Geometry
Difficulty: Medium

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 1

Time allowed: **81 minutes**

Score: **/70**

Percentage: **/100**

Grade Boundaries:

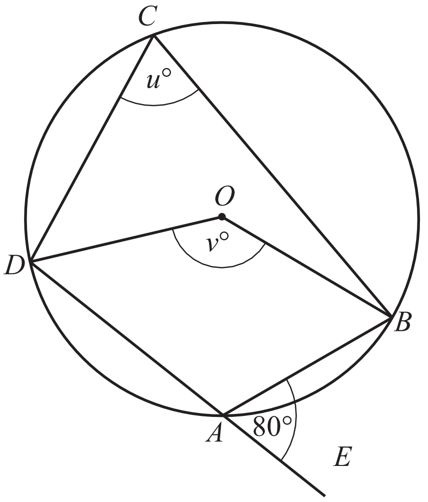
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) ~~Assembled by NS~~

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a)



NOT TO SCALE

A, B, C and D lie on the circle, centre O.
DAE is a straight line.

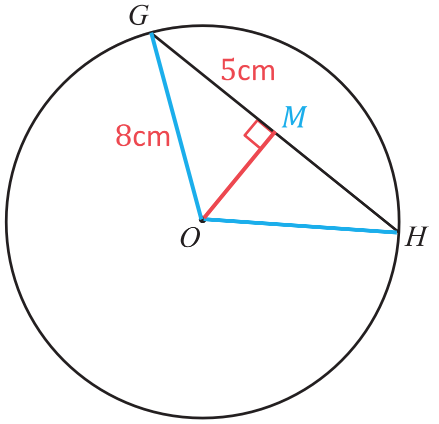
Find the value of u and the value of v. [2]

$D\hat{A}B = 180 - 80 = 100°$ (Angles on a straight line add to 180°)

$u = 180 - 100 = 80°$ (Opposite angles in a Cyclic Quadrilateral add to 180°)

$\boldsymbol{v = 2 \times 80 = 160°}$ (Angle at the Centre is twice the Angle at the Circumference)

(b)



On the diagram, draw the radiuses OG and OH as well as OM, the perpendicular from O to GH.

M is the midpoint of GH and so $GM = 5\text{cm}$.

The diagram shows a circle, centre O, radius 8 cm.
GH is a chord of length 10 cm.

Calculate the length of the perpendicular from O to GH. [3]

OGM is a right-angled triangle so use Pythagoras Theorem:

$$OM^2 + GM^2 = OG^2$$

$$OM^2 + 5^2 = 8^2$$

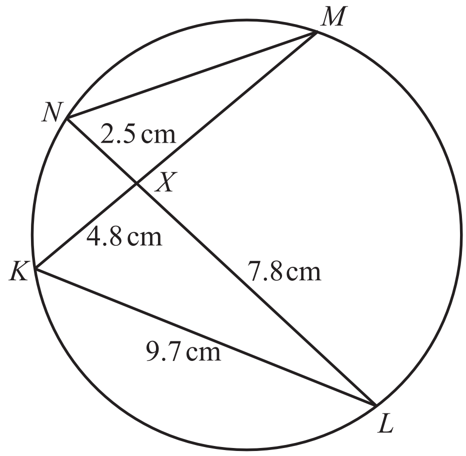
$$OM^2 = 8^2 - 5^2$$

$$OM = \sqrt{8^2 - 5^2}$$

$$\boldsymbol{OM = 6.24\text{cm}} \text{ (to 3sf)}$$

(c) K, L, M and N lie on the circle.
KM and LN intersect at X.
$KL = 9.7$ cm, $KX = 4.8$ cm,
$LX = 7.8$ cm and $NX = 2.5$ cm.

Calculate MN.



NOT TO SCALE

[2]

Spot that $M\widehat{K}L = M\widehat{N}L$ and $N\widehat{L}K = N\widehat{M}K$

(angles in the same segment) and

$M\widehat{X}N = K\widehat{X}L$ (opposite angles).

Therefore the triangles MNX and LKX are similar.

The Scale Factor from LKX to $MNX = \frac{NX}{KX} = \frac{2.5}{4.8}$

so:

$$MN = KL \times \frac{2.5}{4.8}$$

$$MN = 9.7 \times \frac{2.5}{4.8}$$

$\boldsymbol{MN = 5.05}$**cm** (to 3sf)

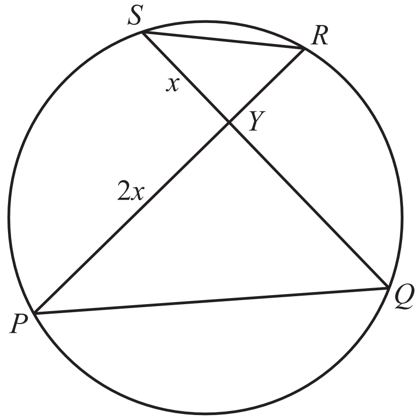
(d) All lengths are in centimetres.

P, Q, R and S lie on the circle.
PR and QS intersect at Y.
$PY = 2x$ and $YS = x$.

The area of triangle $YRS = \frac{5}{12}x(x-1)$.

The area of triangle $YQP = x(x+1)$.

Find the value of x.



NOT TO SCALE

[4]

Again the triangles are similar.

For similar shapes (solids):
$Area\ Factor = Scale\ Factor^2$
$Volume\ Factor = Scale\ Factor^3$

The Scale Factor from YRS to $YQP = \frac{PY}{SY} = \frac{2x}{x} = 2$

So the Area Factor $= 2^2$:

$$Area\ of\ YQP = 2^2 \times Area\ of\ YRS$$

$$x(x+1) = 4 \times \frac{5}{12}x(x-1)$$

Multiply both sides by 3:

$$3x(x+1) = 5x(x-1)$$

Multiply out:

$$3x^2 + 3x = 5x^2 - 5x$$

Put everything on one side:

$$2x^2 - 8x = 0$$

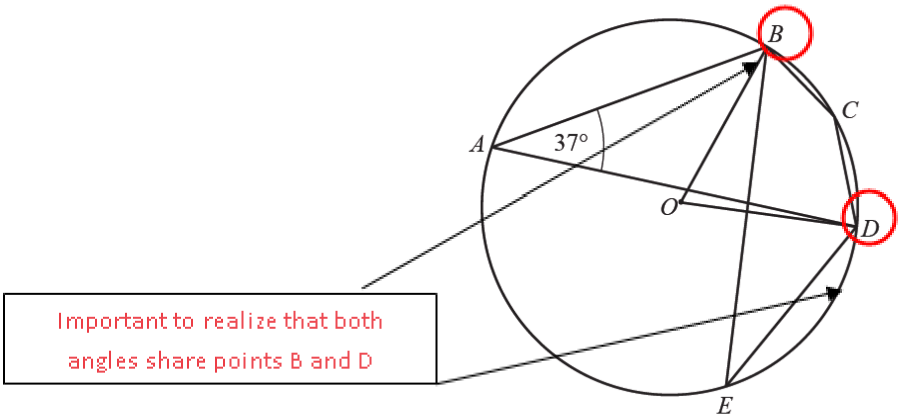
Factorise:

$$2x(x-4) = 0$$

So:

($x = 0$ or) $\boldsymbol{x = 4}$ (x is clearly not zero...)

Question 2



A, B, C, D and E are points on the circle, centre O.
Angle $BAD = 37°$.

Complete the following statements.

(a) Angle BED =

The angle BED is subtended from the same points (B and D) as the angle BAD, so they must have the same size. (Angles in the same segment are equal) [2]

Therefore ***BED*=37°.**

(b) Angle BOD =

The value of BOD will be twice as big as the value of the angle BAD (the angle subtended at the centre of a circle is double the size of the angle subtended at the edge from the same two points). [2]

Therefore we have ***BOD* = 74°.**

(c) Angle BCD =

Opposite angles of a cyclic quad (such as BAD and BCD) are supplementary (add up to 180°). [2]

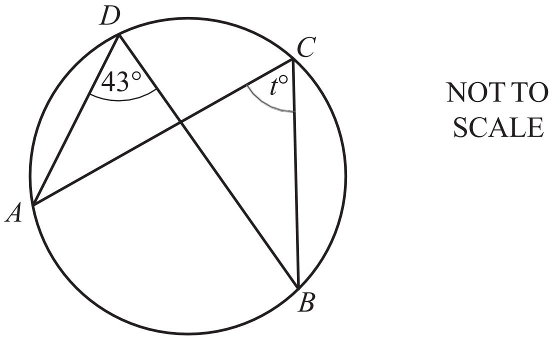
$$180° = BAD + BCD$$

$$180° = 37° + BCD$$

Subtract 37° from both sides to get the value of BCD.

$$\mathbf{BCD = 143°}$$

(a) (i) A, B, C and D lie on the circumference of the circle.

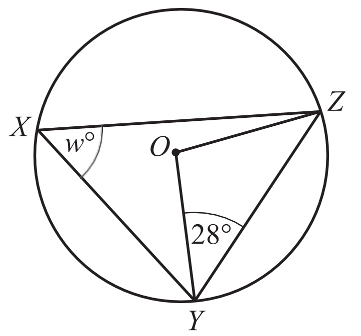


Find the value of t. [1]

The angle ACB is subtended from the same points (A and B) as the angle ADB, so they must have the same size. (Angles in the same segment are equal)

Therefore ***t*°=43°.**

(ii) X, Y and Z lie on the circumference of the circle, centre O.



NOT TO SCALE

Find the value of w, giving reasons for your answer. [3]

Triangle OYZ is isosceles, therefore angles OYZ and OZY have the same size.

$$OZY = 28°$$

The sum of all internal angles of a triangle is 180°.

$$180° = OZY + OYZ + ZOY$$

$$180° = 28° + 28° + ZOY$$

Subtract 56° from both sides of the equation.

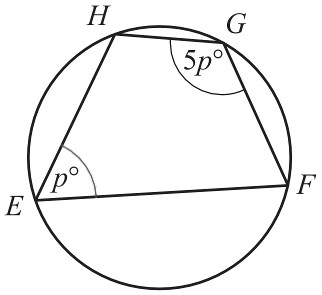
$$ZOY = 124°$$

The value of ZXY will be half as big as the value of the angle ZOY (the angle subtended at the centre of a circle is double the size of the angle subtended at the edge from the same two points).

The two common points for these angles are Z and Y.

Therefore we have ***w* = 62.**

(iii) E, F, G and H lie on the circumference of the circle.



NOT TO SCALE

Find the value of p, giving a reason for your answer. [3]

Opposite angles of a cyclic quadrilateral add up to 180°.

In our case, the opposite angles are HGF and HEF.

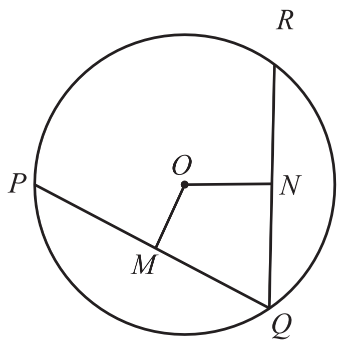
$$180° = HGF + HEF$$

$$180° = 5p° + p°$$

Divide both sides of the equation by 6 to get the value of p.

$$p = \mathbf{30}$$

(b)



NOT TO SCALE

The diagram shows a circle, centre O.
PQ and QR are chords.
OM is the perpendicular from O to PQ.

(i) Complete the statement. [1]

As OM is the perpendicular from O to PQ and PQ is a chord, the length of PQ must be twice the length of PQ (M is the mid-point of PQ).

$$PM : PQ = \mathbf{1 : 2}$$

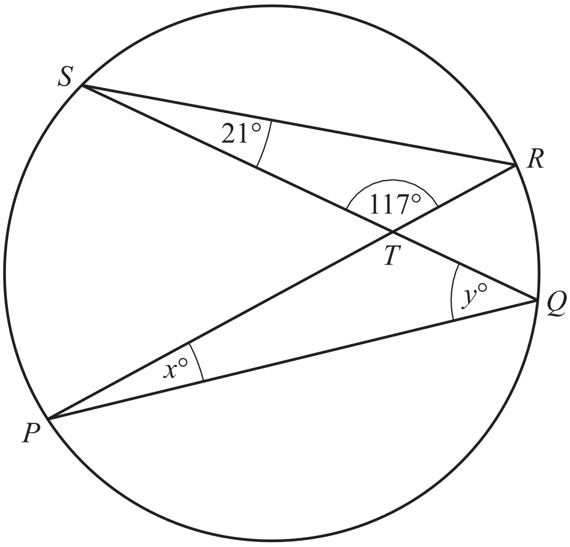
(ii) ON is the perpendicular from O to QR and $PQ = QR$.

Complete the statements to show that triangle OMQ is congruent to triangle ONQ. [4]

As both angles (OMQ and ONQ) have O and Q as their vertices, **OQ is a common side**.

From the fact that M is the midpoint of PQ and N is the midpoint of RQ, we know that **MQ=NQ**

The sizes of MO and NO are the same (**NO=MO**), because they are equal chords **equidistant from the centre O**.



NOT TO SCALE

(a) The chords PR and SQ of the circle intersect at T.
Angle $RST = 21°$ and angle $STR = 117°$.

(i) Find the values of x and y. [2]

$$\boldsymbol{x = 21}$$

$$SRT = 180 - 21 - 117$$

$$= 42$$

$$\rightarrow \boldsymbol{y = 42}$$

(ii) $SR = 8.23$ cm, $RT = 3.31$ cm and $PQ = 9.43$ cm. [2]

Calculate the length of TQ.

The triangles are mathematically similar.

The length scale factor is

$$9.43 \div 8.23$$

$$= 1.1458$$

Hence

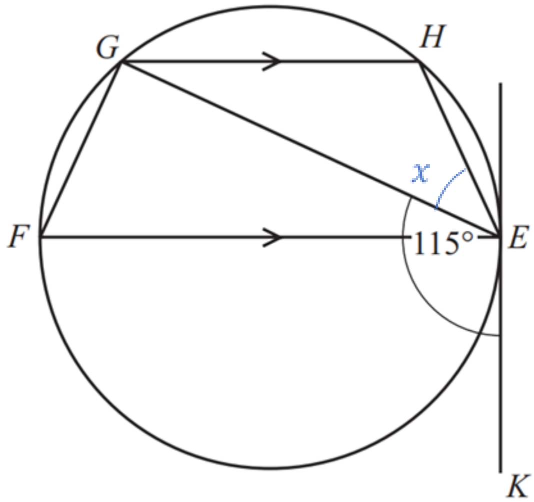
$$TQ = 1.1458 \times 3.31$$

$$= \boldsymbol{3.79}$$

(b) $EFGH$ is a cyclic quadrilateral.
EF is a diameter of the circle.
KE is the tangent to the circle at E.
GH is parallel to FE and angle $KEG = 115°$.

Calculate angle GEH.

[4]



Due to the symmetry of the shape

$$GFE = (115 - 90) + x$$

$$= 25 + x$$

We also have that

$$GFE = 180 - 115$$

$$= 65$$

Hence

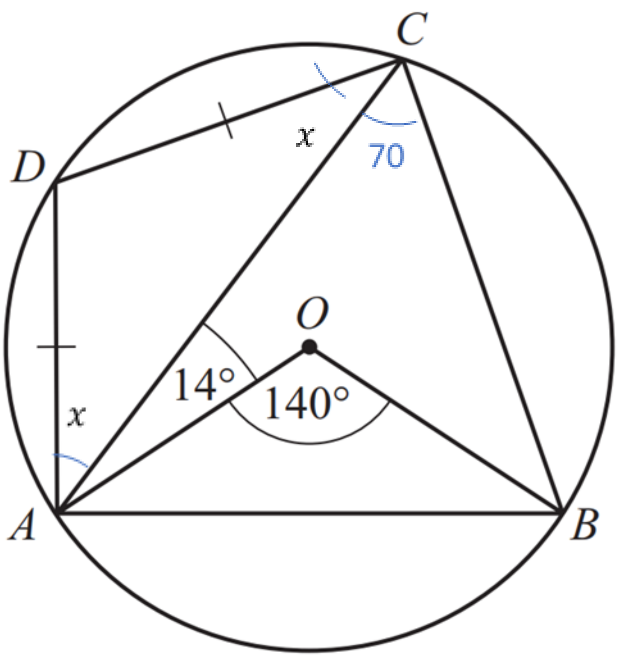
$$25 + x = 65$$

$$\rightarrow \boldsymbol{x = 40}$$

(c) A, B, C and D are points on the circle centre O.
Angle $AOB = 140°$ and angle $OAC = 14°$.
$AD = DC$.

Calculate angle ACD.

[5]



$$OAB = \frac{1}{2}(180 - 140)$$

$$= 20$$

Because it is a cyclic quadrilateral, we have

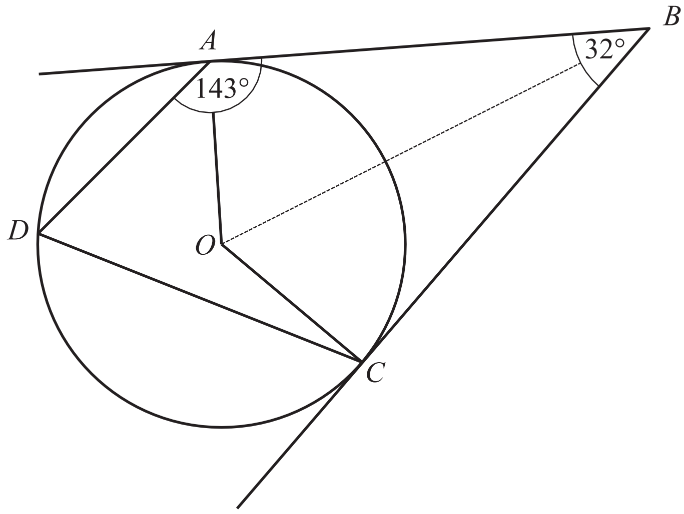
$$x + 14 + 20 + x + 70 = 180$$

$$\rightarrow 2x + 104 = 180$$

$$\rightarrow 2x = 76$$

$$\rightarrow \boldsymbol{x = 38}$$

(a)



NOT TO SCALE

Points A, C and D lie on a circle centre O.
BA and BC are tangents to the circle.
Angle $ABC = 32°$ and angle $DAB = 143°$.

(i) Calculate angle AOC in quadrilateral $AOCB$. [2]

The angle between the tangent and the radial lines are 90˚

This means in quadrilateral OACB we have 2 right angles and one 32˚ angle.

The angles in a quadrilateral must add to 360.

$$AOC = 360 - 32 - 90 - 90$$

$$= \mathbf{148}$$

(ii) Calculate angle ADC. [1]

ADC is half of AOC by circle theorems.

$$ADC = \frac{1}{2} \times 148$$

$$= \mathbf{74}$$

(iii) Calculate angle OCD. [2]

Now look at quadrilateral DABC.

We have three known angles: 143˚, 32˚, and 74˚.

All the angles must add to 360, hence

$$BCD = 360 - 32 - 143 - 74$$

$$= 111$$

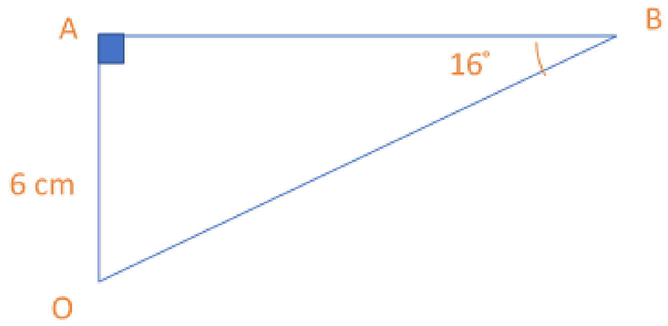
BCD is the angle OCD plus 90˚ so

$$OCD = 111 - 90$$

$$\mathbf{= 21}$$

(iv) $OA = 6$cm.
Calculate the length of AB. [3]

OAB is a right-angle triangle with the following dimensions:



We can use the following trigonometric rules

$$\sin\theta = \frac{opp}{hyp}, \cos\theta = \frac{adj}{hyp}$$

To write

$$hyp \times \sin 16 = 6$$

$$hyp \times \cos 16 = AB$$

If we divide the top equation by the bottom we get

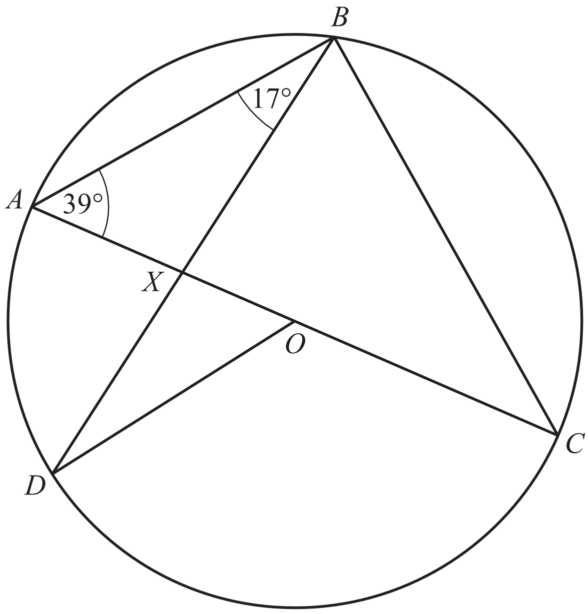
$$\frac{hyp \times \sin 16}{hyp \times \cos 16} = \frac{6}{AB}$$

$$\rightarrow \tan 16 = \frac{6}{AB}$$

$$\rightarrow AB = \frac{6}{\tan 16}$$

$$\mathbf{= 20.9}$$

(b)



NOT TO SCALE

A, B, C and D are on the circumference of the circle centre O.
AC is a diameter.
Angle $CAB = 39°$ and angle $ABD = 17°$.

(i) Calculate angle ACB. [2]

From circle theorems

$$ABC = 90^{\circ}$$

And so, because angles in a triangle add to 180, we have

$$ACB = 180 - 39 - 90$$

$$= \mathbf{51}$$

(ii) Calculate angle BXC. [2]

First, we find XBC

$$XBC = 90 - 17$$

$$= 73$$

Hence

$$BXC = 180 - 73 - 51$$

$$= \mathbf{56}$$

(iii) Give the reason why angle DOA is 34°. [1]

Angle at the centre is twice the angle at the circumference (ADB)

$$ABD = 17$$

$$DOA = 2 \times 17 = 34$$

(iv) Calculate angle BDO. [1]

$$DXO = 180 - BXC$$

$$= 180 - 56$$

$$= 124$$

$$BDO = 180 - DXO - DOA$$

$$= 180 - 124 - 34$$

$$= \mathbf{22}$$

(v) The radius of the circle is 12 cm. Calculate the length of major arc $ABCD$. [3]

The angle DOA is 34, hence the angle of the major arc ABCD is

$$360 - 34$$

$$= 326$$

The length of an arc is

$$r \times \theta$$

Where θ is measured in radians. Converting our angle into radians

$$326 \times \frac{\pi}{180}$$

$$= 5.69$$

Hence

$$ABCD = 5.69 \times 12$$

$$= \mathbf{68.28}$$

Quadrilaterals P and Q each have diagonals which

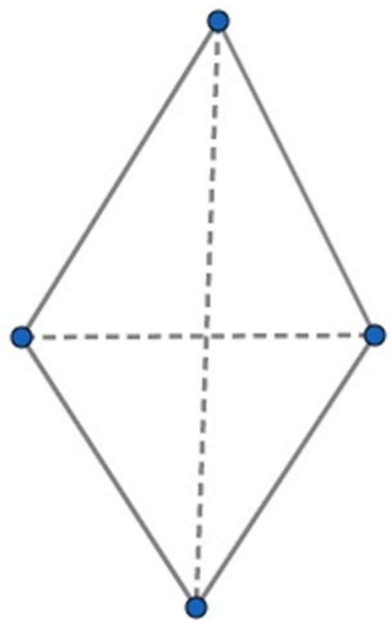
- are unequal,
- intersect at right angles.

P has two lines of symmetry. Q has one line of symmetry.

(a) (i) Sketch quadrilateral P.
Write down its geometrical name. [2]

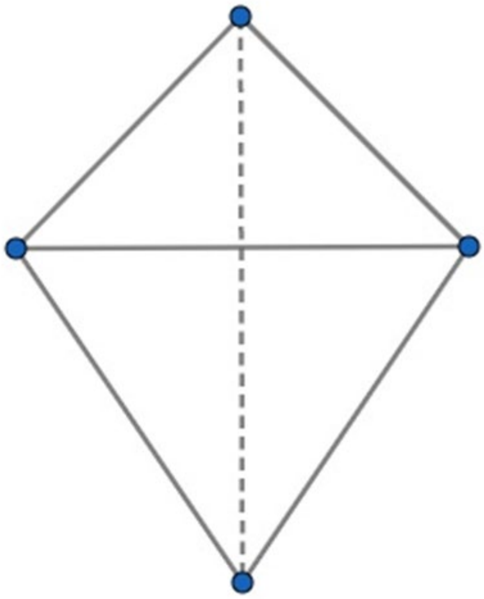
The quadrilaterals with perpendicular diagonals are kites and rhombus.

A rhombus has 2 axis of symmetry, represented in the figure below by 2 dotted lines.



(ii) Sketch quadrilateral Q.
Write down its geometrical name. [2]

A kite has only one line of symmetry, represented in the figure below by a dotted line.

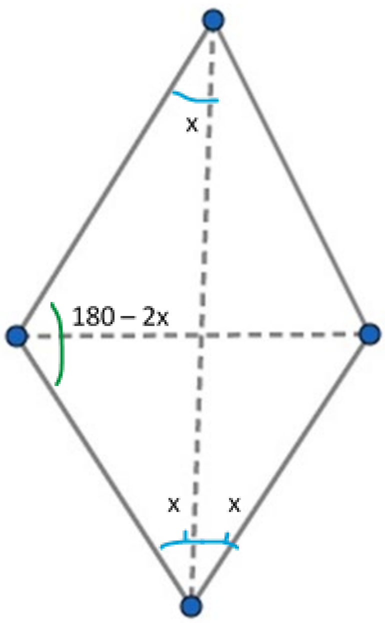


(b) In quadrilateral P, an angle between one diagonal and a side is $x°$.
Write down, in terms of x, the four angles of quadrilateral P. [2]

In a rhombus, a diagonal separates an angle in 2 congruent angles.

Therefore, one of the angles of the rhombus can be expressed as 2x.

If we divide the rhombus into 2 triangles across its vertical symmetry line, the angle will be 180 – 2x. This is because the interior angles in a triangle add up to 180 and the triangle is isosceles, therefore having 2 congruent angles equal to x.



(c) The diagonals of quadrilateral Q have lengths 20 cm and 12 cm.
Calculate the area of quadrilateral Q. [2]

The formula for the area of a kite is:

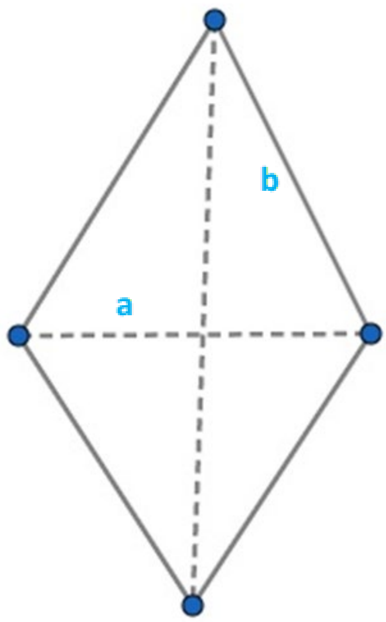
$$\text{Area} = \frac{\text{diagonal 1 x diagonal 2}}{2}$$

$$\text{Area} = \frac{20\text{ cm x }12\text{ cm}}{2}$$

Area = 120 cm^2

(d) Quadrilateral P has the same area as quadrilateral Q.
The lengths of the diagonals and sides of quadrilateral P are all integer values.
Find the length of a side of quadrilateral P. [3]

Area of P = 120 cm^2



In a right-angled triangle, we can use Pythagoras' Theorem to work out the length of one side.

$(a/2)^2 + (b/2)^2 = \text{side}^2$

$a^2 + b^2 = 4\text{side}^2$

The area of the rhombus can be expressed as the product of the diagonals divided by 2.

Area = $120\ \text{cm}^2 = \frac{a \times b}{2}$

$ab = 240\ \text{cm}^2$

The side and the 2 diagonals need to be integers.

⇨ a = 10 cm and b = 24

$100 + 576 = 4\ \text{side}^2$

$\text{Side}^2 = 169$

Side = 13 cm

Geometry
Difficulty: Hard

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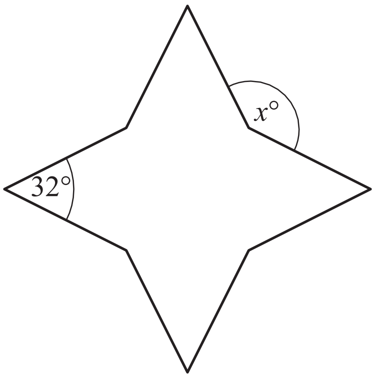


NOT TO SCALE

The diagram shows an octagon.
All of the sides are the same length.
Four of the interior angles are each 32°.
The other four interior angles are equal.

Find the value of x. [4]

Let the reflex interior angles be y, then we have

$x + y = 360$

and

$4 \times 32 + 4y = (8 - 2) \times 180$

$\rightarrow 4y = 1080 - 128$

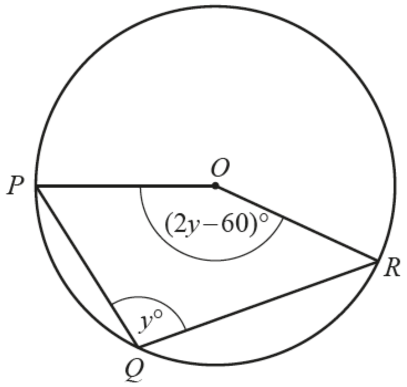
$\rightarrow y = 238$

Hence

$x = 360 - 238$

$= \mathbf{122}$

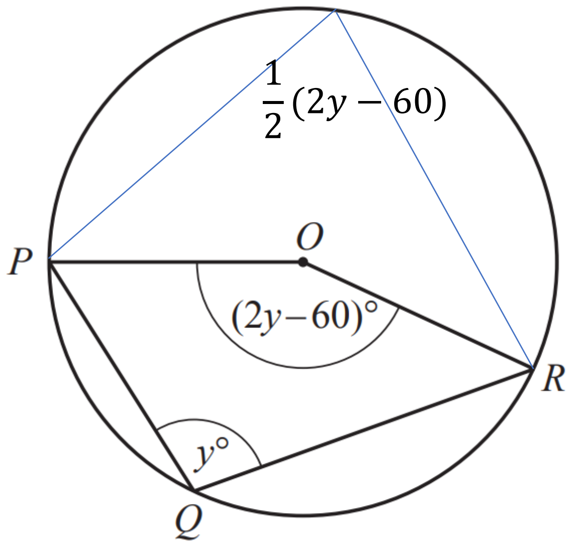
(b)



NOT TO SCALE

P, Q and R lie on a circle, centre O.
Angle $PQR = y°$ and angle $POR = (2y - 60)°$.

Find the value of y. [3]



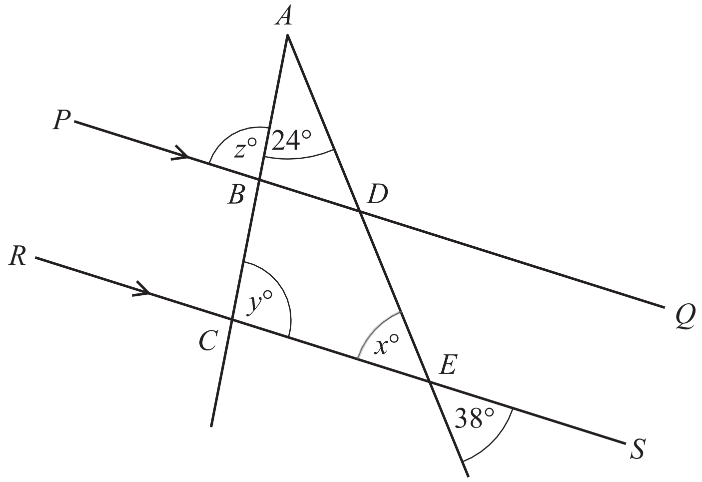
$$\frac{1}{2}(2y-60)+y=180$$

$$\rightarrow 2y-30=180$$

$$\rightarrow y=210 \div 2$$

$$\mathbf{=105}$$

(a)



NOT TO SCALE

PQ is parallel to RS.
ABC and ADE are straight lines.

Find the values of x, y and z. [3]

$$\boldsymbol{x = 38}$$

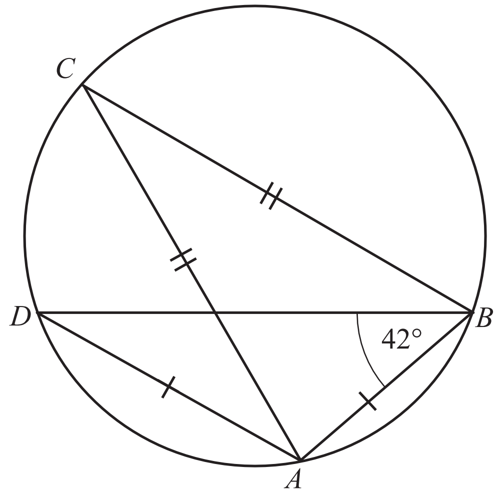
$$y = 180 - 24 - 38$$

$$\boldsymbol{= 118}$$

$$z = 180 - y$$

$$\boldsymbol{= 62}$$

(b)



NOT TO SCALE

The points A, B, C and D lie on the circumference of the circle.
$AB = AD$, $AC = BC$ and angle $ABD = 42°$.

Find angle CAB. [3]

$$ADB = 42$$

Because ABD is an isosceles triangle

$$\rightarrow ACB = 42$$

Because of angles on the same segment.

Angles in a triangle sum to 180 so

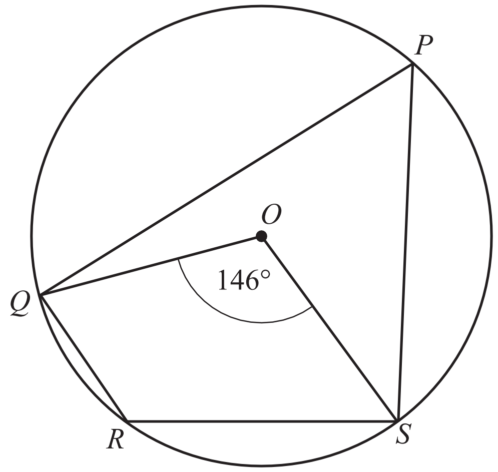
$$ACB + CAB + CBA = 180$$

$$\rightarrow 42 + 2\,CAB = 180$$

$$\rightarrow CAB = \frac{180 - 42}{2}$$

$$= \mathbf{69}$$

(c)



NOT TO
SCALE

The points P, Q, R and S lie on the circumference of the circle, centre O.
Angle $QOS = 146°$.

Find angle QRS. [2]

$$APS = 0.5 \times 146$$

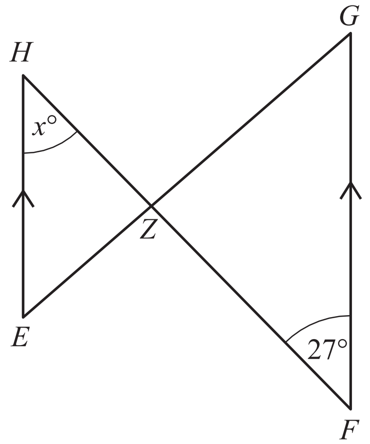
$$= 73$$

Opposite angles in a cyclic quadratic add to 180

$$\rightarrow QRS + 73 = 180$$

$$\rightarrow \boldsymbol{QRS = 107}$$

(a)



NOT TO SCALE

In the diagram, EH is parallel to FG.
The straight lines EG and FH intersect at Z.
Angle $ZFG = 27°$.

(i) Find the value of x. [1]

$$x = 27° \ (Z - angles)$$

(ii) $EH = 5$ cm, $FG = 9$ cm and $ZG = 7$ cm.

Calculate EZ. [2]

To answer this we need to know that similar triangles are related like this:

$$\frac{EZ}{ZG} = \frac{EH}{FG}$$

So:

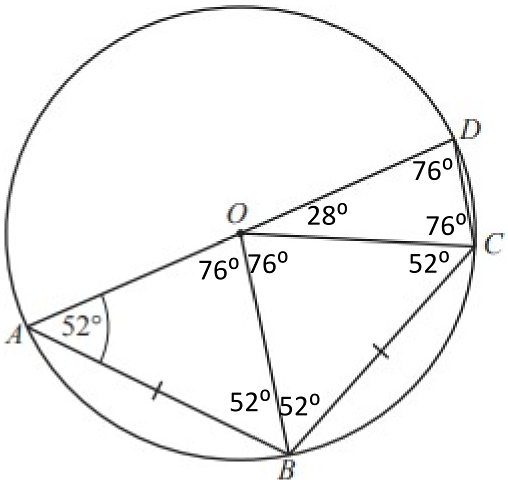
$$EZ = \frac{ZG \times EH}{FG}$$

$$EZ = \frac{7 \times 5}{9}$$

$$EZ = \frac{35}{9}$$

$$EZ = 3.89(3.s.f)$$

(b) The diagram shows points A, B, C and D on the circumference of a circle, centre O.
AD is a straight line, $AB = BC$ and angle $OAB = 52°$.



NOT TO SCALE

Find angle ADC. [3]

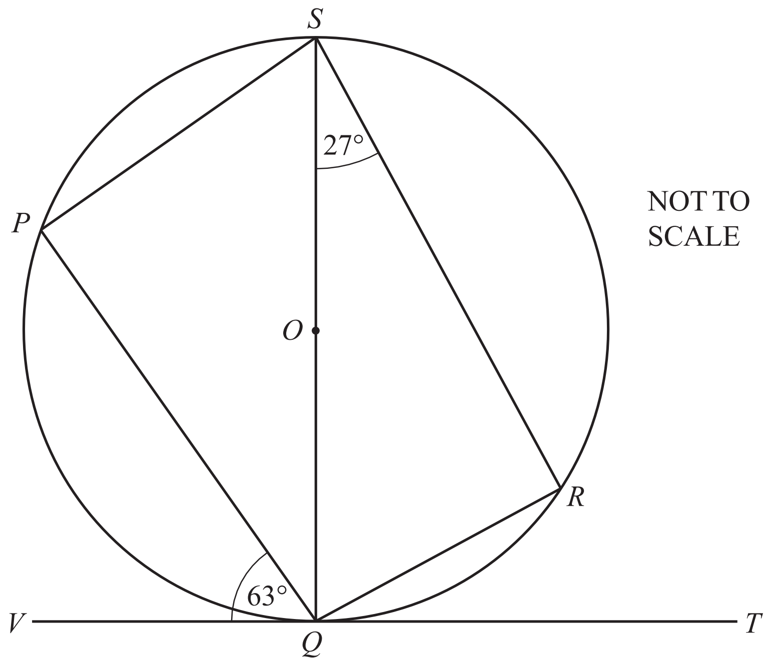
The angle ADC can be found by recognising that the triangles AOB and BOC are identical, and isosceles triangles (as two of the lengths are the radius of the circle). Therefore we can find the interior angles.

After this step, we can find the angle COD, by using the fact that there are $180°$ on a straight line. We also can find that triangle COD is an isosceles triangle, so we can find the remaining triangle.

Therefore:

$$\boldsymbol{ADC = 76°}$$

(c) The diagram shows points P, Q, R and S on the circumference of a circle, centre O.
VT is the tangent to the circle at Q.

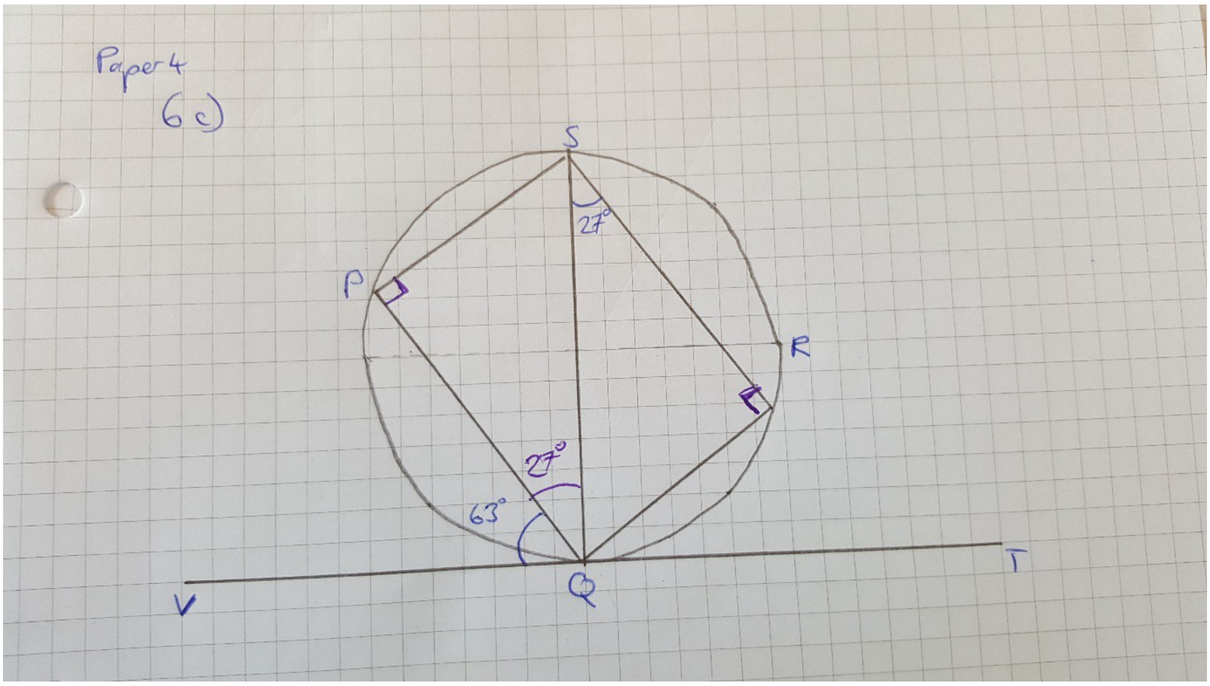


NOT TO SCALE

Complete the statements.

i) Angle QPS=Angle QRS=90^o because of angles in a semicircle. [2]

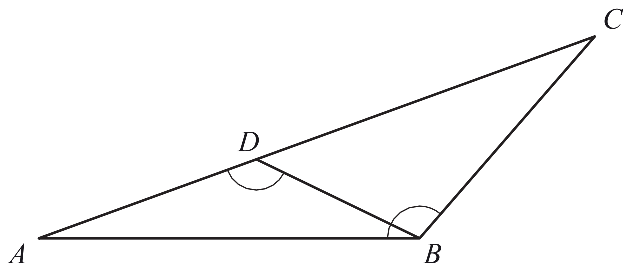
ii) Angle SQP=27^o because of angles at a right angle (tangent to a radius). [2]



(iii) **Part (c)(i)** and **part (c)(ii)** show that [1]

i) The cyclic quadrilateral PQRS is a rectangle.

(a)



NOT TO SCALE

In the diagram, D is on AC so that angle ADB = angle ABC.

(i) Show that angle ABD is equal to angle ACB. [2]

The sum of the interior angles of the two triangles (ABD and ACB) must equal each other (and 180°).

$$ABD + ADB + DAB = ABC + ACB + CAB$$

The angles CAB and DAB are actually one and the same as D line on AC.

$$ABD + ADB = ABC + ACB$$

Angles ADB = angle ABC.

Therefore: $\boldsymbol{ABD = ACB}$

[1]

(ii) Complete the statement.

As the angles in these two triangles are the same, the triangles ABD and ACB are **similar**.

(iii) $AB = 12$ cm, $BC = 11$ cm and $AC = 16$ cm.

Calculate the length of BD. [2]

The angles are similar, therefore the following ratios equal.

$$\frac{BD}{AB} = \frac{BC}{AC}$$

Using the lengths given.

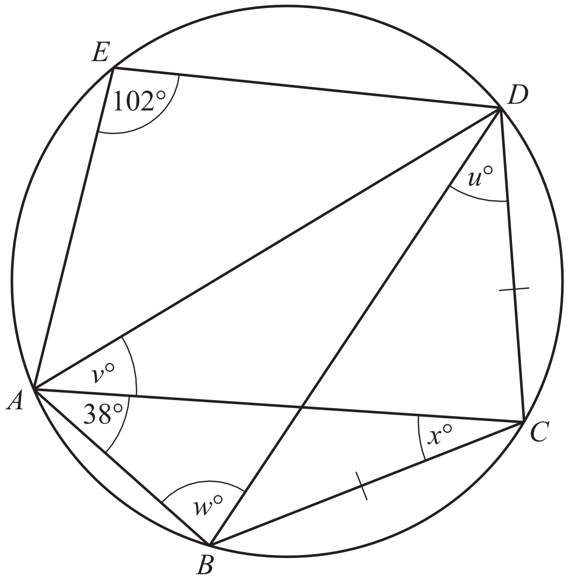
$$\frac{BD}{12\ cm} = \frac{11\ cm}{16\ cm}$$

Multiply both sides by 12cm.

$$BD = \frac{11}{16} \times 12cm$$

$$\boldsymbol{BD = 8.25cm}$$

(b)



A, B, C, D and E lie on the circle.
Angle $AED = 102°$ and angle $BAC = 38°$.
$BC = CD$.
Find the value of

(i) u,

[1]

The angle BDC is subtended by the same points (B and C) as the angle BAC, so they must be the same size. (Angles in the same segment are equal)

Therefore $\boldsymbol{u°=38°}$.

(ii) v,

[1]

As the length of BC and CD are equal, so the angles BAC and CAD are also equal.

Hence:

$$v° = \mathbf{38°}$$

(iii) w,

[1]

Opposite angles of a cyclic quadrilateral add up to 180°.

In this case, the opposite angles are AED and ABD.

$$180° = AED + ABD$$

$$180° = 102° + w°$$

Subtract 102° from both sides of the equation.

$$w = \mathbf{78°}$$

(iv) x.

[1]

The interior angles of any triangle ABC sum to 180°.

The triangle BCD is an isosceles triangle, therefore angles DBC and BDC have the same size 38°.

$$180° = ABC + BCA + CAB$$

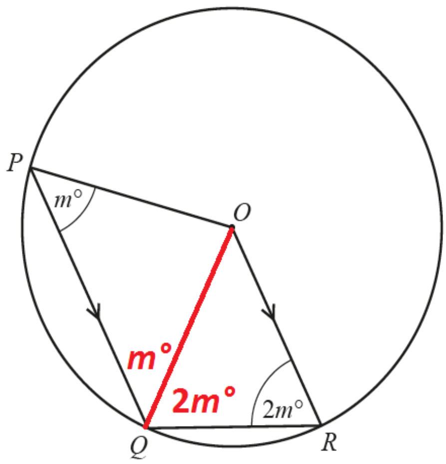
$$180° = (w° + u°) + x° + 38°$$

$$180° = (38° + °78°) + x° + 38°$$

Subtract 154° from both sides of the equation gives:

$$x° = \mathbf{26}°$$

(c)



NOT TO SCALE

In the diagram, P, Q and R lie on the circle, centre O.
PQ is parallel to OR.
Angle $QPO = m°$ and angle $QRO = 2m°$.

Find the value of m.

[5]

The sum of all interior angles of a quadrilateral is 360°.

$$360° = PQR + QRO + ROP + OPQ$$

Two of these angles are known. $POQ = m°$ and $QRO = 2m°$

All length OP, OQ and OR must be equal as they are all radii of the circle. This means that angles POQ and QOR are equilateral triangles.

Therefore we know that the angle OPQ and OQP are the same and also OQR is the same as ORQ.

By summing OQP and OQR, we get the size of angle PQR.

$$PQR = OQP + OQR = OPQ + ORQ$$

$$PQR = m° + 2m° = 3m°$$

As the lines PQ and OR are parallel, the sum of angles at P and O must be the same as the sum of angles at Q and R.

$$ROP + OPQ = PQR + QRO$$

$$ROP + m° = 3m° + 2m°$$

Subtract $m°$ from both sides to get the value of ROP.

$$ROP = 4m°$$

Now we know all four angles of the original equation.

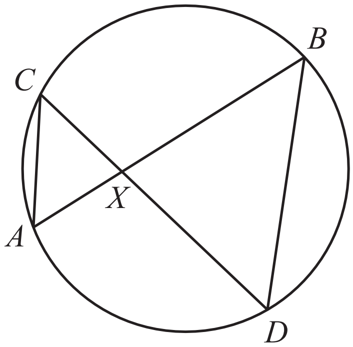
$$360° = 3m° + 2m° + 4m° + m°$$

$$360° = 10m°$$

Divide both sides by 10 to work out the value of m.

$$\boldsymbol{m = 36°}$$

(a) The diagram shows a circle with two chords, AB and CD, intersecting at X.



NOT TO SCALE

(i) Show that triangles ACX and DBX are similar. [2]

Angles in the same segment are equal so:

$$angle\ ACD = angle\ ABD$$

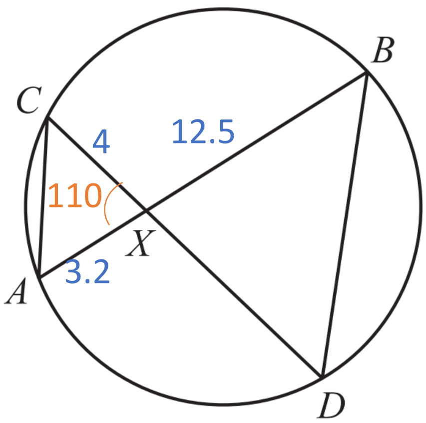
$$angle\ CAB = angle\ CDB$$

Both triangles have **equal angles**, therefore they are mathematically similar.

(ii) $AX = 3.2$ cm, $BX = 12.5$ cm, $CX = 4$ cm and angle $AXC = 110°$.

(a) Find DX. [2]

Mathematically similar and the length scalar is:



$$12.5 \div 4$$

$$= 3.125$$

Hence:

$$DX = 3.2 \times 3.125$$

$$= \mathbf{10}$$

(b) Use the cosine rule to find AC. [4]

Cosine rule is:

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$

Hence:

$$AC^2 = 4^2 + 3.2^2 - 2(3.2)(4)\cos 110$$

$$= 34.9957$$

$$\rightarrow \boldsymbol{AC = 5.92}$$

(c) Find the area of triangle BXD. [2]

Area of AXC is:

$$A = \frac{1}{2}(4)(3.2)\sin 110$$

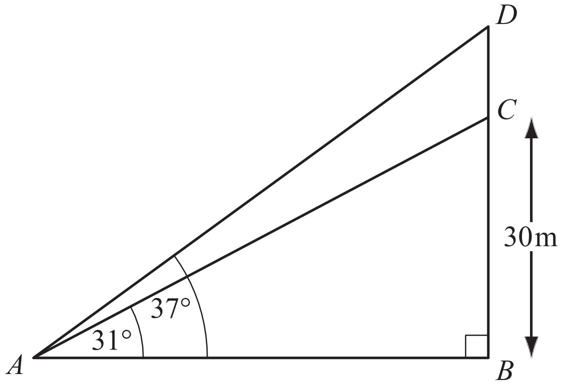
$$= 6.014$$

Area of BXD is therefore

$$6.014 \times 3.125^2$$

$$\boldsymbol{= 58.7}$$

(b)



In the diagram, BC represents a building 30m tall.
A flagpole, DC, stands on top of the building.
From a point, A, the angle of elevation of the top of the building is 31°.
The angle of elevation of the top of the flagpole is 37°.

Calculate the height, DC, of the flagpole. [5]

We can use the trigonometric relation:

$$\tan\theta = \frac{opp}{adj}$$

To find AB as:

$$\tan 31 = \frac{30}{AB}$$

$$\rightarrow AB = \frac{30}{\tan 31}$$

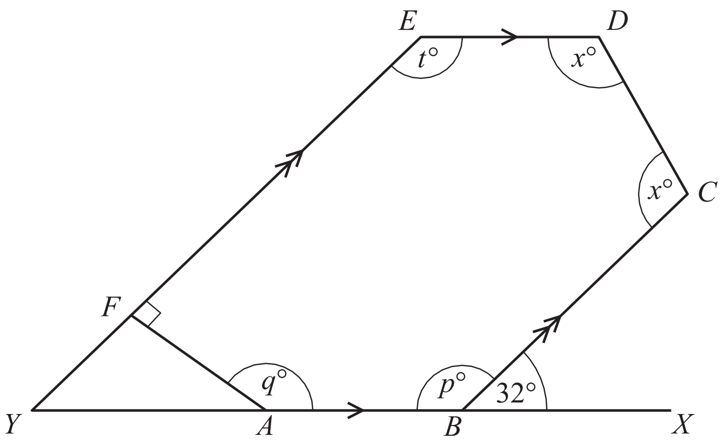
Now we can use this to find DC:

$$\tan 37 = \frac{(30 + DC)}{AB}$$

$$\rightarrow \frac{30}{\tan 31}\tan 37 - 30 = DC$$

$$= 7.62$$

(a)



NOT TO SCALE

ABCDEF is a hexagon.
AB is parallel to *ED* and *BC* is parallel to *FE*.
YFE and *YABX* are straight lines.
Angle $CBX = 32°$ and angle $EFA = 90°$.

Calculate the value of

(i) p, [1]

$\boldsymbol{p = 148}$

(ii) q, [2]

$FYA = 32$

$\rightarrow q = 180 - (180 - 90 - 32)$

$\boldsymbol{= 122}$

(iii) t, [1]

$t = 148$

(iv) x. [3]

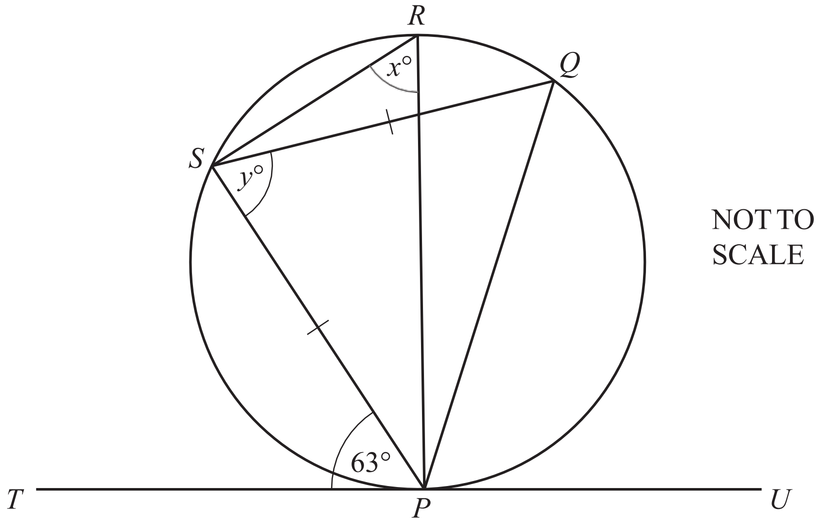
Sum of interior angles is 720

$2x + 148 + 90 + 122 + 148 = 720$

$\rightarrow 2x = 720 - 508$

$\rightarrow \boldsymbol{x = 106}$

(b)



P, Q, R and S are points on a circle and $PS = SQ$.
PR is a diameter and TPU is the tangent to the circle at P.
Angle $SPT = 63°$.

Find the value of

(i) x, [2]

$$RPS = 90 - 63$$

$$= 27$$

$$RSP = 90$$

$$\rightarrow x = 180 - 90 - 27$$

$$\boldsymbol{x = 63}$$

(ii) y. [2]

$$SQP = x$$

$$= 63$$

Angles in a triangle sum to 180

$$y + 2x = 180$$

$$\rightarrow y = 180 - 126 = \mathbf{54}$$

$$= \mathbf{54}$$

(a) One angle of an isosceles triangle is 48°.

Write down the possible pairs of values for the remaining two angles. [2]

An isosceles triangle has 2 same angles out of the 3 angles of the triangle.

Case 1: One of the same angles is 48 degrees

$$Third\ angle = 180° - (48° \times 2)$$

$$= 84°$$

Hence the other 2 angles are **48 degrees and 84 degrees.**

Case 2: The non-same third angle is 48 degrees

$$One\ of\ the\ same\ angles = \frac{180° - 48°}{2}$$

$$= 66°$$

Hence the other 2 angles are **both 66 degrees.**

(b) Calculate the sum of the interior angles of a pentagon. [2]

A pentagon has 5 sides.

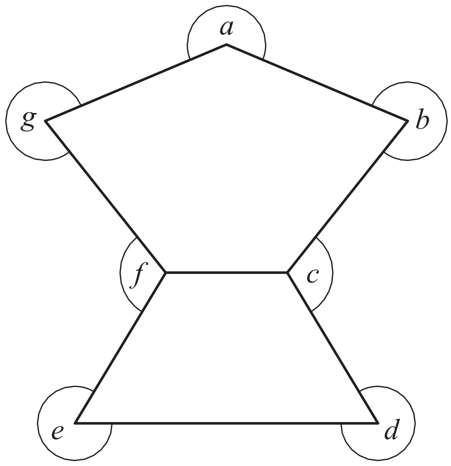
The equation for sum of interior angles is given by:

$$Sum\ of\ interior\ angles = (n - 2) \times 180°$$

$$= (5 - 2) \times 180°$$

$$= \mathbf{540°}$$

(c) Calculate the sum of the angles a, b, c, d, e, f and g shown in this diagram. [2]



NOT TO SCALE

There are 7 angles here:

$$Total = 7 \times 360°$$

$$= 2520°$$

The sum of interior angles of the pentagon:

$$Pentagon\ sum\ of\ interior\ angles = 540°$$

The sum of interior angles of the trapezium:]

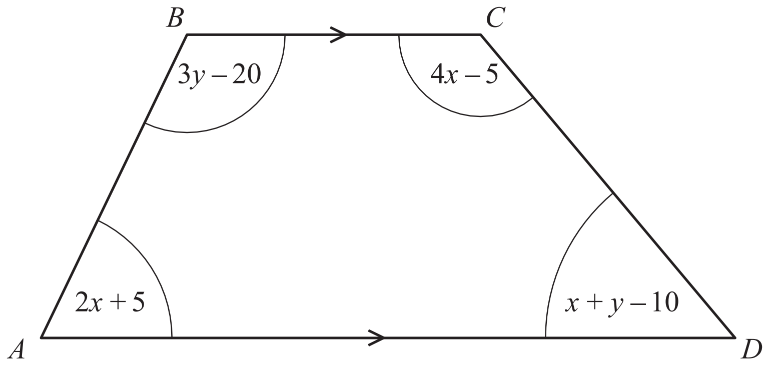
$$Trapezium\ sum\ of\ interior\ angles = 360°$$

Therefore,

$$Sum\ of\ given\ angles = 2520° - 540° - 360°$$

$$= \mathbf{1620°}$$

(d) The trapezium, $ABCD$, has four angles as shown.
All the angles are in degrees.



NOT TO SCALE

(i) Show that $7x + 4y = 390$. [1]

Sum of interior angles of a trapezium is 360 degrees.

$$360°$$

$$= (3y - 20) + (4x - 5) + (2x + 5) + (x + y - 10)$$

$$360 = 7x + 4y - 30$$

$$\mathbf{7x + 4y = 390}\ (shown)$$

(ii) Show that $2x + 3y = 195$. [1]

The sum of angles between 2 parallel lines is 180 degrees:

$$2x + 5 + 3y - 20 = 180$$

$$\mathbf{2x + 3y = 195\ (\mathit{shown})}$$

(iii) Solve these simultaneous equations. [4]

$$7x + 4y = 390 \ldots (1)$$

$$2x + 3y = 195 \ldots (2)$$

From (1):

$$4y = 390 - 7x$$

$$y = \frac{390 - 7x}{4} \ldots (3)$$

Substitute (3) into (2):

$$2x + 3\left(\frac{390 - 7x}{4}\right) = 195$$

$$2x + \left(\frac{1170 - 21x}{4}\right) = 195$$

Multiply throughout by 4:

$$8x + (1170 - 21x) = 780$$

$$13x = 390$$

$$\boldsymbol{x = 30}$$

Sub into (3):

$$y = \frac{390 - 7(30)}{4}$$

$$\boldsymbol{y = 45}$$

(iv) Use your answer to **part (d)(iii)** to find the sizes of all four angles of the trapezium. [1]

$$x = 30$$

$$y = 45$$

$$3y - 20 = 115°$$

$$4x - 5 = 115°$$

$$2x + 5 = 65°$$

$$\boldsymbol{x + y - 10 = 65°}$$

Geometry
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Geometry
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **100 minutes**

Score: **/87**

Percentage: **/100**

Grade Boundaries:

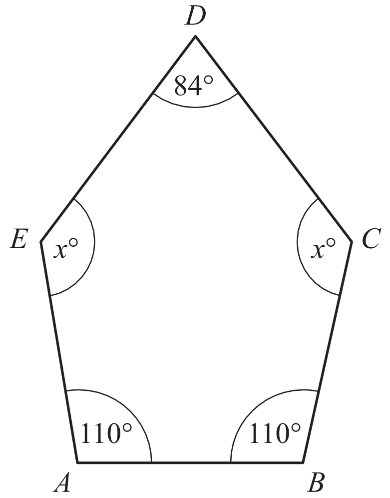
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a)



NOT TO SCALE

In the pentagon $ABCDE$, angle EAB = angle ABC = 110° and angle CDE = 84°.
Angle BCD = angle DEA = x°.

(i) Calculate the value of x. [2]

The sum of the interior angles for a polygon of n sides is given by:

$$I = 180(n-2)$$

So for a pentagon:

$$I = 540$$

Forming an equation for the interior angles.

$$110 + 110 + x + x + 84 = 540$$

Collecting terms.

$$2x + 304 = 540$$

Subtracting 304.

$$2x = 236$$

Dividing by 2. $\boldsymbol{x = 118^o}$

(ii) $BC = CD$.
Calculate angle CBD. [1]

As $BC = CD$ then CBD forms an isosceles triangle, this means angle CBD = angle CDB, and because the interior angles of a triangle sum to 180°, we can form an equation for the interior angles.

$118 + CBD + CDB = 180$

Substituting CDB for CBD as they are equal.

$2CBD + 118 = 180$

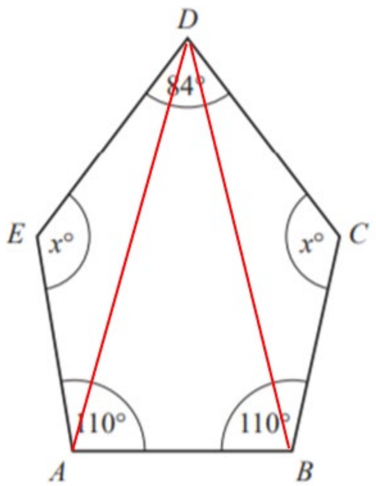
Subtracting 118.

$2CBD = 62$

Dividing by 2.

$\mathbf{CBD = 31}$

(iii) This pentagon also has one line of symmetry.
Calculate angle ADB. [1]



We know the value for CDB, and because it is symmetrical then angle EDA = angle CDB, and since angle ADB and these two known angles sum to 84°, we can form an equation in ADB.

$31 + ADB + 31 = 84$

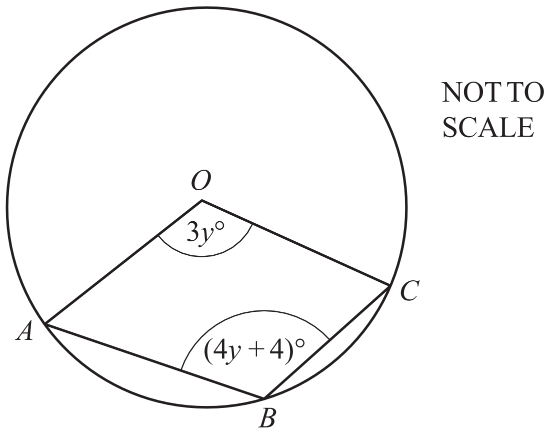
$ADB + 62 = 84$

Subtracting 62.

$\boldsymbol{ADB = 22^o}$

(b) A, B and C lie on a circle centre O.
Angle $AOC = 3y°$ and angle $ABC = (4y + 4)°$.

Find the value of y. [4]



This is a version of the kite theorem although at first it may be unclear. Here angle ABC is half the size of 360 – AOC. Using this, we can form the following equation.

$360 - 3y = 2(4y + 4)$

Expanding brackets.

$360 - 3y = 8y + 8$

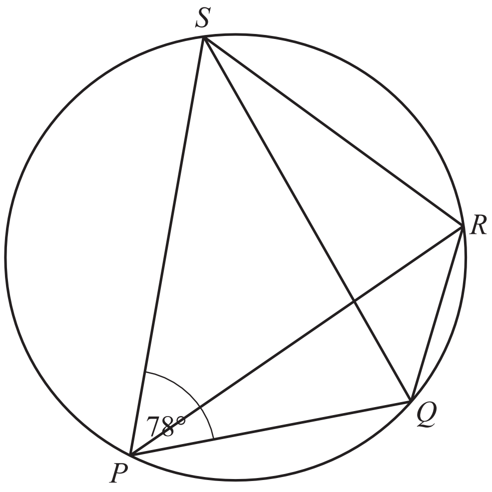
Adding 3y and subtracting 8 from both sides.

$11y = 352$

Dividing by 11.

$\boldsymbol{y = 32^o}$

(c)



NOT TO SCALE

In the cyclic quadrilateral $PQRS$, angle $SPQ = 78°$.

(i) Write down the geometrical reason why angle $QRS = 102°$. [1]

Because in cyclic quadrilaterals, opposite angles sum to 180°

(ii) Angle PRQ : Angle $PRS = 1 : 2$.

Calculate angle PQS. [3]

Here angles PRQ and PRS make up angle QRS, and because we are given the ratio, we know that PRS makes up two thirds of this angle.

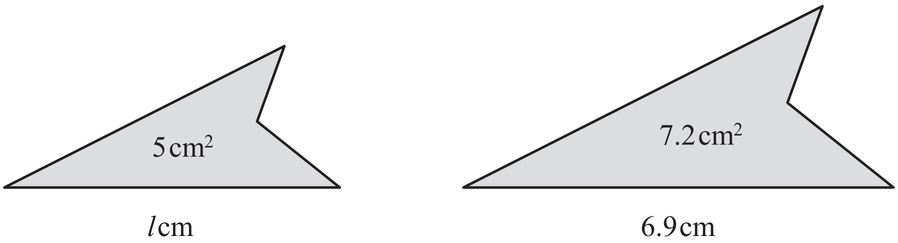
$PRS = \frac{2}{3} * 102$

$= 68^o$

Because triangles PQS and PRS share a chord, angles PRS and PQS must be equal, therefore:

$\boldsymbol{PQS = 68^o}$

(d)



NOT TO SCALE

The diagram shows two similar figures.
The areas of the figures are 5 cm^2 and 7.2 cm^2.
The lengths of the bases are l cm and 6.9 cm.

Calculate the value of l. [3]

First we must find the area scale factor between the two triangles.

$$Area\ scale\ factor = \frac{7.2}{5}$$

$$= 1.44$$

But we need the length scale factor which is the square root of the area scale factor.

$$Length\ scale\ factor = \sqrt{1.44}$$

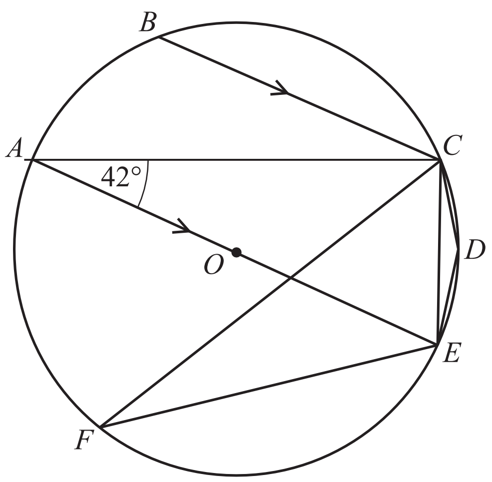
$$= 1.2$$

This means that 6.9cm is 1.2 times bigger than l, hence

$$l = \frac{6.9}{1.2}$$

$$\boldsymbol{l = 5.75cm}$$

(a)



NOT TO SCALE

A, B, C, D, E and F are points on the circumference of a circle centre O.
AE is a diameter of the circle.
BC is parallel to AE and angle $CAE = 42°$.

Giving a reason for each answer, find

(i) angle BCA, [2]

Because of Z angles.

$$BCA = 42$$

(ii) angle ACE, [2]

Because of semicircle

$$ACE = 90$$

(iii) angle CFE, [2]

Because of angles in the same segment

$$CFE = 42$$

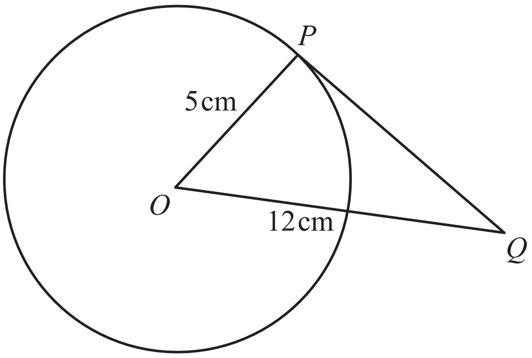
(iv) angle CDE. [2]

Because of cyclic quadrilateral

$$CDE = 180 - 42$$

$$= 138$$

(b)



In the diagram, O is the centre of the circle and PQ is a tangent to the circle at P.
$OP = 5$ cm and $OQ = 12$ cm.

Calculate PQ. [3]

Using Pythagoras'

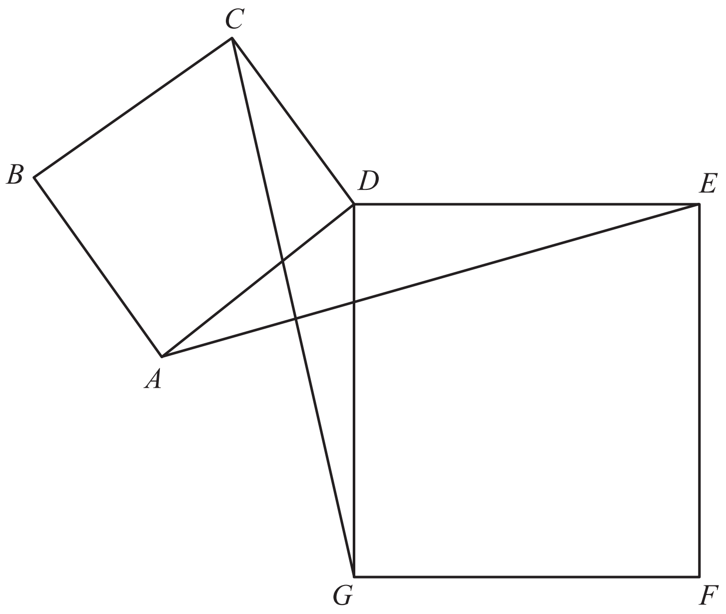
$$PQ^2 + 5^2 = 12^2$$

$$\rightarrow PQ^2 = 144 - 25$$

$$= 119$$

$$\rightarrow \boldsymbol{PQ = 10.9}$$

(c)



NOT TO SCALE

In the diagram, *ABCD* and *DEFG* are squares.

(i) In the triangles *CDG* and *ADE*, explain with a reason which sides and/or angles are equal. [3]

Triangles are mathematically similar so

$$CDG = ADE$$

Also, because they're squares we have

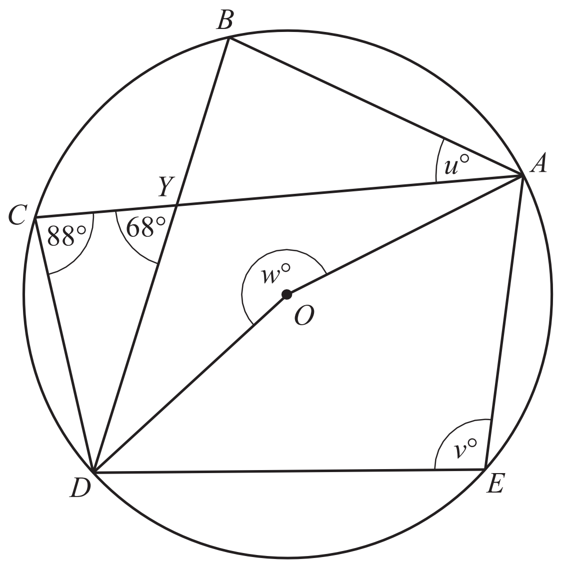
$$AD = CD$$

$$GD = DE$$

(ii) Complete the following statement. [1]

Triangle CDG is **congruent** to triangle ADE

(a)



NOT TO SCALE

A, B, C, D and E lie on the circle, centre O.
CA and BD intersect at Y.
Angle $DCA = 88°$ and angle $CYD = 68°$.
Angle $BAC = u°$, angle $AED = v°$ and reflex angle $AOD = w°$.

[4]

Calculate the values of u, v and w.

From angles in the same segment

$$u = BDC$$

Angles in a triangle sum to 180, hence

$$BDC = 180 - 88 - 68$$

$$\rightarrow \boldsymbol{u = 24}$$

Cyclic quadratic

$$v + 88 = 180$$

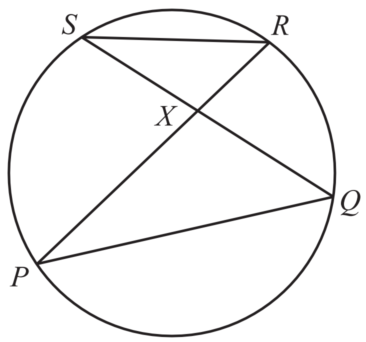
$$\rightarrow \boldsymbol{v = 92}$$

Angle at centre is twice angle on edge

$$w = 2 \times 92$$

$$= \mathbf{184}$$

(b)



NOT TO SCALE

P, Q, R and S lie on the circle. PR and QS intersect at X.
The area of triangle RSX = 1.2 cm^2 and $PX = 3\,SX$.

Calculate the area of triangle PQX. [2]

The length scalar is 3.

If we square the length scale factor, we get the area scale factor.

Hence

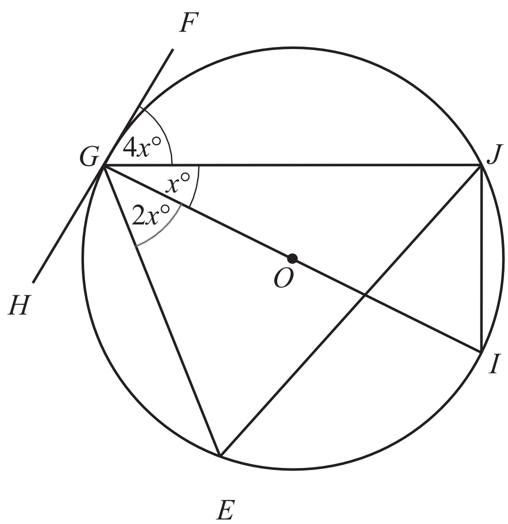
$$A_{PQX} = 3^2 \times 1.2$$

$$= \mathbf{10.8}$$

(c) GI is a diameter of the circle.
FGH is a tangent to the circle at G.
J and K also lie on the circle.
Angle $JGI = x°$, angle $FGJ = 4x°$ and angle $KGI = 2x°$.

Find

(i) the value of x, [2]



NOT TO SCALE

$$4x + x = 90$$

$$\rightarrow x = 90 \div 5$$

$$= \mathbf{18}$$

(ii) the size of angle JKG, [2]

From semi-circle rule we can see that

$$GJI = 90$$

And because angles in a triangle sum to 180

$$x + 90 + JIG = 180$$

$$\rightarrow JIG = 180 - 90 - 18$$

$$= 72$$

Due to angles in same segment

$$JKG = JIG$$

$$\rightarrow \boldsymbol{JKG = 72}$$

(iii) the size of angle GJK. [1]

Angles in a triangle sum to 180.

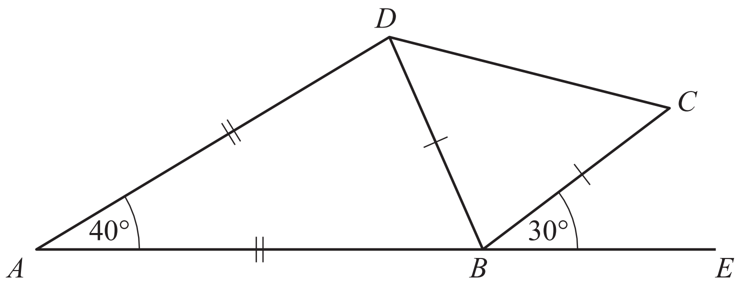
Looking at triangle GKJ

$$3x + 72 + GJK = 180$$

$$\rightarrow GJK = 180 - 72 - 54$$

$$\boldsymbol{= 54}$$

(a)



NOT TO SCALE

ABCD is a quadrilateral with angle *BAD* = 40°.
AB is extended to *E* and angle *EBC* = 30°.
AB = *AD* and *BD* = *BC*.

(i) Calculate angle *BCD*. [3]

AD = AB, therefore the triangle ADB is isosceles.

As a result, the angles ADB and ABD are equal.

The sum of all 3 angles in a triangle is 180°.

We can calculate angle ADB = ABD by using:

180° = 40° + 2 x angle ADB

Angle ADB = 70°.

AE is a straight line, therefore, the sum of the angles: ABD, DBC and CBE is 180°.

Angle BDC = 180° - 70° - 30°

Angle BDC = 80°

BD = BC, therefore, the triangle BDC is isosceles.

The angles BDC and BCD are equal and the sum of all 3

angles in the triangle is 180°.

2 x Angle BCD = 180° - 80°

Angle BCD = 50°

(ii) Give a reason why DC is not parallel to AE. [1]

If the lines were parallel, that would make angles CBE and DCB corresponding angles.

The angles would be equal in this situation.

However, angle CBE = 30° and angle DCB = 50°. The 2 of them are not equal, therefore, the 2 lines, DC and AE, could not be parallel.

(b) A regular polygon has n sides.

Each exterior angle is $\frac{5n}{2}$ degrees.

Find the value of n. [3]

The measure of each exterior angle in a regular polygon with n sides is $\frac{360°}{n}$.

In our case, we also know the size of one exterior angle is $\frac{5n}{2}$.

We obtain the equality:

$$\frac{360°}{n} = \frac{5n}{2}$$

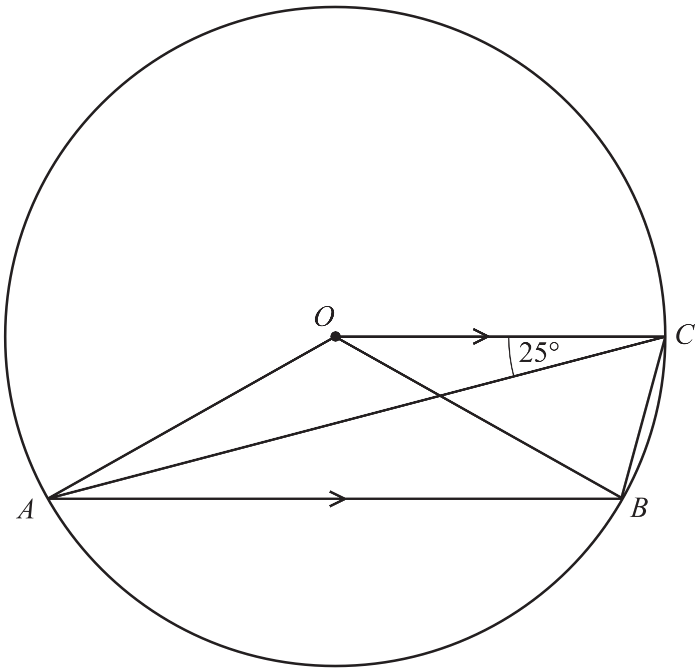
We solve the equality making n the subject.

$5n^2 = 720°$

$n^2 = 144°$

n = 12 sides

(c)



NOT TO SCALE

The diagram shows a circle centre *O*.
A, *B* and *C* are points on the circumference.
OC is parallel to *AB*.
Angle *OCA* = 25°.

Calculate angle *OBC*. [3]

OA = OC as they are both radius in the circle with the centre in O.

Therefore, the triangle OAC is isosceles and the angles OAC is equal to OCA.

The sum of all 3 angles in the triangle OAC is 180°.

180° = 25° x 2 + angle AOC.

Angle AOC = 130°

OC is parallel to AB and OCA and CAB are corresponding angles.

Therefore, OCA = CAB = 25°

Angle OAB = angle CAB + angle CAO

Angle OAB = 2 x 25° = 50°

OA = OB as they are both radius in the circle with the centre in O.

Therefore, the triangle OAB is isosceles and the angles OAB is equal to OBA.

The sum of all 3 angles in the triangle OAB is 180°.

180° = 50° x 2 + angle AOB.

Angle AOB = 80°

Angle COA = angle COB + angle BOA

130° = 80° + angle COB

Angle COB = 50°

OC = OB as they are both radius in the circle with the centre in O.

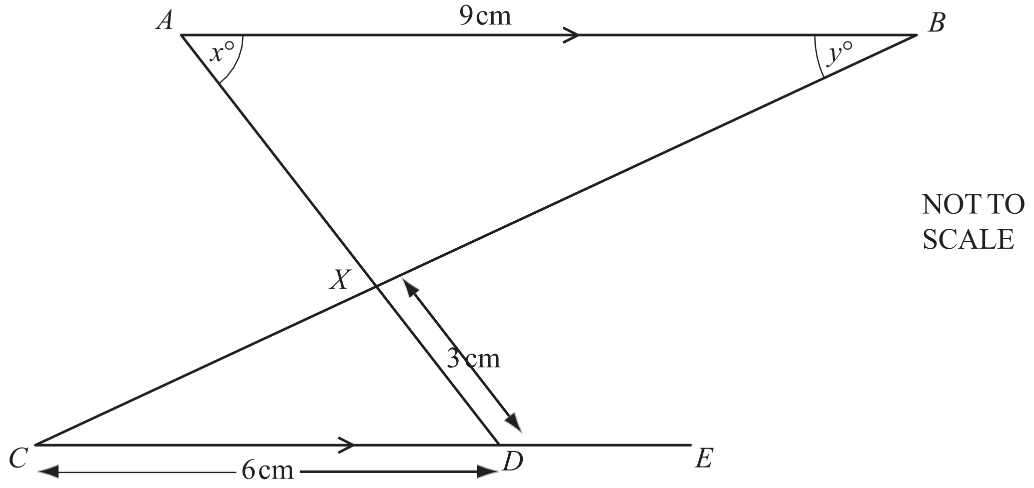
Therefore, the triangle OBC is isosceles and the angles OBC is equal to OCB.

The sum of all 3 angles in the triangle OBC is 180°.

180° = angle OBC x 2 + 50°

Angle OBC = 65°

(a)



The lines AB and CDE are parallel.
AD and CB intersect at X.
$AB = 9$ cm, $CD = 6$ cm and $DX = 3$ cm.

(i) Complete the following statement.

Triangle ABX is .... to triangle DCX. [1]

Similar

(ii) Calculate the length of AX. [2]

ABX is 3/2 times bigger than DCX, hence

$$AX = \frac{3}{2} \times 3$$

$$= \mathbf{\frac{9}{2}}$$

(iii) The area of triangle DCX is 6 cm^2.

Calculate the area of triangle ABX. [2]

Multiply by the length scalar (states in previous answer) squared

$$A = 6 \times \left(\frac{3}{2}\right)^2$$

$$= \mathbf{13.5}$$

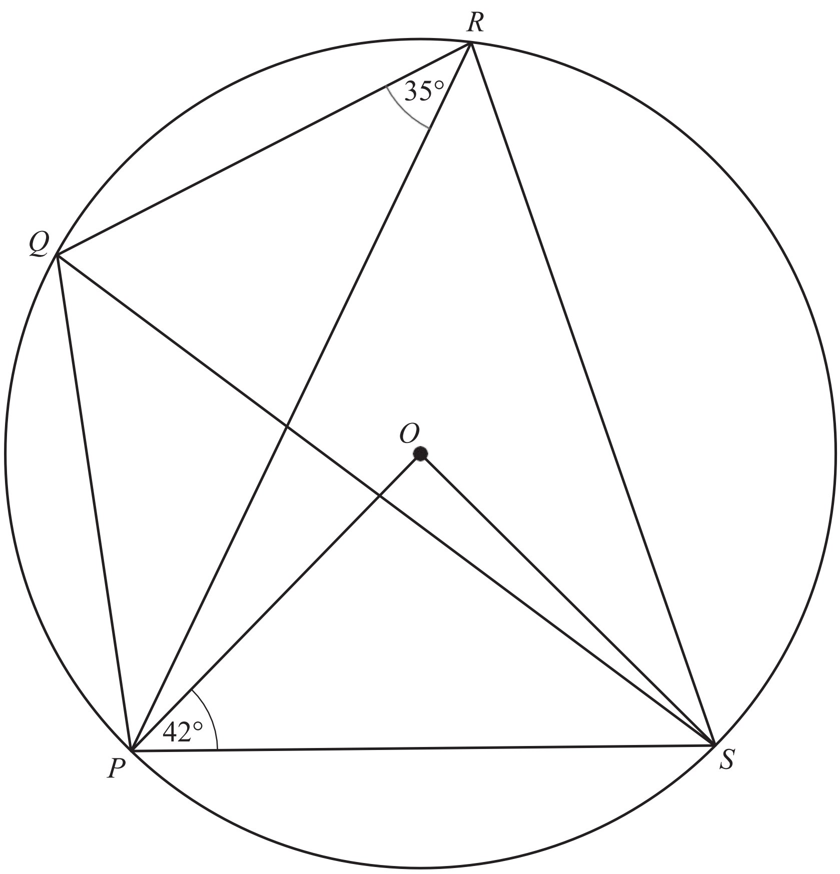
(iv) Angle $BAX = x°$ and angle $ABX = y°$.

Find angle AXB and angle XDE in terms of x and/or y. [2]

$$AXB = 180 - x - y$$

$$XDE = 180 - x$$

(b)



NOT TO SCALE

P, Q, R and S lie on a circle, centre O.
Angle $OPS = 42°$ and angle $PRQ = 35°$.

Calculate

(i) angle POS, [1]

POS is isosceles, hence

$$POS = 180 - 2 \times 42$$

$$= 96$$

(ii) angle PRS, [1]

Angle on edge is half angle in centre, so

$$PRS = \frac{1}{2}POS$$

$$= \mathbf{48}$$

(iii) angle SPQ, [1]

Opposite angles in cyclic quadrilateral sum to 180

$$SPQ + QRS = 180$$

$$\rightarrow SPQ + 35 + 48 = 180$$

$$\rightarrow SPQ = 180 - 83$$

$$= \mathbf{97}$$

(iv) angle PSQ. [1]

Angles in same segment

$$QRS = QSP$$

$$QSP = PSQ$$

$$\rightarrow \boldsymbol{PSQ = 35}$$

(c) The interior angle of a regular polygon is 8 times as large as the exterior angle.

Calculate the number of sides of the polygon. [3]

Let x be the exterior angle and y be the interior angle.

We then have

$x = 180 - y$ (1)

$y = 8x$ (2)

Substituting (2) into (1)

$$\rightarrow x = 180 - 8x$$

$$\rightarrow 9x = 180$$

$$\rightarrow x = 20$$

Each interior angle is 20.

The sum of exterior angles is 360, hence

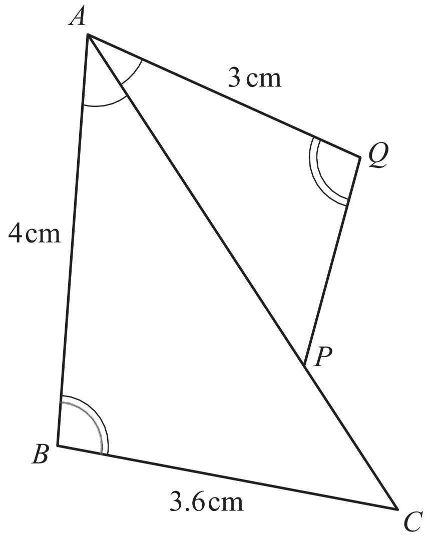
$$20n = 360$$

Where n is the number of sides

$$\rightarrow n = \frac{360}{20}$$

$$= \mathbf{18}$$

(a)



NOT TO SCALE

The diagram shows two triangles ACB and APQ.

Angle PAQ = angle BAC and angle AQP = angle ABC.

AB = 4 cm, BC = 3.6 cm and AQ = 3 cm.

(i) Complete the following statement.

Triangle ACB is .... to triangle APQ. [1]

Triangle ACB is **similar** to triangle APQ

(ii) Calculate the length of PQ. [2]

The two triangles have a length scalar which is equal to

$$AB \div AQ$$

$$= 4 \div 3$$

$$= \frac{4}{3}$$

This means that we have

$$\frac{4}{3} \times PQ = 3.6$$

Divide through by 4/3

$$\rightarrow PQ = 3.6 \div \frac{4}{3}$$

$$= 3.6 \times \frac{3}{4}$$

$$= \mathbf{2.7}$$

(iii) The area of triangle ACB is 5.6 cm^2.

Calculate the area of triangle APQ. [2]

The area scalar is equal to the length scalar squared.

This means that

$$APQ \times \left(\frac{4}{3}\right)^2 = 5.6$$

$$\rightarrow APQ \times \frac{16}{9} = 5.6$$

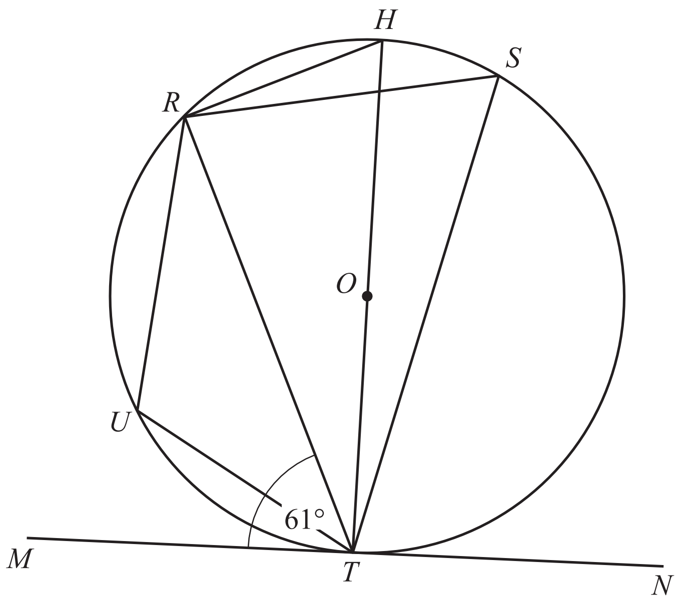
Divide through by 16/9

$$APQ = 5.6 \div \frac{16}{9}$$

$$= 5.6 \times \frac{9}{16}$$

$$= \mathbf{3.15}$$

(b)



NOT TO SCALE

R, H, S, T and U lie on a circle, centre O.
HT is a diameter and MN is a tangent to the circle at T.
Angle $RTM = 61°$.

Find

(i) angle RTH, [1]

$$61 + RTH = 90$$

$$\rightarrow \boldsymbol{RTH = 29}$$

(ii) angle RHT, [1]

Angles in same segment

$$\boldsymbol{RHT = 61}$$

(iii) angle RST, [1]

Angles in same segment

$$\boldsymbol{RST = 61}$$

(iv) angle RUT. [1]

Opposite angles in cyclic quadrilateral add to 180

$$RUT + 61 = 180$$

$$\rightarrow \boldsymbol{RUT = 119}$$

(c) $ABCDEF$ is a hexagon.
The interior angle B is 4° greater than interior angle A.
The interior angle C is 4° greater than interior angle B, and so on, with each of the next interior angles 4° greater than the previous one.

(i) By how many degrees is interior angle F greater than interior angle A? [1]

We have that

$$B = A + 4$$

$$C = B + 4 = A + 8$$

$$\ldots$$

$$F = A + 5 \times 4$$

$$= A + 20$$

So, the answer is

$$\mathbf{20}$$

(ii) Calculate interior angle A. [3]

We have that the sum of the angles must be 720 since

$$sum = (n-2) \times 180$$

$$= 4 \times 180$$

$$= 720$$

Adding all the angles together (bearing in mind that each is 4 greater than the last) gives us

$$(A) + (A+4) + (A+8) + (A+12) + (A+16) + (A+20)$$
$$= 720$$

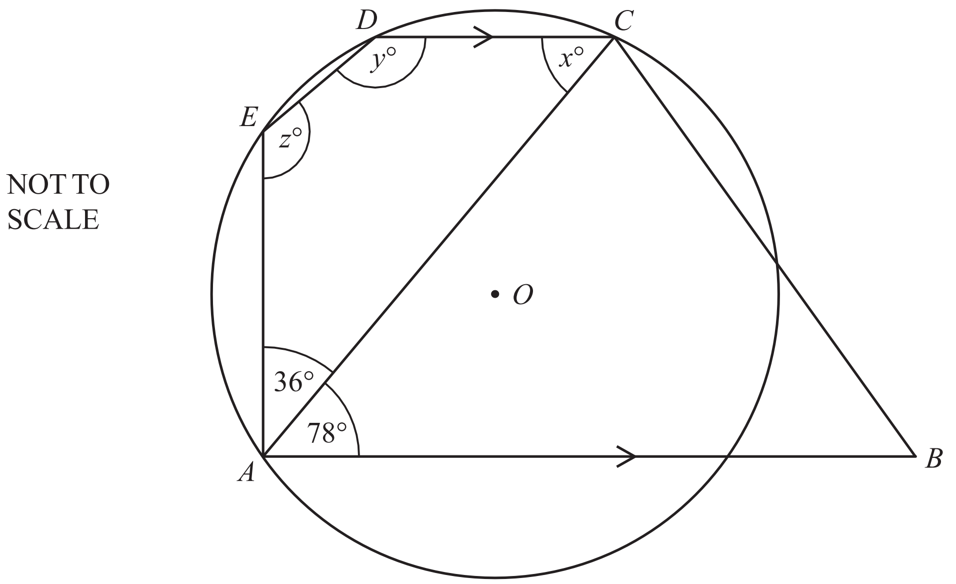
$$\rightarrow 6A + 60 = 720$$

Subtract 60 from both sides

$$6A = 660$$

Divide through by 6

$$\boldsymbol{A = 110}$$



ABCDE is a pentagon.
A circle, centre *O*, passes through the points *A*, *C*, *D* and *E*.
Angle *EAC* = 36°, angle *CAB* = 78° and *AB* is parallel to *DC*.

(a) Find the values of *x*, *y* and *z*, giving a reason for each. [6]

Due to Z angles

$$\boldsymbol{x = 78}$$

Because of opposite angles in a cyclic quadrilateral

$$y = 180 - 36$$

$$\rightarrow \boldsymbol{y = 144}$$

$$z = 180 - 78$$

$$\rightarrow \boldsymbol{z = 102}$$

(b) Explain why *ED* is **not** parallel to *AC*. [1]

$$\boldsymbol{z + 36 = 138}$$

Since this **does not equal 180**, the lines are not parallel.

(c) Find the value of angle *EOC*. [1]

$$EOC = 2 \times 36$$

$$= \mathbf{72}$$

(d) *AB* = *AC*.
Find the value of angle *ABC*. [1]

Angles in a triangle sum to 180.

ABC is isosceles so

$$ABC = ACB$$

$$\rightarrow ABC = \frac{1}{2}(180 - 78)$$

$$= \mathbf{51}$$