2D Perimeters & Areas
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	2D Perimeters & Areas
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **30 minutes**

Score: **/23**

Percentage: **/100**

Grade Boundaries:

Assembled by A S

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

The area of a triangle is 528cm^2.
The length of its base is 33cm.

Calculate the perpendicular height of the triangle. [2]

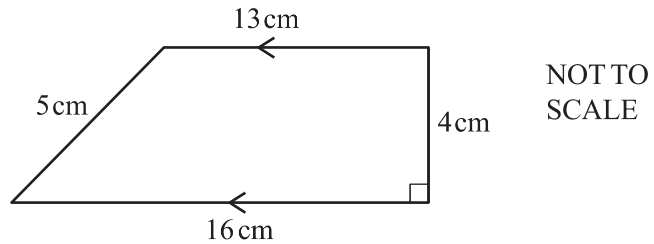
Area of a triangle is

$$A = \frac{1}{2} base \times height$$

$$\rightarrow \frac{1}{2} \times 33 \times height = 528$$

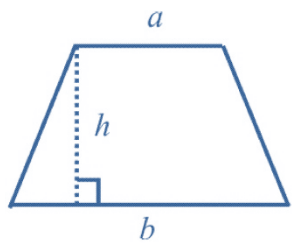
$$\rightarrow height = 2 \times \frac{528}{33}$$

$$= \mathbf{32}$$



Calculate the area of this trapezium. [2]

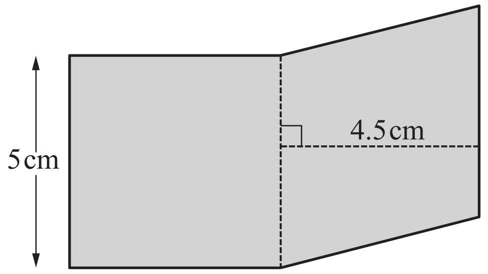
Area of Trapezium $= \frac{1}{2}(a+b) \times h$



Area of Trapezium $= \frac{1}{2}(16+13) \times 4$

$=$ **58cm²**

The shaded shape is made by joining a square and a rhombus.



NOT TO SCALE

Work out

(a) the perimeter of the shaded shape, [1]

Squares and rhombuses both have four sides equal in length (in this case 5cm).

The shaded shape has 6 sides (3 from the square and 3 from the rhombus) so

the Perimeter is just:

$$\text{Perimeter} = 6 \times 5$$

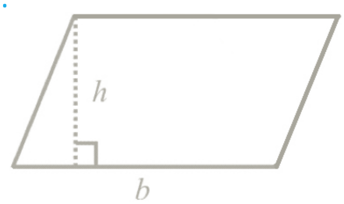
$$= \mathbf{30cm}$$

(b) the area of the shaded shape. [2]

The area of a Square is: Area $= l^2$, where l is the length of one side.

The area of a rhombus is: Area $b \times h$

(a rhombus is a special case of a parallelogram – see diagram)

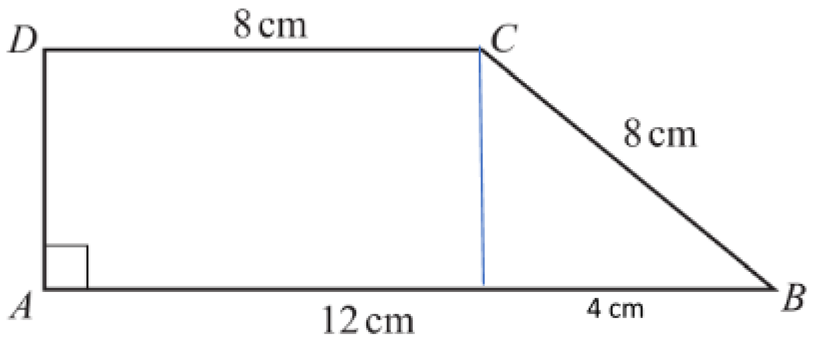


$$\text{Area of shaded shape} = \text{Area of Square} + \text{Area of Rhombus}$$

$$= 5 \times 5 + 5 \times 4.5$$

$$= \mathbf{47.5cm}$$

Question 4



Calculate the area of this trapezium. [4]

We split the trapezium into a rectangle and a triangle (as above) and calculate the length of the blue line using Pythagoras'

$$a^2 + b^2 = c^2$$

$$4^2 + b^2 = 8^2$$

$$16 + b^2 = 64$$

$$b^2 = 48$$

$$b = \sqrt{48}$$

$$b = 4\sqrt{3}$$

The area of the rectangle is

$$A_r = 8 \times 4\sqrt{3}$$

$$A_r = 32\sqrt{3}\ cm^2$$

The area of the triangle is

$$A_t = \frac{1}{2} \times 4 \times 4\sqrt{3}$$

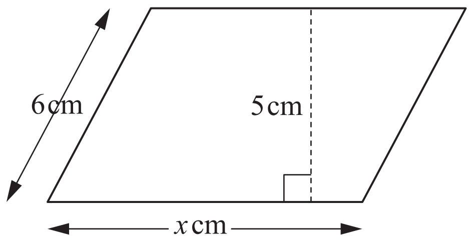
$$A_t = 8\sqrt{3}$$

Total area is then

$$A = 32\sqrt{3} + 8\sqrt{3}$$

$$A = 40\sqrt{3}\ cm^2$$

$$A = \quad \mathbf{69.3\ cm^2}\ (1dp)$$



NOT TO SCALE

The area of this parallelogram is $51.5\,\text{cm}^2$.

Work out the value of x. [2]

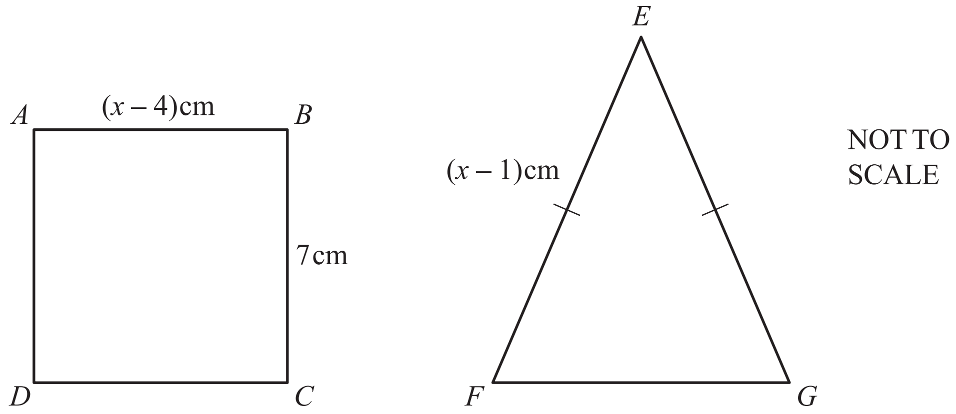
Area of the parallelogram is given by base times perpendicular height

$$5 \times x = 51.5$$

Divide both sides by 5

$$\rightarrow x = 51.5 \div 5$$

$$\rightarrow \boldsymbol{x = 10.3}$$



(a) $ABCD$ is a square.

Find the value of x. [1]

Since ABCD is a square, all sides must have equal length.

$$(x-4)=7$$

Add 4 to both sides of the equation.

$$\boldsymbol{x=11}$$

(b) Square $ABCD$ and isosceles triangle EFG have the same perimeter. [2]

Work out the length of FG.

The triangle FEG is an isosceles triangle, therefore the lengths of FE and GE are the same.

The perimeter of a square is four times the side of BC.

$$FE+GE+FG=4\times BC$$

$$2FE+FG=4\times 7$$

$$2(11-1)+FG=28$$

Subtract 20 from both sides of the equation to get the length of FG.

$$\boldsymbol{FG=8}$$

An equilateral triangle has sides of length 6.2 cm, correct to the nearest millimetre.

Complete the statement about the perimeter, *P* cm, of the triangle. [2]

The side of a triangle is 6.2cm, correct to the nearest millimetre, which means:

- the upper bound is 6.25cm
- the lower bound is 6.15cm.

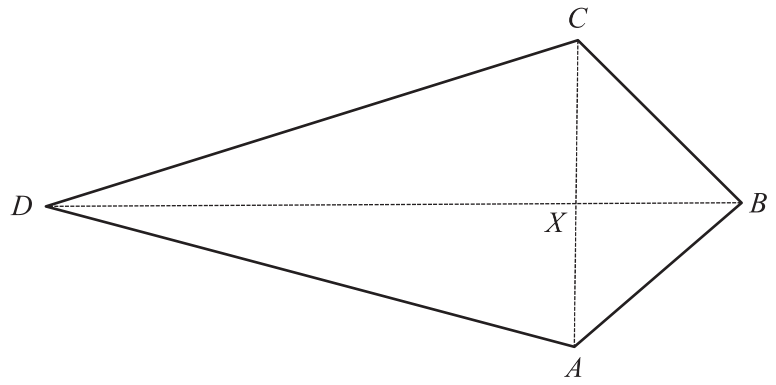
To get the perimeter of the equilateral triangle, we simply multiply its side by 3 (number of sides).

Using the upper bound length, the perimeter is 18.75cm.

Using the lower bound length, the perimeter is 18.45cm.

Therefore it must be true that the length of the perimeter is somewhere between these values.

$$\mathbf{18.45cm \leq P < 18.75cm}$$



NOT TO SCALE

ABCD is a kite.
The diagonals *AC* and *BD* intersect at *X*.
$AC = 12$ cm, $BD = 20$ cm and $DX : XB = 3 : 2$.

(a) Calculate angle *ABC*. [3]

The ratio of DX to XB is 3:2 therefore $DX = 1.5\ XB$

$$BD = DX + XB$$

Substitute 1.5XB for DX:

$$BD = 20 = 2.5XB$$

$$XB = \frac{20}{2.5} = 8\ cm$$

$$DX = 1.5 \times 8 = 12\ cm$$

$$AX = XC = \frac{1}{2}AC = 6\ cm$$

Using trigonometric ratios for triangle ABX we can see that angle ABX is:

$$\tan^{-1}\frac{AX}{BX}$$

$$= \tan^{-1}\left(\frac{6}{8}\right)$$

However, we need angle ABC which is twice angle ABX:

$$angle\ ABC = 2\tan^{-1}\left(\frac{6}{8}\right)$$

$$\boldsymbol{angle\ ABC = 73.7°}$$

(b) Calculate the area of the kite. [2]

The kite is composed of two triangles. And the area of a triangle is $\frac{1}{2} \times base \times height$

$$area\ of\ kite = \frac{1}{2} \times 12 \times 8 + \frac{1}{2} \times 12 \times 12$$

$$\boldsymbol{area\ of\ kite = 120\ cm^2}$$

2D Perimeters & Areas
Difficulty: Easy

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	2D Perimeters & Areas
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 2

Time allowed: **31 minutes**

Score: **/24**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
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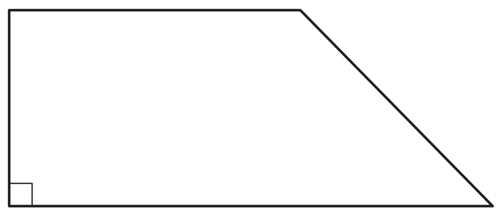
Question 1

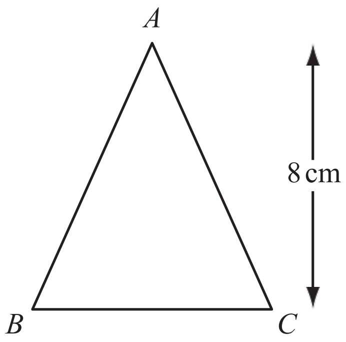


Find the area of the trapezium. [2]

$$Area\ of\ Trapezium = \frac{1}{2} \times (sum\ of\ parallel\ sides) \times height$$

$$= \frac{1}{2} \times (22 + 12) \times 10$$

$$= \mathbf{170 cm^2}$$



NOT TO SCALE

Triangle ABC has a height of 8 cm and an area of 42 cm^2.

Calculate the length of BC. [2]

The formula for the area of a triangle is:

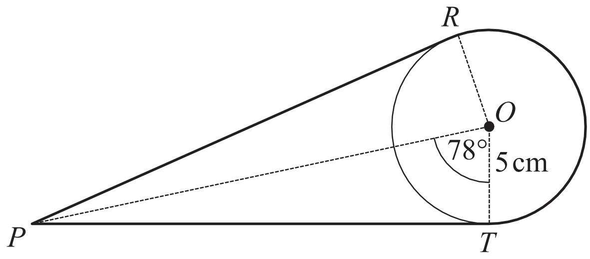
Area = $\frac{1}{2}$ x height x base

In this case, the base is the length of BC.

42 cm^2 = $\frac{1}{2}$ x 8 cm x BC

42 cm^2 = 4 cm x BC

BC = 10.5 cm



NOT TO SCALE

R and T are points on a circle, centre O, with radius 5 cm.
PR and PT are tangents to the circle and angle $POT = 78°$.

A thin rope goes from P to R, around the major arc RT and then from T to P. [6]

Calculate the length of the rope.

We can use the tan rule to calculate PT

$$\tan\theta = \frac{opp}{adj}$$

$$\rightarrow PT = 5\tan 78$$

Note that

$PR = PT$

and

$ROT = POT = 78$

The arc is calculated using

$$l_{arc} = r\frac{\theta\pi}{180}$$

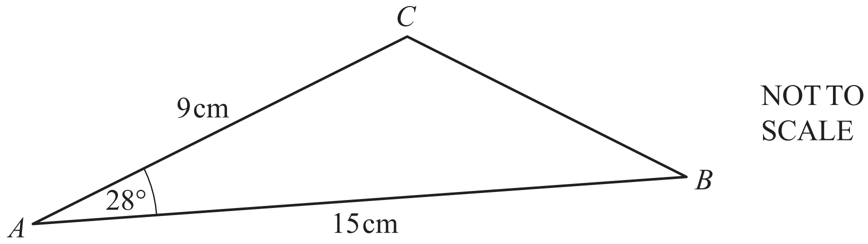
$$= 5 \times (360 - 2 \times 78) \times \frac{\pi}{180}$$

$$= \frac{17\pi}{3}$$

Hence (since $PR = PT$) we have

$$l = \frac{17\pi}{3} + 2 \times 5\tan 78$$

$$= \mathbf{64.8}$$



Calculate the area of triangle ABC. [2]

Area of a triangle is

$$A = \frac{1}{2}ab\sin C$$

$$\rightarrow A = \frac{1}{2}(9)(15)\sin 28$$

$$= \mathbf{31.7}$$

A large rectangular card measures 80 centimetres by 90 centimetres.
Maria uses **all** this card to make small rectangular cards measuring 40 **millimetres** by 15 **millimetres**.
Calculate the number of small cards. [2]

The small cards measure

$$40\ mm \times 15\ mm$$

Converting this to centimetres

$$4\ cm \times 1.5\ cm$$

$$= 6\ cm^2$$

The area of the large card is

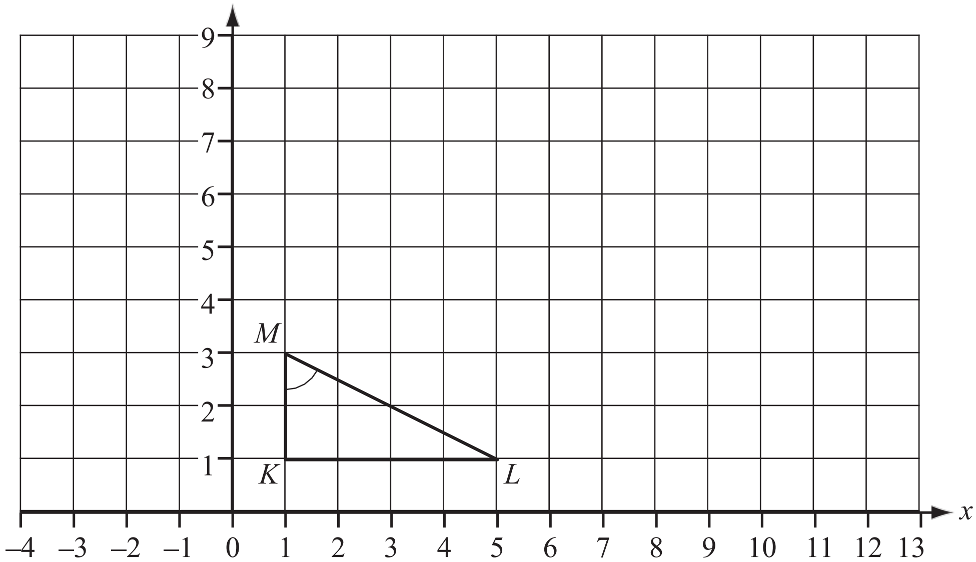
$$80 \times 90$$

$$= 7200\ cm^2$$

Dividing the large card area by the small

$$7200 \div 6$$

$$= \mathbf{1200}$$



The triangle *KLM* is shown on the grid.

(a) Calculate angle *KML*. [2]

$$KL = 4$$

$$KM = 2$$

Using the trigonometric relation

$$\tan\theta = \frac{opp}{adj}$$

we have

$$\tan KML = \frac{KL}{KM}$$

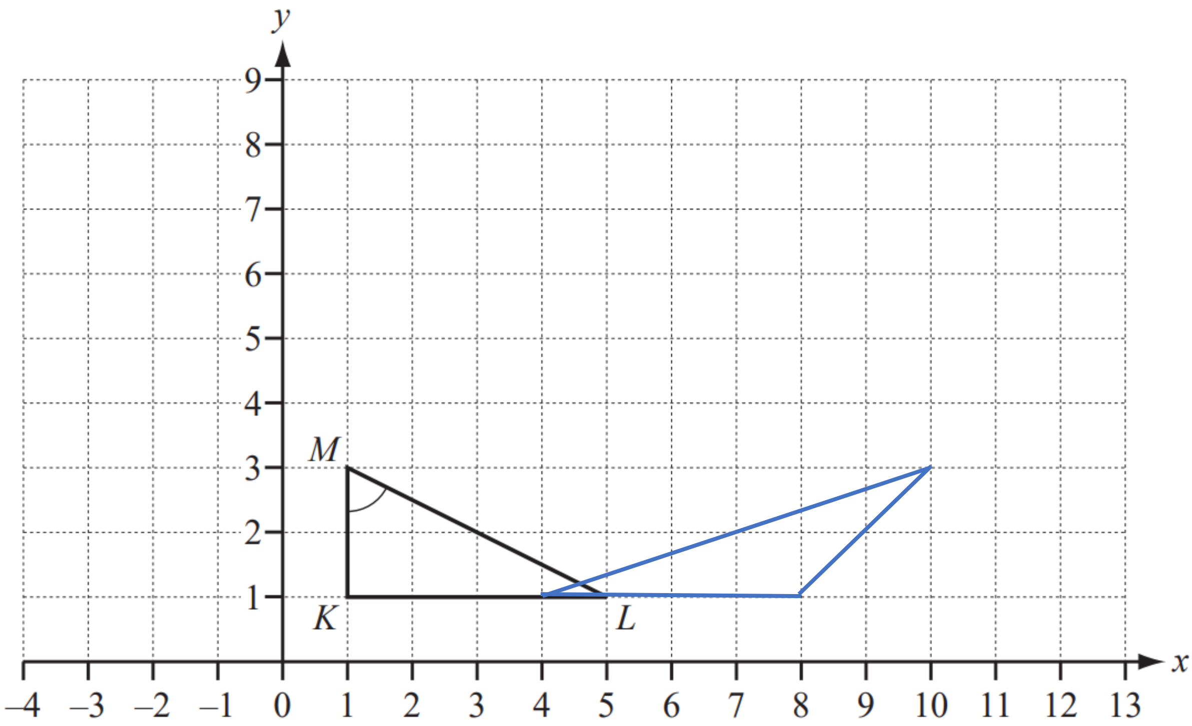
$$= \frac{4}{2}$$

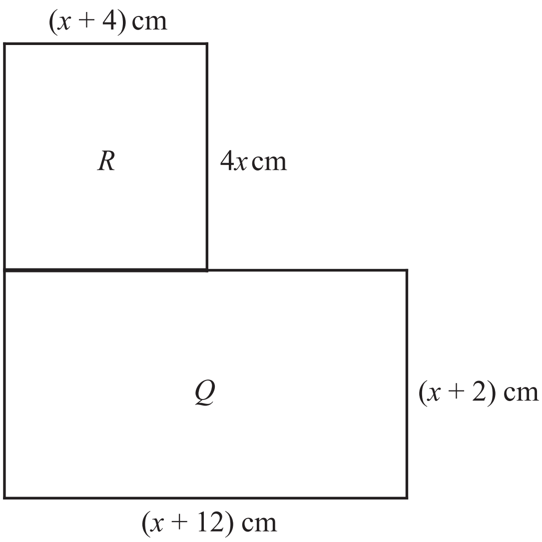
$$= 2$$

$$\rightarrow KML = \tan^{-1} 2$$

$$= 63.4$$

(b) On the grid, draw the shear of triangle KLM, with a shear factor of 3 and the x-axis invariant. [2]





NOT TO SCALE

(a) (i) Write down an expression for the area of rectangle R. [1]

The area of a rectangle is the length multiplied by the width.

A = $4x(x + 4)$

(ii) Show that the total area of rectangles R and Q is $5x^2 + 30x + 24$ square centimetres. [1]

The total area is the sum of the areas of the 2 rectangles.

A = $(x + 2)(x + 12) + 4x(x + 4)$

A = $x^2 + 2x + 12x + 24 + 4x^2 + 16x$

A = $5x^2 + 30x + 24$

(b) The total area of rectangles R and Q is 64 cm^2.
Calculate the value of x correct to 1 decimal place. [4]

A = $5x^2 + 30x + 24 = 64$

$5x^2 + 30x - 40 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our case, a = 5, b = 30 and c = -40

$$x = \frac{-30 \pm \sqrt{30^2 - 4 \, x \, 5 \, x \, (-40)}}{2x5}$$

$$x = \frac{-30 \pm \sqrt{30^2 - 4 \, x \, 5 \, x \, (-40)}}{2x5}$$

$x_1 = 1.1$ and $x_2 = -7.1$

x = 1.1 (because the length of a segment cannot be a negative value)

2D Perimeters & Areas
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	2D Perimeters & Areas
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **32 minutes**

Score: **/25**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

The base of a triangle is 9 cm correct to the nearest cm.
The area of this triangle is 40 cm^2 correct to the nearest 5 cm^2.

Calculate the upper bound for the perpendicular height of this triangle. [3]

The area of a triangle is

$$A = \frac{1}{2} \times base \times height$$

Here we have

$$\sim 40 = \frac{1}{2} \times \sim 9 \times h$$

For the upper bound on height we need the area to be as large as possible and the base to be as short as possible, i.e.

$$A = 42.5$$

$$b = 8.5$$

Hence

$$42.5 = \frac{1}{2} \times 8.5 \times h$$

$$\boldsymbol{h = 10}$$

The scale on a map is 1 : 20 000.
The area of a lake on the map is 1.6 square centimetres.

Calculate the actual area of the lake.
Give your answer in square metres. [3]

The scale is 1 cm on the map is equal to 20000 cm on the ground. This must be converted to m^2. This is done by dividing by 100 followed by squaring the value:

$1cm : 20000\ cm$ Converting to metres $0.01\ m : 200m$

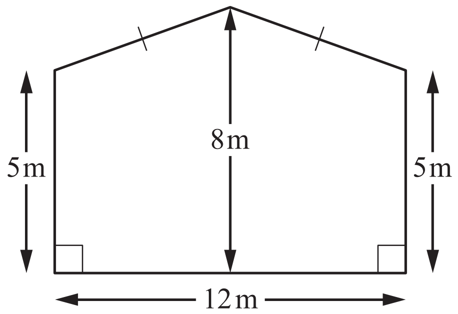
Squaring to get metre square $0.0001\ m^2 : 40000\ m^2$

1.6 cm^2 is divided by 10000 to convert into m^2

Thus, the actual area can then be calculated by:

$$\frac{1.6 \div 10000}{0.0001} \times 40000$$

$$= 64000\ m^2$$



NOT TO
SCALE

The diagram shows the front face of a barn.
The width of the barn is 12 m.
The height of the barn is 8 m.
The sides of the barn are both of height 5 m.

(a) Work out the area of the front face of the barn. [3]

The barn is made out of two trapezoids with sides 5m and 8m long and height 6m (half the base).

The area of a trapezoid of parallel sides *a* and *b* and height *h* is given by:

$$Area\ of\ trapezoid = \frac{(a+b)}{2} \times h$$

In our case *a*=5m, *b*=8m and *h*=6m.

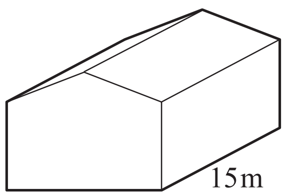
$$Area\ of\ trapezoid = \frac{(5+8)}{2} \times 6 = 39m^2$$

Double the area to get the area of the whole front face:

$$Area = \mathbf{78m^2}$$

(b) The length of the barn is 15 m.

Work out the volume of the barn. [1]

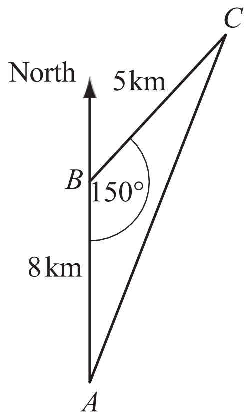


NOT TO
SCALE

The volume of the barn is the area of the front side multiplied by the length of the barn.

$$Volume = 78m^2 \times 15m$$

$$= \mathbf{1170m^3}$$



NOT TO SCALE

A helicopter flies 8 km due north from A to B. It then flies 5km from B to C and returns to A.
Angle $ABC = 150°$.

(a) Calculate the area of triangle ABC. [2]

Area of a triangle is

$$A = \frac{1}{2}ab\sin C$$

$$\rightarrow A = \frac{1}{2}(8)(5)\sin 150$$

$$= \mathbf{10}$$

(b) Find the bearing of B from C. [2]

The bearing of C from B is

$$180 - 150$$

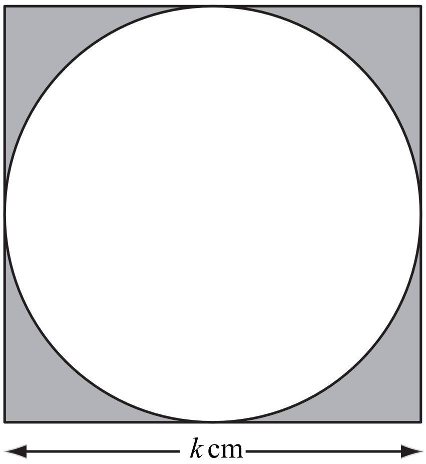
$$= 030$$

Hence, the bearing of B from C is

$$360 - (180 - 30)$$

$$= 360 - 150$$

$$= \mathbf{210}$$



The diagram shows a square of side k cm.

The circle inside the square touches all four sides of the square.

(a) The shaded area is A cm^2.

Show that $4A = 4k^2 - \pi k^2$. [2]

The shaded area is the difference between the area of the square with side *k* and the circle with diameter *k*.

$$A = square\ area - circle\ area$$

Area of a square with side *k*:

$$square\ area = k^2$$

Area of a circle with diameter *k,* and hence radius k/2 gives:

$circle\ area = \pi\left(\frac{k}{2}\right)^2$ (Area of a circle = πr^2)

Hence we have

$$A = k^2 - \pi\left(\frac{k}{2}\right)^2$$

Multiply both sides by 4 to get the final answer.

$$\mathbf{4A = 4k^2 - \pi k^2}$$

(b) Make k the subject of the formula $4A = 4k^2 - \pi k^2$. [3]

Factorise out k^2 on the right hand side:

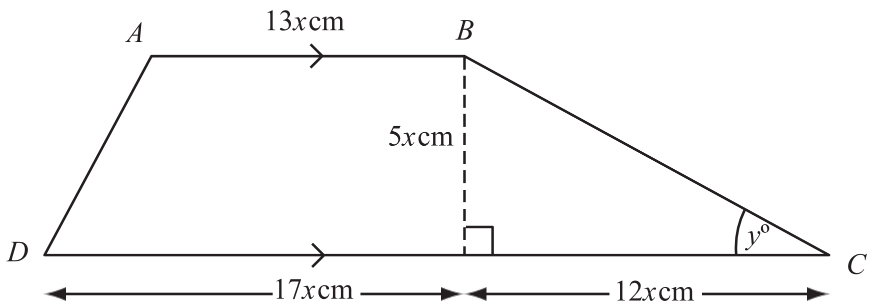
$$4A = (4 - \pi)k^2$$

Divide both sides by the factor (4-π):

$$\frac{4A}{4-\pi} = k^2$$

Now, take the square root of both sides to make k the subject of the formula.

$$\boldsymbol{k = \pm\sqrt{\frac{4A}{4-\pi}}}$$



ABCD is a trapezium.

(a) Find the area of the trapezium in terms of x and simplify your answer. [2]

The formula for the area of a trapezium is:

$A = \frac{(a+b)h}{2}$

where h is the height and a and b represent the 2 bases of the trapezium.

In our case:

CD = 17x + 12x = 29x

$A = \frac{(29x+13x)5x}{2}$

$A = \frac{210x^2}{2}$

$A = 105x^2$

(b) Angle $BCD = y°$. Calculate the value of y. [2]

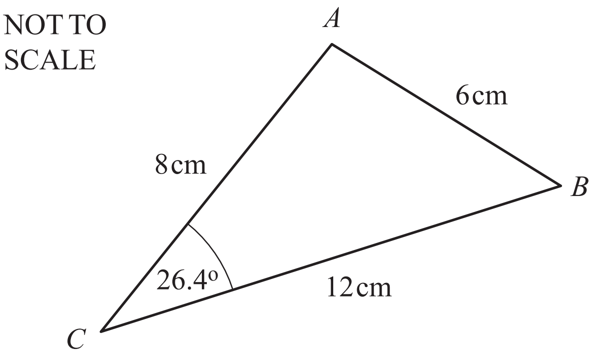
In the right-angled triangle with the hypothenuse BC:

$\tan y = \frac{5x}{12x}$

$\tan y = \frac{5}{12}$

$y = 22.6°$

In triangle ABC, $AB = 6$ cm, $AC = 8$ cm and $BC = 12$ cm. Angle $ACB = 26.4°$.
Calculate the area of the triangle ABC. [2]



We can use the following formula for the area of a triangle:

$$A = \frac{ab \sin C}{2}$$

Where a and b are 2 sides of the triangle and angle C is the angle between them.

In our case:

$$A = \frac{8 \times 12 \times \sin 26.4°}{2}$$

A = 21.3 cm²

Circle Problems
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	Circle Problems
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **37 minutes**

Score: **/29**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

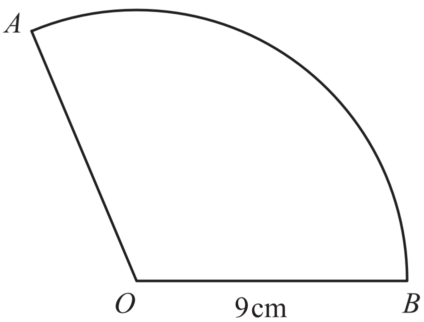
CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

AB is an arc of a circle, centre O, radius 9 cm.
The length of the arc AB is 6π cm.
The area of the sector AOB is $k\pi\,\text{cm}^2$.

Find the value of k.

[3]



The length of an arc is given by

$$l = r\theta$$

Where r is the radius and θ is the angle of the sector. Using this we have

$$6\pi = 9 \times \theta$$

$$\theta = \frac{6\pi}{9}$$

$$\theta = \frac{2}{3}\pi$$

The area of a sector is given by

$$A = \frac{1}{2}r^2\theta$$

So, using our known values, we have

$$A = \frac{1}{2}(9)^2 \times \frac{2}{3}\pi$$

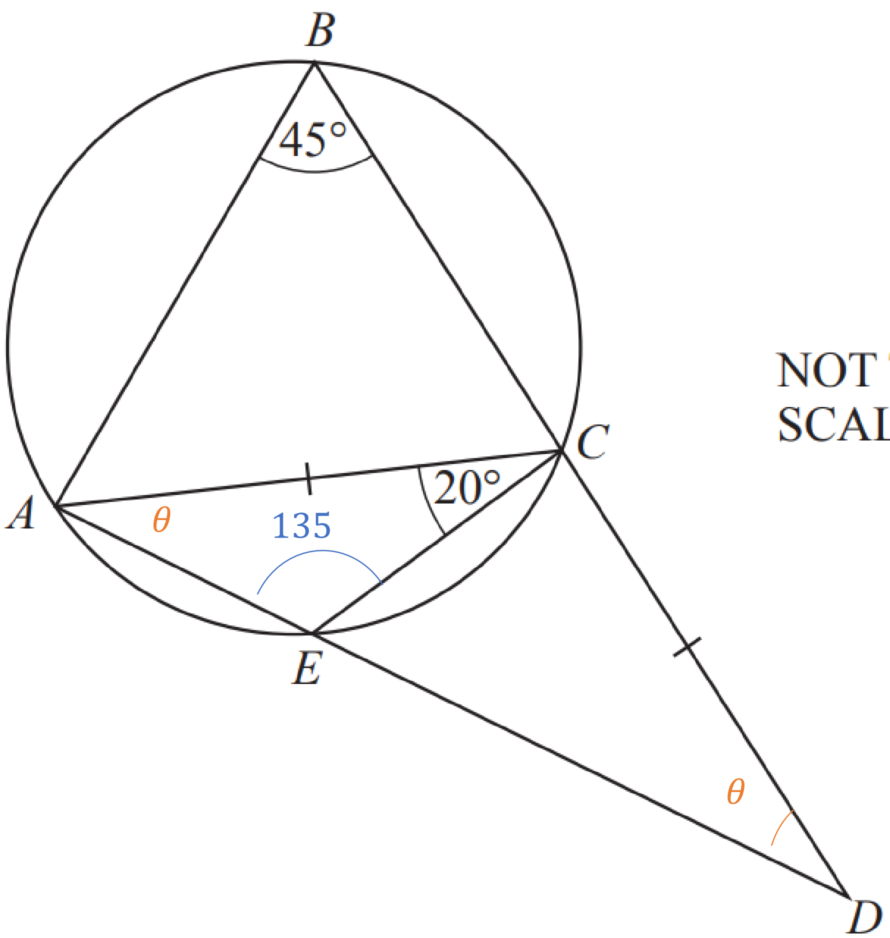
$$A = \frac{1}{2} \times 81 \times \frac{2}{3}\pi$$

$$A = \frac{81}{3}\pi$$

$$A = 27\,\pi$$

$$\boldsymbol{k = 27}$$

Question 2



$ABCE$ is a cyclic quadrilateral.
AED and BCD are straight lines.
$AC = CD$, angle $ABC = 45°$ and angle $ACE = 20°$.

Work out angle ECD. [3]

The interior angles of quadrilateral ABCE must sum to 360°.

The interior angles of any of the triangles must sum to 180°.

Triangle ACD is isosceles so CAD is equal to ADC.

Opposite angles in a cyclic quadrilateral sum to 180

$$ABC + AEC = 180$$

$$\rightarrow AEC = 180 - 45$$

$$\rightarrow AEC = 135°$$

This allows us to find CAE

$$CAE + 20 + 135 = 180°$$

$$\rightarrow CAE = 25°$$

$$\rightarrow ADC = 25°$$

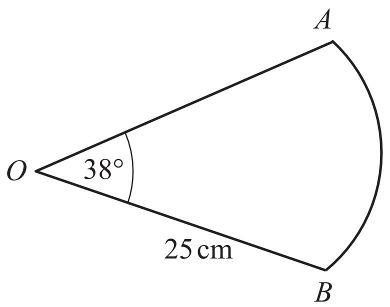
We can now use this to find

$$ADC + CAD + ACD = 180°$$

$$\rightarrow 25 + 25 + (20 + ECD) = 180°$$

$$\rightarrow ECD = 180 - 70$$

$$= \mathbf{110°}$$



The diagram shows a sector of a circle, centre O, radius 25 cm.
The sector angle is 38°.

Calculate the length of the arc AB.
Give your answer correct to 4 significant figures. [3]

The circumference of a circle is $2\pi r$.

Therefore, the circumference of the total circle that this sector has been cut from is:

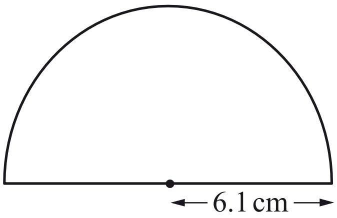
$2\pi(25) = 50\pi\ cm.$

The arc of the sector is a fraction of the circumference of the whole circle.

To work out the fraction:

$$arc\ AB = \frac{angle\ AOB}{total\ angle\ of\ a\ circle} \times 50\pi = \frac{38°}{360°} \times 50\pi$$

$$= \mathbf{16.58\ cm\ (4\ sf)}$$



NOT TO
SCALE

A protractor is a semi-circle of radius 6.1 cm.

Calculate the **perimeter** of the protractor. [3]

The perimeter of a protractor is the sum of two radii and the arc length of the semi-circle.

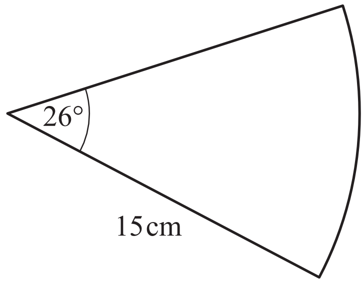
The arc length is half the circumference of the circle ($\frac{1}{2}2\pi r$).

$$r + r + \pi r$$

$$6.1cm + 6.1cm + \pi \times 6.1cm$$

Add the terms to get the perimeter of the protractor.

$$\mathbf{31.4}\boldsymbol{cm}$$



NOT TO SCALE

The diagram shows a sector of a circle with radius 15 cm.

Calculate the perimeter of this sector. [3]

The perimeter of a sector is the sum of two radii and the arc length of the semi-circle.

The arc length is he circumference of the circle multiplied by the ratio of the sector angle and 360°.

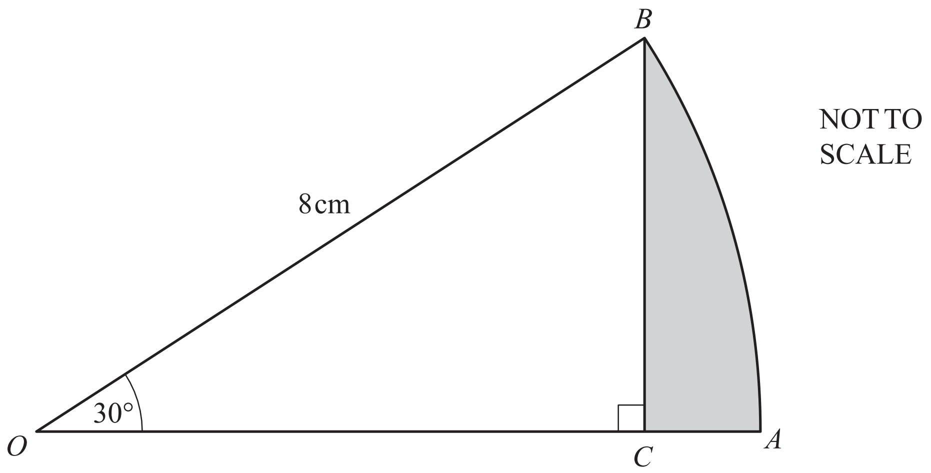
$$(2\pi r \times \frac{sector\ angle}{360°}).$$

$$r + r + 2\pi r \times \frac{sector\ angle}{360°}$$

$$15cm + 15cm + 2\pi \times 15cm \times \frac{26°}{360°}$$

Add the terms to get the perimeter of the sector.

$$\mathbf{36.3cm}$$



OAB is the sector of a circle, centre O, with radius 8 cm and sector angle 30°.
BC is perpendicular to OA.

Calculate the area of the region shaded on the diagram. [5]

The area of the shaded region can be calculated by subtracting the area of the white triangle from the area of the sector.

$$shaded\ area = sector\ area - triangle\ area$$

The area of a sector with radius *r* is given by:

$$sector\ area = \pi r^2 \times \frac{secotr\ angle}{360°}$$

In our case, *r*=8 cm and the sector angle is 30°.

$$sector\ area = \pi(8cm)^2 \times \frac{30°}{360°}$$

$$sector\ area = 16.76\ cm^2$$

The area of a triangle OCB is given by:

$$triangle\ area = \frac{1}{2} BO \times OC \times \sin(BOC)$$

We know BO and the angle BOC, but not OC. The length of this side can be calculated using trigonometry (the triangle is a right-angle triangle).

$$\cos(\mathrm{BOC}) = \frac{OC}{BO}$$

By rearranging: $OC = \mathrm{BO}\ \times \cos(BOC)$

$$triangle\ area = \frac{1}{2}\mathrm{BO}\ \times \cos(BOC) \times BO \times \sin(BOC)$$

$$triangle\ area = \frac{1}{2}\ (8cm) \times \cos(30°) \times (8cm) \times \sin(30°)$$

Use a calculator to find the area of the triangle.

$$triangle\ area = 13.86\ cm^2$$

Subtract the areas to work out the shaded area.

$$shaded\ area = 16.76\ cm^2 - 13.86\ cm^2$$

$$shaded\ area = \mathbf{2.90\ cm^2}$$

The circumference of a circle is 30 cm

(a) Calculate the radius of the circle. [2]

The circumference of a circle is given by 2π*r* where *r* is the radius of the circle.

The circumference of a circle is 30cm.

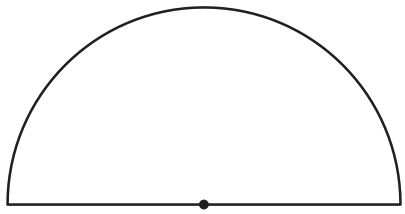
$$30cm = 2\pi r$$

Divide both sides by 2π to get the radius.

$$r = \frac{30}{2\pi} cm$$

$$\boldsymbol{r = 4.77cm}$$

(b)



The length of the arc of the semi-circle is 15cm.

Calculate the area of the semi-circle. [2]

The length of an arc of the semi-circle is π*r* where *r* is the radius of the semi-circle.

The length of the arc is 15cm.

$$15cm = \pi r$$

Divide both sides by π to get the radius.

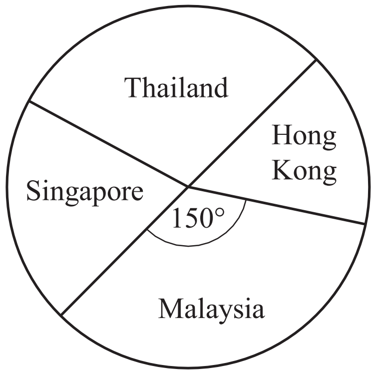
$$r = \frac{15}{\pi} cm$$

The area of the semi-circle is given as: $\frac{1}{2}\pi r^2$

$$Area = \frac{1}{2}\pi\left(\frac{15}{\pi}cm\right)^2$$

Use a calculator to get the value of the area:

$$\boldsymbol{Area = 35.8cm^2}$$



NOT TO SCALE

A travel brochure has 72 holidays in four different countries.
The pie chart shows this information.

(a) There are 24 holidays in Thailand.

Show that the sector angle for Thailand is 120°. [2]

The sum of all the angles around the centre in a pie chart is 360°.

360° correspond to the total number of holidays listed, 72.

24 holidays represent:

$\frac{24}{72} = \frac{1}{3}$ of the total number of holidays.

Therefore, the angle representing 24 holidays would be $\frac{1}{3}$ of the total angle, 360°.

$360° \times \frac{1}{3}$

= **120°**

(b) The sector angle for Malaysia is 150°.
The sector angle for Singapore is twice the sector angle for Hong Kong.

Calculate the number of holidays in Hong Kong. [3]

To calculate the number of holidays in one sector we need to know what proportion that sector represents out of the whole pie chart. To do that, we can work out what proportion does its angle represents out of the total angle of 360°.

We represent with the unknown a, the angle of the sector representing Hong Kong.

The sector representing Singapore is twice the size of the sector representing Hong Kong, therefore, the size of the angle for the Singapore sector will be twice the size of the angle for the sector representing Hong Kong

Singapore angle = 2a

We know that the sum of all the angles is 360°.

We also know from a), that the angle for the sector representing Thailand is 120°.

2a + a + 150° + 120° = 360°

3a = 90°

a = 30°

Thus, the number of holidays in Hong Kong $= \frac{30}{360} \times 72$

$$= 6\ holidays$$

Circle Problems
Difficulty: Easy

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	Circle Problems
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 2

Time allowed: **40 minutes**

Score: **/31**

Percentage: **/100**

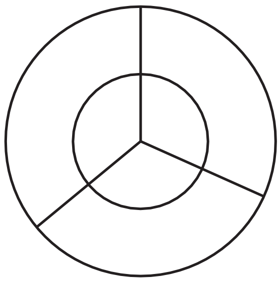
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

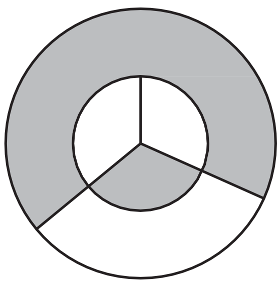
9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



NOT TO SCALE

The diagram shows two concentric circles and three radii.
The diagram has rotational symmetry of order 3.

A club uses the diagram for its badge with some sections shaded.
The radius of the large circle is 6 cm and the radius of the small circle is 4 cm.



NOT TO SCALE

Calculate the total perimeter of the shaded area. [5]

Perimeter is the small circle's circumference, plus 2/3 of the large circle's circumference, plus the connecting lines

$$P = 2\pi(4) + \frac{2}{3} \times 2\pi(6) + 2 \times 6$$

$$= 16\pi + 12$$

$$= \mathbf{62.3}$$

Find the circumference of a circle of radius 2.5 cm. [2]

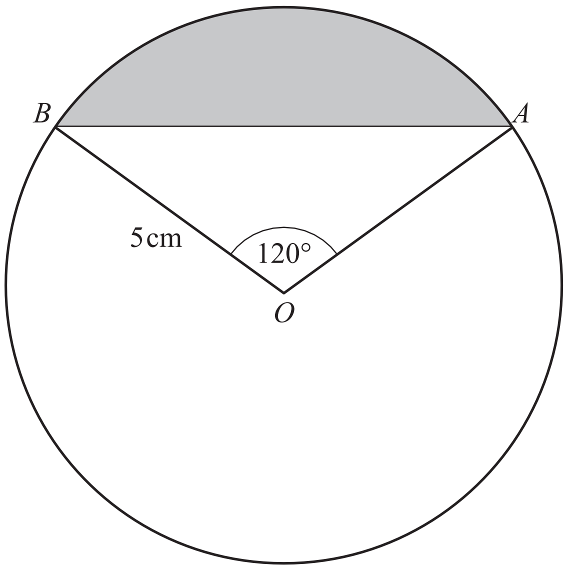
Equation for circumference:

$$2 \times \pi \times r$$

And r = 2.5cm, substitute this value into the formula above.

$$Circumference = 2 \times \pi \times 2.5$$

$$= \mathbf{15.7cm}$$



NOT TO SCALE

A and B lie on a circle centre O, radius 5 cm.
Angle $AOB = 120°$.

Find the area of the shaded segment. [4]

We need to find the area of the entire segment of the circle and subtract the area of the triangle AOB.

The area of the segment:

$$\frac{120°}{360°} \times \pi \times 5^2 = \frac{25}{3}\pi$$

The area of the triangle is $\frac{1}{2}ab\sin(c)$

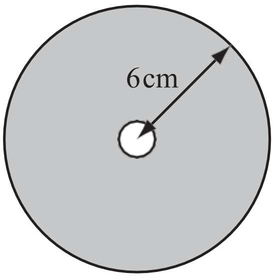
$$= \frac{1}{2} \times 5 \times 5 \times \sin(120)$$

$$= \frac{25\sqrt{3}}{4}$$

Therefore, the shaded area is:

$$= \frac{25}{3}\pi - \frac{25\sqrt{3}}{4}$$

$$= \mathbf{15.4}$$



The diagram shows a circular disc with radius 6 cm.
In the centre of the disc there is a circular hole with radius 0.5 cm.

Calculate the area of the shaded section. [3]

The area of a circle is

$$A = \pi r^2$$

The area of the big circle (without the hole cut out) is

$$A_b = \pi \times 6^2$$

$$= 36\pi$$

The area of the hole is

$$A_h = \pi \times 0.5^2$$

$$= 0.25\pi$$

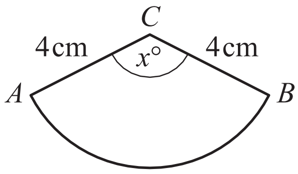
The area of the shaded area is then

$$A = A_b - A_h$$

$$= 36\pi - 0.25\pi$$

$$= 35.75\pi$$

$$= \mathbf{112}$$



NOT TO SCALE

ABC is a sector of a circle, radius 4 cm and centre C.
The length of the arc AB is 8 cm and angle $ACB = x°$.

Calculate the value of x. [3]

The length of an arc is calculated as

$$l_{arc} = r \times \theta$$

Where r is the radius and θ is the angle of the sector measured in radians.

We have

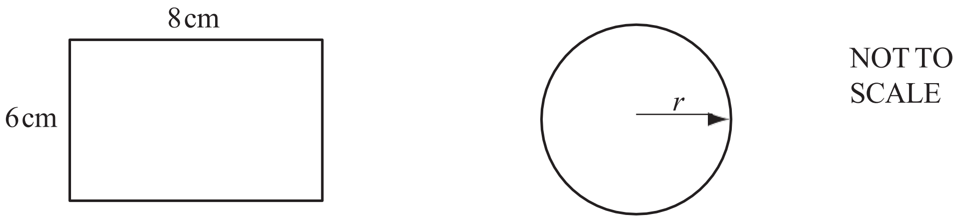
$$8 = 4x$$

$$\rightarrow x = 2\ radians$$

Converting this into degrees

$$x = \frac{2}{\pi} \times 180$$

$$= \mathbf{114.6}$$



The perimeter of the rectangle is the same length as the circumference of the circle.

Calculate the radius, r, of the circle. [3]

Perimeter of the rectangle is

$$6 + 6 + 8 + 8$$

$$= 28$$

Perimeter of the circle is

$$2\pi r = 28$$

$$\rightarrow r = \frac{14}{\pi}$$

$$= \mathbf{4.46}$$

A circle has a radius of 50 cm.

(a) Calculate the area of the circle in cm^2. [2]

The area of a circle is:

A = πr^2

A = $\pi \times 50^2$

A = 2500 π cm^2

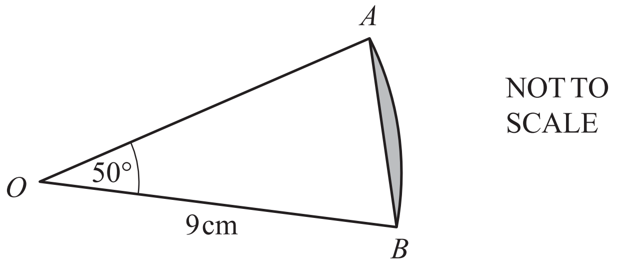
A = 7853 cm^2

(b) Write your answer to part (a) in m^2. [1]

1 m^2 = 10000 cm^2

A = 7853 cm^2

A= 0.7853 m^2



The diagram shows a sector AOB of a circle, centre O, radius 9 cm with angle $AOB = 50°$.

Calculate the area of the segment shaded in the diagram. [4]

The area of the shaded region is the difference between the area of the sector and the area of the triangle AOB.

A sector = $\pi r^2 \frac{AOB}{360°}$

A sector = $\pi 9^2 \frac{50°}{360°}$

A sector = $11.25\,\pi$

A sector = 35.342 cm^2

A triangle = $\frac{\text{ab sin C}}{2}$

Where a and b are 2 sides of the triangle and C is the angle between them.

A triangle = $\frac{9 \times 9 \times \sin 50°}{2}$

sin 50° = 0.766

A triangle = $\frac{81 \times 0.766}{2}$

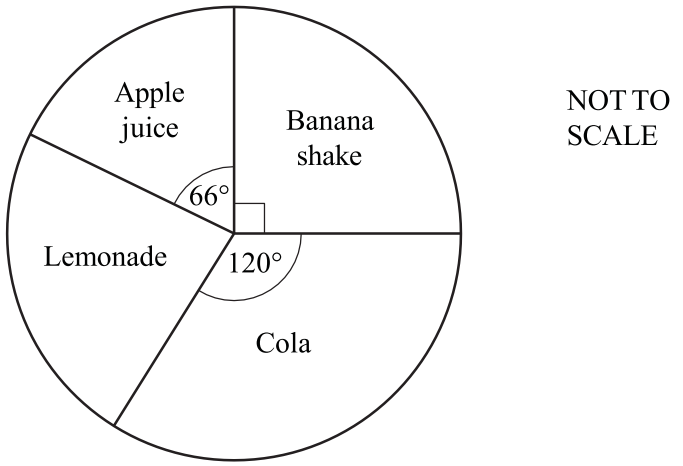
A triangle = 31.023 cm^2

A shaded region = A sector – A triangle

A shaded region = 35.342 cm^2 – 31.023 cm^2

A shaded region = 4.319 cm^2

60 students recorded their favourite drink.
The results are shown in the pie chart.



(a) **Calculate** the angle for the sector labelled Lemonade. [1]

The sum of the four angles around the centre is a full angle (360°).

The square represents a right angle (90°).

$$360° = 66° + 90° + 120° + lemonade$$

Therefore we can calculate the angle for the sector labelled Lemonade.

$$\boldsymbol{lemonade = 84°}$$

(b) Calculate the number of students who chose Banana shake. [1]

The angle for the Banana shake sector is 90°. Divide this angle by the full angle to get the proportion of students who chose Banana shake.

$$Banana\ shake\ proportion = \frac{90°}{360°} = 0.25$$

Multiply the total number of students to get the number of students who chose Banana shake. In total, 60 students recorded their favourite drink.

$$Banana\ shake\ = 60 \times 0.25$$

$$\boldsymbol{Banana\ shake\ = 15}$$

(c) The pie chart has a radius of 3 cm.
Calculate the arc length of the sector representing Cola. [2]

Arc length of a sector or with angle x° for a circle with a radius r is given by:

$$arc\ length = 2\pi r \times \frac{x°}{360°}$$

The sector for Cola has angle

$$x = 120°$$

and our radius is

$$r = 3cm.$$

Using the given values:

$$arc\ length = 2\pi \times 3cm \times \frac{120°}{360°}$$

$$arc\ length = 2\pi\ cm$$

Therefore the arc length of the sector (correct to three significant figures):

$$\boldsymbol{arc\ length = 6.28\ cm}$$

Circle Problems
Difficulty: Easy

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	Circle Problems
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 3

Time allowed: **40 minutes**

Score: **/31**

Percentage: **/100**

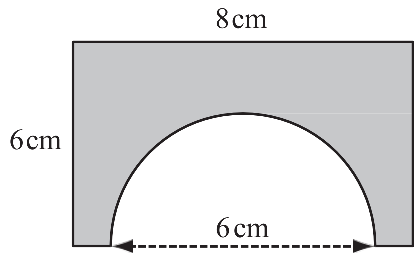
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

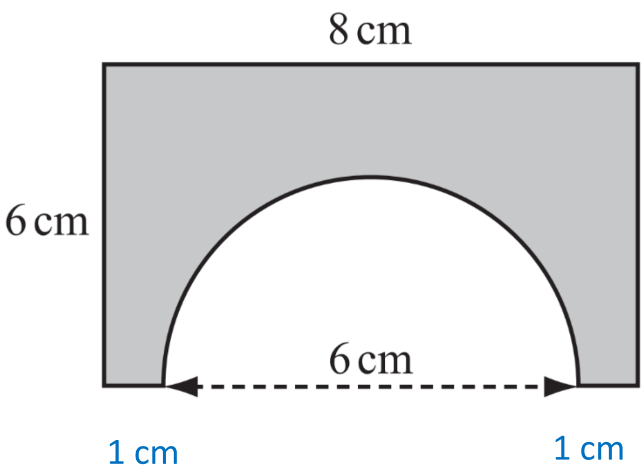
9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



NOT TO SCALE

A semicircle of diameter 6 cm is cut from a rectangle with sides 6 cm and 8 cm.

Calculate the perimeter of the shaded shape, correct to 1 decimal place. [3]



We need to find the length of the arc made by cutting the semi-circle from the rectangle.

The circle has radius 3 cm and an angle of π radians.

The length of an arc is

$$l = r\theta$$

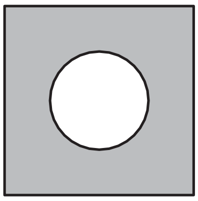
$$\rightarrow l = 3\pi$$

Hence

$$P = 8 + 6 + 1 + 3\pi + 1 + 6$$

$$= 24 + 3\pi$$

$$= \mathbf{31.4}$$



NOT TO
SCALE

The diagram shows a circle of radius 5cm in a square of side 18cm.

Calculate the shaded area. [3]

The shaded area is

$$A_{shaded} = A_{square} - A_{circle}$$

We have

$$A_{square} = 18^2$$

$$= 324$$

and

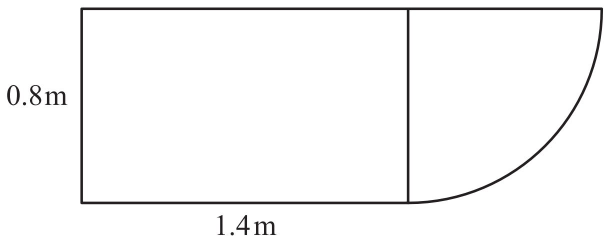
$$A_{circle} = \pi r^2$$

$$= 5^2\pi$$

$$= 78.54$$

Hence

$$\boldsymbol{A_{shaded} = 245}$$



NOT TO SCALE

The top of a desk is made from a rectangle and a quarter circle.
The rectangle measures 0.8m by 1.4m.

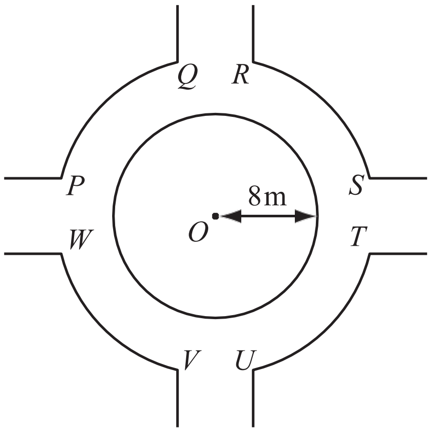
Calculate the surface area of the top of the desk. [3]

The radius of the quarter circle is 0.8 m.

Adding the area of the two shapes together

$$1.4 \times 0.8 + \frac{1}{4} \times \pi(0.8)^2$$

$$= \mathbf{1.62}$$



The diagram shows the junction of four paths.
In the junction there is a circular area covered in grass.
This circle has centre O and radius 8 m.

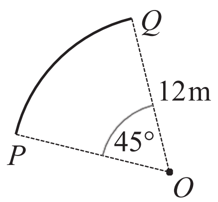
(a) Calculate the area of grass. [2]

$A = \pi r^2$

$= 64\pi$

$= \mathbf{201}$

(b)



The arc PQ and the other three identical arcs, RS, TU and VW are each part of a circle, centre O, radius 12m.
The angle POQ is 45°.
The arcs PQ, RS, TU, VW and the circumference of the circle in part(a) are painted white.
Calculate the total length painted white. [4]

Length of an arc is

$$l = r\theta\frac{\pi}{180}$$

We have four identical arcs plus the circumference of the

inner circle, so the total length is

$$4 \times 12 \times \frac{45\pi}{180} + 2\pi \times 8$$

$$= 28\pi$$

$$= \mathbf{88.0}$$

A spacecraft made 58 376 orbits of the Earth and travelled a distance of 2.656×10^9 kilometres.

(a) Calculate the distance travelled in 1 orbit correct to the nearest kilometre. [2]

$$(2.656 \times 10^9) \div 58\,376$$

$$= \mathbf{45\,498}$$

(b) The orbit of the spacecraft is a circle.

Calculate the radius of the orbit. [2]

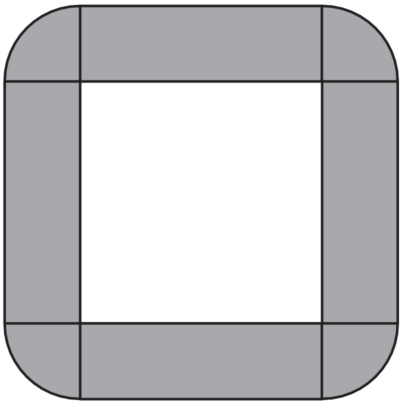
We have that

$$2\pi r = 45498$$

$$\rightarrow r = \frac{45498}{2\pi}$$

$$= \mathbf{7241}$$

A large conference table is made from four rectangular sections and four corner sections.
Each rectangular section is 4 m long and 1.2 m wide.
Each corner section is a quarter circle, radius 1.2 m.



NOT TO SCALE

Each person sitting at the conference table requires one metre of its outside perimeter.
Calculate the greatest number of people who can sit around the **outside** of the table.
Show all your working. [3]

To work out the greatest number of people who can sit around the table we need to calculate the outer length of the conference table and then divide it by the space required by a person, 1 m.

The outer length of the conference table is the sum of one length for each of the 4 rectangles and the arc length of the 4 corner sections.

The angle of one of the arcs is 90°. The length of one of the arcs is:

Arc length = $2\,\pi\,r\,\frac{90°}{360°}$

Arc length = $2.4\,\pi$ x 0.25 m

Arc length = 1.88 m

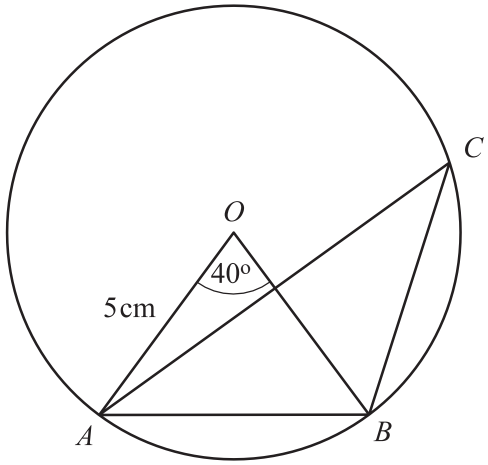
The outer length of the conference table is:

L = 4 x 4 m + 4 x 1.88 m

L = 23.52 m

Keeping in mind that the greatest number of people which can sit around the conference table needs to be a whole number, we work out the answer 23, since there are 23.53 m in total.

The number of people is 23.



NOT TO SCALE

A, B and C are points on a circle, centre O.
Angle $AOB = 40°$.

(a) (i) Write down the size of angle ACB. [1]

In a circle, the central angle Is twice the size of any inscribed angle subtended by the same arc.

In our case, the central angle is 40° and angle CAB is an inscribed angle subtended by the arc AB.

Angle CAB = $\frac{\text{angle OAB}}{2}$

Angle CAB = 20°

(ii) Find the size of angle OAB. [1]

The triangle OAB is isosceles with the radius of the circle OA = OB = 5 cm.

Therefore, the 2 angles OAB and OBA are congruent.

Also, the sum of the interior angles in the triangle OAB is 180°.

40° + 2 x angle OAB = 180°

2 x angle OAB = 140°

Angle OAB = 70°

(b) The radius of the circle is 5 cm.

(i) Calculate the length of the minor arc AB. [2]

The formula for an arc length is:

$$\boxed{\text{Arc length} = 2\,\pi\,r\,\frac{C}{360^\circ}}$$

where C is the central angle and r is the radius of the circle

$$AB = 2\,\pi\,5\,\frac{AOB}{360^\circ}$$

$$AB = \frac{40^\circ}{360^\circ}\,10\,\pi$$

AB = 3.49 cm

(ii) Calculate the area of the minor sector OAB. [2]

The formula for the area of a sector is:

$$\boxed{\text{Sector area} = \pi\,r^2\,\frac{C}{360^\circ}}$$

where C is the central angle and r is the radius of the circle

$$\text{Area} = \pi\,5^2\,\frac{40^\circ}{360^\circ}$$

Area = 8.73 cm^2

The radius of the Earth at the equator is approximately 6.4×10^6 metres.
Calculate the circumference of the Earth at the equator. Give your answer in standard form, correct to 2 significant figures. [3]

Circumference = $2\pi r$

Circumference = 2π x 6.4 x 10^6

Circumference = 40.21 x 10^6

A number in standard form takes up the form: a x 10^n

where n is an integer and 0 < a < 10.

In our case, 40.21 x 10^6 = 4.021 x 10^7

Correct to 2 significant figures

= **4.0 x 10^7**

Circle Problems
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	Circle Problems
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **41 minutes**

Score: **/32**

Percentage: **/100**

Grade Boundaries:

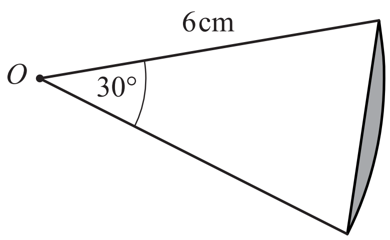
CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

Question 1



NOT TO SCALE

The diagram shows a sector of a circle, centre O and radius 6cm.
The sector angle is 30°.
The area of the shaded segment is $(k\text{r} - c)$ cm^2, where k and c are integers.

Find the value of k and the value of c. [3]

Area of the sector is

$$A_{sector} = \frac{1}{2}\pi r^2 \times \frac{\theta}{180}$$

$$= \frac{1}{12} \times 36 \times \pi$$

$$= 3\pi$$

Using the Sine Rule for the area of the triangle:

$$Area = \frac{1}{2}absinC$$

$$A_{triangle} = \frac{1}{2} \times 6 \times 6 \times \sin 30$$

$$= 9$$

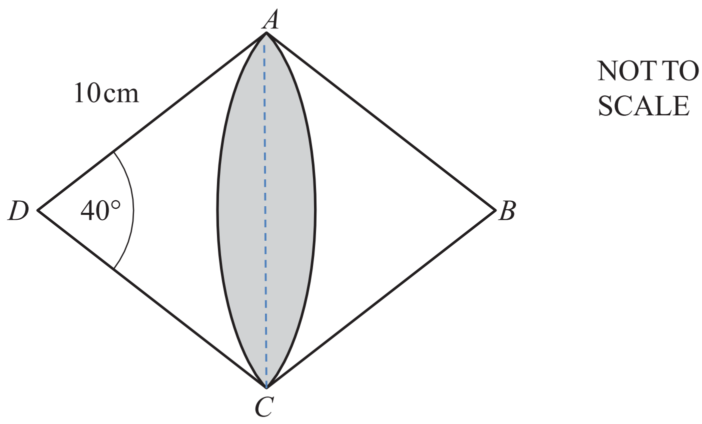
Area of shaded section is then

$$A_{shaded} = 3\pi - 9$$

$$\rightarrow k = 3$$

$$\rightarrow c = 9$$

ABCD is a rhombus with side length 10cm.



Angle *ADC* = 40°.
DAC is a sector of a circle with centre *D*.
BAC is a sector of a circle with centre *B*.

Calculate the shaded area. [4]

The area of a sector is

$$A = \frac{1}{2} \times \frac{\theta}{180} r^2$$

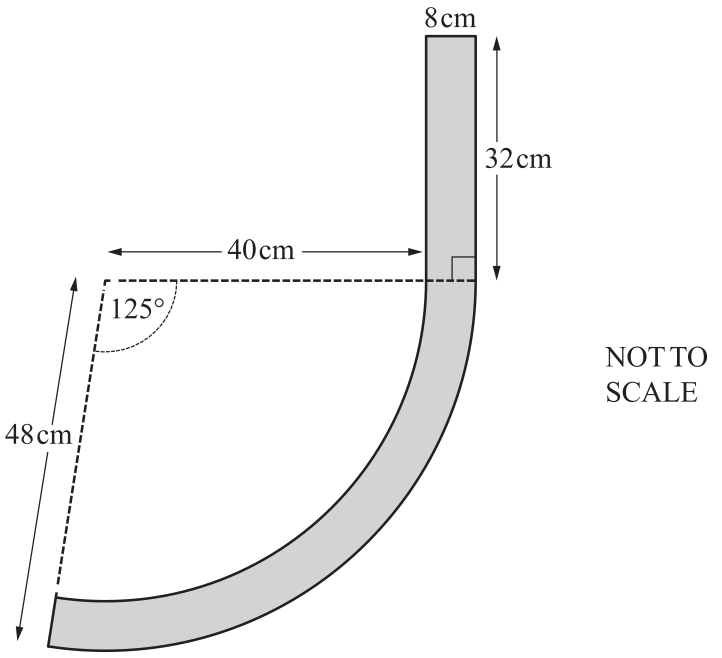
The *Sine Rule* for area of a general triangle is

$$A_t = \frac{1}{2} ab \sin C$$

The area of the shaded can be see as the area of the sector minus the area of the triangle and then multiplied by 2, due to symmetry

$$2 \times \left\{ \left(\frac{40}{360} \times 10^2 \right) - \frac{1}{2} \times 10^2 \times \sin 40 \right\}$$

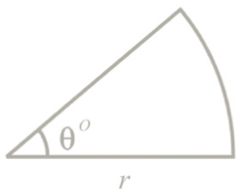
$$= \mathbf{5.53}$$



The diagram shows the cross section of part of a park bench.
It is made from a rectangle of length 32 cm and width 8 cm and a curved section.
The curved section is made from two concentric arcs with sector angle 125°.
The inner arc has radius 40 cm and the outer arc has radius 48 cm.

Calculate the area of the cross section correct to the nearest square centimetre. [5]

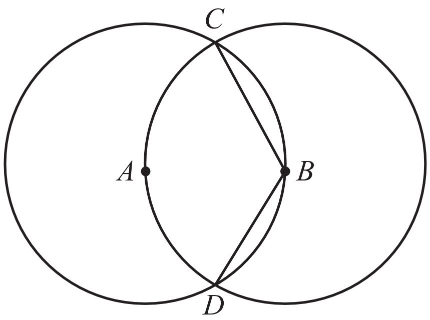
Area of a Sector $= \frac{\theta}{360} \times \pi r^2$



Cross-Section = Rectangle + Bigger Sector − Smaller Sector

$$= 8 \times 32 + \frac{125}{360} \times \pi \times 48^2 - \frac{125}{360} \times \pi \times 40^2$$

$= \mathbf{1024}$ **cm²** (to nearest whole number)



NOT TO SCALE

Two circles, centres A and B, are each of radius 8cm and intersect at C and D.
Each circle passes through the centre of the other circle.

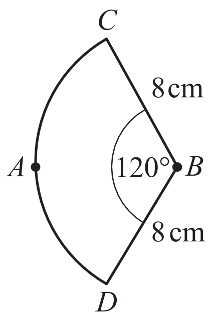
(a) Explain why angle CBD is 120°. [1]

Angle CBD must be 120° because triangles CBA and BDA would both be equilateral triangles. As all angles in an equilateral triangle are equal, they must all be 60° (because the angles inside a triangle always add up to 180°, and 180 ÷ 3 = 60).

We can see by inspection, that angle CBD must be angle CBA plus angle DBA

$$\boldsymbol{CBA + DBA = 60° + 60° = 120°}$$

(b) For the circle, centre B, find the area of the sector BCD. [2]



NOT TO SCALE

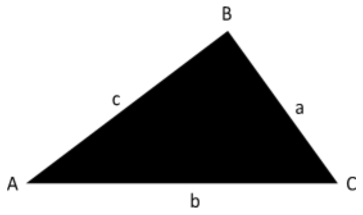
We can use a simple formula to find the area of any sector:

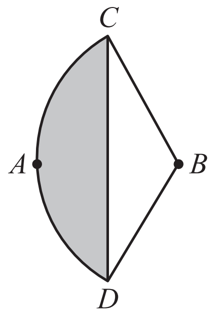
$$Area\ of\ a\ sector = \frac{\theta}{360} \times \pi r^2$$

$$Area_{BCD} = \frac{120}{360} \times \pi(8)^2$$

$$\boldsymbol{Area_{BCD} = 67\ cm^2}$$

(c) (i) Find the area of the shaded segment CAD.





NOT TO SCALE

[3]

We can find the area of the segment CAD (diagram on right) by finding the area of triangle CBD and taking it away from the total area of the sector (from question 19b).

Area of a triangle ABC (diagram on left) $= \frac{1}{2}ab\sin C$

Area of triangle CBD $= \frac{1}{2} \times 8 \times 8 \times \sin 120$

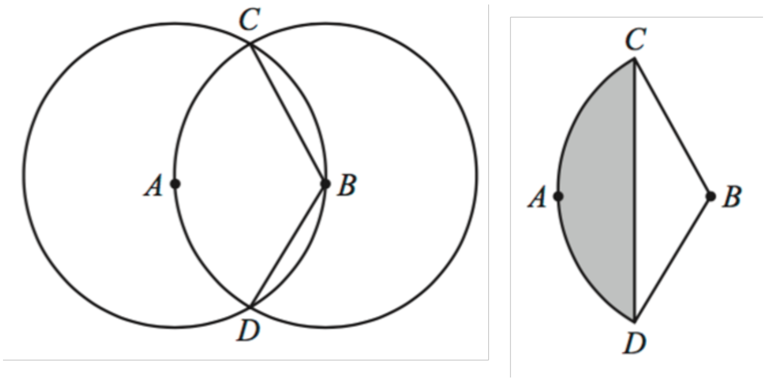
$Area_{CAD} = Area_{sector} - Area_{triangle}$

Therefore thearea of CAD is

$\boldsymbol{Area_{CAD} = 67 - \left(\frac{1}{2} \times 8 \times 8 \times \sin 120\right)}$

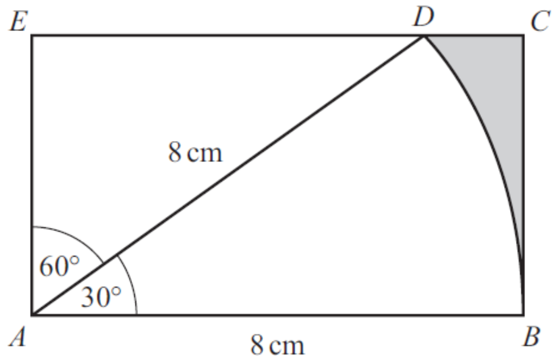
$\boldsymbol{Area_{CAD} = 39.3cm^2}$

(ii) Find the area of overlap of the two circles. [1]



We can see from the 2 diagrams that the area between the 2 circles will be double the area of segment CAD. We calculated CAD in the last question, so we can just use our answer:

$$39.3 \times 2 = 78.6$$



NOT TO SCALE

The diagram shows a rectangle *ABCE*.
D lies on *EC*.
DAB is a sector of a circle radius 8 cm and sector angle 30°.

Calculate the area of the shaded region. [7]

The area of the shaded region can be worked out as the difference between the area of the rectangle ABCE and the sum of the areas representing the sector BAD and the right-angled triangle AED.

The formula for the area of the sector BAD is:

A = $\pi r^2 \frac{angle\ BAD}{360°}$

r = 8 cm

A = $\pi 8^2 \frac{30°}{360°}$ cm^2

A = $\pi 64 \frac{1}{12}$ cm^2

A = 16.76 cm^2

The area of the rectangle ABCE is:

A = length x width

Length = AB

Width = AE

In the right-angled triangle AED:

cos EAD = $\frac{\text{EA}}{\text{AD}}$

cos 60° = $\frac{\text{EA}}{8 \text{ cm}}$

EA = 8 cm x $\frac{1}{2}$

EA = 4 cm

In the right-angled triangle AED:

sin EAD = $\frac{\text{ED}}{\text{AD}}$

sin 60° = $\frac{\text{ED}}{8 \text{ cm}}$

ED = 8 cm x $\frac{\sqrt{3}}{2}$

ED = $4\sqrt{3}$ cm

The area of the right-angled triangle EAD is:

A = $\frac{\text{AE x ED}}{2}$

A = $\frac{4 \text{ cm x } 4\sqrt{3} \text{ cm}}{2}$

A = $8\sqrt{3}$ cm^2

A = 13.86 cm^2

Therefore, the area of the rectangle ABCE is:

$A = AB \times AE$

$A = 8 \text{ cm} \times 4 \text{ cm}$

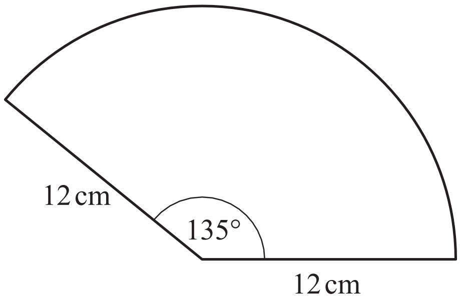
$A = 32 \text{ cm}^2$

The area of the shaded region is:

$A = A_{rectangle} - A_{sector} - A_{triangle}$

$A = 32 \text{ cm}^2 - 16.76 \text{ cm}^2 - 13.86 \text{ cm}^2$

$\mathbf{A = 1.38 \text{ cm}^2}$



NOT TO
SCALE

The diagram shows a sector of a circle of radius 12cm with an angle of 135°.

Calculate the perimeter of the sector. [3]

Here we can use fractions to calculate the perimeter of the sector.

We know that a circle has a total angle of 360°, and here we are looking at a sector of angle 135°. Hence the fraction of the circle we are looking at is

$$\frac{135}{360} = \frac{3}{8}$$

Now we want the perimeter of the total circle – this is an equation you should have memorised.

$$perimeter = circumference = 2\pi r$$

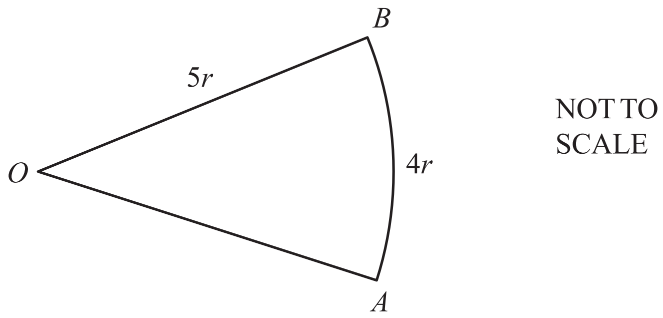
Now we only want the fraction we found of this total result, so we can multiply the two.

$$arc\ perimeter = \frac{3}{8} \times 2\pi r = \frac{3}{4}\pi r$$

We are given $r = 12cm$. We need to remember that the perimeter of this shape also includes two radii, (I.e. Arc length plus two straight sections (radii). Hence our total perimeter becomes:

$$total\ perimeter = \frac{3}{4}\pi r + r + r = \frac{3}{4}\pi r + 2$$

$$\boldsymbol{total\ perimeter = \frac{3}{4}\pi(12) + 2(12) = 52.3cm}$$



The diagram shows a sector of a circle, centre O, radius $5r$.
The length of the arc AB is $4r$.

Find the area of the sector in terms of r, giving your answer in its simplest form. [3]

The length of an arc is

$$l = R\theta$$

Where θ is the angle of the sector measured in radians, and R is the radius.

$$\rightarrow 4r = 5r\theta$$

$$\rightarrow \theta = \frac{4}{5}$$

The area of a sector is

$$A = \frac{1}{2}R^2\theta$$

$$\rightarrow A = \frac{1}{2}(5r)^2 \times \frac{4}{5}$$

$$= \mathbf{10r^2}$$

Circle Problems
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	Circle Problems
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **40 minutes**

Score: **/31**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

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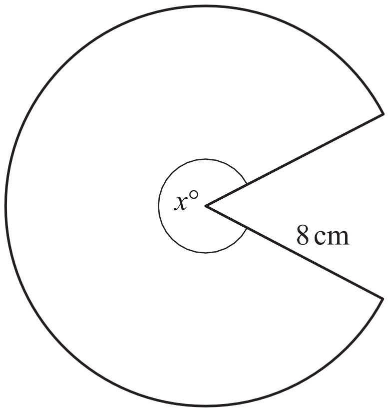
>94%	85%	77%	67%	57%	47%	35%



NOT TO
SCALE

The diagram shows a sector of a circle of radius 8 cm.
The angle of the sector is $x°$.
The perimeter of the sector is $(16 + 14\pi)$ cm.

Find the value of x. [3]

The perimeter of the sector is twice the radius plus the arc length.

Note that the arc length is

$$l = r\theta$$

Hence

$$16 + 8\theta = 16 + 14\pi$$

Where θ is x converted to radians.

Subtracting 16 from both sides gives us

$$8\theta = 14\pi$$

$$\rightarrow \theta = \frac{7}{4}\pi$$

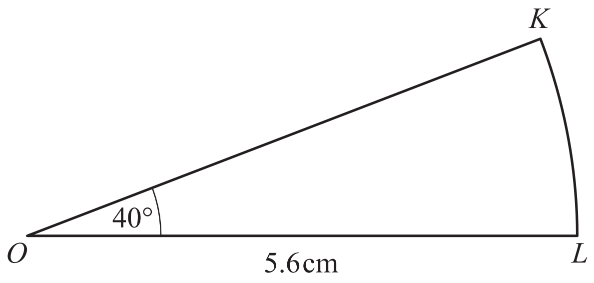
To convert x to radians we did

$$\frac{x}{180} \times \pi = \theta$$

$$\rightarrow \frac{x}{180}\pi = \frac{7}{4}\pi$$

Cancel the π and multiply through by 180

$$\boldsymbol{x = 315}$$



OKL is a sector of a circle, centre *O*, radius 5.6 cm.
Angle *KOL* = 40°.

Calculate

(a) the area of the sector, [2]

Area of a sector is

$$A = \frac{1}{2}\theta r^2$$

Where θ is measured in radians.

Converting our angle to radians we get

$$A = \frac{1}{2} \times \frac{40}{180}\pi \times 5.6^2$$

$$= \mathbf{10.9}$$

(b) the perimeter of the sector. [2]

Arc length is given by

$$KL = r\theta$$

Where, again, θ is measured in radians.

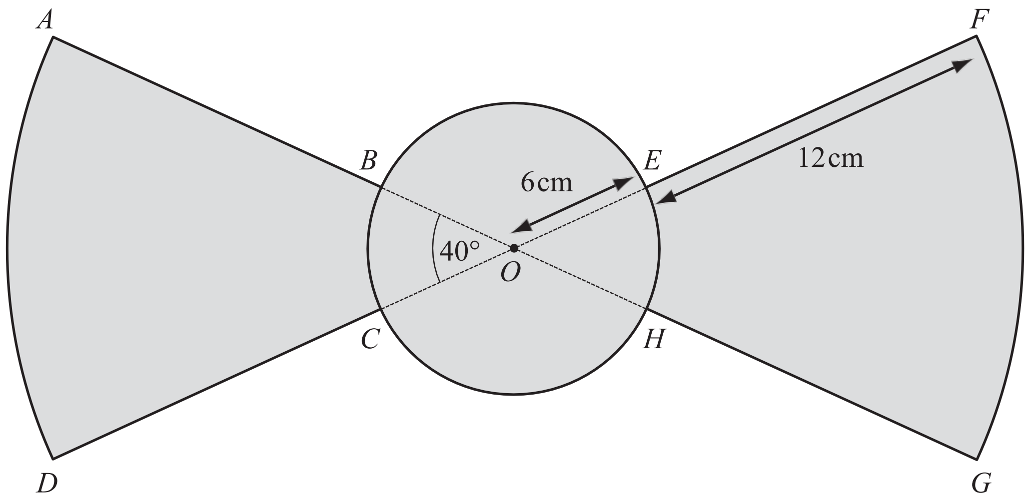
$$\rightarrow KL = 5.6 \times \frac{40}{180} \times \pi$$

$$= 3.9$$

Adding all sides lengths together

$$P = 5.6 + 5.6 + 3.9$$

$$= \mathbf{15.1}$$



The diagram shows part of a fan.
OFG and *OAD* are sectors, centre *O*, with radius 18 cm and sector angle 40°.
B, *C*, *H* and *E* lie on a circle, centre *O* and radius 6 cm.
Calculate the shaded area. [4]

The shaded area is worked out by summing up the area of a circle with the radius 6 cm and the area of the 2 equivalent sectors OFC ang OAD. The areas of the small equivalent sectors EOH and BOC need to be added only once.

The area of the circle is:

$A = \pi 6^2$

A = 113.097 cm^2

The area of one big sector is:

$$A = \pi r^2 \frac{40°}{360°}$$

$$A = \pi(6 + 12)^2 \frac{40°}{360°}$$

A = 113.097 cm^2

The area of one small sector is:

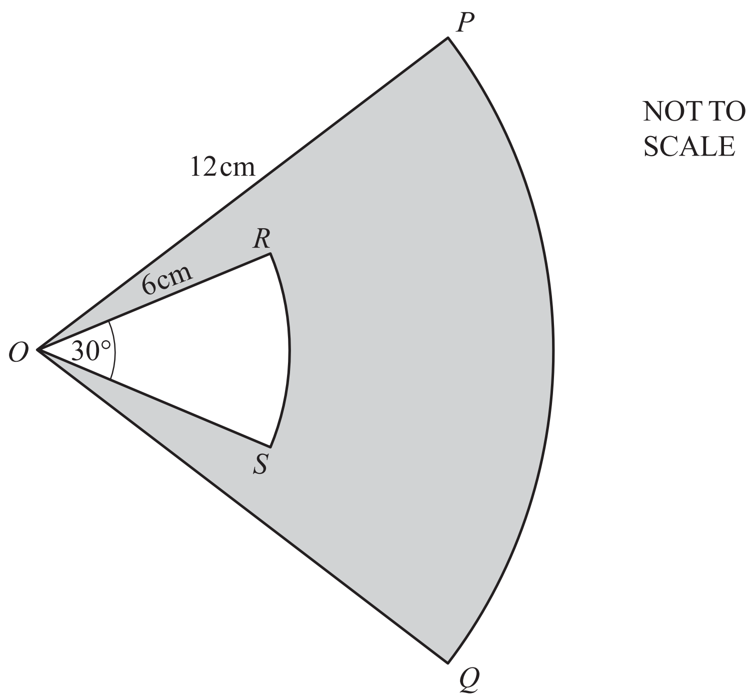
$$A = \pi r^2 \frac{40°}{360°}$$

$$A = \pi 6^2 \frac{40°}{360°}$$

A = 12.56 cm^2

Shaded area = 113.097 cm^2 + 2 x 113.097 cm^2 – 2 x 12.56 cm^2

Shaded area = 314 cm^2



OPQ is a sector of a circle, radius 12 cm, centre O. Angle $POQ = 50°$.

ORS is a sector of a circle, radius 6 cm, also centre O. Angle $ROS = 30°$.

(a) Calculate the shaded area. [3]

The area of a sector is

$$A = \frac{1}{2}r^2\frac{\theta\pi}{180}$$

Hence the area of OPQ is

$$A_{OPQ} = \frac{1}{2}(12)^2 \times \frac{50\pi}{180}$$

$$= 20\pi$$

and the area of ORS

$$A_{ORS} = \frac{1}{2}(6)^2 \times \frac{30\pi}{180}$$

$$= 3\pi$$

So the shaded area is

$$20\pi - 3\pi$$

$$= 17\pi$$

$$= \mathbf{53.4}$$

(b) Calculate the perimeter of the shaded area, *PORSOQP*. [3]

The length of an arc is

$$\boldsymbol{l = r\frac{\theta\pi}{180}}$$

Hence

$$PQ = 12 \times \frac{50\pi}{180}$$

$$= \frac{10}{3}\pi$$

and

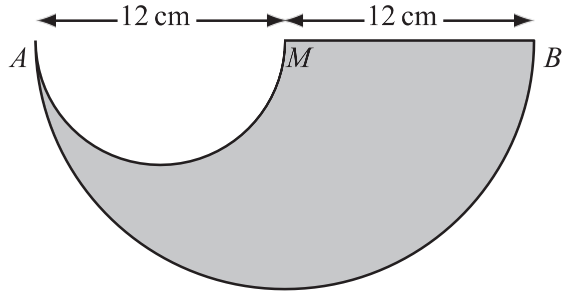
$$RS = 6 \times \frac{30\pi}{180}$$

$$= \pi$$

So, the perimeter is

$$12 + 6 + \pi + 6 + 12 + \frac{10}{3}\pi$$

$$= \mathbf{49.6}$$



The shape above is made by removing a small semi-circle from a large semi-circle.
$AM = MB = 12\,\text{cm}$
Calculate the area of the shape.

[3]

Area of the large semi-circle

$$A_l = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\pi \times 144$$

$$= 72\pi$$

Area of small semi-circle

$$A_s = \frac{1}{2}\pi(6)^2$$

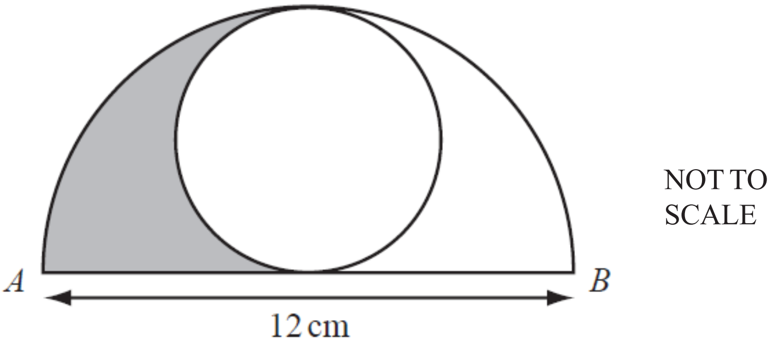
$$= 18\pi$$

Hence, shaded area is

$$72\pi - 18\pi$$

$$= 54\pi$$

$$= \mathbf{170}$$



The largest possible circle is drawn inside a semicircle, as shown in the diagram.
The distance AB is 12 centimetres.

(a) Find the shaded area. [4]

AB is the diameter of the semicircle.

For the small circle inside it, the radius of the semicircle would represent the diameter.

Therefore, the radius of the small circle is AB/4 = 3 cm

The area of the small circle is:

A = πr^2

A = π x 3^2

A = 9 π cm^2

The are of the semicircle is:

A = $\frac{1}{2}\pi$ x 6^2

A = 18 π cm^2

The shaded region is the area of the semicircle minus the area of the small circle inside and divided by 2.

A = (18 π cm^2 - 9π cm^2)/2

A = 9 π cm^2

A = 14.13 cm^2

(b) Find the perimeter of the shaded area. [2]

The perimeter of the shaded area half the perimeter of the semicircle plus half the perimeter of the small circle inside + the radius of the big circle.

The perimeter of a circle is:

P = 2 π r

In our case, the perimeter of the semicircle is:

P = 6π cm

The perimeter of the small circle is:

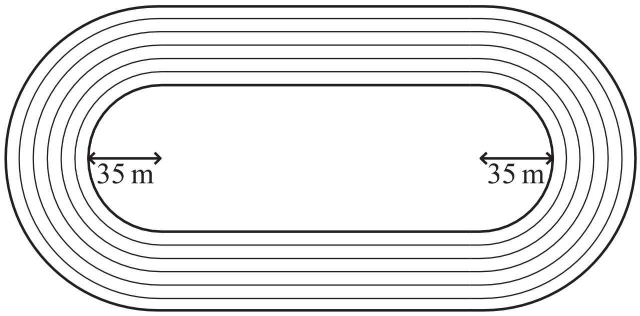
P = 6 π cm

The perimeter of the shaded area is:

P = 3 π cm + 3 π cm + 6 cm (AB/2)

P = 18.8 cm + 6 cm

P = 24.8 cm



NOT TO SCALE

The diagram shows an athletics track with six lanes.
The distance around the inside of the inner lane is 400 metres.
The radius of each semicircular section of the inside of the inner lane is 35 metres.

(a) Calculate the total length of the two straight sections at the inside of the inner lane. [3]

The total length around the inside of the inner lane is 400 m.

From this distance, we need to subtract the length of the 2 semi-circular sections around the inner lane.

The length of the 2 semi-circles is equivalent with the length of one circle of radius 35m.

Length of a circle is:

$L = 2 \pi r$

$L = 70 \pi$ m

$L = 219.91$ m

The length of the 2 straight sections around the inner lane is:

$L = 400$ m – 219.91 m

$L = 180$ m

(b) Each lane is one metre wide.
Calculate the difference in the distances around the outside of the outer lane and the inside of the inner lane. [2]

The distance around the outside of the outer lane is the length of the straight sections work out at point a), which are the same around both the outside of the outer lane and inside of the inner lane, plus the length around the 2 semi-circular sections outside the outer lane.

The length around the two semi-circular sections on the outside of the outer lane is equivalent with the length around a circle with the radius 35 m plus the length added by the 5 lanes.

Each lane has 1 meter, therefore from the inside of the inner lane to the outside of the outer lane are 6 m.

The radius of the large circle will be:

35 m + 6 m = 41 m

The length of the circle is:

$L = 2\,\pi\,41$ m

L = 257.61 m

The length around the outside of the outer lane is:

257.61 m + 180 m= 437.61 m

The difference between the 2 distances is:

437.61 m – 400 m

= **37.61 m**

3D Areas & Volumes
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	3D Areas & Volumes
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **44 minutes**

Score: **/34**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

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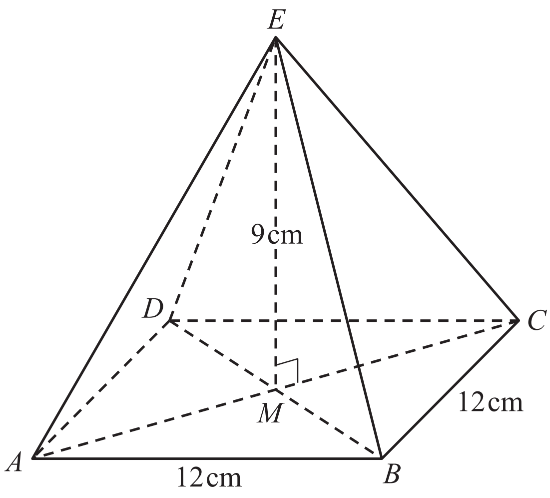


The diagram shows a square-based pyramid *ABCDE*.
The diagonals of the square meet at *M*.
E is vertically above *M*.
$AB = BC = 12$ cm and $EM = 9$ cm.

Calculate the angle between the edge *EC* and the base, *ABCD*, of the pyramid. [4]

We find the length CM using Pythagoras' Theorem

$$CM^2 = 6^2 + 6^2$$

$$= 72$$

The angle ECM can then be found using trigonometry

$$\boldsymbol{\tan\theta = \frac{opp}{adj}}$$

$$\rightarrow \tan ECM = \frac{9}{\sqrt{72}}$$

$$\rightarrow \boldsymbol{ECM = 46.7}$$

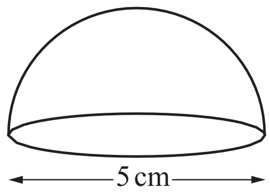
Calculate the volume of a **hemisphere** with radius 3.2 cm.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

$$V = \frac{4}{3}\pi(3.2)^3 \times 0.5$$

$$= \mathbf{68.6}$$

Question 3



NOT TO SCALE

The diagram shows a hemisphere with diameter 5 cm.

Calculate the volume of this hemisphere.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

This is a HEMIsphere so the volume is HALF the volume of a sphere with the same radius.

$$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$$

Note that the diameter in the question had been given rather than the radius.

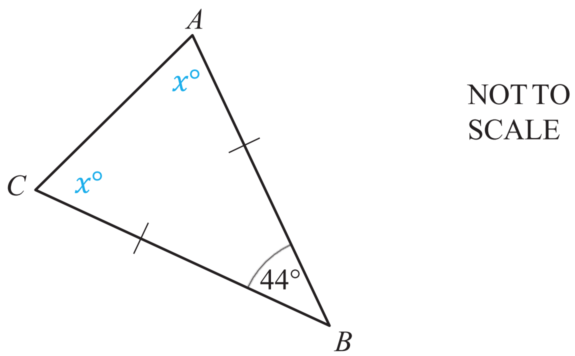
$$r = \frac{1}{2} \times d = \frac{1}{2} \times 5 = 2.5\text{cm}$$

$$V = \frac{1}{2} \times \frac{4}{3} \times \pi \times (2.5)^3$$ (to 3 significant figures)

$$\boldsymbol{V = 32.7\text{cm}^3}$$

Question 4

(a)



Triangle ABC is an isosceles triangle with $AB = CB$.
Angle $ABC = 44°$.

[1]

Find angle ACB.

Base angles of an Isosceles Triangle are equal.

Angles in a triangle add up to 180°.

$$x + x + 44 = 180$$

$$2x = 136$$

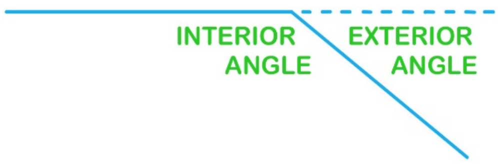
$$\boldsymbol{x = 68°}$$

(b) A regular polygon has an exterior angle of 40°.

Work out the number of sides of this polygon.

[2]

For a regular n sided polygon: Exterior Angle $= \frac{360}{n}$



$$40 = \frac{360}{n}$$

Multiply by n:

$$40n = 360$$

Divide by 40:

$$\boldsymbol{n = 9}$$ **sides**

Calculate the volume of a hemisphere with radius 5cm.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

A hemisphere is a half of a sphere. Therefore the volume of a hemisphere is half the volume of a sphere with the same radius.

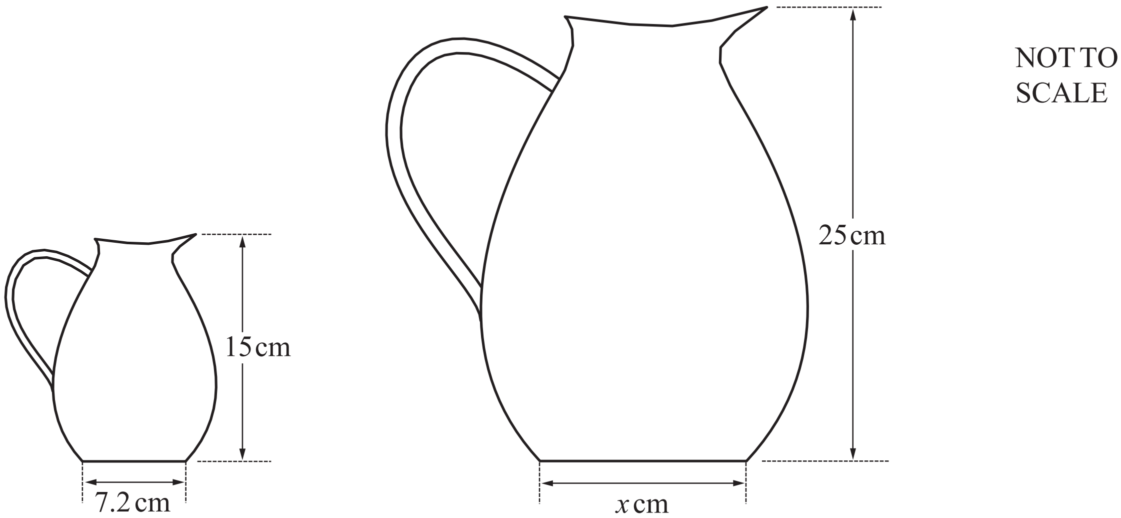
$$Volume = \frac{1}{2} \times \frac{4}{3} \pi r^3$$

In our case the radius *r*=5cm.

$$Volume = \frac{1}{2} \times \frac{4}{3} \pi (5cm)^3$$

$$Volume = \mathbf{261.8\ cm^3}$$

(a)



The diagram shows two jugs that are mathematically similar.

Find the value of x. [2]

Since the jugs are mathematically similar, the ratio of the heights must be the same as the ratio of the widths.

$$\frac{25cm}{15cm} = \frac{x\ cm}{7.2cm}$$

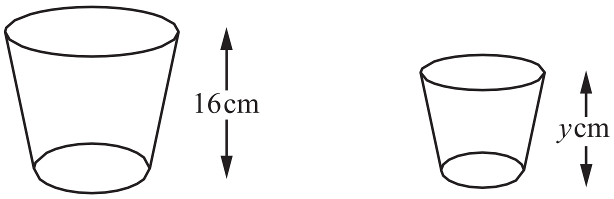
Multiply both sides by 7.2cm.

$$x\ cm = \frac{25cm}{15cm} \times 7.2cm$$

Use calculator to find the value of x.

$$\boldsymbol{x = 12\ cm}$$

(b)



The diagram shows two glasses that are mathematically similar.
The height of the larger glass is 16 cm and its volume is 375 cm^3.
The height of the smaller glass is y cm and its volume is 192 cm^3.

Find the value of y. [3]

Since the volume is in centimetres cubed and the heights are in centimetres (and the glasses are mathematically similar), the ratio of the heights must be the same as the cube root of the ratio of the volumes.

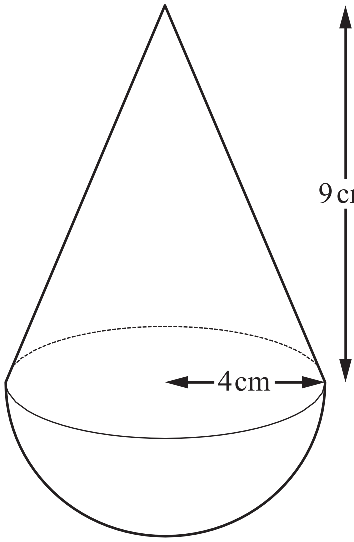
$$\frac{y\ cm}{16cm} = \sqrt[3]{\frac{192\ cm^3}{375\ cm^3}}$$

Multiply both sides by 16cm.

$$y\ cm = \sqrt[3]{\frac{192\ cm^3}{375\ cm^3}} \times 16cm$$

Use calculator to find the value of y.

$$\boldsymbol{y = 12.8\ cm}$$



The diagram shows a toy.
The shape of the toy is a cone, with radius 4 cm and height 9 cm, on top of a hemisphere with radius 4 cm.

Calculate the volume of the toy.
Give your answer correct to the nearest cubic centimetre.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [4]

The total volume is found by summing the volume of the cone and the volume of the hemisphere.

The volume of a hemisphere is half the volume of a sphere with the same radius.

$$volume = cone + hemishpehe$$

$$volume = \frac{1}{3}\pi r^2 h + \frac{1}{2} \times \frac{4}{3}\pi r^3$$

In our case r=4cm (radius of the cone/hemisphere) and h=9cm (height of the cone).

$$volume = \frac{1}{3}\pi (4cm)^2 (9cm) + \frac{2}{3}\pi (4cm)^3$$

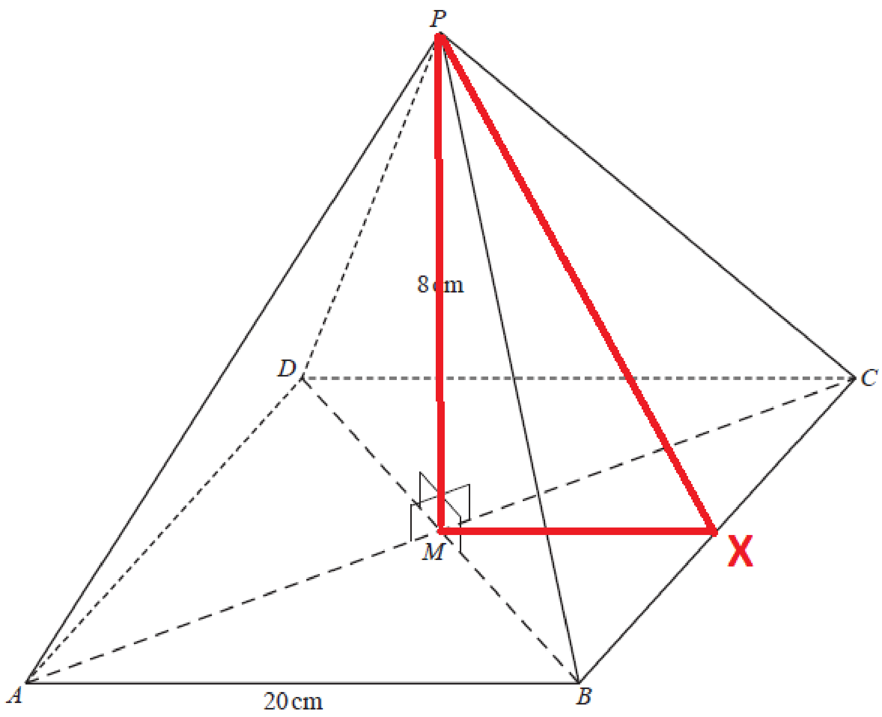
$$volume = 48\pi cm^3 + \frac{128}{3}\pi cm^3$$

Use a calculator to get the total volume.

$$volume = 284.84cm^3$$

Round the volume to the nearest centimetre (round up).

$$volume = \mathbf{285\ cm^3}$$



The diagram shows a solid pyramid on a square horizontal base *ABCD*.
The diagonals *AC* and *BD* intersect at *M*.
P is vertically above *M*.
$AB = 20$ cm and $PM = 8$ cm.

Calculate the total surface area of the pyramid. [5]

The base of the pyramid is a square with side 20cm. The area is therefore:

$$Base\ area = 20cm \times 20cm = 400cm^2$$

To calculate the area of the triangular side of the pyramid, we need to know the length of the bisector of BC passing through P. Let *X* be the point where this bisector meets the line BC.

The length of MX is half the length of AB.

The length of XP (bisector) can be calculated using a Pythagoras' triangle.

$$PX^2 = MX^2 + PM^2$$

$$PX^2 = (10cm)^2 + (8cm)^2$$

$$PX = 12.8cm$$

The area of a triangle can be calculated as half the product of its side and its height.

$$Triangle\ area = \frac{1}{2} \times BC \times PX$$

$$Triangle\ area = \frac{1}{2} \times 20cm \times 12.8cm$$

$$Triangle\ area = 128\ cm^2$$

The pyramid has a base and four triangular sides.

The total surface area of the pyramid:

$$Surface\ area = base\ area + 4 \times triangle\ area$$

$$Surface\ area = 400cm^2 + 4 \times 128\ cm^2$$

$$Surface\ area = \mathbf{912cm^2}$$

The base of a rectangular tank is 1.2 metres by 0.9 metres.
The water in the tank is 53 **centimetres** deep.

Calculate the number of litres of water in the tank. [2]

To calculate the volume of the tank, we multiply the sizes of its three sides.

First, covert the depth from centimetres to metres: 53 $cm = 0.53\ m$

Second, multiply the sides:

$$volume = 0.53m \times 1.2m \times 0.9m$$

$$volume = 0.5724\ m^3$$

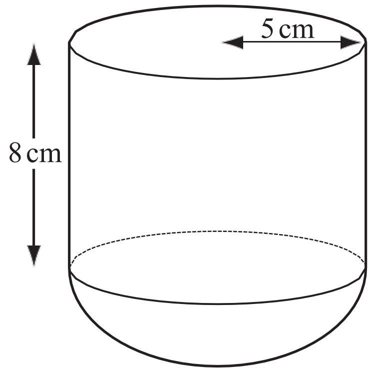
Convert from cubic metres to litres (1 m^3 = 1000 litres)

$$volume = 0.5724\ m^3 \times 1000\ litres\ per\ m^3$$

We get the final answer:

$$\boldsymbol{volume = 572.4\ litres}$$

The diagram shows a child's toy.



NOT TO SCALE

The shape of the toy is a cylinder of radius 5 cm and height 8 cm on top of a hemisphere of radius 5 cm.

Calculate the volume of the toy.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [5]

We can find out the volume of the toy like this:

$$Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3$$

$$Volume\ of\ a\ cylinder = \pi r^2 h$$

$$V_{cyclinder} = \pi \times 5^3 \times 8$$

$V_{hemisphere} = \frac{1}{2}\left(\frac{4}{3} \times \pi \times 5^3\right)$ (remember to multiply by ½ because the toy only has half a sphere) $V_{total} = \left(\frac{1}{2}\left(\frac{4}{3} \times \pi \times 5^3\right)\right) + (\pi \times 5^3 \times 8)$

$$V_{total} = 890cm^3$$

3D Areas & Volumes
Difficulty: Easy

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	3D Areas & Volumes
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 2

Time allowed: **46 minutes**

Score: **/36**

Percentage: **/100**

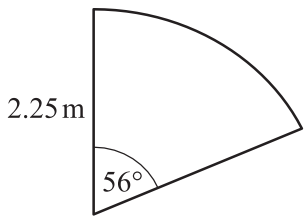
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



NOT TO SCALE

The diagram shows a sand pit in a child's play area.
The shape of the sand pit is a sector of a circle of radius 2.25 m and sector angle 56°.

(a) Calculate the area of the sand pit. [2]

The area of a sector is:

$$A = \frac{1}{2}r^2\theta \times \frac{\pi}{180}$$

$$= 2.25^2 \times \frac{56\pi}{360}$$

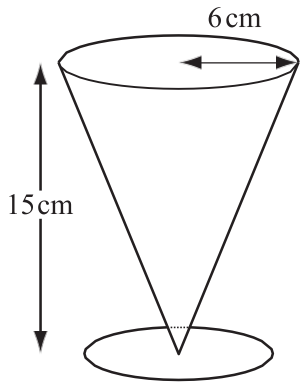
$$= \mathbf{2.47}$$

(b) The sand pit is filled with sand to a depth of 0.3 m.

Calculate the volume of sand in the sand pit. [1]

2.47×0.3

$$= \mathbf{0.742}$$



The diagram shows a glass, in the shape of a cone, for drinking milk.
The cone has a radius of 6 cm and height 15 cm.
A bottle of milk holds 2 litres.

(a) How many times can the glass be completely filled from the bottle?
[The volume, V, of a cone with radius r and height h is $V=\frac{1}{3}\pi r^2 h$.] [4]

Volume of the cone is

$$V=\frac{1}{3}\pi\times 6^2\times 15$$

$$=565.5\ ml$$

The cup can therefore be filled

$$2000\div 565.5$$

$$=3.537$$

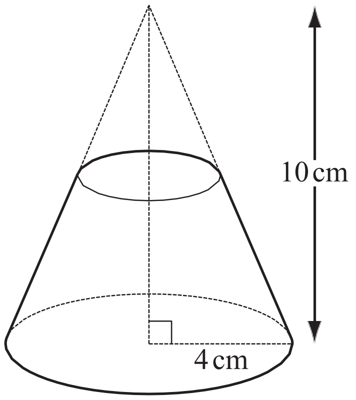
3 whole times

(b) Calculate the volume of milk left in the bottle.
Give your answer in cm^3. [3]

Volume of milk left is

$$2000-3\times 565.5$$

$$=\mathbf{303.5}$$



NOT TO SCALE

A **solid** cone has base radius 4 cm and height 10 cm.
A mathematically similar cone is removed from the top as shown in the diagram.
The volume of the cone that is removed is $\frac{1}{8}$ of the volume of the original cone.

(a) Explain why the cone that is removed has radius 2 cm and height 5 cm. [2]

For 2 similar shapes, the length of one shape is equal to the length of the other shape multiplied by a scale factor, k.

For volumes, the volume of one of the shapes is equal to the volume of the other shaped multiplied by the scale factor cube, k^3

In our case:

$V_{\text{big cone}} = V_{\text{small cone}} \times k^3$

$V_{\text{big cone}} = V_{\text{small cone}} \times \frac{1}{8}$

Therefore:

$k^3 = \frac{1}{8}$

$k = \frac{1}{2}$

The height of the big cone is equal to the height of the small cone multiplied by the scale factor.

The height of the small cone = 10 cm x $\frac{1}{2}$

The height of the small cone = 5 cm

Similarly, **the radius of the base of the small cone = 4 cm x $\frac{1}{2}$**

= 2 cm

(b) Calculate the volume of the remaining solid.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [4]

The volume of the remaining solid is the volume of the big cone minus the volume of the small cone that is being removed.

The volume of the big cone is:

V = $\frac{1}{3}\pi r^2 h$

With r = 4cm and h = 10 cm

V = $\frac{1}{3}\pi 4^2$ x 10

V = $\frac{160\pi}{3}$ cm^3

The volume of the small cone is:

V = $\frac{1}{3}\pi r^2 h$

With r = 2 cm and h = 5 cm

V = $\frac{1}{3}\pi 2^2$ x 5

V = $\frac{20\pi}{3}$ cm^3

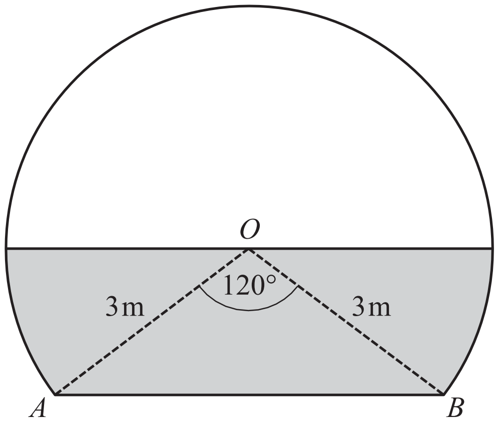
The volume of the remaining solid is:

V = $\frac{160\pi}{3}$ cm^3 - $\frac{20\pi}{3}$ cm^3

V = $\frac{140\pi}{3}$

≈ 147 cm^3

The diagram shows the entrance to a tunnel.
The circular arc has a radius of 3m and centre O. AB is horizontal and angle $AOB = 120°$.



NOT TO SCALE

During a storm the tunnel filled with water, to the level shown by the shaded area in the diagram.

(a) Calculate the shaded area. [4]

The area of shaded region is the sum of the area of the triangle, and the 2 sectors adjacent to the triangle.

$$Area\ of\ triangle = \frac{1}{2} \times a \times b \times \sin(C)$$

$$= \frac{1}{2} \times 3 \times 3 \times \sin(120°)$$

$$= 3.897\ m^2$$

$$Angle\ of\ one\ sector = \frac{360° - 180° - 120°}{2} = 30°$$

$$Area\ of\ sector = \frac{30°}{360°} \times \pi \times 3^2$$

$$= 2.356\ m^2$$

$$Area\ of\ 2\ sectors = 2.356\ m^2 \times 2$$

$$= 4.712\ m^2$$

Therefore,

$$Total\ shaded\ area = 4.712\ m^2 + 3.897\ m^2$$

$$\mathbf{= 8.61\ m^2}$$

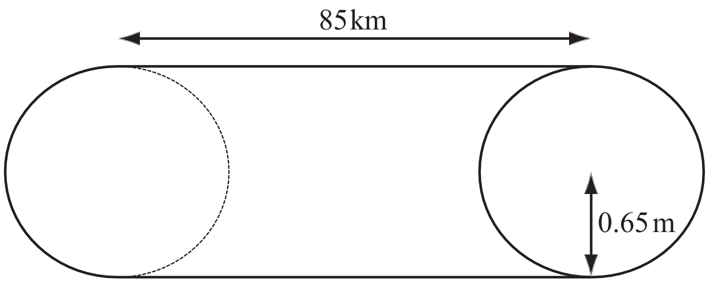
(b) The tunnel is 50 m long.

Calculate the volume of water in the tunnel. [1]

$$Volume = 8.61\ m^2 \times 50\ m$$

$$= 430.5\ m^3$$

$$\mathbf{\approx 431\ m^3}$$



A water pipeline in Australia is a cylinder with **radius** 0.65 **metres** and length 85 **kilometres**.

Calculate the volume of water the pipeline contains when it is full.
Give your answer in cubic metres. [3]

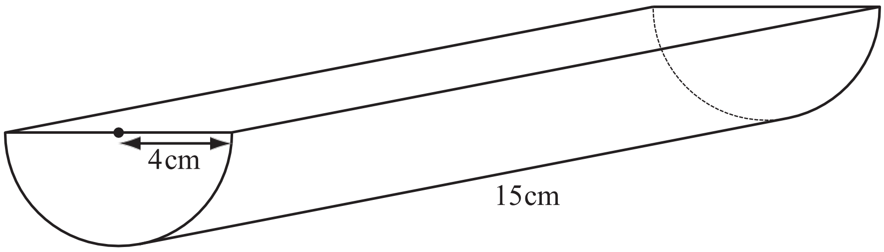
The volume of a cylinder is

$$V = \pi r^2 \times l$$

$$= \pi \times 0.65^2 \times 85000$$

$$= 112822 \approx 113000$$

Question 6



NOT TO SCALE

The diagram shows a solid prism of length 15 cm.
The cross-section of the prism is a semi-circle of radius 4 cm.

Calculate the total surface area of the prism. [4]

Area of the two semi-circular faces is

$$2 \times \frac{1}{2}\pi r^2$$

$$= 16\pi$$

Area of the flat rectangular face is

$$2 \times 4 \times 15$$

$$= 120$$

Area of the long, curved face is

$$\pi r \times 15$$

$$= 60\pi$$

Summing them

$$16\pi + 60\pi + 120$$

$$= 76\pi + 120$$

$$= \mathbf{359}$$

A cylinder has a height of 12 cm and a volume of 920cm^3.

Calculate the radius of the base of the cylinder. [3]

The volume of a cylinder V with height h and radius of its base r is given by:

$$V = \pi r^2 h$$

Divide both sides by πh and take the square root to take r the subject of the formula.

$$\frac{V}{\pi h} = r^2$$

$$r = \sqrt{\frac{V}{\pi h}}$$

We know that:

$$V = 920cm^3$$

$$h = 12cm$$

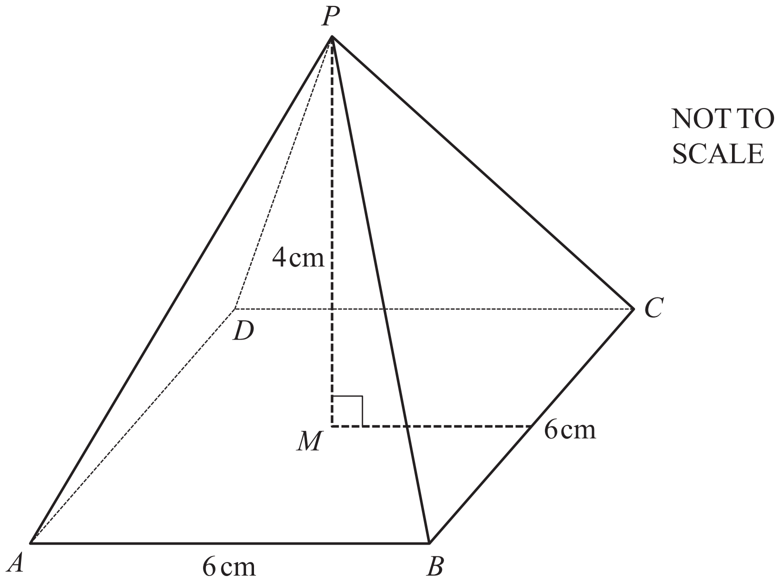
Substitute these values into the formula.

$$r = \sqrt{\frac{920cm^3}{\pi \times 12cm}}$$

Hence we can calculate the radius of the base (correct to 3 significant figures):

$$r = \sqrt{24.404\ cm^2}$$

$$\boldsymbol{r = 4.94\ cm}$$



The diagram shows a pyramid with a square base *ABCD* of side 6 cm.

The height of the pyramid, *PM*, is 4 cm, where *M* is the centre of the base.

Calculate the total surface area of the pyramid. [5]

The pyramid has a base and four triangular sides.

The total surface area of the pyramid:

$$Surface\ area = base\ area + 4 \times triangle\ area$$

The base of the pyramid is a square with side 6cm. As the base is a square, we square the length to get the area:

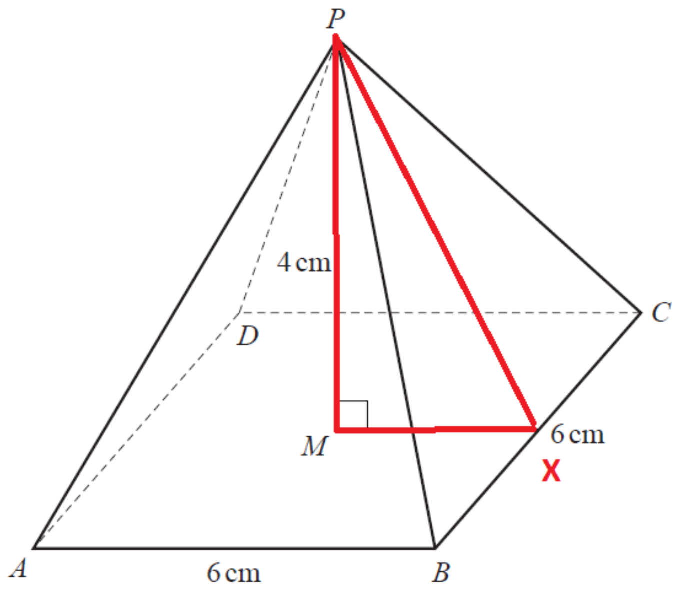
$$Base\ area = (6cm)^2 = 36cm^2$$

To calculate the area of the triangular side of the pyramid, we need to know the length of the bisector of BC passing through P.

Let *X* be the point where this bisector meets the line BC.

The length of MX is half the length of AB.

$$MX = 3cm$$



The length of XP (bisector) can be calculated using a Pythagoras' triangle.

$$PX^2 = MX^2 + PM^2$$

$$PX^2 = (3cm)^2 + (4cm)^2$$

$$PX = 5cm$$

The area of a triangle can be calculated as half the product of its side and its height.

$$Triangle\ area = \frac{1}{2} \times BC \times PX$$

$$Triangle\ area = \frac{1}{2} \times 6cm \times 5cm$$

$$Triangle\ area = 15\ cm^2$$

The total surface area of the pyramid:

$$Surface\ area = base\ area + 4 \times triangle\ area$$

$$Surface\ area = 36cm^2 + 4 \times 15\ cm^2$$

$$\mathbf{Surface\ area = 96\ cm^2}$$

3D Areas & Volumes
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Mensuration (Perimeters, Areas & Volumes)
Sub-Topic	3D Areas & Volumes
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **32 minutes**

Score: **/25**

Percentage: **/100**

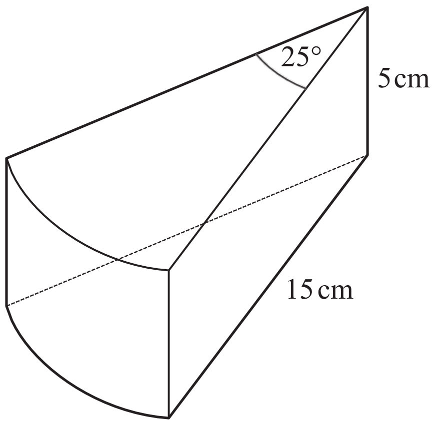
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



NOT TO SCALE

The diagram shows a wooden prism of height 5 cm.
The cross section of the prism is a sector of a circle with sector angle 25°.
The radius of the sector is 15 cm.

Calculate the **total** surface area of the prism. [5]

The total surface area is the sum of all the surfaces:

$$Total\ area = 2 \times sector\ area + 2 \times rectangular\ area + bent\ area$$

The area of a sector of a circle with sector angle 25° and radius 15cm is given as:

$$sector\ area = \pi \times radius^2 \times \frac{sector\ angle}{360°}$$

$$sector\ area = \pi \times (15cm)^2 \times \frac{25°}{360°}$$

$$sector\ area = 49.09cm^2$$

The area of a rectangle is the product of its two sides.

$$rectangular\ area = a \times b$$

$$rectangular\ area = 15cm \times 5cm$$

$$rectangular\ area = 75cm^2$$

The area of the bent part can be calculated as the product of the height of the prism and the length of the arc.

$$bent\ area = height \times arc$$

The length of arc with radius 15cm and sector angle 25° is given as $2\pi \times radius \times \frac{sector\ angle}{360°}$

$$bent\ area = 5cm \times 2\pi \times 15cm \times \frac{25°}{360°}$$

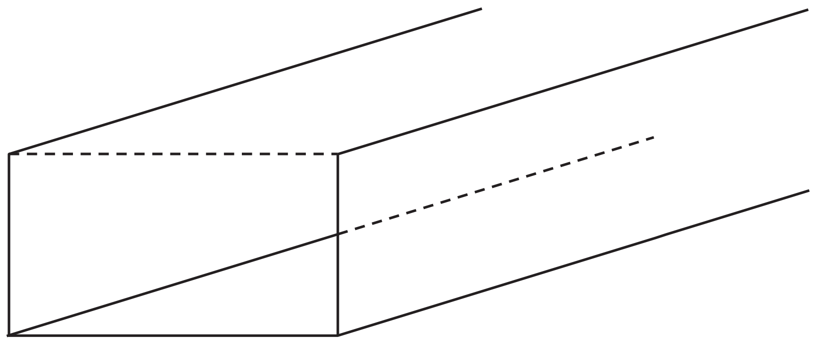
$$bent\ area = 32.72cm^2$$

We can calculate the total surface area of the prism.

$$Total\ area = 2 \times sector\ area + 2 \times rectangular\ area + bent\ area$$

$$Total\ area = 2 \times 49.09cm^2 + 2 \times 75cm^2 + 32.72cm^2$$

$$Total\ area = \mathbf{280.9cm^2}$$



The diagram shows a channel for water.
The channel lies on horizontal ground.
This channel has a constant rectangular cross section with area 0.95 m^2.
The channel is full and the water flows through the channel at a rate of 4 metres/**minute**.

Calculate the number of cubic metres of water that flow along the channel in 3 **hours**. [3]

The amount of water flowing per one minute is found by multiplying the speed of the water and the cross section of the channel.

$$volume\ per\ minute = 4m\ per\ minute \times 0.95m^2$$

$$volume\ per\ minute = 3.8\ m^3\ per\ minute$$

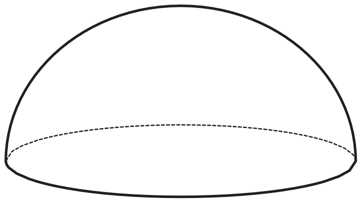
The amount of water that flown is calculated by multiplying the volume flowing per minute by 180 minutes (= 3 hours).

$$volume = 3.8\ m^3\ per\ minute\ \times 180\ minutes$$

We get the amount of water:

$$\boldsymbol{volume = 684\ m^3}$$

The diagram shows a solid hemisphere.

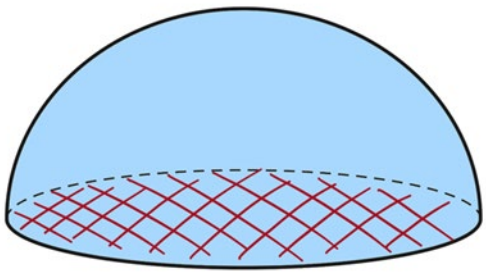


The **total** surface area of this hemisphere is 243π.
The volume of the hemisphere is $k\pi$.

Find the value of k.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [4]

The surface area of a hemisphere (blue) is half that of a sphere **plus** the area of the circle



(red):

$$\frac{1}{2} \times 4\pi r^2 + \pi r^2 = 3\pi r^2$$

Equate this to the surface area of this hemisphere to find

the radius, r:

$$3\pi r^2 = 243\pi$$

$$r^2 = 81$$

$$r = 9$$

The volume of a hemisphere is half that of a sphere:

$$V_{hemisphere} = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$$

Equate this to the volume of this hemisphere:

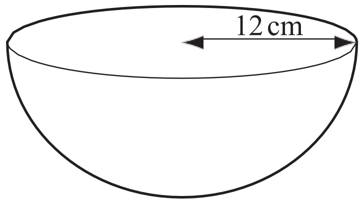
$$\frac{2}{3}\pi r^3 = k\pi$$

Cancel out the π and substitute r = 9:

$$\frac{2}{3} \times 9^3 = k$$

$$\boldsymbol{k = 486}$$

A **hemisphere** has a radius of 12cm.



Calculate its volume.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

The volume of a hemisphere is half the volume of a sphere.

We know the volume of a sphere to be:

$$Vol.\,of\ Sphere = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times 3.142 \times (12^3)$$

$$= 7239cm^3$$

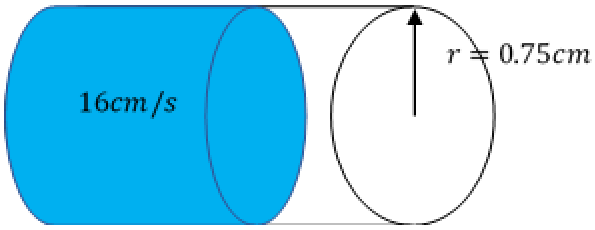
Divide this volume by half to obtain volume of a hemisphere:

$$\frac{7239cm^3}{2} = \mathbf{3620cm^3}$$

$$= \mathbf{3620cm^3}$$

A water pipe has a circular cross section of radius 0.75 cm.
Water flows through the pipe at a rate of 16 cm/s.

Calculate the time taken for 1 litre of water to flow through the pipe. [3]



Here we need to find the time it takes for 1L of water to flow through a pipe of cross-sectional radius $0.75cm$ with a flow rate of $16cm/s$.

First we need to work out the volume of water flowing out of the pipe per second. We know that each second, $16cm$ of water travels out. We need to find the cross-sectional area of this amount of water to calculate the volume.

The area of a circle is given by

$A = \pi r^2$. The volume of a cylinder (prism) is given by $V = AL$, where A is it's circular area and L is its length.

Hence, the volume of this cylinder of water going through the pipe each second is equal to:

$$V = AL = \pi r^2 L = \pi(0.75)^2 \times 16$$

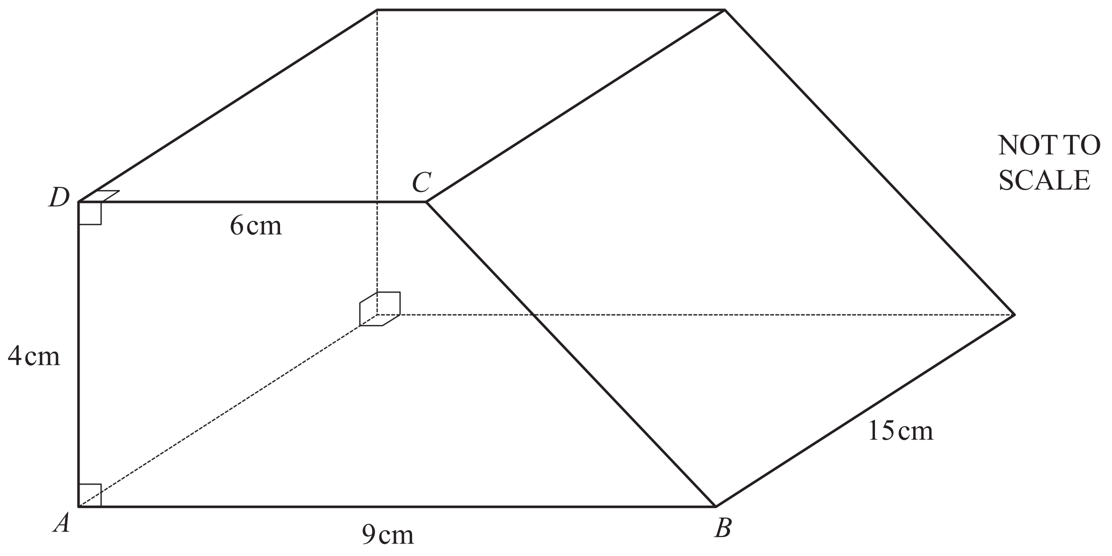
$$V = 9\pi = 28.274 \ldots cm^3/s$$

To find the time it takes to drain 1L of water we must first convert the litre into cm^3. A litre is defined as being $1000cm^3$.

The total time taken is equal to the total volume divided by the volume per second. Written mathematically, this is

$$\boldsymbol{t = \frac{1000}{28.274\ldots}}$$

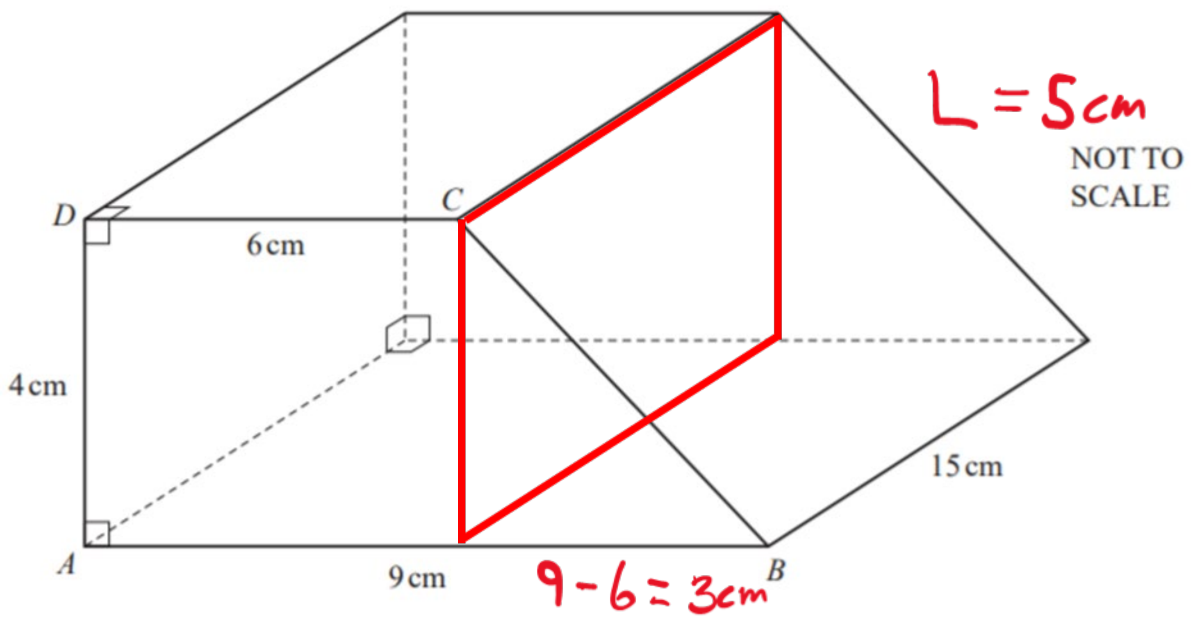
$$\boldsymbol{= 35.4s}$$



The diagram shows a solid prism of length 15 cm.
The cross section of the prism is the trapezium *ABCD*.
Angle *DAB* = angle *CDA* = 90°.
AB = 9 cm, *DC* = 6 cm and *AD* = 4 cm.

Calculate the **total** surface area of the prism. [5]

Here we are given a prism and asked to find the total surface area of it. This is made much simpler by splitting it up as I have below.



Now we have a box and a set of triangles and rectangles to deal with.

Left hand side:

We know all the dimensions of this box, so it is quite straightforward to calculate.

We therefore have:

Two rectangles of area 4×6,

ONE rectangle of area 4×15 (as the red rectangle is not at the surface), and

Two rectangles of area 6×15.

In total the box has surface area:

$$2(4 \times 6) + 1(4 \times 15) + 2(6 \times 15) = 288cm^2$$

Right hand side:

We know the base of the triangles will be $3cm$ as we can compare the lengths of the top and bottom of the box. We know the length of L (labelled above) is equal to $\sqrt{3^2 + 4^2} = 5cm$ due to Pythagoras' theorem.

Hence, we have:

Two triangles of area $\frac{1}{2}(3 \times 4)$One rectangle of area (3×15)

One rectangle of area (15×5)

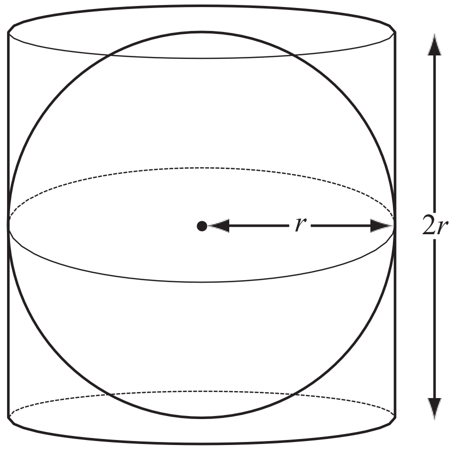
In total the right hand side has a surface area

$$\frac{1}{2} \times 2(3 \times 4) + (3 \times 15) + (15 \times 5) = 132cm^2$$

Adding these two surface areas together we get a total surface area of

$$288cm^2 + 132cm^2 = 420cm^2$$

$$= 420cm^2$$



NOT TO
SCALE

The sphere of radius r fits exactly inside the cylinder of radius r and height $2r$.
Calculate the percentage of the cylinder occupied by the sphere.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [3]

We need to work out the percentage of the cylinder's volume taken up by the sphere's volume:

Therefore, we need to work out both the volume of the sphere and the volume of the cylinder.

V sphere $= \frac{4}{3}\pi r^3$

V cylinder = πr^2h

Where h = 2r

V cylinder = $\pi 2r^3$

We represent the percentage by the unknown x.

$\frac{x}{100} \times \pi 2r^3 = \frac{4}{3}\pi r^3$

Both sides of the equation have as common factor πr^3,

therefore, we can simplify.

$\frac{x}{100}$ x 2 = $\frac{4}{3}$

2x = $\frac{400}{3}$

6x = 400

x = 66.7

The percentage therefore will be 66.7%.