

# Graphs

## Difficulty: Medium

### Question Paper 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Medium
Booklet	Question Paper 1

**Time allowed:** 128 minutes

**Score:** /111

**Percentage:** /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

##### CIE IGCSE Maths (0980) *Assembled by AS*

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

$$f(x) = x^3 - 4x^2 + 15$$

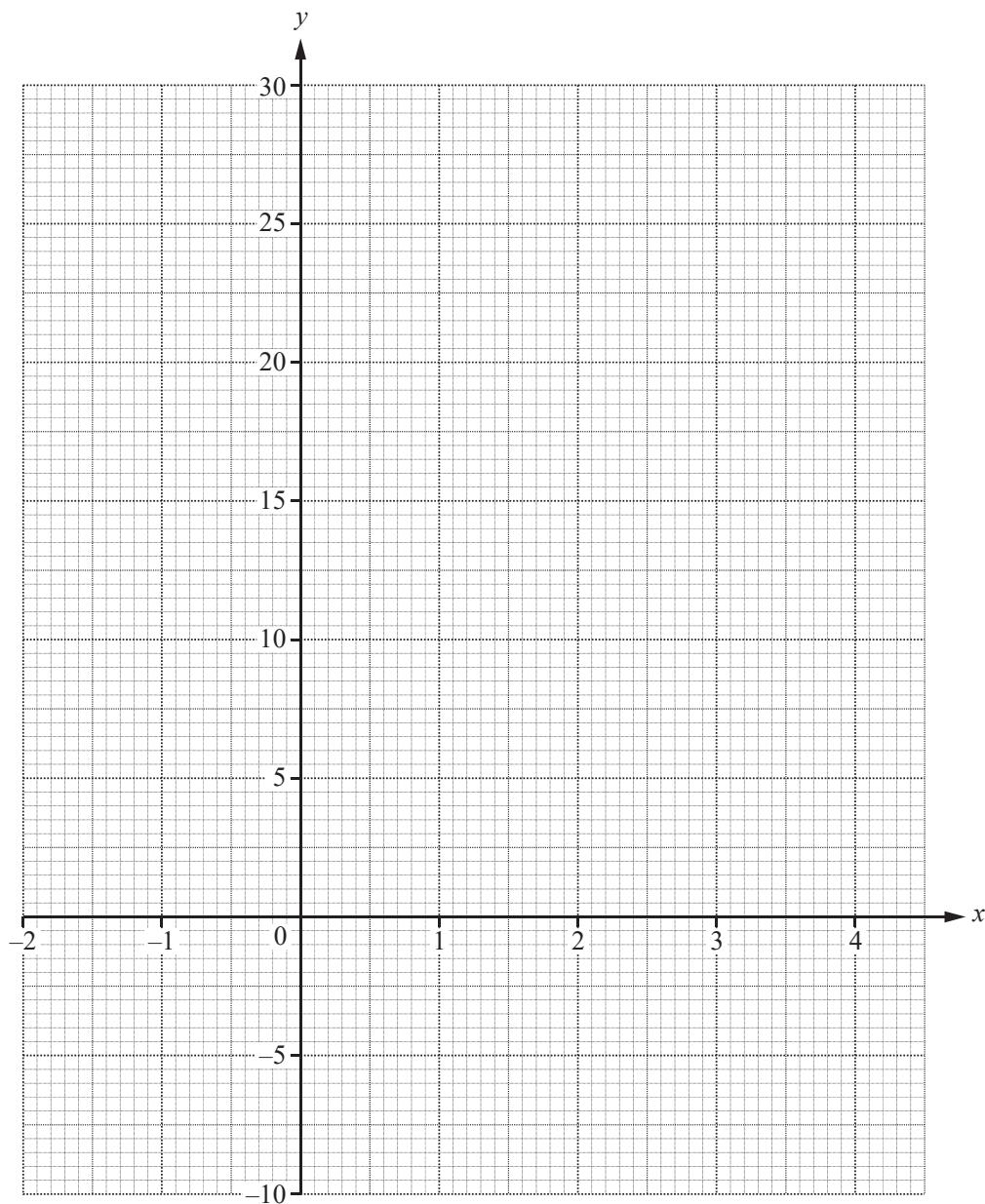
(a) Complete the table of values for  $y = f(x)$ .

[2]

$x$	-2	-1	-0.5	0	1	2	2.5	3	3.5	4	4.5
$y$	-9		13.9	15	12		5.6	6	8.9	15	25.1

(b) On the grid, draw the graph of  $y = f(x)$  for  $-2 \leq x \leq 4.5$ .

[4]



- (c) Use your graph to solve the equation  $f(x) = 0$ . [1]
- (d) By drawing a suitable tangent, estimate the gradient of the graph of  $y = f(x)$  when  $x = 3.5$ . [3]
- (e) By drawing a suitable straight line on the grid, solve the equation  $x^3 - 4x^2 - 2x + 5 = 0$ . [4]

## Question 2

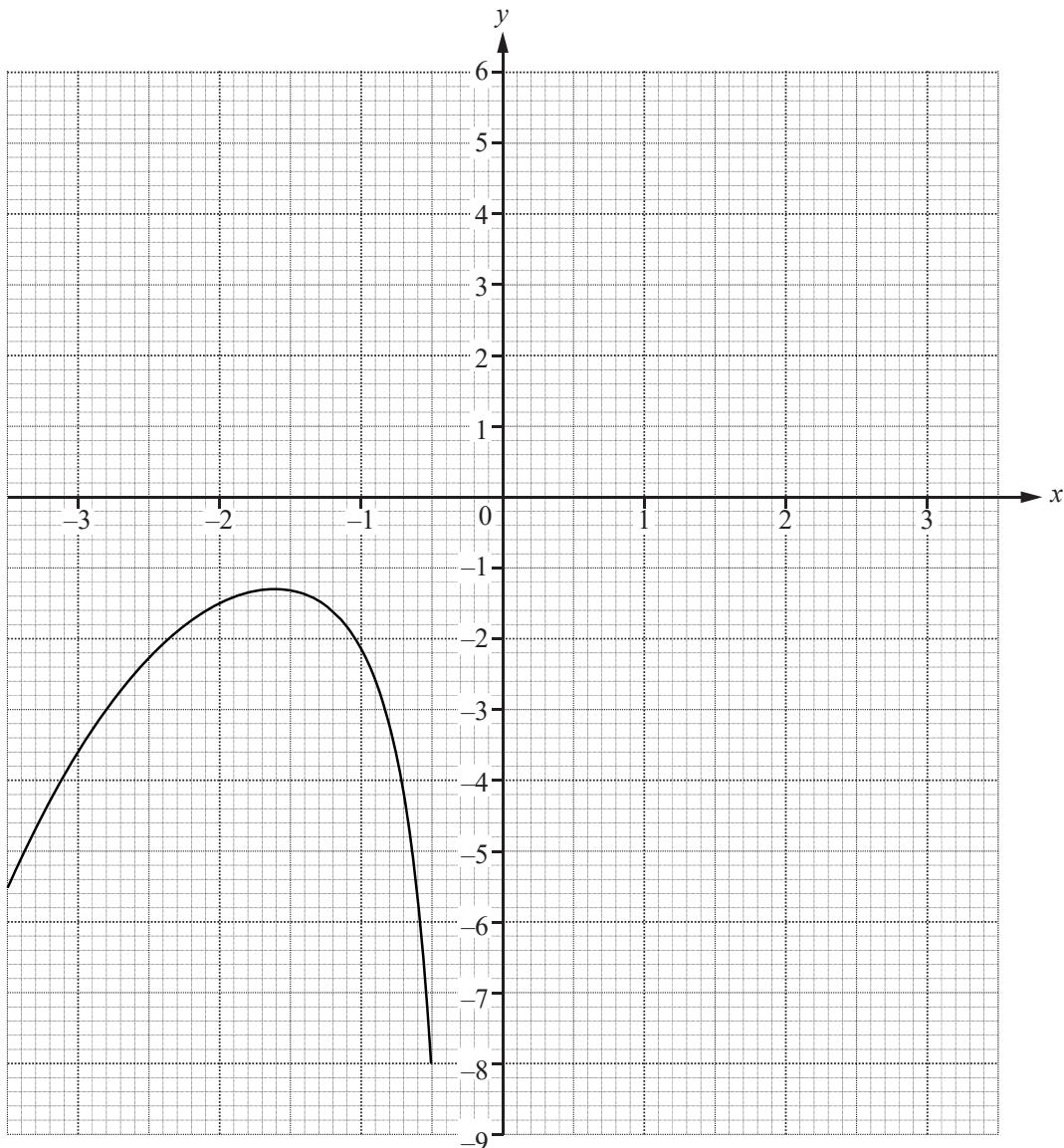
$$y = \frac{x^3}{8} - \frac{2}{x^2}, \quad x \neq 0$$

(a) Complete the table of values.

$x$	0.5	1	1.5	2	2.5	3	3.5
$y$	-8.0	-1.9	-0.5	0.5	1.6		

[2]

(b)



The graph of  $y = \frac{x^3}{8} - \frac{2}{x^2}$  for  $-3.5 \leq x \leq -0.5$  has already been drawn.

On the grid, draw the graph of  $y = \frac{x^3}{8} - \frac{2}{x^2}$  for  $0.5 \leq x \leq 3.5$ .

[4]

(c) Use your graph to solve the equation  $\frac{x^3}{8} - \frac{2}{x^2} = 0$ . [1]

(d)  $\frac{x^3}{8} - \frac{2}{x^2} = k$  and  $k$  is an integer.

Write down a value of  $k$  when the equation  $\frac{x^3}{8} - \frac{2}{x^2} = k$  has

(i) one answer, [1]

(ii) three answers. [1]

(e) By drawing a suitable tangent, estimate the gradient of the curve where  $x = -3$ . [3]

(f) (i) By drawing a suitable line on the grid, find  $x$  when  $\frac{x^3}{8} - \frac{2}{x^2} = 6 - x$ . [3]

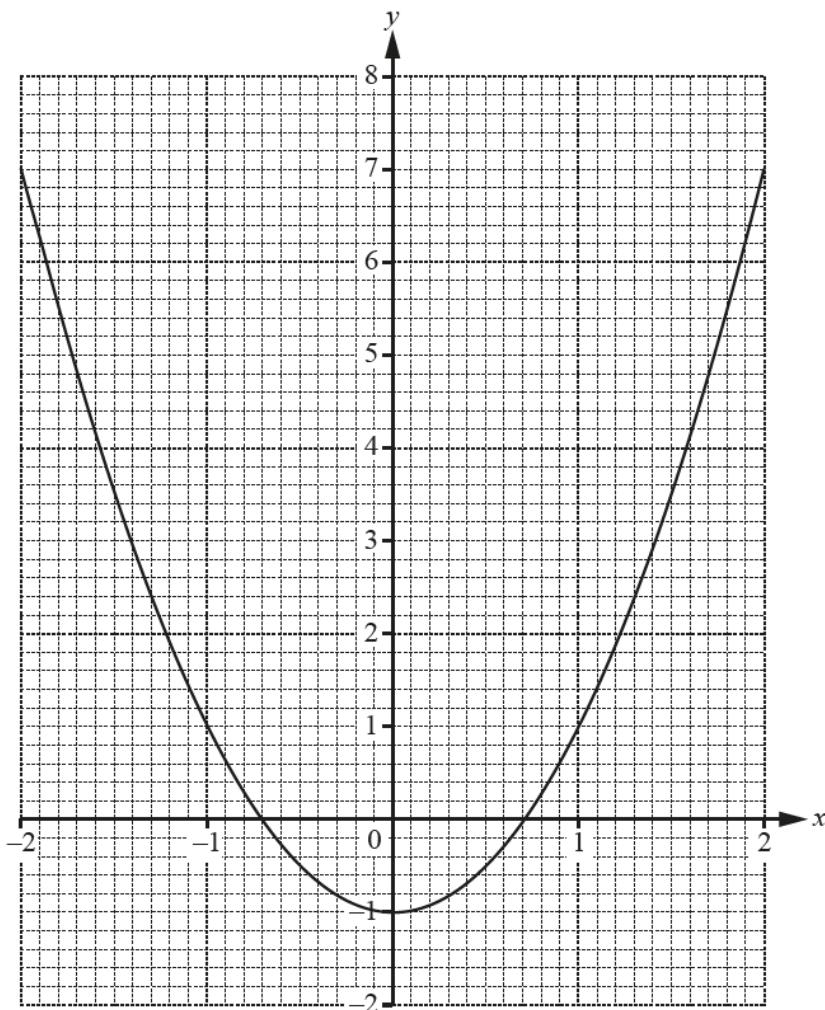
(ii) The equation  $\frac{x^3}{8} - \frac{2}{x^2} = 6 - x$  can be written as  $x^5 + ax^3 + bx^2 + c = 0$ .

Find the values of  $a$ ,  $b$  and  $c$ . [4]

### Question 3

$$f(x) = 2x^2 - 1$$

The graph of  $y = f(x)$ , for  $-2 \leq x \leq 2$ , is drawn on the grid.



- (a) Use the graph to solve the equation  $f(x) = 5$ . [2]
- (b) (i) Draw the tangent to the graph of  $y = f(x)$  at the point  $(-1.5, 3.5)$ . [1]
- (ii) Use your tangent to estimate the gradient of  $y = f(x)$  when  $x = -1.5$ . [2]

(c)  $g(x) = 2^x$

- (i) Complete the table for  $y = g(x)$ .

$x$	- 2	- 1	0	1	2
$y$	0.25	0.5		2	4

[1]

- (ii) On the grid opposite, draw the graph of  $y = g(x)$  for  $-2 \leq x \leq 2$ .

[3]

- (d) Use your graphs to solve

- (i) the equation  $f(x) = g(x)$ ,

[2]

- (ii) the inequality  $f(x) < g(x)$ .

[1]

- (e) (i) Write down the three values.

$g(-3) = .$

$g(-5) = .$

$g(-10) = .$

[1]

- (ii) Complete the statement.

As  $x$  decreases,  $g(x)$  approaches the value .....

[1]

**Question 4**

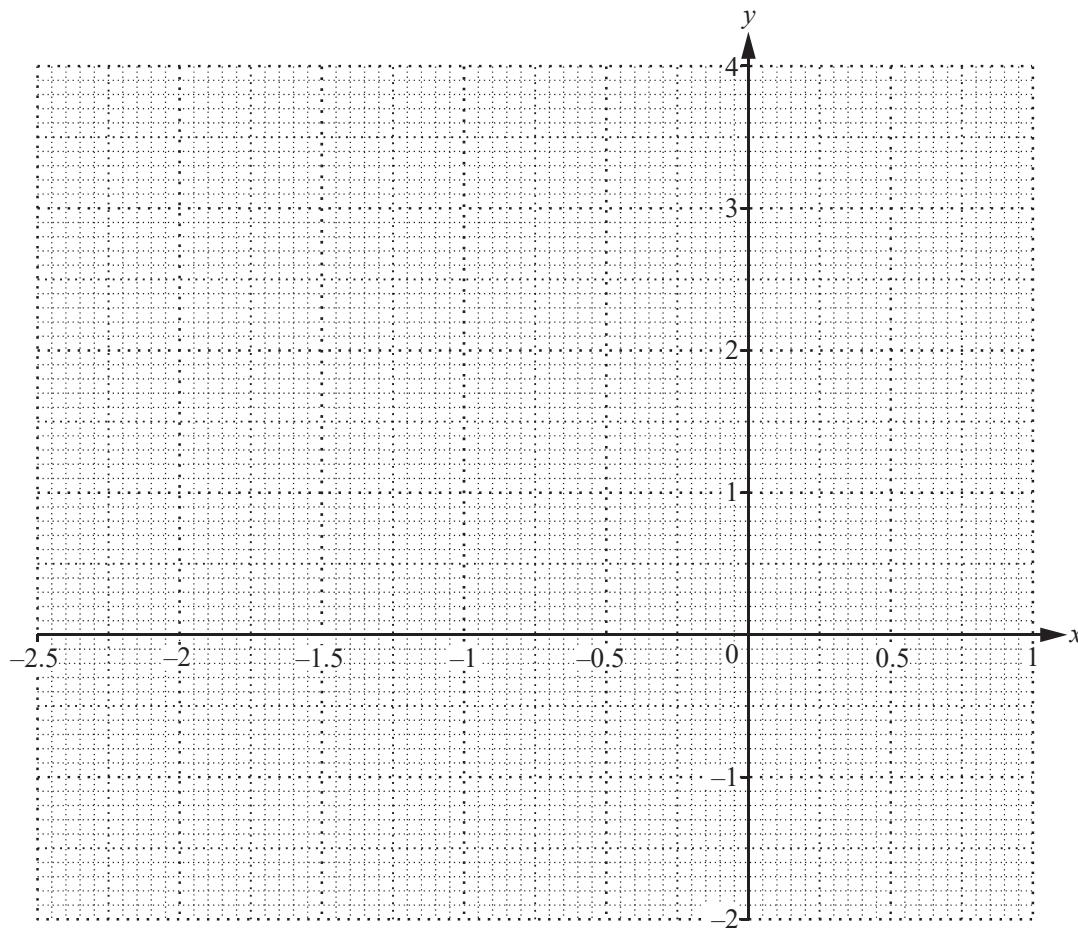
The table shows some values for  $y = 2x^3 + 4x^2$ .

$x$	-2.2	-2	-1.5	-1	-0.5	0	0.5	0.8
$y$	-1.94				0.75	0		3.58

(a) Complete the table.

[4]

(b) Draw the graph of  $y = 2x^3 + 4x^2$  for  $-2.2 \leq x \leq 0.8$ .



[4]

(c) Find the number of solutions to the equation  $2x^3 + 4x^2 = 3$ .

[1]

- (d) (i) The equation  $2x^3 + 4x^2 - x = 1$  can be solved by drawing a straight line on the grid.

Write down the equation of this straight line.

[1]

- (ii) Use your graph to solve the equation  $2x^3 + 4x^2 - x = 1$ .

[3]

- (e) The tangent to the graph of  $y = 2x^3 + 4x^2$  has a negative gradient when  $x = k$ .

Complete the inequality for  $k$ .

[2]

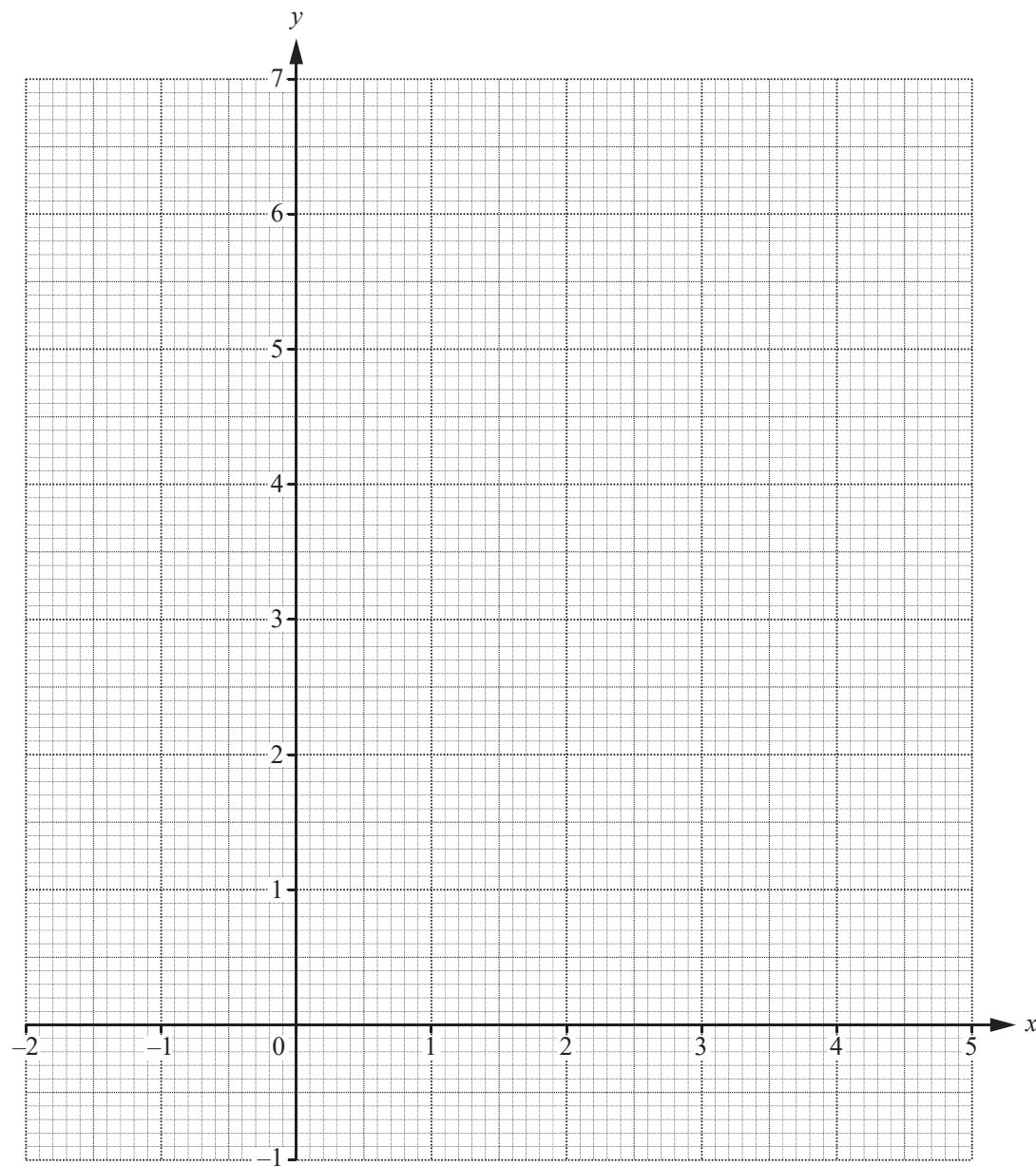
## Question 5

The table shows some values for  $y = 1.5^x - 1$ .

$x$	-2	-1	0	1	2	3	4	5
$y$	-0.56	-0.33				2.38	4.06	6.59

(a) Complete the table. [3]

(b) Draw the graph of  $y = 1.5^x - 1$  for  $-2 \leq x \leq 5$ . [4]



(c) Use your graph to solve the equation  $1.5^x - 1 = 3.5$ . [2]

(d) By drawing a suitable straight line, solve the equation  $1.5^x - x - 2 = 0$ . [3]

(e) (i) On the grid, plot the point  $A$  at  $(5, 5)$ . [1]

(ii) Draw the tangent to the graph of  $y = 1.5^x - 1$  that passes through the point  $A$ . [1]

(iii) Work out the gradient of this tangent. [2]

**Question 6**

$$y = 1 - \frac{2}{x^2}, x \neq 0$$

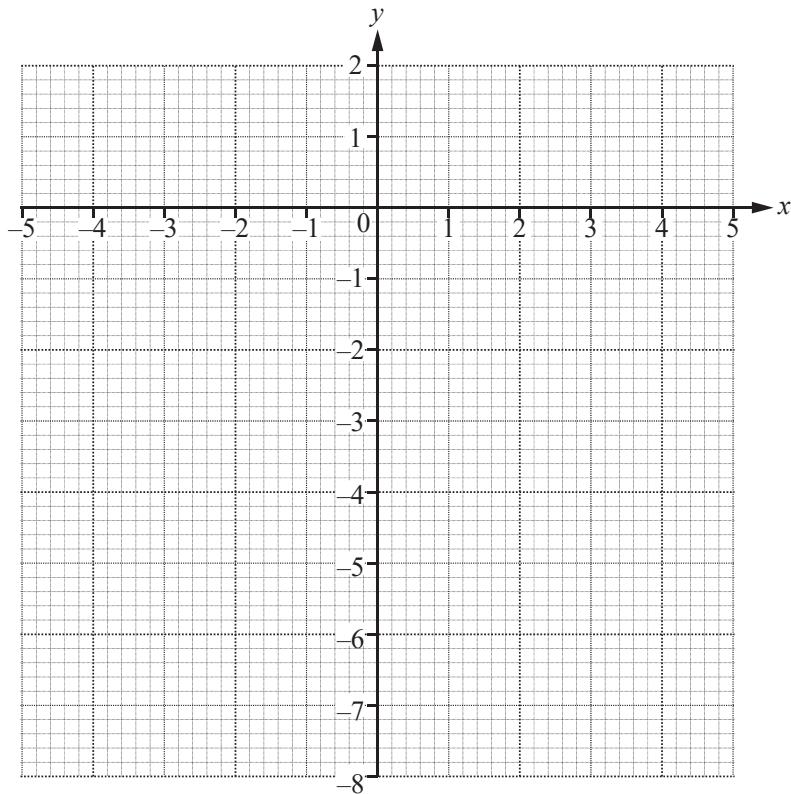
(a) Complete the table.

$x$	-5	-4	-3	-2	-1	-0.5		0.5	1	2	3	4	5
$y$		0.88	0.78			-7		-7			0.78	0.88	

[3]

(b) On the grid, draw the graph of  $y = 1 - \frac{2}{x^2}$  for  $-5 \leq x \leq -0.5$  and  $0.5 \leq x \leq 5$ .

[5]



(c) (i) On the grid, draw the graph of  $y = -x - 1$  for  $-3 \leq x \leq 5$ .

[2]

(ii) Solve the equation  $1 - \frac{2}{x^2} = -x - 1$ .

[1]

- (iii) The equation  $1 - \frac{2}{x^2} = -x - 1$  can be written in the form  $x^3 + px^2 + q = 0$ .

Find the value of  $p$  and the value of  $q$ .

[3]

- (d) The graph of  $y = 1 - \frac{2}{x^2}$  cuts the positive  $x$ -axis at  $A$ .

$B$  is the point  $(0, -2)$ .

- (i) Write down the co-ordinates of  $A$ .

[1]

- (ii) On the grid, draw the straight line that passes through  $A$  and  $B$ .

[1]

- (iii) Complete the statement.

The straight line that passes through  $A$  and  $B$  is a .....

at the point .....

[2]

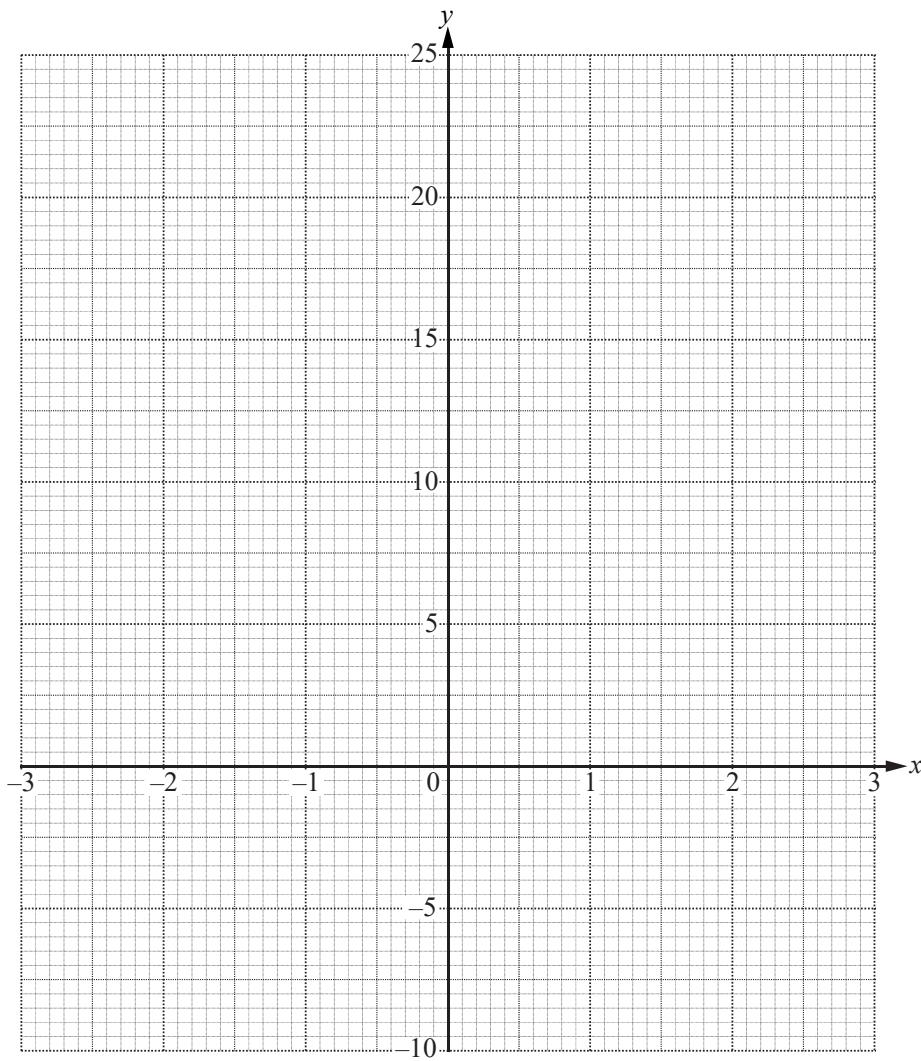
**Question 7**

- (a) Complete the table for  $y = 3x + \frac{2}{x^2} + 1$ ,  $x \neq 0$ .

$x$	-3	-2	-1	-0.5	-0.3		0.3	0.5	1	2	3
$y$	-7.8		0	7.5	22.3		24.1		6	7.5	10.2

[2]

- (b) On the grid, draw the graph of  $y = 3x + \frac{2}{x^2} + 1$  for  $-3 \leq x \leq -0.3$  and  $0.3 \leq x \leq 3$ .



[5]

- (c) Write down the value of the largest integer,  $k$ , so that the equation  $3x + \frac{2}{x^2} + 1 = k$  has exactly one solution.

[1]

(d) (i) By drawing a suitable straight line on the grid, solve  $3x + \frac{2}{x^2} + 1 = 15 - 3x$ . [4]

(ii) The equation  $3x + \frac{2}{x^2} + 1 = 15 - 3x$  can be written in the form  $ax^3 + bx^2 + cx + 2 = 0$ , where  $a, b$  and  $c$  are integers.

Find  $a, b$  and  $c$ .

[3]

# Graphs

## Difficulty: Medium

### Question Paper 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Medium
Booklet	Question Paper 2

**Time allowed:** 121 minutes

**Score:** /105

**Percentage:** /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

##### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

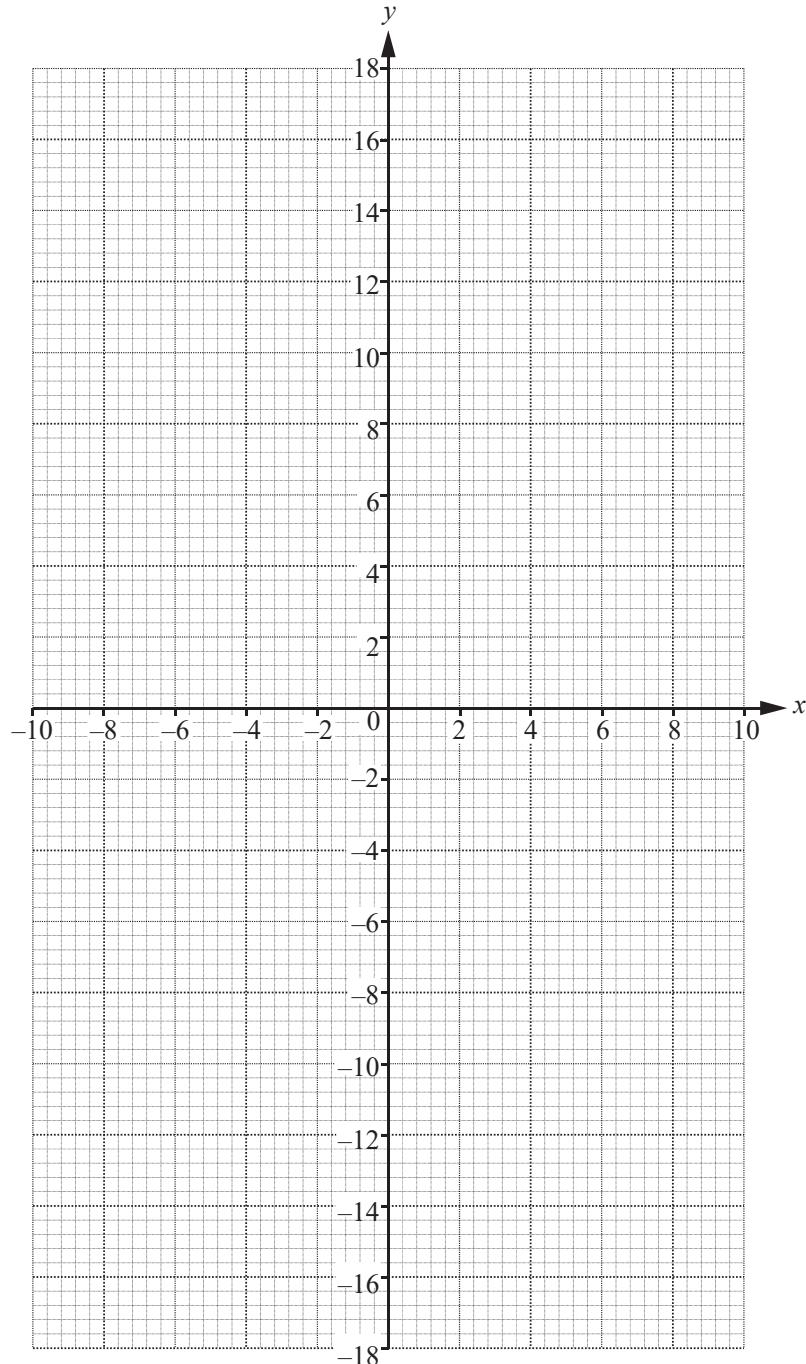
**Question 1**

(a) Complete the table.

$x$	-10	-8	-5	-2	-1.6		1.6	2	5	8	10
$f(x)$	-12	-10.5	-9	-12	-14.1		14.1	12			12

[2]

(b) On the grid, draw the graph of  $y = f(x)$  for  $-10 \leq x \leq -1.6$  and  $1.6 \leq x \leq 10$ .



[5]

(c) Using your graph, solve the equation  $f(x) = 11$ . [2]

(d)  $k$  is a prime number and  $f(x) = k$  has no solutions.

Find the possible values of  $k$ .

[2]

(e) The gradient of the graph of  $y = f(x)$  at the point  $(2, 12)$  is -4.

Write down the co-ordinates of the other point on the graph of  $y = f(x)$  where the gradient is -4.

[1]

(f) (i) The equation  $f(x) = x^2$  can be written as  $x^3 + px^2 + q = 0$ .

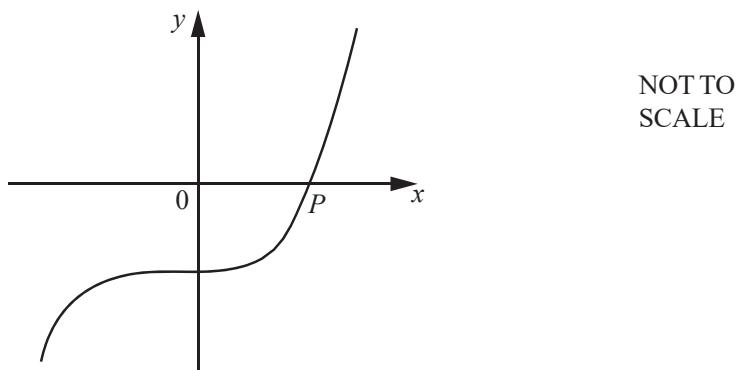
Show that  $p = -1$  and  $q = -20$ .

[2]

(ii) On the grid opposite, draw the graph of  $y = x^2$  for  $-4 \leq x \leq 4$ . [2]

(iii) Using your graphs, solve the equation  $x^3 - x^2 - 20 = 0$ . [1]

(iv)



The diagram shows a **sketch** of the graph of  $y = x^3 - x^2 - 20$ .  
 $P$  is the point  $(n, 0)$ .

Write down the value of  $n$ .

[1]

## Question 2

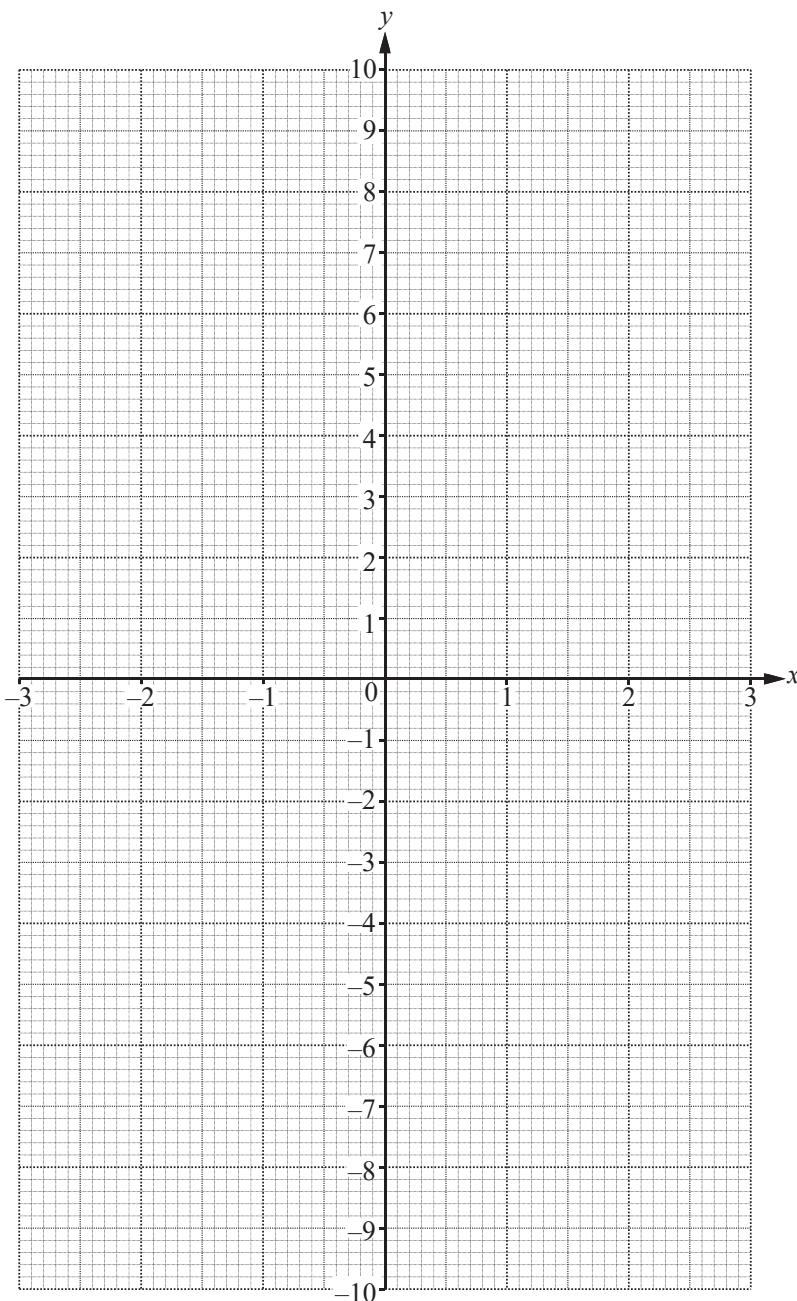
$$f(x) = x^2 - \frac{1}{x} - 4, x \neq 0$$

- (a) (i) Complete the table.

$x$	-3	-2	-1	-0.5	-0.1		0.2	0.5	1	2	3
$f(x)$	5.3	0.5		-1.8	6.0		-9.0	-5.8	-4		4.7

[2]

- (ii) On the grid, draw the graph of  $y = f(x)$  for  $-3 \leq x \leq -0.1$  and  $0.2 \leq x \leq 3$ .



[5]

(b) Use your graph to solve the equation  $f(x) = 0$ . [3]

(c) Find an integer  $k$ , for which  $f(x) = k$  has one solution. [1]

(d) (i) By drawing a suitable straight line, solve the equation  $f(x) + 2 = -5x$ . [4]

(ii)  $f(x) + 2 = -5x$  can be written as  $x^3 + ax^2 + bx - 1 = 0$ . [2]

Find the value of  $a$  and the value of  $b$ .

### Question 3

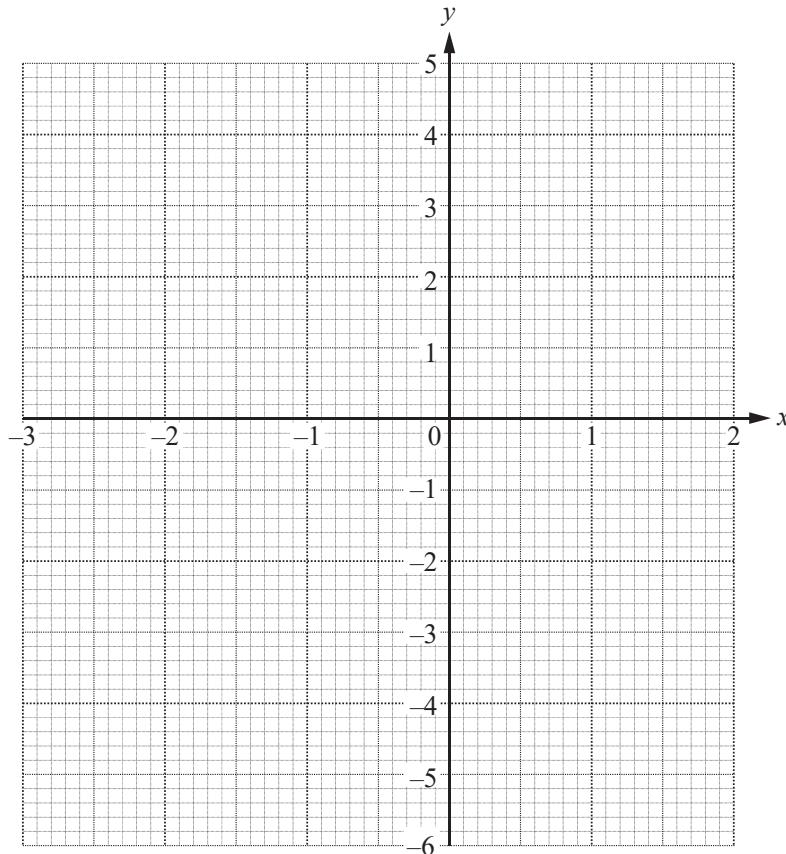
$$f(x) = x - \frac{1}{2x^2}, \quad x \neq 0$$

(a) Complete the table of values.

$x$	-3	-2	-1.5	-1	-0.5	-0.3		0.3	0.5	1	1.5	2
$f(x)$	-3.1	-2.1	-1.7		-2.5	-5.9		-5.3	-1.5		1.3	1.9

[2]

(b) On the grid, draw the graph of  $y = f(x)$  for  $-3 \leq x \leq -0.3$  and  $0.3 \leq x \leq 2$ .



[5]

(c) Use your graph to solve the equation  $f(x) = 1$ .

[1]

(d) There is only one negative integer value,  $k$ , for which  $f(x) = k$  has only one solution for all real  $x$ .

Write down this value of  $k$ .

[1]

(e) The equation  $2x - \frac{1}{2x^2} - 2 = 0$  can be solved using the graph of  $y = f(x)$  and a straight line graph.

(i) Find the equation of this straightline.

[1]

(ii) On the grid, draw this straight line and solve the equation  $2x - \frac{1}{2x^2} - 2 = 0$

[3]

#### Question 4

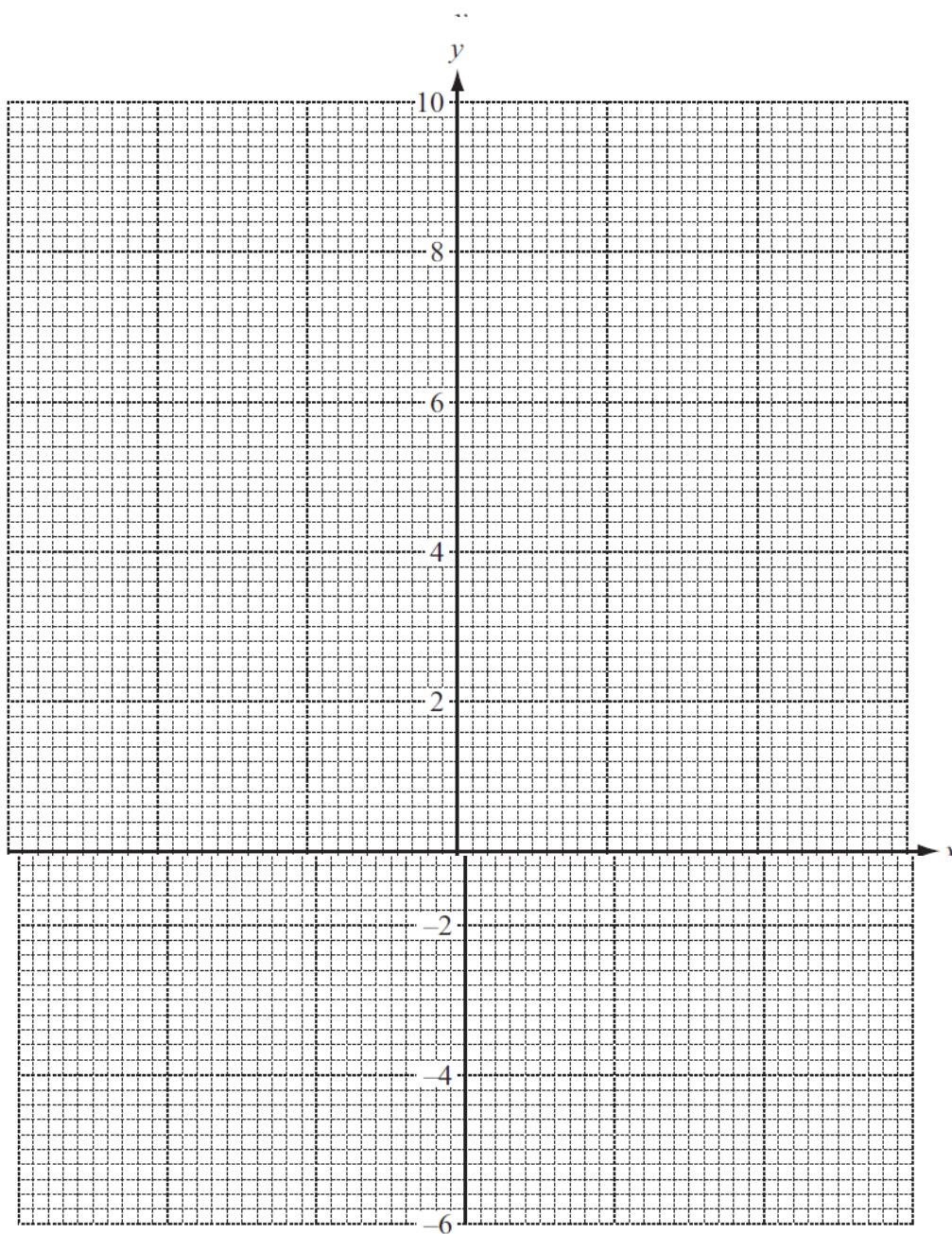
(a) Complete the table of values for  $y = x^2 + \frac{3}{x}$ ,  $x \neq 0$ .

[2]

$x$	-3	-2	-1	-0.5		0.4	0.6	1	1.5	2	3
$y$	8	2.5		-5.8		7.7	5.4	4	4.3		10

(b) Draw the graph of  $y = x^2 + \frac{3}{x}$  for  $-3 \leq x \leq -0.5$  and  $0.4 \leq x \leq 3$ .

[5]



(c) Use your graph to solve the equation  $x^2 + \frac{3}{x} = 5$ . [3]

(d) By drawing a suitable straight line, solve the equation  $x^2 + \frac{3}{x} = x + 5$ . [4]

## Question 5

The table shows some values for the function  $y = 11x - 2x^2 - 12$  for  $1 \leq x \leq 4.5$ .

[3]

$x$	1	1.5	2	2.5	3	3.5	4	4.5
$y$	-3		2	3	3			

(a) Complete the table of values.

(b) On the grid below, draw the graph of  $y = 11x - 2x^2 - 12$  for  $1 \leq x \leq 4.5$ .

[4]



(c) By drawing a suitable line, use your graph to solve the equation  $11x - 2x^2 = 11$ . [2]

(d) The line  $y = mx + 2$  is a tangent to the curve  $y = 11x - 2x^2 - 12$  at the point  $P$ .

By drawing this tangent,

(i) find the co-ordinates of the point  $P$ , [2]

(ii) work out the value of  $m$ . [2]

## Question 6

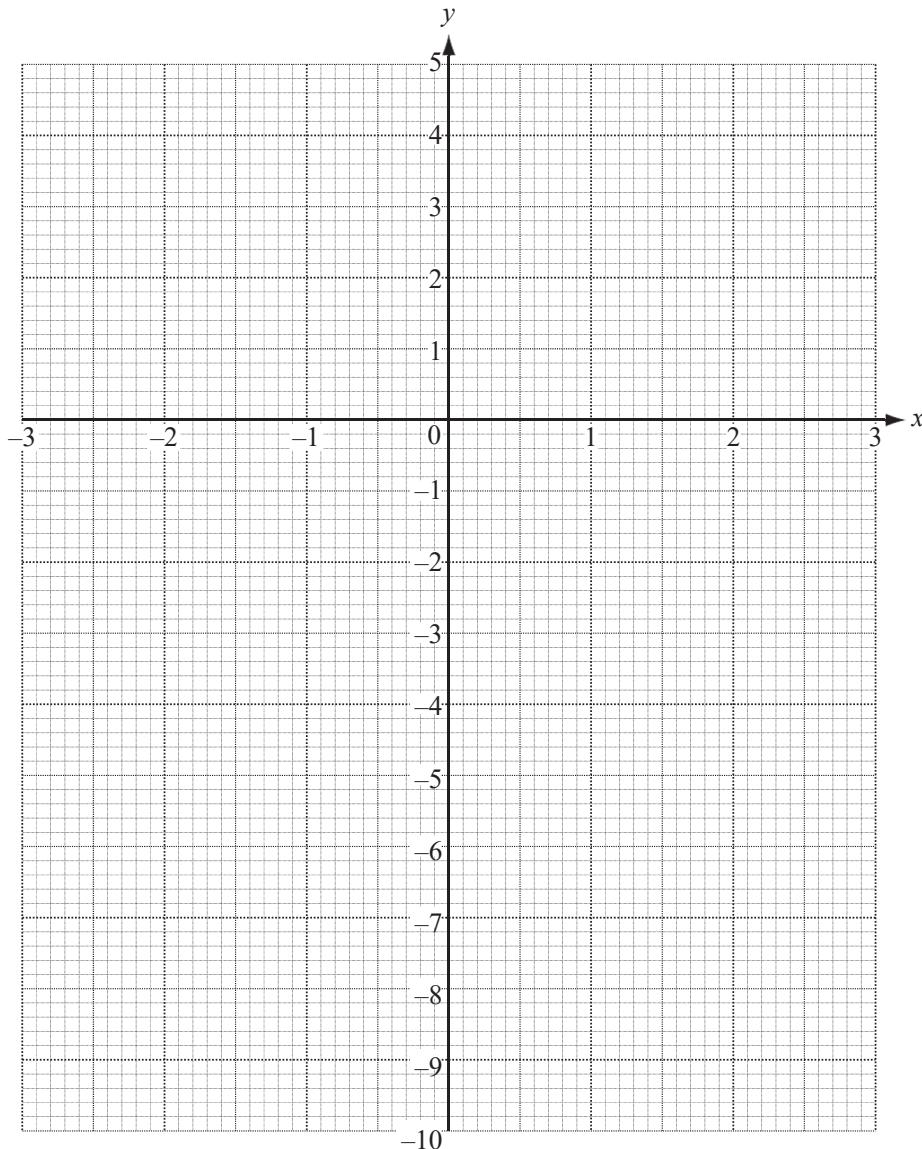
- (a) Complete this table of values for the function  $f(x) = \frac{1}{x} - x^2$ ,  $x \neq 0$ .

[3]

$x$	-3	-2	-1	-0.5	-0.2		0.2	0.5	1	2	3
$f(x)$	-9.33	-4.5	-2	-2.25			4.96			-3.5	-8.67

- (b) Draw the graph of  $f(x) = \frac{1}{x} - x^2$  for  $-3 \leq x \leq -0.2$  and  $0.2 \leq x \leq 3$ .

[5]



(c) Use your graph to solve  $f(x) = -3$ . [3]

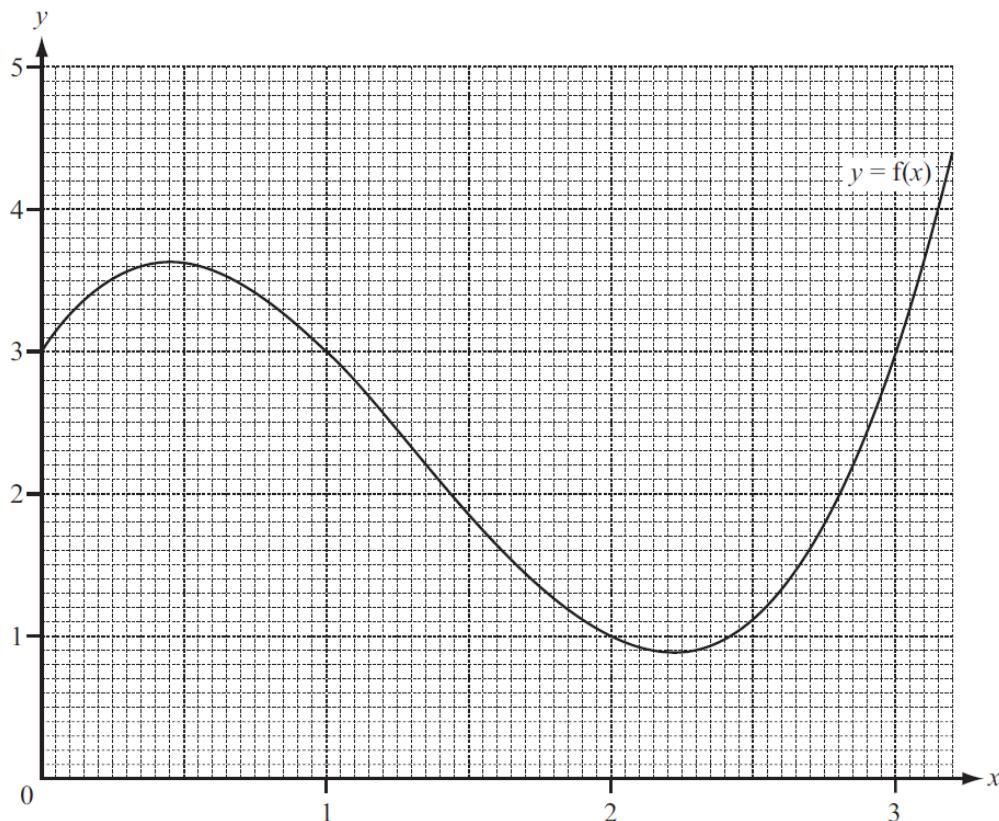
(d) By drawing a suitable line on your graph, solve the equation  $f(x) = 2x - 2$ . [3]

(e) By drawing a suitable tangent, work out an estimate of the gradient of the curve at the point where  $x = -2$ .

You must show your working. [3]

## Question 7

The graph of  $y = f(x)$  is drawn on the grid for  $0 \leq x \leq 3.2$ .



(a) (i) Draw the tangent to the curve  $y = f(x)$  at  $x = 2.5$ . [1]

(ii) Use your tangent to estimate the gradient of the curve at  $x = 2.5$ . [2]

(b) Use the graph to solve  $f(x) = 2$ , for  $0 \leq x \leq 3.2$ . [2]

(c)

$$g(x) = \frac{x}{2} + \frac{2}{x^2} \quad x \neq 0.$$

(i) Complete the table for values of  $g(x)$ , correct to 1 decimal place.

$x$	0.7	1	1.5	2	2.5	3
$g(x)$			1.6		1.6	1.7

[2]

(ii) On the grid opposite, draw the graph of  $y = g(x)$  for  $0.7 \leq x \leq 3$ .

[3]

(iii) Solve  $f(x) = g(x)$  for  $0.7 \leq x \leq 3$ .

[3]

# Graphs

## Difficulty: Medium

### Question Paper 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Medium
Booklet	Question Paper 3

**Time allowed:** 122 minutes

**Score:** /106

**Percentage:** /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

A*	A	B	C	D
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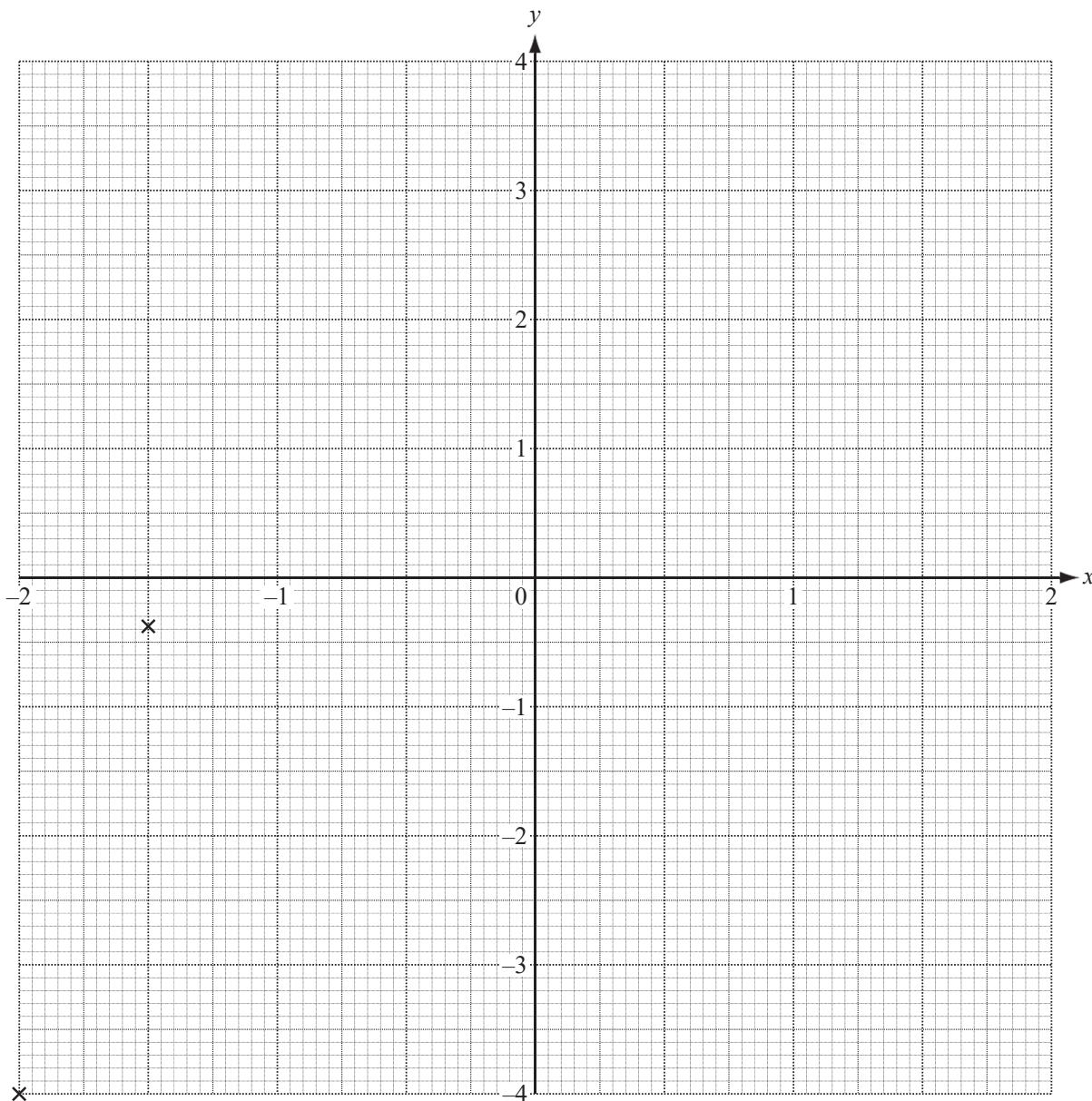
## Question 1

The table shows some values for the equation  $y = x^3 - 2x$  for  $-2 \leq x \leq 2$ .

$x$	-2	-1.5	-1	-0.6	-0.3	0	0.3	0.6	1	1.5	2
$y$	-4	-0.38			0.57		-0.57			0.38	4

(a) Complete the table of values. [3]

(b) On the grid below, draw the graph of  $y = x^3 - 2x$  for  $-2 \leq x \leq 2$ .  
The first two points have been plotted for you.



[4]

(c) (i) On the grid, draw the line  $y = 0.8$  for  $-2 \leq x \leq 2$ . [1]

(ii) Use your graph to solve the equation  $x^3 - 2x = 0.8$ . [3]

(d) By drawing a suitable tangent, work out an estimate for the gradient of the graph of  $y = x^3 - 2x$  where  $x = -1.5$ .

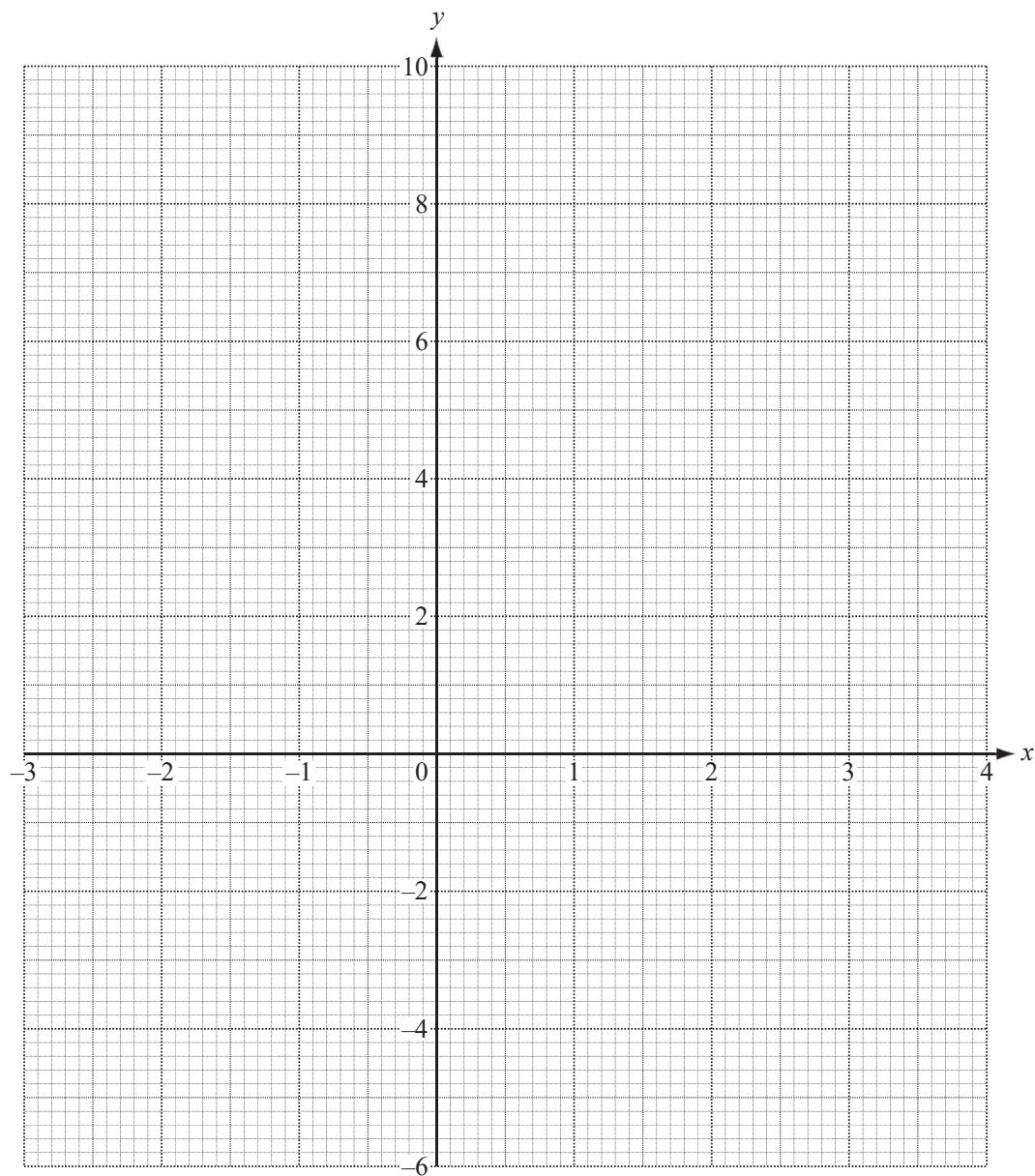
You must show your working. [3]

**Question 2**

(a) Complete the table for the function  $f(x) = \frac{x^3}{2} - 3x - 1$ . [3]

$x$	-3	-2	-1.5	-1	0	1	1.5	2	3	3.5
$f(x)$	-5.5		1.8	1.5		-3.5	-3.8	-3		9.9

(b) On the grid draw the graph of  $y = f(x)$  for  $-3 \leq x \leq 3.5$  [4]



(c) Use your graph to

(i) solve  $f(x) = 0.5$ , [3]

(ii) find the inequalities for  $k$ , so that  $f(x) = k$  has only 1 answer. [2]

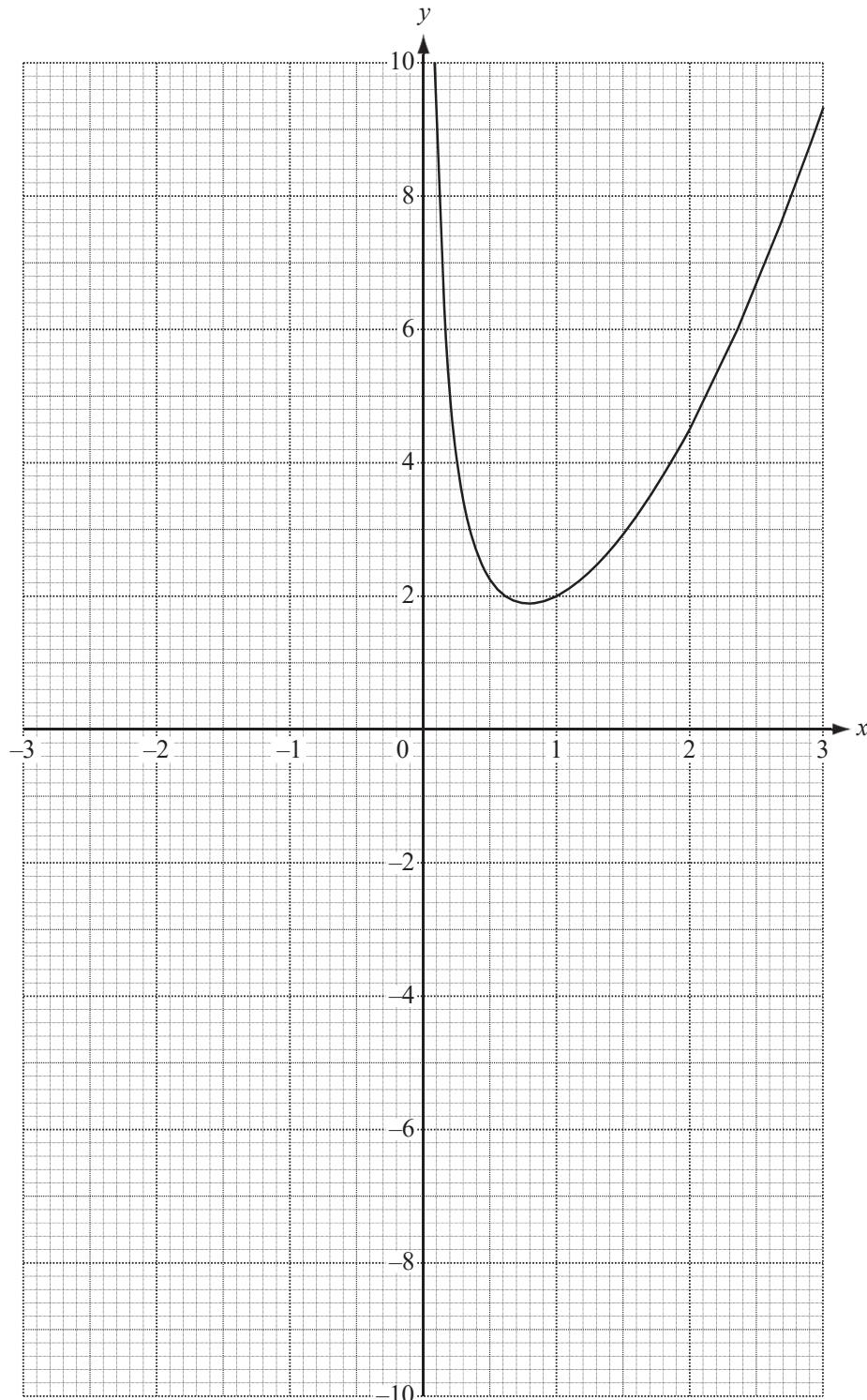
(d) (i) On the same grid, draw the graph of  $y = 3x - 2$  for  $-1 \leq x \leq 3.5$  [3]

(ii) The equation  $\frac{x^3}{2} - 3x - 1 = 3x - 2$  can be written in the form  $x^3 + ax + b = 0$ .  
Find the values of  $a$  and  $b$ . [2]

(iii) Use your graph to find the **positive** answers to  $\frac{x^3}{2} - 3x - 1 = 3x - 2$  for  $-3 \leq x \leq 3.5$ . [2]

### Question 3

The diagram shows the accurate graph of  $y = f(x)$  where  $f(x) = \frac{1}{x} + x^2$  for  $0 < x \leq 3$ .



(a) Complete the table for  $f(x) = \frac{1}{x} + x^2$ . [3]

$x$	-3	-2	-1	-0.5	-0.3	-0.1
$f(x)$		3.5	0	-1.8		

(b) On the grid, draw the graph of  $y = f(x)$  for  $-3 \leq x < 0$ . [3]

(c) By drawing a tangent, work out an estimate of the gradient of the graph where  $x = 2$ . [3]

(d) Write down the inequality satisfied by  $k$  when  $f(x) = k$  has three answers. [1]

(e) (i) Draw the line  $y = 1 - x$  on the grid for  $-3 \leq x \leq 3$ . [2]

(ii) Use your graphs to solve the equation  $1 - x = \frac{1}{x} + x^2$ . [1]

(f) (i) Rearrange  $x^3 - x^2 - 2x + 1 = 0$  into the form  $\frac{1}{x} + x^2 = ax + b$ , where  $a$  and  $b$  are integers. [2]

(ii) Write down the equation of the line that could be drawn on the graph to solve  $x^3 - x^2 - 2x + 1 = 0$ . [1]

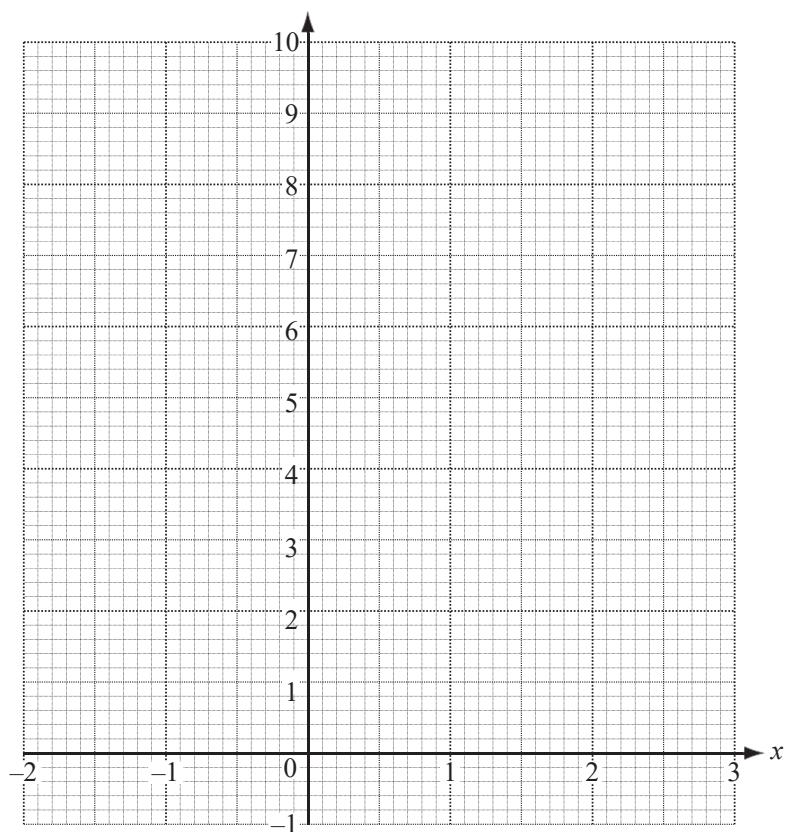
**Question 4**(a) Complete the table of values for  $y = 2^x$ .

[2]

$x$	-2	-1	0	1	2	3
$y$	0.25		1	2		8

(b) On the grid, draw the graph of  $y = 2^x$  for  $-2 \leq x \leq 3$ .

[3]



(c) (i) On the grid, draw the straight line which passes through the points  $(0, 2)$  and  $(3, 8)$ . [1]

(ii) The equation of this line is  $y = mx + 2$ .

Show that the value of  $m$  is 2.

[1]

(iii) One answer to the equation  $2^x = 2x + 2$  is  $x = 3$ .

Use your graph to find the other answer.

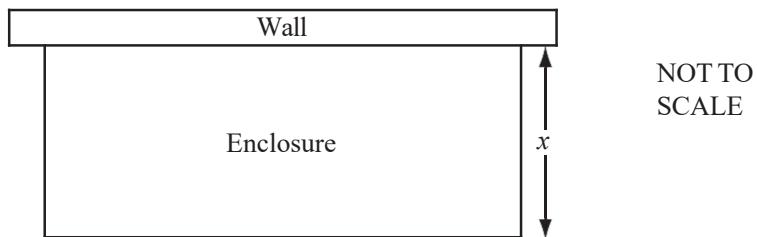
[1]

(d) Draw the tangent to the curve at the point where  $x = 1$ .

Use this tangent to calculate an estimate of the gradient of  $y = 2^x$  when  $x = 1$ .

[3]

## Question 5



A farmer makes a rectangular enclosure for his animals.

He uses a wall for one side and a total of 72 metres of fencing for the other three sides.

The enclosure has width  $x$  metres and area  $A$  square metres.

- (a) Show that  $A = 72x - 2x^2$ . [2]

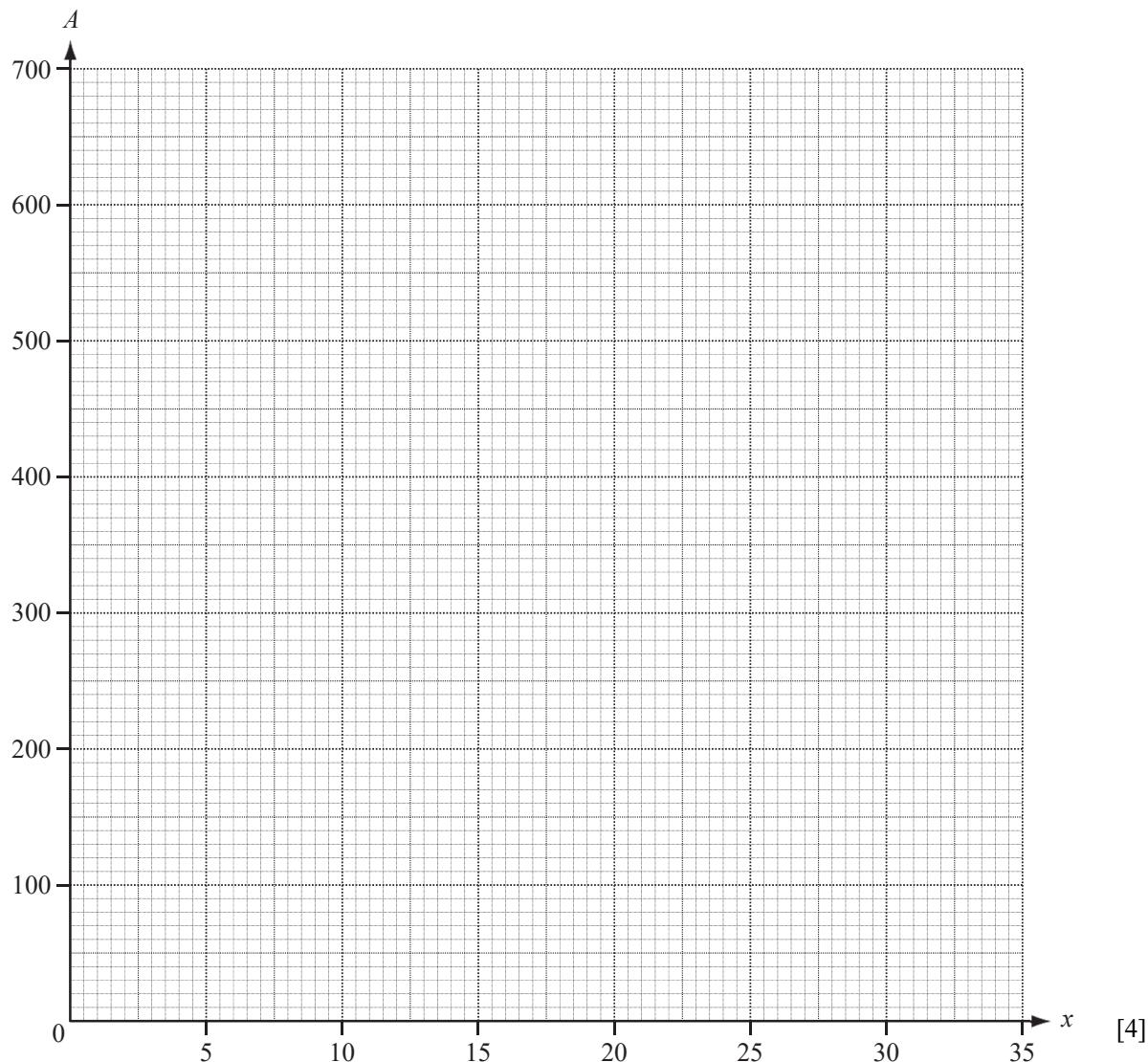
- (b) Factorise completely  $72x - 2x^2$ . [2]

- (c) Complete the table for  $A = 72x - 2x^2$ .

$x$	0	5	10	15	20	25	30	35
$A$	0	310	520			550	360	

[3]

- (d) Draw the graph of  $A = 72x - 2x^2$  for  $0 \leq x \leq 35$  on the grid opposite.



(e) Use your graph to find

(i) the values of  $x$  when  $A = 450$ ,

[2]

(ii) the maximum area of the enclosure.

[1]

(f) Each animal must have at least  $12 \text{ m}^2$  for grazing.

Calculate the greatest number of animals that the farmer can keep in an enclosure which has an area of  $500 \text{ m}^2$ .

[2]

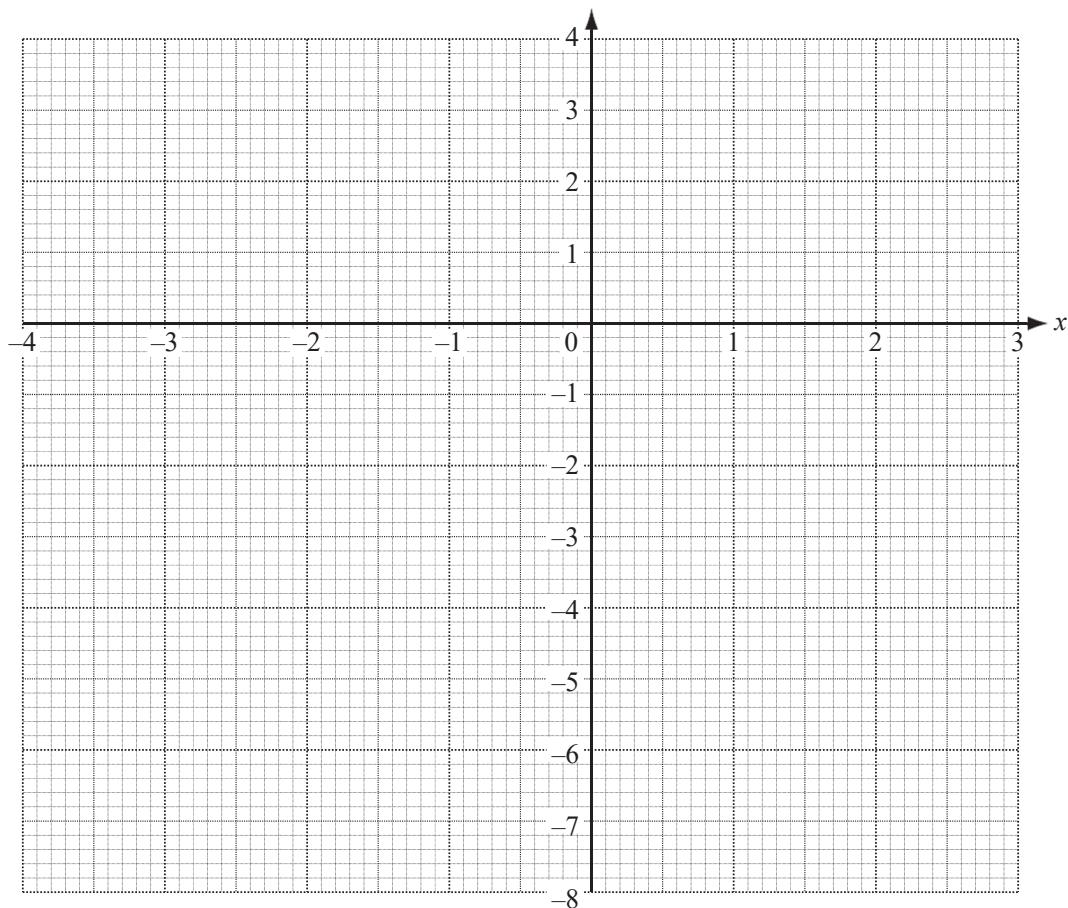
## Question 6

- (a) Complete the table for the function  $f(x) = \frac{x^3}{10} + 1$

$x$	-4	-3	-2	-1	0	1	2	3
$f(x)$		-1.7	0.2	0.9	1	1.1	1.8	

[2]

- (b) On the grid, draw the graph of  $y = f(x)$  for  $-4 \leq x \leq 3$ .



[4]

- (c) Complete the table for the function  $g(x) = \frac{4}{x}, x \neq 0$ .

$x$	-4	-3	-2	-1	1	2	3
$g(x)$	-1	-1.3				2	1.3

[2]

(d) On the grid, draw the graph of  $y = g(x)$  for  $-4 \leq x \leq -1$  and  $1 \leq x \leq 3$ . [3]

(e) (i) Use your graphs to solve the equation  $\frac{x^3}{10} + 1 = \frac{4}{x}$ . [2]

(ii) The equation  $\frac{x^3}{10} + 1 = \frac{4}{x}$  can be written as  $x^4 + ax + b = 0$ .

Find the values of  $a$  and  $b$ . [2]

## Question 7

(a)  $f(x) = 2^x$

Complete the table.

$x$	-2	-1	0	1	2	3	4
$y = f(x)$		0.5	1	2	4		

[3]

(b)  $g(x) = x(4 - x)$

Complete the table.

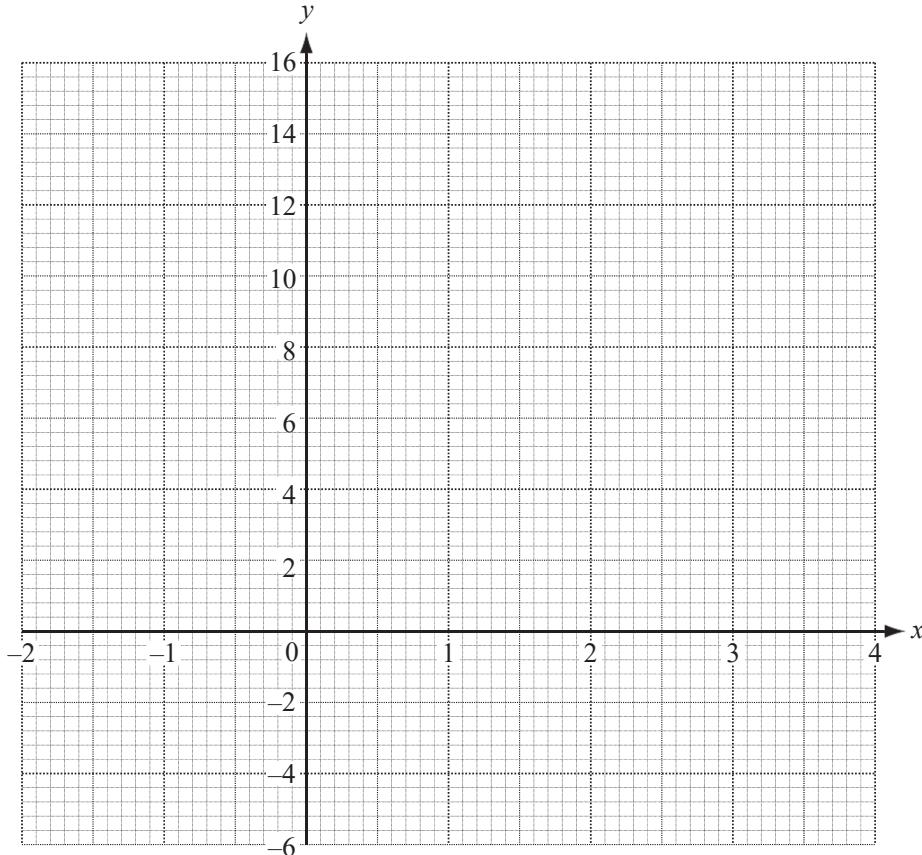
$x$	-1	0	1	2	3	4
$y = g(x)$		0	3		3	0

[2]

(c) On the grid, draw the graphs of

(i)  $y = f(x)$  for  $-2 \leq x \leq 4$ , [3]

(ii)  $y = g(x)$  for  $-1 \leq x \leq 4$ . [3]



(d) Use your graphs to solve the following equations.

(i)  $f(x) = 10$  [1]

(ii)  $f(x) = g(x)$  [2]

(iii)  $\hat{f}(x) = 1.7$  [1]

# Graphs

## Difficulty: Medium

### Question Paper 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Medium
Booklet	Question Paper 4

**Time allowed:** 86 minutes

**Score:** /75

**Percentage:** /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

##### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

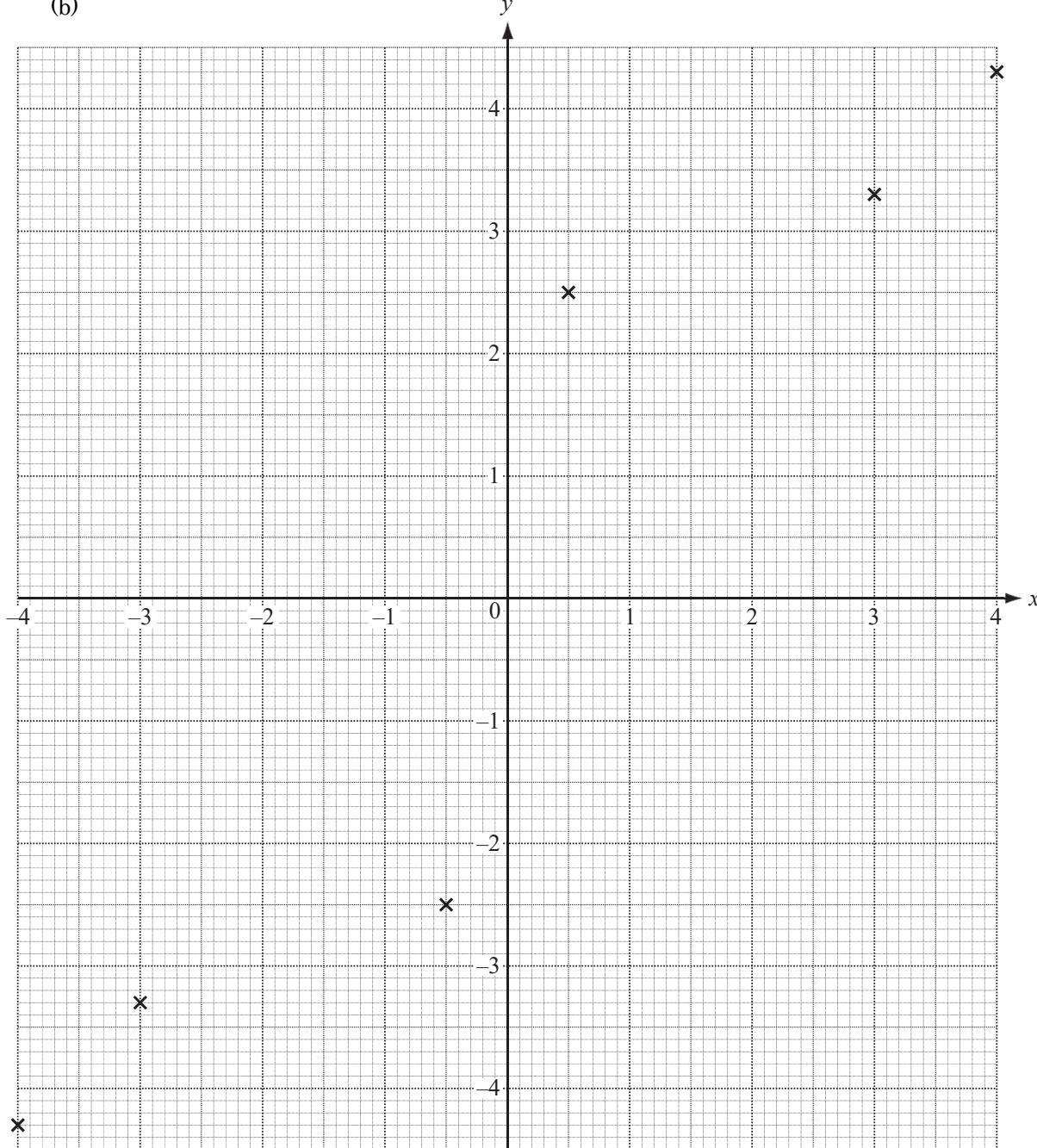
# Question 1

- (a) Complete the table of values for  $y = x + \frac{1}{x}$ .

$x$	-4	-3	-2	-1	-0.5	0.5	1	2	3	4
$y$	-4.3	-3.3			-2.5	2.5			3.3	4.3

[2]

(b)



On the grid, draw the graph of  $y = x + \frac{1}{x}$  for  $-4 \leq x \leq -0.5$  and  $0.5 \leq x \leq 4$ .

Six of the ten points have been plotted for you.

[3]

- (c) There are three integer values of  $k$  for which the equation  $x + \frac{1}{x} = k$  has **no** solutions.

Write down these three values of  $k$ .

[2]

- (d) Write down the ranges of  $x$  for which the gradient of the graph of  $y = x + \frac{1}{x}$  is positive.

[2]

- (e) To solve the equation  $x + \frac{1}{x} = 2x + 1$ , a straight line can be drawn on the grid.

(i) Draw this line on the grid for  $-2.5 \leq x \leq 1.5$ .

[2]

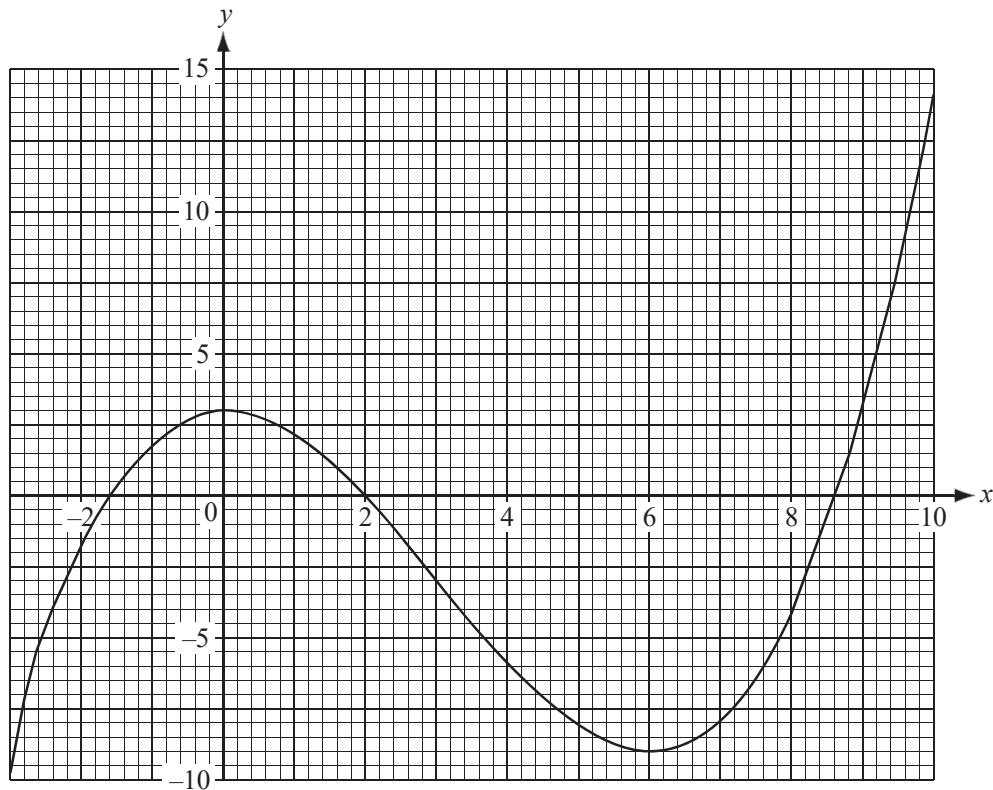
(ii) On the grid, show how you would find the solutions.

[1]

- (iii) Show how the equation  $x + \frac{1}{x} = 2x + 1$  can be rearranged into the form  $x^2 + bx + c = 0$  and find the values of  $b$  and  $c$ .

[3]

## Question 2



The diagram shows the accurate graph of  $y = f(x)$ .

(a) Use the graph to find

(i)  $f(0)$ , [1]

(ii)  $f(8)$ . [1]

(b) Use the graph to solve

(i)  $f(x) = 0$ , [2]

(ii)  $f(x) = 5$ . [1]

(c)  $k$  is an integer for which the equation  $f(x) = k$  has exactly two solutions.

Use the graph to find the two values of  $k$ . [2]

(d) Write down the range of values of  $x$  for which the graph of  $y = f(x)$  has a negative gradient. [2]

(e) The equation  $f(x) + x - 1 = 0$  can be solved by drawing a line on the grid.

(i) Write down the equation of this line. [1]

(ii) How many solutions are there for  $f(x) + x - 1 = 0$ ? [1]

### Question 3

Answer the whole of this question on a sheet of graph paper.

$$f(x) = 3x - \frac{1}{x^2} + 3, \quad x \neq 0.$$

- (a) The table shows some values of  $f(x)$ .

$x$	-3	-2.5	-2	-1.5	-1	-0.5	-0.4	-0.3	0.3	0.4	0.5	1	1.5	2	2.5	3
$f(x)$	$p$	-4.7	-3.3	-1.9	-1	-2.5	-4.5	-9.0	-7.2	-2.1	0.5	$q$	7.1	8.8	10.3	$r$

Find the values of  $p$ ,  $q$  and  $r$ .

[3]

- (b) Draw axes using a scale of 1 cm to represent 0.5 units for  $-3 \leq x \leq 3$  and 1 cm to represent units for  $-10 \leq y \leq 12$ . [1]
- (c) On your grid, draw the graph of  $y = f(x)$  for  $-3 \leq x \leq -0.3$  and  $0.3 \leq x \leq 3$ . [5]

- (d) Use your graph to solve the equations

(i)  $3x - \frac{1}{x^2} + 3 = 0$ , [1]

(ii)  $3x - \frac{1}{x^2} + 7 = 0$ . [3]

(e)  $g(x) = 3x + 3$ .

On the same grid, draw the graph of  $y = g(x)$  for  $-3 \leq x \leq 3$ . [2]

[1]

- (f) (i) Describe briefly what happens to the graphs of  $y = f(x)$  and  $y = g(x)$  for large positive or negative values of  $x$ .

- (ii) Estimate the gradient of  $y = f(x)$  when  $x = 100$ . [1]

## Question 4

Answer the whole of this question on a sheet of graph paper.

- (a) Find the values of  $k$ ,  $m$  and  $n$  in each of the following equations, where  $a > 0$ .

(i)  $a^0 = k$ ,

[1]

(ii)  $a^m = \frac{1}{a}$ ,

[1]

(iii)  $a^n = \sqrt[n]{a}$ .

[1]

- (b) The table shows some values of the function  $f(x) = 2^x$ .

$x$	-2	-1	-0.5	0	0.5	1	1.5	2	3
$f(x)$	$r$	0.5	0.71	$s$	1.41	2	2.83	4	$t$

- (i) Write down the values of  $r$ ,  $s$  and  $t$ .

[3]

- (ii) Using a scale of 2 cm to represent 1 unit on each axis, draw an  $x$ -axis from -2 to 3 and a  $y$ -axis from 0 to 10.

[1]

- (iii) On your grid, draw the graph of  $y = f(x)$  for  $-2 \leq x \leq 3$ .

[4]

- (c) The function  $g$  is given by  $g(x) = 6 - 2x$ .

- (i) On the same grid as part (b), draw the graph of  $y = g(x)$  for  $-2 \leq x \leq 3$ .

[2]

- (ii) Use your graphs to solve the equation  $2 = 6 - 2x$ .

[1]

- (iii) Write down the value of  $x$  for which  $2 < 6 - 2x$  for  $x \in \{\text{positive integers}\}$ .

[1]

## Question 5

Answer the whole of this question on a sheet of graph paper.

The table gives values of  $f(x) = 2^x$ , for  $-2 \leq x \leq 4$ .

$x$	-2	-1	0	1	2	3	4
$f(x)$	$p$	0.5	$q$	2	4	$r$	16

- (a) Find the values of  $p$ ,  $q$  and  $r$ . [3]
- (b) Using a scale of 2 cm to 1 unit on the  $x$ -axis and 1 cm to 1 unit on the  $y$ -axis, draw the graph of  $y = f(x)$  for  $-2 \leq x \leq 4$ . [5]
- (c) Use your graph to solve the equation  $2^x = 7$ . [1]
- (d) What value does  $f(x)$  approach as  $x$  decreases? [1]
- (e) By drawing a tangent, estimate the gradient of the graph of  $y = f(x)$  when  $x = 1.5$ . [3]
- (f) On the same grid draw the graph of  $y = 2x + 1$  for  $0 \leq x \leq 4$ . [2]
- (g) Use your graph to find the non-integer solution of  $2^x = 2x + 1$ . [2]

# Graphs

## Difficulty: Hard

### Question Paper 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Hard
Booklet	Question Paper 1

**Time allowed:** 117 minutes

**Score:** /102

**Percentage:** /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

##### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

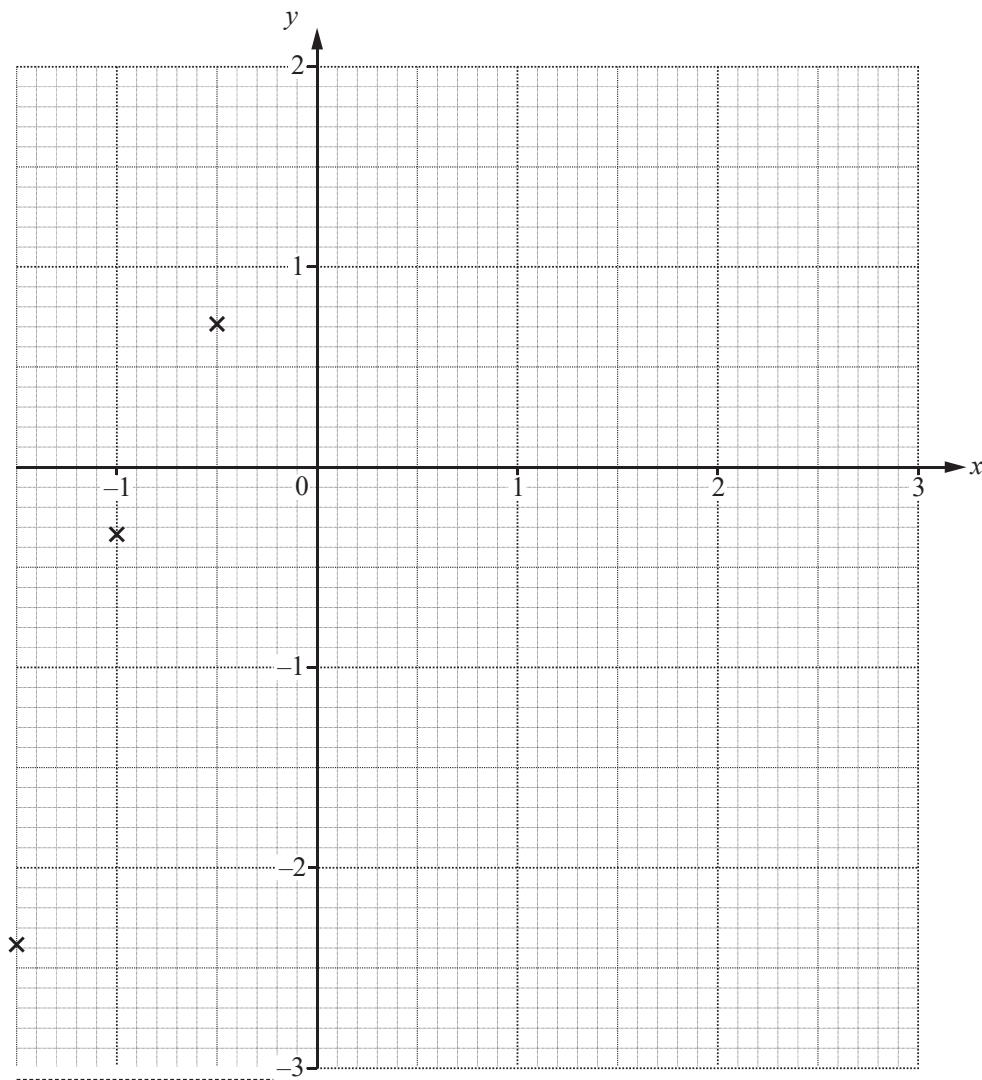
- (a) Complete the table of values for  $y = \frac{x^3}{3} - x^2 + 1$ .

$x$	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
$y$	-2.38	-0.33	0.71		0.79	0.33	-0.13	-0.33	-0.04	

[2]

- (b) Draw the graph of  $y = \frac{x^3}{3} - x^2 + 1$  for  $-1.5 \leq x \leq 3$ .

The first 3 points have been plotted for you.



[4]

(c) Using your graph, solve the equations.

(i)  $\frac{x^3}{3} - x^2 + 1 = 0$  [3]

(ii)  $\frac{x^3}{3} - x^2 + x + 1 = 0$  [2]

(d) Two tangents to the graph of  $y = \frac{x^3}{3} - x^2 + 1$  can be drawn parallel to the  $x$ -axis.

(i) Write down the equation of each of these tangents. [2]

(ii) For  $0 \leq x \leq 3$ , write down the smallest possible value of  $y$ . [1]

**Question 2**

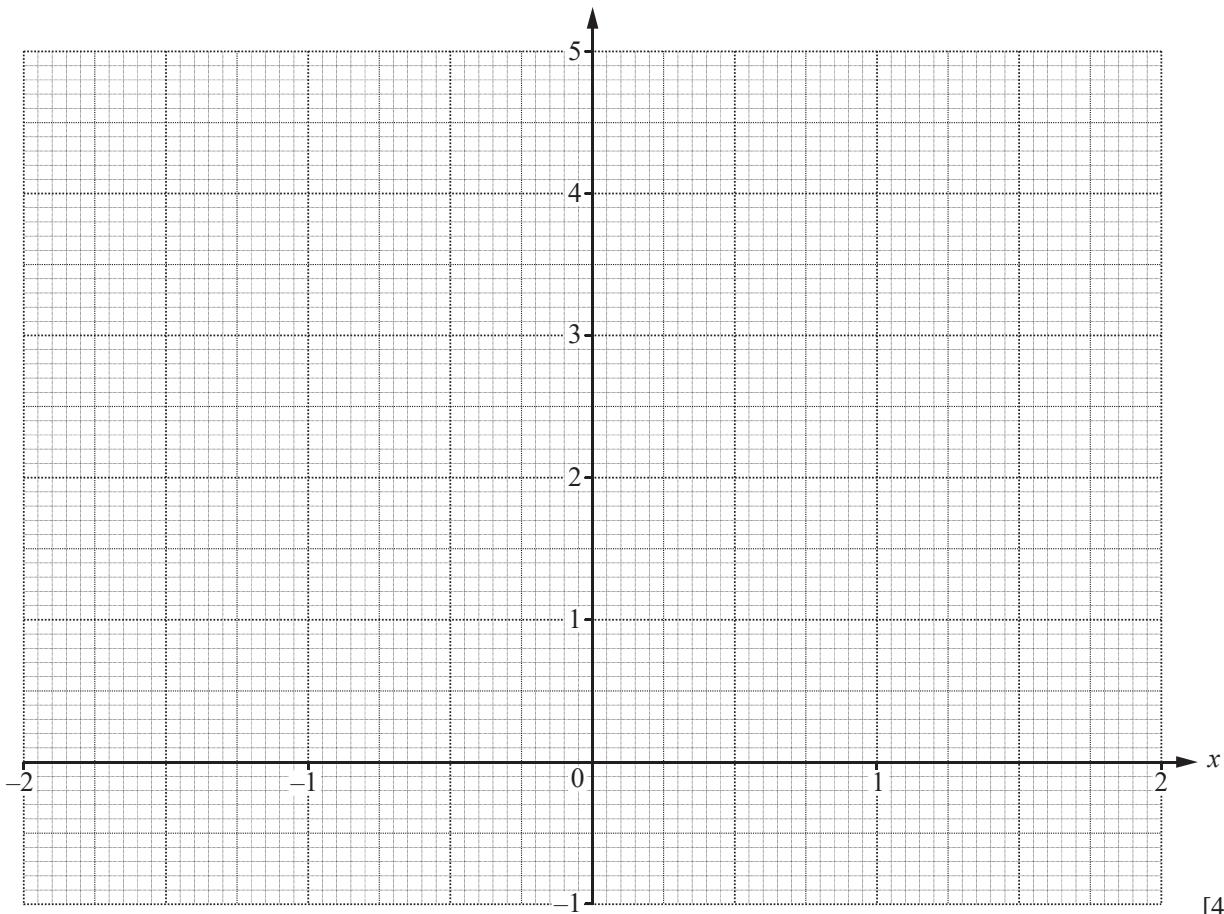
The table shows some values for  $y = x^3 - 3x + 2$ .

$x$	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$y$		3.125		3.375	2		0		4

(a) Complete the table of values.

[4]

(b) On the grid, draw the graph of  $y = x^3 - 3x + 2$  for  $-2 \leq x \leq 2$ .



[4]

(c) By drawing a suitable line, solve the equation  $x^3 - 3x + 2 = x + 1$  for  $-2 \leq x \leq 2$ .

[3]

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where  $x = -1.5$ .

[3]

### Question 3

$$y = x^2 - 2x + \frac{12}{x}, \quad x \neq 0$$

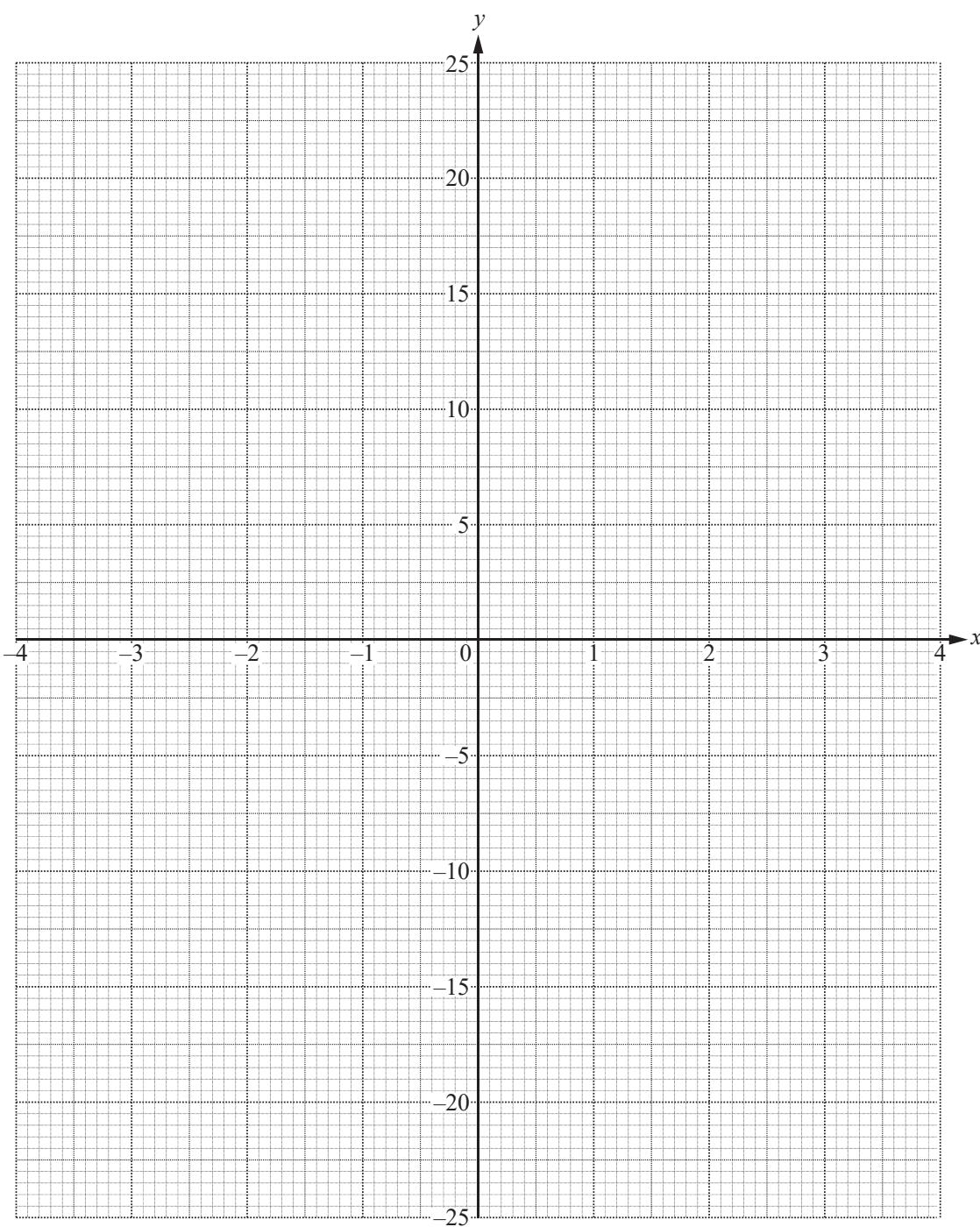
- (a) Complete the table of values.

$x$	-4	-3	-2	-1	-0.5		0.5	1	2	3	4
$y$	21	11		-9	-22.75		23.25	11	6		11

[2]

- (b) On the grid, draw the graph of  $y = x^2 - 2x + \frac{12}{x}$  for  $-4 \leq x \leq -0.5$  and  $0.5 \leq x \leq 4$ .

[5]



(c) By drawing a suitable tangent, find an estimate of the gradient of the graph at the point (1, 11). [3]

(d) The equation  $x^2 - 2x + \frac{12}{x} = k$  has exactly two distinct solutions.

Use the graph to find

(i) the value of  $k$ , [1]

(ii) the solutions of  $x^2 - 2x + \frac{12}{x} = k$ . [2]

(e) The equation  $x^3 + ax^2 + bx + c = 0$  can be solved by drawing the line  $y = 3x + 1$  on the grid.

Find the value of  $a$ , the value of  $b$  and the value of  $c$ . [3]

**Question 4**

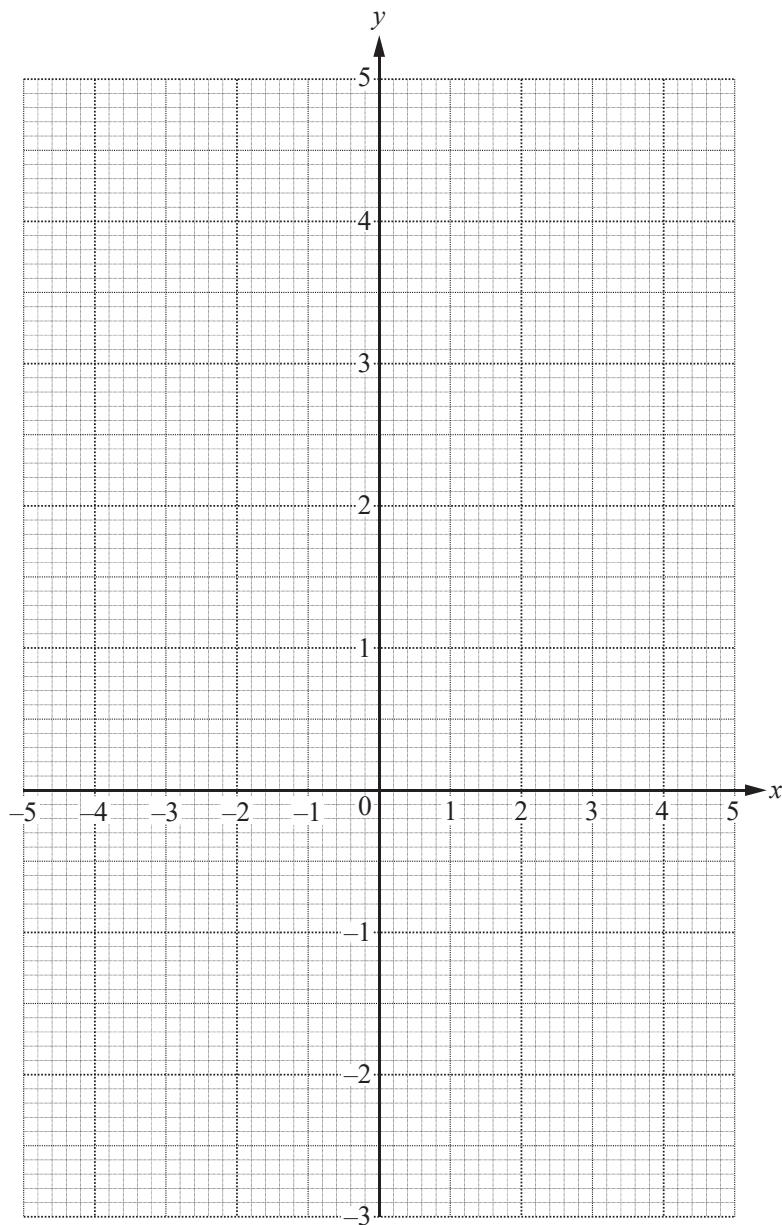
$$f(x) = \frac{8}{x^2} + \frac{x}{2}, \quad x \neq 0.$$

- (a) Complete the table of values for  $f(x)$ .

$x$	-5	-4	-3	-2	-1.5	1.5	2	2.5	3	3.5	4	5
$f(x)$	-2.2	-1.5	-0.6		2.8	4.3		2.5	2.4	2.4		2.8

[3]

- (b) On the grid, draw the graph of  $y = f(x)$  for  $-5 \leq x \leq -1.5$  and  $1.5 \leq x \leq 5$ .



[5]

(c) Solve  $f(x) = 0$ . [1]

(d) By drawing a suitable line on the grid, solve the equation  $f(x) = 1 - x$ . [3]

(e) By drawing a tangent at the point  $(-3, -0.6)$ , estimate the gradient of the graph of  $y = f(x)$  when  $x = -3$ .  
[3]

## Question 5

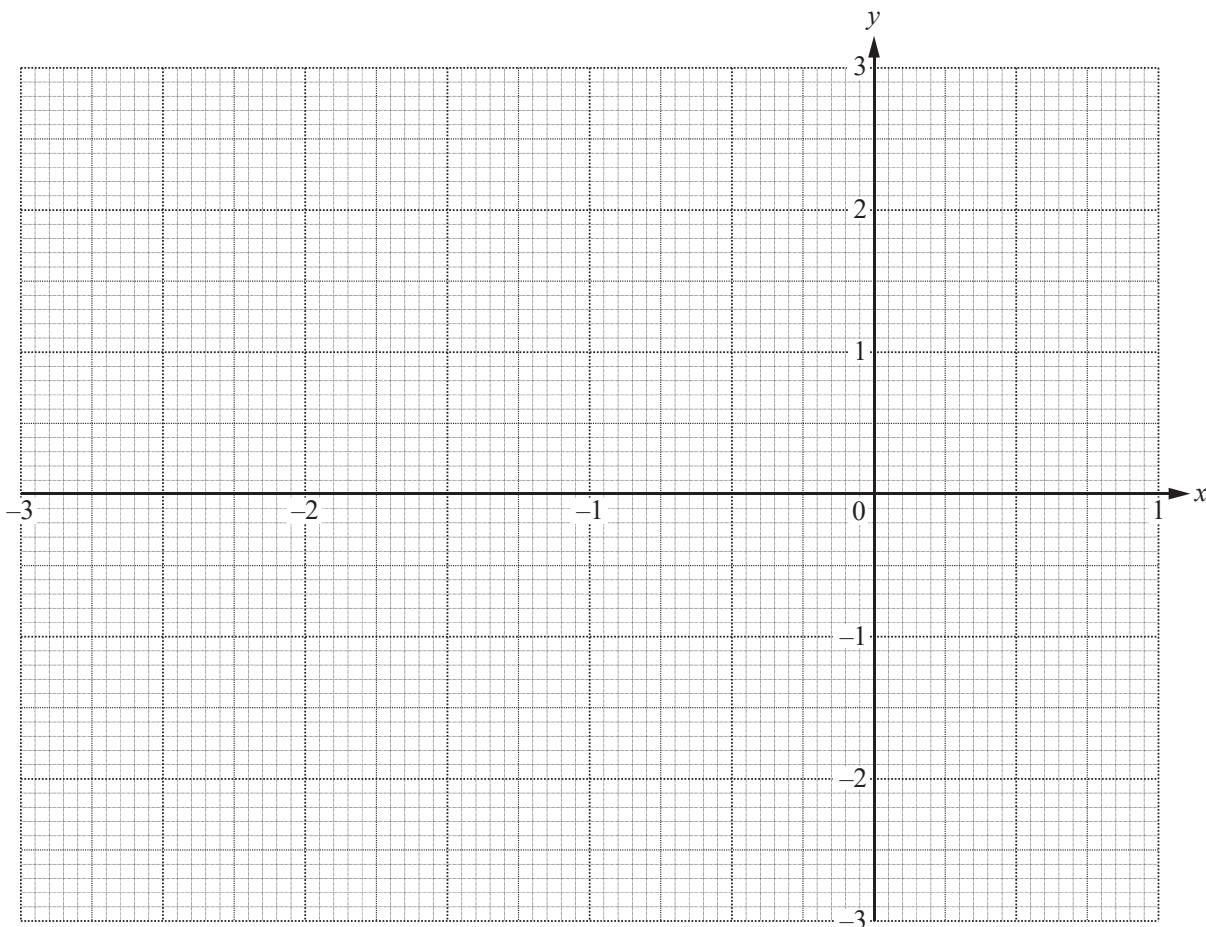
The table shows some values of  $y = x^3 + 3x^2 - 2$ .

$x$	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1
$y$	-2	1.13		1.38		-1.38		-1.13	

(a) Complete the table of values.

[3]

(b) On the grid, draw the graph of  $y = x^3 + 3x^2 - 2$  for  $-3 \leq x \leq 1$ .



[4]

(c) By drawing a suitable line, solve the equation  $x^3 + 3x^2 - 2 = \frac{1}{2}(x + 1)$ .

[4]

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where  $x = -1.75$ .

[3]

## Question 6

$$f(x) = 5x^3 - 8x^2 + 10$$

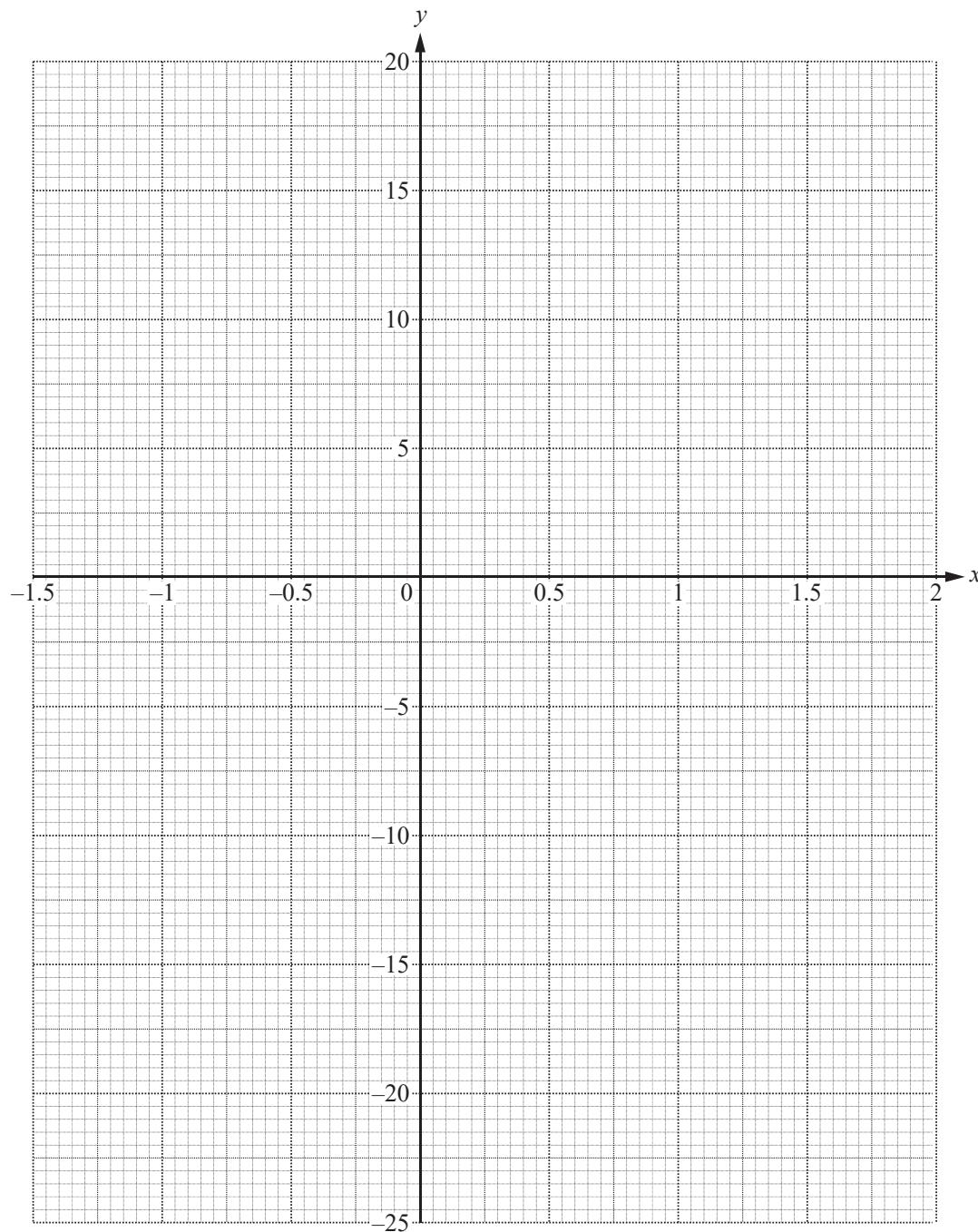
(a) Complete the table of values.

[3]

$x$	-1.5	-1	-0.5	0	0.5	0.75	1	1.5	2
$f(x)$	-24.9			10	8.6	7.6	7		18

(b) Draw the graph of  $y = f(x)$  for  $-1.5 \leq x \leq 2$ .

[4]



- (c) Use your graph to find an **integer** value of  $k$  so that  $f(x) = k$  has
- (i) exactly one solution, [1]
- (ii) three solutions. [1]
- (d) By drawing a suitable straight line on the graph, solve the equation  $f(x) = 15x + 2$  for  $-1.5 \leq x \leq 2$ . [4]
- (e) Draw a tangent to the graph of  $y = f(x)$  at the point where  $x = 1.5$ .  
Use your tangent to estimate the gradient of  $y = f(x)$  when  $x = 1.5$ . [3]

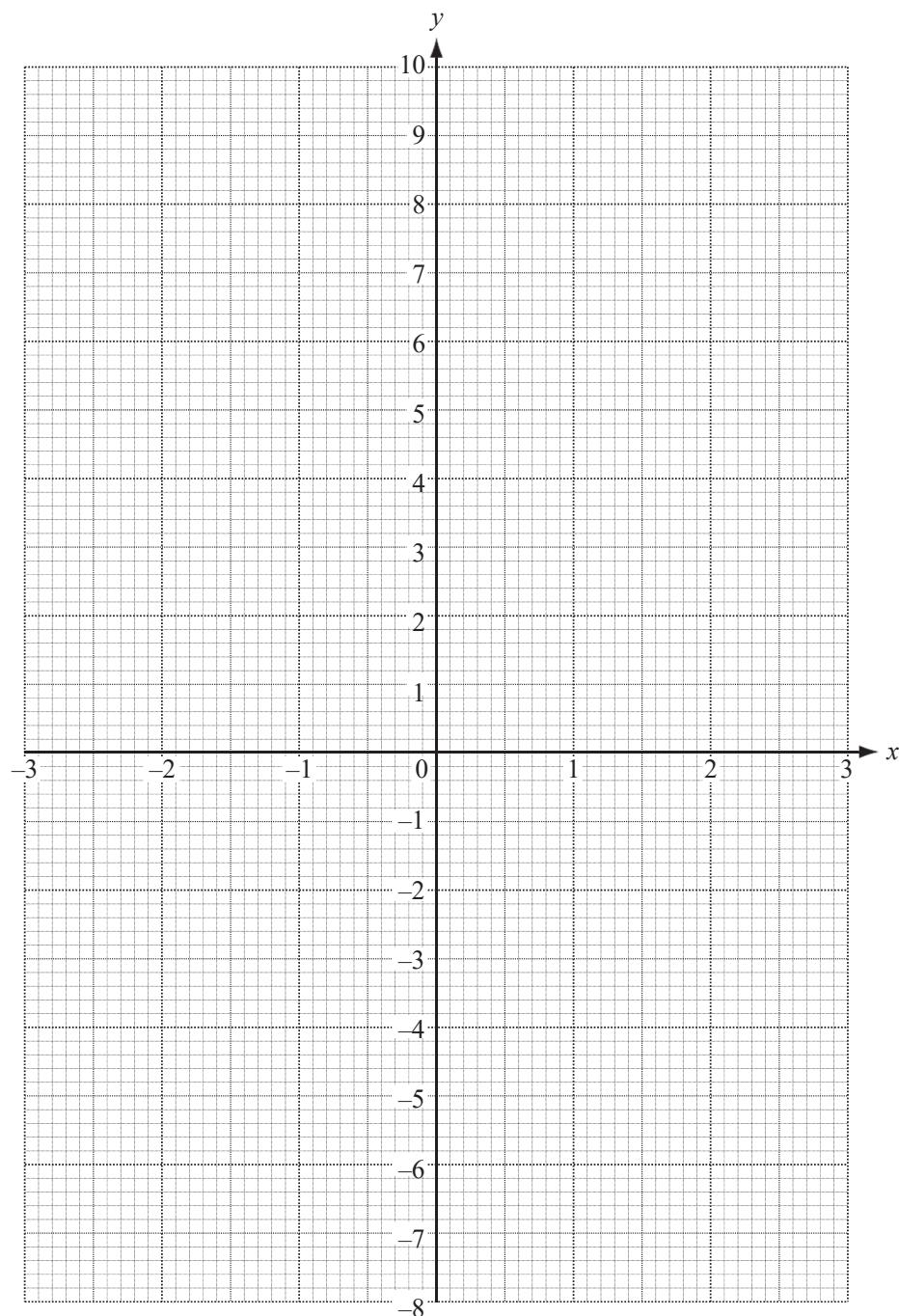
**Question 7**(a) Complete the table of values for  $y = x^3 - 3x + 1$ .

[2]

$x$	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
$y$	-7.125	-1		3		1	-0.375	-1	-0.125	3	9.125

(b) Draw the graph of  $y = x^3 - 3x + 1$  for  $-2.5 \leq x \leq 2.5$ .

[4]



(c) By drawing a suitable tangent, estimate the gradient of the curve at the point where  $x = 2$ . [3]

(d) Use your graph to solve the equation  $x^3 - 3x + 1 = 1$ . [2]

(e) Use your graph to complete the inequality in  $k$  for which the equation

$$x^3 - 3x + 1 = k \text{ has three different solutions.}$$

[2]

# Graphs

## Difficulty: Hard

### Question Paper 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Hard
Booklet	Question Paper 2

**Time allowed:** 135 minutes

**Score:** /117

**Percentage:** /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

##### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

**Question 1**

$$f(x) = \frac{1}{x^2} - 2x, x \neq 0$$

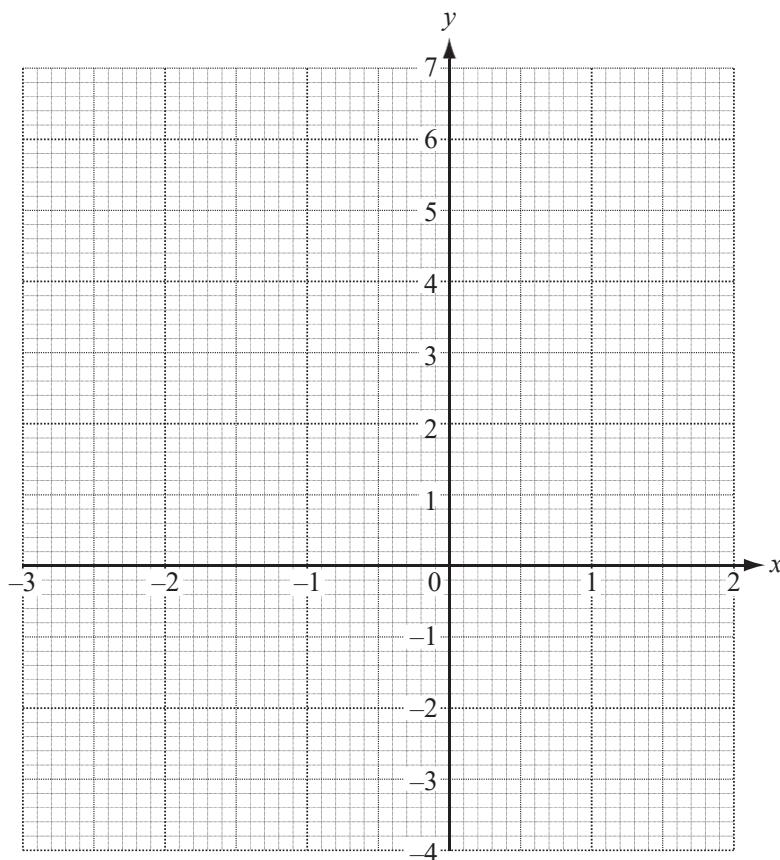
(a) Complete the table of values for  $f(x)$ .

[3]

$x$	-3	-2.5	-2	-1.5	-1	-0.5		0.4	0.5	1	1.5	2
$f(x)$	6.1	5.2	4.3	3.4		5		5.5			-2.6	-3.8

(b) On the grid, draw the graph of  $y = f(x)$  for  $-3 \leq x \leq -0.5$  and  $0.4 \leq x \leq 2$ .

[5]

(c) Solve the equation  $f(x) = 2$ .

[1]

(d) Solve the equation  $f(x) = 2x + 3$ .

[3]

(e) (i) Draw the tangent to the graph of  $y = f(x)$  at the point where  $x = -1.5$ .

[1]

(ii) Use the tangent to estimate the gradient of the graph of  $y = f(x)$  where  $x = -1.5$ .

[2]

**Question 2**

The table shows some values for the function  $y = \frac{1}{x^2} + x$ ,  $x \neq 0$ .

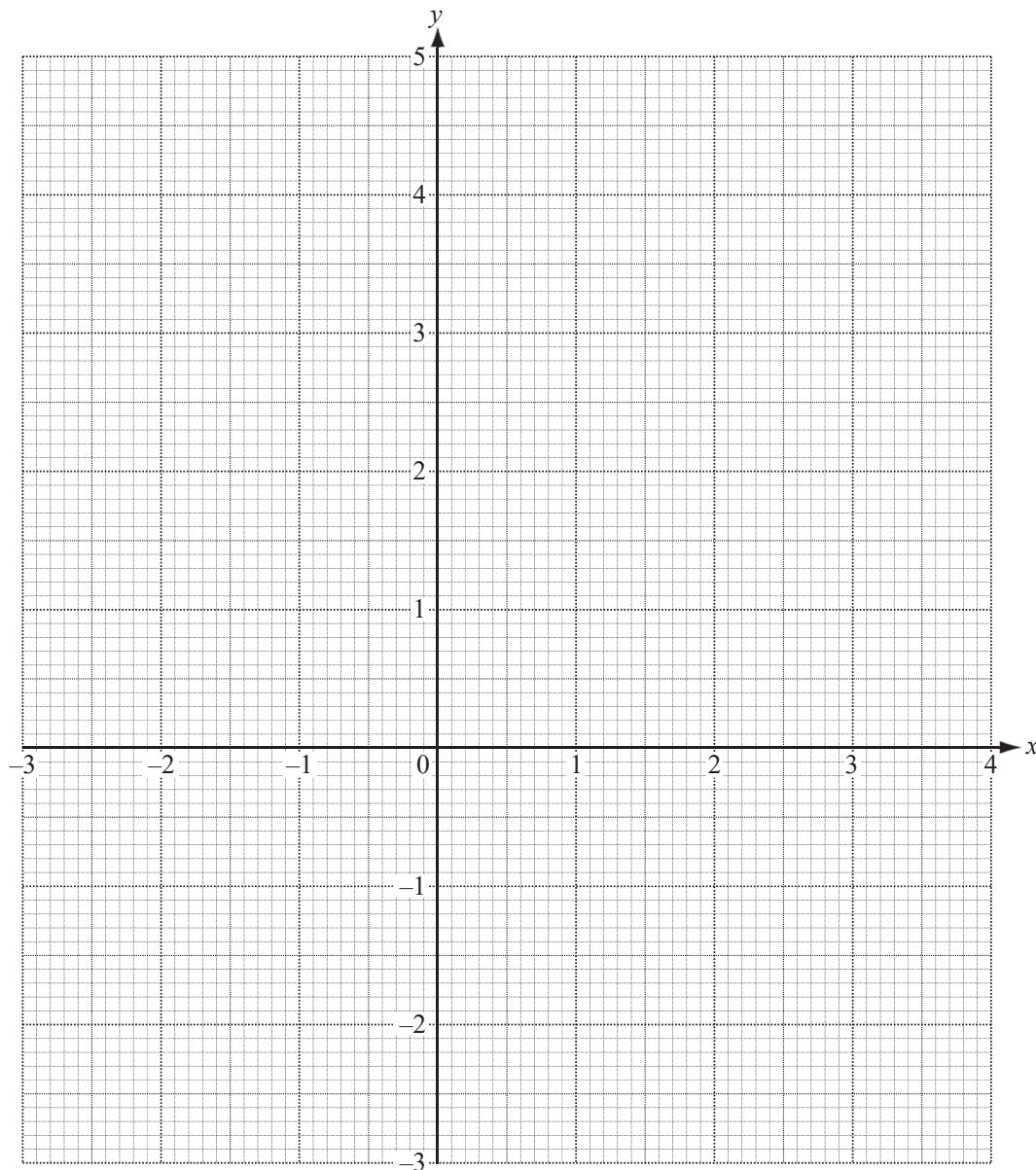
[3]

$x$	-3	-2	-1	-0.5		0.5	1	2	3	4
$y$	-2.89	-1.75		3.5			2	2.25		4.06

(a) Complete the table of values.

(b) On the grid, draw the graph of  $y = \frac{1}{x^2} + x$  for  $-3 \leq x \leq -0.5$  and  $0.5 \leq x \leq 4$ .

[5]



(c) Use your graph to solve the equation  $\frac{1}{x^2} + x - 3 = 0$ . [3]

(d) Use your graph to solve the equation  $\frac{1}{x^2} + x = 1 - x$ . [3]

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where  $x = 2$ .

[3]

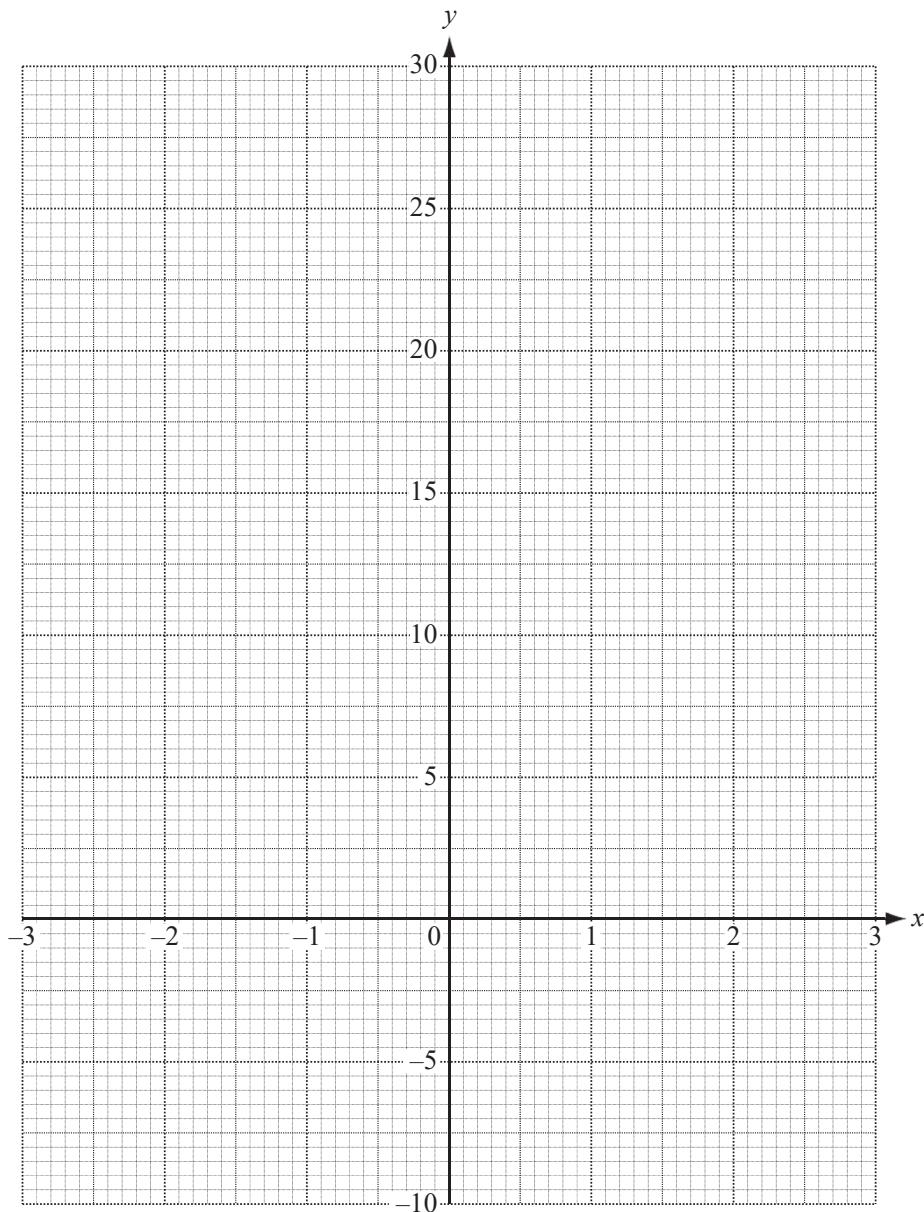
(f) Using algebra, show that you can use the graph at  $y = 0$  to find  $\sqrt[3]{-1}$ . [3]

**Question 3**

(a) Complete the table of values for  $y = \frac{2}{x^2} - \frac{1}{x} - 3x$ . [3]

$x$	-3	-2	-1	-0.5	-0.3		0.3	0.5	1	2	3
$y$	9.6		6		26.5		18.0		-2	-6	-9.1

(b) Draw the graph of  $y = \frac{2}{x^2} - \frac{1}{x} - 3x$  for  $-3 \leq x \leq -0.3$  and  $0.3 \leq x \leq 3$ . [5]



(c) Use your graph to solve these equations.

(i)  $\frac{2}{x^2} - \frac{1}{x} - 3x = 0$  [1]

(ii)  $\frac{2}{x^2} - \frac{1}{x} - 3x - 7.5 = 0$  [3]

(d) (i) By drawing a suitable straight line on the graph, solve the equation  $\frac{2}{x^2} - \frac{1}{x} - 3x = 10 - 3x$ .

[4]

(ii) The equation  $\frac{2}{x^2} - \frac{1}{x} - 3x = 10 - 3x$  can be written in the form  $ax^2 + bx + c = 0$  where  $a, b$  and  $c$  are integers.

Find the values of  $a, b$  and  $c$ . [3]

**Question 4**

$$f(x) = 3 - x - x^2 \quad g(x) = 3^x$$

(a) Complete the tables of values for  $f(x)$  and  $g(x)$ .

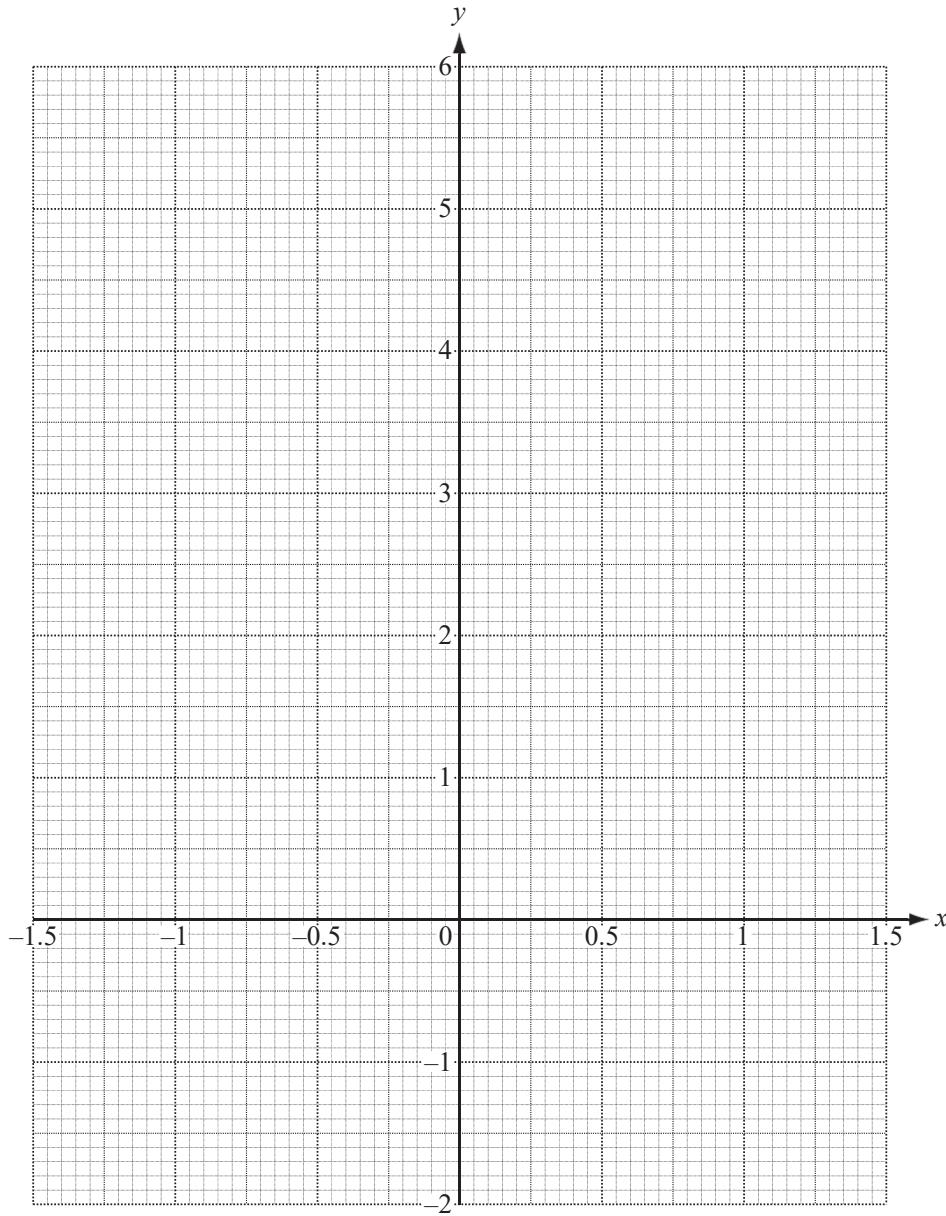
[3]

$x$	-1.5	-1	-0.5	0	0.5	1	1.5
$f(x)$	2.25	3	3.25		2.25	1	-0.75

$x$	-1.5	-1	-0.5	0	0.5	1	1.5
$g(x)$	0.19		0.58		1.73	3	5.20

(b) On the grid, draw the graphs of  $y = f(x)$  and  $y = g(x)$  for  $-1.5 \leq x \leq 1.5$ .

[6]



(c) For  $-1.5 \leq x \leq 1.5$ , use your graphs to solve

(i)  $f(x) = 0$ , [1]

(ii)  $g(x) = 4$ , [1]

(iii)  $f(x) = g(x)$ . [1]

(d) By drawing a suitable tangent, find an estimate of the gradient of the graph of  $y = f(x)$  when  $x = 0.5$ .

[3]

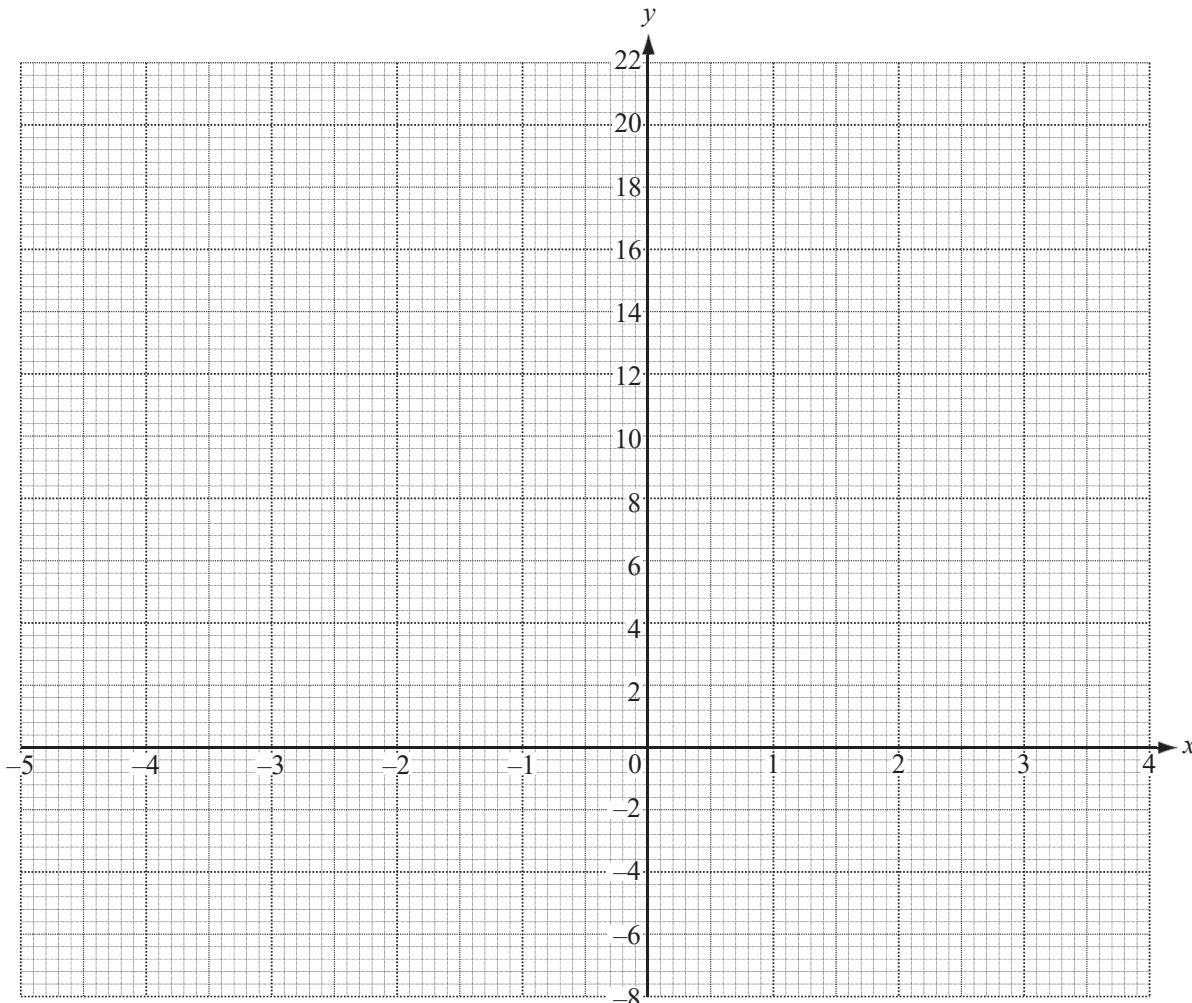
## Question 5

- (a) (i) Complete the table of values for  $y = \frac{1}{2}x^3 + x^2 - 7x$ .

$x$	-5	-4	-3	-2	-1	0	1	2	3	4
$y$	-2.5	12	16.5		7.5	0		-6	1.5	

[3]

- (ii) On the grid, draw the graph of  $y = \frac{1}{2}x^3 + x^2 - 7x$  for  $-5 \leq x \leq 4$ .



[4]

- (b) Use your graph to solve the equation  $\frac{1}{2}x^3 + x^2 - 7x = 2$ .

[3]

(c) By drawing a suitable tangent, calculate an estimate of the gradient of the graph where  $x = 0.4$ .

[3]

(d) (i) On the grid draw the line  $y = 10 - 5x$  for  $-2 \leq x \leq 3$ .

[3]

1  
2

(ii) Use your graphs to solve the equation  $-x^3 + x^2 - 7x = 10 - 5x$ .

[1]

## Question 6

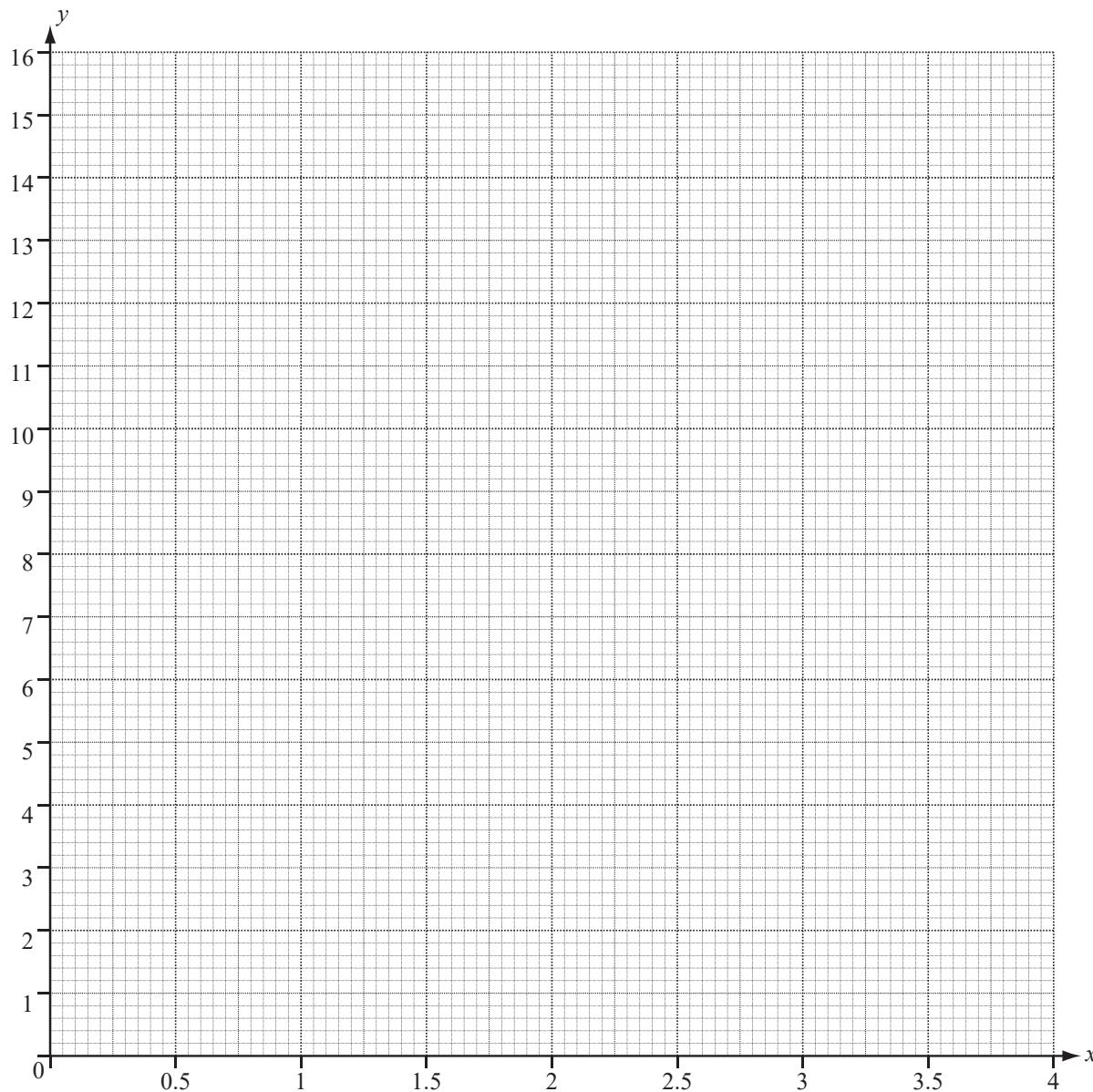
$$f(x) = 2^x$$

(a) Complete the table.

$x$	0	0.5	1	1.5	2	2.5	3	3.5	4
$f(x)$		1.4	2	2.8	4	5.7	8		

[3]

(b) Draw the graph of  $y = f(x)$  for  $0 \leq x \leq 4$ .



[4]

(c) Use your graph to solve the equation  $2^x = 5$ . [1]

(d) Draw a suitable straight line and use it to solve the equation  $2^x = 3x$ . [3]

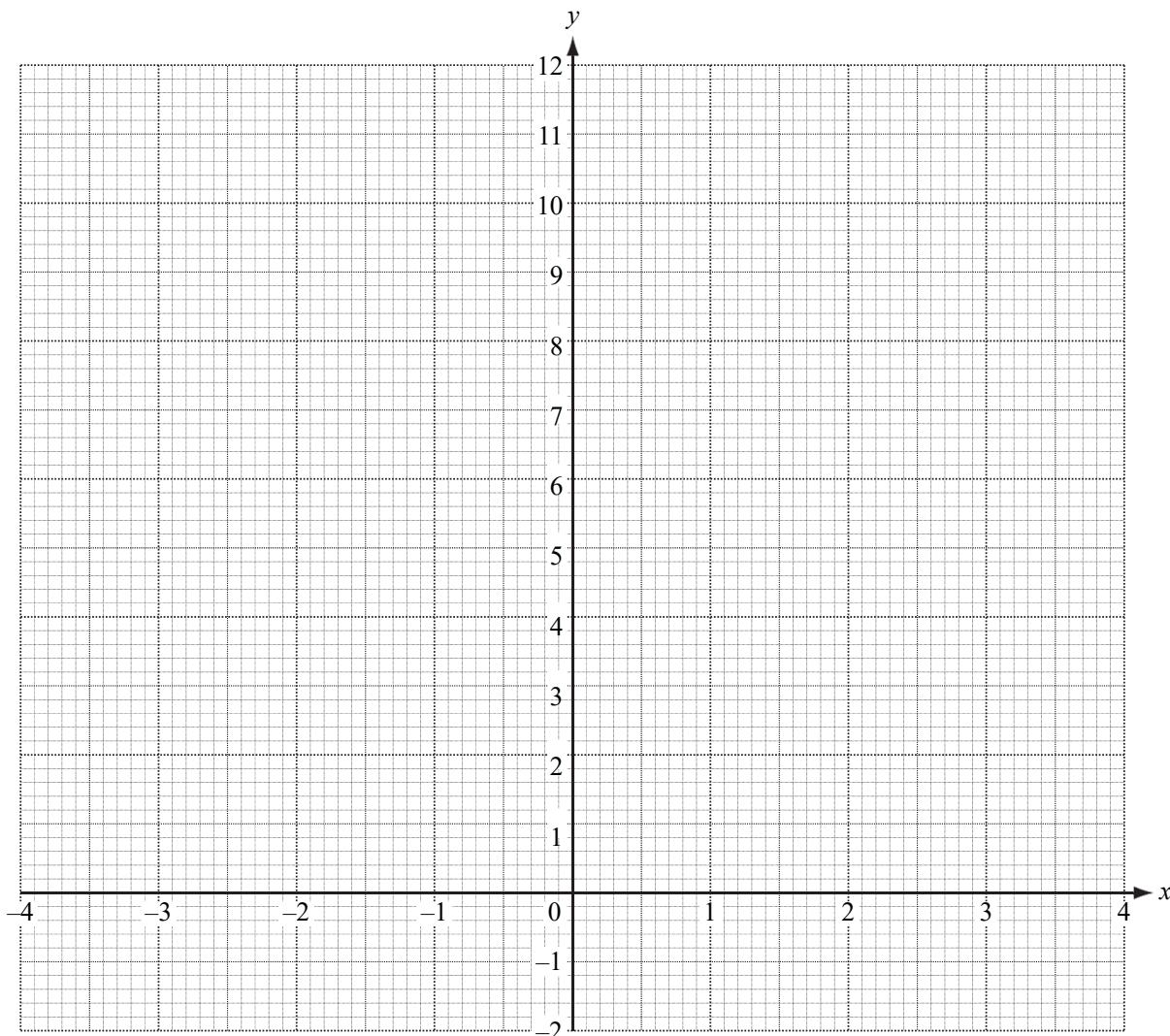
(e) Draw a suitable tangent and use it to find the co-ordinates of the point on the graph of  $y = f(x)$  where the gradient of the graph is 3. [3]

## Question 7

- (a) Complete the table of values for the equation  $y = \frac{4}{x^2}$ ,  $x \neq 0$ . [3]

$x$	- 4	- 3	- 2	- 1	- 0.6		0.6	1	2	3	4
$y$	0.25	0.44			11.11			4.00		0.44	

- (b) On the grid, draw the graph of  $y = \frac{4}{x^2}$  for  $-4 \leq x \leq -0.6$  and  $0.6 \leq x \leq 4$ . [5]



(c) Use your graph to solve the equation  $\frac{4}{x^2} = 6$ . [2]

(d) By drawing a suitable tangent, estimate the gradient of the graph where  $x = 1.5$ . [3]

(e) (i) The equation  $\frac{4}{x^2} - x + 2 = 0$  can be solved by finding the intersection of the graph of  $y = \frac{4}{x^2}$  and a straight line.

Write down the equation of this straight line. [1]

(ii) On the grid, draw the straight line from your answer to part (e)(i). [2]

(iii) Use your graphs to solve the equation  $\frac{4}{x^2} - x + 2 = 0$ . [1]

# Graphs

## Difficulty: Hard

### Question Paper 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Hard
Booklet	Question Paper 3

**Time allowed:** 132minutes

**Score:** /115

**Percentage:** /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

##### CIE IGCSE Maths (0980)

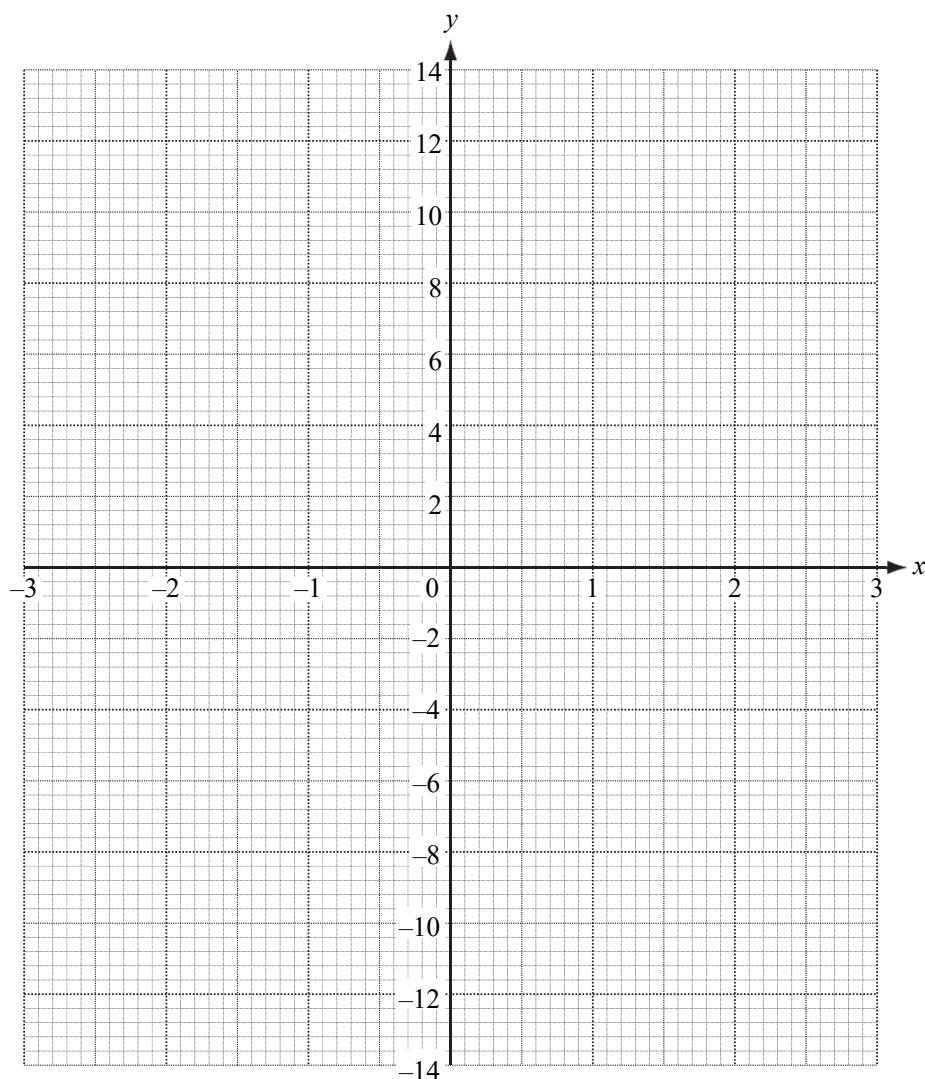
9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

**Question 1**

- (a) Complete the table of values for the function  $y = x^2 - \frac{3}{x}$ ,  $x \neq 0$ . [3]

$x$	- 3	- 2	- 1	- 0.5	- 0.25		0.25	0.5	1	2	3
$y$	10	5.5		6.3	12.1		- 11.9			2.5	8

- (b) Draw the graph of  $y = x^2 - \frac{3}{x}$  for  $-3 \leq x \leq -0.25$  and  $0.25 \leq x \leq 3$ . [5]



(c) Use your graph to solve  $x^2 - \frac{3}{x} = 7$ . [3]

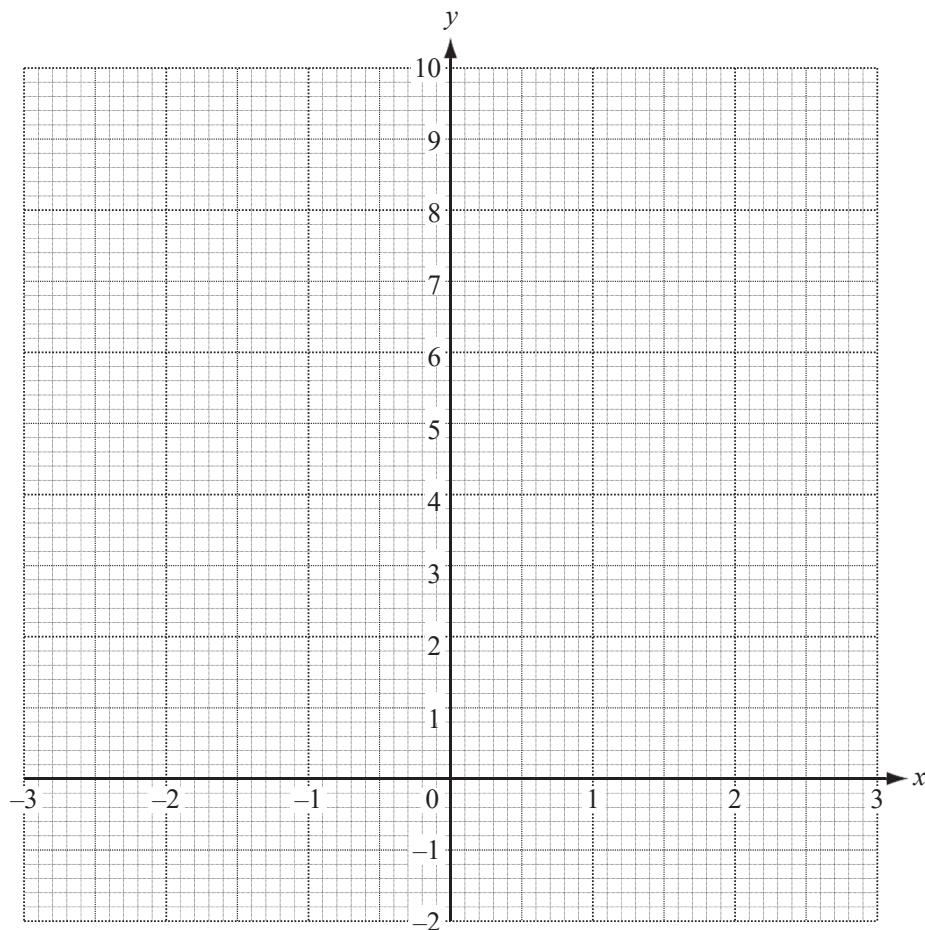
(d) Draw the tangent to the curve where  $x = - 2$ .  
Use the tangent to calculate an estimate of the gradient of the curve where  $x = - 2$ . [3]

**Question 2**

- (a) Complete the table of values for the function  $f(x)$ , where  $f(x) = x^2 + \frac{1}{x^2}$ ,  $x \neq 0$ . [3]

$x$	-3	-2.5	-2	-1.5	-1	-0.5	0.5	1	1.5	2	2.5	3
$f(x)$		6.41		2.69		4.25	4.25		2.69		6.41	

- (b) On the grid, draw the graph of  $y = f(x)$  for  $-3 \leq x \leq -0.5$  and  $0.5 \leq x \leq 3$ . [5]



(c) (i) Write down the equation of the line of symmetry of the graph. [1]

(ii) Draw the tangent to the graph of  $y = f(x)$  where  $x = -1.5$ .  
Use the tangent to estimate the gradient of the graph of  $y = f(x)$  where  $x = -1.5$ . [3]

(iii) Use your graph to solve the equation  $x^2 + \frac{1}{x^2} = 3$ . [2]

(iv) Draw a suitable line on the grid and use your graphs to solve the equation  $x^2 + \frac{1}{x^2} = 2x$ . [3]

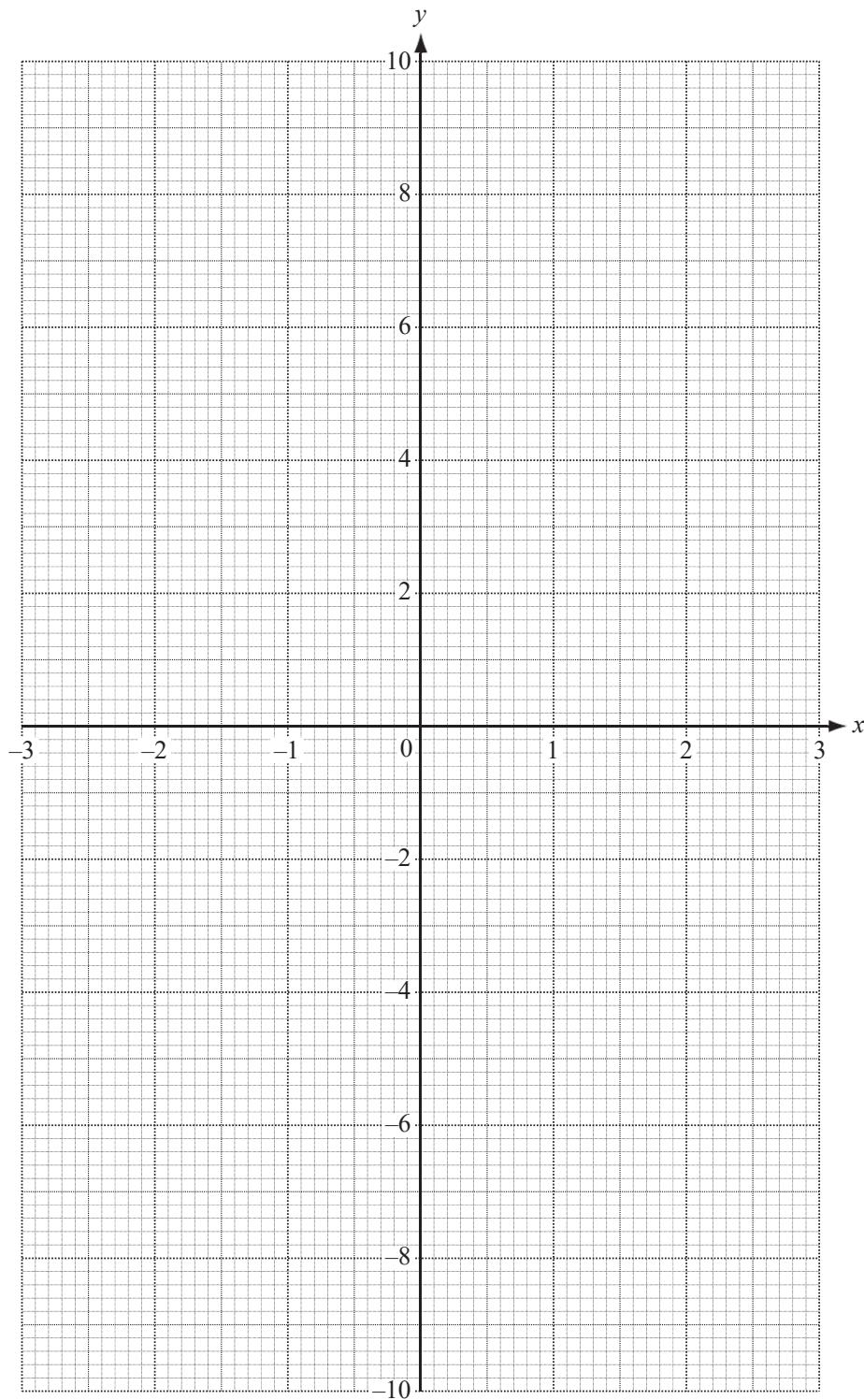
### Question 3

- (a) Complete the table for the function  $f(x) = \frac{2}{x} - x^2$ .

$x$	-3	-2	-1	-0.5	-0.2		0.2	0.5	1	2	3
$f(x)$	-9.7	-5			-10.0		10.0	3.75	1		-8.3

[3]

- (b) On the grid draw the graph of  $y = f(x)$  for  $-3 \leq x \leq -0.2$  and  $0.2 \leq x \leq 3$ .



[5]

(c) Use your graph to

(i) solve  $f(x) = 2$ ,

[1]

(ii) find a value for  $k$  so that  $f(x) = k$  has 3 solutions.

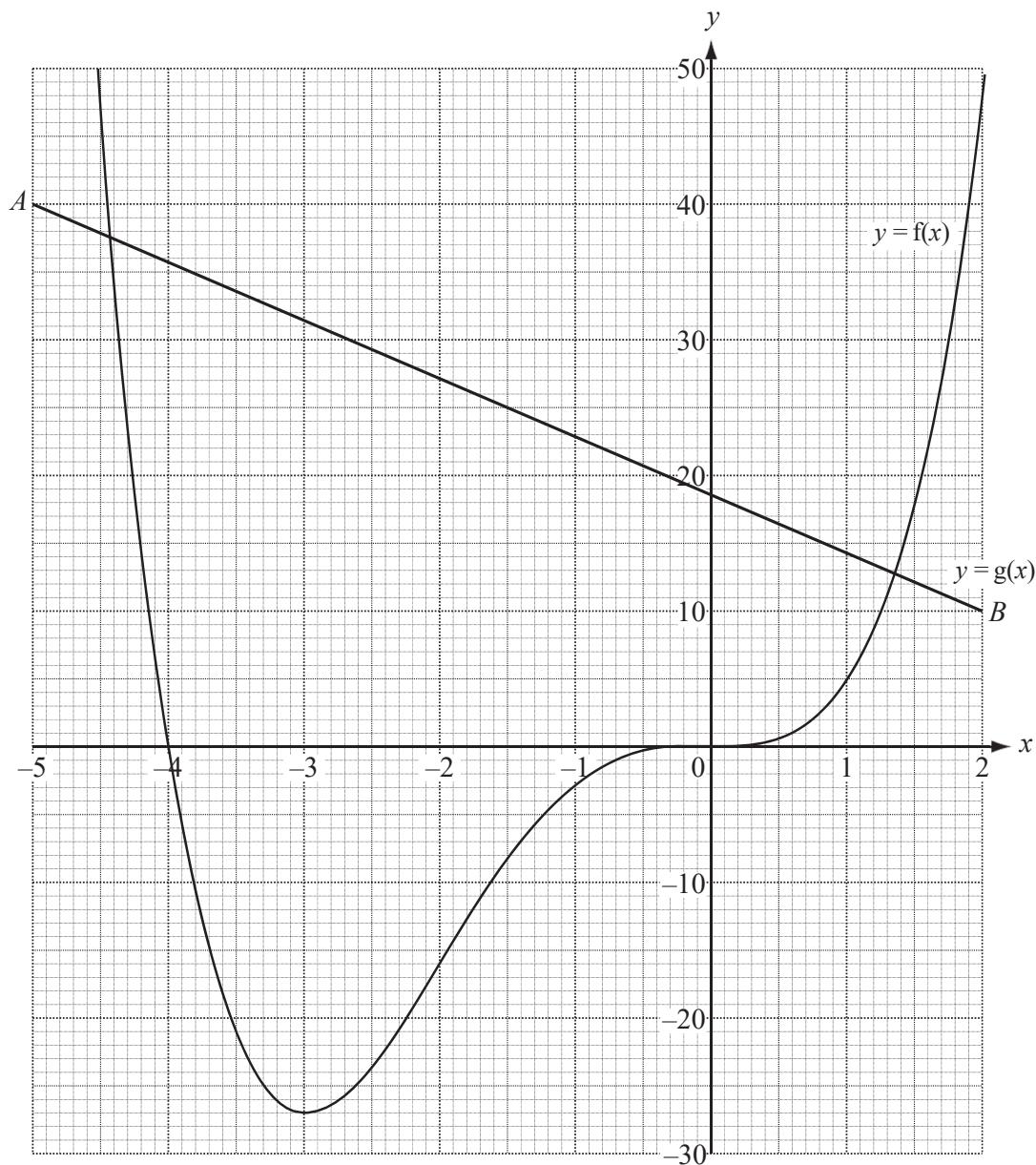
[1]

(d) Draw a suitable line on the grid and use your graphs to solve the equation  $\frac{2}{x} - x^2 = 5x$ . [3]

(e) Draw the tangent to the graph of  $y = f(x)$  at the point where  $x = -2$ .

Use it to calculate an estimate of the gradient of  $y = f(x)$  when  $x = -2$ .

[3]

**Question 4**

The graphs of  $y = f(x)$  and  $y = g(x)$  are shown above.

(a) Find the value of

(i)  $f(-2)$ ,

[1]

(ii)  $g(0)$ .

[1]

(b) Use the graphs to solve

(i) the equation  $f(x) = 20$ , [2]

(ii) the equation  $f(x) = g(x)$ , [2]

(iii) the inequality  $f(x) < g(x)$ . [1]

(c) Use the points  $A$  and  $B$  to find the gradient of  $y = g(x)$  as an exact fraction. [2]

(d) On the grid, draw the graph of  $y = g(x) - 10$ . [2]

(e) (i) Draw the tangent to the graph of  $y = f(x)$  at  $(-3, -27)$ . [1]

(ii) Write down the equation of this tangent. [1]

(f) A region,  $R$ , contains points whose co-ordinates satisfy the inequalities

$$-3 \leq x \leq -2, \quad y \leq 40 \quad \text{and} \quad y \geq g(x).$$

On the grid, draw suitable lines and label this region  $R$ . [2]

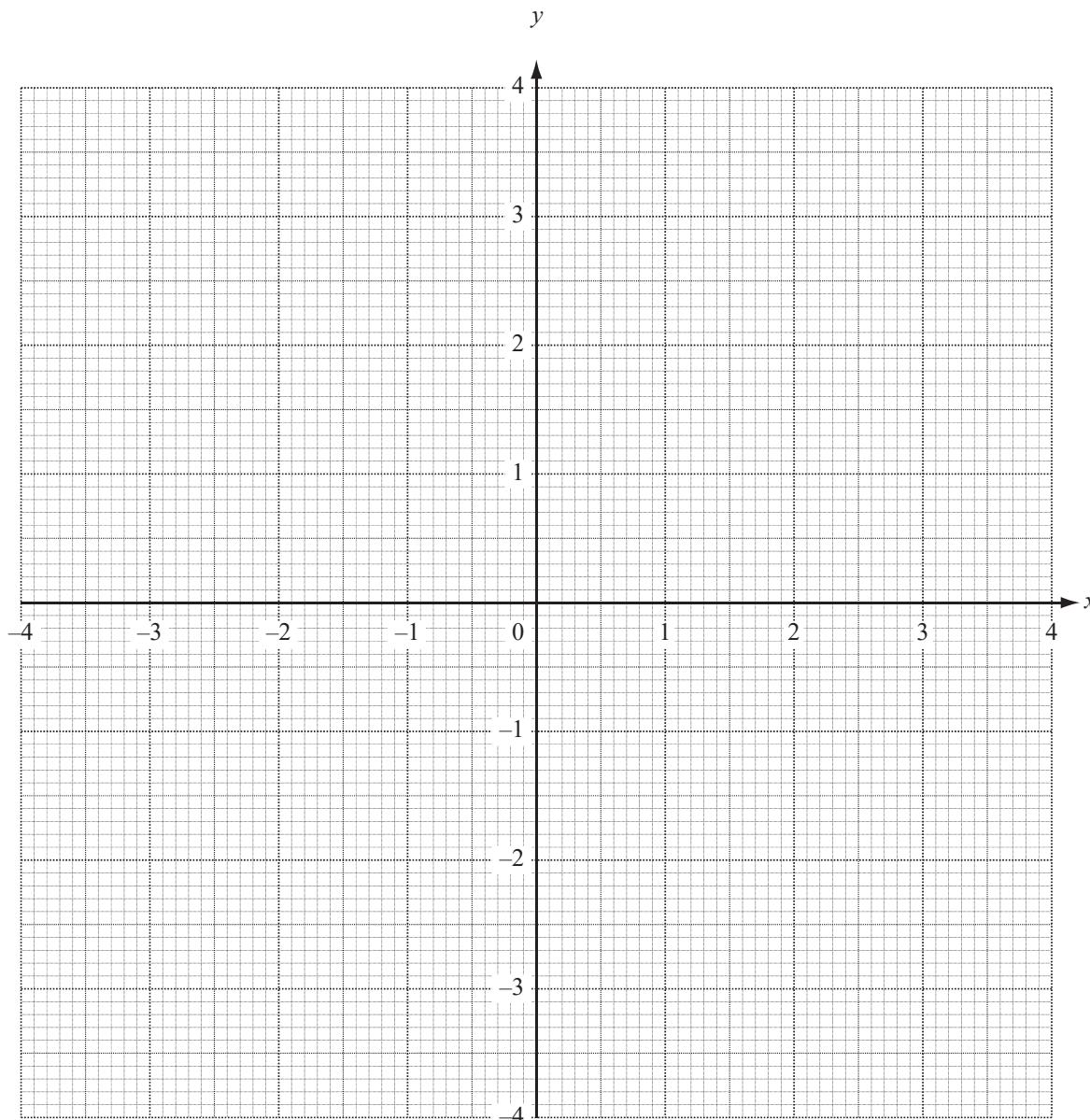
## Question 5

- (a) The table shows some values for the equation  $y = \frac{x}{2} - \frac{2}{x}$  for  $-4 \leq x \leq -0.5$  and  $0.5 \leq x \leq 4$ .

$x$	-4	-3	-2	-1.5	-1	-0.5		0.5	1	1.5	2	3	4
$y$	-1.5	-0.83	0	0.58				-3.75		-0.58	0	0.83	1.5

(i) Write the missing values of  $y$  in the empty spaces. [3]

(ii) On the grid, draw the graph of  $y = \frac{x}{2} - \frac{2}{x}$  for  $-4 \leq x \leq -0.5$  and  $0.5 \leq x \leq 4$ .



[5]

(b) Use your graph to solve the equation  $\frac{x}{2} - \frac{2}{x} = 1$ . [2]

(c) (i) By drawing a tangent, work out the gradient of the graph where  $x = 2$ . [3]

(ii) Write down the gradient of the graph where  $x = -2$ . [1]

(d) (i) On the grid, draw the line  $y = -x$  for  $-4 \leq x \leq 4$ . [1]

(ii) Use your graphs to solve the equation  $\frac{x}{2} - \frac{2}{x} = -x$ . [2]

(e) Write down the equation of a straight line which passes through the origin and does not intersect the graph of  $y = \frac{x}{2} - \frac{2}{x}$ . [2]

## Question 6

Answer the whole of this question on a sheet of graph paper.

The table shows some of the values of the function  $f(x) = x^2 - \frac{1}{x}$ ,  $x \neq 0$ .

$x$	-3	-2	-1	-0.5	-0.2	0.2	0.5	1	2	3
$y$	9.3	4.5	2.0	2.3	$p$	-5.0	-1.8	$q$	3.5	$r$

(a) Find the values of  $p$ ,  $q$  and  $r$ , correct to 1 decimal place. [3]

(b) Using a scale of 2 cm to represent 1 unit on the  $x$ -axis and 1 cm to represent 1 unit on the  $y$ -axis, draw an  $x$ -axis for  $-3 \leq x \leq 3$  and a  $y$ -axis for  $-6 \leq y \leq 10$ .

Draw the graph of  $y = f(x)$  for  $-3 \leq x \leq -0.2$  and  $0.2 \leq x \leq 3$ . [6]

(c) (i) By drawing a suitable straight line, find the three values of  $x$  where  $f(x) = -3x$ . [3]

(ii)  $x^2 - \frac{1}{x} = -3x$  can be written as  $x^3 + ax^2 + b = 0$ .

Find the values of  $a$  and  $b$ . [2]

(d) Draw a tangent to the graph of  $y = f(x)$  at the point where  $x = -2$ .

Use it to estimate the gradient of  $y = f(x)$  when  $x = -2$ . [3]

## Question 7

Answer the whole of this question on a sheet of graph paper.  
Use one side for your working and one side for your graphs.

Alaric invests \$100 at 4% per year compound interest.

- (a) How many dollars will Alaric have after 2 years? [2]

- (b) After  $x$  years, Alaric will have  $y$  dollars.

He knows a formula to calculate  $y$ .

The formula is  $y = 100 \times 1.04^x$

$x$ (Years)	0	10	20	30	40
$y$ (Dollars)	100	$p$	219	$q$	480

Use this formula to calculate the values of  $p$  and  $q$  in the table. [2]

- (c) Using a scale of 2 cm to represent 5 years on the  $x$ -axis and 2 cm to represent \$50 on the  $y$ -axis, draw an  $x$ -axis for  $0 \leq x \leq 40$  and a  $y$ -axis for  $0 \leq y \leq 500$ .

Plot the five points in the table and draw a smooth curve through them. [5]

- (d) Use your graph to estimate

(i) how many dollars Alaric will have after 25 years, [1]

(ii) how many years, to the nearest year, it takes for Alaric to have \$200. [1]

- (e) Beatrice invests \$100 at 7% per year **simple interest**.

(i) Show that after 20 years Beatrice has \$240. [2]

(ii) How many dollars will Beatrice have after 40 years? [1]

(iii) On the **same grid**, draw a graph to show how the \$100 which Beatrice invests will increase during the 40 years. [2]

- (f) Alaric first has more than Beatrice after  $n$  years.

Use your graphs to find the value of  $n$ . [1]

# Graphs

## Difficulty: Hard

### Question Paper 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Graphs
Paper	Paper 4
Difficulty	Hard
Booklet	Question Paper 4

**Time allowed:** 101 minutes

**Score:** /88

**Percentage:** /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

##### CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

## Question 1

Answer the whole of this question on one sheet of graph paper.

$$f(x) = 1 - \frac{1}{x^2}, x \neq 0.$$

(a)

$x$	-3	-2	-1	-0.5	-0.4	-0.3	0.3	0.4	0.5	1	2	3
$f(x)$	$p$	0.75	0	-3	-5.25	$q$	$q$	-5.25	-3	0	0.75	$p$

Find the values of  $p$  and  $q$ .

[2]

(b) (i) Draw an  $x$ -axis for  $-3 \leq x \leq 3$  using 2 cm to represent 1 unit and a  $y$ -axis for  $-11 \leq y \leq 2$  using 1 cm to represent 1 unit.

[1]

(ii) Draw the graph of  $y = f(x)$  for  $-3 \leq x \leq -0.3$  and for  $0.3 \leq x \leq 3$ .

[5]

(c) Write down an integer  $k$  such that  $f(x) = k$  has no solutions.

[1]

(d) On the same grid, draw the graph of  $y = 2x - 5$  for  $-3 \leq x \leq 3$ .

[2]

(e) (i) Use your graphs to find solutions of the equation  $1 - \frac{1}{x^2} = 2x - 5$  [3]

.

(ii) Rearrange  $1 - \frac{1}{x^2} = 2x - 5$  into the form  $ax^3 + bx^2 + c = 0$ , where  $a, b$  and  $c$  are integers. [2]

(f) (i) Draw a tangent to the graph of  $y = f(x)$  which is parallel to the line  $y = 2x - 5$ .

[1]

(ii) Write down the equation of this tangent. [2]

## Question 2

Answer the whole of this question on a sheet of graph paper.

(a) 
$$f(x) = \frac{12}{x+1}$$

$x$	0	1	2	3	4	5	6	7	8	9	10	11
$f(x)$	$p$	6	4	3	2.4	2	1.71	$q$	1.33	$r$	1.09	1

- (i) Calculate the values of  $p$ ,  $q$  and  $r$ . [3]
- (ii) Draw the graph of  $y = f(x)$  for  $0 \leq x \leq 11$ .  
Use a scale of 1cm to 1 unit on each axis. [5]
- (iii) By drawing a suitable line, find an estimate of the gradient of the graph at the point  $(3, 3)$ . [3]
- (b) On the same grid draw the graph of  $y = 8 - x$  for  $0 \leq x \leq 8$ . [2]
- (c) (i) Show that the equation  $f(x) = 8 - x$  simplifies to  $x^2 - 7x + 4 = 0$ . [2]
- (ii) **Use your graph** to solve this equation, giving your answers correct to 1 decimal place. [2]

### Question 3

Answer all of this question on a sheet of graph paper.

(a)  $f(x) = x^2 - x - 3$ .

$x$	-3	-2	-1	0	1	2	3	4
$f(x)$	$p$	3	-1	-3	$q$	-1	3	$r$

(i) Find the values of  $p$ ,  $q$  and  $r$ . [3]

(ii) Draw the graph of  $y = f(x)$  for  $-3 \leq x \leq 4$ .

Use a scale of 1 cm to represent 1 unit on each axis.

[4]

(iii) By drawing a suitable line, estimate the gradient of the graph at the point where  $x = -1$ . [3]

(b)  $g(x) = 6 - \frac{x^3}{3}$ .

$x$	-2	-1	0	1	2	3
$g(x)$	8.67	$u$	$v$	5.67	3.33	-3

(i) Find the values of  $u$  and  $v$ . [2]

(ii) On the same grid as part (a) (ii) draw the graph of  $y = g(x)$  for  $-2 \leq x \leq 3$ . [4]

(c) (i) Show that the equation  $f(x) = g(x)$  simplifies to  $x^3 + 3x^2 - 3x - 27 = 0$ . [1]

(ii) Use your graph to write down a solution of the equation  $x^3 + 3x^2 - 3x - 27 = 0$ . [1]

## Question 4

**Answer the whole of this question on a sheet of graph paper.**

$t$	0	1	2	3	4	5	6	7
$f(t)$	0	25	37.5	43.8	46.9	48.4	49.2	49.6

- (a) Using a scale of 2 cm to represent 1 unit on the horizontal  $t$ -axis and 2 cm to represent 10 units on the  $y$ -axis, draw axes for  $0 \leq t \leq 7$  and  $0 \leq y \leq 60$ .  
Draw the graph of the curve  $y = f(t)$  using the table of values above. [5]
- (b)  $f(t) = 50(1 - 2^{-t})$ .
- (i) Calculate the value of  $f(8)$  and the value of  $f(9)$ . [2]
- (ii) Estimate the value of  $f(t)$  when  $t$  is large. [1]
- (c) (i) Draw the tangent to  $y = f(t)$  at  $t = 2$  and use it to calculate an estimate of the gradient of the curve at this point. [3]
- (ii) The function  $f(t)$  represents the speed of a particle at time  $t$ .  
Write down what quantity the gradient gives. [1]
- (d) (i) On the same grid, draw  $y = g(t)$  where  $g(t) = 6t + 10$ , for  $0 \leq t \leq 7$ . [2]
- (ii) Write down the range of values for  $t$  where  $f(t) > g(t)$ . [2]
- (iii) The function  $g(t)$  represents the speed of a second particle at time  $t$ .  
State whether the first or second particle travels the greater distance for  $0 \leq t \leq 7$ .  
You **must** give a reason for your answer. [2]

## Question 5

Answer the whole of this question on a sheet of graph paper.

$x$	04	03	02	01	0	1	2	3	4
$f(x)$	08	4.5	8	5.5	0	05.5	08	04.5	8

- (a) Using a scale of 2 cm to represent 1 unit on the  $x$ -axis and 2 cm to represent 4 units on the  $y$ -axis, draw axes for  $-4 \leq x \leq 4$  and  $-8 \leq y \leq 8$ .  
Draw the curve  $y = f(x)$  using the table of values given above. [5]

- (b) Use your graph to solve the equation  $f(x) = 0$ . [2]

- (c) On the same grid, draw  $y = g(x)$  for  $-4 \leq x \leq 4$ , where  $g(x) = x + 1$ . [2]

- (d) Write down the value of

- (i)  $g(1)$ ,
- (ii)  $fg(1)$ ,
- (iii)  $g^{01}(4)$ ,
- (iv) the **positive** solution of  $f(x) = g(x)$ .

[4]

- (e) Draw the tangent to  $y = f(x)$  at  $x = 3$ . Use it to calculate an estimate of the gradient of the curve at this point. [3]