Properties of Shapes
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Properties of Shapes
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **20 minutes**

Score: **/16**

Percentage: **/100**

Grade Boundaries:

~~Assembled by A.S.~~

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

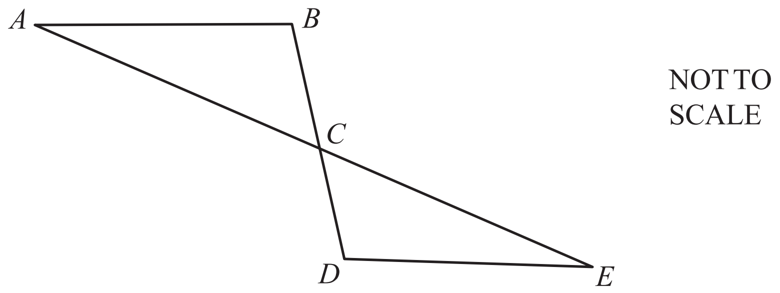
A quadrilateral has rotational symmetry of order 2 and no lines of symmetry.

Write down the mathematical name of this quadrilateral. [1]

Quadrilateral means a four-sided shape. No lines of symmetry mean that the shape cannot be a square or rectangle, for example. Rotational symmetry order 2 means that the shape has 2 positions where it looks the same if it is rotated about its centre through one full turn (360°).

Such a shape is called a

Parallelogram



The diagram shows two straight lines, AE and BD, intersecting at C.
Angle ABC = angle EDC.
Triangles ABC and EDC are congruent.

Write down **two** properties of line segments AB and DE. [2]

From the information we have, we can say that the triangles are the same, just rotated.

Therefore **the length of AB is the same as the length of DE**.

Since the angle ABC is equal to the angle EDC, **the lines AB and DE are parallel**.

ZEBRA

Write down the letters in the word above that have

(a) exactly one line of symmetry, [1]

We can find out which letters in 'ZEBRA' have exactly 1 line of symmetry like this:

Imagine placing a mirror through the centre of each letter at loads of different angles – a line of symmetry is where that mirror would show us the letter we expect to see

For example, if we placed a mirror vertically down the centre of 'A', between the paper and the mirror we would see 'A', so it has a line of symmetry down its centre

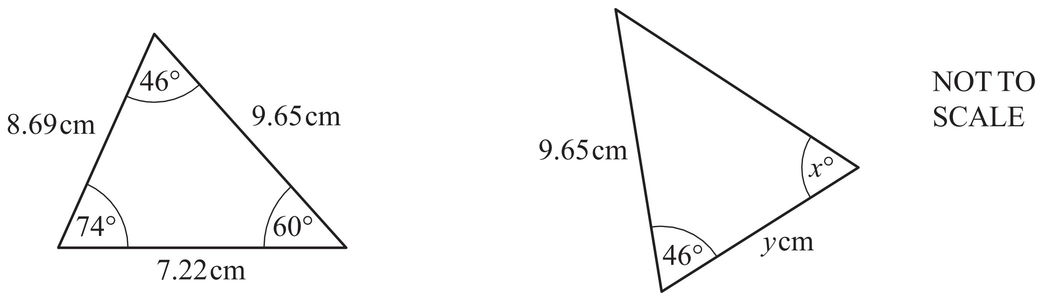
This works for E, B and A, so these 3 letters are the answer

(b) rotational symmetry of order 2. [1]

Rotational symmetry is found by rotating the letter (from the word 'ZEBRA') around an imaginary point, which we place on one of the corners

'Order 2' means that you could rotate the letter around the imaginary point and it would look the same in 2 different positions (see diagram below)

The only letter in 'ZEBRA' for which we can do this is Z – so the answer is Z



These two triangles are congruent.
Write down the value of

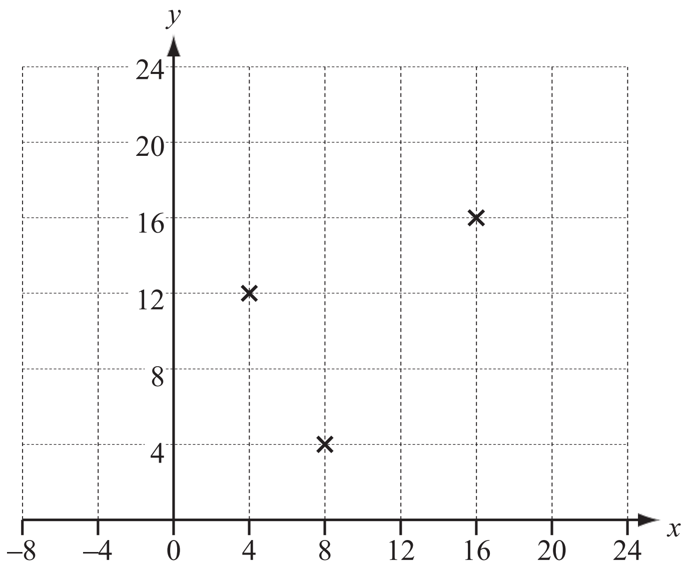
(a) x, [1]

74

(b) y. [1]

8.69

Three of the vertices of a parallelogram are at (4, 12), (8, 4) and (16, 16).



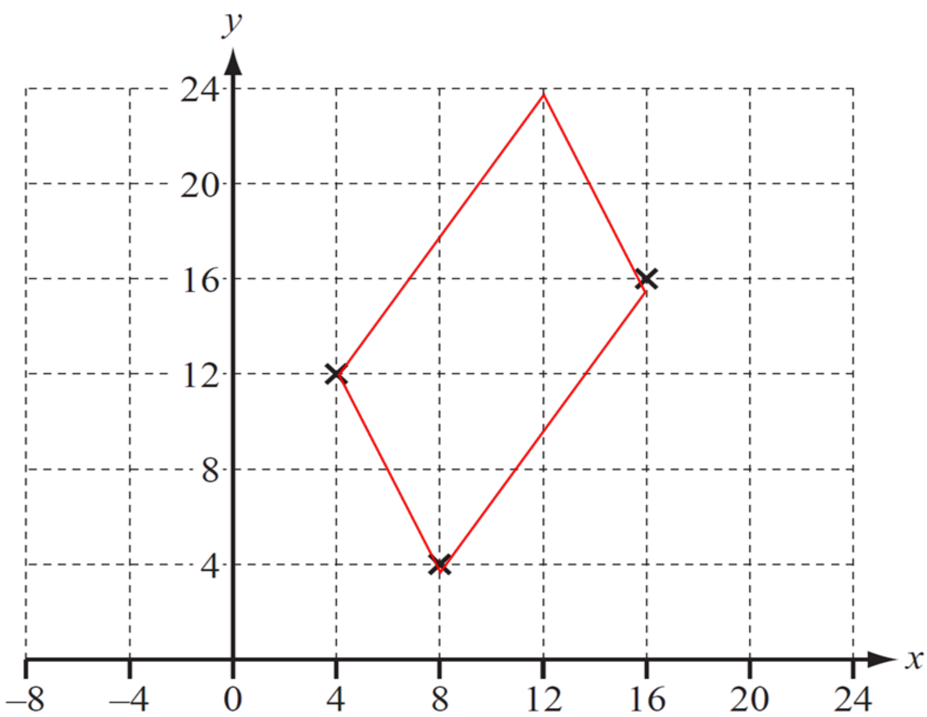
Write down the co-ordinates of two possible positions of the fourth vertex. [2]

Need to ensure that opposite sides of the parallelogram are:

1. Parallel

2. Of the same length

So long as this is achieved, the answer is correct. There are multiple solutions, below is one of them:



Parallelogram drawn in red. The final vertex should be at:

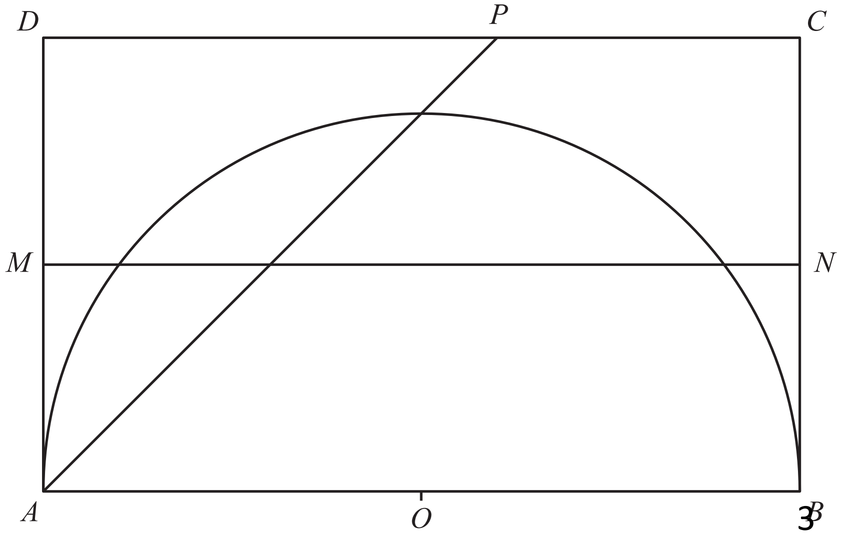
(12, 24)

Other possible solutions: ***(20 ,8)*** *and* ***(-4, 0)***

ABCD is a rectangle with *AB* = 10 cm and *BC* = 6cm. *MN* is the perpendicular bisector of *BC*.

AP is the bisector of angle *BAD*.

O is the midpoint of *AB* and also the centre of the semicircle, radius 5 cm.



Write the letter *R* in the region which satisfies **all** three of the following conditions.

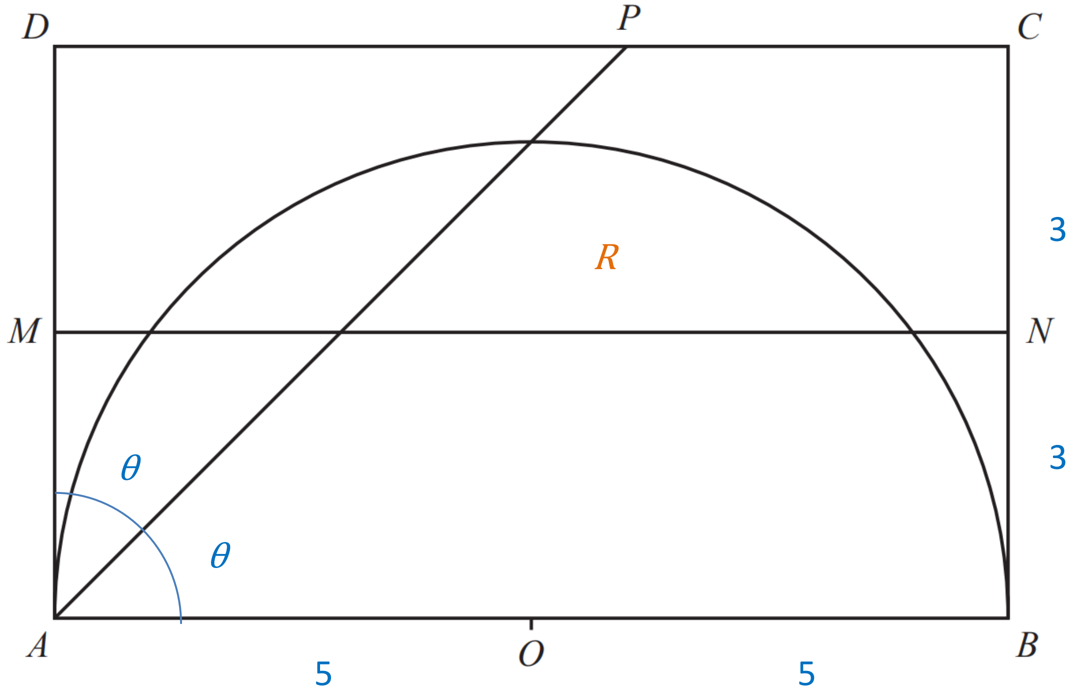
- nearer to *AB* than to *AD*
- nearer to *C* than to *B*
- less than 5 cm from *O*

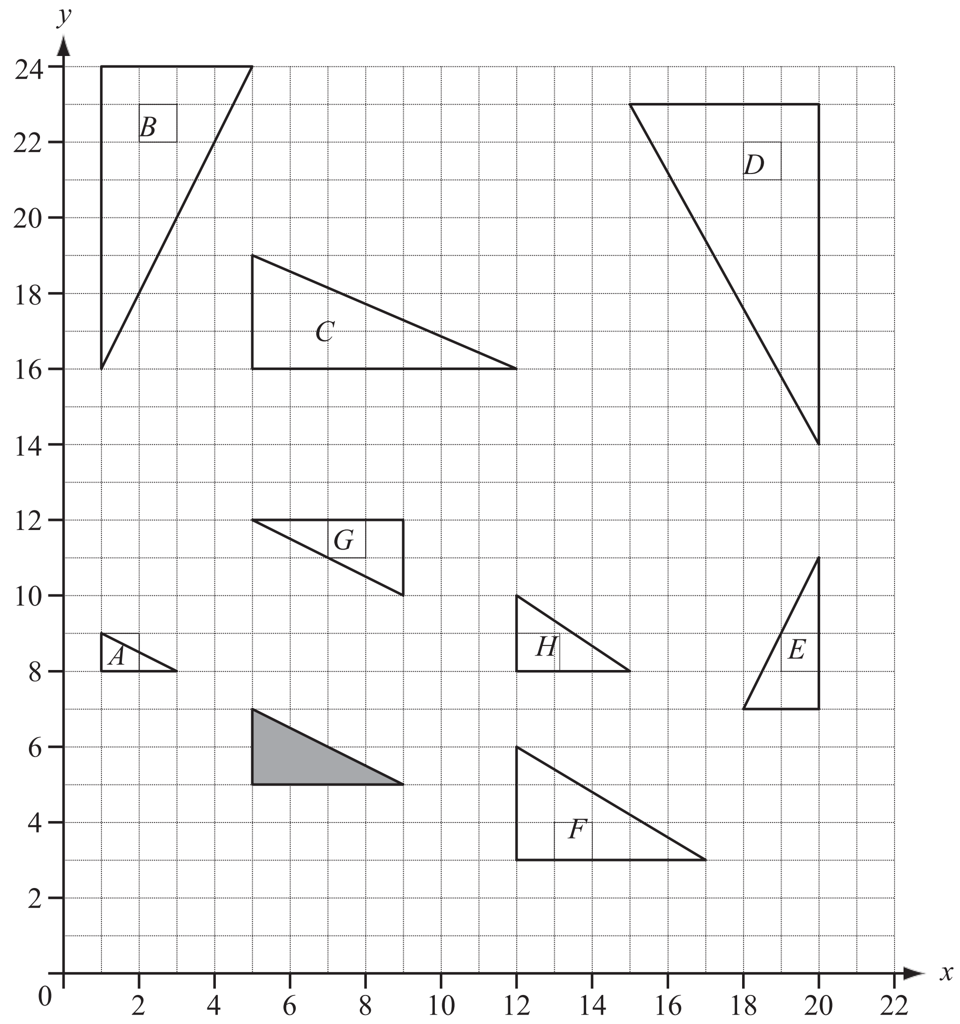
[3]

Region is to the right of AP.

Region is above MN.

Region is within the semicircle.





Write down the letters of all the triangles which are

(a) congruent to the shaded triangle, [2]

E, G

(b) similar, but not congruent, to the shaded triangle. [2]

A, B

Similarity
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Similarity
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **31 minutes**

Score: **/24**

Percentage: **/100**

Grade Boundaries:

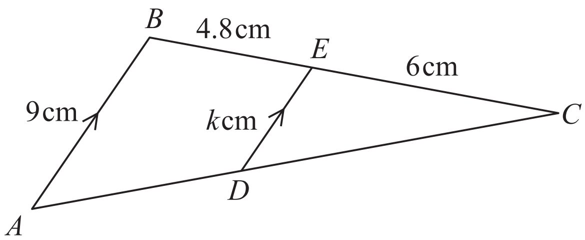
CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

(a)



NOT TO SCALE

Triangles *CBA* and *CED* are similar.
AB is parallel to *DE*.
$AB = 9$ cm, $BE = 4.8$ cm, $EC = 6$ cm and $ED = k$ cm.

Work out the value of k. [2]

If we imagine that triangle ABC was shortened to create triangle CED then the factor by which CB was shortened to create CE is the same factor that shortened AB to make DE.

$$CB = 10.8$$

$$CE = 6$$

Thus the scale factor is

$$\frac{CE}{CB} = \frac{5}{9}$$

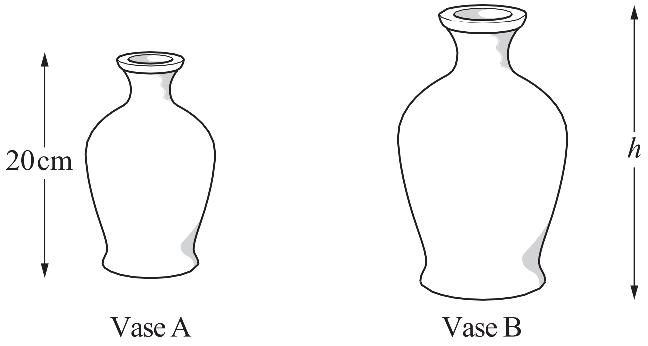
Apply this factor to AB

$$DE = \frac{5}{9} \times AB$$

$$\rightarrow k = 9 \times \frac{5}{9}$$

$$= \mathbf{5}$$

(b)



NOT TO SCALE

The diagram shows two mathematically similar vases.
Vase A has height 20 cm and volume 1500 cm^3.
Vase B has volume 2592 cm^3.

Calculate h, the height of vase B. [3]

The volume scale factor is

$$\frac{2592}{1500} = 1.728$$

This is the volume scale factor is the cube of the length (height) scale factor. The height scale factor is therefore

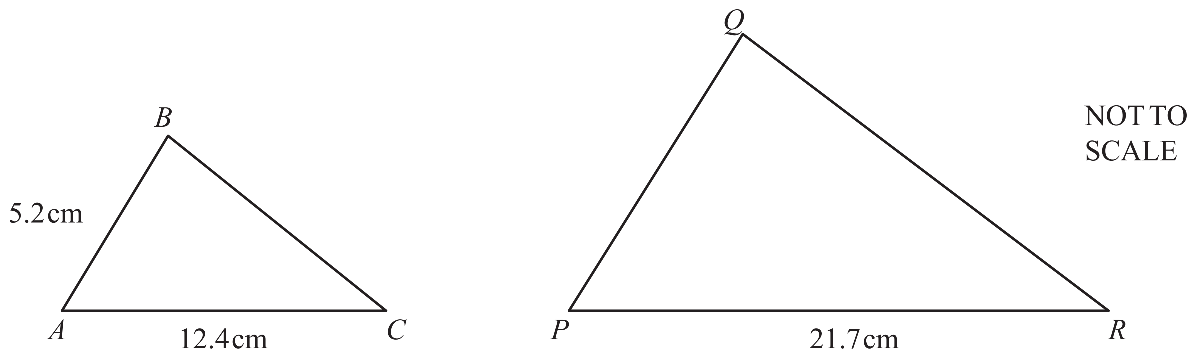
$$\sqrt[3]{1.728} = \frac{6}{5}$$

And hence

$$h_B = \frac{6}{5} \times h_A$$

$$= \mathbf{24}$$

Triangle ABC is similar to triangle PQR.



Find PQ. [2]

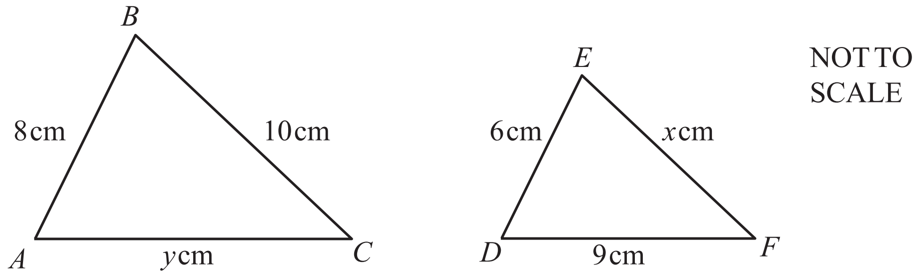
Set up a ratio of lengths between the two triangles to work out the missing length:

$$\frac{12.4}{21.7} = \frac{5.2}{x}$$

$$12.4x = 5.2 \times 21.7$$

$$x = \frac{5.2 \times 21.7}{12.4}$$

$$x = 9.1$$



Triangle ABC is similar to triangle DEF.

Calculate the value of [2]

(a) x,

Since the triangles ABC and DEF are mathematically similar, the ratio of DE to AB must be the same as the ratio EF to BC.

$$\frac{DE}{AB} = \frac{EF}{BC}$$

$$\frac{6\ cm}{8\ cm} = \frac{x\ cm}{10\ cm}$$

Multiply both sides by 10cm.

$$x = 10 \times \frac{6}{8}$$

Use calculator to find the value of x.

$$\boldsymbol{x = 7.5\ cm}$$

(b) y. [2]

Using the same argument, the ratio of AB to DE must be the same as the ratio AC to DF.

$$\frac{AB}{DE} = \frac{AC}{DF}$$

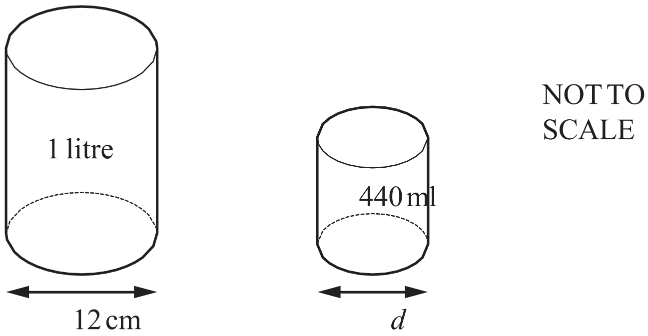
$$\frac{8\ cm}{6\ cm} = \frac{y\ cm}{9\ cm}$$

Multiply both sides by 9cm.

$$y = 9 \times \frac{8}{6}$$

Use calculator to find the value of *y*.

$$\boldsymbol{y = 12\ cm}$$



Two cylindrical cans are mathematically similar.
The larger can has a capacity of 1 litre and the smaller can has a capacity of 440ml.

Calculate the diameter, d, of the 440ml can. [3]

The volume scalar of the two cans is:

$$l^3 = \frac{440}{1000}$$

Where l is the length.

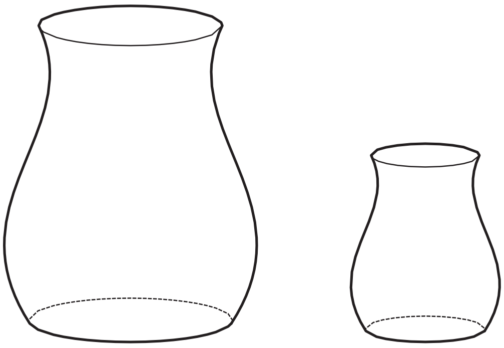
$$\rightarrow l = \sqrt[3]{\frac{440}{1000}}$$

Now we have:

$$d = l \times 12\ cm$$

$$= \sqrt[3]{\frac{440}{1000}} \times 12$$

$$= \mathbf{9.13}$$



The two containers are mathematically similar in shape.
The larger container has a volume of $3456\,cm^3$ and a surface area of $1024\,cm^2$.
The smaller container has a volume of $1458\,cm^3$.

Calculate the surface area of the smaller container. [4]

The volume scale factor will be the cube of the length scale factor.

The area scalar will be the length scalar squared.

Let the length scalar be x.

We have

$$x^3 = \frac{3456}{1458}$$

$$= \frac{64}{27}$$

$$\rightarrow x = \frac{4}{3}$$

We have that the surface area of the small one will be

$$s = \frac{1024}{x^2}$$

$$= 1024 \times \frac{9}{16}$$

=576

The volumes of two similar cones are $36\pi\,\text{cm}^3$ and $288\pi\,\text{cm}^3$.
The base radius of the smaller cone is 3 cm.

Calculate the base radius of the larger cone. [3]

Since the cones are similar the radius of the larger cone is

some multiple, k, of the smaller cone.

$$k = \sqrt[3]{\frac{288\pi}{36\pi}} = 2$$

$$larger\ cone\ radius = 2\ \times 3\ cm = \mathbf{6\ cm}$$

$$= \mathbf{6\ cm}$$

A company sells cereals in boxes which measure 10 cm by 25 cm by 35 cm.

They make a special edition box which is mathematically similar to the original box.

The volume of the special edition box is 15 120 cm^3.

Work out the dimensions of this box. [3]

We can consider each side of the special edition box as a scalar multiple, a, of the original box's sides.

The volume is then

$$10a \times 25a \times 35a = 15\,120$$

$$\rightarrow 8750a^3 = 15120$$

$$\rightarrow a^3 = 1.728$$

$$\rightarrow a = 1.2$$

The sides of the special edition box are then

$$1.2 \times 10$$

$$= \mathbf{12}$$

$$1.2 \times 25$$

$$= \mathbf{30}$$

$$1.2 \times 35$$

$$= \mathbf{42}$$

Similarity
Difficulty: Easy

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Similarity
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 2

Time allowed: **36 minutes**

Score: **/28**

Percentage: **/100**

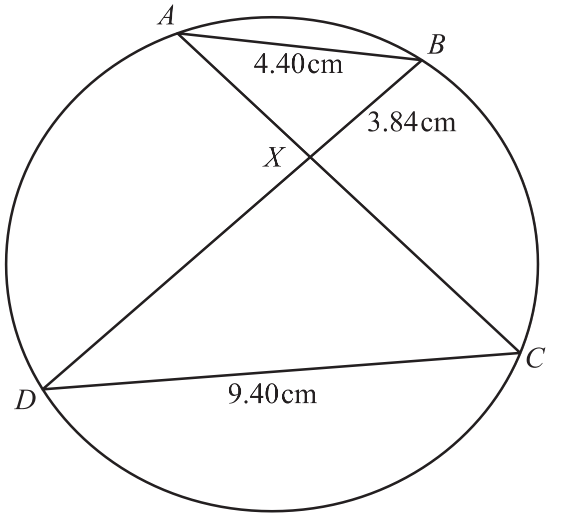
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



NOT TO SCALE

A, B, C and D lie on a circle.
AC and BD intersect at X.

(a) Give a reason why angle BAX is equal to angle CDX. [1]

Angles in the same segment of the circle.

Also:

The 2 angles are equal since the 2 triangles, ABX and CDX, are similar shapes and the angles BAX and CDX are corresponding angles in these shapes.

(i) Calculate the length of CX. [2]

The 2 triangles, ABX and CDX are similar shapes inscribed in the circle.

We know that similar shapes will have the same ratio of their corresponding lengths.

We can write this as:

$$\frac{9.4\ cm}{4.4\ cm} = \frac{\text{CX}}{3.84\ \text{cm}}$$

$$\text{CX} = \frac{3.84\ \text{cm x } 9.4\ \text{cm}}{4.4\ \text{cm}}$$

CX = 8.203 cm

(ii) The area of triangle ABX is 5.41 cm^2.

Calculate the area of triangle CDX. [2]

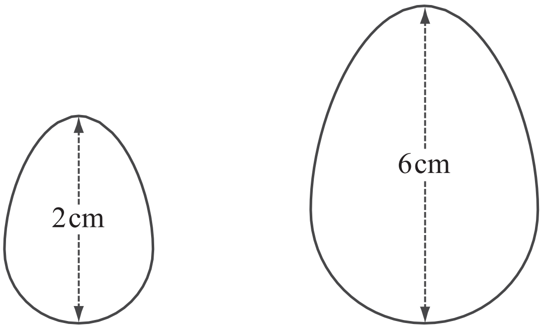
Since the 2 triangles are similar shapes, the ratio of their areas would be

equal to the ratio of their corresponding lengths squared.

We write this is:

$$(\frac{9.4\ cm}{4.4\ cm})^2 = \frac{\text{CX}}{5.41\ \text{cm}^2}$$

CX = 24.7 cm²



A company makes solid chocolate eggs and their shapes are mathematically similar.
The diagram shows eggs of height 2 cm and 6 cm.
The mass of the small egg is 4 g.

Calculate the mass of the large egg. [2]

The height of the bigger egg is 3 times larger than the height of the smaller egg.

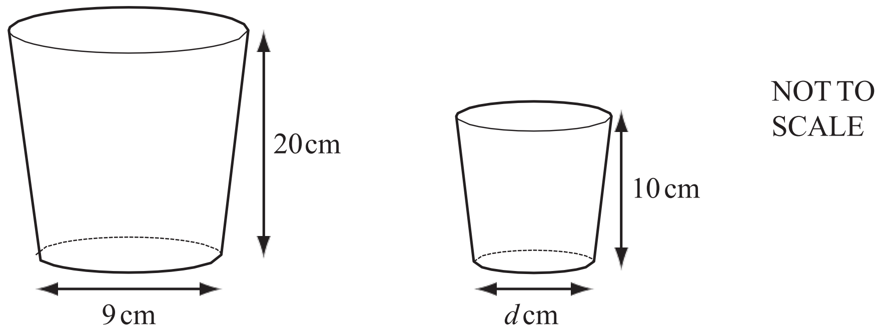
The mass is proportional to volume, which is in turn proportional to the cube of the height of the egg (so that volume is in metres cubed and length is in metres).

To get the mass of the bigger egg, we multiply the mass of the small egg by the ratio of the length cubed.

$$big\ egg\ mass = small\ egg\ mass\ \times height\ ratio^3$$

$$big\ egg\ mass = 4g\ \times 3^3$$

$$\boldsymbol{big\ egg\ mass = 4\ \times 27g = 108g}$$



The diagrams show two mathematically similar containers.
The larger container has a base with diameter 9 cm and a height 20 cm.
The smaller container has a base with diameter *d* cm and a height 10 cm.

(a) Find the value of *d*. [1]

If the containers are mathematically similar, then the ratio of their base to the height must be a constant.

$$\frac{smaller\ base}{smaller\ height} = \frac{larger\ base}{larger\ height} = constant$$

Substitute given values.

$$\frac{d\ cm}{10cm} = \frac{9cm}{20cm}$$

Multiply both sides by 10cm to get the base of the smaller container (*d*).

$$d\ cm = \frac{9cm}{20cm} \times 10cm$$

$$\boldsymbol{d = 4.5}$$

(b) The larger container has a capacity of 1600ml.

Calculate the capacity of the smaller container. [2]

From the first part, we can see that the linear constant of proportionality is:

$$\frac{smaller\ base}{larger\ base} = \frac{10cm}{20cm} = 0.5$$

If we want to convert volume, however, we cannot use linear constant, since the volume is in cubic metres, not metres.

Therefore we need to use the third power of the constant (volume is in meters cubed).

$$\frac{smaller\ volume}{larger\ volume} = (linear\ constnat)^3$$

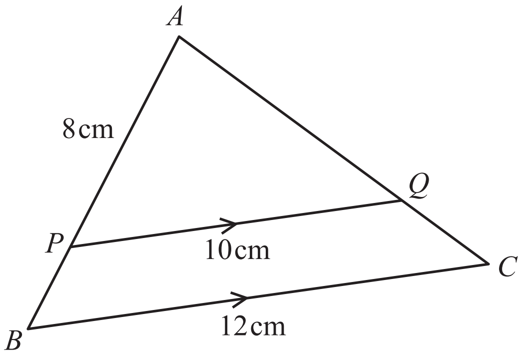
Substitute known values:

$$\frac{smaller\ volume}{1600\ ml} = (0.5)^3$$

Calculate the volume of the smaller container.

$$smaller\ volume = 1600\ ml \times (0.5)^3$$

$$\boldsymbol{smaller\ volume = 200\ ml}$$



APB and AQC are straight lines. PQ is parallel to BC.
$AP = 8$ cm, $PQ = 10$ cm and $BC = 12$ cm.
Calculate the length of AB. [2]

APQ and ABC are similar triangles.

The length scalar is

$$s = \frac{BC}{PQ}$$

$$= \frac{12}{10}$$

$$= 1.2$$

The length AB is then

$$AB = 1.2 \times AP$$

$$= 1.2 \times 8$$

$$= \mathbf{9.6}$$

A cylindrical glass has a radius of 3 centimetres and a height of 7 centimetres.
A large cylindrical jar full of water is a similar shape to the glass.
The glass can be filled with water from the jar exactly 216 times.
Work out the radius and height of the jar. [3]

The volume of the jar is 216 times greater than the volume of the glass, i.e.

$$V_j = s^3 V_g$$

$$s^3 = 216$$

where s^3 is the volume scalar.

If we cube root it, we get s, the length scale factor.

$$s = \sqrt[3]{216}$$

$$= 6$$

Hence the radius of the jar is

$$r_j = 3 \times 6$$

$$= \mathbf{18}$$

and the height is

$$h_j = 7 \times 6$$

$$= \mathbf{42}$$

A car manufacturer sells a similar, scale model of one of its real cars.

(a) The fuel tank of the real car has a volume of 64 litres and the fuel tank of the model has a volume of 0.125 litres.
Show that the length of the real car is 8 times the length of the model car. [2]

Let the volume scale factor of the car and its model be s^3.

We have that

$$0.125s^3 = 64$$

$$\rightarrow s^3 = \frac{64}{0.125}$$

$$= 512$$

If we cube root the volume scale factor, we get the length scalar, s.

Hence

$$s = \sqrt[3]{512}$$

$$= \mathbf{8}$$

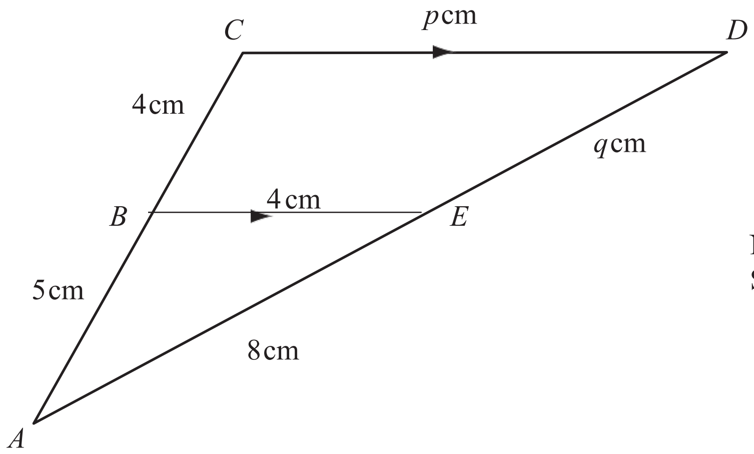
(b) The area of the front window of the model is 0.0175 m^2.
Find the area of the front window of the real car. [2]

The area scalar is the length scalar squared.

$$0.0175 \times 8^2$$

$$= \mathbf{1.12}$$

(a)



NOT TO SCALE

In the diagram triangles *ABE* and *ACD* are similar.
BE is parallel to *CD*.
AB = 5 cm, *BC* = 4 cm, *BE* = 4 cm, *AE* = 8 cm, *CD* = *p* cm and *DE* = *q* cm.
Work out the values of *p* and *q*. [4]

Since the 2 triangles are similar shapes, the ratio of their

corresponding lengths will be equal.

In our case:

$$\frac{5}{9} = \frac{8}{8+q}$$

We solve the equality for q.

63 = 40 + 5q

q = 6.4

$$\frac{5}{9} = \frac{4}{p}$$

We solve the equality for p.

63 = 5p

p = 7.2

(b) A spherical balloon of radius 3 metres has a volume of 36π cubic metres.
It is further inflated until its radius is 12 m.
Calculate its new volume, leaving your answer in terms of π. [2]

The 2 balloons are similar shapes, the ratio of their corresponding

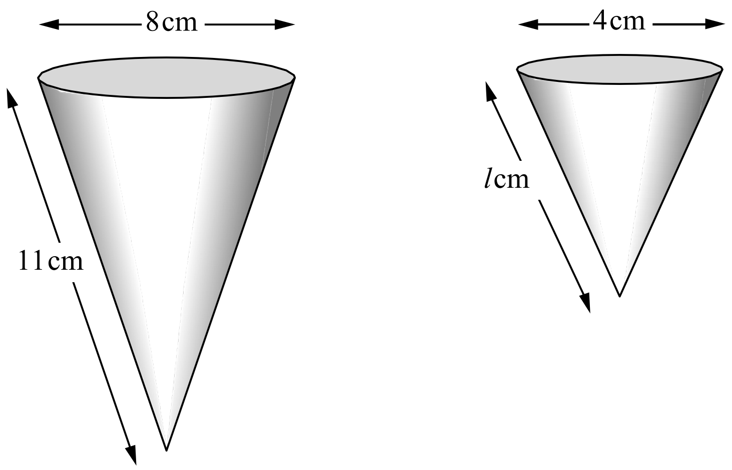
lengths cubes is the ratio of their volumes.

$(\frac{3}{12})^3 = \frac{36\pi}{V}$

We solve the equality for V.

$V = \frac{12^3 x\ 36\pi}{3^3}$

$V = 2304\pi$



NOT TO SCALE

The two cones are similar.

(a) Write down the value of l. [1]

The cones are similar shapes, therefore, the ratios of their corresponding lengths will be equal.

$$\frac{4\ cm}{8\ cm} = \frac{l}{11\ cm}$$

$$l = \frac{11\ cm\ x\ 4\ cm}{8\ cm}$$

l = 5.5 cm

(b) When full, the larger cone contains 172 cm^3 of water.
How much water does the smaller cone contain when it is full? [2]

The ratio of their volumes will also be equal to the ratio of their corresponding lengths cube.

$$\left(\frac{4\ cm}{8\ cm}\right)^3 = \frac{V}{172\ cm^3}$$

Where V represents the volume of the smaller cone.

V = 21.5 cm^3

Similarity
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Similarity
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **26 minutes**

Score: **/20**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
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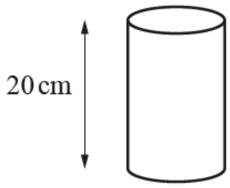
(a)



NOT TO SCALE

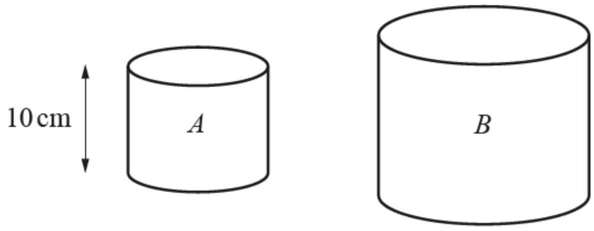
A cylinder has height 20cm.
The area of the circular cross section is 74cm^2.

Work out the volume of this cylinder. [1]

20×74

$= \mathbf{1480}$

(b) Cylinder A is mathematically similar to cylinder B.



NOT TO SCALE

The height of cylinder A is 10 cm and its surface area is 440 cm^2.
The surface area of cylinder B is 3960 cm^2.

Calculate the height of cylinder B. [3]

The length scalar, s, is

$$s = \frac{h_B}{10}$$

$$\rightarrow h_B = 10s$$

Where h_B is the height of cylinder B.

It relates the areas as

$$440s^2 = 3960$$

$$\rightarrow s^2 = \frac{3960}{440}$$

$$= 9$$

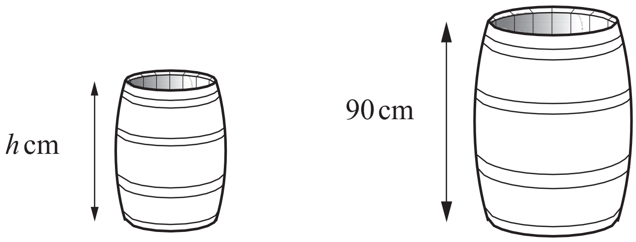
$$\rightarrow s = 3$$

Hence

$$h_B = 10 \times 3$$

$$= \mathbf{30}$$

The two barrels in the diagram are mathematically similar.



NOT TO
SCALE

The smaller barrel has a height of h cm and a capacity of 100 litres.
The larger barrel has a height of 90 cm and a capacity of 160 litres.

Work out the value of h. [3]

We know that there will be some volume scalar, a, that relates the heights as

$$h \times a = 90$$

It will also relate the volumes as

$$100a^3 = 160$$

Divide through by 100

$$\rightarrow a^3 = 1.6$$

cube root both sides

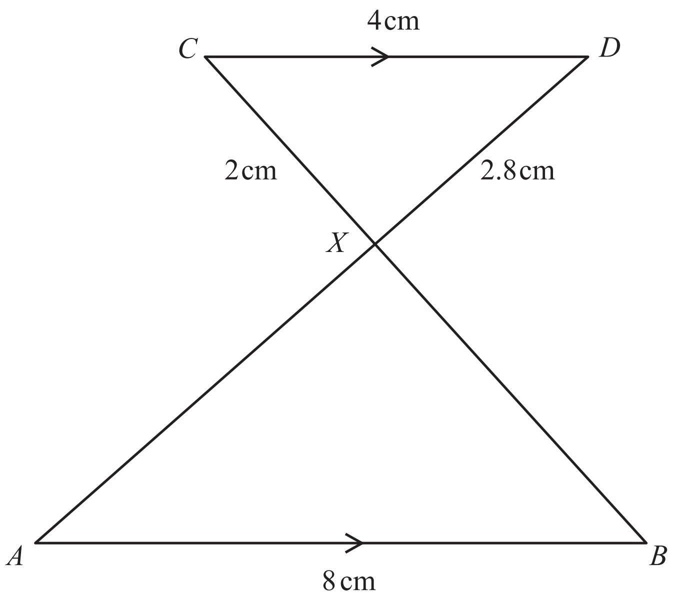
$$\rightarrow a = \sqrt[3]{1.6}$$

Now put this back into the height relation

$$h \times \sqrt[3]{1.6} = 90$$

$$\rightarrow h = 90 \div \sqrt[3]{1.6}$$

$$\approx \mathbf{76.9}$$



NOT TO SCALE

In the diagram, AB and CD are parallel.
AD and BC intersect at X.
$AB = 8$ cm, $CD = 4$ cm, $CX = 2$ cm and $DX = 2.8$ cm.

(a) Complete this mathematical statement. [1]

Triangle ABX is ....**similar**.... to triangle DCX.

(b) Calculate AX. [2]

The length scalar is

$$s = \frac{AB}{CD}$$

$$= \frac{8}{4}$$

$$= 2$$

Hence

$$AX = 2DX$$

$$= 2(2.8)$$

$$= \mathbf{5.6}$$

(c) The area of triangle ABX is $y\text{cm}^2$.

Find the area of triangle DCX in terms of y. [1]

The area scalar will be the length scalar squared, i.e.

$$Area = y \div 2^2$$

$$= \frac{y}{4}$$

Two bottles and their labels are mathematically similar.
The smaller bottle contains 0.512 litres of water and has a label with area 96 cm^2.
The larger bottle contains 1 litre of water.

Calculate the area of the larger label. [3]

The volumes are related by the cube of a scalar, a

$$a^3 \times 0.512 = 1$$

The areas are related by the square of the same scalar

$$a^2 \times 96 = A$$

Where A is the area of the larger label.

By solving the volume relation

$$a^3 = \frac{1}{0.512}$$

$$\rightarrow a = 1.25$$

We can solve for A

$$A = 1.25^2 \times 96$$

$$= \mathbf{150}$$

Two cups are mathematically similar.
The larger cup has capacity 0.5 litres and height 8 cm.
The smaller cup has capacity 0.25 litres.

Find the height of the smaller cup. [3]

Use the capacities to find the Volume Factor from large to small:

$$\text{Volume Factor } = \frac{0.25}{0.5} = \frac{1}{2}$$

The Volume Factor is the cube of the Scale Factor so:

$$\text{Scale Factor } = \sqrt[3]{\frac{1}{2}}$$

$$\text{Height of Smaller Cup } = 8 \times \sqrt[3]{\frac{1}{2}}$$

$$= \mathbf{6.35cm}$$

The length of a backpack of capacity 30 litres is 53 cm.

Calculate the length of a mathematically similar backpack of capacity 20 litres. [3]

Use the capacities to find the Volume Factor from large to small:

Volume Factor $= \frac{20}{30} = \frac{2}{3}$

The Volume Factor is the cube of the Scale Factor so:

Scale Factor $= \sqrt[3]{\frac{2}{3}}$

Length of Smaller Backpack $= 53 \times \sqrt[3]{\frac{2}{3}}$

$= \mathbf{46.3cm}$ (to 3 significant figures)

Similarity
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Similarity
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **26 minutes**

Score: **/20**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

Two containers are mathematically similar.
Their volumes are 54 cm^3 and 128cm^3.
The height of the smaller container is 4.5cm.

Calculate the height of the larger container. [3]

Let the height of the larger container be *y*.

Since the volume is in centimetres cubed and the heights are in centimetres (and the containers are mathematically similar), the ratio of the heights must be the same as the cube root of the ratio of the volumes.

$$\frac{y\ cm}{4.5cm} = \sqrt[3]{\frac{128\ cm^3}{54\ cm^3}}$$

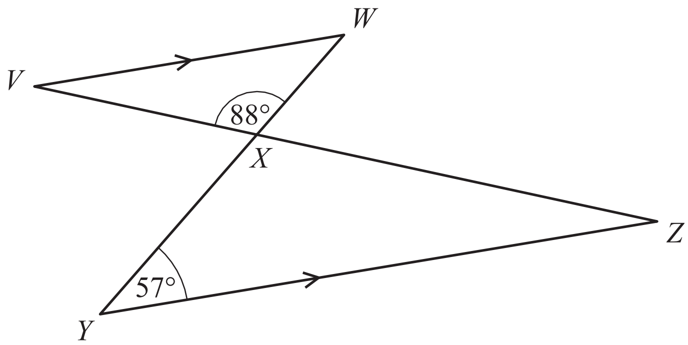
Multiply both sides by 4.5cm.

$$y\ cm = \sqrt[3]{\frac{128\ cm^3}{54\ cm^3}} \times 4.5cm$$

Use calculator to find the height of the larger container.

$$\boldsymbol{y = 6\ cm}$$

(a)



NOT TO SCALE

Two straight lines VZ and YW intersect at X.
VW is parallel to YZ, angle $XYZ = 57°$ and angle $VXW = 88°$.

Find angle WVX. [2]

The angles XYZ and XWV are alternate angles; therefore they have the same size.

$$angle\ XYZ = angle\ XWV = 57°$$

The sum of all interior angles of a triangle must be 180°. This must be true for the triangle WVX.

$$180° = angle\ VXW + angle\ XWV + angle\ WVX$$

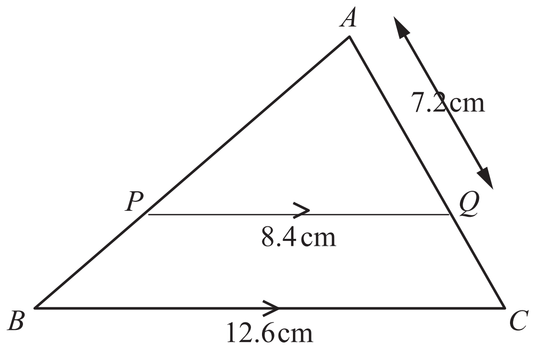
$$180° = 88° + 57° + angle\ WVX$$

$$180° = 145° + angle\ WVX$$

Subtract 145° from both sides to get the final answer.

$$\boldsymbol{angle\ WVX = 35°}$$

(b)



ABC is a triangle and PQ is parallel to BC.
$BC = 12.6$ cm, $PQ = 8.4$ cm and $AQ = 7.2$ cm.

Find AC. [2]

Since the triangles BCA and PQA are similar the ratio of AC to AQ must be the same as the ratio BC to PQ. By putting this into an equation we have:

$$\frac{AC}{AQ} = \frac{BC}{PQ}$$

$$\frac{AC}{7.2\ cm} = \frac{12.6\ cm}{8.4\ cm}$$

Multiply both sides by 7.2.

$$AC = \left(\frac{12.6}{8.4} \times 7.2\right) cm$$

Use calculator to get the length of AC.

$$\boldsymbol{AC = 10.8\ cm}$$

A car, 4.4 metres long, has a fuel tank which holds 65 litres of fuel when full.
The fuel tank of a mathematically similar model of the car holds 0.05 litres of fuel when full.

Calculate the length of the model car in centimetres. [3]

Two shapes are similar if they have the same shape but the size is different. The shapes relate through a scale factor, a number which multiplied by the size of the smaller shape gives the size of the bigger shape.

In our case, the ratio between the length of the bigger car and the length of the smaller car is the scale factor:

$\frac{4.4 \text{ m}}{x} = a$

Where x is the length of the smaller model car and a is the scale factor.

We know that lengths increase by a scale factor, areas increase by a scale factor squared and volumes increase by a scale factor cubed.

Since the 2 cars are mathematically similar models, the ratio of their lengths (the scale factor) cubed is equal to the ratio of their tanks' volumes.

$(\frac{4.4 \text{ m}}{x})^3 = a^3 = \frac{65 \text{ l}}{0.05 \text{ l}}$

Where x is the length of the model car.

$(\frac{4.4 \text{ m}}{x})^3 = 1300$

$1300x^3 = 85.184 \text{ m}^3$

$x^3 = 0.06\ m^3$

$x = 0.403$ m

To convert metres in centimetres, we multiply our result by 100.

The length of the model car is 40.3 cm.

Two similar vases have heights which are in the ratio 3 : 2.

(a) The volume of the larger vase is 1080 cm^3.
Calculate the volume of the smaller vase. [2]

Since the vases are similar, the ratio of their heights cubed will be equal to the ratio of their corresponding volumes.

$$\left(\frac{3}{2}\right)^3 = \frac{1080\ cm^3}{V}$$

Where V represents the volume of the smaller vase.

V = 320 cm^3

(b) The surface area of the smaller vase is 252 cm^2.
Calculate the surface area of the larger vase. [2]

Similarly, since the 2 vases are similar, the ratio of their heights squared will be equal to the ratio of their corresponding surface areas.

$$\left(\frac{3}{2}\right)^2 = \frac{A}{252\ cm^2}$$

Where A represents the surface area of the large vase.

A = 567 cm^2

A statue two metres high has a volume of five cubic metres.
A similar model of the statue has a height of four centimetres.

(a) Calculate the volume of the model statue in cubic centimetres. [2]

The 2 statues are similar shapes.

In this case, the cube of their heights ratio will be equal to the ratio of their corresponding volumes.

We need the result in cubic centimetres.

$5 \text{ m}^3 = 5000000 \text{ cm}^3$

$2 \text{ m} = 200 \text{ cm}$

$$\frac{5000000 \text{ cm}^3}{\text{x}} = \left(\frac{200 \text{ cm}}{4 \text{ cm}}\right)^3$$

Where x represents the volume of the model

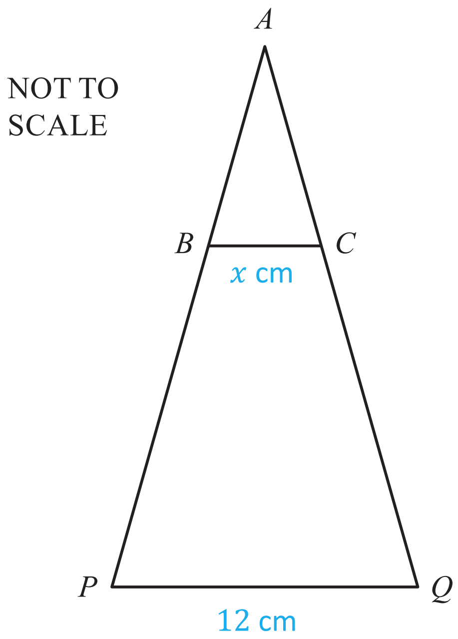
$$\frac{5000000 \text{ cm}^3}{\text{x}} = 50^3$$

$x = 40 \text{ cm}^3$

(b) Write your answer to **part (a)** in cubic metres. [1]

To convert this result in cubic metres we need to divide it by 1000000.

$x = 0.00004 \text{ m}^3$



The area of triangle APQ is 99 cm^2 and the area of triangle ABC is 11 cm^2. BC is parallel to PQ and the length of PQ is 12 cm.

Calculate the length of BC. [3]

Use the Areas given to find the Area Factor from triangle APQ to triangle ABC:

$$\text{Area Factor} = \frac{\text{Area}_{ABC}}{\text{Area}_{APQ}} = \frac{11}{99} = \frac{1}{9}$$

The Scale Factor of an Enlargement is the Square Root of the Area Factor:

$$\text{Scale Factor} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

Now use the Scale Factor to find the required length:

$$BC = \frac{1}{3} \times PQ$$

$$x = \frac{1}{3} \times 12$$

$$\boldsymbol{x = 4 \text{ cm}}$$

Symmetry
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Symmetry
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **31 minutes**

Score: **/24**

Percentage: **/100**

Grade Boundaries:

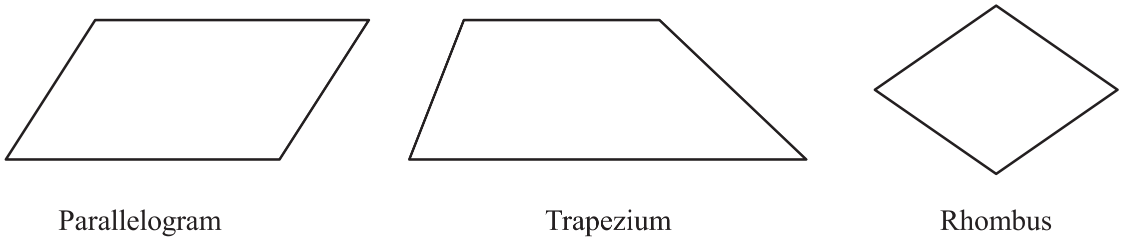
CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

Assembled by A.S

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



Write down which one of these shapes has

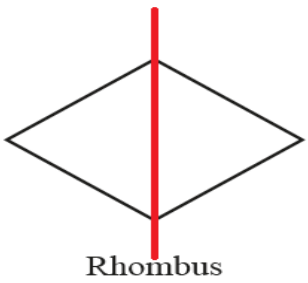
- rotational symmetry of order 2

and

- no line symmetry. [1]

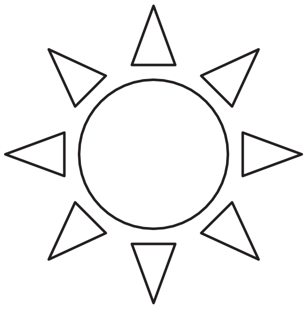
Trapezium does not have a rotational symmetry of order 2 (we would have to rotate it by full 360° to get the same shape).

Rhombus does have a line of symmetry (draw a line through the centre and two of its vertices – image on the right).



Therefore the answer is

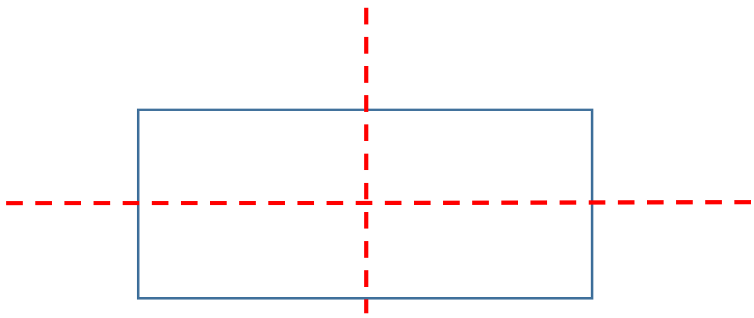
parallelogram.



Write down the order of rotational symmetry of this shape. [1]

=8

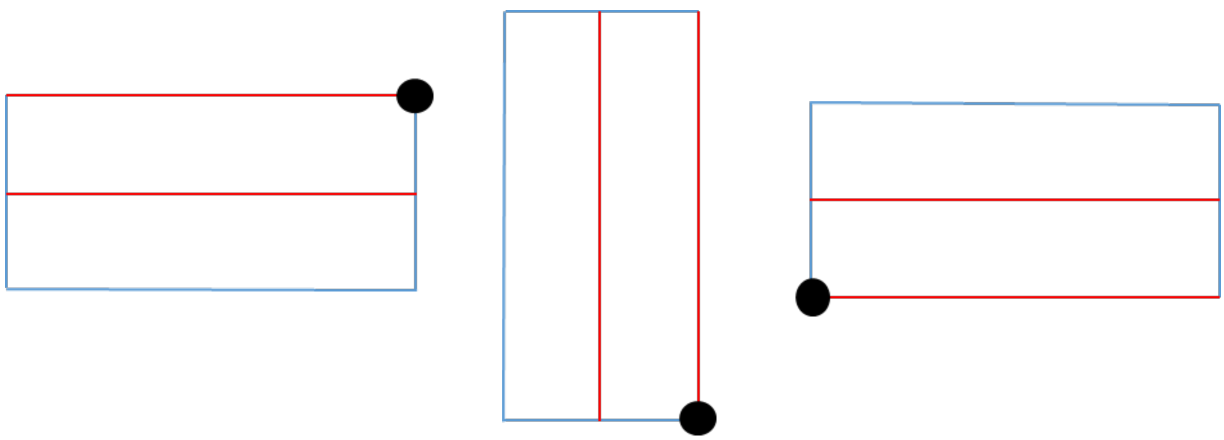
(a) Add **one** line to the diagram so that it has two lines of symmetry. [1]



Forming this shape gives us the 2 lines of symmetry highlighted in red.

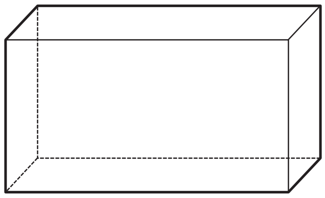
(b) Add **two** lines to the diagram so that it has rotational symmetry of order 2. [1]

For rotational symmetry, you mark a point in the corner of the figure and rotate the figure. Below is one possible answer.



Note that the first and third figures look exactly the same. So, if upon rotation (of which there are 4 orientations), two of the orientations are identical, then the diagram has a rotational symmetry of order 2.

(a) The diagram shows a cuboid.

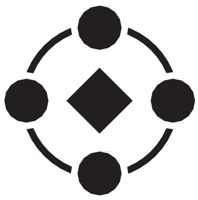


How many planes of symmetry does this cuboid have? [1]

The cuboid can be reflected through the 3 planes that bisect it.

3

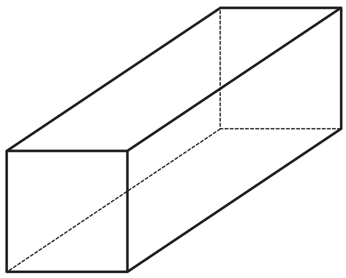
(b) Write down the order of rotational symmetry for the following diagram. [1]



Over a full turn, the shape can be rotated onto itself in 4 different positions.

4

(a)



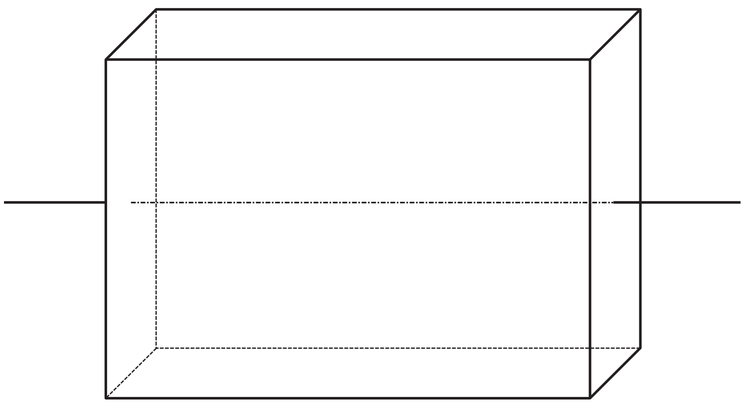
This cuboid has a **square** cross-section.

Write down the number of planes of symmetry. [1]

5.

4 along the length of the cuboid through the square faces and 1 through the middle of the length of the cuboid.

(b)



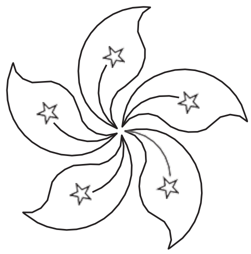
This cuboid has a **rectangular** cross-section.

The axis shown passes through the centre of two opposite faces.

Write down the order of rotational symmetry of the cuboid about this axis. [1]

2.

The original position and the position that's a rotation of π (flipped).



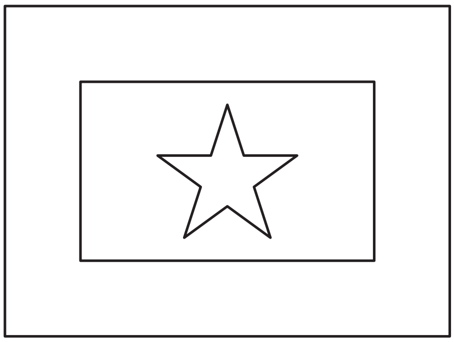
For the diagram, write down

(a) the order of rotational symmetry, [1]

5

(b) the number of lines of symmetry. [1]

0



For the **diagram**, write down

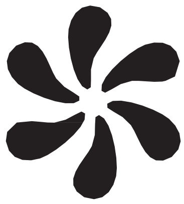
(a) the order of rotational symmetry, [1]

1

(b) the number of lines of symmetry. [1]

1

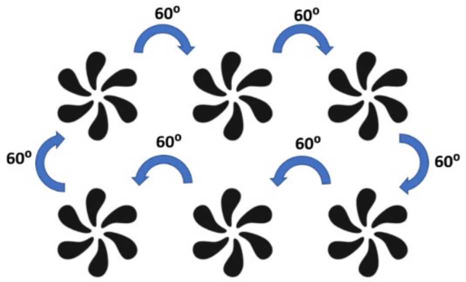
Question 8



For the diagram above write down

(a) the order of rotational symmetry, [1]

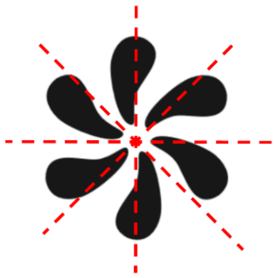
The order of rotational symmetry of a shape is the number of times it can be rotated around a full circle and still look the same. Hence by inspection we can see that:



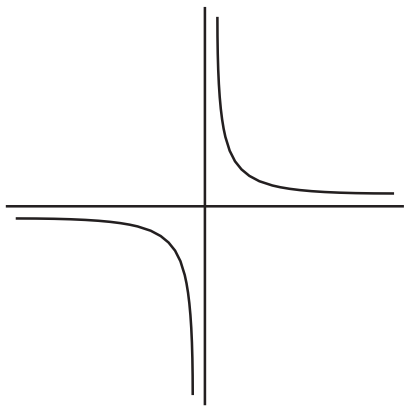
The order of rotational symmetry = 6

(b) the number of lines of symmetry. [1]

A line of symmetry is an imaginary line where you can fold the image and have both halves match exactly. Hence by inspection we can see that there are no lines of symmetry as the image will differ if folded over any imaginary line. 6 such examples are shown below:



The number of lines of symmetry = 0



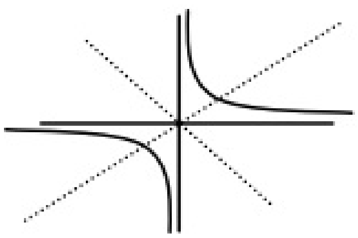
(a) Write down the order of rotational symmetry of the diagram. [1]

The order of rotational symmetry is the number of times the figure can be rotated about itself, up to 360°, so it matches the original figure.

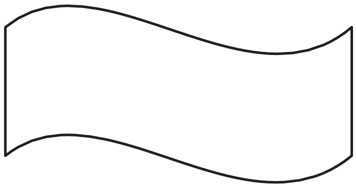
In our case, the order of rotational symmetry is 2.

(b) Draw all the lines of symmetry on the diagram. [1]

A line of symmetry separates the figure in 2 perfectly symmetrical areas.



In our case, there are 2 lines of symmetry about which the figure can be folded to fit perfectly on top.



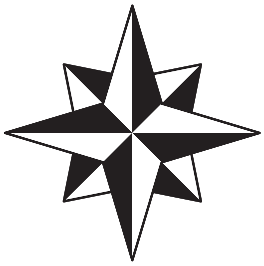
For this diagram, write down

(a) the order of rotational symmetry, [1]

2

(b) the number of lines of symmetry. [1]

0



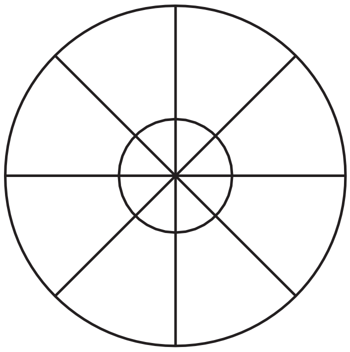
For the diagram, write down

(a) the order of rotational symmetry, [1]

4

(b) the number of lines of symmetry. [1]

0 – this is because the colours are not reflected.



The order of rotational symmetry is the number of times the figure matches its initial shape while rotating it once 360°

For the diagram above write down

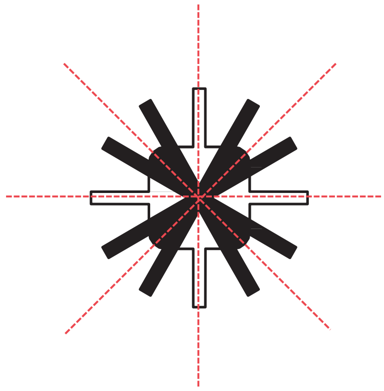
(a) the order of rotational symmetry, [1]

order of rotational symmetry = 4

(b) the number of lines of symmetry. [1]

The line of symmetry is a line over which the figure can be reflected and it will appear unchanged.

number of lines of symmetry = 4



For the shape above, write down

(a) the number of lines of symmetry, [1]

There are four lines of symmetry (mirror lines) indicated on the diagram in red

(b) the order of rotational symmetry. [1]

As the shape is rotated, it will look the same every quarter turn so:

Order of rotational symmetry = 4

Symmetry
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Symmetry
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **26 minutes**

Score: **/20**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
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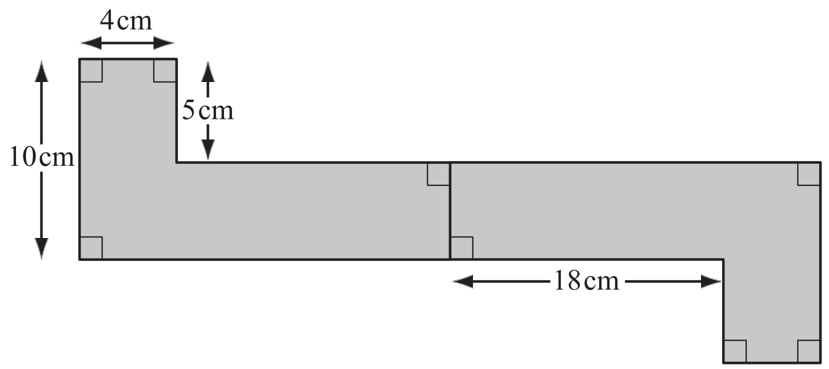
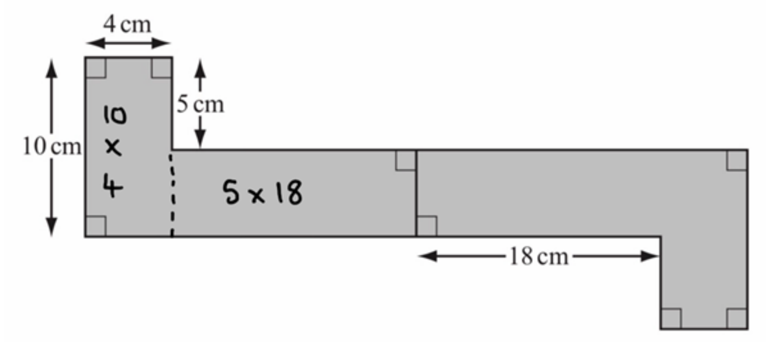


NOT TO SCALE

The shaded shape has rotational symmetry of order 2.

Work out the shaded area. [3]

The shape has rotational symmetry of order 2, therefore each half has the same measurements as shown in the figure.



Therefore, calculate the area of one half and multiply it by two.

$$10 \times 4 + 5 \times 18 = 130\ cm^2$$

$$2 \times 130 = \mathbf{260\ cm^2}$$

$$= \mathbf{260\ cm^2}$$

T R I G O N O M E T R Y

From the above word, write down the letters which have

(a) exactly two lines of symmetry, [1]

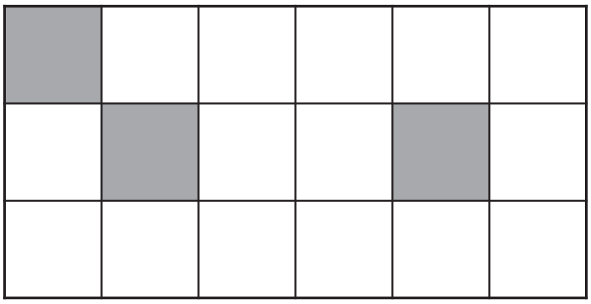
I

(b) rotational symmetry of order 2. [1]

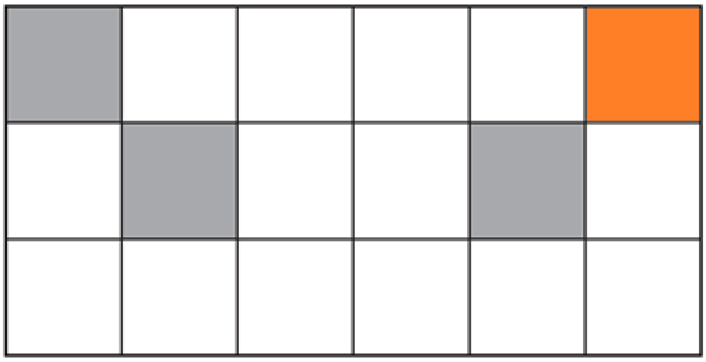
I*, *N

(a) Shade **one** square in each diagram so that there is

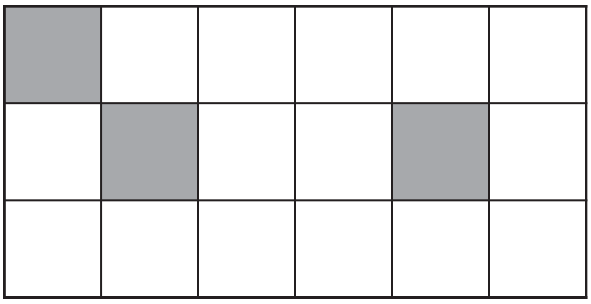
(i) one line of symmetry,



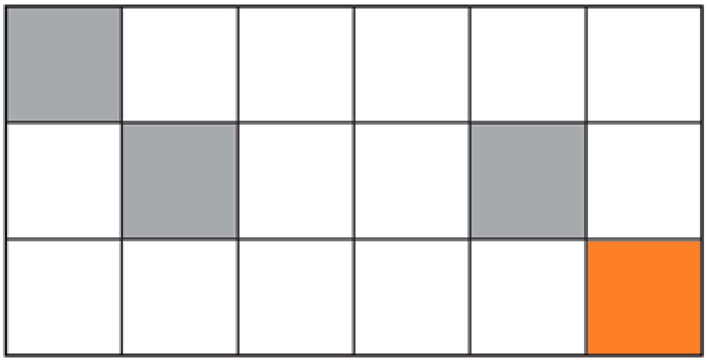
[1]



(ii) rotational symmetry of order 2.

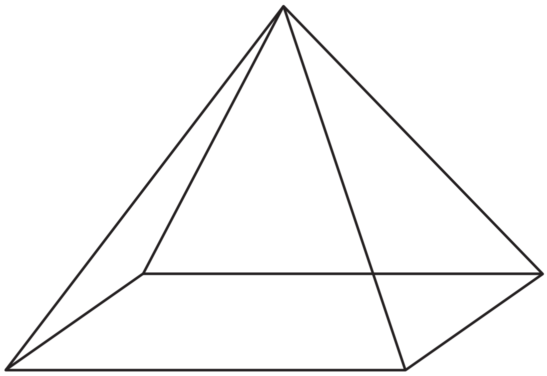


[1]



(b) The pyramid below has a rectangular base.
The vertex of the pyramid is vertically above the centre of the base.

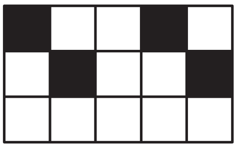
Write down the number of **planes** of symmetry for the pyramid.



[1]

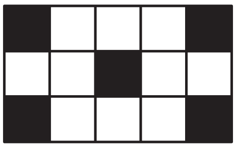
Rectangular base **→ 2 planes of symmetry**

(a) Write down the number of lines of symmetry for the diagram below. [1]



0

(b) Write down the order of rotational symmetry for the diagram below.

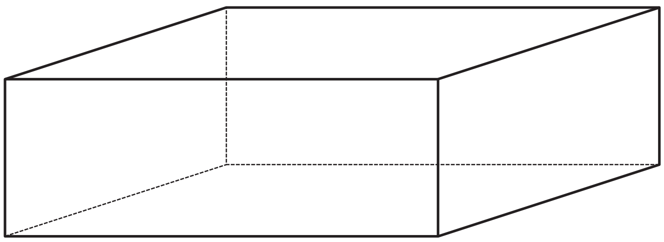


[1]

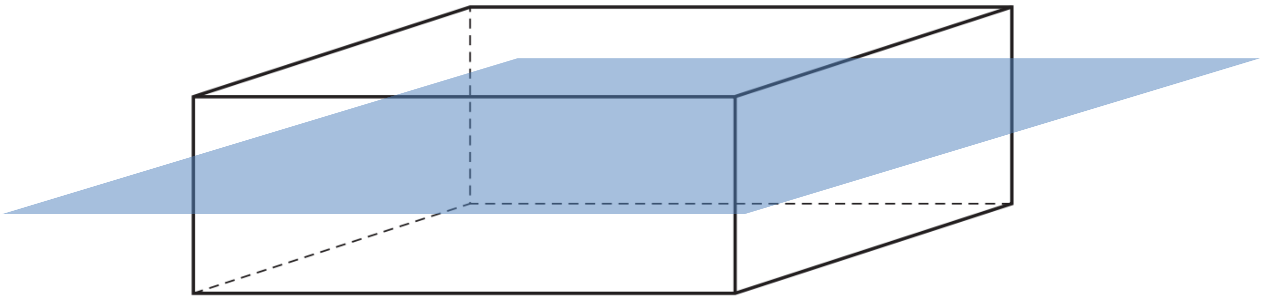
2

(c) The diagram shows a cuboid which has no square faces.

Draw one of the **planes** of symmetry of the cuboid on the diagram.



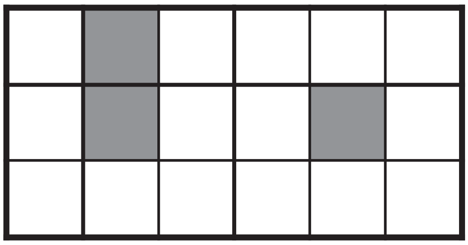
[1]



Question 5

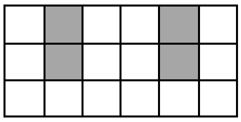
(a) Shade one square in each diagram so that there is

(i) one line of symmetry, [1]

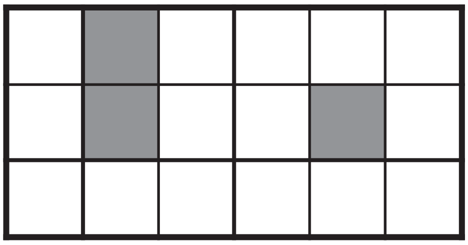


The line of symmetry is a line over which the figure can be reflected

and it will appear unchanged.

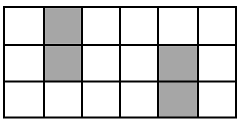


(ii) rotational symmetry of order 2. [1]

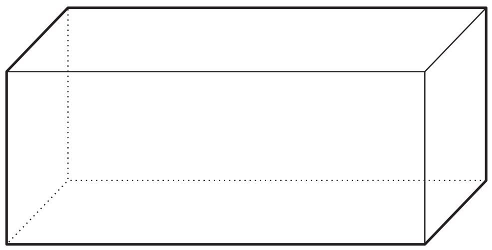


The order of rotational symmetry is the number of times the figure matches

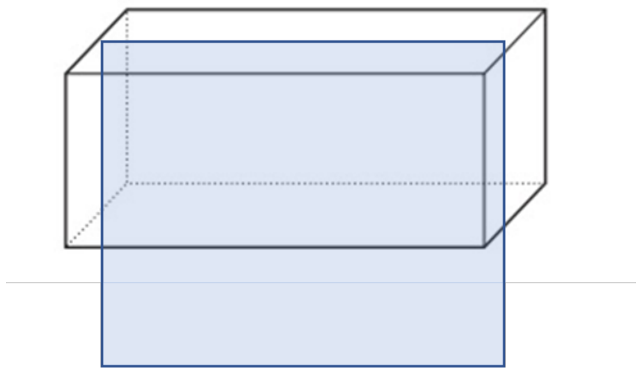
its initial shape while rotating it once 360°



(b) On the diagram below, sketch one of the **planes** of symmetry of the cuboid. [1]

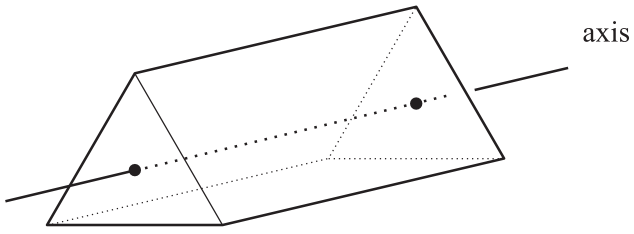


A plane of symmetry separates a shape in half so that each side of the plane is a mirror image of the other side.

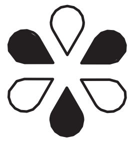


The shape can be cut exactly in half either vertically or horizontally.

(c) Write down the order of rotational symmetry of the equilateral triangular prism about the axis shown. [1]



The order of rotational symmetry around the axis is 3.



(a) Write down the order of rotational symmetry of the diagram. [1]

A shape has rotational symmetry when it is the same after it is rotated. The Order of Symmetry represents how many times the shape looks the same as we rotate it once around. In our case, we can rotate the shape 3 times and still obtain the same image.

The order of rotational symmetry is 3.

(b) Draw the lines of symmetry on the diagram. [1]

A line of symmetry separates the figure in 2 parts which are the same.

The figure above has 3 lines of symmetry, represented in the figure above.

Question 7

(a) Draw a quadrilateral which has rotational symmetry of order 2 and whose diagonals are equal in length.

[2]

(b) Write down the special name of this quadrilateral. [1]

This quadrilateral is a **rectangle**.

The rotational symmetry represents how many times the shape overlaps with the original one if we rotate it once around.

Angles in Polygons
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Angles in Polygons
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **43 minutes**

Score: **/33**

Percentage: **/100**

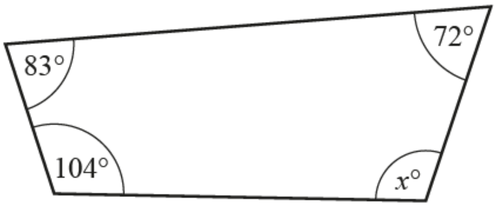
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



NOT TO SCALE

The diagram shows a quadrilateral.

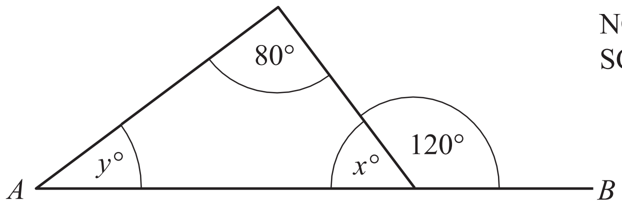
Find the value of x. [1]

The sum of the interior angles of a polygon is 360

$$83 + 104 + x + 72 = 360$$

$$\rightarrow x = 360 - 259$$

$$= \mathbf{101}$$



NOT TO SCALE

In the diagram, AB is a straight line.

Find the value of x and the value of y. [2]

$$x = 180 - 120$$

$$\rightarrow \boldsymbol{x = 60}$$

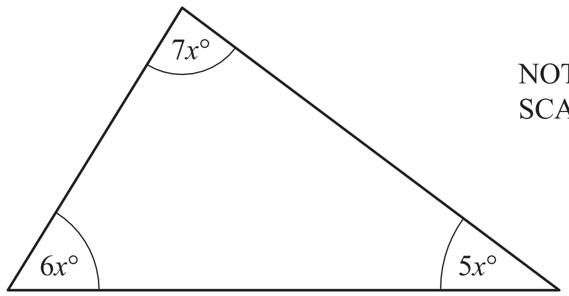
Angles in a triangle sum to 180 hence

$$y = 180 - x - 80$$

$$= 100 - 60$$

$$\rightarrow \boldsymbol{y = 40}$$

The three angles in a triangle are $5x°$, $6x°$ and $7x°$.



(a) Find the value of x. [2]

Angles in a triangle sum to 180

$$\rightarrow 7x + 6x + 5x = 180$$

$$\rightarrow 18x = 180$$

$$\rightarrow \boldsymbol{x = 10}$$

(b) Work out the size of the largest angle in the triangle. [1]

$$\boldsymbol{7x = 70}$$

Five angles of a hexagon are each 115°.

Calculate the size of the sixth angle. [3]

Angles in a hexagon all add to 720. We have

$5\times115+x=720$

Where x is our 6^{th} angle. We rearrange for

$x=720-5\times115$

$x=720-575$

$x=145$

A regular polygon has an interior angle of $172°$.

Find the number of sides of this polygon. [3]

Number of sides of a regular polygon (n) is given by the formula:

$$n = \frac{360}{180 - \theta}$$

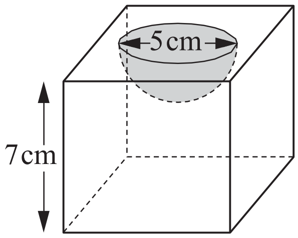
Where θ is the size of the interior angle. Hence

$$n = \frac{360}{180 - 172}$$

$$= \frac{360}{8}$$

$$= \mathbf{45}$$

A solid consists of a metal cube with a hemisphere cut out of it.



NOT TO SCALE

The length of a side of the cube is 7 cm.
The diameter of the hemisphere is 5 cm.

Calculate the volume of this solid.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [3]

The volume of the hemisphere is half that of a sphere

$$V_{hs} = \frac{1}{2} \times \frac{4}{3}\pi r^3$$

Its radius is half the diameter

$$r = \frac{1}{2} \times 5$$

$$= \frac{5}{2}$$

$$V_{hs} = \frac{1}{2} \times \frac{4}{3}\pi \times \left(\frac{5}{2}\right)^3$$

$$= \frac{125}{12}\pi$$

The volume of the cube (without the hemisphere cut out)

is

$$V_s = 7^3$$

$$= 343$$

The total volume is then

$$343 - \frac{125}{12}\pi$$

$$\mathbf{= 310.3}$$

Find the sum of the interior angles of a 25-sided polygon. [2]

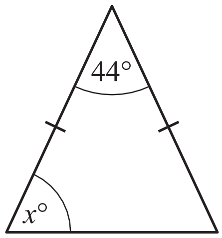
The sum of interior angles of shape with n vertices is given by $(n-2)\times 180°$.

In our case, n=25.

$$(25-2)\times 180° = 23\times 180°$$

$$= \mathbf{4140°}$$

(a)



NOT TO SCALE

The diagram shows an isosceles triangle.

Find the value of x. [1]

The triangle is isosceles, therefore the third angle is also x.

The sum of all three interior angles of a triangle is 180°.

$$180° = 44° + x + x$$

Subtract 44° from both sides.

$$136° = 2x$$

Divide both sides by 2 to get the final answer:

$$\boldsymbol{x = 68°}$$

(b) The exterior angle of a regular polygon is 24°.

Find the number of sides of this regular polygon. [2]

Since the exterior angle is 24°, the interior angle must be 156° (=180°-24°).

The sum of interior angles of shape with n vertices is given by $(n-2)\times 180°$.

The shape is a regular polygon, the sum must also be $n \times (interior\ angle)$, as all interior angles have the same size. We know that this interior angle is 156°.

$$n \times 156° = (n-2)\times 180°$$

$$n \times 156° = n \times 180° - 360°$$

Subtract $n \times 156°$ from both sides.

$$0 = n \times 24° - 360°$$

Add 360° to both sides and divide both sides by 24°.

$$\frac{360°}{24°} = n$$

We get the final answer: $n = 15$.

This regular polygon has

15 sides.

Find the interior angle of a regular polygon with 18 sides. [3]

We know that the sum of the interior angles, x_i, of an n sided polygon is

$$\sum x_i = 180 \times (n - 2)$$

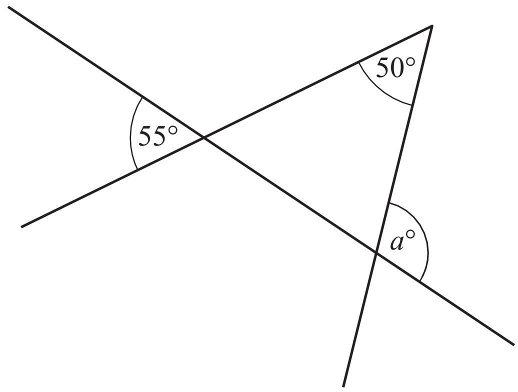
For a regular polygon all the interior angles are the same, so we have

$$18x = 180 \times (18 - 2)$$

$$\rightarrow 18x = 2880$$

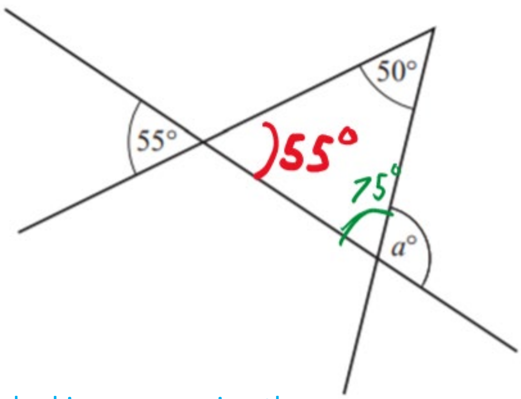
$$\rightarrow x = \frac{2880}{18}$$

$$= 160$$



NOT TO SCALE

Use the information in the diagram to find the value of a. [2]



To solve this we use the information given in the diagram to find a.

First we find the angle marked in red – this must be $55°$, as it is produced by two straight lines intersecting and the opposite angle is given as $55°$.

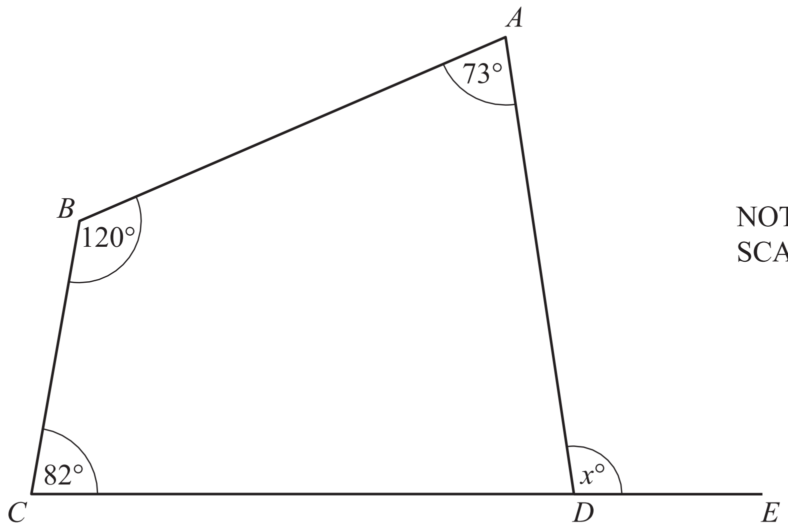
From this we can find the remaining angle in the triangle marked in green, using the rule that the sum of the interior angles of any triangle is $180°$. We know two of the angles, so the third angle is equal to:

$$180 - 50 - 55 = 75°$$

Finally we can work out a. All straight lines have angles adding up to $180°$. Therefore the green angle and a must add up to $180°$. Hence we know that

$$\mathbf{180 - 75 = a = 105°}$$

$$= \mathbf{105°}$$



The diagram shows a quadrilateral *ABCD*.
CDE is a straight line.

Calculate the value of x. [2]

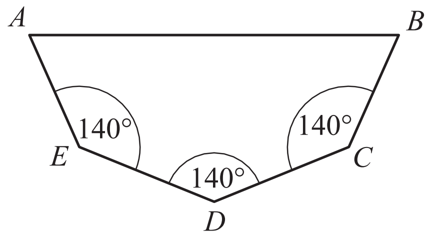
Angles in a quadrilateral sum to 360, so we have that

$$82 + 120 + 73 + (180 - x) = 360$$

$$\rightarrow 455 - x = 360$$

$$\rightarrow x = 455 - 360$$

$$\rightarrow \boldsymbol{x = 95}$$



NOT TO
SCALE

The pentagon has three angles which are each 140°.
The other two interior angles are equal.
Calculate the size of one of these angles. [3]

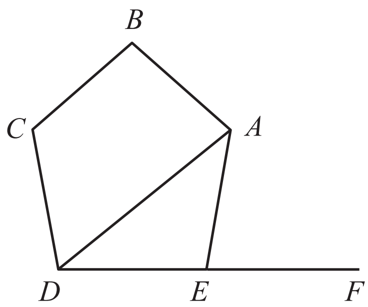
All the interior angles must sum to 540.

Hence

$$A + B + 3 \times 140 = 540$$

$$\rightarrow A = \frac{1}{2}(540 - 3 \times 140)$$

$$= \mathbf{60}$$



NOT TO SCALE

ABCDE is a regular pentagon.
DEF is a straight line.
Calculate

(a) angle *AEF*, [2]

The angle AEF is an exterior angle in the pentagon above.

We also know that the sum of all the exterior angles in a pentagon is 360°.

The size of one of the exterior angles is:

$\frac{360°}{5} = 72°$

Angle AEF = 72°

(b) angle *DAE*. [1]

The sum of the interior angle in a pentagon with its corresponding exterior angle is 180°.

In our case, the interior angle:

AED = 180° - 72°

Angle AED = 108°

The triangle AED is an isosceles triangle with the 2 sides AE and ED equal.

The angles EDA and DAE are also congruent in the triangle AED.

We also know that the sum of the angles of a triangle is 180°

2 x angle DAE = 180° - 108°

2 x angle DAE = 72°

Angle DAE = 36°

Angles in Polygons
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Angles in Polygons
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **26 minutes**

Score: **/20**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

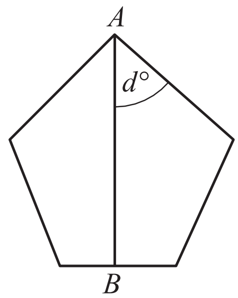
A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

The diagram shows a regular pentagon.
AB is a line of symmetry.

Work out the value of d.



NOT TO SCALE

[3]

The sum of the interior angles of an n sided polygon is **180(n-2)**. Thus the sum of the interior angles of a pentagon is 180 x (5-2) = 540.

One interior angle of a regular pentagon is

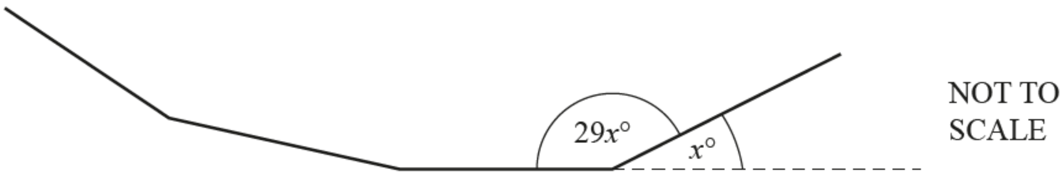
$$540 \div 5$$

$$= 108$$

Split in two halves

$$108 \div 2$$

$$= \mathbf{54}$$



The diagram shows part of a regular polygon.
The exterior angle is $x°$.
The interior angle is $29x°$.

Work out the number of sides of this polygon. [3]

$$29x + x = 180$$

$$\rightarrow 30x = 180$$

$$\rightarrow x = 6$$

Sum of the interior angles of the polygon is

$$(n - 2) \times 180 = 29nx$$

Expand bracket, substitute in $x = 6$

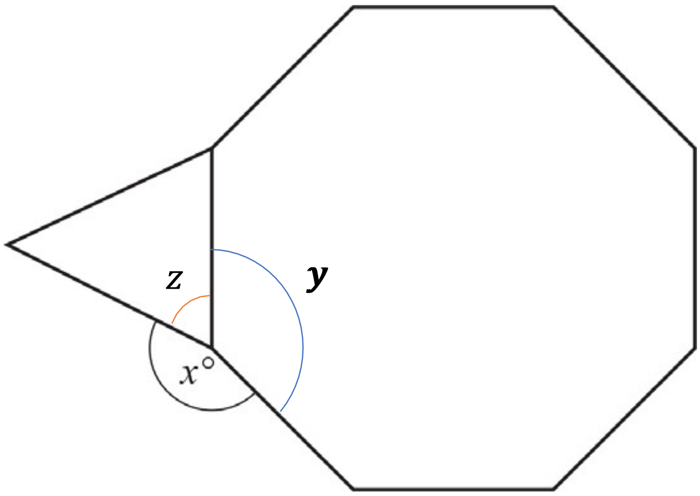
$$\rightarrow 180n - 360 = 174n$$

Subtract $174n$ and add 360 to both sides

$$\rightarrow 6n = 360$$

$$\rightarrow \boldsymbol{n = 60}$$

The diagram shows a regular octagon joined to an equilateral triangle.



NOT TO SCALE

Work out the value of x. [3]

The angles $x, y,$ and z must all sum to 360.

y can be found by using the fact that all the interior angles of a regular octagon sum to

$$(n-2) \times 180$$

$$= 6 \times 180$$

$$= 1080$$

Hence

$$8y = 1080$$

$$\rightarrow y = 135$$

An equilateral triangle has 3 equal angles which all sum to 180, hence

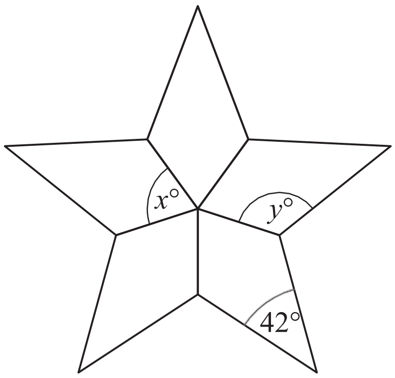
$$3z = 180$$

$$\rightarrow z = 60$$

This gives us

$$x + 60 + 135 = 360$$

$$\rightarrow \boldsymbol{x = 165}$$



NOT TO SCALE

The diagram is made from 5 congruent kites.

Work out the value of

(a) x, [1]

We have

$$5x = 360$$

$$\rightarrow x = \frac{360}{5}$$

$$= \mathbf{72°}$$

(b) y. [2]

Since all angles in quadrilateral sum to 360

$$42 + 2y + x = 360°$$

$$\rightarrow 42 + 2y + 72 = 360°$$

$$\rightarrow 2y = 246$$

$$\rightarrow \boldsymbol{y = 123°}$$

The exterior angle of a regular polygon is 36°.

What is the name of this polygon? [3]

Sum of all interior angles of any figure is given by:

$$(n - 2) \times 180°$$

Since the **exterior** angle is 36 degrees, the interior angle will be:

$$180° - 36° = 144°$$

And the sum of all interior angles is given by:

$$144n$$

Equating the 2 equations and solving:

$$(n - 2) \times 180° = 144n$$

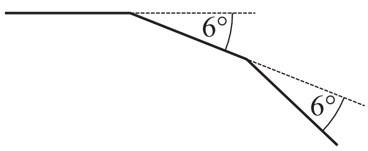
$$180n - 360 = 144n$$

$$36n = 360$$

$$n = 10$$

Hence, this is a

decagon.



NOT TO SCALE

The diagram shows two of the exterior angles of a regular polygon with n sides.
Calculate n. [2]

Each of the interior angles is

$$180 - 6$$

$$= 174°$$

We know that for a regular polygon with n sides the sum of all the angles is

$$174 \times n = (n - 2) \times 180$$

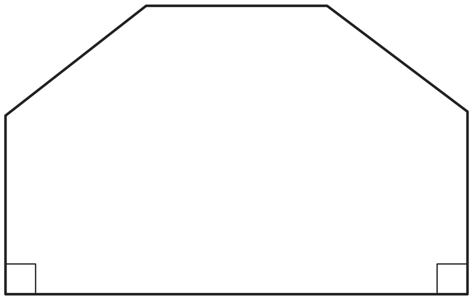
$$\rightarrow 174n = 180n - 360$$

Rearrange for n

$$\rightarrow 360 = 6n$$

$$\rightarrow n = \frac{360}{6}$$

$$= \mathbf{60}$$



The front of a house is in the shape of a hexagon with two right angles.
The other four angles are all the same size.

Calculate the size of one of these angles. [3]

The sum of all the angles in a hexagon is 720°.

We represent with the unknow x the size of one of the

angles which are the same size in the hexagon.

$4x + 2 \times 90° = 720°$

$4x + 180° = 720°$

$4x = 540°$

$x = 135°$

Circle Theorems
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Circle Theorems
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **27 minutes**

Score: **/21**

Percentage: **/100**

Grade Boundaries:

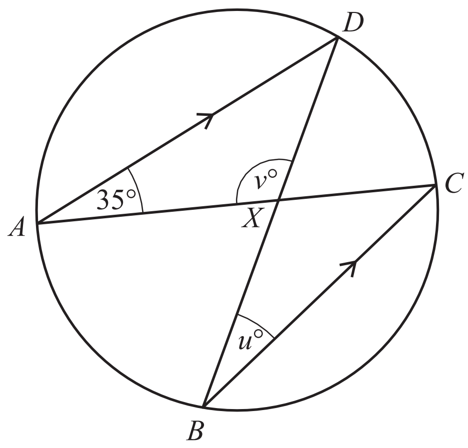
CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

(a)



NOT TO SCALE

A, B, C and D are points on the circle.
AD is parallel to BC.
The chords AC and BD intersect at X.

Find the value of u and the value of v. [3]

Angles in the same segment

$$\boldsymbol{u = 35}$$

Because of Z angles

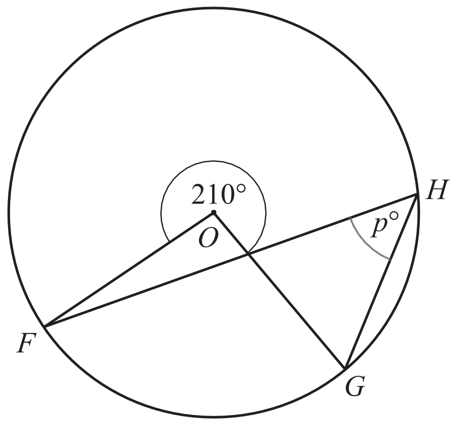
$$BDA = 35$$

Angles in a triangle sum to 180, hence

$$v = 180 - 35 - 35$$

$$\boldsymbol{= 110}$$

(b)



NOT TO SCALE

F, G and H are points on the circle, centre O.

Find the value of p. [2]

We have that

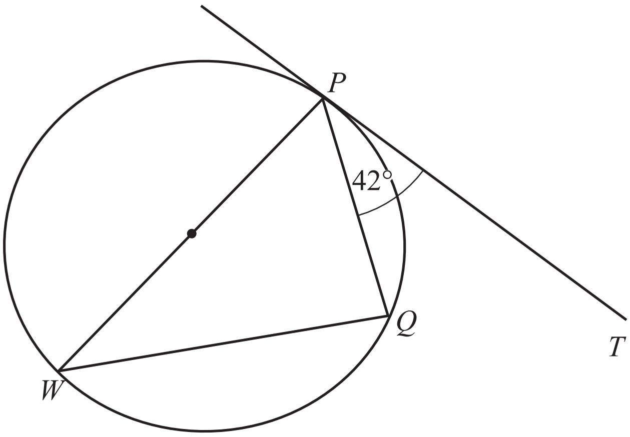
$$FOG = 360 - 210$$

$$= 150$$

Angles on perimeter are half the angle in the centre,

hence

$$\boldsymbol{p = 75}$$



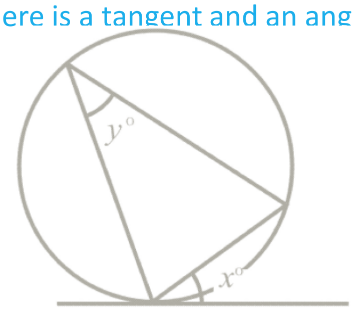
NOT TO SCALE

In the diagram, PT is a tangent to the circle at P.
PW is a diameter and angle $TPQ = 42°$.

Find angle PWQ. [2]

Pick the correct Circle Theorem:

There is a tangent and an angle marked between that and a chord so we use:



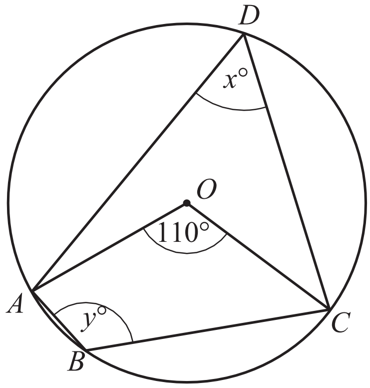
$x = y$

"Angle between Tangent & Chord equals Angle in Alternate Segment"

In this case we see that, with $x = TPQ$ and $y = PWQ$:

$$TPQ = PWQ$$

$$\mathbf{42° = PWQ}$$



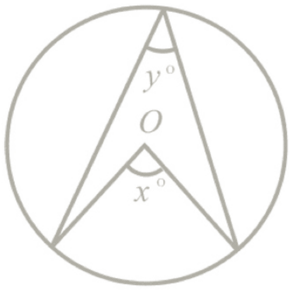
NOT TO
SCALE

A, B, C and D lie on the circle, centre O.

Find the value of x and the value of y. [2]

Pick the correct Circle Theorem to find x:

There is an Angle at the Centre so we use:



$x = 2y$

"Angle at Centre is twice the
Angle at Circumference"

In this case we see that, with $y = ADC$ and $x = AOC$:

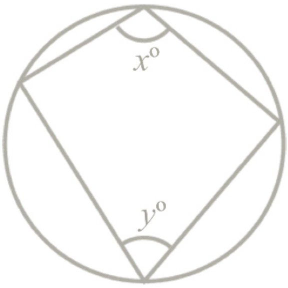
$$AOC = 2 \times ADC$$

$$110 = 2x$$

Dividing by 2: $\boldsymbol{x = 55}$

Pick the correct Circle Theorem to find x:

There is an Angle at the Centre so we use:



$x + y = 180°$

"Opposite Angles in a Cyclic Quadrilateral add to 180º"

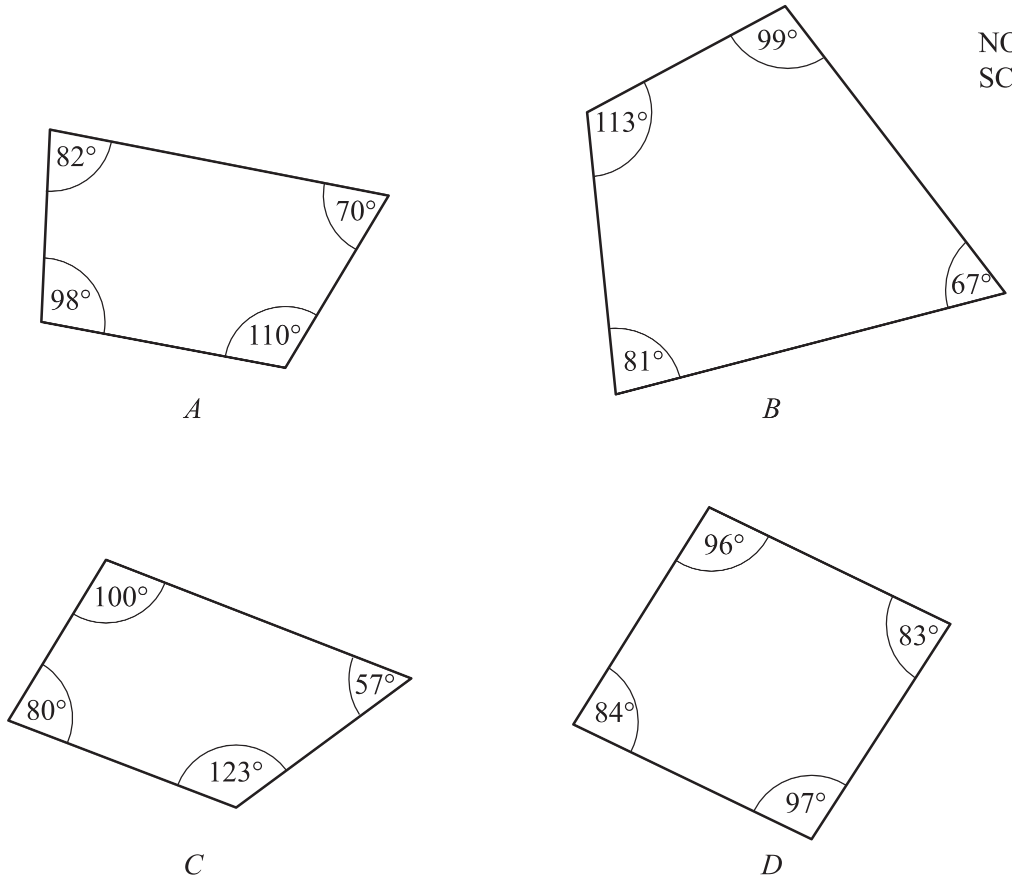
In this case we see that, with $y = ABC$ and $x = ADC$:

$$ADC + ABC = 180$$

$$55 + y = 180$$

Subtracting 55: $\boldsymbol{x = 125°}$

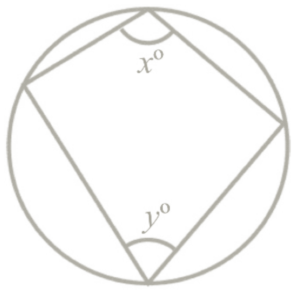
NOT TO SCALE



The diagram shows four quadrilaterals *A*, *B*, *C* and *D*.

Which one of these could be a cyclic quadrilateral? [1]

The relevant Circle Theorem is:

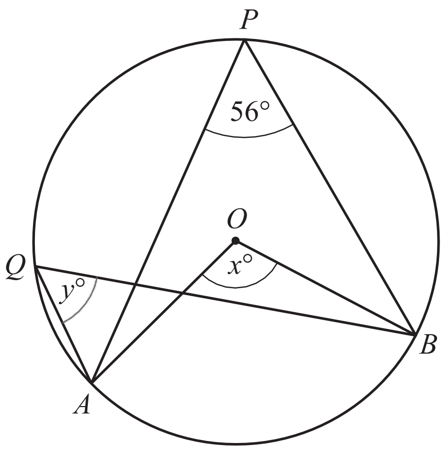


$x + y = 180°$

"Opposite Angles in a Cyclic Quadrilateral add to 180º"

Since $113 + 67 = 180°$ and $81 + 99 = 180°$

***B* could be a Cyclic Quadrilateral**



NOT TO SCALE

A, B, P and Q lie on the circle, centre O.
Angle $APB = 56°$.

Find the value of

(a) x, [1]

From circle theorems (angles subtended at the centre), x must be twice as large as 56°

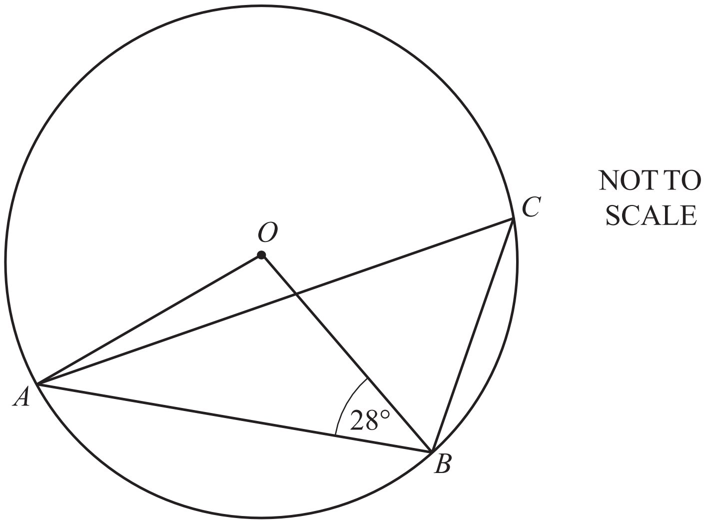
$x = 2 \times 56°$

$= 112°$

(b) y. [1]

Also, from circle theorems (angles subtended from the same chord), y must be the same as 56°

$y = 56°$



In the diagram, A, B and C lie on the circumference of a circle, centre O.

Work out the size of angle ACB.
Give a reason for each step of your working. [4]

Triangle OAB is isosceles with angles OAB and OBA being equal at 28.

The angles in a triangle add to 180, so

$$28 + 28 + AOB = 180$$

$$\rightarrow AOB = 180 - 56$$

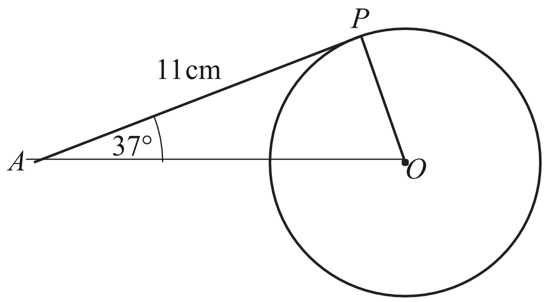
$$\rightarrow AOB = 124$$

From circle theorems the angle at the centre is twice the angle at the circumference,

hence

$$ACB = \frac{1}{2} \times 124$$

$$= \mathbf{62}$$



In the diagram, AP is a tangent to the circle at P.
O is the centre of the circle, angle $PAO = 37°$ and $AP = 11$ cm.

(a) Write down the size of angle OPA. [1]

If AP is a tangent to the circle, then the angle OPA is a right angle.

$$\boldsymbol{OPA = 90°}$$

(b) Work out the radius of the circle. [2]

The radius of the circle (size of PO) can be calculated using trigonometry.

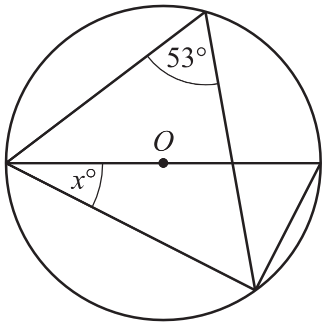
$$\tan(PAO) = \frac{PO}{AP}$$

$$\tan(37°) = \frac{PO}{11cm}$$

Multiply both sides by 11 and use a calculator to work out the value of tan(37°).

$$PO = 11cm \times 0.7336$$

$$\boldsymbol{PO = 8.29cm}$$

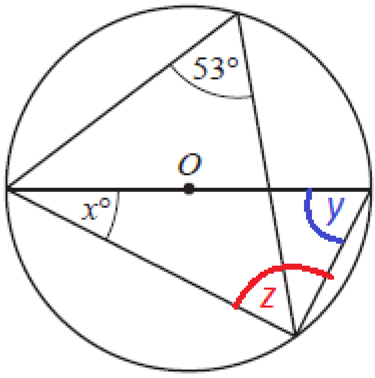


NOT TO SCALE

The diagram shows a circle, centre O.

Find the value of x. [2]

In order to make the analysis easier, we mark angles *y*(blue) and *z*(red) on the diagram.



The angle *y* is subtended from the same points as the angle 53°, so they must have the same size. Therefore *y*=53°.

The line opposite to angle *z* passes through the centre of the circle (diameter), therefore the angle *z* is a right angle (triangle with interior angles *x,y,z* is a right angle triangle). Therefore *z*=90°.

The sum of all three interior angles of a triangle is 180°. From this fact, we can calculate the size of angle *x*.

$$180° = x + y + z$$

$$180° = x + 53° + 90°$$

We get the final answer by subtracting 143°from both sides of the equation.

$$\boldsymbol{x = 37°}$$

Circle Theorems
Difficulty: Easy

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Circle Theorems
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 2

Time allowed: **27 minutes**

Score: **/21**

Percentage: **/100**

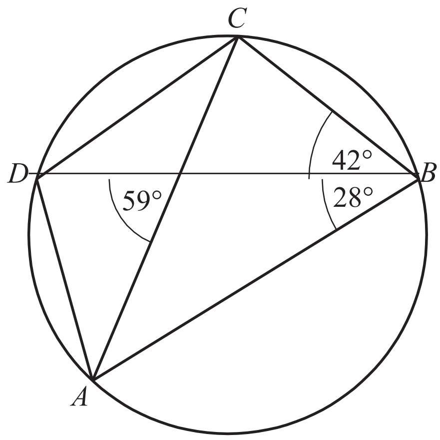
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

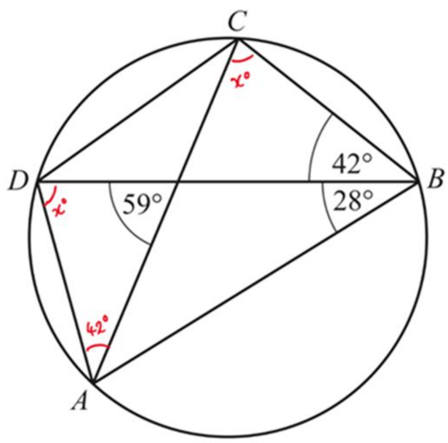


NOT TO SCALE

A, B, C and D lie on the circle.

Find

(a) angle ADC, [1]



The opposite angles in a cyclic quadrilateral sum to 180 degrees. Therefore, angle ABC + angle ADC = 180.

$$42° + 28° + ADC = 180°$$

$$angle\ ADC = 180° - 70°$$

$$\mathbf{angle\ ADC = 110°}$$

(b) angle ADB. [2]

The angles at the circumference subtended by the same arc are equal.

Therefore, angles DAC and DBC are equal:

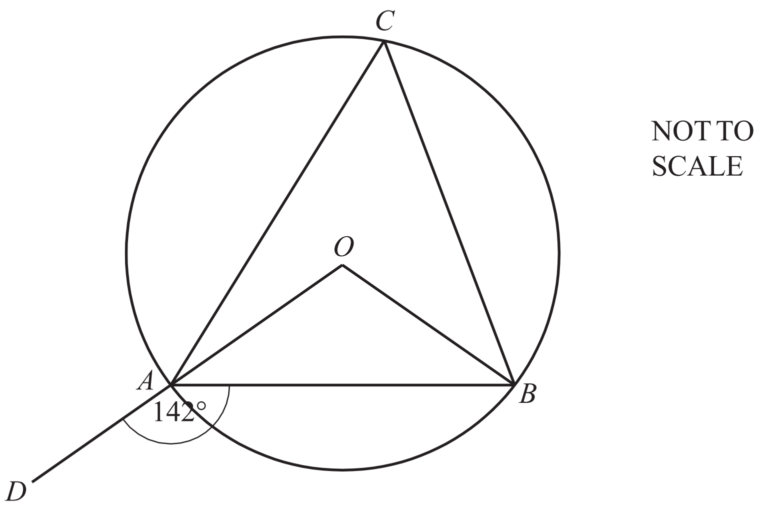
$$angle\ DAC = 42°$$

The sum of the angles in a triangle is equal to 180° :

$$59° + 42° + angle\ ADB = 180°$$

$$angle\ ADB = 180° - 59° - 42°$$

$$\mathbf{angle\ ADB = 79°}$$



A, B and C are points on the circumference of a circle centre O.
OAD is a straight line and angle $DAB = 142°$.

Calculate the size of angle ACB. [3]

Find angle OAB:

$$Angle\ OAB = 180° - 142° = 38°\ (Angle\ of\ a\ straight\ line)$$

Identify correctly Triangle OAB to be an isosceles triangle.

$$Angle\ OAB = Angle\ OBA\ (Isosceles\ triangle)$$

Find Angle AOB:

$$180° - OAB - OBA$$

$$= 180° - 38° - 38°$$

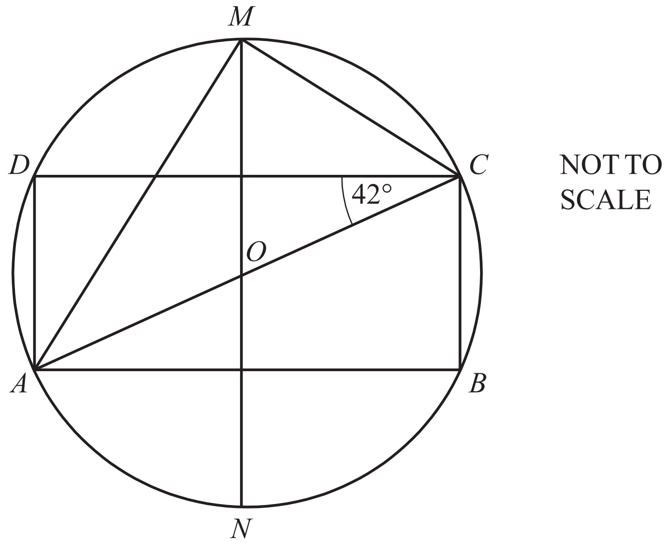
$$= 104°\ (Angle\ sum\ of\ triangle)$$

Finally, find Angle ACB:

$$Angle\ ACB = \frac{AOB}{2}$$

$$= \frac{104°}{2}$$

$$= \mathbf{52°}\ (\boldsymbol{Angle\ at\ the\ center\ theorem})$$



The vertices of the rectangle *ABCD* lie on a circle centre *O*.
MN is a line of symmetry of the rectangle.
AC is a diameter of the circle and angle *ACD* = 42°.

Calculate

(a) angle *CAM*, [2]

$$Angle\ MOC = 90° - 42°$$

$$= 48°\ (Angle\ sum\ of\ triangle)$$

$$Angle\ CAM = \frac{48°}{2}$$

$$= \mathbf{24°\ (Angle\ at\ centre\ of\ circle)}$$

(b) angle DCM. [2]

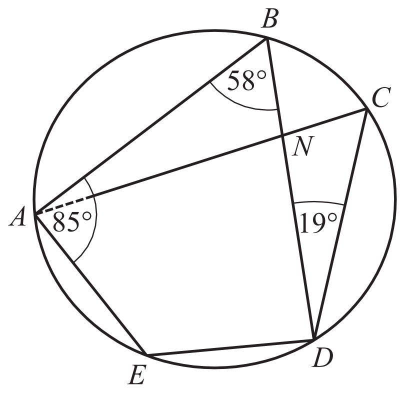
$$Triangle\ AMC\ is\ a\ right\ angled\ triangle\ (Angle\ in\ a\ semi-circle)$$

$$Angle\ MCA = 90° - 24° = 66°\ (angle\ in\ a\ semi-circle)$$

$$Angle\ DCM = Angle\ MCA - Angle\ DCA$$

$$= 66° - 42°$$

$$= \mathbf{24°}$$

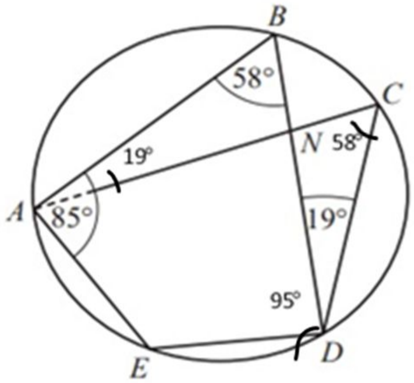


NOT TO SCALE

A, *B*, *C*, *D* and *E* are points on a circle.
Angle *ABD* = 58°, angle *BAE* = 85° and angle *BDC* = 19°.
BD and *CA* intersect at *N*.

Calculate

(a) angle *BDE*, [1]



The quadrilateral ABDE is a cyclic quadrilateral because it is drawn inside of the circle and all the corners are touching the circumference.

The circle theorems states that in a cyclic quadrilateral the opposite angles add up to 180°.

In our case, in the cyclic quadrilateral ABDE, BAE and BDE are opposite angles.

85° + BDE = 180°

BDE = 95°

(b) angle AND. [2]

The angles at the circumference subtended by the same arc are equal.

In our case, this means that both angles CDN and BAN are equal to 19°.

Similarly, the angles BNA and CND are equal, same for the angles BNC and ANC.

These 4 angles are all around the same point, N, meaning that their sum needs to be 360°.

We also know that the sum of the interior angles in a triangle is 180°.

Angle CND = 180° - 19° - 58°

Angle CND = 103°

Since the angle BNA is equal, BNA = 103°.

For the angles around the point N:

Angle CND + angle BNA + angle BNC + angle AND = 360°

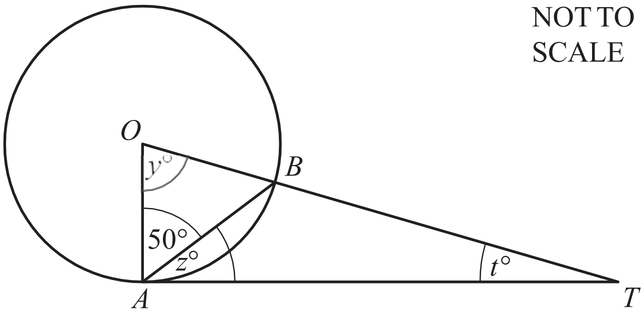
Angle BNC = Angle AND

103° x 2 + angle AND x 2 = 360°

206° + 2 x angle AND = 360°

2 x angle AND = 154°

Angle AND = 77°



TA is a tangent at A to the circle, centre O.
Angle $OAB = 50°$.

Find the value of

(a) y, [1]

In the triangle OAB, both sides OA and OB are a radius in the circle with the centre in O.

Therefore, OA = OB.

This makes the triangle OAB isosceles, having also the 2 angles OAB and OBA congruent.

We know that the sum of all 3 angles in a triangle is 180°.

To work out the size of the angle y, we solve:

y + 2 x angle OAB = 180°

y + 2 x 50° = 180°

y = 80°

(b) z, [1]

TA is the tangent to the circle with the centre in O.

Therefore, the tangent TA is perpendicular on the radius of the circle, OA.

$z + 50° = 90°$

$z = 40°$

(c) t. [1]

We know that the sum of all 3 angles in a triangle is 180°.

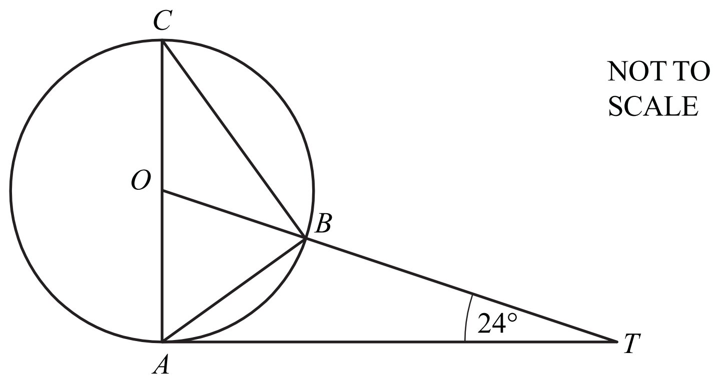
In the triangle OAT, to work out the size of the angle t, we solve:

$t + \text{angle } OAT + y = 180°$

From a), we know that $y = 80°$

$t + 90° + 80° = 180°$

$t = 10°$



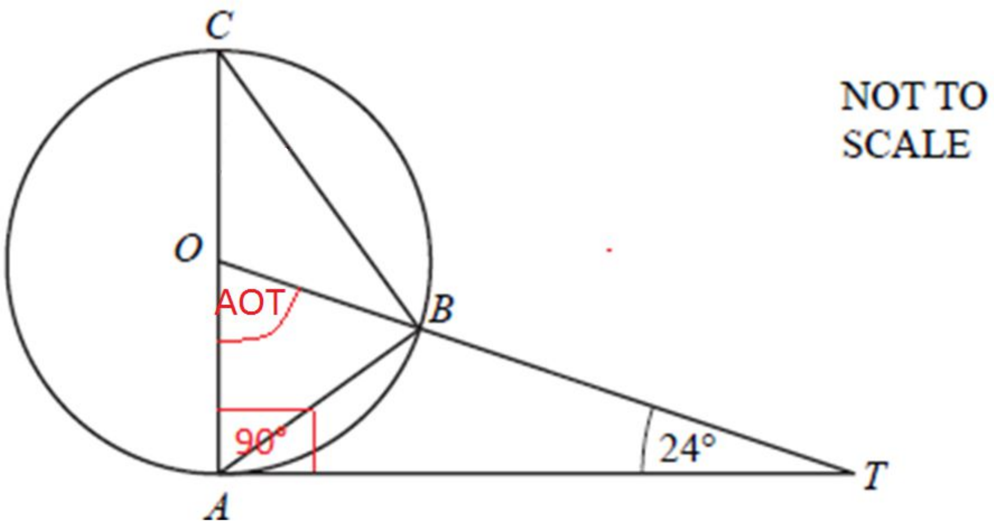
A, B and C are points on a circle, centre O.
TA is a tangent to the circle at A and OBT is a straight line.
AC is a diameter and angle $OTA = 24°$.

Calculate

(a) angle AOT, [2]

Since AT is tangent to the circle and hence perpendicular to radius OA, the triangle OAT is a right angle triangle.

$$angle\ OAT = 90°$$



The interior angles of triangle OAT must sum up to 180°.

Using this result, we can calculate angle AOT:

$$180° = OAT + OTA + AOT$$

$$180° = 90° + 24° + AOT$$

$$\boldsymbol{angle\ AOT = 66°}$$

(b) angle ACB, [1]

The lines OA and OB are both radii and thus equal in length. Hence the angles AOT and AOB are the same as triangle OAB is isosceles/

$$angle\ AOT = angle\ AOB = 66°$$

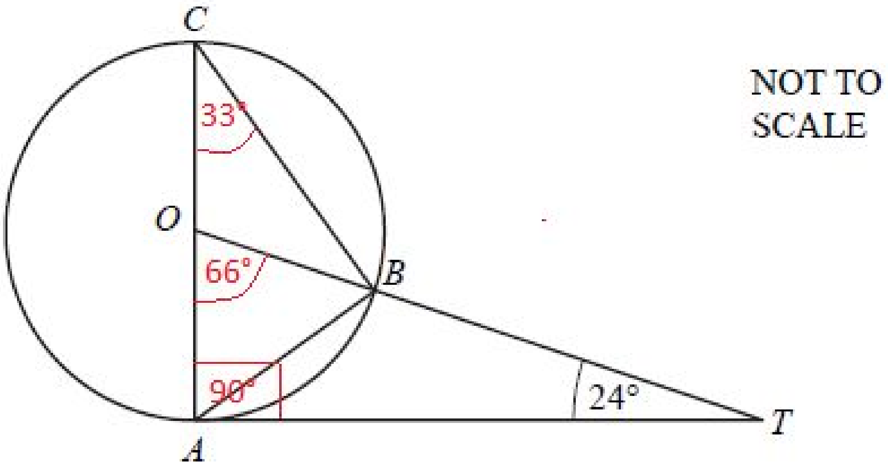
The angle subtended at the centre of a circle (AOB) is twice the angle subtended at the edge (ACB) from the same chord. In our case, the chord is the line AB.

$$2 \times angle\ ACB = angle\ AOB = 66°$$

$$\boldsymbol{angle\ ACB = 33°}$$

(c) angle ABT. [2]

Write down all the angle values we know so far.



The triangle AOB is an isosceles triangle with sides OA and OB of equal length.

Hence angles OAB and OBA are the same. Inner angles of the triangle AOB add up to 180°.

$$180° = AOB + OBA + OAB = AOB + 2 \times OBA$$

$$180° = 66° + 2 \times OBA$$

$$angle\ OBA = 57°$$

OT is a straight line with point B lying on it, so angles OBA and ABT must add to 180° (straight angle).

$$180° = OBA + ABT$$

$$180° = 57° + angle\ ABT$$

$$\boldsymbol{angle\ ABT} = 123°$$

Circle Theorems
Difficulty: Easy

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Circle Theorems
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 3

Time allowed: **26 minutes**

Score: **/20**

Percentage: **/100**

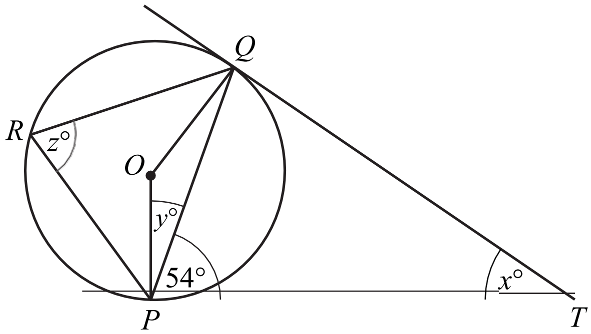
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



The points P, Q and R lie on a circle, centre O.
TP and TQ are tangents to the circle.
Angle $TPQ = 54°$.

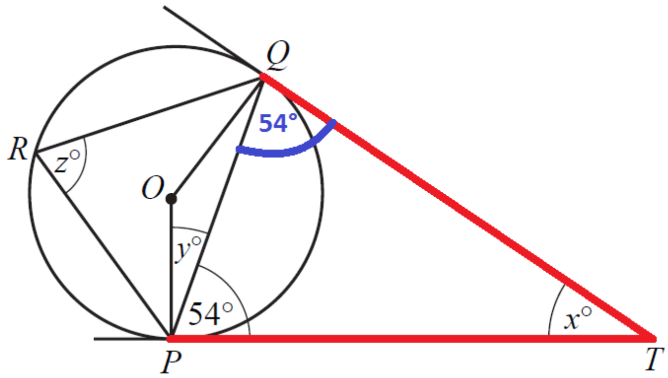
Calculate the value of

(a) x, [1]

Since both PT and QT and tangents and they share a common point T, they are equal in length.

The triangle PTQ is isosceles and so angles QPT and PQT have the same size.

$$angle\ QPT = angle\ PQT = 54°$$



The sum of the interior angles of a triangle must sum up to 180°.

$$180° = QPT + PQT + PTQ$$

$$180° = 54° + 54° + x°$$

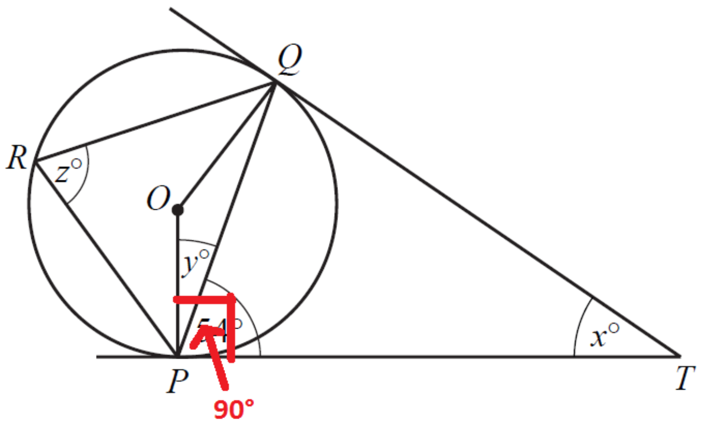
Hence we have the value of *x*.

$$\boldsymbol{x° = 72°}$$

(b) y, [1]

PT is a tangent to the circle so it must be perpendicular to the radius OP.

Therefore the angle OPT is a right angle.



$$angle\ OPT = angle\ OPQ + angle\ QPT$$

$$90° = y° + 54°$$

Hence we have the value of *y*.

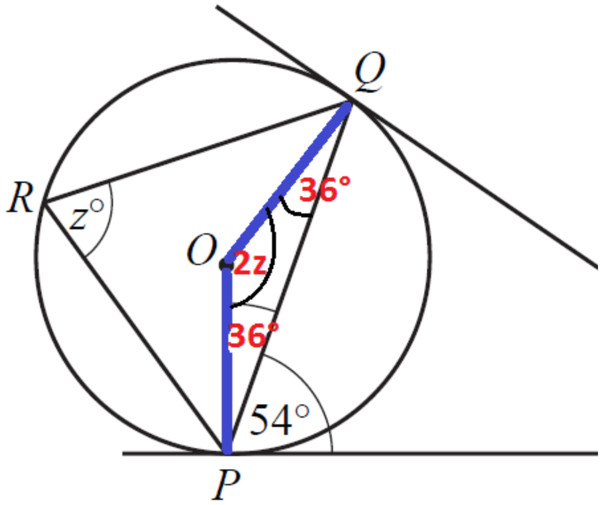
$$\boldsymbol{y° = 36°}$$

(c) z. [2]

The lines OP and OQ are both radii and thus equal in length. Hence

the angles OPQ and OPQ are the same as triangle POQ is isosceles.

$$angle\ OPQ = angle\ OQP = 36°$$



The sum of the interior angles of a triangle must sum up to 180°.

$$180° = OPQ + OQP + POQ$$

$$180° = 36° + 36° + POQ$$

$$POQ = 108°$$

The angle subtended at the centre of a circle (POQ) is twice the angle subtended

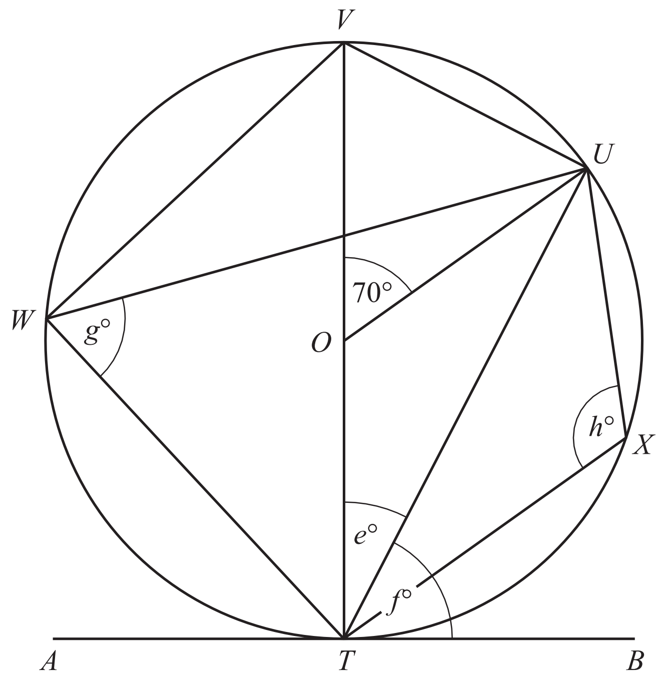
at the edge (PRQ) from the same chord. In our case, the chord is the line PQ.

$$2 \times angle\ PRQ = angle\ POQ = 108°$$

$$angle\ PRQ = 54°$$

Hence we have the value of z.

$$\mathbf{z° = 54°}$$



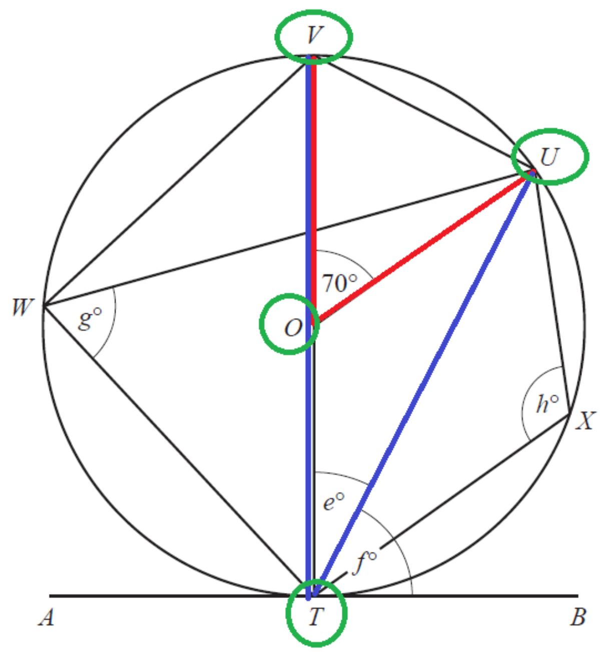
NOT TO SCALE

The diagram shows a circle, centre O.
VT is a diameter and ATB is a tangent to the circle at T.
U, V, W and X lie on the circle and angle $VOU = 70°$.

Calculate the value of

(a) e, [1]

The angle subtended at the centre of a circle (VOU) is twice the angle subtended at the edge (VTU) from the same chord. In our case, the chord is the line VU.



$$2 \times angle\ VTU = angle\ VOU = 70°$$

$$angle\ VTU = 35°$$

Hence we have the value of e.

$$\boldsymbol{e° = 35°}$$

(b) f, [1]

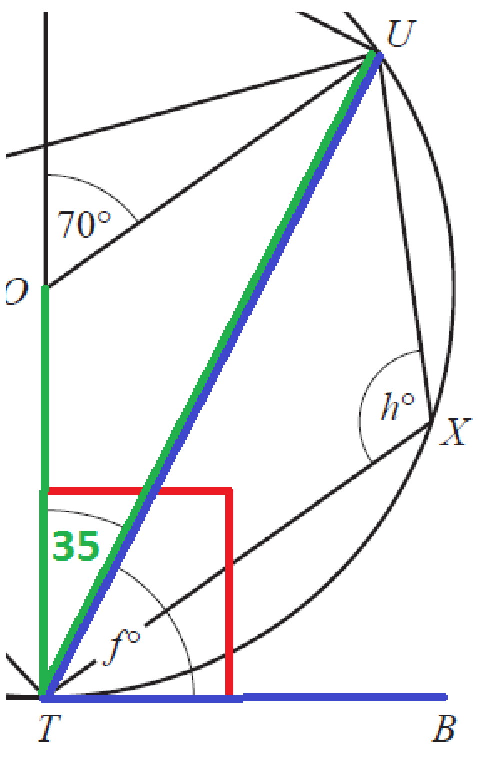
Since AB is a tangent to the circle, angle OTB is a right angle (90°), therefore the angles OTU and UTB must sum up to this value.

$$angle\ OTU + angle\ UTB = angle\ OTB$$

$$35° + f° = 90°$$

Hence we have the value of f.

$$\boldsymbol{f° = 55°}$$



(c) g, [1]

Angle VWU has the same value as angle VTU because:

Both angles are subtended from point V and U

Points W and point T lie on the same circle.

$$angle\ VWU = angle\ VTU = 35°$$

Angle VWT is a right angle (=90°) because:

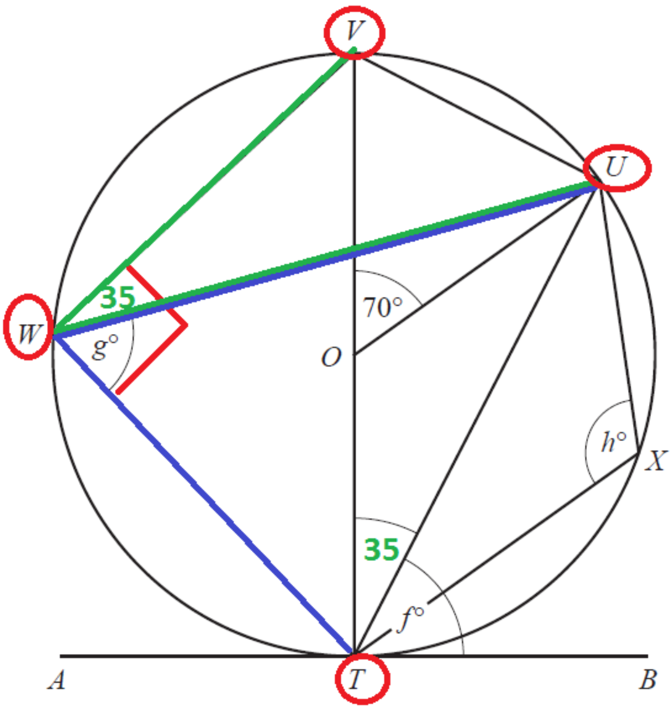
VT is a diameter of the circle

Point W lies on a circumference of the circle.

$$angle\ VWT = 90°$$

Therefore the angles VWT and UWT must sum up to a right angle.

$$angle\ VWT + angle\ UWT = angle\ VWT$$

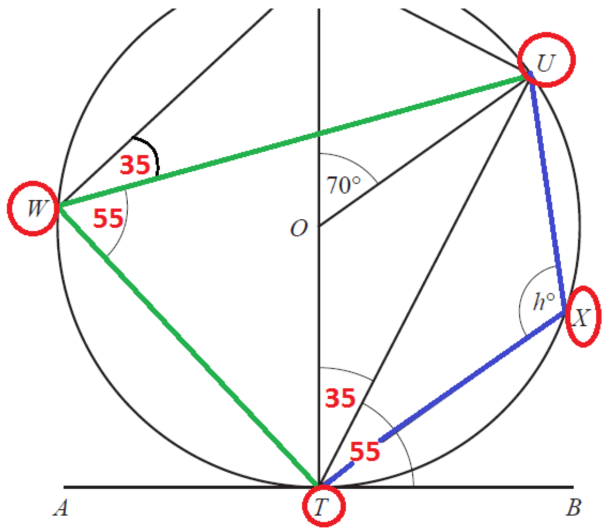


Hence we have the value of g.

$$g° = 55°$$

(d) h. [1]

Angles UWT and UXT lie on the opposite sides of a parallelogram with all four points on a circumference of the circle.

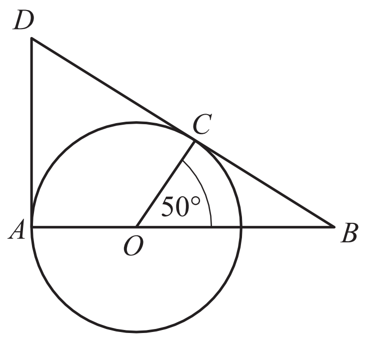


$$angle\ UWT + angle\ UXT = 180°$$

$$55° + h° = 180°$$

Hence we have the value of h.

$$\boldsymbol{h° = 125°}$$



NOT TO SCALE

O is the centre of the circle.
DA is the tangent to the circle at A and DB is the tangent to the circle at C.
AOB is a straight line. Angle $COB = 50°$.
Calculate

(a) angle CBO, [1]

All angles in a triangle add to 180, hence

$$CBO = 180 - 90 - 50$$

$$= \mathbf{40}$$

(b) angle DOC. [1]

Angle DOC is half of angle AOC.

Angle AOC plus angle COB must be 180, so

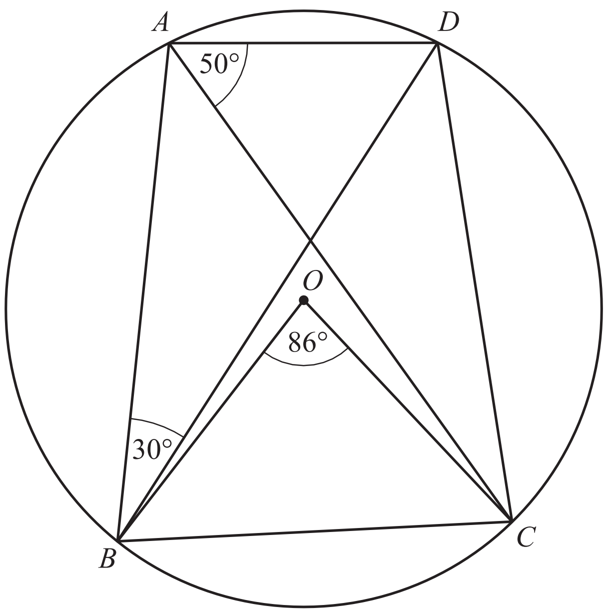
$$AOC = 180 - 50$$

$$= 130$$

Hence

$$DOC = \frac{1}{2} \times 130$$

$$= \mathbf{65}$$



NOT TO SCALE

The points A, B, C and D lie on the circumference of the circle, centre O.

Angle $ABD = 30°$, angle $CAD = 50°$ and angle $BOC = 86°$.

(a) Give the reason why angle $DBC = 50°$. [1]

. DAC and DBC are **angles in the same segment**, they must be equal.

(b) Find

(i) angle ADC, [1]

$ACD = 30$

Angles in a triangle add to 180

$ADC = 180 - 50 - 30$

$= \mathbf{100}$

(ii) angle BDC, [1]

BDC must be half of BOC

$$BDC = 0.5 \times 86$$

$$= \mathbf{43}$$

(iii) angle OBD. [2]

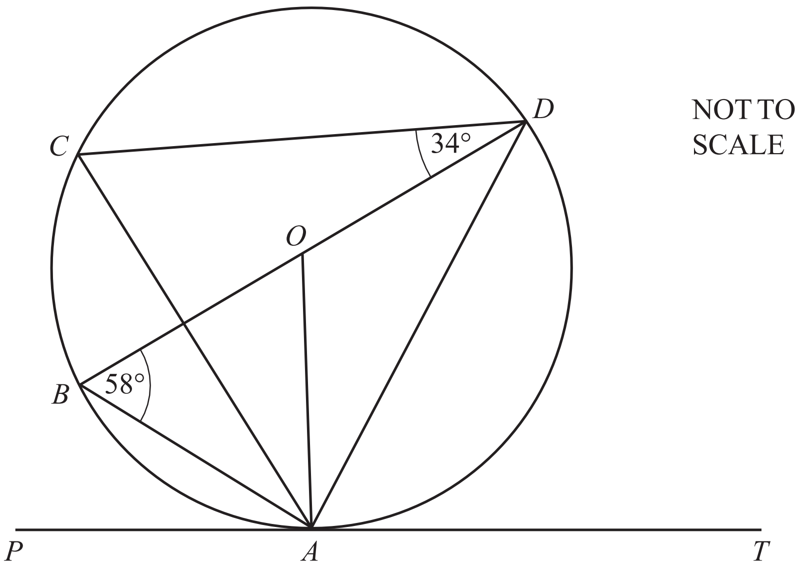
Triangle OBC is isosceles, so angle OBC is

$$\frac{1}{2}(180 - 86)$$

$$= 47$$

$$OBD + 47 = 50$$

$$\rightarrow \boldsymbol{OBD = 3}$$



A, B, C and D lie on the circle, centre O.
BD is a diameter and PAT is the tangent at A.
Angle $ABD = 58°$ and angle $CDB = 34°$.

Find

(a) angle ACD, [1]

Angles DCA and DBA are equal in the circle with centre O.

Angle DCA = Angle DBA = 58°

(b) angle ADB, [1]

OA and OD are both radii in the circle of centre O, therefore, triangle OAD is isosceles with the angles OAD and ODA equal.

2 x Angle ODA + Angle DOA = 180°

Angle DOA = 180° - Angle BOA

OA and OB are both radii in the circle of centre O.

Therefore, the triangle OAB is isosceles, with the angles OAB and OBA equal.

Angle OBA = Angle OAB = 58°

2 x Angle OAB + Angle AOB = 180°

2 x 58° + Angle AOB = 180°

Angle AOB = 64°

Angle DOA = 180° - 64°

Angle DOA = 116°

2 x Angle ODA + 116° = 180°

Angle ODA = 32°

(c) angle DAT, [1]

PAT is a tangent at A and OA is a radius in the circle of centre O.

Therefore, the radius OA is perpendicular on the tangent PAT.

Angle DAT = 90° - Angle OAD

Angle DAT = 90° - 32°

Angle DAT = 58°

(d) angle CAO. [2]

In the circle with centre O, OC and OA are both radii and therefore, equal.

In the triangle OCA, the sides OC and OA are equal, therefore, the triangle is isosceles.

Therefore, the angles OCA and OAC are also equal.

Angle AOB = 180° - 2 x 58°

Angle AOB = 64°

Angle COD = 180° - 2 x 34°

Angle COD = 112°

Angle COB = 180° - 112°

Angle COB = 68°

Angle COA = 64° + 68°

Angle COA = 132°

Angle CAO = Angle OCA = (180° - 132°)/2 = 24°

Circle Theorems
Difficulty: Easy

Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Circle Theorems
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 4

Time allowed: **31 minutes**

Score: **/24**

Percentage: **/100**

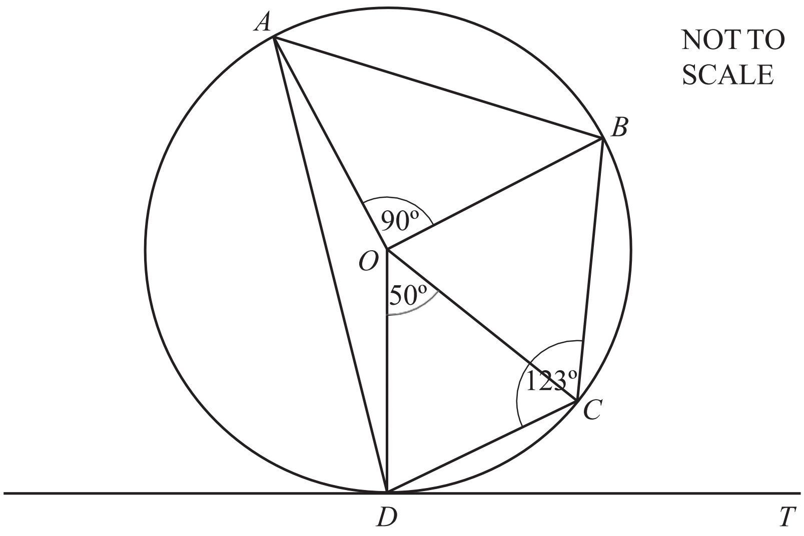
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



The points A, B, C and D lie on a circle centre O.
Angle AOB = 90°, angle COD = 50° and angle BCD = 123°.
The line DT is a tangent to the circle at D.

Find

(a) angle OCD, [1]

ODC is isosceles, hence

$$OCD = \frac{180 - 50}{2}$$

$$= \mathbf{65}$$

(b) angle TDC, [1]

$$TDC + ODC = 90$$

$$\rightarrow TDC = 90 - 65$$

$$= \mathbf{25}$$

(c) angle ABC, [1]

$$OBA = (180 - 90) \div 2$$

$$= 45$$

$$OBC = OCB$$

$$= 123 - 65$$

$$= 58$$

Hence

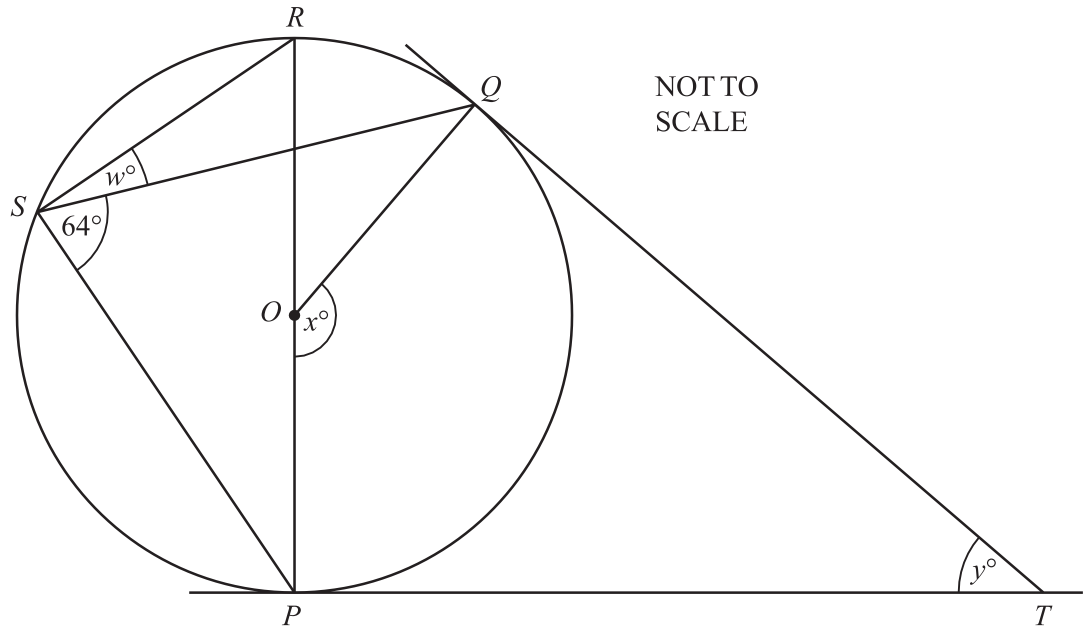
$$ABC = 45 + 58$$

$$= \mathbf{103}$$

(d) reflex angle AOC. [1]

$$AOC = 2 \times 103$$

$$= \mathbf{206}$$



P, Q, R and S lie on a circle, centre O.
TP and TQ are tangents to the circle.
PR is a diameter and angle $PSQ = 64°$.

(a) Work out the values of w and x. [2]

The angle subtended by an arc at the centre is twice the size the angle subtended by an arc at the circumference.

In other words

$x = 2 \times 64°$

Angle $x = 128°$

RS is perpendicular on SP, therefore, we can work out angle w:

$w = 90° - 64°$

$w = 26°$

(b) Showing all your working, find the value of y. [2]

The sum of all the angles in OQTP is 360°

QT is tangent to the circle, therefore, the radius OQ is perpendicular on QT.

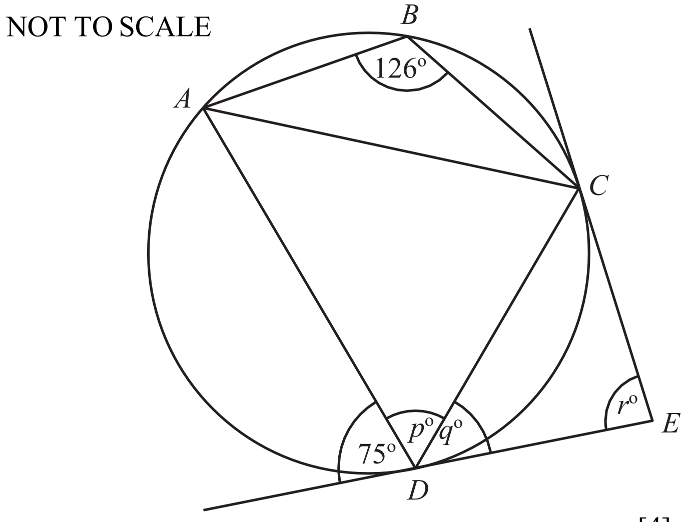
Similarly, OP is perpendicular on PT.

$y + x + 2 \times 90° = 360°$

$x = 128°$

$y = 52°$

$ABCD$ is a cyclic quadrilateral.
The tangents at C and D meet at E.
Calculate the values of p, q and r.



[4]

The opposite angles in a cyclic quadrilateral add up to 180°.

In the cyclic quadrilateral ABCD, ADC and ABC are opposite angles.

ADC = p = 180° - 126°

p = 54°

The tangent DE is a straight line, so the angles p, q and 75°, add up to 180°.

q = 180° - 54° - 75°

q = 51°

Tangents which meet at the same point are equal.

Therefore, CE = DE.

In this case, the triangle CED is isosceles, therefore, angle DCE = q.

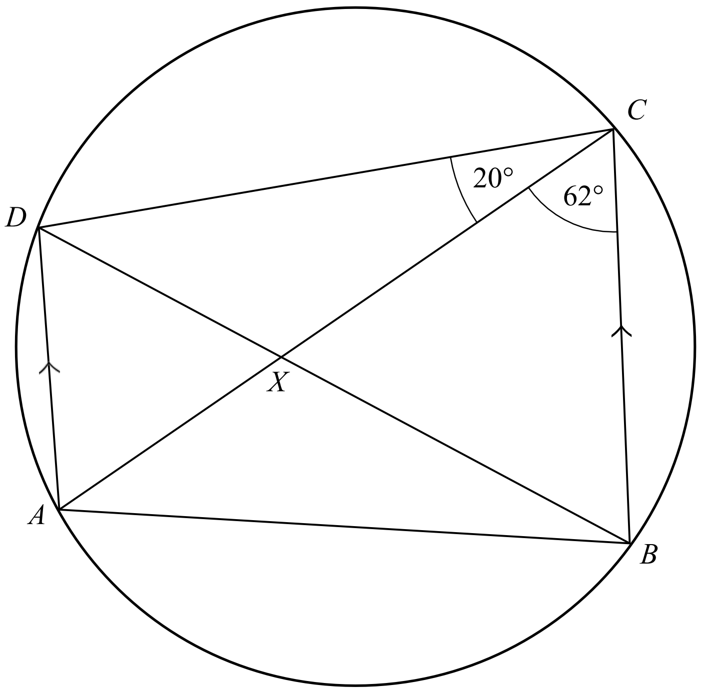
In the triangle DCE, the interior angles add up to 180°

$r + 2q = 180°$

$r + 2 \times 51° = 180°$

$r = 180° - 102°$

$r = 78°$



NOT TO SCALE

ABCD is a cyclic quadrilateral.
AD is parallel to *BC*. The diagonals *DB* and *AC* meet at *X*.
Angle *ACB* = 62° and angle *ACD* = 20°.
Calculate

(a) angle *DBA*, [1]

Angles DCA and DBA are both extended at the circumference by the same arc.

Therefore, **angle DCA = angle DBA = 20°**

(b) angle *DAB*, [1]

In a cyclic quadrilateral, the opposite angles add up to 180°.

Angle DAB = 180° - 20° - 62°

Angle DAB = 98°

(c) angle DAC, [1]

Angles DAC and ACB are corresponding angles with the parallel lines AD and BC.

Therefore, **angle DAC = angle ACB = 62°**

(d) angle AXB, [1]

Angle DAC = 62°

Angle DBA = 20°

Angles CDB and CAB are extended by the same arc at the circumference.

Therefore, angle CDB = angle CAB.

Angle CAB = angle DAB – angle DAC

Angle CAB = 98° - 62°

Angle CAB = 36°

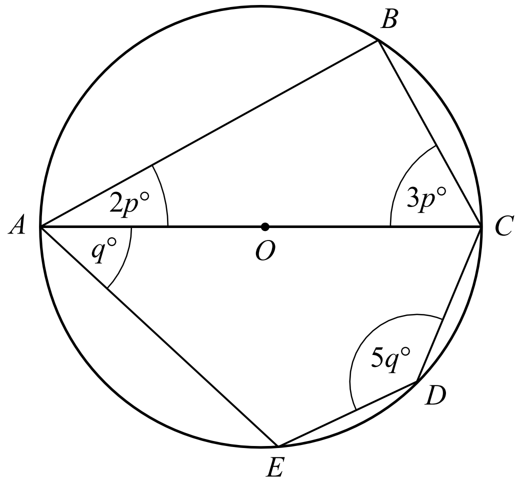
In the triangle AXB, the sum of the angles sum up to 180°.

Angle AXB = 180° - 36° - 20°

Angle AXB = 124°

(e) angle CDB. [1]

Angle CAB = 36°



A, B, C, D and E lie on a circle, centre O. AOC is a diameter.
Find the value of

(a) p, [2]

AOC is the diameter in the circle of centre O.

The angle subtended at the circumference by the diameter is 90°.

$2p + 3p = 180° - 90°$

$5p = 90°$

p = 18°

(b) q. [2]

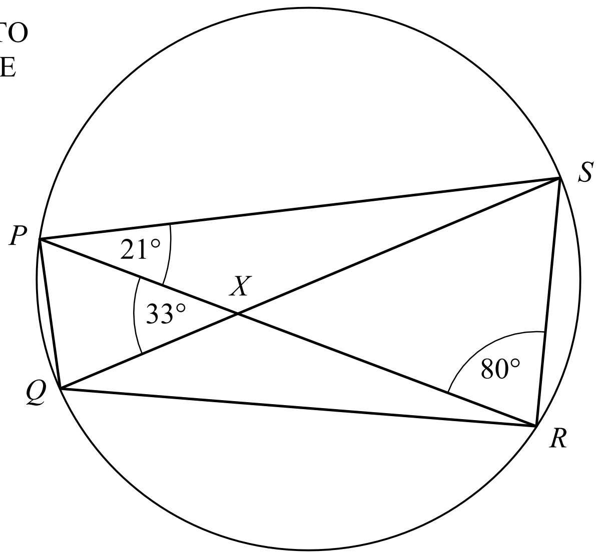
ACDE is a cyclic quadrilateral in the circle of centre O since all the vertex are on the circumference of the circle.

The opposite angles in a cyclic quadrilateral sum up to 180°.

$5q + q = 180°$

q = 30°

NOT TO SCALE



PQRS is a cyclic quadrilateral. The diagonals *PR* and *QS* intersect at *X*.
Angle *SPR* = 21°, angle *PRS* = 80° and angle *PXQ* = 33°.
Calculate

(a) angle *PQS*, [1]

Angle PQS = Angle PRS = 80°

(b) angle *QPR*, [1]

The sum of the angles in triangle PXQ is 180°.

Angle QPR = 180° - angle PQS – angle PXQ

Angle QPR = 180° - 33° - 80°

Angle QPR = 67°

(c) angle *PSQ*. [1]

The sum of the angles in triangle PSQ is 180°.

Angle PSQ = 180° - angle PQS – angle QPR - 21°

Angle PSQ = 180° - 80° - 67°- 21°

Angle PSQ = 12°

Circle Theorems
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Circle Theorems
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **27 minutes**

Score: **/21**

Percentage: **/100**

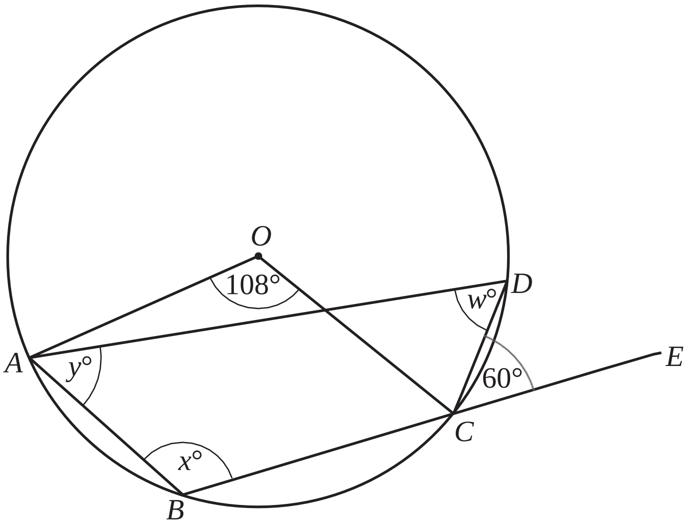
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



NOT TO SCALE

A, B, C and D are points on the circle, centre O.
BCE is a straight line.
Angle AOC = 108° and angle DCE = 60°.

Calculate the values of w, x and y. [3]

$$w = 0.5 \times 108$$

$$\rightarrow \boldsymbol{w = 54}$$

Cyclic quadrilateral, so

$$x + w = 180$$

$$\rightarrow x = 180 - 54$$

$$\rightarrow \boldsymbol{x = 126}$$

$$Angle\ DCB = 180 - 60$$

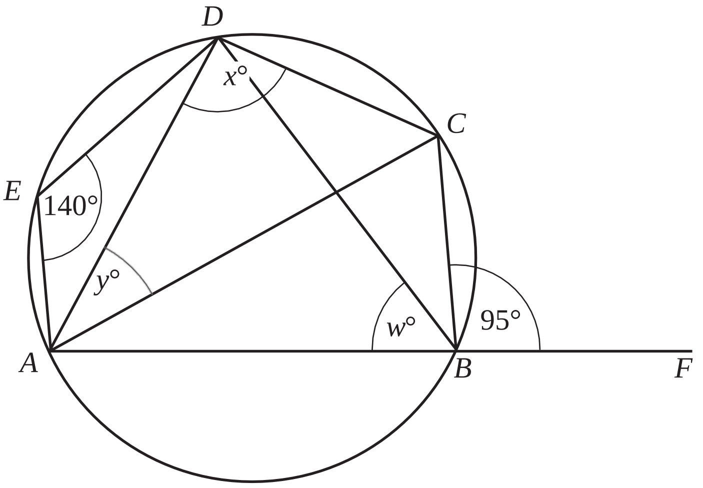
$$= 120$$

Angles in a quadrilateral add to 360, so

$$x + y + w + 120 = 360$$

$$\rightarrow 126 + 54 + y + 120 = 360$$

$$\rightarrow \boldsymbol{y = 60}$$



A, B, C, D and E lie on the circle.
AB is extended to F.
Angle AED = 140° and angle CBF = 95°.

Find the values of w, x and y. [5]

Opposite angles in cyclic quadratic add to 180

$$w + 140 = 180$$

$$\rightarrow \boldsymbol{w = 40}$$

To find x we need angle ABC

$$ABC + 95 = 180$$

$$\rightarrow ABC = 85$$

Hence

$$x + 85 + 180$$

$$\rightarrow \boldsymbol{x = 95}$$

Angle y is equal to DBC because they're angles in the same segment

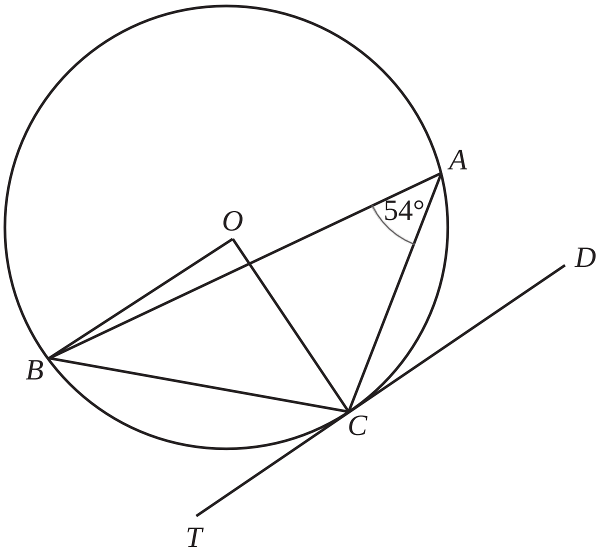
$$DBC = 85 - w$$

$$= 85 - 40$$

$$= 45$$

$$\rightarrow \boldsymbol{y = 45}$$

A, B and C are points on a circle, centre O.
TCD is a tangent to the circle.
Angle $BAC = 54°$.



NOT TO SCALE

(a) Find angle BOC, giving a reason for your answer. [2]

$$\boldsymbol{Angle\ BOC = 108}$$

Because angle at the centre is twice the angle on the circumference.

(b) When O is the origin, the position vector of point C is $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

(i) Work out the gradient of the radius OC. [1]

The gradient is found as:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4 - 0}{3 - 0}$$

$$= -\frac{4}{3}$$

(ii) D is the point $(7, k)$.

Find the value of k. [1]

We know that CD is perpendicular to OC, so its gradient

must be

$$-1 \div \left(-\frac{4}{3}\right)$$

$$= \frac{3}{4}$$

Hence, using the gradient equation:

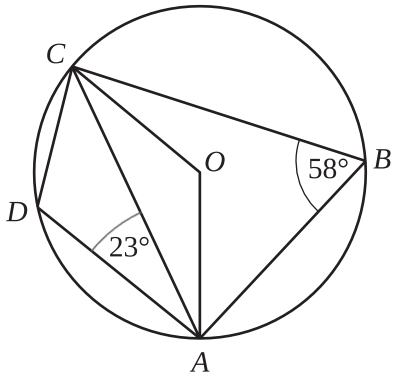
$$\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$\rightarrow \frac{3}{4} = \frac{k - -4}{7 - 3}$$

$$\rightarrow \frac{3}{4} = \frac{k + 4}{4}$$

$$\rightarrow k + 4 = 3$$

$$\rightarrow \boldsymbol{k = -1}$$



NOT TO SCALE

A, *B*, *C* and *D* lie on a circle centre *O*.
Angle *ABC* = 58° and angle *CAD* = 23°.

Calculate

(a) angle *OCA*, [2]

The angle subtended by a chord at the centre of a circle is twice the angle subtended at the circumference.

In our case, the angle COA is twice the size of the angle CBA.

Angle COA = 2 x 58°

Angle COA = 116°

The triangle COA is isosceles since 2 of the sides, OC and OA, are radius of the circle.

Therefore, the sizes of the angles OCA and OAC are equal.

The sum of the angles in a triangle is 180°.

180° = angle COA + 2 x angle OCA

180° = 116° + 2 x angle OCA

Angle OCA = 32°

(b) angle *DCA*. [2]

In our case, the rectangle ABCD is a cyclic quadrilateral since every vertex is on the circle's circumference.

In a cyclic quadrilateral, the opposite angles add up to 180

Therefore, angles CDA and CBA add up to 180°.

Angle CDA + 58° = 180°

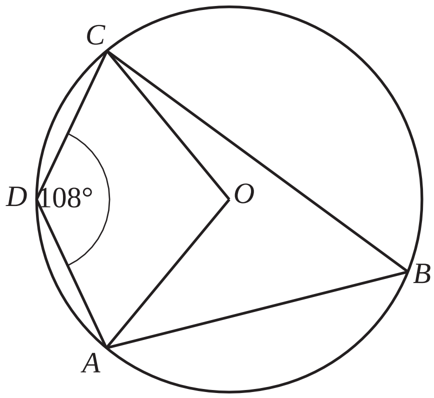
Angle CDA = 122°

In a triangle, all 3 angles add up to 180°.

Therefore, in the triangle CDA:

180° = 23° + 122° + Angle DCA

Angle DCA = 35°



NOT TO SCALE

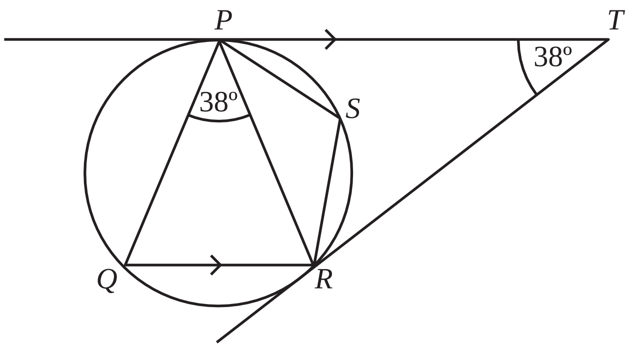
A, *B*, *C* and *D* lie on a circle centre *O*. Angle *ADC* = 108°.

Work out the obtuse angle *AOC*. [2]

Angle ABC is 72 by circle theorems.

The obtuse angle AOC must be twice this (also from circle theorems.

$$= \mathbf{144}$$



NOT TO SCALE

In the diagram *PT* and *QR* are parallel. *TP* and *TR* are tangents to the circle *PQRS*.
Angle *PTR* = angle *RPQ* = 38°.

(a) What is the special name of triangle *TPR*. Give a reason for your answer. [1]

Triangle TPR is isosceles. This is because PT and TR are both tangents to the circle, making them equal.

(b) Calculate

(i) angle *PQR*, [1]

In the triangle PQR, PQ = PR.

The angles PQR and PRQ are therefore equal.

2 x Angle PQR = 180° - 38°

2 x angle PQR = 142°

Angle PQR = 71°

(ii) angle *PSR*. [1]

PQRS is a cyclic quadrilateral since all 4 vertexes are touching the circumference of the circle.

In a cyclic quadrilateral, the opposite angles add up to 180°.

In our case, PQR and PSR are opposite angles in the cyclic quadrilateral.

Angle PSR = 180° - 71°

Angle PSR = 109°

Parallel Lines
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Parallel Lines
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **14 minutes**

Score: **/11**

Percentage: **/100**

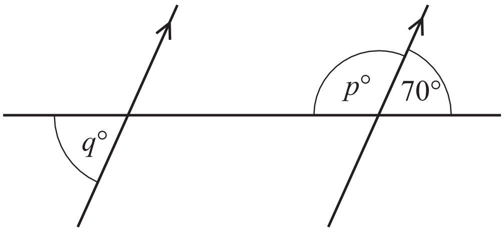
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



NOT TO SCALE

The diagram shows a straight line intersecting two parallel lines.

Find the value of p and the value of q. [2]

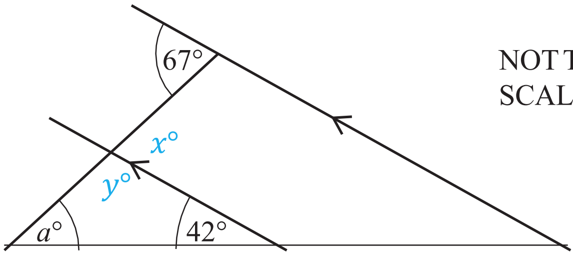
$p = 180 - 70$

$= \mathbf{110}$

$p + q = 180$

$\rightarrow q = 180 - 110$

$= \mathbf{70}$



NOT TO SCALE

Find the value of a. [2]

There is always more than one way to do these… …here's one:

(Give a reason for each step in the working)

$x = 67°$ (Alternate angles in parallel lines are equal)

$y + 67 = 180$ (Angles on a straight line add to 180°)

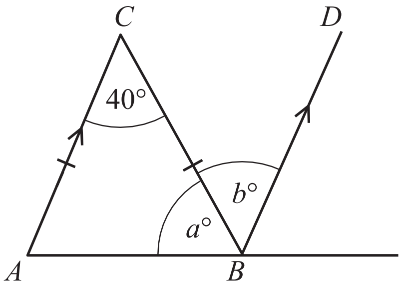
Subtract 67

$y = 113°$

$a + 42 + 113 = 180$ (Angles in a triangle add to 180°)

Subtract 155

$\boldsymbol{a = 25°}$



NOT TO SCALE

Triangle *ABC* is isosceles and *AC* is parallel to *BD*.

Find the value of *a* and the value of *b*. [2]

Because ABC is isosceles we known that angle CAB and angle a are equal. We also know that the sum of the angles is 180˚. We can therefore write

$40+a+a=180$

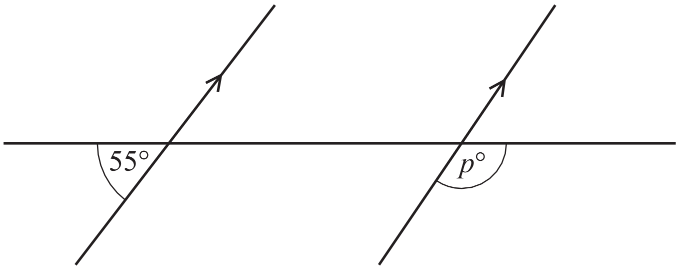
$2a=140$

$a=70$

And because of z angle properties the angle b is equal to angle ACB so

$b=40$

Question 4



NOT TO SCALE

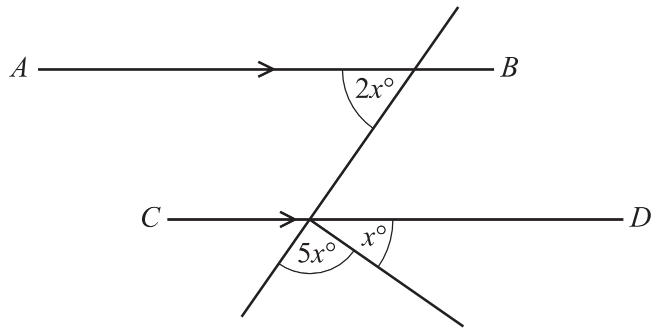
Find the value of p. [2]

Angles on a line add to 180 degrees:

$180° - 55° = 125°$ which we will call angle 'a'

Angles a and p are corresponding angles therefore they are the same:

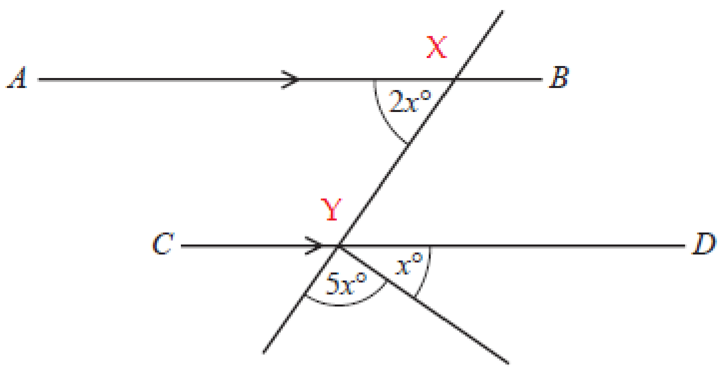
$$p = 125°$$



NOT TO SCALE

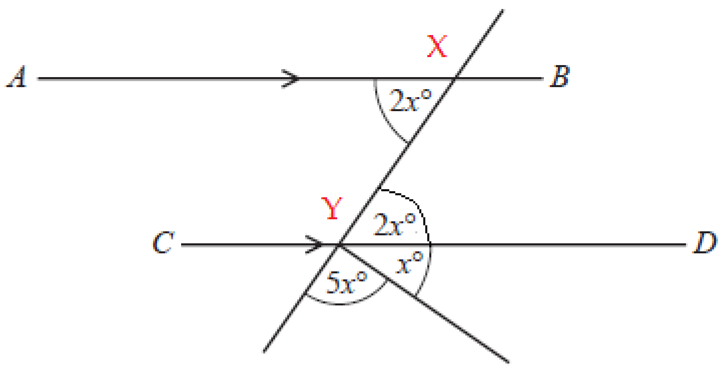
AB is parallel to *CD*.
Calculate the value of *x*. [3]

Label additional points X and Y to make the working clearer.



NOT TO SCALE

Angles AXY and DYX are equal as they are alternate angles; hence both are equal to *2x°*.



The three angles marked at point Y form a straightlineand add up to 180°.

$$180° = 5x° + x° + 2x°$$

$$180° = 8x°$$

$$x = 22.5°$$

Parallel Lines
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Sub-Topic	Parallel Lines
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **30 minutes**

Score: **/23**

Percentage: **/100**

Grade Boundaries:

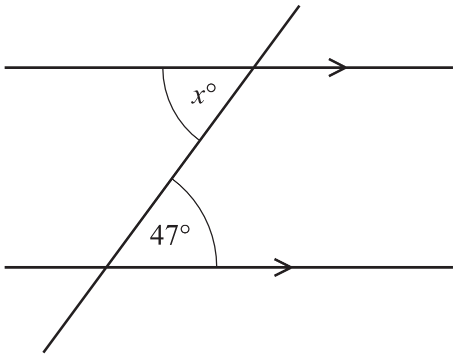
CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

(a) Find the value of x. [1]

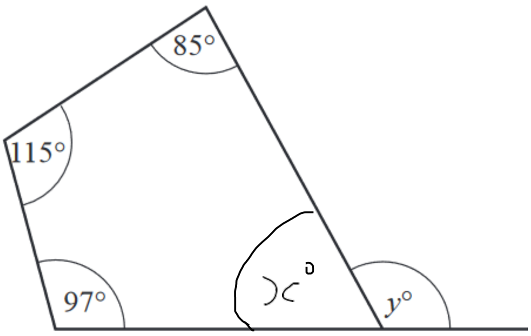


NOT TO SCALE

Using the rules of parallel and intersecting lines, x is alternate to the angle marked.

Hence: x = 47°

(b) Find the value of y. [2]



NOT TO SCALE

x° can be found by finding the difference between the total of the interior angles in the quadrilateral (360°) and the sum of the interior angles already known.

$$x° = 360° - 97° - 115° - 85°$$

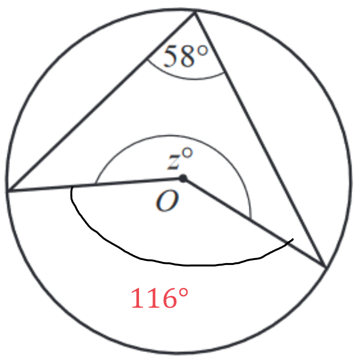
$$= 63°$$

Angles on a straight line add up to 180°, therefore $180° - x° = y°$.

$$180° - 63°$$

$$= 117°$$

(c)



NOT TO SCALE

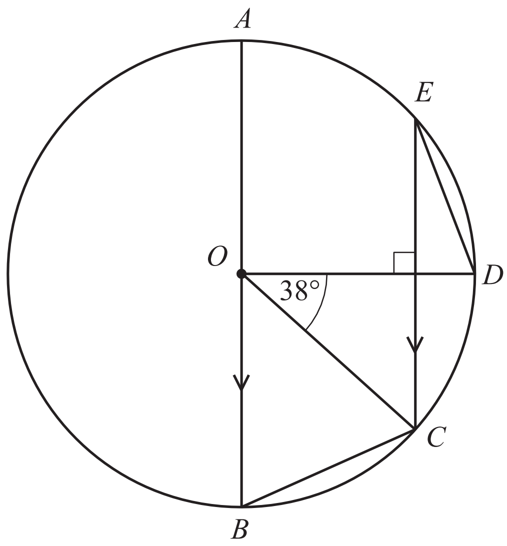
The diagram shows a circle, centre O.

Find the value of z. [2]

Using circle theorems, the angle shown at the centre is twice that of the angle subtended at the circumference (58°). The angle at the centre marked = 2 x 58 = 116 Therefore z is equal to:

$$z° = 360° - 2(58°)$$

$$= 244°$$



NOT TO SCALE

AB is the diameter of a circle, centre O. C, D and E lie on the circle.
EC is parallel to AB and perpendicular to OD. Angle DOC is $38°$.

Work out

(a) angle BOC , [1]

$BOC = 180 - 90 - 38$

$= \mathbf{52}$

(b) angle CBO, [1]

$CBO = (90 + 38) \div 2$

$= \mathbf{64}$

(c) angle EDO . [2]

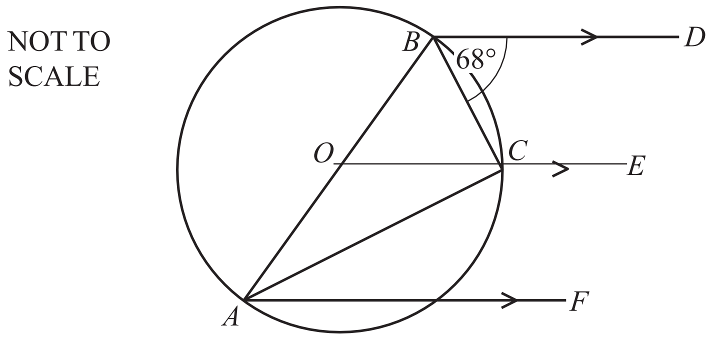
$CED = 38 \div 2$

$= 19$

$\rightarrow EDO = 180 - 90 - 19$

$= \mathbf{71}$

Question 3

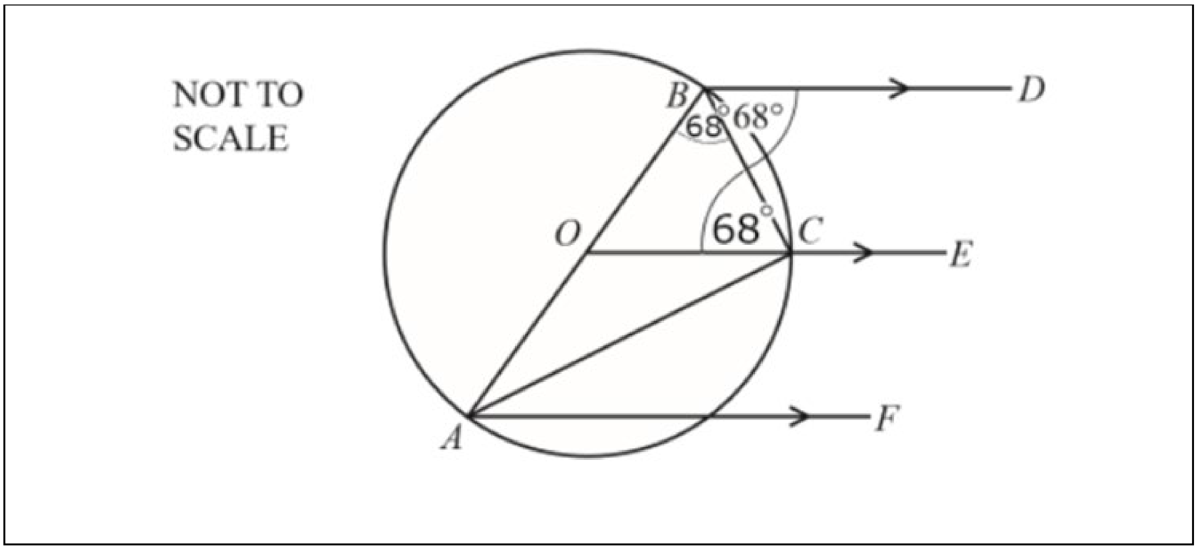


Points *A*, *B* and *C* lie on a circle, centre *O*, with diameter *AB*.
BD, *OCE* and *AF* are parallel lines.
Angle *CBD* = 68°.

Calculate

(a) angle *BOC*, [2]

Using the fact that Angle CBD = Angle OCB = 68^{o}, that the triangle OBC is an isosceles triangle and that the angles in a triangle add up to 180^{o}, we can calculate the Angle BOC:



OBC + BOC + OCB = 180^{o}, as OBC = OCB = 68^{o}

68^{o} + BOC + 68^{o} = 180^{o}

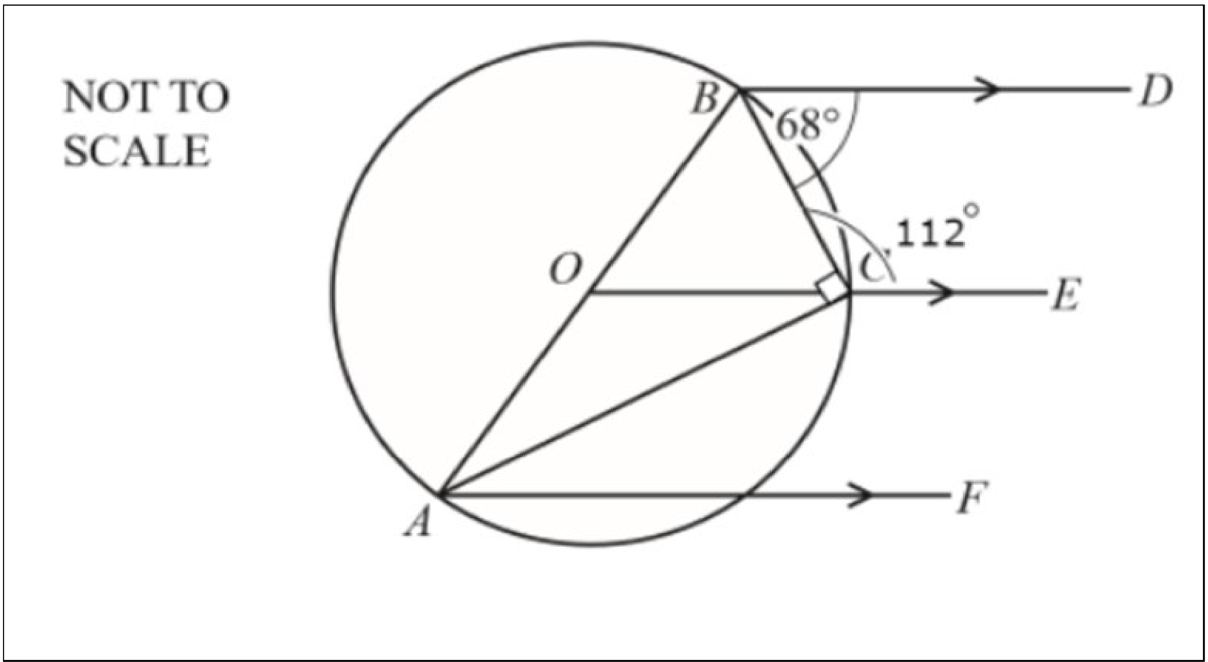
BOC = 44^{o}

(b) angle ACE. [2]

Angle ACB = 90° because of angles in a semicircle

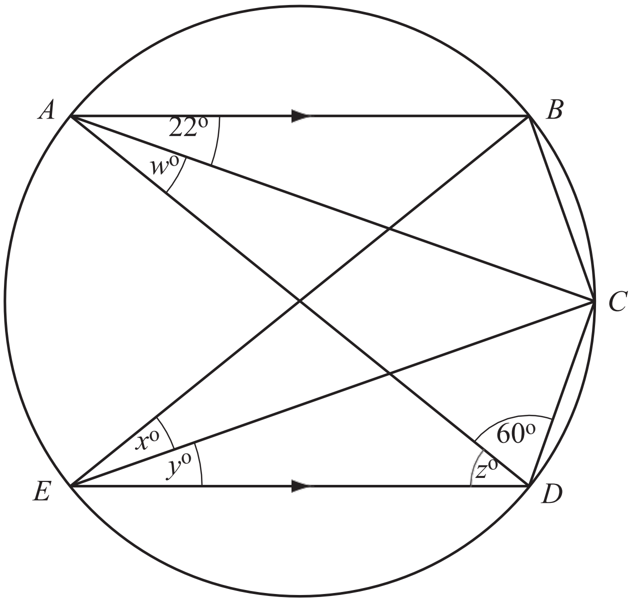
Angle BCE = 112° because Angle BCE & CBD are supplementary, hence the angles sum to 180°

Angle ACE + ACB + BCE = 360°



ACE + 112 + 90 = 360

ACE = 158°



AD is a diameter of the circle *ABCDE*.
Angle *BAC* = 22° and angle *ADC* = 60°.
AB and *ED* are parallel lines.
Find the values of *w*, *x*, *y* and *z*. [4]

BAC and BEC are angles subtended by the same chord, BC, in the circle with centre in O.

Therefore, the 2 angles are equal.

BAC = x = 22°

Similarly, the angles CAD and CED are subtended by the same chord, CD.

w = y

AB and ED are parallel.

BAD and ADE are corresponding angles, so equal.

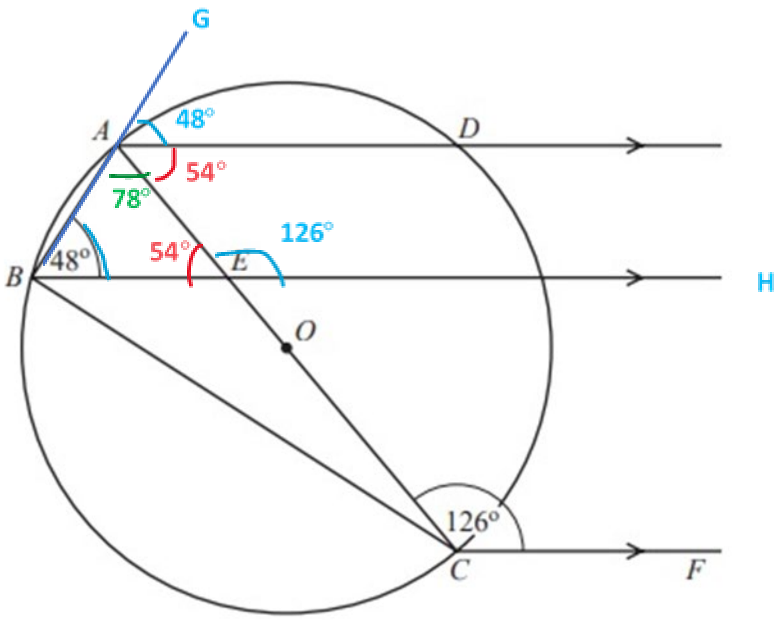
z = 22° + w

AD is the diameter in the circle and ACD is the angle subtended by the diameter at the circumference.

Therefore, angle ACD = 90°

w = 180° - 90° - 60° = 30° = y

z = w + 22° = 30° + 22° = 52°



A, B, C and D lie on a circle centre O. AC is a diameter of the circle.
AD, BE and CF are parallel lines. Angle $ABE = 48°$ and angle $ACF = 126°$.
Find

(a) angle DAE, [1]

BH and CF are 2 parallel lines and the transversal line AC intersects them. In this case, angles FCA and HEA are a pair of corresponding angles, therefore, they are congruent and equal to 126°

BEH is a straight line, therefore the angle BEH is 180°.

180° = angle HEA + angle AEB

180° = 126° + angle AEB

Angle AEB = 54°

BH and AD are 2 parallel lines and the transversal line AC intersects them. In this case, angles DAE and AEB are a pair of corresponding angles, therefore, they are congruent and equal to 54°

Angle DAE = 54°

(b) angle EBC, [1]

In the circle above, ABC is an angle subtended by the diameter of the circle, AC.

Therefore, the size of the angle ABC is 90°.

angle ABC = angle ABE + angle EBC

90° = 48° + angle EBC

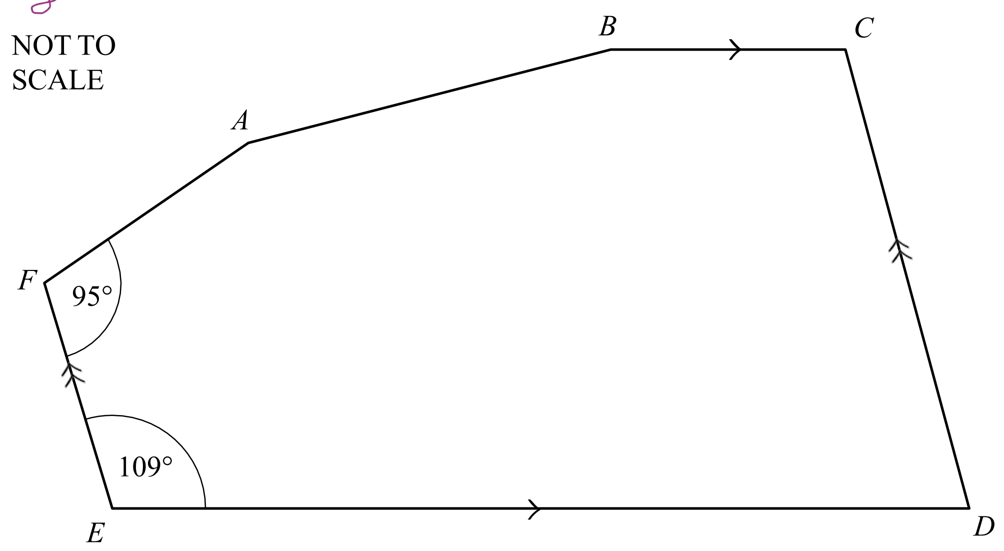
Angle EBC = 42°

(c) angle BAE. [1]

In the triangle AEB, all 3 interior angles add up to 180°.

Angle BAE + 54° + 48° = 180°

Angle BAE = 78°



In the hexagon *ABCDEF*, *BC* is parallel to *ED* and *DC* is parallel to *EF*.
Angle *DEF* = 109° and angle *EFA* = 95°.
Angle *FAB* is equal to angle *ABC*.
Find the size of

(a) angle *EDC*, [1]

For the parallel lines DC and EF, the angles EDC and FED are consecutive interior angles, therefore, their sum is 180°.

Angle EDC = 180° - 109°

Angle EDC = 71°

(b) angle *FAB*. [2]

The sum of all the angles in a hexagon is 720°.

For the parallel lines BC and ED, the angles EDC and BCD are consecutive interior angles, therefore, their sum is 180°.

Angle BCD = 180° - 71°

Angle BCD = 109°

Angle FAB x 2 + Angle EDC + Angle BCD + Angle AFE + Angle FED = 720°

2 x Angle FAB = 180° - 109° - 109° - 71° - 75°

Angle FAB = 168°