EXAMINATIONS COUNCIL OF LESOTHO
Lesotho General Certificate of Secondary Education

CANDIDATE NAME														
CENTRE NUMBER						CANDIDATE NUMBER								

6440464384

MATHEMATICS **0178/02**

Paper 2 (Extended) **October/November 2015**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical Instruments
Tracing Paper (optional)

READ THESE INSTRUCTIONS FIRST

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The total of the marks for this paper is 70.

This document consists of **12** printed pages.

[Turn over

1 The number of students at a college is 1719.

Write this number correct to

(a) the nearest 10,

Answer (a) ..... [1]

(b) two significant figures.

Answer (b) ..... [1]

---

2 21 22 23 24 25 26 27 28 29

From the list of numbers write

(a) a prime number,

Answer (a) ..... [1]

(b) a square number,

Answer (b) ..... [1]

(c) a cube number,

Answer (c) ..... [1]

(d) the square root of 625.

Answer (d) ..... [1]

---

3 Work out $\frac{3}{4} - \frac{1}{2} + \frac{1}{6}$.

Answer ..... [2]

---

4 At noon in Oxbow the temperature was 3 °C.
At midnight the temperature had fallen by 8 °C.

Find the temperature at midnight.

Answer ..... °C [1]

5 Solve the simultaneous equations.

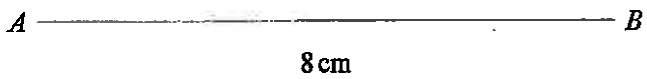
$$5x + 4y = 7$$
$$7x + 4y = 5$$

Answer $x =$ .....

$y =$ ..... [2]

---

6 Triangle ABC is such that $AB = 8$ cm, $AC = 5$ cm and angle $BAC = 60°$.
AB has already been drawn for you.



(a) Construct triangle ABC. [2]

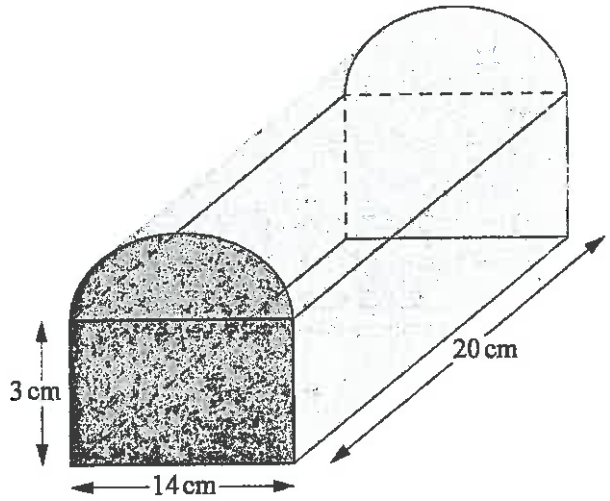
(b) Construct the bisector of angle ABC. [2]

(c) Write down the length of BC.

Answer (c) ..... cm [1]

---

7 The diagram shows a prism.
The cross-section of the prism consists of a rectangle and a semi-circle.



NOT TO SCALE

Work out, leaving your answer in terms of π,

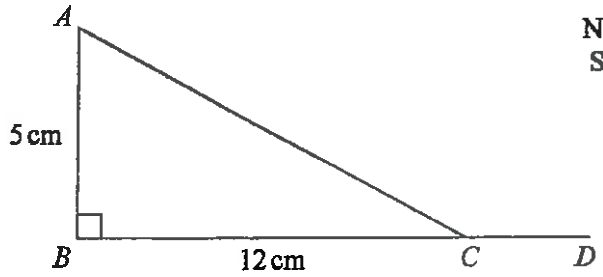
(a) the area of the cross-section,

Answer (a) ..... cm^2 [3]

(b) the volume of the prism.

Answer (b) ..... cm^3 [1]

8 The diagram shows the right-angled triangle ABC.
D is a point on BC produced.



NOT TO SCALE

$AB = 5$ cm and $BC = 12$ cm.

(a) Work out the length AC.

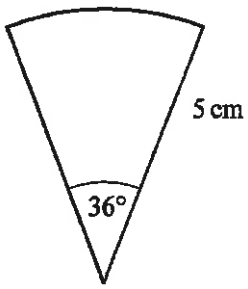
Answer (a) ..... cm [2]

(b) Write down the value of the cosine of angle ACD.

Answer (b) ..... [1]

---

9 The diagram shows a sector of a circle with radius 5 cm and sector angle 36°.



NOT TO SCALE

Calculate the perimeter.
Use $\pi = 3.142$.

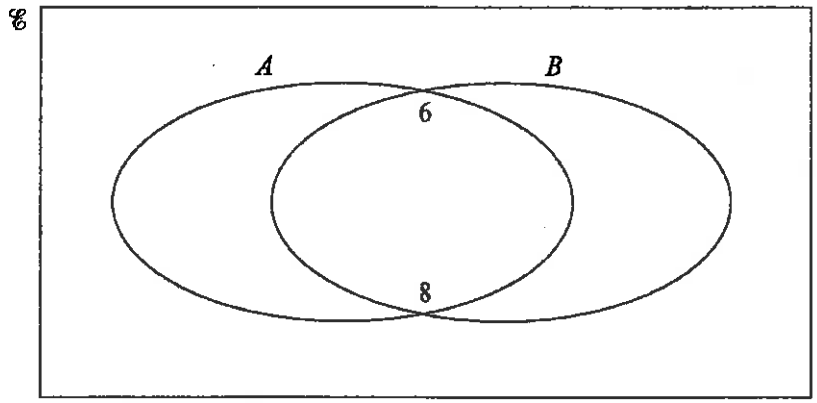
Answer ..... cm [3]

---

10 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$A = \{\text{multiples of two}\}$
$B = \{\text{composite numbers}\}$

(a) Complete the Venn diagram to show this information.



[2]

(b) **(i)** A number is chosen at random from $\mathscr{E}$.

Find the probability that it is a member of $A \cap B$.

Answer (b)(i) ..... [1]

(ii) A composite number is chosen at random.

Find the probability that it is a member of $A \cap B$.

Answer (b)(ii) ..... [1]

---

11 Evaluate the following.

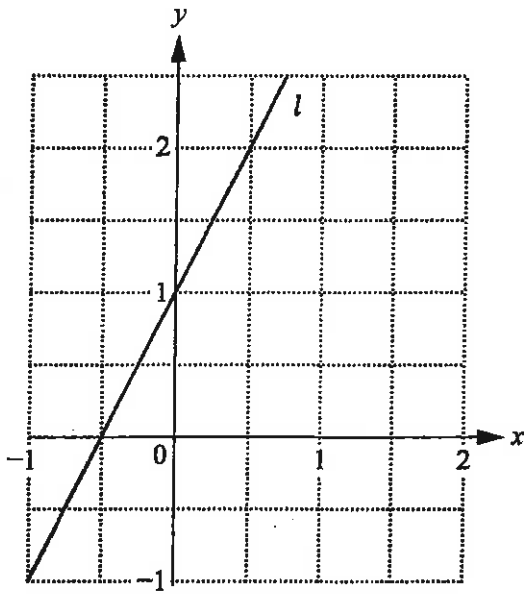
(a) $4^0 + 4^{\frac{1}{2}} + 4^2$

Answer (a) ..... [2]

(b) $27^{-\frac{2}{3}}$

Answer (b) ..... [2]

12

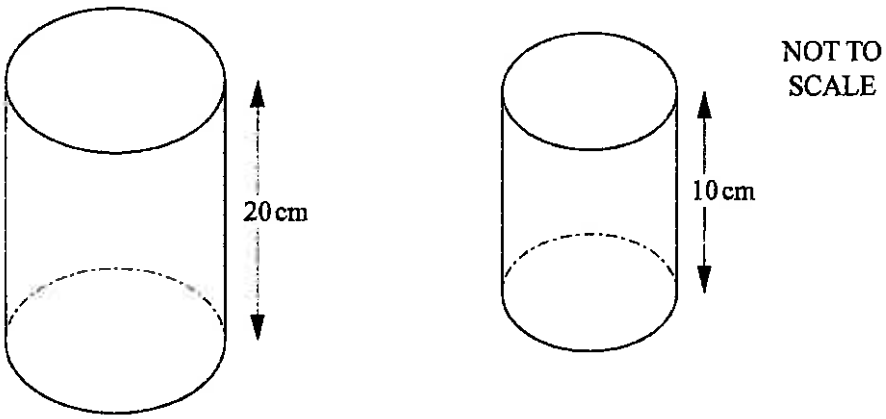


(a) Find the equation of the line l.

Answer (a) $y =$ ..... [2]

(b) Draw the line on the same grid that is parallel to line l passing through the point (1, 1). [1]

13 The diagram shows two similar cylindrical cans.
Their heights are as shown.



(a) If the diameter of the smaller can is 6 cm, calculate the diameter of the larger can.

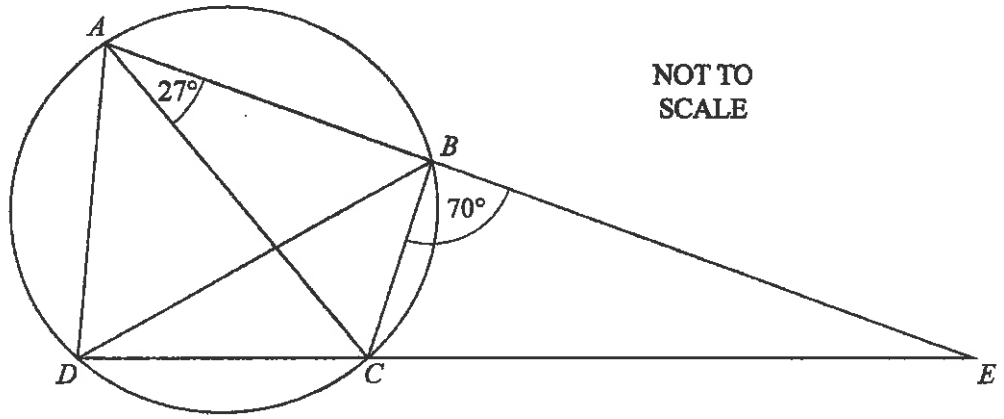
Answer (a) ..... cm [2]

(b) The volume of the smaller can is $90\pi\,\text{cm}^3$.

Find the volume of the larger can, leaving your answer in terms of π.

Answer (b) ..... cm^3 [2]

14



The points A, B, C and D are on the circumference of a circle.
Angle $BAC = 27°$ and angle $EBC = 70°$.
ABE and DCE are straight lines.

Find

(a) angle BDC,

Answer (a) ..... [1]

(b) angle ACB,

Answer (b) ..... [2]

(c) angle ADC.

Answer (c) ..... [1]

---

15 Find the integer values for n which satisfy this inequality.

$$-3 < 2n - 1 \leqslant 5$$

Answer ..... [3]

---

16 $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

(a) Find the matrix **AC**.

Answer (a) ..... [2]

(b) Find $\mathbf{A}^{-1}$.

Answer (b) ..... [2]

---

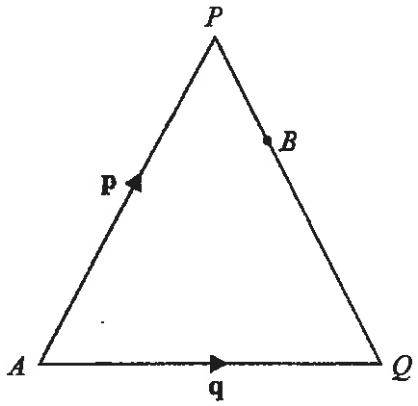
17 x is inversely proportional to $(y - 2)$.
When $y = 6$, $x = 9$.

Find the value of x when $y = 26$.

Answer $x =$ ..... [3]

---

18 The diagram shows triangle APQ.



NOT TO SCALE

$\overrightarrow{AP} = \mathbf{p}$, $\overrightarrow{AQ} = \mathbf{q}$ and $PB = \frac{1}{3}PQ$.

Find $\overrightarrow{AB}$ in terms of **p** and **q**, giving your answer in its simplest form.

Answer ..... [3]

---

19 The diameter of the sun is approximately 1.39×10^6 km.

The diameter of the earth is approximately 1.27×10^4 km.

Leaving your answer in standard form, work out

(a) the radius of the sun,

Answer (a) .....km [2]

(b) by how many kilometres the diameter of the sun is greater than that of the earth.

Answer (b) .....km [2]

Questions 20 and 21 are printed on the next page.

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MATHEMATICS **0178/02**

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1 hour 30 minutes

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1 Lineo spends $7\frac{1}{2}$ hours at the stadium watching games.
She leaves the stadium at 4.20 p.m.

At what time did she arrive at the stadium?

Answer ..... [2]

---

2 **(a)** Express 120 as a product of its prime factors.

Answer (a) ..... [1]

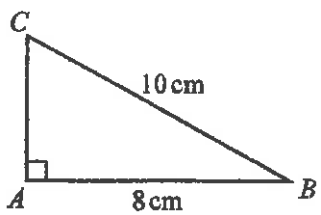
(b) Given that $120k$ is a perfect square, write down the smallest possible value of k.

Answer (b) ..... [1]

(c) Find the highest common factor of 120 and 180.

Answer (c) ..... [1]

3



NOT TO SCALE

Work out the length AC.

Answer ..... cm [3]

---

4 Solve the simultaneous equations.

$3x - 2y = 4$
$5x - 4y = -3$

Answer $x =$ .....

$y =$ ..... [3]

---

5 A polygon has n sides.
Two of its exterior angles are 23° and 85°, while the other $(n - 2)$ exterior angles are 14° each.

Calculate the value of n.

Answer $n =$ ..... [2]

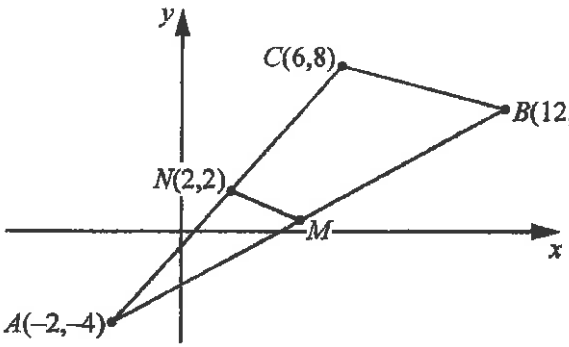
---

6 Find the integer values of x for which $1 - x < 3x + 5 \leqslant x + 9$.

Answer ..... [3]

---

7 In the diagram, M and N are the midpoints of AB and AC respectively.
BC and MN are parallel.



NOT TO SCALE

(a) Find the coordinates of M.

Answer (a) (.....) [2]

(b) Find the gradient of BC.

Answer (b) ..... [1]

(c) Write the equation of MN.

Answer (c) ..... [2]

(d) Find the length of AC.
Leave your answer in the form $\sqrt{r}$.

Answer (d) ..... [2]

8 Factorise completely

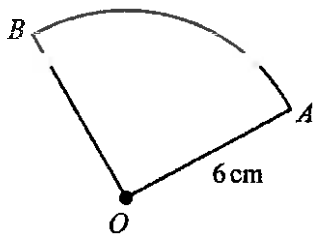
(a) $20ax - by - 5ay + 4bx$,

Answer (a) ..... [2]

(b) $2x^2 - 18$ and **hence** find the prime factors of 182.

Answer (b) prime factors ..... [3]

9



NOT TO SCALE

The area of the sector is $12\pi\,\text{cm}^2$.

Work out the length of the arc AB.
Give your answer in terms of π.

Answer ..... cm [3]

---

10 Khaola makes a map of Mafeteng District and uses a scale of 1 : 50 000.
The area of a village on his map is 8 cm^2.

Calculate, in **square kilometres**, the actual area of the village.

Answer ..... km^2 [2]

---

11 $\mathbf{A} = \begin{pmatrix} -5 & 7 \\ 3 & -4 \end{pmatrix}$

Find $\mathbf{A}^{-1}$.

Answer ..... [2]

---

12 **(a)** y is inversely proportional to x^3.
When $y = 9$, $x = 3$.

Find y when $x = 10$.

Answer (a) ..... [2]

(b) p is directly proportional to q^2.

Find the percentage increase in the value of p when q is increased by 50%.

Answer (b) ..... % [2]

13 **(a)** Simplify $\dfrac{3^{x+3}-3^{x+1}}{3^{x+1}}$.

Answer (a) ..... [2]

(b) Given that $\dfrac{y^4 \times \sqrt{y}}{y^{-3}} = y^n$, find the numerical value of n.

Answer (b) ..... [2]

---

14 The probabilities that three football teams, A, B and C, win their next game are $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively.

Find the probability that

(a) none of the three teams win,

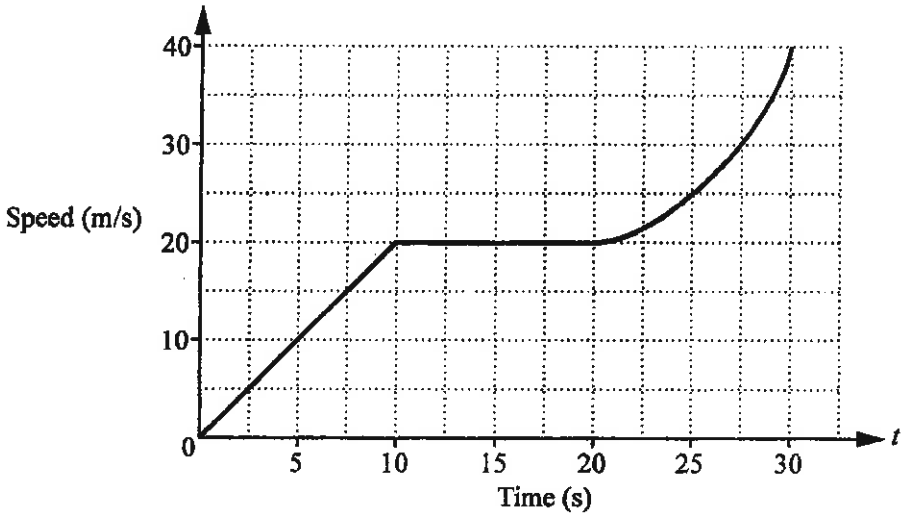
Answer (a) ..... [2]

(b) one of the three teams wins.

Answer (b) ..... [3]

---

15 The diagram shows the speed-time graph for the first 30 seconds of a car journey.



(a) Calculate the acceleration when $t = 8$ seconds.

Answer (a) ..... m/s^2 [1]

(b) Calculate the distance travelled during the first 20 seconds.

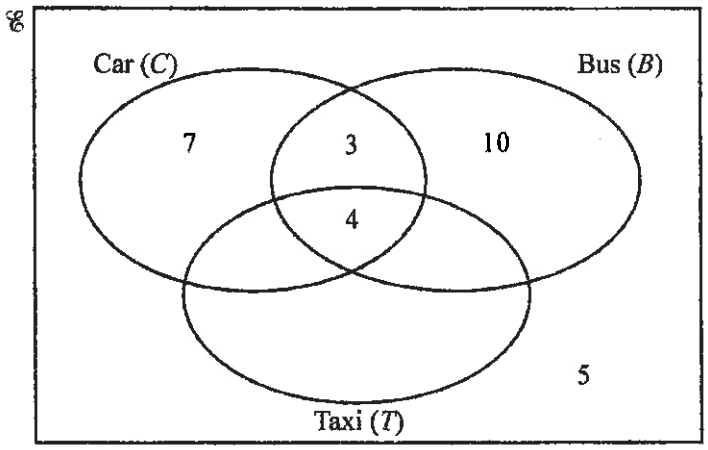
Answer (b) ..... m [2]

(c) When $t = 30$ seconds, the car decelerates at $8\ m/s^2$.

Find the time taken for the car to come to rest.

Answer (c) ..... s [2]

16 In a survey, 40 teachers are asked which forms of transport they regularly use.
The Venn diagram shows part of the information about their responses.



16 teachers use a car.
8 teachers use a taxi **only**.

(a) **(i)** Complete the Venn diagram. [2]

(ii) Find the number of teachers who use both bus and taxi.

Answer (a)(ii) ..... [1]

(b) **(i)** Find $n(C \cap (B \cap T)')$.

Answer (b)(i) ..... [1]

(ii) Describe in **words** what the 5 in the Venn diagram represents.

Answer (b)(ii) .....

.....

..... [1]

17 **(a)** The numbers show goals scored in six football games.

8 x 1 2 11 1

Given that the median is $3\frac{1}{2}$, find the value of x.

Answer (a) $x =$ ..... [2]

(b) The mean of eight numbers is 3.
The mean of a different set of twelve numbers is y.

Given that the mean of these twenty numbers is 9, calculate the value of y.

Answer (b) $y =$ ..... [3]

18 The table shows the length, l mm, of 40 leaves.

Length of a leaf (l mm)	$0 < l \leqslant 10$	$10 < l \leqslant 20$	$20 < l \leqslant 60$	$60 < l \leqslant 80$
Frequency	6	20	4	10

(a) Find the modal class.

Answer (a) ..... [1]

(b) Calculate an estimate of the mean.

Answer (b) ..... mm [4]

(c) On a histogram, the height of the interval $60 < l \leqslant 80$ is 2.5 cm.

Calculate the height of the interval $10 < l \leqslant 20$.

Answer (c) ..... cm [2]

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1 **(a)** Work out.

(i) $1\frac{2}{3} \div 1\frac{3}{4}$

Answer *(a)*(i) ..... [1]

(ii) 0.5×0.007

Answer *(a)*(ii) ..... [1]

(b) Work out.

(i) $2\sqrt{8} \times 4\sqrt{8} + 2$

Answer *(b)*(i) ..... [2]

(ii) $\left(\frac{5}{4^{-1}-9^{-1}}\right)^{\frac{1}{2}}$

Answer *(b)*(ii) ..... [2]

(c) By making suitable approximations, estimate

$$\frac{23.566 \times (5.367)^2}{14.788}.$$

Answer *(c)* ..... [2]

2 $198 = 2 \times 3^2 \times 11$ and $360 = 2^3 \times 3^2 \times 5$.

(a) Find

(i) the highest common factor of 198 and 360,

Answer *(a)*(i) ..... [1]

(ii) the lowest common multiple of 198 and 360.

Answer *(a)*(ii) ..... [1]

(b) Find the smallest value of k such that $\sqrt{360 \times 198 \times k}$ is an integer.

Answer *(b)* ..... [2]

3 (a) Expand and collect like terms.

(i) $4(k - 1) - (3k + 2) + 14$

Answer *(a)*(i) ..... [2]

(ii) $(7x - 2y)(3x + y)$

Answer *(a)*(ii) ..... [2]

(b) (i) Solve the simultaneous equations.

$2x + 2y = 3$
$4x - 5y = 24$

Answer *(b)*(i) $x =$ ..... $y =$ ..... [3]

(ii) Solve.

$3^{x-2} + 3^x = 10$

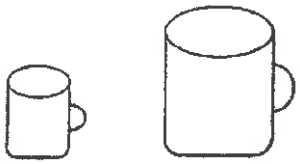
Answer *(b)*(ii) $x =$ ..... [3]

(c) Factorise.

$6x^2 + 5x - 6$

Answer *(c)* ..... [2]

4 The diagram shows two similar coffee mugs.
The large mug holds 500 cm^3 of coffee.



NOT TO SCALE

Small mug Large mug

(a) How many full large mugs would be required to fill a 1 m^3 container?

Answer *(a)* ..... [2]

(b) The small mug holds 32 cm^3 of coffee.

(i) Find the scale factor for the length between the small and large mug.
Give your answer as an exact fraction.

Answer *(b)*(i) ..... [2]

(ii) The height of the large mug is 15 cm.
Work out the height of the small mug.

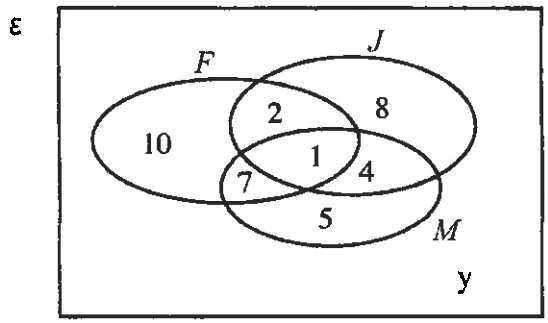
Answer *(b)*(ii) .....cm [2]

(iii) The surface area of the large mug is 350 cm^2.
Work out the surface area of the small mug.

Answer *(b)*(iii) ..... cm^2 [2]

5 A teacher asks 45 students which drink they enjoy.
F = {students who enjoy fizzy drinks}
J = {students who enjoy juice}
M = {students who enjoy milk}

The Venn diagram shows the results.



(a) Use the Venn diagram to find

(i) the number of students who enjoy juice,

Answer *(a)*(i) ..... [1]

(ii) $n(M' \cap F')$,

Answer *(a)*(ii) ..... [1]

(iii) the number of students who enjoy fizzy drinks and juice but not milk.

Answer *(a)*(iii) ..... [1]

(b) Find the value of y.

Answer *(b)* $y =$ ..... [2]

(c) A student is chosen at random.
Find the probability that this student enjoys

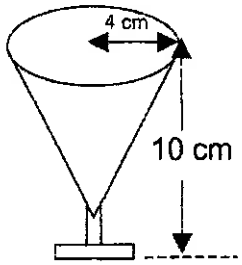
(i) fizzy drinks and juice,

Answer *(c)*(i) ..... [1]

(ii) either milk or juice.

Answer *(c)*(ii) ..... [2]

6 Puleng uses the conically shaped container, as shown in the diagram, to measure morvite (lepoopo).



NOT TO SCALE

The cone has a radius 4 cm and height 10 cm.

The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.

The curved surface area, C, of a cone with radius r and slant height l is $C = \pi rl$.

(a) Work out

the volume of lepoopo that can fill up the container.
Give your answer as a multiple of π.

Answer *(a)* ..... cm^3 [2]

(b) The curved surface area of the container is $k\sqrt{29}$ cm^2.
Find the value of k.

Answer *(b)* k = ... cm^2 [4]

(c) The container holds 30g of lepoopo when it is full.
Work out how much lepoopo 100 full containers can hold.
Give your answer in kg.

Answer *(b)* .....kg [1]

7 **(a)** The determinant of the matrix $\begin{pmatrix} x & x+1 \\ 3x & 4x \end{pmatrix}$ is -2.

(i) Use this information to form an equation and show that it can be reduced to

$x^2 - 3x + 2 = 0$.

Answer *(a)*(i) ..... [3]

(ii) Solve $x^2 - 3x + 2 = 0$.

Answer *(a)*(ii) $x =$ ..... or $x =$ ..... [3]

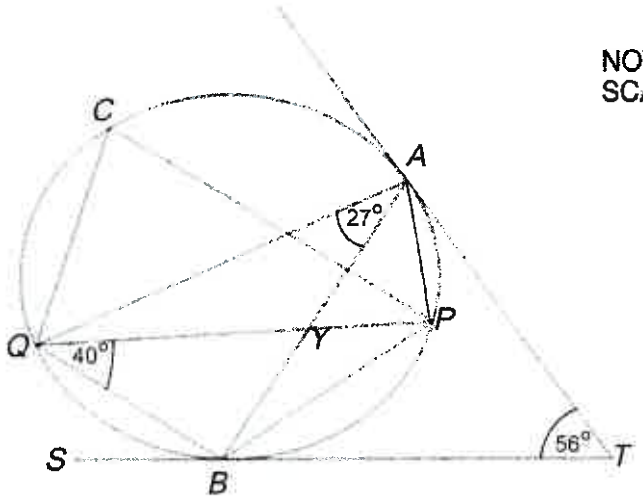
(b) Find the values of x and y.

$$\begin{pmatrix} x \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ y \end{pmatrix}$$

Answer *(b)* $x =$ ..... $y =$ ..... [2]

8 In the diagram, TA and TB are tangents to the circle at the points A and B.

TBS is a straight line.
The straight lines AB and QP intersect at Y.
The angle $QAB = 27°$, angle $PQB = 40°$ and angle $ATB = 56°$.



(a) State with the reasons the values of the angles

PAY,

..... because .....

..... [2]

(b) *QBP*,

..... because .....

..... [2]

(c) *TAP*,

..... because .....

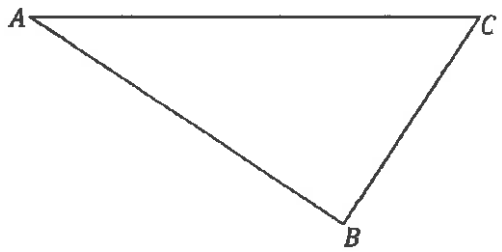
..... [2]

(d) *AYP*,

..... because .....

..... [2]

9 The diagram shows a triangle ABC.



(a) Measure angle ACB.

Answer *(a)* ..... [1]

(b) The point D is above AC, such that AD is 5 cm and CD is 4 cm.
By construction, complete the triangle ADC. [2]

(c) The region, R, lies within the quadrilateral $ABCD$.
The points in R are

- nearer to C than A and
- more than 4 cm from B.

By accurate construction, shade the region R. [4]

95 copies Form E

EXAMINATIONS COUNCIL OF LESOTHO
Lesotho General Certificate of Secondary Education

CANDIDATE NAME													
CENTRE NUMBER						CANDIDATE NUMBER								

MATHEMATICS **0178/02**

Paper 2 (Extended) **October/November 2017**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical Instruments
Tracing Paper (optional)

READ THESE INSTRUCTIONS FIRST

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 70.

This document consists of **13** printed pages and **3** blank pages.

1 Mpho's bank account is overdrawn by M485.

(a) She deposits M230 into the account.

How much does she now owe the bank?

Answer (a) M ..... [1]

(b) What further deposit should she make to give her account a balance of M150?

Answer (a) M ..... [1]

---

2 When written as a product of their prime factors, $a = n \times 3 \times 7$ and $b = n^2 \times 7$.

(a) Express, in terms of n,

(i) the LCM of a and b,

Answer (a)(i) ..... [1]

(ii) the HCF of a and b.

Answer (a)(ii) ..... [1]

(b) Given that the LCM of a and b is 84.

Find the values of n, a and b.

Answer (b) n = ... a = ... b = ... [3]

---

3 On average, the mass of each egg in a tray is 57.2 grams to the nearest tenth of a gram.

(a) Write down the lower bound for the mass.

Answer (a) ..... g [1]

(b) There are 30 eggs in the tray.

Calculate an estimated upper bound for the mass of all eggs in the tray.

Answer (b) ..... g [2]

---

4 **(a)** Show that $\frac{a^{-2}+b^{-2}}{(ab)^{-2}}$ can be simplified to $a^2 + b^2$.

Answer (a) ..... [3]

(b) Find the value of n in the following expression.

$$\left(\frac{2}{3}\right)^{-3} \times \left(\frac{3}{2}\right)^{n} = 1$$

Answer (b) ..... [2]

---

5 A serving of 100 g of powdered milk A provides 2120 KJ of energy.

(a) Express this amount of energy in standard form.

Give your answer in Joules.

Answer (a) .....J [1]

(b) Calculate the amount of energy, in Joules, that can be obtained from 2 kg of powdered milk A.

Give your answer in standard form.

Answer (b) .....J [2]

(c) A serving of 100 g of powdered milk B provides 9.78×10^2 kJ of energy.

Calculate the difference, in Joules, between the amount of energy obtained from powdered milk A and B in serving of 300 g.
Leave your answer in standard form.

Answer (c) .....J [3]

6 The temperature of a substance rises from −3 °C to 12 °C.

Calculate the increase in temperature of the substance.

Answer ..... °C [1]

---

7 Given that $180 = 5 \times p^2 \times q^2$ and that both p and q are both prime.

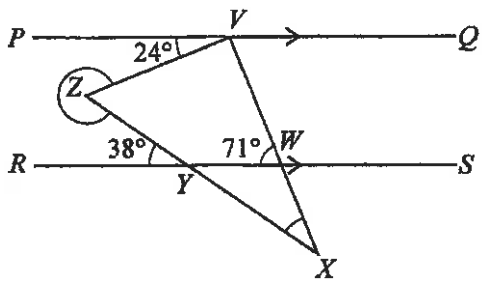
Find the values of p and q.

Answer $p =$ ..... $q =$ ..... [2]

---

8 In the diagram PQ is parallel to RS.

VW and ZY are produced to meet at X.



NOT TO SCALE

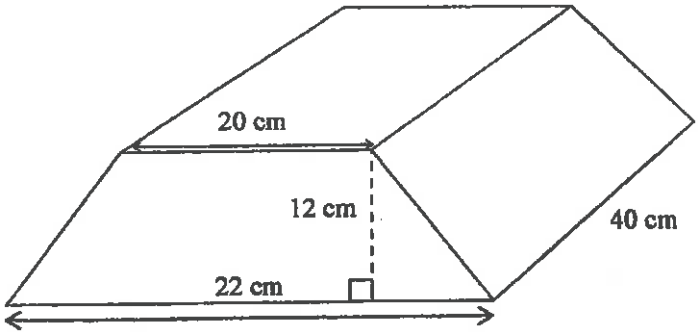
Find the value of angle $W\hat{X}Y$ and the value of **reflex** angle $V\hat{Z}Y$.

Answer $W\hat{X}Y =$ ..... [2]

$V\hat{Z}Y =$ ..... [2]

---

9 The diagram shows a gold bar, which is a prism of length 40 cm.



NOT TO SCALE

(a) Find the area of the cross-section.

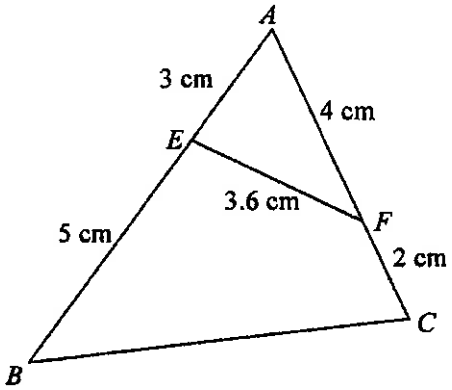
Answer (a) .....cm^2 [2]

(b) Find the volume of the gold bar.

Answer (b) .....cm^3 [1]

10 In the figure, AEB and AFC are straight lines.

$AE = 3$ cm, $EB = 5$ cm, $AF = 4$ cm, $EF = 3.6$ cm and $FC = 2$ cm.



NOT TO SCALE

(a) Stating your reasons clearly, show that triangle ABC and triangle AFE are similar.

Answer (a) .....

.....

..... [3]

(b) Calculate the length of BC.

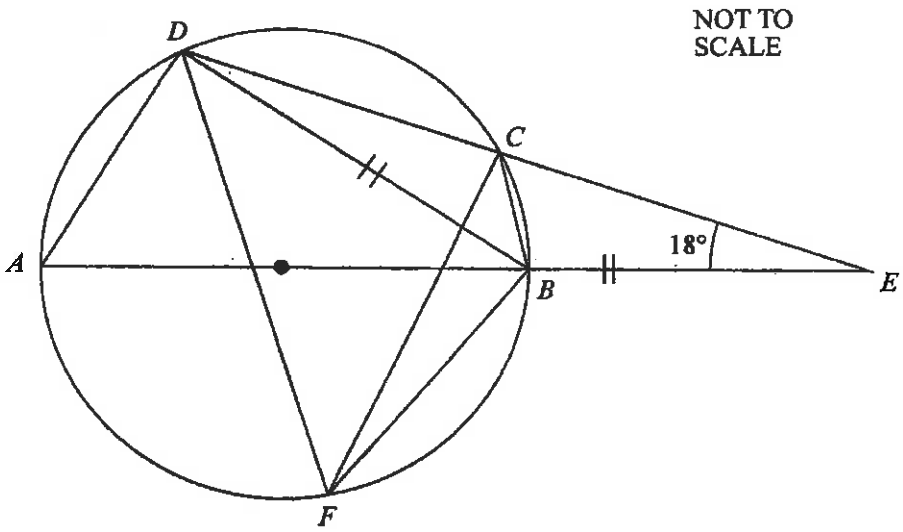
Answer (b) ..... [2]

(c) Find the value of $\dfrac{\textit{Area of quadrilateral } BCFE}{\textit{area of } \triangle ABC}$

Answer (c) ..... [2]

11 In the diagram, A, B, C and F lie on the circumference of the circle.
AB is a diameter of the circle.
AB and DC produced meet at E.

$DB = BE$ and $A\hat{E}D = 18°$.



(a) Find the value of angle ABC.

Answer (a) ..... [2]

(b) Show that BD bisects angle ABC.
Give reasons for each stage of your working.

Answer (b) .....

.....

..... [3]

12 $A = \begin{pmatrix} -1 & 5 \\ 2 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ 2 & -1 \end{pmatrix}$, and $C = \begin{pmatrix} 6 & 8 \\ 3 & t \end{pmatrix}$,

Find

(a) AB,

Answer (a) ..... [2]

(b) the value of t for which C has no inverse.

Answer (b) $t =$ ..... [2]

---

13 In 2014, a street vendor made a profit of M2160.
This was 8% more than the profit made in 2013.

Find the profit made in 2013.

Answer M ..... [2]

---

14 Factorise fully

(a) $3p(a - 8b) - 7q(8b - a)$,

Answer (a) ..... [1]

(b) $2u^2 + 5u - 3$.

Answer (b) ..... [2]

---

15 **(a)** Evaluate $\frac{(5\frac{1}{3})^2}{(1-\frac{1}{3})^3}$.

Answer (a) ..... [3]

(b) Simplify $12(-pt)^2 \times \frac{\sqrt{p^4t^2}}{3} \times \left(\frac{p}{2t}\right)^3$.

Answer (b) ..... [3]

---

16 Given that $S = \dfrac{v^2 - u^2}{2a}$,

(a) find the value of S when $v = 4$, $u = 3$ and $a = 7$.

Answer (a) ..... [2]

(b) make v the subject of the formula.

Answer (b) $v =$ ..... [2]

---

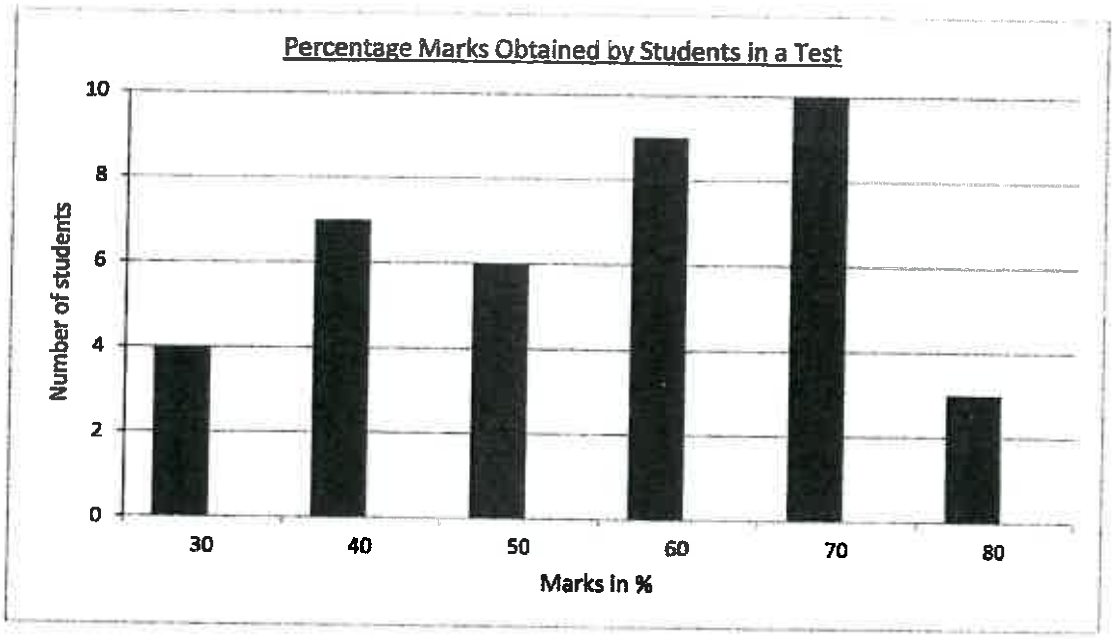
17 By rounding each of the values to 1 significant figure, estimate

$$\frac{5.112 \times 39.997}{0.199}.$$

Answer ..... [2]

---

18 The bar chart shows the marks (%) obtained by some students in a test.



(a) Find the modal mark

Answer (a) .....% [1]

(b) Find the probability that a student scores

(i) more than 60%,

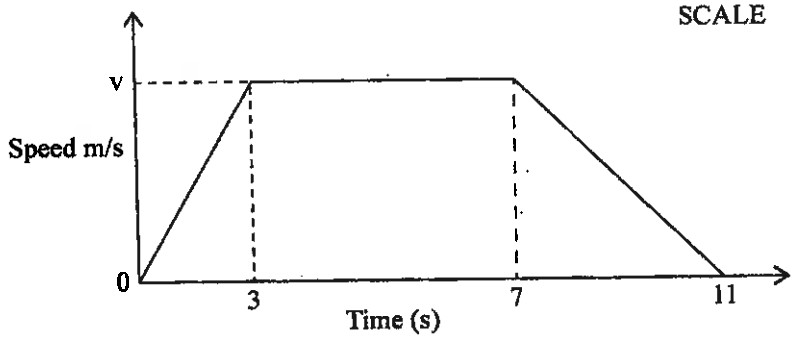
Answer (b)(i) ..... [1]

(ii) 40% or 70%.

Answer (b)(ii) ..... [1]

19 The diagram illustrates the journey of a particle.

NOT TO SCALE



Given that the total distance travelled is 45 m, find the acceleration.

Answer ..... m/s^2 [3]

EXAMINATIONS COUNCIL OF LESOTHO
Lesotho General Certificate of Secondary Education

CANDIDATE NAME	
CENTRE NUMBER	CANDIDATE NUMBER

MATHEMATICS

Paper 2 (Extended)

0178/02

May/June 2018

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical Instruments
Tracing Paper (optional)

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1 Work out.

(a) $-2^3 - {}^{-}9 \times 4 \div (2 \times 3^2)$

Answer (a) ..... [2]

(b) $4^{-\frac{1}{2}} + 16^{\frac{3}{4}}$

Answer (b) ..... [2]

---

2 Given the sequence.

$\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, p, ..., $\frac{21}{22}$, q, ...

Find

(a) the missing terms p and q,

Answer (a) $p =$ ....., $q =$ ..... [2]

(b) the nth term.

Answer (b) ..... [2]

---

3 Temperatures at 04 00 hours and 12 00 hours were −5 °C and 19 °C, respectively.

(a) Find the difference between the two temperatures.

Answer (a) ..... °C [1]

(b) Assuming that the temperature was rising at a steady rate, find

(i) the temperature at 09 30 hours,

Answer (b)(i) ..... °C [3]

(ii) the time when the temperature was 8 °C.

Answer (b)(ii) ..... hours [3]

---

4 **(a)** **(i)** Factorise $a^2 - b^2$.

Answer (a)(i) ..... [1]

(ii) Use your answer to part (a)(i) to evaluate $97^2 - 9$.

Answer (a)(ii) ..... [2]

(b) Factorise **fully** $3x^2 - 13x - 10$.

Answer (b) ..... [2]

5 $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} \frac{1}{4} & k \\ 0 & \frac{1}{3} \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 12 & 0 \\ -9 & m \end{pmatrix}$

Find

(a) $\mathbf{A}^2$,

Answer (a) ..... [2]

(b) k if $\mathbf{AB} = \mathbf{I}$,

Answer (b) $k =$ ..... [2]

(c) m if the determinant of **A** is equal to the determinant of **C**.

Answer (c) $m =$ ..... [2]

6 Solve the following equations.

(a) $3x - 5(3 - x) = 41$

Answer (a) $x =$ ..... [2]

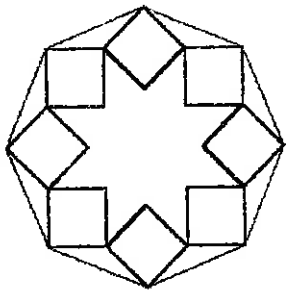
(b) $2^{x+1} - 32 = 0$

Answer (b) $x =$ ..... [2]

(c) $2x + y = 10$
$7x - 3y = 9$

Answer (c) $x =$ ..... $y =$ ..... [3]

7 The diagram shows a design of a tile in the shape of a regular octagon.
The design is made from eight squares all of the same size symmetrically placed inside the octagon as shown.



NOT TO SCALE

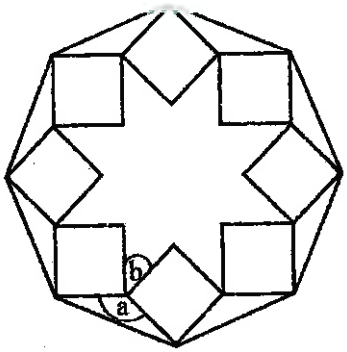
(a) State the number of lines of symmetry of the shape.

Answer (*a*) ..... [1]

(b) Calculate the size of the angle between any two adjacent lines of symmetry.

Answer (*b*) ..... [2]

(c) The letters a and b represent some angles in the diagram.



NOT TO SCALE

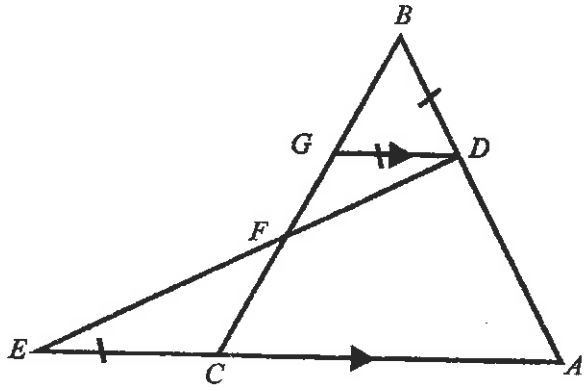
Given that $a = 135°$, calculate the value of b.

Answer (*c*) ..... [2]

8 In the diagram, ABC is an isosceles triangle with $AB = AC$.

ECA and DFE are straight lines.

DG is parallel to AE and $BD = CE = DG$.



NOT TO SCALE

(a) Name the triangle that is similar to ΔABC.

Answer (a) ..... [1]

(b) Show that ΔDGF is congruent to ΔECF.

Answer (b) .....

.....

..... [3]

(c) Given that $BC = 9$ cm and $CF = 3$ cm.

(i) Explain why $BC = 3BG$.

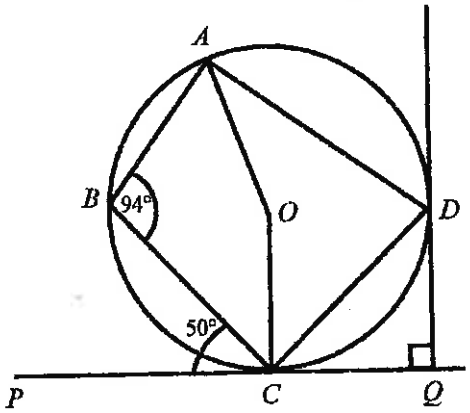
Answer *(c)*(i) ..... [1]

(ii) Find the ratio

area $ADGC$: area ABC

Answer (c)(ii) ..... : ..... [2]

9 In the diagram, A, B, C and D are points on the circumference of the circle with the centre O.
PCQ is a tangent to the circle at C and DQ is a tangent to the circle at D.
Angle $ABC = 94°$ and angle BCP $= 50°$.



NOT TO SCALE

(a) Find

(i) reflex angle AOC,

Answer (*a*)(i) ..... [1]

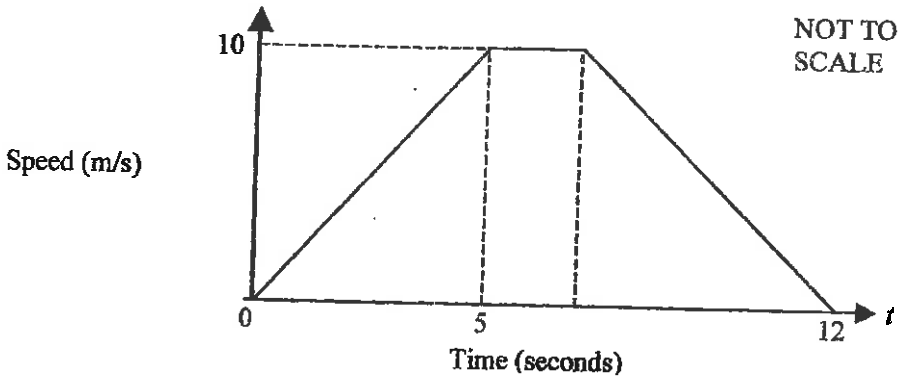
(ii) angle BAO

Answer (*a*)(ii) ..... [2]

(b) If the area of triangle $CDQ = 8$ cm^2, find CD^2.

Answer (*b*) ..... [2]

10 The diagram shows the speed-time graph of a toy-car.



(a) Find the speed of the toy-car when $t = 3$.

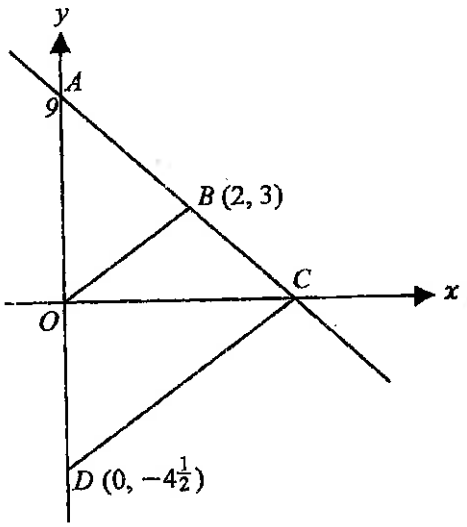
Answer (a) ..... m/s [1]

(b) The toy-car travelled a total distance of 65 m,

Find how long the toy-car took to decelerate.

Answer (b) ..... sec [3]

11 In the diagram, the coordinates of B and D are $(2, 3)$ and $(0, -4\frac{1}{2})$.
OB is parallel to DC.



NOT TO SCALE

(a) Show that the point C is $(3, 0)$.

[2]

(b) Find the equation of line CD.

Answer (b) ..... [2]

(c) Given that the length of BC is $\sqrt{r}$, find the value of r.

Answer (c) $r =$ ..... [2]

12 Given the distribution 12, 10, 8, 15, 18, 6, 8.

(a) (i) Find the lower quartile.

Answer (a)(i) ..... [1]

(ii) Find the inter-quartile range.

Answer (a)(ii) ..... [2]

(b) Calculate the mean.

Answer (b) ..... [2]

---

13 **(a)** Given that $T = \frac{2r\sqrt{p}}{3}$, make p the subject of the formula.

Answer (a) ..... [3]

(b) Find the quadratic equation whose solutions are $x = -3$ and $x = 5$.

Answer (b) ..... [2]

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Lesotho General Certificate of Secondary Education

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MATHEMATICS **0178/02**

Paper 2 (Extended) **October/November 2018**

1 hour 30 minutes

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A001

1 **(a)** The number of people at a football match is 27359.

Write the number to **one** significant figure.

Answer (a) ..... [1]

(b) The number of people who sat in stand A is 2500 correct to two significant figures.

Find the largest number of people that could have sat in stand A.

Answer (b) ..... [1]

---

2 A rectangular painting measures 4.5×10^3 mm by 3×10^2 mm.

Calculate, giving your answer in standard form,

(a) the perimeter of the painting,

Answer (a) ..... mm [2]

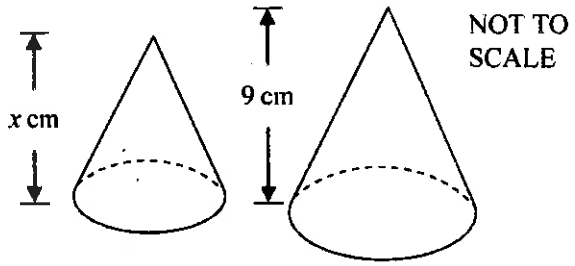
(b) the area of the painting.

Answer (b) ..... mm^2 [2]

---

3 The diagram shows two solid cones which are geometrically similar.

The height of the smaller cone is x cm and that of the larger one is 9 cm.



The volume of the smaller cone is 32 cm^3 and the volume of the larger cone is 108 cm^3.

Calculate the value of x.

Answer ..... [3]

---

4 Given that $\begin{pmatrix} 2 & 4 \\ a & 3 \end{pmatrix} + k\begin{pmatrix} 3 & 1 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -3 & -1 \end{pmatrix}$.

(a) Find the values of a and k.

Answer (a) a = ... k = ... [2]

(b) Given that $\begin{pmatrix} 1 & -2 \\ x & 4 \end{pmatrix}$ has no inverse find the value of x.

Answer (b) x = ... [2]

---

5 **(a)** Express $\frac{x-1}{3}-\frac{2x-1}{2}$ as a single fraction.

Write your answer as simply as possible.

Answer (a) ..... [2]

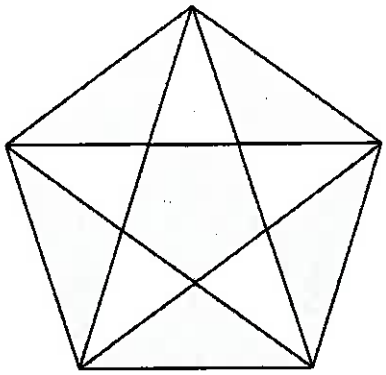
(b) $\frac{1}{x-1}-\frac{3}{x}+1$ can be written as $\frac{x^2+bx+c}{x(x-1)}$.

Find the value of b and the value of c.

Answer (b) b = ...

c = ... [3]

6 The diagram shows a logo in the shape of a regular polygon.



(a) State the name of the regular polygon.

Answer (a) ..... [1]

(b) Write the total number of isosceles triangles that are only white or only grey.

Answer (b) ..... [1]

(c) State the order of rotational symmetry of the logo.

Answer (c) ..... [1]

(d) Find the size of the interior angle of the regular polygon.

Answer (d) ..... [2]

7 **(a)** Solve.

$5 \times 2^y = 320$

Answer (a) $y =$ ..... [2]

(b) Solve the simultaneous equations.

$bx + y = b$
$ax - y = a$

Answer (b) $x =$ ..... $y =$ ..... [3]

8 **(a)** Mpho's salary is M4 400.

(i) Calculate her new salary after a 5% increase.

Answer (a)(i) M ..... [2]

(ii) The M4 400 was a 10% increase on her previous salary.

Calculate Mpho's previous salary.

Answer (a)(ii) M ..... [3]

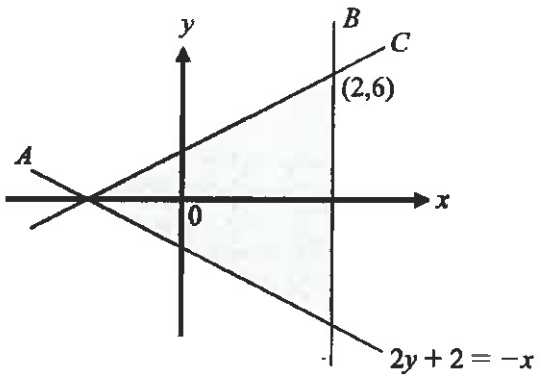
(b) Thato and Pule share M3 500 in the ratio 3:2.

Calculate Pule's share.

Answer (b) M ..... [2]

9 The diagram shows a region enclosed by three straight lines A, B and C.

The equation of line A is $2y + 2 = -x$.



(a) Show that the equation of the line C is $2y = 3x + 6$.

[3]

(b) Write down **three** inequalities which satisfy the shaded region.

Answer (b) .....

.....

..... [3]

(c) For a point (x, y) in the shaded region, find the minimum value of $x - y$.

Answer (c) ..... [2]

10 Here is a list of numbers.

36 29 41 45 15 10 13

(a) Find the median.

Answer (a) ..... [2]

(b) Find the probability that a number chosen at random from this list is prime.

Answer (b) ..... [1]

(c) Thirteen more numbers are added to the list.

The smallest of the thirteen numbers is 14 while the largest number is 48.

(i) Calculate the range of the 20 numbers now in the list.

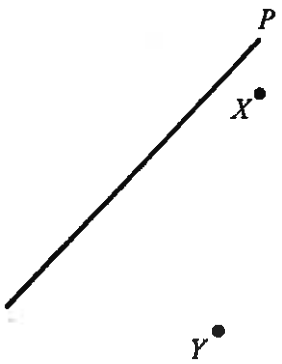
Answer (c)(i) ..... [1]

(ii) The mean of **all the twenty** numbers is 19.2.

Calculate the mean of the added thirteen numbers.

Answer (c)(ii) ..... [3]

11 The diagram shows the position of the two houses X and Y and a path P.



Scale 1 cm represents 25 m

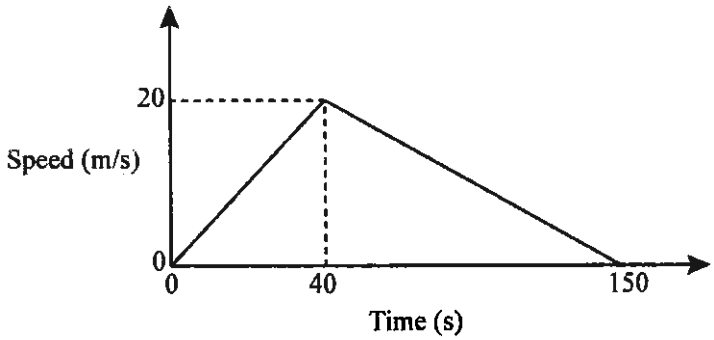
A public phone is installed such that it is

- less than 125 m from X
- nearer to Y than X
- more than 100 m from the path. [4]

Using ruler and compasses only, find the region where the public phone can be placed.

Label it T. [1]

12 The diagram shows the speed-time graph of a toy truck.



Calculate

(a) the acceleration during the first 40 seconds,

Answer (a) ..... [1]

(b) the time, in seconds, it takes to reach the speed of 15 m/s for the first time,

Answer (b) ..... s [2]

(c) the total distance travelled in metres.

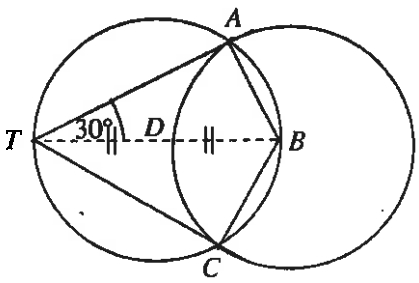
Answer (c) ..... m [2]

13 The diagram shows two circles, one with centre B and another centre D.

The two circles intersect at A and C.

T is the point on one circle from which TA and TC are tangents to the other circle.

Angle $ATB = 30°$



NOT TO SCALE

(a) Explain why the two circles are equal.

Answer (a) .....

..... [1]

(b) Find

(i) angle ATC,

Answer (b)(i) angle ATC = ... [1]

(ii) the reflex angle ABC.

Answer (b)(ii) Reflex angle ABC = ... [2]

14 f is directly proportional to n^2.

(a) **(i)** Given that $f = 6$ when $n = 2$.

Find an equation for f in terms of n.

Answer (a)(i) ..... [2]

(ii) Find f when $n = 8$.

Answer (a)(ii) ..... [1]

(b) Another number, p, is directly proportional to $2n^2$.

Show that f is directly proportional to p.

[3]

EXAMINATIONS COUNCIL OF LESOTHO
Lesotho General Certificate of Secondary Education

CANDIDATE NAME											
CENTRE NUMBER						CANDIDATE NUMBER					

MATHEMATICS **0178/04**

Paper 4 (Extended) **October/November 2015**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic Calculator
Geometrical Instruments
Tracing Paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π use either your calculator value or 3.142.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

This document consists of **20** printed pages.

1 **(a)** Solve the equation.

$(3x-2)(6x+1)=0$

Answer (a) $x =$ .... or $x =$ .... [2]

(b) Factorise completely.

$2a^2+6a-ab-3b$

Answer (b) ..... [2]

(c) Express as a single fraction in its simplest form.
Show all your working.

$\frac{1}{p-2}-\frac{2}{4p+3}$

Answer (c) ..... [3]

(d) Lineo runs at a rate of 170 m/min and walks at a rate of 90 m/min.
Lineo leaves home and takes 6 minutes, by running and walking, to reach a bus stop.
Lineo runs for t minutes.

(i) Find, in terms of t, an expression for

(a) the number of minutes she walks,

Answer (d)(i)(a) ..... min [1]

(b) the distance she runs,

Answer (d)(i)(b) ..... m [1]

(c) the distance to the bus stop.

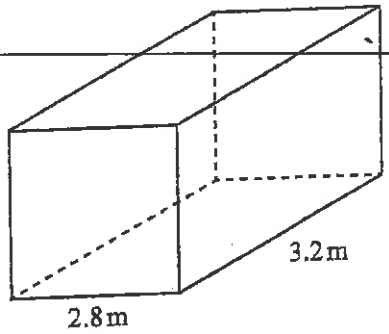
Answer (d)(i)(c) ..... m [2]

(ii) The distance to the bus stop is 740 m.

Find the value of t.

Answer (d)(ii) $t =$ ..... [2]

2 The diagram shows a tank in the shape of a cuboid.



NOT TO SCALE

(a) Calculate the volume of water that can fill up the tank.

Answer (a) ..... m^3 [2]

(b) The outside surface of the tank is to be painted.
(The top and bottom are not included.)

Calculate the area to be painted.

Answer (b) ..... m^2 [3]

(c) 1 litre of paint covers $3\,m^2$.
The cost of a 5 litre container of paint is M90.00.

(i) Find the number of 5 litre containers needed.

Answer (c)(i) ..... [3]

(ii) Calculate the total cost of paint needed.

Answer (c)(ii) M ..... [1]

3 (a) The table below shows the marks scored by a group of students in a test.

Marks	5	6	7	8	9	10
Frequency	2	8	5	6	4	2

Find

(i) the mode,

Answer (a)(i) ..... [1]

(ii) the median,

Answer (a)(ii) ..... [2]

(iii) the mean.

Answer (a)(iii) ..... [3]

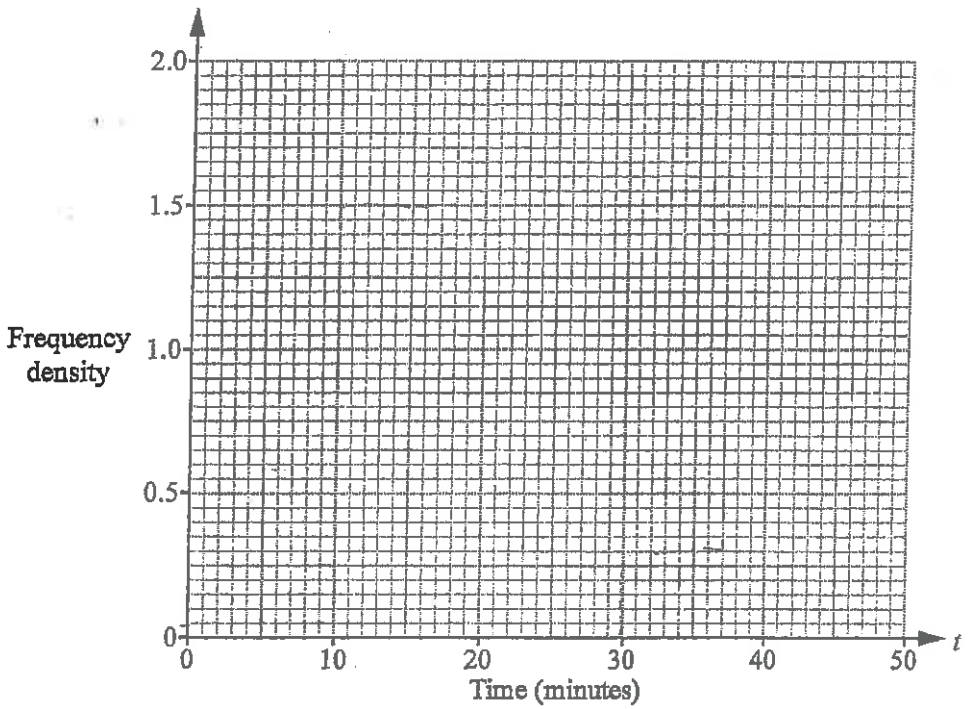
(b) The table below shows the time (t minutes) taken by the students to complete their homework.

Time (t minutes)	$0 < t \leqslant 10$	$10 < t \leqslant 20$	$20 < t \leqslant 30$	$30 < t \leqslant 40$
Frequency	2	15	7	3

(i) Calculate an estimate of the mean.

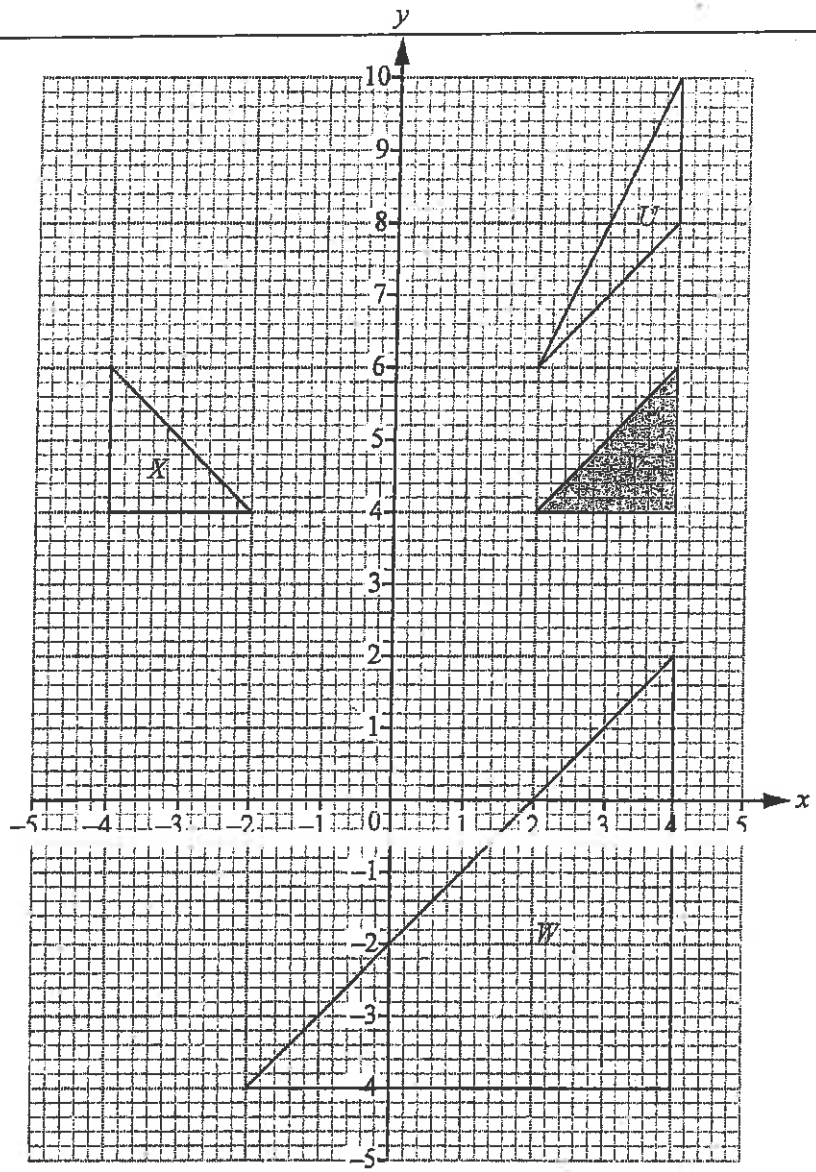
Answer (b)(i) ..... min [4]

(ii) On the grid, draw a histogram to show this information.



[3]

4 The diagram shows triangles *U, V, W, X*.



(a) Describe fully the **single** transformation that maps

(i) triangle V onto triangle X,

Answer(a)(i) .....

..... [2]

(ii) triangle V onto triangle W,

Answer (a)(ii) .....

..... [3]

(iii) triangle V onto triangle U.

Answer (a)(iii) .....

..... [3]

(b) On the same grid,

(i) translate triangle V using the vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and label the image V_1, [2]

(ii) rotate triangle V through 180° about the point (1, 3) and label the image V_2. [2]

(c) Find the matrix which represents the transformation that maps triangle V onto triangle X.

Answer (c) ..... [2]

5 (a) Factorise $x^2 - 18x + 81$.

Answer (a) ..... [1]

(b) Thabo bought x eggs at y maloti per dozen.
He sold each egg for P maloti.

Write an expression, in terms of x, y and P, for the profit he made.

Answer (b) M ..... [2]

(c) Solve the equation $(2x - 3)(x - 4) = 18$.

Answer (c) $x =$ ..... or $x =$ ..... [5]

(d) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

(i) Find R when $R_1 = 4$ and $R_2 = 6$.

Answer (d)(i) R = ... [2]

(ii) Make R_2 the subject of the formula.

Answer (d)(ii) R_2 = ... [3]

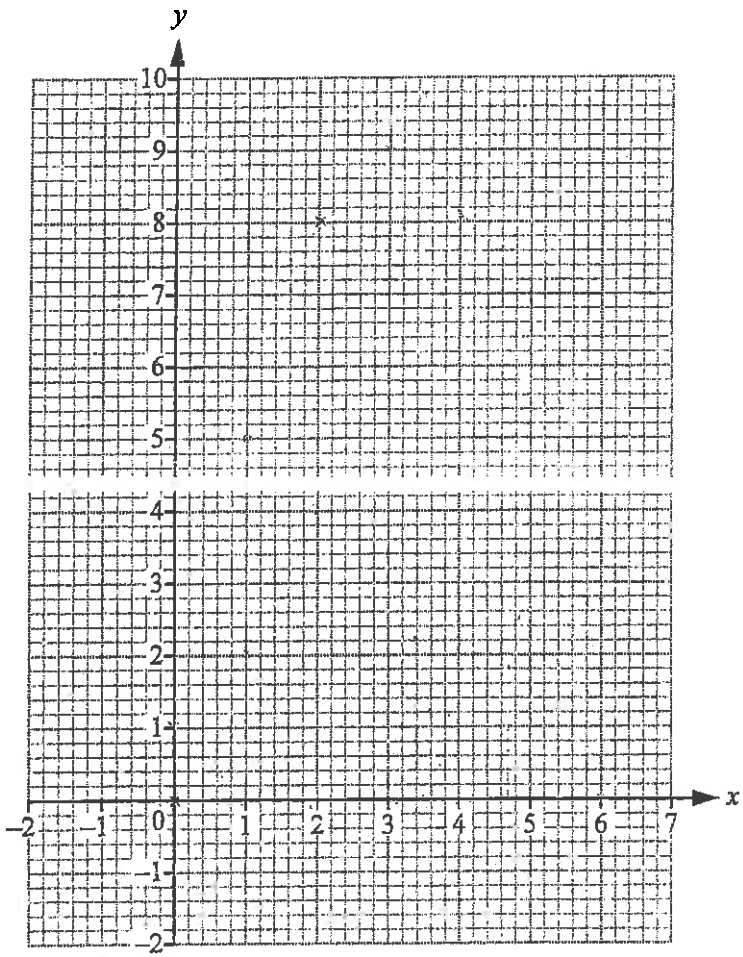
6 (a) $f(x) = 6x - x^2$

(i) Complete the table for $f(x)$.

x	0	1	2	3	4	5	6
$f(x)$	0			9			0

[2]

(ii) On the grid, draw the graph of $y = f(x)$ for $0 \leqslant x \leqslant 6$.



[3]

(iii) Use your graph to solve the equation $f(x) = 6$.

Answer (a)(iii) $x =$ ..... or $x =$ ..... [2]

(iv) By drawing a tangent at the point where $x = 4$, estimate the gradient of the graph of $y = f(x)$ when $x = 4$.

Answer (a)(iv) ..... [3]

(b) $g(x) = 2^x$

(i) Complete the table for $g(x)$.

x	0	1	2	3	3.3
$g(x)$				8	9.8

[2]

(ii) On the same grid opposite, draw the graph of $y = g(x)$ for $0 \leqslant x \leqslant 3.3$. [3]

(c) Use your graphs to find x when $f(x) = g(x)$.

Answer (c) $x =$ ..... or $x =$ ..... [2]

7 (a) A clearance sale at a clothing shop lasts for four days only.
The sale starts on Thursday and ends on Sunday.
For each day of the sale, prices are reduced by 10% of the price on the previous day.

(i) On Wednesday before the start of the sale, a dress was M199.
Thato bought it on Friday.

How much did Thato pay for the dress?

Answer (a)(i) M..... [3]

(ii) Lineo bought the same dress as that of Thato on Sunday.

How much less did Lineo pay than Thato?

Answer (a)(ii) M..... [3]

(iii) On Thursday, Neo paid M360 for his pair of trousers.

What was the price on Wednesday?

Answer (a)(iii) M..... [3]

(iv) How many days would it take for the original prices to be reduced by more than 25%?

Answer (a)(iv) ..... [3]

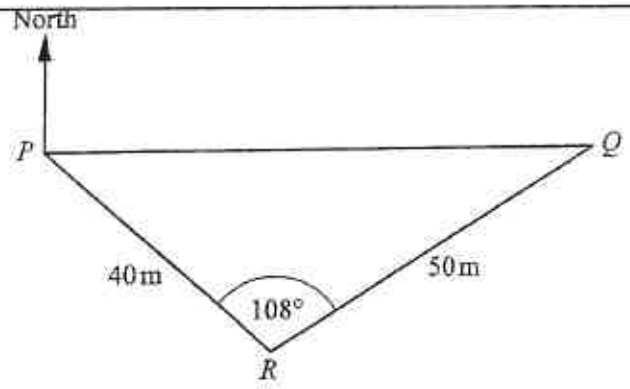
(b) There are two companies from which Neo can borrow money.
Company A: charges 1% compound interest per month.
Company B: charges 12% simple interest per annum.

Neo borrows M800.00 for two years.

Calculate the difference in interest Neo would have to pay if he borrows money in either of the two companies.

Answer (b) M ..... [4]

8 The diagram shows the positions of two houses at P and Q.
The point R represents the position of the electrical pole which is 40 m from P and 50 m from Q.
Angle $PRQ = 108°$.



NOT TO SCALE

(a) Calculate PQ, the distance between the two houses.

Answer (a) ..... m [4]

(b) Calculate the area of triangle PQR.

Answer (b) ..... m^2 [2]

(c) Q is due East of P.

Calculate the bearing of R from P.

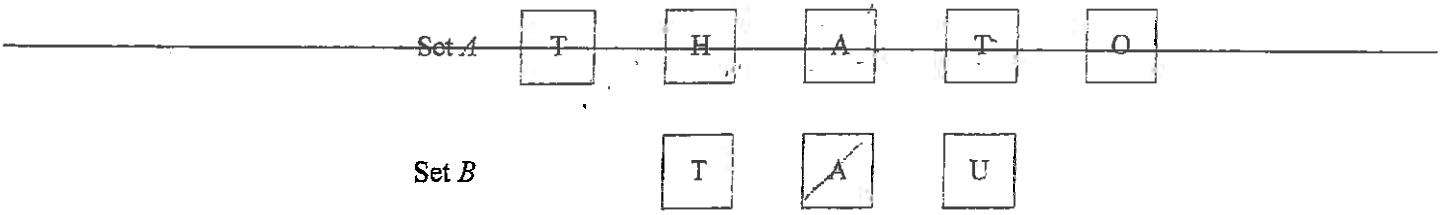
Answer (c) ..... [4]

(d) Another house is constructed along PQ such that it is at the shortest distance from R.

Calculate this shortest distance.

Answer (d) ..... m [2]

9 The diagram shows two sets of cards.



(a) One card is chosen at random from Set *A* and replaced.

(i) Write down the probability that the card chosen shows the letter A.

Answer (a)(i) ..... [1]

(ii) Write down the probability that the card chosen does not show the letter A.

Answer (a)(ii) ..... [1]

(iii) If this is carried out 200 times, write down the expected number of times the card chosen does not show the letter A.

Answer (a)(iii) ..... [1]

(iv) Write down the probability that the card chosen shows the letter H.

Answer (a)(iv) ..... [1]

(b) Two cards are chosen at random, **without** replacement, from Set *A*.

(i) Find the probability that both cards show the letter T.

Answer (b)(i) ..... [2]

(ii) Find the probability that one card shows the letter T and one card shows the letter H.

Answer (b)(ii) ..... [3]

(c) One card is chosen at random from Set *A* and one card is chosen at random from Set *B*.

Find the probability that at least one of the two cards shows the letter T.

Answer (c) ..... [3]

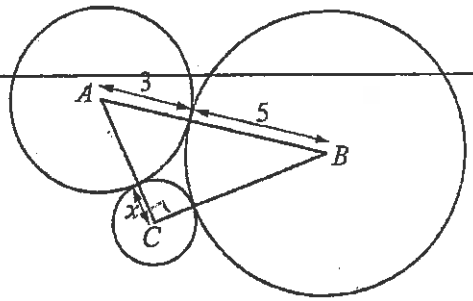
(d) A card is chosen at random, **without** replacement, from Set *B* until the letter shown is T.

Find the probability that this does not happen until the 3rd card is chosen.
Show your working.

Answer (d) ..... [2]

Question 10 is printed on the next page.

10 The diagram shows an arrangement of three circles which are touching.



NOT TO SCALE

The radii of the circles, centre A and centre B, are 3 cm and 5 cm respectively.
The radius of the circle centre C is x cm.

(a) Given that angle $ACB = 90°$, write down an equation in x and show that it reduces to $x^2 + 8x - 15 = 0$.

Answer (a)

[3]

(b) Solve the equation $x^2 + 8x - 15 = 0$.
Show all your working and give each answer correct to 2 decimal places.

Answer (b) $x =$ ..... or $x =$ ..... [4]

(c) Write down the lengths of AC and BC.

Answer (c) $AC =$ ..... cm and $BC =$ ..... cm [2]

EXAMINATIONS COUNCIL OF LESOTHO
Lesotho General Certificate of Secondary Education

CANDIDATE NAME													
CENTRE NUMBER						CANDIDATE NUMBER							

MATHEMATICS **0178/04**

Paper 4 (Extended) **May/June 2017**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic Calculator
Geometrical Instruments
Tracing Paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
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If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π use either your calculator value or 3.142.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

This document consists of **18** printed pages and **2** blank pages.

1 The first three diagrams in a sequence are shown below.
The diagrams are made up of dots and lines.
Each line is one centimetre long.

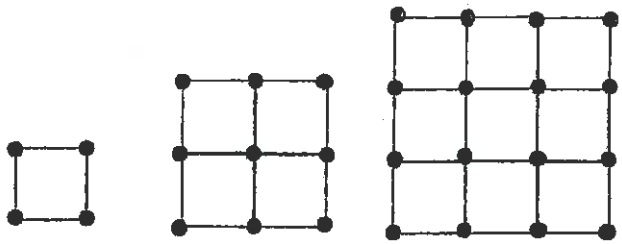


Diagram 1 Diagram 2 Diagram 3 Diagram 4

(a) Draw the next diagram in the sequence. [1]

(b) The table shows some information about the diagrams.

Diagram	1	2	3	4		*n*
Perimeter	4	8	12	*s*		*v*
Area	1	4	9	16		*w*
Number of lines	4	12	24	*t*		*x*

(i) Write down the values of s and t.

Answer (b)(i) s = ...

t = ... [2]

(ii) Write expressions for v, w and x, in terms of n.

Answer (b)(ii) v = ...

w = ...

x = ... [4]

(iii) Find the perimeter of the shape in Diagram 20.

Answer (b)(iii) ..... [1]

(c) The **total number** of lines in the first n diagrams is given by the expression

$$\frac{2}{3}n^3 + hn^2 + kn.$$

(i) Show that

(a) $h + k = \frac{10}{3}$ for $n = 1$,

(b) $4h + 2k = \frac{32}{3}$ for $n = 2$. [1]

[2]

(ii) Find

(a) the values of h and k,

Answer (c)(ii)(a) $h =$ ....

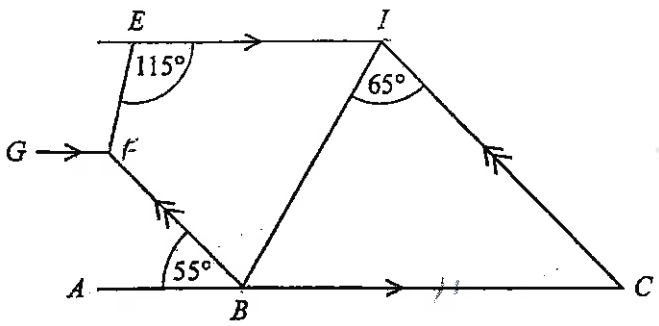
$k =$ .... [3]

(b) the total number of lines in the first 12 diagrams.

Answer (c)(ii)(b) ..... [1]

2 In the diagram, the straight line ABC is parallel to GF and ED.
BF and CD are also parallel.
Angle $ABF = 55°$, angle $DEF = 115°$ and angle $BDC = 65°$.

NOT TO SCALE



(a) State, with reasons, the value of

(i) angle BCD,

.. ....° because ....

..... [2]

(ii) angle ABD,

....° because ....

.... [2]

(iii) angle BFG.

.....° because ....

.. .. [2]

(b) ED is produced to the point H such that $DH = BC$.

State with the reason the name of the quadrilateral $BDHC$.

... because ... .

..... [2]

3 **(a)** Rearrange the formula to make a the subject.

$$P = \frac{y^2 + a}{y + a}$$

Answer (a) $a =$ ..... [3]

(b) Factorise fully.

$$(x^2 - y^2) - (x - y)^2$$

Answer (b) ..... [3]

(c) Simplify.

(i) $\dfrac{y - 2}{2y^2 - 3y - 2}$

Answer (c)(i) ..... [3]

(ii) $\dfrac{ax - ab + bx + b^2}{ax^2 - abx}$

Answer (c)(ii) ..... [3]

4 (a) Neo buys a roof-bike which has a price of M 7200.
He pays 60% of this price and then pays M800 per month for 6 months.

(i) How much does Neo pay altogether?

Answer (a)(i) M ..... [2]

(ii) How much more or less than the original price does Neo pay for the bike?

Answer (a)(ii) M ..... [1]

(iii) Express your answer in **part (a)(ii)** as a percentage of M7200.

Answer (a)(iii) ....% [2]

(b) Tau pays M8075 for a giant-bike in a sale.
The original price had been reduced by 15%.

Calculate the original price of the giant-bike.

Answer (b) M ..... [3]

(c) Two brothers, Mpho and Lefa, invest money at two different Banks, A and B.
Mpho invests M 3000 at Bank A at 5% **simple** interest per year.
Lefa invests M 3000 at Bank B at 4.9% **compounded** interest per year.

(i) How much will Mpho have at the end of 4 years?

Answer (c)(i) M .... [2]

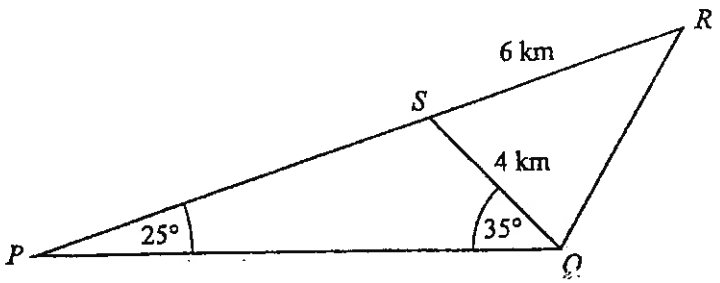
(ii) Who will have more money after 4 years?
Support your answer with working.

Answer (c)(ii) ... . [3]

(iii) How much more will one brother have than the other?

Answer (c)(iii) M ... . [1]

5 The diagram shows the positions P, Q, R and S of four locations on an island.
P, S and R are on a straight line.
$QS = 4$ km, $RS = 6$ km, angle $SPQ = 25°$, angle $PQS = 35°$ and Q is due East of P.



NOT TO SCALE

Calculate

(a) PQ,

Answer (a) ..... km [3]

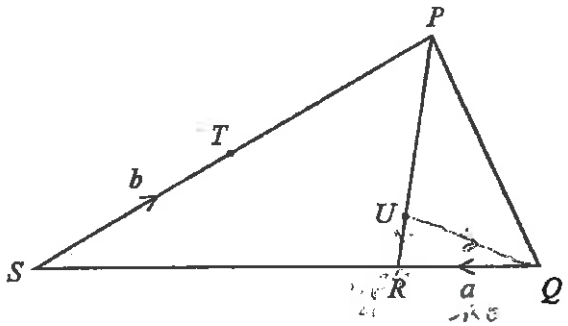
(b) QR

Answer (b) .... km [4]

(c) the area of the triangle QRS.

Answer (c) .... km^2 [2]

6 In the diagram, $\overrightarrow{QR} = \mathbf{a}$ and $\overrightarrow{ST} = \mathbf{b}$.
R is the point on QS such that $\overrightarrow{QS} = 3\overrightarrow{QR}$.
U is the point on RP such that $\overrightarrow{RP} = 4\overrightarrow{RU}$.
T is the midpoint of SP.



(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{RS}$,

Answer (a)(i) .... [1]

(ii) $\overrightarrow{RP}$

Answer (a)(ii) .... [1]

(iii) $\overrightarrow{UQ}$,

Answer (a)(iii) ..... [2]

(iv) $\overrightarrow{TQ}$

Answer (a)(iv) .... [1]

(b) Write down two facts about the points T, U and Q.

Answer (b) .....

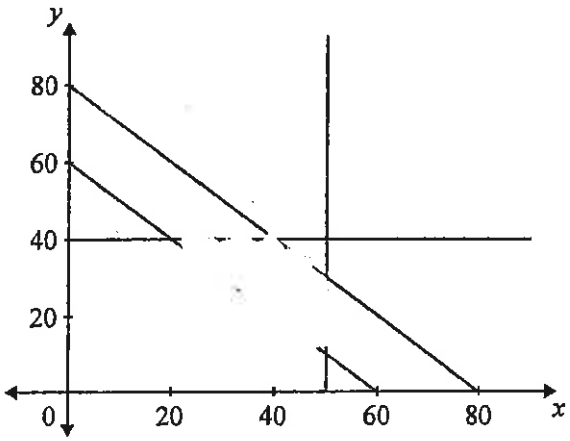
..... [2]

7 A factory makes two types of jeans, type A and type B.
Each month, x of type A and y of type B jeans are made.

The following constraints control the daily production:

- Not more than 50 jeans of type A can be made.
- Not more than 40 jeans of type B can be made.
- The total number of jeans must be at least 60.
- The maximum total number of jeans that can be made is 80

The diagram shows the four constraints.



NOT TO SCALE

One of the constraints is $x + y \leqslant 80$.

(a) Write down in terms of x and/or y the other three constraints.

Answer (a) .....

.....

..... [4]

(b) On the diagram, shade the region that satisfies all the constraints. [1]

(c) $y = -2x + \frac{P}{150}$ is the function that represents the profit, P, in Maloti.

Find the profit of each type of pair of jeans.

Answer (c) Type A. .....

Type B. [2]

(d) How many of each type of jeans should be produced per day to maximise profit?

Answer (d) Type A ....

Type B .... [2]

(e) What is the maximum profit?

Answer (e) M .... [1]

(f) Explain how the profit would be affected if the profit function was $y = -x + \frac{P}{150}$.

Answer (f) ..... [1]

8 The numbers 0, 0, 1, 1, 1, 2, p, 9, 10, 13, q, 16 are in order.
Their mean is 6 and their median is 3.5.

(a) State the mode.

Answer (a) ..... [1]

(b) Find the range.

Answer (b) ..... [1]

(c) Find the value of

(i) p,

Answer (c)(i) $p =$ ..... [1]

(ii) q.

Answer (c)(ii) $q =$ ..... [2]

(d) Find the probability that a number chosen at random is 0 or 1.

Answer (d) ..... [1]

(e) One number is chosen and not replaced.
A second number is then chosen.

Calculate the probability that the two numbers are both 0 or both 1.

Answer (e) ..... [3]

9 The table shows the time (t seconds) taken by 100 candidates to answer a given question.

t	$0 < t \leqslant 20$	$20 < t \leqslant 30$	$30 < t \leqslant 40$	$40 < t \leqslant 50$	$50 < t \leqslant 60$	$60 < t \leqslant 80$
Frequency	10	10	43	22	7	8

(a) Calculate an estimate of the mean time taken.
Show your working.

Answer (a) .... [4]

(b) The data is regrouped to give the following table.

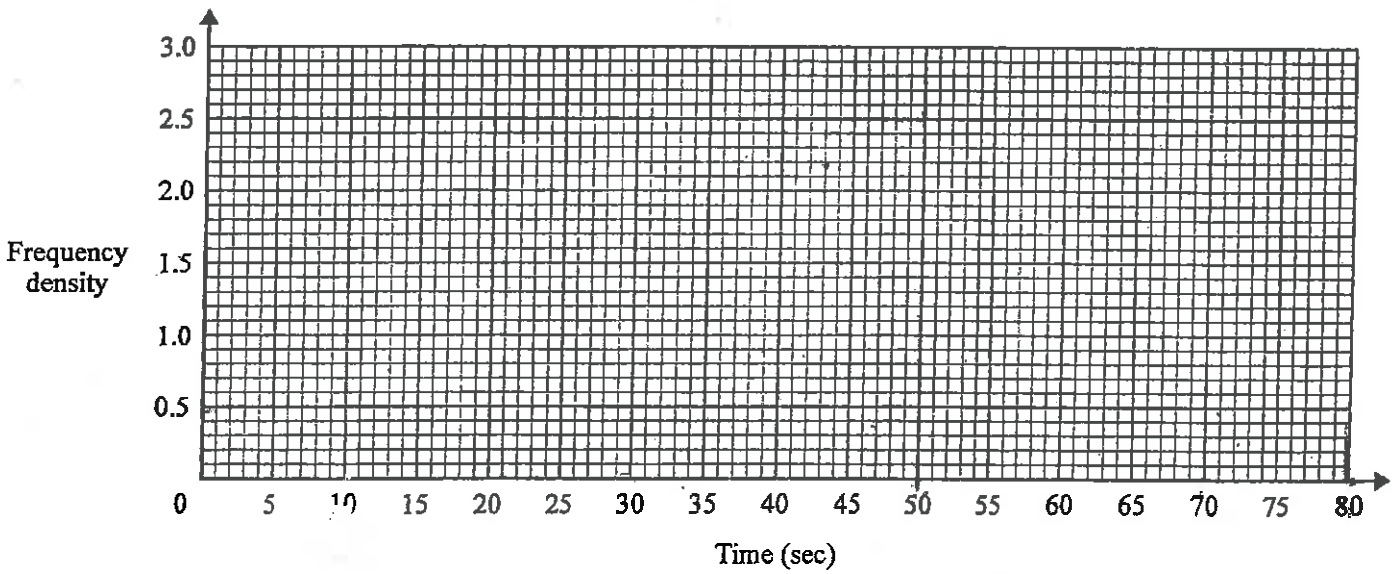
t	$0 < t \leqslant 20$	$20 < t \leqslant 50$	$50 < t \leqslant 80$
Frequency	10	v	w

(i) Write the values of v and w.

Answer (b)(i) v = ...

w = ... [2]

(ii) On the grid, draw a histogram which shows the information in the table in **part (b)**.



[3]

10 Thabiso runs 34 km at an average speed of x km/h.

(a) Write down, in terms of x, an expression for the time taken.

Answer (a) ..... h [1]

(b) Lipuo runs 34 km at an average speed of 2 km/h greater than Thabiso's speed.

Write down, in terms of x, an expression for the time taken by Lipuo.

Answer (b) .... [1]

(c) It is given that Lipuo took **15 minutes** less than Thabiso to compete the 34 km.

Write down an equation in terms of x and show that it simplifies to

$x^2 + 2x - 272 = 0$.

[3]

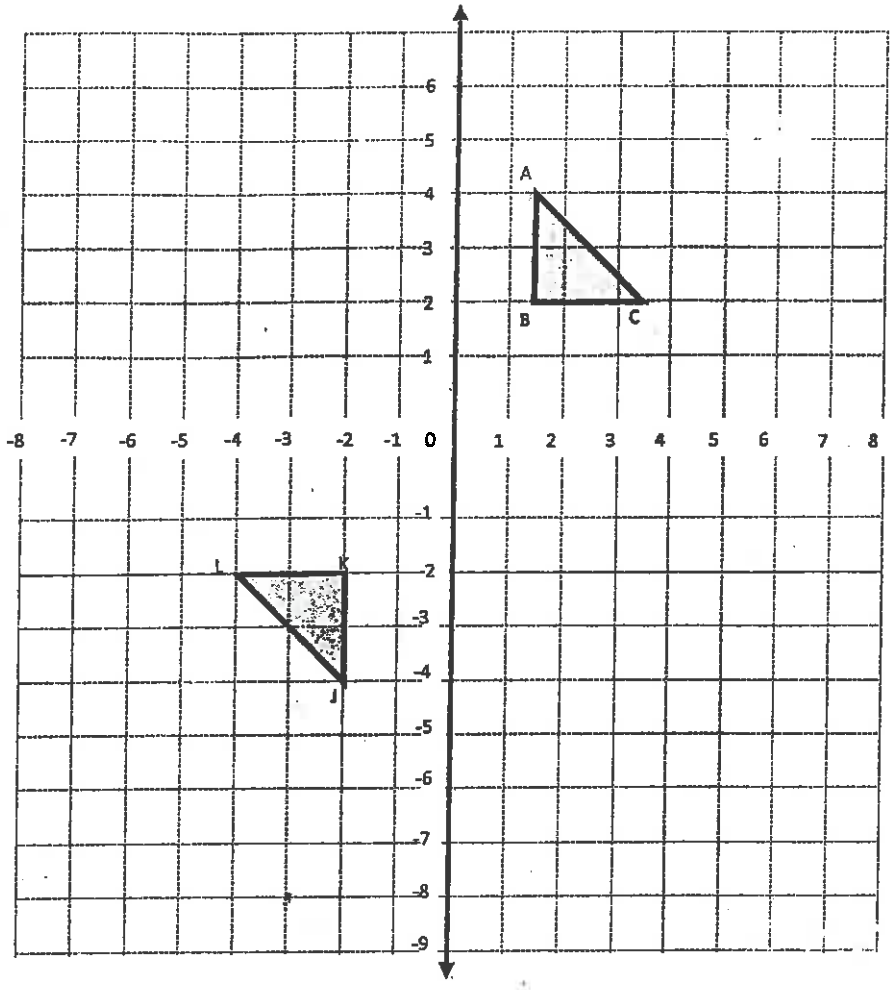
(d) Solve the equation $x^2 + 2x - 272 = 0$.
Give your answers correct to one decimal place

Answer (d) $x =$ .... or $x =$ ... ... [4]

(e) Calculate, in hours and minutes, the time taken by Thabiso to complete the 34 km.

Answer (e) .... h .... min [2]

11



(a) A reflection in the line $x = -1$ maps triangle ABC onto triangle DEF.

Draw and label triangle DEF. [2]

(b) A shear with shear factor 2 and invariant line $y = 2$ maps triangle ABC onto triangle GHI.

Draw and label triangle GHI. [2]

(c) Describe fully the single transformation which maps triangle ABC onto triangle JKL.

Answer (c) .....

.....

.....

..... [3]

(d) Triangle JKL is mapped onto triangle MNP.
The vertices of triangle MNP are $M(-3, -8)$, $N(-3, -4)$ and $P(-5, -4)$.

Find a matrix which fully describes a single transformation which maps triangle JKL onto triangle MNP.

Answer (d) .... [2]

12 The functions f and g are defined as follows:

$$f : x \to \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \; x > 3, \quad \text{and} \quad g : x \to 2x - 3.$$

(a) Show that $f(x) = \frac{1}{x+1}$.

[3]

(b) Find

(i) $f^{-1}(x)$,

Answer (b)(i) .... [2]

(ii) $g^{-1}(x)$.

Answer (b)(ii) .... [2]

(iii) $fg(x)$,

Answer (b)(iii) .... [2]

(iv) $(fg)^{-1}(x)$.

Answer (b)(iv) .... [2]

(c) Solve $\mathrm{fg}(x) = \frac{1}{8}$.

Answer (c) $x =$ .... [2]

(d) Show that $(\mathrm{fg})^{-1}(x) = \mathrm{g}^{-1}\mathrm{f}^{-1}(x)$

[3]

EXAMINATIONS COUNCIL OF LESOTHO
Lesotho General Certificate of Secondary Education

CANDIDATE NAME																	
CENTRE NUMBER						CANDIDATE NUMBER											

MATHEMATICS **0178/04**

Paper 4 (Extended) **May/June 2018**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic Calculator
Geometrical Instruments
Tracing Paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π use either your calculator value or 3.142.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

This document consists of **19** printed pages and **1** blank page.

1 (a) Evaluate $\frac{1}{4}$ + 20%.

Give your answer as a fraction.

Answer (a) ..... [1]

(b) In 2014, the number of people affected by an epidemic was 72 350 600, correct to the nearest hundred.

(i) Write this number 72 350 600 in words.

Answer (b)(i) .....

..... [1]

(ii) Find the lower-bound to the number of people affected by the epidemic in 2014.

Answer (b)(ii) ..... [1]

(iii) Write the number 72 350 600 correct to the nearest hundred thousand.

Answer (b)(iii) ..... [1]

(c) Pule and Thabo receive a sum of money from their father and share it in the ratio 3:2.

Thabo receives M1430.40.

(i) Calculate the total amount they received from their father.

Answer (c)(i) M ..... [2]

(ii) Their father invests M6000 in company A and M5000 in company B.
I Company A pays simple interest at the rate of 4% per year.
II Company B pays compound interest at the rate of 5% per year.

From which company will he get more interest after 5 years and by how much?

Answer (c)(ii) ..... [5]

2 $\mathbf{A} = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix}$ $\mathbf{C} = (2 \quad -3)$.

Find the following matrices.

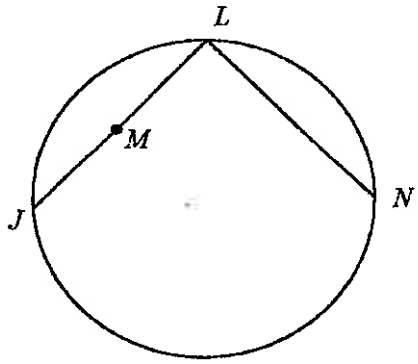
(a) CA

Answer (a) ( ) [2]

(b) $\mathbf{B}^{-1}$

Answer (b) ( ) [2]

3 **(a)** In the diagram, JL and LN are chords of a circle equidistant from the centre.
M is the mid-point of JL.



(i) Use accurate construction to find the centre of a circle and label it C. [2]

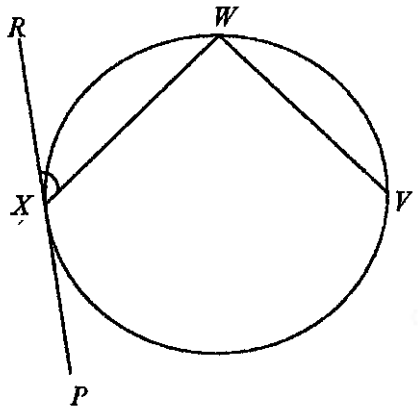
(ii) State the relationship between JM and LN.

Give a reason.

Answer (a)(ii) Relationship: ..... [1]

Reason ..... [1]

(b) The diagram shows a circle with two equal chords VW and WX.
PR is a tangent to the circle at a point X.



NOT TO SCALE

Reflex angle VWX is 270°.

Calculate the size of the marked angle WXR.

Answer (b) $\widehat{WXR}$ = ... [3]

4 (a) Simplify

$$\frac{x^2-1}{2x^2+5x+3}.$$

Answer (a) ..... [4]

(b) y varies inversely as the square root of x.

When $x = 16$, $y = 9$.

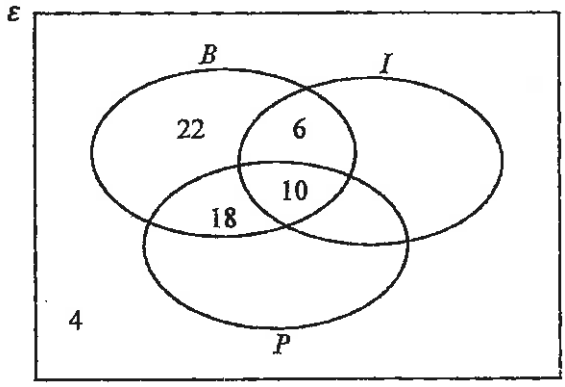
(i) Find the formula for y in terms of x.

Answer (b)(i) ..... [2]

(ii) Find the value of x when $y = 18$.

Answer (b)(ii) ..... [3]

5 In a survey, 60 students are asked whether they have Birth Certificates (B), Identity Documents (I) or Passports (P).
The results are represented in the Venn diagram.



(a) Find the number of students who have

(i) all the three documents,

Answer (a)(i) ..... [1]

(ii) Birth Certificates and Identity Documents only.

Answer (a)(ii) ..... [1]

(b) Find $n(B \cap P \cap I')$

Answer (b) ..... [1]

(c) Use set notation to represent the number of students who have Birth Certificates only.

Answer (c) ..... [1]

6 It is given that f and g are functions of x.

$f(x) = \frac{1}{x+2} + 3$ and $g(x) = 3 - 2x$

(a) Evaluate.

(i) $g(-7)$

Answer (a)(i) ..... [2]

(ii) $fg(3)$

Answer (a)(ii) ..... [3]

(b) Find the value of x for which

$f(x) = 0$.

Answer (b) ..... [2]

(c) Express in terms of x

(i) $g^{-1}(x)$,

Answer (c)(i) ..... [2]

(ii) $g(x)g(x)$.

Answer (c)(ii) ..... [2]

(d) Given that p is another function of x such that $p(x) = x^n$.

Find n if $p(x) = p^{-1}(x)$.

Answer (d) ..... [2]

7 Anna types x words in one minute.

(a) Show that Anna takes $\frac{12000}{x}$ seconds to type 200 words.

Answer (a) ..... [1]

(b) Puleng types 10 fewer words than Anna in one minute.

Write an expression, in terms of x for

(i) the number of words Puleng types in one minute,

Answer (b)(i) ..... [1]

(ii) the number of seconds Puleng takes to type 200 words.

Answer (b)(ii) ..... [1]

(c) It takes Puleng 100 seconds longer to type 200 words than it takes Anna.

(i) Write an equation in x to represent this information and show that it simplifies to $x^2 - 10x - 1200 = 0$.

Answer (c)(i) ..... [3]

(ii) Solve the equation $x^2 - 10x - 1200 = 0$.

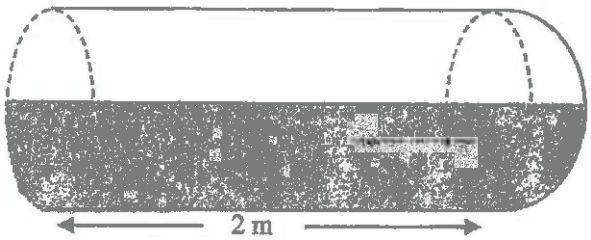
Answer (c)(ii) $x =$ ..... or $x =$ ..... [3]

(iii) Find the time taken, in minutes, for Puleng to type 3510 words.

Answer (c)(iii) ..... min [2]

8 The diagram shows a container made by joining a cylinder of radius 0.3 m and a hemisphere of the same radius.
The length of the cylinder is 2 m.

The container rests on a horizontal surface and it is exactly half filled with water.



NOT TO SCALE

[The volume of a sphere is $\frac{4}{3}\pi r^3$]

[The surface area of a sphere is $4\pi r^2$]

(a) Calculate the surface area of the container that is in contact with the water.

Answer (a) ..... m^2 [4]

(b) The container is now completely filled with water.

Calculate the total volume, in litres, of water in the container.
Note ($1000\ cm^3 = 1$ litre)

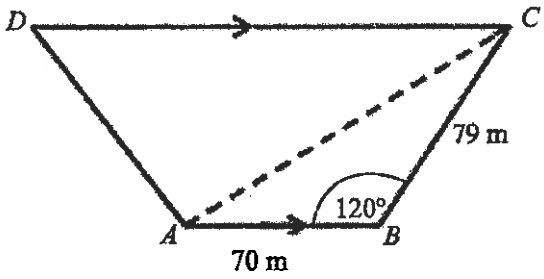
Answer (b) ..... *l* [4]

(c) Water is taken from the container at the constant rate of 10 litres per minute.

Find the length of time needed to draw 300 litres of water from the container.

Answer (c) ..... min [2]

9 The diagram represents a field, $ABCD$ in the shape of a quadrilateral.
AC is a straight footpath across the field and DC is parallel to AB.



NOT TO SCALE

$AB = 70$ m, $BC = 79$ m and $A\hat{B}C = 120°$.

(a) Calculate the distance AC.

Answer (a) ..... m [4]

(b) Calculate the area of triangle ABC.

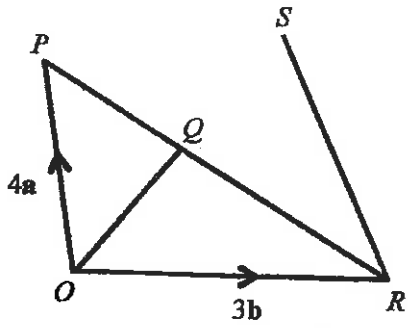
Answer (b) ..... m^2 [2]

(c) A farmer walks the shortest distance from point B to line DC.

Calculate this distance.

Answer (c) ..... m [2]

10 In the diagram $\overrightarrow{OR} = 3\mathbf{b}$, $\overrightarrow{OP} = 4\mathbf{a}$ and $\overrightarrow{RS} = 6\mathbf{a}$.
Q lies on PR such that PQ : QR = 2:3.



NOT TO SCALE

(a) Find the following in terms of **a** and **b**, giving each answer in its simplest form.

(i) $\overrightarrow{PR}$

Answer (a)(i) $\overrightarrow{PR}$ = ... [1]

(ii) $\overrightarrow{OQ}$

Answer (a)(ii) $\overrightarrow{OQ}$ = ... [2]

(b) Find $\overrightarrow{OS}$, in terms of **a** and **b** and hence show that $\overrightarrow{OQ} = \frac{2}{5}\overrightarrow{OS}$.

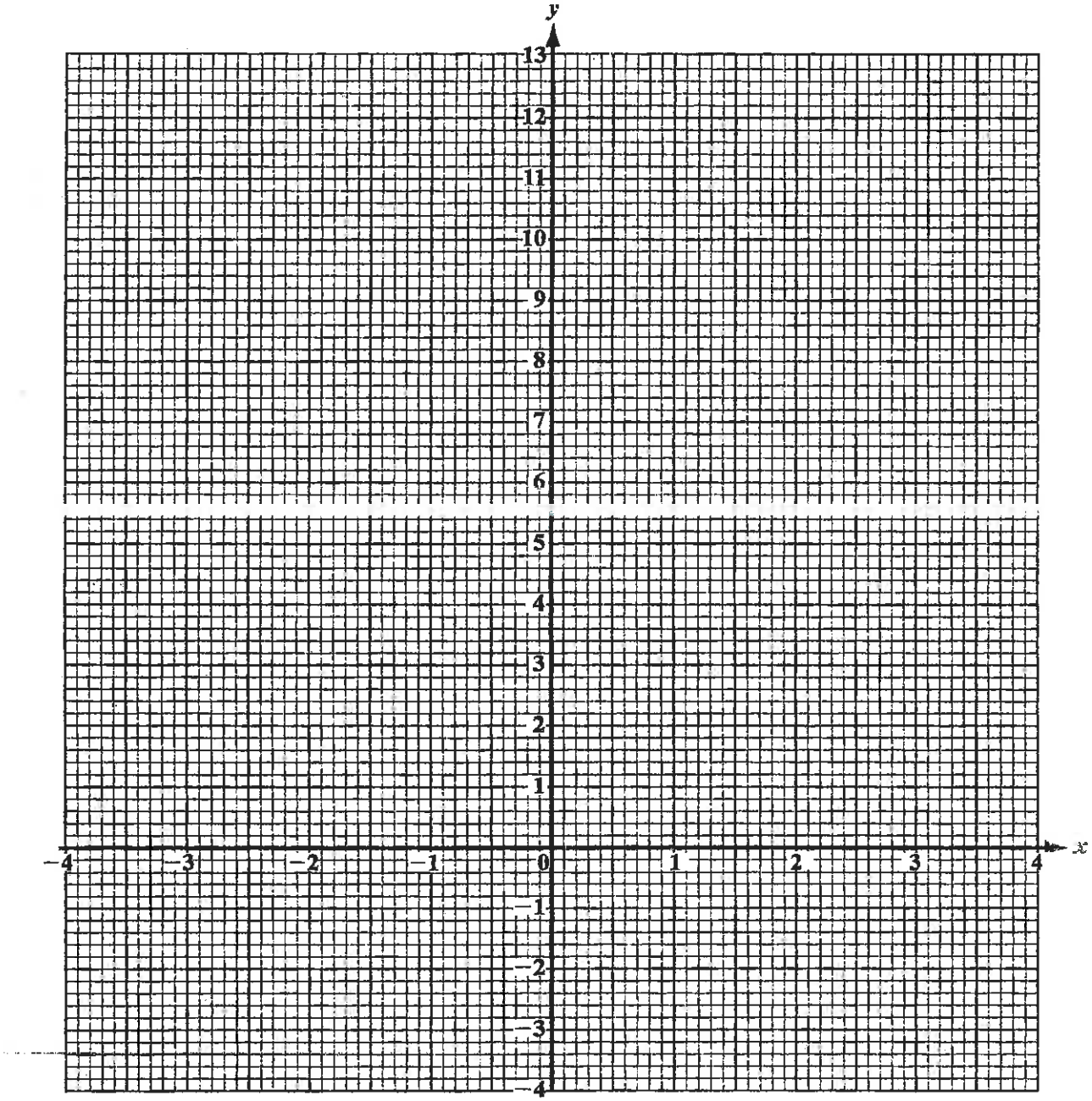
Answer (b)..... [3]

11 (a) Complete the table of values for $y = \frac{1}{3}x^3 + x^2 - 2x$.

x	−4	−3	−2	−1	0	1	2	3
y	2.7		5.3	2.7	0	−0.7		12

[2]

(b) On the grid, draw the graph of $y = \frac{1}{3}x^3 + x^2 - 2x$ for $-4 \leq x \leq 3$.



[4]

(c) Use the graph to solve the equation $\frac{1}{3}x^3+x^2-2x=4$.

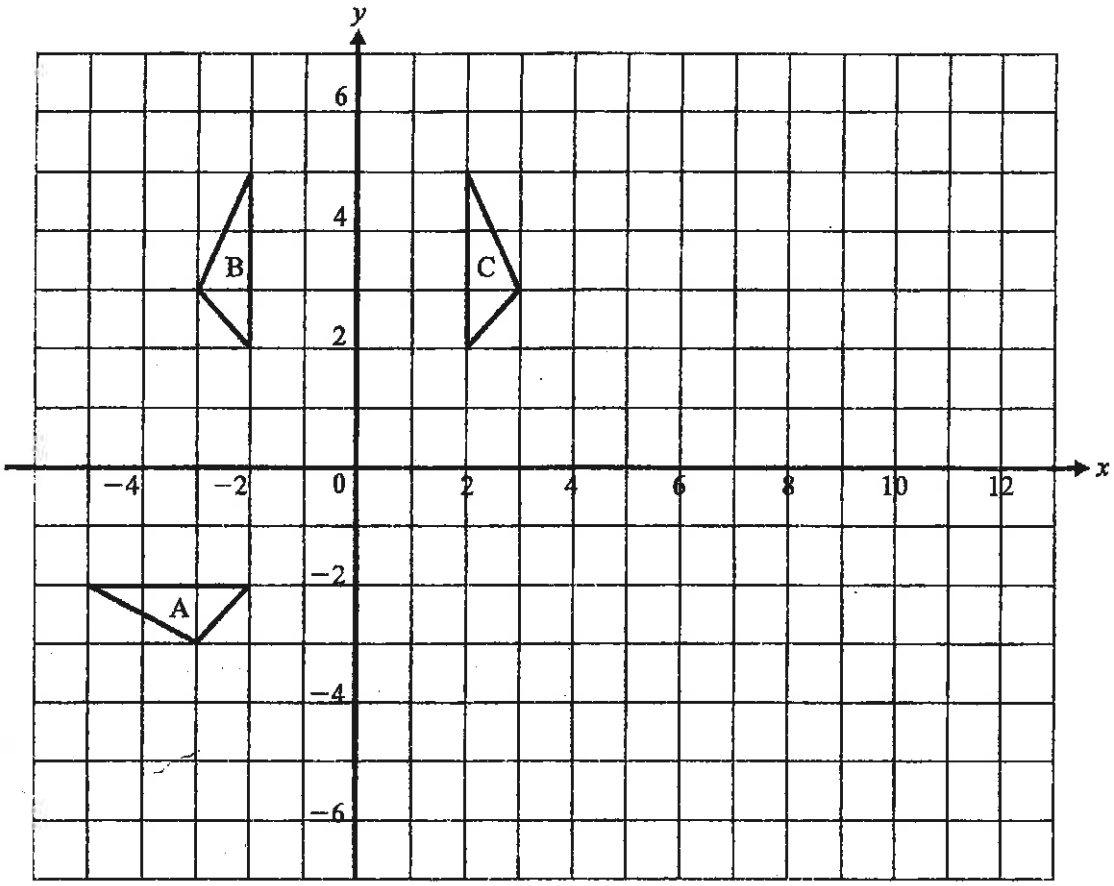
Answer (c) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(d) The equation $\frac{1}{3}x^3+x^2-7=5x$ can be solved by drawing a straight line on the grid.

Find the equation of this straight line in the form of $y = mx + c$.

Answer (d) $y =$ ..... [2]

12 The diagram shows triangles A, B and C.



(a) Describe **fully** the single transformation that maps triangle

(i) A onto C,

Answer (a)(i) .....

..... [2]

(ii) A onto B.

Answer (a)(ii) .....

..... [3]

(b) On the same grid, draw the image of triangle C after a shear, with invariant line $y = 0$ and shear factor 2. [2]

13 (a) There are 25 red marbles and x blue marbles in a box.
One marble is selected at random.

(i) Given that the probability that it is blue is $\frac{1}{6}$, calculate the value of x.

Answer (a)(i) $x =$ ..... [3]

(ii) Find the number of marbles that are in the box.

Answer (a)(ii)..... [1]

(b) In another box there are 27 green marbles and 13 yellow marbles.
Two marbles are selected at random without replacement.

Calculate the probability that

(i) both marbles are green,

Answer (b)(i) ..... [2]

(ii) there is at least one yellow marble.

Answer (b)(ii) ..... [2]

14 The table shows the masses, m grams, of 44 plant seeds.

Mass (g)	$10 \leq m < 20$	$20 \leq m < 25$	$25 \leq m < 35$	$35 \leq m < 50$	$50 \leq m < 55$
Frequency	8	5	11	18	2

(a) (i) Find the modal class.

Answer (a)(i) ..... [1]

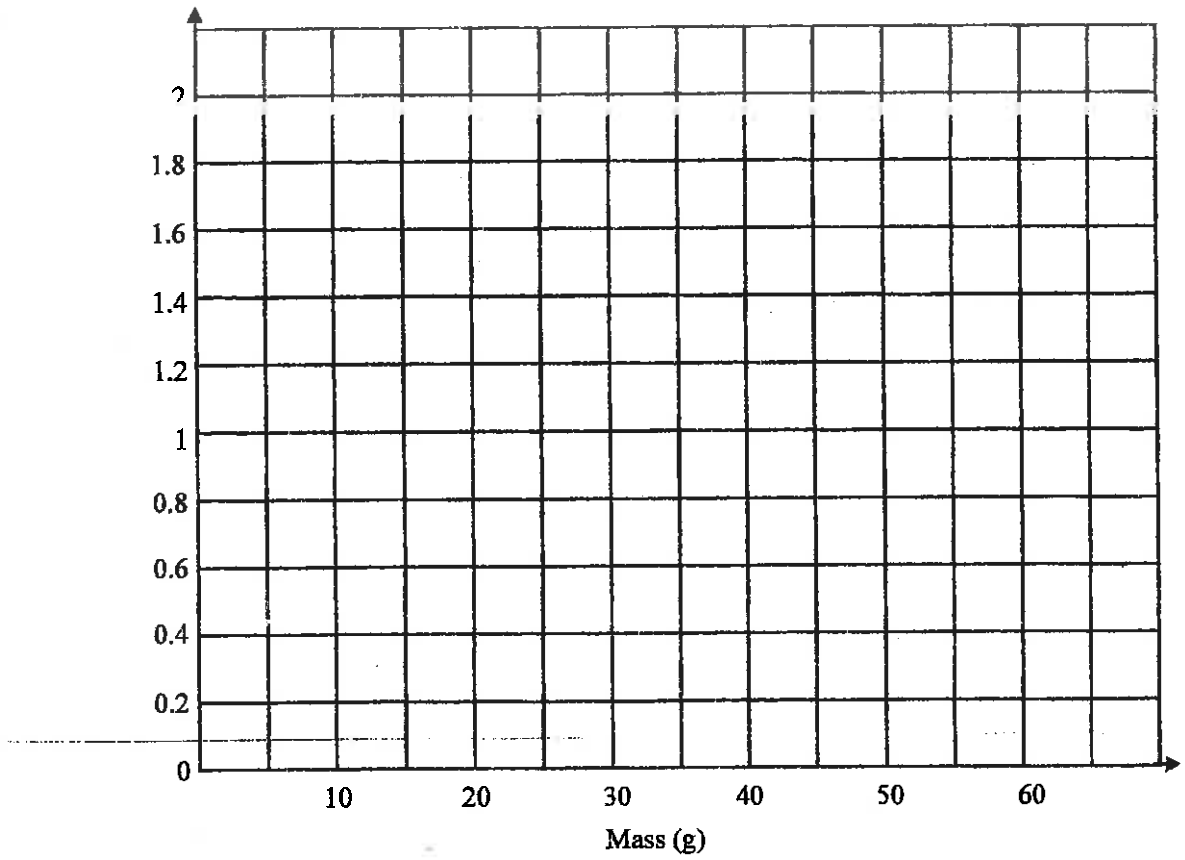
(ii) Find the class that contains the median.

Answer (a)(ii) ..... [1]

(iii) Calculate an estimate of the mean.

Answer (a)(iii) ..... g [4]

(b) Draw a histogram to represent the information in the table.



[4]

15 A school transports learners to a stadium using two types of vehicles (mini-buses and taxis). Each trip should have at least one mini-bus and one taxi but not more than six vehicles altogether.
A mini-bus can carry up to 60 learners while a taxi can carry up to 15 learners.

(a) (i) Given that every vehicle must carry the maximum number of learners, find the minimum number of learners that can be transported in one trip.

Answer (a)(i) ..... [1]

(ii) Taking b for the number of mini-buses used and t for the number of taxis used, write an expression for the number of learners that can be transported.

Answer (a)(ii) ..... [2]

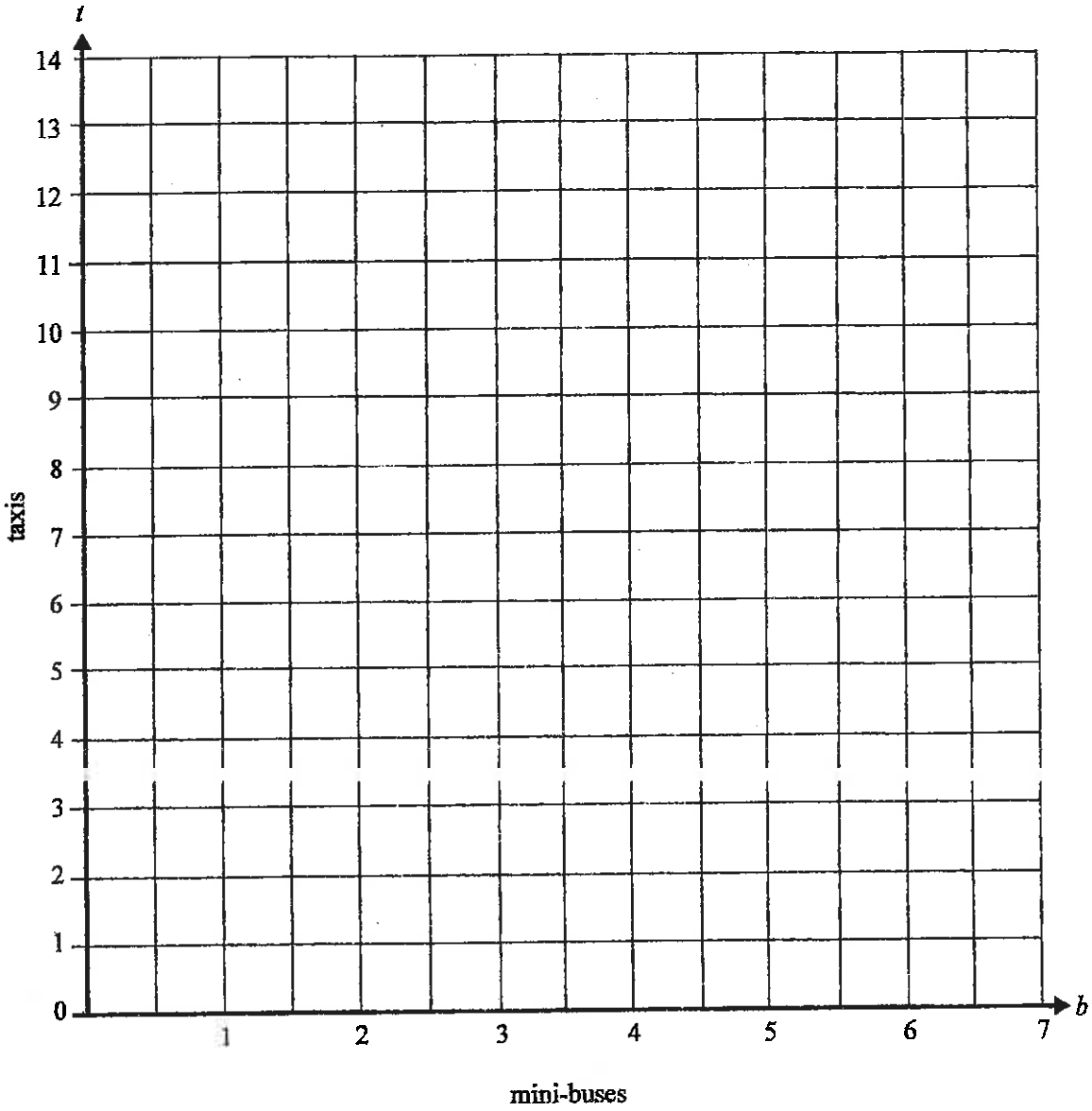
(b) The school has at most 180 learners to transport.

(i) Use this information to form an inequality in b and t

Answer (b)(i) ..... [1]

(ii) Two other inequalities defining region R, of the possible values of b and t are $t \geq 1$, $b \geq 1$ and $b + t \leq 6$.

Use the inequalities to represent the region R on the grid provided.



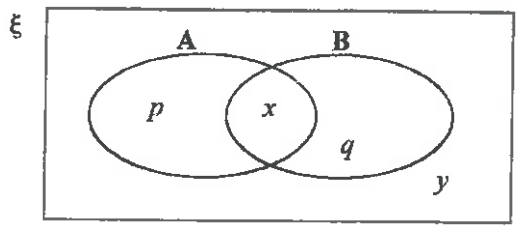
[4]

(c) Transport costs are $M800$ per mini-bus and $M400$ per taxi.

Find the least possible cost, to the school, of transporting the learners.

Answer (c) M ..... [3]

2 Given that $n(\xi) = 200$, $n(A) = 75$ and $n(B) = 35$.



(a) Express p in terms of x.

Answer (a) ..... [1]

(b) Find

(i) the smallest possible value of y,

Answer (b)(i) ..... [2]

(ii) the largest possible value of x,

Answer (b)(ii) ..... [1]

(iii) the value of q if $p = 45$.

Answer (b)(iii) $q =$ ..... [1]

3 **(a)** A car costs M120 000.
Limpho buys the car on hire purchase.
He pays 40% deposit and M1 365 monthly for 60 months.

(i) Find the amount of deposit paid.

Answer (a)(i) M ..... [1]

(ii) Find the extra cost of buying the car on hire purchase.

Answer (a)(ii) M ..... [2]

(iii) Calculate the percentage rate per year of simple interest Limpho is charged on hire purchase for the car.

Answer (a)(iii) ..... [3]

(b) M56 000 is invested at a rate of 3.2% **compound interest each quarter**.

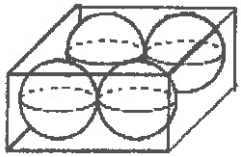
(i) Find, in its simplest form, an expression in terms of n, for the value of the investment after n years.

Answer (b)(i) M ..... [3]

(ii) Find the interest earned after 3 years.

Answer (b)(ii) M ..... [2]

4 The diagram shows 4 identical spherical balls packed into a box that is in the shape of a cuboid. The spheres are packed so that they touch two other spheres and four faces of the box. The radius of each sphere is 3 cm.



NOT TO SCALE

The volume V of a sphere radius r is $V = \frac{4}{3}\pi r^3$.

(a) State the number of vertices a cuboid has.

Answer (a) ..... [1]

(b) Calculate the volume of one ball.

Answer (b) ..... cm^3 [2]

(c) Calculate the volume of the box.

Answer (c) .....cm^3 [2]

(d) Find the volume of unoccupied space in the box.

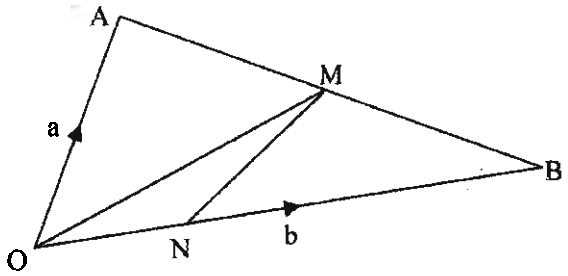
Answer (d) .....cm^3 [1]

(e) Find the percentage of the volume of the box that is **not** occupied by the balls.

Answer (e) ..... % [2]

5 In the diagram, O is the origin $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, and $ON : NB = 1 : 2$.

M is the midpoint of AB.



Express in terms of **a** and/or **b**, in its simplest form

(a) $\overrightarrow{AB}$,

Answer (a) $\overrightarrow{AB}$ = ... [1]

(b) the position vector of M,

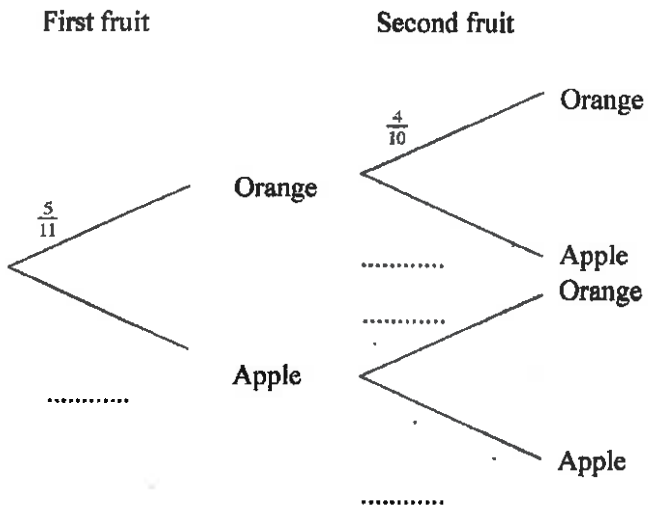
Answer (b) ..... [2]

(c) $\overrightarrow{MN}$.

Answer (c) $\overrightarrow{MN}$ = ... [3]

6 A box contains six apples and five oranges.
Mpho takes a fruit at random from the box without replacement.
He then takes a second fruit at random.

(a) Complete the tree diagram.



[3]

(b) Calculate the probability that Mpho takes

(i) two oranges,

Answer (b)(i) ..... [1]

(ii) at least one apple.

Answer (b)(ii) ..... [2]

(c) Mpho now takes six fruits, at random from the remaining without replacement.

Find the probability **that all** six fruits are oranges.

Answer (c) ..... [1]

7 On the 1st January 2000, Tau was x years old, Pitso was 5 years older than Tau while Neo was twice as old as Tau.

(a) Write expressions, in terms of x, for the ages of Pitso and Neo on the1st January 2000.

Answer (a) Pitso .....

Neo ..... [2]

(b) The product of Neo's age and Tau's age on the **1st January 2002** is the same as the square of Pitso's age on the **1st January 2000**.

Write down an equation in x and show that it simplifies to $x^2 - 4x - 21 = 0$.

Answer (b) ..... [4]

Answer (c)(i) ..... [3]

(ii) How old is Pitso on the 1st January 2002?

Answer (c)(ii) ..... [1]

(d) Neo's height, h metres, is one of the solutions of $h^2 + 8h - 17 = 0$.

(i) Solve $h^2 + 8h - 17 = 0$.

Show all your working and give your answers correct to 2 decimal places.

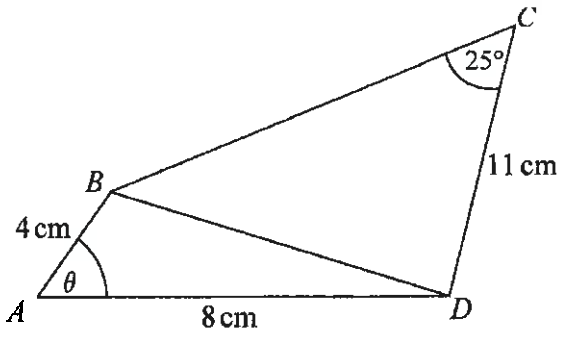
Answer (d)(i) h = ..., h = ... [4]

(ii) Write down Neo's height in centimetres.

Answer (d)(ii) ..... cm [1]

8 The diagram shows a quadrilateral $ABCD$.

$AB = 4\,\text{cm}$, $AD = 8\,\text{cm}$, $CD = 11\,\text{cm}$, $B\hat{C}D = 25°$ and $B\hat{A}D = \theta$.



NOT TO SCALE

(a) Show that $BD = \sqrt{16(5 - 4\cos\theta)}\,\text{cm}$.

[3]

(b) Let $\theta = 40°$.

(i) Find the value of sin CBD.

Answer (b)(i) sin CBD = ... [3]

(ii) Find the value of the acute angle of *CBD*.

Answer (b)(ii) Angle *CBD* = ... [1]

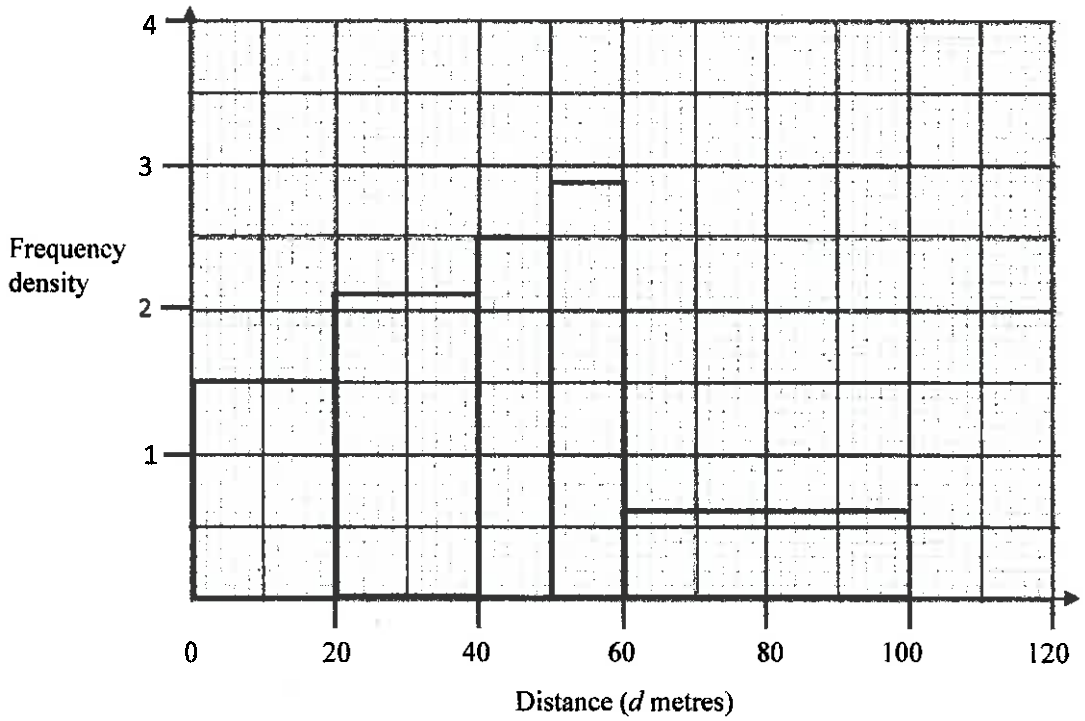
(iii) Work out the perimeter of *ABCD*.

Answer (b)(iii)..... cm [4]

(iv) Find the area of triangle *ABD*.

Answer (b)(iv)..... cm^2 [2]

9 The histogram shows the distance, d metres, ran by 150 students.



(a) Complete the table.

Distance (d m)	$0 < d \leqslant 20$	$20 < d \leqslant 40$	$40 < d \leqslant 50$	$50 < d \leqslant 60$	$60 < d \leqslant 100$
Frequency	30	42		20	

[2]

(b) Calculate an estimate of the mean.

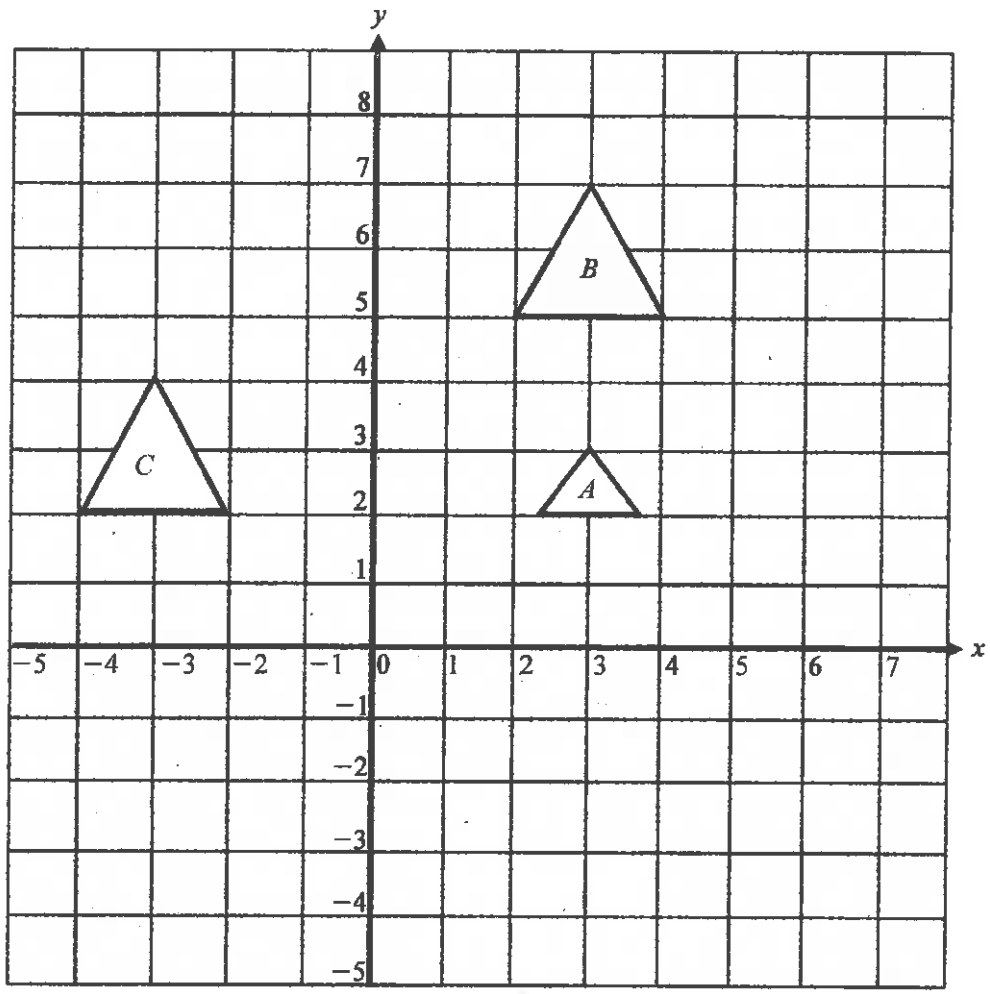
Answer (b) ..... m [4]

(c) 10% of the children ran further than y metres.

Calculate an estimate of y.

Answer (c) ..... [3]

10 The diagram shows triangles A, B and C.



(a) **(i)** Plot and label the point T(5,7) on the grid. [1]

(ii) Rotate the point T through 90° clockwise about the origin.
Label the image T'. [2]

(b) Describe fully the **single** transformation that maps

(i) triangle B onto triangle A,

.....

..... [3]

(ii) triangle B onto triangle C.

.....

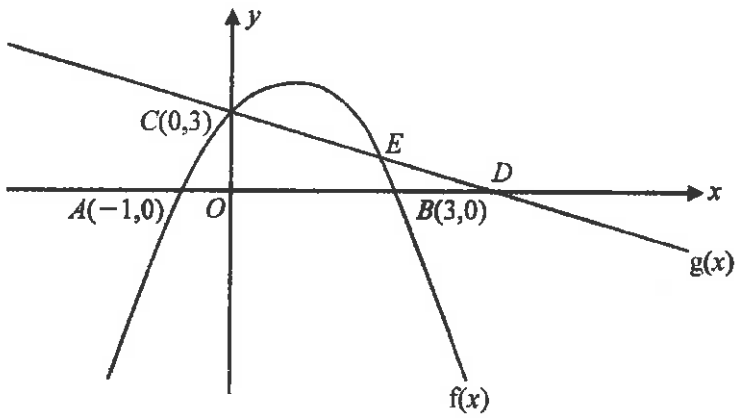
..... [2]

(c) On the grid, draw the image of

(i) triangle B after a reflection in the line $y = 2$, [2]

(ii) Triangle A after a stretch with scale factor 2 and invariant line they-axis. [2]

11 The diagram shows the parabola, f(x), and the straight line, g(x).
Points A, B, C and D are the intercepts on the axes.
E is the point of intersection of the two graphs.



(a) D is the image of B after B has been translated two units to the right.

Write the co-ordinates of point D.

Answer (a) (....., ...) [1]

(b) Find the equation of the straight line through C and D.

Give your answer in the form $y = mx + c$.

Answer (b) $y =$ ..... [2]

(c) Find the equation of the parabola in the form $y = ax^2 + bx + c$.

Answer (c) $y =$ ..... [4]

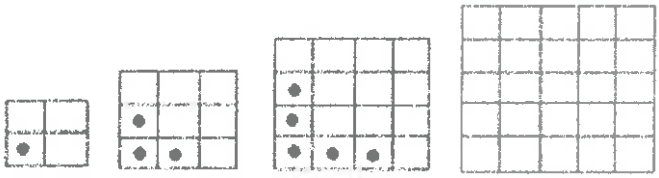
(d) Work out the coordinates of point E.

Answer (d) (....., ...) [4]

(e) Write down the values of x for which $f(x) \geqslant g(x)$.

Answer (e) ..... [2]

12 The diagram shows patterns with dotted and plain tiles.



pattern 1 pattern 2 pattern 3 pattern 4

(a) Complete the 4th pattern. [1]

(b) Complete the table below.

Pattern Number	1	2	3	4	5
Dotted tiles	1	3	5		
Plain tiles	3	6	11		

[2]

(c) Write an expression, in terms of n, for the number of dotted tiles in the nth pattern.

Answer (c) ..... [2]

(d) Find the number of dotted tiles in the 20th pattern.

Answer (d) ..... [1]

(e) How many more dotted tiles are in pattern $n + 7$ than in pattern n?

Answer (e) ..... [2]

(f) Write an expression, in terms of n, for the number of plain tiles in the nth pattern.

Answer (f) ..... [2]

(g) Find

(i) the pattern with 227 plain tiles,

Answer (g)(i) ..... [2]

(ii) the total number of tiles in pattern 65.

Answer (g)(ii) ..... [2]

13 $f(x) = 2^x$ and $g(x) = 2x + 3$.

(a) Find

(i) f(3),

Answer (a)(i) ..... [1]

(ii) $g^{-1}(x)$,

Answer (a)(ii) $g^{-1}(x) =$ ..... [2]

(iii) $gf(x)$, in its simplest form.

Answer (a)(iii) ..... [2]

(b) Solve $f(x) = 8^{(x-1)}$.

Answer (b) $x =$ ..... [3]

(c) Given that $gh(x) = 6x + 1$, find $h(x)$.

Answer (c) ..... [3]

45 Copies Form E

EXAMINATIONS COUNCIL OF LESOTHO
Lesotho General Certificate of Secondary Education

CANDIDATE NAME												
CENTRE NUMBER						CANDIDATE NUMBER						

MATHEMATICS **0178/02**

Paper 2 (Extended) **October/November 2017**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical Instruments
Tracing Paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.

ELECTRONIC CALCULATORS **MUST NOT** BE USED IN THIS PAPER.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 70.

This document consists of **13** printed pages and **3** blank pages.

1 Mpho's bank account is overdrawn by M485.

(a) She deposits M230 into the account.

How much does she now owe the bank?

Answer (a) M ..... [1]

(b) What further deposit should she make to give her account a balance of M150?

Answer (a) M ..... [1]

---

2 When written as a product of their prime factors, $a = n \times 3 \times 7$ and $b = n^2 \times 7$.

(a) Express, in terms of n,

(i) the LCM of a and b,

Answer (a)(i) ..... [1]

(ii) the HCF of a and b.

Answer (a)(ii) ..... [1]

(b) Given that the LCM of a and b is 84.

Find the values of n, a and b.

Answer (b) n = ... a = ... b = ... [3]

---

3 On average, the mass of each egg in a tray is 57.2 grams to the nearest tenth of a gram.

(a) Write down the lower bound for the mass.

Answer (a) ..... g [1]

(b) There are 30 eggs in the tray.

Calculate an estimated upper bound for the mass of all eggs in the tray.

Answer (b) ..... g [2]

---

4 **(a)** Show that $\frac{a^{-2}+b^{-2}}{(ab)^{-2}}$ can be simplified to $a^2 + b^2$.

Answer (a) ..... [3]

(b) Find the value of n in the following expression.

$$\left(\frac{2}{3}\right)^{-3} \times \left(\frac{3}{2}\right)^{n} = 1$$

Answer (b) ..... [2]

---

5 A serving of 100 g of powdered milk A provides 2120 KJ of energy.

(a) Express this amount of energy in standard form.

Give your answer in Joules.

Answer (a) .....J [1]

(b) Calculate the amount of energy, in Joules, that can be obtained from 2 kg of powdered milk A.

Give your answer in standard form.

Answer (b) .....J [2]

(c) A serving of 100 g of powdered milk B provides 9.78×10^2 kJ of energy.

Calculate the difference, in Joules, between the amount of energy obtained from powdered milk A and B in serving of 300 g.
Leave your answer in standard form.

Answer (c) .....J [3]

6 The temperature of a substance rises from −3 °C to 12 °C.

Calculate the increase in temperature of the substance.

Answer ..... °C [1]

---

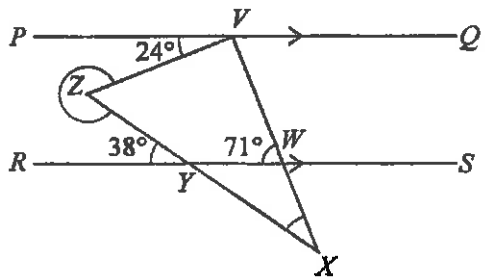
7 Given that $180 = 5 \times p^2 \times q^2$ and that both p and q are both prime.

Find the values of p and q.

Answer $p =$ ..... $q =$ ..... [2]

---

8 In the diagram PQ is parallel to RS.
VW and ZY are produced to meet at X.
$P\hat{V}Z = 24°$, $R\hat{Y}Z = 38°$ and $R\hat{W}V = 71°$.



NOT TO SCALE

Find the value of angle $W\hat{X}Y$ and the value of reflex angle $V\hat{Z}Y$.

Answer $W\hat{X}Y =$ ..... [2]

$V\hat{Z}Y =$ ..... [2]

2015 OCTOBER/NOVEMBER: PAPER 2 SOLUTIONS

1.

a) $1719 = 1720$ $(Correct\ to\ the\ nearest\ 10)$

b) $1719 = 1700$ $(Correct\ to\ two\ significant\ figures)$

2.

a) 23

b) 25

c) 27

d) 25

3. $\frac{3}{4} - \frac{1}{2} + \frac{1}{6}$

$= \frac{9-6+2}{12}$

$= \frac{5}{12}$

4. $Temperature\ at\ midnight = 3° - 8° = -5°$

5. (Use the method of elimination)
(Subtract the second equation from the from the first equation)

$5x + 4y = 7$ ..... [1]
$7x + 4y = 5$ ..... [2]

(After multiplying equation [2] by -1:

$$\begin{array}{r} 5x + 4y = 7 \\ -[\,7x + 4x = 5] \\ \hline -\ 2x + 0 = 2 \end{array}$$

$\Rightarrow -2x = 2$

$\Rightarrow x = \frac{2}{-2} = -1$

(Now solve for y by substituting the value of x in equation [1])

$$5(-1) + 4y = 7$$

$$\Rightarrow -5 + 4y = 7$$

$$\Rightarrow 4y = 7 + 5$$

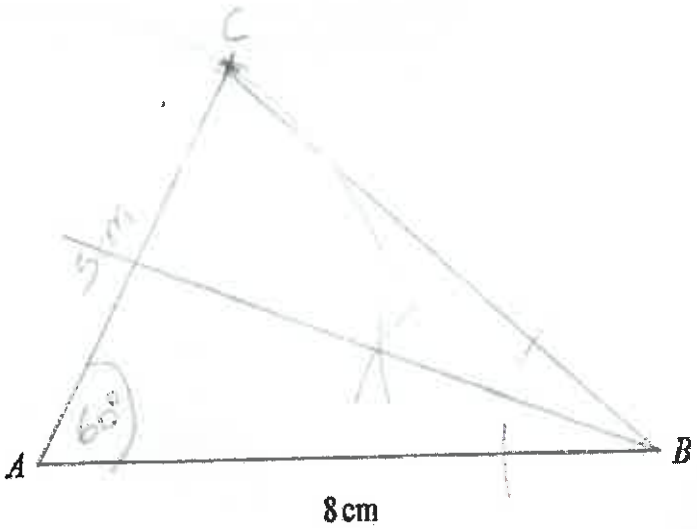
$$\Rightarrow 4y = 12$$

$$\Rightarrow y = \frac{12}{4} = 3$$

6.

a) **Shown on the diagram below**

b) **Shown on the diagram below**



c) **7cm**

11.

a) $4^0 + 4^{\frac{1}{2}} + 4^2$

$= 1 + \sqrt{4} + 16$

$= 19$

b) $27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{\left(27^{\frac{1}{3}}\right)^2} = \frac{1}{\left(\sqrt[3]{27}\right)^2} = \frac{1}{(3)^2} = \frac{1}{9}$

12.

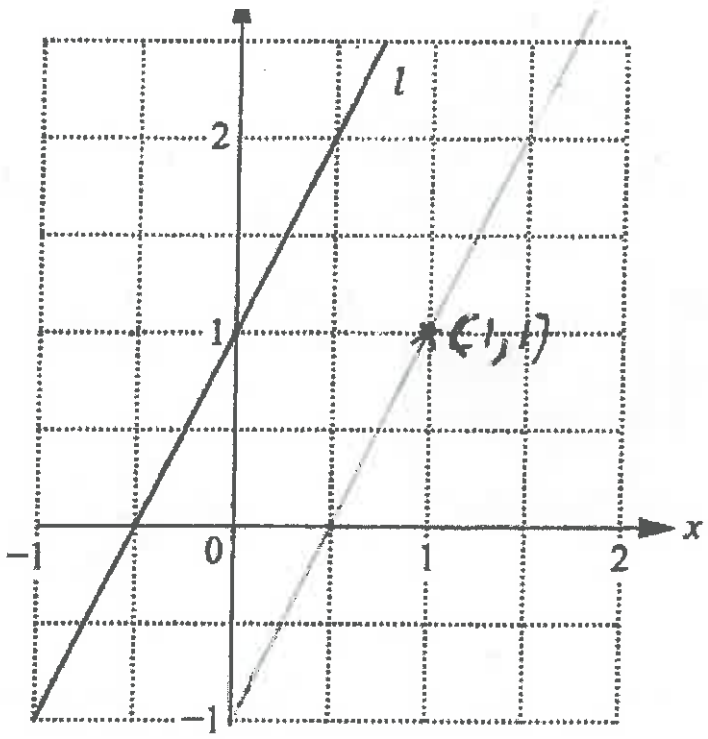
a) (Fist find the y-intercept and gradient and substitute then in $y = mx + c$)]

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{0.5-0} = \frac{1}{0.5} = 2$

$c = 1$

$\therefore y = 2x + 1$

b)



13.

a) (The ratio of heights is equal to the ratio of the diameters)

$$\frac{diameter\ of\ larger\ can}{diameter\ of\ smaller\ can} = \frac{20}{10}$$

$$\Rightarrow \frac{diameter\ of\ larger\ can}{6\ cm} = 2$$

$$\Rightarrow diameter\ of\ larger\ can = 2 \times 6\ cm$$

$$\therefore \boldsymbol{diameter\ of\ larger\ can = 12\ cm}$$

b) (The ratio of volumes is equal to the cube of the ratio of any side lengths)

$$\frac{volume\ of\ larger\ can}{volume\ of\ smaller\ can} = \left(\frac{20}{10}\right)^3$$

$$\Rightarrow \frac{volume\ of\ larger\ can}{volume\ of\ smaller\ can} = \frac{2^3}{1^3}$$

$$\Rightarrow \frac{volume\ of\ larger\ can}{90\pi\ cm^3} = 2^3$$

$$\Rightarrow \boldsymbol{volume\ of\ larger\ can = 2^3 \times 90\pi\ cm^3}$$

$$\therefore \boldsymbol{volume\ of\ larger\ can = 720\pi\ cm^3}$$

14.

a) $\boldsymbol{angle\ BDC = 27°}$

b) $ABC = 180° - 70° = 110°$

$$ACB = 180° - (ABC + BAC)$$

$$= 180° - (110° + 27°)$$

$$= \mathbf{180° - 137°}$$

$$= \mathbf{43°}$$

c) $ADC = 180^0$ - ABC

$= 180^0 - 110^0$

$= 70^0$

15. (First simplify the inequality by performing inverse operations)

$-3 < 2n - 1 \leq 5$

$-2 < 2n \leq 6$ *(After adding 1 in each part of the inequality)*

$-1 < n \leq 6$ *(After dividing by 2 in each part of the inequality)*

∴ ***The integer values are*** $\mathbf{0, 1, 2, 3, 4, 5, 6}$

16.

a) $AC = \begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 8 & + & -2 \\ 10 & + & -3 \end{pmatrix}$

$= \begin{pmatrix} \mathbf{6} \\ \mathbf{7} \end{pmatrix}$

b) $A^{-1} = \frac{1}{2}\begin{pmatrix} \mathbf{3} & \mathbf{-2} \\ \mathbf{-5} & \mathbf{4} \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{2} & \mathbf{-1} \\ -\frac{5}{2} & \mathbf{2} \end{pmatrix}$

17. $x \propto \frac{k}{y-2}$

$\Rightarrow x = \frac{k}{y-2}$

$\Rightarrow k = x(y-2)$

$\Rightarrow k = 9(6-2)$

$\Rightarrow k = 36$

$\therefore x = \frac{36}{y-2}$

when $y = 20$: $x = \frac{36}{20-2} = 2$

18. $\overrightarrow{AB} = \overrightarrow{AP} + \overrightarrow{PB}$

$= \underline{p} + \frac{1}{3}PQ$

$= \underline{p} + \frac{1}{3}(\underline{q} + \underline{p})$

$= \underline{p} + \frac{1}{3}\underline{q} + \frac{1}{3}\underline{p}$

$= \frac{4}{3}\underline{p} + \frac{1}{3}\underline{q}$

19.

a) (Divide diameter by 2)

$(1.39 \times 10^6) \div 2$

$= (139 \times 10^4) \div 2$

$= \frac{139}{2} \times 10^4$

$= 69.5 \times 10^4$

$= \mathbf{6.95 \times 10^5\ km}$

b) (Subtract the diameter of the Earth from that of the sun)

$(1.39 \times 10^6) - (1.27 \times 10^4)$

$= (139 \times 10^4) - (1.27 \times 10^4)$

$= (139 - 1.27) \times 10^4$

$= 137.73 \times 10^4$

$= \mathbf{1.374 \times 10^6\ km}$

20. $V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{9\pi}{2} cm^3 = \frac{4}{3}\pi r^3$ *(After substituting V in the formula)*

$\Rightarrow 6 \times \frac{9\pi}{2} cm^3 = \frac{4}{3}\pi r^3 \times 6$

$\Rightarrow 3 \times 9\pi\ cm^3 = 4\pi r^3 \times 2$

$\Rightarrow 27\pi\ cm^3 = 8\pi r^3$ *(After multiplying by 6 on both sides of the equation)*

$\Rightarrow \frac{27\pi\ cm^3}{8\pi} = r^3$

$\Rightarrow r^3 = \frac{27\ cm^3}{8}$ *(After dividing by 8π on both sides)*

$\Rightarrow r = \sqrt[3]{\frac{27\ cm^3}{8}}$ *(After taking the cube root of both sides)*

$\Rightarrow r = \frac{\sqrt[3]{27}}{\sqrt[3]{8}}\ cm$

$\therefore\ r = \frac{3}{2}\ cm = 1.5\ cm$

2015 OCTOBER/NOVEMBER: PAPER 4 SOLUTIONS

1.

a) (Use the method of factorization to solve this equation)

$(3x-2)(6x+1)=0$

$\Rightarrow (3x-2)=0 \quad or \quad (6x+1)=0$

$\Rightarrow 3x=2 \quad or \quad 6x=-1$

$\therefore \quad x=\frac{2}{3} \quad or \quad x=-\frac{1}{6}$

b) (Use the method of factorization by grouping)

$2a^2+6a-ab-3b$

$=2a(a+3)-b(a+3)$

$=(2a-b)(a+3)$

c) (*The LCM of* $(p-2)$ *and* $(4p+3)$ *is* $(p-2)(4p+)$, *write it in the denomenator*)

$\Rightarrow \frac{1}{p-2}-\frac{2}{4p+3}$

$=\frac{1(4p+3)-2(p-2)}{(p-2)(4p+3)}$

$=\frac{4p+3-2p+4}{(p-2)(4p+3)}$

$=\frac{2p+7}{(p-2)(4p+3)}$

d)

i.

a) $6 - t$

b) $distance = speed \times time$

$= 170m/\ min \ \times \ tmin$

$\therefore$ ***The distance she runs in terms of t is*** $\mathbf{170t}$ ***meters***

c) The distance to the bus stop in terms of t.

$distance = run + walk$

$= Speed \times time + \ Speed \times times$

$= (170\ m\ /\ \ min\ )\ \ t\ min \ \ + \ \ (90\ m\ /\ min)\ (6 - t)min$

$= (170t + 540 - 90t)m$

$= (\mathbf{80t + 540})m$

$\therefore$ [illegible]

$80t = 200$ *(Ater subtracting 540 from bothd sies of the equation)*

$\therefore \ t = 2.5$ *(After dividing by 80 on both sides of the equation)*

2.

a) $Volume\ of\ a\ cuboid = length \times width \times breadth$

$= 2.8m \times 4.5m \times 3.2m$

$= \mathbf{40.32}m^3$

b) $Surface\ area = 2(3.2m \times 4.5m) + 2(2.8m \times 4.8m)$

$= 28.8m^2 \ + \ 26.88m^2$

$= 55.68\ m^2$

c)

i. $1\ liter \rightarrow 3m^2$

$\therefore 5litres \rightarrow 5 \times 3m^2 = 15m^2$

$\therefore \left(\frac{54.78m^2}{15m^2}\right) = \mathbf{3.652 \sim 4\ containers}$

ii. $1\ container \rightarrow M90.00$

$\therefore \mathbf{4\ containers \rightarrow 4 \times M90.00 = M360.00}$

3.

a)

i. (The mode is the mark with the highest frequency)

$\therefore \mathbf{mode = 6\ marks}$

ii. $Median\ Position = \frac{1}{2}(n+1)^{th}\ position$

$= \frac{1}{2}(28)^{th}\ position$

$= 14^{th}\ position$

$\therefore \mathbf{Median = 7}$

iii. $\mathbf{mean = \frac{(5\times2)+(6\times8)+(7\times5)+(8\times6)+(9\times4)+(10\times2)}{2+8+5+6+4+2}}$

$= \frac{\mathbf{197}}{\mathbf{27}}$

$= \mathbf{7.3}$

b)

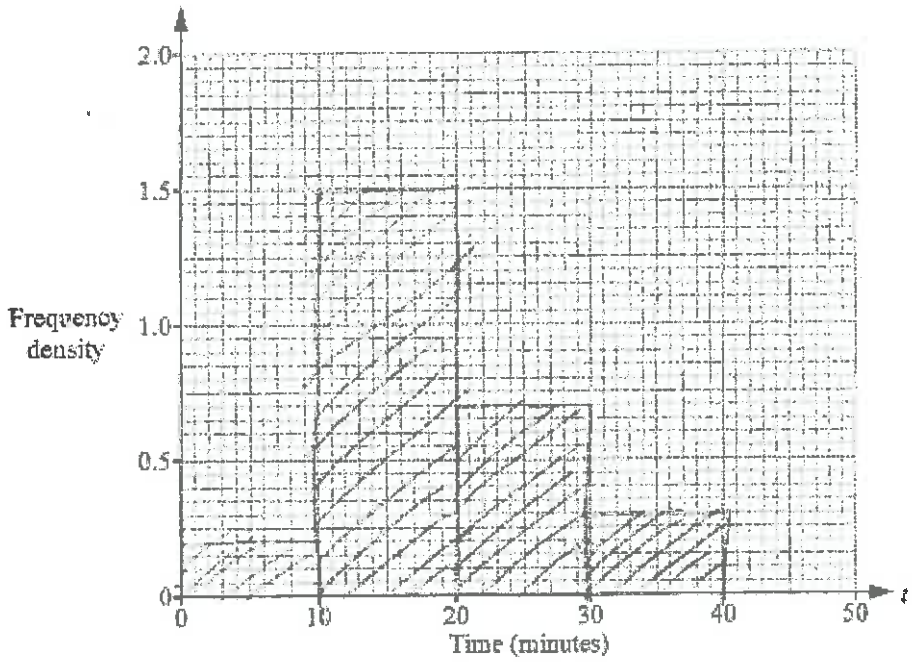
i. $Estimate\ mean = \frac{Sums\ of\ (Frequency\ x\ Class\ mid\text{-}value)}{sum\ of\ frequencies}$

$= \frac{(2\times5)+(15\times15)+(7\times25)+(3\times35)}{2+15+7+3}$

$= \frac{515}{27}$

$= \mathbf{19.07}$

ii.



a)

i. *Reflection in the mirror line,* $x = 0$ *or* $y -$ *axis*

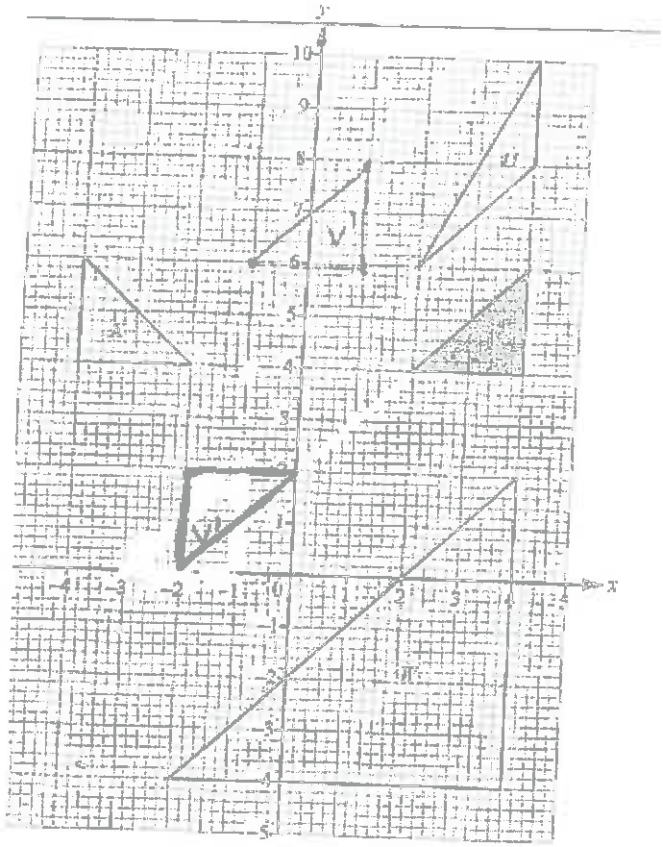
ii. *Enlargement from the centre* (4,8) *with the scale factor of* 3

iii. *Shear with the shear factor of* -1 *and the invariant line is the* $y -$ *axis*

b)

i. Is shown in the diagram below

ii. Is shown in the diagram below



c) $\left(We\ use\ the\ base\ vectors\ \underline{i}\ and\ \underline{j}\ to\ find\ the\ matrix\ of\ transformation\right)$

$\left(These\ base\ vectors\ are\ \underline{i} = \begin{pmatrix}1\\0\end{pmatrix}\ and\ \underline{j} = \begin{pmatrix}0\\1\end{pmatrix}\right)$

$$\therefore \boldsymbol{Matrix\ of\ transformation} = \begin{pmatrix}\mathbf{-1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$$

5.

a) $x^2 - 18x + 81 = (x-9)^2 \qquad (\because a^2 + 2ab + b^2 = (a+b)^2$

b) $Profit = Selling\ price - Cost\ price$

$$= Px - \frac{xy}{12}$$

c) $(2x-3)(x-4)=18$

$\Rightarrow 2x(x-4)-3(x-4)=18$

$\Rightarrow 2x^2-8x-3x+12=18$ *(Collect the like terms)*

$\Rightarrow 2x^2-11x-6=0$ *(Factorize the LHS)*

$\Rightarrow 2x^2-12x+x-6=0$ *(Factorize the LHS by grouping)*

$\Rightarrow 2x(x-6)+1(x-6)=0$ *(Factor out $x-6$)*

$\Rightarrow (x-6)(2x+1)=0$ *(Now apply the zero product rule)*

$\Rightarrow (x-6)=0 \quad or \quad 2x+1=0$

$\therefore \quad x=6 \quad or \quad x=-\frac{1}{2}$

d)

i. $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ *(multiply each side of the equation by R)*

$\Rightarrow 1=\frac{R}{R_1}+\frac{R}{R_2}$ *(After multiplying each side by R)*

$\Rightarrow R_1R_2=RR_2+RR_1$ *(After multiplying each side by R_1R_2)*

$\Rightarrow R_1R_2=R(R_2+R_1)$ *(After factoring out R)*

$\Rightarrow \frac{R_1R_2}{R_2+R_1}=R$ *(After dividing R_2+R_1)*

$\Rightarrow R=\frac{R_1R_2}{R_1+R_2}$

$=\frac{4\times 6}{4+6}$

$=\frac{24}{10}=2.4$

ii. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

(find the LCM of R, R_1, and R_2. It is RR_1R_2)

(Multiply each term of the equation by RR_1R_2)

$$\Rightarrow \frac{1}{R} \times RR_1R_2 = \frac{1}{R_1} \times RR_1R_2 + \frac{1}{R_2} \times RR_1R_2$$

$$\Rightarrow R_1R_2 = RR_2 + RR_1$$

$R_1R_2 - RR_2 = RR_1$ *(After subtracting RR_2 from both sides)*

$R_2(R_1 - R) = RR_1$ *(After factorising the LHS)*

$R_2 = \frac{RR_1}{(R_1 - R)}$ *(After dividing by $R_1 - R$ on both sides)*

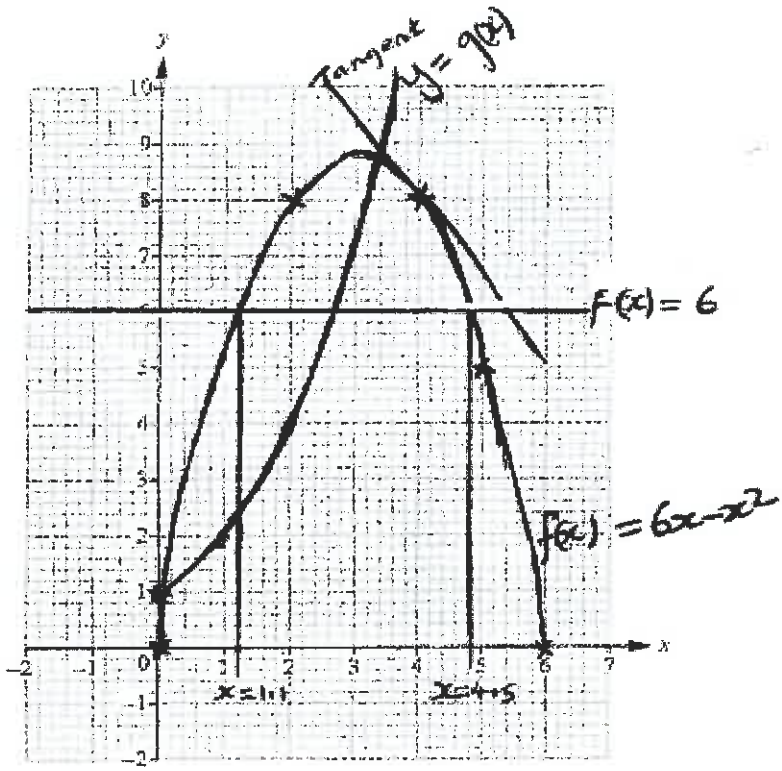
6.

a)

i.

X	0	1	2	3	4	5	6
F(x)	0	**5**	**8**	9	**8**	**5**	0

ii.



iii. $x = 1.1 \; or \; x = 4.5$ *(From the graph above)*

iv. Drawn on the graph above

$\therefore$ *The gradient of the graph of* $y = f(x)$

when $x = 4$ *the gradient of the graph is approximately* -2

b)

i.

X	0	1	2	3	3.3
g(x)	1	2	4	8	9.8

ii. Drawn on the graph above as $y = g(x)$

c) (Find the x-coordinates of points of **intersection** of $g(x) = 2^x$ and $f(x) = 6x - x^2$)

$\therefore$ ***When*** $f(x) = g(x), \quad x = 0.2 \quad or \quad x = 3.2$

7.

a)

i. Money she paid on Thursday

$$\frac{10}{100} \times M199.00 = M19.90$$

$$M199.00 - M19.90 = \boldsymbol{M179.10}$$

Money she paid on Friday

$$\frac{10}{100} \times M179.10 = \boldsymbol{M17.91}$$

$$\boldsymbol{M179.10 - M17.19 = M161.19}$$

$\therefore$ ***Thato paid*** $\boldsymbol{M161.19}$

ii. Money Lineo paid for the dress on Saturday.

$$\frac{10}{100} \times M161.19 = M16.12$$

$$M161.19 - M16.12 = M145.07$$

Money Lineo paid for the dress on Sunday.

$$\frac{10}{100} \times M145.07 = M14.51$$

$$M145.07 - M14.51 = M130.56$$

$\therefore \boldsymbol{M161.19 - M130.56 = M30.63}$

iii. The price on Wednesday

$$x - \frac{10}{100}x = M360.00$$

$$\Rightarrow x\left(1 - \frac{1}{10}\right) = M360.00$$

$$\Rightarrow x\left(\frac{9}{10}\right) = M360.00$$

$$\Rightarrow x = \frac{M360.00}{\frac{9}{10}}$$

$$\Rightarrow x = M360.00 \times \frac{10}{9}$$

$$\Rightarrow x = \frac{\boldsymbol{M3600.00}}{\boldsymbol{9}} = \boldsymbol{M400.00}$$

$$\therefore \quad \boldsymbol{M400.00\ on\ wednesday.}$$

iv. $\frac{25}{100} \times M199.00 = M49.75$

$M199.00 - M49.75 = M149.25$

$$\text{Number of days} = \frac{\boldsymbol{M149.25}}{\boldsymbol{M49.75}} = 3\ DAYS$$

b) $I = \frac{PRT}{100} = M800 \times \frac{12}{100} \times 2 = M192.00$

$$A = P\left(1 + \frac{R}{100}\right)^n = M800(1 + 0.01)^n = M816.08$$

Calculating the interest:

$$Interest = A - P$$

$$= M816.08 - M800$$

$$= M16.08$$

Difference of simple interest and compound interest:

$M192.00 - M16.08 = M175.92$

8.

a) (use the cosine formula to find PQ)

$$(PQ)^2 = (PR)^2 + (RQ)^2 - 2(PR)(QR)cosPQR$$
$$= (40m)^2 + (50m)^2 - 2 \times 40m \times 50m \times cos108°$$
$$= 1600m^2 + 2500m^2 - (-1236.1m^2)$$
$$= 4100m^2 + 1236.1m^2$$
$$= 5336.1m^2$$

$$\therefore \boldsymbol{PQ = \sqrt{5336.1m^2} = 73.05m}$$

b) $Area = \frac{1}{2}\, ab\, sinA$

$$= 0.5 \times 40m \times 50m \times sin108$$
$$= \mathbf{951.1m^2}$$

c) (Q is due East of P , the angle between North line $PQ = 90°$)

(Now use the sine formula to calculate RPQ)

$$\frac{sinRPQ}{50m} = \frac{sin108°}{73.05m}$$

$$\Rightarrow RPQ = Sin^{-1}\left(\frac{sin108°}{73.05m} \times 50m\right) = 40.6$$

$$\therefore \boldsymbol{The\ bearing\ of\ R\ from\ P = 40.6° + 90° = \ 130.6°\sim 131°}$$

d) (This shortest distance is perpendicular to PQ from R, and is apposite to angle RPQ)

Let this distance be x

We know that

$$sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\Rightarrow sin\theta = \frac{x}{PR}$$

$$\Rightarrow x = PR\ Sin\ 41°$$
$$= 40m \sin 41°$$
$$= \mathbf{26.2m}$$

9.

a)

i. $P(letter\ A) = \frac{1}{5}$

ii. $P(not\ letter\ A) = \frac{4}{5}$

iii. $\frac{4}{5} \times 200\ times = 160\ times$

iv. $P(letter\ M) = 0$

b)

i. $\frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$

ii. $\frac{2}{75}$

c) $\frac{3}{5}$

d) $\frac{1}{3}$

10.

a) (Use the Pythagoras rule)

$(3+5)^2 = (5+x)^2 + (3+x)^2$ *(Remove the brackets and simplify)*

$64 = 25 + 10x + x^2 + 9 + 6x + x^2$

$64 = 34 + 10x + x^2 + 6x + x^2$

$64 - 34 = 2x^2 + 16x$

$\Rightarrow 0 = 2x^2 + 16x - 30$

$\Rightarrow 0 = x^2 + 8x - 15$ *(After dividing both sides by 0)*

b) (Use the quadratic formula)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{\left(-8 \pm \sqrt{64-(4\times1\times-15)}\right)}{2\times1}$$

$$\Rightarrow x = \frac{-8 \pm \sqrt{124}}{2}$$

$$\Rightarrow x = \frac{(-8 + \sqrt{124})}{2} \quad or \quad x = \frac{(-8 - \sqrt{124})}{2}$$

$$\Rightarrow \boldsymbol{x = 2.78} \quad \boldsymbol{or} \quad \boldsymbol{x = -9.57}$$

c) **Use 2.75 as length cannot be negative**

$\therefore AC = 3 + 1.56 = \mathbf{4.56}$ *and* $BC = 5 + 1.56 = \mathbf{6.56}$

2018 MAY/JUNE: PAPER 1 SOLUTIONS

1.

a) $6 + 27 \div 3 = 6 + 9 = \mathbf{15}$

b) $28 \div 3\frac{1}{4} = 28 \div \frac{13}{4}$

$$= \frac{28}{1} \times \frac{4}{13}$$

$$= \mathbf{8\frac{8}{13}}$$

c) $\frac{2}{3} - \frac{1}{4}$

$$= \frac{8-3}{12}$$

$$= \mathbf{\frac{5}{12}}$$

2.

a)

i. (First find prime factors of 130 , then out of them, find those that can multiply to give 130)

$130 = 2 \times 65$

$= \mathbf{2 \times 5 \times 13}$

ii. $\mathbf{35 = 5 \times 7}$

$LCM\ of\ 35\ and\ 130 = 2 \times 5 \times 7 \times 13$

$130n = 2 \times 5 \times 7 \times 13$

$$Which\ implies\ that, n = \frac{2 \times 5 \times 7 \times 13}{2 \times 5 \times 13} = \mathbf{7}$$

iii. (Write the set of factors of 48 and those of 84, then identify the highest number that appears in both sets)

$factors\ of\ 48 = \{1,2,3,4,6,8,12,16,24\}$
$factors\ of\ 84 = \{1,2,3,4,6,7,12,14,21,28,42\}$

The HCF is 12

3.

a) (To change a fraction into a percentage, we multiply it by 100%)

$$\frac{9}{20} \times 100\%$$

$$= \frac{9}{2} \times 10\%$$

$$= \frac{90\%}{2}$$

$$= 45\%$$

b) $0.28 = \frac{28}{100} = \frac{7}{25}$

c) $(0.1)^3 = \left(\frac{1}{10}\right)^3 = \frac{1^3}{10^3} = \frac{1}{1000} = 0.001$

4.

a) **2 is the prime number.**
b) $\sqrt{23}$ **is the irrational number.**
c) **1 is the square number.**

5. $least\ possible\ value = 9cm - \frac{degree\ of\ accuracy}{2}$

$$= 9cm - \frac{1cm}{2}$$
$$= 9cm - 0.5cm$$
$$= 8.5cm$$

6.

a) (First convert 1km into centimeters then, write the ratio in the same units and simplify)

1km is equal to 100 000cm

then	$400cm : 1km$	
is the same as	$400cm : 100{,}000\ cm$	
	$400 : 100{,}000$	*(Units have canceled out)*
	$4 : 1000$	*(After dividing by 100 on both sides)*
	$\mathbf{1:250}$	*(After dividing by 4 on both sides)*

b) (Convert all the numbers to decimals then compere their size)

- $\frac{1}{2} = 0.5$
- $0.2^2 = 0.2 \times 0.2 = 0.04$
- 0.25
- $\sqrt{0.04} = \sqrt{\frac{4}{100}} = \frac{\sqrt{4}}{\sqrt{100}} = \frac{2}{10} = 0.2$

Starting with the smallest, the numbers are:

$$\mathbf{0.2^2}\ ,\qquad \mathbf{\sqrt{0.04}}\ ,\qquad \mathbf{0.25}\ ,\qquad \mathbf{\frac{1}{2}}$$

7.

a) (Subtract the area of the small cut out circle from the area of the big circle with radius of 7cm)

$$Area\ of\ a\ circle = \pi r^2$$

$$Area\ of\ the\ ring = \left(\frac{22}{7} \times 7cm \times 7cm\right) - \left(\frac{22}{7} - 3.5cm \times 3.5cm\right)$$

$$= (154cm^2) - (38.5cm^2)$$

$$= \mathbf{115.5cm^2}$$

b) (Add the circumference of the outer part of the ring to the circumference of the inner part)

$$Circumference\ of\ a\ circle = \pi \times diameter$$

$$Total\ perimeter\ of\ the\ ring = \left(\frac{22}{7} \times 14cm\right) + \left(\frac{22}{7} \times 7cm\right)$$

$$= 44cm + 22cm$$

$$= \mathbf{66}cm$$

8.

a) $15^4 \div 15^2 = 15^{4-2} = 15^2 = \boxed{225}$

b) $\left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^1 = \frac{2}{1} = \boxed{2}$

c) $125^{\frac{2}{3}} = \left(\sqrt[3]{125}\right)^2 = 5^2 = \boxed{25}$

9.

a) $f(x) = 3x + 2$

$$f(-2) = 3(-2) + 2$$
$$= -6 + 2$$
$$= -4$$

b) [*Use the general formula for difference of two squares:* $\boldsymbol{a^2 - b^2 = (a - b)(a + b)}$]

$$\mathbf{4x^2 - y^2 = (2x + y)(2x - y)}$$

10.

a) **t = 64**

b)

Row number	Numbers	Sum
5	21,23,25,27,29	125

c)

i. **n**

ii. $\mathbf{n^3}$

11. $Speed = \frac{distance}{time}$

$$time = \frac{distance}{speed} = \frac{50km}{40km/h} = 1\frac{1}{4}\,h = \mathbf{1\ hour\ and\ 15minutes}$$

Now calculating the time the journey finishes:

12:10
+01:15
13:25

$\therefore$ ***The journey was completed at*** $\mathbf{13{:}25hrs}$

12.

a) (The length BC is twice the length BM. First find length BM using the Pythagoras rule)

$$(BA)^2 = (BM)^2 + (AM)^2$$
$$\Rightarrow (BM)^2 = (BA)^2 - (AM)^2$$
$$\Rightarrow BM = \sqrt{(BA)^2 - (AM)^2}$$
$$\Rightarrow BM = \sqrt{(5cm)^2 - (4cm)^2}$$
$$\Rightarrow BM = \sqrt{25cm^2 - 16cm^2}$$
$$\Rightarrow BM = \sqrt{9cm^2}$$
$$\Rightarrow BM = 3cm$$

$\therefore$ ***The length*** $\mathbf{BC = 2 \times 3cm = 6cm}$

b) (Use the trigonometric ratio of the sin)

$$\sin(\theta) = \frac{Opposite}{Hypotenuse}$$

$\therefore \sin ACB = \frac{4}{5}$

13.

a) *The line has the gradient of zero and its y − intercept is 2*

$\therefore$ ***Equation of the line AC is*** $y = 2$

b) (Use the general equation of the straight line: y =mx +c)
(The gradient m, is equal to 2 and the y-intercept c, is equal to 8)

$\therefore$ ***The equation of the line is*** $y = 2x + 8$

c) (At this point the x-coordinates and the y-coordinate of the two lines are equal)
(When can therefore equate the equations of these two lines and solve for x)
(We already know that the x value is 2 at this point)

If we have $, y = 2$ *and* $y = 2x + 8$

then $2 = 2x + 8$

$2 - 8 = 2x$

$-6 = 2x$

$x = -\frac{6}{2} = -3$

$\therefore$ ***The coorainates of A are*** $(-3, 2)$

d) (Use the formula for the area of a triangle)
(The base of the triangle is the length AC and the height of the triangle is the length CB)

$$Area\ of\ triangle = \frac{1}{2} b \times h$$

$$= \tfrac{1}{2} \times AC \times CB$$

$$= \tfrac{1}{2} \times (0 - -3) \times (8 - 2)$$

$$= \tfrac{1}{2} \times 3 \times 6$$

$$= 9$$

14.

a) $\frac{120°}{360°} \times 300\ sweets$

$= \frac{1}{3} \times 300\ sweets$

$= \mathbf{100\ sweets}$

b) $\frac{x°}{360°} \times 300\ sweets = 80\ sweets$ *(Solving this equation for $x°$)*

$x° \times 300\ sweets = 360° \times 80\ sweets$ *(After multiplying by 360° on both sides)*

$x° = \frac{360° \times 80\ sweets}{300\ sweets}$ *(After dividing by 300 sweets on both sides)*

$x° = \frac{360° \times 80}{300}$

$\boldsymbol{x° = 96°}$

15.

a) (3,6)

b) (Image vector = Object vector + Translation vector)

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \boldsymbol{translation\ Vector}$

$= \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

$\therefore \boldsymbol{c(5,4)}$

16.

a) $P(9) = \frac{1}{4}$

b) $P(even\ or\ odd) = P(even) + P(odd)$

$$= \frac{2}{4} + \frac{2}{4}$$

$$= 1$$

c) $P(at\ least\ 4) = \frac{1}{4}$

2016 OCTOBER/NOVEMBER SOLUTIONS

1. (Convert the given times to that of a 24 hour clock and subtract)

$$\begin{array}{r} 1620 \\ -0730 \\ \hline 0850 \end{array}$$

She arrived at the stadium at 8:50 p.m.

2.

a) $120 = 3 \times 40$
$= 3 \times 2 \times 20$
$= 3 \times 2 \times 2 \times 10$
$= 3 \times 2 \times 2 \times 2 \times 5$
$= 2^3 \times 3 \times 5$

b) $k = 3 \times 2 \times 5 = 30$

c) (Write the two numbers as a product of their prime factors)
(Then the HCF is the product of the common bases with their powers)

$180 = 2 \times 90$
$= 2 \times 2 \times 45$
$= 2 \times 2 \times 3 \times 3 \times 5$
$= 2^2 \times 3^2 \times 5$

AND

$120 = 2^3 \times 3 \times 5$

$HCF = 2^2 \times 3 \times 5$

3. $(AC)^2 = (BC)^2 - (AB)^2$
$= (10cm)^2 - (8cm)^2$
$= 100cm^2 - 64cm^2$
$= 36cm^2$

$\therefore AC = \sqrt{36cm^2} = 6cm$

4. (We shall use the method of elimination to solve this pair of equations)

$3x - 2y = 4$ ..... [1]
$5x - 4y = -3$ ..... [2]

(Eliminate x: multiply equation [1] by 5 and equation [2] by 3)

$(3x - 2y = 4) \times 5$
$(5x - 4y = -3) \times 3$

$15x - 10y = 20$ ..... [3]
$15x - 12y = -9$ ..... [4]

(Now subtract equation [4] from equation [3])

$$\begin{array}{l} 15x - 10y = 20 \\ \underline{15x - 12y = -9} \\ \underline{0 + 2y = 29} \end{array}$$

$\Rightarrow 2y = 29$ ..... [5]

$\Rightarrow y = \frac{29}{2}$

$y = 14.5$

(Now substitute the value of y into equation [1])

$3x - 2\left(\frac{29}{2}\right) = 4$

$3x - 29 = 4$

$3x = 33$

$\frac{3x}{3} = \frac{33}{3}$

$\therefore x = 11$

5. $23° + 85° + (n-2)14° = 360°$

$\Rightarrow 23 + 85 + (n-2)14 = 360$

$\Rightarrow 23 + 85 + 14n - 28 = 360$

$\Rightarrow 14n = 360 - 23 - 85 - 23$

$\Rightarrow 14n = 280$

$\Rightarrow \boldsymbol{n = 20}$

6. $1 - x < 3x + 5 \leq x + 9$

$\Rightarrow 1 < 4x + 5 \leq 2x + 9$

$\Rightarrow -1 < x \leq 2$

$\therefore$ ***The Interger values are*** $\mathbf{0, 1}$ ***and*** $\mathbf{2}$

7.

a) $M = \left(\frac{-2+12}{2}, \frac{-4+6}{2}\right)$

$= M\left(\frac{10}{2}, \frac{2}{2}\right)$

$= \boldsymbol{M(5, 1)}$

b) $M_{BC} = \frac{\Delta y}{\Delta x}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\mathbf{6-8}}{\mathbf{12-6}}$

$= -\frac{\mathbf{2}}{\mathbf{6}}$

$= \frac{-1}{3}$

c) (Parallel lines have the same gradient, therefore $M_{BC} = M_{MN}$)

Equation of MN will be of the form:

$$y = Mx + c$$
$$\Rightarrow y = -\frac{1}{3}x + c$$

Now find the y-intercept c: (using the point (2,2)

$$2 = -\frac{1}{3} \times 2 + c$$

$$6 = -2 + 3c$$

$$8 = 3c$$

$$c = \frac{8}{3}$$

$\therefore$ ***Equation of MN is*** $y = \frac{8}{3} - \frac{1}{3}x$

d)

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(6 - (-2))^2 + (8 - (-4))^2}$$
$$= \sqrt{8^2 + 12^2}$$
$$= \sqrt{64 + 144}$$
$$= \sqrt{\mathbf{208}}$$

8.

a) $20ax - by - 5ay + 4bx$
$= 20ax + 4bx - by - 5ay$
$= 4x(5a + b) - y(b + 5a)$
$= 4x(5a + b) - y(5a + b)$
$= (\mathbf{4x - y})(\mathbf{b + 5a})$

b) $2x^2 - 18 = 2(x^2 - 9)$

$= \mathbf{2(x + 3)(x - 3)}$

Now finding prime factors of 182

Assume : $2x^2 - 18 = 182$

$2x^2 = 200$

$x^2 = 100$

$\sqrt{x^2} = \sqrt{100}$

$\boldsymbol{x = 10}$

(Substitute 10 in the expression 2(x + 3) (x-3) to find the prime factors)

$2(x + 3)(x - 3)$

$= 2(10 + 3)(10 - 3)$

$= 2 \times 13 \times 7$

$\therefore$ ***The prime factors of*** **182** ***are*** **; 2 , 13 , 7**

9.

$A = \frac{\theta}{360^\circ} \times \pi r^2$ ……*equation* 1

$arc\ length = \frac{\theta}{360^\circ} \times \pi d$ ..... *equation* 2

from equation 1, $\frac{\theta}{360^\circ} \times \pi = \frac{A}{r^2}$ .....*equation* 3

Now substitute equation 3 into equation 2.

$\Rightarrow$*equation* 2 *becomes*:

$$arc\ length = \frac{A}{r^2} \times d = \frac{12\pi cm^2}{36cm^2} \times 12cm = \mathbf{4\pi}$$

10. $1:50000$

$= 1:2.5 \times 10^9$ *(Ratio of areas)*

$8cm^2 = 2 \times 10^{10} cm$ *(Actual area)*

but $1km^2 = 10^{10} cm^2$

$\Rightarrow 1cm^2 = \frac{1}{10^{10}} km^2$

$\Rightarrow 2 \times 10^{10} = 2 \times 10^{10} \times \frac{1}{10} km^2 = 2km^2$ *(Actual area)*

$\therefore$ ***The actual area of the village is*** $2km^2$

11. $A^{-1} = \frac{1}{det\ A} \begin{pmatrix} -4 & -7 \\ -3 & -5 \end{pmatrix}$

The determinant of A is given by $det(A) = 20 - 21 = -1$

$\therefore A^{-1} = -1 \begin{pmatrix} -4 & -7 \\ -3 & -5 \end{pmatrix} = \begin{pmatrix} 4 & 7 \\ 3 & 5 \end{pmatrix}$

12.

a) $y \propto x^{-3}$

$\Rightarrow y = \frac{k}{x^3}$ *(Where K is the constant of proportionality)*

$\Rightarrow k = x^3 y$

$= 3^3 \times 9$

$= 27 \times 9$

$= 243$

if $x = 10$

$\Rightarrow y = \frac{243}{10^3} = \frac{243}{1000} = 0.243$

b) If p is directly proportional to q^2 ,then:

$\Rightarrow p = kq^2$

Now assume that k=1

$\Rightarrow p = q^2$

Now let $P_1 = 4$

$\Rightarrow q = 2 \qquad (\because \sqrt{4} = 2)$

But now P is increased by 50%

$\therefore 50\%\ 0f\ 2\ is\ 1$

$\therefore q + 1 = 2 + 1 = 3$

$Now\ when\ q = 3$

$\Rightarrow P_2 = 9 \qquad (\because 3^2 = 9)$

$now, p_1 + \frac{t}{100}p_1 = p_2$

$\Rightarrow t\% = \frac{p_2 - p_1}{p_1} = \frac{9-4}{4} = \frac{5}{4} = 1.25 = 125\%$

13.

a) $\frac{(3^{x+3} - 3^{x+1})}{3^{x+1}} = \frac{3^{x+1} \times 3^2 - 3^{3+1}}{3^{x+1}}$

$= \frac{3^{x+1}(3^2 - 1)}{3^{x+1}}$

$= 3^2 - 1$

$= 9 - 1$

$= 8$

b) (Simplify the LHS of the equation first)

$$\frac{y^4 \times \sqrt{y}}{(y^{-3})} = y^n$$

$$\Rightarrow \frac{y^4 \times y^{\frac{1}{2}}}{y^{-3}} = y^n$$

$$\Rightarrow \left(y^4 \times y^{\frac{1}{2}}\right) \times y^3 = y^n$$

$$\Rightarrow y^7 \times y^{\frac{1}{2}} = y^n$$

$$\Rightarrow y^{7\frac{1}{2}} = y^n$$

$$\therefore n = 7\frac{1}{2}$$

14.

a) $P(\text{none of the three teams win}) = \frac{1}{2} \wedge \frac{3}{4} \wedge \frac{4}{5}$

$$= \frac{12}{40}$$

$$= \frac{3}{10}$$

b) $P(one\ of\ them\ wins) = \left(\frac{1}{2} \times \frac{3}{4} \times \frac{4}{5}\right) + \left(\frac{1}{2} \times \frac{1}{4} \times \frac{4}{5}\right) + \left(\frac{1}{2} \times \frac{3}{4} \times \frac{1}{5}\right)$

$$= \frac{12}{40} + \frac{4}{40} + \frac{3}{40}$$

$$= \frac{19}{40}$$

15.

a) $a = \frac{v-u}{t}$ *(Where v is the final speed and u is the initial speed)*

$$= \frac{20\ m/s\ -\ 0\ m/s}{10s}$$

$$= \frac{20m\ /s}{10s}$$

$$= \mathbf{2m/s^2}$$

b) (The area under the speed-time graph is numerically equal to the total distance travelled)

(The area under the graph in the first 20 seconds is the area of a trapezium)

$$\therefore distance = area = \frac{1}{2}(20s + 10s) \times 20\ \text{m/s}$$

$$= \frac{1}{2}(30s) \times 20m\ /\ s$$

$$= 15s \times 20m\ /\ s$$

$$= \mathbf{300m}$$

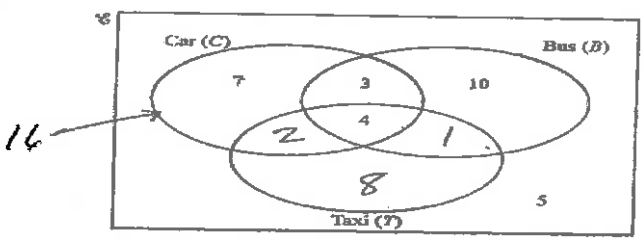
c) $a = \frac{v-u}{t}$

$$\Rightarrow \boldsymbol{t = \frac{v-u}{a}} = \frac{(0-(-40))}{8} = \frac{40}{8} = \mathbf{5\ seconds}$$

16.

a)

i.



ii. $n(B \cap T) = 4 + 1 = 5$

b)

i. $n(C \cap (B \cap T)') = 7 + 5 = 12$

ii. Teacher who use either car, taxi or Bus

17.

a) $\frac{2+x}{2} = 3\frac{1}{2}$

$\Rightarrow \frac{2+x}{2} = \frac{7}{2}$

$\Rightarrow 2 + x = 7$

$\Rightarrow x = 5$

b) The mean of eight numbers is x, [illegible] the sum eight numbers

$\frac{x}{8} = 3$ ..... [1]

The mean of twelve numbers is y. Let z be the sum of the twelve numbers

$y = \frac{z}{12}$ ..... [2]

The mean of the above twenty numbers is 9, calculate the value of y

$\frac{x+z}{20} = 9$ ..... [3]

From equation [1], $x = 24$

From equation [3], $\frac{x+z}{20} = 9 \Rightarrow \frac{24+z}{20} = 9 \Rightarrow z = (20 \times 9) - 24 \Rightarrow z = 156$

(Now substitute the value of z into equation [2] to find the value of y)

$y = \frac{z}{12} = \frac{156}{12}$

$y = 13$

18.

a) $10 < l \leq 20$

b) $Estimate\ of\ the\ mean = \frac{Sum(Frequency \times class\ mid\ value)}{Sum\ of\ frequency}$

$$= \frac{(5\times6)+(15\times20)+(40\times4)+(70\times10)}{6+20+4+10}$$

$$= \frac{1190}{40}$$

$$= 29.75$$

c) $Frequency\ density = \frac{frequency}{class-width}$

$\Rightarrow 2.5 = \frac{10}{x}$

$\Rightarrow x = 4 \qquad (Class - width)$

$\therefore$ ***The height of the interval*** $10 < l \leq 20$ *is* $\frac{20}{2} = 10cm$

2017 MAY/JUNE: PAPER 2 SOLUTIONS

1.

a)

i. $1\frac{2}{3} \div 1\frac{3}{4}$

$= \frac{5}{3} \div \frac{7}{4}$

$= \frac{5}{3} \times \frac{4}{7} = \mathbf{\frac{20}{21}}$

ii. $\mathbf{0.5 \times 0.007 = \frac{5}{10} \times \frac{7}{1000}}$

$= \mathbf{\frac{35}{10{,}000}}$

$= \mathbf{0.0035}$

b)

i. $2\sqrt{8} \times 4\sqrt{8} + 2$

$= 8(\sqrt{8 \times 8}) + 2$

$= 8\sqrt{64} + 2$

$= 8 \times 8 + 2$

$= 64 + 2$

$= 66$

ii. $\left(\frac{5}{4^{-1} - 9^{-1}}\right)^{\frac{1}{2}}$

$= \left(\frac{5}{\frac{1}{4} - \frac{1}{9}}\right)^{\frac{1}{2}}$

$= \left(\frac{5}{\frac{9-4}{36}}\right)^{\frac{1}{2}} = \left(\frac{5}{\frac{5}{36}}\right)^{\frac{1}{2}} = \left(\frac{5}{1} \div \frac{5}{36}\right)^{\frac{1}{2}} = \left(\frac{5}{1} \times \frac{36}{5}\right)^{\frac{1}{2}} = (36)^{\frac{1}{2}} = \sqrt{36} = 6$

c) (To make suitable approximations when estimating the value of a given number, write each number to the nearest 1 significant figure)

$$\frac{23.566 \times 5.367^2}{14.788}$$

$$\approx \frac{20 \times 5^2}{10}$$

$$\approx 2 \times 25$$

$$\approx \mathbf{50}$$

2.

a) $\mathbf{198 = 2 \times 3^2 \times 11}$ $\boldsymbol{and}$ $\mathbf{360 = 2^3 \times 3^2 \times 5}$

(HCF is a product of common bases in their lowest powers)

(Common bases are 2 and 3)

$\therefore \boldsymbol{HCF(198\ and\ 360) = 2 \times 3^2 = 18}$

b) (LCM is a product of all bases with their highest powers)

$$\therefore LCM\ (198\ and\ 360) = 2^3 \times 3^2 \times 5 \times 11$$
$$= 8 \times 9 \times 5 \times 11$$
$$= \mathbf{3960}$$

c) (Find the smallest value of k such that $\sqrt{360 \times 198 \times k}$ is an integer.

$$\sqrt{360 \times 198 \times k}$$

$$= \sqrt{2 \times 3^2 \times 11 \times 2^3 \times\ 3^2 \times 5 \times k}$$

$$= \sqrt{2^4 \times 3^4 \times 5 \times\ 11 \times k}$$

$$= 2^2 \times 3^2 \times \sqrt{5 \times 11 \times k}$$

$\therefore \boldsymbol{k = 5 \times 11 = 55}$ $\boldsymbol{for}$ $\boldsymbol{\sqrt{360 \times 198 \times k}}$ $\boldsymbol{to\ be\ an\ interger}$

3.

a)

i. $4(k-1)-(3k+2)+14$

$= 4k-4-3k-2+14$

$= k-4-2+14$

$= k-6+14$

$= \boldsymbol{k+8}$

ii. $(7x-2y)(3x+y)$

$= 21x^2+7xy-6xy-2y^2$

$= 21x^2+xy-2y^2$

b)

i. (We shall use the method of elimination)

$2x+2y=3$..... [1]

$4x-5y=24$..... [2]

(To eliminate x, multiply equation [1] by 4 and equation [2] by 2)

$8x+8y=12$..... [3]

$8x-10y=48$..... [4]

(Now subtract equation [4] from [3])

$8y-(-10y)=12-48$

$18y=-36$

$\Rightarrow y=-\frac{1}{2}$

(Now find the value of x by substituting the value of y in equation [1]

$$2x + 2\left(-\frac{1}{2}\right) = 3$$

$\Rightarrow \quad 2x - 1 = 3$

$\Rightarrow \quad 2x = 4$

$\Rightarrow \quad x = 2$

$\therefore \quad \boldsymbol{x = 2 \ \ and \ \ y = -\frac{1}{2}}$

ii. (Make the bases to be the same and equate the powers)

$$3^{x-2} + 3^x = 10$$

$\Rightarrow (x - 2) + (x) = 2 + 0$

$\Rightarrow x - 2 + x = 2$

$\Rightarrow 2x = 4$

$\therefore \quad x = 2$

c) (This is a quadratic expression in the form $ax^2 + bx + c$, where $a \neq 1$)
(We shall use the method of grouping)

Now, $6 \times (-6) = -36$

***factors of* 36 = $\{1, -1, 2, -2, 3, -3, 6, -6, 9, -9, 12, -12, 18, -18, 36, -36\}$**

(Note: $9 + (-4) = 5$)

$\therefore \quad 6x^2 + 5x - 6 = 6x^2 + \big(9 + (-4)\big)x - 6$

$= 6x^2 + (9 - 4)x - 6$

$= 6x^2 + 9x - 4x - 6 \quad [Now\ factorize\ by\ grouping]$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2) \quad [After\ factoring\ out\ (2x + 3)]$

4.

a) We know that,

$$100cm = 1m$$

$$\Rightarrow 1,000,000\ cm^3 = 1m^3$$

$$\therefore Number\ of\ full\ large\ mugs\ to\ fill\ 1m^3 = \frac{1{,}000{,}000cm^3}{500cm^3}$$

$$= 2000\ full\ mugs$$

b)

i. $32cm^3 : 500cm^3$

$\Rightarrow$ 32 : 500

$$\therefore\ Scale\ factor\ for\ the\ length = \sqrt[3]{\frac{32}{500}}$$

$$= \sqrt[3]{\frac{8}{125}}$$

$$= \frac{\sqrt[3]{8}}{\sqrt[3]{125}} = \frac{2}{5}$$

ii. 2 : 5 $(Ratio\ of\ sides)$

$$\therefore \frac{2}{5} \times 15 = 2 \times 3 = 6$$

iii. $Ratio\ of\ sides = 2:5$

$\Rightarrow Ratio\ of\ areas = 2^2 : 5^2 = 4 : 25$

$$\therefore \frac{4}{25} \times 350cm^2 = 4 \times 14cm^2 = 56cm^2$$

5.

a)

i. $n(J) = 2 + 1 + 4 + 8 = 15$

ii. $n(M' \cap F) = 10 + 2 = 12$

iii. The number of students who enjoy fizzy drinks and juice but not milk is:

$n([F \cap J] \cap M') = 2$

b) $y = n(\varepsilon) - n(F \cup J \cup M)$

$= 45 - (10 + 2 + 7 + 1 + 8 + 4 + 5)$

$= 45 - 37$

$= 8$

c)

i. $P(F \cap J) = \frac{n(F \cap J)}{n(\varepsilon)} = \frac{3}{45} = \frac{1}{15}$

ii. $P(M \cup J) = \frac{27}{45}$

6.

a) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 16cm^2 \times 10cm$

$= \frac{\mathbf{160\pi cm^3}}{3}$

$= \mathbf{53.3\pi cm^2}$

b) $Curved\ surface\ area = \pi rl = k\pi\sqrt{29}$

$\Rightarrow \pi rl = k\pi\sqrt{29}cm^2$

$\Rightarrow \pi \times 4cm \times \sqrt{(10cm)^2 + (4cm)^2} = k\pi\sqrt{29}\ cm^2$

$\Rightarrow \pi \times 4cm^2 \times \sqrt{116}\ cm = k\pi\sqrt{29}\ cm^2$

$\Rightarrow \pi \times 4\sqrt{116}\ cm^2 = k\pi\sqrt{29}\ cm^2$

$\Rightarrow 4\pi\sqrt{116} = k\pi\sqrt{29}$ $[cm^2\ cancelled\ out]$

$\Rightarrow 4\sqrt{116} = k\sqrt{29}$ $[\pi\ cancelled\ out]$

$\Rightarrow \boldsymbol{k = \frac{4\sqrt{116}}{\sqrt{29}}}$

$= \mathbf{4 \times \sqrt{\frac{116}{29}}}$

$= \mathbf{4 \times \sqrt{4}}$

$= \mathbf{4 \times 2}$

$= \mathbf{8}$

c) $\mathbf{30g \times 100\ full\ containers = 3000g = 3kg}$

7.

a)

i. Given that the determinant of the matrix $\begin{pmatrix} x & x+1 \\ 3x & 4x \end{pmatrix}$ is -2

Reducing to $\boldsymbol{x^2 - 3x + 2 = 0}$

$determinant = x[4x] - [(x+1)(3x)] = -2$

$\Rightarrow 4x^2 - [3x^2 + 3x] = -2$

$\Rightarrow 4x^2 - 3x^2 - 3x = -2$

$\Rightarrow \boldsymbol{x^2 - 3x + 2 = 0}$ **Hence proved.**

ii. (Solve by factorization)

$$x^2 - 3x + 2 = 0$$

$$\Rightarrow (x-2)(x-1) = 0$$

$$\Rightarrow x - 2 = 0 \quad or \quad x - 1 = 0$$

$$\Rightarrow \boldsymbol{x = 2 \quad or \quad x = 1}$$

b) $\begin{pmatrix} x \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ y \end{pmatrix}$

$$\Rightarrow x + (-3) = 2 \quad and \quad 2 + 5 = y$$

$$\Rightarrow \boldsymbol{x = 5 \quad and \quad y = 7}$$

8.

a) $\boldsymbol{\angle PAY = 40°}$

Reason: same segment

b) $\boldsymbol{\angle QBP = 113°}$

Reason: $\boldsymbol{YBP = 27° \ (same\ segment)}$

c) $\boldsymbol{\angle TAP = \frac{180° - 56°}{2} - 40° = 22°}$

Reason: $\boldsymbol{\angle\ sum\ of\ a\ triangle = 180°}$

d) $\boldsymbol{\angle AYP = 180 - (180° - (27 + (113 - 62))) = 78°}$

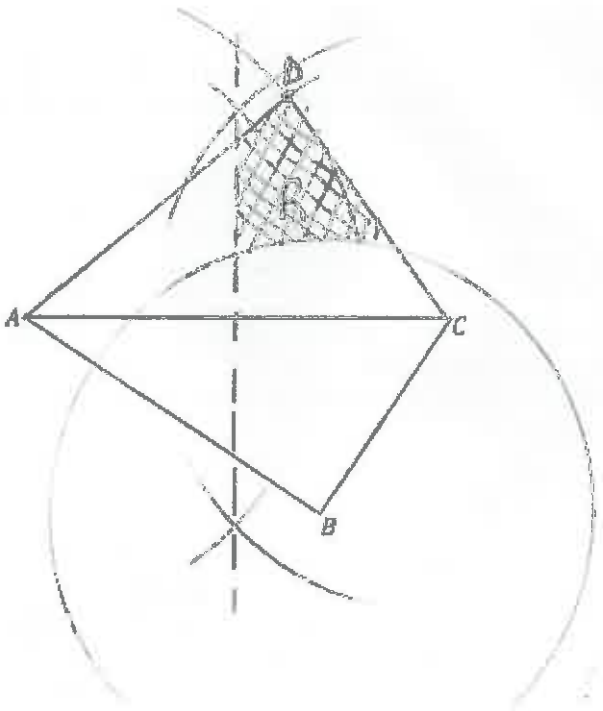
- sum of angles of a triangle and
- angles on a straight line are supplementary.

9.

a) (Use your protractor to measure the angle. You should find $57° \pm 1°$)

b) Shown on the diagram below

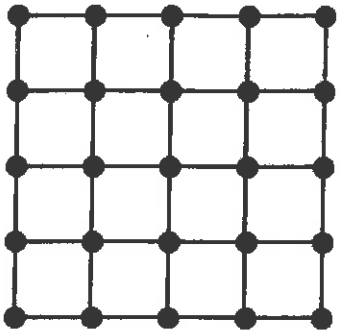
c) Shown on the diagram below



2017 MAY/JUNE: PAPER 4 SOLUTIONS

1.

a)



b)

i. $s = 16 \; and \; t = 40$

ii. $v = 4n$
$w = n^2$
$x = 2n^2 + 2n$

iii. $t_{20} = 4 \times 20 = 80$

c)

i.

a) $\frac{2}{3} \times 1^2 + h \times 1^1 + k \times 1 = 4$

$\Rightarrow \frac{2}{3} + h + k = 4$

$\Rightarrow h + k = 4 - \frac{2}{3}$

$\Rightarrow \boldsymbol{h + k = \frac{10}{3}}$

b) $\frac{2}{3} \times 2^2 + h \times 2^2 + k \times 2 = 16$

$\Rightarrow 5\frac{1}{3} + 4h + 2k = 16$

$\Rightarrow 4h + 2k = 16 - 5\frac{1}{3}$

$\Rightarrow 4h + 2k = \frac{32}{3}$

ii.

a) (Solve the simultaneous equations in (a) and (b) above)

$h + k = \frac{10}{3}$ ..... [1]

$4h + 2k = \frac{32}{3}$ ..... [2]

From the first equation

$h = \frac{10}{3} - k$ ..... [3]

Substitution [3] into [2]

$4\left(\frac{10}{3} - k\right) + 2k = \frac{32}{3}$

$\Rightarrow \frac{40}{3} - 4k + 2k = \frac{32}{3}$

$\Rightarrow \frac{40}{3} - \frac{32}{3} = 2k$

$\Rightarrow \frac{8}{3} = 2k$

$\Rightarrow 8 = 6k$

$\therefore k = 1\frac{1}{3}$

(Find h by substituting k into equation [1])

$$h + 1\frac{1}{3} = \frac{10}{3}$$

$$\Rightarrow h = \frac{10}{3} - 1\frac{1}{3}$$

$$\Rightarrow h = \frac{12}{6}$$

$$\therefore \boldsymbol{h = 2}$$

b) $t_n = \frac{2}{3} \times 12^3 + 2 \times 12^2 + \frac{8}{6} \times 12^2$

$$= \frac{3456}{3} + 288 + 16$$

$$= 1152 + 288 + 16$$

$$= \mathbf{1456}$$

2.

a)

i. $\boldsymbol{\angle BCD = 55°}$

This is because it corresponds with $\angle ABF$

ii. $\boldsymbol{\angle ABD = 120°}$

This is because:

$\boldsymbol{\angle FBD = \angle BDC = 65°}$ **(Alternating angles)**

$\boldsymbol{\angle ABD = 55° + 65° = 120°}$ **(Angle sum)**

iii. $\boldsymbol{\angle BFG = 120°}$

This is because $\angle BFG$ and $\angle ABF$ are interior angles, therefore they are supplementary.

b) **BDHC is a parallelogram**

This is because DH = BC and they are parallel. Then also, BD = HC and they are parallel.

3.

a) $p = \frac{y^2+a}{y+a}$

$\Rightarrow (y+a)p = y^2 + a$ *(After multiplying by $(y+a)$ on both sides)*

$\Rightarrow yp + ap = y^2 + a$

$\Rightarrow ap - a = y^2 - yp$ *(After subtractiing yp and a from both sides)*

$\Rightarrow a(p-1) = y^2 - yp$

$\Rightarrow \boldsymbol{a = \frac{y^2-yp}{p-1}}$

b) $(x^2 - y^2) - (x-y)^2$

$= x^2 - y^2 - [x^2 - 2xy + y^2]$ *(After expanding the second brackets)*

$= x^2 - y^2 - x^2 + 2xy - y^2$

$= 2xy - 2y^2$ *(After collecting like terms and simplifying)*

$= \boldsymbol{2y(x-y)}$

c)

i. $\frac{y-2}{2y^2-3y-2} = \frac{y-2}{(2y+1)(y-2)} = \boldsymbol{\frac{1}{2y+1}}$

ii. $\frac{ax-ab-bx+b^2}{ax^2-abx}$

$= \frac{a(x-b)-b(x-b)}{ax(x-b)}$

$= \frac{(a-b)(x-b)}{ax(x-b)} = \boldsymbol{\frac{a-b}{ax}}$

4.

a)

i. $\frac{60}{100} \times M7200 + M800 \times 6$

$= M4\,320 + M4800$

$= \boldsymbol{M9120}$

ii. $M9120 - M7200 = \boldsymbol{M1920}$

iii. $\frac{1920}{7200} \times \mathbf{100\%} \approx \mathbf{26.7\,\%}$

b) **Let the original price be x**

$\Rightarrow x - \frac{15}{100}x = M8075$

$\Rightarrow 100x - 15x = M807\,500$

$\Rightarrow 85x = M807\,500$

$\Rightarrow x = \frac{M807\,500}{85}$

$\Rightarrow x = M9\,500$

∴ The original price is M9 500

c)

i. $A = P + Prt$

$= M3000 + \left(M3000 \times \frac{5}{100}\right) \times 4$

$= M3000 + 4 \times 150$

$= M3000 + M600$

$= \boldsymbol{M\,3\,600}$

ii. (Find Lefa's accumulated amount after 4 years and compare to that of Mpho)

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$= M3000\left(1 + \frac{4.9}{100}\right)^4$$

$$= M3000 \times 1.049^4$$

$$= M3632.647084$$

$$\approx M3632.65$$

Lefa will have more money after 4 years.

iii. $M3632.65 - M3600 = \boldsymbol{M32.65}$

5.

a) $\angle PSQ = 180° - 35° - 25°$

$= 120°$

(Now use the sine rule to find PQ)

$$\frac{PQ}{sin120°} = \frac{4}{sin25°}$$

$$\Rightarrow PQ = \frac{4}{sin25°} \times sin120°$$

$$\therefore \boldsymbol{PQ = 8.2\ km}$$

b) (First find $\angle RSQ$, then use the cosine rule to find QR)

$\angle PSQ = 180° - 35° - 25°$

$= 120°$

$\Rightarrow \angle RSQ = 180° - 120°$

$= 60°$

Now find QR:

$$(QR)^2 = (SQ)^2 + (SR)^2 - 2(SQ)(SR)cos(\angle RSQ) \qquad [From\ the\ cosine\ rule]$$

$$\Rightarrow (QR)^2 = (4km)^2 + (6km)^2 - 2(4km)(6km)cos60°$$

$$\Rightarrow QR = \sqrt{(4km)^2 + (6km)^2 - 2(4km)(6km)cos60°}$$

$$\Rightarrow QR = \sqrt{(52km^2 + 24km^2)}$$

$$\Rightarrow QR = \sqrt{28km^2}$$

$$\therefore \boldsymbol{QR = 5.29\ km}$$

c) $Area_{\Delta QRS} = \frac{1}{2} \times 4km \times 6km\ sin60°$

$= 2 \times 6 \times sin60°$

$\approx \boldsymbol{10.4km^2}$

6.

a)

i. $\overrightarrow{RS} = 3\overrightarrow{QR} - \overrightarrow{QR}$

$= 3\underline{a} - \underline{a}$

$= \boldsymbol{2\underline{a}}$

ii. $\overrightarrow{RP} = \overrightarrow{RS} + \overrightarrow{SP}$

$= 2\underline{a} + 2\underline{b}$

$= \boldsymbol{2(\underline{a} + \underline{b})}$

iii. $\overrightarrow{UQ} = 4\vec{R} + \overrightarrow{RQ}$

$= -\frac{1}{4}\overrightarrow{RP} - \overrightarrow{QR}$

$= -\frac{1}{4}(2\underline{a} + 2\underline{b}) - \underline{a}$

$= -\frac{1}{2}\underline{a} - \frac{1}{2}\underline{b} - \underline{a}$

$= \boldsymbol{-\frac{3}{2}\underline{a} - \frac{1}{2}\underline{b}}$

iv. $\overrightarrow{TQ} = \overrightarrow{TS} + \overrightarrow{SQ}$

$= -\underline{b} + -3\underline{a}$

$= -\underline{b} - 3\underline{a}$

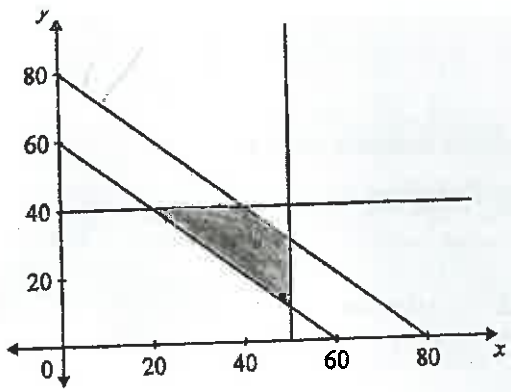
b) T, U and Q are on a straight line, u is half way between T and Q.

7.

a)

- $x \leq 50$
- $y \leq 40$
- $x + y \geq 60$

b)



NOT TO SCALE

c) Type B :

$40 = \frac{P}{150}$

$\Rightarrow P = 150 \times 40$

$\therefore P = 6000$

Type B:

$$0 = -2 \times 50 + \frac{p}{150}$$

$$\Rightarrow 0 = -100 + \frac{p}{150}$$

$$\Rightarrow 100 = \frac{p}{150}$$

$$\Rightarrow P = 100 \times 150$$

$$\therefore \boldsymbol{P = 15000}$$

d) $\boldsymbol{(50, 30)}$

- **Type A: 50 jeans**
- **Type B: 30 jeans**

e) (Substitute 50 and 30 in the profit function)

$$y = -2x + \frac{P}{150}$$

$$\Rightarrow 30 = -2 \times 50 + \frac{P}{150}$$

$$\Rightarrow 30 = -100 + \frac{P}{150}$$

$$\Rightarrow 30 + 100 = \frac{P}{150}$$

$$\Rightarrow 130 = \frac{P}{150}$$

$$\Rightarrow P = 130 \times 150 = 19500$$

The maximum profit is M19 500

f) **The profit in type B would decrease; this would also cause a decrease in maximum profit.**

8.

a) Mode is 1

b) $Range = 16 - 0 = 16$

c)

i. $median = \frac{2+p}{2} = 3.5$

$\Rightarrow 2 + p = 3.5 \times 2$

$\Rightarrow 2 + p = 7$

$\Rightarrow p = 7 - 2$

$\therefore \boldsymbol{p = 5}$

ii. $mean = \frac{1\times3+2+5+9+10+13+q+16}{12} = 6$

$\Rightarrow mean = \frac{58 + q}{12} = 6$

$\Rightarrow 58 + q = 12 \times 6$

$\Rightarrow 58 + q = 72$

$\Rightarrow q = 72 - 58$

$\therefore \boldsymbol{q = 14}$

d) $P(0 \text{ or } 1) = P(0) + P(1)$

$= \frac{2}{12} + \frac{3}{12}$

$= \frac{\mathbf{5}}{\mathbf{12}}$

e) $p(0\ and\ 0\ \ or\ \ 1\ and\ 1) = p(0\ and\ 0) + p(1\ and\ 1)$

$$= \left(\frac{2}{12} \times \frac{1}{11}\right) + \left(\frac{3}{12} \times \frac{2}{11}\right)$$

$$= \frac{2}{132} + \frac{6}{132}$$

$$= \mathbf{\frac{2}{33}}$$

9.

a) $Estimated\ mean = \frac{sum\ of [class-mid\ values \times frequencies]}{sum\ of\ frequencies}$

$$= \frac{10\times10+25\times10+35\times43+45\times22+55\times7+70\times8}{10+10+43+22+7+8}$$

$$= \frac{100+250+1505+990+385+560}{100}$$

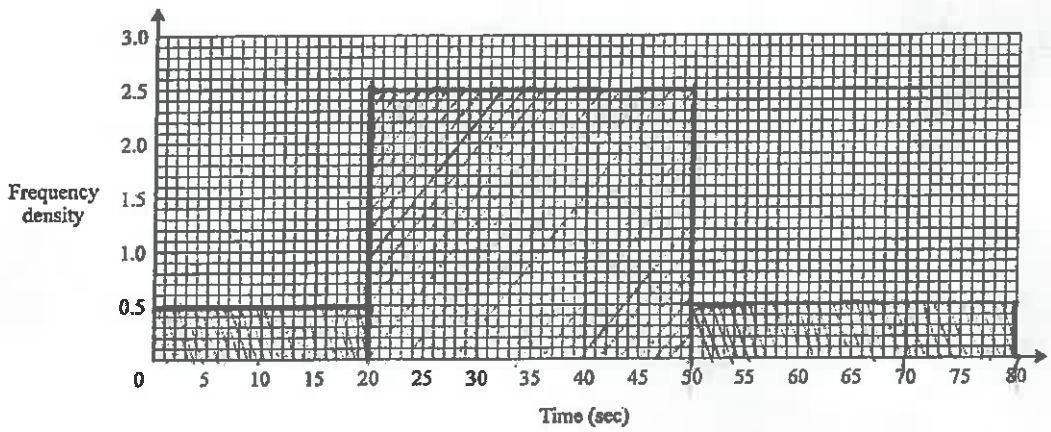
$$= \mathbf{\frac{3790}{100} = 37.9}$$

b)

i.

- $v = 10 + 43 + 22 = 75$
- $w = 7 + 8 = \mathbf{15}$

ii.



10.

a) $time = \frac{distance}{speed}$

$\Rightarrow time = \frac{34}{x}$ $\therefore The\ expression\ is\ \frac{34}{x}$

b) $\frac{34}{2+x}$

c) (There is a mistake is the quadratic formula given)

d) (Use the quadratic formula)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(-2^2 - 4 \times 1 \times -272)}}{2 \times 1}$$

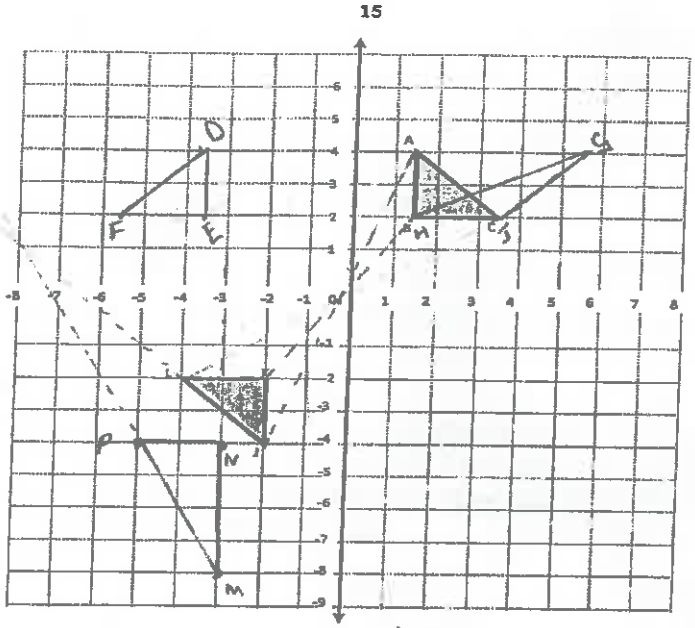
$$x = \frac{-2 + \sqrt{1092}}{2} \quad or \quad x = \frac{-2 - \sqrt{1092}}{2}$$

$$x = 15.5 \quad or \quad x = -17.5$$

e) $\frac{34}{x} = \frac{34}{15.5} = 2.19 = \mathbf{2\ hours\ and\ 11\ minutes}$

11.

a)



b) (The answer is on the grid above)

c) It is a rotation about the center (-1,0) through an angle of 180°

d) $\begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} -3 & -3 \\ -8 & -4 \end{pmatrix}$

$\Rightarrow \quad -\frac{1}{4}\begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix}\begin{pmatrix} -2 & 2 \\ 4 & -2 \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} -3 & -3 \\ -8 & -4 \end{pmatrix}\begin{pmatrix} -2 & 2 \\ 4 & -2 \end{pmatrix}$

$\Rightarrow \quad \begin{pmatrix} a & c \\ b & d \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} -6 & 0 \\ 0 & -8 \end{pmatrix}$

$\therefore \quad \begin{pmatrix} \boldsymbol{a} & \boldsymbol{c} \\ \boldsymbol{b} & \boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathbf{1.5} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} \end{pmatrix}$

12.

a) $f(x) = \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}$

$$= \frac{2(x-1)}{(x-3)(x+1)} - \frac{1}{x-3}$$

$$= \frac{2(x-1)-1(x+1)}{(x-3)(x+1)}$$

$$= \frac{2x-2-x-1}{(x-3)(x+1)}$$

$$= \frac{(x-3)}{(x-3)(x+1)}$$

$$= \frac{1}{x+1}$$

b)

i. Let $f(x) = y$

$\Rightarrow y = \frac{1}{x+1}$ *(Now solve for x)*

$\Rightarrow (x+1)y = 1$

$\Rightarrow xy + y = 1$

$\Rightarrow xy = 1 - y$

$\Rightarrow x = \frac{1-y}{y}$ *(Now exchange the positions of x and y)*

$\therefore f^{-1}(x) = \frac{1-x}{y}$

ii. Let $g(x) = y$

$\Rightarrow y = 2x - 3$ *(Now solve for x)*

$\Rightarrow 2x = y + 3$

$\Rightarrow x = \frac{y+3}{2}$ *(Now exchange the positions of x and y)*

$\Rightarrow y = \frac{x+3}{2}$

$\therefore \boldsymbol{g(x) = \frac{x+3}{2}}$

iii. $fg(x) = f(2x - 3)$

$$= \frac{1}{(2x-3)+1}$$

$$= \frac{\mathbf{1}}{\mathbf{2x-2}}$$

iv. [Find the inverse of $fg(x)$]

Let $fg(x) = y$

$\Rightarrow y = \frac{1}{2x-2}$ *(Solve for x)*

$\Rightarrow y(2x - 2) = 1$

$\Rightarrow 2xy - 2y = 1$

$\Rightarrow 2xy = 1 + 2y$

$\Rightarrow x = \frac{1+2y}{2y}$ *(Exchange the positions x and y)*

$\Rightarrow y = \frac{1+2x}{2x}$

$\therefore \boldsymbol{(fg)^{-1}(x) = \frac{1+2x}{2x}}$

c) $fg(x) = \frac{1}{8}$

$$\Rightarrow \frac{1}{2x-2} = \frac{1}{8}$$

$$\Rightarrow 8 = 2x - 2$$

$$\Rightarrow 2x = 10$$

$$\therefore x = 5$$

d) $(fg)^{-1}(x) = \frac{1+2x}{2x}$

and

$$g^{-1}f^{-1}(x) = g^{-1}\left(\frac{1-x}{x}\right)$$

$$= \frac{\left(\frac{1-x}{x}\right)+3}{2}$$

$$= \frac{1-x}{2x} + \frac{3}{2}$$

$$= \frac{1-x+3x}{2x}$$

$$= \frac{\mathbf{1+2x}}{\mathbf{2x}}$$

$$\therefore \ (fg)^{-1}(x) = g^{-1}f^{-1}(x)$$

Hence proved

2017 OCTOBER/NOVEMBER: PAPER 2 SOLUTIONS

1.

a) $M485 - M230 = M255$

b) $M255 + M150 = M405$

2.

a)

i. $n^2 \times 7 \times 3$ ***OR*** $21n^2$

ii. $7n$

b)

$21n^2 = 84$

$\frac{21n^2}{21} = \frac{84}{21}$

$n^2 = 4$

$n = \pm 2$

Consider +2 to find a

$a = 2 \times 3 \times 7$

$= 6 \times 7$

$= 42$

$b = 2^2 \times 7$

$= 4 \times 7$

$= 28$

$\therefore n = 2, a = 42$ ***and*** $b = 28$

3.

a) $\boldsymbol{Lower\ bound = Value - \frac{limit\ of\ accuracy}{2}}$

The limit of accuracy is a tenth of a gram; 0.1. Therefore the lower bound is:

$$Lower\ bound = 57.2g - \frac{0.1}{2}$$
$$\boldsymbol{Lower\ bound = 57.2g - 0.05g = 57.15g}$$

b) (Find the upper bound for 1 egg, then use it to find upper bound for 30 eggs)

$$Upper\ bound = Value + \frac{limit\ of\ accuracy}{2}$$

$$Upper\ bound = 57.2g + 0.05 = 57.25g$$

$$Upper\ bound\ for\ mass\ of\ 30\ eggs = \ 30 \times 57.25 = \mathbf{1717.5g}$$

4.

a) $\frac{(a^{-2}+b^{-2})}{(ab)^{-2}}$

$$= \left(\frac{1}{a^2} + \frac{1}{b^2}\right) \div \frac{1}{a^2b^2}$$

$$= \left(\frac{1}{a^2} + \frac{1}{b^2}\right) \times a^2b^2$$

$$= \frac{1}{a^2} \times a^2b^2 \ + \ \frac{1}{b^2} \times a^2b^2$$

$$= \boldsymbol{b^2 \ + \ a^2}$$

b) $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{3}{2}\right)^{n} = 1$

$\left(\frac{3}{2}\right)^{3} \times \left(\frac{3}{2}\right)^{n} = 1$

$\left(\frac{3}{2}\right)^{3} \times \left(\frac{3}{2}\right)^{n} = \left(\frac{3}{2}\right)^{0}$

$3 + n = 0$
$n = 0 - 3$
$n = -3$

5.

a) $2120\ KJ = 2\,120\,000\ J = \mathbf{2.12 \times 10^6\ J}$

b) $100g \rightarrow 2.12 \times 10^6 J$
$2000g \rightarrow x$

$x = \frac{2000g \times 2.12 \times 10^6 J}{100g}$

$x = \frac{2000g}{100g} \times 2.12 \times 10^6 J$

$x = \mathbf{4.24 \times 10^7\ J}$

c) $3(2.12 \times 10^6 J - 9.78 \times 10^5 J)$
$= 3(21.2 \times 10^5 J - 9.78 \times 10^5 J)$
$= 3(11.42 \times 10^5 J)$
$= \mathbf{3.426 \times 10^6 J}$

6. $12℃ - -3℃ = \mathbf{15℃}$

7. $180 = 5 \times 36$
$180 = 5 \times 2 \times 18$
$180 = 5 \times 2 \times 2 \times 9$
$180 = 5 \times 2 \times 2 \times 3 \times 3$
$180 = 5 \times 2^2 \times 3^2$

$\therefore p = 2$ *and* $q = 3$

8. $180° - 71° = 109°$

$$W\hat{X}Y = 180° - (109° + 38°)$$
$$= 180° - 147°$$
$$= 33°$$

(Now finding VZY)

$$24 + ZVW + 71 = 180°$$
$$\Rightarrow ZVW = 180° - (71° + 24°)$$
$$\Rightarrow ZVW = 180° - 95°$$
$$\Rightarrow ZVW = 85°$$

$$85° + 33° + XZV = 180°$$
$$\Rightarrow XZV = 180° - (85° + 33°)$$
$$\Rightarrow XZV = 180° - 118°$$
$$\Rightarrow XZV = 62°$$

$$\therefore VZY = 360° - 62° = 298°$$

9.

a) (Use the formula for the area of a trapezium to find the cross-sectional area)

$$Area\ of\ trapezium = \frac{1}{2}\ (sum\ of\ parallel\ sides) \times h$$

$$\therefore Area\ of\ cross\ section = \frac{1}{2}(20cm + 22cm) \times 12cm$$
$$= \frac{1}{2}(42cm) \times 12cm$$
$$= 21cm \times 12cm$$
$$= 252cm^2$$

b) $Volume\ of\ prism = Cross\ sectional\ area \times Length$
$$= 252cm^2 \times 40cm$$
$$= 10\ 080cm^3$$

10.

a) $\hat{BAC} = \hat{EAF}$

$(\boldsymbol{they\ are\ common\ angles})$

$\frac{AB}{AF} = \frac{8}{4} = 2 \quad \boldsymbol{AND} \quad \frac{AC}{AE} = \frac{6}{2} = 2$

(Ratio of corresponding sides is equal)

$\therefore \boldsymbol{triangle\ ABC \sim triangle\ AFE}$

b) $\frac{BC}{3.6cm} = 2$

$\therefore \boldsymbol{BC = 3.2cm \times 2 = 7.2cm}$

c) *Ratio of corresponding sides of the triangles* $=$ $1 : 2$

Ratio of the areas of the triangles $=$ $1^2 : 2^2 = 1 : 4$

$$\therefore \frac{\boldsymbol{Area\ of\ quadrilateral\ BCFE}}{\boldsymbol{Area\ of\ \Delta ABC}} = \frac{4-1}{4} = \frac{3}{4}$$

11.

a) $A\hat{B}C = 180° - D\hat{B}E$

$$\begin{aligned} D\hat{B}E &= 180° - (18° + 18°) \\ &= 180° - 36° \\ &= 144° \end{aligned}$$

$\therefore \boldsymbol{A\hat{B}C = 180° - 144° = 36°}$

b)

- $\angle ABD = 180° - (180° - (18° + 18°))$

  $= 36°$ *(base angles of an isoscles triangle and supplement angles)*

- $\angle DCB = 126°$ *(It is supplementary to $\angle DAB$ wich is $54°$)*

  $\Rightarrow 180° = 126° + 18° + \angle DBC$ *(Sum of angles in a triangle)*

  $\Rightarrow \angle DBC = 180° - (126° + 18°) = 36°$

- $A\hat{B}D = C\hat{B}D = 36°$ *($\therefore$ BD bisects $A\hat{B}C$)*

12. $A = \begin{pmatrix} -1 & 5 \\ 2 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ 2 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 6 & 8 \\ 3 & t \end{pmatrix}$

a) $AB = \begin{pmatrix} -1 & 5 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} -3 + 10 & -2 + -5 \\ 6 + 4 & 4 + -2 \end{pmatrix}$

$= \begin{pmatrix} 7 & -7 \\ 10 & 2 \end{pmatrix}$

b) (When a matrix has no inverse its determinant is equal to zero)

$\boldsymbol{Determinant\ of\ C = (6 \times t - 3 \times 8) = 0}$

(Now solve an equation above for t)

$6t - 24 = 0$

$6t = 24$ *(After adding 24 on both side of the equation)*

$\frac{6t}{6} = \frac{24}{6}$ *(divide by 6 on both sides of the equation)*

$t = 4$

13. *Let x be the the profit made in 2013 then:*

$$x + \frac{8}{100}x = M2160$$

$$x\left(1 + \frac{8}{100}\right) = M2160$$

$$x\frac{27}{25} = M2160$$

$$x = \frac{M2160}{\frac{27}{25}}$$

$$x = M2160 \times \frac{25}{27}$$

$$x = \boldsymbol{M2000}$$

14.

a) $3p(a-8b) - 7q(8b-a)$

$= 3p(a-8b) + 7q(a-8b)$

$= \boldsymbol{(a-8b)(3p+7q)}$

b) $2u^2 + 5u - 3$

$= 2u^2 + 6u - u - 3$

$= 2u(u+3) - 1(u+3)$

$= \boldsymbol{(u+3)(2u-1)}$

15.

a) $\frac{\left(5\frac{1}{3}\right)^2}{\left(1-\frac{1}{3}\right)^3} = \frac{\left(\frac{16}{3}\right)^2}{\left(\frac{2}{3}\right)^3}$

$= \left(\frac{16}{3}\right)^2 \times \left(\frac{3}{2}\right)^3$

$= \frac{16^2}{3^2} \times \frac{3^3}{2^3} = \frac{16\times16}{3\times3} \times \frac{3\times3\times3}{8} = \frac{2\times16}{1} \times \frac{3}{1} = 2\times16\times3 = \boldsymbol{96}$

b) $12(-pt)^2 \times \frac{\sqrt{p^4t^2}}{3} \times \left(\frac{p}{2t}\right)^3$

$= 12p^2t^2 \times \frac{(p^4t^2)^{\frac{1}{2}}}{3} \times \frac{p^3}{2^3t^3}$

$= 12p^2t^2 \times \frac{p^2t}{3} \times \frac{p^3}{8t^3}$

$= \frac{12p^2t^2 \times p^2t \times p^3}{3 \times 8t^3}$

$= \frac{12p^7t^3}{24t^3}$

$= \frac{p^7}{2}$

16.

a) $S = \frac{v^2-u^2}{2a}$

$= \frac{4^2-3^2}{2 \times 7}$

$= \frac{16-9}{14}$

$= \frac{7}{14}$

$= \frac{1}{2}$

b) $S = \frac{v^2-u^2}{2a}$

$2as = v^2 - u^2$ *(After multiplying by 2a on both sides of the equation)*

$2as + u^2 = v^2$ *(After adding u^2 on both sides of the equation)*

$v^2 = 2as + u^2$ *(Now take the square root on both sides)*

$\boldsymbol{v = \pm\sqrt{2as + u^2}}$

17. $\frac{5.112 \times 39.997}{0.199}$

$= \frac{5 \times 40}{0.2} \times \frac{10}{10}$

$= \frac{5 \times 400}{2}$

$= 5 \times 200$

$= 1000$

18.

a) *The modal mark is* 70

b)

i. $P(\textit{more than } 60\%) = \frac{13}{39} = \frac{1}{3}$

ii. $P(40\% \textit{ or } 70\%) = P(40\%) + P(70\%)$

$= \frac{7}{39} + \frac{10}{39}$

$= \frac{17}{39}$

19. (Before we can find the acceleration we have to find the velocity ,v of the particle)

(We can find V from the formula of the area under the graph)

(The area under the graph is numerically the same as the distance travelled)

(Use the formula for the area of a trapezium)

$$Area\ of\ trapezium = \frac{1}{2}(a+b)h$$

$$Distance = \frac{1}{2}(11s + 4s)V$$

$$45m = \frac{15s}{2} \times V$$

$$90m = 15s \times V$$

$$V = \frac{90m}{15s} = 6m/s$$

(Now that we know the maximum speed of the particle, we can find the acceleration, a)

$$\boldsymbol{a = \frac{V-u}{t} = \frac{6m/s\ -\ 0m/s}{3s} = \frac{6m/s}{3s} = 2m/s^2}$$

2018 MAY/JUNE PAPER 2 SOLUTIONS

1.

a) $-2^3 - (-9) \times 4 \div (2 \times 3^2)$

$= -2^3 + 9 \times 4 \div (2 \times 9)$

$= -8 + 9 \times 4 \div 18$

$= -8 + 36 \div 18$

$= -8 + 2$

$= -6$

b) $4^{-\frac{1}{2}} + 16^{\frac{3}{4}} = \frac{1}{\sqrt{4}} + \left(\sqrt[4]{16}\right)^3$

$= \frac{1}{2} + 2^3$

$= \frac{1}{2} + \frac{8}{1}$

$= \frac{1+16}{2}$

$= 8\frac{1}{2}$

2.

a) $let \;\; p = \frac{x}{y} \quad and \quad q = \frac{a}{b}$

Common difference of numerators $= 3 - 1 = 2$

Common difference of denominators $= 4 - 2 = 2$

$\therefore x = 5 + common\ difference$

$= 5 + 2$

$= 7$

And $y = 6 + common\ difference$

$= 6 + 2$

$= 8$

$\therefore p = \frac{7}{8}$

$s = 21 + Common\ difference$

$= 21 + 2$

$= 23$

And $u = 22 + Common\ difference$

$= 22 + 2$

$= 24$

$\therefore\ q = \frac{23}{24}$

b) $t_n = \frac{a+d(n-1)}{x+y(n-1)}$

Where; a and x are first terms and d and y are common differences.

$\therefore\ t_n = \frac{1+2(n-1)}{2+2(n-1)}$

$= \frac{1+2n-2}{2+2n-2}$

$= \frac{2n-1}{2n}$

3.

a) $19℃ - (-5℃) = 24℃$

b)

i. **(Find the rate in degrees per hour and use it to find the temperature)**

$$rate(gradient) = \frac{19° - (-5°)}{1200 - 0400} = \frac{24°}{8\ hours} = 3°\ /\ hour$$

From $y = mx + c$

$\Rightarrow y = 3x + c$

$\Rightarrow T = 3t + c$

Now finding the temperature intercept:

$\Rightarrow\ c = T - 3t$

$\Rightarrow\ c = 19°C - (3°\ /hour\ \times\ 12)$

$\Rightarrow\ c = -17°$

Now finding the Temperature:

$T = 3t + c$

$= 3°C\ /\ hour \times 9.5\ hours\ - 17°$

$= 28.5°C - 17°C$

$= 11.5°$

The temperature at 0930 hours was 11.5°C.

ii. **(Use the straight line equation and rate found in the answer above)**

$T = 3t + c$

$\Rightarrow 8°C = 3°C/hour \times t - 17°$

$\Rightarrow t = \frac{8°C+17°C}{3°\ /\ hour}$

The time when the temperature was 8°C is $0820hrs$.

4.

a)

i. Note that this expression is a difference of two squares.
$a^2 - b^2 = (a - b)(a + b)$

ii. $97^2 - 9$ $(This\ is\ also\ a\ difference\ of\ two\ squares)$

$= (97 - 3)(97 + 3)$

$= 94 \times 100$

$= 9400$

b) $3x^2 - 13x - 10 = (x - 5)(3x + 2)$

5.

a) $A^2 = A \times A$

$$= \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 16+0 & 8+6 \\ 0+0 & 0+9 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 14 \\ 0 & 9 \end{pmatrix}$$

b) We solve the matrix equation $AB = I$ as follows:

$$AB = I$$

$$\Rightarrow \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} \frac{1}{4} & k \\ 0 & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{pmatrix} 1+0 & 4k+\frac{2}{3} \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Using the concept of equal matrices we have,

$$4k + \frac{2}{3} = 0$$

$$\Rightarrow 12k + 2 = 0$$

$$\Rightarrow 12k = -2$$

$$\therefore k = -\frac{2}{12} = -\frac{1}{6}$$

c) $If \det A = \det B$

$$\Rightarrow \begin{vmatrix} 4 & 2 \\ 0 & 3 \end{vmatrix} = \begin{vmatrix} 12 & 0 \\ -9 & m \end{vmatrix}$$

$$\Rightarrow (4 \times 3) - (0 \times 2) = (12 \times m - (-9) \times 0)$$

$$\Rightarrow 12 - 0 = 12m - 0$$

$$\Rightarrow 12 = 12m$$

$$\therefore m = \frac{12}{12} = 1$$

6.

a) $3x - 5(3 - x) = 41$ *(Remove brackets and simplify)*

$\Rightarrow 3x - 15 + 5x = 41$

$\Rightarrow 3x - 15 + 5x - 41 = 0$

$\Rightarrow 8x - 56 = 0$

$\therefore \quad x = \frac{56}{8} = 7$

b) $2^{x+1} - 32 = 0$

$\Rightarrow \quad 2^{x+1} = 32$

$\Rightarrow \quad 2^{x+1} = 2^5 \qquad (\because \ 32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5)$

$\therefore \quad x + 1 = 5 \qquad (\because \ if \ a^x = a^y \ \Rightarrow \ x = y)$

$\therefore \quad x = 4$

c) **Using the method of elimination**

$2x + y = 10$..... [1]

$7x - 3y = 9$..... [2]

Eliminating x to find the value of y:

(Multiply equation [1] by 7 equation [2] by 2)

$14x + 7y = \ 70$..... [3]

$14x - 6y = 18$..... [4]

(Subtract equation [4] from equation [3])

$7y - (-6y) = 70 - 18$

$\Rightarrow 13y = 52$

$\Rightarrow y = \frac{54}{13}$

$\therefore y = 4$

Now using the value of y to find x:

(Substitute 4 into equation [1] then solve for x)

$2x + 4 = 10$

$\Rightarrow \ 2x = 6$

$\Rightarrow \ x = 3$

$\therefore \ x = 3 \quad and \quad y = 4$

7.

a) (The number of sides of any regular polygon is equal to the number of its lines of symmetry)

∴ **This octagon has 8 lines of symmetry.**

b) $Size\ of\ angles\ at\ the\ centre = \frac{360°}{8} = 45°$

$\therefore \frac{45°}{2} = 22.5°$

$\therefore$ *angle betweeen any two adjacent line of symmetry is* $22.5°$

c) $b = 360° - 180° - 135°$

$= 180° - 135°$

$= 45°$

8.

a) Δ ABC $is\ similar\ to\ \Delta DBG$

b) $C\hat{E}F = G\hat{D}F \quad (Alternating\ angles)$

$E\hat{F}C = G\hat{F}D \quad (Vertically\ opposite\ angles)$

$E\hat{C}F = F\hat{G}D \quad (Alternating\ angles)$

$\therefore \ \Delta DGF \sim \Delta ECF$

c)

i. $BG = BC - GC$

$= 9 - 6$

$= 3$

$\because \quad BC = 9cm$

$= 3 \times 3$

$= 3BG$

$\therefore \quad \boldsymbol{BC = 3BG}$

ii. **(Ratio of areas is equal to the square of ratio of sides)**

Ratio of sides:

$\boldsymbol{BG : BC}$

$\boldsymbol{= 1 : 3}$

Ratio of areas:

$\boldsymbol{1^2 : 3^2}$

$\boldsymbol{= 1 \ : \ 9}$

$\therefore \boldsymbol{Area\ of\ _{ADGC} \ : Area\ of\ \Delta ABC}$

$= \boldsymbol{(Area\ of\ \Delta ABC - Area\ \Delta BDG) : Area\ of\ \Delta ABC}$

$= \boldsymbol{(9 - 1) : 9}$

$= \boldsymbol{8 : 9}$

9.

a)

i. (The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle circumference)

$\angle_{AOC} = 2 \times \angle ABC$

$= 2 \times 94°$

$= 188°$

ii. $angle\ BAO = 360° - [(90° - 50°) + 172° + 94]$

$= 360° - (40° + 172° + 94°)$

$= 360° - 306°$

$= \mathbf{54°}$

b) $Area = \frac{1}{2} \times (CQ)^2$

$8cm^2 = \frac{1}{2} \times (CQ)^2$

$16\ cm^2 = (CQ)^2$

$\therefore\ \boldsymbol{CQ = \sqrt{16cm^2} = 4cm}$

$(CD)^2 = (CQ)^2 + (DQ)^2$

$= (4cm)^2 + (4cm)^2$

$= 16\ cm^2 + 16\ cm^2$

$= \mathbf{32cm^2}$

10.

a) $\boldsymbol{a = \frac{v-u}{t} = \frac{10\ m/s\ -0\ m/s}{5s} = 2\ m/\ s^2}$

$\boldsymbol{2m/s^2 = \frac{s}{3sec}}$

$\boldsymbol{\Rightarrow s = 3sec \times 2m/\ s^2}$

$\boldsymbol{\Rightarrow s = 6\ m/\ s}$

b) (Area under speed-time graph is numerically equal to the total distance travelled)

$\Rightarrow Total\ distance\ (Area) = \left[\frac{1}{2} \times 5 \times 10\right] + [10(t-5)] + \left[\frac{1}{2} \times 10 \times (12-t)\right]$

$\Rightarrow\ 65 = 25 + 10t - 50 + 60 - 5t$

$\Rightarrow 65 = 35 + 5t$

$\Rightarrow 30 = 5t$

$\therefore t = 6\ seconds$

$\therefore\ \boldsymbol{interval\ of\ time\ = 12\ sec - 6\ sec = 6\ sec}$

11.

a) $M_{AC} = \frac{3-9}{2-0} = -\frac{6}{2} = -3$

$\therefore -3 = \frac{0-3}{x-2}$

$\Rightarrow -3x + 6 = -3$

$\Rightarrow -3x = -9$

$\Rightarrow \quad x = 3$

$\therefore$ ***Point C is*** $(3, 0)$

b) $M_{CD} = \frac{-4.5-0}{0-3} = \frac{4.5}{3} = 1.5$

$\therefore$ ***equation of CD is*** $y = \frac{3}{2}x - \frac{9}{2}$

c) $BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-3)^2 + (3-2)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

r = 10

12.

a)

i. $lower\ quartile\ position = \frac{1}{4} \times (7+1)^{th}\ position$

$= \left(\frac{8}{4}\right)^{th}$

$= 2^{nd}\ positon$

(Write the distribution in ascending order)

6, 8,8,10,12,15,18

$\therefore L.Q = 8$

ii. $Inter-quartile\ range = U.Q - LQ$

$U.Q\ position = \frac{3}{4}(8)^{th}\ position$

$= (3 \times 2)^{th}\ position$

$= 6^{th}\ position$

$\therefore\ U.Q = 15$ *Therefore inter-quartile range = 15 - 8 = 7*

b) mean = $\frac{12+10+8+15+18+6+8}{7} = \frac{77}{7} = 10$

13.

a) (Make p the subject of the formula)

$$T = \frac{2r\sqrt{p}}{3}$$

$\Rightarrow 3T = 2r\sqrt{p}$ $(After\ multiplying\ both\ sides\ by\ 3)$

$\Rightarrow \frac{3T}{2r} = \sqrt{p}$ $(After\ dividing\ both\ sides\ by\ 2r)$

$\Rightarrow \frac{9T^2}{4r^2} = p$ $(After\ squaring\ both\ sides)$

$\therefore\ p = \frac{9T^2}{4r^2}$

b) $0 = (x+3)(x-5)$

$\Rightarrow 0 = x^2 - 5x + 3x - 15$

$\Rightarrow 0 = x^2 - 2x - 15$

$\therefore$ ***the equation is*** $x^2 - 2x^2 - 15 = 0$

2018 MAY-JUNE PAPER 4 SOLUTIONS

1.

a) $\frac{1}{4}+\frac{1}{5}$

$=\frac{5+4}{20}$

$=\frac{9}{20}$

b)

i. 72 ,350,600 Is words is read as: **Seventy two million, three hundred and fifty thousand, six hundred**.

ii. $\boldsymbol{lower\ bound = 72\ 350\ 600 - \frac{100}{2} = 72\ 350\ 550}$

iii. $\mathbf{72\ 350\ 600 \approx 72\ 400\ 000}$ (To the nearest hundred thousand)

c)

i. **Let x be the total amount received from their father.**

$\frac{2}{3}\ x = M1{,}430.40$

$2x = M7{,}152.00$ (After multiplying by 3 on both sides)

$\boldsymbol{x = M3,576.00}$ (After dividing by 2 on both sides)

ii. Interest made by company A after 5 years.

$$I = \frac{PRT}{100} = M6{,}000.00 \times \frac{4}{100} \times 5\ years = \boldsymbol{M1,200.00}$$

Interest made by company B after 5 years.

$$A = P\left(1 + \frac{R}{100}\right)^n = M5{,}000.00\ \left(1 + \frac{5}{100}\right)^5$$
$$= M5{,}000.00 \times 1.276281562$$
$$= \boldsymbol{M6{,}381.41}$$

$$\therefore I = A - P$$
$$= M6{,}381.41 - M5{,}000.00$$
$$= \boldsymbol{M1{,}381.41} \qquad (For\ company\ B)$$

$We\ conclude\ that \quad I_B > I_A$

Difference of interest of these companies.

$$I_B - I_A = M1{,}381.41 - M1{,}200.00 = M181.41$$

$$\therefore \boldsymbol{Company\ B\ will\ get\ more\ interest\ after\ 5\ years\ by\ M181.4}$$

2.

a) $CA = \begin{pmatrix}1 & -3\end{pmatrix}\begin{pmatrix}-1 & 2\\ 0 & 1\end{pmatrix}$

$= (-2 + 0 \quad 4 + -3)$

$= (\mathbf{-2} \quad \mathbf{1})$

b) $B^{-1} = \frac{1}{detB}\begin{pmatrix}-1 & 2\\ 0 & 1\end{pmatrix}$

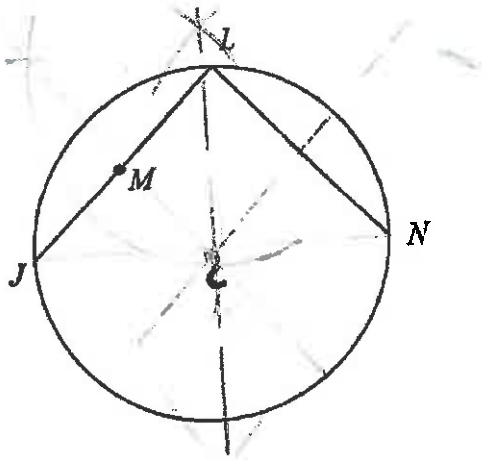
Now finding the determinant of B: $\quad det\ B = 8 - 0 = 8$

$$\therefore \boldsymbol{B^{-1}} = \frac{1}{8}\begin{pmatrix}\mathbf{4} & \mathbf{0}\\ \mathbf{-1} & \mathbf{2}\end{pmatrix} = \begin{pmatrix}\frac{1}{2} & \mathbf{0}\\ -\frac{1}{8} & \frac{1}{4}\end{pmatrix}$$

3.

a)

i. (To find the centre of the circle, draw the perpendicular bisector of the two cords, JL and LN, and where the two bisectors intersect; that is the centre of the circle.)



ii. **JL and LN are two equal chords.**

∴ They are equidistant from the centre.

b) $Acute \angle XWV = |270° - 360°| = 90°$

$Each\ base\ angle\ of\ the\ isosceles\ triangle = 45°$

$\therefore\ \angle WXR = 45°$

4.

a) $\frac{x^2-1}{2x^2+5x+3} = \frac{(x+1)(x-1)}{2x^2+2x+3x+3}$

$$= \frac{(x+1)(x-1)}{2x(x+1)+3(x+1)}$$

$$= \frac{(x+1)(x-1)}{(2x+3)(x+1)}$$

$$= \frac{x-1}{2x+3}$$

b)

i. $y = \frac{k}{\sqrt{x}}$

Finding k

$k = y\sqrt{x}$ $\quad (After\ multiplying\ by\ \sqrt{x}\ on\ both\ sides)$

$= 9\sqrt{16}$

$= 9 \times 4$

$= 36$

$\therefore \boldsymbol{y = \frac{k}{\sqrt{x}}}$ $\quad \boldsymbol{becomes} \quad \boldsymbol{y = \frac{36}{\sqrt{x}} = 36x^{-\frac{1}{2}}}$

ii. (We must make x the subject of the formula first)

$y = \frac{36}{\sqrt{x}}$ $\quad (Multiply\ by\ \sqrt{x}\ \ on\ both\ sides)$

$\Rightarrow \quad y\sqrt{x} = 36$ $\quad (After\ multiplying\ by\ \sqrt{x}\ \ on\ both\ sides)$

$\Rightarrow \quad (y\sqrt{x})^2 = 36^2$

$\Rightarrow \quad xy^2 = 1296$ $\quad (After\ squaring\ both\ sides\ of\ the\ equation)$

$\Rightarrow \quad x = \frac{1296}{y^2}$

$= \frac{1296}{18^2}$ $\quad (Atfer\ Substituting\ 18\ for\ y)$

$= \frac{1296}{324}$

$= 4$

5.

a)

i. (The question requires $n(B \cap I \cap P)$)

$\therefore n(B \cap I \cap P) = 10$

ii. Number of students who have Birth certificate and ID documents only is 6.

b) $n(B \cap P \cap I') = 18$

c) $B \cap I' \cap P'$ *or* $B \cap (I \cup P)'$

6.

a)

i. $g(-7) = 3 - 2(-7)$

$= 3 + 14$

$= 17$

ii. (To find $fg(3)$ first find $fg(x)$)

$fg(x) = f(3 - 2x)$

$= \frac{1}{(3-2x)+2} + \frac{3}{1}$

$= \frac{1}{5-2x} + 3$

$\therefore fg(3) = \frac{1}{5 - 2(3)} + 3$

$= \frac{1}{5-6} + 3$

$= \frac{1}{-1} + 3$

$= 2$

b) $f(x) = \frac{1}{x+2} + 3$

Finding the value of x for which $f(x) = 0$

$$\frac{1}{x+2} + 3 = 0$$

$\Rightarrow \quad 1 + 3(x+2) = 0 \qquad (After\ multiplying\ by\ (x+2) on\ both\ sides)$

$\Rightarrow \quad 1 + 3x + 6 = 0$

$\Rightarrow \quad 3x + 7 = 0$

$\Rightarrow \quad 3x = -7$

$\Rightarrow \quad x = -\frac{7}{3} \qquad (After\ dividing\ by\ 3\ on\ both\ sides)$

$\therefore \quad x = -2\frac{1}{3}$

c)

i. Finding $g^{-1}(x)$

We know that;

$$g(x) = 3 - 2x$$

let $\quad g(x) = y$

then, $\quad y = 3 - 2x$

(From this, make x the subject of the formula.)

$y + 2x = 3 \qquad (After\ adding\ 2x\ on\ both\ sides)$

$2x = 3 - y \qquad (After\ subtracting\ y\ from\ both\ sides)$

$x = \frac{3-y}{2} \qquad (After\ dividing\ by\ 2\ on\ both\ sides)$

Then let $x = f^{-1}(x)$ *and* $y = x$

$\Rightarrow \quad f^{-1}(x) = \frac{3-x}{2}$

ii. $let\ y = x^n$

$\Rightarrow\ x = \sqrt[n]{y}$

$\therefore\ f^{-1}(x) = \sqrt[n]{x}$ ..... [1]

But it's given that

$p(x) = x^n$ ..... [2]

Find n if

$p(x) = p^{-1}(x)$

∴ Equating equation [1] and [2], we obtain

$x^n = \sqrt[n]{x}$

$\Rightarrow\ x^n = x^{\frac{1}{n}}$

(The bases are the same; this implies that the powers are equal)

$n = \frac{1}{n}$ $(Solve\ for\ n)$

$n^2 = 1$ $(After\ multiplying\ n\ on\ both\ sides)$

$\therefore\ \boldsymbol{n = 1 \quad or \quad n = -1}$

7.

a) $x\ words = 60\ seconds$

$\Rightarrow\ 1\ words = \frac{60}{x}\ seconds$

$\therefore\ \boldsymbol{200\ words = 200 \times \frac{60}{x}\ seconds = \frac{12000}{x}\ seconds}$

b)

i. **Puleng types $(x-10)$ words.**

ii. Puleng types $(x-10)$ words per minute

$\Rightarrow\ (x-10)\ words = 60\ seconds$

$\Rightarrow\ 1\ word = \frac{60}{x-10}\ seconds$

$\therefore\ 200\ words = 200 \times \frac{60}{x-10}\ seconds$

$= \frac{12000}{x-10}\ \boldsymbol{seconds}$

c)

i. $\frac{12000}{x} + 100 = \frac{12000}{x-10}$

(Multiply each term by the LCM of the denominators, this is $[x-1]$)

$\frac{12000}{x} \times\ x(x-10)\ +\ 100\,x(x-10) = \frac{12000}{x-10} \times\ x(x-10)$

$12000\,(x-10)\ +\ 100x^2 - 1000x = 12000x$

$12000x - 120\,000\ + 100x^2 - 1000x = 12000x$

$12000x - 120000 + 100x^2 - 1000x - 12000x = 0$

$100x^2 - 1000x - 120000 = 0$

$\boldsymbol{x^2 - 10x - 1200 = 0}$ $\qquad$ $\boldsymbol{(After\ dividing\ by\ 100\ on\ both\ sides)}$

Hence Shown!

ii. (We shall us the method of factorization)

$x^2 - 10x - 1200 = 0$

$\Rightarrow (x-40)(x+30) = 0$

$\Rightarrow\ \boldsymbol{x = 40}\quad \boldsymbol{or}\quad \boldsymbol{x = -30}$

iii. $3510\ words = 3510 \times \frac{60}{x-10} seconds$

$= 3510 \times \frac{60}{30}\ seconds \quad (\because\ x = 40)$

$= 7020\ seconds$

Converting the time to minutes:

$1\ minute = 60\ seconds$

$\Rightarrow\ 1\ second = \frac{1}{60}\ seconds$

$\therefore\ \mathbf{7020\ seconds = 7020 \times \frac{1}{60}\ minutes = 117\ minutes}$

8.

a) (Surface area = area of a halved curved surface + surface area of a halved hemisphere + area of a curved circle)

$Surface\ area = \frac{1}{2}(2\pi rl) + \frac{1}{2}\left(\frac{1}{2} \times 4\pi r^2\right) + \frac{1}{2}\pi r^2$

$= \pi rl + \pi r^2 + \frac{1}{2}\pi r^2$

$= \pi\left(rl + r^2 + \frac{1}{2}r^2\right)$

$= \pi\,(0.3m \times 2m + (0.3m)^2 + 0.5 \times (0.3m)^2)$

$= \pi\,(0.735m^2)$

$= \mathbf{2.31m^2}$

b) $V_{total} = V_{cylinder} + V_{hemisphere}$

$$= \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi\left[(0.3)^2 \times 2m + \frac{2 \times (0.3)^2}{3}\right]$$

$$= \pi\,[\,0.09^2 \times 2m + 0.018m^3\,]$$

$$= \pi\,[\,0.18\,m^3 + 0.018m^3\,]$$

$$= \pi\,[\,0.198m^3\,]$$

$$= \mathbf{0.622m^3}$$

But we know that;

$$1m = 100m$$

$$\Rightarrow 1m^3 = 1\,000\,000cm^3$$

$$\therefore\ 0.622m^3 = 0.622 \times 1\,000\,000cm^3 = 622\,000cm^3$$

But we also know that;

$$1\ litre = 1000cm^3$$

$$\Rightarrow\ 1\ cm^3 = \frac{1}{1000}\ litre$$

$$\therefore\ 622\,000cm^3 = 622\,000 \times \frac{1}{1000}\ litre$$

$$= \mathbf{622\ litres}$$

9.

a) **(We shall use the cosine rule: $c^2 = a^2 + b^2 - 2ab \times cosACB$)**

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC)Cos\hat{ABC}$$
$$= (70m)^2 + (79m)^2 - 2 \times 70m \times 79m \times cos120°$$
$$= 11\,141m^2 - (-5530m^2)$$
$$= 16\,671m^2$$

$$\therefore \boldsymbol{AC = \sqrt{1667m^2} = 129m}$$

b) **If a and b are the arms of a triangle whose vertex is c, then its area is given by the formula:**

$$Area = \frac{1}{2}\, ab \times sinC$$
$$= \frac{1}{2} \times 70m \times 79m \times Sin120°$$
$$= \mathbf{2\,394.6m^2}$$

c) (The shortest distance has to be a perpendicular line to DC from B.)

$$The\ angle\ between\ this\ path\ and\ BC = 120° - 90° = 30°$$

$$Cos30° = \frac{Shortest\ distance}{79m}$$

$$\Rightarrow \boldsymbol{Shortest\ distance = Cos30° \times 79m}$$
$$\boldsymbol{= 68.4m}$$

10.

a)

i. $\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -4\underline{a} + 3\underline{b} = 3\underline{b} - 4\underline{a}$

ii. $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$= 4\underline{a} + \frac{2}{5}(\overrightarrow{PR})$

$= 4\underline{a} + \frac{6}{5}\underline{b} - \frac{8}{5}\underline{a}$

$= \frac{12}{5}\underline{a} + \frac{6}{5}\underline{b}$

$= \frac{6}{5}(2\underline{a} + \underline{b})$

b) (We are to show that $\overrightarrow{OQ} = \frac{2}{5}\overrightarrow{OS}$ .First find $\overrightarrow{OS}$)

$\overrightarrow{OS} = \overrightarrow{OR} + \overrightarrow{RS}$

$= 3\underline{b} + 3\underline{a}$

$= 6\underline{a} + 3\underline{b}$

But from question 10 (a) ii. Above;

$\overrightarrow{OQ} = \frac{12}{5}\underline{a} + \frac{6}{5}\underline{b}$

$= \frac{2}{5}(6\underline{a} + 3\underline{b})$

$= \frac{2}{5}\overrightarrow{OS}$

$\therefore \overrightarrow{OQ} = \frac{2}{5}\overrightarrow{OS}$

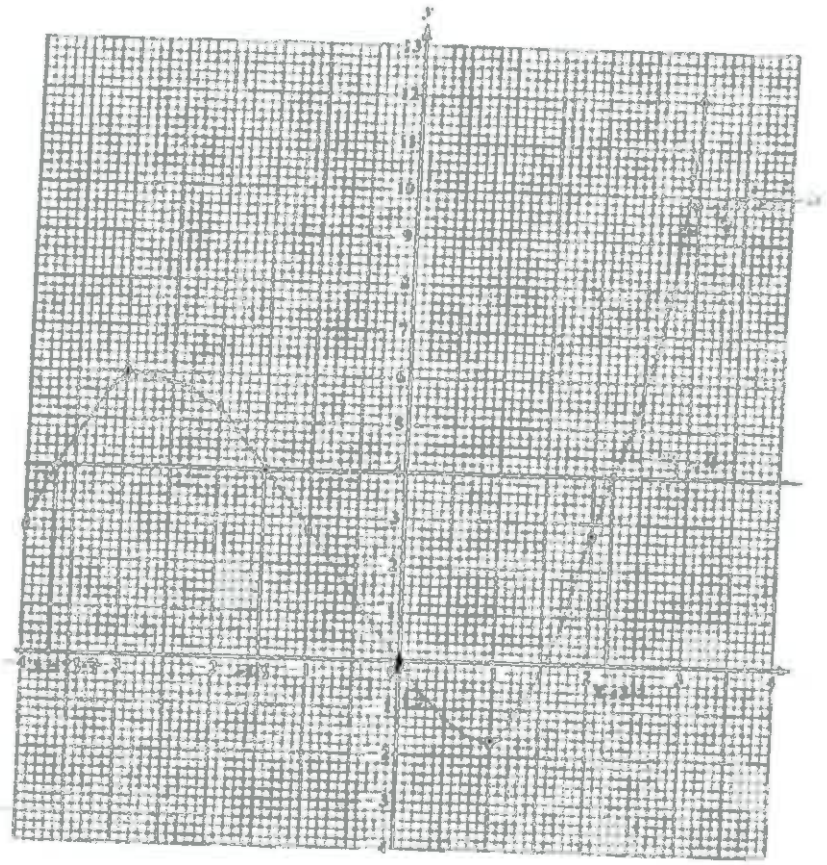
HENCE PROVED

11.

a)

X	-4	-3	-2	-1	0	1	2	3
Y	2.7	6	5.3	2.7	0	-0.7	2.7	12

b)



c) (To solve the equation $\frac{1}{3}x^3 + x^2 - 2x = 4$, we draw the line $y = 4$)

(The x-values at the point of intersection with the graph $y = \frac{1}{3}x^3 + x^2 - 2x$ will be solutions to $\frac{1}{3}x^3 + x^2 - 2x = 4$)

These are: $x = -3.7$ *or* $x = -1.5$ *or* $x = 2.2$

d) $\frac{1}{3}x^3 + x^2 - 7 = 5x$ .....[1]

$\Rightarrow \frac{1}{3}x^3 + x^2 = 5x + 7$ ..... [2]

And

$\frac{1}{3}x^3 + x^2 - 2x = 0$ ..... [3]

$\Rightarrow \frac{1}{3}x^3 + x^2 = 2x$ ..... [4]

Substituting equation [4] into equation [2] we get;

$0 = 3x + 7 \quad \Rightarrow \quad y = 3x + 7$

∴ The equation of the straight line that can be drawn on the grid to solve the equation;

$\frac{1}{3}x^3 + x^2 - 7 = 5x$ **is given by** $y = 3x + 7$

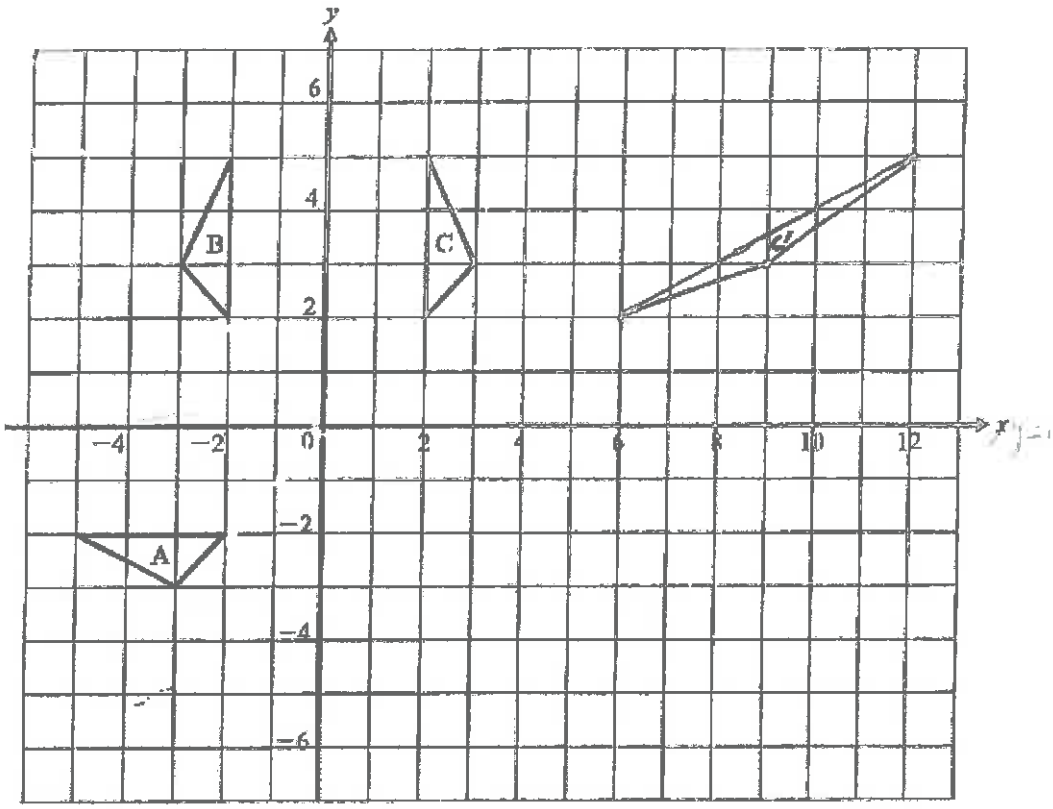
12.

a)

i. **It is a reflection in the mirror line** $y = -x$

ii. **It is a rotation about the origin through an angle of** $-90° \; or \; +270°$

b)



13.

a)

i. $P = \frac{x}{25+x} = \frac{1}{6}$

Solve for x

$$\frac{x}{25+x} = \frac{1}{6}$$

$$\Rightarrow 6x = 25 + x$$

$$\Rightarrow 5x = 25$$

$$\therefore \quad x = 5$$

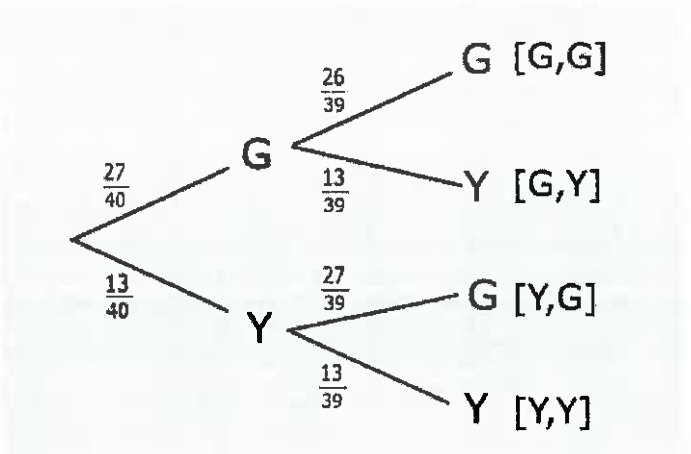
$\therefore$ **5 blue marbles.**

ii. $Number\ of\ marbles\ that\ are\ in\ the\ box = (25 + 5) marbles$

$= 30\ marbles$

b)

i. (Calculating the probability that both marbles are green, using the tree diagram)



$$P(both\ green) = \frac{27}{40} \times \frac{26}{39} = \frac{702}{1560} = \frac{9}{20}$$

ii. $P(at\ least\ one\ yellow) = \frac{1}{1} - \frac{9}{20} = \frac{11}{20}$

14.

a)

i. $35 \leq m < 50$

ii. $median\ position = \frac{1}{2}\, n^{th} = \frac{1}{2} \times 44^{th} = 22^{nd}\ position$

$\therefore The\ median\ position\ is\ 25 \leq m < 35$

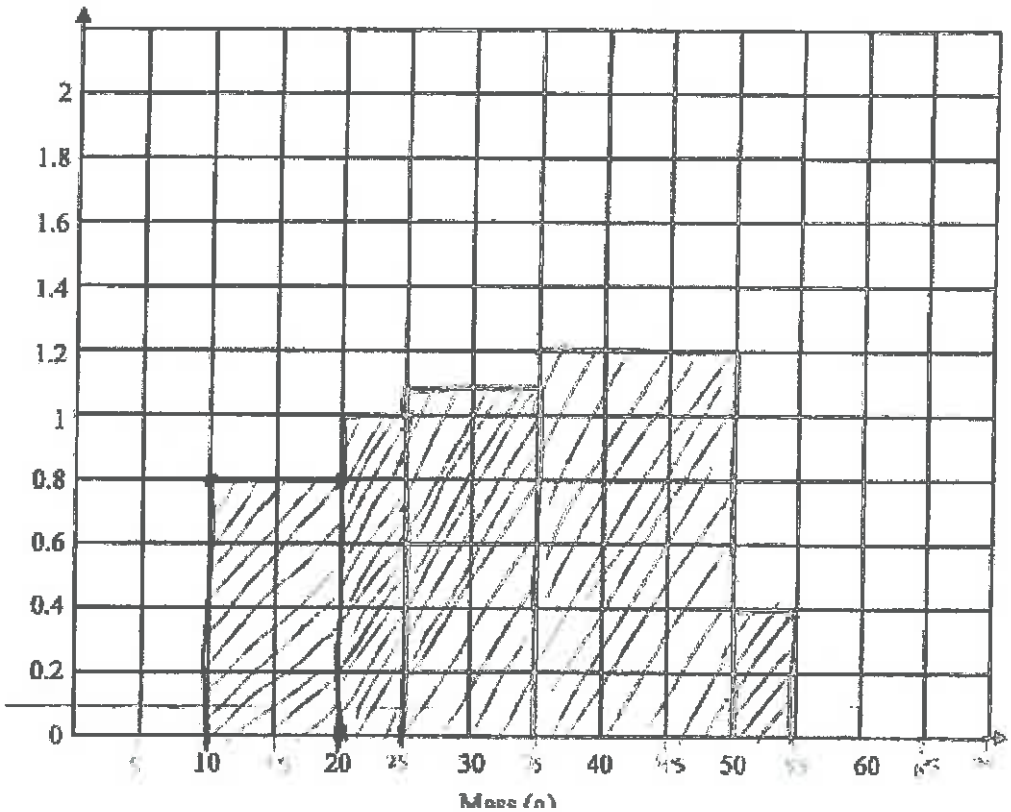
$$Estimated\ mean = \frac{sum(frequency \times variable)}{Sum\ of\ frequency}$$

[Note: Our variable is in ranges (grouped), therefore we must take the mean of each lower and upper limit (mid-class value)]

$$\therefore Estimated\ \ mean = \frac{(15\times8)+(22.5\times5)+(30\times11)+(42.5\times18)+(52.5\times2)}{8+5+11+18+2}$$

$$= \frac{1432.5}{44}$$

$$= \mathbf{32.6}$$



15.

a)

i. The minimum number of learners that can be transported in one trip is :

$\mathbf{60 + 15 = 75}$

ii. **An expression for the number of learners that can be transported is;**

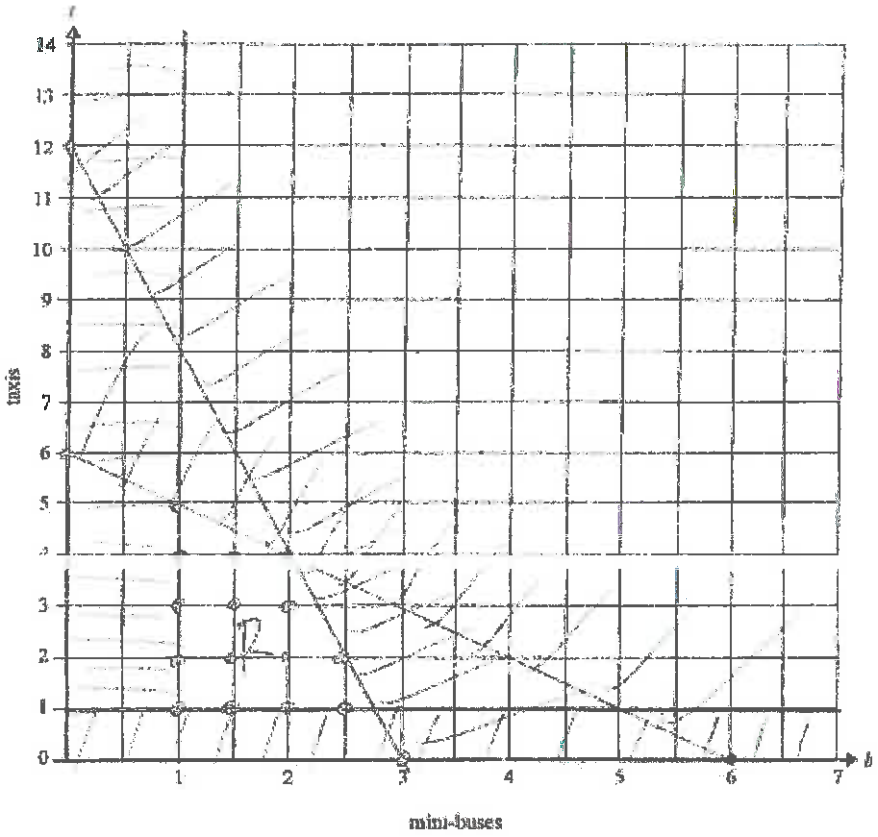
$\mathbf{60}b + \mathbf{15}t$

b)

i. $60b + 15t \leq 180$

$\therefore 4b + t \leq 12$

ii. **We must also draw the boundary line** $t = 6 - b$



c) The minimum value/least possible cost is given by the coordinate (1,1)

$\therefore least\ possible\ cost = M800.00 \times 1 + M400.00 \times 1$

$= M1,200.00$

2018 OCTOBER/NOVEMBER: PAPER 2 SOLUTIONS

1.

a) $27\,359 \approx 30\,000$ **(Correct to one significant figures)**

b) (The question requires us to find the upper bound of the number)

(The second significant figure has a place value of 100; therefore limit of accuracy is 100)

$$Upper\ bound = 2500 + \frac{limit\ of\ accuracy}{2}$$
$$= 2500 + \frac{100}{2}$$
$$= 2500 + 50$$
$$= \mathbf{2550}$$

The largest number of people is 2550.

2.

a) $\boldsymbol{Perimeter = 2a + 2b}$ $(a\ is\ the\ length\ and\ b\ is\ the\ width)$

$$= 2(4.5 \times 10^3) + 2(3 \times 10^2)$$
$$= (9 \times 10^3) + (6 \times 10^2)$$
$$= (90 \times 10^2) + (6 \times 10^2)$$
$$= (90 + 6) \times 10^2$$
$$= 96 \times 10^2$$
$$= \mathbf{9.6 \times 10^3\ mm}$$

b) $Area = length \times width$

$$= (4.5 \times 10^3) mm \times (3 \times 10^2)\ mm$$
$$= [(4.5 \times 3) \times (10^3 \times 10^2)]\ mm^2$$
$$= 13.5 \times 10^5\ mm^2$$
$$= \mathbf{1.35 \times 10^6\ mm^2}$$

3. (For any similar figures, the ratio of volumes is the cube of the ratio of sides)

$Ratio\ of\ heights\ =\ x\ :\ 9$

$Ratio\ of\ volumes =\ x^3 : 9^3$

$= 32 : 108$

(We can cross multiply to find the value of x)

$x^3 \times 108\ =\ 9^3 \times 32$

$\Rightarrow\ x^3 = \frac{9^3 \times\ 32}{108}$ $(After\ dividing\ by\ 108\ on\ both\ sides)$

$\Rightarrow\ x^3\ = \frac{23\ 328}{108} = 216$

$\Rightarrow\ x\ = \sqrt[3]{216} = 6$ $(After\ taking\ the\ cube\ root\ of\ both\ sides)$

4.

a) (Write the matrix equation and solve)

$$\begin{pmatrix} 3 & 4 \\ a & 3 \end{pmatrix} + \begin{pmatrix} 3k & k \\ 0 & -2k \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -3 & -1 \end{pmatrix}$$

$\Rightarrow\ a + 0 = -3$

$\therefore\ a = -3$

$and\quad 2 + 3k = 8$

$\Rightarrow\quad 3k = 6$ $(After\ subtracting\ 2\ from\ both\ sides)$

$\Rightarrow\quad k = 2$ $(After\ dividing\ by\ 3\ on\ both\ sides)$

b) (If the matrix has no inverse, it means its determinant is zero)

$Determinat = (1 \times 4) - (-2x) = 0$

$\Rightarrow\ 4 + 2x = 0$

$\Rightarrow\ x = -2$

5.

a) $\frac{x-1}{3}-\frac{2x-1}{2}$

$=\frac{x-1}{3}-\frac{2x-1}{2}$

$=\frac{2(x-1)-3(2x-1)}{6}$

$=\frac{2x-2-6x+3}{6}$

$=\frac{1-4x}{6}$

b) **(Write $\frac{1}{x-1}-\frac{3}{x}+1$ as a single fraction to find b and c)**

$\frac{1}{x-1}-\frac{3}{x}+1$

$=\frac{x-3(x-1)+x(x-1)}{x(x-1)}$

$=\frac{x-3x+3+x^2-x}{x(x-1)}$

$=\frac{x^2-3x+3}{x(x-1)}$

$\therefore b=-3 \quad and \quad c=3$

6.

a) **Pentagon**

b) ***There are 5 grey isosceles triangles***

c) (For regular polygons, the number of lines of symmetry is equal to order of rotational symmetry)

∴ **Order of rotational symmetry is 5**

d) $size\ of\ interior\ angle=\frac{180°(n-2)}{n}$

$=\frac{180°(5-2)}{5}$

$=\frac{180°\times3}{5}=108°$

7.

a) $5 \times 2^y = 320$ $(Divide\ by\ 5\ on\ both\ sides)$

$\Rightarrow 2^y = 64$ $(Write\ 64\ as\ the\ base\ of\ 2)$

$\Rightarrow 2^y = 2^6$

$\Rightarrow \boldsymbol{y = 6}$

b) $bx + y = b$ .....[1]

$ax - y = a$ ..... [2]

(Add equation [1] to equation [2] to eliminate y)

$(bx + ax) + y + (-y) = b + a$

$\Rightarrow bx + ax = b + a$

$\Rightarrow x(b + a) = b + a$

$\Rightarrow x = \frac{b+a}{b+a}$

$\Rightarrow x = 1$

(Now substitute the value of x into equation [2] to find the value of y)

$a(1) - y = a$

$\Rightarrow a - y = a$

$\Rightarrow a - a = y$

$\Rightarrow y = 0$

$\therefore \boldsymbol{x = 1\ and\ y = 0}$

8.

a)

i. $M4\ 400.00 \times \frac{5}{100} = M220.00$

$New\ salary = M4400.00 + M220.00 = \boldsymbol{M4\ 620.00}$

ii. **Let x be the previous year salary.**

$$previous\ salary\ +\ \frac{10}{100} of\ previous\ salary\ = M4400$$

$$\Rightarrow\ x + \frac{10}{100}x\ = M4400$$

$$\Rightarrow x\left(1 + \frac{1}{10}\right) = M4400$$

$$\Rightarrow 1.1x = M4400$$

$$\Rightarrow x = \frac{\boldsymbol{M4400}}{\boldsymbol{1.1}} = \boldsymbol{M4000}$$

∴ **Mpho's previous salary is M4000**

b) $\frac{2}{5} \times M3500$

$= \frac{M7000}{4}$

$= \boldsymbol{M1400.00}$

9.

a) (First find x-intercept of line A using its equation by substituting y with 0)

$$\boldsymbol{2y\ + 2 = -x}$$

$$\Rightarrow \boldsymbol{2(0) + 2 = -x}$$

$$\Rightarrow \boldsymbol{2 = -x}$$

$$\Rightarrow \boldsymbol{x = -2} \qquad \boldsymbol{(this\ is\ also\ the\ x-intercept\ of\ line\ C)}$$

(Now finding the gradient using the points [-2, 0] and [2, 0])

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{2-(-2)} = \frac{3}{2}$$

(Now finding the y- intercept from the general equation of the straight line)

$$y = mx + c$$

$$\Rightarrow 6 = \frac{3}{2}(2) + c$$

$$\Rightarrow 6 = 3 + c$$

$$\Rightarrow c = 3$$

$$Since\ M = \frac{3}{2}\ \ and\ \ C = 3$$

$$\Rightarrow y = \frac{3}{2}x + 3$$

$$\Rightarrow 2y = 3x + 6 \quad \textbf{\textit{Hence proved}}$$

b) **Line A**

$$y = -\frac{1}{2}x - 2 \qquad (Consider\ [0,0])$$

$$0 \geq -2$$

$$\Rightarrow y \geq -\frac{1}{2}x - 2$$

Line B

$$x = 2 \qquad (Consider\ [0,0])$$

$$x \leq 2$$

Line C

$$2y = 3x + 6 \quad (Consider\ [0,0])$$

$$\Rightarrow 2y \leq 3x + 6$$

The three inequalities are:

$$\boldsymbol{y \geq -\frac{1}{2}x - 2} \quad , \quad \boldsymbol{x \leq 2} \quad , \quad \boldsymbol{2y \leq 3x + 6}$$

c) $2 - 6 = -4$

10.

a) (Rearrange the numbers in ascending order and pick the middle number)

They are: 10,13,15,29,36,41,45

$\therefore \boldsymbol{Median = 29}$

b) (Divide the total number of prime numbers in the distribution, by the total numbers in the distribution)

$$\boldsymbol{P(prime\ number) = \frac{3}{7}}$$

c)

i. $Range = Largest\ number - Smallest\ number$

$= 48 - 10$

$= 38$

ii. (Let z be the sum of 13 added numbers)

Now, $\frac{(36+29+41+45+15+10+13)+z}{20} = 19.2$

$\Rightarrow \frac{189+z}{20} = 19.2$

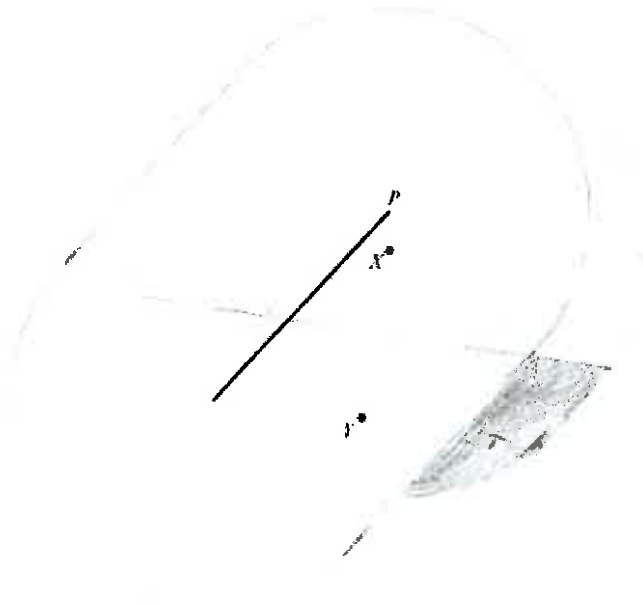
$\Rightarrow z = 195$

$Now\ \frac{195}{13} = 15$

∴ The mean of the added thirteen numbers is 15.

11. *if* $1cm$ *represents* $25m$ *then,* $1m = \frac{1}{25}\ cm$

- *Less than* $125m$ *from* $X \Rightarrow 125m = \frac{1}{25}cm = 5cm$ (Draw a circle of radius 5cm)
- Near to Y than X (Bisect XY)
- More than 100m from the path (Draw locus of points 4cm from the path)



12.

a) $a = \frac{v-u}{t}$

$\Rightarrow a = \frac{20-0}{40} = \frac{20}{40} = 0.5$

Acceleration is 0.5 m/s^2

b) $a = \frac{v-u}{t}$

$\Rightarrow t = \frac{v-u}{a}$

$\Rightarrow t = \frac{15-0}{0.5} = 30sec$

c) (Area under the speed-time graph is numerically the same as the distance travelled)

$Area_{triangle} = \frac{1}{2}bh$

$\Rightarrow Area = \frac{1}{2} \times 150 \times 20$

$= 150 \times 10$

$= 1500m$

∴ The total distance travelled is 1500m.

13.

a) **The diagram shows that the two radii TD and DB of the two circles are equal, which means that the two circles have the same diameter and circumference, therefore they are equal.**

b) $angle\ ATC = 60°$

c) $60° + 90° + 90° = 240°$

$360° - 240° = 120°$

$360° - 120° = \mathbf{240°}$

$\therefore reflex\ angle\ ABC = 240°$

14.

a)

i. If f is directly proportional to $2n^2$

Then $f \propto n^2$

$\Rightarrow f = kn^2$

$\Rightarrow k = \frac{f}{n^2} = \frac{6}{4} = \frac{3}{2}$

$\therefore f = \frac{3}{2}n^2$

ii. $f = \frac{3}{2} \times 8^2$

$= \frac{3}{2} \times 64$

$= 3 \times 32$

$= 96$

b) $p = 2kn^2$

$\Rightarrow kn^2 = \frac{p}{2}$

$but \quad f = kn^2$

$\Rightarrow f = \frac{p}{2}$

$\boldsymbol{Hence\ f \propto p}$

2018 OCTOBER/NOVEMBER PAPER 4 SOLUTIONS

1.

a) $\begin{pmatrix} 2.5 & 3 \\ 2 & 2.5 \\ 3.8 & 3 \end{pmatrix}$

b)

i. (Matrix A shows speeds and matrix B shows the total time for training)

$$\text{Now, } AB = \begin{pmatrix} 2.5 & 3 \\ 2 & 2.5 \\ 3.8 & 3 \end{pmatrix} \quad \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} [2.5 \times 1] + [3 \times 1] & [2.5 \times 3] + [3 \times 2] \\ [2 \times 1] + [2.5 \times 1] & [2 \times 3] + [2.5 \times 2] \\ [3.8 \times 1] + [3 \times 1] & [3.8 \times 3] + [3 \times 2] \end{pmatrix}$$

$$= \begin{pmatrix} 5.5 & 13.5 \\ 4.5 & 11 \\ 6.8 & 17.4 \end{pmatrix}$$

ii. (AB represents distances covered by three students on both running and cycling)

$Atang's\ total\ distance = 5.5km + 13.5km = 19km$

$\boldsymbol{Bene's\ total\ distance = 4.5km + 11km = 15.5km}$

$\boldsymbol{Chabeli's\ total\ distance = 6.8km + 17km = 23.8km}$

2.

a) $\boldsymbol{P = 75 - x}$

b)

i. **(Finding the smallest possible of y)**
(Smallest possible value of y is given by the largest possible value of x)

Now let $x = 1$

$\Rightarrow \quad P = 75 - 1 = 74 \qquad (\because p = 75 - x)$
$and \quad q = 35 - x = 35 - 1 = 34$

Now $y = 200 - [P + x + q]$
$= 200 - [74 + 1 + 34]$
$= 200 - 109$
$= \mathbf{9}$

ii. $\boldsymbol{x\ must\ be\ 34}$

iii. $q = 35 - x$
$= 35 - 30$
$= \mathbf{5}$

3.

a)

i. $\frac{40}{100} \times \boldsymbol{M120\,000 = M48\,000}$

ii. $\boldsymbol{Extra\ cost = M120\,000 - [M48\,000 + \ (60\ months \times M1365)]}$
$\boldsymbol{= M120\,000 - \ M129900}$
$\boldsymbol{= -M9900}$

The extra cost is M9 900

iii. $12\ months = \ 1\ year$
$\Rightarrow 60\ months = 60 \times \frac{1}{12} years = 5\ years$

$$Percentage\ rate = \frac{M9900}{5\ years \times (48\,000 + 60\ months \times M1365)} = \frac{9900}{649500} = \mathbf{2\%}$$

b)

i.

$$A_{quaterly} = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= M56{,}000\left(1 + \frac{0.032}{4}\right)^{4n}$$

$$= \boldsymbol{M56,000(1.0085)^{4n}}$$

ii. $\boldsymbol{A = M56,000(1.0085)^{4\times3} = M61,986.75}$

$\boldsymbol{But\ I = A - P}$

$\boldsymbol{= M61,986.75 - M56,000}$

$\boldsymbol{= M5,986.75}$

4.

a) **12 Vertices**

b) $Volume(1\ ball) = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times (3cm)^3$

$= \boldsymbol{113cm^3}$

c) $Volume\ (Box) = lenght \times breadth \times height$

$= 2d \times 2d \times d$ (d is the diameter of the sphere)

$= (2 \times 6cm) \times (2 \times 6cm) \times 6cm$

$= \boldsymbol{864cm^3}$

d) (Subtract the total volumes of the spheres from the volume of the box)

$$Volume(\ unoccupied) = Volume\ of\ the\ box - Total\ volumes\ of\ the\ sphere$$

$$= 864cm^3 - 113cm^3$$

$$= \boldsymbol{751\ cm^3}$$

e) $Percentage\ of\ unoccupid\ volume = \frac{unoccupied\ volume}{Volume\ of\ Box} \times 100\%$

$= \frac{751\ cm^3}{864 cm^3} \times 100\%$

$= \mathbf{86.9\%}$

5.

a) $\overrightarrow{AB} = -\underline{a} + \underline{b}$

$= \underline{b} - \underline{a}$

b) $\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \underline{a} + \frac{1}{2}(\underline{b} - \underline{a})$

$= a + \frac{1}{2} b - \frac{1}{2} a$

$= \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$

$= \frac{1}{2}(\underline{a} + \underline{b})$

c) $\overrightarrow{MN} = \overrightarrow{NO} + \overrightarrow{OM}$

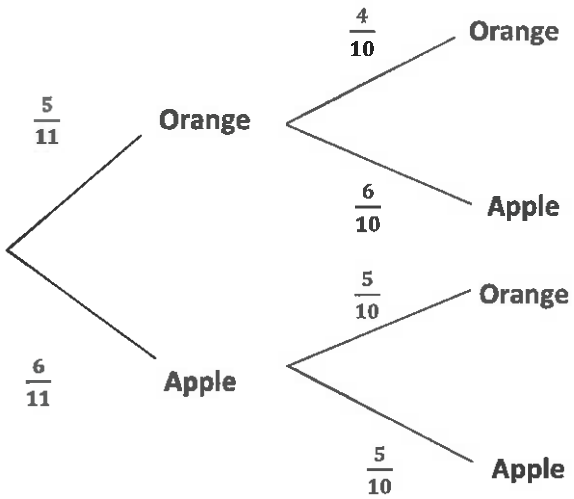
$= -\frac{1}{3}\underline{b} + \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$

$= \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} - \frac{1}{3}\underline{b}$

$= \frac{1}{2}\underline{a} + \frac{1}{6}\underline{b}$

6.

a)



b)

i. $Probability(\ two\ oranges) = \frac{5}{11} \times \frac{4}{10} = \frac{20}{110} = \frac{2}{11}$

ii. $Probability(\ atleast\ one\ apple) = \left(\frac{5}{11} \times \frac{6}{10}\right) + \left(\frac{6}{11} \times \frac{5}{10}\right) + \left(\frac{6}{11} \times \frac{5}{10}\right)$

$$= \frac{3}{11} + \frac{3}{11} + \frac{3}{11}$$

$$= \frac{9}{11}$$

c) $Probability(\ all\ six\ fruits\ are\ oranges) = 0$

7.

a) (It is given that Tau was x years old)

$\therefore$ $\boldsymbol{Pitso\ is\ 5+x, and\ Neo\ is\ 2x\ years\ old}$

b) $(Neo's\ age) \times (Tau'sage) = (Pitso's\ age)^2$

$\Rightarrow (2x+2)(x+2) = (5+x)^2$

$\Rightarrow 2x(x+2) + 2(x+2) = (5+x)(5+x)$

$\Rightarrow 2x^2 + 4x + 2x + 4 = 25 + 10x + x^2$

$\Rightarrow 2x^2 + 4x + 2x + 4 - 25 - 10x - 4x^2 = 0$

$\Rightarrow (2x^2 - 4x^2) + (4x + 2x - 10x) + (4 - 25) = 0$

$\Rightarrow \boldsymbol{x^2 - 4x - 21 = 0}$

$\boldsymbol{Hence\ shown.}$

c)

i. (We shall use the method of factorization)

$x^2 - 4x - 21 = 0$

$\Rightarrow (x-7)(x+4) = 0$

$\Rightarrow (x-7) = 0 \quad OR \quad (x+4) = 0$

$\therefore \boldsymbol{x = 7} \quad \boldsymbol{OR} \quad \boldsymbol{x = -4}$

ii. $x = 7 \quad (In\ 2001, 1^{st}\ January)$

$\Rightarrow Pitso's\ age\ in\ 2002 = 7\ years + 2\ years = \boldsymbol{9\ years}$

d)

i. $h = \frac{(-b \pm \sqrt{b^2-4ac})}{2a}$

$= \frac{\left(-8 \pm \sqrt{8^2-(4\times1\times(-17))}\right)}{2\times1}$

$= \frac{(-8 \pm \sqrt{64+68})}{2}$

$= \frac{(-8 \pm \sqrt{64+68})}{2}$

$\therefore h = \frac{(-8+\sqrt{132})}{2}$ $\quad OR \quad$ $h = \frac{-8-\sqrt{132}}{2}$

$= \frac{-8+11.49}{2}$ $\quad = \frac{-8-11.49}{2}$

$= -9.75$ $\quad = 1.75$

ii. We know that $1m = 100cm$

$\therefore 1.75\, m = 1.75 \times 100cm$

$= 175\ cm$

8.

a) We shall us the cosine rule : $a^2 = b^2 + c^2 - 2ab(cosA)$

$(BD)^2 = (AB)^2 + (AD)^2 - 2(AB)(AD)cos\theta$

$= (4cm)^2 + (8cm)^2 - 2 \times 4\ cm \times 8\ cm \times cos\theta$

$= 16\ cm^2 + 64cm^2 - 64\ cm^2 \cos\theta$

$= 80cm^2 - 64\ cm^2 \cos\theta$

$\therefore BD = \sqrt{80cm^2 - 64cm^2cos\theta}$ $\quad$ ***(After taking the square root of both sides)***

$= \sqrt{16(5 - 4cos\theta)}cm$

Hence shown.

b)

i. (First calculate BD)

$$BD = \sqrt{16(5 - 4cos40)}\ cm$$

$$= \sqrt{80 - 64cos40}\ cm$$

$$= 8.77cm$$

(Now use the sine rule to find sinCBD)

$$\frac{a}{sinA} = \frac{b}{sinB}$$

$$\Rightarrow \quad \frac{sinA}{a} = \frac{sinB}{b}$$

$$\Rightarrow \quad sinA = \frac{sinB \times a}{b}$$

$$\Rightarrow \quad sinCBD = \frac{sin25 \times 8.77cm}{11cm}$$

$$= 0.337$$

ii. $$sinCBD = 0.336942014$$

$$\Rightarrow \quad \sin^{-1}(sinCBD) = \sin^{-1}(0.336942014)$$

$$\Rightarrow \quad CBD = \sin^{-1}(0.336942014)$$

$$\Rightarrow \quad CBD = 19.7°$$

iii. (Calculate angle BDC)

$$angle\ BDC = 180° - (19.7° + 25°)$$

$$= 180° - 42.7°$$

$$= 137.3°$$

NOW, $BC = \frac{8.77}{sin25}° \times sin19.7° = 7cm$

NOW, $$Perimeter = AB + BC + CD + AD$$

$$= 4cm + 7cm + 11cm + 8cm$$

$$= 30cm$$

iv. $Area\ of\ triangle\ ABD = \frac{1}{2}\ ab\ sinC$

$$= \frac{1}{2}\ (AB)(AD) \times sin\theta$$

$$= \frac{1}{2} \times 4cm \times 8cm \sin 40°$$

$$= 2cm \times 8cm \times \sin 40°$$

$$= 10.25\ cm^2$$

9.

a)

Distance(m)	$0 < d \leq 20$	$20 < d \leq 40$	$40 < d \leq 50$	$50 < d \leq 60$	$60 < d \leq 100$
Frequency	30	42	**25**	29	**24**

b) (First calculate the class-mid value)

- $class - midvalue\ of \quad 0 < d \leq 20 = \frac{0+20}{2} = 10$
- $class - midvalue\ of \quad 20 < d \leq 40 = \frac{20+40}{2} = 30$
- $class - midvalue\ of \quad 40 < d \leq 50 = \frac{40+45}{2} = 45$
- $class - midvalue\ of \quad 50 < d \leq 60 = \frac{50+60}{2} = 55$
- $class - midvalue\ of \quad 60 < d \leq 100 = \frac{60+100}{2} = 80$

(Now use the class-midvalues as variables in the equation below)

$$Estimate\ of\ the\ mean = \frac{sum(Frequencies \times Variables)}{sum\ of\ frequencies}$$

$$= \frac{(10\times30)+(30\times42)+(45\times25)+(55\times29)+(80\times24)}{30+42+25+29+24}$$

$$= \frac{6200}{150}$$

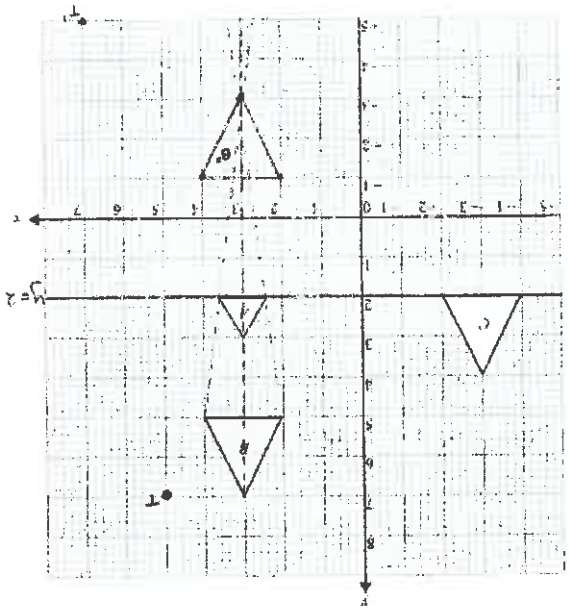
$$= \mathbf{41.33}m$$

10.

a)

i. shown on the diagram below

ii. shown on the diagram below



b)

i. **It is an enlargement from the point (3,-4) with a scale factor $\frac{7}{11}$**

ii. **It is a translation with a translation vector of $\begin{pmatrix}-6\\-3\end{pmatrix}$**

c)

i. **Shown on the diagram above**

ii. **Shown on the diagram above**

11.

a) (We shall use the translation vector equation)

$$\begin{pmatrix}x\\y\end{pmatrix} + Translation\ vector = \begin{pmatrix}x'\\y'\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}x'\\y'\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}5\\0\end{pmatrix} = \begin{pmatrix}x'\\y'\end{pmatrix}$$

$\therefore$ ***The coordinates of a point D , image point of B is*** $(5, 0$

b)

Use the general equation of the straight line $y = mx + c$

Where m is the gradient and c is the y intercept.

$\Rightarrow y = mx + 3$

(Now finding the gradient, m.)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-0}{0-5}$$

$$= -\frac{3}{5}$$

$\therefore$ *the equation of the straight line through C and D is*

$$y = 3 - \frac{3}{5}x$$

c) (This parabola cuts the y-axis at 3, the point (0,3), therefore c=3)

$\Rightarrow \quad y = ax^2 + bx + c$ ..... [1]

Now, in equation [1] above

$y = 0 \quad (Where\ the\ parabola\ cuts\ the\ x-axis)$

$\Rightarrow \quad 0 = ax^2 + bx + c$ ..... [2]

We have two solutions for equation [2]

$x = -1 \quad or \quad x = 3$

$\therefore$ We can have equations of *a* and *b* as shown below.

$0 = 9a + 3b + 3$..... [3]
$0 = a - b + 3$..... [4]

Which are reduced to
$9a + 3b = -3$.....[3]
$a - b = -3$ .....[6]

From equation [6]

$a = b - 3$ ..... [7]

Substituting [7] into [5]

$9(b-3) + 3b = -3$

$9b - 27 + 3b = -3$

$12b = 24$

$b = 2$ ..... [8]

Substituting equation [8] into [6]

$a - 2 = -3$

$\Rightarrow a = -1$

∴ **The equation of the parabola is**

$y = -x^2 + 2x + 3$

d) **(Where $f(x) and\ g(x)$ intersect, the values of y are equal)**

$\because f(x) = g(x)$

$\Rightarrow 3 - \frac{3}{5}x = -x^2 + 2x + 3$

$\Rightarrow 15 - 3x = -5x^2 + 10x + 15$

$\Rightarrow 15 - 3x + 5x^2 - 10x - 15 = 0$

$\Rightarrow 5x^2 - 3x - 10x = 0$

$\Rightarrow 5x^2 - 13x = 0$

$\Rightarrow x(5x - 13) = 0$

$\Rightarrow x = 0 \quad OR \quad x = \frac{13}{5}$

$\therefore$ *the $x-$coordinate of the point* $E = 2\frac{3}{5}$

$$now, use\ y = 3 - \frac{3}{5}x\ to\ find\ the\ y - coodinate\ of\ the\ point\ E$$

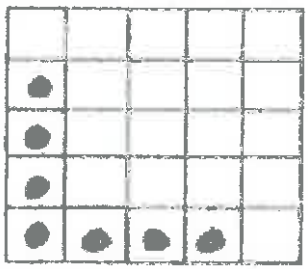
$$\Rightarrow y = 3 - \frac{3}{5} \times \frac{13}{5} = 3 \times \frac{25}{25} - \frac{3}{5} \times \frac{13}{5} = \frac{75}{25} \times \frac{39}{25} = \frac{36}{25}$$

$$\therefore Coordinates\ of\ point\ E\ are\ \left(\frac{13}{5}, \frac{36}{25}\right) = (2.6, 1.4)$$

e) $the\ set\ \{\ldots, -2, -1, 0\}\ shows\ values\ of\ x\ for\ which\ f(x) \geq g(x)$

12.

a) Pattern 4



b)

Pattern Number	1	2	3	4	5
Dotted tiles	1	3	5	7	9
Plain tiles	3	6	11	18	27

c) $n^{th}\ pattern = 2n - 1$

d) From c) above

$$t_n = 2n - 1$$

$$\therefore t_n = 2(20) - 1$$

$$= 40 - 1$$

$$= 39$$

e) $t_n = 2n - 1$

$\Rightarrow t_{n+7} = 2(n+7) - 1$

Now, finding the difference we have

$t_{n+1} - t_n$

$= [2(n+7) - 1] - [2n - 1]$

$= [2n + 14 - 1] - 2n + 1$

$= 2n - 2n + 14 - 1 + 1$

$= 14\ tiles$

The pattern $n + 7$ has 14 more dotted tiles than in pattern n.

f) n^{th} *pattern for plain tiles is*

$t_n = n^2 + 2$

g)

i. $t_n = n^2 + 2 = 227$

$\Rightarrow \quad n^2 + 2 = 227$

$\Rightarrow \quad n^2 = 225$

$\Rightarrow \quad n = 15$

$\therefore 15^{th}$ *pattern has 227 plain tiles.*

ii. n^{th} *pattern for dotted tiles is* $2n - 1$

n^{th} *pattern for plain tiles is* $n^2 + 2$

$\therefore$ *total number of tiles in pattern* 65 *is dotted tiles plus plain tiles.*

$\therefore t_n(total) = (2n - 1) + (n^2 + 2)$

$= 2n - 1 + n^2 + 2$

$= n^2 + 2n + 1$

$\therefore t_{65} = (65)^2 + 2(65) + 1$

$= 4356$

13.

a)

i. $f(x) = 2^x$

$\Rightarrow f(3) = 2^3 = 8$

ii. $g(x) = 2x + 3$

Let $g(x) = y$

$\Rightarrow y = 2x + 3$ *(Make x the subject of the formula)*

$\Rightarrow x = \frac{y-3}{2}$

now, let $g^{-1}(x) = x$ *and* $y = x$

$\therefore g^{-1}(x) = \frac{x-3}{2}$

iii. $gf(x) = g(2^x)$

$= 2(2^x) + 3$

$= 2^{x+1} + 3$

b) $f(x) = 8^{x-1}$

$\Rightarrow 2^x = 8^{x-1}$

$\Rightarrow 2^x = (2^3)^{x-1}$

$\Rightarrow 2^x = 2^{3(x-1)}$

(Since the bases are the same, we may now equate the indices)

$x = 3(x-1)$

$\Rightarrow x = 3x - 3$

$\Rightarrow 3 = 3x - x$

$\Rightarrow 3 = 2x$

$\Rightarrow x = 1\frac{1}{2}$

c) $gh(x) = 6x + 1$

$\Rightarrow 2[h(x)] + 3 = 6x + 1$

$\Rightarrow 2[h(x)] = 6x - 2$

$\Rightarrow h(x) = \frac{6x-2}{2}$

$\Rightarrow h(x) = \frac{2(3x-1)}{2}$

$\therefore \boldsymbol{h(x) = 3x - 1}$