Graphical Inequalities
Difficulty: Medium

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphical Inequalities
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 1

Time allowed: **68 minutes**

Score: **/59**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) Assembled by AS

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Ali buys x rose bushes and y lavender bushes.

He buys:

- at least 5 rose bushes
- at most 8 lavender bushes
- at most 15 bushes in total
- more lavender bushes than rose bushes.

(a) (i) Write down four inequalities, in terms of x and/or y, to show this information. [4]

As we know that x represents rose bushes and y represents lavender bushes, we can turn the following statements into inequalities:

- At least 5 rose bushes

$$\boldsymbol{x \geq 5}$$

- At most 8 lavender bushes

$$\boldsymbol{y \leq 8}$$

- At most 15 bushes in total

$$\boldsymbol{x + y \leq 15}$$

- More lavender bushes than rose bushes

$$\boldsymbol{x < y}$$

(ii) On the grid, show the information in **part (a)(i)** by drawing four straight lines.
Label the region R where all four inequalities are true. [5]

Draw the following four lines.

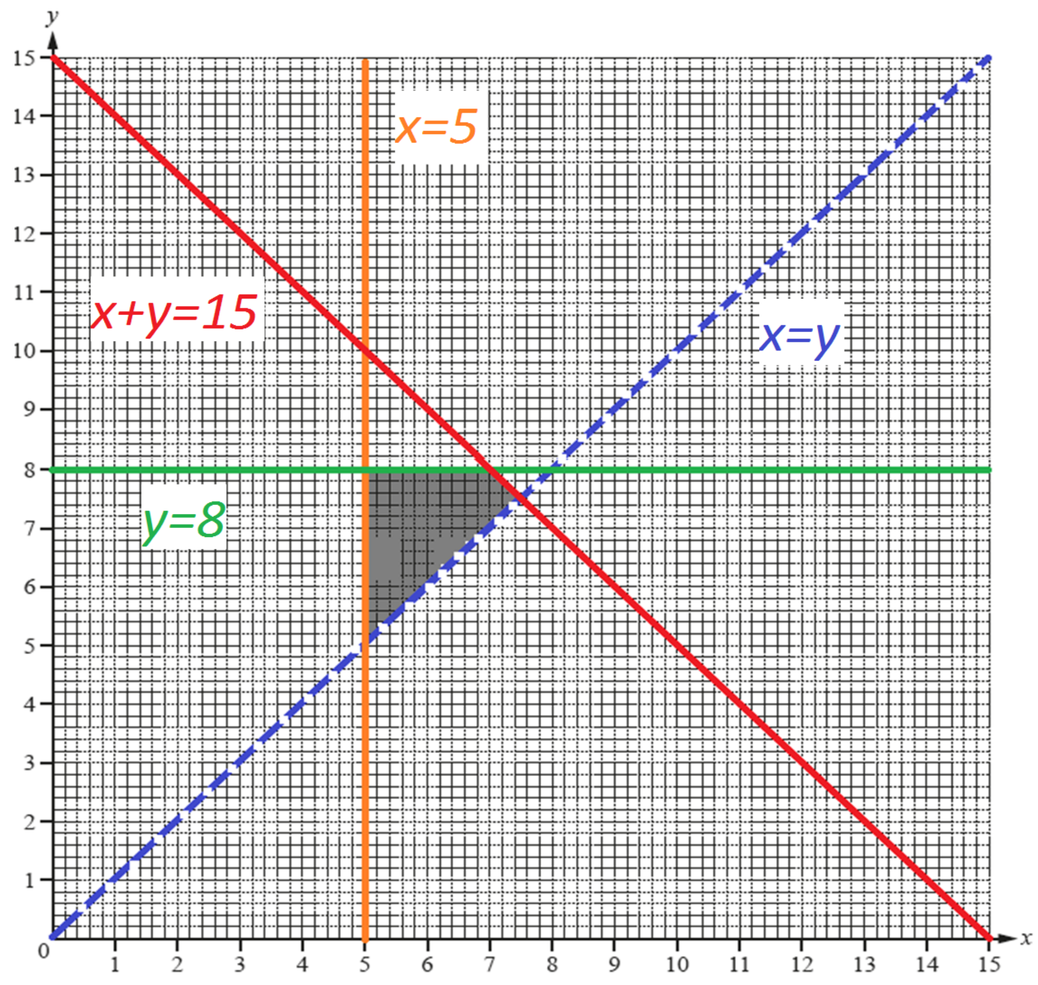
$x = 5$ (orange line – solid, equality allowed)

$y = 8$ (green line – solid, equality allowed)

$x + y = 15$ (red line – solid, equality allowed)

$x = y$(blue line - broken, equality not allowed)

Shade the regions, which does satisfy the inequalities (grey region).



The easiest way to do this is to think of a point which does satisfy the inequality and

find it on the grid to work out the region that does satisfy the inequalities.

(b) Rose bushes cost \$6 each and lavender bushes cost \$4.50 each.

What is the greatest amount of money Ali could spend? [2]

The point in the shaded region, with the highest allowed values of *x* and *y* is (7,8).

(*x*=7, *y*=8)

$$costs = (7 \times \$6 + 8 \times \$4.50)$$

$$\boldsymbol{costs = \$78}$$

The school cook buys potatoes in small sacks, each of mass 4 kg, and large sacks, each of mass 10 kg.
He buys x small sacks and y large sacks.
Today, he buys less than 80 kg of potatoes.

(a) Show that $2x + 5y < 40$. [1]

We want to buy less than 80kg of potatoes.

The cook buys *x* small sacks and *y* large sacks.

$$(mass\ of\ a\ small\ sack\ )x\ +(mass\ of\ a\ large\ sack)y < 80\ kg$$

$$4x\ +10 < 80$$

Divide both sides by 2. We get the final inequality.

$$\mathbf{2x\ +5y < 40}$$

(b) He buys more large sacks than small sacks.
He buys no more than 6 large sacks.

Write down two inequalities to show this information.

[2]

He wants to buy more large sacks than small sacks, so the number of *y* (large sacks) must be bigger than *x* (small sacks).

$$\boldsymbol{y > x}$$

Buying no more than 6 large sacks, means that *y* must be 6 at most (smaller or equal).

$$\boldsymbol{y \leq 6}$$

(c) On the grid, show the information in **part (a)** and **part (b)** by drawing three straight lines and shading the unwanted regions.

[5]

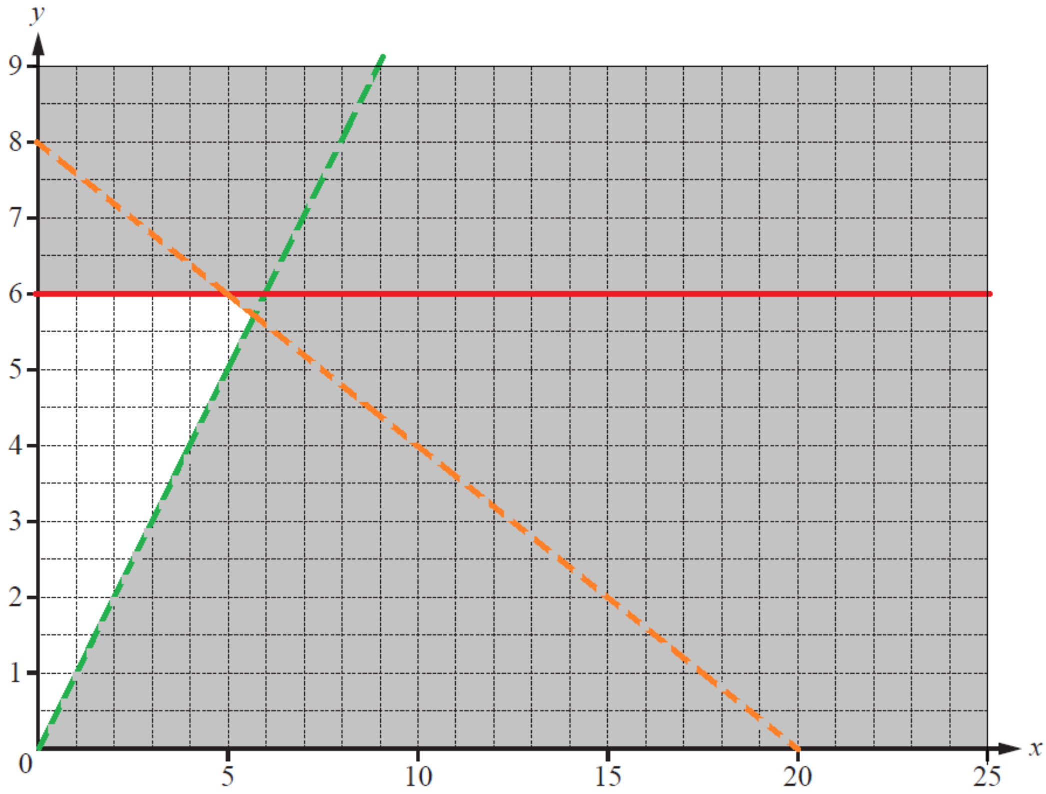
Draw the following three lines.

$2x\ +5y < 40$ (orange line - broken, equality not allowed)

$y > x$ (green line – broken, equality not allowed)

$y \leq 6$ (red line – solid, equality allowed)

Shade the regions, which do not satisfy the inequalities (grey region).



The easiest way to do this is to think of a point which does not satisfy the inequality and find it on the grid to work out the region that does not satisfy the inequalities.

(d) Find the greatest mass of potatoes the cook can buy today. [2]

The point in the white region, with the highest allowed values of x and y is (4,6). (x=4, y=6)

(Note that points (5,5) and (5,6) do not lie in the region due to the dashed lines.)

$$mass = (4 \times 4 + 10 \times 6)kg$$

$$\boldsymbol{mass = 76kg}$$

(a) Luk wants to buy x goats and y sheep.

(i) He wants to buy at least 5 goats.

Write down an inequality in x to represent this condition. [1]

If he wants **at least** 5 goats he wants the goats to be more than or equal to 5:

$$x \geq 5$$

(ii) He wants to buy at least 11 sheep.

Write down an inequality in y to represent this condition. [1]

He wants sheep to be more than or equal to 11:

$$y \geq 11$$

(iii) He wants to buy at least 20 animals.

Write down an inequality in x and y to represent this condition. [1]

He needs the sum of x goats and y sheep to be more than or equal to 20:

$$x + y \geq 20$$

(b) Goats cost \$4 and sheep cost \$8.
The maximum Luk can spend is \$160.

Write down an inequality in x and y and show that it simplifies to $x + 2y \leq 40$. [1]

The costs are \$4 per goat (x) and \$8 per sheep (y) and he must spend less than or equal to \$160 on both

Therefore, $\$4 \times x + \$8 \times y \leq \$160$

$$4x + 8y \leq 160$$

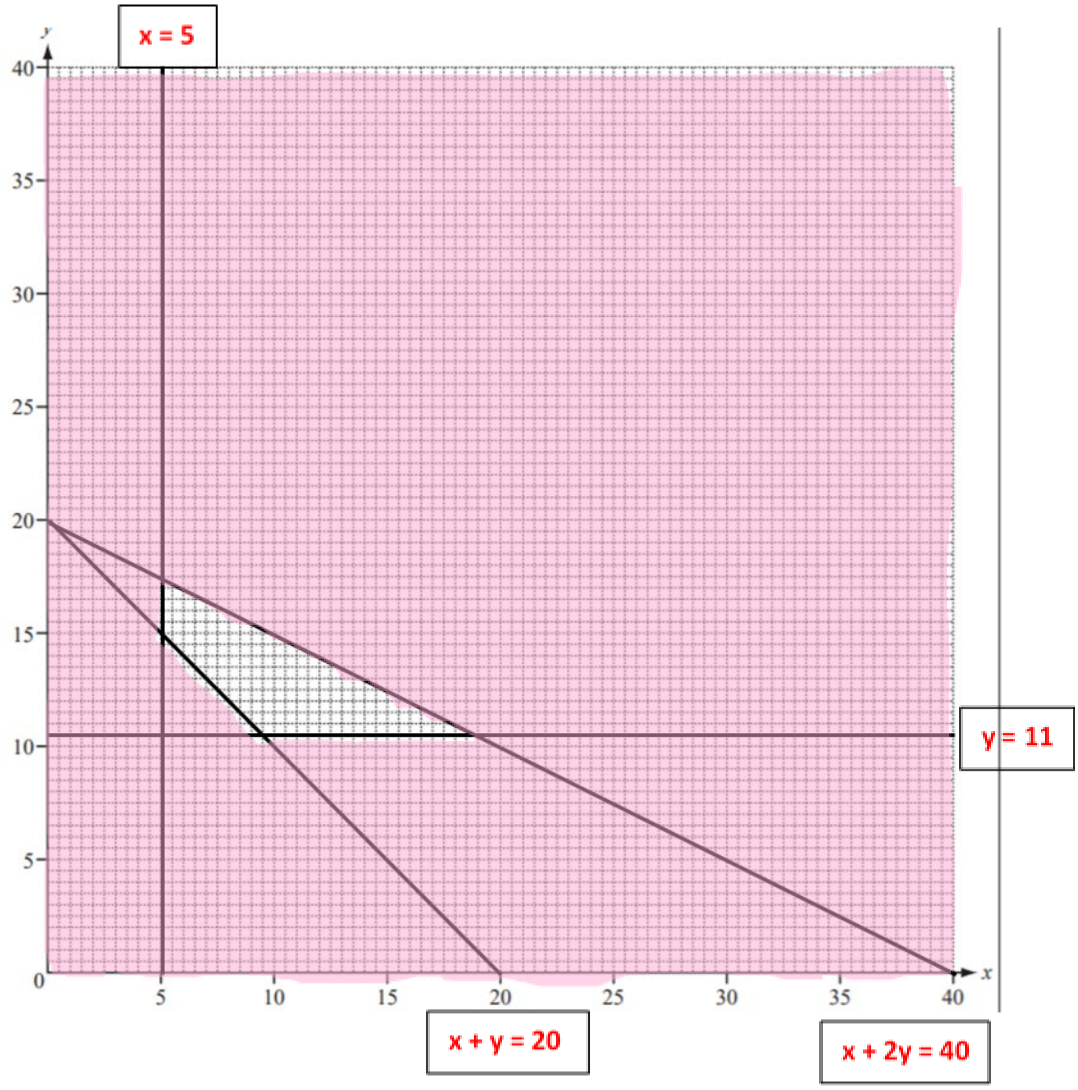
Divide both sides by 4

$$x + 2y \leq 40$$

(c) (i) On the grid below, draw four lines to show the four inequalities and shade the **unwanted** regions.

[7]

Draw on the lines as follows. Use the inequality to choose the side which the unwanted region lies on:



(ii) Work out the maximum number of animals that Luk can buy.

[2]

We must look for the point on the boundary of the "wanted" region where x and y add up to the maximum value.

The top left corner is at (5, 17.5) which adds to 22.5

The bottom right corner is at (18, 11) which adds up to **29** which is our maximum

Jay makes wooden boxes in two sizes. He makes x small boxes and y large boxes.
He makes at least 5 **small** boxes.
The greatest number of **large** boxes he can make is 8.
The greatest total number of boxes is 14.
The number of large boxes is at least half the number of **small** boxes.

(a) (i) Write down four inequalities in x and y to show this information. [4]

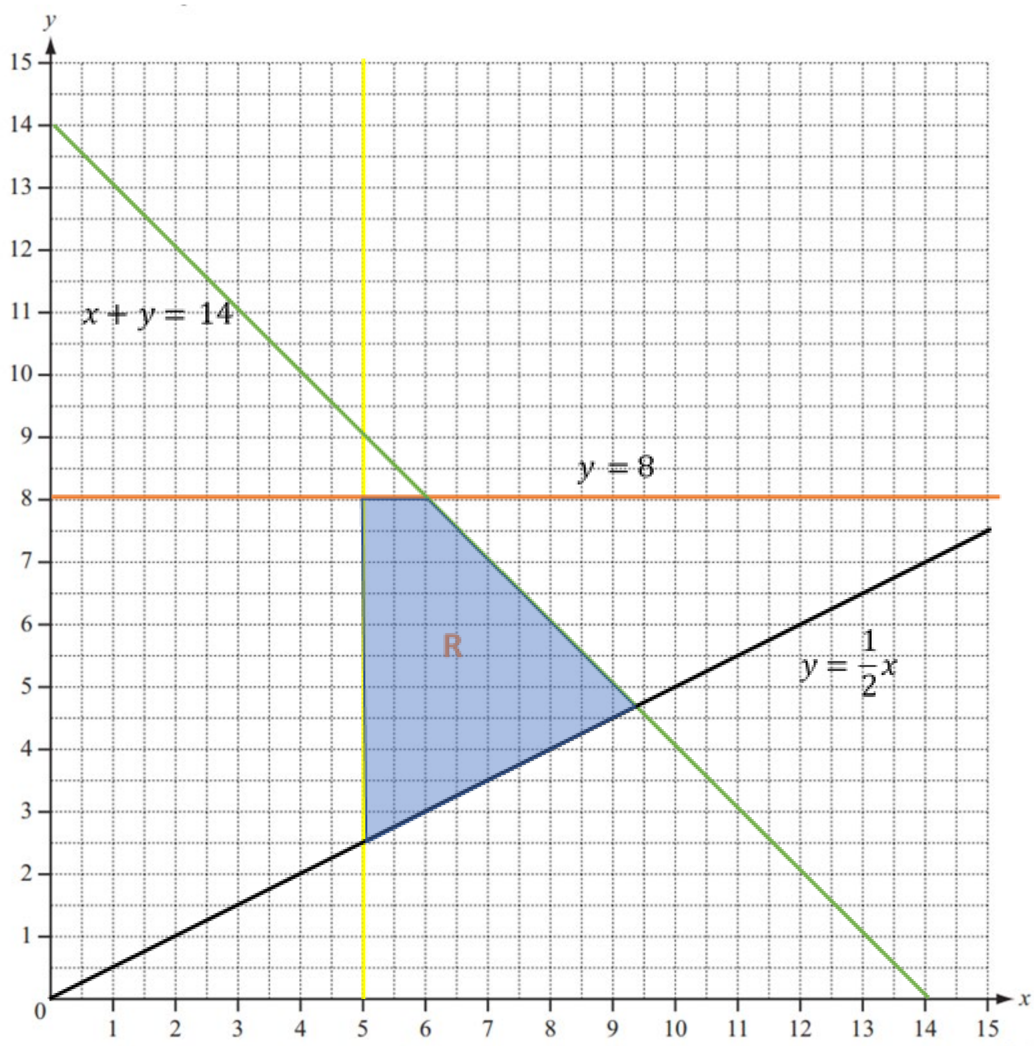
$x \geq 5$ yellow)

$y \leq 8$ (orange)

$x + y \leq 14$ (green)

$y \geq \frac{1}{2}x$ (black)

(ii) Draw four lines on the grid and write the letter R in the region which represents these inequalities. [5]



(b) The price of the small box is \$20 and the price of the large box is \$45.

(i) What is the greatest amount of money he receives when he sells all the boxes he has made? [2]

We want to maximise

$$20x + 45y$$

Within the region R. This will occur at one of the vertices, so we check the two largest. Clearly (5, 8) is smaller than (6, 8) and (5, 2.5) is smaller than the others. This leaves (6, 8) and the intersection of the green and black lines.

$$(6, 8): \;\; 20(6) + 45(8) = 480$$

The other vertex occurs when the following two lines intersect

$$y = 14 - x$$

$$y = \frac{1}{2}x$$

We equate them

$$\rightarrow 14 - x = \frac{1}{2}x$$

Rearrange for x

$$\rightarrow \frac{3}{2}x = 14$$

$$\rightarrow x = \frac{28}{3}$$

Plug into $y = \frac{1}{2}x$ for the y-coordinate

$$\rightarrow y = \frac{14}{3}$$

Hence

$$\left(\frac{28}{3},\frac{14}{3}\right):20\left(\frac{28}{3}\right)+45\left(\frac{14}{3}\right)=396.7$$

So, the other is larger and hence the maximum

=480

(ii) For this amount of money, how many boxes of each size did he make? [1]

He makes

6 small boxes

and

8 large boxes

Peter wants to plant x plum trees and y apple trees.

He wants at least 3 plum trees and at least 2 apple trees.

(a) Write down one inequality in x and one inequality in y to represent these conditions. [2]

We need to express these as inequalities. The plums are represented by x. Peter wants to plant at least 3 plum trees, so x must be 3 or more.

$$x \geq 3$$

Similarly, he wants to plant at least 2 apple trees. The apple trees are represented by y, so y must be 2 or more.

$$y \geq 2$$

(b) There is space on his land for no more than 9 trees.

Write down an inequality in x and y to represent this condition. [1]

The maximum number of trees that he can plant is 9. So the sum of x and y representing the trees must be 9 or less, giving the inequality:

$$x + y \leq 9$$

(c) Plum trees cost \$6 and apple trees cost \$14.

Peter wants to spend no more than \$84.

Write down an inequality in x and y, and show that it simplifies to $3x + 7y \leq 42$. [1]

To get the price, we multiply the number of trees (x or y) by their respective cost per tree. So the total cost is represented as:

$$6x + 14y$$

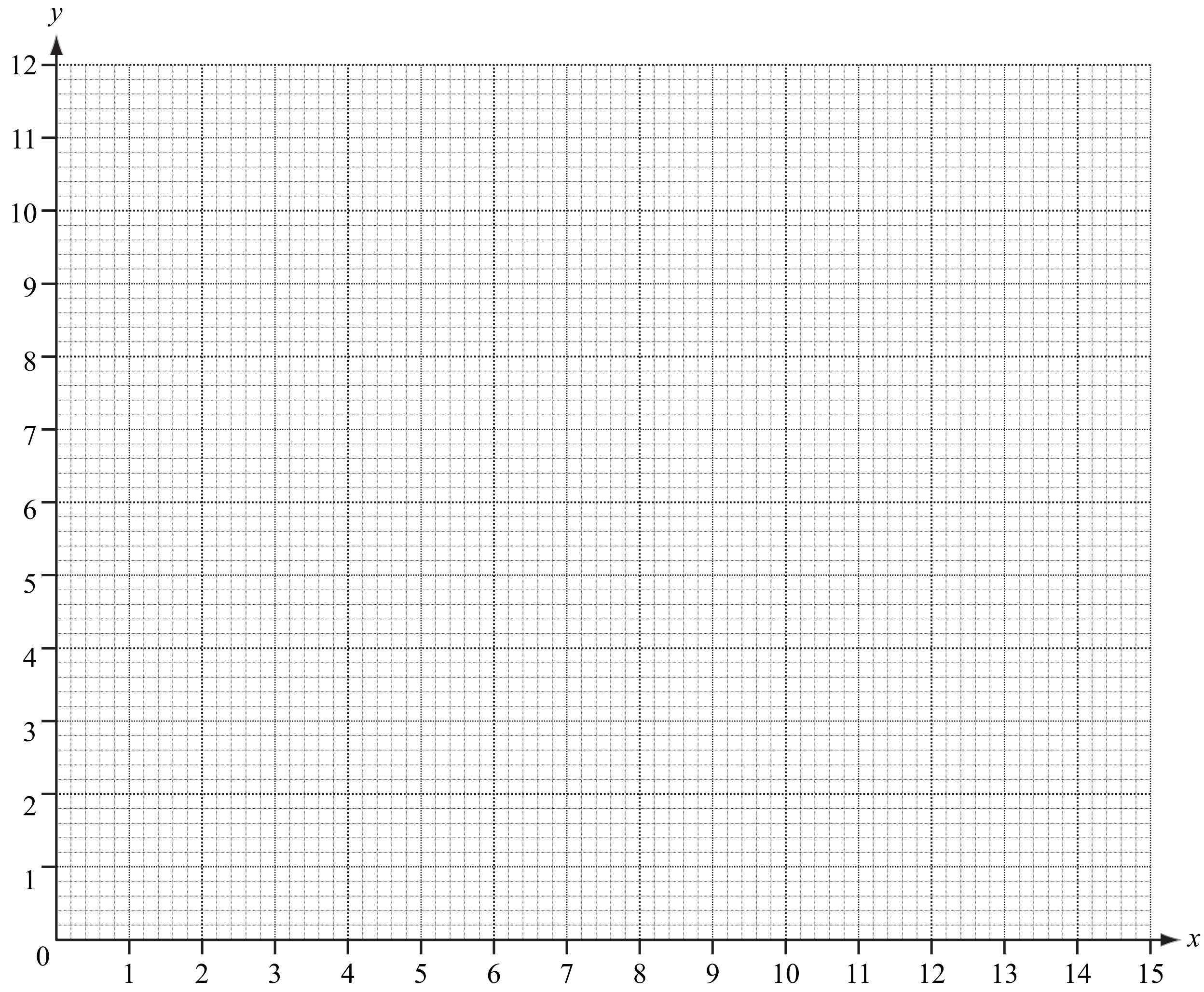
The sum of the total costs must be 84 or less, giving the inequality:

$$6x + 14y \leq 84$$

Divide both sides of the inequality by 2.

$$\mathbf{3x + 7y \leq 42}$$

(d) On the grid, draw four lines to show the four inequalities and shade the unwanted regions. [7]



We plot the inequalities as lines. All lines can be represented as full lines, because all four inequalities give the possibility of both sides being equal (not strictly smaller or greater, in that case, we would have used broken lines).

Draw the following four lines.

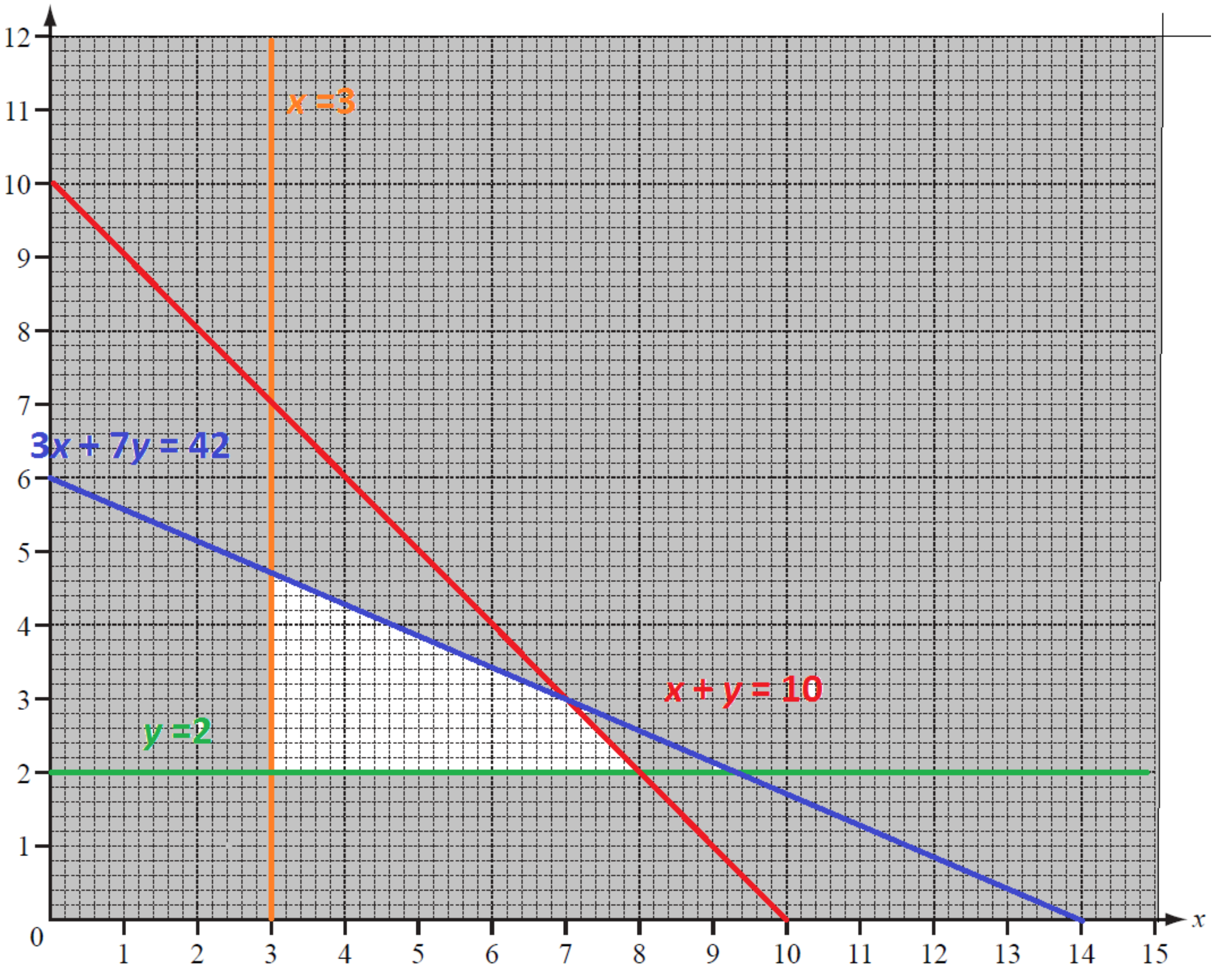
$x = 3$ (orange line)

$y = 2$ (green line)

$x + y = 10$ (red line)

$3x + 7y = 42$(blue line)

Shade the regions, which do not satisfy the inequalities (grey region).



The easiest way to do this is to think of a point which does satisfy the inequality

and find it on the grid to work out the region that does satisfy the inequalities.

(e) Calculate the smallest cost when Peter buys a total of 9 trees. [2]

There are only two points inside the un-shaded region which coordinates sum up

(representing the total number of trees) to 9. These are

$$(7, 2) \text{ } and \text{ } (6,3)$$

We calculate the total price for these two possibilities:

$$cost\ (7,2) = 6 \times (7) + 14 \times (2)$$

$$cost\ (7,2) = 70$$

And the other point

$$cost\ (6,3) = 6 \times (6) + 14 \times (3)$$

$$cost\ (6,3) = 78$$

So the cheapest possibility is if we buy 6 plum trees (x=7) and 3 apple trees (y=2).

The total most is:

$$\boldsymbol{cost\ (x = 7, y = 2) = 70}$$

Graphical Inequalities
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphical Inequalities
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **70 minutes**

Score: **/61**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Pablo plants x lemon trees and y orange trees.

(a) (i) He plants at least 4 lemon trees.

Write down an inequality in x to show this information. [1]

$x \geq 4$

(ii) Pablo plants at least 9 orange trees.

Write down an inequality in y to show this information. [1]

$y \geq 9$

(iii) The greatest possible number of trees he can plant is 20.

Write down an inequality in x and y to show this information. [1]

$x + y \leq 20$

(b) Lemon trees cost \$5 each and orange trees cost \$10 each.

The maximum Pablo can spend is \$170.

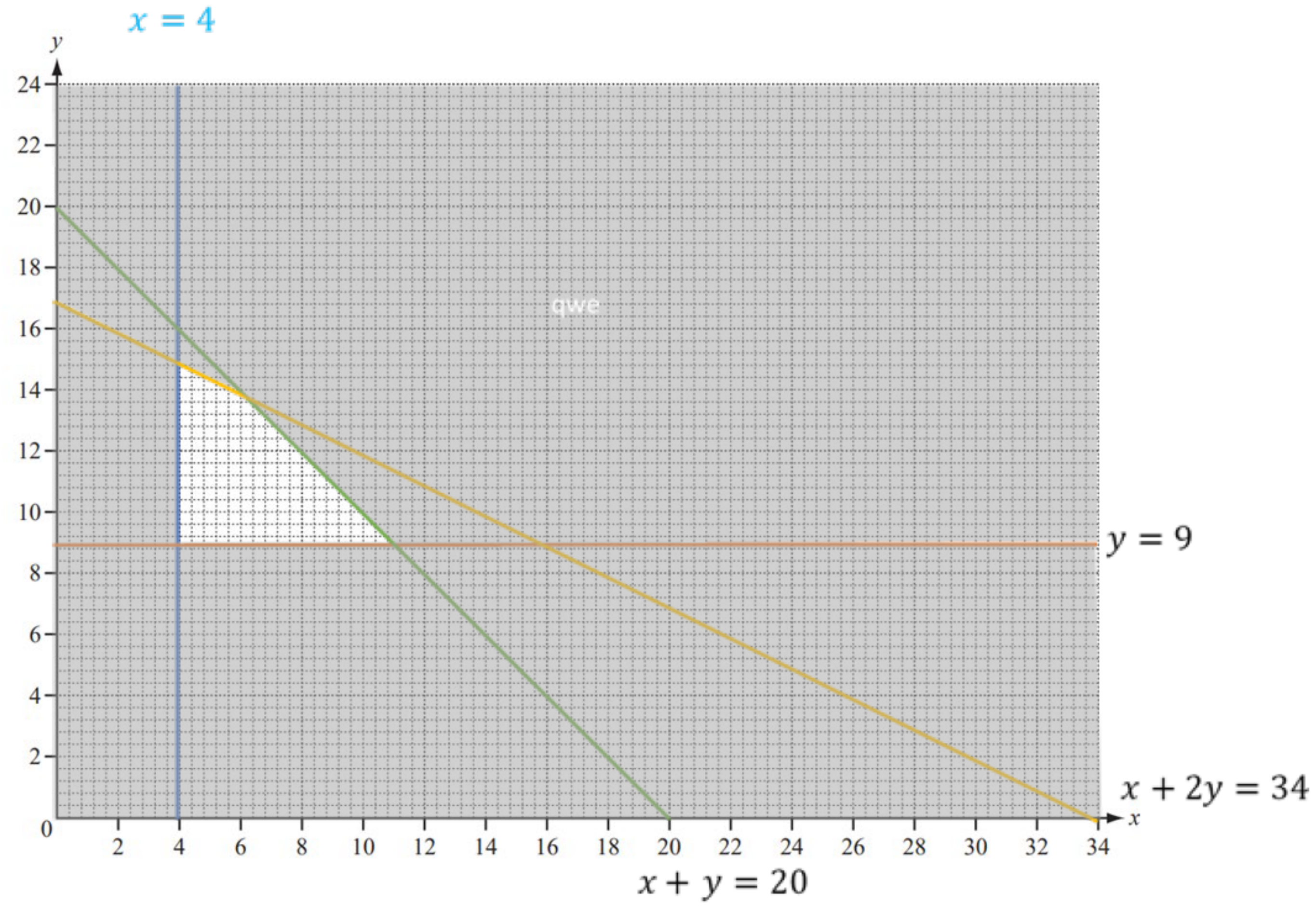
Write down an inequality in x and y and show that it simplifies to $x + 2y \leqslant 34$. [1]

$5x + 10y \leq 170$

Divide through by 5

$x + 2y \leq 34$

(c) (i) On the grid opposite, draw four lines to show the four inequalities and shade the **unwanted** region. [7]



Unwanted region shaded grey.

(ii) Calculate the smallest cost when Pablo buys a total of 20 trees. [2]

Check the vertices of the desired region above (only the

ones touching the green line $x + y = 20$).

It intersects with the orange line, $y = 9$

$$x + 9 = 20$$

$$\rightarrow x = 11$$

Put (11, 9) into the cost equation

$$5(11) + 10(9)$$

$$= 55 + 90$$

$$= 145$$

It also intersects with the yellow line, $x + 2y = 34$

$x + y = 20$ (1)

$x + 2y = 34$ (2)

Split (2) into the following

$$\rightarrow (x + y) + y = 34$$

Now sub in (1)

$$\rightarrow 20 + y = 34$$

$$\rightarrow y = 14$$

And using (1) we get x

$$\rightarrow x = 6$$

Sub into the cost equation

$$5(6) + 10(14)$$

$$= 30 + 140$$

$$= 170$$

Hence the smallest cost is

=145

Mr Chang hires x large coaches and y small coaches to take 300 students on a school trip.
Large coaches can carry 50 students and small coaches 30 students.
There is a maximum of 5 large coaches.

(a) Explain clearly how the following two inequalities satisfy these conditions.

(i) $x \leqslant 5$ [1]

$x \leq 5$

For x – the number of large coaches, the inequality

shows that the maximum number of large coaches is

=5

(ii) $5x + 3y \geqslant 30$ [2]

For x – the number of large coaches and y – the number of small coaches the

total number of students can be written as:

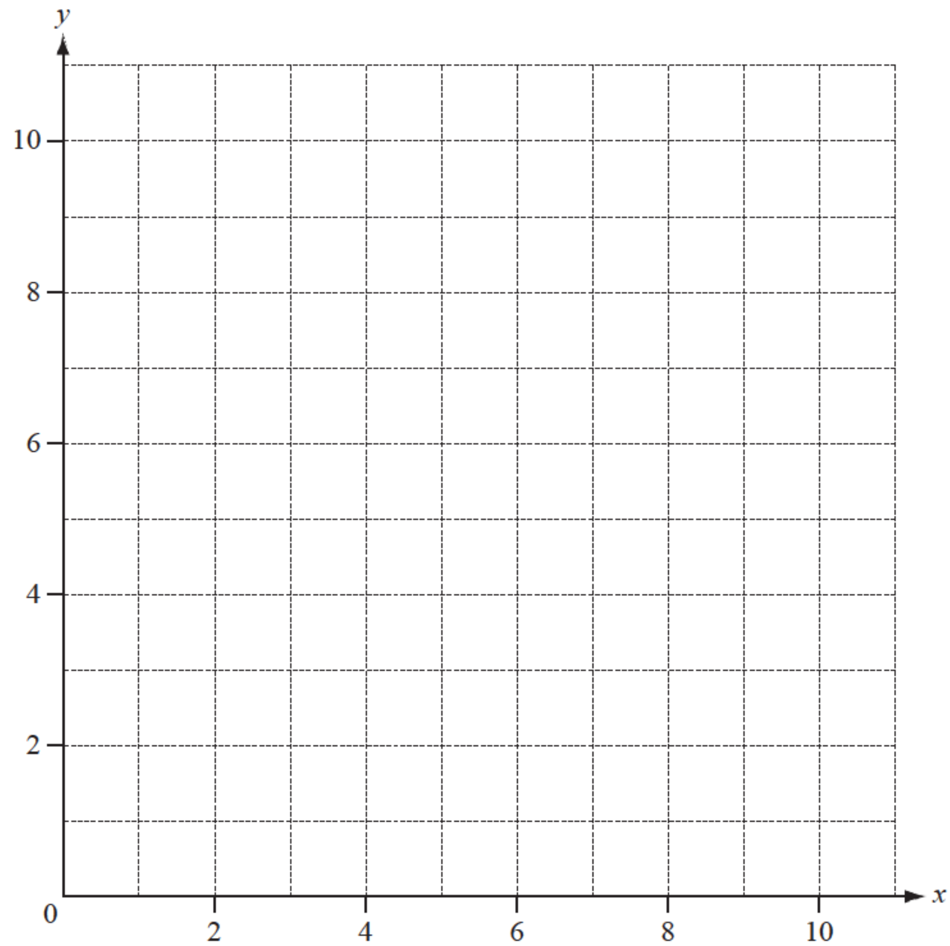
$50x + 30y \geq 300$

Simplified: $5x + 30 \geq 30$, showing that the total number of students that can fit in 50 large coaches and 30 small coaches needs to be at least 300.

Mr Chang also knows that $x + y \leqslant 10$.

(b) On the grid, show the information above by drawing three straight lines and shading the unwanted regions.

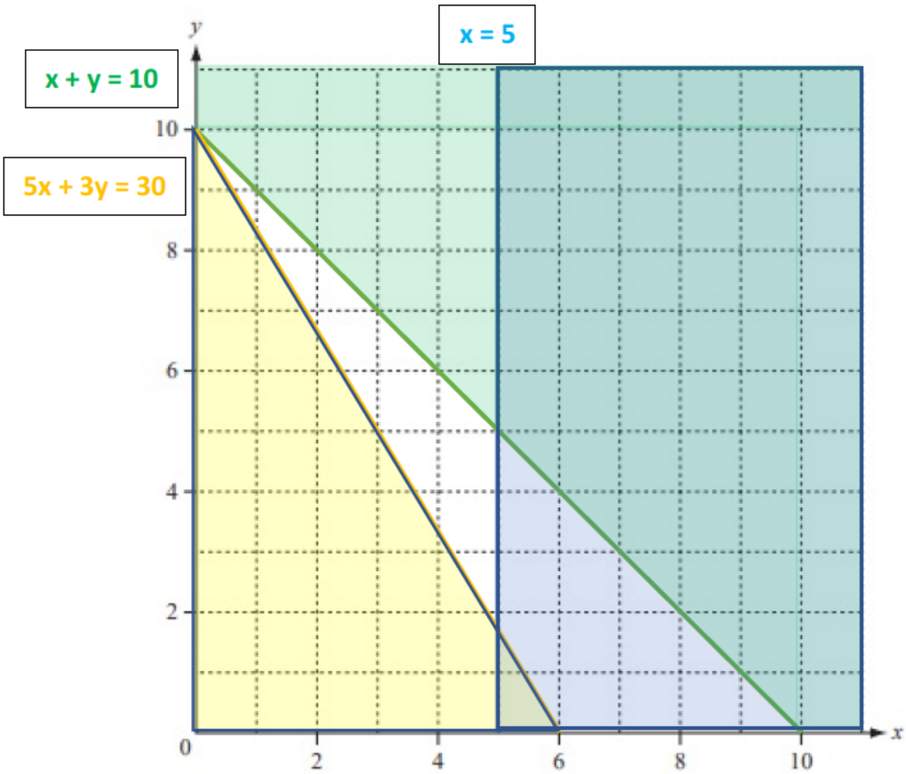
[5]



For the inequality $x \leq 5$, we need to represent the line of equation $x = 5$. The value needs to be less than 5, therefore, we shade the area on the left of the line of equation $x = 5$.

The line of equation $x + y = 10$ is drawn on the figure above by connecting the point of coordinates (0, 10) and (10, 0). The shaded area needs to be on the right side of this line, since the sum needs to be less than 10.

Similarly, for the line of equation $5x + 3y = 30$ we connect the points of coordinates (6, 0) and (0, 10). The shaded area should be on the left side of this line for values greater than 30.



(c) A large coach costs \$450 to hire and a small coach costs \$350.

(i) Find the number of large coaches and the number of small coaches that would give the minimum hire cost for this school trip. [2]

We need to identify 2 values which give the smaller cost while satisfying all the conditions of the inequalities represented above, therefore, these numbers need to be in the triangle indicated on the grid above.

We need to identify the x and y corresponding values for points in that area which give the smallest cost.

Cost = 450x + 350y

(ii) Calculate this minimum cost. [1]

The point with the smallest coordinates in the indicated area is x = 5 and y = 1. We work out these values by looking at the coordinates of the points within the triangle which are the smallest.

For these coordinates, the cost is:

\$450 x 5 + \$350 x 2 = \$2950

Hassan stores books in large boxes and small boxes.
Each large box holds 20 books and each small box holds 10 books.
He has x large boxes and y small boxes.

(a) Hassan must store at least 200 books.

Show that $2x + y \geq 20$. [1]

The number of books which can be stored in a large box is 20x while the number of books which can be stored in a small box is 10y.

The number of total books needs to be higher or equal than 200.

Therefore:

$20x + 10y \geq 200$

(b) Hassan must not use more than 15 boxes.
He must use at least 3 small boxes.
The number of small boxes must be less than or equal to the number of large boxes.

Write down three inequalities to show this information. [3]

The total number of boxes will be the sum of the small and large boxes, where x is the number of large boxes and y is the number of small boxes.

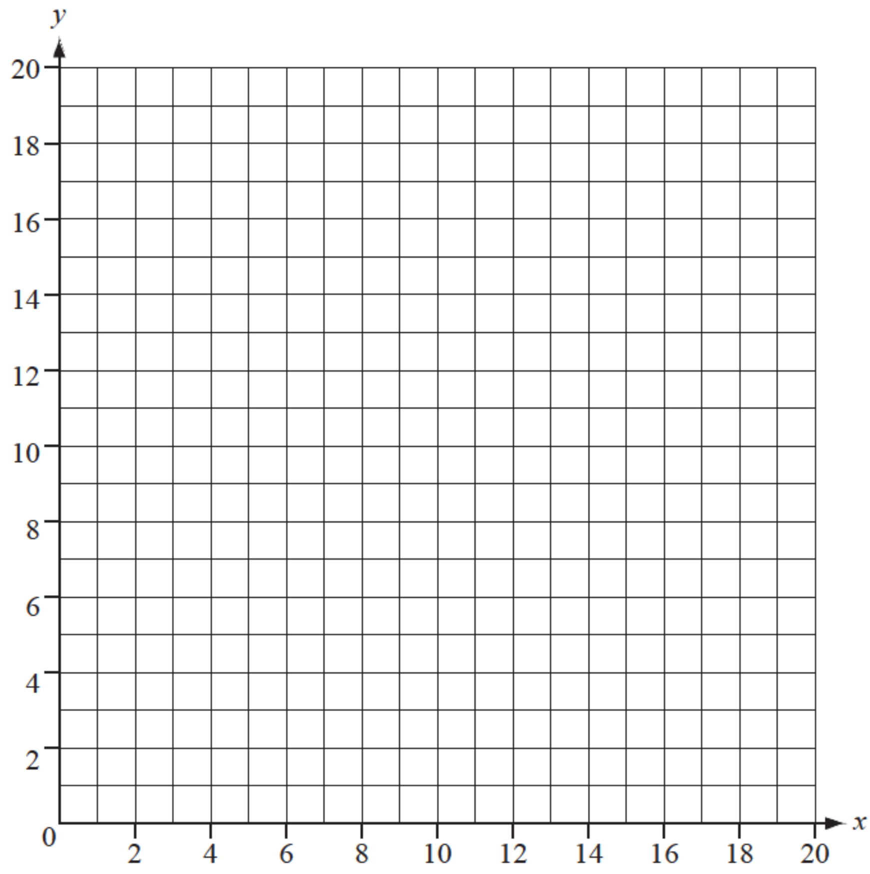
$x + y \leq 15$

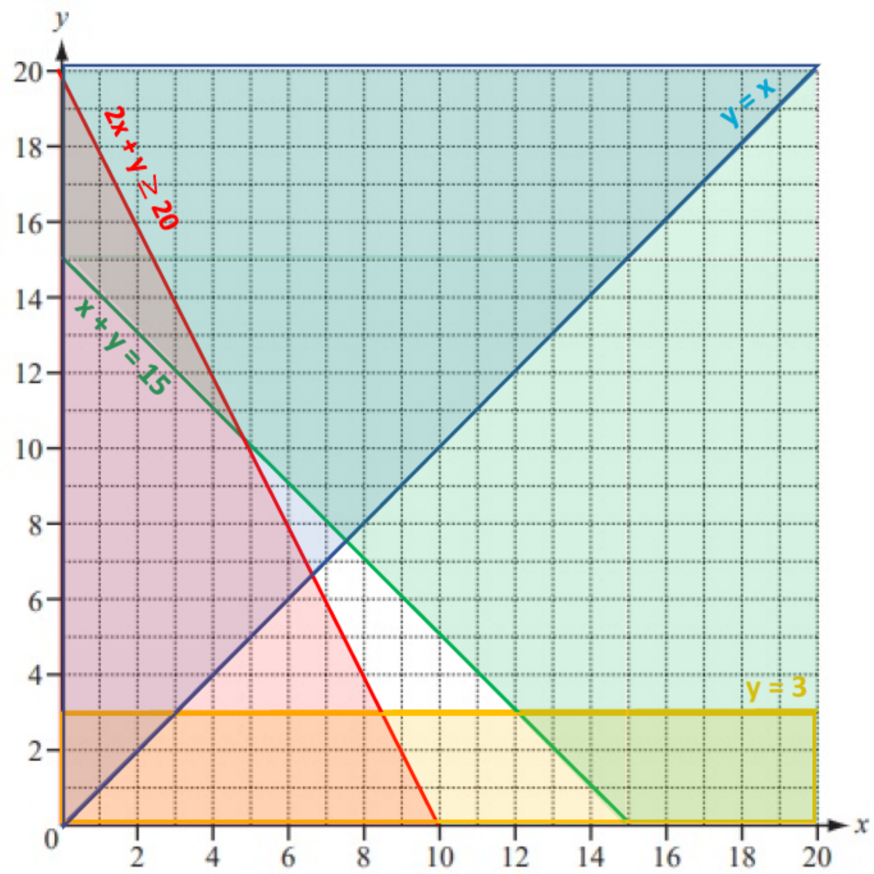
The number of small boxes needs to be greater than or equal to 3.

$y \geq 3$

$y \leq x$

(c) On the grid, show the information in part (a) and part (b) by drawing four straight lines and shading the unwanted regions. [6]





For the inequality $y \geq 3$, we need to represent the line of equation $y = 3$. The values need to be greater than 3, therefore, we shade the area below the line $y = 3$.

Similarly, we draw the line of equation $y = x$ and shade the area for which the y values are greater than the x values, above the line.

The line of equation $x + y = 15$ is drawn on the figure above by connecting the point of coordinates (0, 15) and (15, 0). The shaded area needs to be on the right side of this line.

Similarly, for the line of equation $2x + y = 20$ we connect the points of coordinates (10, 0) and (0, 20). The shaded area should be below this line, since we need values greater than 20.

(d) A large box costs \$5 and a small box costs \$2.

(i) Find the least possible total cost of the boxes. [1]

We need to identify 2 values which give the smaller cost while satisfying all the conditions of the inequalities represented above, therefore, these numbers need to be in the quadrilateral area indicated on the grid above.

We need to identify the x and y corresponding values for points in that area which give the smallest cost.

Cost = $5x + 2y$

The point with the smallest coordinates in the indicated area is $x = 7$ and $y = 6$. We work out these values by looking at the coordinates of the points within the quadrilateral which are the smallest.

For these coordinates, the cost is:

$5 x 7 + $2 x 6 = $47

(ii) Find the number of large boxes and the number of small boxes which give this least possible cost.
[2]

In this case, the number of large boxes would be 7 and the number of small boxes would be 6.

A company has a vehicle parking area of $1200\ \text{m}^2$ with space for x cars and y trucks.

Each car requires $20\ \text{m}^2$ of space and each truck requires $100\ \text{m}^2$ of space.

(a) Show that : $x + 5y \leqslant 60$. [1]

$$20x + 100y \leq 1200$$

Divide through by 20

$$\boldsymbol{x + 5y \leq 60}$$

(b) There must also be space for

(i) at least 40 vehicles, [1]

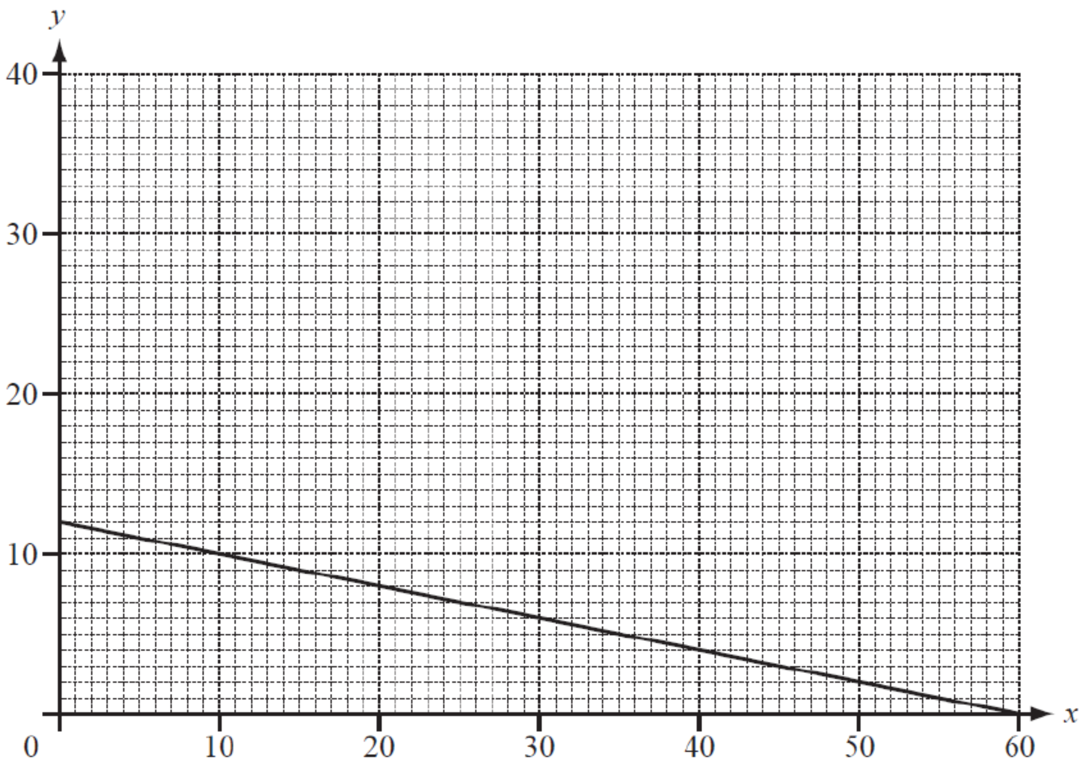
$\boldsymbol{x + y \geq 40}$

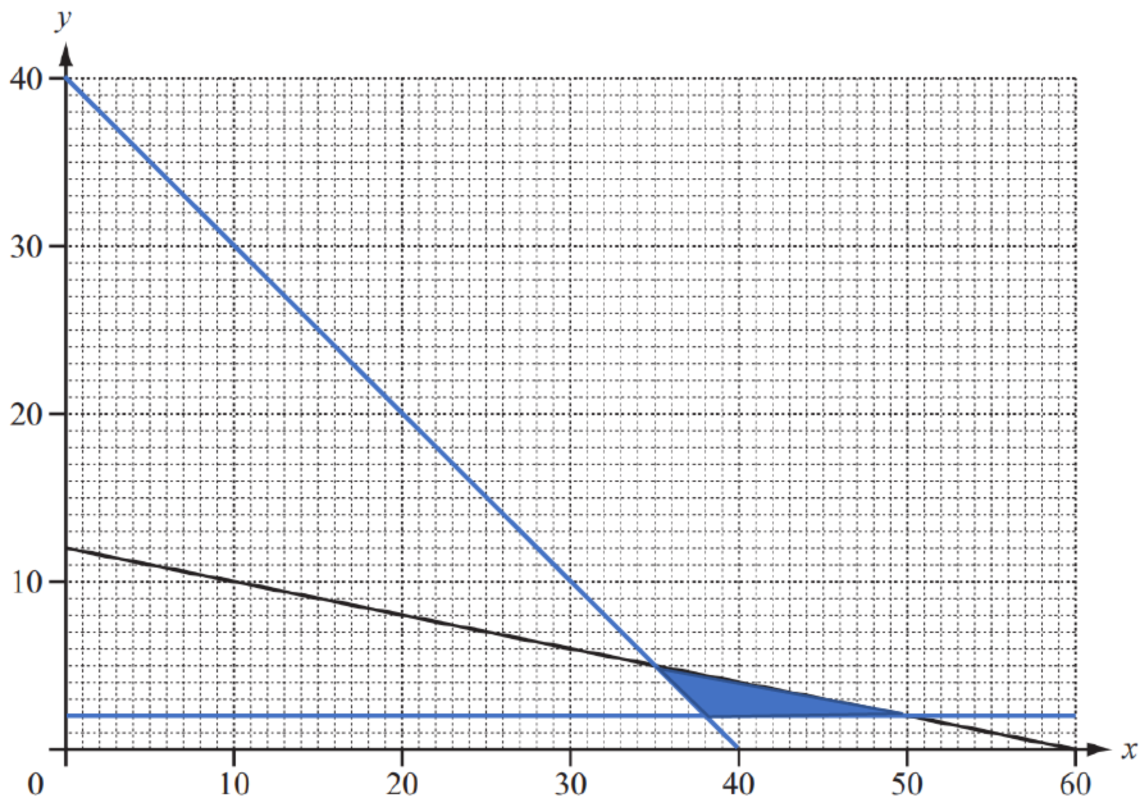
(i) at least 2 trucks.

Write down two more inequalities to show this information. [1]

$\boldsymbol{y \geq 2}$

(c) One line has been drawn for you.
On the grid, show the three inequalities by drawing the other two lines and shading the **unwanted** regions. [4]





(d) Use your graph to find the largest possible number of trucks. [1]

=5

(e) The company charges $5 for parking each car and $10 for parking each truck.
Find the number of cars and the number of trucks which give the company the greatest possible income.

Calculate this income. [3]

Check the vertices of the shape, i.e.

$(38, 2)$ (1)

$(35, 5)$ (2)

$(50, 2)$ (3)

This gives us

$$(1): \ 38 \times 5 + 2 \times 10$$

$$= 210$$

$$(2): 35 \times 5 + 5 \times 10$$

$$= 225$$

$$(3): 50 \times 5 + 2 \times 10$$

$$= 270$$

So, we have

$$\boldsymbol{Number\ of\ cars = 50}$$

$$\boldsymbol{Number\ of\ trucks = 2}$$

$$\boldsymbol{Greatest\ possible\ income = 270}$$

Answer the whole of this question on a sheet of graph paper.

Tiago does some work during the school holidays.
In one week he spends x hours cleaning cars and y hours repairing cycles.
The time he spends repairing cycles is at least equal to the time he spends cleaning cars.
This can be written as $y \geqslant x$.

He spends no more than 12 hours working.
He spends at least 4 hours cleaning cars.

(a) Write down two more inequalities in x and/or y to show this information. [3]

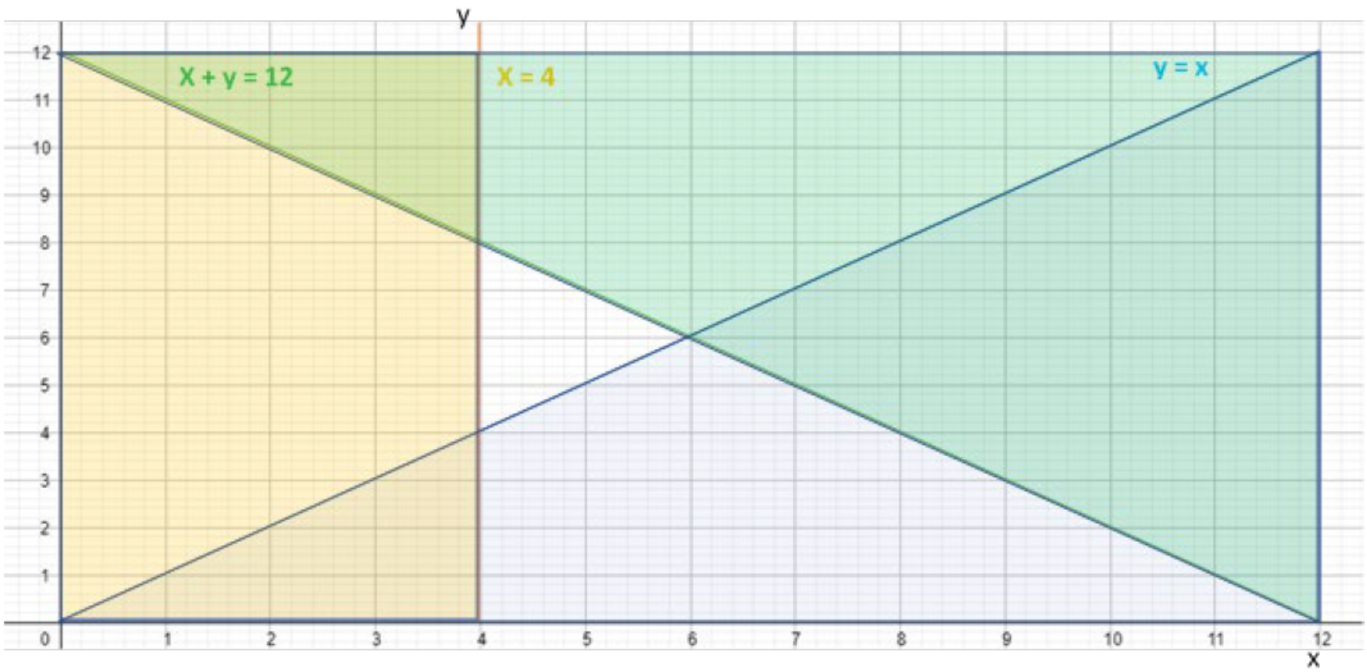
The maximum amount of time he spends both cleaning cars and repairing cycles is 12 hours.

$x + y \leq 12$

$x \geq 4$

(b) Draw x and y axes from 0 to 12, using a scale of 1 cm to represent 1 unit on each axis. [1]

(c) Draw three lines to show the three inequalities. Shade the **unwanted** regions. [5]



We draw the lines of equations: $x = 4$, $y = x$ and $x + y = 12$.

For $y \geq x$, the unwanted region is the one below the line of equation $y = x$.

Similarly, for $x \geq 4$, the unwanted region is at the left of the line of equation $x = 4$.

For $x + y \leq 12$, the unwanted region is above the line of equation $x + y = 12$.

(d) Tiago receives \$3 each hour for cleaning cars and \$1.50 each hour for repairing cycles.

(i) What is the least amount he could receive? [2]

The least amount he could receive is for the lowest corresponding x, y values which can be found within the unshaded region on the graph. The values need to be within that region so that all 3 inequalities are satisfied.

The lowest coordinates within the unshaded triangle are for the point (4, 4)

The least amount he could receive is: 4 x \$3 + 4 x \$1.5 = \$18

(ii) What is the largest amount he could receive? [2]

Similarly, to work out the largest amount he could receive we need to identify the point within the unshaded triangle with the highest x and y coordinates.

This point is: (6, 6)

The largest amount he receives is: \$3 x 6 + \$1.5 x 6 = \$27