Co-ordinate Geometry
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Co-ordinate Geometry
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **78 minutes**

Score: **/68**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) ~~Assembled by NS~~

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

A line joins the points $A(-3,\ 8)$ and $B(2,-\ 2)$.

(a) Find the co-ordinates of the midpoint of AB. [2]

Midpoint is calculated as

$$M = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

$$= \left(\frac{-3+2}{2}, \frac{8-2}{2}\right)$$

$$= \left(-\frac{1}{2}, 3\right)$$

(b) Find the equation of the line through A and B.
Give your answer in the form $y = mx + c$. [3]

Gradient is found as $m = \frac{y_B - y_A}{x_B - x_A}$

$$= \frac{-2-8}{2--3}$$

$$= -\frac{10}{5}$$

$$= -2$$

Now use the straight-line equation with point A

$$y - y_1 = m(x - x_1)$$

$$\rightarrow y - 8 = -2(x+3)$$

$$\rightarrow y = -2x - 6 + 8$$

$$\rightarrow y = -2x + 2$$

(c) Another line is parallel to AB and passes through the point (0, 7).

Write down the equation of this line. [2]

Parallel so it has the same gradient.

Again, using the straight-line equation

$$y - y_1 = m(x - x_1)$$

$$\rightarrow y - 7 = -2(x - 0)$$

$$\rightarrow \boldsymbol{y = -2x + 7}$$

(d) Find the equation of the line perpendicular to AB which passes through the point (1, 5).
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. [4]

Perpendicular so the gradient is

$$m = -\frac{1}{-2}$$

$$= \frac{1}{2}$$

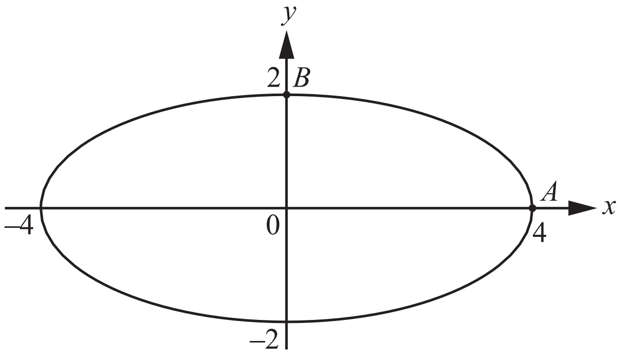
Again, using the straight-line equation

$$y - y_1 = m(x - x_1)$$

$$\rightarrow y - 5 = \frac{1}{2}(x - 1)$$

$$\rightarrow 2y - 10 = x - 1$$

$$\rightarrow \boldsymbol{x - 2y + 9 = 0}$$



NOT TO SCALE

The diagram shows a curve with equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

(a) A is the point (4, 0) and B is the point (0, 2).

(i) Find the equation of the straight line that passes through A and B.
Give your answer in the form $y = mx + c$. [3]

$$m\ (gradient) = \frac{-2}{4}$$

$$= \frac{-1}{2}$$

$$y = \frac{-1}{2}x + 2$$

(ii) Show that $a^2 = 16$ and $b^2 = 4$. [2]

$\underline{When\ x = 0, y = 2}$

$$\frac{0^2}{a^2} + \frac{2^2}{b^2} = 1$$

$$\frac{2^2}{b^2} = 1$$

$$4 = b^2$$

$$\boldsymbol{b^2 = 4}$$

$\underline{When\ x = 4, y = 0}$

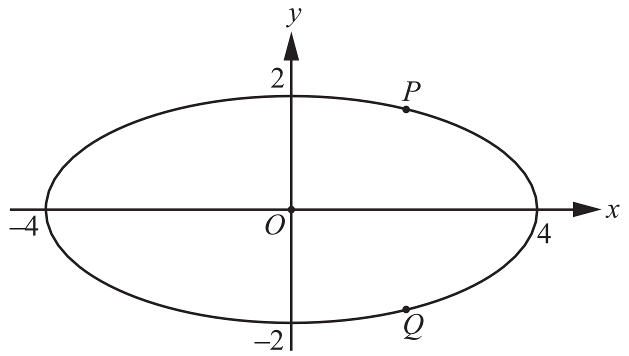
$$\frac{4^2}{a^2} + \frac{0^2}{b^2} = 1$$

$$\frac{4^2}{a^2} = 1$$

$$16 = a^2$$

$$\boldsymbol{a^2 = 16}$$

(b)



NOT TO SCALE

$P\ (2, k)$ and $Q\ (2, -k)$ are points on the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

(i) Find the value of k. [3]

$$\frac{2^2}{16} + \frac{k^2}{4} = 1$$

$$\frac{1}{4} + \frac{k^2}{4} = 1$$

$$k^2 = 3$$

$$\boldsymbol{k = \sqrt{3}}$$

(ii) Calculate angle POQ. [3]

$$\tan x = \frac{\sqrt{3}}{2}$$

$$x = \tan^{-1}\frac{\sqrt{3}}{2}$$

$$Angle\ POQ = 2x$$

$$= \mathbf{81.8°}$$

(c) The area enclosed by a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(i) Find the area enclosed by the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

Give your answer as a multiple of π. [1]

$Area = \pi ab$

$= \pi \times 4 \times 2$

$= \mathbf{8\pi}$

(ii) A curve, mathematically similar to the one in the diagrams, intersects the x-axis at (12, 0) and (-12, 0).

Work out the area enclosed by this curve, giving your answer as a multiple of π. [2]

Original ellipse has been enlarged by a scale factor of 3 to for the new Ellipse. (Eg. (4,0) maps to (12,0)).

Linear scale factor of the enlargement is 3, and hence the area scale factor is $3^2 = 9$)

$Area = area\ of\ original\ ellipse\ x\ 9$

$= 8\pi \times 9$

$= \mathbf{72\pi}$

A line joins the points A (–2, –5) and B (4, 13).

(a) Calculate the length AB. [3]

Length of a line is given by

$$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Here, that is

$$\sqrt{(13+5)^2 + (4+2)^2}$$

$$= \sqrt{324 + 36}$$

$$= \mathbf{18.97}$$

(b) Find the equation of the line through A and B.
Give your answer in the form $y = mx + c$. [3]

The gradient of the line can be found as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{13+5}{4+2}$$

$$= 3$$

Using the straight-line equation

$$y - y_1 = m(x - x_1)$$

With our gradient and the point B we get

$$y - 13 = 3(x - 4)$$

$$\rightarrow y = 3x - 12 + 13$$

$$\rightarrow \boldsymbol{y = 3x + 1}$$

(c) Another line is parallel to AB and passes through the point (0, –5).

Write down the equation of this line. [2]

Parallel means it has the same gradient. This new line, using the same straight-line equation as before, is

$$y + 5 = 3(x - 0)$$

$$\rightarrow \boldsymbol{y = 3x - 5}$$

(d) Find the equation of the perpendicular bisector of AB. [5]

Perpendicular bisector means that it has a perpendicular gradient to line AB and it cuts through the midpoint. The perpendicular gradient is

$$-1 \div 3$$

$$= -\frac{1}{3}$$

The midpoint is

$$M = \left(\frac{4-2}{2}, \frac{13-5}{2}\right)$$

$$= (1, 4)$$

The perpendicular bisector then has the equation

$$y - 4 = -\frac{1}{3}(x - 1)$$

$$\rightarrow 3y - 12 = -x + 1$$

$$\rightarrow \boldsymbol{x + 3y - 13 = 0}$$

A line AB joins the points A (3, 4) and B (5, 8).

(a) Write down the co-ordinates of the midpoint of the line AB. [2]

The midpoint is simply calculated by summing the coordinates of the two points (treating them as vectors) and dividing by 2.

$$\frac{1}{2}\left(\begin{pmatrix}3\\4\end{pmatrix}+\begin{pmatrix}5\\8\end{pmatrix}\right)=\frac{1}{2}\begin{pmatrix}8\\12\end{pmatrix}=\begin{pmatrix}4\\6\end{pmatrix}$$

Therefore the midpoint has co-ordinates **(4 , 6)**

(b) Calculate the distance AB. [3]

First, calculate the line AB as if it was a vector (AB=BO-OA)

$$AB=\begin{pmatrix}5\\8\end{pmatrix}-\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}2\\4\end{pmatrix}$$

Second, we calculate the distance AB by taking the square root of the sum of squares of components of vector AB.

$$|AB|=\sqrt{2^2+4^2}=\sqrt{20}=4.47$$

Hence the distance AB is **4.47**.

(c) Find the equation of the line AB. [3]

The general equation of a line is y=mx+c where m is the gradient and c is a constant.

The gradient is found as the change of y-coordinate over the change of x-coordinate

between points (3,4) and (5,8). Gradient: $m = \frac{8-(4)}{5-(3)} = \frac{4}{2} = 2$.Hence the gradient is 2.

We want the line to pass through point (3,4), therefore the equation must be satisfied:

$$4 = 2 \times 3 + c$$

Subtract 6 from both sides to get the answer:

$$-2 = c$$

Therefore the equation of the line passing through A and B is

$\boldsymbol{y = 2x - 2.}$

(d) A line perpendicular to AB passes through the origin and through the point $(6, r)$.

Find the value of r. [3]

The gradient of line *y=2x-2* is *m*=2 (it is the factor multiplying variable *x*).

The gradient of a perpendicular line *n* is found as negative reciprocal of the original gradient:

$$n = -\frac{1}{m}$$

Therefore the gradient of the new line is:

$$n = -\frac{1}{2}$$

The general equation for a line is $y = -\frac{1}{2}x + p$ where *p* is a constant. This constant is decided by the point through which the equation passes.

We want the new line to pass through the origin (0,0), therefore *p*=0.

$$y = -\frac{1}{2}x$$

When *x*=6, *y*=*r*.

$$r = -\frac{1}{2} \times 6$$

$$= \mathbf{-3}$$

(a) A straight line joins the points (–1, –4) and (3, 8).

(i) Find the midpoint of this line. [2]

The midpoint of a line is

$$\boldsymbol{M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

$$= \left(\frac{-1 + 3}{2}, \frac{-4 + 8}{2}\right)$$

$$= \boldsymbol{(1, 2)}$$

(ii) Find the equation of this line.
Give your answer in the form $y = mx + c$. [3]

Gradient of a line is

$$\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$= \frac{8 - -4}{3 - -1}$$

$$= \frac{12}{4}$$

$$= 3$$

Straight-line equation is

$$y - y_1 = m(x - x_1)$$

$$\rightarrow y - 8 = 3(x - 3)$$

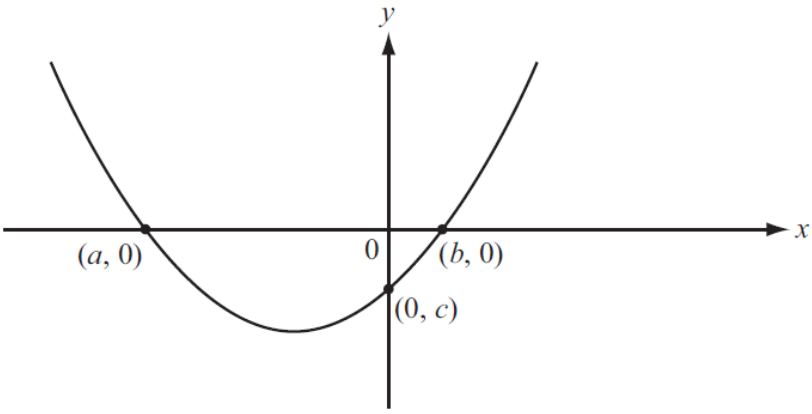
$$\rightarrow y = 3x - 9 + 8$$

$$\rightarrow \boldsymbol{y = 3x - 1}$$

(b) (i) Factorise $x^2 + 3x - 10$. [2]

$(\boldsymbol{x} + 5)(\boldsymbol{x} - 2)$

(ii) The graph of $y = x^2 + 3x - 10$ is sketched below.



NOT TO SCALE

Write down the values of a, b and c. [3]

a and b found by

$$x^2 + 3x - 10 = 0$$

$$\rightarrow (x + 5)(x - 2) = 0$$

$$\rightarrow \boldsymbol{a = -5}, \qquad \boldsymbol{b = 2}$$

c is found by

$$c = 0^2 + 3(0) - 10$$

$$\rightarrow \boldsymbol{c = -10}$$

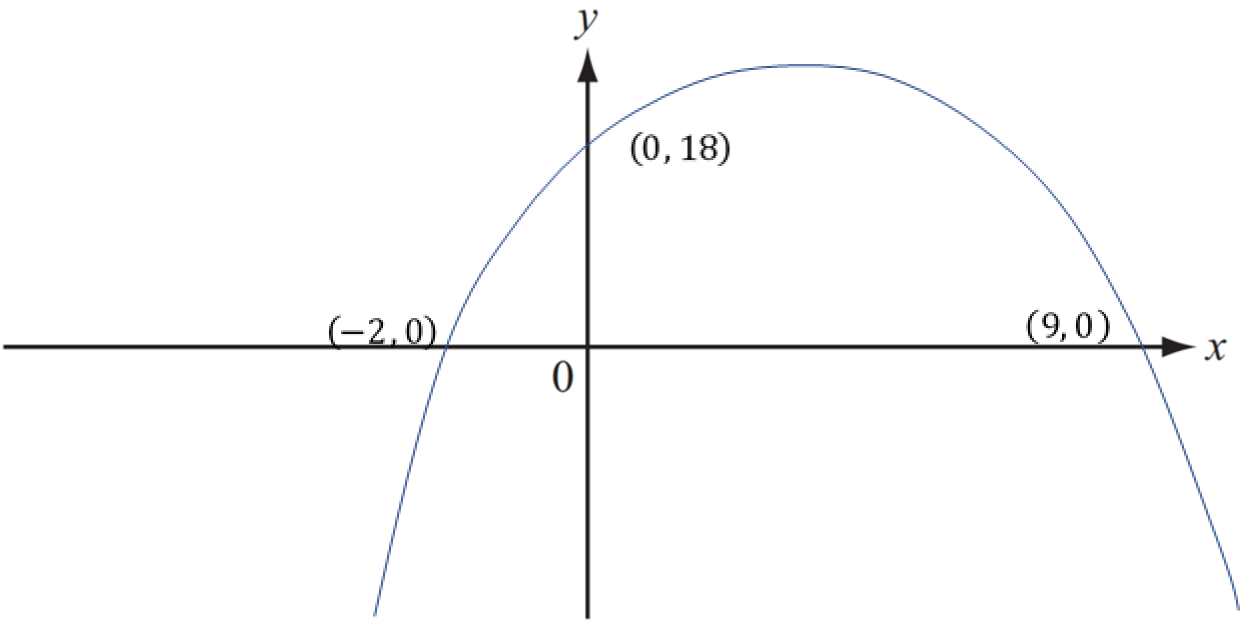
(iii) Write down the equation of the line of symmetry of the graph of $y = x^2 + 3x - 10$. [1]

The line of symmetry will be the midpoint of a and b

$$x = \frac{a + b}{2}$$

$$\rightarrow \boldsymbol{x = -\frac{3}{2}}$$

(c) Sketch the graph of $y = 18 + 7x - x^2$ on the axes below.
Indicate clearly the values where the graph crosses the x and y axes. [4]



Factorise it to find x-axis intercepts

$$-(x^2 - 7x - 18)$$

$$= -(x-9)(x+2)$$

Hence intercepts at $x = 9, x = -2$.

(d) (i) $x^2 + 12x - 7 = (x + p)^2 - q$

Find the value of p and the value of q. [3]

Complete the square on the LHS

$$(x+6)^2 - 36 - 7$$

$$= (x+6)^2 - 43$$

Hence

$$p = 6$$

$$q = 43$$

(ii) Write down the minimum value of y for the graph of $y = x^2 + 12x - 7$. [1]

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