

6. Trigonometry

YOUR NOTES



CONTENTS

6.1 Bearings

6.1.1 Bearings & Scale

6.2 2D Pythagoras & SOHCAHTOA

6.2.1 Pythagoras Theorem

6.2.2 SOHCAHTOA

6.3 Trigonometric Graphs & Equations

6.3.1 Drawing Graphs – Trig Graphs

6.4 Sine & Cosine Rule

6.4.1 Sine & Cosine Rules, Area of Triangle – Basics

6.4.2 Sine & Cosine Rules, Area of Triangle – Harder

6.5 3D Pythagoras & SOHCAHTOA

6.5.1 3D Pythagoras & SOHCAHTOA

6.1 BEARINGS

6.1.1 BEARINGS & SCALE

What are bearings?

- **Bearings** (sometimes referred to as three-figure bearings) are a way of describing and using angles
- The way they are defined means it gives us a precise **location** and/or **direction** which means they have good uses in navigation

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YOUR NOTES



What do I need to know?

Bearings have 3 rules:

1. **They are measured from the North direction**

North is usually straight up in terms of a scale drawing or map drawn on a piece of paper and should be shown somewhere on the diagram

2. **They are measured anti-clockwise (from North)**

If you get muddled up look at a clock on the wall

3. **The angle should always be written (said) with 3 figures**

So angles under 100° should have zeros to start, eg 059, 008

Notice also that the degree symbols are not usually included when talking about bearings



Exam Tip

Make sure you have all the equipment you need for your maths exams, along with a spare pen and pencil. Rubber and pencil sharpener can be essential on these questions as they are all about **accuracy**. Make sure you have compasses that aren't loose and wobbly, make sure you can see and read the markings on your ruler and protractor.

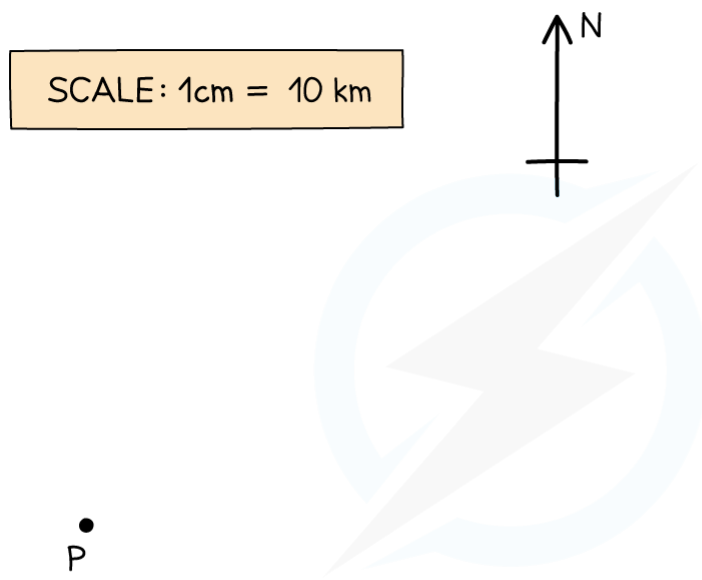
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Worked Example

A ship sets sail from point P, as shown on the map below. It sails on a bearing of 105° until it reaches point Q, 70 km. The ship then changes path, sailing on a bearing of 065° for a further 35 km where its journey finishes. Show on the map below the point Q and the final destination of the ship.



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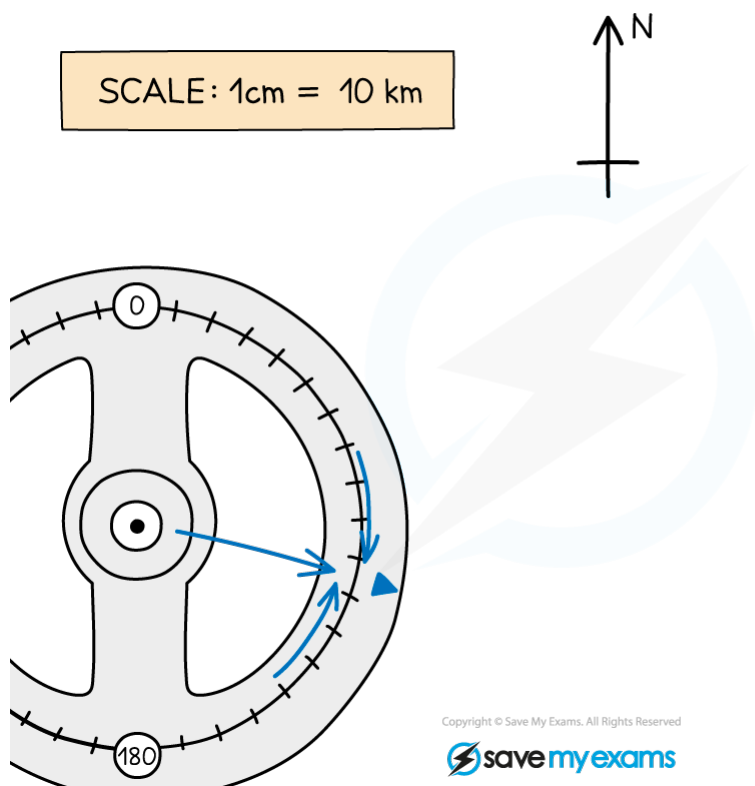
1, 2 – The first thing to do is to measure an angle of 105° from the north direction, anticlockwise

Draw in a north line at P

Be accurate and make a visible mark on your map but avoid big 'blobs'

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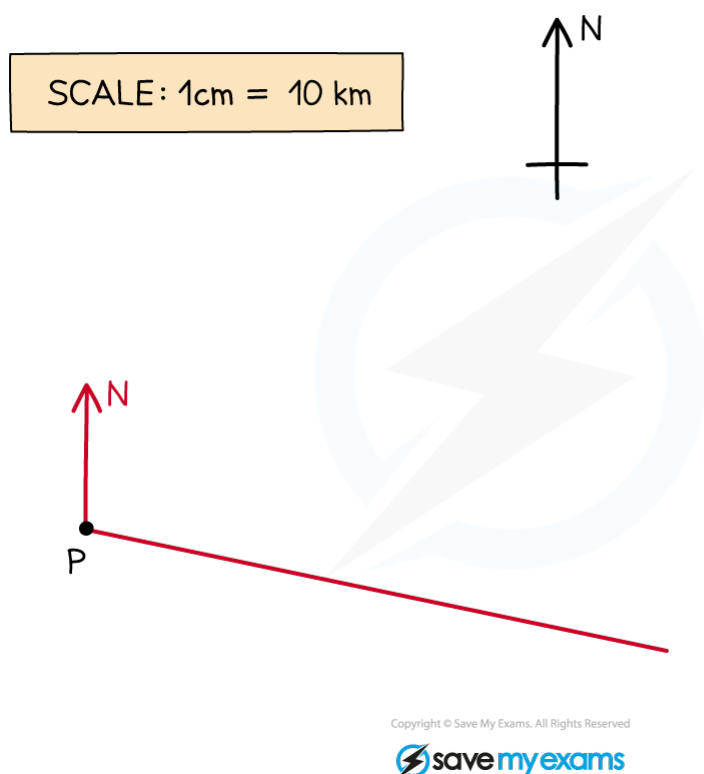
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Now draw a line from P through your angle mark - make this line longer than you'll (probably need it) as in the next stage we need to measure along it accurately

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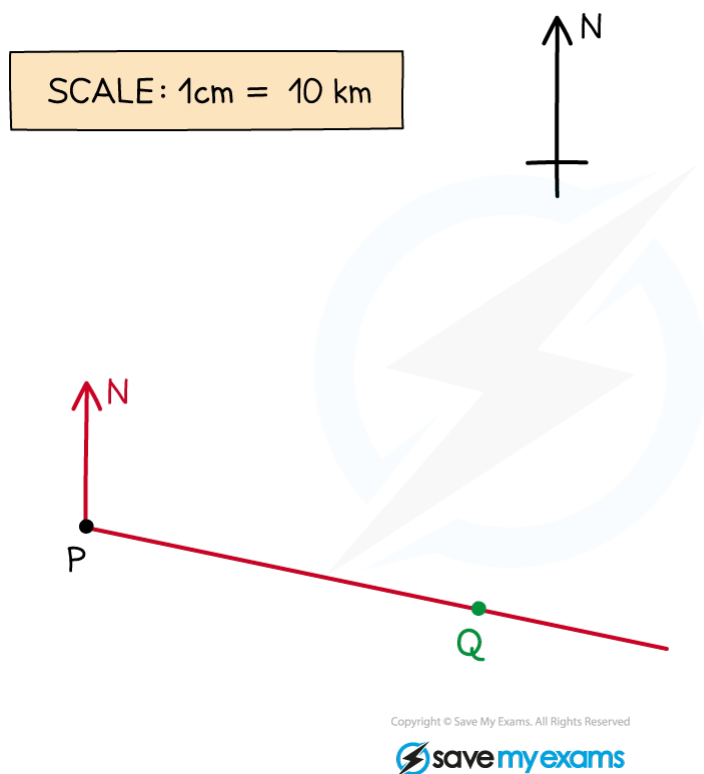
$$70 \text{ km} = 7 \text{ cm}$$

Using the scale 1 cm = 10 km

Measure 7 cm accurate and make a mark on your line, and this will be point Q

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YOUR NOTES



1, 2, 3 – Do not be put off by the 065, it is just 65° as a three-figure bearing

As before measure this anti-clockwise from your point Q and make a mark

Draw a line through Q and your mark

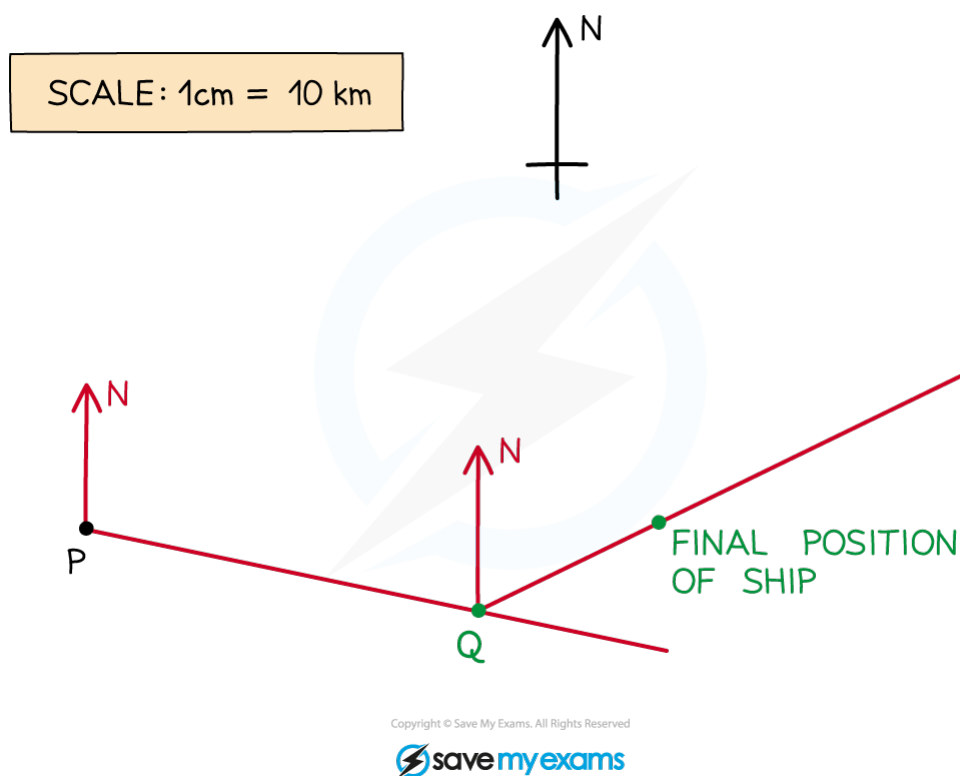
To find the final position of the ship we need to measure along this last line.

35 km = 3.5 cm

Using the scale as before

6. Trigonometry

YOUR NOTES



6. Trigonometry

YOUR NOTES



6.2 2D PYTHAGORAS & SOHCAHTOA

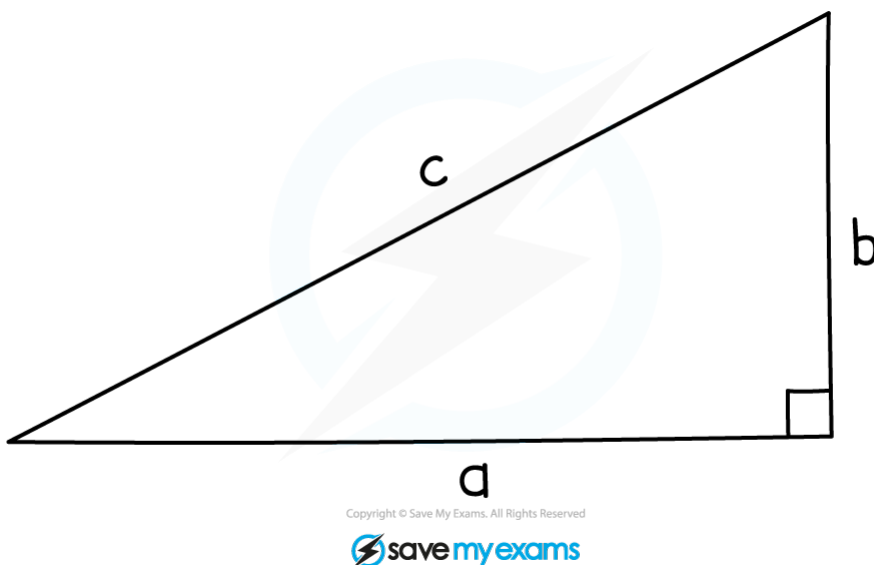
6.2.1 PYTHAGORAS THEOREM

What is Pythagoras?

- Pythagoras was a Greek mathematician who lived over 2500 years ago
- He is most famous for Pythagoras' Theorem, which includes the important formula for right-angled triangles

Pythagoras' theorem?

- The longest side in a right-angled triangle is called the **hypotenuse** - this will always be the side opposite the right angle
- If we label the hypotenuse c , and label the other two sides a and b :



then Pythagoras' Theorem tells us that:

$$a^2 + b^2 = c^2$$

(where a , b , and c are the lengths of the three sides)

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Using Pythagoras' theorem

- To find the length of the **hypotenuse** use:

$$c = \sqrt{a^2 + b^2}$$

(Note that when finding the hypotenuse you **add** inside the square root)

- To find the length of **one of the other sides** use:

$$a = \sqrt{c^2 - b^2} \text{ or } b = \sqrt{c^2 - a^2}$$

Note that when finding one of the other sides you **subtract** inside the square root



Exam Tip

'Reality check' your answers. If the hypotenuse ends up being shorter than another side in your answer, you've made a mistake somewhere!

If your calculator gives you a 'Math ERROR' result, you probably subtracted the wrong way around in the square root.

In questions with multiple steps, don't round off (to 1 decimal place, 3 significant figures, etc) until the very end.

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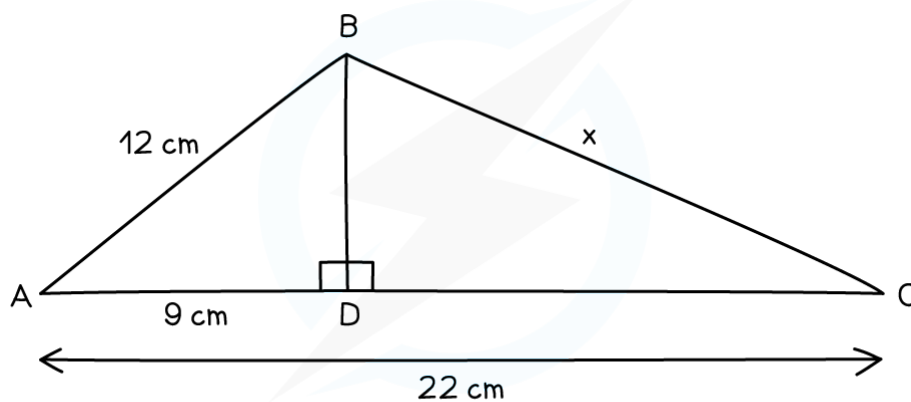


Worked Example

In the following diagram:

$$AB = 12 \text{ cm}$$

AC is a straight line, with $AD = 9 \text{ cm}$ and $AC = 22 \text{ cm}$



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Find x , the length of side BC . Give your answer to 1 d.p.

To find x , we first need to find the length of BD using triangle ABD :

$$2. \quad BD = \sqrt{12^2 - 9^2} = \sqrt{63} = 7.93725\dots$$

Now we can find x using triangle BCD :

$$DC = 22 - 9 = 13 \text{ cm}$$

$$1. \quad x = \sqrt{BD^2 + DC^2} = \sqrt{(\sqrt{63})^2 + 13^2} = \sqrt{63 + 169}$$

$$x = \sqrt{232} = 15.2 \text{ (to 1 d.p.)}$$

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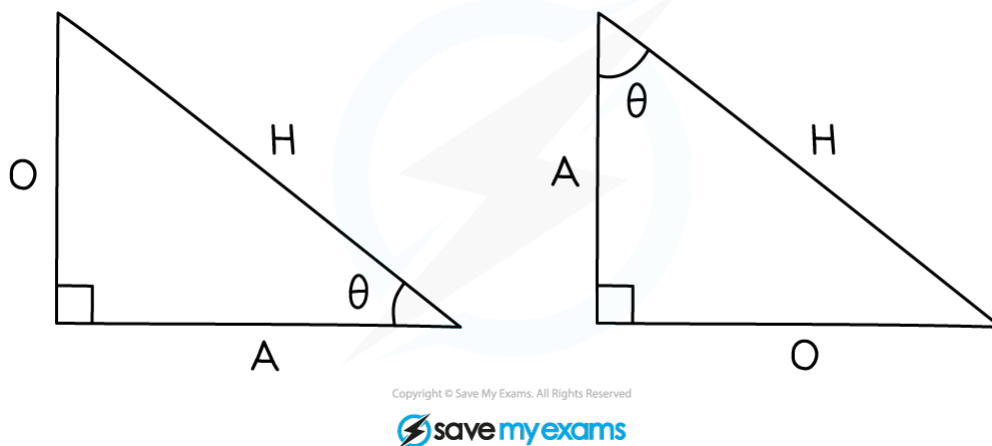
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6.2.2 SOHCAHTOA

What is SOHCAHTOA?

- SOHCAHTOA is a mnemonic – ie a way to remember the 3 important formulas for sin (sine), cos (cosine), and tan (tangent)
- In a right-angled triangle, label one angle (NOT the right angle!), and label the sides of the triangles as follows:



- Note that:
 - θ = Greek letter theta
 - O** = opposite
 - A** = adjacent
 - H** = hypotenuse
 - 'H' is always the same, but 'O' and 'A' depend on which angle we're calling θ
- Using those labels, the three **SOHCAHTOA** equations are:

$$\text{SOH: } \sin \theta = \frac{O}{H}$$

$$\text{CAH: } \cos \theta = \frac{A}{H}$$

$$\text{TOA: } \tan \theta = \frac{O}{A}$$

6. Trigonometry

YOUR NOTES



Using SOHCAHTOA

1. **Label the triangle:** the angle you know (or want to know) is ' θ ' — 'O' and 'A' follow from that
Choose SOH, CAH, or TOA
The letters of the sides you're interested in (ie. know or want to know) tell you which to choose
2. **To find a side:** put and the side you know into the equation from 2, and solve to find the side you don't know
3. **To find an angle:** use the inverse trig functions on your calculator:

$$\theta = \sin^{-1} \left(\frac{O}{H} \right) \quad \text{or} \quad \theta = \cos^{-1} \left(\frac{A}{H} \right) \quad \text{or} \quad \theta = \tan^{-1} \left(\frac{O}{A} \right)$$



Exam Tip

SOHCAHTOA (like Pythagoras) can only be used in right-angles triangles – for triangles that are not right-angled, you will need to use the Sine Rule or the Cosine Rule.

Also, make sure your calculator is set to measure angles in degrees.

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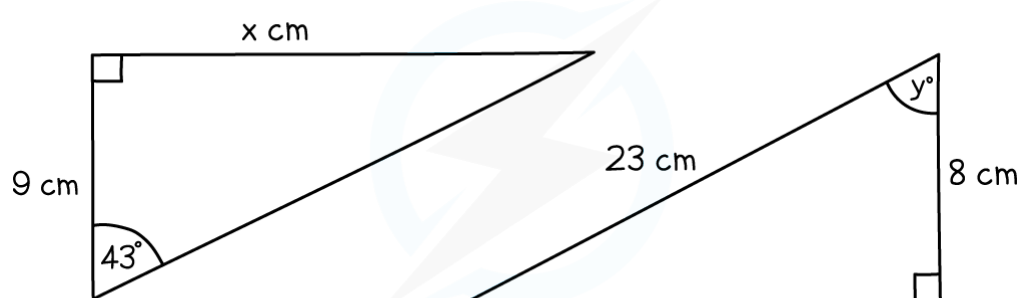
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Worked Example

Find the values of x and y in the following triangles.

Give your answers to 3 significant figures.

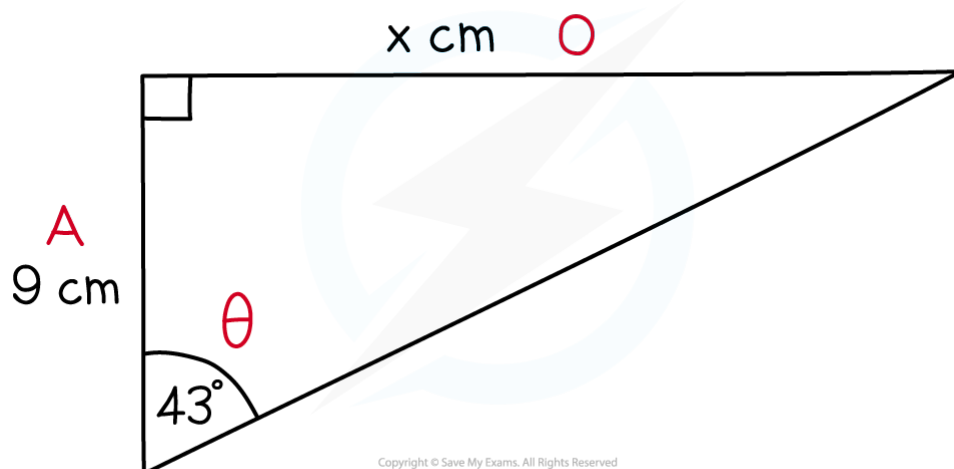


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To find x :

1. Label the triangle



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2. We know A and we want to know O – that's **TOA**.

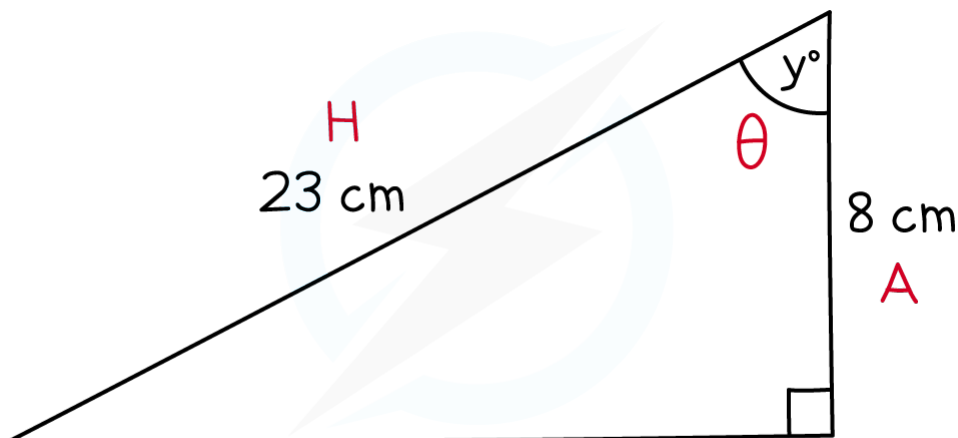
3. $\tan 43 = \frac{x}{9}$

$$x = 9 \times \tan 43 = 8.3926\dots$$

$$x = 8.39 \text{ cm (3 sig. figs.)}$$

To find y:

1. Label the triangle



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2. We know A and H – that's **CAH**.

4. $y = \cos^{-1} \left(\frac{8}{23} \right) = 69.6455\dots$

$$y = 69.6^\circ \text{ (3 sig. figs.)}$$

6. Trigonometry

YOUR NOTES



6.3 TRIGONOMETRIC GRAPHS & EQUATIONS

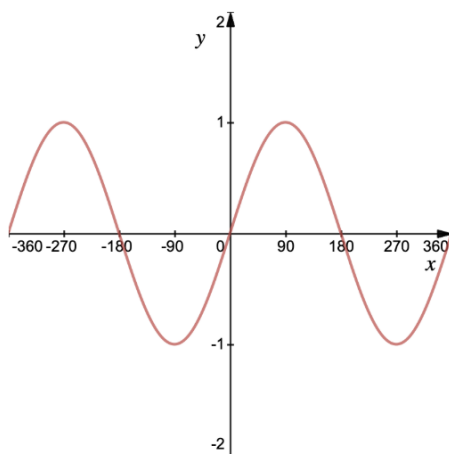
6.3.1 DRAWING GRAPHS - TRIG GRAPHS

Why do we need to know what trigonometric graphs look like?

- Trigonometric Graphs are used in various applications of mathematics – for example, the oscillating nature can be used to model how a pendulum swings

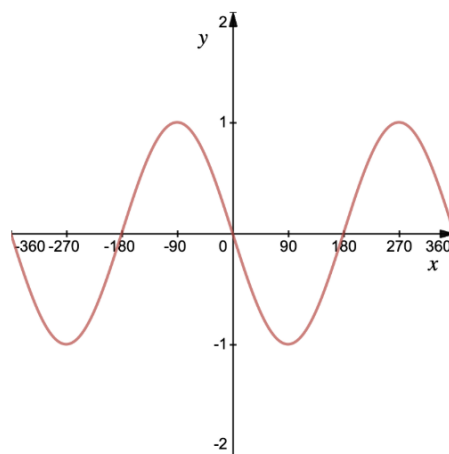
Drawing graphs – trig graphs

- As with other graphs, being familiar with the general style of trigonometric graphs will help you sketch them quickly and you can then use them to find values or angles alongside your calculator
- All trigonometric graphs follow a pattern – a “starting point” and then “something happens every 90° ”.



$y = \sin x$

Starts at $(0, 0)$
Every 90° it cycles
through $1, 0, -1, 0, \dots$

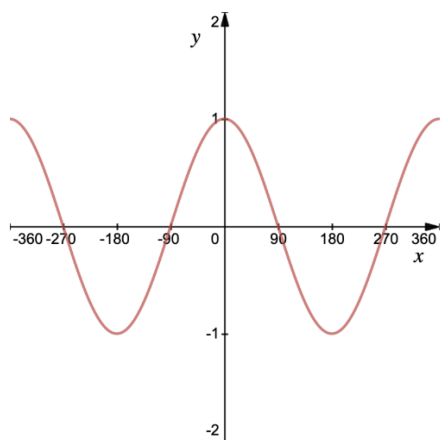


$y = -\sin x$

Learn how this
relates to $y = \sin x$

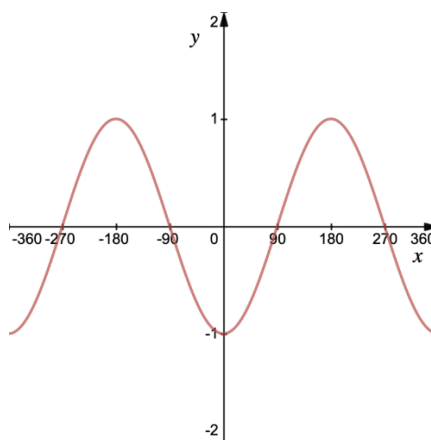
6. Trigonometry

YOUR NOTES



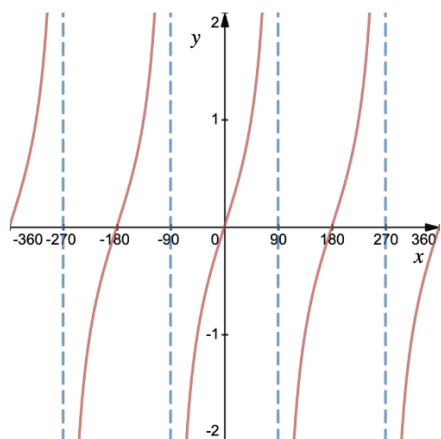
$y = \cos x$

Starts at $(0, 1)$
Every 90° it cycles
through $0, -1, 0, 1, \dots$



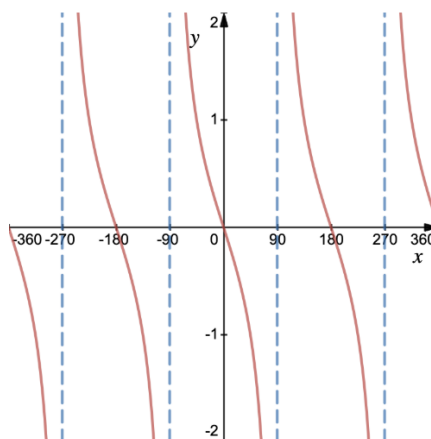
$y = -\cos x$

Learn how this
relates to $y = \cos x$



$y = \tan x$

Starts at $(0, 0)$
Every 90° it is either
 0 or an asymptote (*)



$y = -\tan x$

Learn how this
relates to $y = \tan x$

- (*) An asymptote is a line that a graph gets ever closer to without ever crossing or touching it

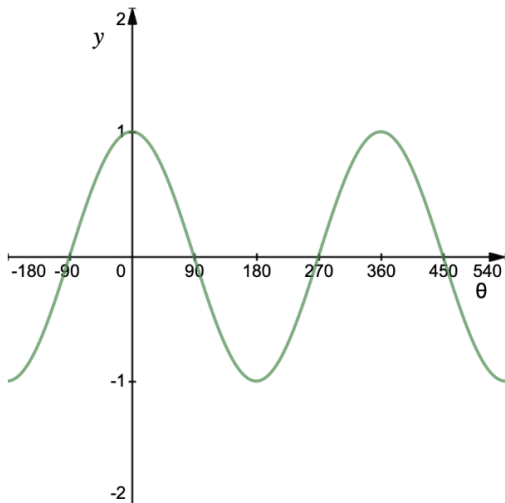
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YOUR NOTES



1. Sketch the graph of $y = \cos \theta$ for $-180^\circ \leq \theta \leq 540^\circ$.

*Do not be put off by the use of θ (theta), it is a Greek letter often used in mathematics to denote angles
Okay we have the positive cos graph, so we start at (0, 1)
and cycle round 0, -1, 0, 1, 0, -1, 0, etc*



Note that it is easier to deal with the positive angles first, then work backwards from (0, 1) to deal with the negative angles

It is easy to see that -180° is linked to "every 90° " but not so obvious that 540° is - just count up in 90° steps until you reach this value or past it.

6. Trigonometry

YOUR NOTES



6.4 SINE & COSINE RULE

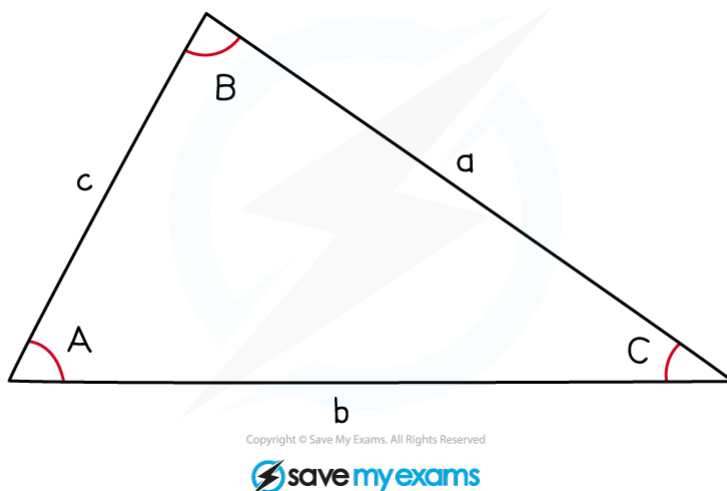
6.4.1 SINE & COSINE RULES, AREA OF TRIANGLE - BASICS

What are the sine & cosine rules, & the area of a triangle formula?

- Remember, **Pythagoras** and **SOHCAHTOA** only work in right-angled triangles
- The **Sine Rule**, **Cosine Rule**, and **Area of a Triangle Formula** allow us to answer triangle questions for ANY triangle

The sine rule, cosine rule, & area of a triangle formula

- In discussing these formulas, we usually label our triangle like this:



- Note: lowercase letters for side lengths, capital letters for angles — and make sure an angle and the side opposite it have the same letter
- The **Sine Rule** tells us that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$\sin 90^\circ = 1$ – so if one of the angles is 90° , this becomes ‘SOH’ from SOHCAHTOA

- The **Cosine Rule** tells us that:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

6. Trigonometry

YOUR NOTES



$\cos 90^\circ = 0$ so if $A = 90^\circ$, this becomes Pythagoras' Theorem

- **The Area of a Triangle Formula tells us that:**

$\text{Area} = \frac{1}{2} ab \sin C$

$\sin 90^\circ = 1$ so if $C = 90^\circ$, this becomes $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

- The **Sine Rule** can also be written 'flipped over':

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- This is more useful when we are using the rule to find angles
- These two versions of the **Cosine Rule** are also valid for the triangle above:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Note that it's always the angle between the two sides in the final term

- These two versions of the **Area of a Triangle Formula** are also valid for the triangle above:

$$\text{Area} = \frac{1}{2} ac \sin B$$

$$\text{Area} = \frac{1}{2} bc \sin A$$

Note that it's always the angle between the two sides on the right-hand side –you can always just relabel the sides and angles of your triangle, though, instead of worrying about alternate versions of the last two formulas

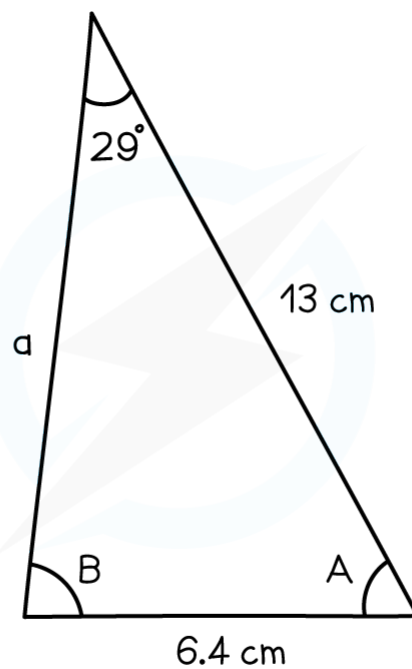
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Worked Example

In the following triangle, all the angles are acute.



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- Use the Sine Rule to find the size of angle B .
- Use the Cosine Rule to find the size of side a .
- Find the area of the triangle.

Give your answers to 3 significant figures.

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YOUR NOTES



- a) Using the 'flipped over' version of the Sine Rule formulas:

$$\frac{\sin B}{13} = \frac{\sin 29}{6.4}$$

$$\sin B = 13 \times \frac{\sin 29}{6.4} = 0.98476\dots$$

$$B = \sin^{-1} \left(13 \times \frac{\sin 29}{6.4} \right) = 79.9873\dots$$

$$B = 80.0^\circ \text{ (3 sig. figs.)}$$

- b) If $B = 80.0^\circ$, then $A = 180 - 29 - 80.0 = 71.0^\circ$ (3 sig. figs.)

Now using the Cosine Rule:

$$a^2 = 13^2 + 6.4^2 - 2 \times 13 \times 6.4 \times \cos 71.0 = 155.7854\dots$$

$$a = \sqrt{13^2 + 6.4^2 - 2 \times 13 \times 6.4 \times \cos 71.0} = 12.4814\dots$$

$$a = 12.5 \text{ cm (3 sig. figs.)}$$

- c) Using the Area of a Triangle Formula:

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\text{Area} = \frac{1}{2} \times 12.5 \times 13 \times \sin 29 = 39.3907\dots$$

$$\text{Area} = 39.4 \text{ cm}^2 \text{ (3 sig. figs.)}$$

6. Trigonometry

YOUR NOTES



6.4.2 SINE & COSINE RULES, AREA OF TRIANGLE - HARDER

Choosing which rule or formula to use

- It is important to be able to decide which Rule or Formula to use to answer a question
- This table summarises the possibilities:

If you know	And you want to know	Use
Two sides and the angle opposite one of the sides	The angle opposite the other side	Sine Rule
Two angles and the side opposite one of the angles	The side opposite the other angle	Sine Rule
Two sides and the angle between them	The third side	Cosine Rule
All three sides	Any angle	Cosine Rule
Two sides and the angle between them	The area of the triangle	Area of a Triangle Formula

Using the cosine rules to find angles

The **Cosine Rule** can be rearranged to give:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

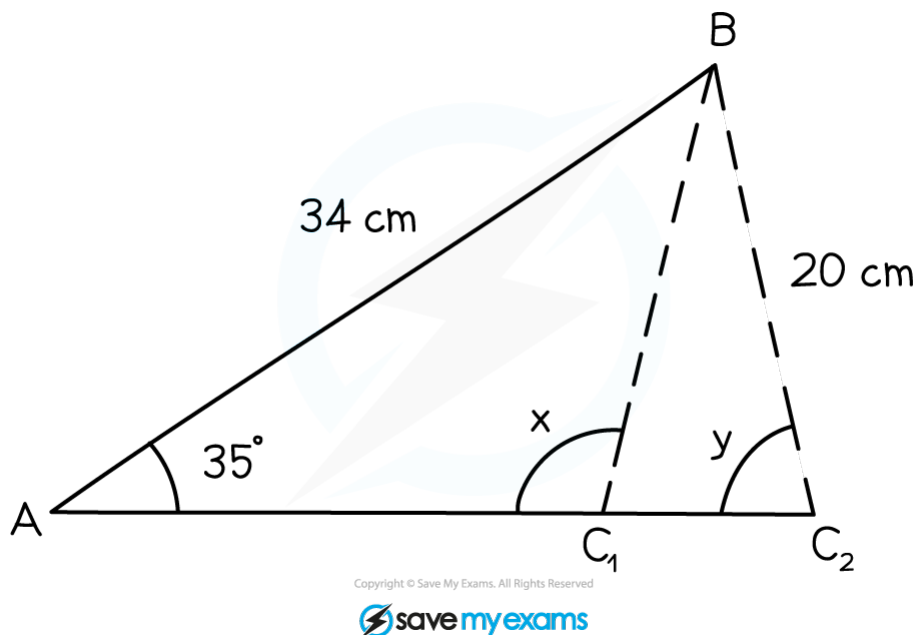
Using the inverse cosine function (ie \cos^{-1}) we can use this to find the size of angle A :

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

This form of the formula is not on your exam formula sheet, so make sure you can do the rearrangement yourself!

6. Trigonometry

YOUR NOTES



If all we know is the lengths of AB and BC , and the size of angle BAC , there are two possible triangles that could be drawn -- one with side BC_1 (and angle $x = 102.8^\circ$), the other with side BC_2 (and angle $y = 77.2^\circ$). Using your calculator and the **Sine Rule** would only find you the possibility with angle y .



Exam Tip

In more involved exam questions, you may have to use both the Cosine Rule and the Sine Rule over several steps to find the final answer.

If your calculator gives you a 'Maths ERROR' message when trying to find an angle using the Cosine Rule, you probably subtracted things the wrong way around when you rearranged the formula.

The Sine Rule can also be written 'flipped over':

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

This is more useful when we are using the rule to find angles

When finding angles with the Sine Rule, use the info in the question to decide whether you have the acute angle case (ie the calculator value) or the obtuse angle case (ie, minus the calculator value).

The Cosine Rule will never give you an ambiguous answer for an angle - as long as you put the right things into the calculator, the answer that comes out will be the correct angle

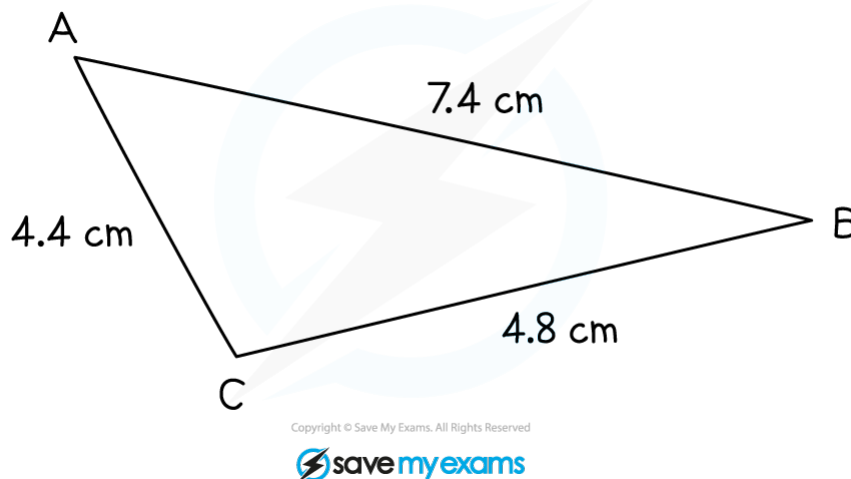
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YOUR NOTES



Worked Example

In the following triangle:



- a) Find the size of angle ABC.
 b) Given that angle ACB is obtuse, use the Sine Rule and your answer from (a) to find the size of angle ABC.
 Give your answers accurate to 1 d.p.

- a) We know all three side lengths and want to find one of the angles – that's a job for the Cosine Rule:

$$4.4^2 = 7.4^2 + 4.8^2 - 2 \times 7.4 \times 4.8 \times \cos(ABC)$$

$$\cos(ABC) = \frac{7.4^2 + 4.8^2 - 4.4^2}{2 \times 7.4 \times 4.8}$$

$$\text{angle } ABC = \cos^{-1}\left(\frac{7.4^2 + 4.8^2 - 4.4^2}{2 \times 7.4 \times 4.8}\right) = 34.6505 \dots$$

$$\text{angle } ABC = 34.7^\circ \text{ (1 d.p.)}$$

- b) Using the 'flipped over' version of the Sine Rule formulas:

$$\frac{\sin ACB}{7.4} = \frac{\sin 34.6505}{4.4}$$

Note that we used the unrounded answer for ABC there, to avoid rounding errors!

$$\sin ACB = 7.4 \times \frac{\sin 34.6505}{4.4}$$

$$ACB = \sin^{-1}\left(7.4 \times \frac{\sin 34.6505}{4.4}\right) = 72.9853$$

But we have been told that ACB is obtuse, so:

$$ACB = 180 - 72.9853 = 107.0147$$

$$\text{angle } ACB = 107.0^\circ \text{ (1 d.p.)}$$

6. Trigonometry

YOUR NOTES



6.5 3D PYTHAGORAS & SOHCAHTOA

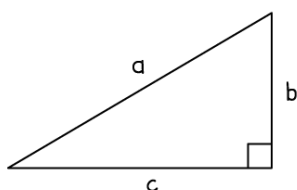
6.5.1 3D PYTHAGORAS & SOHCAHTOA

What are Pythagoras & SOHCAHTOA?

- **Pythagoras' Theorem** helps us find missing side lengths of a right-angled triangle
- It is also frequently used for finding the distance (or length) of a line

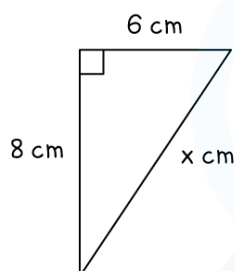
PYTHAGORAS' THEOREM

RIGHT-ANGLED TRIANGLES



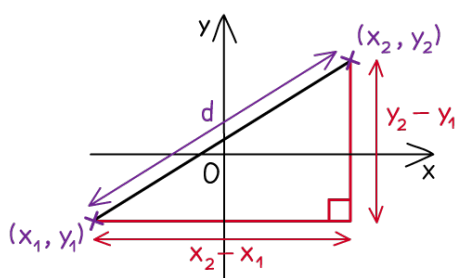
$$a^2 = b^2 + c^2$$

e.g. FIND THE VALUE OF x



$$\begin{aligned} x^2 &= 6^2 + 8^2 \\ x^2 &= 36 + 64 \\ x^2 &= 100 \\ x &= \sqrt{100} \\ x &= 10 \end{aligned}$$

DISTANCE OF A LINE



$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

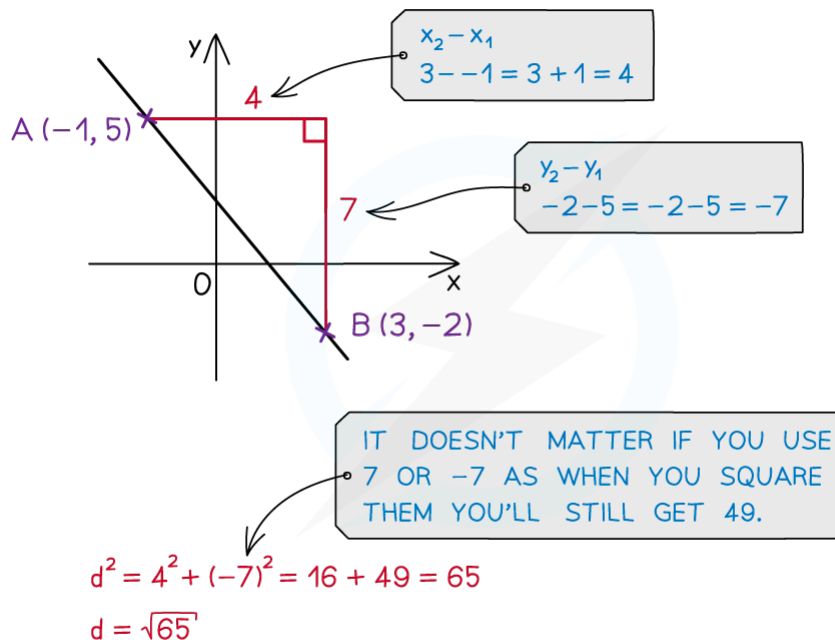
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e.g. FIND THE DISTANCE OF THE LINE AB
IN THE DIAGRAM BELOW



LENGTH OF THE LINE AB IS 8.1
TO ONE DECIMAL PLACE

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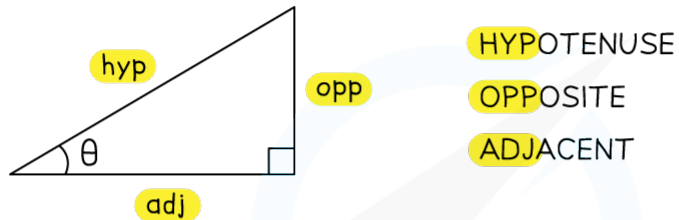
- **SOHCAHTOA** is an acronym for the three trigonometric ratios that connect angles (θ) and sides (Opposite, Hypotenuse and Adjacent) in a right-angled triangle
 - Sine - SOH - $\sin \theta = O \div H$
 - Cosine - CAH - $\cos \theta = A \div H$
 - Tangent - TOA - $\tan \theta = O \div A$

6. Trigonometry

YOUR NOTES



SOHCAHTOA



$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

SOH



CAH



TOA



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e.g. FIND THE VALUE OF x

NOT GIVEN
NOT WANTED

SOH
 $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

$\sin 41^\circ = \frac{x}{8}$

$x = 8 \sin 41^\circ$

$x = 5.25 \text{ cm} \quad (\text{TO } 2 \text{ DP})$

MAKE SURE
CALCULATOR
IS IN DEGREES
MODE

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6. Trigonometry

YOUR NOTES

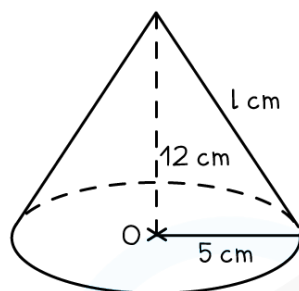


How does Pythagoras work in 3D?

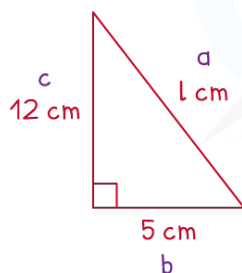
- 3D shapes can often be broken down into several 2D shapes
 - For example nets and surface area
- With **Pythagoras' Theorem** problems you will be specifically looking for right-angled triangles
 - The right-angled triangles you need will have two known sides and one unknown side

PYTHAGORAS' THEOREM IN 3D

e.g. A CONE HAS BASE RADIUS 5 cm AND A PERPENDICULAR HEIGHT OF 12 cm.



FIND THE LENGTH, l cm, OF THE SLANTED HEIGHT OF THE CONE



DRAW THE 2D TRIANGLE OUT TO MAKE IT EASIER TO SEE.

$$a^2 = b^2 + c^2$$

$$l^2 = 5^2 + 12^2$$

$$l^2 = 25 + 144$$

$$l^2 = 169$$

$$l = 13 \text{ cm}$$

USE PYTHAGORAS' THEOREM TO SOLVE THE PROBLEM.

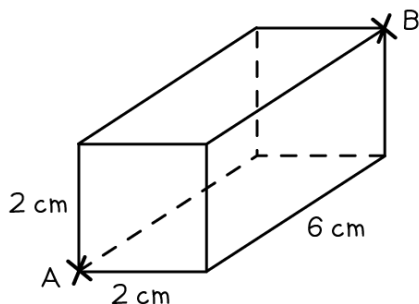
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6. Trigonometry

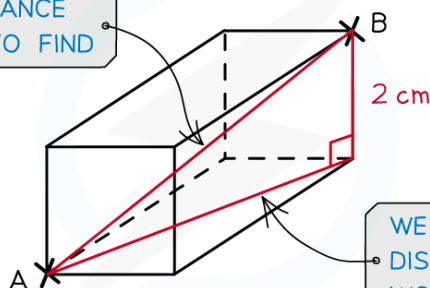
YOUR NOTES



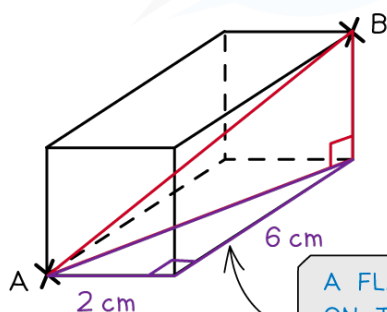
e.g. FIND THE DISTANCE BETWEEN A AND B.



THIS IS THE DISTANCE WE ARE TRYING TO FIND



WE DON'T KNOW THIS DISTANCE BUT CAN WORK IT OUT

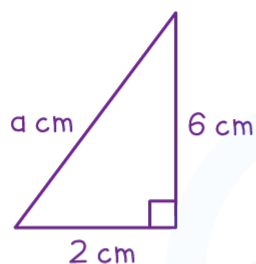


A FLAT RIGHT-ANGLED TRIANGLE ON THE BASE ALLOWS US TO FIND THE MISSING LENGTH OF THE REAL TRIANGLE

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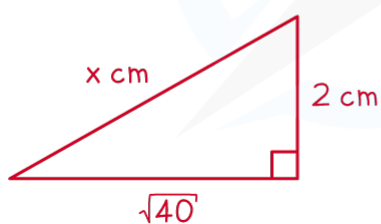
6. Trigonometry

YOUR NOTES



NO NEED TO SQUARE ROOT – IT WILL GET SQUARED AGAIN SOON, ANYWAY

$$a^2 = 2^2 + 6^2 = 4 + 36 = 40$$



$$x^2 = (\sqrt{40})^2 + 2^2$$

$$x^2 = 40 + 4$$

$$x^2 = 44$$

$$x = 6.63 \quad (2 \text{ DP})$$

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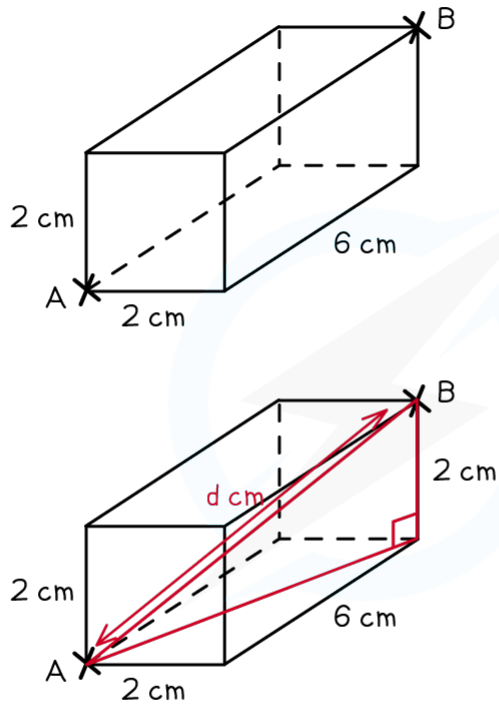
- There is a 3D version of the Pythagoras' Theorem formula
 - $d^2 = x^2 + y^2 + z^2$
- However it is usually far easier to see a problem by splitting it into two or more 2D problems

6. Trigonometry

YOUR NOTES



e.g. FIND THE DISTANCE BETWEEN A AND B.



$$d^2 = 2^2 + 6^2 + 2^2 = 44$$

LENGTH OF AB IS $\sqrt{44} = 6.63$ (2 DP)

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How does SOHCAHTOA work in 3D?

- Again look for a combination of right-angled triangles that would lead to the missing angle or side
- The angle you are working with can be awkward in 3D
 - The angle between a **line** and a **plane** is not obvious
 - If unsure, put a point on the line and draw a new line to the plane
This should create a right-angled triangle

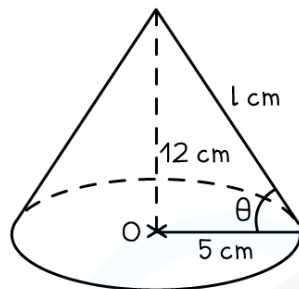
6. Trigonometry

YOUR NOTES

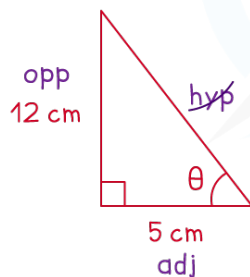


SOHCAHTOA IN 3D

e.g. A CONE HAS BASE RADIUS 5 cm AND A PERPENDICULAR HEIGHT OF 12 cm.



FIND THE ANGLE θ , GIVING YOUR ANSWER TO ONE DECIMAL PLACE.



DRAW THE 2D TRIANGLE OUT TO MAKE IT EASIER TO SEE.

$$\text{TOA} - \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{12}{5} = 2.4$$

$$\theta = \tan^{-1}(2.4)$$

$$\theta = 67.4 \text{ (1 DP)}$$

REMEMBER TO USE SHIFT WHEN FINDING ANGLES

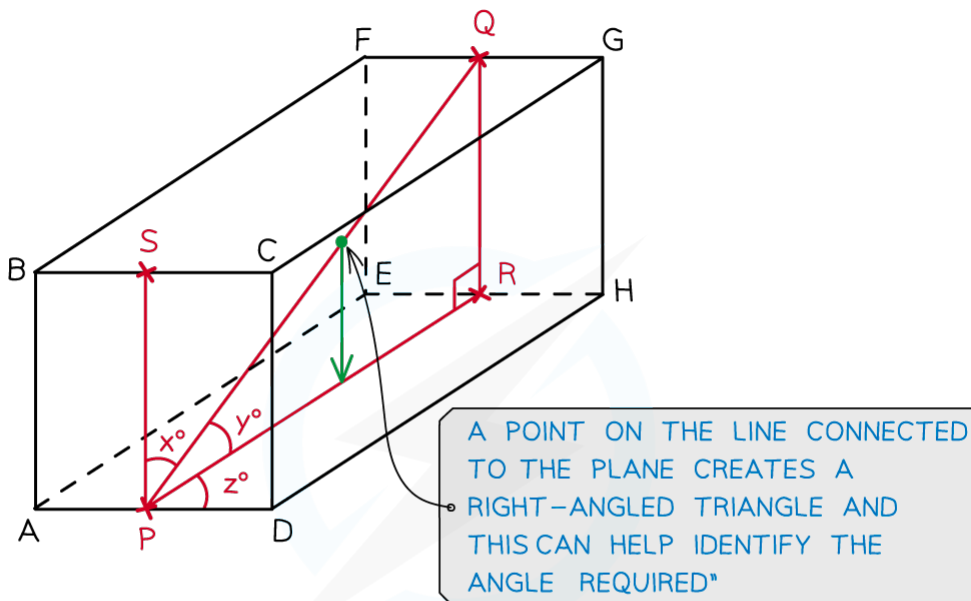
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6. Trigonometry

YOUR NOTES



ANGLE BETWEEN A LINE AND A PLANE



x° IS THE ANGLE BETWEEN THE LINE PQ AND THE PLANE ABCD (LINE PS)

y° IS THE ANGLE BETWEEN THE LINE PQ AND THE PLANE AEHD (LINE PR)

z° IS THE ANGLE BETWEEN THE LINE PR AND THE LINE AD

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- Once you have your 2D triangle(s) you can begin to solve problems

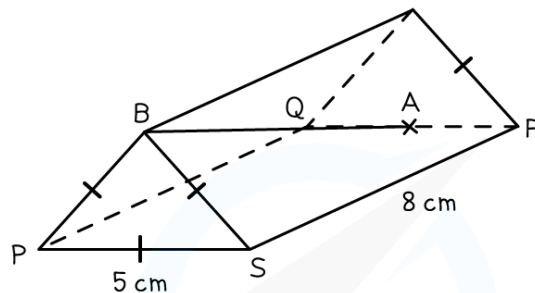
6. Trigonometry

YOUR NOTES

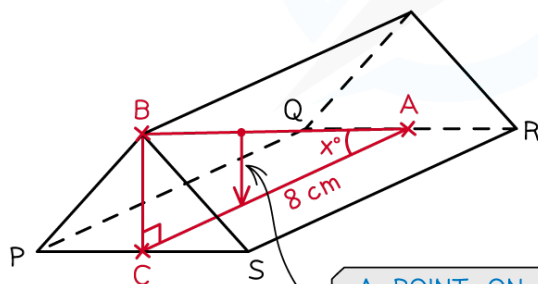


PROBLEM SOLVING

e.g. IN THE TRIANGULAR PRISM BELOW, POINT A IS THE MID-POINT OF QR AND B IS A VERTEX OF THE FACE PBS WHICH IS AN EQUILATERAL TRIANGLE.



FIND THE ANGLE BETWEEN THE LINE AB AND THE PLANE PQRS.



DRAW IN A RIGHT ANGLED TRIANGLE AND DETERMINE ANY KNOWN DISTANCES, AND THE ANGLE BEING ASKED FOR.

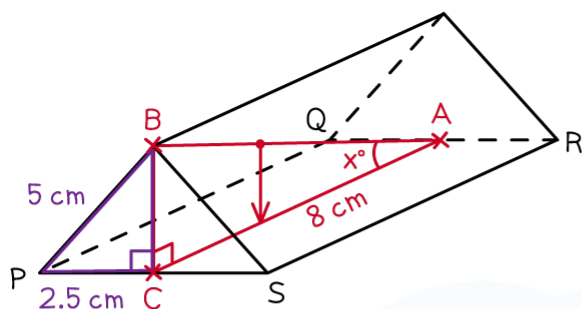
A POINT ON THE LINE AB CONNECTED TO THE PLANE PQRS CREATES A RIGHT ANGLED TRIANGLE

ONLY ONE SIDE OF TRIANGLE ABC IS KNOWN BUT BC CAN BE CALCULATED BY PYTHAGORAS' THEOREM.

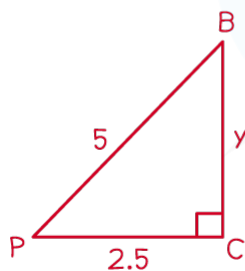
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6. Trigonometry

YOUR NOTES



PB = 5 cm AS TRIANGLE PBS IS EQUILATERAL
PC = 2.5 cm AS A IS MIDPOINT OF QR,
SO C IS MIDPOINT OF PS.



DRAW THE 2D TRIANGLES
SEPARATELY TO HELP KEEP
THINGS CLEAR

$$y^2 = 5^2 - 2.5^2$$

$$y^2 = 18.75$$

LEAVE AS y^2 FOR NOW TO
AVOID ROUNDING

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6. Trigonometry

YOUR NOTES



opp
y cm

hyp

DON'T KNOW AND NOT BEEN ASKED TO FIND

8 cm
adj

x°

$y^2 = 18.75$

“TOA – $\tan \theta = \frac{\text{opp}}{\text{adj}}$ ”

$\tan x^\circ = \frac{y}{8}$

$x = \tan^{-1}\left(\frac{\sqrt{18.75}}{8}\right)$

$x = 28.4^\circ$ (1 DP)

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Exam Tip

Add lines/triangles/etc. to any given diagram to help you see the problem and draw any 2D triangles separately as a 3D diagram can get hard to follow.

6. Trigonometry

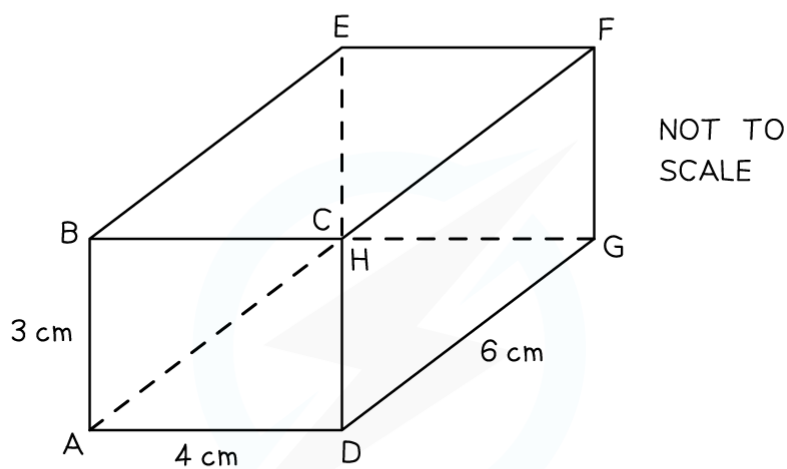
YOUR NOTES



Worked Example



A pencil is being put into a cuboid shaped box which has dimensions 3 cm by 4 cm by 6 cm.

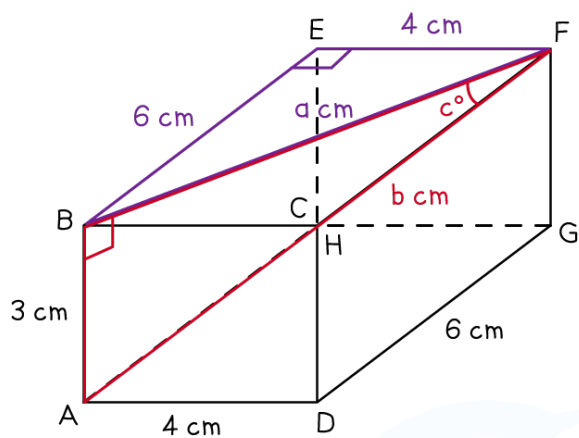


- Find the length of the longest pencil that can fit inside the box.
- Find the angle that the pencil would make with the top of the box (ie the plane BEFC).

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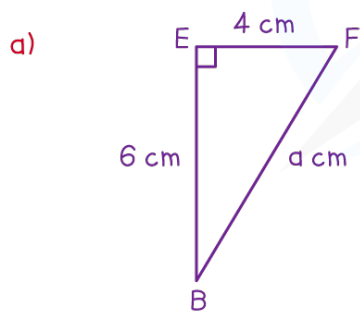
6. Trigonometry

YOUR NOTES



ADD TRIANGLES
AND LINES, LABELS
TO THE DIAGRAM
SO YOU CAN SEE
WHAT IS HAPPENING

THE LONGEST POSSIBLE PENCIL WILL BE FROM OPPOSITE
VERTICES – A "BOTTOM LEFT" AND F "TOP RIGHT"

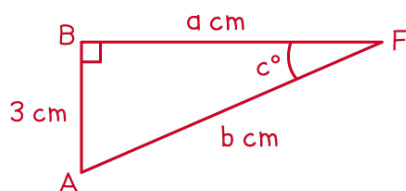


$$a^2 = 4^2 + 6^2$$

$$a^2 = 16 + 36$$

$$a^2 = 52$$

LEAVE AS a^2
FOR NOW



$$b^2 = a^2 + 3^2$$

$$b^2 = 52 + 9$$

$$b^2 = 61$$

$$b = 7.8 \text{ cm (1 DP)}$$

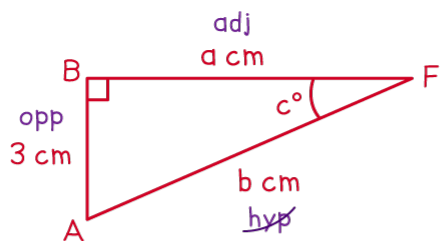
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6. Trigonometry

YOUR NOTES



b)



c WAS IDENTIFIED AS
ANGLE REQUIRED ON
ORIGINAL 3D DIAGRAM

WE KNOW ALL 3 SIDES BUT
WE SHALL CHOOSE TO USE 3
AND a ON THIS OCCASION

"TOA" - $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$$\tan c^\circ = \frac{3}{\sqrt{52}}$$

$$c = \tan^{-1}\left(\frac{3}{\sqrt{52}}\right)$$

$$c = 22.6^\circ \text{ (1 DP)}$$

AVOID ROUNDING UNTIL FINAL
ANSWER WHENEVER POSSIBLE

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