8. Probability

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8.1 PROBABILITY

8.1.1 BASIC PROBABILITY

What do we mean by basic probability?

- There are both simple concepts and very hard topics in probability
- Probability is used in many areas and industries, eg. insurance costs and the rate of interest on loans

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What do I need to know?

- Be aware of how we write and use **probability notation:**

1. Event

- When we have a **trial** or **experiment** there may be several outcomes
- We use capital letters to denote an event – this is something that might happen in our trial
- For example, A could be the event of getting an even number when rolling a dice
- Note that it is more common to see reference to "outcomes" rather than "events" but they broadly mean the same thing.

2. "A-dash"

- If A is an event, then A' (spoken: "A-dash") is the event "not A"
- Notice we're not necessarily interested in what has happened, just that A hasn't

3. Probability of …

- **P(A)** means the probability that event A happens (eg. rolling a six on a dice)
- **P(A')** means the probability that event A does not happen (eg. getting any other number when rolling a dice)
- The letter **n** is often used to talk about the number of times A happens (or might happen) if the trial is **repeated** several times

4. Total probability

- All the different events for a trial have a total probability of 1 (**certainty**)
- This should make sense in that something will happen from a trial (eg. when rolling a (normal) dice it is certain you will get a number between 1 and 6)

5. Experimental probability

- Also known as theoretical probability
- This is where the probability of an event can be determined by considering all the possible outcomes (eg. on a dice we would say the probability of getting each individual number is 1/6) without performing any trials
- However sometimes we don't know this (maybe because we know we have a **biased** dice but don't know **how** it is biased) so we can only talk about probabilities once we have done some trials:

$$P\ (A) = \frac{Number\ of\ times\ A\ happens}{Total\ number\ of\ trials\ (n)}$$

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- The more trials that are done, the more the experimental probability will reflect the true probability of the event

6. Number of outcomes

- If we want to **estimate** the number of times an event will happen out of a total of n trails we calculate:

No. of times A happens=n ×P(A)

- You will sometimes see/hear this being called the "**expected** number of times A happens"

7. Mutually exclusive **(OR** means **+)**

- Mutually exclusive events cannot happen at the same time, rolling a 2 on a dice and rolling a 4 on a dice
- This leads to the result that

$$P\ (2)\ \ OR\ P\ (4) = P\ (2) + P\ (4) = \tfrac{1}{6} + \tfrac{1}{6} = \tfrac{2}{6}$$

8. Independent events (**AND** means)

- This is the first situation where we might combine different events from different trials
- For example:
  A is the event "Rolling a 3" and B is the event "Flipping heads"
  These events are **independent** because one does not affect (the probability of) the other
  So to find the probability of both events happening we multiply their individual probabilities together:

$$P\ (A)\ \ AND\ P\ (B) = \tfrac{1}{6} \times \tfrac{1}{2} = \tfrac{1}{12}$$

- Note that this is **NOT** the opposite of mutually exclusive – because both often crop up together it is easy to think they must be linked

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Exam Tip

It is unusual in probability questions that you will be asked to simplify fractions – so don't, in case you mess it up! You could use your calculator to do it automatically but this topic can appear on all papers.
In probability questions, it is usually easiest to use whatever number format the question does. Probabilities can be fractions, decimals or percentages (nothing else!). If no format is indicated in the question then fractions are normally best.

Worked Example

1. Emilia is using a spinner that has outcomes and probabilities as shown in the table below.

Outcome	Blue	Yellow	Green	Red	Purple
Probability		0.2	0.1		0.4

The probability of spinning a Blue is twice the probability of spinning a Red

(a) Complete the probability table.

(b) Emilia spins the spinner twice. Work out the probability she gets a Yellow on the first spin and a Green on the second spin.

(c) Emilia spins the spinner 150 times. Find an estimate for the number of times the spinning lands on Yellow or Purple.

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(a)

If we say $P\ (Blue) = x$ and $P\ (Red) = y$ then

$x = 2y$

Also

$x + 0.2 + 0.1 + y + 0.4 = 1$

$x + y = 1 - 0.7$

$x + y = 0.3$

Then

$3y = 0.3$

$y = 0.1$

So $x = 0.2$

4 – This is a very formal way of solving this problem and you don't need to do it like this

However many mistakes are made by not taking the time to ensure things like "blue is double red"

The key here is all probabilities add up to 1

Outcome	Blue	Yellow	Green	Red	Purple
Probability	0.2	0.2	0.1	0.1	0.4

(b)

$P\ (Yellow)\ AND\ P\ (Green) = 0.2 \times 0.1$

$= 0.02$

8 – "**AND** means $\times$ "

Getting yellow on the first spin does not affect the probability of getting green on the second spin

(c)

$P\ (Yellow)\ OR\ P\ (Purple) = 0.2 + 0.4 = 0.6$

$150 \times 0.6 = 90$

7 – "**OR** means +"

6 – Expected number of outcomes

2. Jake is throwing a biased coin. He throws it 200 times and it lands on heads 145 times.
   (a) Estimate the probability of getting a tails with this coin.
   (b) Comment on the reliability of this estimate.

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(a)

$P\,(Tails) \approx \frac{200-145}{200}$

$= \frac{55}{200}$

5 – **Experimental Probability**

Notice they ask for tails!

No need to simplify but if on calculator it would do it for you ($\frac{11}{40}$)

(b)

As Jake has thrown the coin a large number of times, 200, this is a reliable estimate for the probability of throwing tails with it.

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8.1.2 PROBABILITY - VENN DIAGRAMS

What is a venn diagram?

- **Venn diagrams** allow us to show **two** (or more) characteristics of a situation where there is overlap between the characteristics
- For example, students in a VI Form can study Biology or Chemistry but there may be students who study both

What do I need to know?

- You can be asked to draw a Venn diagram and/or interpret a Venn diagram
- Strictly speaking the rectangle (box) is always **essential** on a Venn diagram as it represents everything that can happen in the situation
- You may see the letters ε or ζ written inside or just outside the box – this means "the set of all possible outcomes" – basically it just means "everything"!
- The words **AND** and **OR** become very important in both drawing and interpreting Venn diagrams
- You will need to be familiar with the symbols ∩ and ∪ – **intersection** and **union**, loosely speaking these mean **AND** and **OR** (respectively)

1. Drawing a Venn Diagram

- You'll need a "box" and overlapping "bubbles" depending on how many characteristics you are dealing with
- You will not normally be given all the values for every section in your diagram
- You will be expected to work out missing information in order to complete your Venn diagram
- Remember to consider **AND** and **OR**

2. Interpretation

- This is where "not A" (A') and similar probability that event A does not happen can get confusing
- The symbols ∩ and ∪ are often used here too

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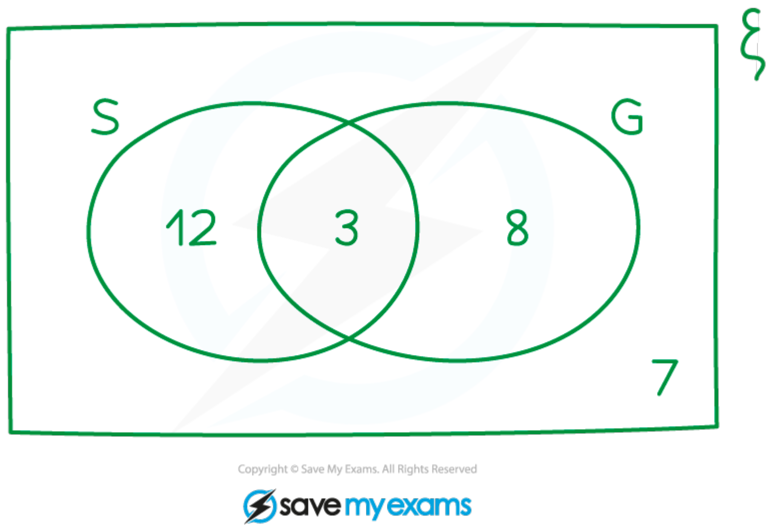
Exam Tip

Lightly highlighting the part of the Venn diagram you need can help but make sure you can still read the whole diagram for later parts of the question if you do this.

Worked Example

1. In a class of 30 students, 15 study Spanish, 3 of whom also study German.
   7 students study neither Spanish nor German.
   (a) Draw a Venn diagram to show this information
   (b) Use your Venn diagram to find the probability that a student, selected at random from the class, studies Spanish but not German.

(a)



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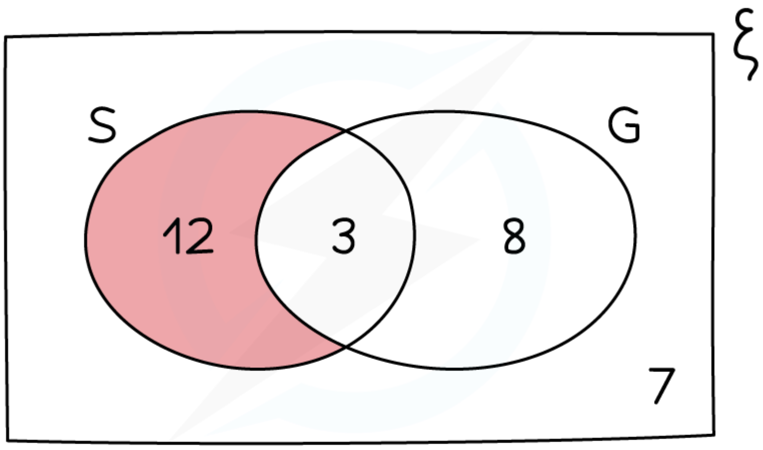
1 – You should start with the 3 in the overlap ("middle") then deduce that the "Spanish only" bubble will have 12 in it. 7 needs to be outside both bubbles but within the box. With a total of 30 required you can now work out how many study "German only" and complete the diagram.

(b)



2 – Highlight the part of the Venn diagram you need ("Spanish only")

Students studying Spanish only = 12

Total number of students = 30

Pick out the numbers you need carefully

P (random student studies S only) = $\frac{12}{30}$

2. Given the Venn diagram below answer the following questions:

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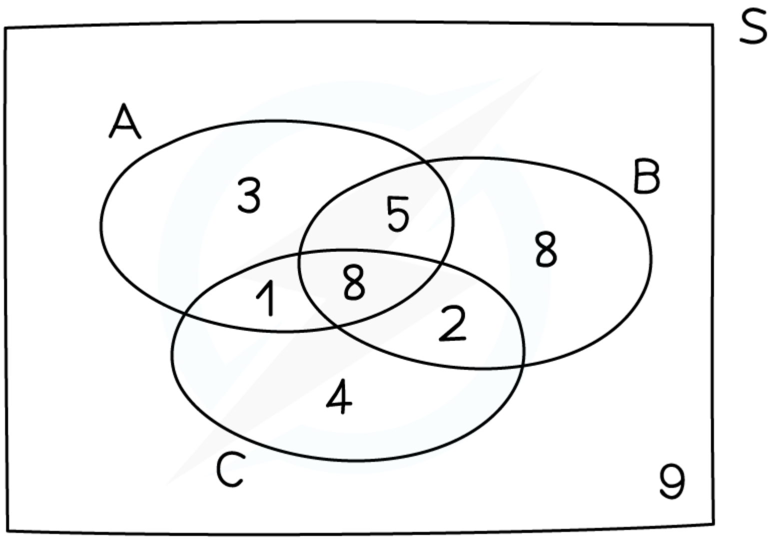

(a) $P(A)$

(b) $P(A \cap B \cap C)$

(c) $P(B' \cap C)$

(d) $P(A \cup B)$

(e) $P(A \cup B \cup C)$

(f) $P(A' \cup B')$

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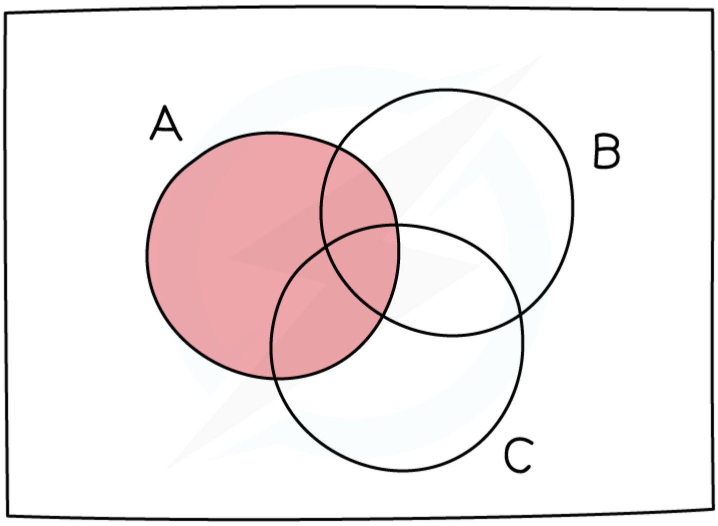
(a)



Draw a quick sketch of the diagram without the details when multiple parts to a question

See these types of questions as "ways to win" – here if you are in "bubble A" you "win", B and C don't come into it at all

$Total\ in\ A = 3+5+1+8 = 17$

$Total = 3+5+1+8+2+4+8+9 = 40$

$P(A) = \frac{17}{40}$

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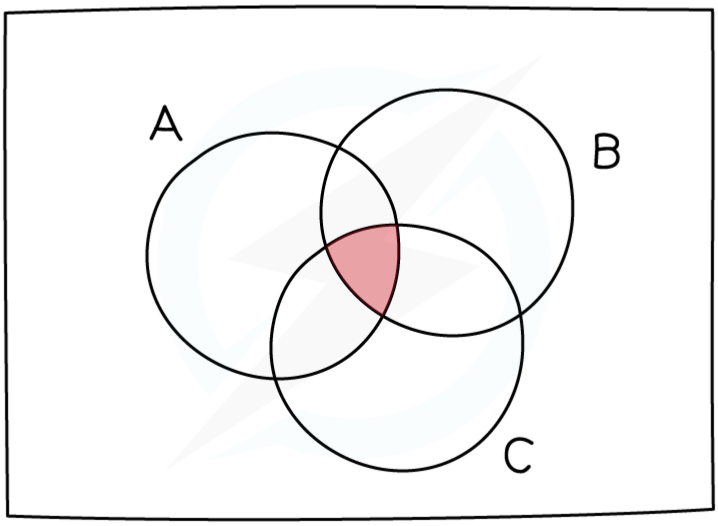
(b)



∩ - intersection – AND – you "win" if "in A" AND "in B" AND "in C"

$Total\ in\ A, B\ and\ C = 8$

$Total = 40$ Worked out in part (a)

$P(A \cap B \cap C) = \frac{8}{40}$

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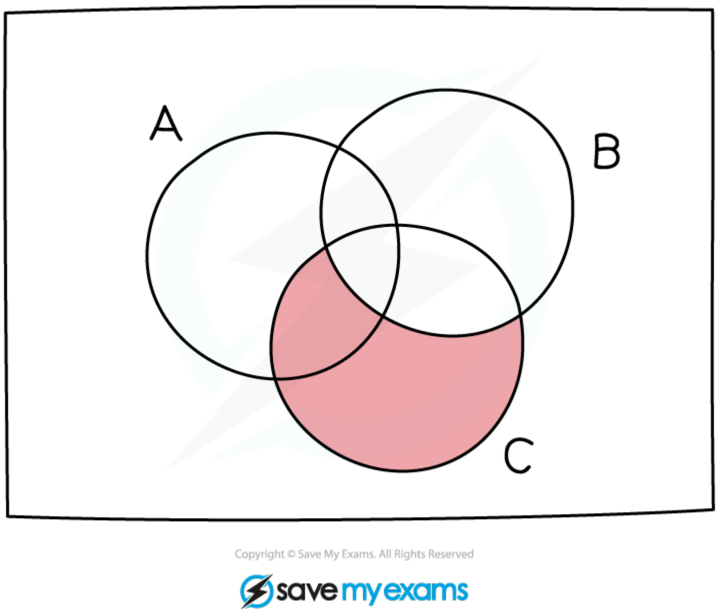
(c)



∩ - intersection – AND – you "win" if "not in B" AND "in C"

$Total\ in\ not\ B\ and\ C = 1 + 4 = 5$

$Total = 40$

$P(B' \cap C) = \frac{5}{40}$

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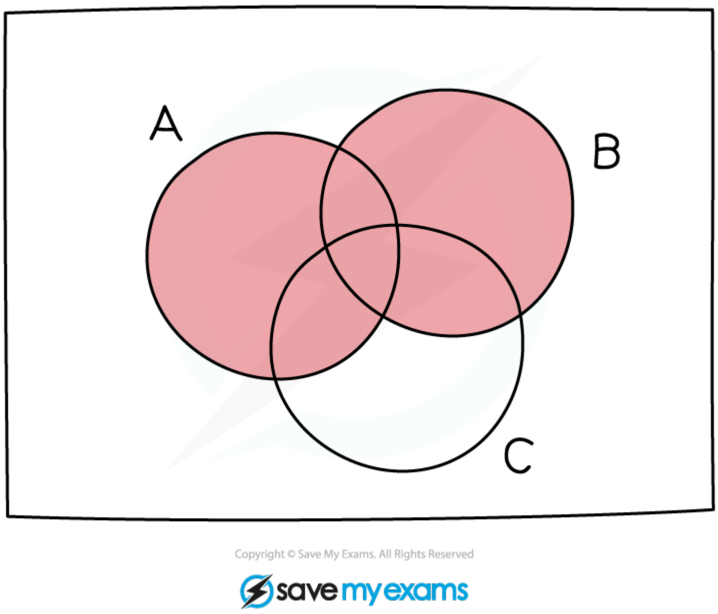
(d)



∪ - union – OR – you "win" if "in A" OR "in B"

$Total\ in\ A\ or\ B = 3 + 1 + 8 + 5 + 2 + 8 = 27$

$Total = 40$

$P(A \cup B) = \frac{27}{40}$

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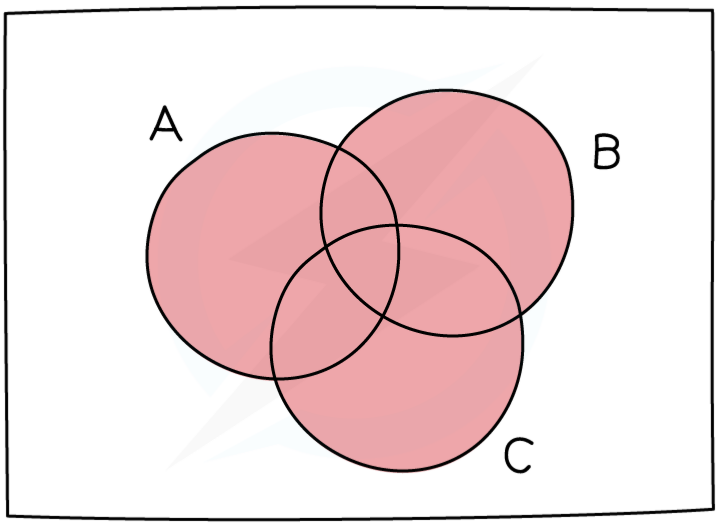
(e)



∪ - union – OR – you "win" if "in A" OR "in B" OR "in C"

$Total\ in\ A, B\ or\ C = 40 - 9 = 31$ Easy to subtract from whole in this case

$Total = 40$

$P(A \cup B \cup C) = \frac{31}{40}$

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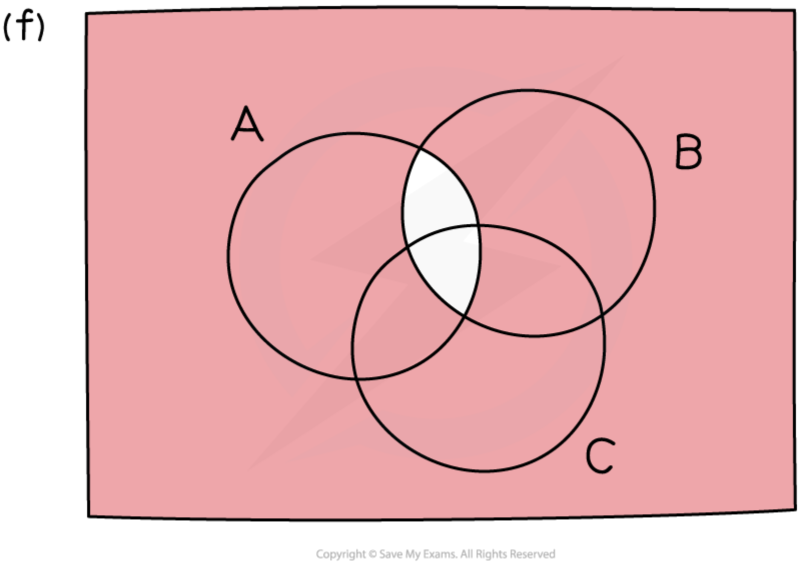
(f)



∪ - union – OR – you "win" if "not in A" OR "not in B"

This one is particular difficult to see without a friend!

$Total\ in\ A'\ OR\ B' = 40 - 8 - 5 = 27$ Easier to subtract again

$Total = 40$

$P(A' \cup B') = \frac{27}{40}$

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8.1.3 PROBABILITY - TWO WAY TABLES

What are two-way tables?

- While **Venn diagrams** are great at showing **overlap** they can only show one feature (**characteristic**) of a situation at one time
- In the notes on Venn diagrams we had an example talking about students studying either Spanish or German, or both
  However we may **also** be interested in how many **boys** and **girls** were studying Spanish and/or German as well
- This is where we need a two-way table – one of the **characteristics** will be the columns and the other will be presented in rows
- Once we have our table we can use the numbers within to determine probabilities

What do I need to know?

- You'll need to be able to construct a two-way table from information given in words and then use a table to calculate probabilities
- So you'll need to be familiar with the basics and notation around probability

1. Total row/column

- It may not be obvious from the wording but a total row and column can be really helpful in two-way table questions
- If they're not mentioned, or included when given a table, add them in

2. Completing a table

- It may not be possible to add numbers to the table from every sentence, one at a time
- You will usually have to combine one piece of information with another in order to fully complete a table

3. Conditional probability

- Two-way tables in particular give rise to using **conditional probability**
- This can get complicated but with two-way tables, it is usually straightforward to see which parts of the table the question is referring to

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Worked Example

At an art group children are allowed to choose between four activities:

'Colouring', 'Painting', 'Clay Modelling' and 'Sketching'.

There is a total of 60 children attending the art group.

12 of the boys chose the activity 'Colouring'.

A total of 20 children chose 'Painting' and a total of 15 chose 'Clay Modelling'.

13 girls chose to do 'Clay Modelling'.

8 of the 30 boys chose 'Sketching', and did 4 of the girls.

(a) Construct a two-way table to show this information.

(b) Find the probability that:

(i) a randomly selected child chose 'Colouring',

(ii) a randomly selected child is a boy who chose 'Sketching',

(iii) a randomly selected child is a boy, given that they chose 'Painting',

(iv) a randomly selected child chose 'Clay Modelling', given that they're a girl

(a)

	Colouring	Painting	Clay Modelling	Sketching	Total
Boys	12	8	2	8	30
Girls	1	12	13	4	30
Total	13	20	15	12	60

1, 2 - Construct the table carefully, including total row and column

The values highlighted you should've been able to complete from the information given in the question

Work your way round the table, not necessarily in order – if you get to a stage where you can't complete a value then you would've missed something from the question so go back and have another look

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(b)

(i) $P(Colouring) = \frac{13}{60}$

Total colouring ÷ Total Children

(ii) $P(Boy\ AND\ Sketching) = \frac{8}{60}$

Boy & Sketching ÷ Total Children

(iii) $P(Boy\ GIVEN\ THAT\ Painting) = \frac{8}{20}$

3 – Conditional probability

Boy & Painting ÷ Total Painting

(iv) $P(Clay\ Modelling\ GIVEN\ THAT\ Girl) = \frac{13}{30}$

3 – Conditional probability

Girl & Clay Modelling ÷ Total Girls

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8.1.4 COMBINED PROBABILITY - BASICS

What do we mean by combined probabilities?

- This can mean lots of things as you'll see over these notes and the next set
- In general it means we have more than one 'thing' (trial/event) to bear in mind and these things may be **independent**, **mutually exclusive** or may involve an event that follows on from a previous event drawing a second counter from a bag

1. Tree diagrams

- Especially useful when we have more than one trial but are only concerned with two outcomes from each
- Even more useful when probabilities change for the second experiment

2. Replacement

- Are items being selected at random replaced or not?
- If not then numbers will decrease as the situation progresses and so probabilities change – this is often called conditional probability

3. AND's and **OR**'s

- **AND** means for **independent** events
- **OR** means for **mutually exclusive** events

4. Sum of all probabilities is 1

- This is a very basic fact that gets lost along the way in more complicated probability questions – but it is one of the best 'tricks' you can use!
- A good example of its use is when you want the probability of something being "non zero": $P(x \geq 1) = 1 - P(x = 0)$

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Worked Example

1. A box contains 3 blue counters and 8 red counters.

A counter is taken at random and **not** replaced.

Work out the probability:

(a) Both counters are red,

(b) There is one of each colour,

(c) Both are the same colour

Do note, as far as the maths is concerned, taking a counter out at random, and not replacing it, is the same as taking two counters out at the same time.

(a)

$P(1^{st}\ is\ red) = \frac{8}{11}$ There's 8 reds to start with, 8 + 3 = 11 in total

$P(2^{nd}\ is\ red) = \frac{7}{10}$ 2 – not replaced so **reds** and **total** decrease by 1

$P(both\ red) = \frac{8}{11} \times \frac{7}{10}$ 3 – a hidden AND – red AND red

$= \frac{56}{110}$ When values get big it may be wise to simplify, particularly if you're likely to use them later

(b)

$P(red/blue\ OR\ blue/red) = P(red\ AND\ blue)\ OR\ P(blue\ AND\ red)$

3 – a hidden AND **and** OR question!

$= \frac{8}{11} \times \frac{3}{10} + \frac{3}{11} \times \frac{8}{10}$

1 – you may prefer a tree diagram here

$= \frac{24}{110} + \frac{24}{110}$

$= \frac{48}{110}$

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(c)

$$P(\text{both blue } OR \text{ both red}) = P(\text{blue } AND \text{ blue}) \; OR \; P(\text{red } AND \text{ red})$$

3 – a hidden AND **and** OR question!

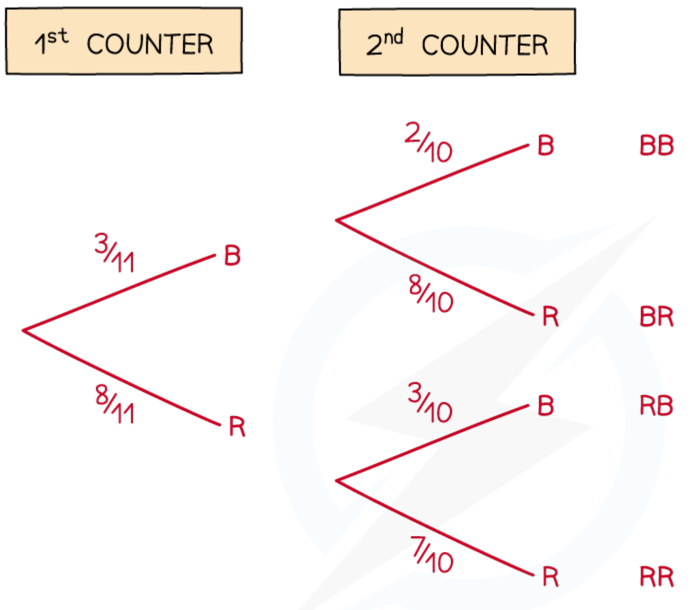
$$= \frac{3}{11} \times \frac{2}{10} + \frac{8}{11} \times \frac{7}{10}$$

1 – you may prefer a tree diagram here

$$= \frac{6}{110} + \frac{56}{110}$$

$$= \frac{62}{110}$$

Note that you could draw a tree diagram to help with this question – particularly parts (b) and (c). The diagram below is what we would describe as complete, but you don't need to go that far in the exam – draw as much or as little of the tree diagram as you need to help you understand and see your way through the problem



$\frac{3}{11} \times \frac{2}{10} = \frac{6}{110}$ †

$\frac{3}{11} \times \frac{8}{10} = \frac{24}{110}$ *

$\frac{8}{11} \times \frac{3}{10} = \frac{24}{110}$ *

$\frac{8}{11} \times \frac{7}{10} = \frac{56}{110}$ †

* (b) ONE OF EACH COLOUR: $\frac{24}{110} + \frac{24}{110} = \frac{48}{110}$

† (c) BOTH SAME COLOUR: $\frac{6}{110} + \frac{56}{110} = \frac{62}{110}$

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2. The probability of winning a fairground game is known to be 26%.

If the game is played 4 times what is the probability that there is **at least one** win.

Write down one assumption you have made.

$P(at\ least\ one\ win) = 1 - P(zero\ wins)$ 4 – Use $P(lose) = 1 - P(win)$

$= 1 - (0.74)^4$ $1 - 0.26 = 0.74$

3 – 'Loose' AND 'Lose' AND 'Lose' AND 'Lose'

$= 0.7001$ **(4 decimal places)**

I have made the assumption that each attempt at the fairground game is independent – that the outcome of one game does not affect the outcome of the next.

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8.1.5 COMBINED PROBABILITY - HARDER

What do I need to know?

- Be aware of the basics, ie. tree diagrams, replacement, AND and OR, the sum of all probabilities is 1 (see Combined Probability – Basics)
- Note that there may be some problem-solving and algebra involved (see question 2 of the Worked Example)

Worked Example

1. José passes two sets of traffic lights on his commute to work.

The probability of him being stopped at the first set of lights is 0.3.

If he **is stopped** at the first set of lights the probability of him being stopped at the second set is 0.6.

If he **is not stopped** at the first set, the probability of José being stopped at the second set is 0.2.

Find the probability that José is stopped at **exactly** one set of traffic lights.

A complicated question with lots of words – so use a tree diagram!

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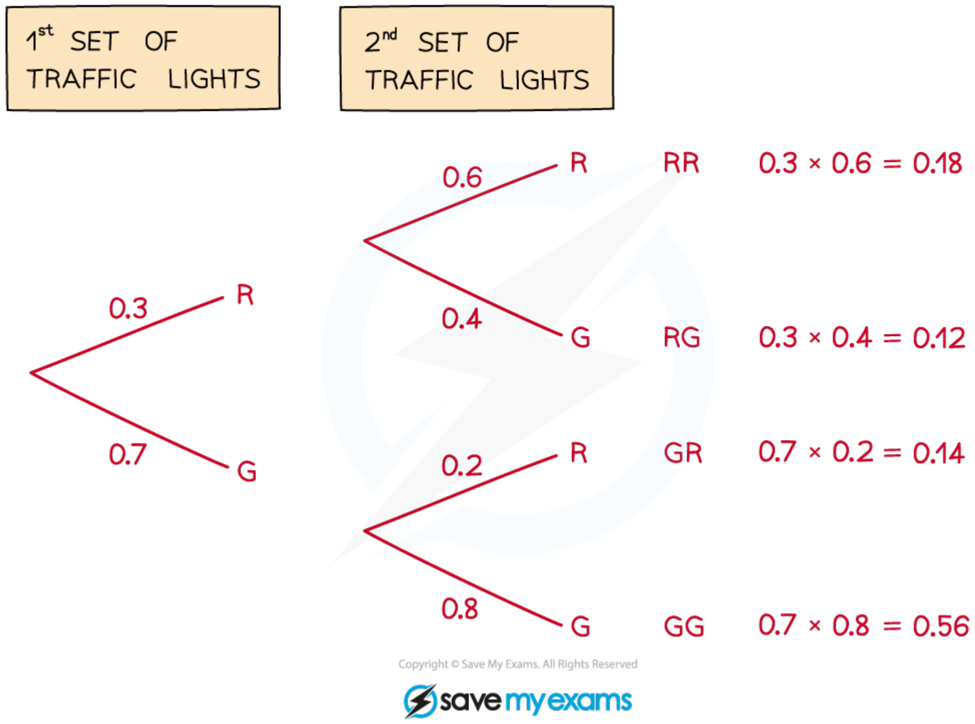

We've used R and G for red and green on traffic lights (we can ignore amber for the purposes of this question!) but you can use any system you like but it's usually easiest to stick to letters (An alternative could be S for stopped, N for not stopped.)

$P(1^{st}\ set\ is\ green) = 1 - 0.3 = 0.7$ 4 – The good old "1 –" trick!

$P\left(2^{nd} set\ is\ green\ given\ 1^{st}\ is\ red\right) = 1 - 0.6 = 0.4$

$P\left(2^{nd} set\ is\ green\ given\ 1^{st}\ is\ green\right) = 1 - 0.2 = 0.8$

Now we have all the probabilities to complete the diagram

$P(stopped\ at\ exactly\ one\ set\ of\ lights) = P(R\ AND\ G)\ OR\ P(G\ AND\ R)$

Make sure those ANDs and ORs are the right way round by using words if need be

$= 0.3 \times 0.4 + \ 0.7 \times 0.2$

If you've written these on your diagram (like above) there is no need to write them again

$= 0.12 + \ 0.14$

$= \mathbf{0.26}$

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2. A bag contains 7 red counters and b blue counters.

Two counters are taken at random from the bag.

The probability they are both red is $\frac{7}{40}$.

(a) Show that $b^2 + 13b - 198 = 0$

(b) Find b and hence find the probability that the two counters are of a different colour.

(a)

$P(R\ AND\ R) = P(R) \times P(R) = \frac{7}{40}$

You can use a combination of words and letters to help you understand the problem

So, $\frac{7}{b+7} \times \frac{6}{b+6} = \frac{7}{40}$

There are b+7 counters to start with,

$\frac{42}{(b+7)(b+6)} = \frac{7}{40}$

this decreases by one once the first counter

$\frac{42 \times 40}{7} = (b+7)(b+6)$

is selected so there are b+6 counters when

$240 = b^2 + 13b + 42$

the second one is selected

$b^2 + 13b - 198 = 0$

Lots of algebra here but should be straightforward once you get started

(b)

$(b+22)(b-9) = 0$

Solve the quadratic, use the formula if you cannot do the factorising

$\cancel{b = -22}$ or $b = 9$

We reject b = -22 as we cannot have a negative number of counters

$b = 9$

$P(one\ of\ each\ colour) = P(R\ AND\ B)\ OR\ P(B\ AND\ R)$

As you do more of these you will get quicker at them – you could always use a tree diagram here

$= \frac{7}{16} \times \frac{9}{15} + \frac{9}{16} \times \frac{7}{15}$

$= \frac{126}{240}$

If you do this on your calculator you will automatically get the rounded answer of $\frac{21}{40}$

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8.1.6 CONDITIONAL PROBABILITY

What is conditional probability?

- **Conditional probability** refers to situations where the **probability** of an **event changes** or is **dependent** on **other events** having already **happened**
- For example, names drawn from a hat, **without** replacement, containing 10 different names
  - The first name drawn out of the hat has a 1/10 of being a particular name
  - The second name drawn would have a 1/9 chance of being a particular name (or probability zero if the particular name was first out)
  - The probability has changed depending on what has happened already
- Make sure you understand the basics of probability before continuing

e.g. TWO DICE ARE ROLLED AND THEIR SCORES ADDED TOGETHER.
GIVEN THAT THE TOTAL SCORE WAS 8, FIND THE PROBABILITY THAT ONE OF THE DICE LANDED ON 3.

- There are formal ways of writing condition probability but this is not necessary for IGCSE
- **Conditional probability** questions can occur from **Venn diagrams**, **tree diagrams** and **tables**
- Some questions may be given in words only, but it is usually easier to understand what is happening by drawing one of these three diagrams

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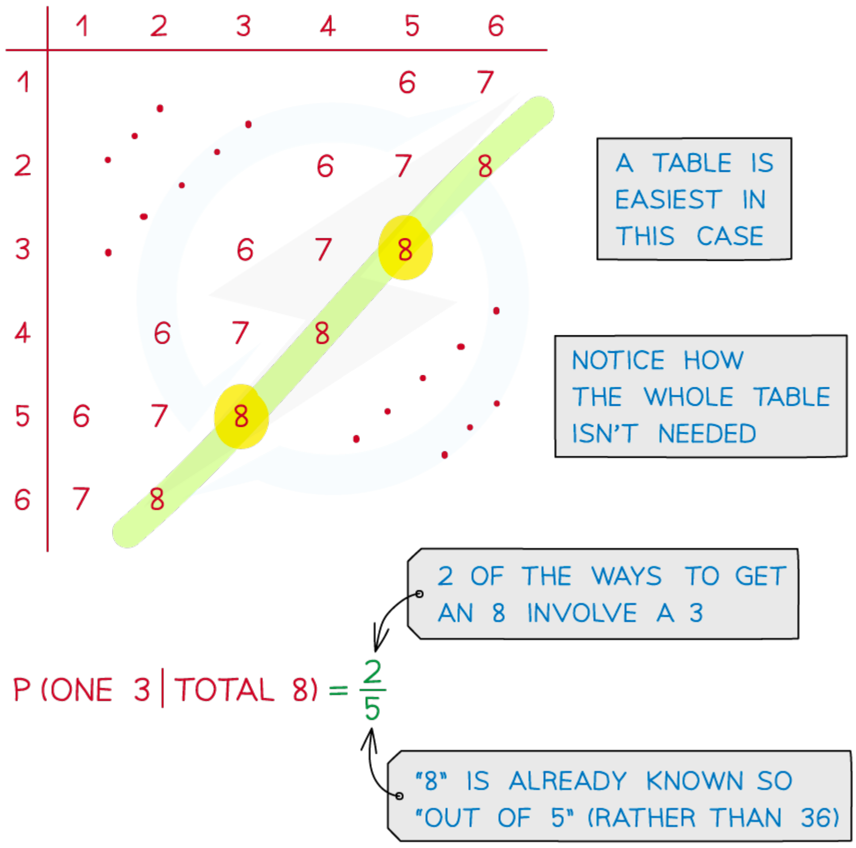
e.g. TWO DICE ARE ROLLED AND THEIR SCORES ADDED TOGETHER.
GIVEN THAT THE TOTAL SCORE WAS 8, FIND THE PROBABILITY THAT ONE OF THE DICE LANDED ON 3.



- Conditional probability questions are often called "given that" questions
  - eg Find the probability of it raining today given that it rained yesterday
- The phrase "given that" is not always used in questions
  - You will need to look out for alternative phrases and interpret them

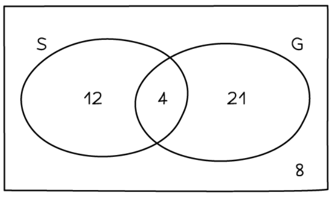
8. Probability

YOUR NOTES

Venn diagrams and conditional probability

- Ensure you are familiar with Venn Diagrams

e.g. THE VENN DIAGRAM BELOW SHOWS THE NUMBER OF STUDENTS AT A COLLEGE STUDYING SPANISH (S) AND GERMAN (G)



A STUDENT IS CHOSEN AT RANDOM.

FIND:

a) THE PROBABILITY THEY STUDY SPANISH BUT NOT GERMAN

b) THE PROBABILITY THEY STUDY NEITHER SPANISH NOR GERMAN

c) THE PROBABILITY THEY STUDY SPANISH, GIVEN THAT THEY STUDY GERMAN

d) THE PROBABILITY THEY STUDY GERMAN, GIVEN THAT THEY STUDY SPANISH

a) $\frac{12}{45}$ — $\frac{12}{12+4+21+8}$ BASIC PROBABILITY

b) $\frac{8}{45}$ — THE VALUE 8 IS OUTSIDE BOTH "BUBBLES"

c) $\frac{4}{25}$ — 4 OF THOSE STUDYING GERMAN, ALSO STUDY SPANISH

"GIVE THAT" IN THE QUESTION TELLS US IT IS ONLY "OUT OF" THOSE STUDYING GERMAN, $4+21$

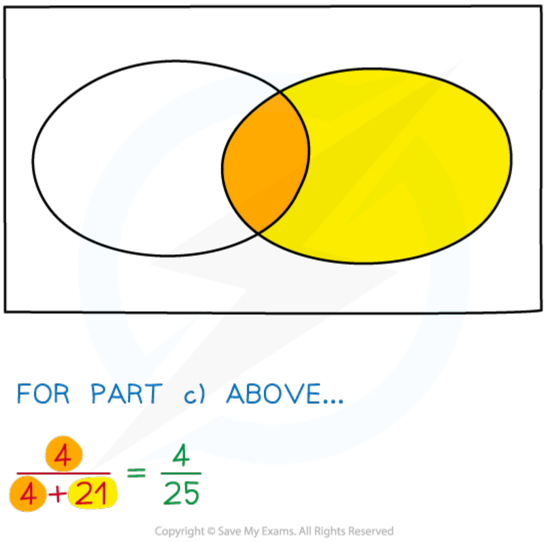
d) $\frac{4}{16}$ — 4 OF THOSE STUDYING SPANISH, ALSO STUDY GERMAN

"GIVE THAT" IN THE QUESTION TELLS US IT IS ONLY "OUT OF" THOSE STUDYING SPANISH, $12+4$

8. Probability

YOUR NOTES

- Shading can help with Venn diagrams
    - Draw a “mini-version” of the Venn diagram rather than use the original as you may need that later

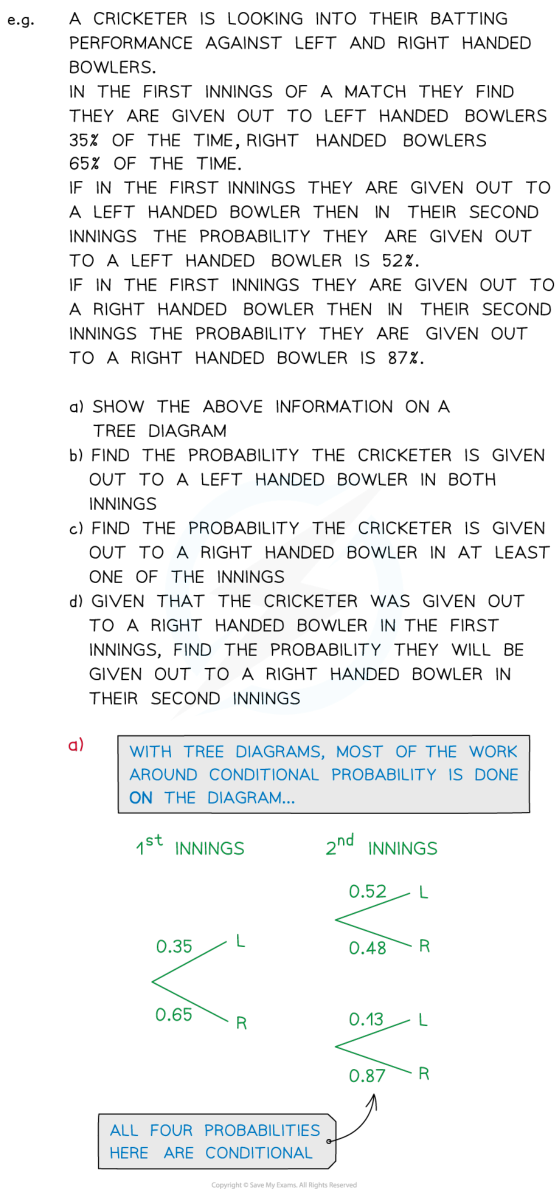


8. Probability

YOUR NOTES
⬇

Tree diagrams and conditional probability

- Ensure you are familiar with Tree Diagrams



8. Probability

YOUR NOTES
⬇

b) $P(LL) = 0.35 \times 0.52$

$= 0.182 \qquad (18.2\%)$

c) $P(LR)$ OR $P(RL)$ OR $P(RR)$

WHICH IS THE SAME AS

$1 - P(LL) = 1 - 0.182$

$= 0.818 \qquad (81.8\%)$

d) $P(R|R) = 0.87 \qquad (87\%)$

MEANS "GIVEN THAT"

THE "WORK" HERE HAD ALREADY BEEN DONE ON THE DIAGRAM SO ANSWER CAN BE WRITTEN STRAIGHT DOWN

- With tree diagrams the "conditional" probabilities are on the diagram
    - Look at the second set of branches
    - These are "conditional" on what happened in the first branch

8. Probability

YOUR NOTES
⬇

Tables and conditional probability

- Ensure you are familiar with Two-way Tables

e.g. A SURVEY IS TAKEN OF THE TYPE AND FUEL TYPE OF VEHICLES IN A CAR PARK.
SOME OF THE RESULTS ARE SHOWN IN THE TABLE BELOW.

	Compact	Mid-size	Large	Sport	Other	Total
Petrol	36		8		4	84
Diesel	31	39		4	3	89
Hybrid		5	0	0	0	
Electric	7		0	0	0	9
Other	1	0	0	1	0	2
Total	86		20	11	7	200

a) COMPLETE THE TABLE.

A VEHICLE FROM THE SURVEY IS SELECTED AT RANDOM.

b) FIND THE PROBABILITY THE VEHICLE IS LARGE.

c) GIVEN THAT THE SELECTED VEHICLE IS DIESEL POWERED, FIND THE PROBABILITY THAT IT IS COMPACT

d) GIVEN THAT THE SELECTED VEHICLE IS MID-SIZED, FIND THE PROBABILITY IT IS A HYBRID

e) THE SELECTED VEHICLE IS A PETROL POWERED CAR. WHAT IS THE PROBABILITY IT IS NOT A LARGE CAR.

a)

	Compact	Mid-size	Large	Sport	Other	Total
Petrol	36	30	8	6	4	84
Diesel	31	39	12	4	3	89
Hybrid	11	5	0	0	0	16
Electric	7	2	0	0	0	9
Other	1	0	0	1	0	2
Total	86	76	20	11	7	200

b) $P(\text{LARGE}) = \frac{20}{200} = \frac{1}{10}$

20 LARGE CARS "OUT OF" 200 IN TOTAL

c) $P(\text{COMPACT} \mid \text{DIESEL}) = \frac{31}{89}$

31 OF THE 89 DIESEL CARS ARE COMPACT

d) $P(\text{HYBRID} \mid \text{MID-SIZE}) = \frac{5}{76}$

5 OUT OF THE 76 MID-SIZED CARS ARE HYBRIDS

e) $P(\text{NOT LARGE} \mid \text{PETROL}) = \frac{76}{84}$

76 OUT OF THE 84 PETROL CARS ARE NOT LARGE

8. Probability

YOUR NOTES
⬇

Exam Tip

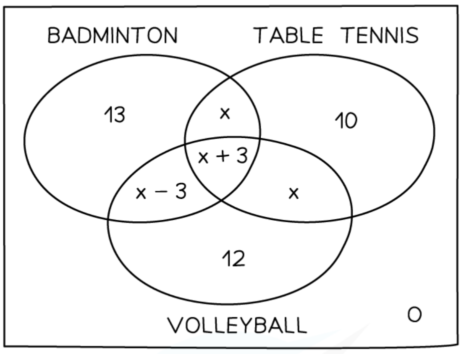
Take time and care with diagrams for probability as many of the calculations involved rely on these being correct.
In general use whatever the question does for probabilities – decimals, fractions or percentages. The only exception to this is that it can often be easier to change percentages to decimals – especially if multiplication is required.

Worked Example

8. Probability

YOUR NOTES
⬇

The Venn diagram below shows how 55 members of a sports club chose different sports to play one day.



(a) Show that $x = 5$.

One of the 55 members is chosen at random.

(b) Find the probability the selected member did not choose volleyball.

(c) Find the probability the selected member chose badminton, given that they had already chosen table tennis.

(d) Find the probability the selected member chose volleyball, given that they had already chosen the other two sports.

(e) Find the probability the selected member chose badminton given that they only chose one sport.

8. Probability

YOUR NOTES
⬇

a) $13 + 10 + 12 + x + x + 3 + x - 3 + x + 0 = 55$

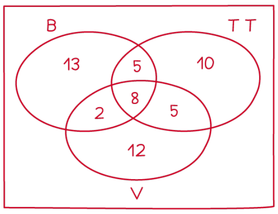
TOTAL MEMBERS

$4x + 35 = 55$

$4x = 20$

$x = 5$

VENN DIAGRAM NOW LOOKS LIKE



ADD TO THE ORIGINAL DIAGRAM ON THE EXAM PAPER

"TOTAL" – "VOLLEYBALL"

b) $P(\text{NOT VOLLEYBALL}) = \dfrac{55 - (2 + 8 + 5 + 12)}{55}$

$= \dfrac{28}{55}$

BOTH BADMINTON & TABLE TENNIS

c) $P(\text{BADMINTON} \mid \text{TABLE TENNIS}) = \dfrac{5 + 8}{5 + 8 + 5 + 10}$

BADMINTON "GIVEN THAT" THEY'VE CHOSEN TABLE TENNIS

$= \dfrac{13}{28}$

"OUT OF" TABLE TENNIS

8 CHOSE ALL 3

d) $P(\text{VOLLEYBALL} \mid \text{OTHER TWO}) = \dfrac{8}{8 + 5}$

8 + 5 CHOSE BADMINTON & TABLE TENNIS

$= \dfrac{8}{13}$

13 BADMINTON ONLY

e) $P(\text{BADMINTON} \mid \text{ONE SPORT}) = \dfrac{13}{13 + 10 + 12}$

THIS QUESTION SHOWS YOU'RE NOT ALWAYS LOOKING FOR AN OVERLAP!

$= \dfrac{13}{35}$

13 + 10 + 12 CHOSE ONLY ONE SPORT