Sequences
Difficulty: Medium

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 1

Time allowed: **63 minutes**

Score: **/55**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) Assembled by AS

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a) The nth term of a sequence is $8n - 3$.

(i) Write down the first two terms of this sequence. [1]

$$1st: \ 8(1) - 3 = \mathbf{5}$$

$$2nd: \ 8(2) - 3 = \mathbf{13}$$

(ii) Show that the number 203 is not in this sequence. [2]

Try to solve

$$8n - 3 = 203$$

Add 3 to both sides

$$8n = 206$$

Divide through by 8

$$\boldsymbol{n = 25.75}$$

n must be an integer

(b) Find the nth term of these sequences.

(i) 13, 19, 25, 31, … [2]

$$\boldsymbol{nth}: \ \mathbf{6}\boldsymbol{n} + \mathbf{7}$$

(ii) 4, 8, 14, 22, … [2]

$$\boldsymbol{nth}: \ \boldsymbol{n}^{\mathbf{2}} + \boldsymbol{n} + \mathbf{2}$$

(c) $\ldots\ ,\quad 20,\quad 50,\quad \ldots$

The second term of this sequence is 20 and the third term is 50.
The rule for finding the next term in this sequence is subtract y then multiply by 5.

Find the value of y and work out the first term of this sequence. [4]

We have that

$5(a-y)=20$ (1)

$5(20-y)=50$ (2)

Divide (2) through by 5

$$\rightarrow \boldsymbol{y=10}$$

$$20-y=10$$

Plug this into (1)

$$5(a-10)=20$$

Divide through by 5

$$a-10=4$$

add 10 to both sides

$$\boldsymbol{a=14}$$

(a) Complete the table for the four sequences A, B, C and D.

	Sequence				Next term	nth term
A	2	5	8	11	**14**	$\mathbf{3n - 1}$
B	20	14	8	2	**−4**	$\mathbf{-6n + 26}$
C	1	4	9	16	**25**	$\mathbf{n^2}$
D	0	2	6	12	**20**	$\mathbf{n^2 - n}$

[10]

Working:

A: There is a common difference between terms of $+3$ so the formula will be of the form $n^{th}\ term = 3n + c$. Looking at $n = 1$ we see that $c = -1$.

B: There is a common difference between terms of -6 so the formula will be of the for $n^{th}\ term = -6n + c$. Looking at $n = 1$ we see that $c = 26$.

C: These should be recognized as the square numbers: $1^2,\ 2^2,\ 3^2,\ 4^2$. The next term will therefore be 5^2 and so the n^{th} term will be n^2.

D: For sequences like this, where the differences go up by 2 each time, the "next term" should be straightforward (add 2 more than the previous difference). For the n^{th} term it is easiest if a relationship with the n^2 sequence above can be spotted. A little thought shows that the first four terms are $1^2 - 1,\ 2^2 - 2,\ 3^2 - 3,\ 4^2 - 4$ and so the n^{th} term will be $n^2 - n$.

(b) The sum of the first n terms of a sequence is $\dfrac{n(3n+1)}{2}$.

(i) When the sum of the first n terms is 155, show that $3n^2 + n - 310 = 0$. [2]

$$\frac{n(3n+1)}{2} = 155$$

Multiply by 2:

$$n(3n+1) = 310$$

Expand the bracket:

$$3n^2 + n = 310$$

Subtract 310 from both sides:

$$\mathbf{3n^2 + n - 310 = 0}$$

(ii) Solve $3n^2 + n - 310 = 0$. [3]

Factorise (or use Quadratic Formula):

$$(3n + 31)(n - 10) = 0$$

And so:

$$n = -\frac{31}{3} \text{ or } n = 10$$

(iii) Complete the statement.

The sum of the first ......10...... terms of this sequence is 155. [1]

Since n must be positive we ignore the other root $\left(^{-31}/_{3}\right)$ in (ii)

The first three diagrams in a sequence are shown below.
The diagrams are made by drawing lines of length 1 cm.

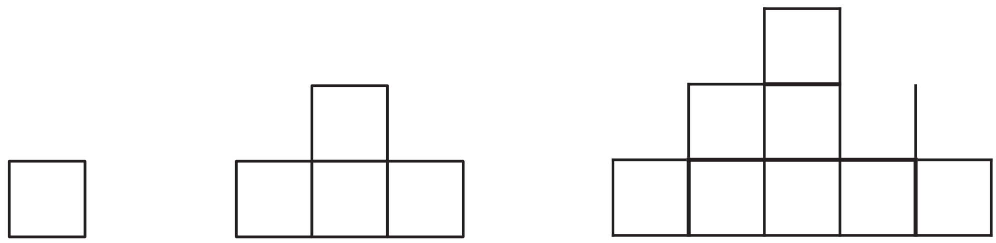


Diagram 1 Diagram 2 Diagram 3

(a) The areas of each of the first three diagrams are shown in this table.

Diagram	1	2	3
Area (cm^2)	1	4	9

(i) Find the area of Diagram 4. [1]

The area is the square of the base of the diagram.

For Diagram 4:

$$area = 4^2 = \mathbf{16}$$

(ii) Find, in terms of n, the area of Diagram n. [1]

As mentioned in part a)i), the area is the square of the base.

For Diagram n:

$$area = \boldsymbol{n^2}$$

(b) The numbers of 1 cm lines needed to draw each of the first three diagrams are shown in this table.

Diagram	1	2	3
Number of 1 cm lines	4	13	26

(i) Find the number of 1 cm lines needed to draw Diagram 4. [1]

Between Diagram 1 and Diagram 2, there is a difference of 9 lines.

Between Diagram 2 and Diagram 3, there is a difference of 13 lines, which is more by 4 lines.

To get the next difference, add 4. We get that the difference between Diagram 3 and Diagram 4 should be 17.

$$26 + 17 = 43$$

Number of 1 cm lines needed to draw Diagram 4 is **43**.

(ii) In which diagram are 118 lines of length 1 cm needed? [1]

To get the lines in Diagram 5, we add 21 (=17+4) to the number of lines in Diagram 4.

$$Diagram\ 5 = 43 + 21 = 64$$

To get the lines in Diagram 6, we add 25 (=21+4) to the number of lines in Diagram 5.

$$Diagram\ 6 = 64 + 25 = 89$$

To get the lines in Diagram 7, we add 29 (=25+4) to the number of lines in Diagram 6.

$$Diagram\ 7 = 89 + 29 = 118$$

There are 118 lines of length 1cm needed to draw **Diagram 7**.

(c) The **total** number of 1 cm lines needed to draw both Diagram 1 and Diagram 2 is 17.
The **total** number of 1 cm lines needed to draw all of the first n diagrams is

$$\frac{2}{3}n^3 + an^2 + bn.$$

Find the value of a and the value of b.
Show all your working. [6]

To draw Diagram 1 (n=1), 4 lines are needed.

$$\frac{2}{3}(1)^3 + a(1)^2 + b \times 1 = 4$$

To draw Diagrams 1 and 2 (n=2), 17 lines are needed.

$$\frac{2}{3}(2)^3 + a(2)^2 + b \times 2 = 17$$

We have two simultaneous equations:

$$\frac{2}{3} + a + b = 4$$

$$\frac{16}{3} + 4a + 2b = 17$$

Subtract twice the first equation from the second equation.

$$\frac{16}{3} + 4a + 2b - 2 \times \left(\frac{2}{3} + a + b\right) = 17 - 2 \times 4$$

$$\frac{12}{3} + 2a = 9$$

Subtract 4 from both sides of the equation.

$$2a = 5$$

Divide both sides by 2 to get the value of a.

$$\boldsymbol{a = 2.5}$$

Insert the value of *a* into the first equation.

$$\frac{2}{3} + 2.5 + b = 4$$

$$\frac{19}{6} + b = 4$$

Subtract 19/6 from both sides and find the value of *b*. (Use a calculator)

$$\boldsymbol{b = \frac{5}{6}}$$

Complete the table for each sequence.

Sequence	1st term	2nd term	3rd term	4th term	5th term	6th term		nth term
A	15	8	1	–6				
B	$\frac{5}{18}$	$\frac{6}{19}$	$\frac{7}{20}$	$\frac{8}{21}$				
C	2	5	10	17				
D	2	6	18	54				

[11]

It may be easier to work out the *n*th term of a sequence and then try *n*=5 and *n*=6.

- Sequence A:

The sequence is a simple arithmetic sequence with first term *a*=15 and a common difference *d*=-7.

The *n*th term of an arithmetic sequence with first term *a* and common difference *d* is given as:

$$A(n) = a + (n-1)d$$

In our case:

$$A(n) = 15 + (n-1) \times (-7)$$

$$A(n) = \mathbf{22 - 7n}$$

Apply *n*=5 and *n*=6 to get 5th and 6th term.

$$A(5) = 22 - 7 \times (5) = \mathbf{-13}$$

$$A(6) = 22 - 7 \times (6) = \mathbf{-20}$$

- Sequence B:

Both the numerator and the denominator of this sequence increase as an arithmetic with a common difference 1.

The first term of the sequence in the numerator is 5.

The first term of the sequence in the denominator is 18.

Use the same principles as we used for sequence A.

$$B(n) = \frac{5 + (n-1) \times (1)}{18 + (n-1) \times (1)} = \frac{\mathbf{4 + n}}{\mathbf{17 + n}}$$

Apply n=5 and n=6 to get 5th and 6th term.

$$B(5) = \frac{4+5}{17+5} = \frac{\mathbf{9}}{\mathbf{22}}$$

$$B(6) = \frac{4+6}{17+6} = \frac{\mathbf{10}}{\mathbf{23}}$$

- Sequence C:

We may notice that the terms of this sequence are actually very similar to squares of integer numbers, only greater by 1 in each case.

This observation tells us that the general equation for this sequence is:

$$C(n) = \boldsymbol{n^2 + 1}$$

Apply n=5 and n=6 to get 5th and 6th term.

$$C(5) = (5)^2 + 1 = \mathbf{26}$$

$$C(6) = (6)^2 + 1 = \mathbf{37}$$

- Sequence D:

This is a simple geometric sequence with common ratio r=3 and the first term a=2

Th nth term of a geometric sequence with first term a and common ratio r is given as:

$$D(n) = a \times r^{(n-1)}$$

In our case:

$$D(n) = \mathbf{2} \times \mathbf{3}^{(\boldsymbol{n}-\mathbf{1})}$$

Apply *n*=5 and *n*=6 to get 5th and 6th term.

$$D(5) = 2 \times 3^{(5-1)} = \mathbf{162}$$

$$D(6) = 2 \times 3^{(6-1)} = \mathbf{486}$$

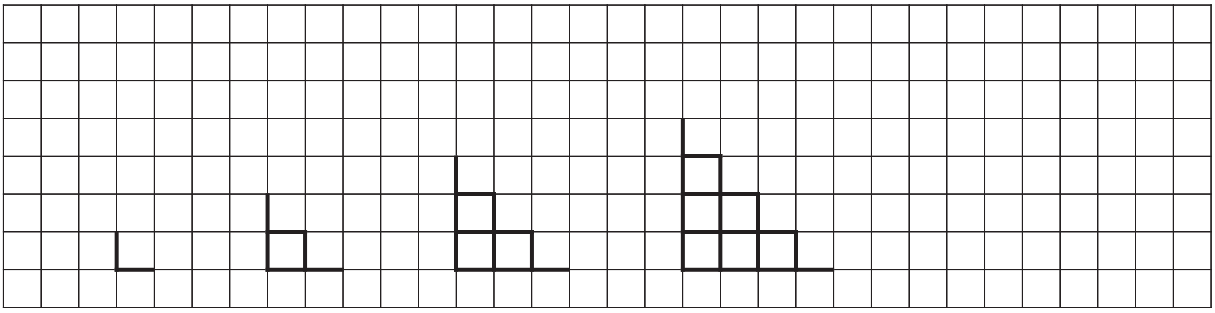


Diagram 1 Diagram 2 Diagram 3 Diagram 4 Diagram 5

Diagram 1 shows two lines of length 1 unit at right angles forming an .

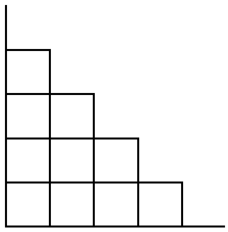
Two s are added to Diagram 1 to make Diagram 2. This forms one small square.

Three s are added to Diagram 2 to make Diagram 3. This forms three small squares.
The sequence of Diagrams continues.

[1]

(a) Draw Diagram 5.

To draw Diagram 5, we add 5 to Diagram 4.



(b) Complete the table. [2]

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5
Number of lines of length 1 unit	2	6	12	20	
Number of small squares	0	1	3	6	

We notice the pattern to complete the table.

In the first row, the factor we add to a number of lines to get the next number is increasing by 2.

In the second how, this factor is simply increasing by 1 each time.

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5
Number of lines of length 1 unit	2	+4 6	+6 12	+8 20	+10 30
Number of small squares	0	+1 1	+2 3	+3 6	+4 10

(c) Find an expression, in terms of n, for the number of lines of length 1 unit in Diagram n. [2]

The relation is clearly not linear (the factor we have to add changes every time), so we try quadratic relation with the general formula $an^2 + bn + c = number\ of\ lines$

Use data for n=1, n=2 and n=3 from the table above.

$$n = 1:\ \ a + b + c = 2$$

$$n = 2:\ \ 4a + 2b + c = 6$$

$$n = 3:\ \ 9a + 3b + c = 12$$

By solving these three simultaneous equations, we get that *a=1 , b=1, c=0.*

The general formula:

$$number\ of\ lines = \boldsymbol{n^2 + n}$$

(To check if the relations is correct, we can plug in the values for n=4 and n=5.)

(d) Find an expression, in terms of n, for the number of small squares in Diagram n. [2]

We similarly guess that the relation is quadratic and use the same method.

Use data for n=1, n=2 and n=3 from the table above.

$$n = 1:\ \ a + b + c = 0$$

$$n = 2:\ \ 4a + 2b + c = 1$$

$$n = 3:\ \ 9a + 3b + c = 3$$

By solving these three simultaneous equations, we get that *a= 1/2 , b=-1/2, c=0.*

The general formula:

$$number\ of\ squares = \boldsymbol{\frac{1}{2}n^2 - \frac{1}{2}n}$$

(To check if the relations is correct, we can plug in the values for n=4 and n=5.)

Sequences
Difficulty: Medium

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 2

Time allowed: **66 minutes**

Score: **/57**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
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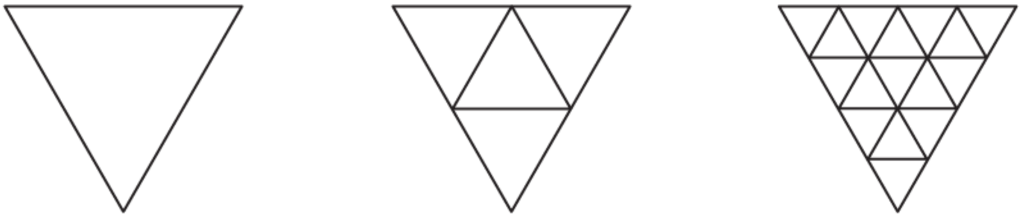

The first three diagrams in a sequence are shown above.
Diagram 1 shows an equilateral triangle with sides of length 1 unit.

In Diagram 2, there are 4 triangles with sides of length $\frac{1}{2}$ unit.

In Diagram 3, there are 16 triangles with sides of length $\frac{1}{4}$ unit.

(a) Complete this table for Diagrams 4, 5, 6 and n. [6]

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5	Diagram 6		Diagram n
Length of side	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$		$\frac{1}{2^{n-1}}$
Length of side as a power of 2	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}		2^{1-n}

(b) (i) Complete this table for the number of the smallest triangles in Diagrams 4, 5 and 6. [2]

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5	Diagram 6
Number of smallest triangles	1	4	16	64	256	1024
Number of smallest triangles as a power of 2	2^0	2^2	2^4	2^6	2^8	2^{10}

(ii) Find the number of the smallest triangles in Diagram n, giving your answer as a power of2. [1]

$\mathbf{2^{2n-2}}$

(c) Calculate the number of the smallest triangles in the diagram where the smallest triangles have sides of Length $\frac{1}{128}$ unit. [2]

$$\frac{1}{128} = 2^{-7}$$

$$\rightarrow n - 1 = 7$$

$$\rightarrow n = 8$$

Hence

$$2^{2n-2}$$

$$= 2^{14}$$

$$\mathbf{= 16\,384}$$

Complete the table for the following sequences.
The first row has been completed for you.

	Sequence	Next two terms	nth term	
	1 5 9 13	17 21	4n-3	
(a)	12 21 30 39	***48 57***	***9n+3***	[3]
(b)	80 74 68 62	***56 50***	***86 – 6n***	[3]
(c)	1 8 27 64	***125 216***	***n^3***	[2]
(d)	2 10 30 68	***130 222***	***n^3+n***	[2]

(a) Each number is 9 greater than the previous. If we subtract 3 from each number, we see a pattern divisible by 9. Hence 9n+3 is the general form.

(b) Each number is less than the previous by 6. Since the first number is when n=1, then we set 86 – 6n to be the first term, and the subtraction follows on.

(c) The key here is to identify a power trend. The first term does not give us much information, but if take 8 = 2^3, and 27 = 3^3, and 64 = 4^3, we can predict the trend. The nth term is simple here as simply n^3.

(d) Note the similarity between this pattern and the one in (c). It is consistently greater by 3.

(a) The nth term of a sequence is $n(n+1)$.

(i) Write the two missing terms in the spaces. 2, 6, ……. , 20, ……. [2]

The nth term = n(n + 1)

2 is the first term in the sequence: 1 x (1 + 1) = 2

6 is the second term: 2 x (2 + 1) = 6

The third term will be: 3 x (3 + 1)

= 12

20 is the forth term: 4 x (4 + 1) = 20

The fith term: 5 x (5 + 1)

= 30

(ii) Write down an expression in terms of n for the $(n + 1)$th term. [1]

Following the pattern of the nth term, we work out the

term in position (n + 1):

The n + 1 term will be: (n + 1)(n + 1 + 1)

= (n + 1)(n + 2)

(iii) The difference between the nth term and the $(n + 1)$th term is $pn + q$.

Find the values of p and q. [2]

The nth term = n(n + 1)

The (n + 1)th term = (n + 1)(n + 2)

(n + 1)(n + 2) – n(n + 1) = pn + q

To solve the equlity, we can use (n + 1) as common factor on the left side of the equation:

$(n + 1)(n + 2 – n) = pn + q$

$2(n + 1) = pn + q$

We multiply the bracket by (-2) to obtain the terms in the same form on both sides.

$2n + 2 = pn + q$

By having the equality written in this form, we can deduce that

$p = q = 2$

(iv) Find the positions of the two consecutive terms which have a difference of 140. [2]

The difference between 2 terms will be 2n + 2.

$2n + 2 = 140$

$n \geqslant 3.$

$n = 69$

(b) A sequence u_1, u_2, u_3, u_4, ... is given by the following rules. [1]

$u_1 = 2$, $u_2 = 3$ and $u_n = 2u_{n-2} + u_{n-1}$ for

For example, the third term is u_3 and $u_3 = 2u_1 + u_2 = 2 \times 2 + 3 = 7$.
So, the sequence is 2, 3, 7, u_4, u_5, ...

(i) Show that $u_4 = 13$.

Following the pattern, we observe that:

$u_4 = 2u_2 + u_3$

$u_4 = 2 \times 3 + 7$

$u_4 = 13$

(ii) Find the value of u_5. [1]

Following the pattern, we observe that:

$u_5 = 2u_3 + u_4$

$u_5 = 2 \times 7 + 13$

$u_5 = 27$

(iii) Two consecutive terms of the sequence are 3413 and 6827 .

Find the term before and the term after these two given terms. [2]

We represent the term before with the unknown a and the term after with the unknown b.

a, 3413, 6827, b

We write the numbers we know according to the formula used above:

6827 = 2a + 3413

a = 1707

b = 2 x 3413 + 6827

b = 13653

(a) The first five terms P_1, P_2, P_3, P_4 and P_5 of a sequence are given below.

1	$= 1 = P_1$
$1 + 2$	$= 3 = P_2$
$1 + 2 + 3$	$= 6 = P_3$
$1 + 2 + 3 + 4$	$= 10 = P_4$
$1 + 2 + 3 + 4 + 5$	$= 15 = P_5$

(i) Write down the next term, P_6, in the sequence 1, 3, 6, 10, 15… [1]

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(ii) The formula for the nth term of this sequence is

$$P_n = \frac{1}{2}n(n+1).$$

Show this formula is true when $n = 6$. [1]

$$P_6 = \frac{1}{2}(6)(6+1)$$

$$= 3 \times 7$$

$$= \mathbf{21}$$

(iii) Use the formula to find P_{50}, the 50th term of this sequence. [1]

$$P_{50} = \frac{1}{2}(50)(51)$$

$$= 25 \times 51$$

$$= \mathbf{1275}$$

(iv) Use your answer to **part (iii)** to find 3 + 6 + 9 + 12 +15 + ………… + 150. [1]

Firstly, we can factor out 3 from the sum

$$3(1+2+3+4+5+\cdots+50)$$

We can see that this is just

$$3 \times P_{50}$$

$$= 3 \times 1275$$

$$= \mathbf{3825}$$

(v) Find 1 + 2 + 3 + 4 + 5 +………… + 150. [1]

This is just the 150th term

$$P_{150} = \frac{1}{2}(150)(151)$$

$$= 75 \times 151$$

$$= \mathbf{11325}$$

(vi) Use your answers to **parts (iv)** and (v) to find the sum of the numbers less than 150 which are **not** multiples of 3. [1]

The sum of all the numbers less than 150 minus the multiples of 3.

This is just

$$(1+2+3+\cdots+150) - (3+6+9+\cdots 150)$$

$$= P_{150} - 3 \times P_{50}$$

$$= 11325 - 3825$$

$$= \mathbf{7500}$$

(b) The first five terms, S_1, S_2, S_3, S_4 and S_5 of a different sequence are given below.

(1×1)	$= 1 = S_1$
$(1 \times 2) + (2 \times 1)$	$= 4 = S_2$
$(1 \times 3) + (2 \times 2) + (3 \times 1)$	$= 10 = S_3$
$(1 \times 4) + (2 \times 3) + (3 \times 2) + (4 \times 1)$	$= 20 = S_4$
$(1 \times 5) + (2 \times 4) + (3 \times 3) + (4 \times 2) + (5 \times 1)$	$= 35 = S_5$

(i) Work out the next term, S_6, in the sequence 1, 4, 10, 20, 35… [2]

$$S_6 = (1 \times 6) + (2 \times 5) + (3 \times 4) + (4 \times 3) + (5 \times 2) + (6 \times 1)$$

$$= \mathbf{56}$$

(ii) The formula for the nth term of this sequence is

$$S_n = \frac{1}{6} n(n+1)(n+2).$$

Show this formula is true for $n = 6$. [1]

$$S_6 = \frac{1}{6}(6)(6+1)(6+2)$$

$$= 7 \times 8$$

$$= \mathbf{56}$$

(iii) Find $(1 \times 20) + (2 \times 19) + (3 \times 18) \ldots\ldots\ldots\ldots + (20 \times 1)$. [1]

This is just

$$S_{20} = \frac{1}{6}(20)(21)(22)$$

$$= \mathbf{1540}$$

(c) Show that $S_6 - S_5 = P_6$, where P_6 is your answer to **part (a)(i).** [1]

$$S_6 - S_5 = 56 - 35$$

$$\mathbf{= 21 = P_6}$$

(d) Show by algebra that $S_n - S_{n-1} = P_n$. $[P_n = \frac{1}{2}n(n+1)]$ [3]

$$S_n - S_{n-1}$$

$$= \frac{1}{6}n(n+1)(n+2) - \frac{1}{6}(n-1)(n-1+1)(n-1+2)$$

$$= \frac{1}{6}n(n+1)(n+2) - \frac{1}{6}n(n-1)(n+1)$$

Factorise out the $\frac{1}{6}n(n+1)$

$$\frac{1}{6}n(n+1)\{(n+2)-(n-1)\}$$

$$= \frac{1}{6}n(n+1)(3)$$

$$\boldsymbol{= \frac{1}{2}n(n+1) = P_n}$$

In all the following sequences, after the first two terms, the rule is to add the previous two terms to find the next term.

(a) Write down the next two terms in this sequence.

1 1 2 3 5 8 13 .... [1]

21 34

(b) Write down the first two terms of this sequence.

.... 3 11 14 [2]

-5 8

(c) (i) Find the value of d and the value of e.

2 d e 10 [3]

$d + e = 10$ (1)

$2 + d = e$ (2)

Sub (2) into (1) to get

$$\rightarrow d + (2 + d) = 10$$

$$\rightarrow 2d + 2 = 10$$

Minus 2 from both sides and then divide through by 2

$$\rightarrow \boldsymbol{d = 4}$$

Sub this into (2) to get

$$\rightarrow \boldsymbol{e = 6}$$

(ii) Find the value of x, the value of y and the value of z.

-33	x	y	z	18

[5]

$-33 + x = y$ (1)

$x + y = z$ (2)

$y + z = 18$ (3)

Rearrange (3) for

$y = 18 - z$ (4)

and sub this into (2)

$$x + 18 - z = z$$

$$\rightarrow x + 18 = 2z$$

$\rightarrow x = 2(z - 9)$ (5)

sub (4) into (1) for

$$-33 + x = 18 - z$$

$\rightarrow x = 51 - z$ (6)

Equate (5) and (6)

$$51 - z = 2(z - 9)$$

$$\rightarrow 3z = 69$$

$$\rightarrow \boldsymbol{z = 23}$$

Sub into (4)

$$\boldsymbol{y = -5}$$

Sub into (1)

$$-33 + x = -5$$

$$\rightarrow \boldsymbol{x = 28}$$

Sequences
Difficulty: Medium

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 3

Time allowed: **59 minutes**

Score: **/51**

Percentage: **/100**

Grade Boundaries:

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A*	A	B	C	D
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CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

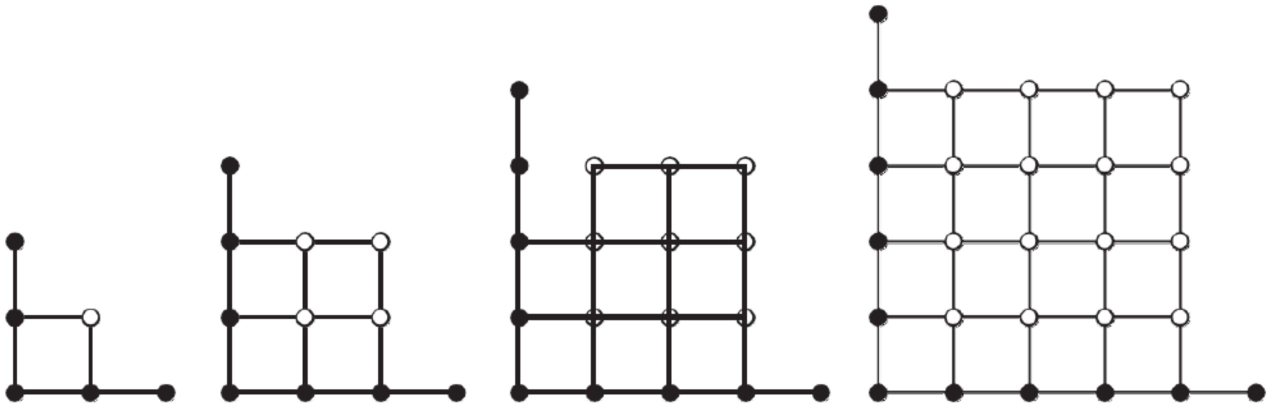


Diagram 1	Diagram 2	Diagram 3	Diagram 4
1 white dot	4 white dots	9 white dots	16 white dots
5 black dots	7 black dots	9 black dots	11 black dots
6 lines	14 lines	26 lines	42 lines

The four diagrams above are the first four of a pattern.

(a) Diagram 5 has been started below.
Complete this diagram and write down the information about the numbers of dots and lines.

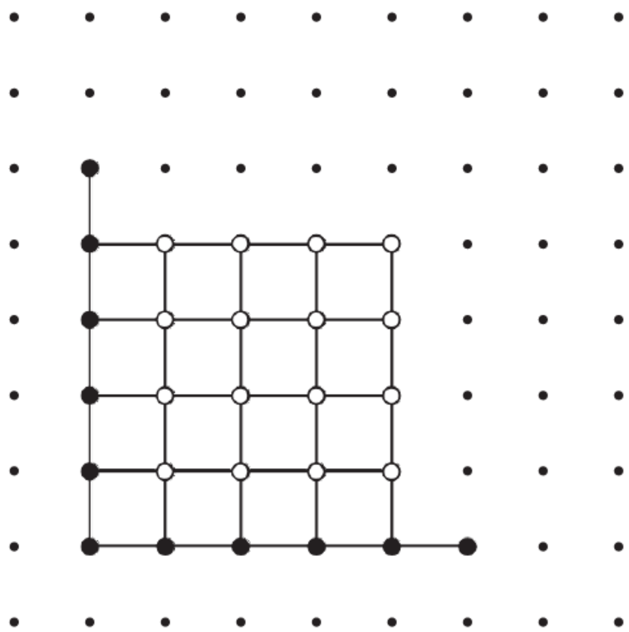


Diagram 5

.... white dots

.... black dots

.... lines [4]

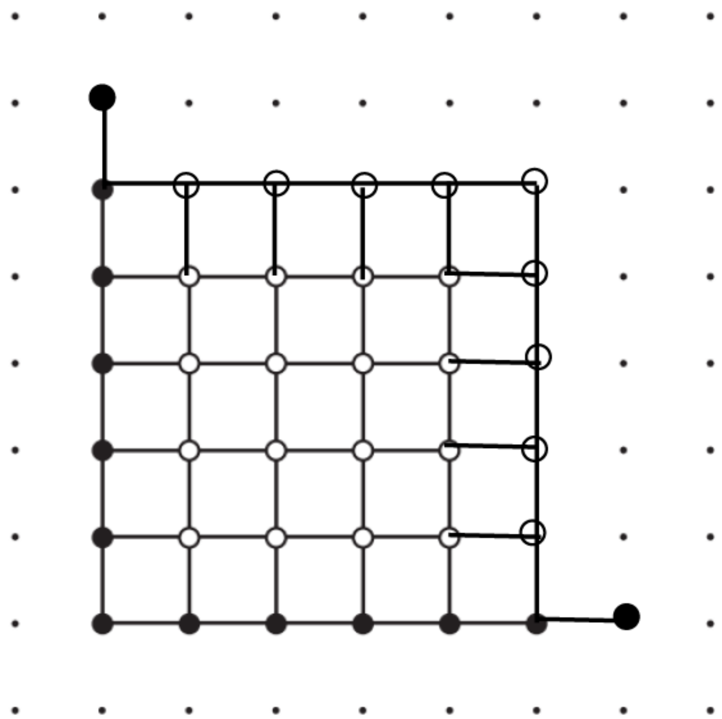


Diagram 5

...25... white dots

...13... black dots

...62... lines

(b) Complete the information about the number of dots and lines in Diagram 8. [3]

64 white dots

19 black dots

146 lines

(c) Complete the information about the number of dots in Diagram n.
Give your answers in terms of n. [2]

n^2 white dots

$2n + 3$ black dots

(d) The number of lines in diagram n is $k(n^2 + n + 1)$.

Find

(i) the value of k, [1]

Check white diagram 1

$$k(1^2 + 1 + 1) = 6$$

$$\rightarrow 3k = 6$$

$$\rightarrow \boldsymbol{k = 2}$$

(ii) the number of lines in Diagram 100. [1]

$$2(100^2 + 100 + 1)$$

$$= 2(10101)$$

$$= \mathbf{20202}$$

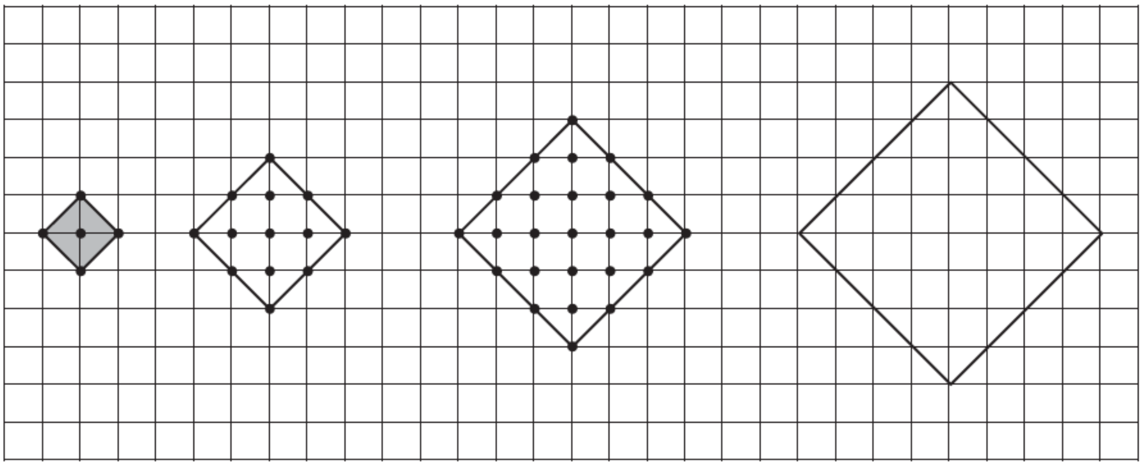


Diagram 1 Diagram 2 Diagram 3 Diagram 4

The diagrams show squares and dots on a grid.

Some dots are on the sides of each square and other dots are inside each square.

The area of the square (shaded) in Diagram 1 is 1 unit^2.

(a) Complete Diagram 4 by marking all the dots. [1]

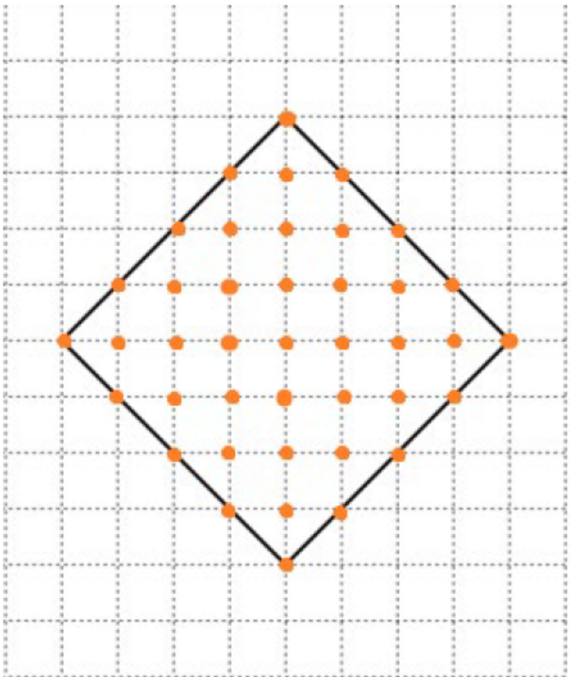


Diagram 4

(b) Complete the columns in the table below for Diagrams 4, 5 and n.

Diagram	1	2	3	4	5	-------	n
Number of units of area	1	4	9			-------	
Number of dots inside the square	1	5	13			-------	$(n-1)^2 + n^2$
Number of dots on the sides of the square	4	8	12			-------	
Total number of dots	5	13	25			-------	

[7]

Diagram	1	2	3	4	5	...	n
Number of units of area	1	4	9	**16**	**25**	...	$\boldsymbol{n^2}$
Number of dots inside the square	1	5	13	**25**	**41**	...	$(n-1)^2 + n^2$
Number of dots on the sides of the square	4	8	12	**16**	**20**	...	$\boldsymbol{4n}$
Total number of dots	5	13	25	**41**	**61**	...	$\boldsymbol{n^2 + (n+1)^2}$

(c) For Diagram 200, find the number of dots

(i) inside the square, [1]

$$(200 - 1)^2 + 200^2$$

$$= \mathbf{79\ 601}$$

(ii) on the sides of the square. [1]

$$4 \times 200$$

$$= \mathbf{800}$$

(d) Which diagram has 265 dots inside the square? [1]

$$(n - 1)^2 + n^2 = 265$$

Trial and error yields

$$\boldsymbol{n = 12}$$

		Total
Row 1	1	$= 1$
Row 2	$3 + 5$	$= 8$
Row 3	$7 + 9 + 11$	$= 27$
Row 4	$13 + 15 + 17 + 19$	$= 64$
Row 5		
Row 6		

The rows above show sets of consecutive odd numbers and their totals.

(a) Complete Row 5 and Row 6. [2]

Add 3 and increase number of figures

Row 5 – 21 + 23 + 25 + 27 + 29 = 125

Row 6 – 31 + 33 + 35 + 37 + 39 + 41 = 216

(b) What is the special name given to the numbers 1, 8, 27, 64…? [1]

Cubes

(c) Write down in terms of n,

(i) how many consecutive odd numbers there are in Row n, [1]

n

(ii) the total of these numbers. [1]

n^3

(d) The first number in Row n is given by $n^2 - n + 1$.

Show that this formula is true for Row 4. [1]

$n = 4$

$4^2 - 4 + 1 = 13$

(e) The total of Row 3 is 27. This can be calculated by $(3 \times 7) + 2 + 4$.

The total of Row 4 is 64. This can be calculated by $(4 \times 13) + 2 + 4 + 6$. The total of Row 7 is 343. Show how this can be calculated in the same way. [1]

total = (7 x 43) + 2 + 4 + 6 + 8 + 10 + 12

= 343

(f) The total of the first n **even** numbers is $n(n + 1)$.

Write down a formula for the total of the first $(n - 1)$ **even** numbers. [1]

$n(n - 1)$

(g) Use the results of **parts (d)**, **(e)** and **(f)** to show clearly that the total of the numbers in Row n gives your answer to **part (c)(ii)**. [2]

First number in row n = $(n^2 - n + 1) \times n$ and sum of the rest of the numbers in row n will be $(n - 1) \times n$

Row n = $n(n^2 - n + 1) + n(n - 1)$

$= n^3 - n^2 + n + n^2 - n$

$= n^3$

$$1 + 2 + 3 + 4 + 5 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}$$

(a) (i) Show that this formula is true for the sum of the first 8 natural numbers. [2]

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

$$\mathbf{= 36}$$

$$\frac{8(8+1)}{2}$$

$$= 4 \times 9$$

$$\mathbf{= 36}$$

(ii) Find the sum of the first 400 natural numbers. [1]

$$\frac{400(400+1)}{2}$$

$$\mathbf{= 80\ 200}$$

(b) (i) Show that $2 + 4 + 6 + 8 + \ldots\ldots\ldots\ldots\ldots + 2n = n(n+1)$. [1]

We have that

$$2 + 4 + 6 + 8 + \cdots + 2n$$

$$= 2(1 + 2 + 3 + 4 + \cdots + n)$$

$$= 2 \times \frac{n(n+1)}{2}$$

$$\boldsymbol{= n(n+1)}$$

(ii) Find the sum of the first 200 even numbers. [1]

$$200(200+1)$$

$$= \mathbf{40\ 200}$$

(iii) Find the sum of the first 200 odd numbers. [1]

$$80\ 200 - 40\ 200$$

$$= \mathbf{40\ 000}$$

(c) (i) Use the formula at the beginning of the question to find the sum of the first $2n$ natural numbers. [1]

$$\frac{2n(2n+1)}{2}$$

$$= \boldsymbol{n(2n+1)}$$

(ii) Find a formula, in its simplest form, for

$$1 + 3 + 5 + 7 + 9 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + (2n - 1).$$

Show your working. [2]

This is the sum of the odd numbers up to $2n$.

This is the sum of the natural number up to $2n$

$$n(2n+1)$$

minus the sum of the even numbers up until $2n$

$$n(n+1)$$

Hence

$$n(2n+1) - n(n+1)$$

$$= 2n^2 + n - n^2 - n$$

$$= \boldsymbol{n^2}$$

(a) Write down the 10th term and the nth term of the following sequences.

(i) 1, 2, 3, 4, 5 …, …, [1]

10th term = 10

Nth term = n

(ii) 7, 8, 9, 10, 11 …, …, [1]

1st term: 7 = 1 + 6

2nd term: 8 = 2 + 6

…

10th term = 10 + 6 = 16

Nth term = n + 6

(iii) 8, 10, 12, 14, 16 …, … . [3]

1st term: 8 = 2 x 1 + 6

2nd term: 10 = 2 x 2 + 6

…

10th term = 2 x 10 + 6 = 26

Nth term = 2n + 6

(b) Consider the sequence

$1(8-7), 2(10-8), 3(12-9), 4(14-10), \ldots\ldots\ldots, \ldots\ldots\ldots .$

(i) Write down the next term and the 10th term of this sequence in the form $a(b-c)$ where a, b and c are integers. [3]

We notice that the sequence is formed by multiplying the sequence from a) i) by the difference between the sequences a) iii) and a) ii). The next and 10th term are worked out by using the terms at a) i), ii) and iii).

next term: 5(16 – 11)

10th term: 10(26 – 16)

(ii) Write down the nth term in the form $a(b-c)$ and then simplify your answer. [2]

The nth term is formed by using the nth term of the previous 3 sequences.

Nth term = $n((2n+6)-(n+6)) = n(2n+6-n-6) = n^2$

Sequences
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **91 minutes**

Score: **/79**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

The table shows the first four terms in sequences A, B, C and D.

Complete the table.

Sequence	1st term	2nd term	3rd term	4th term	5th term		nth term
A	16	25	36	49			
B	5	8	11	14			
C	11	17	25	35			
D	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$			

[12]

Sequence	1st term	2nd term	3rd term	4th term	5th term	nth term
A	16	25	36	49	**64**	$(n+3)^2$
B	5	8	11	14	**17**	$3n+2$
C	11	17	25	35	**47**	$(n+3)^2-(3n+2)$
D	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{7}{6}$	$\frac{n+2}{n+1}$

The table shows the first five terms of sequences A, B and C.

Sequence	1st term	2nd term	3rd term	4th term	5th term	6th term
A	3	4	5	6	7	
B	0	1	4	9	16	
C	–3	–3	–1	3	9	

(a) Complete the table for the 6th term of each sequence. [2]

A increases as an arithmetic sequence with a common difference 1.

$$\boldsymbol{A(6th) = 8}$$

The differences between terms of B sequence changes as a arithmetic sequence with a common difference 2. The difference between 5th and 4th term is 7, so the next difference will be 9.

$$\boldsymbol{B(6th) = B(5th) + 9 = 16 + 9 = 25}$$

Sequence C is dependent of sequences A and B. (C=B-A)

$$\boldsymbol{C(6th) = B(6th) - A(6th) = 25 - 8 = 17}$$

(b) Write down the nth term of sequence A. [1]

As mentioned before, the common difference of sequence A is 1 and the first term is 3.

Therefore the nth term will be:

$$\boldsymbol{A(nth) = 3 + (n - 1) = 2 + n}$$

(c) (i) Find the nth term of sequence B. [2]

The relation is clearly not linear (the factor we have to add changes every time), so we try quadratic relation with the general formula $an^2 + bn + c = B(nth)$

Use data for n=1, n=2 and n=3 from the table above.

$$n = 1: \;\; a + b + c = 0$$

$$n = 2: \;\; 4a + 2b + c = 1$$

$$n = 3: \;\; 9a + 3b + c = 4$$

By solving these three simultaneous equations, we get that *a=1* , *b=-2, c=1.*

The general formula:

$$B = \boldsymbol{n^2 - 2n + 1}$$

(equivalently) $B = \boldsymbol{(n-1)^2}$

(To check if the relations is correct, we can plug in the values for n=4 and n=5.)

(ii) Find the value of n when the nth term of sequence B is 8281. [2]

The *n* the value of B is 8281.

$$8281 = (n-1)^2$$

Take square root of both sides of the equation.

$$91 = n - 1$$

Add 1 to both sides to calculate the value of *n*.

$$n = \mathbf{92}$$

(d) (i) Find the nth term of sequence C in its simplest form. [2]

We can find the *n*-th term by using the relation between sequence C and sequences A and B mentioned earlier.

$$C = B - A = (n-1)^2 - (n+2)$$

Multiply out the brackets.

$$C = n^2 - 2n + 1 - n - 2$$

The *n*th term of the sequence in its simplest form:

$$C(n) = \boldsymbol{n^2 - 3n - 1}$$

(ii) Find the 8th term of sequence C. [1]

To find the 8th term of sequence C, set *n*=8.

$$C(8) = 8^2 - 3 \times 8 - 1$$

$$C(8) = \mathbf{39}$$

(e) The nth term of another sequence D is $\left(-\frac{1}{2}\right)^{n-1}$.

Complete the table for the first four terms of sequence D. [3]

Sequence	1st term	2nd term	3rd term	4th term
D				

Work out the terms by using a calculator and setting n=1,2,3,4

1st term

$$D(1) = \left(-\frac{1}{2}\right)^{1-1} = \mathbf{1}$$

2nd term

$$D(2) = \left(-\frac{1}{2}\right)^{2-1} = -\frac{\mathbf{1}}{\mathbf{2}}$$

3rd term

$$D(3) = \left(-\frac{1}{2}\right)^{3-1} = \frac{\mathbf{1}}{\mathbf{4}}$$

4th term

$$D(4) = \left(-\frac{1}{2}\right)^{4-1} = -\frac{\mathbf{1}}{\mathbf{8}}$$

The first four terms of sequences A, B, C and D are shown in the table.

Sequence	1st term	2nd term	3rd term	4th term	5th term		*n*th term
A	$\frac{1}{3}$	$\frac{2}{4}$	$\frac{3}{5}$	$\frac{4}{6}$			
B	3	4	5	6			
C	–1	0	1	2			
D	–3	0	5	12			

(a) Complete the table. [8]

It may be easier to work out the *n*th term of a sequence and then try *n*=5.

- **Sequence A:**

Both the numerator and the denominator of this sequence increase as an arithmetic with a common difference 1.

The sequence is a simple arithmetic sequence with first term *a*=15 and a common difference *d*=-7.

The *n*th term of an arithmetic sequence with first term *a* and common difference *d* is given as:

$$A(n) = a + (n-1)d$$

The first term of the sequence in the numerator is 1.

The first term of the sequence in the denominator is 3.

$$A(n) = \frac{1 + (n-1) \times (1)}{3 + (n-1) \times (1)} = \frac{\boldsymbol{n}}{\boldsymbol{3+n}}$$

Apply *n*=5 to get 5th

$$A(5) = \frac{5}{2+5} = \frac{\mathbf{5}}{\mathbf{7}}$$

- **Sequence B:**

The sequence is a simple arithmetic sequence with first term *a*=3 and a common difference *d*=1.

Use the same principles as we used for sequence A. In our case:

$$B(n) = 3 + (n - 1) \times (1)$$

$$B(n) = \mathbf{2 + n}$$

Apply *n*=5 to get 5th

$$B(5) = 2 + (5) = \mathbf{7}$$

- **Sequence C:**

The sequence is a simple arithmetic sequence with first term *a*=-1 and a common difference *d*=1.

Use the same principles as we used for sequence A. In our case:

$$C(n) = -1 + (n - 1) \times (1)$$

$$C(n) = \mathbf{-2 + n}$$

Apply *n*=5 to get 5th

$$C(5) = -2 + (5) = \mathbf{3}$$

- **Sequence D:**

We may notice that the terms of this sequence are actually very similar to squares of integer numbers, only smaller by 4 in each case.

This observation tells us that the general equation for this sequence is:

$$D(n) = \boldsymbol{n^2 - 4}$$

Apply *n*=5 and *n*=6 to get 5th and 6th term.

$$D(5) = (5)^2 - 4 = \mathbf{21}$$

(b) Which term in sequence A is equal to $\frac{36}{37}$? [2]

We want to satisfy the equation:

$$\frac{n}{2+n} = \frac{36}{37}$$

As the difference between the numerator and the denominator must be 2, multiply both the numerator and the denominator of the fraction on the right hand side by 2.

$$\frac{n}{2+n} = \frac{72}{74}$$

The answer is now easy to work out.

$$\boldsymbol{n = 72}$$

(c) Which term in sequence D is equal to 725? [2]

Again, we want to satisfy the following equation:

$$725 = n^2 - 4$$

Add 4 to both sides of the equation.

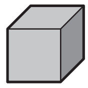
$$729 = n^2$$

Take square root of both sides (take positive value of (*n*)

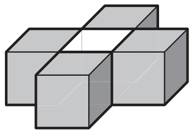
$$\boldsymbol{n = 27}$$

Question 4

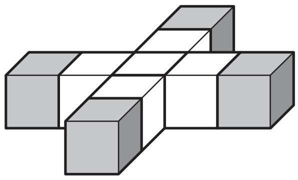
Layer 1



Layer 2



Layer 3



The diagrams show layers of white and grey cubes.
Khadega places these layers on top of each other to make a tower.

(a) Complete the table for towers with 5 and 6 layers. [4]

Number of layers	1	2	3	4	5	6
Total number of white cubes	0	1	6	15	**28**	**45**
Total number of grey cubes	1	5	9	13	**17**	**21**
Total number of cubes	1	6	15	28	**45**	**66**

(b) (i) Find, in terms of n, the **total** number of **grey** cubes in a tower with n layers. [2]

Increasing by 4 so we have:

$$grey\ cubes = 4n + c$$

Input test value:

$$4(1) + c = 1$$

$$\rightarrow c = -3$$

Hence:

$$\mathbf{4n - 3}$$

(ii) Find the total number of grey cubes in a tower with 60 layers. [1]

$4(60) - 3$

$= \mathbf{237}$

(iii) Khadega has plenty of white cubes but only 200 grey cubes.
How many layers are there in the highest tower that she can build? [2]

$$4n - 3 = 200$$

$$\rightarrow 4n = 203$$

$$\rightarrow n = 50.75$$

n must be integer, so:

$$\boldsymbol{n = 50}$$

(c) The expression for the **total** number of **white** cubes in a tower with n layers is $pn^2 + qn + 3$.

Find the value of p and the value of q.
Show all your working. [5]

Input test values:

$$p(1)^2 + q(1) + 3 = 0$$

$\rightarrow p + q = -3 \quad (1)$

$$p(2)^2 + q(2) + 3 = 1$$

$\rightarrow 4p + 2q = -2 \quad (2)$

Multiply (1) by 2

$$2p + 2q = -6$$

Now subtract it from (2)

$$2p = 4$$

$$\rightarrow \boldsymbol{p = 2}$$

Substitute this value into (1)

$$2 + q = -3$$

$$\rightarrow \boldsymbol{q = -5}$$

(d) Find an expression, in terms of n, for the **total** number of cubes in a tower with n layers.
Give your answer in its simplest form. [2]

Add white cubes to grey cubes

$$2n^2 - 5n + 3 + (4n - 3)$$

$$= \boldsymbol{2n^2 - n}$$

(a)

1	= 1
1 + 2	= 3
1 + 2 + 3	= 6
1 + 2 + 3 + 4	= 10

(i) Write down the next line of this pattern. [1]

$$\mathbf{1 + 2 + 3 + 4 + 5 = 15}$$

(i) The sum of the first n integers is $\frac{n}{k}(n+1)$.

Show that $k = 2$. [2]

$$\boldsymbol{Sum\ of\ first\ n\ integers = \frac{n}{k}(n+1)}$$

n=1, sum=1,

$$\frac{1}{k}(1+1) = 1$$

$$\frac{2}{k} = 1$$

$$\boldsymbol{k = 2}$$

n=2, sum=3,

$$\frac{2}{k}(2+1) = 3$$

$$\frac{6}{k} = 3$$

$$\boldsymbol{k = 2}$$

k=2 for all n.

(iii) Find the sum of the first 60 integers. [1]

Let n = 60,

$$\frac{60}{2}(60+1) = \mathbf{1830}$$

(iv) Find n when the sum of the first n integers is 465. [2]

Formulate the equation:

$$\frac{n}{2}(n+1) = 465$$

Solve:

$$\frac{n^2}{2} + \frac{n}{2} = 465$$

$$n^2 + n - 930 = 0$$

$$n = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-930)}}{2(1)}$$

$$= \mathbf{30\ or - 31}\ (\boldsymbol{N.A.})$$

Therefore, n=30 (answer)

(v) $1 + 2 + 3 + 4 + \ldots\ldots + x = \dfrac{(n-8)(n-7)}{2}$

Write x in terms of n. [1]

Let x=n and for:

$$\frac{n}{2}(n+1) = sum$$

If we let x be (n-8),

$$\frac{(n-8)}{2}(n-8+1) = \frac{x}{2}(x+1)$$

(b)

1^3	$= 1$
$1^3 + 2^3$	$= 9$
$1^3 + 2^3 + 3^3$	$= 36$
$1^3 + 2^3 + 3^3 + 4^3$	$= 100$

(i) Complete the statement.

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225 = 15^2$$ [2]

(ii) The sum of the first n integers is $\frac{n}{2}(n+1)$.

Find an expression, in terms of n, for the sum of the first n cubes. [1]

Earlier in part (a), we saw that 1 + 2 + 3 + 4 + 5 = 15,

Since taking the sum of the cube of each of the terms is 15^2,

The expression for the sum will be the square of the expression earlier.

$$\left(\frac{n}{2}(n+1)\right)^2 = \frac{n^2}{4}(n+1)^2$$

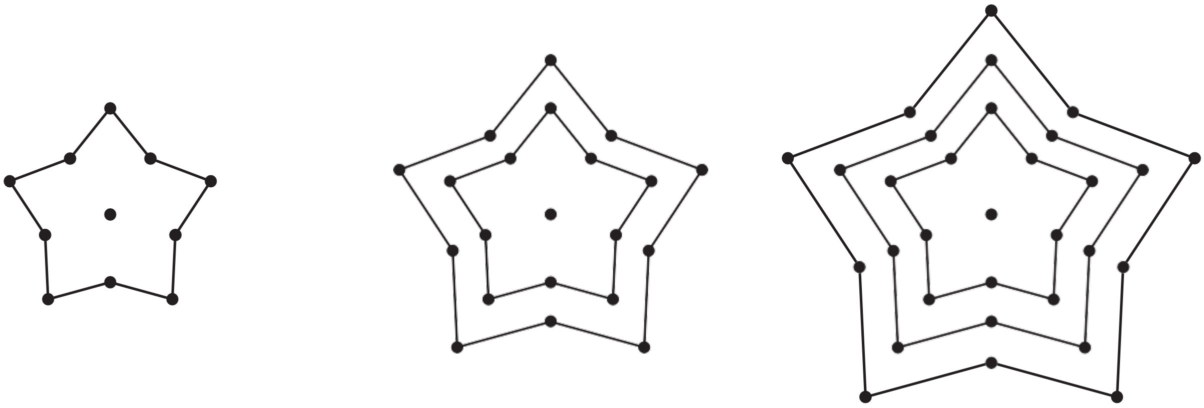
(iii) Find the sum of the first 19 cubes. [2]

n=19,

$$\frac{n^2}{4}(n+1)^2 = \frac{19^2}{4}(19+1)^2$$

$$= 36100$$

Question 6



Star 1 Star 2 Star 3

The diagrams show a sequence of stars made of lines and dots.

[4]

(a) Complete the table for Star 5, Star 7 and Star n.

Here we can see that the number of lines is 10 times greater than the star number and the number of dots is one greater than this.

So the number of lines, l, can be given by:

$$l = 10n$$

Hence the number of dots, d, is given by:

$$d = 10n + 1$$

Using this we can substitute in the unknowns and complete the table.

	Star 1	Star 2	Star 3	Star 4	Star 5	Star 7	Star n
Number of dots	10	20	30	40	**50**	**70**	**10n**
Number of lines	11	21	31	41	**51**	**71**	**10n + 1**

(b) The sums of the number of dots in two consecutive stars are shown in the table.

Star 1 and Star 2	Star 2 and Star 3	Star 3 and Star 4
32	52	72

Find the sum of the number of dots in

(i) Star 10 and Star 11, [1]

Here we can see that the difference between each star pair is 20, meaning if you use the lowest star number in each pair as n, the coefficient of n must be 20. But by multiplying each value for n by 20 you come 12 short, hence 12 must be added. This gives the nth term as:

$D = 20n + 12$

Substituting 10.

$\boldsymbol{D = 212}$

(ii) Star n and Star $(n + 1)$, [1]

We worked this out in the first part as:

$\boldsymbol{D = 20n + 12}$

(iii) Star $(n + 7)$ and Star $(n + 8)$. [1]

As n + 7 is the lowest term, we simply substitute this in for n.

$D = 20(n + 7) + 12$

Expanding brackets.

$D = 20n + 140 + 12$

$\boldsymbol{D = 20n + 152}$

(c) The **total number of dots** in the first n stars is given by the expression $5n^2 + 6n$.

(i) Show that this expression is correct when $n = 3$.

[2]

Number of dots in first three stars:

$D = 11 + 21 + 31$

$D = 63$

Substituting 3 into the equation for n.

$D = 5(3)^2 + 6(3)$

$= 45 + 18$

$= 63$

(ii) Find the total number of dots in the first 10 stars. [1]

Substituting 10 into equation.

$D = 5(10)^2 + 6(10)$

$D = 500 + 60$

$\boldsymbol{D = 560}$

(d) The total number of dots in the first n stars is $5n^2 + 6n$.
The number of dots in the $(n + 1)$th star is $10(n + 1) + 1$.

Add these two expressions to show that the total number of dots in the first $(n + 1)$ stars is

$$5(n + 1)^2 + 6(n + 1) .$$

You must show each step of your working. [4]

Adding the two expressions.

$5n^2 + 6n + 10(n + 1) + 1$

Expanding brackets:

$5n^2 + 6n + 10n + 10 + 1$

As a $6(n + 1)$ we must take this out before we take the quadratic

$5n^2 + 10n + 5 + (6n + 6)$

Factorising 5 from the quadratic part and 6 from the linear part.

$5(n^2 + 2n + 1) + 6(n + 1)$

Factorising quadratic as required.

$\mathbf{5(n + 1)^2 + 6(n + 1)}$

Sequences
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **94 minutes**

Score: **/82**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Question 1

Consecutive integers are set out in rows in a grid.

(a) This grid has 5 columns.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35

a		b
	n	
c		d

The shape drawn encloses five numbers 7, 9, 13, 17 and 19. This is the $n = 13$ shape.

In this shape, $a = 7$, $b = 9$, $c = 17$ and $d = 19$.

(i) Calculate $bc - ad$ for the $n = 13$ shape. [1]

$$bc - ad$$

$$= (9)(17) - (7)(19)$$

$$= 153 - 133$$

$$\mathbf{= 20}$$

(ii) For the 5 column grid, $a = n - 6$.

Write down b, c and d in terms of n for this grid. [2]

$$\boldsymbol{b = n - 4}$$

$$\boldsymbol{c = n + 4}$$

$$\boldsymbol{d = n + 6}$$

(iii) Write down bc - ad in terms of n. [2]
Show clearly that it simplifies to 20.

$$bc - ad$$

$$= (n-4)(n+4) - (n-6)(n+6)$$

Expand out the brackets

$$= n^2 - 4n + 4n - 16 - (n^2 - 6n + 6n - 36)$$

$$= n^2 - 16 - n^2 + 36$$

$$= \mathbf{20}$$

(b) This grid has 6 columns. The shape is drawn for $n = 10$.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

a		b
	n	
c		d

(i) Calculate the value of bc - ad for n = 10. [1]

$$bc - ad$$

$$= (5)(15) - (3)(17)$$

$$= 75 - 51$$

$$= \mathbf{24}$$

(ii) Without simplifying, write down bc - ad in terms of n for this grid. [2]

$$a = n - 7$$

$$b = n - 5$$

$$c = n + 5$$

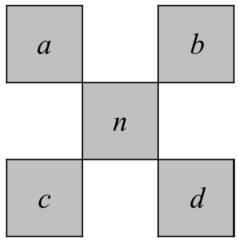
$$d = n + 7$$

$$\rightarrow bc - ad$$

$$= \mathbf{(n-5)(n+5) - (n-7)(n+7)}$$

(c) This grid has 7 columns.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35



Show clearly that $bc - ad = 28$ for $n = 17$. [1]

For n = 17 we have

$$a = 9$$

$$b = 11$$

$$c = 23$$

$$d = 25$$

$$\rightarrow bc - ad$$

$$= (11)(23) - (9)(25)$$

$$= 253 - 225$$

$$= \mathbf{28}$$

(d) Write down the value of *bc* - *ad* when there are *t* columns in the grid. [1]

20 for 5 columns.

24 for 6 columns.

28 for 7 columns.

t columns:

$$\rightarrow \boldsymbol{bc - ad = 4t}$$

(e) Find the values of *c*, *d* and *bc* - *ad* for this shape. [2]

2	3	4
	16	
c		*d*

Let t be the number of columns.

We have

$$3 + t = 16$$

$$\rightarrow t = 13$$

Hence

$$c = 2 + 13 + 13$$
$$= \mathbf{28}$$

$$d = 4 + 13 + 13$$

$$= \mathbf{30}$$

$$bc - ad = 4t$$

$$= 4(13)$$

$$= \mathbf{52}$$

(a) Complete the table for the 6 th term and the nth term in each sequence. [12]

	Sequence	6th term		nth term
A	11, 9, 7, 5, 3	**1**		$13-2n$
B	1, 4, 9, 16, 25	**36**		n^2
C	2, 6, 12, 20, 30	**42**		**n(n+1)**
D	3, 9, 27, 81, 243	**729**		3^n
E	1, 3, 15, 61, 213	**687**		$3^n-n(n+1)$

Sequence C is sequence B plus the nth term, hence n^2+n

Sequence E is Sequence D minus Sequence C, hence $3^n-(n^2+n)$

(b) Find the value of the 100 th termin

(i) Sequence A, [1]

$13-2(100)$

$=-187$

(ii) Sequence C. [1]

$(100)^2+100$

$=10100$

(c) Find the value of n in Sequence D when the n th term is equal to 6561. [1]

$3^n = 6561$

$\rightarrow \boldsymbol{n = 8}$

(d) Find the value of the 10 th term in Sequence E. [1]

$3^{10} - 10(10 + 1)$

$= 59049 - 110$

$= \mathbf{58939}$

Question 3

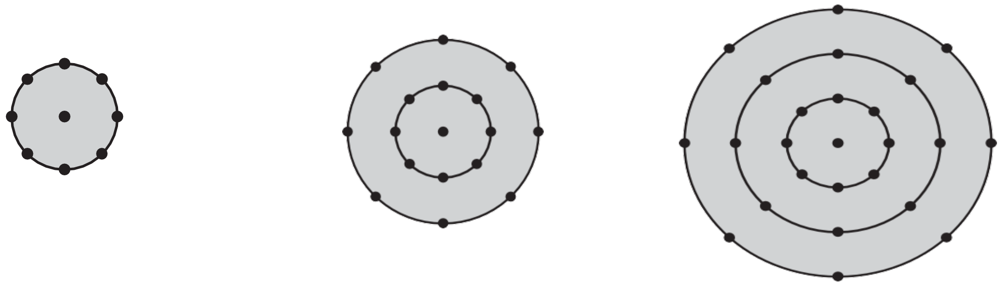


Diagram 1 Diagram 2 Diagram 3

The diagrams show a sequence of dots and circles.
Each diagram has one dot at the centre and 8 dots on each circle.
The radius of the first circle is 1 unit.
The radius of each new circle is 1 unit greater than the radius of the previous circle.

(a) Complete the table for diagrams 4 and 5. [4]

Diagram	1	2	3	4	5
Number of dots	9	17	25	**33**	**41**
Area of the largest circle	π	4π	9π	$\mathbf{16\pi}$	$\mathbf{25\pi}$
Total length of the circumferences of the circles	2π	6π	12π	$\mathbf{20\pi}$	$\mathbf{30\pi}$

(b) (i) Write down, in terms of n, the number of dots in diagram n. [2]

$\mathbf{8n + 1}$

(ii) Find n, when the number of dots in diagram n is 1097. [2]

$1097 = 8n + 1$

$\rightarrow 8n = 1096$

$\rightarrow \mathbf{n = 137}$

(c) Write down, in terms of n and π, the area of the largest circle in [1]

(i) diagram n,

$n^2\pi$

(ii) diagram $3n$. [1]

$(3n)^2\pi$

$= \mathbf{9n^2\pi}$

(d) Find, in terms of n and π, the total length of the circumferences of the circles in diagram n. [2]

$(n^2 + n)\pi$

The first and the *n*th terms of sequences *A*, *B* and *C* are shown in the table below.

(a) Complete the table for each sequence. [5]

	1st term	2nd term	3rd term	4th term	5th term	*n*th term
Sequence *A*	1					n^3
Sequence *B*	4					$4n$
Sequence *C*	4					$(n+1)^2$

We simply plug the integers (n=2,3,4,5) into the formulas for general n which are given on the right.

	1st term	2nd term	3rd term	4th term	5th term	*n*th term
Sequence *A*	1	8	27	64	125	n^3
Sequence *B*	4	8	12	16	20	$4n$
Sequence *C*	4	9	16	25	36	$(n+1)^2$
		$n = 1$	$n = 2$	$n = 3$	$n = 4$	

(b) Find

(i) the 8th term of sequence *A*, [1]

We want to calculate the value of

$$n^3$$

for n=8:

$$(8)^3 = 512$$

(ii) the 12th term of sequence C. [1]

We want to calculate the value of

$$(n+1)^2$$

for n=12:

$$(12+1)^2 = (13)^2$$

$$\mathbf{(12+1)^2 = 169}$$

(c) (i) Which term in sequence A is equal to 15 625? [1]

We want to find out for what value of n is:

$$n^3 = 15625$$

Take the cube root of both sides to get the value of n:

$$n = \sqrt[3]{15625}$$

$$\boldsymbol{n = 25}$$

(ii) Which term in sequence C is equal to 10 000? [1]

We want to find out for what value of n is:

$$(n+1)^2 = 10\,000$$

Take the square root of both sides.

$$n + 1 = \sqrt{10\,000}$$

$$n + 1 = 100$$

Subtract 1 from both sides of the equation.

$$\boldsymbol{n = 99}$$

(d) The first four terms of sequences D and E are shown in the table below.

Use the results from **part (a)** to find the 5th and the nth terms of the sequences D and E. [4]

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence D	5	16	39	80		
Sequence E	0	1	4	9		

From the part (a) , we can see that sequence D is the sum of

the terms of sequence A and sequence B. The 5th term of D:

$$D(n = 5) = A(n = 5) + B(n = 5)$$

$$D(n = 5) = 125 + 20$$

$$\boldsymbol{D(n = 5) = 145}$$

For general n:

$$D(n) = A(n) + B(n)$$

$$\boldsymbol{D(n) = n^3 + 4n}$$

Similarly, we can observe that sequence Ecan be obtained by subtracting the terms of sequence B from sequence C. The 5th term of E:

$$E(n = 5) = C(n = 5) - B(n = 5)$$

$$E(n = 5) = 36 - 20$$

$$\boldsymbol{E(n = 5) = 16}$$

For general *n*:

$$E(n) = C(n) - B(n)$$

$$\boldsymbol{E(n) = (n + 1)^2 - 4n}$$

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence D	5	16	39	80	**145**	$\boldsymbol{n^3 + 4n}$
Sequence E	0	1	4	9	**16**	$\boldsymbol{(n+1)^2 - 4n}$

(a) (i) Work out the first 3 terms of the sequence whose nth term is $n(n + 2)$. [2]

We plug in the values n = 1, 2, 3 to find the first three terms of the sequence.

For n=1:

$$S(n = 1) = 1(1 + 2)$$

$$\boldsymbol{S(n = 1) = 2}$$

For n=2:

$$S(n = 2) = 2(2 + 2)$$

$$\boldsymbol{S(n = 2) = 8}$$

For n=3:

$$S(n = 3) = 3(3 + 2)$$

$$\boldsymbol{S(n = 3) = 15}$$

(ii) Which term in this sequence is equal to 168? [3]

Set the term equal to 168 and solve the quadratic equation for n.

$$168 = n(n + 2)$$

$$168 = n^2 + 2n$$

N = 12

(b) Find a formula for the *n*th term of the following sequences.

(i) 5 8 11 14 17 …… [2]

The first term of the sequence is 5. The common difference between the terms is 3.

Hence we can immediately recognize this series as a arithmetic series with the general form:

$$a + d(n - 1)$$

where *a* is the first term and *d* is the common difference. Therefore in our case where

$$a = 5$$

$$d = 3$$

We can write the *n*th term:

$$5 + 3(n - 1)$$

$$\mathbf{2 + 3n}$$

(ii) 1 2 4 8 16 …… [2]

The terms do not have a common difference so it is not an arithmetic series, but each next term is formed by multiplying the previous one by 2, so they have a common ratio of *r*.

We can recognize this as a geometric series with first term 1 and common ratio 2. The general form:

$$a \times r^{n-1}$$

where *a* is the first term and *r* is the common ratio. We have:

$$a = 1$$

$$r = 2$$

Hence we can write the nth term:

$$1 \times 2^{n-1}$$

$$\mathbf{2^{n-1}}$$

(c)

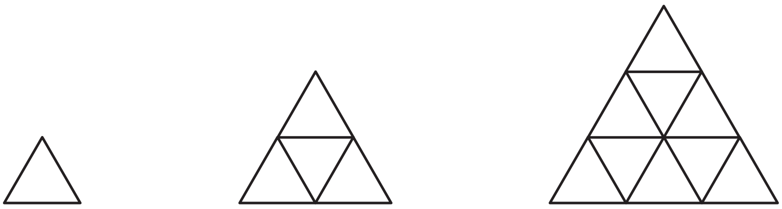


Diagram 1 Diagram 2 Diagram 3

A sequence of diagrams is formed by drawing equilateral triangles each of side one centimetre.
Diagram 1 has 3 one centimetre lines.
Diagram 2 has 9 one centimetre lines.

The formula for the **total** number of one centimetre lines needed to draw all of the first n **diagrams** is

$$an^3 + bn^2 + n.$$

Find the values of a and b.

[6]

We need to realize that the formula described total number of lines to draw all of the first n diagrams, not just the one corresponding to the number n. So for n=2, we sum the number of lines in Diagram 1 and Diagram 2.

This way, we form two equations. They are for n=1 and 2. (Let this sequence be called B)

$$B(n=1) = a \times 1^3 + b \times 1^2 + 1 = 3$$

$$B(n=2) = a \times 2^3 + b \times 2^2 + 2 = 3 + 9$$

So we have two equations of two unknowns.

$$a + b + 1 = 3$$

$$8a + 4b + 2 = 12$$

Multiply the first equation by 4 and subtract it from the second equation to eliminate *b*.

$$8a + 4b + 2 - (4a + 4b + 4) = 12 - 12$$

$$4a + 2 - 4 = 12 - 12$$

$$4a - 2 = 0$$

We have one equation of one unknown, so we simply solve for *a*.

$$\boldsymbol{a = \frac{1}{2}}$$

Now we know the value of *a*, we substitute it back into the first original

equation to solve for *b*.

$$\frac{1}{2} + b + 1 = 3$$

$$\frac{3}{2} + b = 3$$

Subtract 3/2 from both sides of the equation and find the value of *b*.

$$\boldsymbol{b = \frac{3}{2}}$$

(a) (i) The first three positive integers 1, 2 and 3 have a sum of 6.

Write down the sum of the first 4 positive integers. [1]

The sum of the first 4 positive integers:

$$sum(4) = 1 + 2 + 3 + 4$$

$$\boldsymbol{sum(4) = 10}$$

(ii) The formula for the sum of the first n integers is $\frac{n(n+1)}{2}$.

Show the formula is correct when $n = 3$. [1]

Use the formula for n=3:

$$formula(n = 3) = \frac{3(3+1)}{2}$$

$$formula(n = 3) = 6$$

This result agrees with the given value.

The formula is correct when *n*=3.

(iii) Find the sum of the first 120 positive integers. [1]

Use the formula to find the sum of the first 120 positive integers (n=120).

$$formula(n = 120) = \frac{120(120+1)}{2}$$

$$\boldsymbol{formula(n = 120) = 7260}$$

(iv) Find the sum of the integers

$121 + 122 + 123 + 124 + \ldots\ldots + 199 + 200.$ [2]

$$sum = 121 + 122 + 123 + \cdots + 199 + 200$$

The sum of the integers goes from 121 to 200.

This sum is equal to the sum of all integers up to 200 without the sum of all integers up to 120 (which are not included in out sum).

$$sum = formula(n = 200) - formula(n = 120)$$

Use the formula and the answer from the previous part.

$$sum = \frac{200(200+1)}{2} - 7260$$

$$sum = 20100 - 7260$$

$$\mathbf{sum = 12\,840}$$

(v) Find the sum of the even numbers

$2 + 4 + 6 + \ldots\ldots + 800.$ [2]

The sum of all even numbers:

$$sum(even) = 2 + 4 + 6 + \cdots + 800$$

Factorize 2 from all terms (they are even so 2 is their factor).

$$sum(even) = 2(1 + 2 + 3 \ldots + 400)$$

Now we just have a simple sum of integers up to 400 multiplied by a factor. We can use our formula.

$$sum(even) = 2 \times formula(n = 400)$$

$$sum(even) = 2 \times \frac{400(400+1)}{2}$$

Hence we have the sum of the even numbers:

$$\boldsymbol{sum(even) = 160\,400}$$

(b) (i) Complete the following statements about the sums of cubes and the sums of integers. [2]

$1^3 = 1$	$1 = 1$
$1^3 + 2^3 = 9$	$1 + 2 = 3$
$1^3 + 2^3 + 3^3 =$	$1 + 2 + 3 =$ ....
$1^3 + 2^3 + 3^3 + 4^3 =$	$1 + 2 + 3 + 4 =$ ....

Use your calculator to find the values.

Sums of cubes:

$$\boldsymbol{1^3 + 2^3 + 3^3 = 36}$$

$$\boldsymbol{1^3 + 2^3 + 3^3 + 4^3 = 100}$$

Sums of integers are known from part a).

$$\boldsymbol{1 + 2 + 3 = 6}$$

$$\boldsymbol{1 + 2 + 3 + 4 = 10}$$

(ii) The sum of the first 14 integers is 105.

Find the sum of the first 14 cubes. [1]

The sum of the first 14 cubes (using calculator):

$$1^3 + 2^3 + 3^3 + \cdots + 13^3 + 14^3$$

$$= \mathbf{11\,025}$$

(iii) Use the formula in part(a)(ii) to write down a formula for the sum of the first n cubes. [1]

We can notice from the pattern in b)i) and b)ii) that the sum of cubes up to number *n* is the same as the sum of integers up to number *n* squared (the whole sum is squared).

Hence the formula for the sum of the first *n* cubes:

$$sum(n, cubes) = (sum(n, integers))^2$$

Using the formula from part a)ii):

$$sum(n, cubes) = (formula(n))^2$$

$$\boldsymbol{sum(n, cubes) = \left(\frac{n(n+1)}{2}\right)^2}$$

(iv) Find the sum of the first 60 cubes. [1]

Sum of the first cubes (use *n*=60).

$$sum(n = 60, cubes) = \left(\frac{60(60+1)}{2}\right)^2$$

$$sum(n = 60, cubes) = (1830)^2$$

Sum of the first 60 cubes:

$$\boldsymbol{sum(n = 60, cubes) = 3\ 348\ 900}$$

(v) Find n when the sum of the first n cubes is 278784. [2]

Use general expression for the sum of the cubes, equate it to 278 784 and solve for *n*

$$sum(n, cubes) = 278\ 784$$

$$\left(\frac{n(n+1)}{2}\right)^2 = 278\ 784$$

Take square root of both sides:

$$\frac{n(n+1)}{2} = 528$$

Multiply both sides by 2:

$$n(n+1) = 1056$$

Hence we have a quadratic equation:

$$n^2 + n - 1056 = 0$$

The positive solution of this equation is:

$$\boldsymbol{n = 32}$$

Sequences
Difficulty: Hard

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 3

Time allowed: **93 minutes**

Score: **/81**

Percentage: **/100**

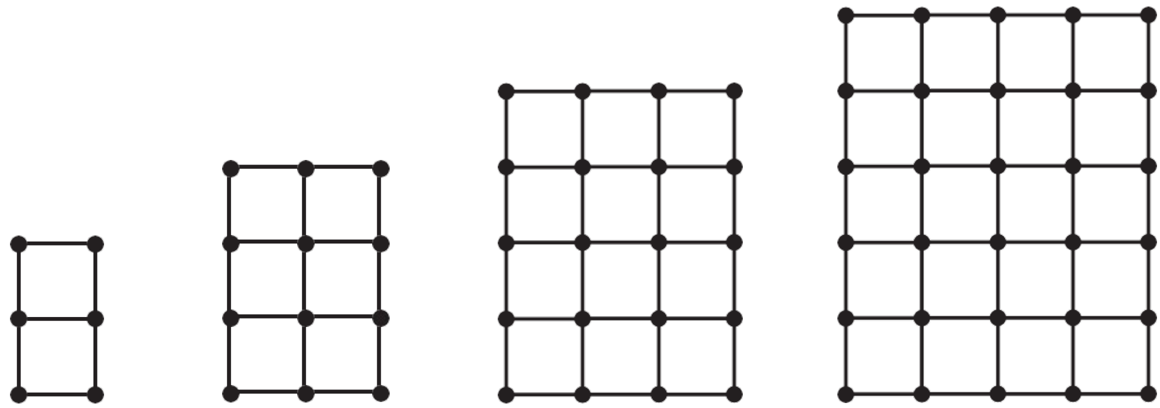
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



The first four Diagrams in a sequence are shown above.
Each Diagram is made from dots and one centimetre lines.
The area of each small square is 1 cm^2.

(a) Complete the table for Diagrams 5 and 6.

Diagram	1	2	3	4	5	6
Area (cm^2)	2	6	12	20		
Number of dots	6	12	20	30		
Number of one centimetre lines	7	17	31	49		

[4]

Diagram	1	2	3	4	5	6
Area (cm^2)	2	6	12	20	**30**	**42**
Number of dots	6	12	20	30	**42**	**56**
Number of one-centimetre lines	7	17	31	49	**71**	**97**

(b) The area of Diagram n is $n(n+1)\,\text{cm}^2$.

(i) Find the area of Diagram 50. [1]

$$A = 50(50+1)$$

$$= 50 \times 51$$

$$= \mathbf{2550}$$

(ii) Which Diagram has an **area** of 930 cm^2? [1]

$$n(n+1) = 930$$

Trial and error to find

$$\boldsymbol{n = 30}$$

(c) Find, in terms of n, the number of **dots** in Diagram n. [1]

$$\boldsymbol{(n+1)(n+2)}$$

(d) The number of one centimetre lines in Diagram n is $2n^2 + pn + 1$.

(i) Show that $p = 4$. [2]

Check for n = 1

$$2n^2 + pn + 1 = 7$$

$$\rightarrow 2 + p + 1 = 7$$

$$\rightarrow p + 3 = 7$$

$$\rightarrow \boldsymbol{p = 4}$$

(ii) Find the number of one centimetre lines in Diagram 10. [1]

$$2(10)^2 + 4(10) + 1$$

$$= 200 + 40 + 1$$

$$= \mathbf{241}$$

(iii) Which Diagram has 337 one centimetre lines? [3]

$$2n^2 + 4n + 1 = 337$$

$$\rightarrow 2n^2 + 4n - 336 = 0$$

$$\rightarrow n^2 + 2n - 168 = 0$$

Using the quadratic equation

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 + 672}}{2}$$

$$\rightarrow n = 12, n = -14$$

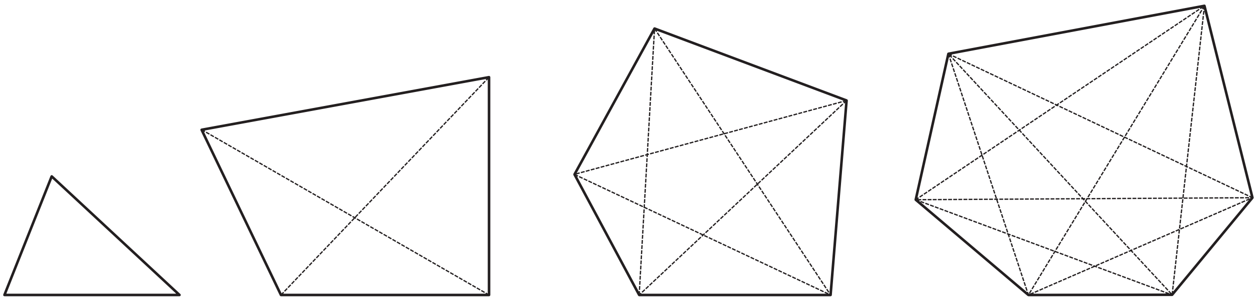
n must be positive so

$$\boldsymbol{n = 12}$$

(e) For **each** Diagram, the number of squares of area $1\ \text{cm}^2$ is A, the number of dots is D and the number of one centimetre lines is L.

Find a connection between A, D and L that is true for each Diagram. [1]

$$L = A + D - 1$$



The diagrams show some polygons and their diagonals.

(a) Complete the table.

Number of sides	Name of polygon	Total number of diagonals
3	triangle	0
4	quadrilateral	2
5		5
6	hexagon	9
7	heptagon	14
8		

[3]

Number of sides	Name of polygon	Total number of diagonals
3	Triangle	0
4	Quadrilateral	2
5	**Pentagon**	5
6	Hexagon	9
7	Heptagon	14
8	**Octagon**	**20**

(b) Write down the total number of diagonals in

(i) a decagon (a 10-sided polygon), [1]

35

(ii) a 12-sided polygon. [1]

54

(c) A polygon with n sides has a total of $\frac{1}{p}n(n-q)$ diagonals, where p and q are integers.

(i) Find the values of p and q. [3]

Substituting in values for the triangle and the quadrilateral gives us

$$\frac{1}{p}(3)(3-q)=0$$

$$\frac{1}{p}(4)(4-q)=2$$

Solving the equation for the triangle gives us either

$$p=\infty, \qquad q=3$$

Of which only q = 3 is physical, hence

$$\boldsymbol{q=3}$$

Substituting this into the quadrilateral equation gives us

$$\frac{1}{p}(4)(4-3)=2$$

$$\rightarrow \frac{4}{p}=2$$

$$\rightarrow \boldsymbol{p=2}$$

(ii) Find the total number of diagonals in a polygon with 100 sides. [1]

Using our equation

$$\frac{1}{2}(100)(100-3)$$

$$= 50 \times 97$$

$$= \mathbf{4850}$$

(iii) Find the number of sides of a polygon which has a total of 170 diagonals. [2]

We have

$$\frac{1}{2}n(n-3) = 170$$

Expanding the bracket and moving everything to one side

$$\frac{1}{2}n^2 - \frac{3}{2}n - 170 = 0$$

Multiply through by 2

$$\rightarrow n^2 - 3n - 340 = 0$$

Factorise

$$(n-20)(n+17) = 0$$

$$\rightarrow n = 20, \qquad n = -17$$

Shape must have positive number of sides, hence

$$\boldsymbol{n = 20}$$

(d) A polygon with $n + 1$ sides has 30 more diagonals than a polygon with n sides.

[1]

Find n.

Writing the problem mathematically gives us

$$\frac{1}{2}(n+1)(n+1-3) = \frac{1}{2}n(n-3) + 30$$

$$\rightarrow \frac{1}{2}(n+1)(n-2) = \frac{1}{2}n(n-3) + 30$$

Multiplying through by 2 and expanding the brackets

$$(n+1)(n-2) = n(n-3) + 60$$

$$\rightarrow n^2 - n - 2 = n^2 - 3n + 60$$

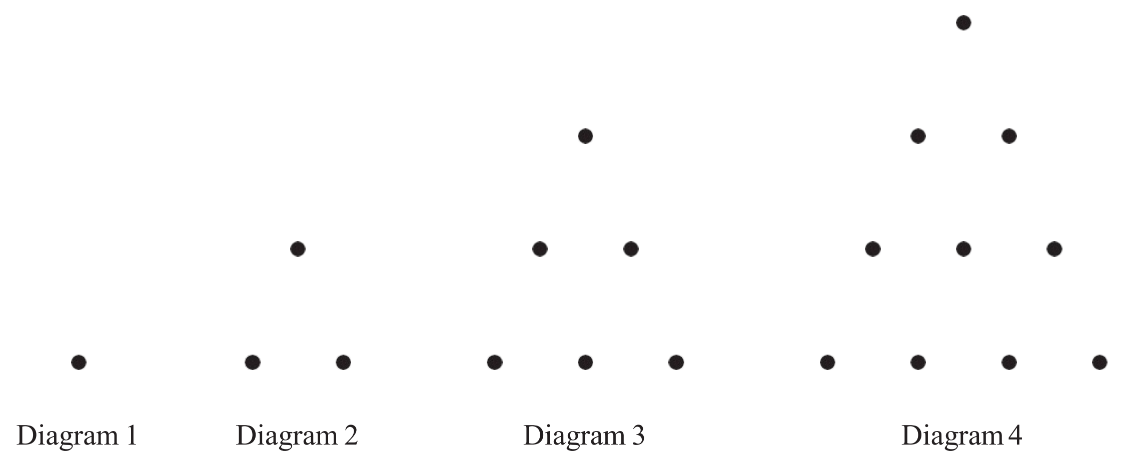
Adding $3n$ and 2 to both sides and cancelling the n^2

terms

$$2n = 62$$

Divide through by 2

$$\boldsymbol{n = 31}$$



The first four terms in a sequence are 1, 3, 6 and 10.
They are shown by the number of dots in the four diagrams above.

(a) Write down the next four terms in the sequence. [2]

We notice that the first terms in the sequence are calculated as follows:

The first term: 1

The second term: 1 + 2 = 3

The third term: 3 + 3 = 6

The forth term: 6 + 4 = 10

…

Therefore, the next 4 terms will be:

The 5^{th} term: 10 + 5 = 15

The 6^{th} term: 15 + 6 = 21

The 7^{th} term: 21 + 7 = 28

The 8^{th} term: 28 + 8 = 36

(b) (i) The sum of the two consecutive terms 3 and 6 is 9. The sum of the two consecutive terms 6 and 10 is 16.

Complete the following statements using different pairs of terms.

The sum of the two consecutive terms .... and .... is ....

The sum of the two consecutive terms .... and .... is .... [1]

We select 2 random consecutive terms from the sequence described at point a).

The sum of the 2 consecutive terms 15 and 21 is: 15 + 21 = 36

The sum of the 2 consecutive terms 21 and 28 is: 21 + 28 = 49

(ii) What special name is given to these sums? [1]

We notice that the 2 sums obtained at point b)i) are **square numbers.**

(c) (i) The formula for the nth term in the sequence 1, 3, 6, 10… is $\dfrac{n(n+1)}{k}$, where k is an integer.

Find the value of k. [1]

To work out the integer k we apply it for one of the known terms in the sequence.

We randomly choose the second term, 3, for n = 2.

n(n + 1)/k = 3

2(2 + 1)/k = 3

6/k = 3

k = 2

(ii) Test your formula when $n = 4$, **showing your working.** [1]

We substitute the values n = 4 and k = 2 and solve the equation.

n(n + 1)/2 = 4(4 + 1)/2

n(n + 1)/2 = 20/2

n(n + 1)/2 = 10

(iii) Find the value of the 180th term in the sequence. [1]

the 180th term in the sequence is worked out by substituting the value

n = 180 in the formula above.

n(n + 1)/2 = 108(180 + 1)/2

n(n + 1)/2 = 180 x 181/2

n(n + 1)/2 = 16 290

(d) (i) Show clearly that the sum of the nth and the $(n + 1)$th terms is $(n + 1)^2$. [3]

nth term = n(n + 1)/2

(n + 1)th term = (n + 1)(n + 1 + 1)/2

(n + 1)th term = (n + 1)(n + 2)/2

The sum of the 2 terms is:

Nth term + (n + 1)th term = n(n + 1)/2 + (n + 1)(n + 2)/2

We notice that (n + 1) is the common factor for both terms in the sum above.

Nth term + (n + 1)th term = (n + 1)(n + n + 2)/2

Nth term + (n + 1)th term = (n + 1)(2n + 2)/2

We multiply the 2 brackets to simplify.

Nth term + (n + 1)th term = $(2n^2 + 2n + 2n + 2) = 2$

Nth term + (n + 1)th term = $n^2 + 2n + 1$

We use the formula:

$(a + b)^2 = a^2 + 2ab + b^2$

In our case, a = n and b = 1.

Nth term + (n + 1)th term = $(n + 1)^2$

(ii) Find the values of the two consecutive terms which have a sum of 3481. [2]

From d)i) we know that the sum of 2 consecutive terms is equal to $(n + 1)^2$

n + n +

$n(n + 1)/2 + (n + 1)(n + 2)/2 = 3481$

$n + 1 = \pm 59$

We select the positive value, 59, since the sequence contains positive terms.

The 59th term is one of the 2 consecutive terms which add up to 3481, the second term being 58th.

58th term = $58(58 + 1)/2$

58th term = 1711

59th term = $59(59 + 1)/2$

59th term = 1770

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

A 3 by 3 square

x	b	c
d	e	f
g	h	i

can be chosen from the 6 by 6 grid above.

(a) One of these squares is

8	9	10
14	15	16
20	21	22

.

In this square, $x = 8$, $c = 10$, $g = 20$ and $i = 22$.

For this square, calculate the value of

(i) $(i - x) - (g - c)$, [1]

$$(22 - 8) - (20 - 10)$$

$$= 14 - 10$$

$$= \mathbf{4}$$

(ii) $cg - xi$. [1]

$$(10)(20) - (8)(22)$$

$$= 200 - 176$$

$$= \mathbf{24}$$

(b)

x	b	c
d	e	f
g	h	i

(i) $c = x + 2$. Write down g and i in terms of x. [2]

$$\boldsymbol{g = x + 12}$$

$$\boldsymbol{i = x + 14}$$

(ii) Use your answers to **part(b)(i)** to show that $(i - x) - (g - c)$ is constant. [1]

$$(x + 14 - x) - \big(x + 12 - (x + 2)\big)$$

$$= 14 - 10$$

$$\boldsymbol{= 4}$$

(iii) Use your answers to **part(b)(i)** to show that $cg - xi$ is constant. [2]

$$(x + 2)(x + 12) - x(x + 14)$$

$$= x^2 + 14x + 24 - x^2 - 14x$$

$$\boldsymbol{= 24}$$

(c) The 6 by 6 grid is replaced by a 5 by 5 grid as shown.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

A 3 by 3 square

x	b	c
d	e	f
g	h	i

can be chosen from the 5 by 5 grid.

For any 3 by 3 square chosen from this 5 by 5 grid, calculate the **value** of

(i) $(i - x) - (g - c)$, [1]

$$c = x + 2$$

$$g = x + 10$$

$$i = x + 12$$

Hence

$$(i - x) - (g - c)$$

$$= 12 - 8$$

$$\mathbf{= 4}$$

(ii) $cg - xi$. [1]

$$(x + 2)(x + 10) - x(x + 12)$$

$$\mathbf{= 20}$$

(d) A 3 by 3 square is chosen from an n by n grid.

(i) Write down the value of $(i - x) - (g - c)$. [1]

In general we have

$$c = x + 2$$

$$g = x + 2n$$

$$i = x + 2n + 2$$

Hence

$$(i - x) - (g - c)$$

$$= 2n + 2 - (2n - 2)$$

$$\mathbf{= 4}$$

(ii) Find g and i in terms of x and n. [2]

$$\boldsymbol{g = x + 2n}$$

$$\boldsymbol{i = x + 2n + 2}$$

(iii) Find $cg - xi$ in its simplest form. [1]

$$(x + 2)(x + 2n) - x(x + 2n + 2)$$

$$= x^2 + (2n + 2)x + 4n - x^2 - 2nx - 2x$$

$$\boldsymbol{= 4n}$$

The table shows some terms of several sequences.

Term	1	2	3	4		8	
Sequence P	7	5	3	1		p	
Sequence Q	1	8	27	64		q	
Sequence R	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$		r	
Sequence S	4	9	16	25		s	
Sequence T	1	3	9	27		t	
Sequence U	3	6	7	-2		u	

(a) Find the values of p, q, r, s, t and u. [6]

$$p = -7$$

$$q = 512$$

$$r = \frac{8}{9}$$

$$s = 81$$

$$t = 2187$$

$$u = -2106$$

(b) Find the nth term of sequence

(i) P, i) $-2n + 9$ [1]

(ii) Q, ii) n^3 [1]

(iii) R, iii) $\frac{n}{n+1}$ [1]

(iv) S, iv) $(n + 1)^2$ [1]

(v) T, v) 3^{n-1} [1]

(vi) U. vi) $(n + 1)^2 - 3^{n-1}$ [1]

(c) Which term in sequence **P** is equal to -777? [2]

$$-2n + 9 = -777$$

$$\rightarrow -2n = -786$$

$$\rightarrow \boldsymbol{n = 393}$$

(d) Which term in sequence **T** is equal to 177 147? [2]

$$3^{n-1} = 177\ 147$$

Trial and error give us

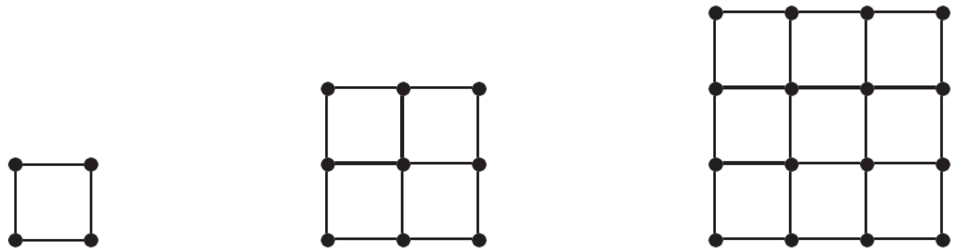
$$\rightarrow n - 1 = 11$$

$$\rightarrow \boldsymbol{n = 12}$$

Correct way to solve this is using logs, i.e.

$$\log_3 3^{n-1} = \log_3(177147)$$

$$\rightarrow n - 1 = \log_3(177147)$$



The first three diagrams in a sequence are shown above.

The diagrams are made up of dots and lines. Each line is one centimetre long.

(a) Make a sketch of the next diagram in the sequence. [1]

The first diagram is a square with the side 1 unit, the second diagram has the side 2 units and the third one has the side 3 units. The next unit will be a square with 4 units side.

(b) The table below shows some information about the diagrams.

Diagram	1	2	3	4	---------	n
Area	1	4	9	16	---------	x
Number of dots	4	9	16	p	---------	y
Number of one centimetre lines	4	12	24	q	---------	z

(i) Write down the values of p and q. [2]

Looking at the diagrams we work out p = 25 and q = 40

(ii) Write down each of x, y and z in terms of n. [4]

Following the pattern, we notice that:

$1 = 1^2$

$4 = 2^2$

…

$x = n^2$

$4 = (1 + 1)^2$

$9 = (2 + 1)^2$

$16 = (3 + 1)^2$

$y = (n + 1)^2$

$z = x + y - 1$

$4 = 1^2 + (1 + 1)^2 - 1$

$12 = 2^2 + (2 + 1)^2 - 1$

…

$z = n^2 + (n + 1)^2 - 1$

(c) The **total** number of one centimetre lines in the first n diagrams is given by the expression

$$\frac{2}{3}n^3 + fn^2 + gn.$$

(i) Use $n = 1$ in this expression to show that $f + g = \frac{10}{3}$. [1]

We substitute n = 1 in the expression:

For n = 1, the number of one cm lines is 4.

$\frac{2}{3}1^3 + f \times 1^2 + g \times 1 = 4$

$\frac{2}{3} + f + g = 4$

$\mathbf{f + g = \frac{10}{3}}$

(ii) Use $n = 2$ in this expression to show that $4f + 2g = \frac{32}{3}$. [2]

For n = 2, the number of one cm lines is 4. The number of lines in the first 2 diagrams is 4 + 12 = 16.

$\frac{2}{3}2^3 + f \times 2^2 + 2g = 16$

$\mathbf{4f + 2g = \frac{32}{3}}$

(iii) Find the values of f and g. [3]

$f + g = \frac{10}{3}$

$4f + 2g = \frac{32}{3}$

We multiply the first equation by 2 to obtain 2g in both

equations and then subtract them to work out f.

$2f + 2g = \frac{20}{3}$

$4f + 2g = \frac{32}{3}$

$2f = 4$

$f = 2$

$2 + g = \frac{10}{3}$

$g = \frac{4}{3}$

(iv) Find the total number of one centimetre lines in the first 10 diagrams. [1]

We substitute in the expression f = 2, n = 10 and g = 4/3.

$\frac{2}{3}10^3 + 2 \times 10^2 + 10 \times \frac{4}{3}$

$=$ **880**