Vectors
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors and transformations
Sub-Topic	Vectors
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **41 minutes**

Score: **/32**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

Assembled by N.S

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

(a) D is the point $(2, -5)$ and $\overrightarrow{DE} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$.

Find the co-ordinates of the point E. [1]

$$E = \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{9} \\ \mathbf{-4} \end{pmatrix}$$

So coordinates are: (9,-4)

(b) $\mathbf{v} = \begin{pmatrix} t \\ 12 \end{pmatrix}$ and $|\mathbf{v}| = 13$.

Work out the value of t, where t is negative. [2]

We have that

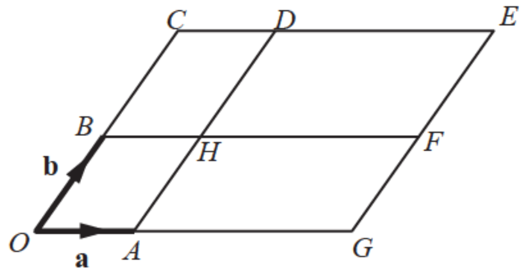
$$|\vec{v}| = \sqrt{t^2 + 12^2} = 13$$

$$\rightarrow t^2 = 13^2 - 12^2$$

$$= 25$$

$$\rightarrow \boldsymbol{t = -5}$$

The diagram shows a parallelogram *OCEG*.



NOT TO SCALE

O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
BHF and *AHD* are straight lines parallel to the sides of the parallelogram.
$\overrightarrow{OG} = 3\overrightarrow{OA}$ and $\overrightarrow{OC} = 2\overrightarrow{OB}$.

(a) Write the vector $\overrightarrow{HE}$ in terms of **a** and **b**. [1]

$\overrightarrow{HE} = \overrightarrow{HD} + \overrightarrow{DE}$

$= \vec{b} + 2\vec{a}$

(b) Complete this statement.

$\mathbf{a} + 2\mathbf{b}$ is the position vector of point [1]

D

(c) Write down two vectors that can be written as $3\mathbf{a} - \mathbf{b}$. [2]

$\overrightarrow{CF}$ and $\overrightarrow{BG}$

(a) $\overrightarrow{GH} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

Find

(i) $5\overrightarrow{GH}$, [1]

$$\begin{pmatrix} \mathbf{30} \\ \mathbf{-20} \end{pmatrix}$$

(ii) $\overrightarrow{HG}$. [1]

$$\begin{pmatrix} \mathbf{-6} \\ \mathbf{4} \end{pmatrix}$$

(b) $\begin{pmatrix} 6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

Find the value of y. [1]

$$7 + y = 3$$

Take 7 from both sides

$$\rightarrow \boldsymbol{y = -4}$$

$\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \overrightarrow{BA} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$

(a) Find $\overrightarrow{CA}$. [2]

$$\overrightarrow{CA} = \overrightarrow{CB} + \overrightarrow{BA}$$

$$\overrightarrow{CA} = -\overrightarrow{BC} + \overrightarrow{BA}$$

$$\overrightarrow{CA} = -\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 6 \end{pmatrix}$$

$$\overrightarrow{CA} = \begin{pmatrix} -2-5 \\ -3+6 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-7} \\ \mathbf{3} \end{pmatrix}$$

(b) Work out $|\overrightarrow{BA}|$. [2]

To find the magnitude of a vector use Pythagoras Theorem:

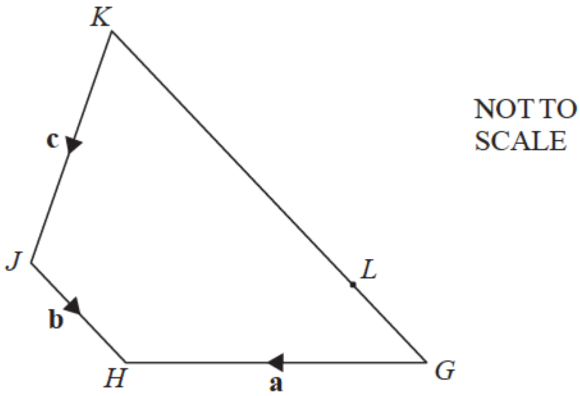
$$\left|\begin{pmatrix} x \\ y \end{pmatrix}\right| = \sqrt{x^2 + y^2}$$

$$|\overrightarrow{BA}| = \left|\begin{pmatrix} -5 \\ 6 \end{pmatrix}\right| = \sqrt{(-5)^2 + 6^2}$$

$$|\overrightarrow{BA}| = \sqrt{61}$$

(to 3 significant figures)

$$= \mathbf{7.81}$$



$GHJK$ is a quadrilateral.
$\overrightarrow{GH} = \mathbf{a}$, $\overrightarrow{JH} = \mathbf{b}$ and $\overrightarrow{KJ} = \mathbf{c}$.
L lies on GK so that $LK = 3GL$.

Find an expression, in terms of **a**, **b** and **c**, for $\overrightarrow{GL}$. [2]

$\overrightarrow{GL}$ is a quarter of $\overrightarrow{GK}$, which can be written as

$$\overrightarrow{GL} = \frac{1}{4}\overrightarrow{GK}$$

$$= \frac{1}{4}(\vec{a} - \vec{b} - \vec{c})$$

Question 6

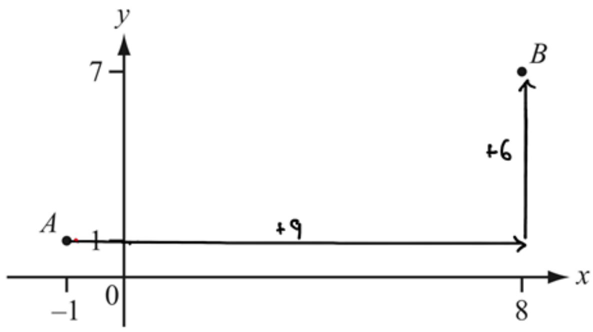
NOT TO SCALE

A is the point (–1, 1) and B is the point (8, 7).

(a) Write $\overrightarrow{AB}$ as a column vector. [1]

To get from A to B we move +9 units along the x axis $(8 - -1 = 9)$ and

+6 units along the y axis $(7 - 1 = 6)$.



Therefore, the column vector for AB is:

$$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

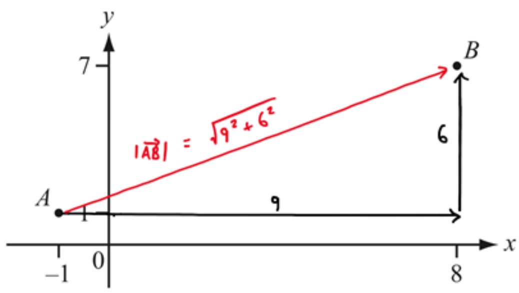
(b) Find $|\overrightarrow{AB}|$. [2]

The magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is given by $\sqrt{x^2 + y^2}$

$$\left|\begin{pmatrix} 9 \\ 6 \end{pmatrix}\right| = \sqrt{9^2 + 6^2}$$

$$= \sqrt{81 + 36}$$

$$= \sqrt{117}$$



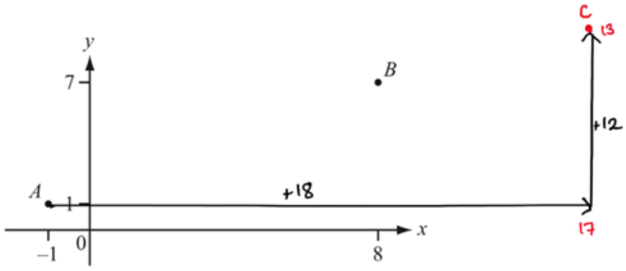
(c) $\overrightarrow{AC} = 2\overrightarrow{AB}$.

Write down the co-ordinates of C. [1]

$$\overrightarrow{AC} = 2\overrightarrow{AB}$$

$$= 2\begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 18 \\ 12 \end{pmatrix}$$



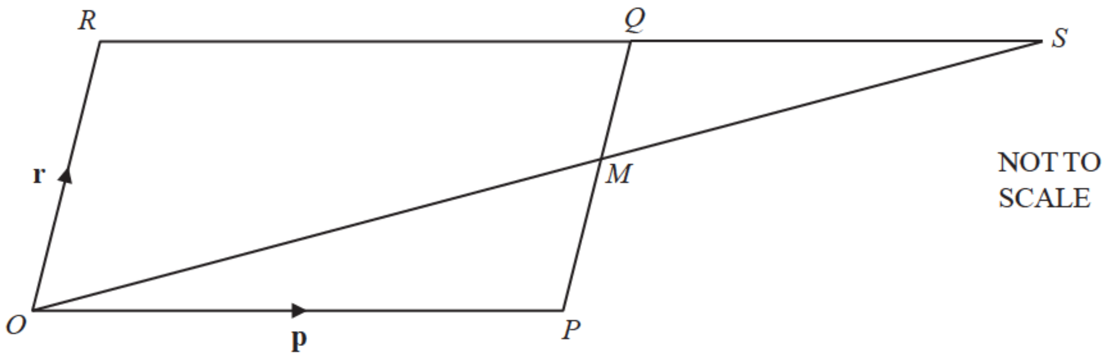
The x coordinate of C is 18 units from the x coordinate of A

$$x\ coordinate\ of\ C = -1 + 18 = 17$$

The y coordinate of C is 12 units from the y coordinate of A

$$y\ coordinate\ of\ C = 1 + 12 = 13$$

$$\boldsymbol{Coodinates\ of\ C\!:\ (17, 13)}$$



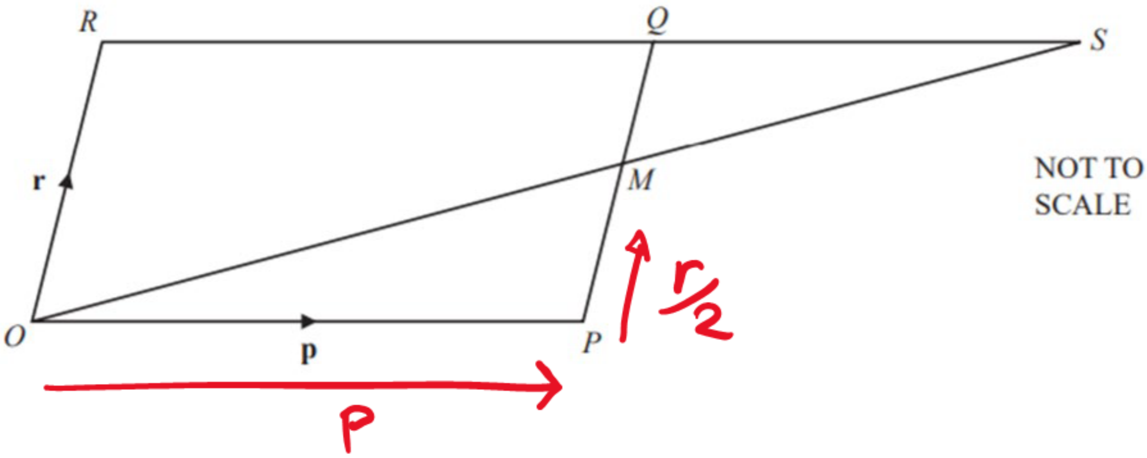
$OPQR$ is a parallelogram, with O the origin.
M is the midpoint of PQ.
OM and RQ are extended to meet at S.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.

(a) Find, in terms of $\mathbf{p}$ and $\mathbf{r}$, in its simplest form, [1]

(i) $\overrightarrow{OM}$,

We are told that $OPQR$ is a parallelogram, so we know that the vectors $\boldsymbol{r}$ and $\boldsymbol{p}$ travel the same distance and the same direction along each of the sides OR and PQ respectively. Hence $\overrightarrow{PQ} = \overrightarrow{OR}$ and $\overrightarrow{PM} = \frac{r}{2}$.

To get from O to M we take the path shown below.



Hence the simplest path is $\overrightarrow{OM}$

$$= \boldsymbol{p} + \frac{1}{2}\boldsymbol{r}$$

(ii) the position vector of S. [1]

The line OS is double of the line OM and the points O, M and S are colinear (on a straight line).

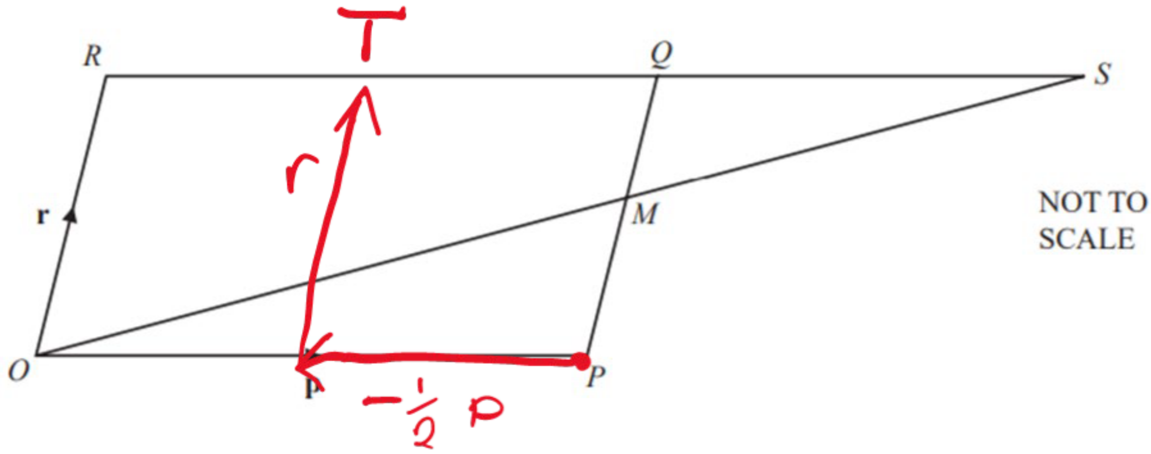
Hence we just double vector $\overrightarrow{OM}$ to get $\overrightarrow{OS}$

$$2(\overrightarrow{OM}) = \overrightarrow{OS} = 2\left(\boldsymbol{p} + \frac{1}{2}\boldsymbol{r}\right)$$

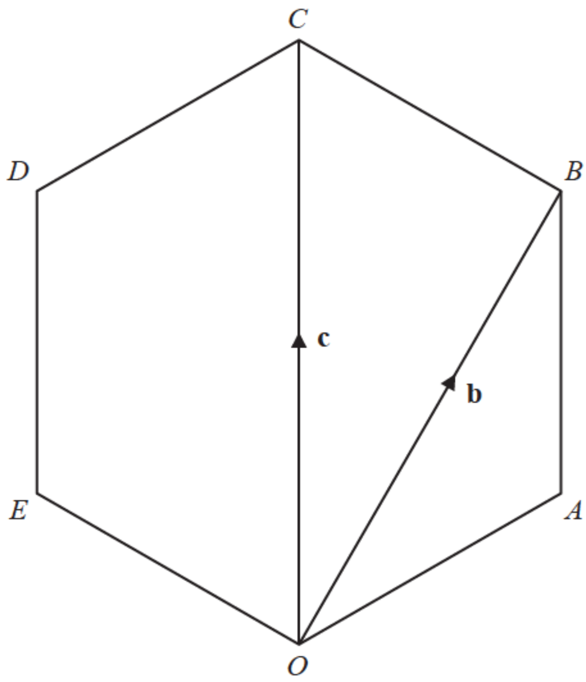
$$\overrightarrow{\boldsymbol{OS}} = \boldsymbol{2p + r}$$

(b) When $\overrightarrow{PT} = -\frac{1}{2}\mathbf{p} + \mathbf{r}$, what can you write down about the position of T? [1]

Here we can draw a diagram to help us.



From this we can see that T is the midpoint of RQ.



OABCDE is a regular polygon.

(a) Write down the geometrical name for this polygon. [1]

The shape has 6 sides therefore it is a **hexagon**

(b) O is the origin. $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

Find, in terms of **b** and **c**, in their simplest form,

(i) $\overrightarrow{BC}$, [1]

To go from B to C we must go backwards along $\overrightarrow{OB}$ and forwards along $\overrightarrow{OC}$

$$\overrightarrow{BC} = -\overrightarrow{OB} + \overrightarrow{OC}$$

$$\overrightarrow{BC} = -\boldsymbol{b} + \boldsymbol{c}$$

(ii) $\overrightarrow{OA}$, [2]

Going from O to A is equal to $\overrightarrow{OB} + \overrightarrow{BA}$

$$\overrightarrow{BA} = \overrightarrow{CB} = -\overrightarrow{BC} = \boldsymbol{b} - \boldsymbol{c}$$

$$\overrightarrow{OA} = \boldsymbol{b} + \boldsymbol{b} - \boldsymbol{c}$$

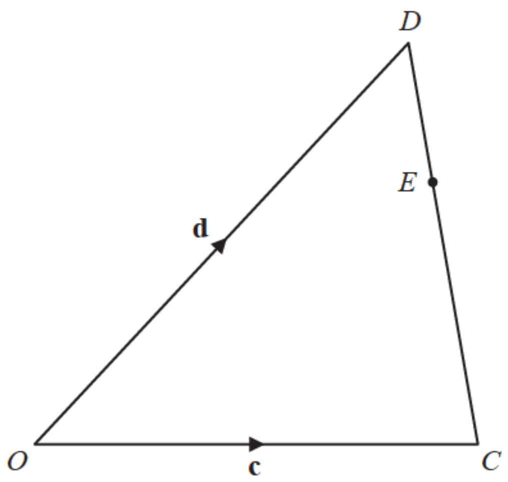
$$= 2\boldsymbol{b} - \boldsymbol{c}$$

$$= \boldsymbol{b} - \frac{1}{2}\boldsymbol{c}$$

(iii) the position vector of E. [1]

The position vector of E is the same as C

$$-\boldsymbol{b} + \boldsymbol{c}$$



NOT TO SCALE

In the diagram, O is the origin.
$\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OD} = \mathbf{d}$.
E is on CD so that $CE = 2ED$.

Find, in terms of c and d, in their simplest forms, [2]

(a) $\overrightarrow{DE}$,

Note that

$$\overrightarrow{DE} = \frac{1}{3}\overrightarrow{DC}$$

So, we need

$$\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$$

$$= -\overrightarrow{OD} + \overrightarrow{OC}$$

$$= -\vec{d} + \vec{c}$$

Hence

$$\overrightarrow{DE} = \frac{1}{3}(\vec{c} - \vec{d})$$

(b) the position vector of E. [2]

The position vector of E is

$$\vec{e} = \overrightarrow{OE}$$

$$= \overrightarrow{OD} + \overrightarrow{DE}$$

$$= \vec{d} + \frac{1}{3}(\vec{c} - \vec{d})$$

$$= \frac{1}{3}\vec{c} + \frac{2}{3}\vec{d}$$

Vectors
Difficulty: Easy

Model Answers 2

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Exam Board	CIE
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Sub-Topic	Vectors
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 2

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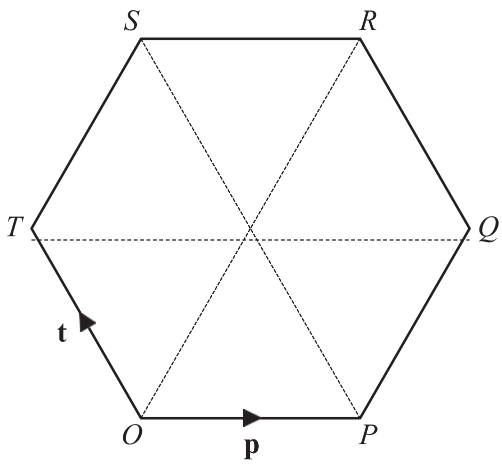


O is the origin and $OPQRST$ is a regular hexagon.

$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OT} = \mathbf{t}$.

Find, in terms of **p** and **t**, in their simplest forms,

(a) $\overrightarrow{PT}$, [1]

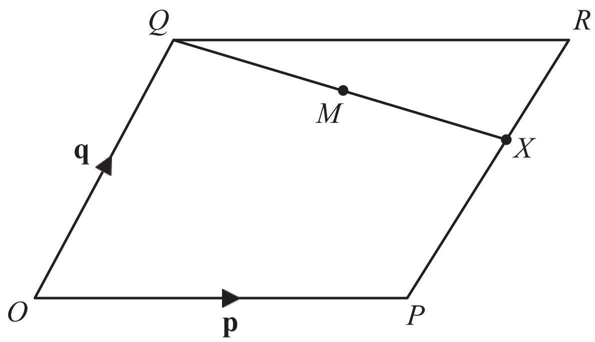
$\vec{t} - \vec{p}$

(b) $\overrightarrow{PR}$, [2]

$2\vec{t} + \vec{p}$

(c) the position vector of R. [2]

$2\vec{p} + 2\vec{t}$



NOT TO SCALE

O is the origin and $OPRQ$ is a parallelogram.
The position vectors of P and Q are p and q.
X is on PR so that $PX = 2XR$.

Find, in terms of p and q, in their simplest forms

(a) ,$\overrightarrow{QX}$ [2]

$$\overrightarrow{QX} = \overrightarrow{QO} + \overrightarrow{OP} + \overrightarrow{PX}$$

$$= -\vec{q} + \vec{p} + \frac{2}{3}\vec{q}$$

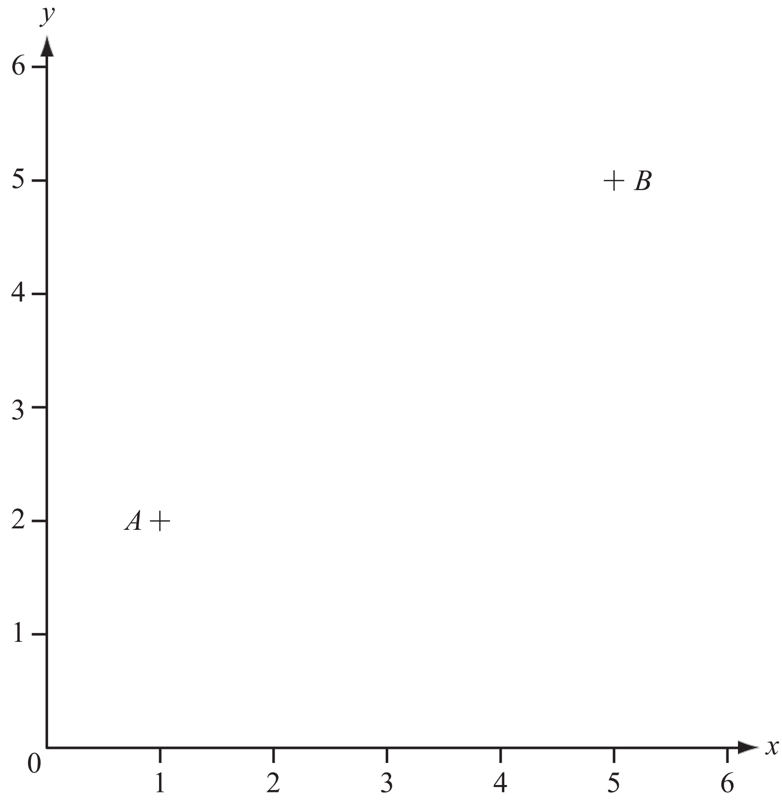
$$= \vec{p} - \frac{1}{3}\vec{q}$$

(b) the position vector of M, the midpoint of QX. [2]

$$\overrightarrow{OM} = \overrightarrow{OQ} + \overrightarrow{QM}$$

$$= \vec{q} + \frac{1}{2}\left(\vec{p} - \frac{1}{3}\vec{q}\right)$$

$$= \frac{5}{6}\vec{q} + \frac{1}{2}\vec{p}$$



The points $A(1, 2)$ and $B(5, 5)$ are shown on the diagram .

(a) Work out the co-ordinates of the midpoint of AB. [1]

We represent the mid-point of AB with the point $M(x_M, y_M)$.

We know that:

$$x_M = \frac{xA + xB}{2}$$

$$x_M = \frac{1+5}{2}$$

$$x_M = 3$$

$y_M = \frac{yA+yB}{2}$

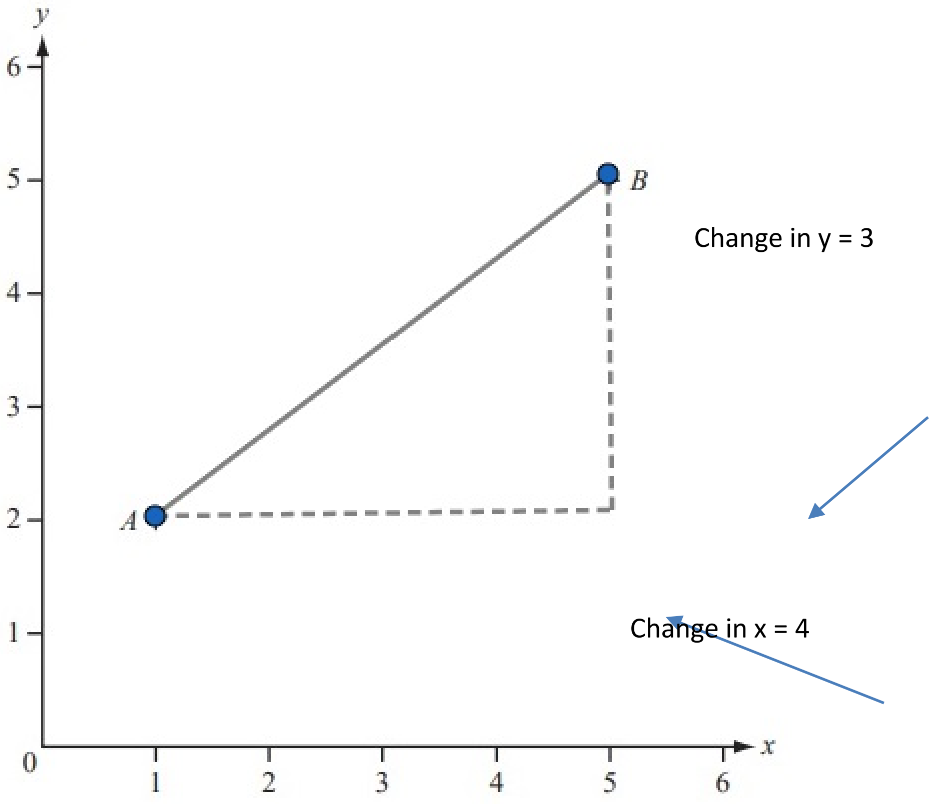
$y_M = \frac{2+5}{2}$

$y_M = 3.5$

The mid-point is:

M(3, 3.5)

(b) Write down the column vector $\overrightarrow{AB}$. [1]

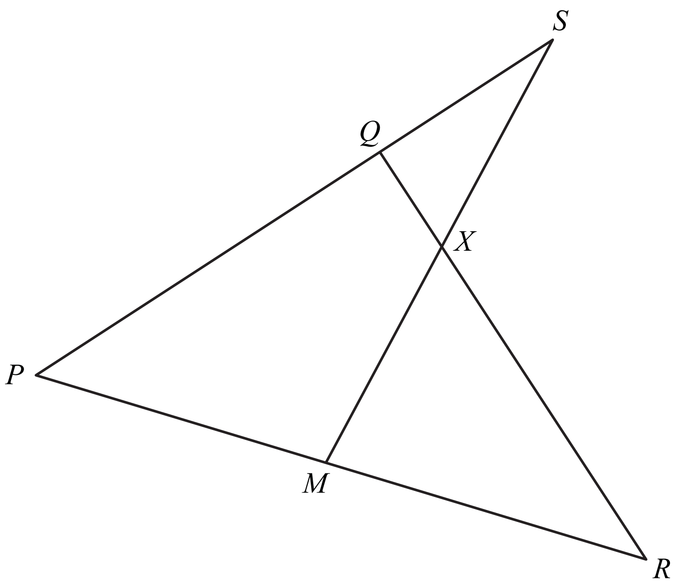


The column vector for the vector $\overrightarrow{AB}$ has as top value the change in x

and as bottom value the change in y.

$\overrightarrow{AB} = \begin{pmatrix} change\ in\ x \\ change\ in\ y \end{pmatrix}$

$\overrightarrow{\boldsymbol{AB}} = \begin{pmatrix} \mathbf{4} \\ \mathbf{3} \end{pmatrix}$



In the diagram, *PQS*, *PMR*, *MXS* and *QXR* are straight lines.

$PQ = 2\ QS$.
M is the midpoint of *PR*.
$QX : XR = 1 : 3$.

$\overrightarrow{PQ} = \mathbf{q}$ and $\overrightarrow{PR} = \mathbf{r}$.

(a) Find, in terms of q and r,

(i) $\overrightarrow{RQ}$, [1]

The vector RQ can be split into two vectors.

$$\overrightarrow{RQ} = \overrightarrow{RP} + \overrightarrow{PQ}$$

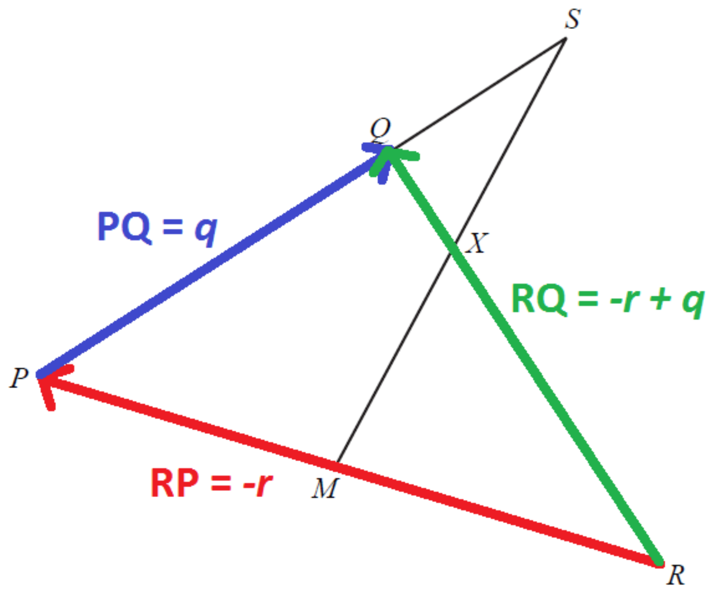
It is important to remember that for vectors:

$\overrightarrow{RP} = -\overrightarrow{PR}$.

Using this fact:

$$\overrightarrow{RQ} = -\overrightarrow{PR} + \overrightarrow{PQ}$$

$$\boldsymbol{\overrightarrow{RQ} = -r + q}$$



(ii) $\overrightarrow{MS}$. [1]

The vector MS can be split into two vectors.

$$\overrightarrow{MS} = \overrightarrow{MP} + \overrightarrow{PS} = -\overrightarrow{PM} + \overrightarrow{PS}$$

Since M is the midpoint of PR, we have

$$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PR}$$

We also have

$$\overrightarrow{PQ} = 2\overrightarrow{QS}$$

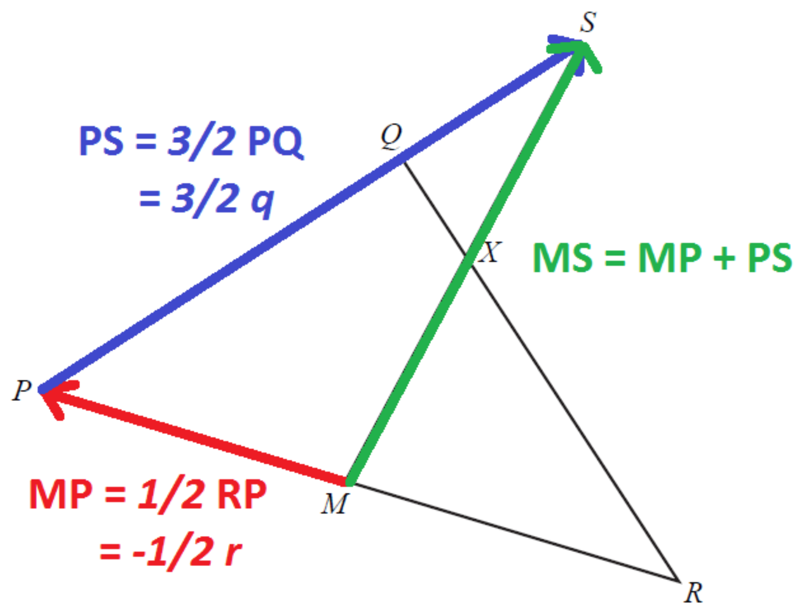
Hence

$$\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS} = \frac{3}{2}\overrightarrow{PQ}$$

Therefore

$$\overrightarrow{MS} = -\frac{1}{2}\overrightarrow{PR} + \frac{3}{2}\overrightarrow{PQ}$$

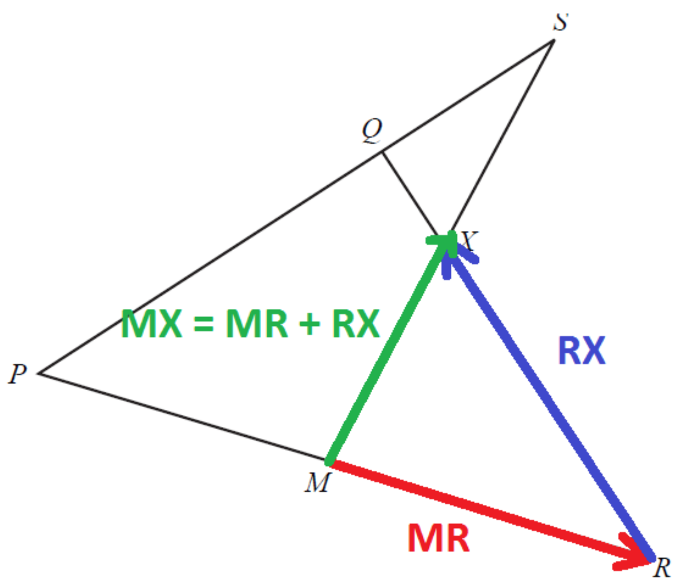
$$\boldsymbol{\overrightarrow{MS} = \frac{1}{2}(-r + 3q)}$$



(b) By finding $\overrightarrow{MX}$, show that X is the midpoint of MS. [3]

The vector MX can be split into two vectors.

$$\overrightarrow{MX} = \overrightarrow{MR} + \overrightarrow{RX}$$



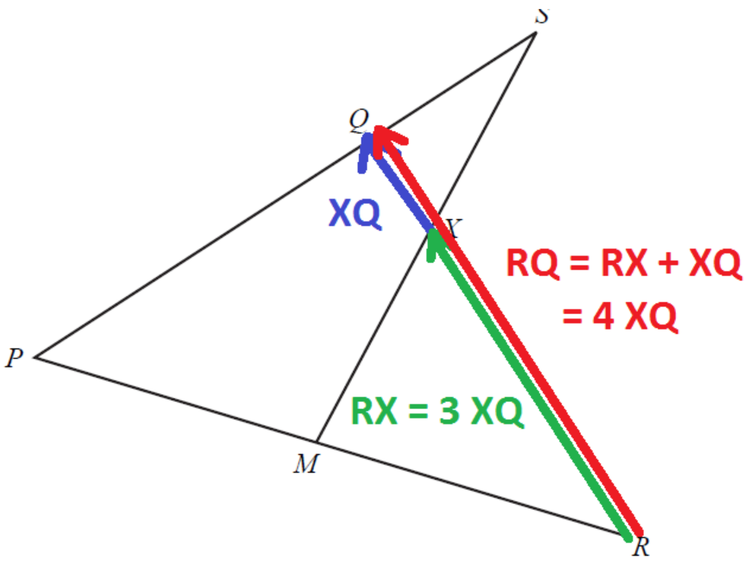
Since M is the midpoint of PR, we have

$$\overrightarrow{MR} = \frac{1}{2}\overrightarrow{PR}$$

The ratio QX : XR is given as 1 : 3. Therefore the ratio RQ :

RX is 4 : 3.

$$\overrightarrow{RX} = \frac{3}{4}\overrightarrow{RQ}$$



We have:

$$\overrightarrow{MX} = \frac{1}{2}\overrightarrow{PR} + \frac{3}{4}\overrightarrow{RQ}$$

$$\overrightarrow{MX} = \frac{1}{2}r + \frac{3}{4}(-r + q)$$

Combine the vectors.

$$\overrightarrow{MX} = -\frac{1}{4}r + \frac{3}{4}q$$

We can see that:

$$\overrightarrow{MX} = \frac{1}{2}\left(\frac{1}{2}(-r + 3q)\right)$$

$$\boldsymbol{\overrightarrow{MX} = \frac{1}{2}\overrightarrow{MS}}$$

Hence proving that X is the midpoint of MS.

The position vector r is given by $\mathbf{r} = 2\mathbf{p} + t(\mathbf{p} + \mathbf{q})$.

(a) Complete the table below for the given values of t.
Write each vector in its simplest form.
One result has been done for you.

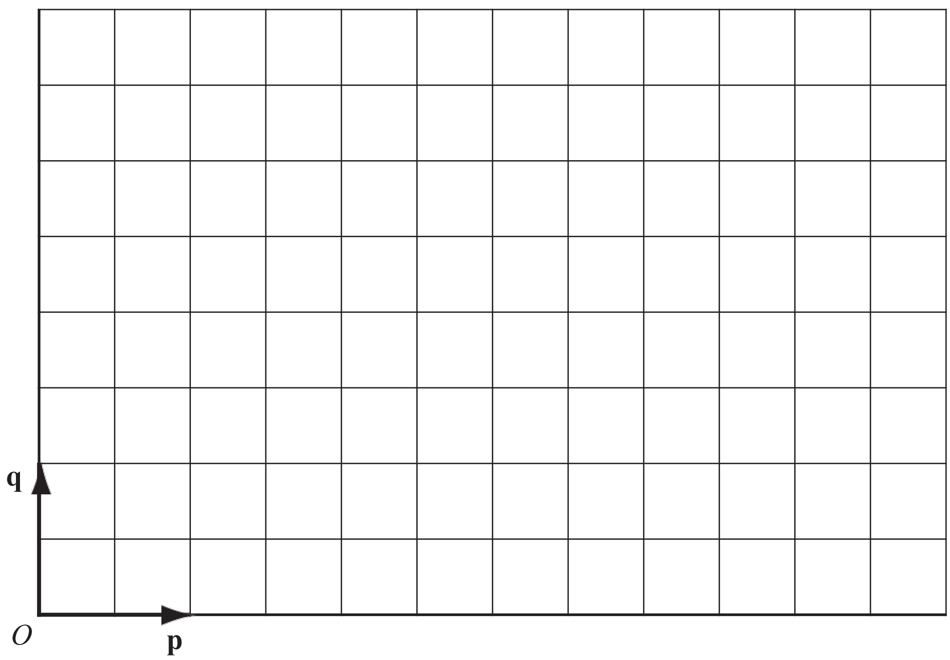
t	0	1	2	3
r			$4\mathbf{p} + 2\mathbf{q}$	

[3]

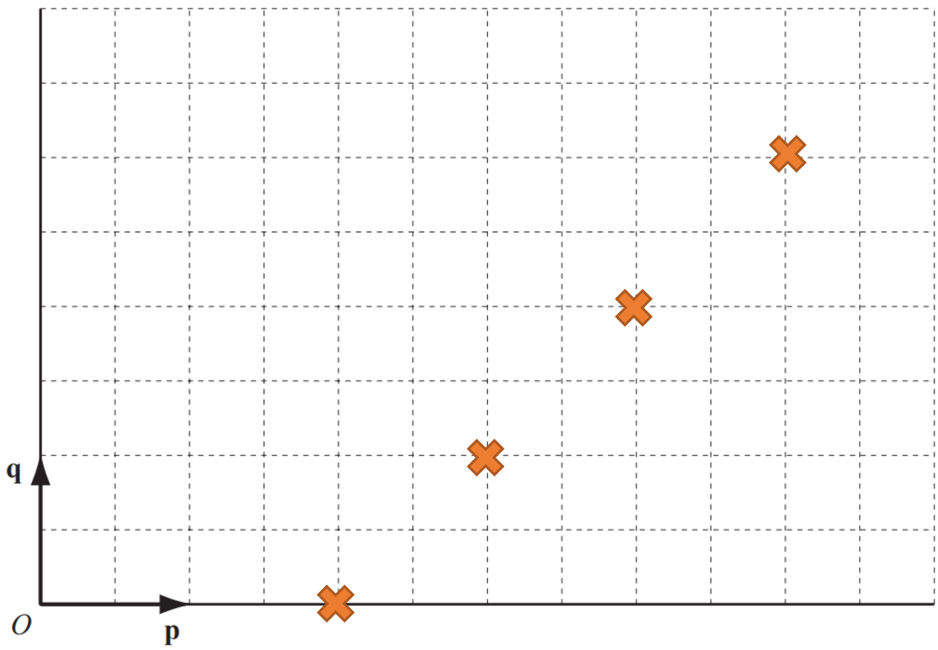
t	0	1	2	3
$\vec{r}$	$2\vec{p}$	$3\vec{p} + \vec{q}$	$4\vec{p} + 2\vec{q}$	$5\vec{p} + 3\vec{q}$

(b) O is the origin and **p** and **q** are shown on the diagram.

(i) Plot the 4 points given by the position vectors in the table.

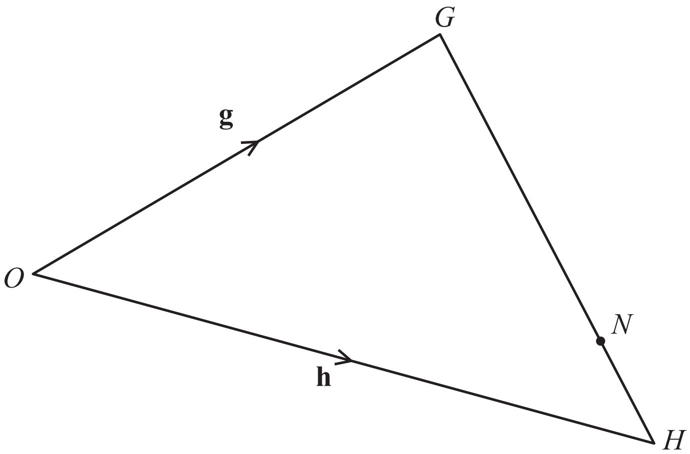


[2]



(ii) What can you say about these four points? [1]

They create a **straight line (colinear)**.



NOT TO SCALE

In triangle OGH, the ratio $GN : NH = 3 : 1$.

$\overrightarrow{OG} = \mathbf{g}$ and $\overrightarrow{OH} = \mathbf{h}$.

Find the following in terms of $\mathbf{g}$ and $\mathbf{h}$, giving your answers in their simplest form.

(a) $\overrightarrow{HG}$ [1]

$$\overrightarrow{HG} = \overrightarrow{HO} + \overrightarrow{OG}$$

$$= -\overrightarrow{OH} + \vec{g}$$

$$= \vec{\boldsymbol{g}} - \vec{\boldsymbol{h}}$$

(b) $\overrightarrow{ON}$ [2]

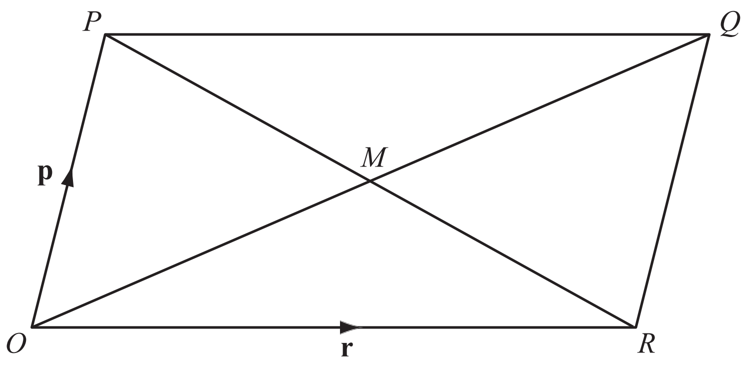
$$\overrightarrow{ON} = \overrightarrow{OG} + \overrightarrow{GN}$$

$$= \vec{g} + \frac{3}{4}\overrightarrow{GH}$$

$$= \vec{g} + \frac{3}{4}(-\overrightarrow{HG})$$

$$= \vec{g} + \frac{3}{4}(\vec{h} - \vec{g})$$

$$= \frac{\mathbf{1}}{\mathbf{4}}\vec{\boldsymbol{g}} + \frac{\mathbf{3}}{\mathbf{4}}\vec{\boldsymbol{h}}$$



O is the origin and $OPQR$ is a parallelogram whose diagonals intersect at M.

The vector $\overrightarrow{OP}$ is represented by p and the vector $\overrightarrow{OR}$ is represented by r.

(a) Write down a single vector which is represented by

(i) $\mathbf{p} + \mathbf{r}$, [1]

$\underline{p} + \underline{r} = \overrightarrow{OQ}$

(ii) $\frac{1}{2}\mathbf{p} - \frac{1}{2}\mathbf{r}$. [1]

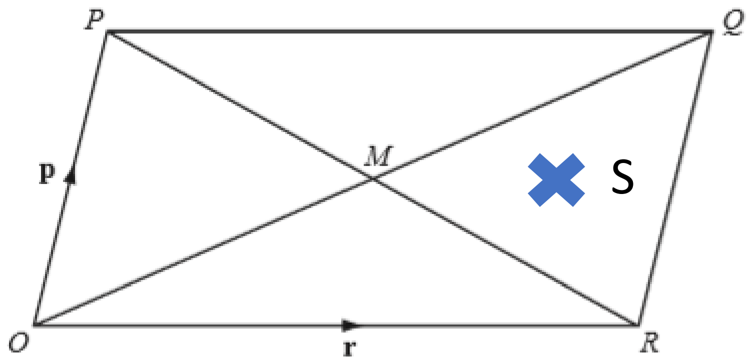
$\frac{1}{2}\underline{p} + \frac{1}{2}\underline{r} = \overrightarrow{RP} = \overrightarrow{RM}$

(b) On the diagram, mark with a cross (x) and label with the letter S the point with position vector

$\frac{1}{2}\mathbf{p} + \frac{3}{4}\mathbf{r}$. [2]

The point S is halfway up the parallelogram and $\frac{3}{4}$ along from O, given by

$\frac{1}{2}\underline{p} + \frac{3}{4}\underline{r} = \overrightarrow{OS}$



Vectors
Difficulty: Easy

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
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Sub-Topic	Vectors
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Difficulty	Easy
Booklet	Model Answers 3

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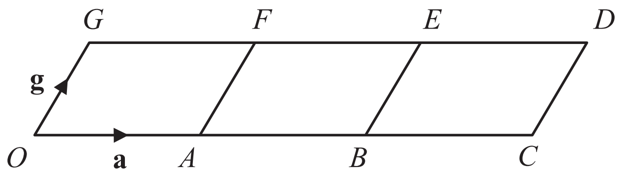


The diagram is made from three identical parallelograms.

O is the origin. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OG} = \mathbf{g}$.

Write down in terms of **a** and **g**

(a) $\overrightarrow{GB}$, [1]

$\overrightarrow{GB} = \overrightarrow{OB} + \overrightarrow{GO}$

$\overrightarrow{OB} = \overrightarrow{AB} + \overrightarrow{OA} = 2\mathrm{a}$

$\overrightarrow{GO} = -\overrightarrow{OG} = -\mathrm{g}$

$\boldsymbol{\overrightarrow{GB} = 2\mathrm{a} - \mathrm{g}}$

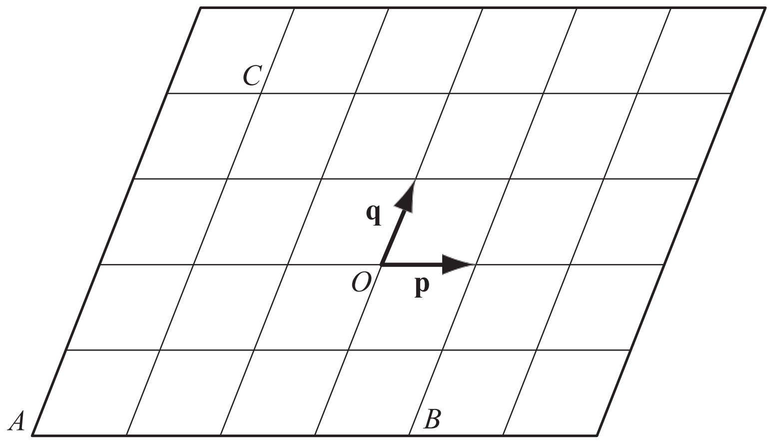
(b) the position vector of the centre of the parallelogram $BCDE$. [1]

The centre of the parallelogram BCDE is the intersection of its diagonals, BD and CE.

The intersection of the 2 diagonals is the mid-point of each of the 2 diagonals.

Therefore, the mid-point of EC has the position:

Centre of the parallelogram $= \frac{1}{2}\overrightarrow{BE} + \frac{1}{2}\overrightarrow{CD} = 2\,\frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{g}$



O is the origin. Vectors p and q are shown in the diagram.

(a) Write down, in terms of p and q, in their simplest form

(i) the position vector of the point A, [1]

$$\overrightarrow{OA} = -3\vec{p} - 2\vec{q}$$

(ii) $\overrightarrow{BC}$, [1]

$$\overrightarrow{BC} = -3\vec{p} + 4\vec{q}$$

(iii) $\overrightarrow{BC} - \overrightarrow{AC}$. [2]

$$\overrightarrow{AC} = \vec{p} + 4\vec{q}$$

Hence

$$\overrightarrow{BC} - \overrightarrow{AC}$$

$$= 4\vec{q} - 3\vec{p} - \vec{p} - 4\vec{q}$$

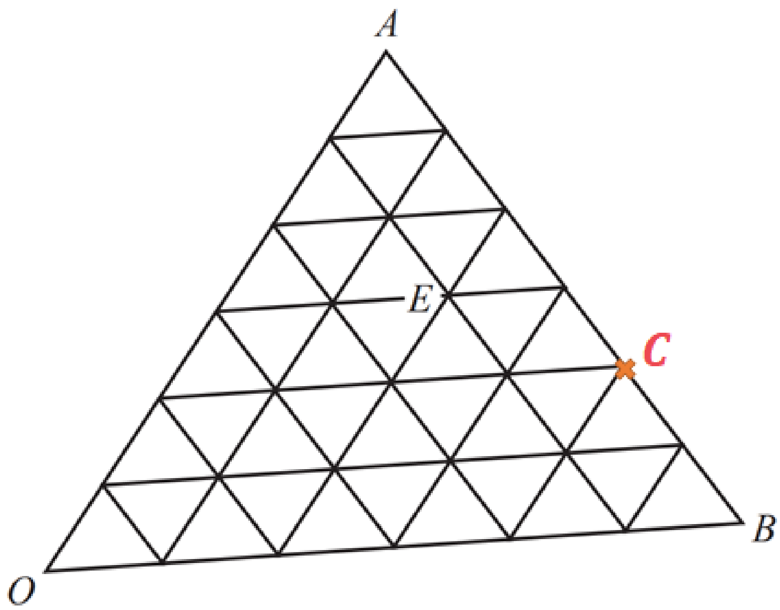
$$= -4\vec{p}$$

(b) If $|\,\mathrm{p}\,| = 2$, write down the value of $|\,AB\,|$. [1]

The length of $\vec{p}$ is 2, hence

$$|\overrightarrow{AB}| = 4 \times 2$$

$$= 8$$



O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) C has position vector $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$.

Mark the point C on the diagram. [1]

(b) Write down, in terms of **a** and **b**, the position vector of the point E. [1]

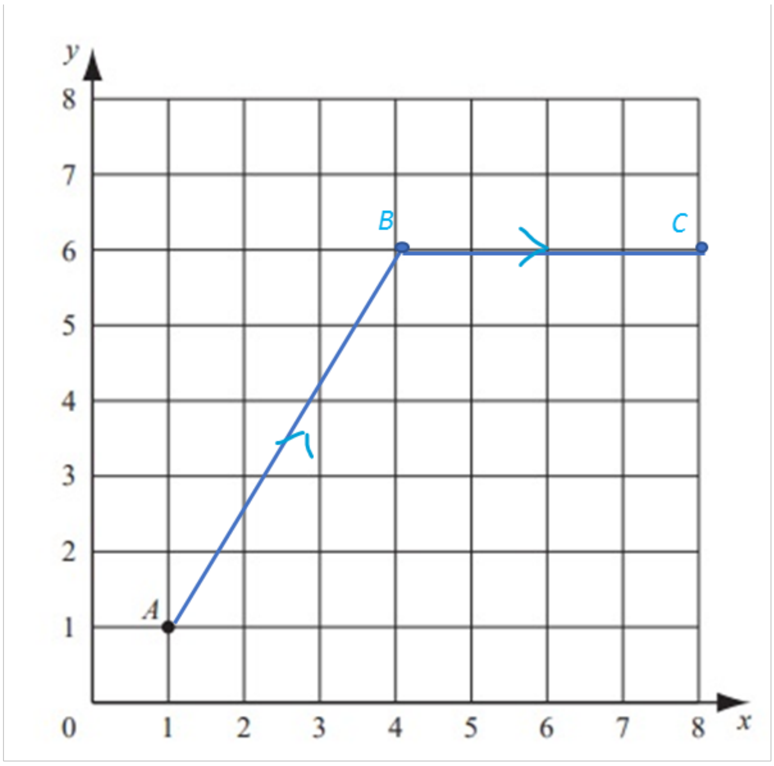
$$\overrightarrow{OE} = \frac{1}{2}\vec{a} + \frac{1}{3}\vec{b}$$

(c) Find, in terms of **a** and **b**, the vector $\overrightarrow{EB}$. [2]

$$\overrightarrow{EB} = -\overrightarrow{OE} + \overrightarrow{OB}$$

$$= -\frac{1}{2}\vec{a} - \frac{1}{3}\vec{b} + \vec{b}$$

$$= -\frac{1}{2}\vec{a} + \frac{2}{3}\vec{b}$$



(a) Using a scale of 1cm to represent 1 unit, draw the vectors

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ on the grid above. [2]

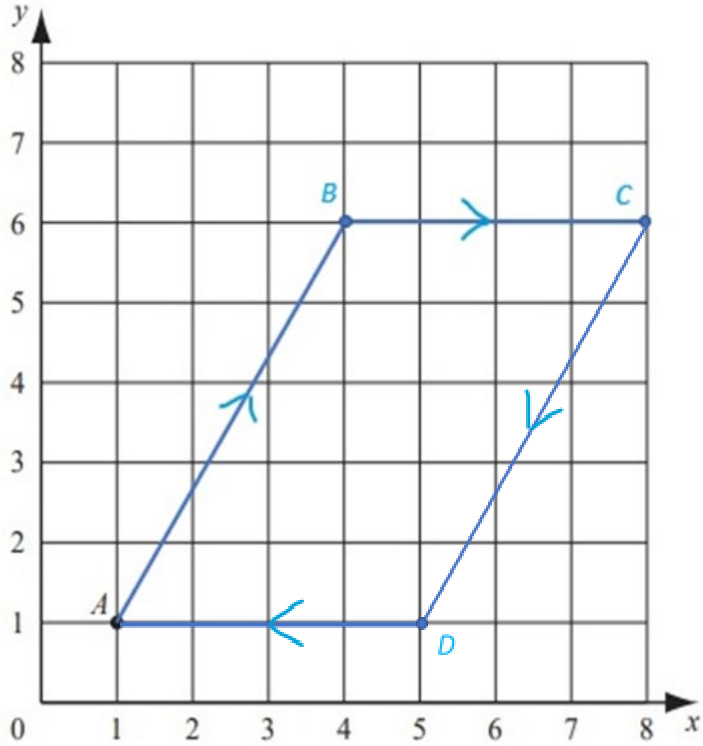
In a column vector, the top number represents the number of units moved on the x-axis and the bottom number represents the number of units moved on the y-axis.

In our case, the vector $\overrightarrow{AB}$ has 3 units on the x-axis and 5 units on the y-axis. Similarly, $\overrightarrow{BC}$ has only 4 units on the x-axis, therefore it is a straight line.

(b) *ABCD* is a parallelogram.
Write down the coordinates of *D*. [2]

Since ABCD is a parallelogram, AB will be parallel with CD and BC will be parallel with AD.

Using the grid above, we can draw the parallelogram ABCD.



According to the grid, D has the coordinates D (5, 1).

(c) Calculate $|\overrightarrow{AB}|$. [2]

The modulus of a vector represents its magnitude.

For a column vector $\overrightarrow{AB} = \binom{x}{y}$, the modulus of the vector will be:

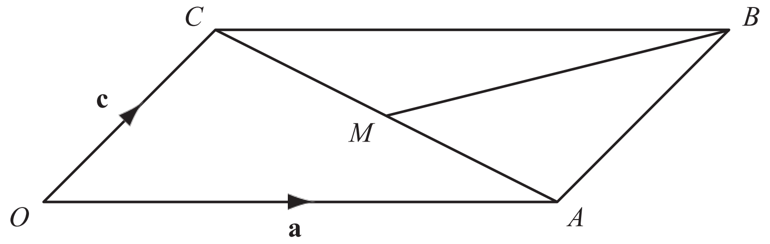
$|\overrightarrow{AB}| = \sqrt{x^2 + y^2}$

In our case, x = 3 and y = 5.

$\left|\overrightarrow{AB}\right| = \sqrt{3^2 + 5^2}$

$\left|\overrightarrow{AB}\right| = \sqrt{34}$

$\left|\overrightarrow{\boldsymbol{AB}}\right|$ **= 5.83**



$OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and M is the mid-point of CA.
Find in terms of a and c

(a) $\overrightarrow{OB}$, [1]

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OA} + \overrightarrow{OC}$$

$$= \boldsymbol{a} + \boldsymbol{c}$$

(b) $\overrightarrow{CA}$, [1]

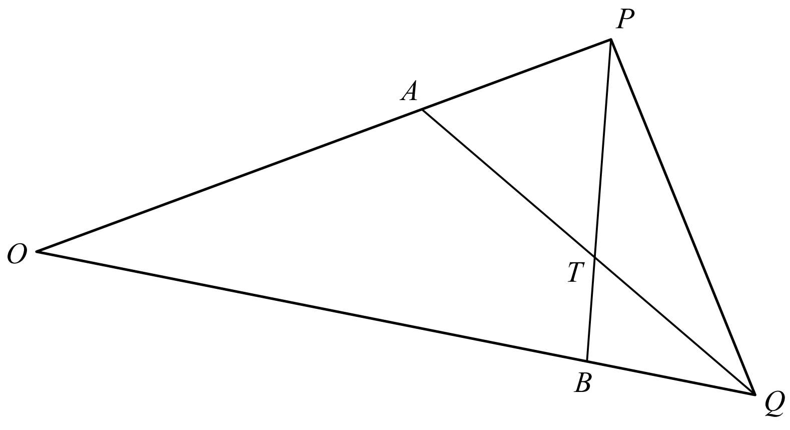
$$\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA} = -\boldsymbol{c} + \boldsymbol{a}$$

$$= \boldsymbol{a} - \boldsymbol{c}$$

(c) $\overrightarrow{BM}$. [2]

$$\overrightarrow{BM} = \overrightarrow{BC} + \overrightarrow{CM} = \overrightarrow{AO} + \frac{1}{2}\overrightarrow{CA} = -\boldsymbol{a} + \frac{1}{2}(\boldsymbol{a} - \boldsymbol{c})$$

$$= -\frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{c}$$



NOT TO SCALE

In the diagram $OA = \frac{2}{3}OP$ and $OB = \frac{3}{4}OQ$.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.

(a) Find in terms of **p** and **q**

(i) $\overrightarrow{AQ}$, [2]

$\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ}$

$OA = \frac{2}{3}OP$

Therefore: $\overrightarrow{OA} = \frac{2}{3}\overrightarrow{OP}$

$\overrightarrow{AO} = -\overrightarrow{OA}$

$\overrightarrow{AO} = -\frac{2}{3}\overrightarrow{OP} = -\frac{2}{3}\mathrm{p}$

$\boldsymbol{\overrightarrow{AQ} = -\frac{2}{3}\mathbf{p} + \mathbf{q}}$

[2]

(ii) $\overrightarrow{BP}$.

$\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB}$

$\overrightarrow{PO} = -\overrightarrow{OP} = -\text{p}$

$\text{OB} = \frac{3}{4}\text{OQ}$

$\overrightarrow{OB} = \frac{3}{4}\overrightarrow{OQ} = \frac{3}{4}\text{q}$

$\overrightarrow{PB} = \text{p} - \frac{3}{4}\text{q}$

(b) AQ and BP intersect at T.
$BT = \frac{1}{3}BP$.
Find $\overrightarrow{QT}$ in terms of **p** and **q**, in its simplest form. [2]

$\overrightarrow{QT} = \overrightarrow{TB} + \overrightarrow{BQ}$

$\overrightarrow{TB} = -\overrightarrow{BT}$

$\text{TB} = \frac{1}{3}\text{BP}$

$\overrightarrow{TB} = \frac{1}{3}\overrightarrow{BP}$

$\overrightarrow{BP} = -\overrightarrow{PB}$

$\overrightarrow{PB} = -\text{p} + \frac{3}{4}\text{q}$

$\overrightarrow{BP} = \text{p} - \frac{3}{4}\text{q}$

$\overrightarrow{TB} = \frac{1}{3}(\text{p} - \frac{3}{4}\text{q})$

$\overrightarrow{OB} = \frac{3}{4}\overrightarrow{OQ}$

Therefore: $\overrightarrow{BQ} = \frac{1}{4}\overrightarrow{OQ} = \frac{1}{4}$ q

$\overrightarrow{QT} = \overrightarrow{TB} - \overrightarrow{BQ}$

$\overrightarrow{QT} = \frac{1}{3}(\text{p} - \frac{3}{4}\text{q}) - \frac{1}{4}\text{q}$

$\overrightarrow{QT} = \frac{1}{3}\text{p} - \frac{1}{4}\text{q} - \frac{1}{4}\text{q}$

$\boldsymbol{\overrightarrow{QT} = \frac{1}{3}\textbf{p} - \frac{1}{2}\textbf{q}}$

$\mathbf{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ Find $3\mathbf{a} - 2\mathbf{b}$. [2]

$3a - 2b = 3\begin{pmatrix} 2 \\ -3 \end{pmatrix} - 2\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$3a - 2b = \begin{pmatrix} 6 \\ -9 \end{pmatrix} - \begin{pmatrix} 10 \\ -2 \end{pmatrix}$

$\mathbf{3a - 2b = \begin{pmatrix} -4 \\ -7 \end{pmatrix}}$

Vectors
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors and transformations
Sub-Topic	Vectors
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **28 minutes**

Score: **/22**

Percentage: **/100**

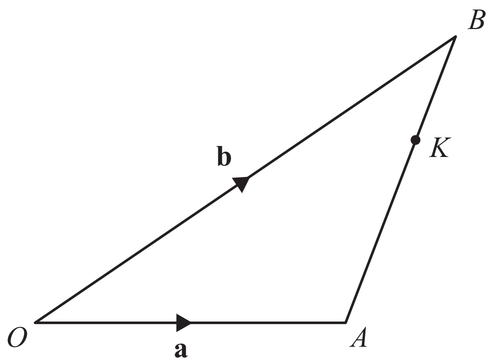
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



NOT TO SCALE

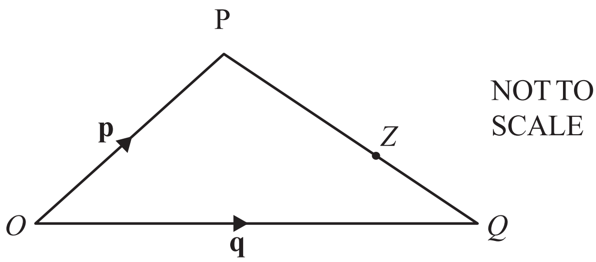
O is the origin and K is the point on AB so that $AK : KB = 2 : 1$.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

Find the position vector of K.
Give your answer in terms of **a** and **b** in its simplest form. [3]

$$\overrightarrow{OK} = \vec{a} + \frac{2}{3}\overrightarrow{AB}$$

$$= \vec{a} + \frac{2}{3}(\vec{b} - \vec{a})$$

$$= \frac{1}{3}\vec{a} + \frac{2}{3}\vec{b}$$



O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.
Z is a point on PQ such that $PZ : ZQ = 5 : 2$.

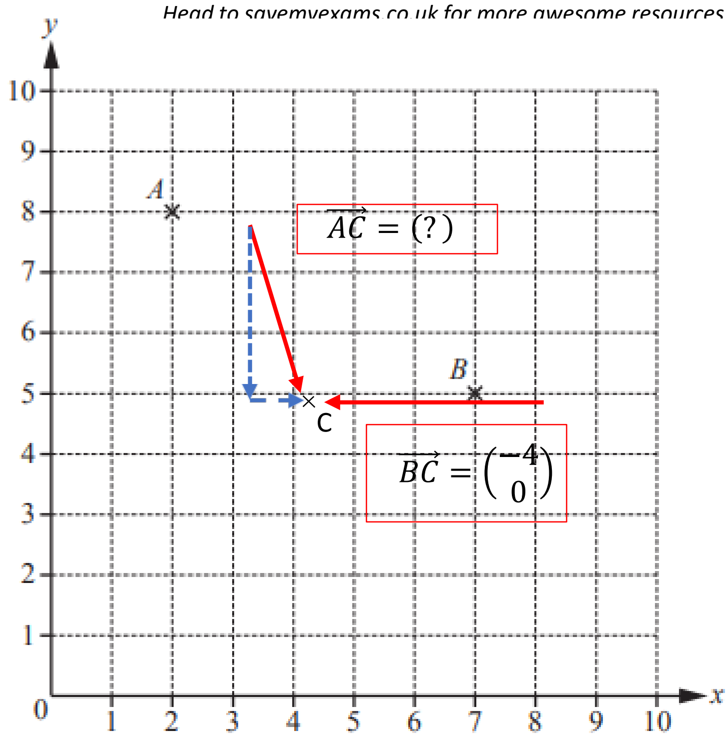
Work out, in terms of $\mathbf{p}$ and $\mathbf{q}$, the position vector of Z.
Give your answer in its simplest form. [3]

$$\overrightarrow{OZ} = \overrightarrow{OP} + \overrightarrow{PZ}$$

$$= \vec{p} + \frac{5}{7}\overrightarrow{PQ}$$

$$= \vec{p} + \frac{5}{7}(\vec{q} - \vec{p})$$

$$= \frac{2}{7}\vec{p} + \frac{5}{7}\vec{q}$$



Points A and B are marked on the grid.

$\overrightarrow{BC} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

(a) On the grid, plot the point C. [1]

(b) Write $\overrightarrow{AC}$ as a column vector. [1]

To get from A to C, you need to go down 3 units, and across 1 units. This looks like this:

$$\overrightarrow{AC} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

(c) $\overrightarrow{DE}$ is a vector that is perpendicular to $\overrightarrow{BC}$.
The magnitude of $\overrightarrow{DE}$ is equal to the magnitude of $\overrightarrow{BC}$.

Write down a possible column vector for $\overrightarrow{DE}$. [2]

BC is horizontal. As DE is perpendicular to BC, we know that it is vertical. If the magnitude is the same, then the vector crosses the same number of units, either upwards or downwards. An option is

$$\overrightarrow{DE} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

There is another option of:

$$\overrightarrow{DE} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

Work out

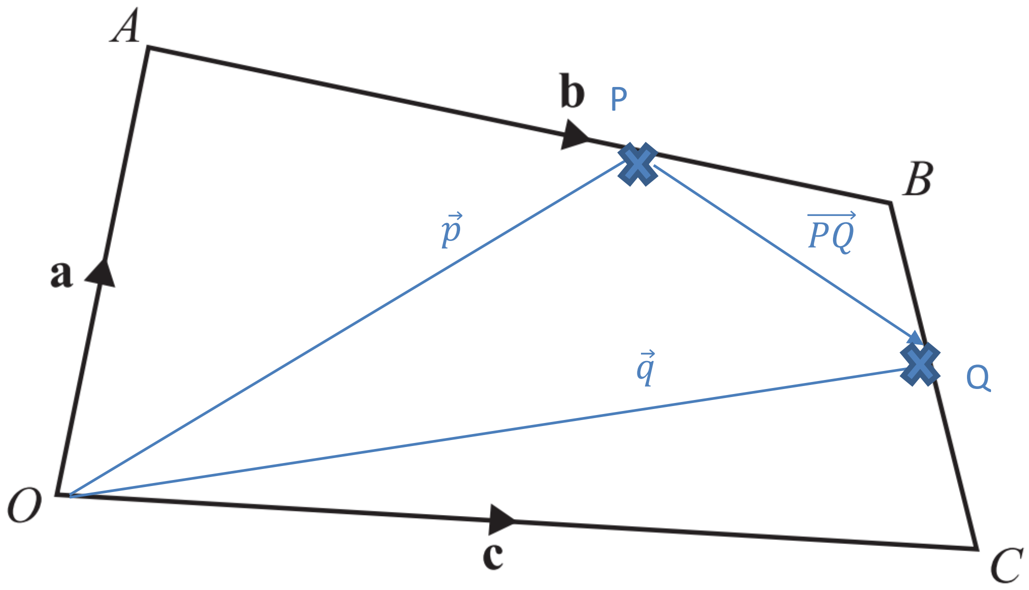
$$2\begin{pmatrix}3\\5\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}$$

[1]

$$2\times\begin{pmatrix}3\\5\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}2\times 3\\2\times 5\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}$$

$$=\begin{pmatrix}2\times 3-1\\2\times 5-2\end{pmatrix}$$

$$=\begin{pmatrix}\mathbf{5}\\\mathbf{8}\end{pmatrix}$$



NOT TO SCALE

In the diagram, O is the origin, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{AB} = \mathbf{b}$.
P is on the line AB so that $AP : PB = 2 : 1$.
Q is the midpoint of BC.

Find, in terms of **a**, **b** and **c**, in its simplest form

(a) $\overrightarrow{CB}$, [1]

$$\overrightarrow{CB} = \vec{a} + \vec{b} - \vec{c}$$

(b) the position vector of Q, [2]

$$\vec{q} = \vec{c} + \frac{1}{2}\overrightarrow{CB}$$

$$= \vec{c} + \frac{1}{2}(-\vec{c} + \vec{a} + \vec{b})$$

$$= \frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$$

(c) $\overrightarrow{PQ}$. [2]

$$\overrightarrow{PQ} = -\vec{p} + \vec{q}$$

$$\vec{p} = \vec{a} + \frac{2}{3}\vec{b}$$

$$\rightarrow \overrightarrow{PQ} = -\vec{a} - \frac{2}{3}\vec{b} + \frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$$

$$= -\frac{1}{2}\vec{a} - \frac{1}{6}\vec{b} + \frac{1}{2}\vec{c}$$

$$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

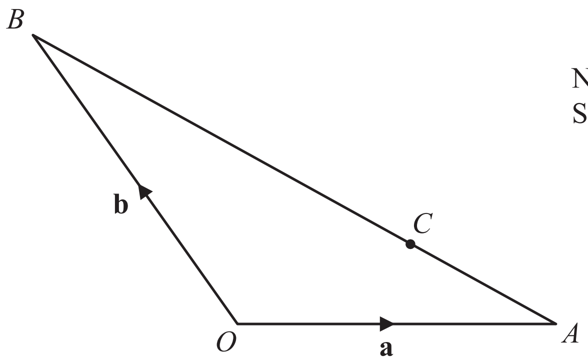
Find $|\overrightarrow{AB}|$. [2]

The modulus of a vector is found by square-rooting the sum of the squares of its elements.

$$|\overrightarrow{AB}| = \left|\begin{matrix} -3 \\ 5 \end{matrix}\right| = \sqrt{(-3)^2 + 5^2} = \sqrt{34}$$

$$|\overrightarrow{AB}|$$

$$= \mathbf{5.83}$$



In the diagram, O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
C is on the line AB so that $AC : CB = 1 : 2$.

Find, in terms of **a** and **b**, in its simplest form,

(a) $\overrightarrow{AC}$, [2]

Since AC:CB = 1:2, we can say that $\overrightarrow{AB} = 3\,\overrightarrow{AC}$

The vector AB can be split into two vectors.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

It is important to remember that for vectors: $\overrightarrow{AO} = -\overrightarrow{OA}$.

Using this fact:

$$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$$

$$\overrightarrow{AB} = -\boldsymbol{a} + \boldsymbol{b}$$

We know that:

$$\overrightarrow{AB} = 3\,\overrightarrow{AC} = -\boldsymbol{a} + \boldsymbol{b}$$

Divide both sides by 3 to get the AC:

$$\overrightarrow{AC} = \frac{-\boldsymbol{a} + \boldsymbol{b}}{\mathbf{3}}$$

(b) the position vector of C. [2]

The position vector of C (OC) can be split into two vectors.

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$$

We know AC from part a)

$$\overrightarrow{OC} = a + \frac{-a+b}{3}$$

$$\overrightarrow{OC} = \frac{3a}{3} + \frac{-a+b}{3}$$

Add the fractions together.

$$\overrightarrow{OC} = \frac{\mathbf{2a+b}}{\mathbf{3}}$$

Vectors
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors and transformations
Sub-Topic	Vectors
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **34 minutes**

Score: **/26**

Percentage: **/100**

Grade Boundaries:

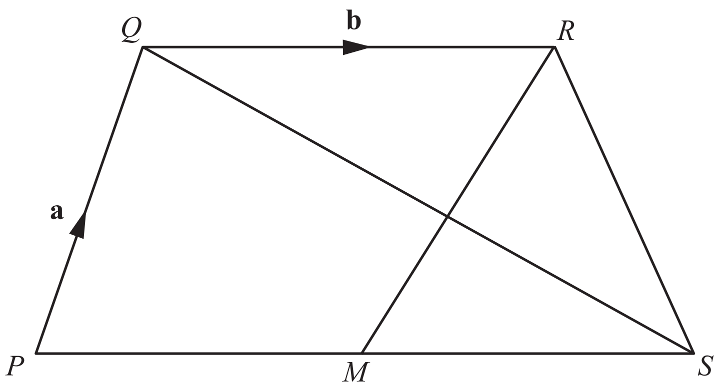
CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

Assembled by A-S



NOT TO SCALE

PQRS is a quadrilateral and *M* is the midpoint of *PS*.

$\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{QR} = \mathbf{b}$ and $\overrightarrow{SQ} = \mathbf{a} - 2\mathbf{b}$.

(a) Show that $\overrightarrow{PS} = 2\mathbf{b}$. [1]

The vector PS can be split into two vectors.

$$\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS}$$

It is important to remember that for vectors: $\overrightarrow{AB} = -\overrightarrow{BA}$.

Using this fact:

$$\overrightarrow{PS} = \overrightarrow{PQ} - \overrightarrow{SQ}$$

$$\overrightarrow{PS} = a - (a - 2b)$$

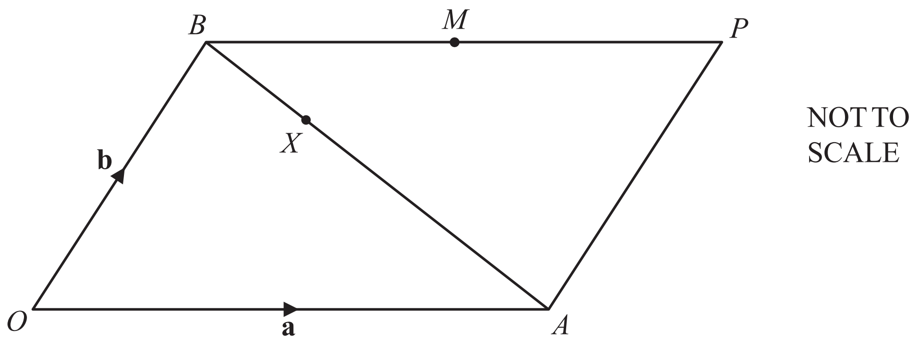
$$\boldsymbol{\overrightarrow{PS} = 2b}$$

(b) Write down the mathematical name for the quadrilateral *PQRM*, giving reasons for your answer. [2]

The mathematical name for PQRM is **a parallelogram**.

We know this because **QR and PM are equal in length and parallel to each other**.

(M is a midpoint of PS, therefore $\overrightarrow{PS} = \frac{1}{2}(2a) = a = \overrightarrow{QR}$)



$OAPB$ is a parallelogram.
O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
M is the midpoint of BP.

(a) Find, in terms of **a** and **b**, giving your answer in its simplest form,

(i) $\overrightarrow{BA}$, [1]

The vector BA can be split into two vectors.

$$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$$

It is important to remember that for vectors: $\overrightarrow{BO} = -\overrightarrow{OB}$.

Using this fact:

$$\overrightarrow{BA} = -\overrightarrow{OB} + \overrightarrow{OA}$$

$$\overrightarrow{BA} = -\boldsymbol{b} + \boldsymbol{a}$$

(ii) the position vector of M. [1]

Since M is the midpoint of BP, BM=1/2 BP.

OAPB is a parallelogram, therefore $\overrightarrow{OA} = \overrightarrow{BP}$.

This means that $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{OA} = \frac{1}{2}a$

The position vector of M can be split into two vectors.

$$\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$$

$$\overrightarrow{OM} = \boldsymbol{b} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{a}$$

(b) X is on BA so that $BX : XA = 1 : 2$.

Show that X lies on OM. [4]

Since BX:XA = 1:2, the length of BX is one third of the length of BA.

$$\overrightarrow{BX} = \frac{1}{3}\overrightarrow{BA}$$

The vector OX can be split into two vectors:

$$\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{BX} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA}$$

Using part a)i)

$$\overrightarrow{OX} = b + \frac{1}{3}(-b + a)$$

$$\overrightarrow{OX} = \frac{2}{3}b + \frac{1}{3}a$$

From part a) ii), we know that $\overrightarrow{OM} = b + \frac{1}{2}a$

Multiply both sides by 2/3.

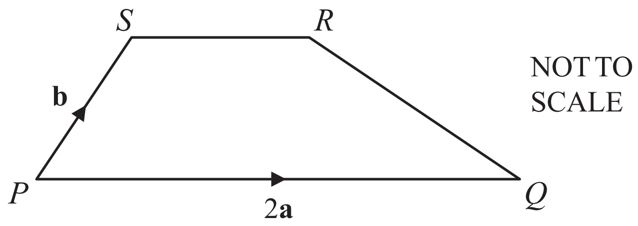
$$\frac{2}{3}\overrightarrow{OM} = \frac{2}{3}\left(b + \frac{1}{2}a\right)$$

$$\frac{2}{3}\overrightarrow{OM} = \frac{2}{3}b + \frac{1}{3}a$$

$$\frac{2}{3}\overrightarrow{OM} = \overrightarrow{OX}$$

This means that **X lies on OM**.

(a)



$PQRS$ is a trapezium with $PQ = 2SR$.
$\overrightarrow{PQ} = 2\mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$.

Find $\overrightarrow{QR}$ in terms of **a** and **b** in its simplest form. [2]

The vector QR can be split into more vectors.

$$\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PS} + \overrightarrow{SR}$$

We use the fact that PQ=2SR (so SR= ½ PQ)

$$\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PS} + \frac{1}{2}\overrightarrow{PQ}$$

It is important to remember that for vectors: $\overrightarrow{QP} = -\overrightarrow{PQ}$.

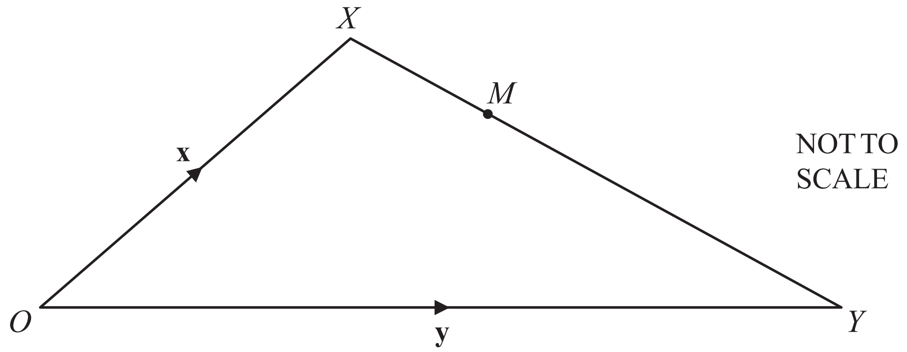
$$\overrightarrow{QR} = -\overrightarrow{PQ} + \overrightarrow{PS} + \frac{1}{2}\overrightarrow{PQ}$$

Use known values for the vectors. $\overrightarrow{PQ} = 2a$ and $\overrightarrow{PS} = b$

$$\overrightarrow{QR} = -2a + b + \frac{1}{2} \times 2a$$

$$\boldsymbol{\overrightarrow{QR} = -a + b = b - a}$$

(b)



$\overrightarrow{OX} = \mathbf{x}$ and $\overrightarrow{OY} = \mathbf{y}$.
M is a point on XY such that $XM : MY = 3 : 5$.

Find $\overrightarrow{OM}$ in terms of $\mathbf{x}$ and $\mathbf{y}$ in its simplest form. [2]

The vector OM can be split into more vectors.

$$\overrightarrow{OM} = \overrightarrow{OX} + \overrightarrow{XM}$$

Since XM:MY = 3:5, we can say that XM:XY = 3:8.

$$\overrightarrow{OM} = \overrightarrow{OX} + \frac{3}{8}\overrightarrow{XY}$$

Split vector XY into two vectors. It is important to remember that for vectors: $\overrightarrow{XO} = -\overrightarrow{OX}$.

$$\overrightarrow{OM} = \overrightarrow{OX} + \frac{3}{8}(\overrightarrow{XO} + \overrightarrow{OY})$$

$$\overrightarrow{OM} = \overrightarrow{OX} + \frac{3}{8}(-\overrightarrow{OX} + \overrightarrow{OY})$$

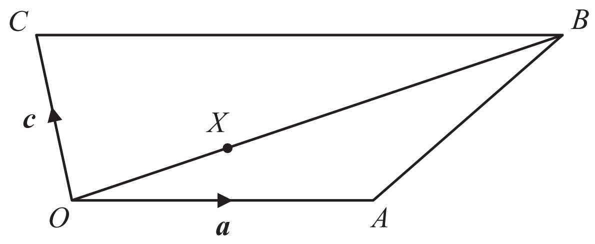
Use known values for the vectors. $\overrightarrow{OX} = x$ and $\overrightarrow{OY} = y$

$$\overrightarrow{OM} = x + \frac{3}{8}(-x + y)$$

$$\overrightarrow{OM} = x - \frac{3}{8}x + \frac{3}{8}y$$

We get the final answer:

$$\overrightarrow{OM} = \frac{5}{8}\boldsymbol{x} + \frac{3}{8}\boldsymbol{y}$$



NOT TO SCALE

The diagram shows a quadrilateral $OABC$.
$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{CB} = 2\mathbf{a}$.
X is a point on OB such that $OX : XB = 1 : 2$.

(a) Find, in terms of **a** and **c**, in its simplest form

(i) $\overrightarrow{AC}$, [1]

$$\overrightarrow{AC} = -\overrightarrow{OA} + \overrightarrow{OC}$$

$$= \vec{c} - \vec{a}$$

(ii) $\overrightarrow{AX}$. [3]

$$\overrightarrow{AX} = -\overrightarrow{OA} + \overrightarrow{OX}$$

$$= -\vec{a} + \frac{1}{3}\overrightarrow{OB}$$

We need to find OB:

$$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$$

$$= \vec{c} + 2\vec{a}$$

Hence:

$$\overrightarrow{AX} = -\vec{a} + \frac{1}{3}(\vec{c} + 2\vec{a})$$

$$= \frac{1}{3}\vec{c} - \frac{1}{3}\vec{a}$$

(b) Explain why the vectors $\overrightarrow{AC}$ and $\overrightarrow{AX}$ show that C, X and A lie on a straight line. [2]

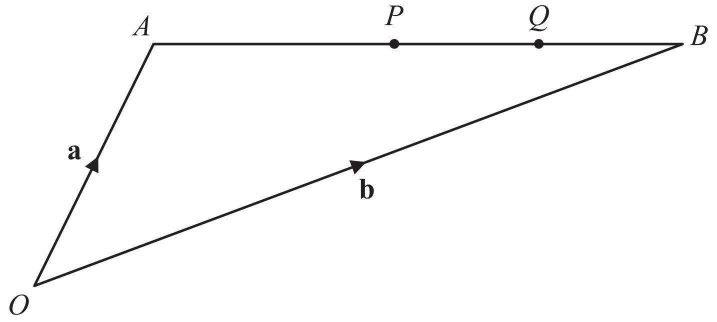
We can see that $\overrightarrow{AX}$ **is a scalar multiple of** $\overrightarrow{AC}$**:**

$$\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AC}$$

This means that they are **parallel**.

They also share a **common point, A**, therefore A, C, and X

lie on a straight line.



The diagram shows two points, P and Q, on a straight line AB.
P is the midpoint of AB and Q is the midpoint of PB.
O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

Write down, in terms of **a** and **b**, in its simplest form [2]

(a) $\overrightarrow{AP}$,

$$\overrightarrow{AP} = \frac{1}{2}\overrightarrow{AB}$$

Need to find $\overrightarrow{AB}$

$$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$$

$$= \vec{b} - \vec{a}$$

Hence

$$\overrightarrow{AP} = \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a}$$

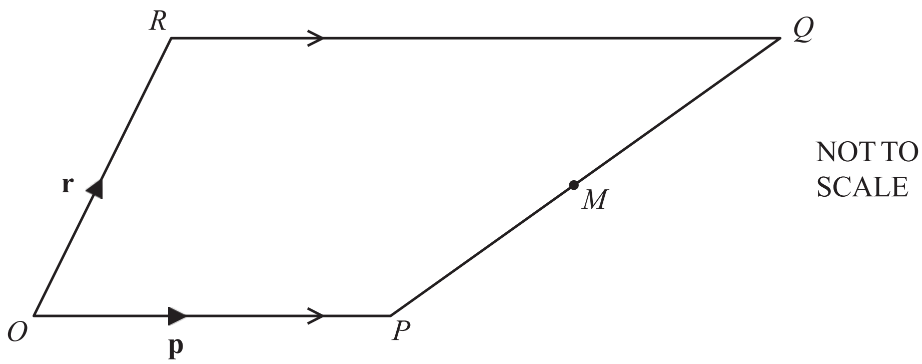
(b) the position vector of Q. [2]

$$\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$$

$$= \vec{a} + \frac{3}{4}\overrightarrow{AB}$$

$$= \vec{a} + \frac{3}{4}(\vec{b} - \vec{a})$$

$$= \boldsymbol{\frac{1}{4}\vec{a} + \frac{3}{4}\vec{b}}$$



$OPQR$ is a trapezium with RQ parallel to OP and $RQ = 2OP$.
O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.
M is the midpoint of PQ.

Find, in terms of **p** and **r**, in its simplest form [1]

(a) $\overrightarrow{PQ}$,

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OR} + \overrightarrow{RQ}$$

$$= -\vec{p} + \vec{r} + 2\vec{p}$$

$$\vec{r} + \vec{p}$$

(b) $\overrightarrow{OM}$, the position vector of M. [2]

$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM}$$

$$= \vec{p} + \frac{1}{2}(\vec{r} + \vec{p})$$

$$\frac{1}{2}\vec{r} + \frac{3}{2}\vec{p}$$

Vectors
Difficulty: Hard

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors and transformations
Sub-Topic	Vectors
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 3

Time allowed: **39 minutes**

Score: **/30**

Percentage: **/100**

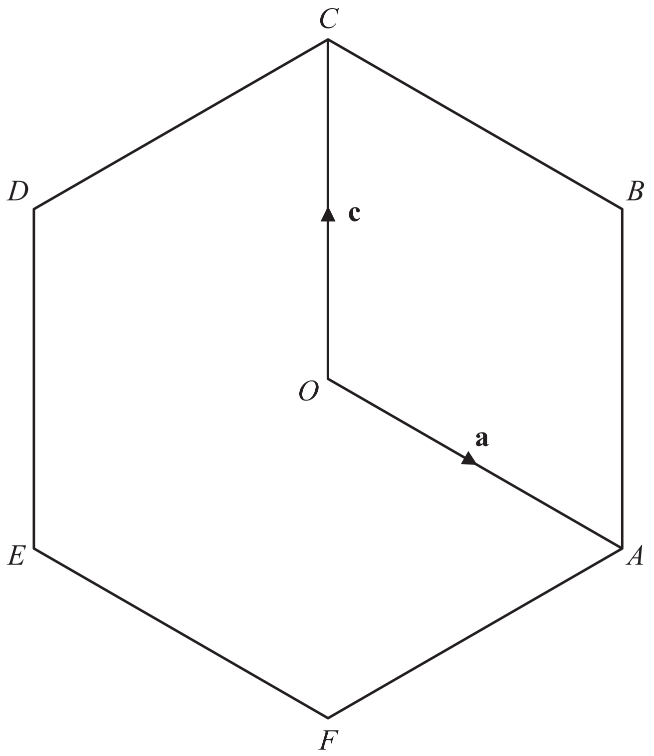
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



O is the origin.
$ABCDEF$ is a regular hexagon and O is the midpoint of AD.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, in their simplest form

(a) $\overrightarrow{BE}$, [2]

We know that:

$$OA + AB = OB$$

And also that AB is equal to OC since they are parallel

$$\overrightarrow{OC} = \overrightarrow{AB} = \boldsymbol{c}$$

But BO is in the opposite direction to OB, so:

$$\overrightarrow{BO} = -\overrightarrow{OB}$$

And since O is midpoint of AD, and this is a regular hexagon, O must be midpoint of BE as well, making BO half of BE:

$$2\overrightarrow{BO} = \overrightarrow{BE}$$

Therefore,

$$\overrightarrow{OB} = \boldsymbol{a} + \boldsymbol{c}$$

$$\overrightarrow{BO} = -(\boldsymbol{a} + \boldsymbol{c})$$

$$\overrightarrow{BE} = -2(\boldsymbol{a} + \boldsymbol{c})$$

$$\overrightarrow{\boldsymbol{BE}} = -2\boldsymbol{a} - 2\boldsymbol{c}$$

(b) $\overrightarrow{DB}$, [2]

We formulate first the equation containing DB:

$$\overrightarrow{ED} + \overrightarrow{DB} = \overrightarrow{EB}$$

Rearrange to make DB the subject:

$$\overrightarrow{DB} = \overrightarrow{EB} - \overrightarrow{ED}$$

We can easily find EB from previously, just flip the sign:

$$\overrightarrow{BE} = -2\boldsymbol{a} - 2\boldsymbol{c}$$

$$\overrightarrow{EB} = 2\boldsymbol{a} + 2\boldsymbol{c}$$

ED is parallel to AB and is equivalent as they are both sides of a regular hexagon:

$$\overrightarrow{ED} = \overrightarrow{AB} = \boldsymbol{c}$$

Hence,

$$\overrightarrow{DB} = \overrightarrow{EB} - \overrightarrow{ED}$$

$$= 2\boldsymbol{a} + 2\boldsymbol{c} - \boldsymbol{c}$$

$$= 2\boldsymbol{a} + \boldsymbol{c}$$

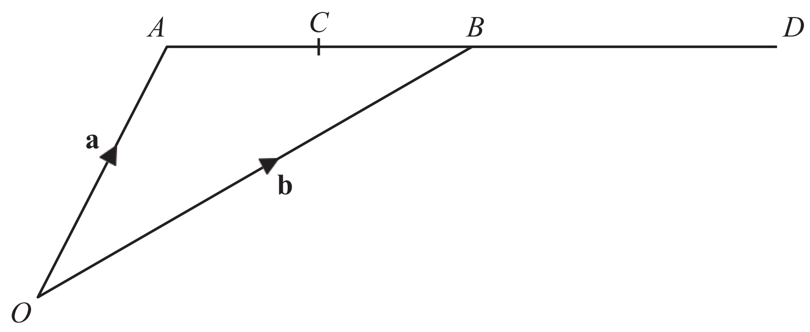
(c) the position vector of E. [2]

The position vector of E is given by the vector of E from origin, OE.

In this case, for a regular hexagon, OE will be half of BE, since O is the midpoint.

$$\overrightarrow{OE} = \frac{\overrightarrow{BE}}{2}$$

$$= -\boldsymbol{a} - \boldsymbol{c}$$



A and B have position vectors **a** and **b** relative to the origin O.
C is the midpoint of AB and B is the midpoint of AD.

Find, in terms of **a** and **b**, in their simplest form

(a) the position vector of C, [2]

C is the mid-point of $\overrightarrow{AB}$, therefore the position of C is half of $\overrightarrow{AB}$.

Therefore:

$\overrightarrow{AB} = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

(b) the vector $\overrightarrow{CD}$. [2]

B is the mid-point of AD, therefore:

$\overrightarrow{BA} = \overrightarrow{BD} = \frac{1}{2}\overrightarrow{AD}$

C is the mid-point of AB, therefore:

$\overrightarrow{AC} = \overrightarrow{CB} = \frac{1}{2}\overrightarrow{AB}$

By looking at the diagram, we see that $\overrightarrow{CD} = \overrightarrow{BD} + \overrightarrow{CB}$

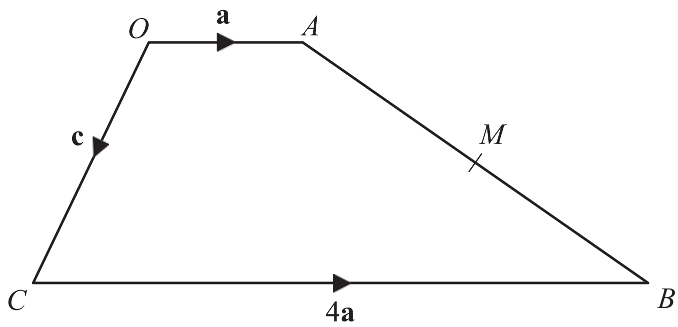
$\overrightarrow{CD} = \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{AB}$

We know that AB = $\frac{1}{2}$ AD

$\overrightarrow{CD}$ = $1\frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{AB}$ = **b - a**

$\boldsymbol{\overrightarrow{CD}}$ **= $1\frac{1}{2}$ b - $1\frac{1}{2}$ a**



O is the origin, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{CB} = 4\mathbf{a}$.
M is the midpoint of AB.

(a) Find, in terms of **a** and **c**, in their simplest form

(i) the vector $\overrightarrow{AB}$, [2]

By looking at the diagram, we can see that in the

trapezium OABC, the vector AB will be:

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{AC} + \overrightarrow{CB}$

$\overrightarrow{OA}$ = a => $\overrightarrow{AO}$ = -a

$\overrightarrow{AC}$ = c

$\overrightarrow{CB}$ = 4a

$\overrightarrow{AB}$ = -a + c + 4a

$\overrightarrow{AB}$ = 3a + c

(ii) the position vector of M. [2]

M is the mid-point of the vector $\overrightarrow{AB}$, therefore its position will be half of

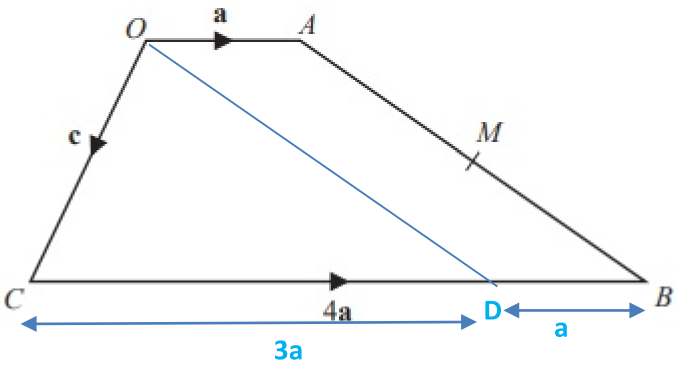
the position of $\overrightarrow{AB}$.

From i), we know that $\overrightarrow{AB}$ = 3**a** + **c**

M will have the position: $\overrightarrow{OA} + \overrightarrow{AB}$ = **a** + $\frac{1}{2}$(3**a**) + $\frac{1}{2}$**c**

= $2\frac{1}{2}$a + $\frac{1}{2}$c

(b) Mark the point D on the diagram where $\overrightarrow{OD} = 3\mathbf{a} + \mathbf{c}$. [2]



$\overrightarrow{CB}$ = 4a

We mark a point D on the vector $\overrightarrow{CB}$ which is $\frac{3}{4}\overrightarrow{CB}$.

Therefore:

$\overrightarrow{CD} = \frac{3}{4} \times 4a$

$\overrightarrow{CD}$ = 3a

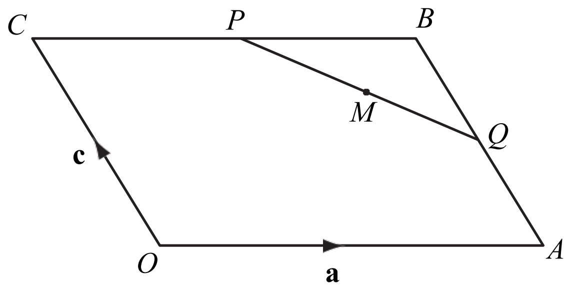
For the point D marked on the diagram above, the vector

$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$

$\overrightarrow{OC}$ = c

$\overrightarrow{CD}$ = 3a

$\overrightarrow{OD}$ = 3a + c



NOT TO SCALE

O is the origin and $OABC$ is a parallelogram.
$CP = PB$ and $AQ = QB$.

$\overrightarrow{OA}$ = a and $\overrightarrow{OC}$ = c .
Find in terms of a and c, in their simplest form,

(a) $\overrightarrow{PQ}$, [2]

The vector PQ can be split into two vectors.

$$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ}$$

It is important to remember that for vectors:

$\overrightarrow{OC} = -\overrightarrow{CO}$.

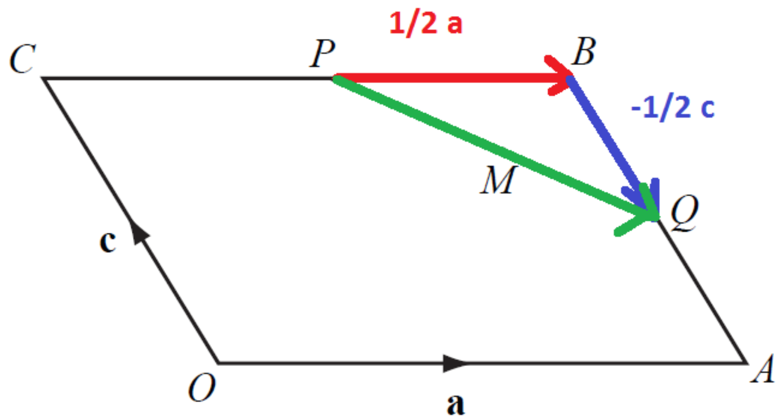
Since OABC is a parallelogram, we have:

$$\overrightarrow{OC} = \overrightarrow{AB} \quad and \quad \overrightarrow{OA} = \overrightarrow{CB}$$

From the fact that CP = PB and AQ = QB, we deduce that P and Q are midpoints, hence:

$$\overrightarrow{PB} = \frac{1}{2}\overrightarrow{CB} = \frac{1}{2}\overrightarrow{OA} = \frac{1}{2}\boldsymbol{a}$$

$$\overrightarrow{QB} = \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}\overrightarrow{OC} = \frac{1}{2}\boldsymbol{c}$$



Putting all this together:

$$\overrightarrow{PQ} = \overrightarrow{PB} - \overrightarrow{QB}$$

$$\overrightarrow{PQ} = \frac{1}{2}a - \frac{1}{2}c$$

(b) the position vector of M, where M is the midpoint of PQ. [2]

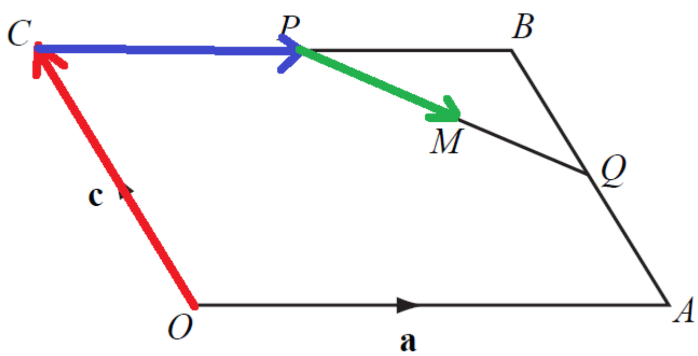
M is the midpoint of PQ, implying:

$$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ}$$

The position vector of M can be expressed as a sum of multiple vectors (we gradually expand to end up with vectors that we know):

$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM}$$

$$\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CP} + \overrightarrow{PM}$$



We substitute using the previous known equations (including CP = PB).

$$\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{PB} + \frac{1}{2}\overrightarrow{PQ}$$

Express the vectors in terms of **a** and **c**.

$$\overrightarrow{OM} = \boldsymbol{c} + \frac{1}{2}\boldsymbol{a} + \frac{1}{2}(\frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{c})$$

$$\boldsymbol{\overrightarrow{OM} = \frac{3}{4}a + \frac{3}{4}c}$$

$\overrightarrow{AB} = \mathbf{a} + t\mathbf{b}$ and $\overrightarrow{CD} = \mathbf{a} + (3t - 5)\mathbf{b}$ where t is a number.

Find the value of t when $\overrightarrow{AB} = \overrightarrow{CD}$. [2]

$$\vec{a} + t\vec{b} = \vec{a} + (3t - 5)\vec{b}$$

Comparing coefficients, we have that

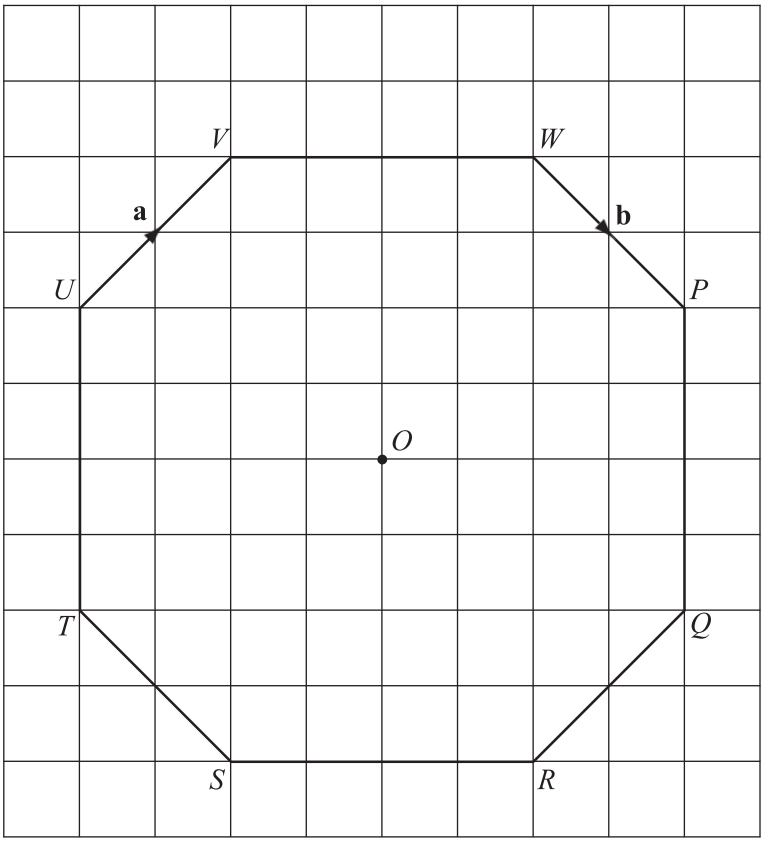
$$t = 3t - 5$$

Add 5 to both sides and subtract t from both sides

$$\rightarrow 2t = 5$$

Divide through by 2

$$\rightarrow \boldsymbol{t = \frac{5}{2} = 2\frac{1}{2} = 2.5}$$



The origin O is the centre of the octagon $PQRSTUVW$.
$\overrightarrow{UV} = \mathbf{a}$ and $\overrightarrow{WP} = \mathbf{b}$.

(a) Write down in terms of **a** and **b**

(i) $\overrightarrow{VW}$, [1]

$\overrightarrow{VW} = \overrightarrow{UV} + \overrightarrow{WP}$

$\overrightarrow{VW} = \mathbf{a} + \mathbf{b}$

(ii) $\overrightarrow{TU}$, [1]

$|\mathbf{a} - \mathbf{b}|$.

$\overrightarrow{TU} = \overrightarrow{UV} + \overrightarrow{PW}$

$\overrightarrow{PW} = -\overrightarrow{WP} = -\mathbf{b}$

$\overrightarrow{TU} = \mathbf{a} - \mathbf{b}$

(iii) $\overrightarrow{TP}$, [1]

$\overrightarrow{TP} = \overrightarrow{TU} + \overrightarrow{UV} + \overrightarrow{VW} + \overrightarrow{WP}$

$\overrightarrow{TP}$ = a – b + a + a + b + b

$\overrightarrow{TP}$ = 3a + b

(iv) the position vector of the point P. [1]

The position vector of point P is $\overrightarrow{OP}$.

$\overrightarrow{OP} = \frac{1}{2}\overrightarrow{TP}$

$\overrightarrow{OP} = \frac{1}{2}$(3a + b)

$\overrightarrow{OP} = \frac{3}{2}$a + $\frac{1}{2}$b

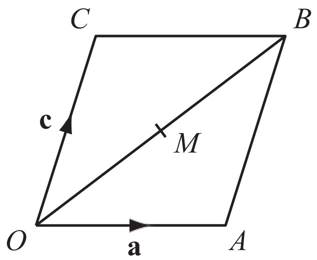
(b) In the diagram, 1 centimetre represents 1 unit.
Write down the value of $|\mathbf{a} - \mathbf{b}|$. [2]

$\overrightarrow{TU}$ = a – b

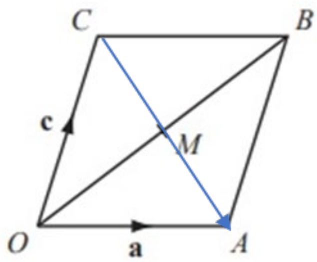
$|\overrightarrow{TU}| = |a - b|$

= 4

(1 cm = 1 unit)



$OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.
M is the mid-point of OB.
Find $\overrightarrow{MA}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.



[2]

Using the triangle rule, we subtract the vectors a and c to obtain the vector $\overrightarrow{CA}$.

a – c = $\overrightarrow{CA}$

M is the mid-point of the diagonal OB in the parallelogram ABCO. Therefore, M is also the mid-point of the diagonal CA.

$\overrightarrow{MA} = \frac{\overrightarrow{CA}}{2}$

$\overrightarrow{MA} = \frac{\mathbf{a}-\mathbf{c}}{2}$

$$= \frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{c}$$

Transformations
Difficulty: Easy

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors and transformations
Sub-Topic	Transformations
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 1

Time allowed: **30 minutes**

Score: **/23**

Percentage: **/100**

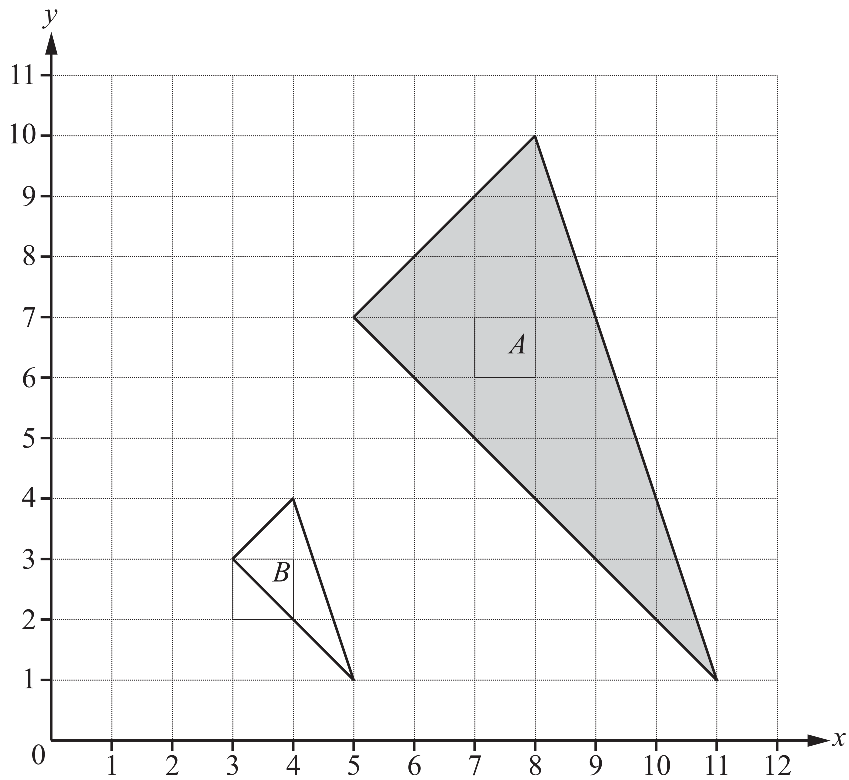
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%



Describe fully the **single** transformation that maps triangle A onto triangle B. [3]

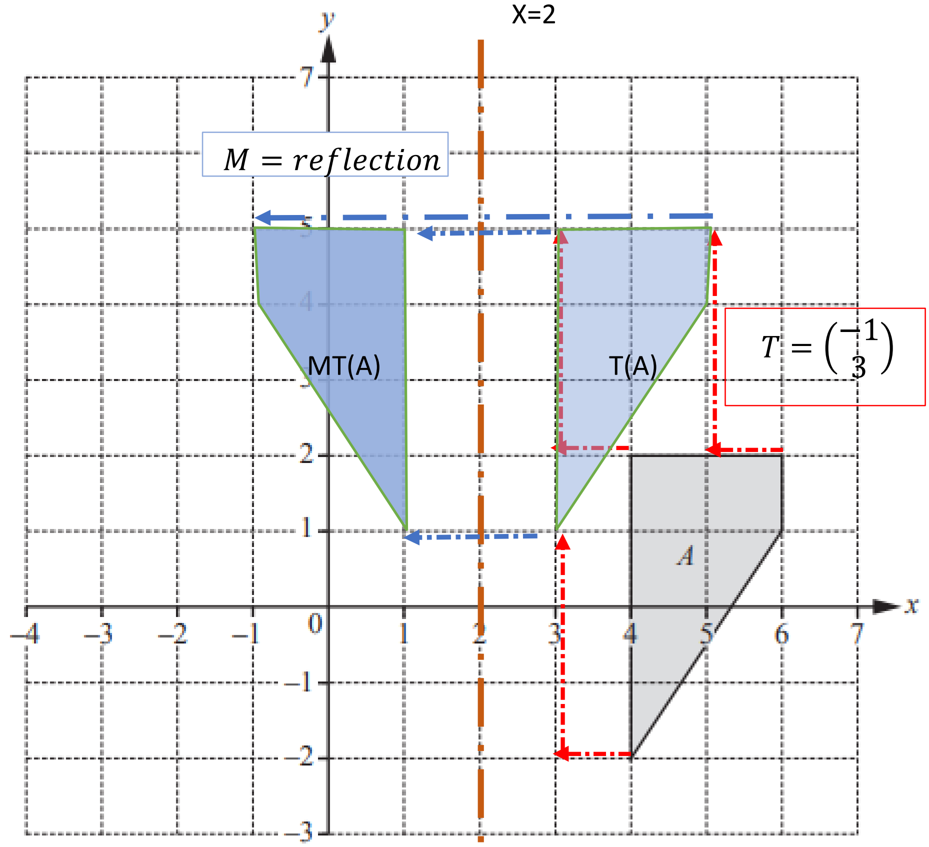
Enlargement, scale factor 1/3, centre (2, 1).

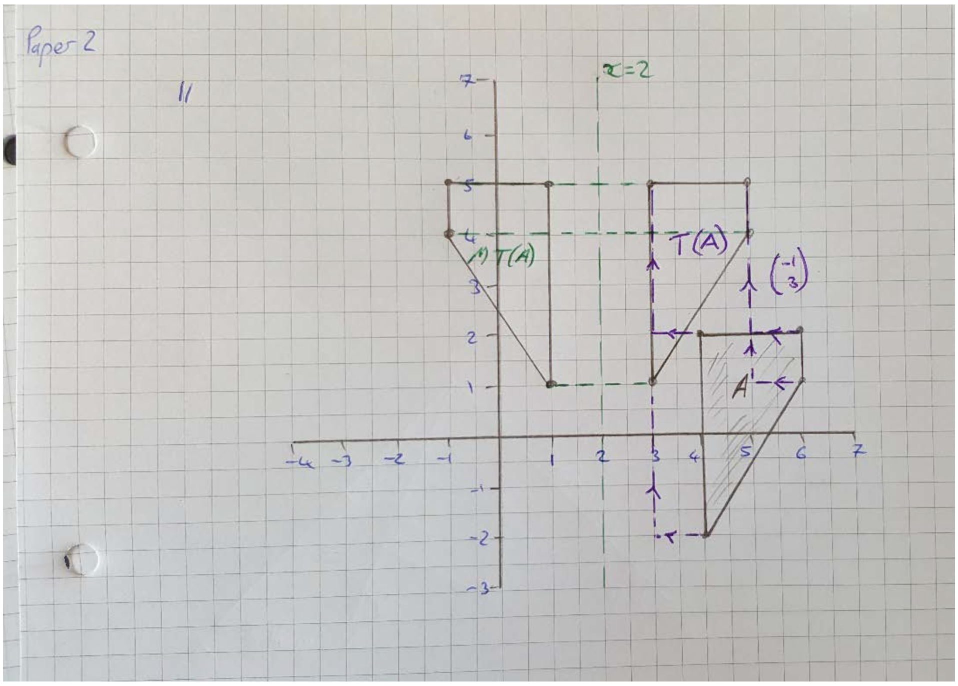
T(X) is the image of the shape X after translation by the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

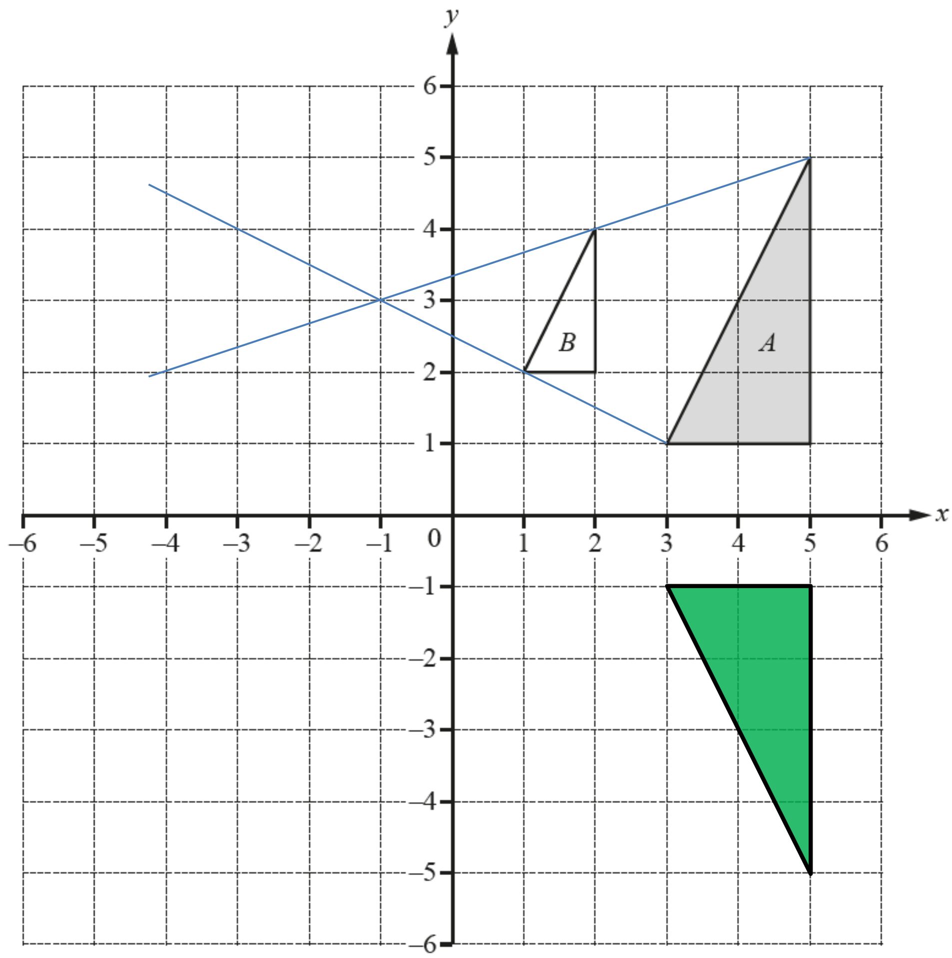
M(Y) is the image of the shape Y after reflection in the line $x = 2$.

On the grid, draw MT(A), the image of shape A after the transformation MT. [3]

In this question we need to first translate, and then reflect the shape







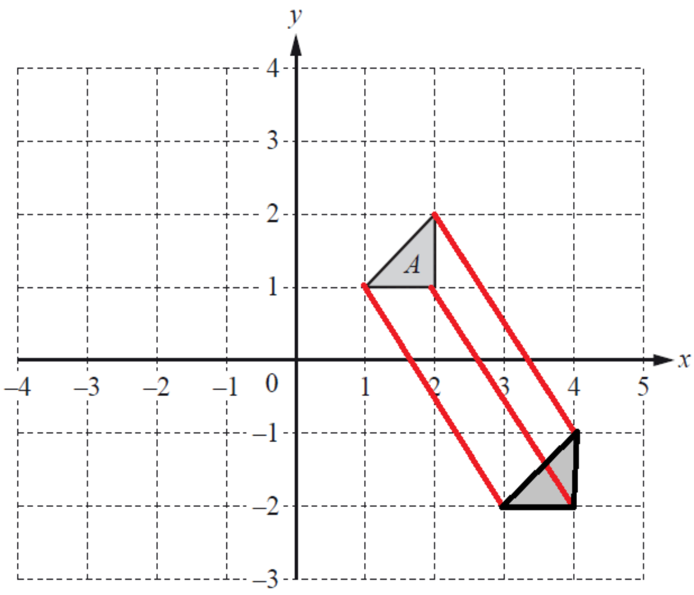
(a) Describe fully the **single** transformation that maps triangle A onto triangle B. [3]

Change in size so it's an Enlargement – need to find Centre and Scale Factor.

Draw lines through equivalent points on A and B – where they cross is Centre: $(-1, 3)$

Use equivalent sides to find Scale Factor $= \frac{Side\ of\ B}{Side\ of\ A} = \frac{2}{4} = \frac{1}{2}$

Enlargement, Centre $(-1, 3)$, Scale Factor 2



Draw the image of shape *A* after a translation by the vecto $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$. [2]

This translation represents a shift by 2 units in the positive *x* direction and by 3 units in the negative *y* direction.

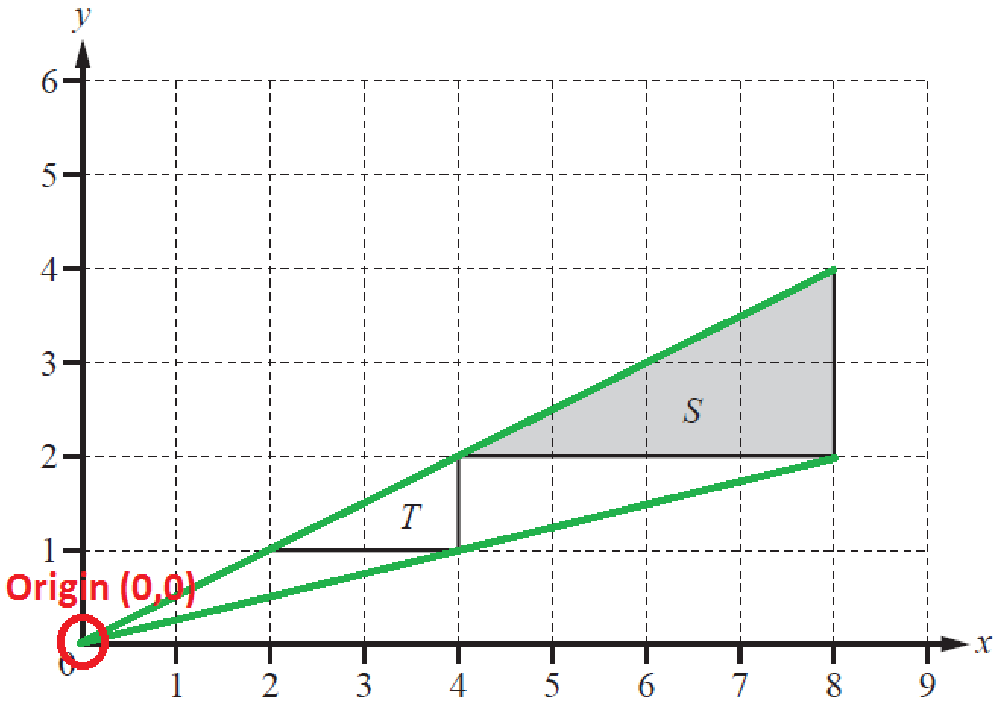
The new triangle has vertices **(3,-2), (4,-2) and (4,-1)**.

(a) Describe fully the **single** transformation that maps triangle *S* onto triangle *T*. [3]

When we join the corresponding vertices of triangles S and T, the lines meet at the origin (0,0).

The distance from (0,0) to a vertex of triangle T is half the distance from (0,0) to a corresponding vertex of triangle S.

This suggests that the scale factor of the enlargement is 1/2.



The transformation is an enlargement with centre (0,0) and the scale factor ½ .

(b) Find the matrix which represents the transformation that maps triangle S onto triangle T. [2]

The value of every coordinate has to be halved.

This can be achieved by a matrix $\begin{pmatrix} \mathbf{1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{1/2} \end{pmatrix}$

Example: $\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2}x \\ \frac{1}{2}y \end{pmatrix}$

Find the 2×2 matrix that represents a rotation through 90° clockwise about (0, 0). [2]

A general matrix for rotation looks like $\begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$ where x is an angle of anticlockwise rotation.

This matrix becomes $\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$ for x=- 90°(minus sign because we want a clockwise rotation).

This is a matrix that represents a rotation through 90° clockwise about (0,0)

(p, q) is the image of the point (x, y) under this combined transformation.

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

(a) Draw the image of the triangle under the combined transformation. [3]

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

Multiply the matrix with x and y,

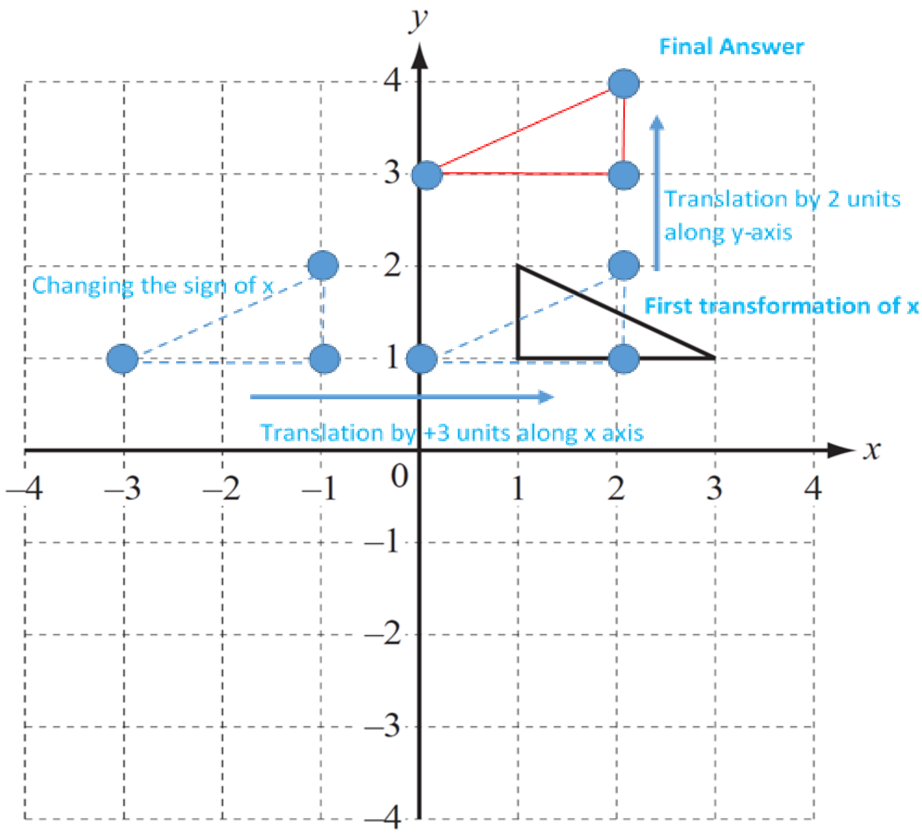
$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{-x+3} \\ \boldsymbol{y+2} \end{pmatrix}$$

Therefore the transformations are:

1. Changing the sign of x and translating it by 3 units along the x axis

2. Translating 2 units along the y axis



(b) Describe fully the **single** transformation represented by $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. [2]

It is a reflection on the y-axis.

Transformations
Difficulty: Easy

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors and transformations
Sub-Topic	Transformations
Paper	Paper 2
Difficulty	Easy
Booklet	Model Answers 2

Time allowed: **39 minutes**

Score: **/30**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
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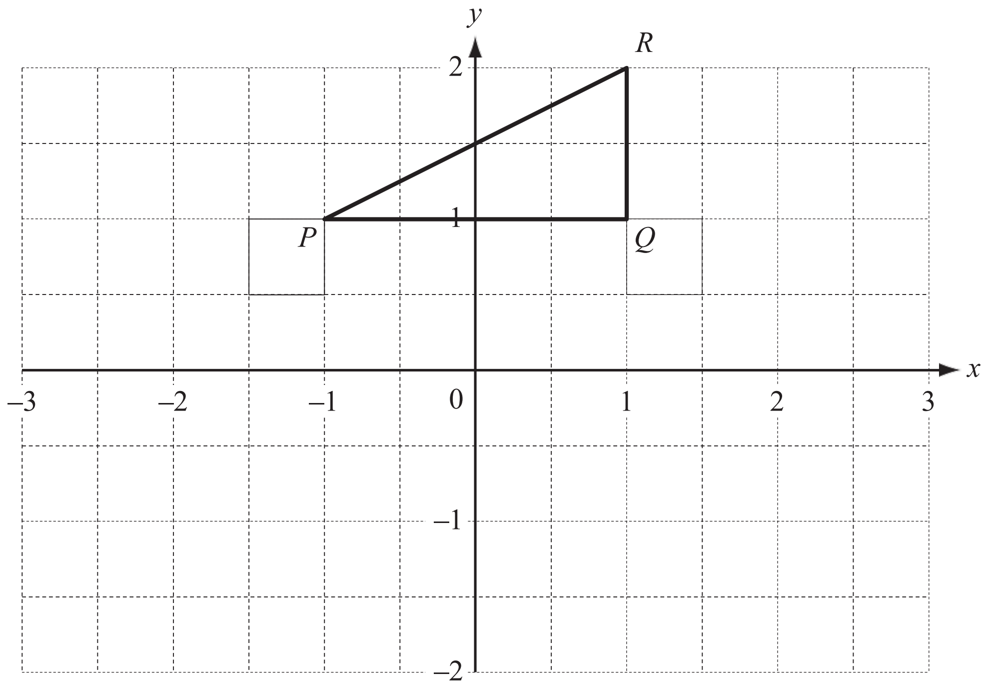
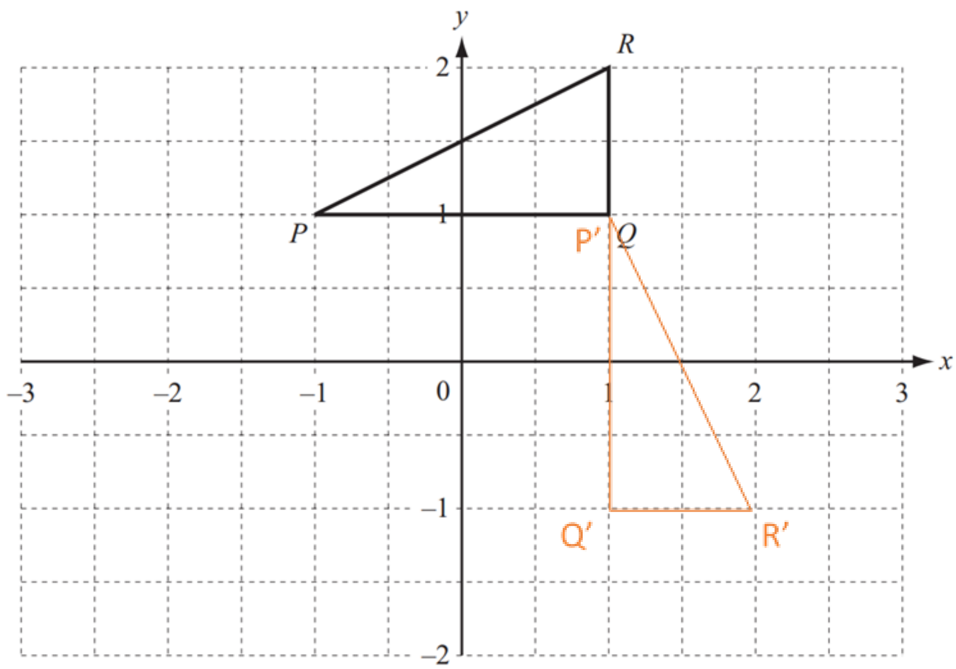
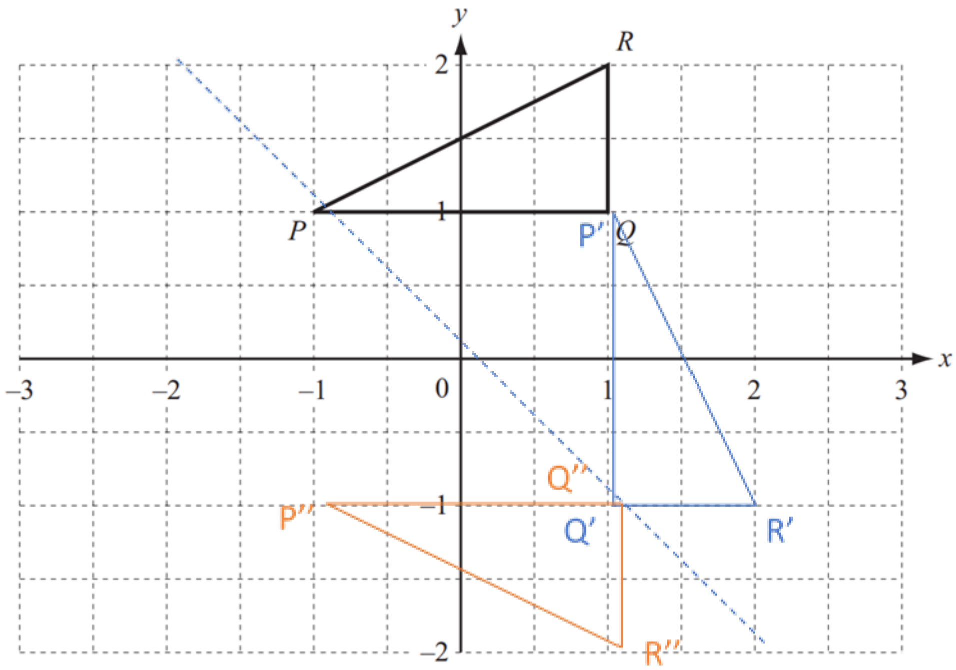


The triangle PQR has co-ordinates $P(-1, 1)$, $Q(1, 1)$ and $R(1, 2)$.

(a) Rotate triangle PQR by 90° clockwise about (0, 0).
Label your image $P'Q'R'$. [2]

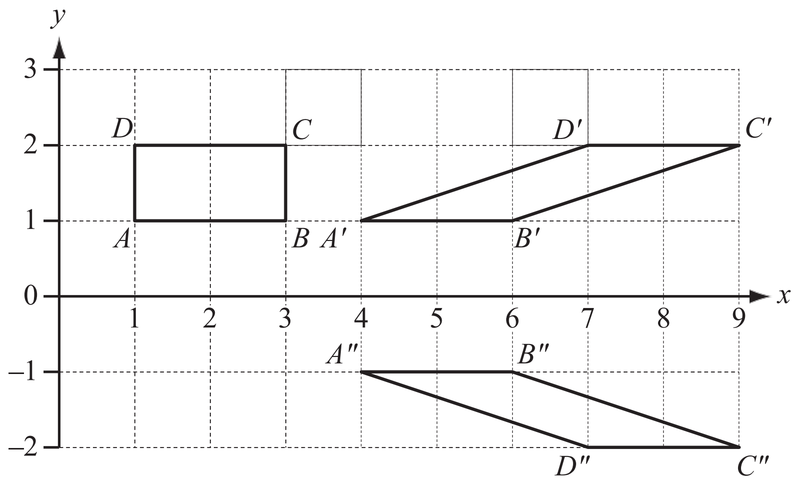


(b) Reflect your triangle $P'Q'R'$ in the line $y = -x$.
Label your image $P''Q''R''$. [2]



(c) Describe fully the single transformation which maps triangle PQR onto triangle $P''Q''R''$. [2]

A reflection in the line $y = 0$ (the x-axis)



(a) Describe the single transformation which maps $ABCD$ onto $A'B'C'D'$. [3]

It is a **shear** of **scale factor 3** with **x-axis invariant.**

(b) A single transformation maps $A'B'C'D'$ onto $A''B''C''D''$. [2]
Find the matrix which represents this transformation.

It is a reflection in the x-axis, i.e.

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ -y \end{pmatrix}$$

This is represented by

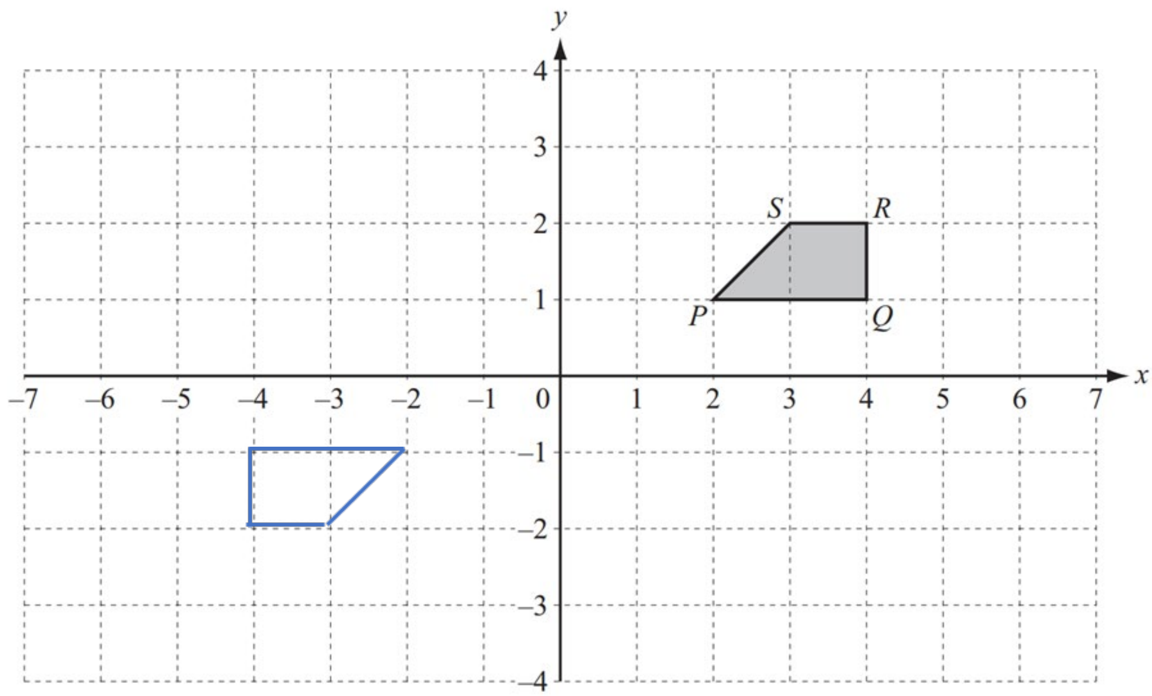
$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix}$$

We can see this by

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

On the grid on the next page, draw the image of $PQRS$ after the transformation represented by $\mathbf{BA}$. [5]



We apply A first, so we have

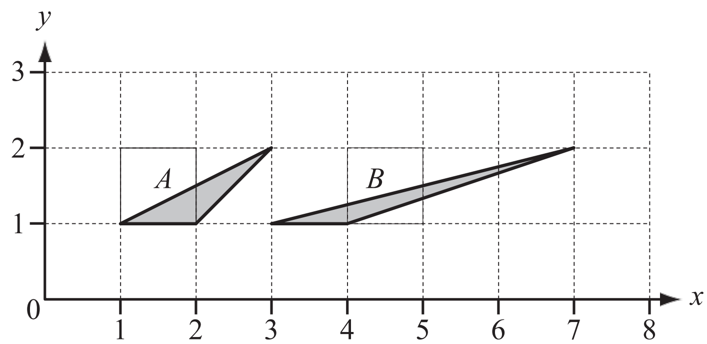
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

A reflection in the line $y = x$. Now apply B

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$$

From the original shape this is a reflection in x-axis and a reflection in the y-axis

(drawn in blue).



(a) Describe fully the single transformation that maps triangle A onto triangle B. [3]

Shear, scale factor 2, x axis invariant.

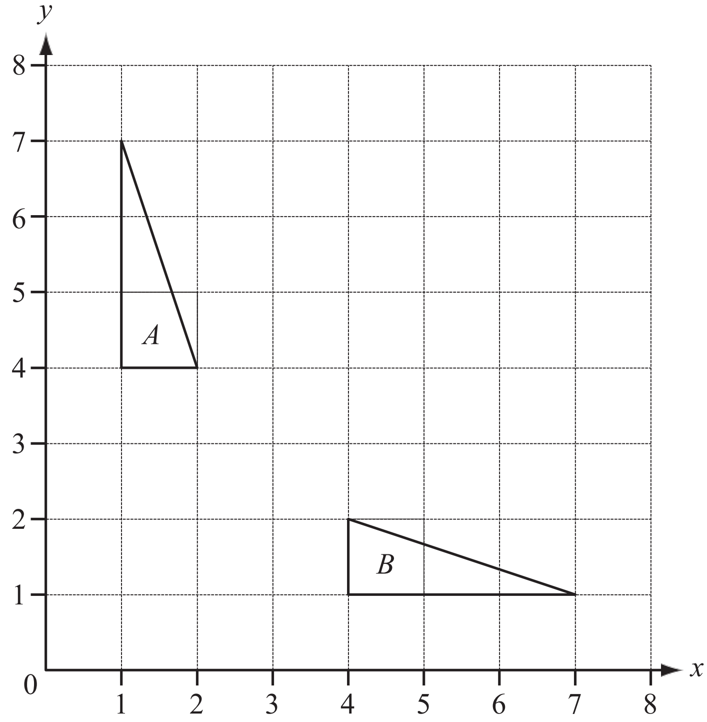
(b) Find the 2 × 2 matrix which represents this transformation. [2]

We require

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x + 2y \\ y \end{pmatrix}$$

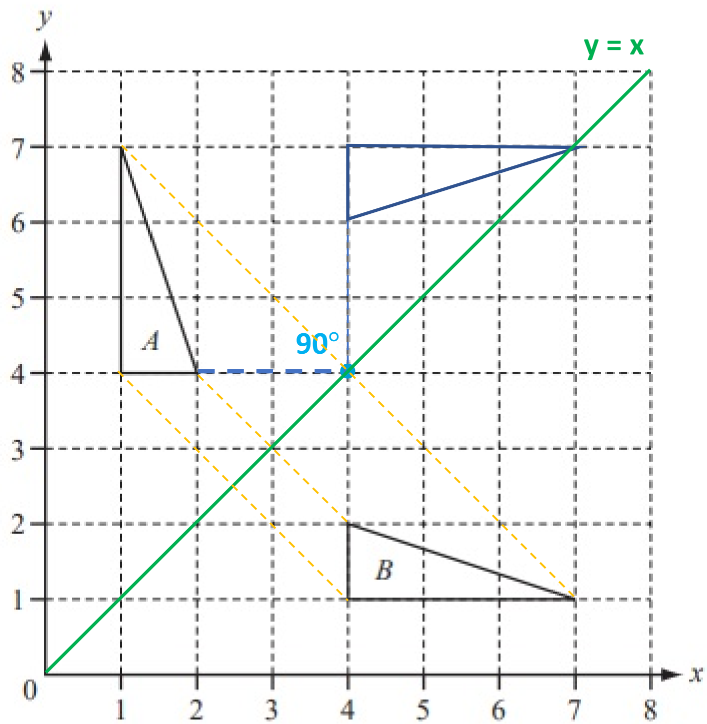
This is done using the matrix

$$\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$



(a) Describe fully the **single** transformation which maps triangle A onto triangle B. [2]

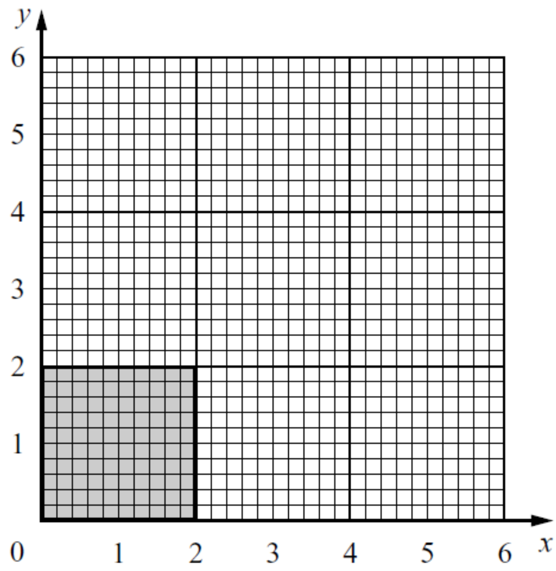
The single transformation which maps triangle A onto triangle B is a reflection with the mirror line the line of equation y = x, represented on the figure below in green. The distance from each point of the object and corresponding point of the image to the mirror line is equal. The distances are also perpendicular on the mirror lir



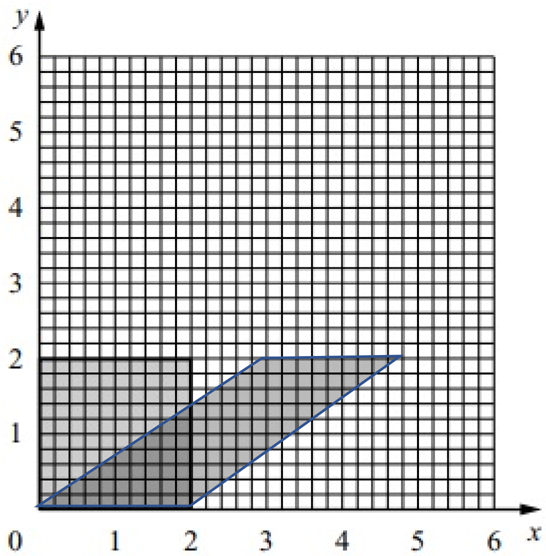
(b) On the grid, draw the image of triangle A after rotation by 90° clockwise about the point (4, 4). [2]

The point of coordinates (4, 4) is marked on the figure above in blue. The figure is rotated about this point clockwise, to the right, and by 90°. In other words, the distance from a point of the object to this point and the distance from the corresponding point of the image to this point form a 90° angle and are equal in length.

(a)

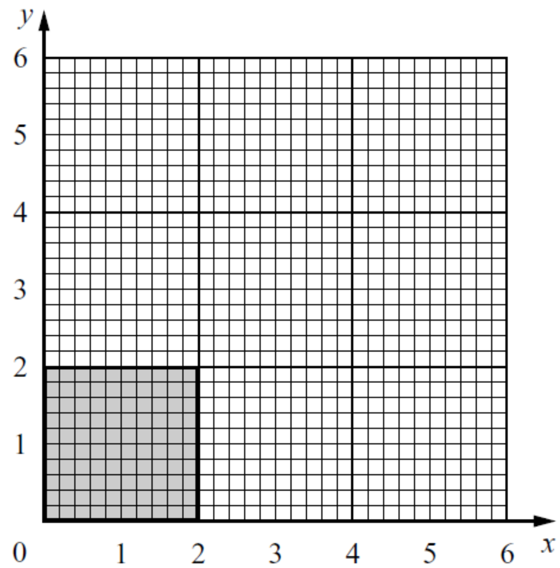


Draw the shear of the shaded square with the x-axis invariant and the point (0, 2) mapping onto the point (3, 2). [2]

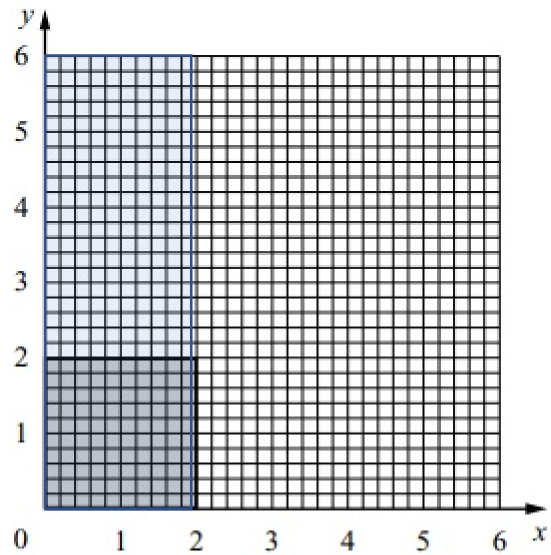


x-axis invariant – the shear happens parallel to the x-axis

(b)



(i) Draw the one-way stretch of the shaded square with the x-axis invariant and the point (0, 2) mapping onto the point (0, 6). [2]



x-axis invariant – so the x coordinates of each point remains the same while the y coordinates change, parallel to the y-axis.

Point (0, 2) is mapped onto point (0, 6), therefore, the sides are stretch in the vertical direction by 3.

(ii) Write down the matrix of this stretch. [1]

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 6 & 6 \end{pmatrix}$$

The matrix for this stretch multiplied by the matrix representing the object results in the matrix representing the image.

The matrix giving this stretch is:

$$\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{3} \end{pmatrix}$$

Transformations
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Vectors and transformations
Sub-Topic	Transformations
Paper	Paper 2
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **31 minutes**

Score: **/24**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D	E
>88%	76%	63%	51%	40%	30%

CIE IGCSE Maths (0980)

9	8	7	6	5	4	3
>94%	85%	77%	67%	57%	47%	35%

Question 1

(a) $\mathbf{N} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

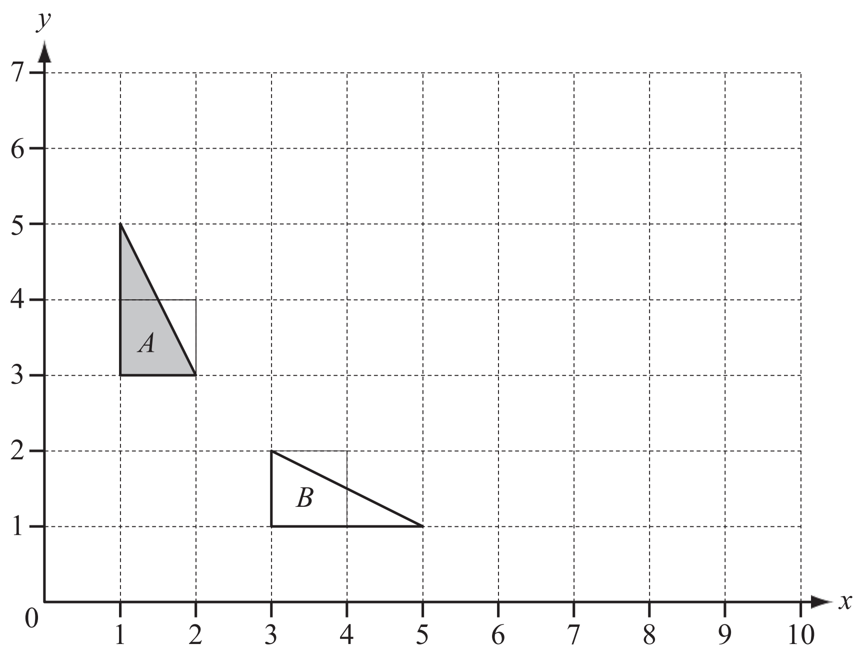
Describe fully the **single** transformation represented by **N**. [3]

We have that

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix}$$

Hence N represents a **rotation, 90° clockwise, about the origin.**

(b) Find the matrix which represents the **single** transformation that maps triangle A onto triangle B. [2]

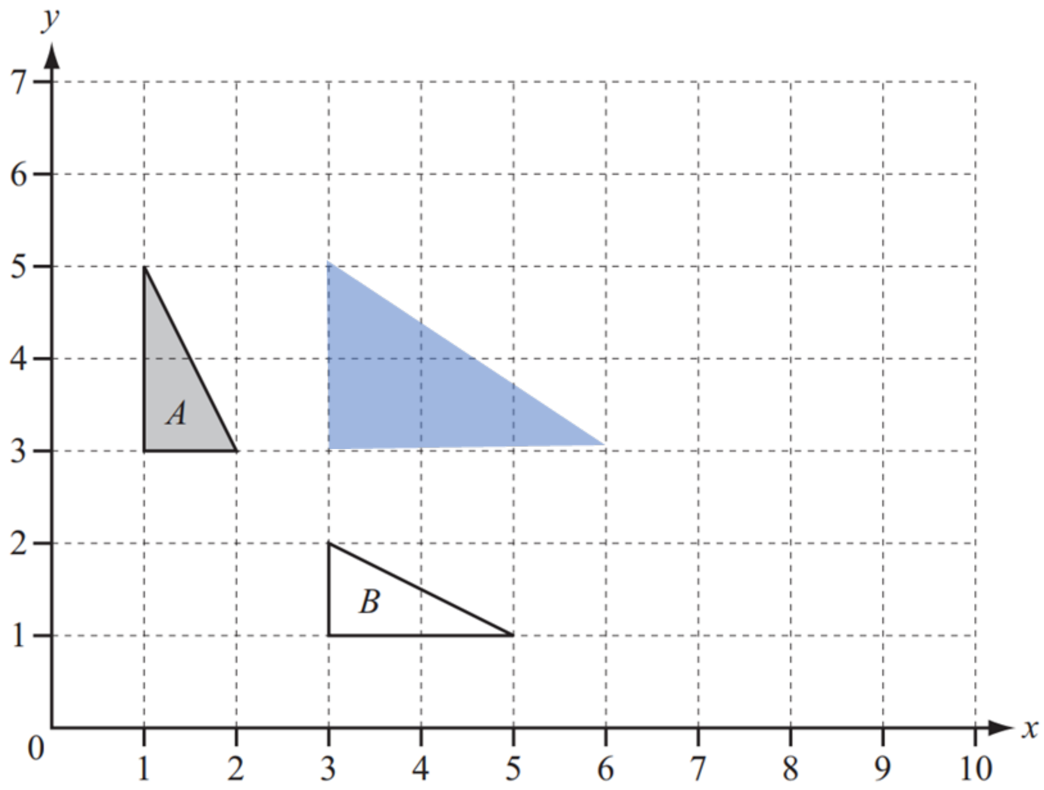


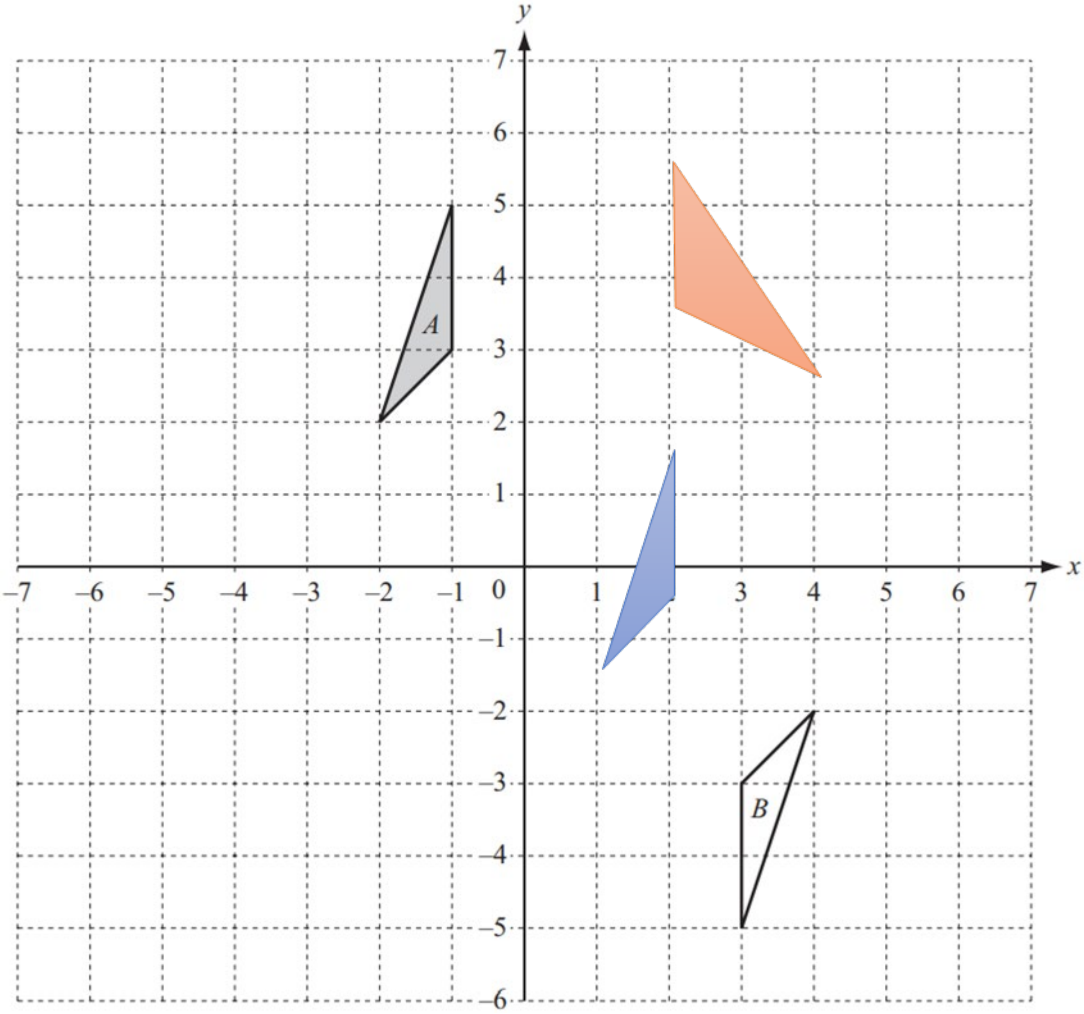
That is a reflection through the line $y = x$, hence we have the matrix representation

$$\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$$

(c) On the grid, draw the image of triangle A under a stretch, factor 3, with the y-axis invariant. [2]

Blue triangle drawn on graph below.





(a) Draw the image of triangle A after a translation by the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. [2]

The blue shape above.

(b) Describe fully the **single** transformation which maps triangle A onto triangle B. [3]

A rotation, centre (1, 0), of 180˚.

(c) Draw the image of triangle A after the transformation represented by the matri $\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$. [3]

We have

$$\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2x \\ y \end{pmatrix}$$

This is the orange shape drawn above.

Question 3

Find the matrix which represents the combined transformation of a reflection in the x axis **followed** by a reflection in the line $y = x$. [3]

Reflection in the x-axis means that

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$$

Which is represented by

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Reflection in the line y = x means that

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ x \end{pmatrix}$$

Which is represented by

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The x-axis reflection is done first, so we have the transformation

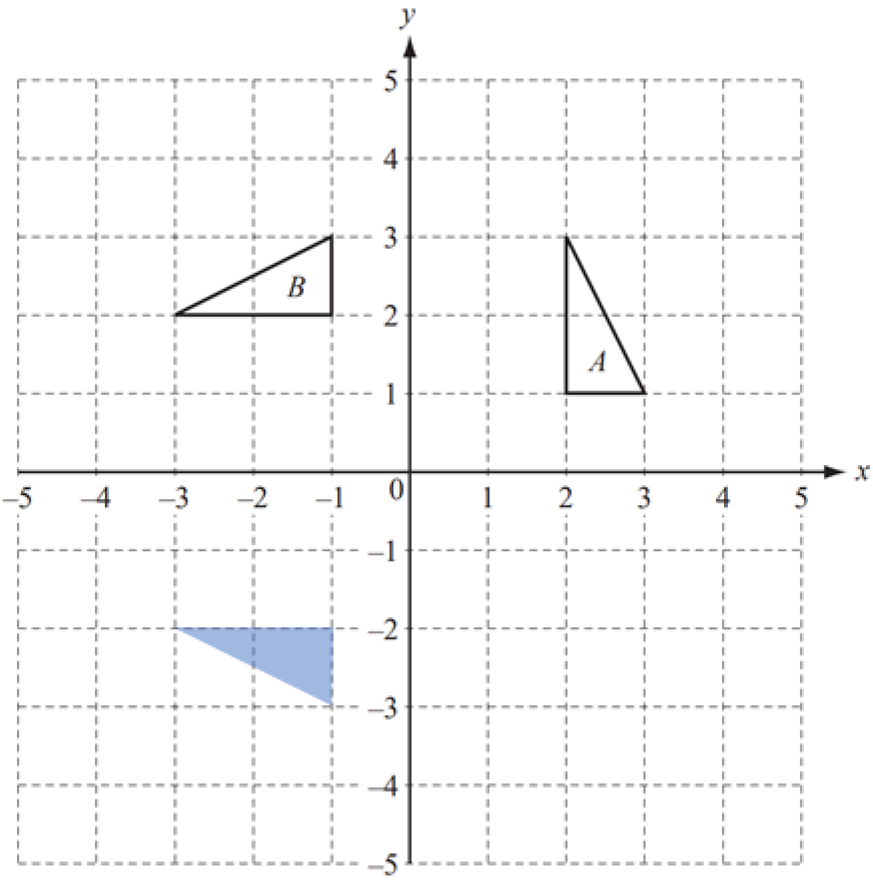
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

Multiplying the two 2 x 2 matrices together will gives us the matrix representation of the total transformation.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \times 1 + 1 \times 0 & 0 \times 0 + 1 \times -1 \\ 1 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times -1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$$



(a) A transformation is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

(i) **On the grid above**, draw the image of triangle A after this transformation. [2]

The blue triangle on the graph below

We have that

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ -x \end{pmatrix}$$

(ii) Describe fully this transformation. [2]

It is a reflection in the line $y = -x$

(b) Find the 2 by 2 matrix representing the transformation which maps triangle A onto triangle B. [2]

We require

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x \end{pmatrix}$$

Which is done by the matrix

$$\begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$$