3. Coordinate Geometry

CONTENTS

3.1 COORDINATE GEOMETRY

3.1.1 COORDINATES

What are coordinates?

- When we want to plot a point on a graph we need to know where to put it
- If the horizontal axis is labelled **x** and the vertical axis is labelled **y**, then the **x** and **y coordinates** are how far we go along the **x** and **y** axes to plot the point

What can we do with coordinates?

- If we have two points with coordinates (x_1, y_1) and (x_2, y_2) then we should be able to find
    - the **gradient** of the line through them
    - the **midpoint** of the two points
    - the **distance** between the two points

- Don't get fazed by the horrid notation (x_1, y_1) – this is just "point 1" and the other is "point 2" so we put the little numbers (subscripts) in so that we know which coordinate we are referring to. In questions there'll be lots of nice numbers
Here's how we do each of those:

3. Coordinate Geometry

1. **GRADIENT** $= \dfrac{RISE}{RUN} = \dfrac{y_2 - y_1}{x_2 - x_1}$

2. **MIDPOINT** is the "Average Point": $\left(\dfrac{x_1 + x_2}{2} , \dfrac{y_1 + y_2}{2} \right)$

3. **DISTANCE** is found using Pythagoras Theorem:

$$Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- You should also know that three points A, B and C lie on the same STRAIGHT LINE if AB and AC (or BC) have the same gradient

Exam Tip

If in doubt, SKETCH IT!

A quick, reasonably accurate sketch can make things a lot clearer.

Worked Example

1. *Three points A, B and C have coordinates* $(5, 2), (-1, 4)$ *and* $(-2, 6)$ *respectively.*
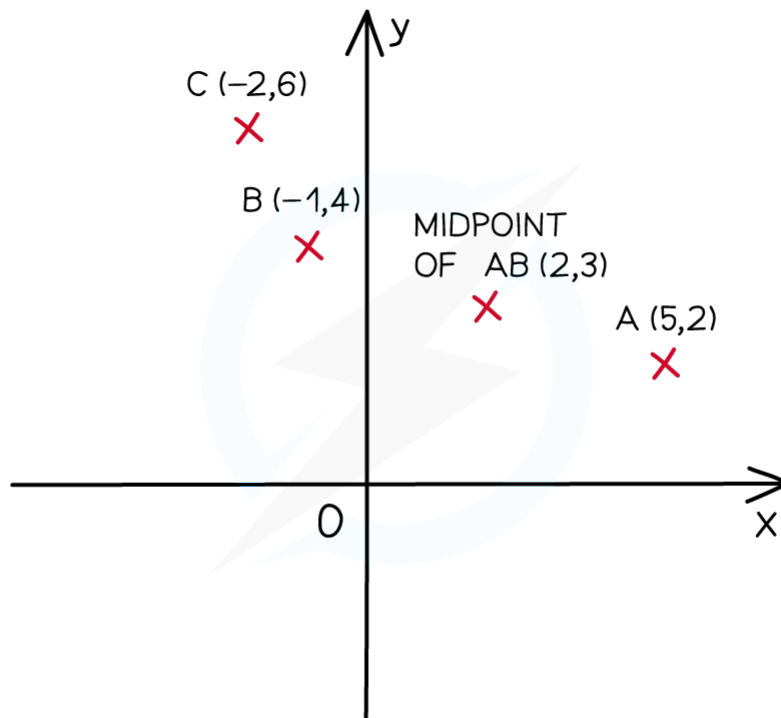
    (a) *Find the midpoint of AB,*

    (b) *Find the distance between the midpoint of AB and point C*

    (c) *Show that AB and BC are not part of the same straight line.*

    Exam Tip − Sketch the points, from which you should be able

    see roughly where the midpoint should go and so is a quick

    check to see if your answer to (a) is about right.

    Add to your diagram as you work through the question

3. Coordinate Geometry

C (−2,6)
✗

B (−1,4)
✗

MIDPOINT
OF AB (2,3)
✗

A (5,2)
✗

y

O

x

(a) *Midpoint is* $\left(\frac{5+(-1)}{2}, \frac{2+4}{2}\right)$

$= (2, 3)$

2 − Find the midpoint of AB

(b) *Distance* $= \sqrt{(6-3)^2 + ((-2)-2)^2}$

$= \sqrt{25}$

$= 5$

3 − Distance between$(2, 3)$ *and* $(-2, 6)$

(Positive square root only as it is a distance)

(c) *Gradient of AB* $= \frac{4-2}{(-1)-5} = \frac{2}{-6} = -\frac{1}{3}$

1 − Showing that gradients are unequal will

Gradient of BC $= \frac{6-4}{(-2)-(-1)} = \frac{2}{-1} = -2$

be enough to show AB and BC are two

Gradients of AB and BC are not

equal and so AB and BC are not part

of the same straight line.

different lines.

3. Coordinate Geometry

3.1.2 STRAIGHT LINES - FINDING EQUATIONS

Why do we want to know about straight lines and their equations?

- Straight Line Graphs (Linear Graphs) have lots of uses in mathematics – one use is in navigation
- We may want to know the equation of a straight line so we can program it into a computer that will plot the line on a screen, along with several others, to make shapes and graphics

How do we find the equation of a straight line?

- The general EQUATION of a straight line is **y = mx + c**
  - where **m** is the gradient
  - **c** is the **y**-axis intercept

- To find the EQUATION of a straight line you need TWO things:
  - the gradient, m
  - any point on the line

- You might find these things from a graph, another equation or two points

- You may be asked to give the equation in the form **ax + by + c = 0**
  (especially if **m** is a fraction)
  If in doubt, SKETCH IT!
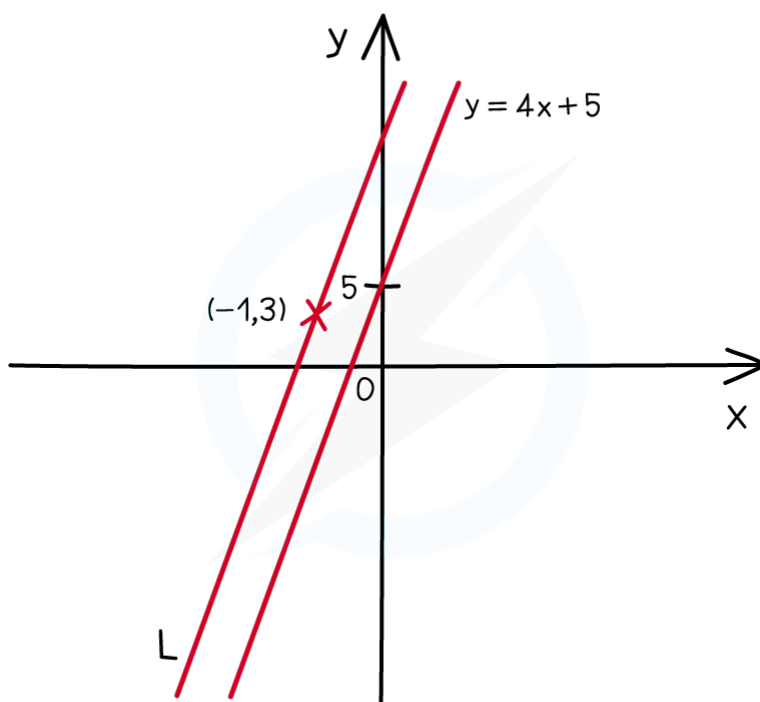
3. Coordinate Geometry

Worked Example

1. *The line P has equation $y = 4x + 5$.*
   Find the equation of the line L parallel to P which passes through the point $(-1, 3)$.
   Give your answer in the form $ax + by + c = 0$

   Sketch not essential but if it doubt ...
   Sketching the given line and roughly where the new lineL
   should be will give you a good idea if your final
   answer is realistic or not

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$y = mx + c$ $1 - Using\ the\ general\ equation\ of\ a\ straight\ line$

$m = 4$ $2 - Since\ the\ line\ L\ is\ parallel\ to\ y = 4x + 5$

$y = 4x + c$ $We\ now\ know\ that\ the\ equation\ of\ L\ looks\ like\ this$

$3 = 4 \times (-1) + c$ $2 - The\ second\ thing\ we\ need\ is\ a\ point$

$c = 7$ $Use\ the\ point\ (-1,\ 3)\ -\ ie\ x = -1,\ y = 3\ to\ find\ c$

$y = 4x + 7$ $3 - This\ is\ the\ equation\ of\ L\ but\ it's\ not\ in\ the\ correct$

$0 = 4x - y + 7$ $format\ so\ rearrange,\ in\ stages\ if\ need\ be,\ until\ it\ is.$

$4x - y + 7 = 0$

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3. Coordinate Geometry

3.1.3 STRAIGHT LINES - DRAWING GRAPHS

How do we draw the graph of a straight line from an equation?

- Before you start trying to draw a straight line, make sure you understand how to find the equation of a straight line – that will help you understand this

- How we draw a straight line depends on what form the equation is given in

- There are two main forms you might see:
  y = mx + c and **ax + by = c**

- Different ways of drawing the graph of a straight line:

1. From the form **y = mx + c**
   (you might be able to rearrange to this form easily)
   plot **c** on the **y**-axis
   go 1 across, **m** up (and repeat until you can draw the line)

2. From **ax + by = c**
   put **x = 0** to find **y**-axis intercept
   put **y = 0** to find **x**-axis intercept
   (You may prefer to rearrange to **y = mx + c** and use above method)

Exam Tip

It might be easier just to plot ANY two points on the line (a third one as a check is not a bad idea either) or use the TABLE function on your calculator.
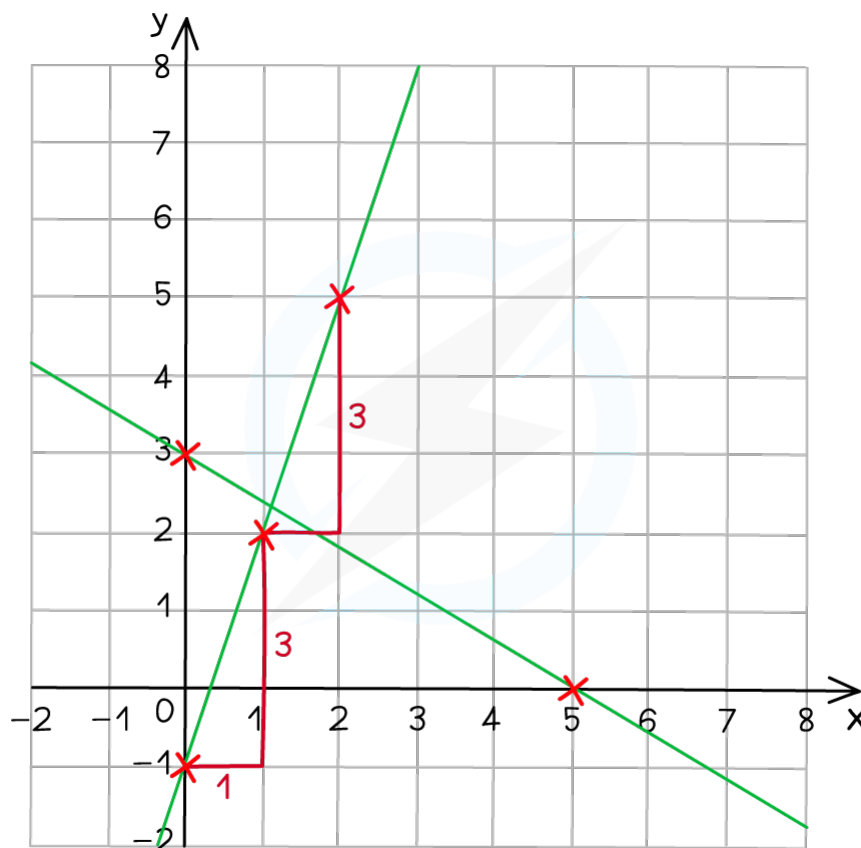
3. Coordinate Geometry

Worked Example

1. *On the axes below draw the lines $y = 3x - 1$ and $3x + 5y = 15$*

$1 - y = 3x - 1$ *First plot c, which is* -1 *on the* $y - axis$
Then go across 1, m, which is 3, up $-$ *this has been done twice on the diagram*
Doing the second part twice gives you three points that are on the line and so you can be confident you have the right answer

$2 - 3x + 5y = 15$ *When* $x = 0, 5y = 15$ *and so* $y = 3$*, therefore* $(0, 3)$ *is on the line*
When $y = 0, 3x = 15$ *and so* $x = 5$*, therefore* $(5, 0)$ *is on the line*
Plot the two points $(0, 3)$ *and* $(5, 0)$ *and join them up to get your line*

3. Coordinate Geometry

3.1.4 PERPENDICULAR LINES

What are perpendicular lines?

- You should already know that PARALLEL lines have **equal** gradients

- PERPENDICULAR LINES do meet each other and where they do the two lines form a right angle – ie they meet at 90°

What's the deal with perpendicular gradients (and lines)?

- Before you start trying to work with perpendicular gradients and lines, make sure you understand how to find the equation of a straight line – that will help you do the sorts of questions you will meet

- Gradients **m¹** and **m²** are PERPENDICULAR if $m_1 \times m^2 = -1$

- We can use **m² = -1 ÷ m²** to find a perpendicular gradient
(This is called the NEGATIVE RECIPROCAL)
If in doubt, SKETCH IT!

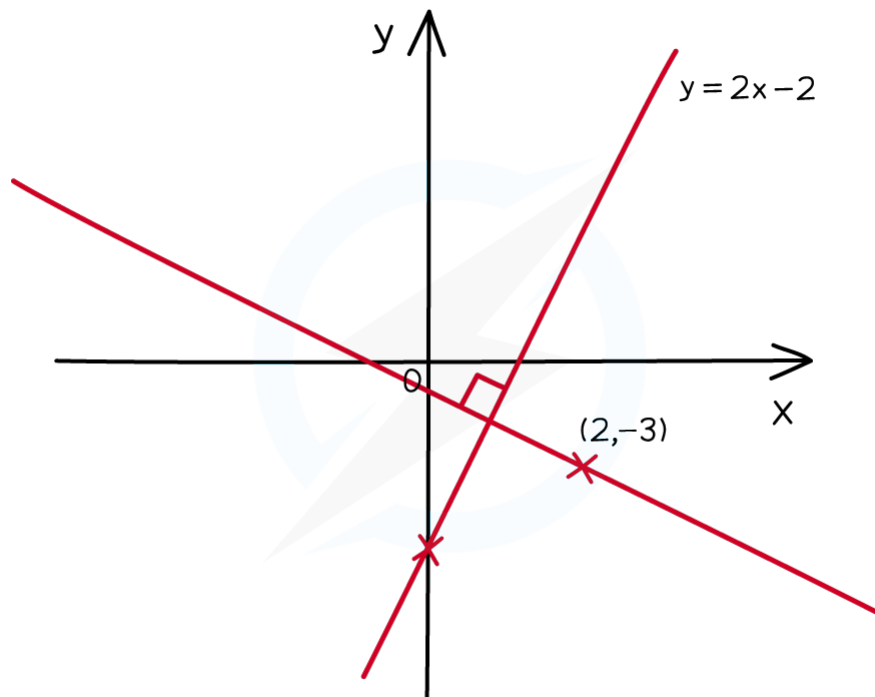## Worked Example

1. *The line L has equation $y = 2x - 2$.*
   Find an equation of the line perpendicular to L which passes through the point $(2, -3)$.
   Leave your answer in the form $ax + by + c = 0$ where a, b and c are integers.

   Sketch is not essential but if it helps or you are
   stuck then draw one

3. Coordinate Geometry

$y = 2x - 2$

$(2, -3)$

$m_1 = 2$

L is in the form $y = mx + c$ so we can see that its gradient is 2

$m_2 = -1 \div 2$

$m_2 = -\frac{1}{2}$

1, 2 − The gradient of the line perpendicular to L will be the negative reciprocal of 2

See Finding Equations of Straight Lines

$y = -\frac{1}{2}x + c$

$-3 = -\frac{1}{2} \times 2 + c$

$c = -2$

We know the gradient of the line we're after but we still need to find c

Find c by substituting the point $(2, -3)$ in :

$y = -\frac{1}{2}x - 2$

$2y = -x - 4$

$x + 2y + 4 = 0$

This is the line we want but it is not in the correct form

Multiply by 2 to get rid of fractions (GROF) and rearrange until we do have it in the required format