Probability
Difficulty: Medium

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Probability
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 1

Time allowed: **92 minutes**

Score: **/80**

Percentage: **/100**

Grade Boundaries:

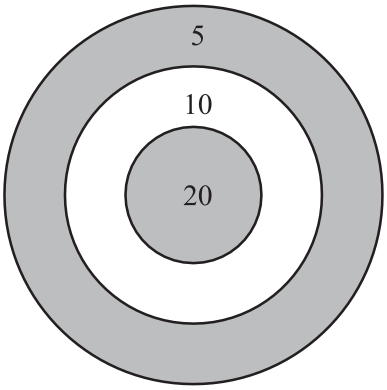
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) ~~Assembled by AS~~

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Kiah plays a game.
The game involves throwing a coin onto a circular board.
Points are scored for where the coin lands on the board.



If the coin lands on part of a line or misses the board then 0 points are scored.
The table shows the probabilities of Kiah scoring points on the board with one throw.

Points scored	20	10	5	0
Probability	x	0.2	0.3	0.45

(a) Find the value of x. [2]

All the probabilities must sum to 1

$$x + 0.2 + 0.3 + 0.45 = 1$$

$$\rightarrow \boldsymbol{x = 0.05}$$

(b) Kiah throws a coin fifty times.

Work out the expected number of times she scores 5 points. [1]

We expect her to score 5 points with a probability of 0.3. Out of fifty throws this is

$$50 \times 0.3$$

$$= \mathbf{15}$$

(c) Kiah throws a coin two times.

Calculate the probability that

(i) she scores either 5 or 0 with her first throw, [2]

The probability that she scores a 5 or a 0 is the two probabilities added

$$0.3 + 0.45$$

$$= \mathbf{0.75}$$

(ii) she scores 0 with her first throw and 5 with her second throw, [2]

This is conditional probability. She scores a 5 given that she has scored a zero. We multiply them

$$0.3 \times 0.45$$

$$= 0.135$$

(iii) she scores a total of 15 points with her two throws. [3]

In her two throws she can score 15 only by scoring a 5 then a 10 *or* a 10 then a 5.

This probability is found by

$$0.2 \times 0.3 + 0.3 \times 0.2$$

$$= \mathbf{0.12}$$

(d) Kiah throws a coin three times.

Calculate the probability that she scores a total of 10 points with her three throws. [5]

To score 10 she can throw

$$5, 5, 0$$

$$5, 0, 5$$

$$0, 5, 5$$

$$10, 0, 0$$

$$0, 10, 0$$

$$0, 0, 10$$

We need to add all these probabilities

$$3 \times 0.3^2 \times 0.45 + 3 \times 0.2 \times 0.45^2$$

$$= \mathbf{0.243}$$

Question 2

A A A A B B C

(a) One of these 7 cards is chosen at random.

Write down the probability that the card

(i) shows the letter A, [1]

There are 4 cards with the letter A and 7 cards in total. The probability of picking a card which shows the letter A is therefore:

$$\frac{4}{7}$$

(ii) shows the letter A or B, [1]

There are 6 cards with the letter A or B (4 with A and 2 with B) and 7 cards in total.

The probability of picking a card which shows the letter A or B is therefore:

$$\frac{6}{7}$$

(iii) does not show the letter B. [1]

There are 5 cards with letter different from B (4 with A and 1 with C) and 7 cards in total.

The probability of picking a card which does not show the letter B is therefore:

$$\frac{5}{7}$$

(b) Two of the cards are chosen at random, without replacement.

Find the probability that

(i) both show the letter A, [2]

When picking the first cars, we can use the probability found in part a) i).

$$first\ is\ A = \frac{4}{7}$$

However, when we are picking the second cars, both the total number of cards and the number of cards with the letter A decreases (because one card was already picked).

The probability of picking a card with letter A in second round (3 cards with A, 6 in total):

$$second\ is\ A = \frac{3}{6}$$

To get the probability that both will show the letter A, multiply the probabilities together.

$$both\ are\ A = \frac{4}{7} \times \frac{3}{6}$$

$$= \mathbf{\frac{12}{42}}$$

(ii) the two letters are different. [3]

The easiest way to calculate the probability that the two letters are different is to calculate the probability that they are the same (i.e. complete opposite) and then subtract this probability from 1.

$$both\ different = 1 - both\ same = 1 - both\ are\ A - both\ are\ B$$

We do not have to include the probability that both are C, since there is only one card showing C.

We already know the probability that both are A, therefore we only need to calculate the probability that both cards show the letter B. Use the same method as in part b) i).

When picking the card, there are 2 cards with B and 7 cards in total: $first\ is\ B = \frac{2}{7}$

The probability of picking a card with letter B in second round (1 card with B, 6 in total):

$$second\ is\ B = \frac{1}{6}$$

To get the probability that both will show the letter A, multiply the probabilities together.

$$both\ are\ B = \frac{2}{7} \times \frac{1}{6} = \mathbf{\frac{2}{42}}$$

Therefore:

$$both\ different = 1 - \frac{12}{42} - \frac{2}{42}$$

$$= \mathbf{\frac{28}{48}}$$

(c) Three of the cards are chosen at random, without replacement.

Find the probability that the cards do not show the letter C. [2]

During the first round, there are 7 cards in total and 6 of them do not show the letter C.

The probability of picking one of them is $\frac{6}{7}$.

In the second round, both the total number of cards and the number of cards not showing C decreases (the cards are not replaced). There are 6 cards and 5 of them do not show C.

The probability of picking one of them is $\frac{5}{6}$.

Similar thing applies for the third round as well. Now, there are 5 cards and 4 of them do not show C.

The probability of picking one of them is $\frac{4}{5}$.

The get the probability that all three cards do not show the letter C, we multiply the previous probabilities.

$$\frac{6}{7} \times \frac{5}{6} \times \frac{4}{5}$$

$$= \mathbf{\frac{120}{210}}$$

In this question write any probability as a fraction.

Navpreet has 15 cards with a shape drawn on each card.
5 cards have a square, 6 cards have a triangle and 4 cards have a circle drawn on them.

(a) Navpreet selects a card at random.

Write down the probability that the card has a circle drawn on it.

[1]

There are 4 cards with circle. Since Navpreet has 15 cards in total, the probability of picking a card with a circle drawn on it is:

$$\frac{cards\ with\ circles}{total\ cards}$$

$$= \frac{\mathbf{4}}{\mathbf{15}}$$

(b) Navpreet selects a card at random and replaces it.
She does this 300 times.

Calculate the number of times she expects to select a card with a circle drawn on it.

[1]

The probability of picking a card with a circle was found in part a).

To calculate the expected number of cards with this circle, we need to multiply the number of times Navpreet selects a card with this probability.

$$300\ cards\ \times \frac{4}{15}$$

$$= \mathbf{80}$$

(c) Navpreet selects a card at random, replaces it and then selects another card.

Calculate the probability that

(i) one card has a square drawn on it and the other has a circle drawn on it, [3]

Navpreet can either get the card with a square first and then the card with a circle or the other way around. To account for this, we multiply the probability of picking square and then circle by 2.

$$pick\ square \times pick\ circle = \frac{squares}{total} \times \frac{circles}{total}$$

$$= \frac{5}{15} \times \frac{4}{15}$$

$$pick\ square \times pick\ circle = \frac{20}{225}$$

Now multiply by two (to include both scenarios).

$$probability = \mathbf{\frac{40}{225}}$$

(ii) neither card has a circle drawn on it.

[3]

There are 11 cards with some other shape than circle.

The probability of picking one of these cards is:

$$\frac{not\ circle}{total} = \frac{11}{15}$$

To get the probability that neither card has a circle drawn on it, we multiply the probability of a card not having a circle (11/15) with itself (since two cards are picked).

$$neither\ with\ a\ circle = not\ circle\ \times not\ circle$$

$$neither\ with\ a\ circle = \frac{11}{15} \times \frac{11}{15}$$

$$neither\ with\ a\ circle$$

$$= \mathbf{\frac{121}{225}}$$

(d) Navpreet selects two cards at random, without replacement.

Calculate the probability that

(i) only one card has a triangle drawn on it, [3]

Now the cards are not replaced, therefore the total number of cards decreases with every draw.

There are two possible scenarios:

- Picking a triangle, then not picking a triangle

$$\frac{triangle}{total} \times \frac{not\ triangle}{total\ without\ one} = \frac{6}{15} \times \frac{9}{14} = \frac{54}{210}$$

- Not picking a triangle, then picking a triangle

$$\frac{not\ triangle}{total} \times \frac{triangle}{total\ without\ one} = \frac{9}{15} \times \frac{6}{14} = \frac{54}{210}$$

To get the probability that only one card has a triangle, we add the probabilities of these two scenarios:

$$\frac{54}{210} + \frac{54}{210}$$

$$= \mathbf{\frac{108}{210}}$$

(ii) the two cards have different shapes drawn on them. [4]

There are three possible scenarios:

- Triangle and not triangle

$$\frac{triangle}{total} \times \frac{not\ triangle}{total\ without\ one} = \frac{6}{15} \times \frac{9}{14} = \frac{54}{210}$$

- Square and not square

$$\frac{square}{total} \times \frac{not\ square}{total\ without\ one} = \frac{5}{15} \times \frac{10}{14} = \frac{50}{210}$$

- Circle and not circle

$$\frac{circle}{total} \times \frac{not\ circle}{total\ without\ one} = \frac{4}{15} \times \frac{11}{14} = \frac{44}{210}$$

To get the probability that the two cards have different shapes, we add the probabilities of these three scenarios:

$$\frac{54}{210} + \frac{50}{210} + \frac{44}{210}$$

$$= \mathbf{\frac{148}{210}}$$

(a) A square spinner is biased.
The probabilities of obtaining the scores 1, 2, 3 and 4 when it is spun are given in the table.

Score	1	2	3	4
Probability	0.1	0.2	0.4	0.3

(i) Work out the probability that on one spin the score is 2 or 3. [2]

$0.2 + 0.4$

$= \mathbf{0.6}$

(ii) In 5000 spins, how many times would you expect to score 4 with this spinner? [1]

5000×0.3

$= \mathbf{1500}$

(iii) Work out the probability of scoring 1 on the first spin and 4 on the second spin. [2]

0.1×0.3

$= \mathbf{0.03}$

(b) In a bag there are 7 red discs and 5 blue discs.
From the bag a disc is chosen at random and not replaced.
A second disc is then chosen at random.

Work out the probability that at least one of the discs is red.
Give your answer as a fraction. [3]

The possible combinations are:

$$RB$$

$$BR$$

$$RR$$

Hence, we have:

$$\frac{7}{12}\times\frac{5}{11}+\frac{5}{12}\times\frac{7}{11}+\frac{7}{12}\times\frac{6}{11}$$

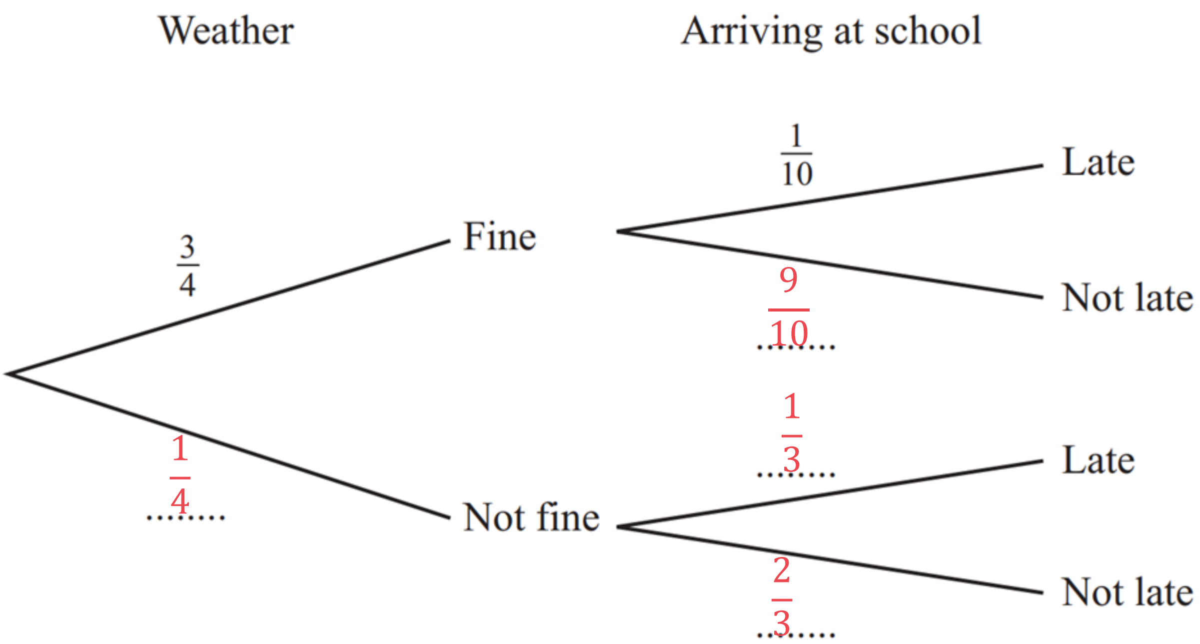
$$=\frac{112}{132}$$

$$=\mathbf{\frac{28}{33}}$$

Question 5

If the weather is fine the probability that Carlos is late arriving at school is $\frac{1}{10}$.
If the weather is not fine the probability that he is late arriving at school is $\frac{1}{3}$.
The probability that the weather is fine on any day is $\frac{3}{4}$.

(a) Complete the tree diagram to show this information. [3]



(b) In a school term of 60 days, find the number of days the weather is expected to be fine. [1]

$\frac{3}{4} \times 60$

$= \mathbf{45}$

(c) Find the probability that the weather is fine and Carlos is late arriving at school. [2]

$$\frac{3}{4} \times \frac{1}{10}$$

$$= \mathbf{\frac{3}{40}}$$

(d) Find the probability that Carlos is not late arriving at school. [3]

$$\frac{3}{4} \times \frac{9}{10} + \frac{1}{4} \times \frac{2}{3}$$

$$= \frac{27}{40} + \frac{2}{12} = \mathbf{\frac{101}{120}}$$

$$= \mathbf{\frac{101}{120}}$$

(e) Find the probability that the weather is not fine on at least one day in a school week of 5 days. [2]

Probability of weather being fine all week is

$$\left(\frac{3}{4}\right)^5$$

Hence

$$1 - \left(\frac{3}{4}\right)^5$$

$$= \mathbf{\frac{781}{1024}}$$

In this question, give all your answers as fractions.

N	A	T	I	O	N

The letters of the word **NATION** are printed on 6 cards.

(a) A card is chosen at random.

Write down the probability that

(i) it has the letter **T** printed on it, [1]

$\frac{1}{6}$

(ii) it does not have the letter **N** printed on it, [1]

$\frac{2}{3}$

(iii) the letter printed on it has no lines of symmetry. [1]

$\frac{1}{3}$

(b) Lara chooses a card at random, replaces it, then chooses a card again.

Calculate the probability that only **one** of the cards she chooses has the letter **N** printed on it. [3]

$$\frac{2}{6} \times \frac{4}{6} \times 2$$

$$= \frac{16}{36} = \frac{8}{18}$$

$$= \frac{8}{18}$$

(c) Jacob chooses a card at random and does not replace it.
He continues until he chooses a card with the letter **N** printed on it.

Find the probability that this happens when he chooses the 4th card. [3]

$$\frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$$

$$= \frac{48}{360}$$

$$= \frac{2}{15}$$

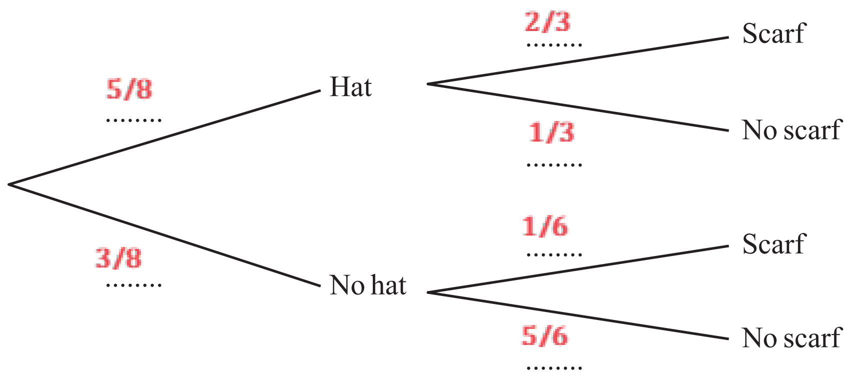
In this question, give all your answers as fractions.

When Ivan goes to school in winter, the probability that he wears a hat is $\frac{5}{8}$.

If he wears a hat, the probability that he wears a scarf is $\frac{2}{3}$.

If he does not wear a hat, the probability that he wears a scarf is $\frac{1}{6}$.

(a) Complete the tree diagram. [3]



(b) Find the probability that Ivan

(i) does not wear a hat and does not wear a scarf, [2]

$$P(\text{no hat no scarf}) = \frac{3}{8} \times \frac{5}{6}$$

$$= \frac{15}{48}$$

(ii) wears a hat but does not wear a scarf, [2]

$$P(\text{hat but no scarf}) = \frac{5}{8} \times \frac{1}{3}$$

$$= \frac{5}{24}$$

(iii) wears a hat or a scarf but not both. [2]

$$\mathbf{P(either\ hat\ or\ scarf\ but\ not\ both)}$$

$$= \mathbf{P(hat\ and\ no\ scarf) + P(no\ hat\ and\ scarf)}$$

$$= \left(\frac{5}{8} \times \frac{1}{3}\right) + \left(\frac{3}{8} \times \frac{1}{6}\right)$$

$$= \frac{13}{48}$$

(c) If Ivan wears a hat and a scarf, the probability that he wears gloves is $\frac{7}{10}$.
Calculate the probability that Ivan does **not** wear all three of hat, scarf and gloves. [3]

$$P(hat\ and\ scarf\ and\ gloves) = \left(\frac{5}{8} \times \frac{2}{3} \times \frac{7}{10}\right)$$

$$= \frac{7}{24}$$

$$P(no\ hat\ no\ scarf\ and\ no\ gloves) = 1 - \frac{7}{24}$$

$$= \frac{17}{24}$$

Probability
Difficulty: Medium

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Probability
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 2

Time allowed: **77 minutes**

Score: **/67**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

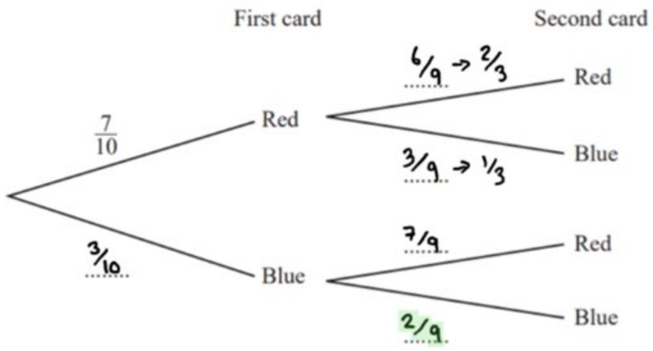
9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Question 1

In a box there are 7 red cards and 3 blue cards.
A card is drawn at random from the box and is not replaced.
A second card is then drawn at random from the box.

(a) Complete this tree diagram. [3]

If a card is taken without replacement, then the probability of drawing the second card will be $\frac{x}{9}$



(b) Work out the probability that the two cards are of different colours.
Give your answer as a fraction. [3]

Either we pick red and (x) blue or (+) blue and (x) red:

$$\frac{7}{10} \times \frac{1}{3} + \frac{3}{10} \times \frac{7}{9} = \mathbf{\frac{7}{15}}$$

$$= \mathbf{\frac{7}{15}}$$

Question 2

In all parts of this question give your answer as a fraction in its lowest terms.

(a) (i) The probability that it will rain today is $\frac{1}{3}$.

What is the probability that it will not rain today? [1]

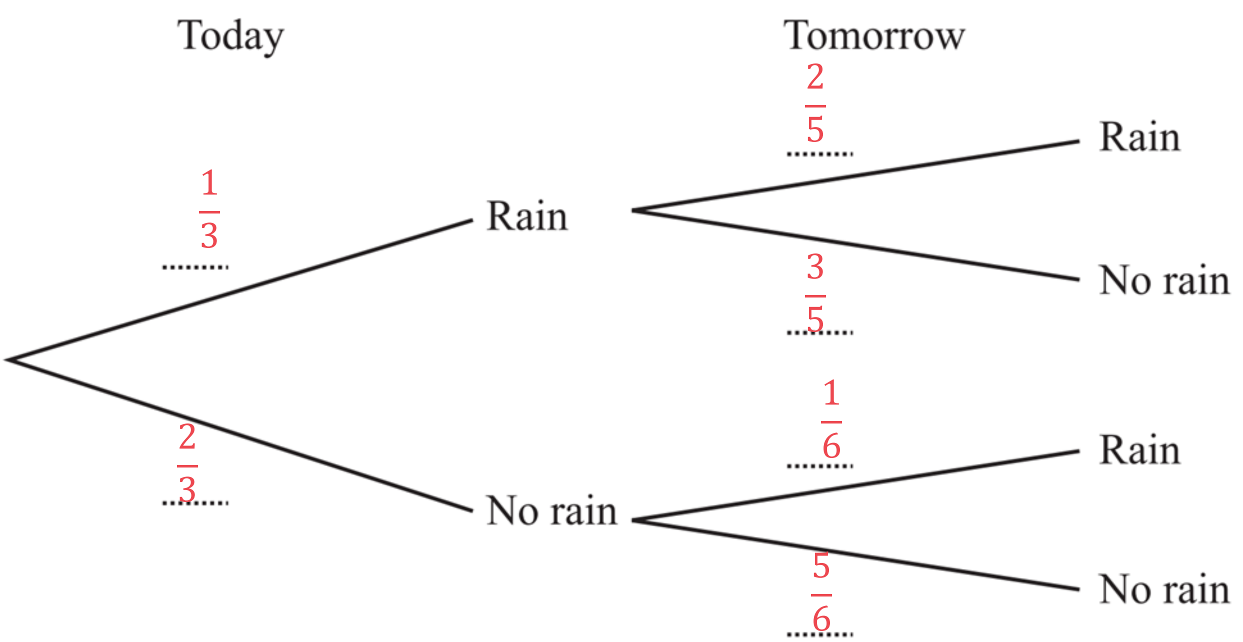
$$1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

(ii) If it rains today, the probability that it will rain tomorrow is $\frac{2}{5}$.

If it does not rain today, the probability that it will rain tomorrow is $\frac{1}{6}$.

Complete the tree diagram. [2]



(b) Find the probability that it will rain on at least one of these two days. [3]

Probability of no rain is

$$\frac{2}{3} \times \frac{5}{6}$$

$$= \frac{10}{18}$$

$$= \frac{5}{9}$$

Hence probability of at least one day of rain

$$1 - \frac{5}{9}$$

$$= \mathbf{\frac{4}{9}}$$

(c) Find the probability that it will rain on only one of these two days. [3]

$$\frac{1}{3} \times \frac{3}{5} + \frac{2}{3} \times \frac{1}{6}$$

$$= \frac{1}{5} + \frac{1}{9}$$

$$= \mathbf{\frac{14}{45}}$$

Question 3

Set A	S	U	M	S	
Set B	M	I	N	U	S

The diagram shows two sets of cards.

(a) One card is chosen at random from Set A and replaced.

(i) Write down the probability that the card chosen shows the letter M. [1]

Probability = $\frac{\text{nr.of favourable cases}}{\text{nr.of total cases}}$

In this situation, the number of total cases is 4, since there are 4 cards in total in Set A.

There is only one card with the letter M, therefore, we have only one favourable case, being the one in which that card with M is picked.

Probability = $\frac{1}{4}$

(ii) If this is carried out 100 times, write down the expected number of times the card chosen shows the letter M. [1]

The card is always replaced, therefore, the number of total cases will always be 4. Moreover, if the card with the letter M is picked, it will also be replaced.

This means that the probability to pick the card with the letter M form Set A will always be the same.

The expected number of times to pick the card M is:

$\frac{1}{4}$ x 100 = 25 times

(b) Two cards are chosen at random, **without** replacement, from Set A.

Find the probability that both cards show the letter S. [2]

For both cards to show the letter S, the first card picked and the second card picked needs to have the letter S.

The probability that the first card has the letter S is:

$P = \frac{2}{4}$

The card is not replaced, therefore, the number of total cards is now 3. If the first card picked has the letter S, there will be only one card with the letter S left.

The probability that the second card picked has also the letter S is:

$P = \frac{1}{3}$

For both cards to have the letter S, the 2 events for which we calculated the probability above need to happen simultaneously.

$P = \frac{2}{4} \times \frac{1}{3}$

$\mathbf{P = \frac{2}{12}}$

(c) One card is chosen at random from Set A and one card is chosen at random from Set B.

Find the probability that exactly one of the two cards shows the letter U. [3]

There are 2 possibilities to pick exactly one card showing the letter U.

1. The card showing the letter U is from Set A and the card from Set B shows any other letter, but U.
2. The card showing the letter U is from Set B and the card from Set A shows any other letter, but U.

The probability to choose a card with the letter U from Set A is P = $\frac{1}{4}$, while the probability that the card picked from Set A does not show the letter U is P = $\frac{3}{4}$

Similarly, the probability that the card picked from Set B does not show the letter U is P = $\frac{4}{5}$ and the probability that the card has the letter U is P = $\frac{1}{5}$.

For each of the 2 possibilities, the 2 events need to happen simultaneously (both choosing the card with the letter U from one set and NOT choosing the card with the letter U from the other set). Therefore, the 2 probabilities of the 2 separate events will be multiplied.

For the first possibility, the probability is: P = $\frac{1}{4}$ x $\frac{4}{5}$ = $\frac{1}{5}$

For the second possibility, the probability is: P = $\frac{3}{4}$ x $\frac{1}{5}$ = $\frac{3}{20}$

The probability to pick exactly one card with the letter U will be the sum of the probabilities for the 2 ways in which this can happen. We add them up because the 2 possibilities cannot happen simultaneously.

$$P = \frac{3}{20} + \frac{1}{5}$$

$$\mathbf{P = \frac{7}{20}}$$

(d) A card is chosen at random, **without** replacement, from Set B until the letter shown is either I or U.

Find the probability that this does not happen until the 4th card is chosen. [2]

Firstly, we need to keep in mind that for each card picked the number of total cards left is reduced.

For the 4th card to show either the letter I or U, the first 3 cards drawn need to show any of the other 3 letters in Set B.

For the first card drawn, the probability of picking a card other than I or U is:

$$P = \frac{3}{5}$$

The card is not replaced, therefore, for the second card drawn, the probability of picking a card other then I or U is:

$$P = \frac{2}{4}$$

Similarly, the probability for the third card picked to show a letter other than I or U is:

P = $\frac{1}{3}$

At this point, the 4th card picked will be either I or U since the 2 are the only ones left in Set B, therefore the probability of this event is P = 1 = $\frac{2}{2}$

The probability that this happens is the product of all the probabilities of the events listed above, since they need to happen simultaneously.

P = $\frac{3}{5}$ x $\frac{2}{4}$ x $\frac{1}{3}$ x 1

P = $\boldsymbol{\frac{6}{60}}$

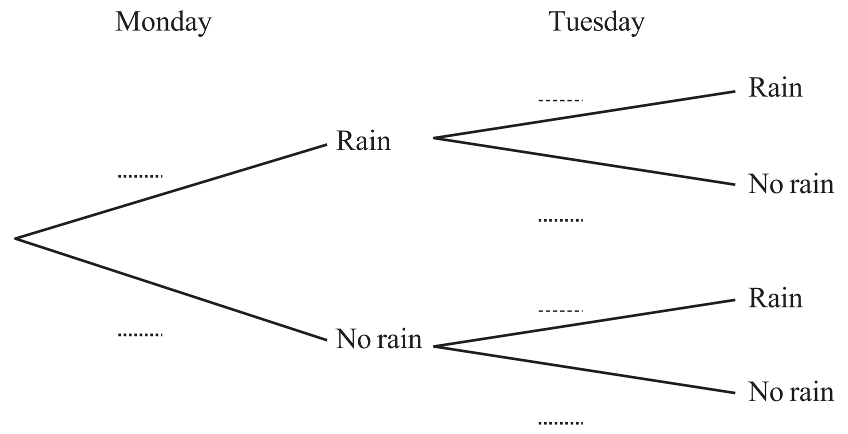
In this question give all your answers as fractions.

The probability that it rains on Monday is $\frac{3}{5}$.

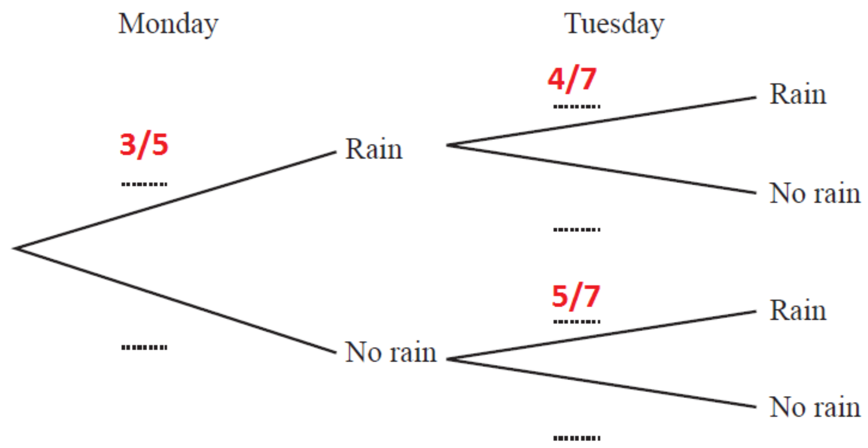
If it rains on Monday, the probability that it rains on Tuesday is $\frac{4}{7}$.

If it does not rain on Monday, the probability that it rains on Tuesday is $\frac{5}{7}$.

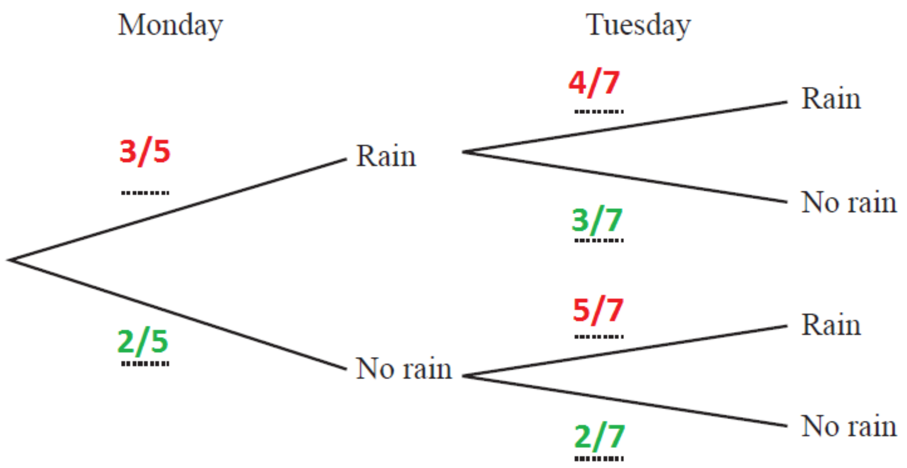
(a) Complete the tree diagram. [3]



We fill in the given probabilities:



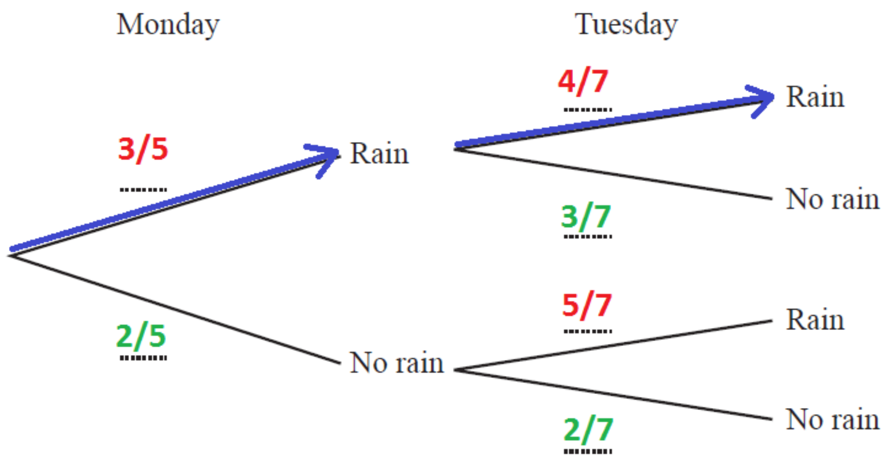
We need to remember that the probabilities (fractions) branching from a given point must always add up to 1. With this in mind, we can finish the diagram.



(b) Find the probability that it rains

(i) on both days, [2]

To find the probability that it will rain on both days we follow this outcome along the branches of the diagram and multiply the probabilities to together.



$$Rain\ on\ both\ days$$

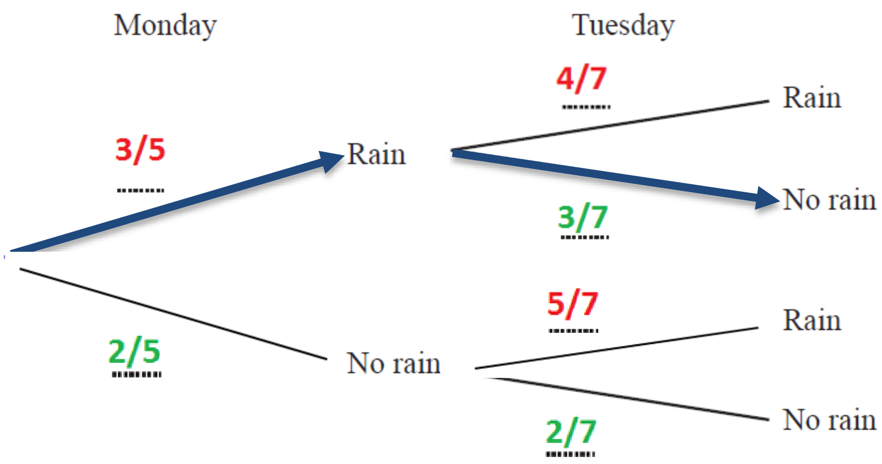
$$= Rain\ on\ Monday \times Rain\ on\ Tuesday\ (given\ rain\ on\ Monday)$$

$$Rain\ on\ both\ days = \frac{3}{5} \times \frac{4}{7}$$

Hence the probability that it will rain on both days:

$$\boldsymbol{Rain\ on\ both\ days = \frac{12}{35}}$$

(ii) on Monday but not on Tuesday, [2]



$Mon\text{: } rain, Tue\text{: no } rain$

$= rain\ on\ Monday \times no\ rain\ on\ Tuesday\ (given\ rain\ on\ Monday)$

$$Mon\text{: } rain, Tue\text{: } no\ rain = \frac{3}{5} \times \frac{3}{7}$$

Hence the probability that it will rain on Monday but not on Tuesday:

$\boldsymbol{Mon\text{: } rain, Tue\text{: } no\ rain = \frac{9}{35}}$ **(only rain on Monday)**

(iii) on only one of the two days. [2]

Add the probabilities together.

$$Rain\ only\ on\ one\ day = Rain\ only\ on\ Monday + Rain\ only\ on\ Tuesday$$

$$Rain\ only\ on\ one\ day = \frac{9}{35} + \frac{10}{35}$$

$$\boldsymbol{Rain\ only\ on\ one\ day = \frac{19}{35}}$$

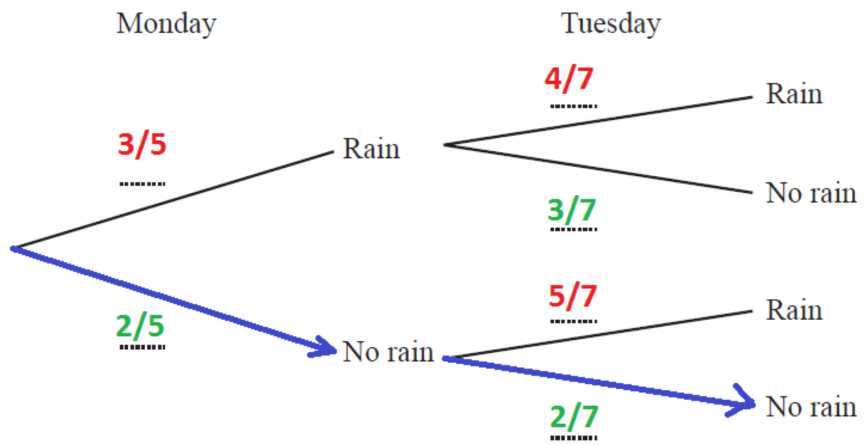
(c) If it does not rain on Monday and it does not rain on Tuesday, the probability that it does not rain on Wednesday is $\frac{1}{4}$.

Calculate the probability that it rains on at least one of the three days. [3]

Calculating the probability that it rains on at least one of the three days can seem very complicated as there are many possible scenarios. This problem, however, can be made very simple by considering that the total probability must always sum up to 1.

Therefore we only need to find the probability that it does not rain at all during the three days and then subtract it from 1, because in any other scenario, it rains at least once.

Find probability that it does not rain on Monday and Tuesday:



$$No\ rain\ on\ Mon\ and\ Tue = No\ rain\ Monday + No\ rain\ Tuesday\ (given\ no\ rain\ Mon)$$

$$No\ rain\ on\ Mon\ and\ Tue = \frac{2}{5} \times \frac{2}{7}$$

$$No\ rain\ on\ Mon\ and\ Tue = \frac{4}{35}$$

Multiply this by the probability that it does not rain on Wednesday (given no rain Monday and Tuesday) to get the probability that it does not rain during the three days.

$$No\ rain\ at\ all = \frac{4}{35} \times \frac{1}{4}$$

$$No\ rain\ at\ all = \frac{1}{35}$$

As mentioned before, to get the probability that it rains at least once, we subtract the probability that it does not rain at all from 1.

$$Rains\ at\ least\ once = 1 - No\ rain\ at\ all$$

$$Rains\ at\ least\ once = 1 - \frac{1}{35}$$

$$\mathbf{Rains\ at\ least\ once = \frac{34}{35}}$$

Katrina puts some plants in her garden.

The probability that a plant will produce a flower is $\frac{7}{10}$.

If there is a flower, it can only be red, yellow or orange.

When there is a flower, the probability it is red is $\frac{2}{3}$ and the probability it is yellow is $\frac{1}{4}$.

(a) Draw a tree diagram to show **all** this information.

Label the diagram and write the probabilities on each branch. [5]

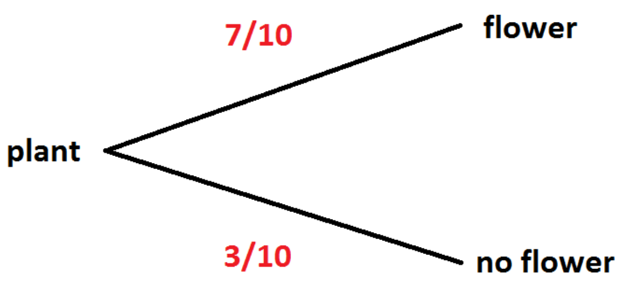
The first branching point of the tree diagram is whether the plant will produce a flower or not. The probability that it will is 7/10.

The total probability must sum up to 1 at every branching point. Hence the probability that the plant will not produce a flower is 3/10 so that:

$$p(flower) + p(no\ flower) = 1$$

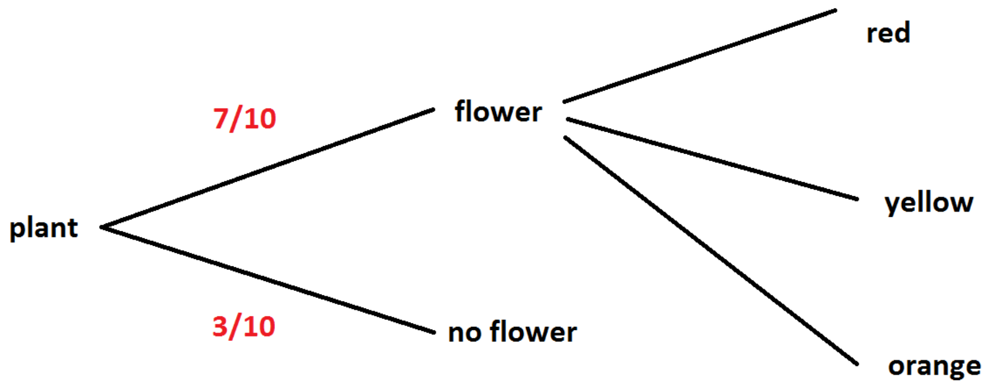
$$\frac{7}{10} + \frac{3}{10} = 1$$

$$\therefore\ p(no\ flower) = \frac{3}{10}$$



If the plant does not product a flower, there is nothing that can happen, the branch ends.

If the plant however produces a flower, the flower can have one of three different colours – red, yellow and orange. This is another branching point after the flower has been produced.



Given that the plant has produced a flower, the probability that it is red is 2/3 and the probability that it is yellow is 1/4.

$$p(red|flower) = \frac{2}{3}$$

$$p(yellow|flower) = \frac{1}{4}$$

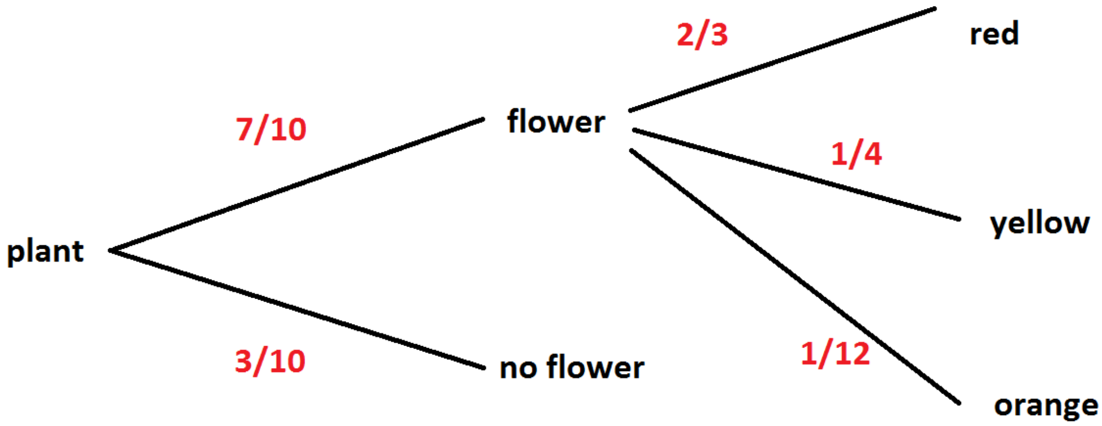
The probability for the flower to be orange (given the flower has produced a plant) can be calculated using the fact that the probabilities sum to 1 at every branching point.

$$p(red|flower) + p(yellow|flower) + p(orange|flower) = 1$$

$$\frac{2}{3} + \frac{1}{4} + p(orange|flower) = 1$$

$$p(orange|flower) = 1 - \frac{2}{3} - \frac{1}{4}$$

$$\therefore\ p(orange|flower) = \frac{1}{12}$$



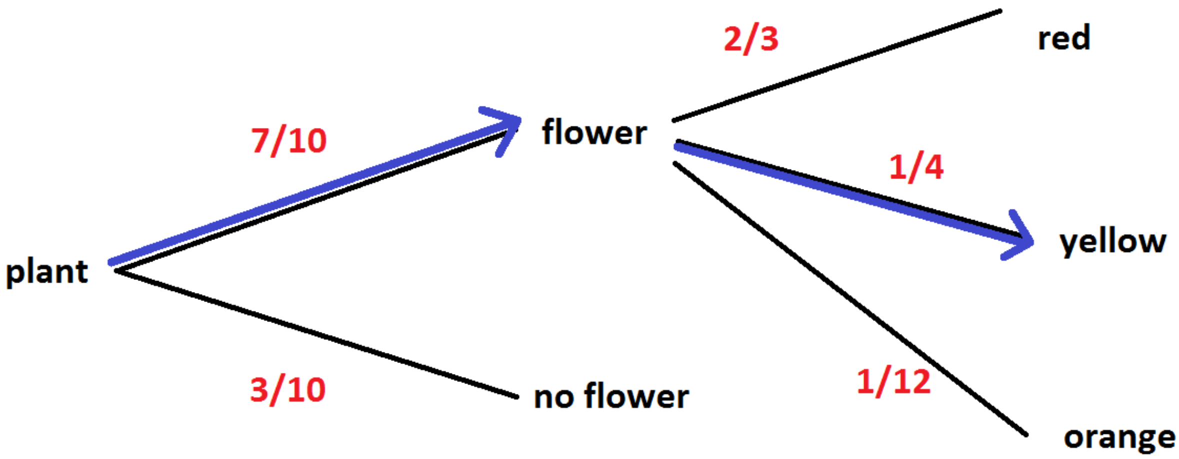
(b) A plant is chosen at random.

Find the probability that it will **not** produce a yellow flower. [3]

We want to find the probability that a randomly selected plant will not produce a yellow flower. The easiest way to find this is to find the probability that it will produce a yellow flower and then subtract from the total probability 1.

$$p(not\ yellow) = 1 - p(yellow)$$

To obtain the probability of getting a yellow flower, we follow the tree diagram to this outcome and multiply any probabilities that we encounter along the path (all these branches much be taken).



$$p(yellow) = p(flower) \times p(yellow|flower)$$

$$p(yellow) = \frac{7}{10} \times \frac{1}{4}$$

$$p(yellow) = \frac{7}{40}$$

Hence we can calculate the probability that the plant will not produce yellow flower.

$$p(not\ yellow) = 1 - p(yellow)$$

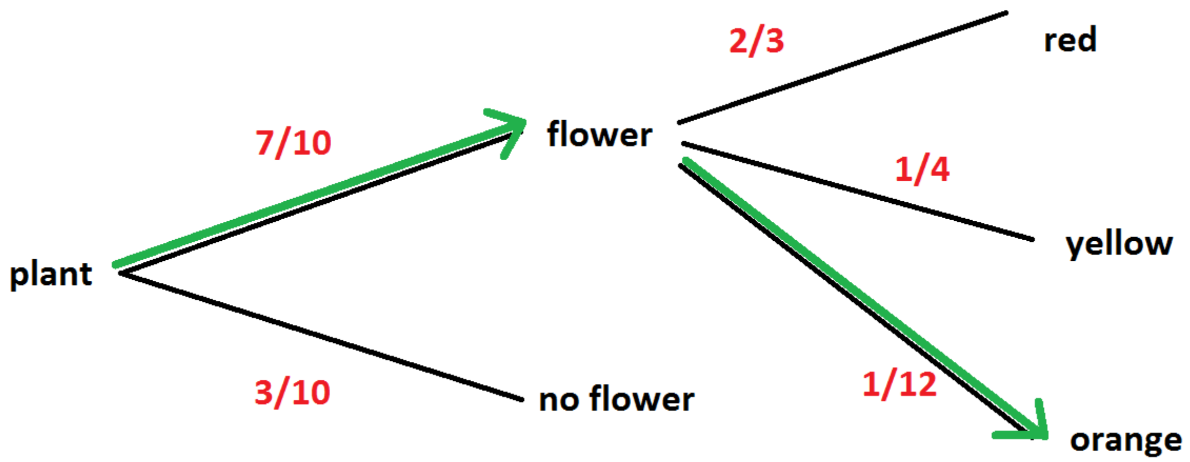
$$p(not\ yellow) = 1 - \frac{7}{40}$$

$$\boldsymbol{p(not\ yellow) = \frac{33}{40}}$$

(c) If Katrina puts 120 plants in her garden, how many orange flowers would she expect? [2]

First, we need to find the probability that a plant will produce orange flower (and then multiply by the number of plants to get the expected value).

Following a similar method as before, we follow the tree diagram (multiply probabilities):



$$p(orange) = p(flower) \times p(orange|flower)$$

$$p(yellow) = \frac{7}{10} \times \frac{1}{12}$$

$$p(yellow) = \frac{7}{120}$$

Multiply the probability by the number of plants Katrina plants to get the expected value.

$$Exp(orange) = p(orange) \times total$$

$$Exp(orange) = \frac{7}{120} \times 120$$

$$\boldsymbol{Exp(orange) = 7}$$

Question 6

Sacha either walks or cycles to school.

On any day, the probability that he walks to school is $\frac{3}{5}$.

(a) (i) A school term has 55 days.

Work out the expected number of days Sacha walks to school. [1]

$$\frac{3}{5} \times 55$$

$$= \mathbf{33}$$

(ii) Calculate the probability that Sacha walks to school on the first 5 days of the term. [2]

5 consecutive days of walking is

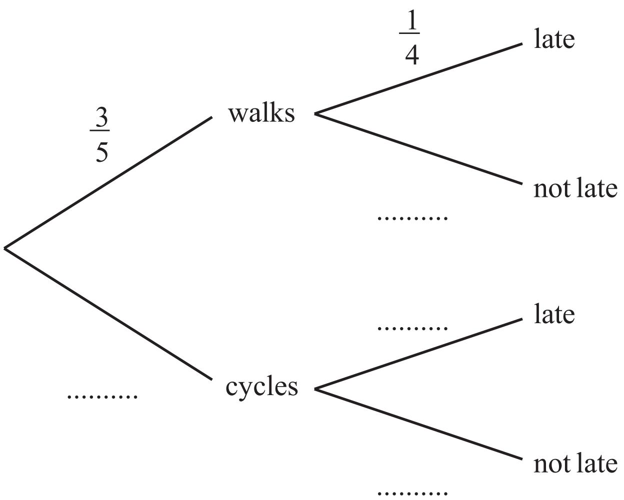
$$\left(\frac{3}{5}\right)^5 = \frac{\mathbf{243}}{\mathbf{3125}}$$

$$= \mathbf{0.07776}$$

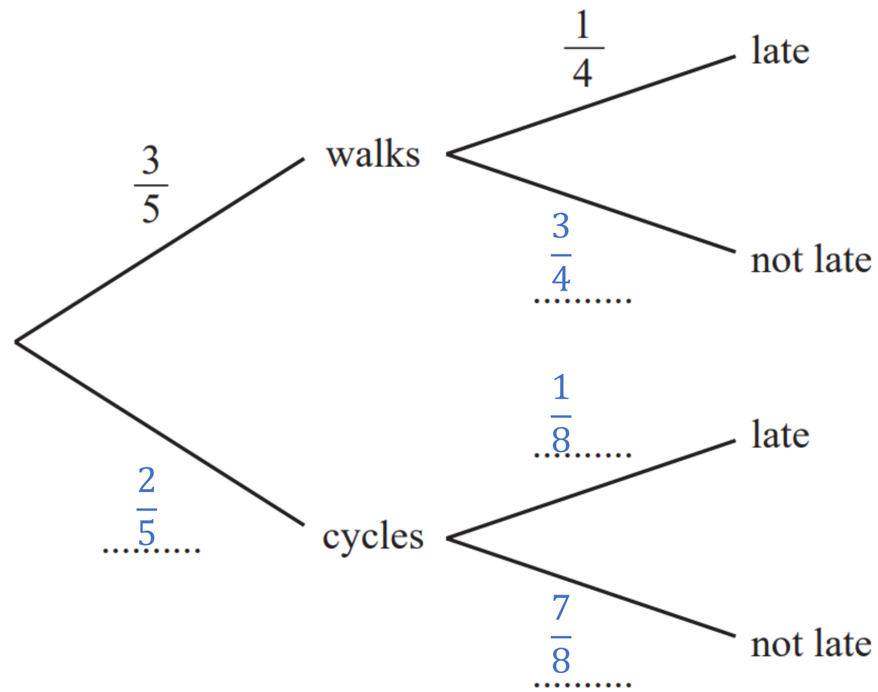
(b) When Sacha walks to school, the probability that he is late is $\frac{1}{4}$.

When he cycles to school, the probability that he is late is $\frac{1}{8}$.

(i) Complete the tree diagram by writing the probabilities in the four spaces provided.



[3]



(ii) Calculate the probability that Sacha cycles to school and is late. [2]

$$\frac{2}{5} \times \frac{1}{8} = \frac{1}{20}$$

(iii) Calculate the probability that Sacha is late to school. [2]

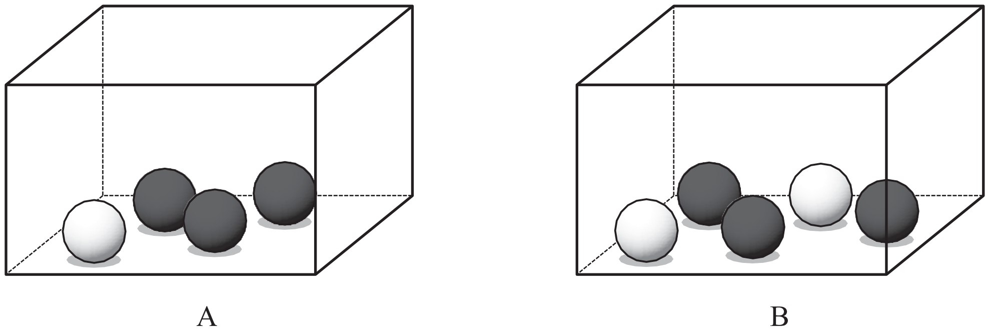
$$\frac{3}{5} \times \frac{1}{4} + \frac{1}{20}$$

$$= \frac{4}{20}$$

$$= \frac{1}{5}$$

$$= 0.2$$

Question 7



Box A contains 3 black balls and 1 white ball.
Box B contains 3 black balls and 2 white balls.

(a) A ball can be chosen at random from either box.
Complete the following statement.

There is a greater probability of choosing a white ball from Box .

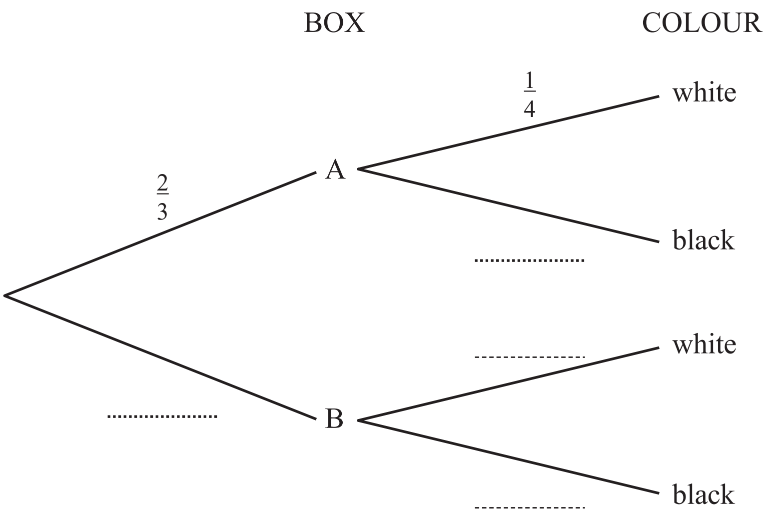
Explain your answer. [1]

B because the probability of white in B is $\frac{2}{5}$ which is larger than the probability of a white from A, $\frac{1}{4}$.

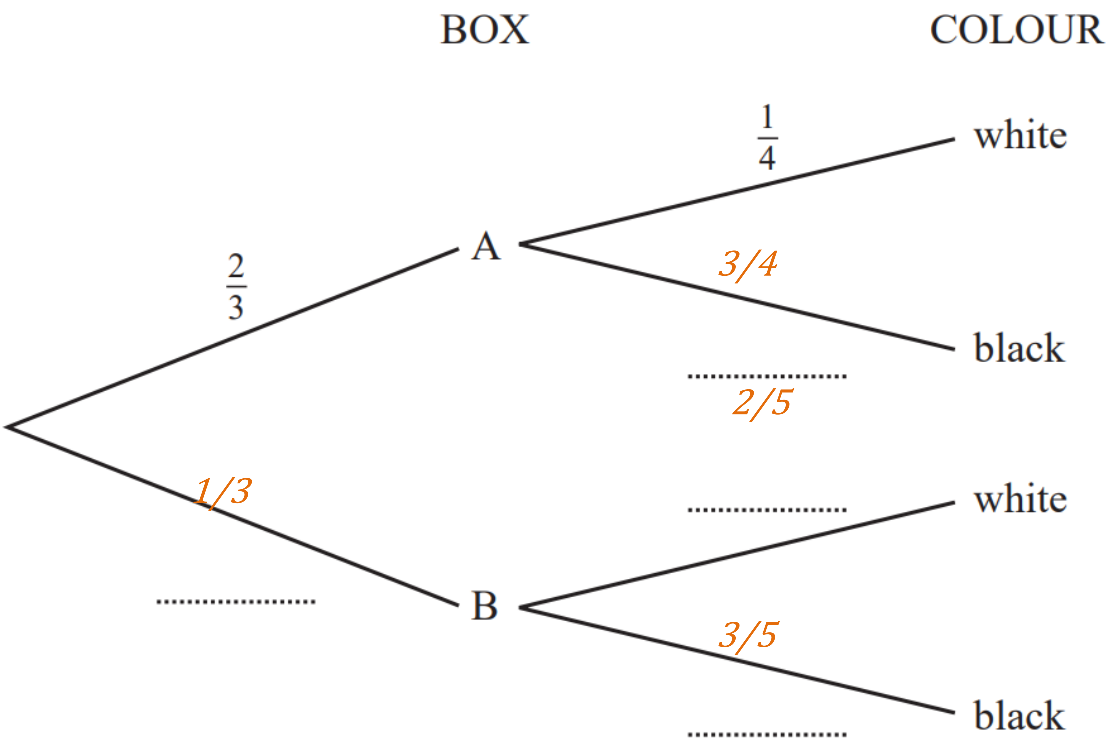
(b) Abdul chooses a box and then chooses a ball from this box at random.

The probability that he chooses box A is $\frac{2}{3}$.

(i) Complete the tree diagram by writing the four probabilities in the empty spaces.



[4]



(ii) Find the probability that Abdul chooses box A and a black ball. [2]

Multiply the probabilities together

$$\frac{2}{3} \times \frac{3}{4}$$

$$= \frac{6}{12}$$

$$= \mathbf{\frac{1}{2}}$$

(iii) Find the probability that Abdul chooses a black ball. [2]

Add the probability of a black ball from A to the probability of a black ball from B

$$\frac{2}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{3}{5}$$

$$= \frac{1}{2} + \frac{1}{5}$$

$$= \mathbf{\frac{7}{10}}$$

(c) Tatiana chooses a box and then chooses **two** balls from this box at random (without replacement).

The probability that she chooses box A is $\frac{2}{3}$.

Find the probability that Tatiana chooses two white balls. [2]

For box A we have

$$1st\ white = \frac{1}{4}$$

$$2nd\ white = 0$$

For box B we have

$$1st\ white = \frac{2}{5}$$

$$2nd\ white = \frac{1}{4}$$

Hence, probability of 2 white balls is

$$\frac{2}{3}\times\frac{1}{4}\times 0+\frac{1}{3}\times\frac{2}{5}\times\frac{1}{4}$$

$$=\frac{2}{60}$$

$$=\mathbf{\frac{1}{30}}$$

Probability
Difficulty: Medium

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Probability
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 3

Time allowed: **94 minutes**

Score: **/82**

Percentage: **/100**

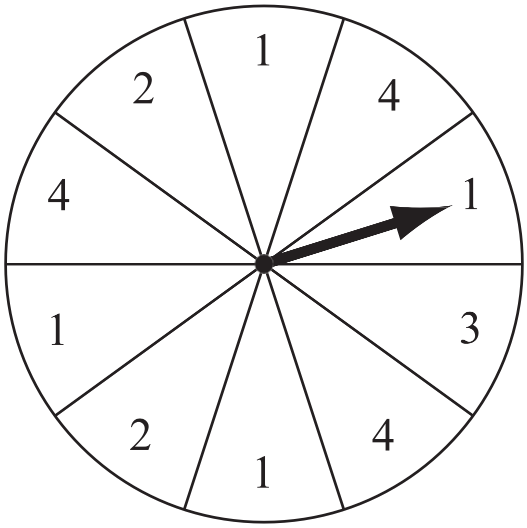
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



The diagram shows a circular board, divided into 10 numbered sectors.

When the arrow is spun it is equally likely to stop in any sector.

(a) Complete the table below which shows the probability of the arrow stopping at each number.

Number	1	2	3	4
Probability		0.2		0.3

[1]

Number	1	2	3	4
Probability	**0.4**	0.2	**0.1**	0.3

(b) The arrow is spun once.

Find

(i) the most likely number, [1]

1

(ii) the probability of a number less than 4. [1]

0.7

(c) The arrow is spun twice.

Find the probability that

(i) both numbers are 2, [1]

0.2×0.2

$= \mathbf{0.04}$

(ii) the first number is 3 and the second number is 4, [2]

0.1×0.3

$= \mathbf{0.03}$

(iii) the two numbers add up to 4. [3]

Possible combinations are

$1 \textit{ then } 3$

$3 \textit{ then } 1$

$2 \textit{ then } 2$

The probabilities are then

$0.4 \times 0.1 + 0.1 \times 0.4 + 0.2 \times 0.2$

$= 0.04 + 0.04 + 0.04$

$= \mathbf{0.12}$

(d) The arrow is spun several times until it stops at a number 4.

[2]

Find the probability that this happens on the third spin.

We need to not hit 4 in the first 2 spins and then hit 4 on the third.

This gives us a probability of

$$0.7 \times 0.7 \times 0.3$$

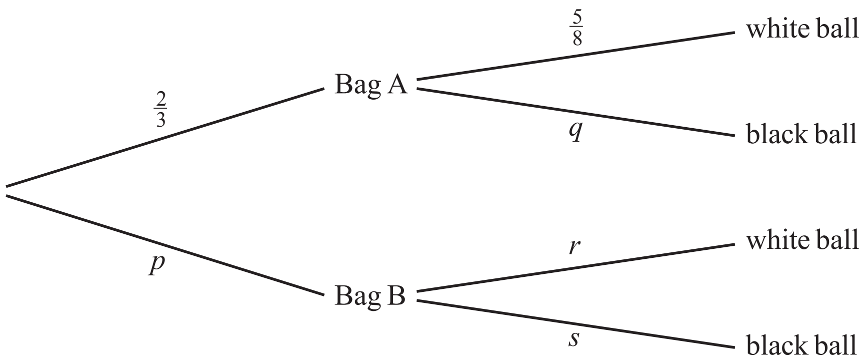
$$= \mathbf{0.147}$$

(a)

Nadia must choose a ball from Bag A or from Bag B.
The probability that she chooses Bag A is $\frac{2}{3}$.

Bag A contains 5 white and 3 black balls.
Bag B contains 6 white and 2 black balls.

The tree diagram below shows some of this information.



(i) Find the values of p, q, r and s. [3]

$$p = \frac{1}{3} \quad q = \frac{3}{8} \quad r = \frac{6}{8} \quad s = \frac{2}{8}$$

(ii) Find the probability that Nadia chooses Bag A and then a white ball. [2]

$$\frac{2}{3} \times \frac{5}{8}$$

$$= \frac{10}{24}$$

$$= \frac{5}{12}$$

(iii) Find the probability that Nadia chooses a white ball. [2]

Add the two white ball branches together

$$\frac{2}{3} \times \frac{5}{8} + \frac{1}{3} \times \frac{6}{8}$$

$$= \frac{5}{12} + \frac{3}{12}$$

$$= \mathbf{\frac{2}{3}}$$

(b) Another bag contains 7 green balls and 3 yellow balls.
Sani takes three balls out of the bag, without replacement.

(i) Find the probability that all three balls he chooses are yellow. [2]

$$\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}$$

$$= \frac{6}{720}$$

$$\mathbf{\frac{1}{120}}$$

(ii) Find the probability that at least one of the three balls he chooses is green. [1]

We take 1 minus the probability of no green balls (i.e. all yellow balls)

$$1 - \frac{1}{120}$$

$$= \mathbf{\frac{119}{120}}$$

Question 3

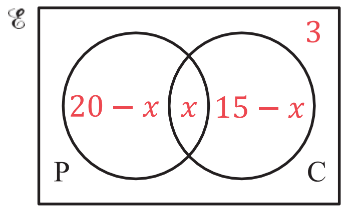
(a) There are 30 students in a class.
20 study Physics, 15 study Chemistry and 3 study neither Physics nor Chemistry.

(i) **Copy and complete** the Venn diagram to show this information. [2]

(ii) Find the number of students who study both Physics **and** Chemistry. [1]

Taking parts (i) and (ii) together:

Call the number of students who do both Physics and Chemistry x and fill in the Venn Diagram:



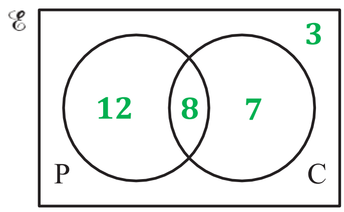
Use the total number of students to form and solve an equation to find x:

$$(20-x)+x+(15-x)+3=30$$

$$38-x=30$$

$$\boldsymbol{x=8}$$

And complete the Venn Diagram:



(iii) A student is chosen at random. Find the probability that the student studies Physics but not Chemistry. [2]

We use $\mathrm{n}(A)$ to mean the number of elements in Set A:

$$\mathrm{P}(P\cap C')=\frac{\mathrm{n}(P\cap C')}{\mathrm{n}(\mathcal{E})}$$

$$\mathrm{P}(\boldsymbol{P}\cap\boldsymbol{C}')=\frac{\mathbf{12}}{\mathbf{30}}$$

(iv) A student who studies Physics is chosen at random. Find the probability that this student does not study Chemistry. [2]

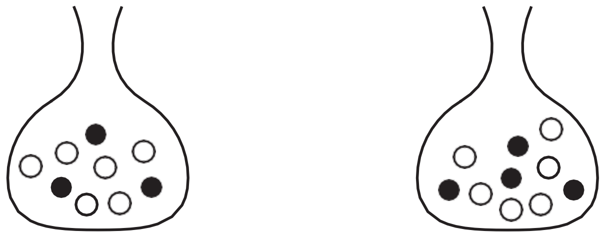
We use $\mathrm{P}(A|B)$ to mean the Probability that an element is in Set A given that

it is in Set B:

$$\mathrm{P}(C'|P)=\frac{\mathrm{n}(C'\cap P)}{\mathrm{n}(P)}$$

$$\mathrm{P}(\boldsymbol{P}\cap\boldsymbol{C}')=\frac{\mathbf{12}}{\mathbf{20}}$$

(b)



A B

Bag A contains 6 white beads and 3 black beads.
Bag B contains 6 white beads and 4 black beads.
One bead is chosen at random from each bag.
Find the probability that

In Probability:
"AND means $\times$
OR means $+$"

(i) both beads are black, [2]

$$\mathrm{P}(Black_A \text{ AND } Black_B) = \frac{3}{9} \times \frac{4}{10} = \mathbf{\frac{12}{90}}$$

(ii) at least one of the two beads is white. [2]

The quickest way to do "at least one" problems is usually to use:

$$P(At\ least\ one) = 1 - P(None)$$

In this case:

$$P(At\ least\ one) = 1 - P(Black_A \text{ AND } Black_B)$$

$$P(At\ least\ one) = 1 - \frac{12}{90}$$

$$\boldsymbol{P(At\ least\ one) = \frac{78}{90}}$$

The beads are not replaced.
A second bead is chosen at random from each bag.
Find the probability that

For non-replacement
"Reduce the top by one,
Reduce the bottom by one"

(iii) all four beads are white, [3]

$$\mathrm{P}(All\ four\ beads\ white) = \mathrm{P}(White_A \text{ AND } White_A \text{ AND } White_B \text{ AND } White_B)$$

$$\mathrm{P}(All\ four\ beads\ white) = \frac{6}{9} \times \frac{5}{8} + \frac{6}{10} \times \frac{5}{9} = \mathbf{\frac{900}{6480}}$$

(iv) the beads are not all the same colour. [3]

$$\mathrm{P}(All\ four\ not\ same\ colour) = 1 - \mathrm{P}(All\ White \text{ OR } All\ Black)$$

$$\mathrm{P}(All\ four\ not\ same\ colour) = 1 - \left(\frac{900}{6480} + \frac{3}{9} \times \frac{2}{8} + \frac{4}{10} \times \frac{3}{9}\right)$$

$$\boldsymbol{\mathrm{P}(All\ four\ not\ same\ colour) = \frac{5508}{6480}}$$

Question 4

A D A M   D A N I E L

Adam writes his name on four red cards and Daniel writes his name on six white cards.

(a) One of the ten cards is chosen at random. Find the probability that

(i) the letter on the card is **D**, [1]

Probability = $\frac{\text{nr.of favourable cases}}{\text{nr.of total cases}}$

Here, the number of favourable cases is the number of cards with the letter D and the number of total cases represents the total number of cards, 10.

P = $\frac{2}{10} = \frac{1}{5}$ **= 0.2**

(ii) the card is red, [1]

Here, the number of favourable cases is the number of red cards, 4, and the number of total cases represents the total number of cards, 10.

P = $\frac{4}{10} = \frac{2}{5}$ **= 0.4**

(iii) the card is red **or** the letter on the card is **D**, [1]

There are 4 red cards and 2 cards with the letter D, out of which one is red. This sums up 5 possible cards.

P = $\frac{5}{10} = \frac{1}{2} = \mathbf{0.5}$

(iv) the card is red **and** the letter on the card is **D**, [1]

There is only one red card with the letter D.

P = $\frac{1}{10} = 0.1$

(v) the card is red **and** the letter on the card is **N**. [1]

There are no red cards with the letter N.

P = 0

(b) Adam chooses a card at random and then Daniel chooses one of the remaining 9 cards at random.
Giving your answers as fractions, find the probability that the letters on the two cards are

(i) both **D**, [2]

The cards picked are not replaced, therefore, the number of total cards and the number of cards with the letter D on them are reduced by 1 every time a card is chosen.

The probability that the first card has the letter D:

P = $\frac{2}{10}$

The probability that the second card has the letter D:

P = $\frac{1}{9}$

The probability that both cards have the letter D is:

The probability that the first card has the letter D:

P = $\frac{2}{10}$ x $\frac{1}{9} = \frac{2}{90}$

P = $\frac{1}{45}$

(ii) both **A**, [2]

The probability that the first card has the letter D:

P = $\frac{3}{10}$

The probability that the second card has the letter D:

P = $\frac{2}{9}$

The probability that both cards have the letter D is:

The probability that the first card has the letter D:

P = $\frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$

P = $\frac{1}{15}$

(iii) the same, [2]

The only 2 letters which are found at least twice are A and D.

To have the 2 cards the same, they can be with either one of these letters:

P = $\frac{1}{45} + \frac{1}{15}$

P = $\frac{4}{45}$

(iv) different. [2]

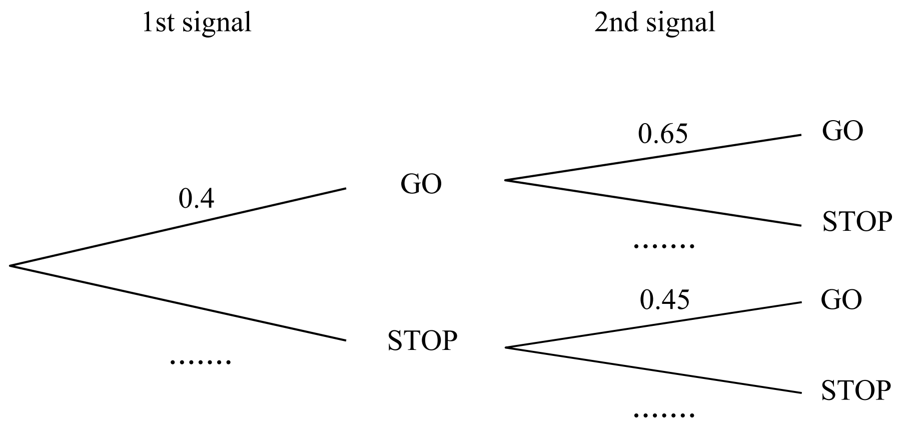
The probability that the 2 cards are different is 1 minus the probability that the 2 cards are the same.

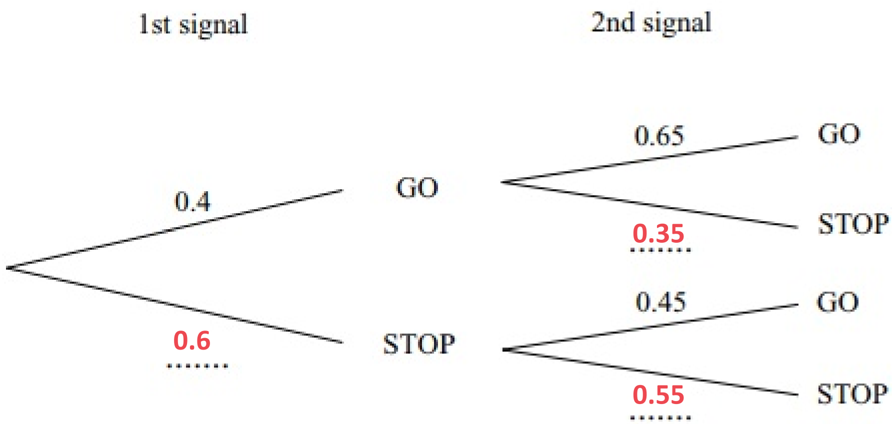
$P = 1 - \frac{4}{45}$

$\mathbf{P = \frac{41}{45}}$

There are 2 sets of road signals on the direct 12 kilometre route from Acity to Beetown.
The signals say either “GO” or “STOP”.
The probabilities that the signals are “GO” when a car arrives are shown in the tree diagram.

(a) Copy and complete the tree diagram for a car driver travelling along this route. [3]





1st signal STOP: P = 1 – 0.4 = 0.6

1st signal GO, 2nd signal STOP: P = 1 – 0.65 = 0.35

1st signal STOP, 2nd signal STOP: P = 1 – 0.45 = 0.55

(b) Find the probability that a car driver

(i) finds both signals are "GO", [2]

The 2 events need to happen simultaneously:

P = 0.4 x 0.65

P = 0.26

(ii) finds exactly one of the two signals is "GO", [3]

P = 0.4 x 0.35 + 0.6 x 0.45

P = 0.41

(iii) does not find two "STOP" signals. [2]

He can only find one STOP signal, which has the same probability as finding exactly

one GO signal. P = 0.41

or none: P = 0.26 (equivalent with the probability that both signals are GO)

P = 0.26 + 0.41

P = 0.67

(c) With no stops, Damon completes the 12 kilometre journey at an average speed of 40 kilometres per hour.

(i) Find the time taken in **minutes** for this journey. [1]

speed = distance/time

Time = 12 km/40km/h

Time = 0.3 h

We convert the time in minutes:

Time = 0.3 h x 60 minutes/h

Time = 18 minutes

(ii) When Damon has to stop at a signal it adds 3 minutes to this journey time.

Calculate his average speed, in **kilometres per hour**, if he stops at both road signals. [2]

If he stops twice, then the time will be increased by 6 minutes.

Total time = 24 minutes

We convert the time in h.

24 minutes/60 minutes/h = 0.4 h

Speed = distance/time

Speed = 12 km/0.4 h

Speed = 30 km/h

(d) Elsa takes a different route from Acity to Beetown.
This route is 15 kilometres and there are no road signals.
Elsa's average speed for this journey is 40 kilometres per hour.
Find

(i) the time taken in **minutes** for this journey, [1]

speed = distance/time

Time = 15 km/40 km/h

Time = 0.375 h

We convert the time in minutes:

Time = 0.375 h x 60 minutes/h

Time = 22.5 minutes

(ii) the probability that Damon takes more time than this on his 12 kilometrejourney. [2]

The 12 km journey will take longer than this one only if there are 2

STOP signals on the way.

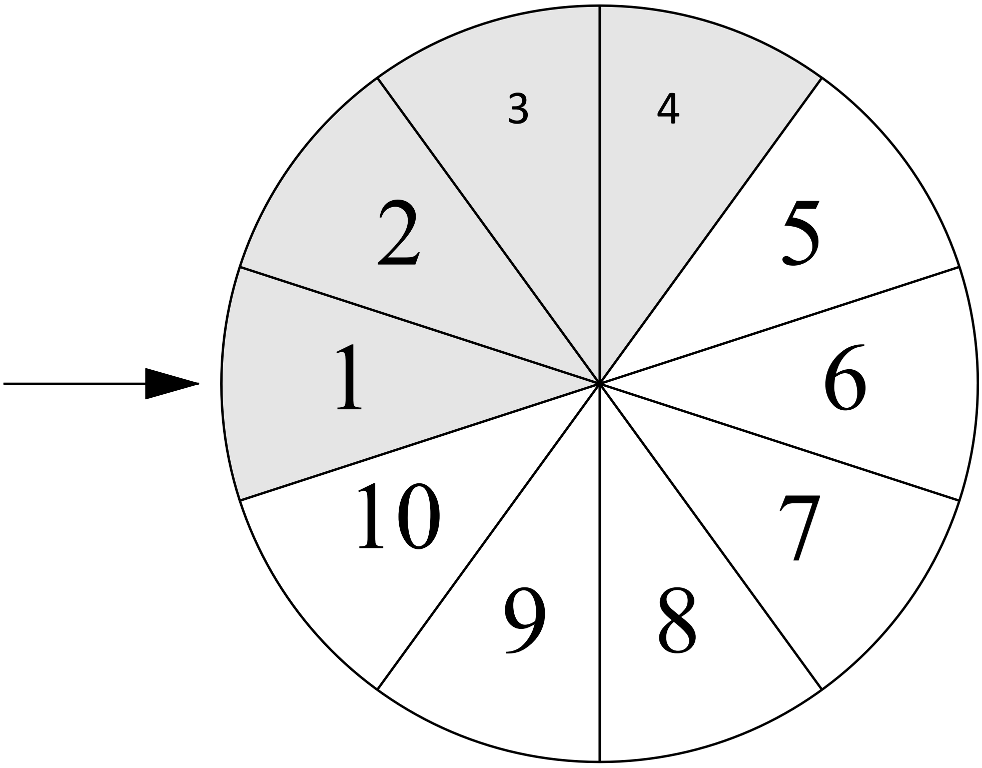
The probability that both signals are GO is:

P = 0.67

The probability that both signals are STOP is:

P = 1 – 0.67

P = 0.33



A wheel is divided into 10 sectors numbered 1 to 10 as shown in the diagram.
The sectors 1, 2, 3 and 4 are shaded.
The wheel is spun and when it stops the fixed arrow points to one of the sectors.
(Each sector is equally likely.)

(a) The wheel is spun once so that one sector is selected. Find the probability that

(i) the number in the sector is even, [1]

Probability = $\frac{\text{nr.of favourable cases}}{\text{nr.of total cases}}$

Here, the number of favourable cases is the number of sectors with an even number and the number of total cases represents the total number of sectors, 10.

$P = \frac{5}{10} = \frac{1}{2}$

(ii) the sector is shaded, [1]

There are 4 shaded sectors in total.

$P = \frac{4}{10} = \frac{2}{5}$

(iii) the number is even **or** the sector is shaded, [1]

There are 4 shaded sectors, 2 of which contain even numbers and there are 5 even number in total. The number of possible sectors is 7.

P = $\frac{7}{10}$

(iv) the number is odd **and** the sector is shaded. [1]

There are 4 shaded sectors, 2 of which contain odd numbers.

P = $\frac{2}{10} = \frac{1}{5}$

(b) The wheel is spun twice so that each time a sector is selected. Find the probability that

(i) both sectors are shaded, [2]

The number of total sectors remains the same after each selection, therefore:

The probability that the 1st selection gives a shaded sector is: P = $\frac{4}{10}$ and the probability that the 2nd selection gives a shaded sector is: P = $\frac{4}{10}$

The probability that both are shaded is: **P = 0.4 x 0.4 = 0.16**

(ii) one sector is shaded and one is not, [2]

The probability that a sector is shaded is: P = $\frac{4}{10}$ = 0.4 and the probability that the sector is not shaded is: P = $\frac{6}{10}$ = 0.6.

The probability that only one of the selections are a shaded sector is:

P = 0.4 x 0.6 x 2 = 0.48

(iii) the sum of the numbers in the two sectors is greater than 20, [2]

The maximum sum of 2 numbers from the ones on the sectors is: 10 + 10 = 20 for the case in which both selections show number 10. There are no numbers higher, therefore, the probability that the sum is greater than 20 is **P = 0.**

(iv) the sum of the numbers in the two sectors is less than 4, [2]

The possibilities for a sum less than 4 is: 1 + 1 = 2, 1 + 2 = 3, 2 + 1 = 3.

There are 3 possibilities for a sum less than 4 and the total number of possibilities is 10 x 10 = 100.

$$\mathbf{P = \frac{3}{100} = 0.03}$$

(v) the product of the numbers in the two sectors is a square number. [3]

The possibilities for the product to be a square number are:

1 x 1 = 1, 2 x 2 = 4, 3 x 3 = 9 … 10 x 10 = 100 – 10 possibilities

1 x 4 = 4, 1 x 9 = 9, 2 x 8 = 16, 9 x 4 = 36 – 4 possibilities

4 x 1 = 4, 9 x 1 = 9, 8 x 2 = 16, 4 x 9 = 36 – 4 possibilities

$$\mathbf{P = \frac{3}{100} = 0.18}$$

Probability
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Probability
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **83 minutes**

Score: **/72**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a) A bag contains red beads and green beads.
There are 80 beads altogether.
The probability that a bead chosen at random is green is 0.35 .

(i) Find the number of red beads in the bag. [2]

Probability of red is

$$1 - 0.35$$

$$= 0.65$$

So, number of reds is

$$0.65 \times 80$$

$$= \mathbf{52}$$

(ii) Marcos chooses a bead at random and replaces it in the bag.
He does this 240 times.

Find the number of times he would expect to choose a green bead. [1]

$$240 \times 0.35$$

$$= \mathbf{84}$$

(b) A different bag contains 2 blue marbles, 3 yellow marbles and 4 white marbles.
Huma chooses a marble at random, notes the colour, then replaces it in the bag.
She does this three times.

Find the probability that

(i) all three marbles are yellow, [2]

$$\left(\frac{3}{9}\right)^3$$

$$= \frac{27}{729} = \mathbf{\frac{1}{27}}$$

(ii) all three marbles are different colours. [3]

Multiply the probability of each colour times the number of ways of ordering the 3 colours, which is denoted as

$$3! = 3 \times 2 \times 1$$

Hence

$$3! \times \frac{2}{9} \times \frac{3}{9} \times \frac{4}{9}$$
$$= \frac{144}{729}$$
$$= \mathbf{\frac{16}{81}}$$

(c) Another bag contains 2 green counters and 3 pink counters.
Teresa chooses three counters at random **without** replacement.

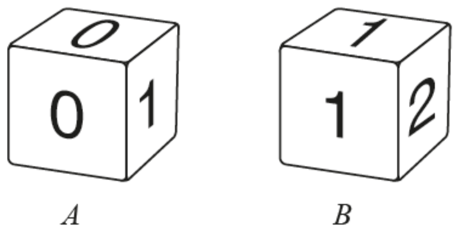
Find the probability that she chooses more pink counters than green counters. [4]

Probability of 2 pink counters and 1 green plus the probability of 3 pink counters.

$$P(2p\ 1g) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times 3$$
$$= \frac{12}{20}$$
$$= \frac{6}{10}$$
$$P(3p) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}$$
$$= \frac{6}{60}$$
$$= \frac{1}{10}$$

Hence

$$\boldsymbol{P} = \mathbf{\frac{7}{10}}$$



The diagram shows two fair dice.
The numbers on dice A are 0, 0, 1, 1, 1, 3.
The numbers on dice B are 1, 1, 2, 2, 2, 3.
When a dice is rolled, the score is the number on the top face.

(a) Dice A is rolled once.

Find the probability that the score is not 3. [1]

On Dice A, there are 5 ways of rolling "not 3" and 6 outcomes in total:

$$\boldsymbol{P(not\ 3) = \frac{5}{6}}$$

(b) Dice A is rolled twice.

Find the probability that the score is 0 both times. [2]

In probability "and" means "×", "or" means "+".

On Dice A, there are 2 ways of rolling 0 and 6 outcomes in total:

$$P(0\ and\ 0) = \frac{2}{6} \times \frac{2}{6}$$

$$\boldsymbol{P(0\ and\ 0) = \frac{4}{36} = \frac{1}{9}}$$

(c) Dice A is rolled 60 times.

Calculate an estimate of the number of times the score is 0. [1]

The number of times an event E might happen in n trials is

$$n(E) = n \times P(E)$$

On Dice A, there are 2 ways of rolling 0 and 6 outcomes in total:

$$n(0) = 60 \times \frac{2}{6}$$

$$\boldsymbol{n(0) = 20}$$

(d) Dice A and dice B are each rolled once.
The product of the scores is recorded.

(i) Complete the possibility diagram.

For each space on the diagram multiply the number on Dice A by the Number on Dice B

Dice B \ Dice A	0	0	1	1	1	3
3	0	0	**3**	**3**	**3**	**9**
2	0	0	**2**	**2**	**2**	**6**
2	0	0	**2**	**2**	**2**	**6**
2	0	0	**2**	**2**	**2**	**6**
1	0	0	**1**	**1**	**1**	**3**
1	0	0	1	1	1	3

[2]

(ii) Find the probability that the product of the scores is

(a) 2, [1]

There are 9 ways of getting a product of 2 and 36 outcomes in total:

$$P(2) = \frac{9}{36} = \frac{1}{4}$$

(b) greater than 3. [1]

There are 4 ways of getting a product of more than 3 (ie 4 or more) and 36 outcomes in total:

$$P(>3) = \frac{4}{36} = \frac{1}{9}$$

(e) Eva keeps rolling dice B until 1 is scored.

Find the probability that this happens on the 5th roll. [2]

In probability "and" means "×", "or" means "+".

$$P(not\ 1\ and\ not\ 1\ and\ not\ 1\ and\ not\ 1\ and\ 1) = \frac{4}{6} \times \frac{4}{6} \times \frac{4}{6} \times \frac{4}{6} \times \frac{2}{6}$$

$$P(first\ 1\ on\ the\ 5th\ roll) = \frac{16}{243}$$

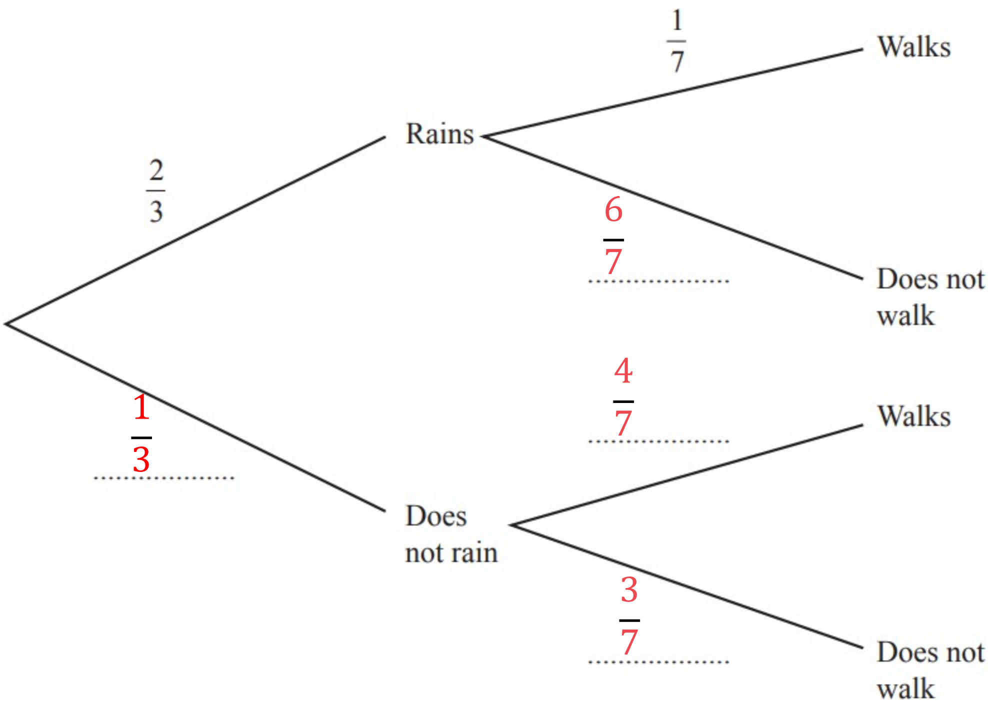
Question 3

Each morning the probability that it rains is $\frac{2}{3}$.

If it rains, the probability that Asha walks to school is $\frac{1}{7}$.

If it does not rain, the probability that Asha walks to school is $\frac{4}{7}$.

(a) Complete the tree diagram.



[2]

(b) Find the probability that it rains and Asha walks to school. [2]

Multiply the probabilities together

$$\frac{2}{3} \times \frac{1}{7}$$

$$= \mathbf{\frac{2}{21}}$$

(c) **(i)** Find the probability that Asha does not walk to school. [3]

Add the two `does not walk to school` branches together

$$\frac{2}{3} \times \frac{6}{7} + \frac{1}{3} \times \frac{3}{7}$$

$$= \frac{12}{21} + \frac{3}{21}$$

$$= \frac{15}{21}$$

$$= \mathbf{\frac{5}{7}}$$

(ii) Find the expected number of days Asha does not walk to school in a term of 70 days. [2]

Multiply probability of not walking by number of days

$$E(x) = \frac{5}{7} \times 70$$

$$= \mathbf{50}$$

(d) Find the probability that it rains on exactly one morning in a school week of 5 days. [2]

We need to multiply the probability of not raining on 4 days with raining on one day and multiply by 5 since any one of the 5 days could not have rain

$$\left(\frac{1}{3}\right)^4 \times \frac{2}{3} \times 5$$

$$= \frac{\mathbf{10}}{\mathbf{243}}$$

Ravi spins a biased 5-sided spinner, numbered 1 to 5.
The probability of each number is shown in the table.

Number	1	2	3	4	5
Probability	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	x	x

(a) Find the value of x. [3]

The total probability has to add up to 1, so we can say that:

$$\frac{1}{6}+\frac{1}{4}+\frac{1}{3}+x+x=1$$

$$\frac{3}{4}+2x=1$$

$$2x=\frac{1}{4}$$

$$\boldsymbol{x=\frac{1}{8}}$$

(b) Ravi spins the spinner once.

Find the probability that the number is 2 or 3. [2]

The probability of either a 2 or a 3 is:

$$P(2\ or\ 3)=\frac{1}{4}+\frac{1}{3}$$

$$\boldsymbol{P(2\ or\ 3)=\frac{7}{12}}$$

(c) Ravi spins the spinner twice.

Find the probability that

(i) the number is 2 both times, [2]

The probability of both spins being a 2 is:

$$P(2\ and\ 2) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

(ii) the sum of the numbers is 3. [3]

The probability of the total sum being 3 means that we either obtain (2 then 1) , or (1 then 2). As these are independent of each other, we can say that:

$$P(2\ then\ 1) + P(1\ then\ 2) = 2 \times P(1\ and\ 2)$$

$$P(2\ then\ 1) + P(1\ then\ 2) = 2 \times \frac{1}{6} \times \frac{1}{4}$$

$$\boldsymbol{P(2\ then\ 1) + P(1\ then\ 2) = \frac{1}{12}}$$

(d) Ravi spins the spinner 72 times.

Calculate how many times he expects the number 1. [1]

Spinning 72 times, we would expect 1 to come up one sixth of the times:

$$N = \frac{1}{6} \times 72$$

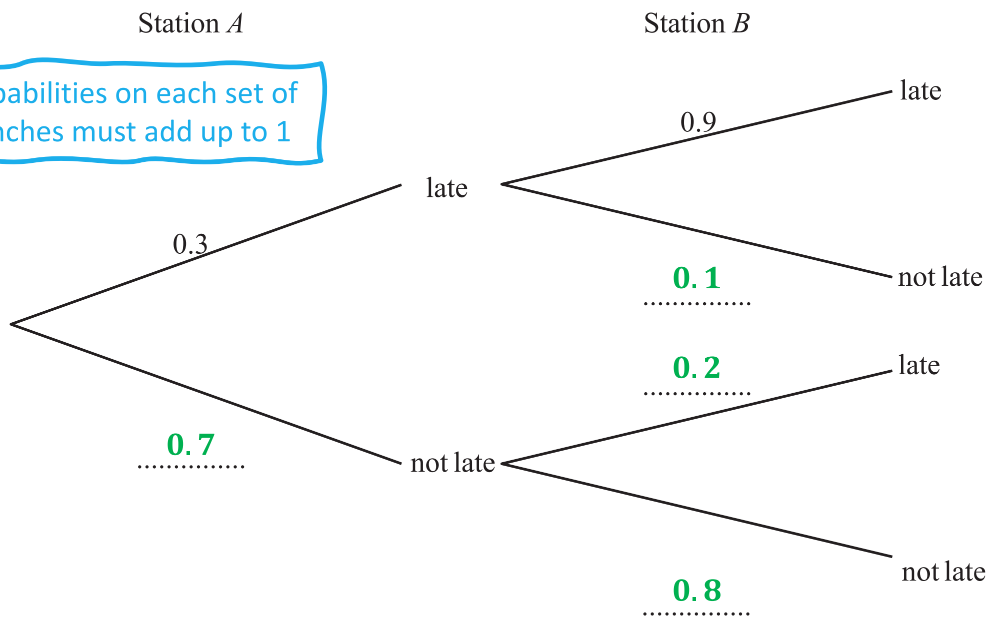
$$\boldsymbol{N = 12}$$

Question 5

A train stops at station A and then at station B.
If the train is late at station A, the probability that it is late at station B is 0.9 .
If the train is not late at station A, the probability that it is late at station B is 0.2 .
The probability that the train is late at station A is 0.3 .

(a) Complete the tree diagram. [2]

Probabilities on each set of branches must add up to 1



(b) **(i)** Find the probability that the train is late at one or both of the stations. [3]

It is often quicker to use the complement of an event and subtract from 1:

$$\mathrm{P}(\textit{Late at least one station}) = 1 - \mathrm{P}(\textit{Not Late at either station})$$

$$\mathrm{P}(\textit{Late at least one station}) = 1 - \mathrm{P}(\textit{Not Late } \mathrm{AND} \textit{ Not Late})$$

$$\mathrm{P}(\textit{Late at least one station}) = 1 - 0.7 \times 0.8$$

$$\mathrm{P}(\textbf{\textit{Late at least one station}}) = \mathbf{0.44}$$

In Probability:
"AND means ×
OR means +"

(ii) This train makes 250 journeys.

Find the number of journeys that the train is expected to be late at one or both of the stations. [1]

$$\textit{Expected Value} = \textit{Number of Trials} \times \textit{Probability}$$

$$\textit{Expected Value} = 250 \times 0.44$$

$$\textbf{\textit{Expected Value}} = \mathbf{110}$$

(c) The train continues to station C.
The probability that it is late at all 3 stations is 0.27 .

Describe briefly what this probability shows. [1]

Since $\mathrm{P}(Late\ at\ A\ \text{AND}\ Late\ at\ B) = 0.3 \times 0.9 = 0.27$

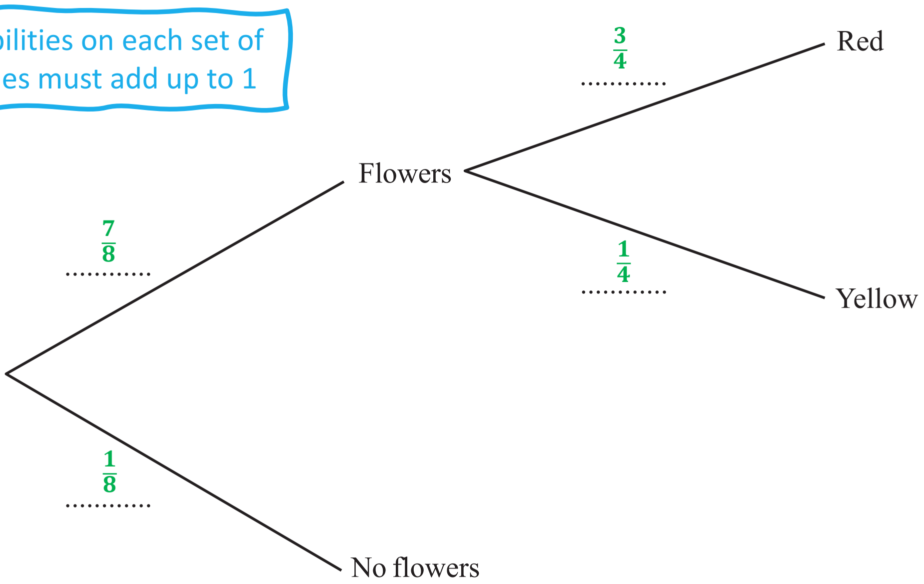
and $\mathrm{P}(Late\ at\ A\ \text{AND}\ Late\ at\ B\ \text{AND}\ Late\ at\ C) = 0.27$

that means **if the train is late at the first two stations it is certain to be late at the third.**

The probability that a plant will produce flowers is $\frac{7}{8}$.
The flowers are either red or yellow.
If the plant produces flowers, the probability that the flowers are red is $\frac{3}{4}$.

(a) **(i)** Complete the tree diagram by writing a probability beside each branch. [2]

Probabilities on each set of branches must add up to 1



(ii) Calculate the probability that a plant, chosen at random, will produce red flowers. [2]

$$P(Flowers \text{ AND } Red) = \frac{7}{8} \times \frac{3}{4}$$

$$P(Flowers \text{ AND } Red) = \frac{21}{32}$$

In Probability:
"AND means ×
OR means +"

(iii) Two plants are chosen at random.

Calculate the probability that both will produce red flowers. [2]

$$\text{P}(Red \text{ AND } Red) = \frac{21}{32} \times \frac{21}{32}$$

$$\text{P}(\boldsymbol{Both\ Red}) = \frac{\mathbf{441}}{\mathbf{1024}}$$

(b) Alphonse buys 200 of these plants.

Calculate the number of plants that are expected to produce flowers. [2]

$$Expected\ Value = Number\ of\ Trials\ \times\ Probability$$

$$Expected\ Number\ of\ Plants\ with\ Flowers = 200 \times \frac{7}{8}$$

$$\boldsymbol{Expected\ Number of\ Plants\ with\ Flowers = 175}$$

(c) Gabriel has 1575 plants with red flowers.

Estimate the total number of plants that Gabriel has. [2]

$$Expected\ Value = Number\ of\ Trials\ \times\ Probability$$

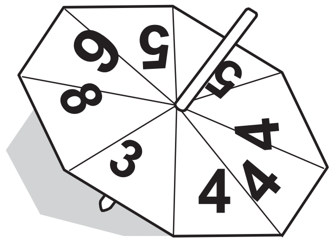
$$1575 = Number\ of\ Plants\ \times\ \frac{21}{32}$$

Multiply by $\frac{32}{21}$

$$1575 \times \frac{32}{21} = Number\ of\ Plants$$

$$\boldsymbol{Number of\ Plants = 2400}$$

Sandra has a fair eight-sided spinner.
The numbers on the spinner are 3, 4, 4, 4, 5, 5, 6 and 8.
Sandra spins the spinner twice and records each number it lands on.



In Probability:
"AND means ×
OR means +"

Find the probability that

(a) both numbers are 8, [2]

$$P(8\ AND\ 8) = \frac{1}{8} \times \frac{1}{8}$$

$$\mathbf{P(8\ AND\ 8) = \frac{1}{64}}$$

(b) the two numbers are not both 8, [1]

$$P(Not\ both\ 8) = 1 - P(both\ 8)$$

$$P(Not\ both\ 8) = 1 - \frac{1}{64}$$

$$\mathbf{P(Not\ both\ 8) = \frac{63}{64}}$$

(c) one number is odd and one number is even, [2]

There are two different orders in which this can happen:

$$P(One\ odd, one\ even) = P(Odd\ AND\ Even\ OR\ Even\ AND\ Odd)$$

$$P(One\ odd, one\ even) = \frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{3}{8}$$

$$\mathbf{P(One\ odd, one\ even) = \frac{30}{64}}$$

(d) the total of the two numbers is at least 13, [3]

To get a total of at least 13 she must spin an 8 and a 5 or 6 (there are two different orders in which this can happen) OR two 8s:

$$\mathrm{P}(Total \geq 13) = \mathrm{P}(8\ AND\ 5\ or\ 6\ OR\ 5\ or\ 6\ AND\ 8\ OR\ 8\ AND\ 8)$$

$$\mathrm{P}(Total \geq 13) = \frac{1}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{1}{8} + \frac{1}{8} \times \frac{1}{8}$$

$$\mathbf{P}(\boldsymbol{Total} \geq \mathbf{13}) = \frac{\mathbf{7}}{\mathbf{64}}$$

(e) the second number is bigger than the first number.

[3]

To get a total of at least 13 she must spin an 8 and a 5 or 6 (there are two different orders in which this can happen) OR two 8s:

$$\mathrm{P}(Second\ Number > First\ Number) = \mathrm{P}(8\ AND < 8\ OR\ 6\ AND < 6\ OR\ 5\ AND < 5\ OR\ 4\ AND\ 3)$$

$$\mathrm{P}(Second\ Number > First\ Number) = \frac{1}{8} \times \frac{7}{8} + \frac{1}{8} \times \frac{6}{8} + \frac{2}{8} \times \frac{4}{8} + \frac{3}{8} \times \frac{1}{8}$$

$$\mathbf{P}(\boldsymbol{Second\ Number} > \boldsymbol{First\ Number}) = \frac{\mathbf{24}}{\mathbf{64}}$$

Probability
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Probability
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **90 minutes**

Score: **/78**

Percentage: **/100**

Grade Boundaries:

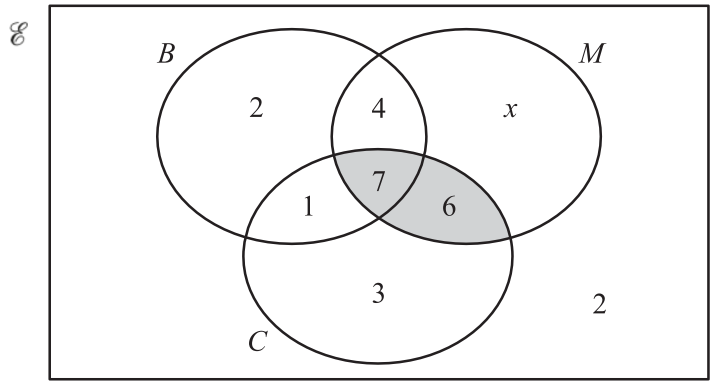
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

30 students were asked if they had a bicycle (B), a mobile phone (M) and a computer (C).
The results are shown in the Venn diagram.



(a) Work out the value of x. [1]

There are 30 students in total. Therefore the sum of all numbers in the diagram must be 30.

$$30 = 2 + 4 + 7 + 1 + x + 6 + 3 + 2$$

$$30 = 25 + x$$

Subtract 25 from both sides of the equation.

$$\boldsymbol{x = 5}$$

(b) Use set notation to describe the shaded region in the Venn diagram. [1]

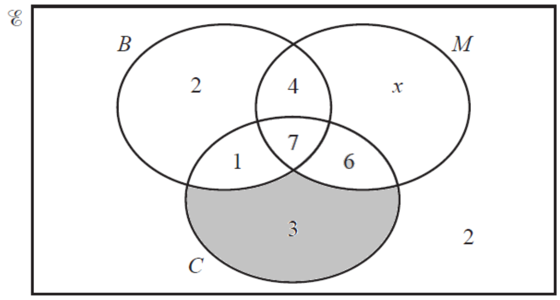
The shaded region is the intersection of set C and set M.

$$\boldsymbol{C \cap M}$$

(c) Find $n(C \cap (M \cup B)')$. [1]

We are looking for a region, which is the intersection of C and the region, which is not in M or B. This is the region, which belongs to C only.

$$n(C \cap (M \cup B)') = \mathbf{3}$$



(d) A student is chosen at random.

(i) Write down the probability that the student is a member of the set M [1]

Begin a member of M' means not being one of those in set M.

Total number of students in set M: n(M)=4+7+6+5=22.

Therefore there are 8 students who are members of M' (=30-22).

To find the probability of being in M' divide the number of students in this set by the total number of students (30 students).

$$\mathbf{\frac{8}{30}}$$

(ii) Write down the probability that the student has a bicycle. [1]

The probability of having a bicycle is the same as being a member of set B.

There are 14 students in set B (=2+1+7+4).

The probability of having a bicycle:

$$\frac{\mathbf{14}}{\mathbf{30}}$$

(e) Two students are chosen at random from the students who have computers.

Find the probability that each of these students has a mobile phone but no bicycle. [3]

Having a computer and mobile phone, but no bicycle is the same as being a member of set $C \cap M \cap B'$. There are 6 students who are members of this set.

When choosing the first students there are 17 students to choose from (those who have computer) and 6 of those who are members of the set $C \cap M \cap B'$. The probability of picking one of these students is therefore:$\frac{6}{17}$

When picking the second student, there are only 16 students left and 5 are members of the set $C \cap M \cap B'$. The probability of picking the right second student is therefore:$\frac{5}{16}$

Multiply the probabilities to get the probability that each of these students (who have a computer) has a mobile phone but no bicycle.

$$\frac{6}{17} \times \frac{5}{16}$$

$$= \frac{\mathbf{30}}{\mathbf{272}}$$

Gareth has 8 sweets in a bag.
4 sweets are orange flavoured, 3 are lemon flavoured and 1 is strawberry flavoured.

(a) He chooses two of the sweets at random.

Find the probability that the two sweets have different flavours. [4]

It is easier to find the probability of picking two sweets of the same flavour and then subtracting that probability from 1 (=100%).

We have to keep in mind that sweets are not replaces, so the number of sweets in a bag decreases between picks.

Picking two orange

In the first step, he has 8 sweets to choose from and 4 are orange. In the second step, he has 7 to choose from and 3 of them are orange (one orange was already picked).

$$\frac{4}{8} \times \frac{3}{7}$$

$$= \frac{12}{56}$$

- **Picking two lemon**

  In the first step, he has 8 sweets to choose from and 3 of those are lemon flavoured. In the second step, he has 7 to choose from and 2 of them are lemon (one already chosen).

$$\frac{3}{8} \times \frac{2}{7}$$

$$= \frac{6}{56}$$

- **Picking two strawberry flavoured**

  Not possible since there is only one.

Add the probabilities together to get the probability of picking two sweets of the same flavour.

$$\frac{12}{56}+\frac{6}{56}=\frac{18}{56}$$

Subtract this probability from 1 to get the probability that two sweets have different flavours.

$$different\ flavours = 1-\frac{18}{56}$$

$$= \mathbf{\frac{38}{56}}$$

(b) Gareth now chooses a third sweet.

Find the probability that **none** of the three sweets is lemon flavoured. [2]

In the first step, there are 8 sweets in total and 5 are not lemon flavoured.

$$probability = \frac{5}{8}$$

In the first step, there are 7 sweets in total and 4 are not lemon flavoured (one already picked).

$$probability = \frac{4}{7}$$

In the first step, there are 6 sweets in total and 3 are not lemon flavoured (two already picked).

$$probability = \frac{3}{6}$$

Multiply the probabilities to find out the probability that none is lemon flavoured:

$$\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}=\mathbf{\frac{60}{336}}$$

$$= \mathbf{\frac{5}{28}}$$

Kenwyn plays a board game.
Two cubes (dice) each have faces numbered 1, 2, 3, 4, 5 and 6.
In the game, a **throw** is rolling the **two** fair 6-sided dice and then adding the numbers on their top faces.
This total is the number of spaces to move on the board.
For example, if the numbers are 4 and 3, he moves 7 spaces.

(a) Giving each of your answers as a fraction in its simplest form, find the probability that he moves

(i) two spaces with his next throw, [2]

To move two spaces, he must score 1 on both die{

$$\frac{1}{6} \times \frac{1}{6}$$

$$= \mathbf{\frac{1}{36}}$$

(ii) ten spaces with his next throw. [3]

There are 3 ways of making 10 with two dice:

$$(4, 6), (6, 4), (5, 5)$$

Hence:

$$3 \times \left(\frac{1}{6} \times \frac{1}{6}\right)$$

$$= \frac{3}{36}$$

$$= \mathbf{\frac{1}{12}}$$

(b) What is the most likely number of spaces that Kenwyn will move with his next throw?
Explain your answer. [2]

7 because **it has the greatest number of combinations.**

(c)

95	96	97	98	99	100
				Go back 3 spaces	WIN

To win the game he must move **exactly** to the 100th space.
Kenwyn is on the 97th space.
If his next throw takes him to 99, he has to move back to 96.
If his next throw takes him over 100, he stays on 97.

Find the probability that he reaches 100 in either of his next two throws. [5]

Probability of throwing a 3 (and hence finishing on the first throw) is:

$$2 \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{2}{36}$$

$$= \frac{1}{18}$$

He could succeed on his second throw by first throwing above 3 with a probability of:

$$1 - \frac{2}{36} - \frac{1}{36}$$

$$= \frac{33}{36}$$

$$= \frac{11}{12}$$

and then throwing a 3 with his second throw giving us:

$$\frac{11}{12} \times \frac{1}{18}$$

$$= \frac{11}{216}$$

Or he could throw a 2 with probability:

$$\frac{1}{36}$$

and then throw a 4 with probability:

$$3 \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{12}$$

Giving us:

$$\frac{1}{36} \times \frac{1}{12}$$

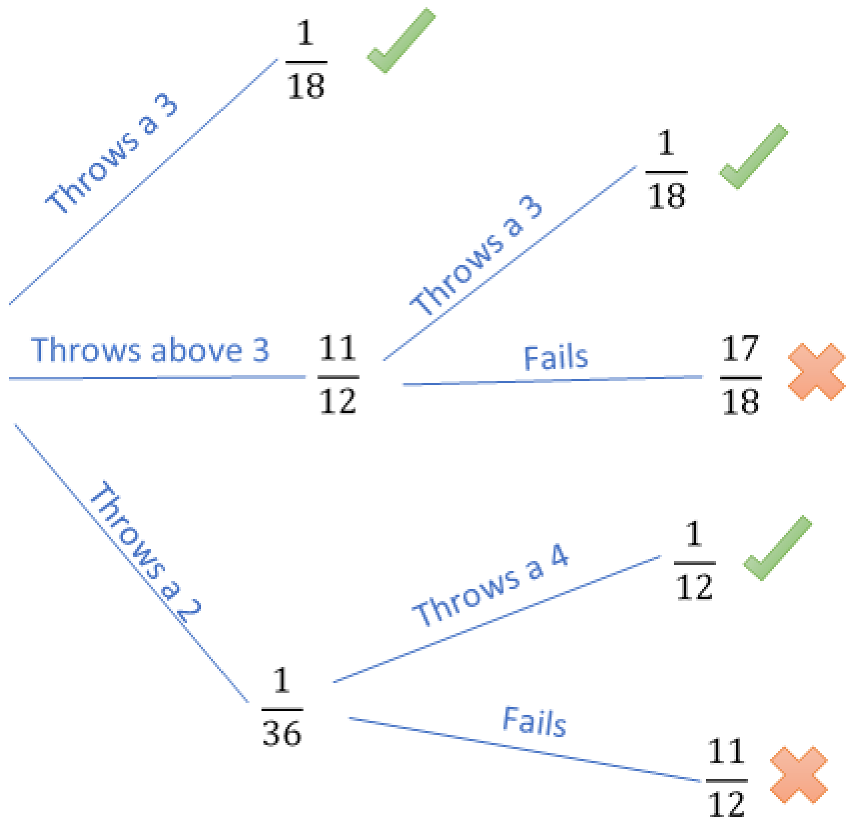
$$= \frac{1}{432}$$

Adding these together:

$$\frac{1}{18} + \frac{1}{432} + \frac{11}{216}$$

$$= \mathbf{\frac{47}{432}}$$

Visualise using a tree diagram:



E	N	L	A	R	G	E	M	E	N	T

Prettie picks a card at random from the 11 cards above and does not replace it.
She then picks a second card at random and does not replace it.

(a) Find the probability that she picks

(i) the letter L and then the letter G, [2]

There are a total of 11 letters.

There is only 1 L amongst all 11 letters, the probability she picks it is:

$$P(L) = \frac{1}{11}$$

Now that L has been picked, there are 10 letters left. The probability G is picked (G is the only letter in the remaining cards):

$$P(G) = \frac{1}{10}$$

Now, combining them as she picked them in sequence (L then G), the probability is:

$$P(L\ then\ G) = \frac{1}{11} \times \frac{1}{10}$$

$$= \mathbf{\frac{1}{110}}$$

(ii) the letter E twice, [2]

There are a total of 3 Es in the 11 cards. She picks E twice, note there that the total number of Es remaining will reduce every time she picks it:

$$P(E\ then\ E) = \frac{3}{11} \times \frac{2}{10}$$

$$= \mathbf{\frac{3}{55}}$$

(iii) two letters that are the same. [2]

The key here is to first note that only 2 letters appear more than once in this word. The letters are E and N. We already have the calculations for probability for E. Now compute for N.

$$P(N\ and\ then\ N) = \frac{2}{11} \times \frac{1}{10}$$

$$= \frac{1}{55}$$

To find probability for both E and N, sum them up:

$$P(E\ and\ E\ or\ N\ and\ N) = \frac{3}{55} + \frac{1}{55}$$

$$= \mathbf{\frac{4}{55}}$$

(b) Prettie now picks a third card at random.

Find the probability that the three letters

(i) are all the same, [2]

The only case is for E, as there are 3 Es only. No other letter has 3 of the same kind.

$$P(E, E, E) = \frac{3}{11} \times \frac{2}{10} \times \frac{1}{9}$$

$$= \mathbf{\frac{1}{165}}$$

(ii) **do not** include a letter E, [2]

Now group all non-Es into one group, and Es into another group. We have:

8 non-Es

$$P(non\ E, non\ E, non\ E) = \frac{8}{11} \times \frac{7}{10} \times \frac{6}{9}$$

$$= \mathbf{\frac{56}{165}}$$

(iii) include exactly two letters that are the same. [5]

Recall that only E and N can repeat:

$$P(2\ letters\ same) = 3\left(\frac{3}{11} \times \frac{2}{10} \times \frac{8}{9}\right) + 3\left(\frac{2}{11} \times \frac{1}{10} \times \frac{9}{9}\right)$$

$$= \mathbf{\frac{1}{5}}$$

We are trying to find a combination of Es or Ns within the 3 cards drawn. The 3 comes about from:

$$3\ choose\ 2 = 3$$

Question 5

(a)

Two discs are chosen at random **without** replacement from the five discs shown in the diagram.

(i) Find the probability that both discs are numbered 2 . [2]

The probability that the first disc is a 2

$$p_1 = \frac{2}{5}$$

The probability that the second disk chosen is a 2

$$p_2 = \frac{1}{4}$$

Hence, the probability that they are both twos

$$p = p_1 \times p_2$$

$$= \frac{2}{20}$$

$$= \mathbf{\frac{1}{10}}$$

(ii) Find the probability that the numbers on the **two** discs have a total of 5 . [3]

The ways we can make 5 is to pick

$$1 + 4$$

$$2 + 3$$

$$3 + 2$$

$$4 + 1$$

The probabilities of each paring are

$$\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$$

$$\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$$

$$\frac{1}{5} \times \frac{2}{4} = \frac{1}{10}$$

$$\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$$

Adding these probabilities together

$$\frac{1}{20} + \frac{1}{10} + \frac{1}{10} + \frac{1}{20}$$

$$= \mathbf{\frac{3}{10}}$$

(iii) Find the probability that the numbers on the two discs do **not** have a total of 5. [1]

If the probability of getting 5 is 0.3 then the probability of not getting 5 must be

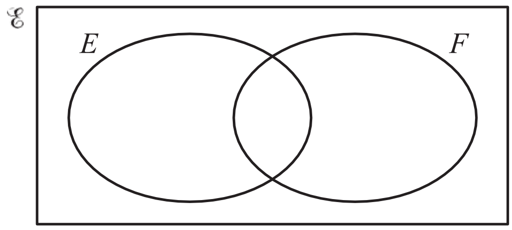
$$1 - \frac{3}{10}$$

$$= \mathbf{\frac{7}{10}}$$

(b) A group of international students take part in a survey on the nationality of their parents.

E = {students with an English parent}
F = {students with a French parent}

$n(\mathscr{E}) = 50$, $n(E) = 15$, $n(F) = 9$ and $n(E \cup F)' = 33$.



(i) Find $n(E \cap F)$.

[1]

There are 50 total students and 33 of them have neither an English or a French parent

$$n(E \cup F)' = 33$$

$$50 - 33 = 17$$

So, there are 17 that do.

15 have an English parent and 9 have a French parent.

$$15 + 9$$

$$= 24$$

This means there must be an overlap of

$$24 - 17$$

$$= 7$$

Hence

$$\boldsymbol{n(E \cap F) = 7}$$

(ii) Find $n(E' \cup F)$.

[1]

$n(E' \cup F)$

$= 33 + 9$

$= \mathbf{42}$

(iii) A student is chosen at random.
Find the probability that this student has an English parent and a French parent. [1]

$$\boldsymbol{p = \frac{7}{50}}$$

(iv) A student who has a French parent is chosen at random.
Find the probability that this student also has an English parent. [1]

$$p = \frac{7}{9}$$

(a) Emile lost 2 blue buttons from his shirt.

A bag of spare buttons contains 6 white buttons and 2 blue buttons.

Emile takes 3 buttons out of the bag at random **without replacement**.

Calculate the probability that

(i) all 3 buttons are white, [3]

The buttons are not replaced, meaning that the number of total buttons in the bag will decrease by 1 for each pick. Also, the number of white buttons will decrease each time there is a white button picked.

Probability = $\frac{\text{number of favourbale cases}}{\text{number of total cases}}$

The number of total cases here is the total number of buttons in the bag, while the number of favourable cases is the number of white buttons in the bag.

To pick 3 white buttons:

The probability that the first one picked is white is:

$P = \frac{6}{8}$

The probability that the second one picked is white is:

$P = \frac{5}{7}$

The probability that the third one picked is white is:

$P = \frac{4}{6}$

The 3 separate events above need to happen simultaneously to obtain 3 white buttons, therefore, we need to multiply their probabilities to obtain the probability to obtain 3 white buttons.

$P = \frac{6}{8} x \frac{5}{7} x \frac{4}{6}$

$\mathbf{P = \frac{120}{336}}$

(ii) exactly one of the 3 buttons is blue. [3]

There are 3 possibilities to obtain exactly one blue button:

1. The first button picked is blue and the other 2 are white
2. The second button picked is blue and the other 2 are white
3. The third button picked is blue and the other 2 are white

The buttons are not replaced, so their number will be reduced after every pick.

1. The probability that the first button picked is blue is: $\frac{2}{8}$, while the probability that the second one picked is white is: $\frac{6}{7}$ and the third one picked is white is: $\frac{5}{6}$

   These 3 events need to happen simultaneously to obtain only one blue button out of 3, considering that the first one picked was the blue one:

   $P = \frac{2}{8} x \frac{6}{7} x \frac{5}{6}$

   $P = \frac{60}{336}$

2. Similarly, the probability that the first button picked is white is: $\frac{6}{8}$, while the probability that the second one picked is blue is: $\frac{2}{7}$ and the third one picked is white is: $\frac{5}{6}$

These 3 events need to happen simultaneously to obtain only one blue button out of 3, considering that the second one picked was the blue one:

$$P = \frac{6}{8} \times \frac{2}{7} \times \frac{5}{6}$$

$$P = \frac{60}{336}$$

3. Similarly, the probability that the first button picked is white is: $\frac{6}{8}$, while the probability that the second one picked is white is: $\frac{5}{7}$ and the third one picked is blue is: $\frac{2}{6}$

These 3 events need to happen simultaneously to obtain only one blue button out of 3, considering that the second one picked was the blue one:

$$P = \frac{6}{8} \times \frac{5}{7} \times \frac{2}{6}$$

$$P = \frac{60}{336}$$

Finally, the 3 possibilities cannot happen simultaneously, therefore, we need to add them up to obtain the probability that exactly one of them is blue.

$$P = 3 \times \frac{60}{336}$$

$$\mathbf{P = \frac{180}{336}}$$

(b) There are 25 buttons in another bag.

This bag contains x blue buttons.

Two buttons are taken at random **without replacement**.

The probability that they are both blue is $\frac{7}{100}$

(i) Show that $x^2 - x - 42 = 0$. [4]

2 buttons are picked at random:

The probability that the first one is blue is:

$P = \frac{x}{25}$

The buttons are not replaced, therefore, the number of blue buttons in the bag is decreasing, together with the number of total buttons.

The probability that the second one is blue is:

$P = \frac{x-1}{24}$

The 2 events need to happen simultaneously to have both buttons blue, therefore, we need to multiply their probabilities:

$P = \frac{x}{25} \times \frac{x-1}{24}$

The probability that both buttons picked are blue is: $P = \frac{7}{100}$

$\frac{x}{25} \times \frac{x-1}{24} = \frac{7}{100}$

$\frac{x(x-1)}{600} = \frac{7}{100}$

$100x(x-1) = 4200$

We divide both sides by 100 to simplify.

$x^2 - x = 42$

We move all terms to one side.

$x^2 - x - 42 = 0$

(ii) Factorise $x^2 - x - 42$. [2]

$x^2 - x - 42 = 0$

We rewrite the second order equation above to factorise it:

$x^2 + 6x - 7x - 42 = 0$ (42 = 6 x 7)

$x^2 + 6x - 7x - 42 = 0$ (42 = 6 x 7)

For the first 2 terms, the common factor is x while for the last 2 terms, the common factor is 7.

$x(x + 6) - 7(x + 6) = 0$

For these 2 last terms, the common factor is (x + 6).

$(x - 7)(x + 6) = 0$

(iii) Solve the equation $x^2 - x - 42 = 0$. [1]

$x^2 - x - 42 = 0$

is equivalent to:

$(x - 7)(x + 6) = 0$ (from ii))

Having the equation in this form, we can see that the equation can be 0

only if either of the 2 brackets is equal to 0.

Either:

$x + 6 = 0 \Rightarrow x = -6$

or

$x - 7 = 0 \Rightarrow x = 7$

The 2 solutions are:

$x = -6$ and $x = 7$

(iv) Write down the number of buttons in the bag which are **not** blue. [1]

The number of blue buttons in the bag cannot be negative, therefore, we

select the solution $x = 7$.

The number of buttons which are not blue is:

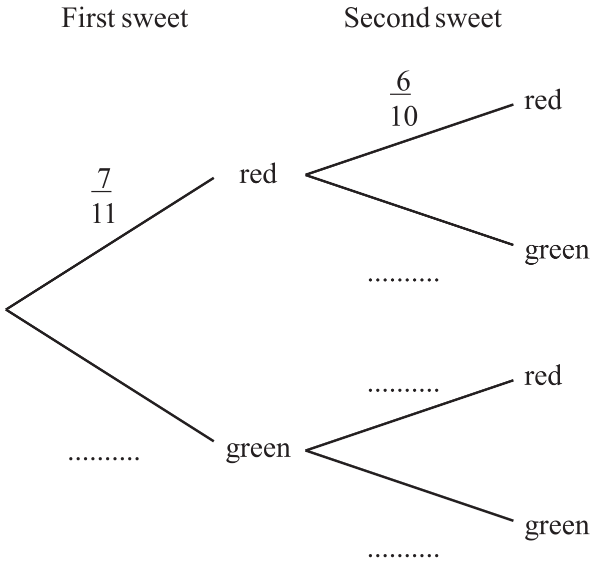
$25 - 7$

= 18 buttons which are not white.

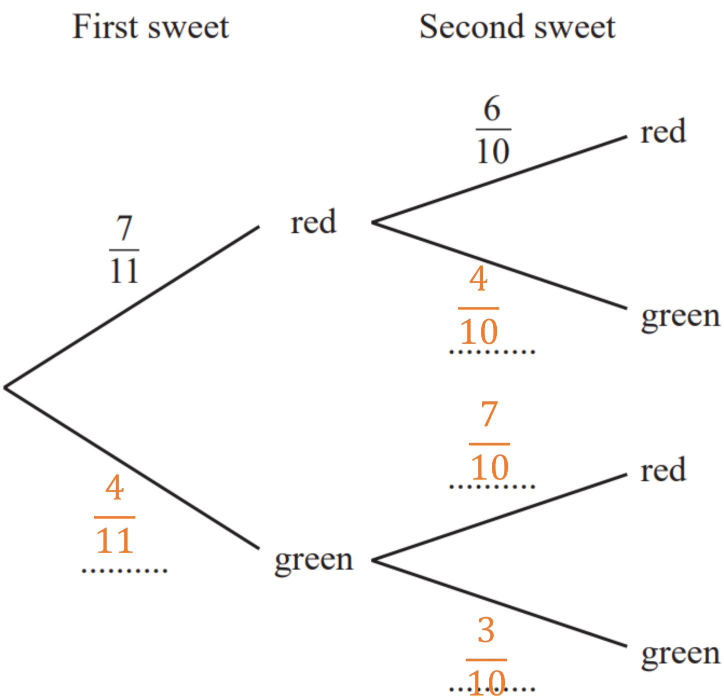
Question 7

A bag contains 7 red sweets and 4 green sweets.
Aimee takes out a sweet at random and eats it.
She then takes out a second sweet at random and eats it.

(a) Complete the tree diagram.



[3]



(b) Calculate the probability that Aimee has taken

(i) two red sweets, [2]

Two red sweets is

$$\frac{7}{11} \times \frac{6}{10}$$

$$= \frac{\mathbf{42}}{\mathbf{110}} = \frac{\mathbf{21}}{\mathbf{55}}$$

(ii) one sweet of each colour. [3]

We can have green then red or red then green

$$P = \frac{7}{11} \times \frac{4}{10} + \frac{4}{11} \times \frac{7}{10}$$

$$= \frac{\mathbf{56}}{\mathbf{110}} = \frac{\mathbf{28}}{\mathbf{55}}$$

(c) Aimee takes a third sweet at random.
Calculate the probability that she has taken

(i) three red sweets, [2]

Three red sweets is

$$\frac{7}{11} \times \frac{6}{10} \times \frac{5}{9}$$

$$= \mathbf{\frac{210}{990}} = \mathbf{\frac{7}{33}}$$

(ii) at least one red sweet. [3]

At least one red sweet means she does not get 3 green sweets.

The probability of 3 green sweets is

$$\frac{4}{11} \times \frac{3}{10} \times \frac{2}{9}$$

$$= \frac{4}{165}$$

So, the probability of at least 1 red is

$$1 - \frac{4}{165}$$

$$= \mathbf{\frac{161}{165}}$$

Probability
Difficulty: Hard

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Probability
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 3

Time allowed: **83 minutes**

Score: **/72**

Percentage: **/100**

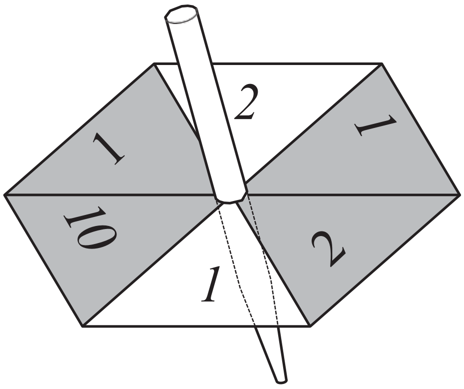
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



The diagram shows a spinner with six numbered sections.
Some of the sections are shaded.
Each time the spinner is spun it stops on one of the six sections.
It is equally likely that it stops on any one of the sections.

(a) The spinner is spun once.

Find the probability that it stops on

(i) a shaded section, [1]

$\frac{4}{6}$

(ii) a section numbered 1, [1]

$\frac{3}{6}$

(iii) a shaded section numbered 1, [1]

$\frac{2}{6}$

(iv) a shaded section or a section numbered 1. [1]

$\frac{5}{6}$

(b) The spinner is now spun twice.

Find the probability that the total of the two numbers is [2]

(i) 20,

Need two 10s in a row

$$\frac{1}{6} \times \frac{1}{6}$$

$$= \mathbf{\frac{1}{36}}$$

(ii) 11. [2]

10 and 1 in any order

$$\frac{1}{6} \times \frac{3}{6} + \frac{3}{6} \times \frac{1}{6}$$

$$= \mathbf{\frac{6}{36}}$$

(c) (i) The spinner stops on a shaded section.

Find the probability that this section is numbered 2. [1]

$$\mathbf{\frac{1}{4}}$$

(ii) The spinner stops on a section numbered 2.

Find the probability that this section is shaded. [1]

$$\mathbf{\frac{1}{2}}$$

(d) The spinner is now spun until it stops on a section numbered 2.

The probability that this happens on the nth spin is $\frac{16}{243}$.

Find the value of n. [2]

Probability of not landing on 2 is 4/6.

We don't land on 2 for n – 1 spins.

$$\left(\frac{4}{6}\right)^{n-1} \times \frac{2}{6} = \frac{16}{243}$$

Using trial and error we solve this for

$$\boldsymbol{n = 5}$$

1	1	6	7	11	12

Six cards are numbered 1, 1, 6, 7, 11 and 12.

In this question, give all probabilities as fractions.

(a) One of the six cards is chosen at random.

(i) Which number has a probability of being chosen of $\frac{1}{3}$? [1]

Divide total number of cards by 3 = 2.

There are 2 cards numbered '1' and so

1 is the correct answer.

(ii) What is the probability of choosing a card with a number which is smaller than **at least three of the other numbers**? [1]

We must first note what the three highest numbered cards are, in this case, they are 7, 11 and 12.

Next, we need to find the probabilities of each remaining card respectively. The remaining numbered cards are 1, 1 and 6. We already know the probability of choosing a 1 is 1/3 from (i), and to get our answer we add this to the probability of choosing a 6, which is 1/6 (remember when adding fractions, all denominators must be the same, so in this case we represent 1/3 as 2/6).

2/6 + 1/6 = **3/6**

(b) Two of the six cards are chosen at random, without replacement.

Find the probability that

(i) they are both numbered 1, [2]

The probability of choosing a 1 is 1/3. Because the card is not replaced, the probability of choosing the second card is 1/5, since there is only a single card numbered 1 left and 5 cards in total to choose from.

Multiply,

2/6 x 1/5

= **2/30**

(ii) the total of the two numbers is 18, [3]

There are four ways of getting a sum of 18 from two numbered cards:

1. 7 and 11
2. 11 and 7
3. 12 and 6
4. 6 and 12

The individual probability of each method can be worked out like this:

1/6 x 1/5 = 1/30

Each individual probability is the same. Therefore, we simply multiply by 4 to get our answer:

1/30 x 4

= **4/30**

(iii) the first number is **not** a 1 and the second number is a 1. [2]

We can work out the probability that the first number is not a 1 by using the probability that the first number is a 1, which we know is 2/6 (or 1/3).

We subtract this from 1:

1 – 2/6 = 4/6

Without replacement, the probability of getting a 1 on 2nd selection is 2/5.

We multiply these probabilities together to get our final answer:

4/6 x 2/5

= **8/30**

(c) Cards are chosen, without replacement, until a card numbered 1 is chosen.

Find the probability that this happens before the third card is chosen. [2]

There are two ways in which a card numbered 1 can be chosen before the third card is drawn:

1. A 1 is chosen on 1st selection
2. A 1 is not chosen on 1st selection, but is chosen on 2nd selection

The probability of choosing a 1 on 1st selection is 2/6. The probability of choosing a different number on 1st selection and 1 on 2nd selection is 8/30.

We add these probabilities together to get our answer:

2/6 + 8/30

= **18/30**

(d) A seventh card is added to the six cards shown in the diagram.
The mean value of the seven numbers on the cards is 6.

Find the number on the seventh card. [2]

The sum of the card numbers and unknown value divided by 7 equals 6.

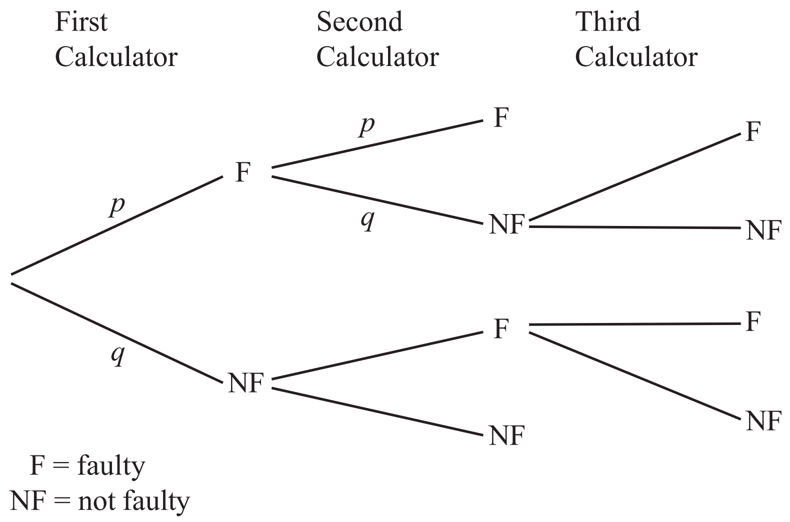
We can work out the unknown value (x) like this:

$(1 + 1 + 6 + 7 + 11 + 12 + x)/7 = 6$

$38 + x = 42$

$x = 42 - 38$

$x = 4$



The tree diagram shows a testing procedure on calculators, taken from a large batch.

Each time a calculator is chosen at random, the probability that it is faulty (F) is $\frac{1}{20}$.

(a) Write down the values of p and q. [1]

The sum of the 2 probabilities, p and q, is equal to 1.

p + q = 1

p is the probability that a randomly chosen calculator is

1/20.

p = 1/20

q = 1 – 1/20

q = 19/20

(b) Two calculators are chosen at random.

Calculate the probability that

(i) both are faulty, [2]

The probability that a randomly chosen calculator is faulty is p = 1/20.

The probability that 2 calculators are faulty is:

P = p x p

P = 1/20 x 1/20

P = 1/400

(ii) **exactly one** is faulty. [2]

There are 2 possibilities to have exactly one of the calculators faulty. Either only the first one is faulty or only the second one is faulty.

The probability that the first one is faulty and the second one is not is:

P = p x q

P = 1/20 x 19/20

P = 19/400

The probability the exactly one of them is faulty is

P = 19/400 + 19/400

P = 38/400

(c) If **exactly one** out of two calculators tested is faulty, then a third calculator is chosen at random.

Calculate the probability that exactly one of the first two calculators is faulty **and** the third one is faulty. [2]

The probability that exactly one of the 2 calculators is faulty is 38/400

The probability that the third calculator chosen at random is faulty is p = 1/20

The probability that exactly one of the first 2 is faulty and then the third one is faulty as well is:

P = 38/400 x 1/20

P = 38/8000

(d) The whole batch of calculators is rejected

either if the first two chosen are both faulty
or if a third one needs to be chosen and it is faulty.

Calculate the probability that the whole batch is rejected. [2]

The probability that both calculators from the 2 first 2 chosen are faulty is:

P = 1/400 (b) i))

If probability that the third one needs to be selected and it is also faulty is:

P = 38/8000 (c))

The probability that the batch is rejected is:

P = 1/400 + 38/8000

P = 58/8000

(e) In one month, 1000 batches of calculators are tested in this way.

How many batches are expected to be rejected? [1]

The probability that one batch is rejected is: 58/8000.

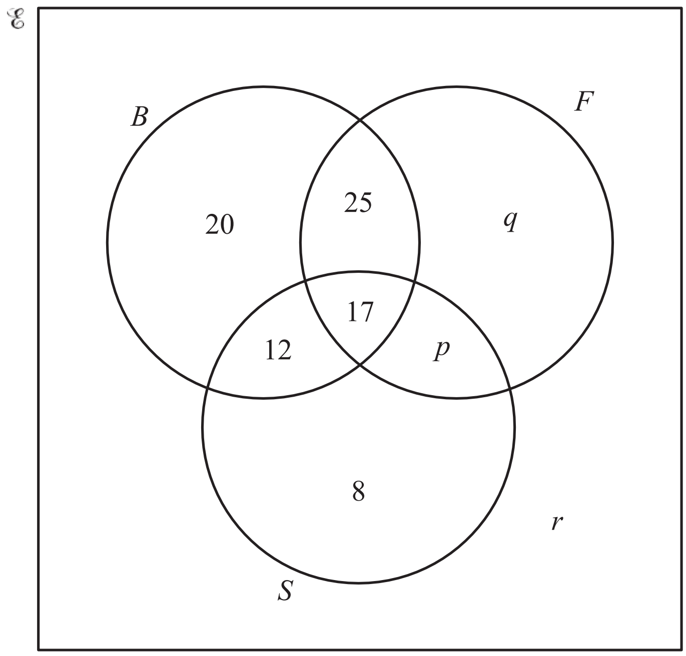
Out of 1000 batches, the probability that one is rejected is:

P = 58/8000 x 1000

P = 7.25

In a survey, 100 students are asked if they like basketball (B), football (F) and swimming (S).

The Venn diagram shows the results.



42 students like swimming.

40 students like exactly one sport.

(a) Find the values of p, q and r. [3]

We have that 42 students like swimming

$$8 + 12 + 17 + p = 42$$

$$\rightarrow p + 37 = 42$$

$$\rightarrow \boldsymbol{p = 5}$$

40 students like exactly 1 sport

$$20 + q + 8 = 40$$

$$\rightarrow 28 + q = 40$$

$$\rightarrow \boldsymbol{q = 12}$$

100 students in total

$$r = 100 - 20 - 12 - 25 - 17 - p - q - 8$$

$$\rightarrow \boldsymbol{r = 1}$$

(b) How many students like

(i) all three sports, [1]

17

(ii) basketball and swimming but not football? [1]

12

(c) Find

(i) $n(B')$, [1]

Number of elements not in B

$$q + p + 8 + r$$

$$= \mathbf{26}$$

(ii) $n((B \cup F) \cap S')$. [1]

Number of elements in the intersection of the union of B and F, with not S

$$20 + 25 + q$$

$$= \mathbf{57}$$

(d) One student is chosen at random from the 100 students. Find the probability that the student

(i) only likes swimming, [1]

$$\frac{8}{100} = \frac{\mathbf{2}}{\mathbf{25}}$$

(ii) likes basketball but not swimming. [1]

$$\frac{45}{100} = \frac{\mathbf{9}}{\mathbf{20}}$$

(e) Two students are chosen at random from those who like basketball.

Find the probability that they each like exactly one other sport. [3]

$$20 + 25 + 12 + 17$$

$$= 74$$

74 students in total like basketball.

The probability of like exactly on other sport is

$$\frac{12}{74} + \frac{25}{74}$$

$$= \frac{37}{74}$$

Hence the probability of both students liking one other sport is

$$\frac{37}{74} \times \frac{36}{73}$$

$$= \mathbf{\frac{18}{73}}$$

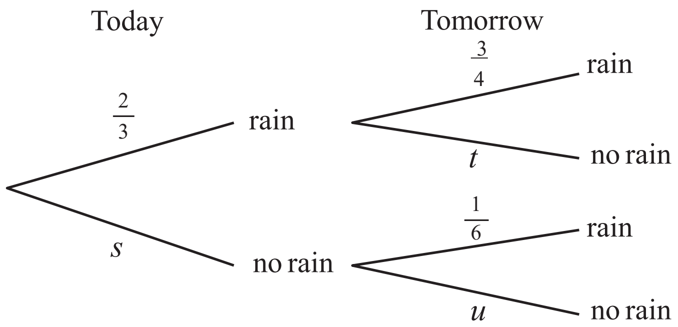
Give your answers to this question as fractions.

(a) The probability that it rains today is $\frac{2}{3}$.

If it rains today, the probability that it will rain tomorrow is $\frac{3}{4}$.

If it does not rain today, the probability that it will rain tomorrow is $\frac{1}{6}$.

The tree diagram below shows this information.



(i) Write down, as fractions, the values of s, t and u. [3]

The sum of the probabilities is 1.

$s = 1 - \frac{2}{3}$

$\mathbf{s = \frac{1}{3}}$

$t = 1 - \frac{3}{4}$

$\mathbf{t = \frac{1}{4}}$

$u = 1 - \frac{1}{6}$

$\mathbf{u = \frac{5}{6}}$

(ii) Calculate the probability that it rains on both days. [2]

To obtain the probability that it rains on both days we need to multiply the probability that it rains for each day.

$P = \frac{3}{4} \times \frac{2}{3}$

$\mathbf{P = \frac{1}{2}}$

(iii) Calculate the probability that it will not rain tomorrow. [2]

The probability that it rains today and will not rain tomorrow is:

$P = \frac{1}{4} \times \frac{2}{3}$

$P = \frac{1}{6}$

The probability that it does not rain today and will not rain tomorrow is:

$P = \frac{5}{6} \times \frac{1}{3}$

$P = \frac{5}{18}$

By adding up the probabilities for the 2 possibilities we obtain:

$P = \frac{5}{18} + \frac{1}{6}$

$\mathbf{P = \frac{4}{9}}$

(b) Each time Christina throws a ball at a target, the probability that she hits the target is $\frac{1}{3}$.

She throws the ball three times.
Find the probability that she hits the target

(i) three times, [2]

To hit the target 3 times, the probability will be:

$P = \frac{1}{3} \text{ x } \frac{1}{3} \text{ x } \frac{1}{3}$

$\mathbf{P = \frac{1}{27}}$

(ii) at least once. [2]

The probability to hit the target at least once means the target can be hit either once, twice or all 3 times.

If the probability that she hits the target once is $\frac{1}{3}$, then the probability that she does not hit the target is: $\frac{2}{3}$

$P = 1 - \frac{2}{3} \text{ x } \frac{2}{3} \text{ x } \frac{2}{3}$

$\mathbf{P = \frac{19}{27}}$

(c) Each time Eduardo throws a ball at the target, the probability that he hits the target is $\frac{1}{4}$.

He throws the ball until he hits the target.
Find the probability that he **first** hits the target with his

(i) 4th throw, [2]

We consider the probability of not hitting the target for the first 3 throws, $\frac{3}{4}$, and then the probability that he does hit the target for his 4^{th} throw, $\frac{1}{4}$.

$P = \frac{3}{4} \text{ x } \frac{3}{4} \text{ x } \frac{3}{4} \text{ x } \frac{1}{4}$

$\mathbf{P = \frac{27}{256}}$

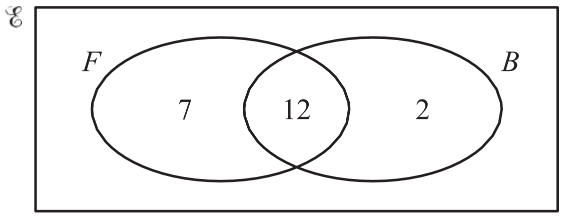
(ii) nth throw. [1]

Similarly, we consider the probability of not hitting the target for the first n-1 throws, $\frac{3}{4}$, and then the probability that he does hit the target for his nth throw, $\frac{1}{4}$.

$$P = \left(\frac{3}{4}\right)^{n-1} \times \frac{1}{4}$$

Question 6

(a) All 24 students in a class are asked whether they like football and whether they like basketball.
Some of the results are shown in the Venn diagram below.



$\mathscr{E}$ = {students in the class}.
F = {students who like football}.
B = {students who like basketball}.

(i) How many students like both sports? [1]

There are 12 students who like both sports. This is represented by the elements in both Set F and Set B.

(ii) How many students do not like either sport? [1]

The number of students which like football, basketball or both is: 7 + 2 + 12 = 21 students.

The number of students who don't like either sport is: 24 – 21 = 3 students.

(iii) Write down the value of $n(F \cup B)$. [1]

n(F ∪ B) – represents the number of elements in the union of the 2 sets so all numbers inside F or B .

n(F ∪ B) = 7+12+2=21

(iv) Write down the value of $n(F' \cap B)$. [1]

n(F' ∩ B) – represents the number of elements in the intersection of all the elements which are not in F and the elements in Set B. The elements which are not in Set F are the 2 elements in Set B and the 3 elements which are not in either Set B or Set F. The intersection of these 5 elements and the elements in Set B is 2.

n(F' ∩ B) = 2

(v) A student from the class is selected at random.
What is the probability that this student likes basketball? [1]

probability = $\frac{\text{number of favourable cases}}{\text{nunmber of total cases}}$

In this case, the number of total cases represents the total number of students in the class, 24.

The number of favourable cases represent the number of students which like basketball, 14.

P = $\frac{14}{24}$

Simplified: **P = $\frac{7}{12}$**

(vi) A student who likes football is selected at random. What is the probability that this student likes basketball? [1]

probability = $\frac{\text{number of favourable cases}}{\text{nunmber of total cases}}$

In this case, the number of total cases represents the number of students which like football, 19.

The number of favourable cases represent the number of students which like both football and basketball, 12.

P = $\frac{12}{19}$

(b) Two students are selected at random from a group of 10 boys and 12 girls.
Find the probability that

(i) they are both girls, [2]

In this case, the total number of students is 10 boys + 12 girls = 22.

The probability the first student picked is a girl is:

$P = \frac{12}{22}$

If the first student picked is a girl, the number of girls left in the group selected will now be 11. Also, after the first student is picked the total number of students in the group selected is 21.

The probability that the second student picked is also a girl is:

$P = \frac{11}{21}$

The probability of the 2 events happening simultaneously is:

$P = \frac{12}{22} \times \frac{11}{21}$

$\mathbf{P = \frac{132}{462}}$

(ii) one is a boy and one is a girl. [3]

In this case, the total number of students is 10 boys + 12 girls = 22.

There are 2 possibilities to pick a girl and a boy: either the first student picked is a girl and the second one is a boy or the other way around.

The probability the first student picked is a girl is:

$P = \frac{12}{22}$

After the first student is picked the total number of students left in the group selected is 21.

The probability that the second student picked is a boy is:

$P = \frac{10}{21}$

The probability of the 2 events happening simultaneously is:

$P = \frac{12}{22} \times \frac{10}{21}$

$P = \frac{120}{462}$

A second possibility would be that the first student picked is a boy and the second student picked is a girl.

The probability of the first student picked being a boy is:

$P = \frac{10}{22}$

After the first student is picked the total number of students left in the group selected is 21.

The probability that the second student picked is a girl is:

$P = \frac{12}{21}$

The probability of the 2 events happening simultaneously is:

$P = \frac{10}{22} \times \frac{12}{21}$

$P = \frac{120}{462}$

The probability of picking one boy and one girl is the sum of the probabilities for the 2 possibilities of this event happening.

$P = \frac{120}{462} + \frac{120}{462}$

$\mathbf{P} = \frac{240}{462}$