60 and Ursula receives \$40. [2]

There are

$$3 + 2 + 5$$

$$= 10$$

total parts and Thomas receives 3 parts whilst Ursula receives 2 parts.

$$\text{Thomas} = \frac{3}{10} \times 200$$

$$= 60$$

$$\text{Ursula} = \frac{2}{10} \times 200$$

$$= 40$$

- (b) Thomas buys a book for \$21.  
What percentage of his \$60 does Thomas have left? [2]

Thomas has

$$60 - 21$$

$$= 39$$

left.

As a percentage this is

$$\frac{39}{60} \times 100\%$$

$$= 65\%$$

- (c) Ursula buys a computer game for \$36.80 in a sale.  
The sale price is 20% less than the original price.  
Calculate the original price of the computer game. [3]

$$36.80 \div 0.8$$

$$= 46$$

- (d) Vanessa buys some books and some pencils.  
Each book costs \$12 **more** than each pencil.  
The total cost of 5 books and 2 pencils is \$64.20.  
Find the cost of one pencil. [3]

Let the book price be  $b$  and the pencil price be  $p$ .

We have

$$5b + 2p = 64.20 \quad (1)$$

$$b = p + 12 \quad (2)$$

Sub (2) into (1) for

$$5(12 + p) + 2p = 64.2$$

$$\rightarrow 60 + 7p = 64.2$$

Minus 60 from both sides

$$7p = 4.2$$

Divide through by 7

$$p = 0.6$$

## Question 4

Alberto and Maria share \$240 in the ratio 3 : 5.

- (a) Show that Alberto receives \$90 and Maria receives \$150. [1]

For Alberto

$$240 \times \frac{3}{8}$$

$$= 90$$

For Maria

$$240 \times \frac{5}{8}$$

$$= 150$$

- (b) (i) Alberto invests his \$90 for 2 years at  $r\%$  per year **simple** interest.

At the end of 2 years the amount of money he has is \$99.

Calculate the value of  $r$ .

[2]

He has gained

$$99 - 90$$

$$= \$9$$

over two years, that means

$$9 \div 2$$

$$= \$4.5$$

each year.

As a percentage this is

$$\frac{4.5}{90} \times 100$$

$$= 5\%$$

- (ii) The \$99 is 60% of the cost of a holiday. [2]  
Calculate the cost of the holiday.

$$99 \div 0.6$$

$$= 165$$

- (c) Maria invests her \$150 for 2 years at 4% per year **compound** interest. [2]  
Calculate the exact amount Maria has at the end of 2 years.

Multiply by 1.04 for each year

$$150 \times 1.04^2$$

$$= 162.24$$

- (d) Maria continues to invest her money at 4% per year **compound** interest.  
After 20 years she has \$328.67.
- (i) Calculate exactly how much more this is than \$150 invested for 20 years at 4% per year **simple** interest. [3]

At simple interest we have

$$150 + 0.04 \times 150 \times 20$$

$$= 270$$

The difference is

$$328.67 - 270$$

$$= 58.67$$

- (ii) Calculate \$328.67 as a percentage of \$150. [2]

$$(328.67 \div 150) \times 100$$

$$= 219$$

## Question 5

Daniella is 8 years old and Edward is 12 years old.

- (a) Their parents give them some money in the ratio of their ages.

(i) Write the ratio      Daniella's age : Edward's age      in its simplest form.      [1]

$$8 : 12$$

Divide through by 4

$$= 2 : 3$$

- (ii) Daniella receives \$30.

Show that Edward receives \$45.

[1]

$$30 \times \frac{3}{2}$$

$$= 45$$

- (iii) What percentage of the total amount of money given by their parents does Edward receive? [2]

Total money is

$$30 + 45$$

$$= 75$$

As a percentage, Edward's amount is

$$\frac{45}{75} \times 100$$

$$= 60\%$$

- (b) Daniella invests her \$30 at 3% per year, **compound** interest.  
Calculate the amount Daniella has after 2 years.  
Give your answer correct to 2 decimal places. [3]

Compound interest of 3% means multiply by 1.03 each year

$$30 \times 1.03^2$$

$$= 31.83$$

- (c) Edward also invests \$30.  
He invests this money at a rate of  $r\%$  per year, **simple** interest.  
After 5 years he has a total amount of \$32.25.  
Calculate the value of  $r$ . [2]

Edward has gained

$$32.25 - 30$$

$$= 2.25$$

In 5 years.

Per year, this is

$$2.25 \div 5$$

$$= 0.45$$

As a percentage this is

$$\frac{0.45}{30} \times 100$$

$$= 1.5\%$$

## Question 6

Marcus receives \$800 from his grandmother.

- (a) He decides to spend \$150 and to divide the remaining \$650 in the ratio  
savings : holiday = 9 : 4.

Calculate the amount of his savings.

[2]

The ratio represents the amount of money for both savings and holiday divided by their highest common factor. We represent the highest common factor with the unknown  $x$ .

$$\text{Savings} = 9x$$

$$\text{Holiday} = 4x$$

$$9x + 4x = \$650$$

$$x = \$50$$

$$\text{savings} = 9 \times \$50$$

$$\text{savings} = \$450$$

- (b) (i) He uses 80% of the \$150 to buy some clothes.

Calculate the cost of the clothes.

[2]

$$\frac{80}{100} \times \$150$$

$$= \$120$$

- (ii) The money remaining from the \$150 is  $37\frac{1}{2}\%$  of the cost of a day trip to Cairo.

Calculate the cost of the trip.

[2]

$$\text{The money remaining} = \$150 - \$120 = \$30$$

$$\text{Cost of the day trip} \times \frac{37.5}{100} = \$30$$

$$37.5 \times \text{cost of the day trip} = \$3000$$

$$\text{Cost of the day trip} = \$80$$

- (c) (i) Marcus invests \$400 of his savings for 2 years at 5 % per year **compound** interest.

Calculate the amount he has at the end of the 2 years.

[2]

We use the formula for calculating the compound interest:

$$\text{Initial amount} \times (1 + \text{rate})^n = \text{final amount}$$

where n represents the number of years

$$\$400 \times (1 + \frac{5}{100})^2$$

$$= \$441$$

- (ii) Marcus's sister also invests \$400, at  $r$  % per year **simple** interest.  
At the end of 2 years she has exactly the same amount as Marcus.

Calculate the value of  $r$ .

[3]

For simple interest, the amount of money she earns is:

$$\$400 \times \frac{r}{100} \times 2 = \$441 - \$400$$

$$8r = \$41$$

$$r = 5.125$$

## Question 7

Beatrice has an income of \$40 000 in one year.

(a) She pays:

- no tax on the first \$10 000 of her income;
- 10% tax on the next \$10 000 of her income;
- 25 % tax on the rest of her income.

Calculate

(i) the total amount of tax Beatrice pays,

[2]

Beatrice pays 10% tax on 10000 and 25% tax on 20000.

$$0.1 \times 10000 + 0.25 \times 20000$$

$$= \$6000$$

(ii) the total amount of tax as a percentage of the \$40000.

[2]

As a percentage this is

$$\frac{6000}{40000} \times 100\%$$

$$= 15\%$$

- (b) Beatrice pays a yearly rent of \$10 800.

After she has paid her tax, rent and bills, she has \$12 000.

Calculate how much Beatrice spends on bills.

[1]

Beatrice has spent

$$40000 - 12000$$

$$= 28000$$

in total.

Hence

$$\text{bills} + \text{tax} + \text{rent} = 28000$$

$$\rightarrow 10800 + 6000 + \text{bills} = 28000$$

$$\rightarrow \text{bills} = 28000 - 10800 - 6000$$

$$= \$11200$$

- (c) Beatrice divides the \$12 000 between shopping and saving in the ratio

$$\text{shopping : saving} = 5 : 3.$$

- (i) Calculate how much Beatrice spends on shopping in one year. [2]

$$12000 \times \frac{5}{8}$$

$$= \$7500$$

- (ii) What fraction of the original \$40000 does Beatrice **save**?

Give your answer in its lowest terms. [1]

The amount saved is

$$12000 - 7500$$

$$= 4500$$

As a fraction

$$\frac{4500}{40000}$$

$$= \frac{9}{80}$$

- (d) The rent of \$10800 is an increase of 25 % on her previous rent.

Calculate her previous rent. [2]

$$10800 \div 1.25$$

$$= \$8640$$

# Number

## Difficulty: Medium

### Model Answers 5

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 5

**Time allowed:** 77 minutes

**Score:** /67

**Percentage:** /100

**Grade Boundaries:**

#### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

#### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

Each year a school organises a concert.

- (a) (i) In 2004 the cost of organising the concert was \$ 385.

In 2005 the cost was 10% less than in 2004.

Calculate the cost in 2005.

[2]

$$385 \times 0.9$$

$$= \$346.50$$

- (ii) The cost of \$ 385 in 2004 was 10% more than the cost in 2003.

Calculate the cost in 2003.

[2]

$$385 \div 1.1$$

$$= \$350$$

- (b) (i) In 2006 the number of tickets sold was 210.

The ratio

Number of adult tickets : Number of student tickets was 23 : 19.

How many adult tickets were sold?

[2]

$$210 \times \frac{23}{23 + 19}$$

$$= 115$$

- (ii) Adult tickets were \$ 2.50 each and student tickets were \$ 1.50 each.

Calculate the **total amount received** from selling the tickets.

[2]

Number of student tickets is

$$210 - 115$$

$$= 95$$

Hence, total amount is

$$115 \times 2.5 + 95 \times 1.5$$

$$= \$430$$

(iii) In 2006 the cost of organising the concert was \$ 410.

Calculate the percentage profit in 2006.

[2]

Profit is

$$430 - 410$$

$$= 20$$

as a percentage, this is

$$\frac{20}{410} \times 100$$

$$= 4.88$$

(c) In 2007, the number of tickets sold was again 210.

Adult tickets were \$ 2.60 each and student tickets were \$ 1.40 each.

The total amount received from selling the 210 tickets was \$ 480.

How many student tickets were sold?

[4]

Let the number of student tickets sold be  $x$ .

Hence

$$2.6(210 - x) + 1.4x = 480$$

$$\rightarrow 546 - 1.2x = 480$$

$$\rightarrow 1.2x = 546 - 480$$

$$\rightarrow x = \frac{66}{1.2}$$

$$= 55$$

## Question 2

Maria, Carolina and Pedro receive \$800 from their grandmother in the ratio

$$\text{Maria: Carolina: Pedro} = 7:5:4.$$

- (a) Calculate how much money each receives. [3]

The ratio represents the actual amount of money divided by a common factor.

We represent the common factor with the unknown  $x$ .

Maria received  $7x$ , Carolina received  $5x$  and Pedro received  $4x$ .

$$7x + 5x + 4x = 800$$

$$x = 50$$

Maria received  $7 \times 50 = 350$ , Carolina received  $5 \times 50 = 250$  and Pedro received

$$4 \times 50$$

$$= 200$$

- (b) Maria spends  $\frac{2}{7}$  of her money and then invests the rest for two years  
at 5% per year simple interest.  
How much money does Maria have at the end of the two years? [3]

We work out the amount of money that Maria invests:

$$350 - \frac{2}{7} \times 350 = 350 - 100 = 250$$

The rate is 5% per year simple interest.

$$250 + \frac{2 \times 5 \times 250}{100} = 250 + 25$$

$$= 275$$

- (c) Carolina spends all of her money on a hi-fi set and two years later sells it at a loss of 20%.  
How much money does Carolina have at the end of the two years? [2]

She loses 20% of her initial amount:

$$\text{Final amount} = 250 - \frac{20 \times 250}{100}$$

$$= 200$$

- (d) Pedro spends some of his money and at the end of the two years he has \$100.  
Write down and simplify the ratio of the amounts of money Maria, Carolina and Pedro have at the end of the two years. [2]

$$\text{Maria : Carolina : Pedro} = 275 : 200 : 100$$

We can simplify the ratio by 25.

$$\text{Maria : Carolina : Pedro} = 11 : 8 : 4$$

- (e) Pedro invests his \$100 for two years at a rate of 5% per year **compound interest**.  
Calculate how much money he has at the end of these two years. [2]

The rate is 5% per year **compound interest**.

$$100 \times \left(1 + \frac{5}{100}\right)^2 = 100 + 10.25$$

$$= 110.25$$

### Question 3

A Spanish family went to Scotland for a holiday.

- (a) The family bought 800 pounds (£) at a rate of £1 = 1.52 euros (€).  
How much did this cost in euros? [1]

$$\text{£1} = \text{€ } 1.52 \text{ €}$$

The amount of money payed in euros for £800 is:

$$\text{£800} \times \text{€ } 1.52/\text{£}$$

$$= \text{€ } 1216$$

- (b) The family returned home with £118 and changed this back into euros.  
They received €173.46.  
Calculate how many euros they received for each pound. [1]

To work out the number of euros received for each pound, we divide the  
number of euros obtained by the number of pounds.

$$\frac{\text{€ } 173.46}{\text{£ } 118} = 1.47$$

The answer is they received 1.47 euros for each pound.

- (c) A toy which costs €11.50 in Spain costs only €9.75 in Scotland.  
Calculate, as a percentage of the cost in Spain, how much less it costs in Scotland. [2]

The difference between the 2 prices of the toy is:

$$\text{€ } 11.50 - \text{€ } 9.75 = \text{€ } 1.75$$

The toy costs € 1.75 less in Scotland than in Spain.

We need to write this number as a percentage of the cost in Spain.

$$\frac{x}{100} \times \text{€ } 11.50 = \text{€ } 1.75$$

$$\text{€ } 11.50x = \text{€ } 175$$

$$x = 15.21\%$$

- (d) The total cost of the holiday was €4347.00.  
In the family there were 2 adults and 3 children.  
The cost for one adult was double the cost for one child.  
Calculate the cost for one child. [2]

We note with  $a$  the cost for one adult and with  $c$  the cost for one child.

$$a = 2c$$

$$2a + 3c = \text{€}4347$$

We substitute the first equation into the second one to obtain an equation with only one unknown.

$$4c + 3c = \text{€}4347$$

$$7c = \text{€}4347$$

$$c = \text{€}621$$

- (e) The original cost of the holiday was reduced by 10% to €4347.00.  
Calculate the original cost. [2]

$$\text{€}4347 = \text{original cost} - \frac{10}{100} \times \text{original cost}$$

$$\text{€}4347 = 90 \times \text{original cost}$$

$$\text{Original cost} = \text{€}4830$$

- (f) The plane took 3 hours 15 minutes to return to Spain.

The length of this journey was 2350 km.

Calculate the average speed of the plane in

- (i) kilometres per hour,

[2]

To express the result in km per hour, we firstly need to have the time, 3 hours in

15 minutes, only in hours.

$$\frac{15 \text{ minutes}}{60 \text{ minutes}} = 0.25$$

$$\text{Time} = 3.25 \text{ hours}$$

To work out the speed, we divide the distance by time.

$$\text{Speed} = \frac{2350 \text{ km}}{3.25 \text{ hours}}$$

$$\text{Speed} = 723.07 \text{ km/hour}$$

- (ii) metres per second.

[1]

To express the result in meter per second we need to have the distance in meters

and the time in seconds.

$$\text{Distance} = 2350000 \text{ m}$$

$$\text{Time} = 3.25 \text{ hours} \times 3600$$

$$\text{Time} = 11700 \text{ s}$$

$$\text{Speed} = \frac{2350000 \text{ m}}{11700 \text{ seconds}}$$

$$\text{Speed} = 200.9 \text{ m/s}$$

## Question 4

The population of Newtown is 45 000.  
The population of Villeneuve is 39 000.

- (a) Calculate the ratio of these populations in its simplest form.

[1]

The ratio is:

$$\text{Newton : Villeneuve} = 45\,000 : 39\,000$$

We simplify this by a common factor.

In this case, the highest common factor is 3000.

$$\text{Newton : Villeneuve} = 15 : 13$$

- (b) In Newtown, 28% of the population are below the age of twenty.  
Calculate how many people in Newtown are below the age of twenty.

[2]

The number of people below the age of twenty is:

$$\frac{28}{100} \times 45\,000 = 12\,600$$

- (c) In Villeneuve, 16 000 people are below the age of twenty.  
Calculate the percentage of people in Villeneuve below the age of twenty.

[2]

We note this percentage with the unknown x.

$$\frac{x}{100} \times 39\,000 = 16\,000$$

$$x = 41.02\%$$

- (d) The population of Newtown is 125% **greater** than it was fifty years ago.  
Calculate the population of Newtown fifty years ago.

[2]

We note the population from fifty years ago with the unknown  $y$ .

$$y + \frac{125y}{100} = 45\ 000$$

$$\frac{225y}{100} = 45\ 000$$

$$y = 20\ 000$$

- (e) The two towns are combined and made into one city called Monocity.  
In Monocity the ratio of men : women : children is 12 : 13 : 5.  
Calculate the number of children in Monocity.

[2]

The total population of the city Monocity is the sum of the populations  
of the 2 previous cities.

$$\text{Monocity population} = 45\ 000 + 39\ 000$$

$$\text{Monocity population} = 84\ 000$$

The ratio of men : women : children is represented by the actual  
number of men, women and children divided by the highest common  
factor of the 3 numbers.

We note this common factor with the unknown  $x$ .

$$\text{The number of men} = 12x$$

$$\text{The number of women} = 13x$$

$$\text{The number of children} = 5x$$

The total population will be:

$$12x + 13x + 5x = 84\ 000$$

$$30x = 84\ 000$$

$$x = 2800$$

The number of children in Monocity =  $5x$

The number of children in Monocity =  $5 \times 2800$

**The number of children in Monocity = 14000**

## Question 5

- (a) The technical data of a car includes the following information.

Type of road	Petrol used per 100 km
Main roads	9.2 litres
Other roads	8.0 litres

- (i) How much petrol is used on a journey of 350 km on a main road?

[1]

We work out how many litres are needed by knowing that for main roads there are 9.2 litres used per 100 km.

$$\frac{9.2 \text{ litres}}{100 \text{ km}} \times 350 \text{ km}$$

$$= 32.2 \text{ litres}$$

- (ii) On other roads, how far can the car travel on 44 litres of petrol?

[1]

We work out how many km can the car go using 44 litres by knowing that for other roads there are 8 litres used per 100 km.

$$\frac{100 \text{ km}}{8 \text{ litres}} \times 44 \text{ litres}$$

$$= 550 \text{ km}$$

- (iii) A journey consists of 200 km on a main road and 160 km on other roads.

- (a) How much petrol is used?

[2]

On a main road, 200 km use:

$$200 \text{ km} \times \frac{9.2 \text{ litres}}{100 \text{ km}} = 18.4 \text{ litres}$$

On other roads, 160 km use:

$$160 \text{ km} \times \frac{8 \text{ litres}}{100 \text{ km}} = 12.8 \text{ litres}$$

The whole journey will need:

18.4 litres + 12.8 litres

= **31.2 litres**

(b) Work out the amount of petrol used per 100 km of this journey.

[1]

The total distance travelled is:

200 km + 160 km = 360 km

31.2 litres are used for 360 km. For 100 km of the same journey:

$$100 \text{ km} \times \frac{31.2 \text{ litres}}{360 \text{ km}}$$

= **8.7 litres**

(b) A model of a car has a scale of 1 : 25.

(i) The length of the car is 3.95 m.

Calculate the length of the model.  
Give your answer in centimetres.

[3]

A scale of 1 : 25 represents that 1 m in the model is equal to 25 m in reality.

For a real length of 3.95 m, the model length will be:

$$\frac{3.95 \text{ m} \times 1 \text{ m}}{25 \text{ m}} = 0.158 \text{ m}$$

We convert this length in centimetres.

**0.158 m = 15.8 cm**

- (ii) The painted surface area of the model is  $128 \text{ cm}^2$ .  
Calculate the painted surface area of the car, giving your answer in square centimetres. [2]

In this case, the scale represents that  $1 \text{ cm}^2$  in the model is equivalent with  $25 \text{ cm}^2$ . Thus the area scale factor is  $25^2$ .

The surface area of the car is:

$$A = 25^2 \times 128 \text{ cm}^2$$

$$\mathbf{A = 80000 \text{ cm}^2}$$

- (iii) The size of the luggage space of the car is 250 litres.  
Calculate the size of the luggage space of the model, giving your answer in millilitres. [3]

For a linear scale factor of  $1 : 25$ , the scale factor for volume is  $25^3$ . The

volume of the luggage space in the model will be:

$$V = \frac{250 \text{ litres}}{25^3}$$

$$V = 0.016 \text{ litres}$$

$$\mathbf{V = 16 \text{ cm}^3}$$

## Question 6

- (a) At an athletics meeting, Ben's time for the 10 000 metres race was 33 minutes exactly and he finished at 15 17.

- (i) At what time did the race start?

[1]

**The race started at 14:44.**

- (ii) What was Ben's average speed for the race? Give your answer in kilometres per hour.

[2]

$$\text{speed} = \text{distance}/\text{time}$$

We convert 33 minutes in hours.

$$\text{Time} = 33 \text{ minutes}/60 \text{ minutes/h}$$

$$\text{Time} = 0.55 \text{ h}$$

$$\text{Distance} = 10 \text{ 000 metres} = 10 \text{ km}$$

$$\text{Speed} = 10 \text{ km}/0.55 \text{ km/h}$$

**Speed = 18.18 h**

- (iii) . The winner finished 51.2 seconds ahead of Ben.

How long did the winner take to run the 10 000 metres?

[1]

**The winner took: 33 minutes – 51.2 seconds = 32 minutes and 8.8 seconds.**

- (b) The winning distance in the javelin competition was 80 metres.  
Otto's throw was 95% of the winning distance.  
Calculate the distance of Otto's throw.

[2]

$$\text{Distance} = \frac{95 \times 80 \text{ metres}}{100}$$

$$\text{Distance} = 76 \text{ m}$$

- (c) Pamela won the long jump competition with a jump of 6.16 metres.  
This was 10% further than Mona's jump.  
How far did Mona jump?

[2]

We represent Mona's jump with the unknown x and

Pamela's jump with the unknown y.

$$x + \frac{10x}{100} = 6.16 \text{ m}$$

$$\frac{110x}{100} = 6.16 \text{ m}$$

$$x = 5.6 \text{ m}$$

# Number

## Difficulty: Hard

### Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 1

**Time allowed:** 114 minutes

**Score:** /99

**Percentage:** /100

**Grade Boundaries:**

#### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

#### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

- (a) A library has a total of 10 494 fiction and non-fiction books.  
The ratio fiction books : non-fiction books = 13 : 5.

Find the number of non-fiction books the library has.

[2]

$$10494 \times \frac{5}{18}$$

$$= 2915$$

- (b) The library has DVDs on crime, adventure and science fiction.  
The ratio crime : adventure : science fiction = 11 : 6 : 10.  
The library has 384 **more** science fiction DVDs than adventure DVDs.

Calculate the number of crime DVDs the library has.

[2]

Let the number of adventure DVDs be  $x$ .

Then

$$\frac{x}{6} = \frac{x + 384}{10}$$

Multiply through by 6

$$\rightarrow x = \frac{6}{10}x + \frac{2304}{10}$$

$$\rightarrow x - \frac{6}{10}x = \frac{2304}{10}$$

$$\rightarrow \frac{4}{10}x = \frac{2304}{10}$$

$$\rightarrow x = \frac{2304}{10} \div \frac{4}{10}$$

$$= 576$$

Now we let the number of crime DVDs by  $y$ , and we have

$$y = 576 \times \frac{11}{6}$$

$$= 1056$$

- (c) Every Monday, Sima travels by car to the library.  
The distance is 20km and the journey takes 23 minutes.

- (i) Calculate the average speed for the journey in kilometres per hour.

[2]

23 minutes in hours is

$$\frac{23}{60}$$

Hence speed is

$$20 \div \frac{23}{60}$$

$$= \frac{1200}{23}$$

$$= 52.2$$

- (ii) One Monday, she is delayed and her average speed is reduced to 32km/h.

Calculate the percentage increase in the journey time.

[5]

We have the speed distance time relation

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\rightarrow \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{20}{32}$$

$$= \frac{5}{8} \text{ hours}$$

$$= 37.5 \text{ minutes}$$

This is an increase of

$$\frac{37.5 - 23}{23} \times 100$$

$$= 63\%$$

- (d) In Spain, the price of a book is 11.99 euros.

In the USA, the price of the same book is \$12.99 .

The exchange rate is \$1 = 0.9276 euros.

Calculate the difference between these prices.

Give your answer in dollars, correct to the nearest cent.

[3]

Divide by exchange rate for price in dollars

$$11.99 \div 0.9276$$

$$= 12.9258301$$

Difference in price is then

$$12.99 - 12.9258301$$

$$= 0.06$$

- (e) 7605 books were borrowed from the library in 2016.

This was 22% less than in 2015.

Calculate the number of books borrowed in 2015.

[3]

$$7605 \div 0.78$$

$$= 9750$$

## Question 2

(a) Alex has \$20 and Bobbie has \$25.

- (i) Write down the ratio Alex's money : Bobbie's money in its simplest form. [1]

Ratios can be cancelled just like fractions/equations:

Alex : Bobby

20 : 25

Divide both sides by 5 to get 4 : 5

- (ii) Alex and Bobbie each spend  $\frac{1}{5}$  of their money.

Find the ratio Alex's remaining money : Bobbie's remaining money in its simplest form.

[1]

If they spend  $\frac{1}{5}$  they are left with  $\frac{4}{5}$ :

Alex : Bobby

$\frac{4}{5} \times 20 : \frac{4}{5} \times 25$

16 : 20

Divide both sides by 4 to get 4 : 5 (Could you have predicted that?)

- (iii) Alex and Bobbie then each spend \$4.

Find the new ratio Alex's remaining money : Bobbie's remaining money in its simplest form.

[2]

Now subtract 4 from the 16 and 20:

Alex : Bobby

16 - 4 : 20 - 4

12 : 16

Divide both sides by 4 to get 3 : 4

- (b) (i) The population of a town in the year 1990 was 15 600.  
The population is now 11 420.

Calculate the percentage decrease in the population.

[3]

We can always use: "After" is a percentage of "Before"

$$11420 = p \times 15600$$

Divide both sides by 15600:  $p = \frac{11420}{15600}$

$$p = 0.732 \dots$$

Interpret this as a percentage:  $p = 73.2\%$

This is what is left so to find the decrease subtract from 100%:

$$\text{Percentage Decrease} = 100 - 73.2$$

$$\text{Percentage Decrease} = 26.8\%$$

- (ii) The population of 15 600 was 2.5% less than the population in the year 1980.

Calculate the population in the year 1980.

[3]

We can always use: "After" is a percentage of "Before"

$$P_{1990} \text{ is } 97.5\% \text{ of } P_{1980}$$

Use the decimal form of 97.5%:  $15600 = 0.975 \times P_{1980}$

Divide both sides by 0.975:  $P_{1980} = \frac{15600}{0.975}$

$$P_{1980} = 16000$$

- (c) Chris invests \$200 at a rate of  $x\%$  per year simple interest.  
At the end of 15 years the total interest received is \$48.

Find the value of  $x$ . [2]

For Simple Interest the amount received each year is the same:

$$\text{Yearly interest} = \frac{48}{15} = \$3.20$$

We now express this as a percentage of \$200 to find  $x$ .

$$\begin{aligned}x &= \frac{3.20}{200} \times 100 \\&= 1.6\%\end{aligned}$$

- (d) Dani invests \$200 at a rate of  $y\%$  per year compound interest.  
At the end of 10 years the value of her investment is \$256.

Calculate the value of  $y$ , correct to 1 decimal place. [3]

For Compound Interest the amount received each year is not the same:

To increase by  $y\%$  for  $n$  years, multiply by  $1.0y^n$  (if  $y > 10$ , use  $1.y^n$ )

$$200 \times 1.0y^{10} = 256$$

$$1.0y^{10} = \frac{256}{200}$$

$$1.0y = \sqrt[10]{\frac{256}{200}}$$

$$1.0y = 1.02499\dots$$

$$y = 2.499\dots\%$$

$$y = 2.5\% \quad (\text{to 1 decimal place})$$

### Question 3

An energy company charged these prices in 2013.

Electricity price	Gas price
23.15 cents per day plus 13.5 cents for each unit used	24.5 cents per day plus 5.5 cents for each unit used

- (a) (i) In 90 days, the Siddique family used 1885 units of **electricity**.

Calculate the total cost, in dollars, of the electricity they used.

[2]

**90 days plus 1885 units**

$$90 \times 23.15 + 1885 \times 13.5$$

$$= 27531 \text{ cents}$$

$$= \$275.31$$

- (ii) In 90 days, the **gas** used by the Khan family cost \$198.16 .

Calculate the number of units of gas used.

[3]

**Let the number of units used be  $u$ .**

$$90 \times 24.5 + 5.5u = 19816$$

$$\rightarrow 2205 + 5.5u = 19816$$

**Subtract 2205 from both sides**

$$5.5u = 17611$$

**Divide through by 5.5**

$$\mathbf{u = 3202}$$

- (b) In 2013, the price for each unit of electricity was 13.5 cents.

Over the next 3 years, this price increased exponentially at a rate of 8% per year.

Calculate the price for each unit of electricity after 3 years.

[2]

$$13.5 \times 1.08^3$$

$$\mathbf{= 17.01}$$

(c) Over these 3 years, the price for each unit of gas increased from 5.5 cents to 7.7 cents.

(i) Calculate the percentage increase from 5.5 cents to 7.7 cents.

[3]

$$\frac{(7.7 - 5.5)}{5.5} \times 100\% \\ = 40\%$$

(ii) Over the 3 years, the 5.5 cents increased exponentially by the same percentage each year to 7.7 cents.

Calculate the percentage increase **each year**.

[3]

$$5.5x^3 = 7.7$$

Divide through by 5.5

$$x^3 = 1.4$$

Cube root

$$x = 1.119$$

Hence, each year the percentage increase is

$$11.9\%$$

(d) In 2015, the energy company divided its profits in the ratio

shareholders : bonuses : development = 5 : 2 : 6.

In 2015, its profits were \$390 million.

Calculate the amount the company gave to shareholders.

[2]

$$390 \times \frac{5}{5 + 2 + 6} \\ = 390 \times \frac{5}{13} \\ = 150 (\text{million})$$

- (e) The share price of the company in June 2015 was \$258.25 .  
This was an increase of 3.3% on the share price in May 2015.

Calculate the share price in May 2015.

[3]

The share price in May is m

$$m \times 1.033 = 258.25$$

Divide through by 1.033

$$258.25 \div 1.033$$

$$= 250$$

## Question 4

(a) Annie and Dermot share \$600 in the ratio 11 : 9.

- (i) Show that Annie receives \$330.

[1]

$$600 \times \frac{11}{11 + 9}$$

$$= \frac{6600}{20}$$

$$= 330$$

- (ii) Find the amount that Dermot receives.

[1]

$$600 - 330$$

$$= 270$$

- (b) (i) Annie invests \$330 at a rate of 1.5% per year compound interest.

Calculate the amount that Annie has after 8 years.

Give your answer correct to the nearest dollar.

[3]

$$330 \times 1.015^8$$

$$= 371.74$$

$$= 372$$

- (ii) Find the amount of **interest** that Annie has, after the 8 years, as a percentage of the \$330. [2]

The amount of interest she has is

$$372 - 330$$

$$= 42$$

As a percentage this is

$$\frac{42}{330} \times 100\%$$

$$= 12.7\%$$

- (c) Dermot has \$70 to spend.  
He spends \$24.75 on a shirt.
- (i) Find \$24.75 as a fraction of \$70.  
Give your answer in its lowest terms. [1]

$$24.75 \div 70$$

$$= \frac{99}{280}$$

- (ii) The \$24.75 is the sale price after reducing the original price by 10%.
- Calculate the original price. [3]

This price is 90% of the original price, hence the original price is

$$24.75 \div 0.9$$

$$= 27.50$$

- (d) After one year, the value of Annie's car had reduced by 20%.  
At the end of the second year, the value of Annie's car had reduced by a further 15% of its value at the end of the first year.
- (i) Calculate the overall percentage reduction after the two years. [2]

The price drop is

$$(1 - 0.2) \times (1 - 0.15) \times x$$

Where  $x$  is the initial value

$$= 0.8 \times 0.85 \times x$$

$$= 0.68x$$

So, total percentage reduction is

$$100 - 68$$

$$= 32$$

- (ii) After three years the overall percentage reduction in the value of Annie's car is 40.84%.

Calculate the percentage reduction in the third year.

[2]

We have that

$$0.68 \times y = 1 - 0.4084$$

Where  $y$  is the value reduction in the third year

$$\rightarrow 0.68y = 0.5916$$

Divide through by 0.68

$$\rightarrow y = 0.87$$

So, the percentage reduction in the third year is

$$100 - 87$$

$$= 13$$

## Question 5

(a) In 2016, a company sold 9600 cars, correct to the nearest hundred.

- (i) Write down the lower bound for the number of cars sold. [1]

**9550**

- (ii) The average profit on each car sold was \$2430, correct to the nearest \$10.

Calculate the lower bound for the total profit.

Write down the exact answer. [2]

**Lower bound for 1 car**

**2425**

**Lower bound for total profit**

**$9550 \times 2425$**

**= 23 158 750**

- (iii) Write your answer to part (a)(ii) correct to 4 significant figures. [1]

**23 160 000**

- (iv) Write your answer to part (a)(iii) in standard form. [1]

**$2.316 \times 10^7$**

- (b) In April, the number of cars sold was 546.

This was an increase of 5% on the number of cars sold in March.

Calculate the number of cars sold in March. [3]

**$546 \div 1.05$**

**= 520**

- (c) The price of a new car grows exponentially by 3% per year.

A new car has a price of \$3000 in 2013.

Find the price of a new car 4 years later. [2]

**$3000 \times 1.03^4$**

**= 3376.53**

*Assessed by A/S*

## Question 6

The Smith family paid \$5635 for a holiday in India.

The total cost was divided in the ratio travel : accommodation : entertainment = 10 : 17 : 8.

- (a) Calculate the percentage of the total cost spent on entertainment.

[2]

The percentage of the total cost spent on entertainment can be worked out like this:

$$\text{Percentage cost} = \frac{\text{Entertainment cost}}{\text{Total cost}} \times 100\%$$

$$\text{Percentage cost} = \frac{8}{10 + 17 + 8} \times 100\%$$

$$\text{Percentage cost} = \frac{8}{35} \times 100\%$$

$$\text{Percentage cost} = 0.2285714 \dots \times 100\%$$

$$\text{Percentage cost} = 22.9\%$$

- (b) Show that the amount spent on accommodation was \$2737.

[2]

The amount spent on accommodation can be worked out like this

$$\text{Accommodation cost} = \text{Accommodation Ratio} \times \text{total cost}$$

$$\text{Accommodation cost} = \frac{17}{35} \times 5635$$

$$\text{Accommodation cost} = 2737$$

- (c) The \$5635 was the total amount Mr Smith received from an investment he made 5 years ago. Compound interest at a rate of 2.42% per year was paid on this investment.

Calculate the amount he invested 5 years ago.

[3]

If Mr Smith invested a specific amount ( $x$ ) 5 years ago, at 1.24% interest, and now has

\$5635, we can say that

$$x \times 1.0242^5 = 5635$$

$$x = \frac{5635}{1.0242^5}$$

$$\mathbf{x = \$5000}$$

- (d) Mr Smith, his wife and their three children visit a theme park.  
The tickets cost 2500 Rupees for an adult and 1650 Rupees for a child.

Calculate the total cost of the tickets.

[2]

We can work out the total price of the admission to the theme park like this:

$$\textit{Price for admission} (\$)$$

$$= (\textit{number of adults} \times 2500) + (\textit{number of children} \times 1650)$$

We know that there are 2 adults and 3 children, so

$$\textit{Price} = 2(2500) + 3(1650)$$

$$\mathbf{\textit{Price} = \$9950}$$

- (e) One day the youngest child spent 130 Rupees on sweets.  
On this day the exchange rate was 1 Rupee = \$0.0152.

Calculate the value of the sweets in dollars, correct to the nearest cent.

[2]

In order to change from Rupees to Dollars, we can do this:

$$1 \text{ Rupee} = \$0.0152$$

$$130 \text{ Rupees} = \$0.0152 \times 130$$

$$130 \text{ Rupees} = \$1.976$$

But we need this to the nearest cent

$$\mathbf{130 \text{ Rupees} = \$1.98 \text{ (3.s.f)}}$$

## Question 7

- (a) (i) Each year the value of a car decreases by 15% of its value at the beginning of that year. Alberto buys a car for \$18000.

Calculate the value of Alberto's car after 3 years.

[2]

For Compound Interest multiply by the scale factor to the power of the number of years:

$$\text{Value after 3 years} = 18000 \times 0.85^3$$

$$\text{Value} = \$11054.25$$

- (ii) Belinda bought a car one year ago.  
The value of this car has decreased by 15% to \$14025.

Calculate how much Belinda paid for the car.

[3]

Write a statement down in the form: "After is a percentage of Before":

*Current Value is 85% of Price Paid*

Translate this into an equation ("is" means "=", "of" means "×"):

$$14025 = 0.85 \times \text{Price Paid}$$

$$\text{Price Paid} = \frac{14025}{0.85}$$

$$\text{Answer} = \$16500$$

- (b) Chris invested some money at a rate of 5% per year compound interest.  
After 2 years the value of this investment is \$286.65 .

Calculate how much Chris invested.

[2]

Write down an equation in the form: "After is a (compound) percentage of before"

$$\text{Current Value} = 1.05^2 \times \text{Amount Invested}$$

$$286.65 = 1.05^2 \times \text{Amount Invested}$$

$$\text{Amount Invested} = \frac{286.65}{1.05^2}$$

$$\text{Amount Invested} = \$260$$

- (c) Dani invested \$200 and after 2 years the value of this investment is \$224.72 .

Calculate the rate of interest per year when the interest is

- (i) simple,

[3]

$$\text{Interest for 1 year} = \frac{224.72 - 200}{2} = \$12.36$$

which, expressed as a percentage of \$200, is

$$\frac{12.36}{200} \times 100 = 6.18\% \text{ rate of interest}$$

- (ii) compound.

[3]

Write down an equation in the form: "After is a (compound) percentage of before"

$$\text{Current Value} = x^2 \times \text{Amount Invested}$$

(where  $x$  is the Scale Factor for one year's interest)

$$224.72 = x^2 \times 200$$

$$x^2 = \frac{224.72}{200}$$

$$x = \sqrt{1.1236} = 1.06$$

This is the scale factor for a 6% rate of interest

# Number

## Difficulty: Hard

### Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 2

**Time allowed:** 104 minutes

**Score:** /90

**Percentage:** /100

**Grade Boundaries:**

#### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

#### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

- (a) \$1 = 3.67 dirhams

Calculate the value, in dollars, of 200 dirhams.  
Give your answer correct to 2 decimal places.

[2]

Use ratios for currency conversion questions: create as many columns and rows as you need and use Scale Factors to move between them:

Dollars : Dirhams

$$\begin{array}{rcl} 1 & : & 3.67 \\ x & : & 200 \end{array} \quad \text{Multiply by } \frac{200}{3.67} \quad x = 1 \times \frac{200}{3.67} = \$54.50$$

- (b) (i) Write as a single fraction, in its simplest form.

$$\frac{1000}{x} - \frac{1000}{x+1}$$

[3]

$$\frac{1000}{x} - \frac{1000}{x+1} = \frac{1000(x+1) - 1000x}{x(x+1)}$$

$$= \frac{1000x + 1000 - 1000x}{x(x+1)}$$

$$= \frac{1000}{x(x+1)}$$

- (ii) One day in 2014, 1 euro was worth  $x$  rand.  
One year later, 1 euro was worth  $(x+1)$  rand.

Winston changed 1000 rand into euros in both years.  
In 2014 he received 4.50 euros more than in 2015.

Write an equation in terms of  $x$  and show that it simplifies to

[3]

$$9x^2 + 9x - 2000 = 0.$$

$$\text{Number of Euros in 2014} = \frac{1000}{x}; \text{Number of Euros in 2015} = \frac{1000}{x+1}$$

$$\frac{1000}{x} - \frac{1000}{x+1} = 4.50$$

$$\frac{1000}{x(x+1)} = 4.5$$

Multiply by  $x(x + 1)$ :

$$1000 = 4.5x(x + 1)$$

Multiply out the brackets:

$$1000 = 4.5x^2 + 4.5x$$

Multiply by 2 and subtract 2000:

$$0 = 9x^2 + 9x - 2000$$

- (iii) Use the quadratic formula to solve the equation  $9x^2 + 9x - 2000 = 0$ .  
Show all your working and give your answers correct to 2 decimal places. [4]

Use  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  with  $a = 9$ ,  $b = 9$  and  $c = -2000$ .

It is a good idea to work out  $b^2 - 4ac$  (the discriminant) first:

$$b^2 - 4ac = 9^2 - 4 \times 9 \times (-2000) = 72081$$

Now put the right hand side of

$$x = \frac{-9 \pm \sqrt{72081}}{2 \times 9}$$

into a calculator (first with the  $+$  and then with the  $-$ ) to get:

$$x = -15.42 \text{ or } x = 14.42$$

- (iv) Calculate the number of euros Winston received in 2014.  
Give your answer correct to 2 decimal places. [2]

Since the number of Rand must be positive, use  $x = 14.42$  from (iv) in the formula:

$$\text{Number of Euros in 2014} = \frac{1000}{x}$$

$$\text{Number of Euros in 2014} = \frac{1000}{14.42}$$

$$\text{Number of Euros in 2014} = €69.37$$

## Question 2

- (a) A jigsaw puzzle has edge pieces and inside pieces.  
The ratio edge pieces : inside pieces = 3 : 22.

(i) There are 924 inside pieces.

Calculate the total number of pieces in the puzzle.

[2]

When using ratios, create as many columns and rows as you

need and use Scale Factors to move between them:

Edge : Inside : Total

$$\begin{array}{ccc} 3 & : & 22 & : & 25 \\ (3 \times 42) & : & 924 & : & 25 \times 42 \end{array}$$

Multiply by  $\frac{924}{22} = 42$

**Total Number of Pieces = 1050**

- (ii) Find the percentage of the total number of pieces that are edge pieces.

[1]

$$\text{Percentage} = \frac{3 \times 42}{1050} \times 100 = 12\%$$

- (iii) Anjum and Betty spent a total of 9 hours completing the puzzle.  
The ratio Anjum's time : Betty's time = 7 : 5.

Work out how much time Anjum spent on the puzzle.

[2]

Anjum : Betty : Total

$$\begin{array}{ccc} 7 & : & 5 & : & 12 \\ 7 \times \frac{9}{12} & : & (7 \times \frac{9}{12}) & : & 9 \end{array}$$

Multiply by  $\frac{9}{12}$

**Anjum's time = 5.25 hours**

- (b) The price of the puzzle was \$15.99 in a sale.  
This was 35% less than the original price.

Calculate the original price of the puzzle.

[3]

Write a statement down in the form: "After is a percentage of Before":

Sale Price is 65% of Original Price

Translate this into an equation ("is" means "=", "of" means "×"):

*Sale Price = 0.65 × Original Price*

Put in the numbers:

$$15.99 = 0.65 \times \text{Original Price}$$

Solve (in this case by dividing by 0.65):

$$\text{Original Price} = \frac{15.99}{0.65} = \$24.60$$

- (c) Betty takes a photograph of the completed puzzle.  
The photograph and the completed puzzle are mathematically similar.

The area of the photograph is  $875 \text{ cm}^2$  and the area of the puzzle is  $2835 \text{ cm}^2$ .

The length of the photograph is 35 cm.

Work out the length of the puzzle.

For similar shapes:

$$\text{Area Factor} = \text{Scale Factor}^2$$

[3]

$$\text{Area Factor} = \frac{2835}{875} \Rightarrow \text{Scale Factor} = \sqrt{\frac{2835}{875}}$$

$$\text{Length of puzzle} = 35 \times \sqrt{\frac{2835}{875}}$$

**Length of puzzle = 63cm**

- (d) (i) The area of another puzzle is  $6610 \text{ cm}^2$ .

Change  $6610 \text{ cm}^2$  into  $\text{m}^2$ .

[1]

There are  $100 \times 100 = 10000 \text{ cm}^2$  in  $1 \text{ m}^2$  so

$$6610 \text{ cm}^2 = \frac{6610}{10000}$$

$$= 0.661 \text{ m}^2$$

- (ii) The cost price of this puzzle is \$12.50 .  
The selling price is \$18.50 .

Calculate the percentage profit.

[3]

$$\text{Percentage Profit} = \frac{\text{Actual Profit}}{\text{Cost Price}} \times 100$$

$$\text{Percentage Profit} = \frac{18.50 - 12.50}{12.50} \times 100$$

$$\text{Percentage Profit} = 48\%$$

### Question 3

- (a) Kristian and Stephanie share some money in the ratio 3 : 2.  
Kristian receives \$72.

(i) Work out how much Stephanie receives. [2]

$$\begin{array}{ccc} K & : & S \\ 3 & : & 2 \\ \times 24 & & \times 24 \end{array}$$

Work out the multiplier between  
3 and 72 and multiply result by 2

$$72 : ?$$

$$72 \div 3 = 24$$

$$2 \times 24 = 48$$

Answer is \$48.00

- (ii) Kristian spends 45% of his \$72 on a computer game.

Calculate the price of the computer game.

[1]

$$72 \times 0.45 = 32.4$$

45% means  $45 \div 100$  ie 0.45

Answer is \$32.40

- (iii) Kristian also buys a meal for \$8.40 .

Calculate the fraction of the \$72 Kristian has left after buying the computer game and the meal.  
Give your answer in its lowest terms.

[2]

$$72 - 32.4 - 8.4 = 31.2$$

Step 1: Work out how much Kristian has left

$$\frac{31.2}{72} = \frac{13}{20}$$

Step 2: Write this as a fraction

Step 3: Use fraction button on calculator to  
simplify

- (iv) Stephanie buys a book in a sale for \$19.20.  
This sale price is after a reduction of 20%.

Calculate the original price of the book.

[3]

$$19.2 \div 0.8 = 24$$

Reverse percentage - new price is 80% of the original

Answer is \$24

- (b) Boris invests \$550 at a rate of 2% per year simple interest.

Calculate the amount Boris has after 10 years.

[3]

$$0.02 \times 550 \times 10 = 110$$

2% of \$550 for 10 years

$$110 + 550 = 660$$

Answer is \$660

- (c) Marlene invests \$550 at a rate of 1.9% per year compound interest.

Calculate the amount Marlene has after 10 years.

[2]

$$550 \times 1.019^{10} = 663.9$$

Using compound interest formula:

$$A = P \left(1 + \frac{r}{100}\right)^n$$

Increase of 1.9% means new amount is now 101.9%

Answer is \$663.90

- (d) Hans invests \$550 at a rate of  $x\%$  per year compound interest.  
At the end of 10 years he has a total amount of \$638.30, correct to the nearest cent.

Find the value of  $x$ .

[3]

$$550 \times x^{10} = 638.3$$

Using compound interest formula

$$x^{10} = \frac{638.3}{550}$$

Divide both sides by 550

$$x = \sqrt[10]{\frac{638.3}{550}}$$

Inverse of power of 10 is the 10th root

$$x = 1.5$$

## Question 4

A football club sells tickets at different prices dependent on age group.

- (a) (i) At one game, the club sold tickets in the ratio

$$\text{under 18 : 18 to 60 : over 60} = 2 : 7 : 3.$$

There were 6100 tickets sold for people aged under 18.

Calculate the **total** number of tickets sold for the game. [3]

Adding the ratios there are

$$2 + 7 + 3$$

$$= 12 \text{ parts}$$

Total 'parts' of this fraction. 2 parts were sold to under 18s, written as so

$$\frac{2}{12} \times \text{total tickets} = 6100$$

$$\rightarrow \frac{1}{6} \times T = 6100$$

Multiply both sides by 6

$$T = 6 \times 6100$$

$$= 36\,600$$

- (ii) Calculate the percentage of tickets sold for people aged under 18. [1]

The percentage is found as

$$\frac{6100}{36600} \times 100\%$$

$$= 16.7\% \text{ or } 16\frac{2}{3}\%$$

(b) The table shows the football ticket prices for the different age groups.

Age	Price
Under 18	\$15
18 to 60	\$35
Over 60	\$18

At a **different** game there were 42 600 tickets sold.

- 14% were sold to people aged under 18
- $\frac{2}{3}$  of the tickets were sold to people aged 18 to 60
- The remainder were sold to people aged over 60

Calculate the total amount the football club receives from ticket sales for this game.

[5]

14% of 42600 sold to under 18s. This is

$$0.14 \times 42600$$

$$= 5964$$

$\frac{2}{3}$  sold to 18 to 60s, this is

$$\frac{2}{3} \times 42600$$

$$= 28400$$

The remainder sold to over 80s, this is

$$42600 - 5964 - 28400$$

$$= 8236$$

The money made is then

$$5964 \times 15 + 28400 \times 35 + 8236 \times 18$$

$$= 1\,231\,708$$

- (c) In a sale, the football club shop reduced the price of the football shirts to \$23.80 .  
An error was made when working out this sale price.  
The price was reduced by 30% instead of 20%.

Calculate the correct sale price for the football shirt.

[5]

23.80 is the price when its reduced by 30%. The original price is then

$$23.80 \div 0.7$$

$$= 34$$

Now we reduce this by 20%

$$34 \times 0.8$$

$$= 27.2$$

## Question 5

Aasha, Biren and Cemal share \$640 in the ratio 8 : 15 : 9.

- (a) Show that Aasha receives \$160.

[1]

Aasha's share represents 8 parts out of the total of 32 because

$$32 = 8 + 16 + 9$$

To get the amount Aasha receives, we multiply the total amount of money by her share (8 parts) and divide by the total 32 parts.

$$\text{Aasha receives} = \$640 \times \frac{8 \text{ parts}}{32 \text{ parts}}$$

$$\text{Aasha receives} = \$160$$

- (b) Calculate the amount that Biren and Cemal receive.

[2]

We calculate the amount that Biren received using an identical method. We know that Biren's share represents 15 parts of the total of 32 parts.

$$\text{Biren receives} = \$640 \times \frac{15 \text{ parts}}{32 \text{ parts}}$$

$$\text{Biren receives} = \$300$$

Then Cemal will receive the rest. We subtract Aasha's and Biren's share from the total.

$$\text{Cemal receives} = \$640 - \$160 - \$300$$

$$\text{Cemal receives} = \$180$$

- (c) Aasha uses her \$160 to buy some books.  
Each book costs \$15.25 .

Find the greatest number of books that she can buy.

[2]

Divide the amount Aasha receives by the cost of one book to obtain the number of books she could buy.

$$\frac{\$160}{\$15.25} = 10.4918..$$

However since she can only buy a natural number of books, round down to the closest whole number.

**Aasha buys 10 books.**

- (d) Biren spends  $\frac{3}{8}$  of his share on clothes and  $\frac{1}{3}$  of his share on a computer.

Find the fraction of his share that he has left.

Write your fraction in its lowest terms.

[3]

Add the two fractions to get the total fraction Biren has spent.

$$\frac{3}{8} + \frac{1}{3}$$

Denominators of the two fractions must be the same in order to add them.

$$\text{fraction spent} = \frac{3}{8} \times \frac{3}{3} + \frac{1}{3} \times \frac{8}{8}$$

$$\text{fraction spent} = \frac{9}{24} + \frac{8}{24} = \frac{17}{24}$$

Subtract the fraction he spent to get the fraction of his share that he has left.

$$\text{fraction left} = 1 - \text{fraction spent}$$

$$\text{fraction left} = 1 - \frac{17}{24} = \frac{24}{24} - \frac{17}{24}$$

Hence we have the fractions he has left.

$$\text{fraction left} = \frac{7}{24}$$

## Question 6

- (a) Meena sells her car for \$6000.  
This is a loss of 4% on the price she paid.

Calculate the price Meena paid for the car.

[3]

If Meena sells her car for \$6000 and this is a loss of 4%, then this amount represents 96% of the original price.

$$96\% = 100\% - 4\%$$

Divide the amount she received by 0.96 (=96%) to get the price Meena paid for the car.

$$\$6000 \div 0.96 = \$6250$$

- (b) Eisha changes some euros (€) into dollars (\$) when the exchange rate is €1 = \$1.351 .  
She receives \$6000.

Calculate how many euros Eisha changes.  
Give your answer correct to the nearest euro.

[3]

To convert from dollars to euros, divide the amount in dollars by the exchange rate of dollars per one euro.

$$\frac{\$6000}{\$1.351 \text{ per } €1} = € 4441.15..$$

We round up the amount to the nearest euro.

$$€ 4441$$

(c) Meena and Eisha both invest their \$6000.

Meena invests her \$6000 at a rate of 1.5% per year compound interest.

Eisha invests her \$6000 in a bank that pays simple interest.

After 8 years, their investments are worth the same amount.

Calculate the rate of simple interest per year that Eisha received.

[5]

The rate of 1.5% per year (compound interest) means that the amount of money next year will be 1.015 times the previous amount.

$$1.015 = 1 + \frac{1.5\%}{100\%}$$

We want to find out the amount after 8 years, so the initial amount has to be multiplied by the factor 1.05 eight times.

$$\$6000 \times 1.015^8 = \$6758.96..$$

Hence the interest is:

$$\$6758.96.. - \$6000 = \$758.96..$$

This should be same as applying simple interest of  $x\%$  to \$6000 for 8 years.

$$\$6000 \times \frac{x\%}{100\%} \times 8 = \$758.96..$$

Divide both sides by 8.

$$60 \times x = 94.87..$$

Divide both sides by 60 to find the simple interest.

$$x \% = 1.58 \%$$

## Question 7

A film company uses 512 actors in a film.  
The actors are in the ratio men : women : children = 7 : 11 : 14.

- (a) (i) Show that there are 224 children in the film.

[2]

When the total number of actors is split into 32 (=7+11+14) pieces, children represent 14 of them. The total is 512 actors.

We divide the total by 32 and multiply by 14 to get the number of children in the film:

$$512 \times \frac{14}{32}$$

$$= 224$$

- (ii) Find the number of men in the film.

[1]

We use the same method, but instead multiply by 7 (proportion of men in the film).

$$512 \times \frac{7}{32}$$

$$= 112$$

- (b) Every working day, each child is given \$1 to spend.  
Each child works for 45 days.

Calculate the total amount that the film company gives the children to spend.  
Give your answer correct to the nearest \$100.

[2]

The number of children acting in the film was found in part a) i). Multiply the number of children by the amount of money they are given per day and by the number of days they are working.

$$224 \times \$1 \text{ per day} \times 45 \text{ days} = \$10\,080$$

Round the number up to the nearest \$100.

The amount that the company gives the children to spend: **\$10 100**

- (c) The children have lessons every day in groups of no more than 12.

Calculate the smallest possible number of groups.

[2]

Divide the total number of children by the (maximum) number of children in the group.

$$\frac{224}{12} = 18.66$$

By rounding to a whole group, we work out that the smallest possible number of groups is 19.

- (d) The film costs four million and ninety three thousand dollars to make.

- (i) Write this number in figures.

[1]

Four million and ninety three thousand dollars

$$4 \times 1\,000\,000 + 93 \times 1\,000$$

$$= 4\,093\,000$$

- (ii) Write your answer to part (d)(i) in standard form.

[1]

In order to rewrite this into a standard form, we need to realize that there are six digits in after the first digit 4 (this will be the exponent of 10).

Therefore the standard form is

$$4.093 \times 10^6.$$

- (e) A DVD copy of the film costs \$2.75 to make.  
The selling price is \$8.20 .

Calculate the percentage profit.

[3]

To calculate the profit, subtract the costs from the selling price.

$$\text{profit} = \text{selling price} - \text{costs}$$

$$\text{profit} = \$8.20 - \$2.75 = \$5.45$$

We calculate the percentage profit by dividing the profit by the film costs (and multiplying by 100%).

$$\frac{\$5.45}{\$2.75} \times 100\%$$

$$= 198\%$$

# Number

## Difficulty: Hard

### Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 3

**Time allowed:** 93 minutes

**Score:** /81

**Percentage:** /100

**Grade Boundaries:**

#### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

#### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

12000 vehicles drive through a road toll on one day.  
The ratio cars:trucks:motorcycles = 13:8:3.

- (a) (i) Show that 6500 cars drive through the road toll on that day. [1]

When the total number of vehicles is split into 24 (=13+8+3) pieces, cars represent 13 of them. The total is 12 000 vehicles.

We divide the total by 24 and multiply by 13 to get the number of cars:

$$12\ 000 \times \frac{13}{24} \\ = 6\ 500$$

- (ii) Calculate the number of trucks that drive through the road toll on that day. [1]

Trucks represent 8 of the total number of pieces. Use similar method as in part a)i), but multiply by 8 instead.

$$12\ 000 \times \frac{8}{24} = 4000$$

**There are 4000 trucks that drive through the road toll on that day.**

- (b) The toll charges in 2014 are shown in the table.

Vehicle	Charge
Cars	\$2
Trucks	\$5
Motorcycles	\$1

Show that the **total** amount paid in tolls on that day is \$34500. [2]

To work out the amount paid by those driving cars, multiply the number of cars (found in part a)i)) by \$2.

$$6500 \times \$2 = \$13\ 000$$

Each truck has to pay \$5. Multiply this number by the number of trucks (found in part a)ii))

$$4000 \times \$5 = \$20\ 000$$

The number of motorcycles can be found by subtracting the number of cars and trucks from the total number of vehicles. Multiply this number by \$1.

$$(12\ 000 - 6500 - 4000) \times \$1 = \$1\ 500$$

Add the numbers together to get the total amount paid in tolls on that day:

$$\$13\ 000 + \$20\ 000 + \$1\ 500$$

$$= \$34\ 500$$

- (c) This total amount is a decrease of 8% on the total amount paid on the same day in 2013.

Calculate the total amount paid on that day in 2013.

[3]

If 34 500 is decrease of 8% on the total amount paid, then it must represent 92% (=100%-8%) of the total amount paid on the same day in 2013.

Divide the number by  $0.92 = (92\% / 100\%)$  to get the total amount paid on that day in 2013.

$$\frac{34\ 500}{0.92} = 37\ 500$$

The total amount paid on that day in 2013 was **\$37 500**.

- (d) 2750 of the 6500 car drivers pay their toll using a credit card.

Write down, in its simplest terms, the fraction of car drivers who pay using a credit card.

[2]

We write the number as a fraction.

$$\frac{2750}{6500}$$

Both the numerator and the denominator are divisible by 50. Divide both numbers by 50.

$$\frac{2750 \div 50}{6500 \div 50} = \frac{55}{130}$$

Numbers 55 and 130 share a common factor 5. Divide both of them by this factor.

$$\frac{55 \div 5}{130 \div 5} = \frac{11}{26}$$

These two numbers do not share a common factor (11 is a prime number), therefore this is the fraction of car drivers who pay using a credit card in its simplest form:

$$= \frac{11}{26}$$

- (e) To the nearest thousand, 90000 cars drive through the road toll in one week.

Write down the lower bound for this number of cars.

[1]

To get the lower bound, we have to subtract half the ‘value of precision’ (1000 in our case as the number is correct to nearest thousand).

$$90\,000 - \frac{1000}{2}$$

$$= 89\,500$$

## Question 2

- (a) Last year a golf club charged \$1650 for a family membership.  
This year the cost increased by 12%.

Calculate the cost of a family membership this year.

[2]

Since the cost increase by 12%, this year will represent 112% of last year's membership.

Multiply the cost for last year by 1.12 (=112/100) to get the cost of a family membership this year:

$$\$1650 \times 1.12$$

$$= \$1848$$

- (b) The golf club runs a competition.  
The total prize money is shared in the ratio 1st prize:2nd prize = 9:5.  
The 1st prize is \$500 more than the 2nd prize.

- (i) Calculate the total prize money for the competition.

[2]

The difference between the first and the second prize \$500 represents 4 parts of the total prize

$$(=9-5).$$

Divide the difference by 4 and multiply by factor 14 (=9+5) to get the total prize money.

$$\frac{\$500}{4} \times 14$$

$$= \$1750$$

- (ii) What percentage of the total prize money is given as the 1st prize?

[1]

The amount of money given as the 1<sup>st</sup> prize:

$$\frac{\$1750 - \$500}{2} + \$500 = \$1125$$

To work out the percentage of the total prize money given as the 1<sup>st</sup> prize, divide the first prize by the total prize money and multiply by 100%.

$$\frac{1125}{1750} \times 100$$

$$= 64.3\%$$

- (c) For the members of the golf club the ratio men : children = 11 : 2.  
The ratio women : children = 10 : 3.

(i) Find the ratio men:women. [2]

Multiply the ratio of men : children by 3 and the ratio women : children by 2 to get a equal ratio of children in both cases.

$$\text{men : children} = 33:6$$

$$\text{women : children} = 20:6$$

We can now combine the ratios:

$$\text{men : women} = 33:20$$

- (ii) The golf club has 24 members who are children.

Find the total number of members. [3]

The ratio men : women : children

$$\text{men : women : children} = 33 : 20 : 6$$

When we sum the factors, we get that when we divide the total number of members into 59 groups (=33+20+6), children will represent 6 of them.

Therefore to get the total number of members, divide the number of child members by 6 and multiply the number by 59.

$$\frac{24}{6} \times 59$$

$$= 236$$

- (d) The club shop sold a box of golf balls for \$20.40 .  
The shop made a profit of 20% on the cost price.

Calculate the cost price of the golf balls.

[3]

Divide the price of a box of gold balls by 1.2 (=120/100) to calculate the cost price.

If the profit is 20% on the cost price, then the retail price is 120% of the cost price.

$$\text{cost price} = \frac{20.40}{1.2}$$

$$= \$17$$

### Question 3

Jaideep builds a house and sells it for \$450000.

- (a) He pays a tax of 1.5% of the selling price of the house.

Show that he pays \$6750 in tax.

[1]

To find 1.5% of \$450 000, we multiply the number by 0.015 (=1.5%=1.5/100).

$$\$450\ 000 \times 0.015$$

$$= \$6750$$

- (b) \$6750 is 12.5% more than the tax Jaideep paid on the first house he built.

Calculate the tax Jaideep paid on the first house he built.

[3]

If \$6750 is 12.5% more than the tax of the first house, then this amount represents

112.5% of the original tax. We want to find the original tax (100% if \$6750 represents 112.5%)

$$\frac{\$6750}{112.5\%} = \frac{\text{first house tax}}{100\%}$$

Multiply both sides by 100%.

$$\text{first house tax} = \frac{\$6750}{112.5\%} \times 100\%$$

Use a calculator to get the tax he paid on the first house.

$$\text{first house tax}$$

$$= \$6000$$

- (c) The house is built on a rectangular plot of land, 21 m by 17 m, both correct to the nearest metre.

Calculate the upper bound for the area of the plot.

[2]

The area of a rectangle with sides  $a$  and  $b$  is given as:

$$\text{Area} = a \times b$$

In order to find the upper bound for the area, we take the highest possible values of  $a$  and  $b$  (sides of the rectangle). The multiple of two large numbers produce a large number.

The side  $a$  is 21m, correct to the nearest metre, so the upper bound is 21.5 m

The side  $b$  is 17m, correct to the nearest metre, so the upper bound is 17.5 m

The upper bound for the area is found by multiplying the upper bounds for the sides of the rectangle.

$$\text{Area} = 21.5m \times 17.5m$$

$$\text{Area} = 376.25 \text{ m}^2$$

- (d) On a plan of the house, the area of the kitchen is 5.6 cm<sup>2</sup>.  
The scale of the plan is 1:200.

Calculate the actual area of the kitchen in square metres.

[2]

First, the scale of the plan is a linear conversion (one dimensional), but area has two dimensions, so we multiply the area on the plan by the square of this factor in order to get the actual area.

$$5.6 \text{ cm}^2 \times 200^2 = 224\,000 \text{ cm}^2$$

Second, we convert the area from square centimetres to square metres.

$$1 \text{ m}^2 = 100\text{cm} \times 100\text{cm} = 10\,000 \text{ cm}^2$$

So the actual area of the kitchen is:

$$\frac{224\,000 \text{ cm}^2}{10\,000 \text{ cm}^2 \text{ per m}^2}$$

$$= 22.4 \text{ m}^2$$

- (e) The house was built using cuboid blocks each measuring 12 cm by 16 cm by 27 cm.

Calculate the volume of one block.

[2]

The volume of one block can be found by multiplying the sizes of its three sides.

$$\text{Volume} = 12\text{cm} \times 16\text{cm} \times 27\text{cm}$$

Use a calculator to find the volume of one block.

$$\text{Volume of one block} = 5184 \text{ cm}^3$$

- (f) Jaideep changes \$12000 into euros (€) to buy land in another country.  
The exchange rate is €1 = \$1.33 .

Calculate the number of euros Jaideep receives.

Give your answer correct to the nearest euro.

[3]

The relation between euros and dollars is €1 = \$1.33. By inverting this relation (dividing both sides of the equation by 1.33), we get that

$$\$1 = \text{€} \frac{1}{1.33}$$

To convert from dollars to euros, we multiply the amount in dollars by the exchange rate of euros per a dollar.

$$\$12\,000 \times \frac{1}{1.33} \text{€ per \$} = \text{€}9022.56$$

Rounding to the nearest euro, we obtain that \$12 000 will be exchanged for

**€9023.**

## Question 4

- (a) Alfonso has \$75 to spend on the internet.  
He spends some of the money on music, films and books.

- (i) The money he spends on music, films and books is in the ratio

$$\text{music : films : books} = 5 : 3 : 7.$$

He spends \$16.50 on music.

Calculate the **total** amount he spends on music, films and books.

[3]

Let the amount Alfonso spends on music, films, and books be  $x$ .

We know that:

$$\frac{5}{5+3+7} \times x = 16.5$$

$$\rightarrow \frac{5}{15}x = 16.5$$

$$\rightarrow \frac{1}{3}x = 16.5$$

$$\rightarrow x = 3 \times 16.5$$

$$= 49.50$$

- (ii) Find this total amount as a percentage of the \$75.

[1]

$$(49.5 \div 75) \times 100\%$$

$$= 66\%$$

- (b) The download times for the music, films and books are in the ratio

$$\text{music : films : books} = 2 : 9 : 1.$$

The **total** download time is 3 hours and 33 minutes.

Calculate the download time for the films.

Give your answer in hours, minutes and seconds.

[3]

**Converting the time to just minutes:**

$$3 \text{ hours } 33 \text{ mins} = 213 \text{ mins}$$

**Now finding the time for films:**

$$213 \times \frac{9}{12}$$

$$= 159\frac{3}{4} \text{ mins}$$

$$= 159 \text{ mins } 45 \text{ secs}$$

$$= \mathbf{2 \text{ hours } 39 \text{ mins } 45 \text{ secs}}$$

- (c) The cost of \$16.50 for the music was a reduction of 12% on the original cost.

Calculate the original cost of the music.

[3]

$$16.5 \div 0.88$$

$$= \mathbf{18.75}$$

## Question 5

There are three different areas, A, B and C, for seating in a theatre.  
The numbers of seats in each area are in the ratio A:B:C = 11:8:7.  
There are 920 seats in area B.

[1]

- (a) (i) Show that there are 805 seats in area C.

We have the relation between the total number of seats, S, and the number of seats in area B and C as

$$S \times \frac{8}{26} = 920$$

$$S \times \frac{7}{26} = S_C$$

If we divide the bottom equation by the top, we get

$$\frac{S_C}{920} = \frac{7}{26} \div \frac{8}{26}$$

$$\rightarrow S_C = 920 \times \frac{7}{8}$$

$$= 805$$

- (ii) Write the number of seats in area B as a percentage of the total number of seats.

[2]

Total number of seats is

$$S = \frac{26}{8} \times 920$$

$$= 2990$$

As a percentage we get

$$\frac{920}{2990} \times 100$$

$$= 30.8$$

- (b) The cost of a ticket for a seat in each area of the theatre is shown in the table.

Area A	\$11.50
Area B	\$15
Area C	\$22.50

For a concert 80% of area B tickets were sold and  $\frac{3}{5}$  of area C tickets were sold.  
The total amount of money taken from ticket sales was \$35834.

Calculate the number of area A tickets that were sold.

[5]

The amount of money from B is

$$0.8 \times 920 \times 15$$

$$= 11040$$

The amount of money made from C is

$$\frac{3}{5} \times 805 \times 22.50$$

$$= 10867.50$$

Hence the amount of money made from A must be

$$35834 - 11040 - 10867.50$$

$$= 13926.50$$

Divide by price per ticket

$$13926.50 \div 11.50$$

$$= 1211$$

- (c) The total ticket sales of \$35834 was 5% less than the ticket sales at the previous concert.

Calculate the ticket sales at the previous concert.

[3]

$$35834 \div 0.95$$

$$= 37720$$

## Question 6

Last year Mukthar earned \$18900.  
He did not pay tax on \$5500 of his earnings.  
He paid 24% tax on his remaining earnings.

- (a) (i) Calculate how much tax Mukthar paid last year. [2]

Since \$5500 of his earnings were not taxed,

$$\text{Taxable Income} = \$18900 - \$5500$$

$$= \$13400$$

Since 24% of this amount was then taxed:

$$24\% \times \$13400 = \$3216$$

$$= \$3216$$

- (ii) Calculate how much Mukthar earned each month after tax had been paid. [2]

Income over the year:

$$\$18900 - \$3216 = \$15684$$

One year has twelve months:

$$\text{Monthly Income} = \frac{\$15684}{12} = \$1307$$

$$= \$1307$$

- (b) This year Mukthar now earns \$19750.50.

Calculate the percentage increase from \$18900. [2]

Percentage change formula:

$$\frac{\text{new value} - \text{original value}}{\text{original value}} \times 100$$

$$\frac{19750.50 - 18900}{18900} \times 100 = 4.5\%$$
  
$$= 4.5\%$$

(c) Mukthar has \$1500 to invest in one of the following ways.

- **Account A** paying **simple** interest at a rate of 4.1% per year
- **Account B** paying **compound** interest at a rate of 3.3% per year

Which account will be worth more after **3 years** and by how much?

[5]

**Account A:**

$$\text{Interest} = \$1500 \times 0.041 = \$61.50$$

$$\text{Over 3 years: } \$61.50 \times 3 = \$184.50$$

$$\text{Sum} = \$184.50 + \$1500 = \$1684.50$$

**Account B:**

$$1500 \times 1.033^3 = 1653.45$$

Therefore, the difference in between the accounts:

$$\$1684.50 - \$1653.45 = \$31.05$$

Therefore, Account A is **better and by \$31.05**.

## Question 7

- Noma flies from Johannesburg to Hong Kong.  
Her plane leaves Johannesburg at 18 45 and arrives in Hong Kong 13 hours and 25 minutes later.  
The local time in Hong Kong is 6 hours ahead of the time in Johannesburg.

- (a) At what time does Noma arrive in Hong Kong? [2]

By Johannesburg time:

Break 13 hours 25 mins into: 6 hours and 7 hours 25 mins

If we add 6 hours to 1845, we get 0045hrs on a new day, now it is easier to add  
the 7 hours to get 0745hrs. Finally we add the 25 mins and we get 0810hrs.

Now in HK time, since HK is +6hrs Johannesburg time:

**Final answer: 0810hrs + 6hrs = 1410hrs or 2.10pm**

- (b) Noma sleeps for part of the journey.  
The time that she spends sleeping is given by the ratio

$$\text{sleeping} : \text{awake} = 3 : 4.$$

Calculate how long Noma sleeps during the journey.  
Give your answer in hours and minutes. [2]

Convert 13 hours 25 mins into all minutes:

Each hour has 60 mins, therefore 13 hours 25 mins = 805 minutes

Take:

$$\frac{805 \text{ mins}}{7} \times 3 = 345 \text{ mins (spent sleeping)}$$

Now convert back into hours and minutes by dividing by 60:

$$\frac{345}{60} = 5.75 \text{ hours} = 5 \text{ hours and .75 hours}$$

$$= 5 \text{ hours } 45 \text{ minutes}$$

- (c) (i) The distance from Hong Kong to Johannesburg is 10 712 km.  
 The time taken for the journey is 13 hours and 25 minutes.

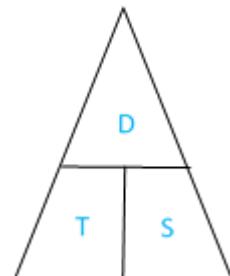
Calculate the average speed of the plane for this journey.

[2]

Divide the distance by the minutes, since distance divide

by time equals speed:

$$\frac{10712\text{km}}{805\text{mins}} = 13.31\text{km/min}$$



$$\frac{13.31\text{km}}{\text{min}}$$

$$= 798.4\text{km/hr}$$

- (ii) The plane uses fuel at the rate of 1 litre for every 59 metres travelled.

Calculate the number of litres of fuel used for the journey from Johannesburg to Hong Kong.  
 Give your answer in standard form.

[4]

$$\text{Convert distance to metres: } 10712\text{km} \times 1000 \frac{\text{m}}{\text{km}} = 10712000\text{m}$$

$$= 1.0712 \times 10^7\text{m}$$

$$\text{Litres of fuel needed} = \frac{1.0712 \times 10^7\text{m}}{\frac{59\text{m}}{l}}$$

$$= 181559\text{ l}$$

$$= 1.82 \times 10^5$$

- (d) The cost of Noma's journey is 10148 South African Rand (R).  
This is an increase of 18% on the cost of the journey one year ago.

Calculate the cost of the same journey one year ago.

[3]

Taking the cost 1 year ago as the base,

$$118\% = 10148 \text{ Rand}$$

$$1\% = \frac{10148}{118} = 86 \text{ Rand}$$

$$100\% = 86 \times 100$$

$$= 8600 \text{ Rand}$$

# Number

## Difficulty: Hard

### Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 4

**Time allowed:** 93 minutes

**Score:** /81

**Percentage:** /100

**Grade Boundaries:**

#### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

#### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

- (a) (i) In a camera magazine, 63 pages are used for adverts.  
The ratio number of pages of adverts : number of reviews = 7:5.

Calculate the number of pages used for reviews. [2]

7 parts constitute pages used for adverts, and this

amounts to 63.

5 parts for reviews.

Calculate pages in 1 part:

$$1 \text{ part} = \frac{63}{7}$$

$$= 9$$

$$5 \text{ parts} = 9 \times 5$$

$$= 45$$

- (ii) In another copy of the magazine, 56 pages are used for reviews and for photographs.  
The ratio number of pages of reviews : number of pages of photographs = 9:5.

Calculate the number of pages used for photographs. [2]

Now, there are a total of 56 pages. Adding the parts

together gives 14.

5 parts are for photographs.

Similarly, find pages in 1 part:

$$1 \text{ part} = \frac{56}{14}$$

$$= 4$$

$$5 \text{ parts} = 4 \times 5$$

$$= 20$$

- (iii) One copy of the magazine costs \$4.90 .  
An annual subscription costs \$48.80 for 13 copies.

Calculate the percentage discount by having an annual subscription.

[3]

We know that 1 copy is \$4.90.

Calculate cost per copy if annual subscription is chosen:

$$\text{Cost per copy (annual subscription)} = \frac{\$48.80}{13}$$

$$= \$3.75$$

Now, calculate this cost difference:

$$\text{Cost difference per copy} = \$4.90 - \$3.75$$

$$= \$1.15$$

Finally, compute this as a percentage discount over the non-annual subscription option:

$$\% \text{ discount} = \frac{\$1.15}{\$4.90} \times 100\%$$

$$= 23.4\%$$

- (b) In a car magazine, 25% of the pages are used for selling second-hand cars,  $62\frac{1}{2}\%$  of the **remaining** pages are used for features, and the other 36 pages are used for reviews.

Work out the total number of pages in the magazine.

[4]

Find percentage used for reviews:

Note here that 62.5% is of the REMAINING pages, which is 75% of the total number of pages as 25% of the magazine has gone to second-hand cars.

Thus, multiply 62.5% by 75%.

$$\% \text{ reviews} = 100\% - 25\% - (62.5\% \times 0.75)$$

$$= 28.125\%$$

$$28.125\% = 36 \text{ pages}$$

$$1\% = \frac{36}{28.125}$$

$$= 1.28 \text{ pages}$$

Therefore, since total is 100%,

$$\mathbf{100\% = 128 \text{ pages}}$$

## Question 2

A factory produces bird food made with sunflower seed, millet and maize.

- (a) The amounts of sunflower seed, millet and maize are in the

ratio sunflower seed:millet:maize = 5 : 3 : 1 .

- (i) How much millet is there in 15 kg of bird food?

[2]

Millet, as part of the bird food, is

$$\frac{3}{5 + 3 + 1}$$

$$= \frac{3}{9}$$

$$= \frac{1}{3}$$

of it. Hence, in 15kg, there is

$$\frac{1}{3} \times 15\text{kg}$$

$$= 5\text{kg}$$

of millet.

- (ii) In a small bag of bird food there is 60 g of sunflower seed.

What is the mass of bird food in a small bag?

[2]

Sunflower seeds make up

$$\frac{5}{5 + 3 + 1}$$

$$= \frac{5}{9}$$

of the bird food. We have a small bag of bird food that weighs  $m$ , and we know that

$$\frac{5}{9} \times m = 60g$$

Hence

$$m = \frac{9}{5} \times 60$$

$$= 108$$

- (b) Sunflower seeds cost \$204.50 for 30 kg from Jon's farm or €96.40 for 20 kg from Ann's farm.  
The exchange rate is \$1 = €0.718.

Which farm has the cheapest price per kilogram?

You must show clearly all your working.

[4]

Let us consider the cost of 60kg from both farms.

$$\text{Jon's: } 2 \times \$204.50 = \$409.00$$

$$\text{Ann's: } 3 \times €96.40 = €289.20$$

Divide Ann's quote by the exchange rate to convert it to dollars

$$289.20 \div 0.718 = \$402.79$$

Hence the cheapest farm is **Ann's farm**.

- (c) Bags are filled with bird food at a rate of 420 grams per second.

How many 20 kg bags can be **completely** filled in 4 hours?

[3]

Convert the time into seconds

$$4 \text{ h} \times 60 \times 60$$

$$= 14400 \text{ s}$$

Times the rate by the time to get the weight of seeds

$$14400 \times 420$$

$$= 6048000 \text{ g}$$

Convert to kg by dividing by 1000

$$6048000 \div 1000$$

$$= 6048 \text{ kg}$$

Divide by the size of the bags (20 kg) to get the number of bags

$$6048 \div 20$$

$$= 302.4$$

Ignore the partial filling of a bag, so we get

**=302**

*Assembled by NS*

- (d) Brian buys bags of bird food from the factory and sells them in his shop for \$15.30 each.  
He makes 12.5% profit on each bag.

How much does Brian pay for each bag of bird food? [3]

The price Brian sells the food for is 12.5% bigger than the price he pays.

If we let the price he pays for the bags be  $p$ , then we have

$$1.125 \times p = 15.30$$

Divide through by 1.125

$$p = 15.30 \div 1.125$$

$$= 13.60$$

- (e) Brian orders 600 bags of bird food.

The probability that a bag is damaged is  $\frac{1}{50}$ .

How many bags would Brian expect to be damaged? [1]

Times the number of bags by the probability

$$600 \times \frac{1}{50}$$

$$= 12$$

### Question 3

- (a) The Martinez family travels by car to Seatown.  
The distance is 92 km and the journey takes 1 hour 25 minutes.

- (i) The family leaves home at 0750.  
Write down the time they arrive at Seatown.

[1]

$$7:50 + 1\text{hr } 25\text{m}$$

$$= 8\text{hr } 75\text{m}$$

$$= 9\text{hr } 15\text{m}$$

$$= \mathbf{09:15}$$

- (ii) Calculate the average speed for the journey.

[2]

Convert the time into just units of hours

$$1\text{hr } 25\text{m} = 1\text{hr} + \frac{25}{60}\text{hr}$$

$$= \frac{85}{60}\text{hr}$$

$$= \frac{17}{12}\text{hr}$$

Speed distance time relation

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\rightarrow \text{speed} = 92\text{km} \div \frac{17}{12}$$

$$= \mathbf{64.9\ kmh^{-1}}$$

- (iii) During the journey, the family stops for 10 minutes.

[1]

Calculate 10 minutes as a percentage of 1 hour 25 minutes.

Convert time to just minutes

$$1 \text{ hr } 25m = 60m + 25m$$

$$= 85m$$

10m as a percentage is then

$$\frac{10}{85} \times 100\%$$

$$= 11.8\%$$

- (iv) 92 km is 15% more than the distance from Seatown to Deecity.

[3]

Calculate the distance from Seatown to Deecity.

If D is the distance from Seatown to Deecity then

$$1.15 \times D = 92$$

Since 92 is 15% greater than D. Now divide through by

1.15 for

$$\rightarrow D = \frac{92}{1.15}$$

$$= 80$$

- (b) The Martinez family spends \$150 in the ratio

$$\text{fuel : meals : gifts} = 11 : 16 : 3.$$

- (i) Show that \$15 is spent on gifts.

[2]

There are

$$11 + 16 + 3$$

$$= 30$$

total parts and 3 of those parts are spent on gifts. Hence

$$\frac{3}{30} \times \$150$$

$$= \$15$$

- (ii) The family buys two gifts.  
The first gift costs \$8.25.

Find the ratio

$$\text{cost of first gift : cost of second gift.}$$

[2]

Give your answer in its simplest form.

The cost of the second gift is

$$15 - 8.25$$

$$= 6.75$$

Giving us

$$\text{first gift : second gift} = 8.25 : 6.75$$

$$= 1 : \frac{6.75}{8.25}$$

$$= 1 : \frac{9}{11}$$

$$= 11 : 9$$

## Question 4

- (a) In Portugal, Miguel buys a book about planets.  
The book costs €34.95.  
In England the same book costs £27.50.  
The exchange rate is £1 = €1.17.

Calculate the difference in pounds (£) between the cost of the book in Portugal and England. [2]

Cost in pounds for the book in Portugal

$$34.95 \div 1.17$$

$$= 29.87$$

Hence, difference is

$$29.87 - 27.50$$

$$= 2.37$$

- (b) In the book, the distance between two planets is given as  $4.07 \times 10^{12}$  kilometres.  
The speed of light is  $1.1 \times 10^9$  kilometres per hour.

Calculate the time taken for light to travel from one of these planets to the other. [3]  
Give your answer in days and hours.

Speed distance time relation is

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\rightarrow 1.1 \times 10^9 = \frac{4.07 \times 10^{12}}{\text{time}}$$

$$\rightarrow \text{time} = \frac{4.07 \times 10^{12}}{1.1 \times 10^9}$$

$$= 3.7 \times 10^3 \text{ hours}$$

Convert to days

$$= 3.7 \times 10^3 \div 24 \text{ days}$$

$$= 154\frac{1}{6} \text{ days}$$

$$= 154 \text{ days} \frac{1}{6} \times 24 \text{ hours}$$

$$= \mathbf{154 \text{ days } 4 \text{ hours}}$$

- (c) In one of the pictures in the book, a rectangle is drawn.  
The rectangle has length 9.3 cm and width 5.6cm, both correct to one decimal place. [1]
- (i) What is the lower bound for the length?

$$\mathbf{9.25}$$

- (ii) Work out the lower and upper bounds for the area of the rectangle. [2]

Lower bound is

$$9.25 \times 5.55$$

$$= \mathbf{51.3375}$$

Upper bound is

$$9.35 \times 5.65$$

$$= \mathbf{52.8275}$$

## Question 5

- (a) Abdullah and Jasmine bought a car for \$9000.  
Abdullah paid 45% of the \$9000 and Jasmine paid the rest.

(i) How much did Jasmine pay towards the cost of the car?

[2]

We first need to work out the amount paid by Abdullah.

$$\frac{45}{100} \times \$9000 = \$4050$$

Jasmine pays the rest:

$$\$9000 - \$4050 = \$4950$$

- (ii) Write down the ratio of the payments Abdullah : Jasmine in its simplest form.

[1]

$$\text{Abdullah : Jasmine} = \$4050 : \$4950$$

The highest common factor we can divide these numbers  
by is 450.

Therefore, the simplest form of the ratio would be:

$$9 : 11$$

- (b) Last year it cost \$2256 to run the car.  
Abdullah, Jasmine and their son Henri share this cost in the ratio 8:3:1.  
Calculate the amount each paid to run the car.

[3]

A ratio represents the actual amount of money paid by  
each simplified with the heights common factor.

We represent this factor as the unknown x.

Therefore: Abdullah part =  $8x$

Jasmine part =  $3x$

Henri part =  $x$

By adding up these 3 amounts we obtain the total cost.

$$8x + 3x + x = \$2256$$

$$12x = \$2256$$

$$x = \$188$$

Therefore: Abdullah pays  $8 \times \$188 = \$1504$

Jasmine pays  $3 \times \$188 = \$564$

Henri pays =  $\$188$

- (c) (i) A new truck costs \$15 000 and loses 23% of its value each year.  
Calculate the value of the truck after three years.

[3]

We can apply here the compound interest formula since

each year the value decreases by 23% from the value in

that specific year.

The value decreases by 23%, meaning that after each year

the amount remaining is:

$100\% - 23\% = 77\%$  of the initial price

$$77\% = \frac{77}{100} = 0.77$$

The value after 3 years will be:

\$15000 x (0.77)<sup>n</sup> where n is the number of years.

$$\$15000 \times (0.77)^3$$

$$= \$6847.99$$

(ii) Calculate the overall percentage loss of the truck's value after three years. [3]

To calculate the overall loss percentage, we need to work out what percentage does \$6947.99 represent out of the initial value.

We represent this percentage through the unknown y.

$$\frac{y}{100} \times \$15000 = \$6847.99$$

$$y = 45.65\%$$

The remaining value is 45.6% of the initial value,

therefore the loss is represented by:

$$100\% - 45.65\%$$

$$= 54.35\%$$

## Question 6

(a)  $72 = 2 \times 2 \times 2 \times 3 \times 3$  written as a product of prime factors.

(i) Write the number 126 as a product of prime factors.

[2]

**126 = 2 x 63** (63 is not divisible by 2, so we divide it further by 3, the next smallest prime factor)

**126 = 2 x 3 x 21** (21 = 3 x 7, both prime factors)

**126 = 2 x 3 x 3 x 7**

(ii) Find the value of the highest common factor of 72 and 126.

[1]

**126 = 2 x 3 x 3 x 7**

**72 = 2 x 2 x 2 x 3 x 3**

By looking at the 2 numbers written in this form, we see that their common prime factors are:  $2 \times 3 \times 3$

Therefore, their highest common factor is:  $2 \times 3 \times 3$

**= 18.**

(iii) Find the value of the lowest common multiple of 72 and 126.

[2]

Lowest common multiple (LCM) of a and b is the smallest integer which is evenly divisible by both a and b.

We can calculate it by looking at the prime factorisation of the 2 numbers and picking each of the prime factors at their highest power and multiplying them.

e.g.

$$126 = 2 \times 3 \times 3 \times 7 = 2^1 \times 3^2 \times 7^1$$

$$72 = 2^3 \times 3^2$$

The highest power of 2 in both numbers is  $2^3$ , the highest power of 3 is  $3^2$  and for 7 is  $7^1$

The LCM will be:

$$\text{LCM}(126, 72) = 2^3 \times 3^2 \times 7^1$$

$$= 504$$

- (b) John wants to estimate the value of  $\pi$ .

He measures the circumference of a circular pizza as 105 cm and its diameter as 34 cm, both correct to the nearest centimetre.

Calculate the lower bound of his estimate of the value of  $\pi$ .  
Give your answer correct to 3 decimal places.

[4]

The lower bound is the smallest number which would round up to the estimated value.

$$\text{Circumference} = \text{diameter} (2r) \times \pi$$

Where  $r$  is the radius of the circle.

In our case:

$$105 \text{ cm} = 34 \text{ cm} \times \pi$$

Both numbers are correct to the nearest centimetre, therefore:

$$104.5 \text{ cm} \leq \text{circumference} < 105.5$$

$$33.5 \text{ cm} \leq \text{diameter} < 34.5$$

To obtain the lower bound of  $\pi$ , we select the lowest possible value for the circumference (the numerator) and the highest possible value for the diameter (the denominator).

$$\pi = \frac{104.5 \text{ cm}}{34.5 \text{ cm}}$$

$$\pi = 3.029$$

- (c) The volume of a cylindrical can is  $550 \text{ cm}^3$ , correct to the nearest  $10 \text{ cm}^3$ .  
The height of the can is  $12 \text{ cm}$  correct to the nearest centimetre.

Calculate the upper bound of the radius of the can.  
Give your answer correct to 3 decimal places.

[5]

$$V = \pi r^2 h$$

$$h = 12 \text{ cm}$$

$$V = 550 \text{ cm}^3$$

Both numbers are correct to the nearest cm and  $\text{cm}^3$ ,  
therefore:

$$545 \text{ cm}^3 \leq V < 555 \text{ cm}^3$$

$$11.5 \text{ cm} \leq h < 12.5 \text{ cm}$$

To obtain the upper bound of  $r$ , we select the highest possible value for  $V$  (the numerator) and the lowest possible value for  $h$  (the denominator).

$$r^2 = \frac{V}{h \times \pi}$$

$$r^2 = \frac{555 \text{ cm}^3}{11.5 \text{ cm} \times \pi}$$

$$r^2 = 15.3619 \text{ cm}^2$$

$$r = 3.919 \text{ cm}$$

## Question 7

A school has 220 boys and 280 girls.

- (a) Find the ratio of boys to girls, in its simplest form.

[1]

$$220 \div 280$$

$$= \frac{11}{14}$$

Hence

*Boys : Girls*

$$= \frac{11}{14} : 1$$

$$= 11 : 14$$

- (b) The ratio of students to teachers is 10 : 1.  
Find the number of teachers.

[2]

Total number of students is

$$220 + 280$$

$$= 500$$

There are 10 times as many students, hence

$$50$$

- (c) There are 21 students on the school's committee.  
The ratio of boys to girls is 3 : 4.  
Find the number of girls on the committee.

[2]

There are

$$\frac{3}{7} \times 21$$

$$= 9 \text{ boys}$$

Hence there are

$$21 - 9$$

$$= 12 \text{ girls}$$

- (d) The committee organises a disco and sells tickets.  
35% of the school's students each buy a ticket. Each ticket costs \$1.60.  
Calculate the total amount received from selling the tickets.

[3]

Number of students to buy a ticket

$$0.35 \times 500$$

$$= 175$$

Money made is

$$175 \times 1.6$$

$$= 280$$

- (e) The cost of running the disco is \$264.

This is an increase of 10% on the cost of running last year's disco.

Calculate the cost of running last year's disco.

[2]

We have that

$$264 = c \times 1.1$$

Where  $c$  is the cost of last year's disco

$$\rightarrow c = 264 \div 1.1$$

$$= 240$$

# Number

## Difficulty: Hard

### Model Answers 5

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Number
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 5

**Time allowed:** **78 minutes**

**Score:** **/68**

**Percentage:** **/100**

**Grade Boundaries:**

**CIE IGCSE Maths (0580)**

A*	A	B	C	D
>83%	67%	51%	41%	31%

**CIE IGCSE Maths (0980)**

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

Chris goes to a shop to buy meat, vegetables and fruit.

- (a) (i) The costs of the meat, vegetables and fruit are in the ratio

$$\text{meat : vegetables : fruit} = 2 : 2 : 3.$$

The cost of the meat is \$2.40.

Calculate the **total** cost of the meat, vegetables and fruit. [2]

$$1 \text{ share} = 2.40/2 = \$1.20$$

Next, we multiply by each of the ratios and add together

to get the total cost.

$$(2 \times 1.20) + (2 \times 1.20) + (3 \times 1.20)$$

$$= \$8.40$$

- (ii) Chris pays with a \$20 note.

What percentage of the \$20 has he spent? [2]

$$(8.40/20) \times 100$$

$$= 42\%$$

- (b) The masses of the meat, vegetables and fruit are in the ratio

$$\text{meat : vegetables : fruit} = 1 : 8 : 3.$$

The total mass is 9 kg.

Calculate the mass of the vegetables. [2]

$$1+8+3 = 12$$

$$9/12 = 0.75\text{kg}$$

Next multiply this by the share value:

$$0.75 \times 8$$

$$= 6\text{kg}$$

- (c) Calculate the cost per kilogram of the fruit. [3]

We can use our values for both cost and mass that we have

previously worked out in questions a) and b) to help us here. So

first, we work out the cost and mass of the fruit:

$$\text{Cost} \rightarrow 3 \times 1.20 = \$3.60$$

$$\text{Mass} \rightarrow 0.75 \times 3 = 2.25\text{kg}$$

Divide the cost by the mass:

$$3.60 / 2.25 = \$1.60$$

So the cost is **\$1.60 per kilogram.**

- (d) The cost of the meat, \$2.40, is an increase of 25% on the cost the previous week.

Calculate the cost of the meat the previous week.

[2]

Because \$2.40 is a 25% increase, we can say that \$2.40 represents 125% of

the original price and work it out like this:

$$1\% = 2.40 / 125 = 0.0192$$

Multiply by 100:

$$0.0192 \times 100$$

$$= \$1.92$$

## Question 2

Vreni took part in a charity walk.  
She walked a distance of 20 kilometres.

- (a) She raised money at a rate of \$12.50 for each kilometre.

- (i) How much money did she raise by walking the 20 kilometres?

[1]

$$20 \times 12.5$$

$$= 250$$

- (ii) The money she raised in **part (a)(i)** was  $\frac{5}{52}$  of the total money raised.

Work out the total money raised.

[2]

$$250 \times \frac{52}{5}$$

$$= 2600$$

- (iii) In the previous year the total money raised was \$2450.

Calculate the percentage increase on the previous year's total.

[2]

$$\frac{2600}{2450} \times 100$$

$$= 106.1\%$$

Hence, an increase of

$$6.1\%$$

- (b) Part of the 20 kilometres was on a road and the rest was on a footpath.  
The ratio road distance : footpath distance was 3:2.

- (i) Work out the road distance.

[2]

$$20 \times \frac{3}{3+2}$$

$$= 12$$

- (ii) Vreni walked along the road at 3 km/h and along the footpath at 2.5 km/h.  
How long, in hours and minutes, did Vreni take to walk the 20 kilometres?

[2]

Footpath distance is

$$20 - 12$$

$$= 8$$

We know that

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\rightarrow \text{time} = \frac{\text{distance}}{\text{speed}}$$

Adding the two different travel times together gives us

$$\text{time} = \frac{12}{3} + \frac{8}{2.5}$$

$$= \frac{20}{5} + \frac{16}{5}$$

$$= 7.2 \text{ hours}$$

$$= 7 \text{ hours } 12 \text{ minutes}$$

- (iii) Work out Vreni's average speed.

[1]

Average speed is

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\rightarrow \text{speed} = \frac{20}{7.2}$$

$$= 2.78$$

(iv) Vreni started at 08 55. At what time did she finish?

[1]

$$08\ 55 + 07\ 12$$

$$= 15\ 67$$

$$= \mathbf{16\ 07}$$

(c) On a map, the distance of 20 kilometres was represented by a length of 80 centimetres.

The scale of the map was  $1:n$ .Calculate the value of  $n$ .

[2]

Convert everything to metres.

$$80\ cm = 0.8\ m$$

$$20\ km = 20\ 000\ m$$

Hence, map scale is

$$0.8 : 20\ 000$$

$$\rightarrow 1 : \frac{20000}{0.8}$$

$$\rightarrow 1 : 25\ 000$$

$$\rightarrow n = \mathbf{25\ 000}$$

### Question 3

- (a) The scale of a map is 1:20 000 000.

On the map, the distance between Cairo and Addis Ababa is 12 cm.

- (i) Calculate the distance, in kilometres, between Cairo and Addis Ababa.

[2]

The scale shows that 1 cm on the map represents 20 000 000 cm in reality.

If the distance on the map is 12 cm, then the distance in reality will be:

$$12 \text{ cm} \times 20 000 000 = 240 000 000 \text{ cm}$$

We convert cm in km:

$$\text{Distance in km} = 2 400 \text{ km.}$$

- (ii) On the map the area of a desert region is 13 square centimetres.

Calculate the actual area of this desert region, in square kilometres.

[2]

Using the scale of the map, we deduce that 1 cm<sup>2</sup> on the map represents 20 000 000<sup>2</sup> cm<sup>2</sup> in reality.

$$4 \times 10^{14} \text{ cm}^2 = 40 000 \text{ km}^2 \text{ representing } 1 \text{ cm}^2$$

For 13 cm<sup>2</sup> on the map, the real area will be:

$$40 000 \text{ km}^2/\text{cm}^2 \times 13 \text{ cm}^2$$

$$= 520 000 \text{ km}^2$$

(b) (i) The actual distance between Cairo and Khartoum is 1580 km.

On a different map this distance is represented by 31.6 cm.

Calculate, in the form  $1:n$ , the scale of this map.

[2]

31.6 cm on the map represents 1580 km in reality.

We convert the distance from km to cm.

$$1580 \text{ km} = 158\,000\,000 \text{ cm}$$

Therefore, 1 cm on the map will represent:

$$\frac{158\,000\,000 \text{ cm} \times 1 \text{ cm}}{31.6 \text{ cm}} = 5\,000\,000 \text{ cm}$$

$$1:n = 1:5\,000\,000$$

(ii) A plane flies the 1580 km from Cairo to Khartoum.

It departs from Cairo at 11 55 and arrives in Khartoum at 1403.

Calculate the average speed of the plane, in kilometres per hour.

[4]

$$\text{Speed} = \text{distance}/\text{time}$$

The total time travelled is:  $T = 2$  hours and 8 minutes.

The result needs to be in km/h, so we need to convert the time only in hours.

60 minutes represent 1 hour, so 8 minutes will be:

$$\frac{8 \text{ minutes} \times 1 \text{ hour}}{60 \text{ minutes}} = 0.133 \text{ hours}$$

The total time travelled is:  $T = 2.133$  hours.

$$\text{Speed} = 1580 \text{ km}/2.133 \text{ hours}$$

$$\text{Speed} = 740.74 \text{ km/h}$$

## Question 4

Hassan sells fruit and vegetables at the market.

- (a) The mass of fruit and vegetables he sells is in the ratio  
fruit : vegetables = 5 : 7.

Hassan sells 1.33 tonnes of vegetables.

How many kilograms of fruit does he sell?

[3]

Fruit: vegetables = 5 : 7

This ratio represents the amount of fruit and vegetables sold simplified by a common factor we note with x.

In this case, the amount of fruit would be  $5x$  and the amount of vegetables would be  $7x$ .

We know that the amount of vegetables is 1.33 tonnes.

We convert tonnes in kilograms to have both amounts expressed in the same unit.

$$1.33 \text{ tonnes} = 1330 \text{ kg}$$

$$1330 \text{ kg} = 7x$$

$$x = 190 \text{ kg}$$

The amount of fruit is:

$$\text{fruit} = 5 \times 190 \text{ kg}$$

$$\text{fruit} = 950 \text{ kg}$$

- (b) The amount of money Hassan receives from selling fruit and vegetables is in the ratio  
fruit : vegetables = 9 : 8.

Hassan receives a **total** of \$765 from selling fruit and vegetables.  
Calculate how much Hassan receives from selling fruit. [2]

Fruit: vegetables = 9 : 8

This ratio represents the amount of money received from selling fruit and vegetables simplified by a common factor we note with  $y$ .

In this case, the amount of money received for fruit would be  $9y$  and the amount of money received from vegetables would be  $8y$ .

We know that the total amount of money received for both is \$765.

This can be written in the form of the expression:

$$9y + 8y = \$765$$

$$17y = \$765$$

$$y = \$45$$

The amount of money received for fruit is:

$$\text{fruit} = 9y$$

$$\text{fruit} = 9 \times \$45$$

$$\text{fruit} = \$405$$

$$\text{vegetables} = 8y$$

$$\text{vegetables} = 8 \times \$45$$

$$\text{vegetables} = \$360$$

- (c) Calculate the average price of Hassan's fruit, in dollars per kilogram.

[2]

To work out the average price for fruit we need to divide the amount of money received from selling fruit by the total amount of fruit which has been sold.

$$\text{Price} = \frac{\$405}{950 \text{ kg}}$$

$$\text{Price} = 0.43 \text{ dollars/kg}$$

- (d) (i) Hassan sells oranges for \$0.35 per kilogram.

He reduces this price by 40%.

Calculate the new price per kilogram.

[2]

The initial price is \$0.35/kg.

40% of this amount is:

$$\frac{40}{100} \times \$0.35/\text{kg} = \$0.14/\text{kg}$$

If the initial price is reduced by \$0.14/kg, the new price

will be:

$$\$0.35/\text{kg} - \$0.14/\text{kg} = \$0.21/\text{kg}$$

- (ii) The price of \$0.35 per kilogram of oranges is an increase of 25% on the previous day's price.

Calculate the previous day's price.

[2]

We note the previous day's price with the unknown x.

$$x + \frac{25x}{100} = \$0.35/\text{kg}$$

$$\frac{125x}{100} = \$0.35/\text{kg}$$

$$x = \frac{\$35/\text{kg}}{125}$$

$$x = \$0.28/\text{kg}$$

## Question 5

Fatima and Mohammed each buys a bike.

- (a) Fatima buys a city-bike which has a price of \$120.  
She pays 60 % of this price and then pays \$10 per month for 6 months.  
(i) How much does Fatima pay altogether? [2]

Remembering that % ("percent") means " $\div 100$ " so  $60\% = 60 \div 100 = 0.6$

$$\text{Amount Fatima pays} = 0.6 \times 120 + 10 \times 6 = \$132$$

- (ii) Work out your answer to part (a)(i) as a percentage of the original price of \$120. [2]

Divide the answer to (a)(i) by 120 and write as a percentage

$$\text{Percentage} = \frac{132}{120} = 1.1 = 110\%$$

- (b) Mohammed pays \$159.10 for a mountain-bike in a sale.  
The original price had been reduced by 14 %.  
Calculate the original price of the mountain-bike. [2]

Write a statement down in the form: "After is a percentage of Before":

Sale Price is 86% of Original Price (A 14% reduction means that 86% is left)

Translate this into an equation ("is" means "=", "of" means " $\times$ "):

$$\text{Sale Price} = 0.86 \times \text{Original Price}$$

Put in the numbers:

$$159.10 = 0.86 \times \text{Original Price}$$

Solve (in this case by dividing by 0.86):

$$\text{Original Price} = \frac{159.1}{0.86} = \$185$$

- (c) Mohammed's height is 169 cm and Fatima's height is 156 cm.  
The frame sizes of their bikes are in the same ratio as their heights.  
The frame size of Mohammed's bike is 52 cm.  
Calculate the frame size of Fatima's bike.

[2]

When using ratios, create as many columns and rows as you

need and use Scale Factors to move between them:

$$\begin{array}{ccc}
M & : & F \\
\text{Height} & 169 & : \quad 156 \\
\text{Frame Size} & 52 & : \quad ? \\
& & \curvearrowright \text{ Multiply by } \frac{52}{169} \\
& ? = 156 \times \frac{52}{169} = 48 \text{ cm}
\end{array}$$

- (d) Fatima and Mohammed are members of a school team which takes part in a bike ride for charity.

- (i) Fatima and Mohammed ride a total distance of 36 km.

The ratio distance Fatima rides : distance Mohammed rides is 11 : 9.  
Work out the distance Fatima rides.

[2]

$$11 + 9 = 20,$$

$$\frac{36}{20} = \frac{9}{5},$$

$$\frac{9}{5} \times 11 \text{ for Fatima} = \frac{99}{5} = 19.8 \text{ km}$$

- (ii) The distance of 36 km is only  $\frac{2}{23}$  of the total distance the team rides.

Calculate this total distance.

[2]

Translate the information given into an equation ("is" means "=" , "of" means "×"):

$$36 \text{ km} = \frac{2}{23} \times \text{Total distance}$$

Solve (in this case by multiplying by  $\frac{23}{2}$ ):

$$\text{Total Distance} = 36 \times \frac{23}{2} = 414 \text{ km}$$

## Question 6

A train starts its journey with 240 passengers.  
144 of the passengers are adults and the rest are children.

- (a) Write the ratio Adults : Children in its lowest terms. [2]

The number of children is:  $240 - 144 = 96$

The ratio will be:

Adults: Children = 144 : 96

To obtain the simplest form, we need to divide the numbers by their highest common factor, 48.

Adults: Children = 3 : 2

- (b) At the first stop,  $37\frac{1}{2}\%$  of the adults and  $\frac{1}{3}$  of the children get off the train.  
20 adults and  $x$  children get onto the train.  
The total number of passengers on the train is now 200.

- (i) How many children got off the train? [1]

The number of children who get off is:  $\frac{1}{3} \times 96$

= 32

- (ii) How many adults got off the train? [1]

The number of children who got off is:  $(37\frac{1}{2} \times 144)/100$

= 54

- (iii) How many **adult** passengers are **on** the train as it sets off again? [1]

The initial number of adults is 144, 54 get off and then 20 get on:

The final number will be:  $144 - 54 + 20$

= 110 adults

(iv) What is the value of  $x$ ?

[1]

The total number of passengers is 200:

$$x = 200 - 110 \text{ adults}$$

$$x = 90 \text{ children which are on the train.}$$

The initial number of children is 96, and  $1/3$  get off:

$$96 - 1/3 \times 96 = 64$$

$$x = 90 - 64$$

**= 26 children**

(c) After a second stop, there are 300 passengers on the train and the ratio Men :

Women : Children is 6 : 5 : 4.

Calculate the number of children now on the train.

[2]

The ratio represents the actual number of men, women and children divided by their

highest common factor which we consider in this case as the unknown  $x$ .The number of men =  $6x$ The number of women =  $5x$ The number of children =  $4x$ 

The total number of passengers = 300

$$6x + 4x + 5x = 300$$

$$15x = 300$$

$$x = 20$$

The number of men =  $6 \times 20 = 120$ The number of women =  $5 \times 20 = 100$ **The number of children =  $4 \times 20 = 80$**

- (d) On Tuesday the train journey took 7 hours and 20 minutes and began at 13 53.

- (i) At what time did the train journey end?

[1]

The journey finished at

: 21.13

- (ii) Tuesday's time of 7 hours 20 minutes was 10% more than Monday's journey time. How many minutes longer was Tuesday's journey?

[2]

We convert the time only in minutes, since the result is required in minutes.

$$7 \times 60 + 20 = 440 \text{ minutes}$$

The difference between the 2 journeys is of 10%.

$$\frac{100 \times 440 \text{ minutes}}{110}$$

$$= 40 \text{ minutes}$$