# Functions
# Difficulty: Medium

## Model Answers 1

| | |
|---|---|
| Level | IGCSE |
| Subject | Maths (0580/0980) |
| Exam Board | CIE |
| Topic | Functions |
| Paper | Paper 4 |
| Difficulty | Medium |
| Booklet | Model Answers 1 |

**Time allowed:** **72 minutes**

**Score:** **/63**

**Percentage:** **/100**

**Grade Boundaries:**

**CIE IGCSE Maths (0580)**

| A* | A | B | C | D |
|---|---|---|---|---|
| >83% | 67% | 51% | 41% | 31% |

**CIE IGCSE Maths (0980)** ~~Assembled by AS~~

| 9 | 8 | 7 | 6 | 5 | 4 |
|---|---|---|---|---|---|
| >95% | 87% | 80% | 69% | 58% | 46% |

$\mathrm{f}(x) = 3 - 2x$ $\qquad$ $\mathrm{g}(x) = \frac{4}{x}, x \neq 0$ $\qquad$ $\mathrm{h}(x) = 4^x$

(a) Find f(5). [1]

$f(5) = 3 - 2(5)$

$= \mathbf{-7}$

(b) Find gh(3). [2]

$h(3) = 4^3$

$= 64$

$\rightarrow gh(3) = g(64)$

$= \mathbf{\frac{4}{64}}$

(c) Find $\mathrm{f}^{-1}(x)$. [2]

Let $y = f(x)$, then $x = f^{-1}(y)$

$y = 3 - 2x$

$\rightarrow 2x = 3 - y$

$\rightarrow x = \frac{3 - y}{2}$

$\rightarrow \boldsymbol{f^{-1}(x) = \frac{3 - x}{2}}$

(d) Show that $hf(x) = \frac{64}{16^x}$. [3]

$$hf(x) = 4^{f(x)}$$

$$= 4^{3-2x}$$

$$= \frac{4^3}{4^{2x}}$$

$$= \mathbf{\frac{64}{16^x}}$$

(e) Find the value of $x$ when $h(x) = g(0.5)$. [2]

$$g(0.5) = 8$$

$$4^x = 8$$

$$\rightarrow \boldsymbol{x = 1.5}$$

$f(x) = 3x - 2$ $\quad$ $g(x) = x^2$ $\quad$ $h(x) = 3^x$

(a) Find f(-3). [1]

$$f(-3) = 3(-3) - 2$$

$$= -9 - 2$$

$$= \mathbf{-11}$$

(b) Find the value of $x$ when $f(x) = 19$. [2]

$$3x - 2 = 19$$

Add 2 to both sides

$$3x = 21$$

Divide through by 3

$$\boldsymbol{x = 7}$$

(c) Find fh(2). [2]

$$fh(x) = 3h(x) - 2$$

$$= 3 \times 3^x - 2$$

$$= 3^{x+1} - 2$$

$$\rightarrow fh(2) = 3^3 - 2$$

$$= 27 - 2$$

$$= \mathbf{25}$$

(d) Find $\text{gf}(x) + \text{f}(x) + x$.
Give your answer in its simplest form. [3]

$$gf(x) = \left(f(x)\right)^2$$

$$= (3x - 2)^2$$

$$= 9x^2 - 12x + 4$$

$$\rightarrow gf(x) + f(x) + x = 9x^2 - 12x + 4 + 3x - 2 + x$$

$$= \mathbf{9x^2 - 8x + 2}$$

(e) Find $\text{f}^{-1}(x)$. [2]

Let $y = f(x)$ then $x = f^{-1}(y)$

$$y = 3x - 2$$

Add 2 to both sides

$$y + 2 = 3x$$

Divide through by 3

$$x = \frac{y+2}{3} = f^{-1}(y)$$

$$\rightarrow \boldsymbol{f^{-1}(x) = \frac{x+2}{3}}$$

(a) $y = \frac{3}{x} + 2, \quad x \neq 0$

(i) Find the value of $y$ when $x = -6$. [1]

$$y = \frac{3}{-6} + 2 = -\frac{1}{2} + 2 = \mathbf{1.5}$$

(ii) Find $x$ in terms of $y$. [3]

$$y = \frac{3}{x} + 2$$

Multiply by $x$:

$$xy = 3 + 2x$$

Subtract x from both sides:

$$xy - 2x = 3$$

Factorise:

$$x(y - 2) = 3$$

Divide by $(y - 2)$:

$$\boldsymbol{x = \frac{3}{y - 2}}$$

(b) $g(x) = 2 - x \qquad h(x) = 2^x$

(i) Find g(5). [1]

$$g(5) = 2 - 5 = \mathbf{-3}$$

(ii) Find hhh(2). [2]

$$\text{hhh}(2) = \text{h}\Big(\text{h}\big(\text{h}(2)\big)\Big)$$

$$= \text{h}\big(\text{h}(2^2)\big)$$

$$= \text{h}(2^4)$$

$$= 2^{16}$$

$$\mathbf{hhh(2) = 65536}$$

(iii) Find $x$ when $g(x) = h(3)$. [2]

$g(x) = h(3)$

$2 - x = 2^3$

$-x = 8 - 2$

$-x = 6$

x = -6

(iv) Find $x$ when $g^{-1}(x) = -1$. [1]

The easiest way to solve $g^{-1}(x) = -1$ is to rewrite it as:

$$x = g(-1)$$

$$x = 2 - (-1)$$

$$\boldsymbol{x = 3}$$

$f(x) = 2 - 3x$ $\qquad$ $g(x) = 7x + 3$

(a) Find

(i) $f(-3)$, [1]

We find the value *f*(-3) by plugging -3 into the function *f*.

$$f(-3) = 2 - 3 \times (-3)$$

$$f(-3) = 2 + 9$$

$$\boldsymbol{f(-3) = 11}$$

(ii) $g(2x)$. [1]

We find the value *g*(2x) by plugging 2*x* into the function *g*.

$$g(2x) = 7(2x) + 3$$

$$\boldsymbol{g(2x) = 14x + 3}$$

(b) Find $gf(x)$ in its simplest form. [2]

We find function $gf(x)$by "plugging" function *f(x)* into function *g(x)*. (Essentially rewriting *x* in *g(x)* into f*(x)*).

$$g(f(x)) = 7f(x) + 3$$

$$g(f(x)) = 7(2 - 3x) + 3$$

Multiply out the bracket.

$$g(f(x)) = 14 - 21x + 3$$

$$\boldsymbol{g(f(x)) = 17 - 21x}$$

(c) Find $x$ when $3f(x) = 7$. [3]

Solve

$$3(f(x)) = 7$$

$$3(2 - 3x) = 7$$

Multiply out the bracket.

$$6 - 9x = 7$$

Subtract 6 from both sides of the equation.

$$-9x = 1$$

Divide both sides by -9 to find the value of *x*.

$$\boldsymbol{x = -\frac{1}{9}}$$

(d) Solve the equation.

$$f(x + 4) - g(x) = 0$$ [3]

We find the value *f*(x+4) by plugging *(x+4)* into the function *f*.

$$f(x + 4) = 2 - 3(x + 4)$$

$$f(x + 4) = -10 - 3x$$

Subtract function g(x).

$$f(x + 4) - g(x) = -10 - 3x - (7x + 3)$$

We require that this expression should be equal to 0.

$$-10 - 3x - 7x - 3 = 0$$

$$-13 - 10x = 0$$

Add 13 to both sides of the equation.

$$-10x = 13$$

Divide both sides by -10 to solve for *x*.

$$x = -\frac{13}{10}$$

$$\boldsymbol{x = -1.3}$$

$$f(x) = 2x - 1 \qquad g(x) = \frac{1}{x},\quad x \neq 0 \qquad h(x) = 2^x$$

(a) Find h(3). [1]

We find the value *h*(3) by inserting *x*=3 into the function *h*.

$$h(3) = 2^3$$

$$h(3) = \mathbf{8}$$

(b) Find fg(0.5). [2]

Start by finding the value of *g*(0.5).

$$g(0.5) = \frac{1}{0.5}$$

$$g(0.5) = 2$$

Now find the value of the function *f* at *x*=2 to work out *fg*(0.5).

$$f(2) = f\big(g(0.5)\big)$$

$$fg(0.5) = 2 \times (2) - 1$$

$$fg(0.5) = \mathbf{3}$$

(c) Find $f^{-1}(x)$. [2]

The easiest way to find the value of $f^{-1}(x)$ is to write down the original function $f = 2x + 5$

and now write *x* instead of *f* and $f^{-1}$ instead of *x*.

$$x = 2f^{-1}(x) - 1$$

Add 1 to both sides of the equation.

$$x + 1 = 2f^{-1}(x)$$

Divide both sides by 2 to get the final answer.

$$f^{-1}(x) = \frac{\boldsymbol{x+1}}{\mathbf{2}}$$

(d) Find ff($x$), giving your answer in its simplest form. [2]

We find function $ff(x)$by "plugging" function *f(x)* into another function *f(x)*.

(Essentially rewriting *x* in *f(x)* into *f(x)*).

$$f\big(f(x)\big) = 2\big(f(x)\big) - 1$$

$$f\big(h(x)\big) = 2(2x - 1) - 1$$

We multiply the bracket:

$$f\big(h(x)\big) = 4x - 2 - 1$$

Therefore the function in its simplest for is:

$$f\big(h(x)\big) = \mathbf{4x - 3}$$

(e) Find $(f(x))^2 + 6$, giving your answer in its simplest form. [2]

In this case, we have to square the whole function *f(x)*.

$$(f(x))^2 + 6 = (2x - 1)^2 + 6$$

Square the bracket.

$$(f(x))^2 + 6 = 4x^2 - 4x + 1 + 6$$

We get the function in its simplest form:

$$(f(x))^2 + 6 = \mathbf{4x^2 - 4x + 7}$$

(f) Simplify $hh^{-1}(x)$. [1]

This expression is equal to *x* as it represents applying function to its inverse and that is an identity.

(Essentially applying the function and then "undoing" the function by applying the inverse)

$$hh^{-1}(x) = \boldsymbol{x}$$

(g) Which of the following statements is true?

$f^{-1}(x) = f(x)$

$g^{-1}(x) = g(x)$

$h^{-1}(x) = h(x)$ [1]

Applying an inverse to function *g(x)* will not change it as it is a self inverse function (apply the function twice and you get the original input).

By using the method mentioned in part c)

$$x = \frac{1}{g^{-1}(x)}$$

Invert both sides and observe that this is equivalent to:

$$g^{-1}(x) = \frac{1}{x} = g(x)$$

The answer is:

$$\boldsymbol{g^{-1}(x) = g(x)}$$

(h) Use two of the functions f($x$), g($x$) and h($x$) to find the composite function which is equal to $2^{x+1} - 1$.

[1]

As powers are involved, we certainly have to use function *h*($x$).

The expression also includes a absolute term (-1) therefore one of the equations should be a polynomial function, i.e. *f(x)*.

After inspection (or trial an error), we work out the order in which the functions are applied.

$$fh(x) = 2 \times h(x) - 1 = 2 \times 2^x - 1$$

$$\boldsymbol{fh(x)} = 2^{x+1} - 1$$

$\mathrm{f}(x) = 2x - 1$ $\qquad$ $\mathrm{g}(x) = x^2 + x$ $\qquad$ $h(x) = \frac{2}{x}, x \neq 0$

(a) Find ff(3). [2]

To find the value of $ff(3)$, we first need to work out the value of function *f* for *x*=3 and then work out the value of function *f* for this value.

$$f(3) = 2 \times (3) - 1 = 5$$

We now find the value of *f*(5)

$$f\big(f(3)\big) = f(5) = 2 \times (5) - 1$$

$$f\big(f(3)\big) = f(5) = 10 - 1$$

We have to final answer:

$$\boldsymbol{ff(3) = 9}$$

(b) Find gf($x$), giving your answer in its simplest form. [3]

We find function $gf(x)$by inserting function *f(x)* into function *g(x)*. (Essentially rewriting *x* in *g(x)* into *f(x)*).

$$g\big(f(x)\big) = \big(f(x)\big)^2 + f(x)$$

$$g\big(f(x)\big) = (2x - 1)^2 + (2x - 1)$$

We multiply the bracket:

$$g\big(f(x)\big) = 4x^2 - 4x + 1 + 2x - 1$$

Therefore the function in its simplest form is:

$$g\big(f(x)\big) = \boldsymbol{4x^2 - 2x}$$

(c) Find $f^{-1}(x)$. [2]

The easiest way to find the value of $f^{-1}(x)$ is to write down the original function $f = 5x - 3$ and now write $x$ instead of $f$ and $f^{-1}$ instead of $x$.

$$x = 2f^{-1}(x) - 1$$

Now, we want to rearrange the function.

Add 1 to both sides of the equation.

$$x + 1 = 2f^{-1}(x)$$

Divide both sides by 2

$$\frac{x+1}{2} = f^{-1}(x)$$

We get the final answer.

$$f^{-1}(x) = \frac{\mathbf{x+1}}{\mathbf{2}}$$

(d) Find h($x$) + h($x$ + 2), giving your answer as a single fraction. [4]

We start with:

$$h(x) + h(x+2) = \frac{2}{x} + \frac{2}{x+2}$$

Multiply top and bottom of the first fraction by (x+2). Do the same with x and the second term.

$$\frac{2(x+2)}{x(x+2)} + \frac{2x}{(x+2)x}$$

The fractions have the same denominator, therefore they can be added.

$$\frac{2(x+2)+2x}{x(x+2)}$$

Multiply out the brackets in the numerator.

$$\frac{2x+4+2x}{x(x+2)}$$

Add relevant terms to get the final form of the expression.

$$\boldsymbol{\frac{4x+4}{x(x+2)}}$$

# Functions
# Difficulty: Medium

## Model Answers 2

| | |
|---|---|
| Level | IGCSE |
| Subject | Maths (0580/0980) |
| Exam Board | CIE |
| Topic | Functions |
| Paper | Paper 4 |
| Difficulty | Medium |
| Booklet | Model Answers 2 |

**Time allowed:** **82 minutes**

**Score:** **/71**

**Percentage:** **/100**

**Grade Boundaries:**

**CIE IGCSE Maths (0580)**

| A* | A | B | C | D |
|---|---|---|---|---|
| >83% | 67% | 51% | 41% | 31% |

**CIE IGCSE Maths (0980)**

| 9 | 8 | 7 | 6 | 5 | 4 |
|---|---|---|---|---|---|
| >95% | 87% | 80% | 69% | 58% | 46% |

$f(x) = 5x - 2$ $\quad g(x) = \dfrac{7}{x - 3}, x \neq 3$ $\quad h(x) = 2x^2 + 7x$

(a) Work out

(i) f(2), [1]

$$f(2) = 5(2) - 2$$

$$= \mathbf{8}$$

(ii) hg(17). [2]

$$hg(x) = 2\big(g(x)\big)^2 + 7g(x)$$

$$= 2\left(\frac{7}{x-3}\right)^2 + \frac{49}{x-3}$$

$$\rightarrow hg(17) = 2\left(\frac{7}{14}\right)^2 + \frac{49}{14}$$

$$= \frac{1}{2} + \frac{7}{2}$$

$$= \mathbf{4}$$

(b) Solve $g(x) = x + 3$. [3]

Need to solve:

$$\frac{7}{x-3} = x + 3$$

Multiply through by $(x - 3)$:

$$7 = (x+3)(x-3)$$

$$\rightarrow 7 = x^2 - 9$$

Add 9 to both sides:

$$\rightarrow x^2 = 16$$

$$\rightarrow \boldsymbol{x = \pm 4}$$

(c) Solve h($x$) = 11, showing all your working and giving your answers correct to 2 decimal places.

[5]

Need to solve

$$2x^2 + 7x = 11$$

$$\rightarrow 2x^2 + 7x - 11 = 0$$

Use the quadratic formula

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\rightarrow x = \frac{-7 \pm \sqrt{49 + 88}}{4}$$

$$\rightarrow \boldsymbol{x = 1.18, \qquad x = -4.68}$$

(d) Find $f^{-1}(x)$. [2]

Let $y = f(x)$, then $x = f^{-1}(y)$

$$y = 5x - 2$$

Add 2 to both sides:

$$5x = y + 2$$

Divide through by 5:

$$x = \frac{y + 2}{5} = f^{-1}(y)$$

$$\rightarrow \boldsymbol{f^{-1}(x) = \frac{x + 2}{5}}$$

(e) Solve $g^{-1}(x) = -0.5$. [1]

$$g^{-1}(x) = -0.5$$

$$\rightarrow x = g(-0.5)$$

$$= \frac{7}{-0.5 - 3}$$

$$= \mathbf{-2}$$

$\mathrm{f}(x) = \frac{1}{x}, x \neq 0$ $\qquad$ $\mathrm{g}(x) = 1 - x$ $\qquad$ $\mathrm{h}(x) = x^2 + 1$

(a) Find $\mathrm{fg}\left(\frac{1}{2}\right)$. [2]

$$g\left(\frac{1}{2}\right) = 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

$$\rightarrow fg\left(\frac{1}{2}\right) = f\left(\frac{1}{2}\right)$$

$$= \frac{1}{\frac{1}{2}}$$

$$\mathbf{= 2}$$

(b) Find $\mathrm{g}^{-1}(x)$, the inverse of $\mathrm{g}(x)$. [1]

Let $y = g(x)$, then $x = g^{-1}(y)$

$$y = 1 - x$$

$$\rightarrow x = 1 - y$$

$$\rightarrow \boldsymbol{g^{-1}(x) = 1 - x}$$

$$\boldsymbol{= 1 - x}$$

(c) Find hg($x$), giving your answer in its simplest form. [3]

$$hg(x) = [g(x)]^2 + 1$$

$$= (1 - x)^2 + 1$$

$$= \boldsymbol{x^2 - 2x + 2}$$

(d) Find the value of $x$ when g($x$) = 7 . [1]

$$1 - x = 7$$

$$\rightarrow \boldsymbol{x = -6}$$

(e) Solve the equation h($x$) = 3$x$.
Show your working and give your answers correct to 2 decimal places. [4]

$$x^2 + 1 = 3x$$

$$\rightarrow x^2 - 3x + 1 = 0$$

Use the quadratic formula

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

$$\rightarrow \boldsymbol{x = 2.62, \qquad x = 0.38}$$

(f) A function k($x$) is its own inverse when $k^{-1}(x) = k(x)$.

For which of the functions f($x$) , g($x$) and h($x$) is this true? [1]

$\boldsymbol{f(x)}$ **and** $\boldsymbol{g(x)}$

$f(x) = 4 - 3x$ $\qquad$ $g(x) = 3^{-x}$

(a) Find f(2$x$) in terms of $x$. [1]

$f(2x) = 4 - 3(2x)$

$= \mathbf{4 - 6x}$

(b) Find ff($x$) in its simplest form. [2]

$ff(x) = 4 - 3f(x)$

$= 4 - 3(4 - 3x)$

$= \mathbf{9x - 8}$

(c) Work out gg(–1).
Give your answer as a fraction. [3]

$g(-1) = 3^1$

$= 3$

$\rightarrow gg(-1) = g(3)$

$= 3^{-3}$

$$= \frac{\mathbf{1}}{\mathbf{27}}$$

(d) Find $f^{-1}(x)$, the inverse of f($x$). [2]

Let $y = f(x)$, then $x = f^{-1}(y)$

$y = 4 - 3x$

$\rightarrow 3x = 4 - y$

$$\rightarrow x = \frac{4 - y}{3}$$

$$\rightarrow \boldsymbol{f^{-1}(x) = \frac{4 - x}{3}}$$

(e) Solve the equation gf($x$) = 1. [3]

$$gf(x) = 3^{-f(x)}$$

$$= 3^{3x-4}$$

Need to solve

$$3^{3x-4} = 1$$

$$\rightarrow 3x - 4 = 0$$

$$\rightarrow \boldsymbol{x = \frac{4}{3} = 1\frac{1}{3}}$$

$f(x) = 4x + 3$ $\quad$ $g(x) = \dfrac{7}{x+1}\ (x \neq -1)$ $\quad$ $h(x) = x^2 + 5x$

(a) Work out

(i) h(–3), [1]

Substitute x = -3 into h(x),

$$h(x) = x^2 + 5x$$

$$h(-3) = (-3)^2 + 5(-3)$$

$$= 9 - 15$$

$$= \mathbf{-6}$$

(ii) hg(13). [2]

First obtain hg(x) by substituting g(x) into h(x). This means that all the x terms in h(x) will be replaced by g(x).

$$hg(x) = \left(\frac{7}{x+1}\right)^2 + 5\left(\frac{7}{x+1}\right)$$

$$hg(13) = \left(\frac{7}{13+1}\right)^2 + 5\left(\frac{7}{13+1}\right)$$

$$= \mathbf{2.75}$$

(b) Find $f^{-1}(x)$. [2]

The inverse can be obtained by letting f(x) be y, and then swapping the x and y terms around.

Let f(x) be y,

$$f(x) = y = 4x + 3$$

Invert by swapping x and y,

$$x = 4y + 3$$

Rearrange for y,

$$4y = x - 3$$

$$\boldsymbol{y = \frac{x - 3}{4}}$$

(c) (i) Solve the equation f($x$) = 23. [2]

Substitute f(x) = 23

$$23 = 4x + 3$$

$$4x = 20$$

$$\boldsymbol{x = 5}$$

(ii) Solve the equation h($x$) = 7.

Show all your working and give your answers correct to 2 decimal places. [5]

For h(x) = 7,

$$7 = x^2 + 5x$$

$$x^2 + 5x - 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)}$$

$$\boldsymbol{x = 1.14 \text{ or } x = -6.14}$$

$f(x) = x^2 + x - 1$ $\quad$ $g(x) = 1 - 2x$ $\quad$ $h(x) = 3^x$

(a) Find the value of hg(–2). [2]

To work out hg(-2) we initially calculate g(-2) and then use

this value as x for the function h(x) again.

g(-2) = 1 – 2 x (-2)

g(-2) = 5

hg(-2) = $3^5$

**hg(-2) = 243**

(b) Find $g^{-1}(x)$. [2]

g(x) = 1 – 2x

To work out the inverse of the function g(x), we equal it to the variable y and

then make x the subject of the function.

1 – 2x = y

2x = 1 – y

x = $\frac{1-y}{2}$

Therefore, we obtain the inverse function $g^{-1}$ (y) = $\frac{1-y}{2}$

We substitute y with the variable x to obtain:

**$g^{-1}$ (x) = $\frac{1-x}{2}$**

(c) Solve the equation $f(x) = 0$.
Show all your working and give your answers correct to 2 decimal places. [4]

$f(x) = x^2 + x - 1 = 0$

We use the following formula to work out the solutions of the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our case, a = 1, b = 1 and c = -1

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \, x \, (-1)}}{2x1}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

**$X_1$ = -1.62 and $X_2$ = 0.62**

(d) Find $fg(x)$.
Give your answer in its simplest form. [3]

Similar to point a), to work out fg(x) we use the function g(x) as the x value for the function f(x).

$g(x) = 1 - 2x$

$fg(x) = (1 - 2x)^2 + 1 - 2x - 1$

$fg(x) = 1 - 4x + 4x^2 + 1 - 2x - 1$

We simplify to obtain the following second order equation.

**$fg(x) = 4x^2 - 6x + 1$**

(e) Solve the equation $h^{-1}(x) = 2$. [1]

$h^{-1}(x) = 2$

$h(x) = 3^x$

We need to solve the equation h(x) = y, making x the subject.

$h(x) = 3^x = y$

$x = \log_3 y$

The inverse function $h^{-1}(x)$ will be:

$h^{-1}(x) = \log_3 x = 2$

$\log_3 x = 2$

**$x = 9$**

$\mathrm{f}(x) = 6 + x^2$ $\qquad\qquad$ $\mathrm{g}(x) = 4x - 1$

(a) Find

(i) g(3), [1]

$$g(3) = 4(3) - 1$$

$$= 12 - 1$$

$$= \mathbf{11}$$

(ii) f (–4 ). [1]

$$f(-4) = 6 + (-4)^2$$

$$= 6 + 16$$

$$= \mathbf{22}$$

(b) Find the inverse function $\mathrm{g}^{-1}(x)$. [2]

Let $y = g(x)$ then, if we rearrange, $x = g^{-1}(y)$

$$y = 4x - 1$$

Add 1 to both sides

$$4x = y + 1$$

Divide both sides by 4

$$x = \frac{y+1}{4}$$

$$\rightarrow \boldsymbol{g^{-1}(x) = \frac{x+1}{4}}$$

(c) Find fg($x$) in its simplest form. [3]

$$fg(x) = 6 + [g(x)]^2$$

$$= 6 + (4x - 1)^2$$

$$= 6 + 16x^2 - 8x + 1$$

$$\mathbf{= 16x^2 - 8x + 7}$$

(d) Solve the equation gg($x$) = 3. [3]

$$gg(x) = 4g(x) - 1$$

$$= 4(4x - 1) - 1$$

$$= 16x - 5$$

$$gg(x) = 3 \rightarrow 16x - 5 = 3$$

Add 5 to both sides

$$16x = 8$$

Divide through by 16

$$\mathbf{x = 0.5 = \frac{1}{2}}$$

# Functions
# Difficulty: Hard

## Model Answers 1

| | |
|---|---|
| Level | IGCSE |
| Subject | Maths (0580/0980) |
| Exam Board | CIE |
| Topic | Functions |
| Paper | Paper 4 |
| Difficulty | Hard |
| Booklet | Model Answers 1 |

**Time allowed:** **84 minutes**

**Score:** **/73**

**Percentage:** **/100**

**Grade Boundaries:**

**CIE IGCSE Maths (0580)**

| A* | A | B | C | D |
|---|---|---|---|---|
| >83% | 67% | 51% | 41% | 31% |

**CIE IGCSE Maths (0980)**

| 9 | 8 | 7 | 6 | 5 | 4 |
|---|---|---|---|---|---|
| >95% | 87% | 80% | 69% | 58% | 46% |

$\mathrm{f}(x) = 2x + 1$ $\mathrm{g}(x) = x^2 + 4$ $\mathrm{h}(x) = 2^x$

(a) Solve the equation $\mathrm{f}(x) = \mathrm{g}(1)$. [2]

$$2x + 1 = 1^2 + 4$$

$$2x + 1 = 5$$

$$2x = 4$$

$$\boldsymbol{x = 2}$$

(b) Find the value of fh(3). [2]

$$fh(x) = 2(2^x) + 1$$

$$fh(3) = 2(2^3) + 1$$

$$= 2 \times 8 + 1$$

$$\boldsymbol{= 17}$$

**(c)** Find $\mathrm{f}^{-1}(x)$. [2]

Let $x = 2y + 1$ and change the subject of the formula:

$$x = 2y + 1$$

$$2y = x - 1$$

$$y = \frac{x-1}{2}$$

$$\boldsymbol{f^{-1}(x) = \frac{x-1}{2}}$$

(d) Find gf($x$) in its simplest form. [3]

$$gf(x) = (2x + 1)^2) + 4$$

$$= 4x^2 + 4x + 1 + 4$$

$$\boldsymbol{= 4x^2 + 4x + 5}$$

(e) Solve the equation $h^{-1}(x) = 0.5$ . [1]

$$h(x) = 2^x$$

$$x = 2^{0.5}$$

$$\boldsymbol{x = \sqrt{2}}$$

(f) $\dfrac{1}{h(x)} = 2^{kx}$

Write down the value of $k$. [1]

$$\frac{1}{2^x} = 2^{kx}$$

$$2^{-x} = 2^{kx}$$

$$\boldsymbol{k = -1}$$

$f(x) = 5x + 7$ $\qquad$ $g(x) = \dfrac{4}{x - 3},\ x \neq 3$

(a) Find

(i) fg(1), [2]

We apply f(x) to g(x) like so

$$fg(x) = 5g(x) + 7$$

$$= \frac{20}{x-3} + 7$$

$$fg(1) = \frac{20}{1-3} + 7$$

$$= -10 + 7$$

$$\mathbf{= -3}$$

(ii) gf(x), [2]

We apply the function g to the output of function f giving:

$$gf(x) = \frac{4}{f(x) - 3}$$

$$= \frac{4}{5x + 7 - 3}$$

$$= \frac{\mathbf{4}}{\mathbf{5x + 4}}$$

(iii) $g^{-1}(x)$, [3]

Let $y = g(x)$. If we rearrange for $x = f(y)$ then that function of y will be $g^{-1}(y)$.

$$y = \frac{4}{x-3}$$

Multiply both sides by $x - 3$

$$y(x-3) = 4$$

Divide both sides by y

$$x - 3 = \frac{4}{y}$$

Add 3 to both sides

$$x = \frac{4}{y} + 3 = g^{-1}(y)$$

$$\rightarrow \boldsymbol{g^{-1}(x) = \frac{4}{x} + 3}$$

(iv) $f^{-1}f(2)$. [1]

Inverse function applied to the function reverses it's effect, so

$$\boldsymbol{f^{-1}f(2) = 2}$$

(b) $f(x) = g(x)$

(i) Show that $5x^2 - 8x - 25 = 0$. [3]

We have

$$5x + 7 = \frac{4}{x - 3}$$

Multiply both sides by $x - 3$

$$(5x + 7)(x - 3) = 4$$

Expand

$$5x^2 + 7x - 15x - 21 = 4$$

Rearrange and simplify forming a quadratic equation that equals zero:

$$\mathbf{5x^2 - 8x - 25 = 0}$$

(ii) Solve $5x^2 - 8x - 25 = 0$.
Show all your working and give your answers correct to 2 decimal places. [4]

We use the quadratic formula, given as

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (where $ax^2 + bx + c = 0$}

Substitute for a=5, b=-8, c=-25

$$x = \frac{8 \pm \sqrt{64 + 20 \times 25}}{10}$$

$$= \frac{8 \pm 2\sqrt{141}}{10}$$

$$= \mathbf{3.17, -1.57}$$

$\mathrm{f}(x) = 2x + 5$ $\qquad$ $\mathrm{g}(x) = 2^x$ $\qquad$ $\mathrm{h}(x) = 7 - 3x$

(a) Find

(i) f(3), [1]

We find the value *f*(3) by plugging *x*=3 into the function *f*.

$$f(3) = 2 \times (3) + 5$$

$$f(3) = 6 + 5$$

$$f(3) = \mathbf{11}$$

(ii) gg(3). [2]

Start by finding the value of *g*(3).

$$g(3) = 2^{(3)}$$

$$g(3) = 8$$

Now find the value of the function *g* at *x*=8 to work out *gg*(3).

$$g(8) = g(g(3))$$

$$gg(3) = 2^{(8)}$$

$$gg(3) = \mathbf{256}$$

(b) Find $\mathrm{f}^{-1}(x)$. [2]

The easiest way to find the value of $f^{-1}(x)$ is to write down the original function $f = 2x + 5$ and now write *x* instead of *f* and $f^{-1}$ instead of *x*.

$$x = 2f^{-1}(x) + 5$$

Subtract 5 from both sides of the equation.

$$x - 5 = 2f^{-1}(x)$$

Divide both sides by 2 to get the final answer.

$$f^{-1}(x) = \frac{\boldsymbol{x - 5}}{\mathbf{2}}$$

(c) Find fh($x$), giving your answer in its simplest form. [2]

We find function $fh(x)$by "plugging" function *h(x)* into function *f(x)*. (Essentially rewriting *x* in *f(x)* into *h(x)*).

$$f(h(x)) = 2(h(x)) + 5$$

$$f(h(x)) = 2(7 - 3x) + 5$$

We multiply the bracket:

$$f(h(x)) = 14 - 6x + 5$$

Therefore the function in its simplest for is:

$$f(h(x)) = \mathbf{19 - 6x}$$

(d) Find the integer values of $x$ which satisfy this inequality. [3]

$$1 < \mathrm{f}(x) \leqslant 9$$

Start with the inequalities:

$$1 < 2x + 5 \leq 9$$

Subtract 5 from all terms.

$$-4 < 2x \leq 4$$

Divide all terms by 2.

$$-2 < x \leq 2$$

By solving this simple inequality for integer values, we have solutions:

$$\boldsymbol{x = -1, 0, 1, 2}$$

$f(x) = 1 - 2x$ $\qquad$ $g(x) = \dfrac{1}{x}, x \neq 0$ $\qquad$ $h(x) = x^3 + 1$

(a) Find the value of

(i) gf(2), [2]

$$f(2) = 1 - 2(2)$$

$$= -3$$

Hence

$$gf(2) = g(-3)$$

$$= \frac{1}{-3}$$

$$= -\frac{\mathbf{1}}{\mathbf{3}}$$

(ii) h(–2). [1]

$$h(-2) = (-2)^3 + 1$$

$$= -8 + 1$$

$$= \mathbf{-7}$$

(b) Find fg($x$).
Write your answer as a single fraction. [2]

$$fg(x) = 1 - 2g(x)$$

$$= 1 - \frac{2}{x}$$

$$= 1 \times \frac{x}{x} - \frac{2}{x}$$

$$= \frac{\boldsymbol{x - 2}}{\boldsymbol{x}}$$

(c) Find $h(x)^{-1}$, the inverse of h($x$). [2]

Let $y = h(x)$ then $x = h^{-1}(y)$

$$y = x^3 + 1$$

Subtract 1 from both sides

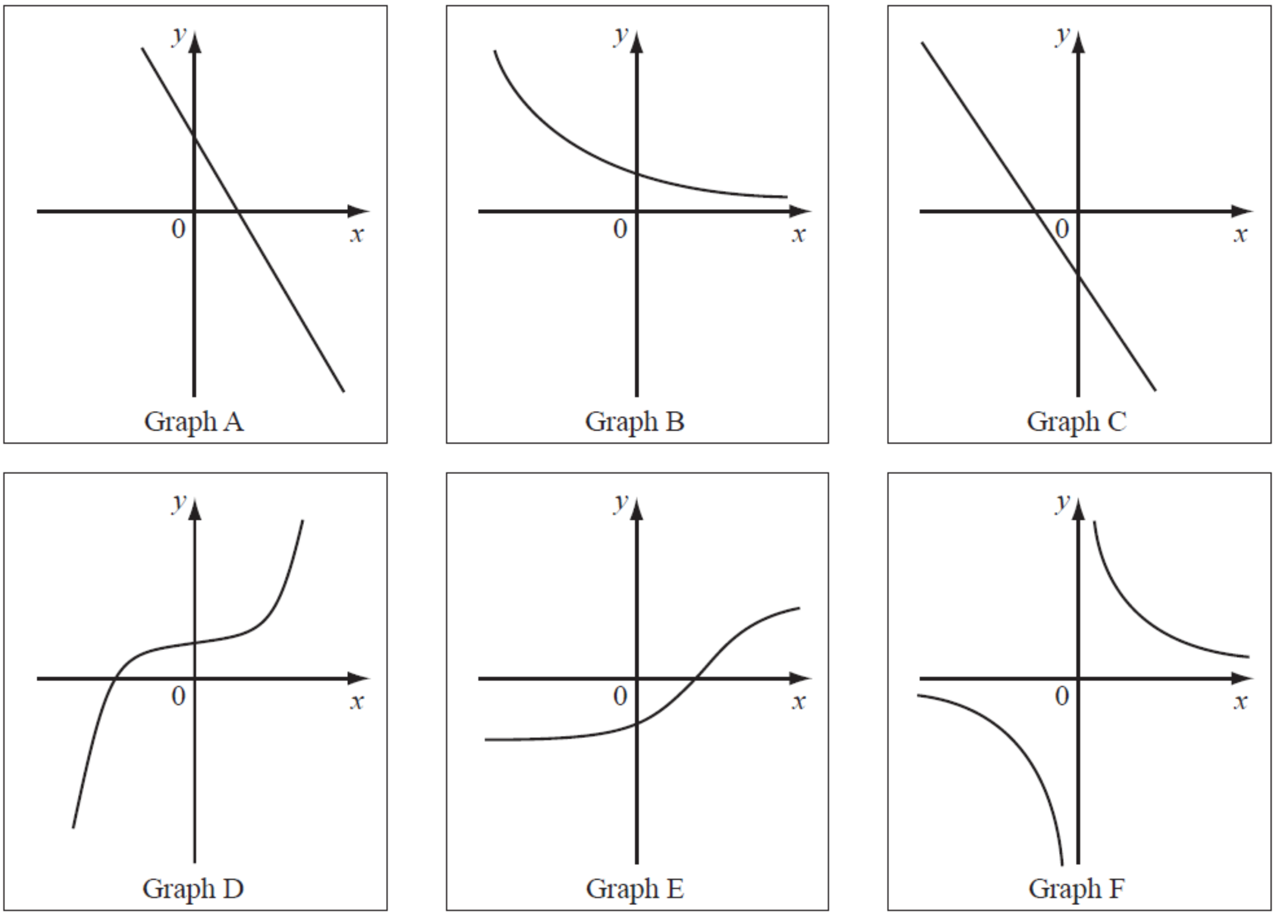
$$y - 1 = x^3$$

cube root both sides

$$x = \sqrt[3]{y-1}$$

$$\rightarrow \boldsymbol{h^{-1}(x) = \sqrt[3]{x-1}}$$

(d) Write down which of these sketches shows the graph of each of $y = f(x)$, $y = g(x)$ and $y = h(x)$. [3]



$\boldsymbol{y = f(x)}$ **is Graph A**

$\boldsymbol{y = g(x)}$ **is Graph F**

$\boldsymbol{y = h(x)}$ **is Graph D**

(e) $k(x) = x^5 - 3$

Solve the equation $k^{-1}(x) = 2$. [2]

$k^{-1}(x) = 2$

$\rightarrow x = k(2)$

$= 2^5 - 3$

$\boldsymbol{= 29}$

$$f(x) = 4x - 2$$

$$g(x) = \frac{2}{x} + 1$$

$$h(x) = x^2 + 3$$

(a) (i) Find the value of hf(2). [2]

f(x) = 4x -2

$g(x) = \frac{2}{x} + 1$

$h(x) = x^2 + 3$

The function hf(x) is equal to h(f(x))

To work out hf(2) we initially calculate f(2) and then we use this value as x for h(x).

f(2) = 4 x 2 – 2

f(2) = 6

$h(f(2)) = 6^2 + 3$

**h(f(2)) = 39**

(ii) Write fg($x$) in its simplest form. [2]

Similar to a), to work out fg(x) we calculate f(g(x)).

$g(x) = \frac{2}{x} + 1$

$fg(x) = 4 \times (\frac{2}{x} + 1) - 2$

$fg(x) = \frac{8}{x} + 4 - 2$

$fg(x) = \frac{8}{x} + 2$

To have this result in its simplest form we can factorise,

using 2 as a common factor for the 2 terms.

**$fg(x) = 2(\frac{4}{x} + 1)$**

(b) Solve $g(x) = 0.2$. [2]

$g(x) = \frac{2}{x} + 1 = 0.2 \frac{2}{x} = -0.8$

$x = \frac{2}{-0.8}$

**$x = -2.5$**

(c) Find the value of $gg(3)$. [2]

Similar to point a), to work out gg(3) we initially calculate g(3) and then use

this value as x for the function g(x) again.

$g(3) = \frac{2}{3} + 1$

$g(3) = 1.666$

$gg(3) = \frac{2}{1.666} + 1$

**$gg(3) = 2.204$**

(d) (i) Show that f($x$) = g($x$) can be written as $4x^2 - 3x - 2 = 0$. [1]

f(x) = g(x)

$4x - 2 = \frac{2}{x} + 1$

We multiply both sides by x to have all terms in the same form.

$4x^2 - 2x = 2 + x$

We move all the terms on one side.

**$4x^2 - 3x - 2 = 0$**

(ii) Solve the equation $4x^2 - 3x - 2 = 0$.

Show all your working and give your answers correct to 2 decimal places. [4]

$4x^2 - 3x - 2 = 0$

We use the following formula to solve the second order equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a = 4, b = -3 and c = -2

We substitute these values to work out x.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4x4x(-2)}}{2x4}$$

$$x = \frac{3 \pm \sqrt{41}}{8}$$

$$x = \frac{3 \pm 6.4}{8}$$

$x_1 = \frac{3+6.4}{8}$ $x_2 = \frac{3-6.4}{8}$

$x_1 = 1.175$ $x_2 = -0.425$

Correct to 2 decimal places:

**$x_1 = 1.18$ $x_2 = -0.43$**

$f(x) = 3x + 1 \qquad g(x) = (x+2)^2$

(a) Find the values of

(i) gf(2), [2]

The simplest way to evaluate *gf*(2) is to find the value of the inner function f(x) at *x*=2 and then find the value of *g*(y) at the point *y*=*f*(2).

$$f(2) = 3 \times (2) + 1$$

$$f(2) = 7$$

Substitute this value into the outer function.

$$g(f(2)) = g(7)$$

$$g(f(2)) = ((7) + 2)^2$$

$$\boldsymbol{g(f(2)) = 81}$$

(ii) ff(0.5). [2]

Follow the same method again but with *ff*(0.5). In this case, the inner and the outer functions are the same.

$$f(0.5) = 3 \times (0.5) + 1$$

$$f(0.5) = 2.5$$

Substitute this value into the outer function *f*.

$$f(f(0.5)) = f(2.5)$$

$$f(f(0.5)) = 3 \times 2.5 + 1$$

$$\boldsymbol{f(f(0.5)) = 8.5}$$

(b) Find $f^{-1}(x)$, the inverse of $f(x)$. [2]

The easiest way to find the value of $f^{-1}(x)$ is to write down the original function $f = 3x + 1$ and now write $x$ instead of $f$ and $f^{-1}$ instead of $x$.

$$x = 3f^{-1}(x) + 1$$

Subtract 1 from both sides of the equation.

$$x - 1 = 3f^{-1}(x)$$

Divide both sides by 3 to get the final answer.

$$\boldsymbol{f^{-1}(x) = \frac{x-1}{3}}$$

(c) Find $fg(x)$.

Give your answer in its simplest form. [2]

We find function $fg(x)$ by "substituting" function $g(x)$ into another function $f(x)$. (Essentially rewriting $x$ in $f(x)$ into $g(x)$).

$$f\big(g(x)\big) = 3g(x) + 1$$

$$f\big(g(x)\big) = 3(x+2)^2 + 1$$

Multiply out the bracket and simplify.

$$f\big(g(x)\big) = 3(x^2 + 4x + 4) + 1$$

$$\boldsymbol{f\big(g(x)\big) = 3x^2 + 12x + 13}$$

(d) Solve the equation $x^2 + f(x) = 0$.

Show all your working and give your answers correct to 2 decimal places. [4]

We want to solve the equation.

$x^2 + f(x) = 0.$

Hence the quadratic equation we have is:

$x^2 + 3x + 1 = 0.$

Therefore we use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a$ = 1, $b$= 3 and $c$ = 1 (from $ax^2 + bx + c = 0$).

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

By using a calculator the answers are (correct to two decimal places):

$$\boldsymbol{x = -2.62 \text{ and}}$$

$$\boldsymbol{x = -0.38\ (2\ dp)}$$

# Functions
# Difficulty: Hard

## Model Answers 2

| | |
|---|---|
| Level | IGCSE |
| Subject | Maths (0580/0980) |
| Exam Board | CIE |
| Topic | Functions |
| Paper | Paper 4 |
| Difficulty | Hard |
| Booklet | Model Answers 2 |

**Time allowed:** **81 minutes**

**Score:** **/70**

**Percentage:** **/100**

**Grade Boundaries:**

**CIE IGCSE Maths (0580)**

| A* | A | B | C | D |
|---|---|---|---|---|
| >83% | 67% | 51% | 41% | 31% |

**CIE IGCSE Maths (0980)**

| 9 | 8 | 7 | 6 | 5 | 4 |
|---|---|---|---|---|---|
| >95% | 87% | 80% | 69% | 58% | 46% |

(a) $f(x) = 2x - 1$ $g(x) = x^2$

Work out

(i) f(2), [1]

$f(2) = 2(2) - 1$

$= 4 - 1$

$= \mathbf{3}$

(ii) g( – 2), [1]

$g(-2) = (-2)^2$

$= \mathbf{4}$

(iii) ff($x$) in its simplest form, [2]

$f\big(f(x)\big) = 2f(x) - 1$

$= 2(2x - 1) - 1$

$= \mathbf{4x - 3}$

(iv) $f^{-1}(x)$, the inverse of f($x$), [2]

Let $y = f(x)$, then $x = f^{-1}(y)$.

$y = 2x - 1$

Add 1 to both sides

$y + 1 = 2x$

Divide through by 2

$$\frac{y+1}{2} = x = f^{-1}(y)$$

$$\rightarrow \boldsymbol{f^{-1}(x) = \frac{x+1}{2}}$$

(v) $x$ when gf($x$) =4. [4]

$$gf(x) = [f(x)]^2$$

$$= (2x-1)^2$$

$$= 4x^2 - 4x + 1$$

We have that $gf(x) = 4$ so

$$4x^2 - 4x + 1 = 4$$

$$\rightarrow 4x^2 - 4x - 3 = 0$$

$$\rightarrow (2x-3)(2x+1) = 0$$

$$\rightarrow \boldsymbol{x = \frac{3}{2}, x = -\frac{1}{2}}$$

(b) $y$ is **inversely** proportional to $x$ and $y = 8$ when $x = 2$.

Find,

(i) an equation connecting $y$ and $x$, [2]

We have

$$y \propto \frac{1}{x}$$

$$\rightarrow y = \frac{k}{x}$$

Sub in our known values

$$8 = \frac{k}{2}$$

$$\rightarrow k = 16$$

Hence

$$\boldsymbol{y = \frac{16}{x}}$$

(ii) $y$ when $x = \dfrac{1}{2}$. [1]

Sub in the x-value

$$y = \frac{16}{0.5}$$

$$= \mathbf{32}$$

$$f(x) = 2x - 1 \qquad g(x) = x^2 + 1 \qquad h(x) = 2^x$$

(a) Find the value of

(i) $f\left(-\frac{1}{2}\right)$, [1]

f(x) = 2x – 1

f(-1/2) = 2 x (-1/2) – 1

**f(-1/2) = -2**

(ii) g (-5) [1]

g(x) = $x^2$ + 1

g(-5) = $(-5)^2$ + 1

**g(-5) = 26**

(iii) h (–3). [1]

h(x) = $2^x$

h(-3) = $2^{-3}$

h(-3) = $1/2^3$

**h(-3) = 1/8**

(b) Find the inverse function $f^{-1}(x)$. [2]

To work out the inverse of the function f(x) we solve the equation f(x) = y for x, where y is a random output.

f(x) = y

$2x - 1 = y$

$2x = y + 1$

$x = (y + 1)/2$

Therefore:

$f^{-1}(y) = (y + 1)/2$

We substitute the variable y with x to work out the

inverse of f(x).

**$f^{-1}(x) = (x + 1)/2$**

(c) $g(x) = z$.
Find $x$ in terms of $z$. [2]

g(x) = z

$x^2 + 1 = z$

$x^2 = z - 1$

**$x = \sqrt{z - 1}$**

(d) Find gf($x$), in its simplest form. [2]

gf(x) = g(f(x))

$f(x) = 2x - 1$

We consider f(x) as the x variable for the function g(x)

$g(x) = x^2 + 1$

$g(f(x)) = (2x - 1)^2 + 1$

$g(f(x)) = 4x^2 - 4x + 1 + 1$

$g(f(x)) = 4x^2 - 4x + 2$

We factorise using 2 as the common factor for all 3 terms

to simplify the function gf(x).

**$gf(x) = 2(2x^2 - 2x + 1)$**

(e) $h(x) = 512$.
Find the value of $x$. [1]

$h(x) = 512$

$h(x) = 2^x$

$2^x = 512$

$x = \log_2 512$

**$x = 9$**

(f) Solve the equation $2f(x) + g(x) = 0$, giving your answers correct to 2 decimal places. [5]

$2(f(x)) + g(x) = 0$

$2(2x - 1) + x^2 + 1 = 0$

$4x - 2 + x^2 + 1 = 0$

$x^2 + 4x - 1 = 0$

We use the following formula to solve the second order equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our case, a = 1, b = 4 and c = -1

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \: x \: 1 \: x (-1)}}{2 \: x \: 1}$$

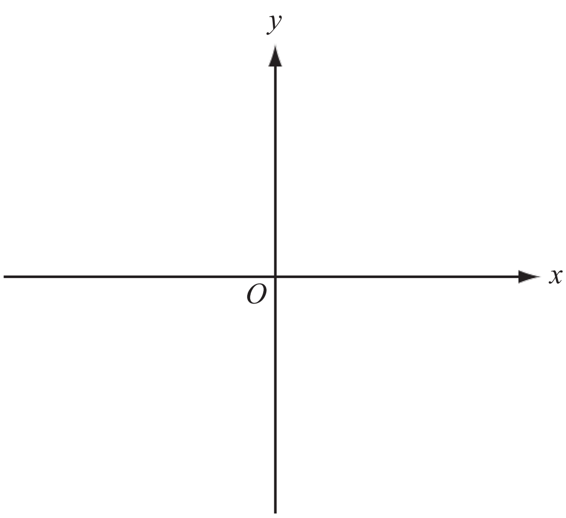
$$x = \frac{-4 \pm \sqrt{20}}{2}$$

**x = -4.24 and x = 0.24**

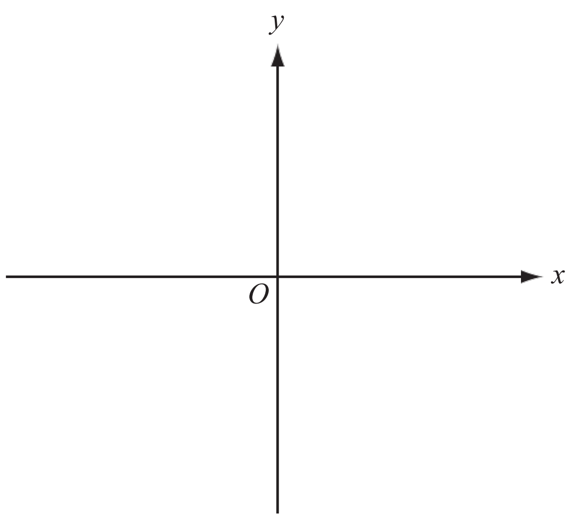
(g) Sketch the graph of

*(i)* $y = \mathrm{f}(x)$,

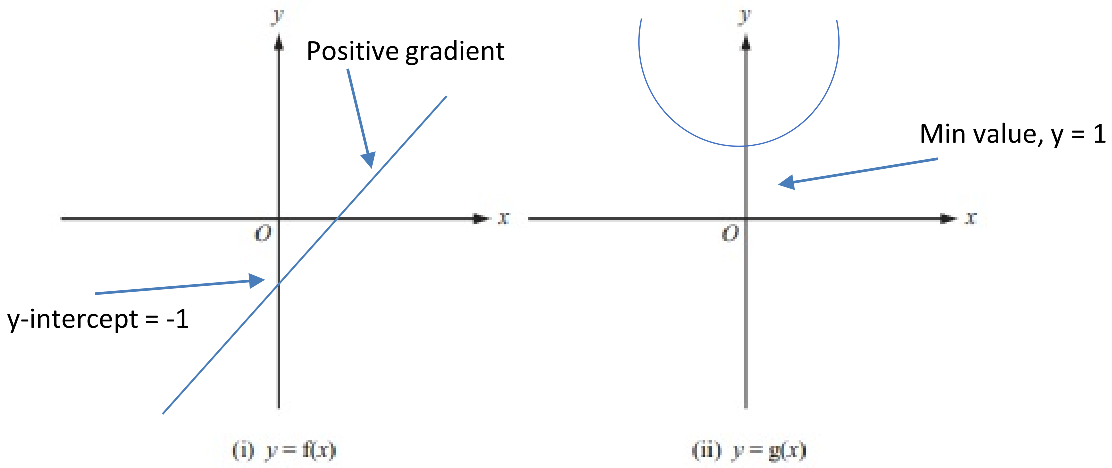
*(ii)* $y = \mathrm{g}(x)$.




(i) $y = \mathrm{f}(x)$




(ii) $y = \mathrm{g}(x)$ [3]




The equation of a line takes up the form:

$y = mx + n$

where m is the gradient and n is the y-intercept.

i) $f(x) = 2x - 1$

In this case, the gradient is m = 2 and the y-intercept is n = -1

i) $g(x) = x^2 + 1$

We know that the value $x^2$ can have only positive values with the minimum value possible g(x) = 1 for x = 0. Therefore, the graph will be a U-shaped parabola with the minimum value 1.

$f(x) = 2x - 1,$ $\quad g(x) = \frac{3}{x} + 1,$ $\quad h(x) = 2^x.$

(a) Find the value of fg(6). [1]

$$g(6) = \frac{3}{6} + 1$$

$$= \frac{3}{2}$$

Hence

$$fg(6) = f\left(\frac{3}{2}\right)$$

$$= 2\left(\frac{3}{2}\right) - 1$$

$$= \mathbf{2}$$

(b) Write, as a **single fraction**, gf($x$) in terms of $x$. [3]

$$gf(x) = \frac{3}{f(x)} + 1$$

$$= \frac{3}{2x - 1} + 1$$

$$= \frac{3}{2x - 1} + \frac{2x - 1}{2x - 1}$$

$$= \frac{\mathbf{2x + 2}}{\mathbf{2x - 1}}$$

(c) Find $g^{-1}(x)$. [3]

Let $y = g(x)$, then $x = g^{-1}(y)$

$$y = \frac{3}{x} + 1$$

$$\rightarrow \frac{3}{x} = y - 1$$

$$\rightarrow \frac{x}{3} = \frac{1}{y-1}$$

$$\rightarrow x = \frac{3}{y-1}$$

$$\rightarrow \boldsymbol{g^{-1}(x) = \frac{3}{x-1}}$$

(d) Find hh(3). [2]

$$h(3) = 2^3$$

$$= 8$$

$$\rightarrow hh(3) = h(8)$$

$$= 2^8$$

$$= \mathbf{256}$$

(e) Find $x$ when $h(x) = g\left(-\frac{24}{7}\right)$ [2]

$$g\left(-\frac{24}{7}\right) = 3 \times \left(-\frac{7}{24}\right) + 1$$

$$= -\frac{21}{24} + \frac{24}{24}$$

$$= \frac{3}{24}$$

$$= \frac{1}{8}$$

Hence, we need to solve

$$h(x) = \frac{1}{8}$$

$$\rightarrow 2^x = \frac{1}{8}$$

$$\rightarrow \boldsymbol{x = -3}$$

$$f(x) = x^2 - 4x + 3 \qquad \text{and} \qquad g(x) = 2x - 1.$$

(a) Solve $f(x) = 0$. [2]

$f(x) = x^2 - 4x + 3$

$f(x) = 0$

$x^2 - 4x + 3 = 0$

We can re-write the equation as:

$x^2 - x - 3x + 3 = 0$

For the first 2 terms we use x as a common factor and for the last 2

factors we use -3 as a common factor.

$x(x - 1) - 3(x - 1) = 0$

In this form, $(x - 1)$ is a common factor.

$(x - 3)(x - 1) = 0$

Therefore, x takes the values:

**$x = 1$ and $x = 3$.**

(b) Find $g^{-1}(x)$. [2]

$g(x) = 2x - 1$

To work out the inverse function of g(x), $g^{-1}(x)$, we note with y the output of this function.

To work out the inverse function of f(x) we note with y the output of this function.

$g(x) = y$

$2x - 1 = y$

We need to rearrange this equation, making x the subject.

$x = \frac{y+1}{2}$

To obtain the inverse function we re-write this expression replacing y with x.

$y = \frac{x+1}{2}$

$\mathbf{g^{-1}(x) = \frac{x+1}{2}}$

(c) Solve $f(x) = g(x)$, giving your answers correct to 2 decimal places.

[5]

$x^2 - 4x + 3 = 2x - 1$

$x^2 - 6x + 4 = 0$

The quadratic equation formula is:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

In this case, a = 1, b = -6 and c = 4.

We use these values in the quadratic equation formula to work out x.

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 4}}{2 \times 1}$$

$$x = \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$x = \frac{6 \pm \sqrt{20}}{2}$$

$$x = \frac{6 \pm 4.47}{2}$$

$x_1 = \frac{6+4.47}{2}$ and $x_2 = \frac{6-4.47}{2}$

**$x_1 = 5.235$ and $x_2 = 0.765$**

**x = 5.24 (correct to 2 decimal places) and x = 0.77 (correct to 2 decimal places)**

(d) Find the value of gf(–2). [2]

gf(-2)

The composition of the 2 functions, g and f, can be

written as: g(f(x))

We initially need to work out the value of f(-2) and then

use this value as x for g(x).

f(-2) = $(-2)^2$ – 4(-2) + 3

f(-2) = 4 + 8 + 3

f(-2) = 15

g(f(x)) = g(15)

g(15) = 2 x 15 – 1

**g(15) = 29**

(e) Find fg($x$). Simplify your answer. [3]

fg(-2)

The composition of the 2 functions, g and f, can be

written as: f(g(x))

$g(x) = 2x – 1$

We use this function as x for the function f(x)

$f(x) = x^2 – 4x + 3$

$fg(x) = (2x – 1)^2 – 4(2x – 1) + 3$

We simplify this form by using the formula:

$(a + b)^2 = a^2 + 2ab + b^2$

$fg(x) = 4x^2 – 4x + 1 – 8x + 4 + 3$

**$fg(x) = 4x^2 – 12x + 8$**

(a) $f(x) = 2 - 3x$ and $g(x) = x^2$.

(i) Solve the equation $f(x) = 7 - x$. [2]

f(x) = 2 – 3x

2 – 3x = 7 – x

-2x = 5

$x = \frac{-5}{2} = -2.5$

(ii) Find $f^{-1}(x)$. [2]

f(x) = 2 – 3x

To work out the inverse function of f(x), $f^{-1}(x)$, we note with y the output of this function.

To work out the inverse function of f(x) we note with y the output of this function.

f(x) = y

2 – 3x = y

We need to rearrange this equation, making x the subject.

2 – y = 3x

$x = \frac{2-y}{3}$

To obtain the inverse function we re-write this expression replacing y with x.

$y = \frac{2-x}{3}$

$\mathbf{f^{-1}(x) = \frac{2-x}{3}}$

(iii) Find the value of gf(2) – fg(2). [3]

gf(2)

The composition of the 2 functions, g and f, can be written as: g(f(x))

We initially need to work out the value of f(2) and then use this value as x for g(x).

f(2) = 2 – 3 x 2

f(2) = -4

g(f(x)) = g(-4)

g(-4) = $(-4)^2$

**g(-4) = 16**

gf(-2)

The composition of the 2 functions, g and f, can be written as: f(g(x))

We initially need to work out the value of g(-2) and then use this value as x for f(x).

g(2) = $2^2$

g(2) = 4

f(g(x)) = 2 – 3 x 4

f(4) = -10

**gf(2) – fg(2) = 16 – (-10) = 26**

(iv) Find fg($x$). [1]

The composition of the 2 functions, g and f, can be

written as: f(g(x))

$g(x) = x^2$

We use this function as x for the function f(x)

$f(x) = 2 - 3x$

**$fg(x) = 2 - 3x^2$**

(b) $h(x) = x^x$.

(i) Find the value of h(2). [1]

$h(x) = x^x$

For x = 2:

**$h(2) = 2^2 = 4$**

(ii) Find the value of h(−3), giving your answer as a fraction. [1]

For x = -3:

$h(-3) = (-3)^{-3}$

$h(-3) = \frac{1}{(-3)^3}$

**$h(-3) = \frac{1}{-27}$**

(iii) Find the value of h(7.5), giving your answer in standard form. [2]

For x = 7.5:

$h(7.5) = 7.5^{7.5}$

h(7.5) = 3655606.7

In standard form, **h(7.5) = 3.65 x $10^6$.**

(iv) h(−0.5) is not a real number. Explain why. [1]

For x = -0.5:

$h(-0.5) = (-0.5)^{-0.5}$

$h(-0.5) = \frac{1}{(-0.5)^{0.5}}$

$-0.5^{0.5} = \sqrt{-0.5}$

**The square root of a negative number it is not a real number.**

(v) Find the integer value for which h($x$) = 3125. [1]

h(x) = 3125

$x^x = 3125$

$x = \log_x 3125$

**The integer in this case is x = 5.**