Perimeters, Area and Volumes
Difficulty: Medium

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Perimeters, Area and Volumes
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 1

Time allowed: **107 minutes**

Score: **/93**

Percentage: **/100**

Grade Boundaries:

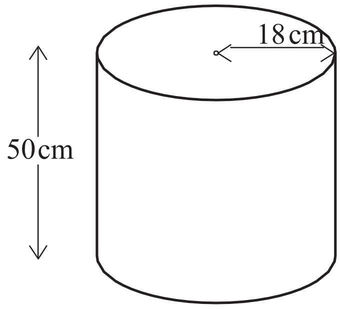
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) ~~Assembled by AS~~

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a) The diagram shows a cylindrical container used to serve coffee in a hotel.



NOT TO SCALE

The container has a height of 50 cm and a radius of 18 cm.

(i) Calculate the volume of the cylinder and show that it rounds to 50 900 cm^3, correct to 3 significant figures.
[2]

Volume of a cylinder is

$$V = \pi r^2 l$$

Where r is the radius of the circular face and l is the length (or height)

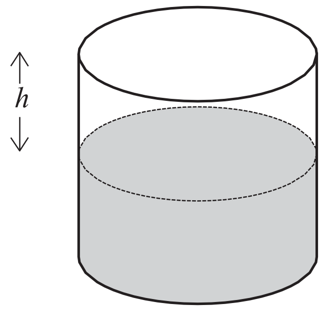
$$\rightarrow V = \pi \times 18^2 \times 50$$

$$= 50893.80099$$

$$= \mathbf{50900} \ (3sf)$$

(ii) 30 litres of coffee are poured into the container.

Work out the height, h, of the empty space in the container. [3]



We know that

$$1\ litre = 1000\ cm^3$$

Hence the volume of the empty space is

$$50900 - 30000$$

$$= 20900$$

And the equation is

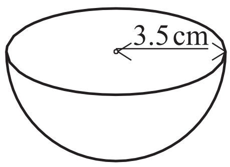
$$18^2\pi h = 20900$$

Dividing through by $18^2\pi$

$$h = \frac{20900}{18^2\pi}$$

$$= \mathbf{20.5}$$

(iii) Cups in the shape of a hemisphere are filled with coffee from the container.
The radius of a cup is 3.5 cm.



NOT TO SCALE

Work out the maximum number of these cups that can be completely filled from the 30 litres of coffee in the container.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [4]

The volume of one cup is

$$\frac{1}{2} \times \frac{4}{3}\pi(3.5)^3$$

$$= 89.8cm^3$$

$$= 0.0898\ litres$$

Divide the volume of coffee by this volume

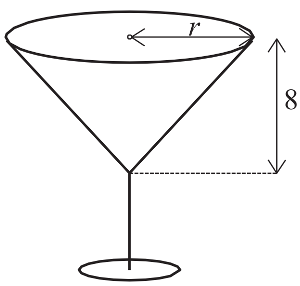
$$30 \div 0.0898$$

$$= 334.0757$$

So, number of filled cups is

$$\mathbf{334}$$

(b) The hotel also uses glasses in the shape of a cone.



NOT TO SCALE

The capacity of each glass is $95\,\text{cm}^3$.

(i) Calculate the radius, r, and show that it rounds to 3.3cm, correct to 1 decimal place.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [3]

$$95 = \frac{1}{3}\pi r^2 \times 8.4$$

Times through by 3 and divide through by 8.4π

$$r^2 = \frac{35}{\pi}$$

Now square root

$$r = 3.33779$$

$$= \mathbf{3.3}$$

(ii) Calculate the curved surface area of the cone.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r^2 h$.] [4]

Slant length is found using Pythagoras'

$$l^2 = r^2 + h^2$$

$$= 3.3^2 + 8.4^2$$

$$= 81.45$$

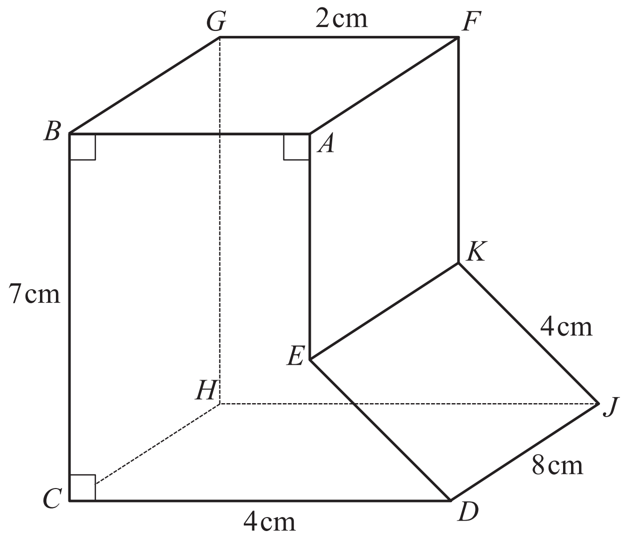
$$\rightarrow l = 9.02$$

Hence

$$A = \pi(3.3)(9.02)$$

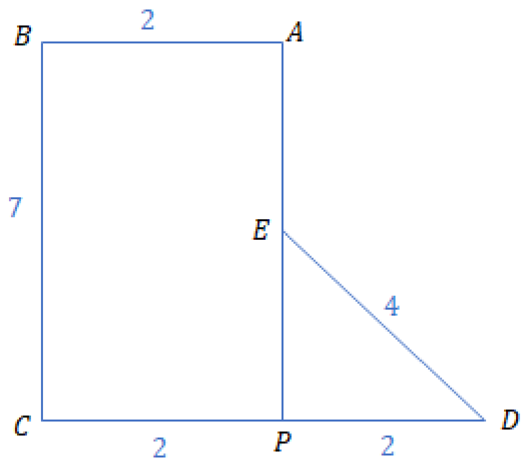
$$= \mathbf{93.5}$$

(a) The diagram shows a solid metal prism with cross section $ABCDE$.



NOT TO SCALE

(i) Calculate the area of the cross section $ABCDE$. [6]



The area ABCDE can be split into a rectangle and a triangle like above.

The height of the triangle is found with Pythagoras'

$$DE^2 = PD^2 + PE^2$$

$$\rightarrow 4^2 = 2^2 + h^2$$

$$\rightarrow 16 - 4 = h^2$$

$$\rightarrow h = \sqrt{12}$$

Hence $$erea = 2 \times 7 + \frac{1}{2} \times 2 \times \sqrt{12}$$

$$= 14 + \sqrt{12}$$

$$= \mathbf{17.46}$$

(ii) The prism is of length 8cm.

Calculate the volume of the prism. [1]

$$V = 8 \times 17.46$$

$$= \mathbf{139.7}$$

(b) A cylinder of length 13 cm has volume 280 cm^3.

(i) Calculate the radius of the cylinder. [3]

Volume of a cylinder is

$$V = \pi r^2 \times l$$

Hence

$$280 = \pi r^2 \times 13$$

$$\rightarrow r^2 = \frac{280}{13\pi}$$

$$= 6.856$$

$$\rightarrow \boldsymbol{r} = \mathbf{2.62}$$

(ii) The cylinder is placed in a box that is a cube of side 14cm.

Calculate the percentage of the volume of the box that is occupied by the cylinder. [3]

Volume of box is

$$14^3 = 2744$$

Hence percentage occupied by cylinder is

$$\frac{280}{2744} \times 100\%$$

$$= \mathbf{10.2}$$

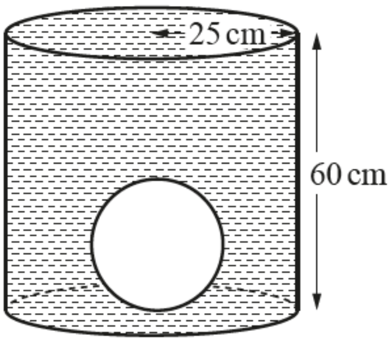
(a) Calculate the volume of a metal sphere of radius 15 cm and show that it rounds to 14 140 cm^3, correct to 4 significant figures.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

$$V = \frac{4}{3} \times \pi \times 15^3$$

$$= 14137.166 \ldots$$

$$= \mathbf{14140\ to\ 4\ sig\ figs}$$

(b) (i) The sphere is placed inside an empty cylindrical tank of radius 25 cm and height 60 cm. The tank is filled with water.



NOT TO SCALE

Calculate the volume of water required to fill the tank. [3]

$$Volume\ of\ cylinder = \pi \times r^2 \times h$$

$$= \pi \times 25^2 \times h$$

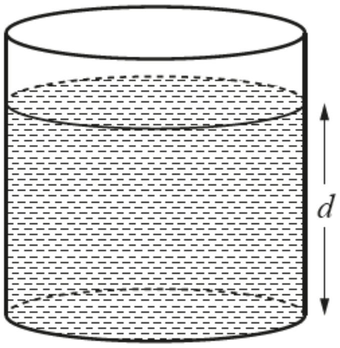
$$= 37500\pi$$

$$Volume\ of\ water = Volume\ of\ sphere - Volume\ of\ cylinder$$

$$= 103672.557 \ldots$$

$$= \mathbf{103700\ to\ 4\ sig\ figs}$$

(ii) The sphere is removed from the tank.



Calculate the depth, d, of water in the tank. [2]

$V = \pi r^2 h$

$\pi \times r^2 \times h = 33000\pi$

$\pi \times 25^2 \times d = 33000\pi$

$d = 33000 \div 25^2$

$\boldsymbol{d = 52.8}$

(c) The sphere is melted down and the metal is made into a solid cone of height 54cm.

(i) Calculate the radius of the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [3]

$Volume\ of\ sphere = 4500\pi$

$\frac{1}{3}\pi r^2 h = 4500\pi$

$\frac{1}{3}\pi r^2 \times 54 = 4500\pi$

$18r^2 = 4500$

$r = \sqrt{\frac{4500}{18}}$

$r = \sqrt{250}$

$\boldsymbol{r = 15.8cm}$

(ii) Calculate the **total** surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.] [4]

$$l = \sqrt{r^2 + l^2}$$

$$l = \sqrt{250 + 54^2}$$

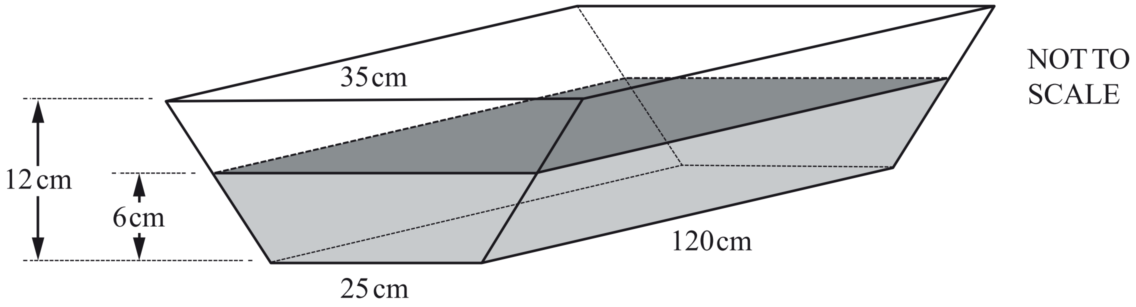
$$l = \sqrt{3166}$$

$$A = \pi rl + \pi r^2$$

$$A = \pi \times \sqrt{250} \times \sqrt{3166} + \pi \times 250$$

$$\boldsymbol{A = 3580cm^2}$$

The diagram shows a horizontal water trough in the shape of a prism.



The cross section of this prism is a trapezium.
The trapezium has parallel sides of lengths 35 cm and 25 cm and a perpendicular height of 12 cm.
The length of the prism is 120 cm.

(a) Calculate the volume of the trough. [3]

The volume of the trough is given as the product of the area of cross section (the trapezium face) and the length of the prism.

The area of a trapezium with parallel sides 35 cm and 25 cm and height 12 cm is given as:

$$Trapezium = \frac{(35cm + 25cm)}{2} \times 12cm$$

$$Trapezium = 360\ cm^2$$

The volume of the trough:

$$Volume = trapezium \times length$$

$$Volume = 360\ cm^2 \times 120cm$$

$$Volume = \mathbf{43\ 200\ cm^3}$$

(b) The trough contains water to a depth of 6 cm.

(i) Show that the volume of water is 19 800 cm^3. [2]

Since the water is half way between the top and the bottom, the top and bottom and the width of the trapezium increases linearly, it must be right in between 25cm and 35cm. Therefore the top side of the "water trapezium" is 30cm (=(35+25)/2) .

The height of the trapezium is 6cm.

Calculate the volume using the same method as in part a)

$$Volume = water\ trapezium \times length$$

$$Volume = \frac{(30cm + 25cm)}{2} \times 6cm \times 120cm$$

$$Volume = \mathbf{19\ 800\ cm^3}$$

(ii) Calculate the percentage of the trough that contains water. [1]

Divide the volume of water by the total volume of the trough (and multiply by 100%) to get the percentage of the trough that contain water.

$$\frac{19\ 800\ cm^3}{43\ 200\ cm^3} \times 100\%$$

$$= \mathbf{45.8}\%$$

(c) The water is drained from the trough at a rate of 12 litres per hour.

Calculate the time it takes to empty the trough.
Give your answer in hours and minutes. [4]

First, we convert the volume from cubic centimetres into litres.

One cubic centimetre is equal to one millilitre.

$$19\,800\ cm^3 = 19\,800\ ml$$

Divide the number by 1000 to get the volume in litres (1l=1000ml)

$$19.8\ l$$

Second, we divide the volume by the rate to get the time needed to empty the trough.

$$\frac{19.8\ l}{12\ l\ per\ hour} = 1.65\ hours$$

Finally, we convert from hours into hours and minutes (by multiplying the rest by 60).

$$1hour + 0.65hours \times 60minutes\ per\ hour = 1hour + 39\ minutes$$

It will take **1 hours and 39 minutes** to empty the trough.

(d) The water from the trough just fills a cylinder of radius *r* cm and height 3*r* cm.

Calculate the value of *r*. [3]

The volume of a cylinder with radius *r* and height *h* is given as:

$$V = \pi r^2 h$$

In our case, V=19 800cm^3, h=3r.

$$19\,800 = \pi r^2(3r)$$

$$19\,800 = 3\pi r^3$$

Divide both sides by 3π and take a cube root of both sides of the equation.

$$r = \sqrt[3]{\frac{19800}{3\pi}}$$

Use a calculator to calculate the value of *r*.

$$r = \mathbf{12.8\ cm}$$

(e) The cylinder has a mass of 1.2 kg.
1 cm^3 of water has a mass of 1 g.

Calculate the total mass of the cylinder and the water.
Give your answer in kilograms. [2]

The total mass of the cylinder and the water (1 kg=1000 g)

$$mass = cyliner + volume\ of\ water \times density$$

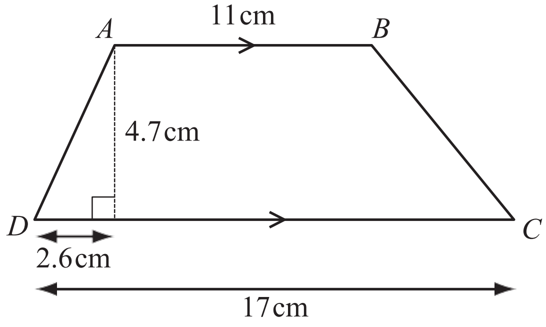
$$mass = 1.2kg + 19\,800\ cm^3 \times 1g\ per\ cm^3$$

$$mass = 1.2kg + 19.8\ kg$$

Total mass of the cylinder with water:

$$mass = \mathbf{21\ kg}$$

(a) $ABCD$ is a trapezium.



NOT TO SCALE

(i) Calculate the length of AD. [2]

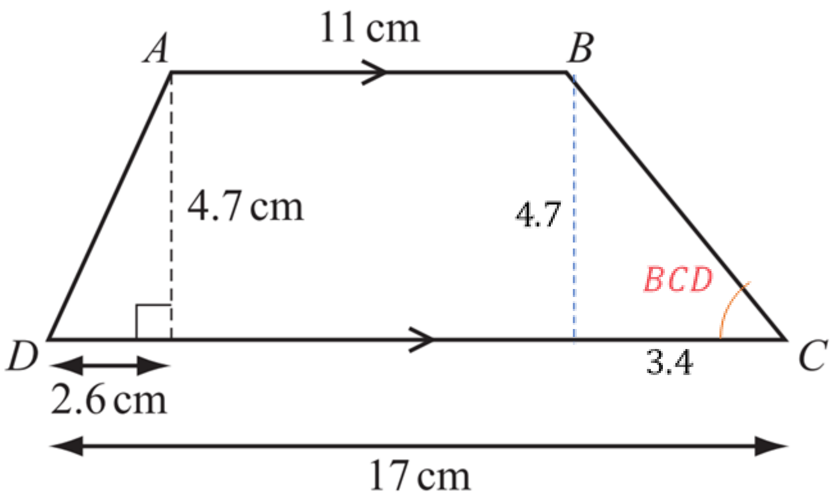
Use Pythagoras' Theorem

$$AD^2 = 2.6^2 + 4.7^2$$

$$= 28.85$$

$$\rightarrow \boldsymbol{AD = 5.37}$$

(ii) Calculate the size of angle BCD. [3]



We can find BCD using the trigonometric relation

$$\boldsymbol{\tan\theta = \frac{opp}{adj}}$$

$$\rightarrow \tan BCD = \frac{4.7}{3.4}$$

$$\rightarrow \boldsymbol{BCD = 54.1}$$

(iii) Calculate the area of the trapezium $ABCD$. [2]

Add the areas of the two triangles on the side with the rectangle in the middle.

Area of a triangle is

$$A = \frac{1}{2} base \times height$$

Hence

$$Area = \frac{1}{2} \times 2.6 \times 4.7 + 11 \times 4.7 + \frac{1}{2} \times 3.4 \times 4.7$$

$$= 65.8$$

(b) A **similar** trapezium has perpendicular height 9.4 cm.

Calculate the area of this trapezium. [3]

The length scalar is

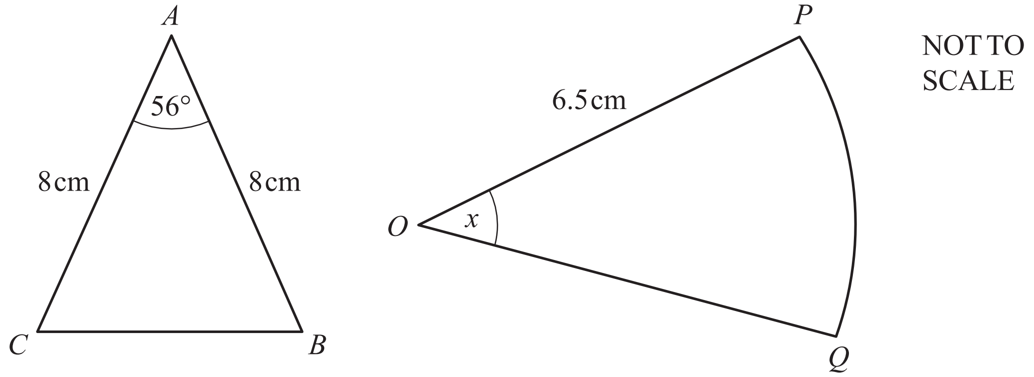
$$l = 9.4 \div 4.7$$

$$= 2$$

Square this for the area scalar, giving us

$$Area = 65.8 \times 2^2$$

$$=263.2$$



The diagram shows a triangle and a sector of a circle.
In triangle ABC, $AB = AC = 8$ cm and angle $BAC = 56°$.
Sector OPQ has centre O, sector angle x and radius 6.5cm.

(a) Show that the area of triangle ABC is 26.5cm^2 correct to 1 decimal place. [2]

Area of a triangle is

$$\boldsymbol{A = \frac{1}{2}ab\sin C}$$

$$= \frac{1}{2} \times 8 \times 8 \times \sin 56$$

$$= 26.529$$

$$= \mathbf{26.5\ (1dp)}$$

(b) The area of sector OPQ is equal to the area of triangle ABC.

(i) Calculate the sector angle x. [3]

Area of a sector is

$$\boldsymbol{A = \frac{1}{2}r^2\theta \times \frac{\pi}{180}}$$

Hence

$$26.5 = \frac{(6.5)^2 x\pi}{360}$$

Multiply through by 360, divide through by $6.5^2\pi$

$$x = \frac{26.5 \times 360}{6.5^2\pi}$$

$$= \mathbf{71.87}$$

(ii) Calculate the perimeter of the sector OPQ. [3]

The length of the arc is

$$l = \frac{rx\pi}{180}$$

$$= 6.5 \times \frac{71.87\pi}{180}$$

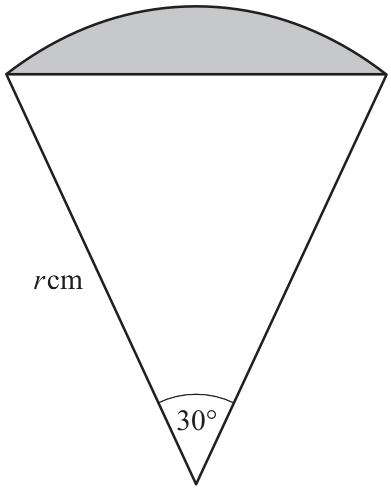
$$= 8.15$$

Hence the perimeter is

$$2 \times 6.5 + 8.15$$

$$= \mathbf{21.15}$$

(c) The diagram shows a sector of a circle, radius r cm.



NOT TO SCALE

(i) Show that the area of the shaded segment is $\frac{1}{4}r^2(\frac{1}{3}\pi - 1)$ cm^2. [4]

The area of the shaded area is the area of the sector

minus the area of the triangle

$$A = \frac{1}{2}r^2\frac{30\pi}{180} - \frac{1}{2}r^2 \sin 30$$

$$= \frac{1}{2}r^2 \times \frac{1}{6}\pi - \frac{1}{2}r^2 \times \frac{1}{2}$$

$$= \frac{1}{2}r^2\left(\frac{1}{6}\pi - \frac{1}{2}\right)$$

$$= \boldsymbol{\frac{1}{4}r^2\left(\frac{1}{3}\pi - 1\right)}$$

(ii) The area of the segment is 5 cm^2.

Find the value of r. [3]

We have that

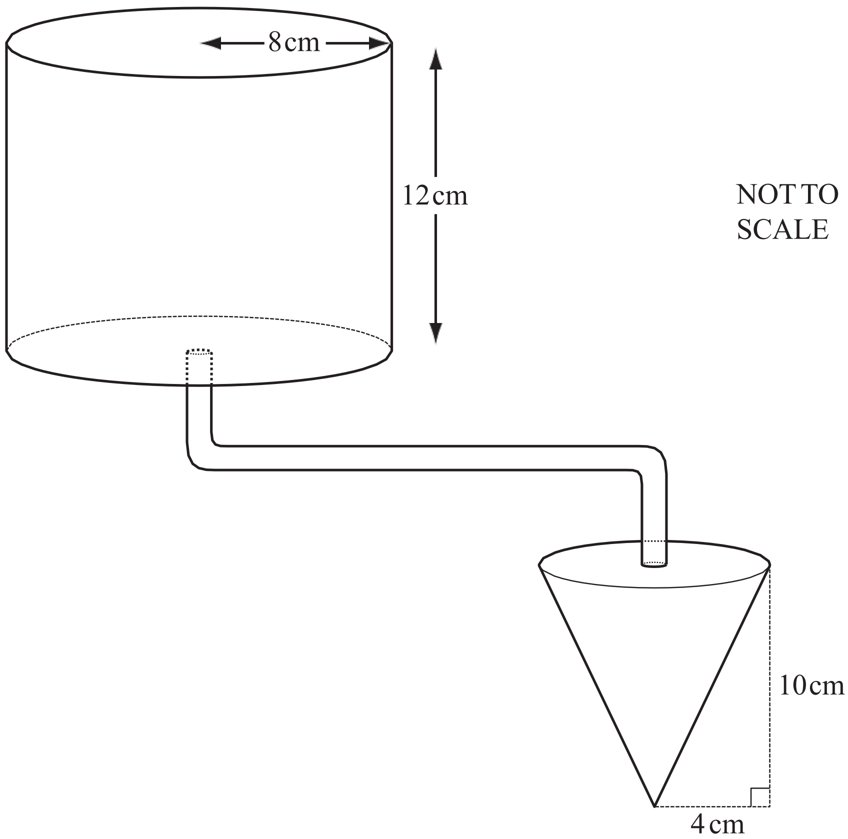
$$\frac{1}{4}r^2\left(\frac{1}{3}\pi - 1\right) = 5$$

Multiply through by 4 and divide through by $\left(\frac{1}{3}\pi - 1\right)$

$$\rightarrow r^2 = \frac{20}{\frac{1}{3}\pi - 1}$$

$$= 423.75$$

$$\rightarrow \boldsymbol{r = 20.6}$$



The diagram shows a cylinder with radius 8 cm and height 12 cm which is full of water.
A pipe connects the cylinder to a cone.
The cone has radius 4 cm and height 10 cm.

(a) (i) Calculate the volume of water in the cylinder.
Show that it rounds to 2410 cm^3 correct to 3 significant figures. [2]

Volume of a cylinder is

$$\boldsymbol{A = \pi r^2 \times h}$$

Where r is the radius of the circular face and h is the height.

Thus

$$A = \pi \times 8^2 \times 12$$

$$= 2412.743$$

$$\boldsymbol{= 2410 \ (3sf)}$$

(ii) Change 2410 cm^3 into litres. [1]

We have

$$1\ cm^3 = 1\ ml$$

Hence

$$2410\ cm^3 = 2410\ ml$$

$$= \mathbf{2.41\ l}$$

(b) Water flows from the cylinder along the pipe into the cone at a rate of 2 cm^3 per second.

Calculate the time taken to fill the empty cone.
Give your answer in minutes and seconds correct to the nearest second.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [4]

The volume of the cone is

$$V = \frac{1}{3}\pi(4^2)(10)$$

$$= \frac{160}{3}\pi$$

The time to fill it is then

$$\frac{160}{3}\pi \div 2$$

$$= \frac{160}{6}\pi$$

$$= 83.7758\ s$$

Nearest second

$$= 60\ s + 24\ s$$

$$= \mathbf{1\ mins\ 24\ secs}$$

(c) Find the number of empty cones which can be filled completely from the full cylinder. [3]

Volume of cone is

$$\frac{160}{3}\pi = 167.55$$

Number of cones is

$$2410 \div 167.55$$

$$= 14.38$$

Full cones

$$= \mathbf{14}$$

Perimeters, Area and Volumes
Difficulty: Medium

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Perimeters, Area and Volumes
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 2

Time allowed: **94 minutes**

Score: **/82**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

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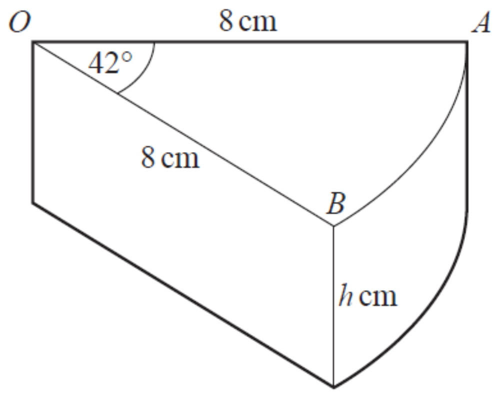


NOT TO SCALE

A wedge of cheese in the shape of a prism is cut from a cylinder of cheese of height h cm.
The radius of the cylinder, OA, is 8 cm and the angle $AOB = 42°$.

(a) (i) The volume of the wedge of cheese is 90cm^3.

Show that the value of h is 3.84 cm correct to 2 decimal places. [4]

The equation for volume of this wedge:

$$Volume = Area\ of\ sector \times Height\ of\ wedge$$

$$Area\ of\ sector = \frac{42°}{360°} \times Area\ of\ circle$$

$$= \frac{42°}{360°} \times (\pi \times 8^2)$$

$$= 23.46cm^2$$

Since volume is given as 90 cm^3,

$$90 = 23.46 \times Height\ of\ wedge$$

$$Height = \frac{90}{23.46}$$

$$= \mathbf{3.84cm\ (shown)}$$

(ii) Calculate the **total** surface area of the wedge of cheese. [5]

The surface area will be the sum of all sides of the figure. We first look at the curved surface:

$$Curved\ surface\ area = \frac{42}{360} \times Circumference\ of\ circle \times Height$$

$$= \frac{42}{360} \times 2 \times \pi \times 8 \times 3.84$$

$$= 22.52\ cm^2$$

There are 2 rectangular sides:

$$Sum\ of\ 2\ rectangular\ sides = 2 \times (8 \times 3.84)$$

$$= 61.44\ cm^2$$

There are 2 areas of sector:

$$Sum\ of\ 2\ sector\ areas = 23.46 \times 2$$

$$= 46.92\ cm^2$$

Therefore:

$$Total\ surface\ area = 22.52 + 61.44 + 46.92$$

$$= \mathbf{130.9\ cm^2}$$

(b) A mathematically similar wedge of cheese has a volume of 22.5 cm³.

Calculate the height of this wedge. [3]

Here we want to scale this wedge down to accommodate a smaller volume of 22.5 cm³.

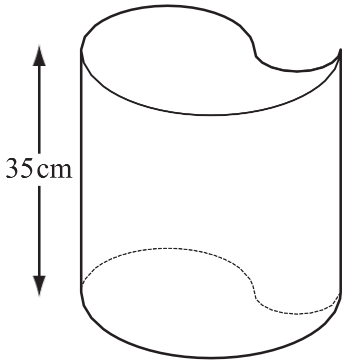
Conducting scaling:

$$\frac{3.84^3}{h^3} = \frac{90}{22.5}$$

$$\boldsymbol{h = 2.42cm}$$

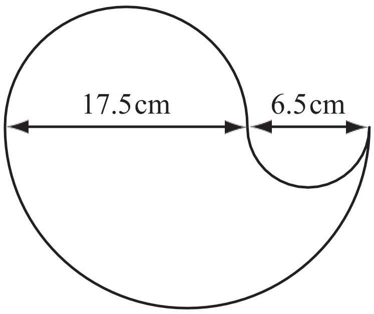
The heights have to be cubed here because for unit balance, they have to match the cubed units of the volume. Another way to put it is that the volume of the wedge is a function of the cube of a unit length.

Sandra has designed this open container.
The height of the container is 35 cm.



NOT TO SCALE

The cross section of the container is designed from three semi-circles with diameters 17.5 cm, 6.5 cm and 24 cm.



NOT TO SCALE

(a) Calculate the area of the cross section of the container. [3]

The area of the cross section can be calculated by subtracting the area of the smallest semicircle from the large bottom semicircle, and then adding that with the area of medium sized semicircle.

$$Area, smallest\ semicircle = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \pi \times \left(\frac{6.5}{2}\right)^2$$

$$= 16.6 \text{ cm}^2$$

$$Area, medium\ semicircle = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \pi \times \left(\frac{17.5}{2}\right)^2$$

$$= 120.26\ cm^2$$

$$Area, largest = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \pi \times \left(\frac{17.5 + 6.5}{2}\right)^2$$

$$= 226.19\ cm^2$$

$$Cross\ sectional\ area = 226.19\ cm^2 - 16.6\ cm^2 + 120.26\ cm^2$$

$$= \mathbf{329.9\ cm^2}$$

(b) Calculate the external surface area of the container, including the base. [4]

$$Largest\ curved\ surface\ area$$

$$= circumference\ of\ semicircle \times height$$

$$= \pi \times r \times h$$

$$= \pi \times \left(\frac{17.5 + 6.5}{2}\right) \times 35$$

$$= 1319.5\ cm^2$$

$$Smallest\ curved\ surface\ area = \pi \times r \times h$$

$$= \pi \times \left(\frac{6.5}{2}\right) \times 35$$

$$= 357.36\ cm^2$$

$$Medium\ curved\ surface\ area = \pi \times r \times h$$

$$= \pi \times \left(\frac{17.5}{2}\right) \times 35$$

$$= 962.11\ cm^2$$

Remember to add the area of the bottom face:

(We do not add the top face as this is a container, there has to be an opening!)

$$Total\ surface\ area = 1319.5 + 357.36 + 962.11 + (329.9)$$

$$= \mathbf{2970\ cm^2}$$

(c) The container has a height of 35 cm.

Calculate the capacity of the container.
Give your answer in litres. [3]

$$Volume = 329.9\ cm^2 \times 35cm$$

$$= 11546.5\ cm^3$$

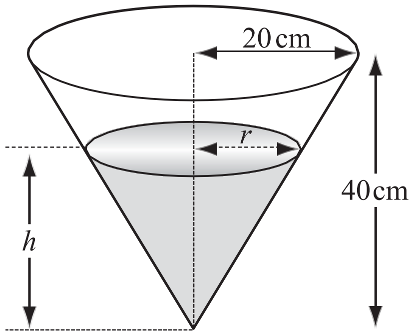
Convert to litres:

1 litre is 1000 cm^3

$$Capacity\ in\ litres = \frac{11546.5\ cm^3}{1000\,\frac{cm^3}{l}}$$

$$= \mathbf{11.5\ l}$$

(d) Sandra's container is completely filled with water.
All the water is then poured into another container in the shape of a cone.
The cone has radius 20 cm and height 40 cm.



NOT TO SCALE

(i) The diagram shows the water in the cone.

Show that $r = \frac{h}{2}$. [1]

By similar triangles principle:

$$\frac{r}{h} = \frac{20}{40}$$

$$\boldsymbol{r = \frac{h}{2}\ (shown)}$$

(ii) Find the height, h, of the water in the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [3]

$$Volume = \frac{1}{3} \times \pi \times r^2 \times h = 11546.5\ cm^3$$

$$r = \frac{h}{2}$$

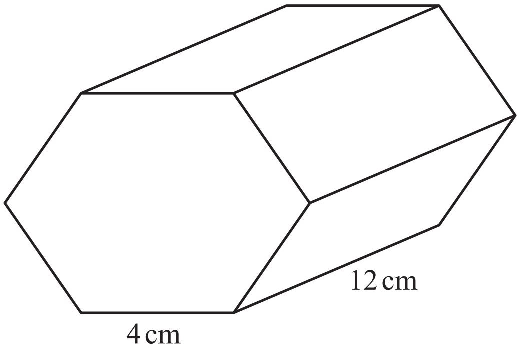
Therefore,

$$\frac{1}{3} \times \pi \times \left(\frac{h}{2}\right)^2 \times h = 11546.5$$

$$\frac{\pi}{3} \times \left(\frac{h^3}{4}\right) = 11546.5$$

$$\boldsymbol{h = 35.3\ cm}$$

(a)



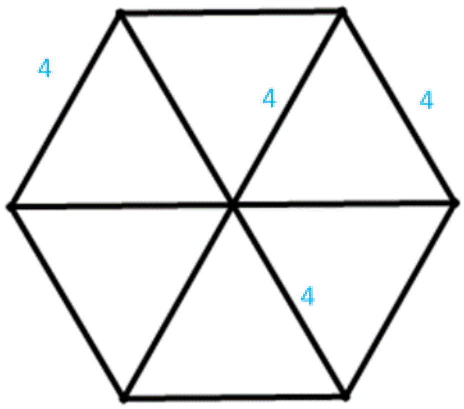
NOT TO SCALE

The diagram shows a prism of length 12 cm.
The cross section is a regular hexagon of side 4 cm.

Calculate the total surface area of the prism. [4]

Need to calculate area of a regular hexagon side.

First identify that the hexagon has 6 sides, and it can be divided into 6 equilateral triangles, with angles 60 degrees.



The area can be obtained by:

$$\text{Areas of 6 triangles} = 6 \times \frac{1}{2} ab \sin C$$

$$= 6 \times \frac{1}{2} \times 4 \times 4 \times \sin(60°)$$

$$= 41.569\ cm^2$$

The prism has 2 such hexagon surfaces,

$$Sum\ of\ areas\ of\ hexagon\ sides = 41.569 \times 2$$

$$= 83.138\ cm^2$$

Now account for 6 rectangles around the prism:

$$Sum\ of\ 6\ rectangles = 12 \times 4 \times 6$$

$$= 288\ cm^2$$

Hence,

$$Total\ area = 288 + 83.138$$

$$= 371\ cm^2$$

(b) Water flows through a cylindrical pipe of radius 0.74 cm.
It fills a 12 litre bucket in 4 minutes.

(i) Calculate the speed of the water through the pipe in centimetres per minute. [4]

$$Volumetric\ flowrate\ of\ water = \frac{12}{4} l/min$$

$$= 3l/min$$

$$= 3000cm^3/min$$

Note to convert litres to cm^3 so that the units are consistent!

Now, we know that we can relate volumetric flowrate to the speed times the area.

$$Volumetric\ flowrate\ \left(\frac{cm^3}{min}\right) = speed\ \left(\frac{cm}{min}\right) \times area\ (cm^2)$$

$$Speed = \frac{Volumetric\ flowrate}{area}$$

$$= \frac{3000}{\pi \times r^2}$$

$$= \frac{3000}{\pi \times (0.74^2)}$$

$$= 1740\ cm/min$$

(ii) When the 12 litre bucket is emptied into a circular pool, the water level rises by 5 **millimetres**.

Calculate the radius of the pool correct to the nearest centimetre. [5]

This pool is cylindrical, because it is circular and has a height.

First, convert 5mm into cm.

$$5mm = 0.5cm$$

$$Volume\ of\ pool = Base\ Area \times Height = Volume\ emptied$$

$$= 12000cm^3$$

$$12000 = Base\ Area \times Height$$

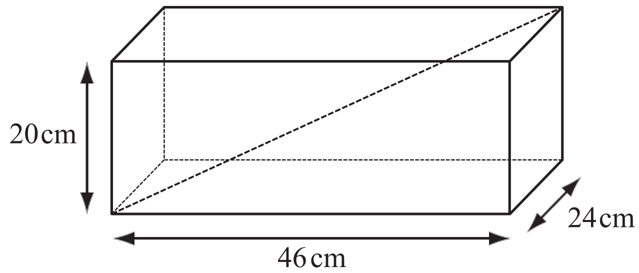
$$12000 = Base\ Area \times 0.5cm$$

$$12000 = \pi \times r^2 \times 0.5$$

$$r = \sqrt{\frac{24000}{\pi}}$$

$$= 87\ cm\ (nearest\ centimetre)$$

(a)

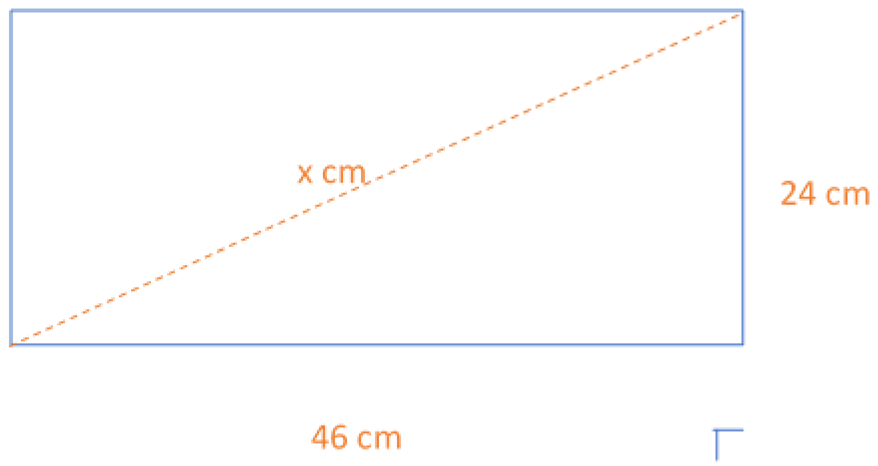


NOT TO SCALE

Jose has a fish tank in the shape of a cuboid measuring 46 cm by 24 cm by 20 cm.

Calculate the length of the diagonal shown in the diagram. [3]

First, consider the bottom of the fish tank.



Find the diagonal using Pythagoras' Theorem

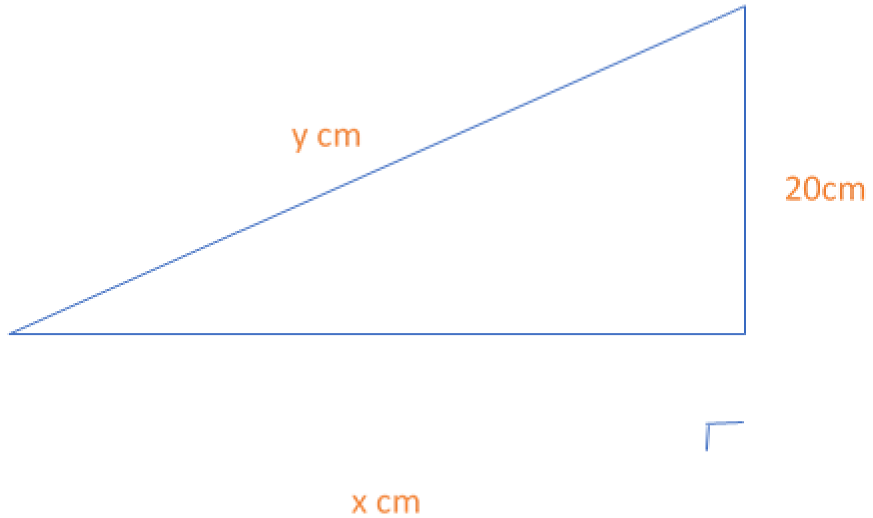
$$x^2 = 46^2 + 24^2$$

$$= 2692$$

$$\rightarrow x = \sqrt{2692}$$

$$= 2\sqrt{673}$$

Now consider the diagonal of interest, y



Find this, again, using Pythagoras'

$$y^2 = x^2 + 20^2$$

$$= 2692 + 400$$

$$= 3092$$

$$\rightarrow y = \sqrt{3092}$$

$$= \mathbf{55.6}$$

(b) Maria has a fish tank with a volume of 20 000 cm^3.

Write the volume of Maria's fish tank as a percentage of the volume of Jose's fish tank. [3]

The volume of Jose's fish tank is

$$20 \times 46 \times 24$$

$$= 22080\ cm^3$$

Maria's volume as a percentage of Jose's is

$$\frac{20000}{22080} \times 100\%$$

$$= \mathbf{90.6\%}$$

(c) Lorenzo's fish tank is mathematically similar to Jose's and double the volume.

Calculate the dimensions of Lorenzo's fish tank. [3]

Mathematically similar means that its sides are the same ratio to each other. We can imagine that each side is multiplied by some scalar a. So, we may write

$$20a \times 46a \times 24a = 2 \times 22080$$

$$\rightarrow 22080a^3 = 44160$$

$$\rightarrow a^3 = 2$$

$$\rightarrow a = \sqrt[3]{2}$$

Hence

$$height = 20 \times \sqrt[3]{2}$$

$$= \mathbf{25.2}$$

$$length = 46 \times \sqrt[3]{2}$$

$$= \mathbf{58.0}$$

$$depth = 24 \times \sqrt[3]{2}$$

$$= \mathbf{30.2}$$

(d) A sphere has a volume of 20 000 cm^3. Calculate its radius.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [3]

We have

$$\frac{4}{3}\pi r^3 = 20000$$

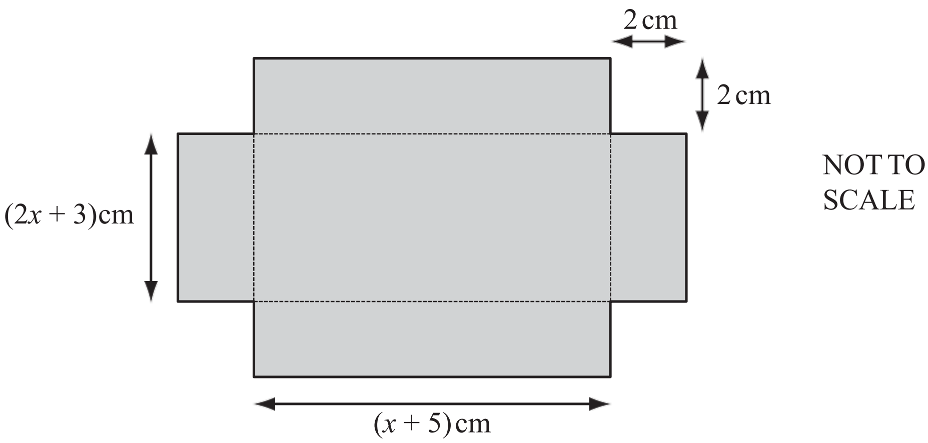
Rearranging for r

$$r^3 = \frac{3}{4} \times \frac{20000}{\pi}$$

$$\rightarrow r = \sqrt[3]{\frac{15000}{\pi}}$$

$$= \mathbf{16.8}$$

A rectangular piece of card has a square of side 2 cm removed from each corner.



(a) Write expressions, in terms of x, for the dimensions of the rectangular card before the squares are removed from the corners. [2]

The width is

$$x + 5 + 2 + 2$$

$$= \boldsymbol{x + 9}$$

The height is

$$2x + 3 + 2 + 2$$

$$= \boldsymbol{2x + 7}$$

(b) The diagram shows a net for an open box.
Show that the volume, V cm^3, of the open box is given by the formula $V = 4x^2 + 26x + 30$. [3]

The box will have sides of length $x + 5$ and $2x + 3$ and a depth of 2. This gives a volume of

$$V = 2 \times (x + 5) \times (2x + 3)$$

$$= 2(2x^2 + 10x + 3x + 15)$$

$$= \boldsymbol{4x^2 + 26x + 30}$$

(c) (i) Calculate the values of x when $V = 75$.
Show all your working and give your answers correct to two decimal places. [5]

We have

$$4x^2 + 26x + 30 = 75$$

$$\rightarrow 4x^2 + 26x - 45 = 0$$

Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\rightarrow x = \frac{-26 \pm \sqrt{26^2 - 4(4)(-45)}}{8}$$

$$= \mathbf{1.42, -7.92}$$

(ii) Write down the length of the longest edge of the box. [1]

The sides of the box are either

$$2x + 3$$

$$= 2(1.42) + 3$$

$$= 5.84$$

or

$$x + 5$$

$$= 1.42 + 5$$

$$= \mathbf{6.42}$$

Which is clearly larger.

A metal cuboid has a volume of 1080 cm^3 and a mass of 8 kg.

(a) Calculate the mass of one cubic centimetre of the metal.
Give your answer in grams. [1]

Divide the mass by the volume for the mass of one cubic centimetre

$$\frac{8kg}{1080cm^3} = \frac{1}{135} kgcm^{-3}$$

For the answer in grams we must multiply by 1000

$$\frac{1}{135} \times 1000$$

$$= \frac{200}{27} g$$

$$= \mathbf{7.4g}$$

(b) The base of the cuboid measures 12 cm by 10 cm.

Calculate the height of the cuboid. [2]

We have that

$$12 \times 10 \times h = 1080$$

Where h is the height.

Divide through by 10×12

$$\rightarrow h = \frac{1080}{12 \times 10}$$

$$= \mathbf{9cm}$$

(c) The cuboid is melted down and made into a sphere with radius r cm.

(i) Calculate the value of r.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [3]

The volume of the sphere is equal to the volume of the cuboid

$$\frac{4}{3}\pi r^3 = 1080$$

$$\rightarrow r^3 = \frac{3}{4} \times \frac{1080}{\pi}$$

$$= 257.83$$

$$\rightarrow r = \sqrt[3]{257.83}$$

$$= \mathbf{6.36cm}$$

(ii) Calculate the surface area of the sphere.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.] [2]

We have

$$A = 4\pi \times (6.36)^2$$

$$= \mathbf{508\ cm^2}$$

(d) A larger sphere has a radius R cm.
The surface area of this sphere is double the surface area of the sphere with radius r cm in part (c).
Find the value of $\frac{R}{r}$. [2]

The larger sphere's surface area is

$$\boldsymbol{A_2 = 4\pi R^2}$$

It is also twice the area of the smaller sphere

$$A_2 = 2 \times A_1$$

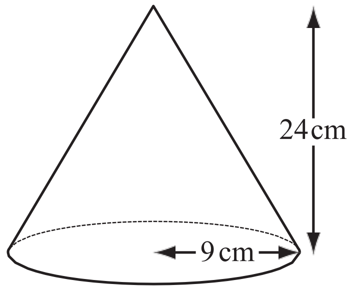
$$= 8\pi r^2$$

$$\rightarrow 4\pi R^2 = 8\pi r^2$$

Cancel the π on both sides, divide through by $4r^2$

$$\rightarrow \frac{R^2}{r^2} = 2$$

$\rightarrow \boldsymbol{\frac{R}{r} = \sqrt{2}}$**=1.41…**



NOT TO SCALE

A solid metal cone has base radius 9 cm and vertical height 24 cm.

(a) Calculate the volume of the cone.

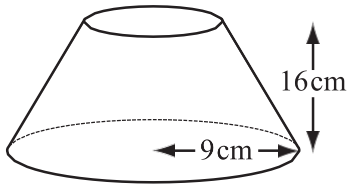
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [2]

$$V = \frac{1}{3}\pi(9)^2 \times 24$$

$$= 648\pi$$

$$= \mathbf{2036}$$

(b)



NOT TO SCALE

A cone of height 8 cm is removed by cutting parallel to the base, leaving the solid shown above. Show that the volume of this solid rounds to 1960 cm^3, correct to 3 significant figures. [4]

The upper radius is

$$9 \times \frac{8}{24}$$

$$= 3$$

The volume of the missing cone is

$$V_c = \frac{1}{3}\pi(3)^2 \times 8$$

$$= 24\pi$$

Hence the volume of the solid is

$$648\pi - 24\pi$$

$$= 624\pi$$

$$= \mathbf{1960}$$

(c) The 1960 cm^3 of metal in the solid in part (b) is melted and made into 5 identical cylinders, each of length 15 cm.
Show that the radius of each cylinder rounds to 2.9 cm, correct to 1 decimal place. [4]

The volume of each cylinder is

$$1960 \div 5$$

$$= 392$$

The volume of a cylinder is

$$V_{cyl} = \pi r^2 \times l$$

$$\rightarrow 392 = \pi r^2 \times 15$$

$$\rightarrow \pi r^2 = \frac{392}{15}$$

$$\rightarrow r = \sqrt{\frac{392}{15\pi}}$$

$$= \mathbf{2.9}$$

Perimeters, Area and Volumes
Difficulty: Medium

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Perimeters, Area and Volumes
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 3

Time allowed: **105 minutes**

Score: **/91**

Percentage: **/100**

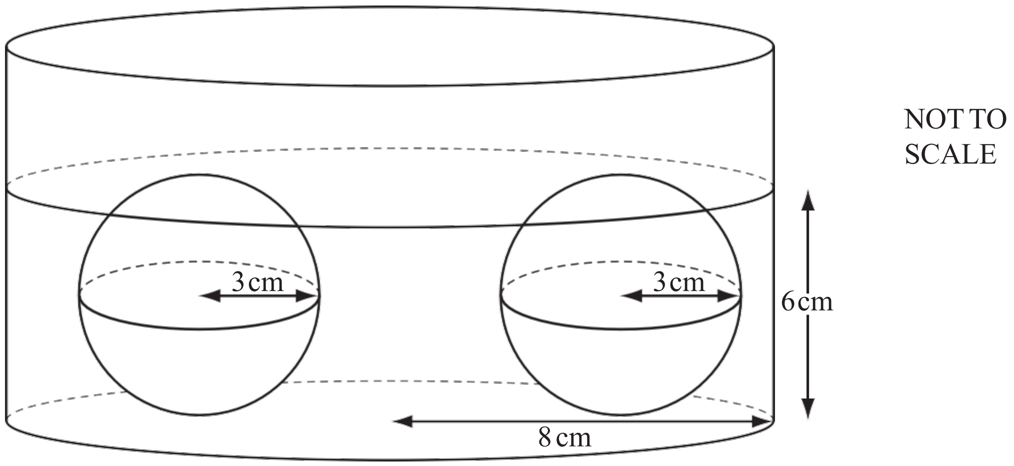
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



The diagram shows two solid spheres of radius 3 cm lying on the base of a cylinder of radius 8 cm.

Liquid is poured into the cylinder until the spheres are just covered.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

(a) Calculate the volume of liquid in the cylinder in

(i) cm^3, [4]

Volume of the container filled (without the spheres) is

$$\pi \times 8^2 \times 6$$

$$= 384\pi$$

Volume of the spheres is

$$2 \times \frac{4}{3}\pi \times 3^3$$

$$= 72\pi$$

Hence, volume of liquid is

$$384\pi - 72\pi$$

$$= 312\pi$$

$$\mathbf{= 980}$$

(ii) litres. [1]

We have that

$$1\ cm^3 = 1\ ml$$

Hence

$$980\ cm^3 = 980\ ml$$

$$= \mathbf{0.98\ l}$$

(b) One cubic centimetre of the liquid has a mass of 1.22 grams.

Calculate the mass of the liquid in the cylinder.

Give your answer in kilograms. [2]

Mass in grams is

$$980 \times 1.22$$

$$= 1195.6\ g$$

$$= \mathbf{1.2\ kg}$$

(c) The spheres are removed from the cylinder.

Calculate the new height of the liquid in the cylinder. [2]

We have that same volume of liquid.

The volume of the container filled is

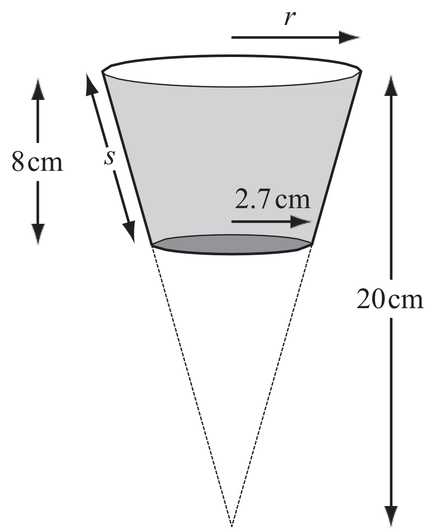
$$\boldsymbol{V = \pi r^2 h}$$

$$= 64h\pi$$

$$= 980$$

$$\rightarrow h = \frac{980}{64\pi}$$

$$= \mathbf{4.87}$$



The diagram shows a plastic cup in the shape of a cone with the end removed.
The vertical height of the cone in the diagram is 20 cm.
The height of the cup is 8 cm.
The base of the cup has radius 2.7 cm.

(a) (i) Show that the radius, r, of the circular top of the cup is 4.5cm. [2]

The initial big cone and the small cone which is being removed to form the cup are similar shapes.

Therefore, the ratios of their corresponding lengths are equal.

Looking at the figure, we can see that the height of the big cone is 20 cm, while the height of the small cone is 20 cm – 8 cm (the height of the cup) = 12 cm.

The radius of the base for the small cone is 2.7 cm, while the radius for the base of the big cone is r.

We can write this as:

$$\frac{2.7\text{ cm}}{r} = \frac{12\text{ cm}}{20\text{ cm}}$$

$$r = \frac{2.7\text{ cm} \times 20\text{ cm}}{12\text{ cm}}$$

r = 4.5 cm

(ii) Calculate the volume of water in the cup when it is full.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [4]

The volume of the cup is equal to the volume of the big cone minus the volume of the small cone which is taken out to make the cup.

The volume of the big cone is:

V = $\frac{1}{3}\pi r^2$h

Where r = 4.5 cm and h = 20 cm

V = $\frac{1}{3}\pi 4.5^2$x 20

V = 424.11 cm^3

The volume of the small cone is:

V = $\frac{1}{3}\pi r^2$h

Where r = 2.7 cm and h = 12 cm

V = $\frac{1}{3}\pi 2.7^2$x 12

V = 91.6 cm^3

The volume of the cup is:

V = 424.11 cm^3 – 91.6 cm^3

V = 332.51 cm^3

(b) (i) Show that the slant height, s, of the cup is 8.2 cm. [3]

We can use Pythagoras' Theorem in the right-angled triangle with the hypothenuse s and the height of the cup perpendicular on the base.

$s^2 = 8^2 + (4.5\text{ cm} - 2.7\text{ cm})^2$

s = 8.2 cm

(ii) Calculate the curved surface area of the outside of the cup.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.] [5]

The curved surface area of the outside of the cup can be worked out as the curved surface area of the big cone minus the curved surface area of the small cone.

We need to work out that slant height for both the big and the small cone.

For the small cone, we can work out the slant height by using Pythagoras' Theorem in the right-angled triangle with the slant height as hypothenuse.

Slant height$^2 = 12^2 + 2.7^2$

Slant height = 12.3 cm

The slant height of the big cone represents the slant height of the small cone added up with the slant height of the cup.

Slant height = 8.2 cm + 12.3 cm = 20.5 cm

The curved surface area for the big cone is:

A = $\pi r l$

Where r = 4.5 cm and l = 20.5 cm

A = π x 4.5 cm x 20.5 cm

A = 289.81cm^2

The curved surface area for the small cone is:

A = $\pi r l$

Where r = 2.7 cm and l = 12.3 cm

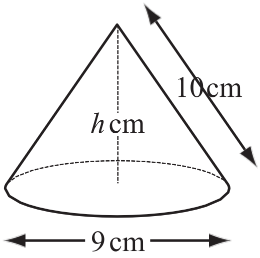
A = π x 2.7 cm x 12.3 cm

A = 104.33 cm^2

The curved area of the cup is:

A = 289.81 cm^2 – 104.33 cm^2

A = 185.5 cm^2



NOT TO SCALE

A solid cone has diameter 9 cm, slant height 10 cm and vertical height h cm.

(a) (i) Calculate the curved surface area of the cone.
[The curved surface area, A, of a cone, radius r and slant height l is $A = \pi r l$.] [2]

We substitute r and l in the formula for the curved surface area of a cone:

The diameter is 9 cm, therefore, the radius is 4.5 cm.

$A = \pi \times r \times l$

$A = \pi \times 4.5 \text{ cm} \times 10 \text{ cm}$

$A = 141.3 \text{ cm}^2$

(ii) Calculate the value of h, the vertical height of the cone. [3]

The vertical height is perpendicular on the base, therefore perpendicular on the radius of the circle, forming a right-angled triangle with the slant height as hypothenuse.

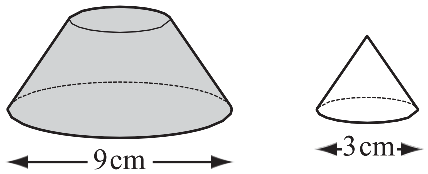
Using Pythagoras' Theorem, we can work out the height.

$h^2 + 4.5^2 = 10^2$

$h^2 = 100 - 20.25$

$h = 8.93 \text{ cm}$

(b)



NOT TO SCALE

Sasha cuts off the top of the cone, making a smaller cone with diameter 3 cm.
This cone is **similar** to the original cone.

(i) Calculate the **vertical** height of this small cone. [2]

Since the 2 cones are similar shapes, the ratio of their corresponding lengths should be equal.

We can write this as:

$$\frac{\text{h small cone}}{8.93 \text{ cm}} = \frac{3 \text{ cm}}{9 \text{ cm}}$$

$$\text{h small cone} = \frac{8.93 \text{ cm x } 3 \text{ cm}}{9 \text{ cm}}$$

h small cone = 2.97 cm

(ii) Calculate the curved surface area of this small cone. [2]

To calculate the curved surface area of the small cone we need to work out first the length of the slant height.

We can do this by using the fact that the small cone and the initial cone are similar shapes, therefore, the ratio of their corresponding length is equal.

$$\frac{3 \text{ cm}}{9 \text{ cm}} = \frac{\text{l small cone}}{10 \text{ cm}}$$

$$\text{l small cone} = \frac{10 \text{ cm x } 3 \text{ cm}}{9 \text{ cm}}$$

l small cone = 3.33 cm

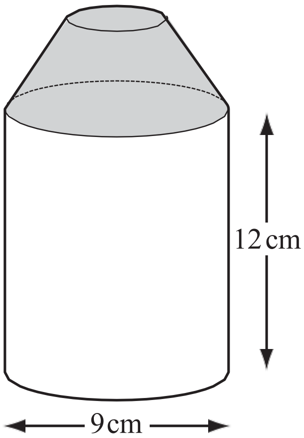
The formula for the curved surface area of a cone is:

A = π x r x l

A = π x 1.5 cm x 3.33 cm

A = 15.69 cm^2

(c)



NOT TO SCALE

The shaded solid from **part (b)** is joined to a solid cylinder with diameter 9 cm and height 12 cm.
Calculate the **total** surface area of the whole solid. [5]

The surface area of the new solid will be equal with the curved surface area of the cylinder plus the surface area of the initial cone with the surface area of the small cone taken out. The area of a circle is added as the bottom of the solid (radius = 4.5 cm) and the area of a small circle of radius 1.5 cm is added as the top of the solid.

A solid = A curved cylinder + A initial cone – A small cone + A circle

The curved surface area of a cylinder has the formula:

A = $2\pi rh$

A = $2\pi \ x \ 4.5 \ cm \ x \ 12 \ cm$

A = 339.29 cm^2

The area of the top circle is:

A = πr^2

A = $\pi \ x \ 1.5^2$

A = 7.068 cm^2

The area of the bottom circle is:

A = πr^2

A = $\pi \ x \ 4.5^2$

A = 63.61 cm^2

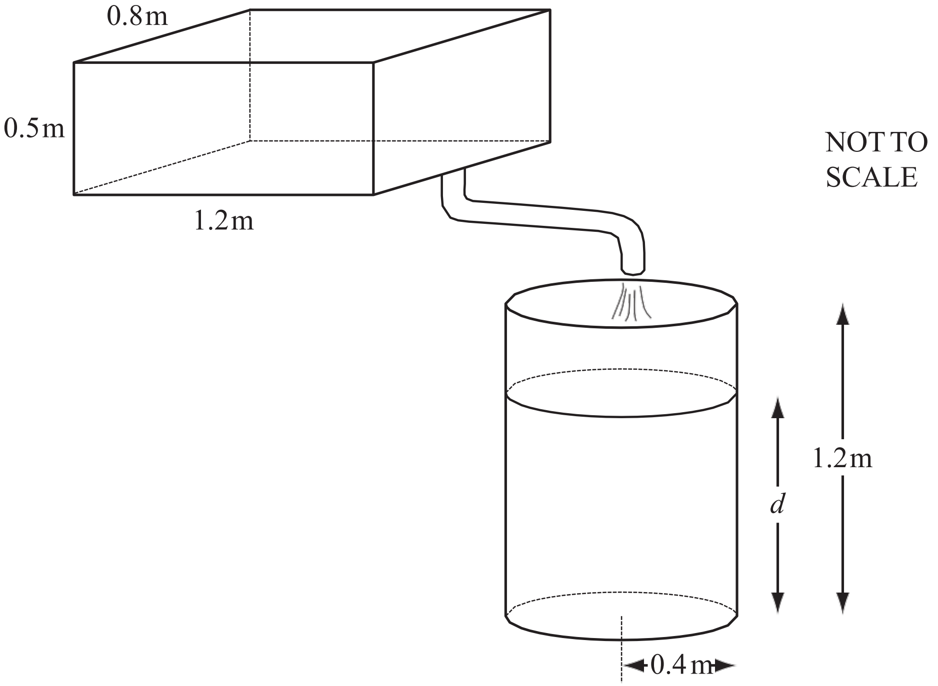
The curved area of the shaded solid from part b) is:

Area curved initial cone – Area curved small cone = 141 cm^2 – 15.7 cm^2 = 125.3 cm^2

The area of the whole solid formed is:

A = 63.61 cm^2 + 125.3 cm^2 + 339.29 cm^2 + 7.068 cm^2

A = 535.26 cm^2



A rectangular tank measures 1.2 m by 0.8 m by 0.5 m.
(a) Water flows from the full tank into a cylinder at a rate of 0.3 m^3/min.

Calculate the time it takes for the full tank to empty.
Give your answer in minutes and seconds. [3]

We need to work out the volume of water which needs to be moved and then divide it by the rate at which the tank is emptied to work out the time.

The volume of the rectangular tank is:

V = 1.2 m x 0.8 m x 0.5 m

V = 0.48 m^3

The rate is: 0.3 m^3/min

Time = $\frac{0.48 \text{ m}^3}{0.3 \text{ m}^3/min}$

Time = 1.6 minutes

1 minute represents 60 seconds, therefore, 0.6 minutes represent:

0.6 minutes x 60 s/minute = 36 seconds

The total time is 1 minute and 36 seconds.

(b) The radius of the cylinder is 0.4 m.

Calculate the depth of water, *d*, when all the water from the rectangular tank is in the cylinder.

[3]

The formula for the volume of a cylinder is:

$V = \pi r^2 h$

Where r is the radius and h is the height of the cylinder, in this case

the depth of the water, d

The radius of the cylinder is r = 0.4 m

The volume will be equal to the volume of water in the first tank.

$V = 0.48\ m^3 = \pi\ 0.4^2\ d$

$d = \frac{0.48\ m^3}{0.502\ m^2}$

d = 0.956 m

(c) The cylinder has a height of 1.2 m and is open at the top.
The inside surface is painted at a cost of $2.30 per m^2.

Calculate the cost of painting the inside surface. [4]

The inside area is equivalent with the surface area of the cylinder minus

the area of the circle at the top, since the cylinder is open.

The formula for the surface area of a cylinder is:

A = $2\pi rh$

Where r is the radius, r = 0.4 m

A = 2π 0.4 x 1.2 m^2

A = 3.0159 m^2

The formula for the area of a circle is:

A = πr^2

Where r is the radius, r = 0.4 m

A = π x 0.16 m^2

A = 0.502 m^2

The area that needs to be painted is the surface area of the cylinder plus the area of one of the circles (the bottom, since the top is open).

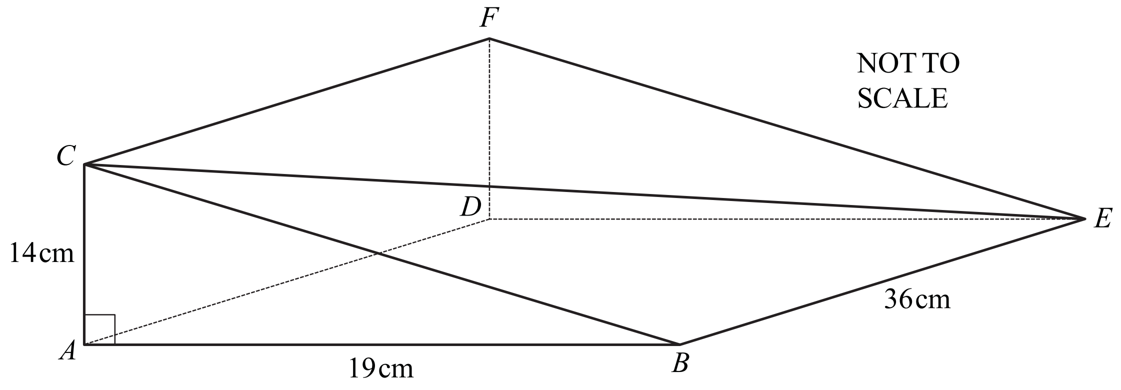
A = 3.0159 m^2 + 0.502 m^2

A = 3.5179 m^2

The cost is \$2.30/m^2, therefore, the total cost is:

Cost = 3.5179 m^2 x \$2.30/m^2

Cost = \$8.091



In the diagram, *ABCDEF* is a prism of length 36 cm.
The cross-section *ABC* is a right-angled triangle.
AB = 19 cm and *AC* = 14 cm.

Calculate

(a) the length *BC*, [2]

The triangle ABC is a right angle triangle. Hence the length of BC can be determined using Pythagoras theorem:

$$CB^2 = AB^2 + CA^2$$

Substitute the given values:

$$CB^2 = (19cm)^2 + (14cm)^2$$

$$CB^2 = 557cm^2$$

Take the square root of both sides to get the length of CB (3sf).

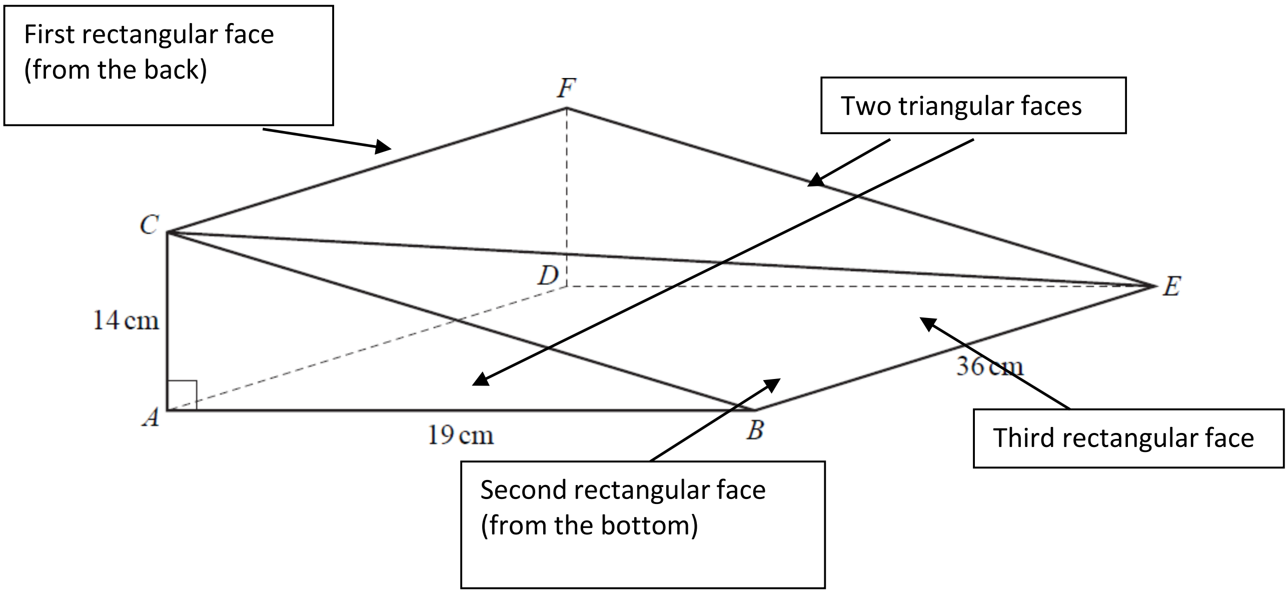
$$\mathbf{CB = 23.6cm}$$

(b) the total surface area of the prism, [4]

The prism has 5 faces.

- Two faces are identical triangles with perpendicular edges of length 14cm and 19cm
- A rectangular face with edges of length 14cm and 36m

- A rectangular face with edges of length 19cm and 36cm
- A rectangular face with edges of length 36cm and 23.6cm



The area of a rectangle is the product of its sides.

The area of a right angle triangle is half the product of its perpendicular sides.

$$Area$$
$$= 2 \times triangle + first\ rectangel + second\ rectange + third\ rectangle$$

Plug in given values:

$$Area$$
$$= 2 \times \frac{1}{2} \times (14cm) \times (19cm) + (14cm) \times (36cm) + (19cm) \times (36cm)$$
$$+ (23.6cm) \times (36cm)$$

Calculate the total surface area of the prism.

$$Area = 266cm^2 + 504cm^2 + 684cm^2 + 849.6cm^2$$

$$\mathbf{Area = 2303.6\ cm^2}$$

(c) the volume of the prism, [2]

The volume of the prism is given as the product of the area of the cross-section triangle and its width.

The area of triangle from the previous part of the question:

$$Triangle\ area = \frac{1}{2} \times (14cm) \times (19cm)$$

$$Triangle\ area = 133\ cm^2$$

The width of the prism (length of the edge perpendicular to the triangle) is 36cm.

$$Volume = Triangle\ area \times width$$

$$Volume = 133\ cm^2 \times 36cm$$

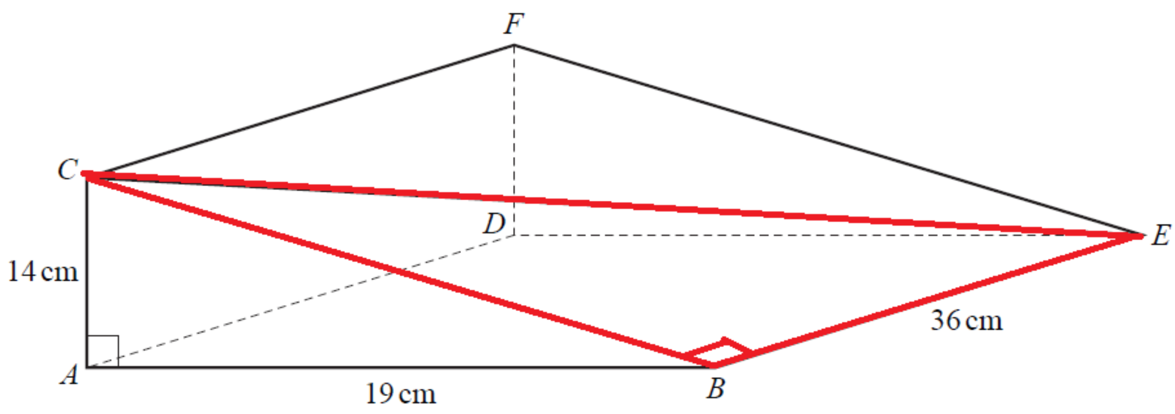
Hence we have the volume of the prism:

$$\boldsymbol{Volume = 4788\ cm^3}$$

(d) the length CE, [2]

The triangle BCE is a right angle triangle. Hence the length of CE can be determined using Pythagoras theorem:

$$CE^2 = CB^2 + BE^2$$



Substitute the given values:

$$CE^2 = (23.6cm)^2 + (36cm)^2$$

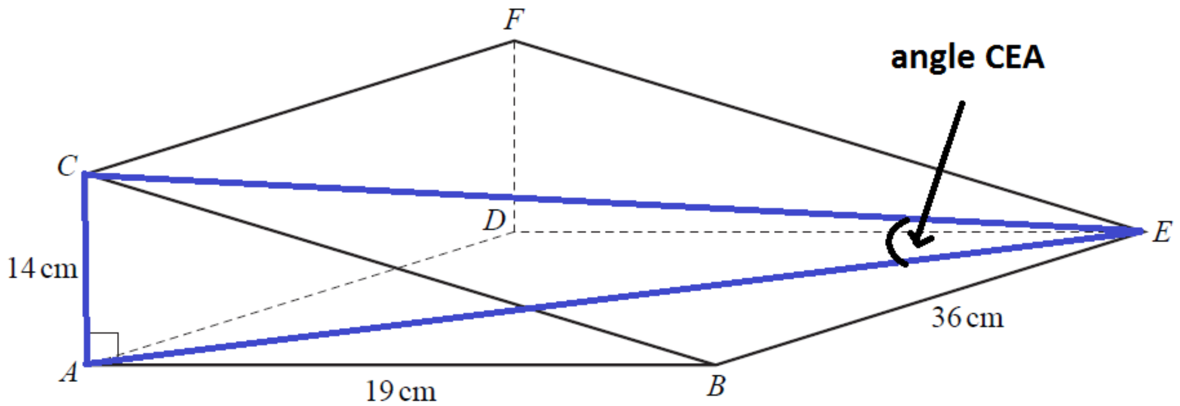
$$CE^2 = 1852.96\ cm^2$$

Take the square root of both sides to get the length of CE (3sf).

$$\boldsymbol{CE = 43.0cm}$$

(e) the angle between the line CE and the base $ABED$. [3]

Consider a right-angle triangle CAE.



The angle between the line CE and base ABED is the angle CEA. We can calculate this angle using sine function (opposite over hypotenuse):

$$\sin(CEA) = \frac{CA}{CE}$$

Plug in known values:

$$\sin(CEA) = \frac{14cm}{43.0cm}$$

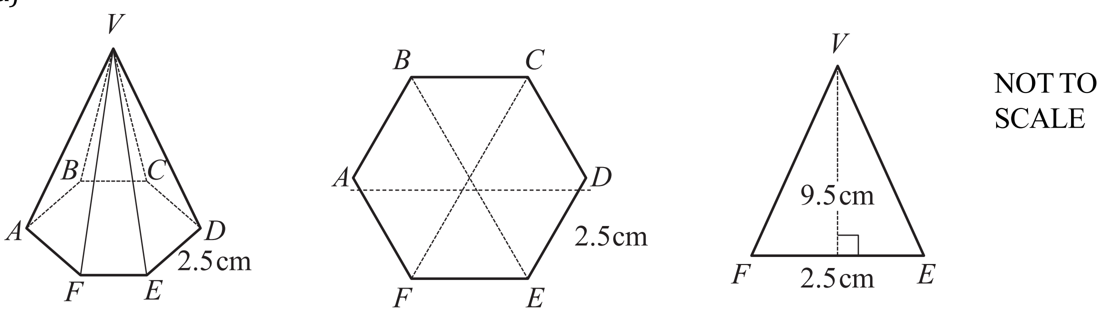
$$\sin(CEA) = 0.3256\ldots$$

Apply inverse sin function to both sides of the equation to find the angle CEA.

$$\text{angle CEA} = \arcsin(0.3256\ldots)$$

$$\textbf{angle CEA} = \mathbf{19.0^\circ}$$

(a)



A solid pyramid has a **regular hexagon** of side 2.5 cm as its base.
Each sloping face is an isosceles triangle with base 2.5 cm and height 9.5 cm.

Calculate the **total** surface area of the pyramid. [4]

The surface area of the pyramid is made us of 6 isosceles triangles and the hexagonal base.

$$surface\ area = 6 \times triangle\ area + hexagon\ base\ area$$

The area of a triangle with base *a* and height *b* is given by:

$$triangle\ area = \frac{1}{2}ab$$

The area of a hexagon with base *c* is the same as the area of 6 equilateral triangles with side *c*.

$$hexagon\ area = 6\ equilateral\ triangle\ area$$

$$hexagon\ area = 6\ \times \frac{1}{2}c^2\sin(60°)$$

$$hexagon\ area = \frac{3\sqrt{3}}{2}c^2$$

In our case, the base of a triangle and the base of a hexagon are the same.

$$a = c = 2.5cm$$

and the height of the triangle is:

$$b = 9.5cm$$

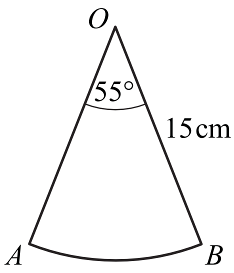
Plug these values into the formula for the pyramid area to find the answer:

$$surface\ area = 6 \times \frac{1}{2} \times (2.5cm)(9.5cm) + \frac{3\sqrt{3}}{2}(2.5cm)^2$$

$$surface\ area = 71.25\ cm^2 + 16.24\ cm^2$$

$$\mathbf{surface\ area = 87.5\ cm^2}$$

(b)



NOT TO SCALE

A sector OAB has an angle of 55° and a radius of 15 cm.

Calculate the area of the sector and show that it rounds to 108 cm^2, correct to 3 significant figures. [3]

The area of a sector with radius *r* is given by:

$$sector\ area = \pi r^2 \times \frac{secotr\ angle}{360°}$$

In our case, *r*= 15 cm and the sector angle is 55°.

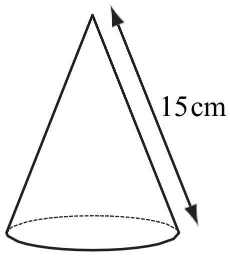
$$sector\ area = \pi(15cm)^2 \times \frac{55°}{360°}$$

$$sector\ area = 107.99\ cm^2$$

Correct to three significant figures:

$$\mathbf{sector\ area = 108\ cm^2}$$

(c)



NOT TO SCALE

The sector radii *OA* and *OB* in **part (b)** are joined to form a cone.

(i) Calculate the base radius of the cone.
[The curved surface area, *A*, of a cone with radius *r* and slant height *l* is $A = \pi r l$.] [2]

We are given the surface area of a cone with radius *r* and slant height *l*.

$$area = \pi r l$$

We already know the area from the previous questions. Hence we just solve for *r*.

$$108\ cm^2 = \pi \times (15cm) \times r$$

Divide both sides by $\pi \times (15cm)$ and find the value of *r* (3sf).

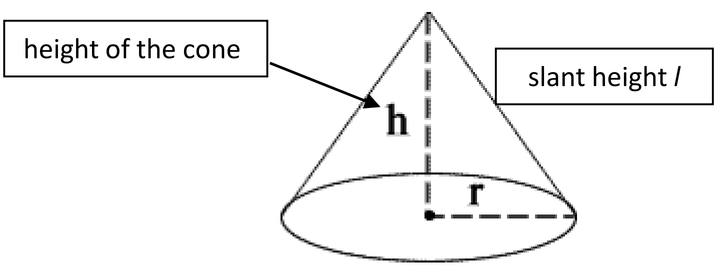
$$r = \frac{108\ cm^2}{\pi \times (15cm)}$$

$$\boldsymbol{r = 2.29\ cm}$$

(ii) Calculate the perpendicular height of the cone. [3]

The height *h* of the cone can be found from Pythagoras triangle

$$l^2 = h^2 + r^2$$



Subtracting r^2 from both sides

$$h^2 = l^2 - r^2$$

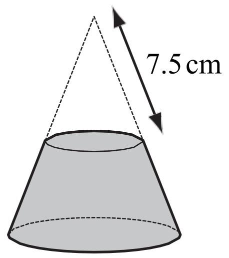
Use the value of r from the previous part of the question.

$$h^2 = (15cm)^2 - (2.29cm)^2$$

Taking square root of both sides, we get the final answer:

$$\boldsymbol{h = 14.8\ cm}$$

(d)



NOT TO SCALE

A solid cone has the same dimensions as the cone in **part (c)**.
A small cone with slant height 7.5 cm is removed by cutting parallel to the base.

Calculate the volume of the remaining solid.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [3]

The volume of the remaining solid is equal to the volume of the original cone (with slant height 15cm) when we subtract the volume of the removed cone (slant height l).

$$volume = \frac{1}{3}\pi r^2 h - \frac{1}{3}\pi R^2 H$$

(capital letters denote the smaller cone which was removed)

These two cones we are talking about are mathematically similar, hence the ratio of their respective dimensions is a constant.

$$\frac{big\ cone\ slant\ height}{small\ cone\ sland\ height} = \frac{big\ cone\ radius}{small\ cone\ radius} = \frac{big\ cone\ height}{small\ cone\ height} = 2$$

We see that this ratio is 2 as the slant height of the original cone is twice as long as the slant height of the removed (small) cone. Therefore we can deduce:

$$R = \frac{r}{2} = 1.145cm$$

$$H = \frac{h}{2} = 7.4cm$$

Substituting all values into the formula for the volume of the remaining solid:

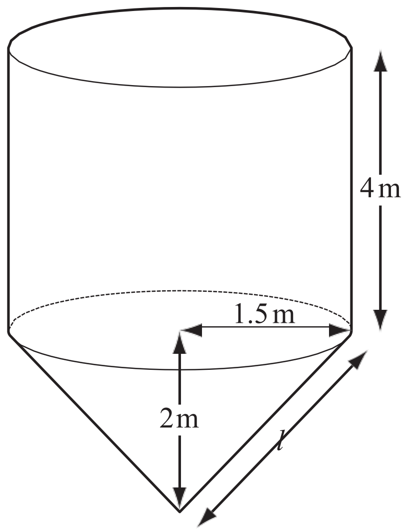
$$volume = \frac{1}{3}\pi r^2 l - \frac{1}{3}\pi R^2 L$$

$$volume = \frac{1}{3}\pi(2.29cm)^2 \times (14.8cm) - \frac{1}{3}\pi(1.145cm)^2 \times (7.4cm)$$

$$volume = 81.276cm^2 - 10.159cm^2$$

We get the volume of the remaining shape correct to 3 significant figures.

$$\boldsymbol{volume = 71.1cm^2}$$



NOT TO SCALE

An open water storage tank is in the shape of a cylinder on top of a cone.
The radius of both the cylinder and the cone is 1.5 m.
The height of the cylinder is 4 m and the height of the cone is 2 m.

(a) Calculate the total surface area of the outside of the tank.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.] [6]

We have that the total area is the cone plus the cylinder (do not include the circle on top because it is an **open** cylinder)

$$\boldsymbol{A = \pi r l + 2\pi r h}$$

l can be found using Pythagoras'

$$l^2 = 2^2 + 1.5^2$$

$$\rightarrow l = 2.5$$

$$A = \pi \times 1.5 \times 2.5 + 2 \times \pi \times 1.5 \times 4$$

$$\boldsymbol{= 49.5}$$

(b) The tank is completely full of water.

(i) Calculate the volume of water in the tank and show that it rounds to 33 m^3, correct to the nearest whole number.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [4]

The volume is

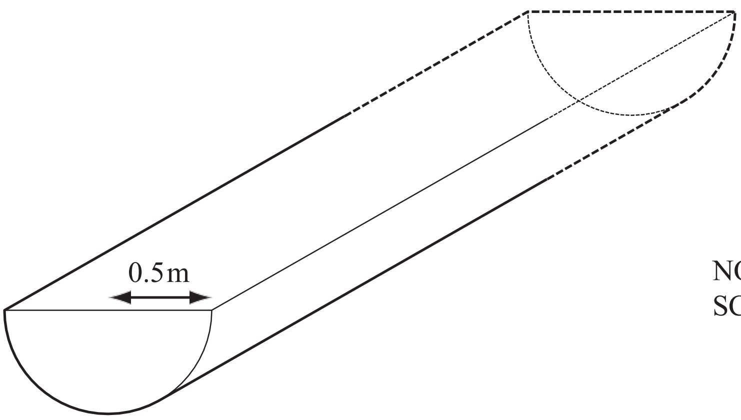
$$V = \frac{1}{3}\pi r^2 h + \pi r^2 h$$

$$= \pi\left(\frac{1}{3} \times 1.5^2 \times 2 + 1.5^2 \times 4\right)$$

$$= 10.5\pi$$

$$= 33$$

(ii)



The cross-section of an irrigation channel is a semi-circle of radius 0.5 m.
The 33 m^3 of water from the tank completely fills the irrigation channel.

Calculate the length of the channel. [3]

The volume of the channel is

$$V = \frac{1}{2} \times \pi r^2 \times l$$

Where l is the length of the channel and r is the radius.

$$\rightarrow 33 = \frac{1}{2}\pi \times 0.5^2 \times l$$

$$\rightarrow 33 = \frac{1}{8}\pi \times l$$

Divide through by π and multiply through by 8 for

$$\rightarrow l = \frac{264}{\pi}$$

$$\mathbf{= 84.0}$$

(c) (i) Calculate the number of litres in a full tank of 33 m^3. [1]

1 metre cubed is 1000 litres

$$\rightarrow 33\ m^3$$

$$\mathbf{= 33\ 000\ \mathit{l}}$$

(ii) The water drains from the tank at a rate of 1800 litres per minute.

Calculate the time, in minutes and seconds, taken to empty the tank. [2]

The time (in minutes) is

$$33000 \div 1800$$

$$= \frac{55}{3}\ minutes$$

$$= 18\frac{1}{3}\ minutes$$

Convert to minutes and seconds

$$= 18\ min + \frac{1}{3} \times 60\ s$$

$$\mathbf{= 18\ \mathit{min}\ 20\mathit{s}}$$

Perimeters, Area and Volumes
Difficulty: Medium

Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Perimeters, Area and Volumes
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 4

Time allowed: **108 minutes**

Score: **/94**

Percentage: **/100**

Grade Boundaries:

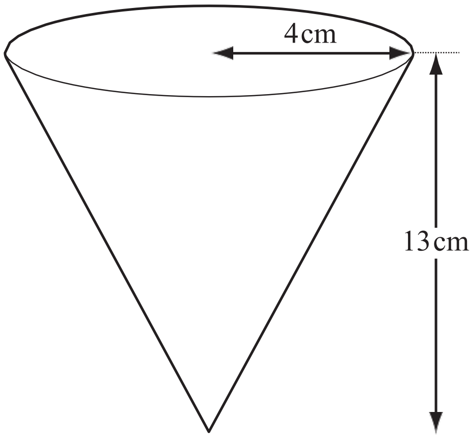
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a)



NOT TO SCALE

The diagram shows a cone of radius 4 cm and height 13 cm.
It is filled with soil to grow small plants.
Each cubic centimetre of soil has a mass of 2.3g.

(i) Calculate the volume of the soil inside the cone.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [2]

$$V = \frac{1}{3}\pi(4)^2(13)$$

$$= \mathbf{217.8}$$

(ii) Calculate the mass of the soil. [1]

Multiply density by volume to get mass

$$217.8 \times 2.3$$

$$= \mathbf{501.0}$$

(iii) Calculate the greatest number of these cones which can be filled completely using 50 kg of soil. [2]

Convert the mass to grams

$$50 \times 1000 = 50000g$$

Divide this by the mass in 1 cone to get number of cones

$$\frac{50000}{501}$$

$$= \mathbf{99.8}$$

So, we have **99** filled cones.

(b) A similar cone of height 32.5 cm is used for growing larger plants.

Calculate the volume of soil used to fill this cone. [3]

The length scalar is

$$\frac{32.5}{13} = 2.5$$

The volume scalar is then

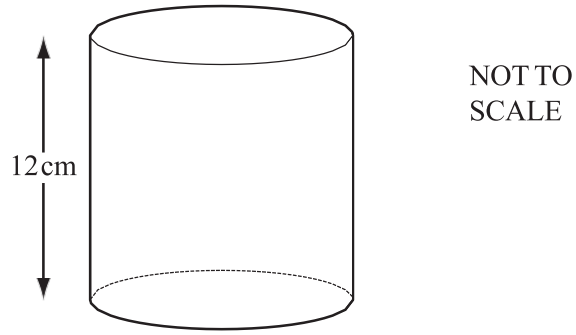
$$2.5^3$$

So, we have a volume of

$$217.8 \times 2.5^3$$

$$= \mathbf{3403}$$

(c)



Some plants are put into a cylindrical container with height 12 cm and volume 550 cm^3.

Calculate the radius of the cylinder. [3]

Volume of a cylinder is

$$\boldsymbol{V = \pi r^2 \times h}$$

So, we have

$$550 = \pi r^2 \times 12$$

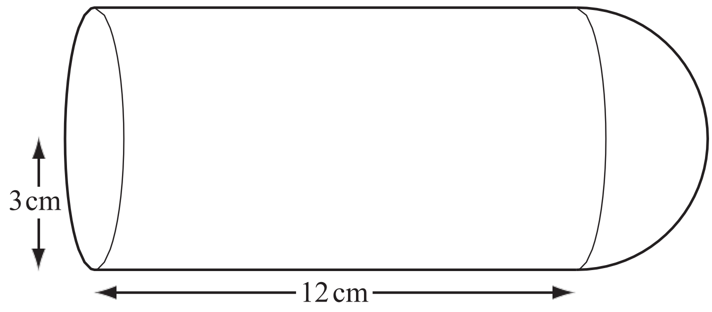
Divide through by 12π

$$\frac{550}{12\pi} = r^2$$

Square root

$$r = \sqrt{\frac{550}{12\pi}}$$

$$= \mathbf{3.82}$$



NOT TO SCALE

The diagram shows a solid made up of a hemisphere and a cylinder.
The radius of both the cylinder and the hemisphere is 3 cm.
The length of the cylinder is 12 cm.

(a) (i) Calculate the volume of the solid.

[The volume, V, of a **sphere** with radius r is $V = \frac{4}{3}\pi r^3$.] [4]

Add the volumes of the two shapes together

$$V = \pi r^2 \times l + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

Note that we have halved the given formula because we

have a hemisphere, not a full sphere.

$$= \pi(3)^2(12) + \frac{1}{2} \times \frac{4}{3}\pi(3)^3$$

$$= 126\pi$$

$$= \mathbf{396}$$

(ii) The solid is made of steel and 1 cm^3 of steel has a mass of 7.9 g.
Calculate the mass of the solid.
Give your answer in kilograms. [2]

The mass in grams is

$$395.8 \times 7.9$$

$$= 3128.4$$

Converting to kilos by dividing by 1000

$$= \mathbf{3.13\ kg}$$

(iii) The solid fits into a box in the shape of a cuboid, 15 cm by 6 cm by 6 cm.
Calculate the volume of the box **not** occupied by the solid. [2]

The cuboid box has a volume

$$15 \times 6 \times 6$$

$$= 540$$

Hence the volume not occupied is

$$540 - 396$$

$$= \mathbf{144}$$

(b) (i) Calculate the total surface area of the solid.
You must show your working.
[The surface area, A, of a **sphere** with radius r is $A = 4\pi r^2$.] [5]

Add the surface area of the two shapes

$$A = \pi r^2 + 2\pi r \times l + \frac{1}{2}(4\pi r^2)$$

We have added the circular face, the curved surface, and

the hemisphere.

$$= 3^2\pi + 2(3)(12)\pi + 2(3)^2\pi$$

$$= 99\pi$$

$$= \mathbf{311}$$

(ii) The surface of the solid is painted.
The cost of the paint is \$0.09 per millilitre.
One millilitre of paint covers an area of 8 cm^2.
Calculate the cost of painting the solid. [2]

The amount of paint needed is

$$311 \div 8$$

$$= 38.88\ ml$$

The cost is

$$38.88 \times 0.09$$

$$= \pounds\mathbf{3.50}$$

A spherical ball has a radius of 2.4 cm.

(a) Show that the volume of the ball is 57.9 cm^3, correct to 3 significant figures.

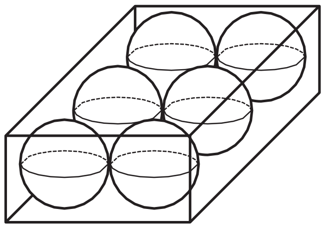
[The volume V of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.] [2]

$$V = \frac{4}{3}\pi(2.4)^3$$

$$= 57.9058$$

$$= \mathbf{57.9}$$

(b)



NOT TO SCALE

Six spherical balls of radius 2.4 cm fit exactly into a **closed** box.
The box is a cuboid.

Find

(i) the length, width and height of the box, [3]

Height must be twice the radius

$$height = 2 \times 2.4$$

$$= \mathbf{4.8}$$

Width is 2 balls, or 4 times the radius

$$width = 4 \times 2.4$$

$$= \mathbf{9.6}$$

Length is 3 balls, or 6 times the radius

$$length = 6 \times 2.4$$

$$= \mathbf{14.4}$$

(ii) the volume of the box, [1]

$14.4 \times 9.6 \times 4.8$

$= 663.552$

$= \mathbf{663.6}$

(iii) the volume of the box **not** occupied by the balls, [1]

Volume occupied by balls is

6×57.9

$= 347.4$

Hence, volume not occupied is

$663.6 - 347.4$

$= \mathbf{316.2}$

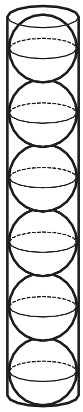
(iv) the surface area of the box. [2]

2 long faces, 2 short faces, and the top and bottom.

$2 \times 14.4 \times 4.8 + 2 \times 4.8 \times 9.6 + 2 \times 9.6 \times 14.4$

$= \mathbf{507}$

(c)



The six balls can also fit exactly into a **closed** cylindrical container, as shown in the diagram.

Find

(i) the volume of the cylindrical container, [3]

The area of the circular face is

$$\pi \times 2.4^2$$

$$= 18.1$$

and the length is

$$12 \times 2.4$$

$$= 28.8$$

Hence, the volume is

$$18.1 \times 28.8$$

$$= \mathbf{521}$$

(ii) the volume of the cylindrical container **not** occupied by the balls, [1]

Volume not occupied is

$$521 - 6 \times 57.9$$

$$= \mathbf{174}$$

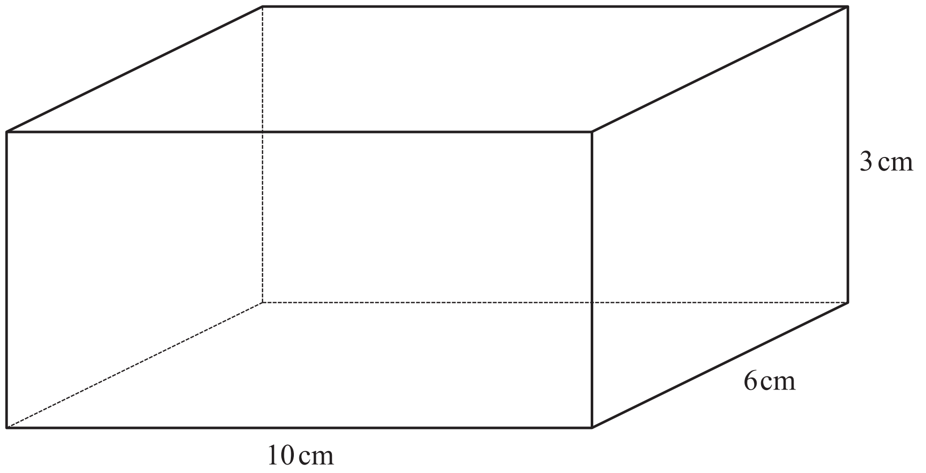
(iii) the surface area of the cylindrical container. [3]

Add the two circular faces to the long, curved surface

$$A = 2 \times \pi r^2 + 2\pi r \times l$$

$$= 2(2.4)^2\pi + 2(2.4)(28.8)\pi$$

$$= \mathbf{470}$$



A solid metal cuboid measures 10 cm by 6 cm by 3 cm.

(a) Show that 16 of these solid metal cuboids will fit exactly into a box which has internal measurements 40 cm by 12 cm by 6 cm. [2]

Boxes can stack in 2s since

$$6 \div 3$$

$$= 2$$

They can fit 2 abreast

$$12 \div 6$$

$$= 2$$

And can fit 4 along

$$40 \div 10$$

$$= 4$$

Hence, we can fit

$$4 \times 2 \times 2$$

$$= \mathbf{16}$$

(b) Calculate the volume of one metal cuboid. [1]

Volume of one box is

$$10 \times 6 \times 3$$

$$= \mathbf{180}$$

(c) One cubic centimetre of the metal has a mass of 8 grams.
The box has a mass of 600 grams.

Calculate the **total** mass of the 16 cuboids **and** the box in

(i) grams, [2]

Each box has a weight of

$$180 \times 8$$

$$= 1440$$

Hence, total mass is

$$600 + 16 \times 1440$$

$$= \mathbf{23640}$$

(ii) kilograms. [1]

$$23640 \div 1000$$

$$= \mathbf{23.64}$$

(d) (i) Calculate the surface area of **one** of the solid metal cuboids. [2]

Add all the faces together

$$2 \times (6 \times 3 + 3 \times 10 + 6 \times 10)$$

$$= \mathbf{216}$$

(ii) The surface of each cuboid is painted. The cost of the paint is $25 per **square metre**.

Calculate the cost of painting all 16 cuboids. [3]

Converting into square metres

$$216 \div 100^2$$

$$= 0.0216$$

Surface area of all 16 cuboids

$$0.0216 \times 16$$

$$= 0.3456$$

Hence the cost is

$$0.3456 \times 25$$

$$= \mathbf{8.64}$$

(e) One of the solid metal cuboids is melted down.
Some of the metal is used to make 200 identical solid spheres of radius 0.5 cm.

Calculate the volume of metal from this cuboid which is **not** used.

[The volume, V, of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.] [3]

Recall that volume of one cuboid is 180.

Volume of 200 spheres is

$$\frac{4}{3}\pi \times 0.5^3 \times 200$$

$$= 104.7$$

Hence the unused volume is

$$180 - 104.7$$

$$= \mathbf{75.3}$$

(f) 50 cm^3 of metal is used to make 20 identical solid spheres of radius r.

Calculate the radius r. [3]

We have that

$$20 \times \frac{4}{3}\pi r^3 = 50$$

Divide through by 20

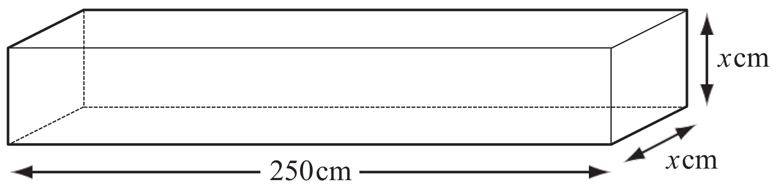
$$\frac{4}{3}\pi r^3 = \frac{5}{2}$$

Divide through by $\frac{4}{3}\pi$

$$r^3 = \frac{15}{8\pi}$$

$$= 0.5968$$

$$\rightarrow \boldsymbol{r = 0.842}$$



NOT TO SCALE

A solid metal bar is in the shape of a cuboid of length of 250 cm.
The cross-section is a square of side x cm.
The volume of the cuboid is 4840 cm^3.

(a) Show that $x = 4.4$. [2]

The volume of a cuboid is:

V = length x width x height

In our case, the width and height are both equal to x.

$4840 \text{ cm}^3 = 250x^2 \text{ cm}^3$

$x^2 = 19.36 \text{ cm}^2$

x = 4.4 cm

(b) The mass of 1 cm^3 of the metal is 8.8 grams.
Calculate the mass of the whole metal bar in kilograms. [2]

The weight of the bar is:

$4840 \text{ cm}^3 \times 8.8\text{g} = 42\,592 \text{ g}$

We convert this amount in kg by dividing by 1000.

42 592 g

= **42.592 kg**

(c) A box, in the shape of a cuboid measures 250 cm by 88 cm by h cm.
120 of the metal bars fit exactly in the box. [2]
Calculate the value of h.

The volume of the box will be equal to the volume of 120 metal bars.

V = 120 x 4840 cm^3

V = 580 800 cm^3

The box is also a cuboid, therefore its volume is:

V = length x width x height

580 800 cm^3 = 250 cm x 88 cm x h

h = 26.4 cm

(d) One metal bar, of volume 4840 cm^3, is melted down to make 4200 identical small spheres.

All the metal is used.

(i) Calculate the radius of each sphere. Show that your answer rounds to 0.65 cm, correct to 2 decimal places.
[The volume, V, of a sphere, radius r, is given by $V = \frac{4}{3}\pi r^3$.] [4]

The volume of one sphere is:

V = 4840 cm^3 / 4200

V = 1.152 cm^3

V = $\frac{4}{3}\pi r^3$

1.152 cm^3 = $\frac{4}{3}\pi r^3$

r = 0.6503 cm, rounds to 0.65 cm.

(ii) Calculate the surface area of each sphere, using 0.65 cm for the radius.
[The surface area, A, of a sphere, radius r, is given by $A = 4\pi r^2$.] [1]

$A = 4\pi r^2$

r = 0.65 cm

$A = 4\pi 0.65^2$

A = 5.31 cm²

(iii) Calculate the total surface area of all 4200 spheres as a percentage of the surface area of the metal bar. [4]

The surface area of one sphere is: A = 5.31 cm^2

The surface area of 4200 spheres is:

A = 4200 x 5.31 cm^2

A = 22.302 cm^2

The surface area of the metal bar is equal to 2 x the area of a square with the side = 4.4 cm plus 4x the area of a rectangle with the measures 250 cm and 4.4 cm.

A = 2 x 4.4 cm x 4.4 cm + 4 x 4.4 cm x 250 cm

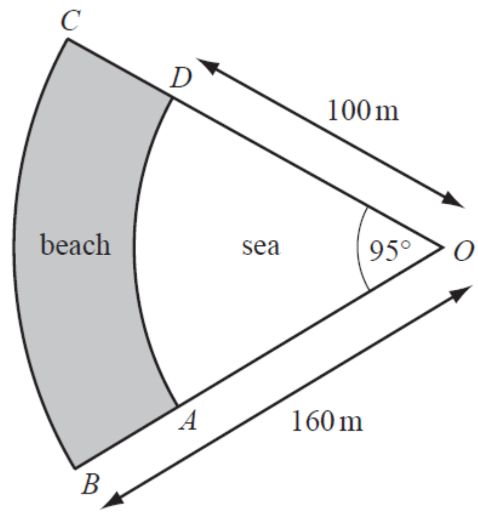
A = 4438.72 cm^2

The area of the spheres as a percentage of the area of the metal bar is:

$\frac{22.302 \text{ cm}^2}{4438.72 \text{ cm}^2}$ x 100 = 5.02 x 100

= **502 %**

The shaded area shows a beach.
AD and BC are circular arcs, centre O.
$OB = 160$ m, $OD = 100$ m and angle $AOD = 95°$.



(a) Calculate the area of the beach $ABCD$ in square metres. [3]

Area of a sector is:

$$A = \frac{1}{2} r^2 \frac{\theta\pi}{180}$$

Hence:

$$ABCD = \frac{1}{2}(160)^2 \frac{95\pi}{180} - \frac{1}{2}(100)^2 \frac{95\pi}{180}$$

$$= \frac{95\pi}{360} \times (160^2 - 100^2)$$

$$= 12933 \approx 12900$$

(b) The beach area is covered in sand to a depth of 1.8 m.

Calculate the volume of the sand in cubic metres. [1]

$$12900 \times 1.8$$

$$= 23279 \approx 23300$$

(c) Write both the following answers in standard form.

(i) Change your answer to part(b) into cubic millimetres. [1]

$23279 \times (1000)^3$

$= \mathbf{2.33 \times 10^{13}}$

(ii) Each grain of sand has a volume of 2 mm^3 correct to the nearest mm^3.

Calculate the maximum possible number of grains of sand on the beach. [2]

For maximum number of grains, we need minimal volume per grain, which is

$$1.5\ mm^3$$

Hence

$$2.33 \times 10^{13} \div 1.5$$

$$= \mathbf{1.55 \times 10^{13}}$$

[The surface area of a sphere of radius r is $4\pi r^2$ and the volume is $\frac{4}{3}\pi r^3$.]

(a) A solid metal sphere has a radius of 3.5 cm.

One cubic centimetre of the metal has a mass of 5.6 grams.

Calculate

(i) the surface area of the sphere, [2]

$$A_S = 4\pi(3.5)^2$$

$$= \mathbf{153.9}$$

(ii) the volume of the sphere, [2]

$$V_S = \frac{4}{3}\pi(3.5)^3$$

$$= \mathbf{179.6}$$

(iii) the mass of the sphere. [2]

$$179.6 \times 5.6$$

$$= \mathbf{1005.7}$$

(b)

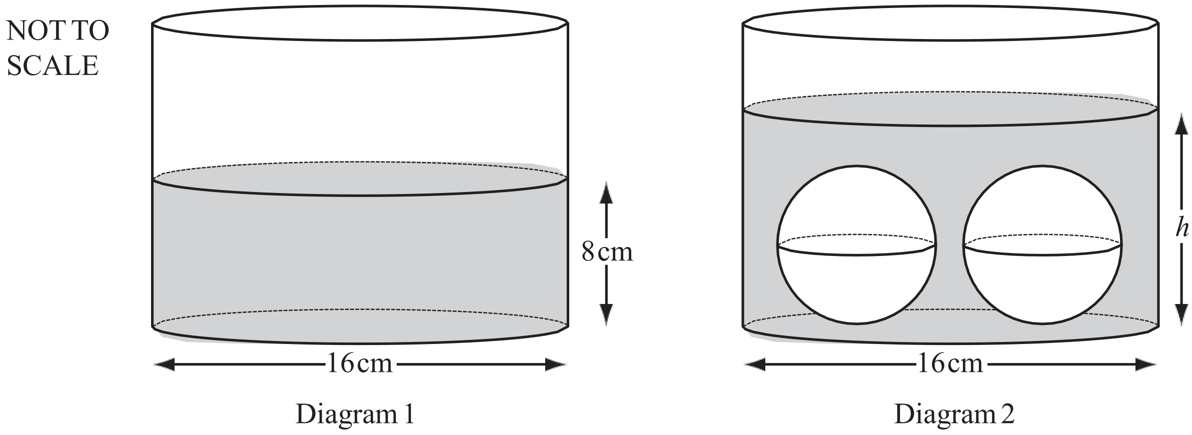


Diagram 1 shows a cylinder with a **diameter** of 16 cm.

It contains water to a depth of 8 cm.

Two spheres identical to the sphere in **part (a)** are placed in the water. This is shown in Diagram 2.

Calculate h, the new depth of water in the cylinder. [4]

Volume of water in cylinder is

$$V = \pi r^2 \times h$$

$$= 8^2\pi \times 8$$

$$= 1608.5$$

Two spheres are place in, both with volume of 179.6.

Hence, we now have

$$8^2\pi h - 2 \times 179.6 = 1608.5$$

$$\rightarrow 64\pi h = 1967.7$$

$$\rightarrow h = \frac{1967.7}{64\pi}$$

$$= \mathbf{9.79}$$

(c) A different metal sphere has a mass of 1 kilogram.

One cubic centimetre of this metal has a mass of 4.8 grams.

Calculate the radius of this sphere. [3]

The mass is

$$m = V \times \rho$$

Where ρ is the density, and V is the volume.

Hence

$$1000 = \frac{4}{3}\pi r^3 \times 4.8$$

$$\rightarrow r^3 = \frac{3}{4} \times \frac{1000}{4.8\pi}$$

$$= 49.736$$

$$\rightarrow \boldsymbol{r = 3.68}$$

Perimeters,Area and Volumes
Difficulty: Medium

Model Answers 5

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Perimeters,Area and Volumes
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 5

Time allowed: **89 minutes**

Score: **/77**

Percentage: **/100**

Grade Boundaries:

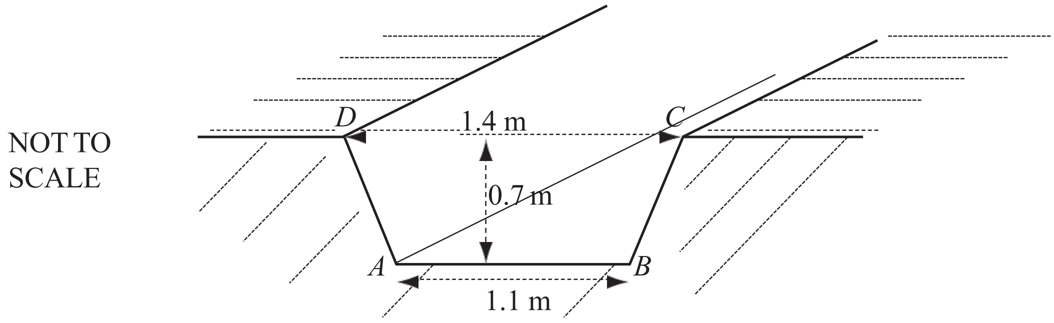
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Workmen dig a trench in level ground.



(a) The cross-section of the trench is a trapezium *ABCD* with parallel sides of length 1.1 m and 1.4 m and a vertical height of 0.7 m.
Calculate the area of the trapezium. [2]

The formula for the area of a trapezium is:

$A = \frac{(a+b)h}{2}$

Where a and b represent the 2 bases of the trapezium

and h is the height.

In our case:

$A = \frac{(1.4+1.1)\text{x } 0.7}{2}$

$A = 0.875\ m^2$

(b) The trench is 500 m long.
Calculate the volume of soil removed. [2]

The volume is the area of the trapezium multiplied by the length.

$V = 0.875\ m^2 \text{ x } 500\ m$

$V = 437.5\ m^3$

(c) One cubic metre of soil has a mass of 4.8 tonnes.
Calculate the mass of soil removed, giving your answer in tonnes and in standard form. [2]

If 1 m^3 weights 4.8 tonnes, then the volume taken out will weight:

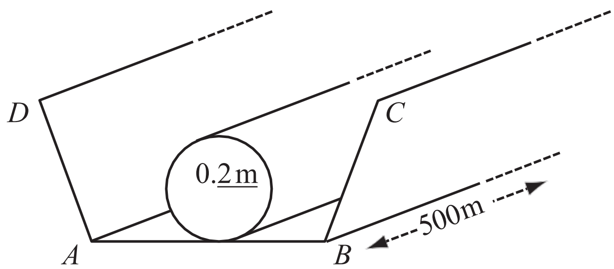
437.5 m^3 x 4.8 tonnes/m^3 = 2100 tonnes

A number in standard form is: $a \times 10^n$ where n is an integer and $0 < a < 10$.

In our case, 2100 tonnes = 2.1×10^3

(d) Change your answer to **part (c)** into grams. [1]

NOT TO SCALE



1 tonne = 1 000 000 grams = 10^6 grams

2.1×10^3 tonnes = $2.1 \times 10^3 \times 10^6$ grams

2.1×10^3 tonnes = 2.1×10^9 grams

(e) The workmen put a cylindrical pipe, radius 0.2 m and length 500 m, along the bottom of the trench, as shown in the diagram.
Calculate the volume of the cylindrical pipe. [2]

The formula for the volume of a cylinder is:

V = $\pi r^2 h$

h = 500 m and r = 0.2 m

V = = $\pi \ x \ 0.2^2 x \ 500 \ m$

V = 62.83 m^3

(f) The trench is then refilled with soil.
Calculate the volume of soil put back into the trench as a percentage of the original amount of soil removed. [3]

The volume of soil added is represented by the total volume taken out minus the volume of the cylinder.

V of soil added = 437.5 m^3 - 62.83 m^3

V of soil added = 374.67 m^3

We represent this as a percentage of the initial amount of soil removed.

374.67 m^3 = 437.5 m^3 x $\frac{x}{100}$

Where x is the percentage.

x = 85.63%

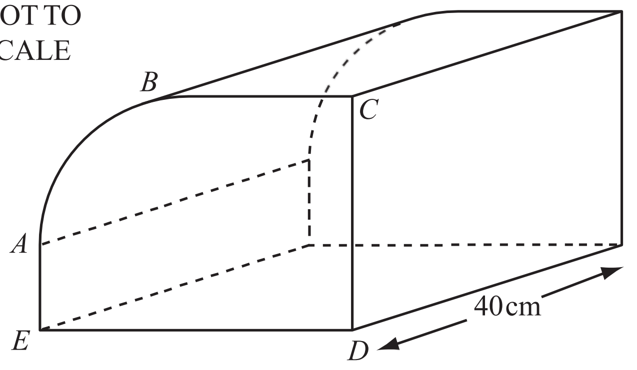


Diagram 1

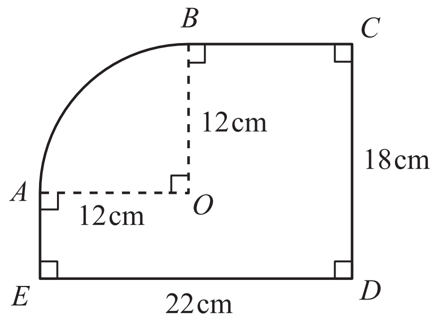


Diagram 2

Diagram 1 shows a closed box. The box is a prism of length 40 cm.
The cross-section of the box is shown in Diagram 2, with all the right-angles marked.
AB is an arc of a circle, centre *O*, radius 12 cm.
ED = 22 cm and *DC* = 18 cm.

Calculate

(a) the perimeter of the cross-section, [3]

The perimeter = arc length + BC + CD + DE + EA

Arc length = $2\pi r$ x $\frac{arc\ angle}{360°}$

Arc length = 24π x $\frac{90°}{360°}$

Arc length = 18.8 cm

Perimeter = 18.8 cm + 22 + 18 + (18 – 12) + (22 – 12)

Perimeter = 74.8 cm

(b) the area of the cross-section, [3]

The area of the cross section will be the sector area plus the areas of the 2

small rectangles which can be formed.

Sector area = πr^2 x $\frac{sector\ angle}{360°}$

Sector area = $\pi 12^2$ x $\frac{90°}{360°}$

Sector area = 113.09 cm^2

Cross section area = 113.09 cm^2 + 6 cm x 12 cm + 10 cm x 12 cm

Cross section area = 365.09 cm^2

(c) the volume of the box, [1]

Volume = area cross section x length

Volume = 365.09 cm^2 x 40 cm

Volume = 14603.6 cm^3

(d) the **total** surface area of the box. [4]

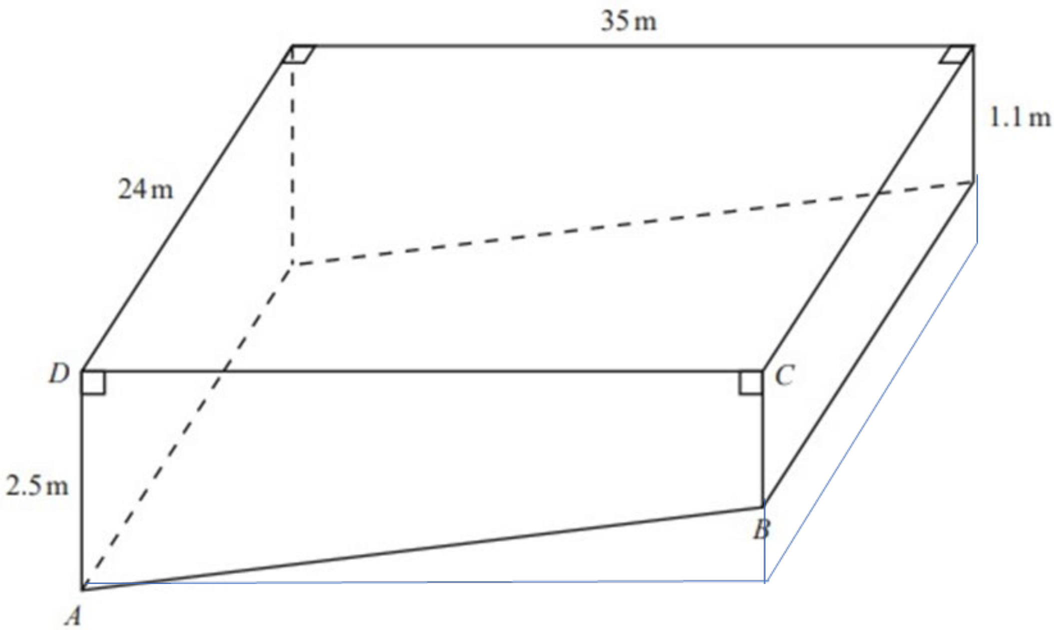
The total surface area is: 2 x cross section area + area of rectangle (40 x 18) + area of rectangle (6 x 40) + area of rectangle (10 x 40) + area of base rectangle (22 x 40) + curved area

Curved surface area = 40 cm x arc length

Curved surface area = 40 cm x 18.8 cm = 752 cm^2

Total surface area = 730 cm^2 + 720 cm^2 + 240 cm^2 + 400 cm^2 + 752 cm^2 + 880 cm^2

Total surface area = 3722 cm^2



The diagram shows a swimming pool of length 35 m and width 24 m.
A cross-section of the pool, *ABCD*, is a trapezium with *AD* = 2.5 m and *BC* = 1.1 m.

(a) Calculate

(i) the area of the trapezium *ABCD*, [2]

The formula for the area of a trapezium is:

Area = $\frac{1}{2}$ x h x (a + b)

Where h is the height of the trapezium and a and b are the 2 bases.

In our case, h = DC, a = CB and b = AD.

The area of the trapezium ABCD is:

A = $\frac{1}{2}$ x 35 m x (2.5 m + 1.1 m)

A = 17.5 m x 3.6 m

A = 63 m^2

(ii) the volume of the pool, [2]

The volume of the shape describing the swimming pool is the area of the base, in this case the trapezium ABCD, multiplied by the height of the shape, 24 m.

From a) i), we know that the area of the base of the shape is 63 m^2.

Volume = 24 m x 63 m^2

Volume = 1512 m^3

(iii) the number of litres of water in the pool, when it is full. [1]

1 dm^3 = 1 L

The volume of the pool is 1512 m^3.

1512 m^3 = 1512000 dm^3

Therefore, the pool can fit 1512000 L.

(b) AB = 35.03 m correct to 2 decimal places.
The sloping rectangular floor of the pool is painted.
It costs \$2.25 to paint one square metre.

(i) Calculate the cost of painting the floor of the pool. [2]

The area of the rectangle describing the floor of the pool is:

A = 35.03 m x 24 m

A = 840.72 m^2

For the cost of \$2.25 per square meter, the cost of painting the floor of the pool is:

\$2.25/m^2 x 840.72 m^2 = \$1891.62

(ii) Write your answer to **part (b)(i)** correct to the nearest hundred dollars. [1]

The cost correct to the nearest hundred is: 1900

The digit in the hundredth place in the number 1891.62 is 9. This digit is greater than 5, so we increase the preceding digit by 1 and replace the rest of the digits with 0.

(c) (i) Calculate the volume of a cylinder, radius 12.5 cm and height 14 cm. [2]

The formula for the volume of a cylinder is:

$V = \pi r^2 h$

where r is the radius of the base and h is the height of the cylinder

$V = \pi \times 12.5^2 \text{ cm}^2 \times 14 \text{ cm}$

$V = 6872.23 \text{ cm}^3$

(ii) When the pool is emptied, the water flows through a cylindrical pipe of radius 12.5 cm. The water flows along this pipe at a rate of 14 centimetres per second.
Calculate the time taken to empty the pool.
Give your answer in days and hours, correct to the nearest hour. [4]

The volume of water in the pool is 1512 m^3 and the volume of a cylinder is 6872.23 cm^3.

We convert m^3 to cm^3 to have both values with the same unit.

$1512 \text{ m}^3 = 1512000000 \text{ cm}^3$

The volume of a cylinder of water is emptied from the pool per second. We divide the total amount of water in the pool by the volume of a cylinder to work out the number of seconds necessary to empty the pool.

$\frac{1512000000\ cm^3}{6872.23\ cm^3}$ = 220015.919 s

We divide the time in seconds by 3600 to work out the number of hours.

$\frac{220015.919\ s}{3600\ s/hour}$ = 61.11 hours

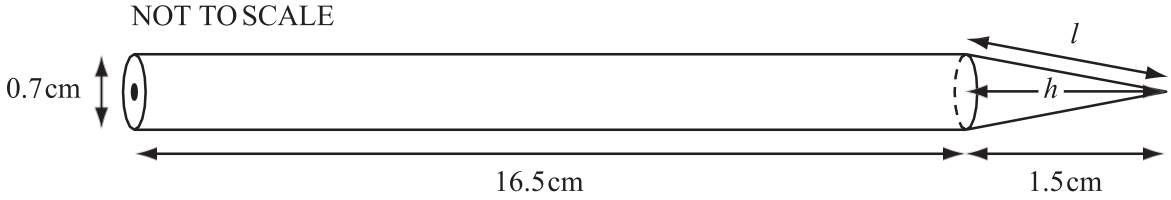
We divide this time by 24 to work out the time in days and hours.

$\frac{61.11\ hours}{24\ hours/day}$ = 2.54 days

2 days represent 48 hours.

61.11 hours – 48 hours = 13.11 hours

The total time taken to empty the pool is 2 days and 13 hours.



The diagram shows a pencil of length 18 cm.
It is made from a cylinder and a cone.
The cylinder has **diameter** 0.7 cm and length 16.5 cm.
The cone has **diameter** 0.7 cm and length 1.5 cm.

(a) Calculate the volume of the pencil.

[The volume, V, of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.] [3]

The volume of the pencil is the sum of the volume of the cone and the volume of the cylinder.

The formula for the volume of a cone is:

$V = \frac{1}{3}\pi r^2 h$

The cone has the diameter d = 0.7 cm.

Therefore, the radius is r = 0.35 cm.

The height of the cone is h = 1.5 cm.

$V = \frac{1}{3}\pi \times 0.35^2 \text{ cm}^2 \times 1.5 \text{ cm}$

$V = 0.192 \text{ cm}^3$

The formula for the volume of a cone is:

$V = \pi r^2 h$

The base of the cylinder has the diameter d = 0.7 cm.

Therefore, the radius is r = 0.35 cm.

The height of the cylinder is h = 16.5 cm.

$V = \pi \times 0.35^2 \text{ cm}^2 \times 16.5 \text{ cm}$

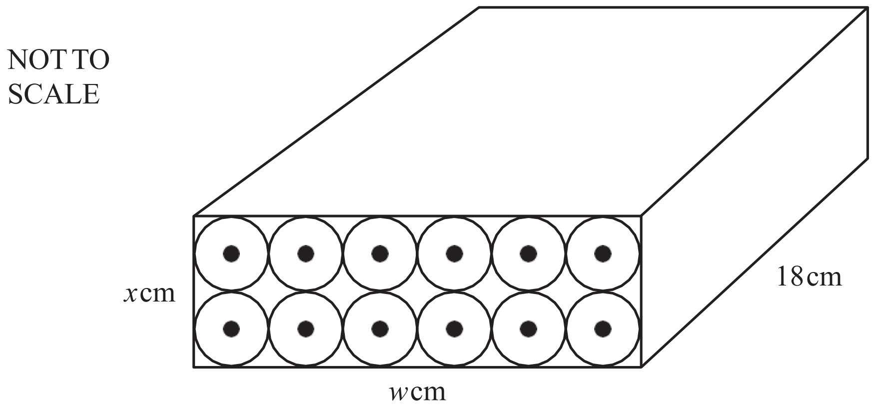
$V = 6.349 \text{ cm}^3$

The volume of the pencil is:

$V = 6.349\ cm^3 + 0.192\ cm^3$

$V = 6.541\ cm^3$

(b)



Twelve of these pencils just fit into a rectangular box of length 18 cm, width w cm and height x cm.
The pencils are in 2 rows of 6 as shown in the diagram.

(i) Write down the values of w and x. [2]

The size x is twice the diameter of the base of a pencil.

Therefore, x = 2 x 0.7 cm

x = 1.4 cm

The size x is six times the diameter of the base of a

pencil.

Therefore, x = 6 x 0.7 cm

x = 4.2 cm

(ii) Calculate the volume of the box. [2]

The volume of the rectangular box is:

V = length x width x height

$V = 4.2 \text{ cm} \times 1.4 \text{ cm} \times 18 \text{ cm}$

$V = 105.84 \text{ cm}^3$

(iii) Calculate the percentage of the volume of the box occupied by the pencils. [2]

The box has 12 pencils inside.

The total volume occupied by the pencils in this case is 12 multiplied by the volume of one pencil. We worked out the volume of one pencil at a).

$V = 12 \times 6.541 \text{ cm}^3$

$V = 78.492 \text{ cm}^3$

We need to express this volume as a percentage of the volume of the whole box. We note this percentage with the unknown x.

$\frac{x}{100} \times 105.84 \text{ cm}^3 = 78.492 \text{ cm}^3$

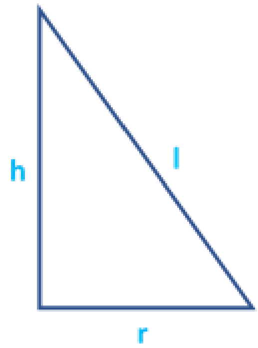
$x = 74.16\%$

(c) Showing all your working, calculate

(i) the slant height, l, of the cone, [2]

The height of the cone, h, is perpendicular on the base of the plane and therefore perpendicular on the radius of the base.

In the right-angled triangle showed below, we can use Pythagoras' Theorem to work out the length of the hypothenuse, l.



$r^2 + h^2 = l^2$

$l^2 = 0.35^2\ cm^2 + 1.5^2\ cm^2$

$l^2 = 2.25\ cm^2$

$l = 1.54$ cm

(ii) the **total** surface area of **one** pencil, giving your answer correct to 3 significant figures.
[The curved surface area, A, of a cone of radius r and **slant** height l is given by $A = \pi r l$.] [6]

$r^2 + h^2 = l^2$

$l^2 = 0.35^2\ cm^2 + 1.5^2\ cm^2$

$l^2 = 2.25\ cm^2$

$l = 1.54$ cm

For the curved surface area of a cylinder:

$A = 2\pi rh$

$A = 2\pi \times 0.35\ cm \times 16.5\ cm$

$A = 36.285\ cm^2$

The formula for the area of a circle is:

$A = \pi r^2$

$A = \pi\ 0.35^2\ \text{cm}^2$

$A = 0.385\ \text{cm}^2$

The surface area of a pencil is:

$A = 0.385\ \text{cm}^2 + 36.285\ \text{cm}^2 + 1.69\ \text{cm}^2$

$A = 38.3\ \text{cm}^2$, correct to 3 significant figures.

Water flows through a pipe into an empty cylindrical tank.
The tank has a radius of 40 cm and a height of 110 cm.

(a) Calculate the volume of the tank. [2]

The formula for the volume of a cylinder is:

$V = \pi r^2 h$

$V = \pi \times 40^2 \text{ cm}^2 \times 110 \text{ cm}$

$V = 552920.3 \text{ cm}^3$

(b) The pipe has a cross-sectional area of 1.6 cm^2.
The water comes out of the pipe at a speed of 14 cm/s.
How long does it take to fill the tank?
Give your answer in hours and minutes, correct to the nearest minute. [4]

The total volume of water in the tank is 552920.3 cm^3.

We firstly multiply the cross-sectional area of the pipe by the speed of the water to work out the amount of water coming out through it in a second.

$14 \text{ cm/s} \times 1.6 \text{ cm}^2 = 22.4 \text{ cm}^3/\text{s}$

This volume is divided by this value to work out the total time taken.

$\text{Time} = \frac{552920.3 \, cm^3}{22.4 \, cm^3/s}$

Time = 24683.9 s

We convert this time into hours.

$\text{Time} = \frac{24683.9 \, s}{3600 \, s/hour}$

Time = 6.85 hours

0.85 hours represent 0.85 hours x 60 minutes/hour = 51 minutes

The answer is: Time = 6 hours and 51 minutes

(c) All the water from the tank is added to a pond which has a surface area of 70 m^2.
Work out the increase in the depth of water in the pond.
Give your answer in millimetres, correct to the nearest millimetre. [4]

We convert the volume of water in cm^3 in m^3 to have all values in the same unit.

552920.3 cm^3 = 552.92 m^3

The volume represents the surface area multiplies by the height.

We divide this volume by the surface area of the pond to work out the depth of the water.

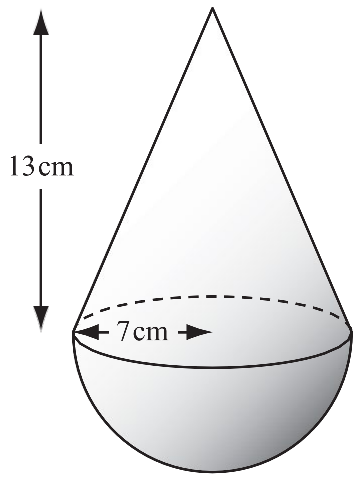
Depth = $\frac{552.92 \text{ m}^3}{70 \text{ m}^2}$

Depth = 7.89 m

We convert this value in millimetres.

7.89 m = 7890 mm

Question 6



NOT TO SCALE

The diagram shows a solid made up of a hemisphere and a cone.
The base radius of the cone and the radius of the hemisphere are each 7 cm.
The height of the cone is 13 cm.

[The volume of a hemisphere of radius r is given by $V = \frac{2}{3}\pi r^3$.]

[The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.]

(a) (i) Calculate the total volume of the solid. [2]

$$Volume = Volume\ of\ Hemisphere + Volume\ of\ Cone$$

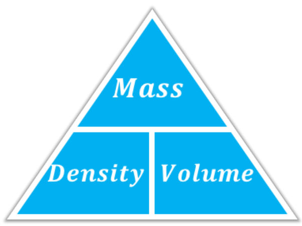
$$Volume = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$Volume = \frac{2}{3} \times \pi \times 7^3 + \frac{1}{3} \times \pi \times 7^2 \times 13$$

$$\boldsymbol{Volume} = \mathbf{1385cm^3}$$ (to nearest whole number)

(ii) The solid is made of wood and 1 cm^3 of this wood has a mass of 0.94 g.
Calculate the mass of the solid, in kilograms, correct to 1 decimal place. [3]

From the blue triangle on the right: $Mass = Density \times Volume$



$$Mass = 0.94 \times 1385 \textbf{ grams}$$

Divide by 1000 to get kilograms:

$$\boldsymbol{Mass} = \mathbf{1.3\ kg}$$

(b) Calculate the curved surface area of the cone.
[The curved surface area of a cone of radius r and sloping edge l is given by $A = \pi rl$.] [3]

$$Curved\ Surface\ Area = \pi rl$$

$$Curved\ Surface\ Area = \pi \times 7 \times \sqrt{7^2 + 13^2}$$

$$\boldsymbol{Curved\ Surface\ Area} = \mathbf{325cm^2}$$

l is found using Pythagoras Theorem:

$$l^2 = r^2 + h^2$$

$$l = \sqrt{7^2 + 13^2}$$

(c) The cost of covering all the solid with gold plate is $411.58.
Calculate the cost of this gold plate per square centimetre.

[The curved surface area of a **hemisphere** is given by $A = 2\pi r^2$.] [5]

$$Total\ Surface\ Area = \pi rl + 2\pi r^2$$

$$Total\ Surface\ Area = 325 + 2 \times \pi \times 7^2$$

$$Total\ Surface\ Area = 623.57\ldots\ \text{cm}^2$$

A small amount of thought about the phrase "cost per square centimetre" gives:

$$Cost\ per\ cm^2 = \frac{Total\ Cost}{Total\ Surface\ Area}$$

$$Cost\ per\ cm^2 = \frac{411.58}{623.57\ \ldots}$$

$\boldsymbol{Cost\ per\ cm^2 = \$0.65}$ **or 65 cents** (to the nearest cent)

Perimeters, Area and Volumes
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Perimeters, Area and Volumes
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **93 minutes**

Score: **/81**

Percentage: **/100**

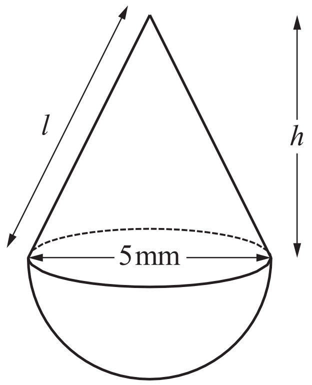
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



NOT TO SCALE

The diagram shows a solid made from a hemisphere and a cone.
The base diameter of the cone and the diameter of the hemisphere are each 5 mm.

(a) The total surface area of the solid is $\frac{115\mathrm{r}}{4}$ mm^2.

Show that the slant height, l, is 6.5 mm.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \mathrm{r}rl$.]
[The surface area, A, of a sphere with radius r is $A = 4\mathrm{r}r^2$.] [4]

Surface area of the hemisphere is

$$A_{hs} = 2\pi 2.5^2$$

$$= 12.5\pi$$

Surface area of the cone part is

$$2.5\pi l$$

Hence

$$12.5\pi + 2.5\pi l = \frac{115}{4}\pi$$

$$\rightarrow 2.5l = \frac{65}{4}$$

$$\rightarrow \boldsymbol{l = 6.5}$$

(b) Calculate the height, h, of the cone. [3]

Using Pythagoras' Theorem

$$2.5^2 + h^2 = 6.5^2$$

$$\rightarrow h^2 = 6.5^2 - 2.5^2$$

$$= 36$$

$$\rightarrow \boldsymbol{h = 6}$$

(c) Calculate the volume of the solid.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [4]

$$V_{cone} = \frac{1}{3}\pi(2.5)^2 \times 6$$

$$= \frac{25}{2}\pi$$

$$V_{hsphere} = \frac{1}{2} \times \frac{4}{3}\pi(2.5)^3$$

$$= \frac{125}{12}\pi$$

Hence

$$V = \left(\frac{125}{12} + \frac{25}{2}\right)\pi$$

$$= \frac{275}{12}\pi$$

$$= \mathbf{72}$$

(d) The solid is made from gold.

1 **cubic centimetre** of gold has a mass of 19.3 grams.
The value of 1 gram of gold is \$38.62.

Calculate the value of the gold used to make the solid. [3]

The volume is in mm^3, so we have

$$72\ mm^3 = 0.072\ cm^3$$

The mass is

$$0.072 \times 19.3$$

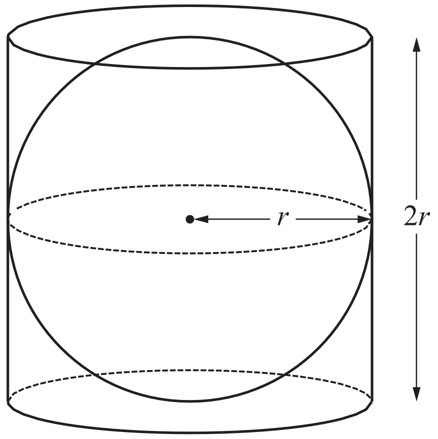
$$= 1.3896\ g$$

And the value is

$$1.3896 \times 38.62$$

$$= \mathbf{53.67}$$

(a)



NOT TO SCALE

A sphere of radius r is inside a closed cylinder of radius r and height $2r$.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\mathrm{r}r^3$.]

(i) When $r = 8$ cm, calculate the volume inside the cylinder which is **not** occupied by the sphere. [3]

The empty space is found by subtracting the volume of the sphere from the volume of the cylinder:

$$\text{Empty Space} = \pi r^2 h - \frac{4}{3}\pi r^3$$

$$= \pi \times 8^2 \times 2 \times 8 - \frac{4}{3} \times \pi \times 8^3$$

$$= \frac{1024}{3}\pi \text{ cm}^2$$ (this answer is an exact value which is fine)

$$= \mathbf{1072\ cm^2}$$ (to the nearest whole number)

(ii) Find r when the volume inside the cylinder **not** occupied by the sphere is 36cm^3. [3]

Using the formula for empty space in (i) and $h = 2r$:

$$\text{Empty Space} = \pi r^2 \times 2r - \frac{4}{3}\pi r^3 = 36$$

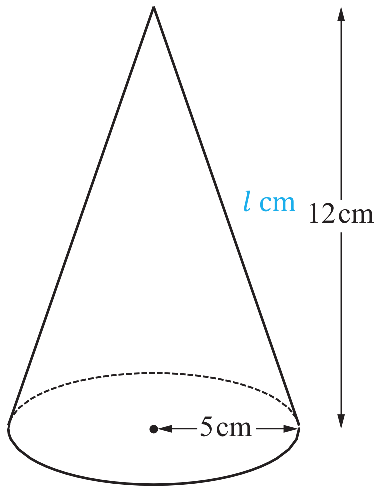
$$\frac{2}{3}\pi r^3 = 36$$ since $\left(2 - \frac{4}{3}\right) = \frac{2}{3}$

Divide by $\frac{2}{3}\pi$ $\quad r^3 = \frac{3\times36}{2\pi}$

Cube root $\quad r = \sqrt[3]{\frac{3\times36}{2\pi}}$

$\boldsymbol{r} = \mathbf{2.58\ cm}$ (to 3 significant figures)

(b)



NOT TO SCALE

The diagram shows a solid cone with radius 5 cm and perpendicular height 12cm.

(i) The **total** surface area is painted at a cost of \$0.015 per cm^2.

Calculate the cost of painting the cone.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.] [4]

Cost = 0.015 × Total Surface Area

= 0.015 × (Curved Surface Area + Base)

$$\text{Cost} = 0.015 \times (\pi rl + \pi r^2)$$

We know r but need to find l which we do using Pythagoras Theorem:

$$l^2 = 5^2 + 12^2$$

$$l = \sqrt{169}$$

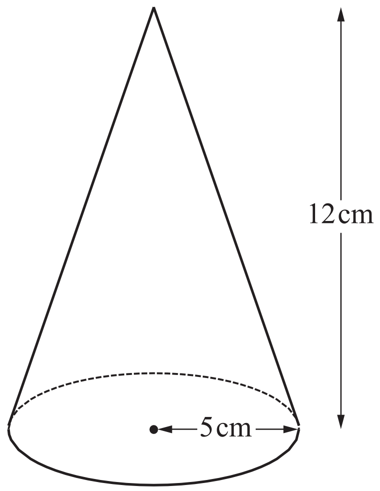
$$l = 13 \text{ cm}$$

Using the formula for Cost from above:

$$\text{Cost} = 0.015 \times (\pi \times 5 \times 13 + \pi \times 5^2)$$

Cost = \$4.24 (to nearest cent)

(b)



(ii) The cone is made of metal and is melted down and made into smaller solid cones with radius 1.25 cm and perpendicular height 3 cm.
Calculate the number of smaller cones that can be made. [3]

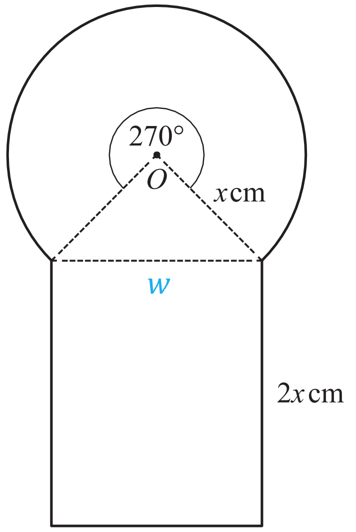
Volume of a Cone $= \pi r^2 h$

To find the Number of Smaller Cones divide Volume of the Large Cone by the

Volume of one Smaller Cone:

Number of Smaller Cones $= \dfrac{\pi \times 5^2 \times 12}{\pi \times 1.25^2 \times 3}$

Number of Smaller Cones = 64



NOT TO SCALE

The diagram shows a sector of a circle, a triangle and a rectangle.
The sector has centre O, radius x cm and angle $270°$.
The rectangle has length $2x$ cm.

The total area of the shape is kx^2 cm^2 .

(a) Find the value of k. [5]

Area is made up of a rectangle, a right-angled triangle and ¾ of a circle.

Find the width, w, of the rectangle using Pythagoras Theorem in the right-angled triangle.

$$w^2 = x^2 + x^2$$

$$w = \sqrt{2x^2} = \sqrt{2}x$$

Now use: Total Area = Rectangle + Right-angled triangle + ¾ of a circle.

$$\text{Total Area} = w \times l + \tfrac{1}{2} \times b \times h + \tfrac{3}{4} \times \pi \times r^2$$

$$\text{Total Area} = \sqrt{2}x \times 2x + \tfrac{1}{2} \times x \times x + \tfrac{3}{4} \times \pi \times x^2$$

$$\text{Total Area} = \left(2\sqrt{2} + \tfrac{1}{2} + \tfrac{3}{4}\pi\right)x^2$$

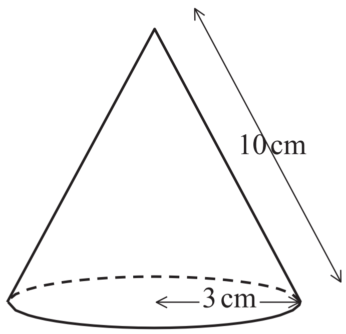
$$\boldsymbol{k = \left(2\sqrt{2} + \tfrac{1}{2} + \tfrac{3}{4}\pi\right) = 5.68}$$

(b) Find the value of x when the total area is 110 cm^2. [2]

$$\text{Total Area} = 5.68x^2 = 110$$

Dividing by 5.68 and square rooting gives $x = \sqrt{\frac{110}{5.68}}$

which gives $\boldsymbol{x = 4.40}$



NOT TO SCALE

The diagram shows a hollow cone with radius 3 cm and slant height 10 cm.

(a) (i) Calculate the curved surface area of the cone.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.] [2]

$$A = \pi(3)(10)$$

$$= 30\pi$$

$$= \mathbf{94.2}$$

(ii) Calculate the perpendicular height of the cone. [3]

The height can be found using Pythagoras'

$$h^2 + 3^2 = 10^2$$

$$\rightarrow h^2 = 100 - 9$$

$$= 91$$

$$\rightarrow \boldsymbol{h} = \mathbf{9.54}$$

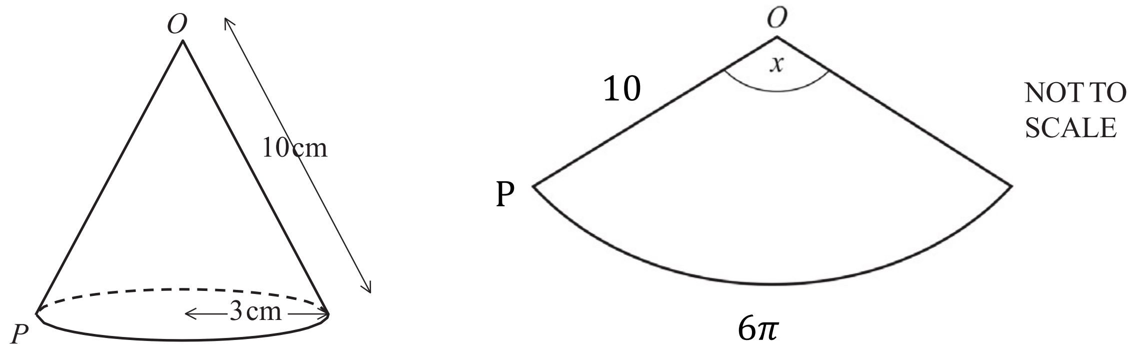
(iii) Calculate the volume of the cone.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [2]

$$V = \frac{1}{3}\pi(3)^2(9.54)$$

$$= \mathbf{89.9}$$

(b)



The cone is cut along the line *OP* and is opened out into a sector as shown in the diagram.

Calculate the sector angle *x*. [4]

The perimeter of the circular base of the cone is now the length of the arc.

Perimeter of a circle is

$$2\pi r = 2\pi \times 3$$

$$= 6\pi$$

The length of an arc is

$$l = r\theta$$

If θ is measured in radians.

Hence

$$6\pi = 10x$$

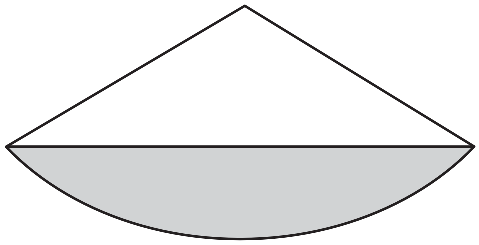
$$\rightarrow x = 0.6\pi \ radians$$

Converting to degrees

$$x = 0.6\pi \times \frac{180}{\pi}$$

$$= \mathbf{108}$$

(c)



The diagram shows the same sector as in **part (b)**.

Calculate the area of the shaded segment. [4]

The shaded area is the area of the sector minus the area of the unshaded triangle.

The area of the sector is

$$A = \frac{\pi}{360} x r^2$$

$$= 30\pi$$

$$= 94.248$$

The area of the triangle is

$$A_t = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 10^2 \sin 108$$

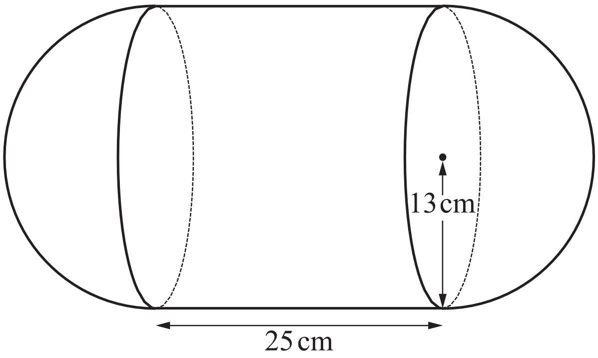
$$= 47.553$$

Hence the area of the segment is

$$94.248 - 47.553$$

$$= \mathbf{46.7}$$

(a)



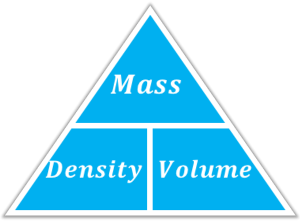
NOT TO SCALE

The diagram shows a solid made up of a cylinder and two hemispheres.
The radius of the cylinder and the hemispheres is 13 cm.
The length of the cylinder is 25 cm.

(i) One cubic centimetre of the solid has a mass of 2.3 g.

Calculate the mass of the solid.
Give your answer in kilograms. [The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

[4]



Two hemispheres make one sphere so:

$$Total\ Volume = Volume\ of\ Cylinder + Volume\ of\ Sphere$$

$$Total\ Volume = \pi r^2 h + \tfrac{4}{3}\pi r^3$$

Using the blue triangle on the right:

$$Mass = Density \times Volume$$

$$Mass = 2.3 \times \left(\pi \times 13^2 \times 25 + \tfrac{4}{3} \times \pi \times 13^3\right)$$

$$Mass = 51694.802\ \ldots\ \textbf{grams}$$

Dividing by 1000 gives:

$$\boldsymbol{Mass = 51.7}\ \textbf{kg}$$

(ii) The surface of the solid is painted at a cost of \$4.70 per square metre.

Calculate the cost of painting the solid.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.] [4]

Two hemispheres make one sphere so:

$$Total\ Surface\ Area = Curved\ Surface\ Area\ of\ Cylinder + Surface\ Area\ of\ Sphere$$

$$Total\ Surface\ Area = 2\pi rh + 4\pi r^2$$

A small amount of thought tells us:

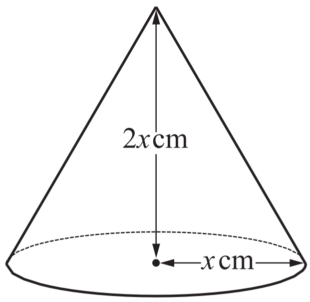
$$Total\ Cost = Total\ Surface\ Area \times Cost\ per\ Square\ Metre$$

(NB we must work in **Metres**):

$$Total\ Cost = (2 \times \pi \times 0.13 \times 0.25 + 4 \times \pi \times 0.13^2) \times 4.70$$

$$\boldsymbol{Total\ Cost} = \$\mathbf{1.96}$$

(b)



NOT TO SCALE

The cone in the diagram has radius x cm and height $2x$ cm.
The volume of the cone is 500 cm^3.

Find the value of x.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [3]

$$Volume = \frac{1}{3}\pi r^2 h$$

$$500 = \frac{1}{3} \times \pi \times x^2 \times 2x$$

$$\frac{3}{2\pi} \times 500 = x^3$$

$$x = \sqrt[3]{\frac{1500}{2\pi}}$$

$$\boldsymbol{x = 6.20\ \text{cm}}\ \text{(to 3 sf)}$$

(c) Two mathematically similar solids have volumes of 180 cm^3 and 360 cm^3.
The surface area of the smaller solid is 180 cm^2.

Calculate the surface area of the larger solid. [3]

$$Volume\ Factor = \frac{360}{180} = 2$$

$$Scale\ Factor = \sqrt[3]{Volume\ Factor} = \sqrt[3]{2}$$

$$Area\ Factor = Scale\ Factor^2 = \left(\sqrt[3]{2}\right)^2$$

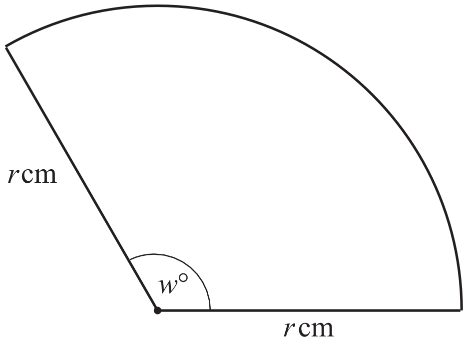
$$Larger\ Area = Smaller\ Area \times Scale\ Factor$$

$$Larger\ Area = 180 \times \left(\sqrt[3]{2}\right)^2$$

$$\boldsymbol{Larger\ Area = 286\text{cm}^2}\ \text{(to 3 sf)}$$

$$\boxed{\begin{array}{l}\text{For similar shapes (solids):} \\ Area\ Factor = Scale\ Factor^2 \\ Volume\ Factor = Scale\ Factor^3\end{array}}$$

(a)



NOT TO
SCALE

The area of this sector is r^2 square centimetres.

Find the value of w.

For a Sector of a Circle:
$Sector\ Area = \frac{\theta}{360} \times \pi r^2$
$Arc\ Length = \frac{\theta}{360} \times 2\pi r$

[3]

$$Sector\ Area = \frac{\theta}{360} \times \pi r^2$$

$$r^2 = \frac{w}{360} \times \pi r^2$$

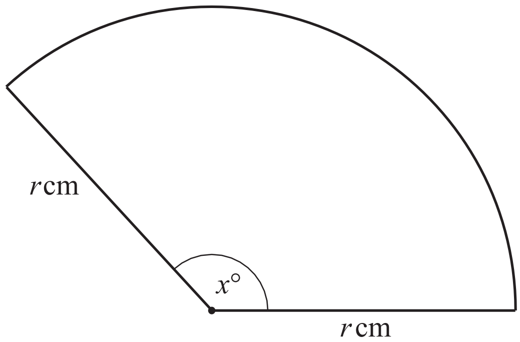
Cancel the r^2s:

$$1 = \frac{w}{360} \times \pi$$

Multiply by $\frac{360}{\pi}$:

$$\boldsymbol{w} = \frac{360}{\pi} = \mathbf{115°} \text{ (to the nearest degree)}$$

(b)



NOT TO
SCALE

The perimeter of this sector is $2r + \frac{7\pi r}{10}$ centimetres.

Find the value of x.

[3]

$$Perimeter = 2r + \frac{\theta}{360} \times 2\pi r$$

$$2r + \frac{7\pi r}{10} = 2r + \frac{x}{360} \times 2\pi r$$

Subtract the $2r$s:

$$\frac{7\pi r}{10} = \frac{x}{360} \times 2\pi r$$

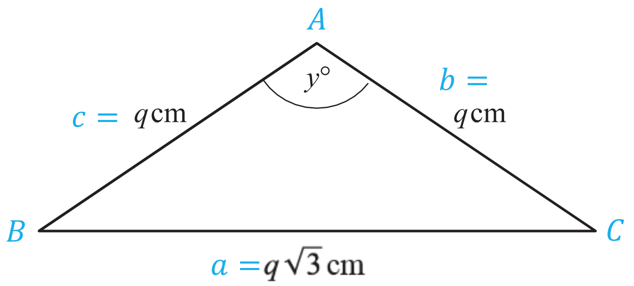
Cancel the πrs:

$$\frac{7}{10} = \frac{x}{360} \times 2$$

Multiply by $\frac{360}{2}$:

$$\boldsymbol{x} = \mathbf{126°}$$

(c)



NOT TO SCALE

The perimeter of the isosceles triangle is $2q + q\sqrt{3}$ centimetres.

Find the value of y. [4]

Using the Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\left(q\sqrt{3}\right)^2 = q^2 + q^2 - 2 \times q \times q \times \cos y$$

$$3q^2 = 2q^2 - 2q^2\cos A$$

Cancel the q^2s:

$$3 = 2 - 2\cos A$$

Subtract 2:

$$3 = -2\cos A$$

Divide by -2:

$$\cos A = -\tfrac{3}{2}$$

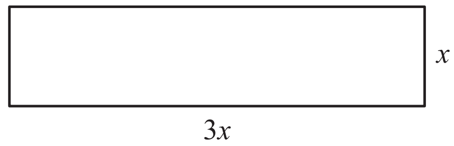
$$\boldsymbol{A} = \cos^{-1}\left(-\tfrac{3}{2}\right) = \mathbf{120°}$$

Question 7

The **perimeter** of each of the three shapes is 60cm.

Find x in each part.

(a) Rectangle



Rectangle:
$Perimeter = 2 \times (l + w)$

[2]

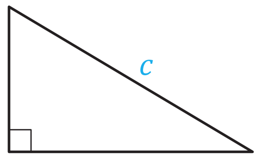
$$60 = 2 \times (3x + x)$$

$$60 = 8x$$

Divide by 8:

$$\boldsymbol{x = 7.5}\textbf{cm}$$

(b) Triangle $a = 3x$



$b = 4x$

Right-angled Triangle:
Pythagoras Theorem
$a^2 + b^2 = c^2$

Using Pythagoras: $c^2 = (3x)^2 + (4x)^2 = 25x^2$ so $c = 5x$

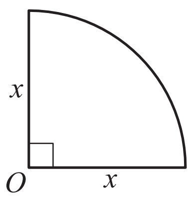
[3]

$$Perimeter = 3x + 4x + 5x = 60$$

$$12x = 60$$

$$\boldsymbol{x = 5}\textbf{cm}$$

(c) Sector



Sector of a circle (angle θ):
$Arc\ Length = \frac{\theta}{360} \times 2\pi r$

In this sector $\theta = 90°$ so $Arc\ Length = \frac{90}{360} \times 2 \times \pi \times x = \frac{1}{2}\pi x$

[3]

$$Perimeter = 2x + \tfrac{1}{2}\pi x = 60$$

Factorise:

$$x\left(2 + \tfrac{1}{2}\pi\right) = 60$$

Divide by $\left(2 + \frac{1}{2}\pi\right)$:

$$x = \frac{60}{2 + \frac{1}{2}\pi}$$

$\boldsymbol{x = 16.8}$**cm** (to 3sf)

Perimeters,Area and Volumes
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Perimeters,Area and Volumes
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **92 minutes**

Score: **/80**

Percentage: **/100**

Grade Boundaries:

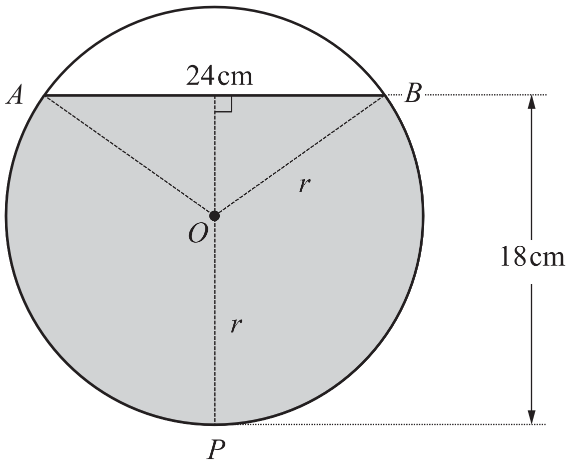
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Question 1

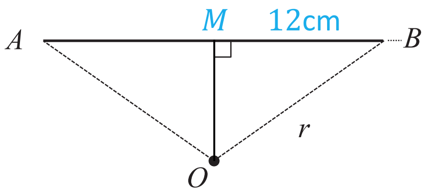


NOT TO SCALE

Right-angled Triangle:
Pythagoras Theorem
$a^2 + b^2 = c^2$

The diagram shows the cross section of a cylinder, centre O, radius r, lying on its side.
The cylinder contains water to a depth of 18 cm.
The width, AB, of the surface of the water is 24 cm.

(a) Use an algebraic method to show that $r = 13$ cm. [4]



Look at the triangle AOB. The vertical line through P and O meets AB at M, the midpoint of AB.

So $MB = 12$cm. From the main diagram $PM = 18$cm and $PO = r$cm so $OM = (18 - r)$cm.

Using Pythagoras Theorem on the triangle OMB:

$$OM^2 + MB^2 = OB^2$$

$$(18 - r)^2 + 12^2 = r^2$$

$$324 - 36r + r^2 + 144 = r^2$$

Cancel the r^2 and simplify:

$$468 - 36r = 0$$

$$\boldsymbol{r = \frac{468}{36} = 13\text{cm}}$$

(b) Show that angle $AOB = 134.8°$, correct to 1 decimal place. [2]

Use SOHCAHTOA to find $M\hat{O}B$:

$$\sin M\hat{O}B = \frac{12}{13}$$

$$\Rightarrow M\hat{O}B = \sin^{-1}\left(\frac{12}{13}\right) = 67.38\ldots°$$

$\boldsymbol{M\hat{O}B} = 2 \times M\hat{O}B = 2 \times 67.38\ldots = \boldsymbol{134.8}°$ (to 1dp)

(c) (i) Calculate the area of the major sector $OAPB$. [3]

Sector of a circle (angle θ):

$Sector\ Area = \frac{\theta}{360} \times \pi r^2$

$$Sector\ Area = \frac{\theta}{360} \times \pi r^2$$

$$Sector\ Area = \frac{360-134.8}{360} \times \pi \times 13^2$$

$$\boldsymbol{Sector\ Area} = \mathbf{332cm^2}\ \text{(to 3sf)}$$

(ii) Calculate the area of the shaded segment APB. [3]

$$Area\ of\ Shaded\ Segment = Area\ of\ Sector\ OAPB + Area\ of\ Triangle\ AOB$$

$$Area\ of\ Shaded\ Segment = 332 + 2 \times \frac{1}{2} \times MB \times OM$$

$$Area\ of\ Shaded\ Segment = 332 + 2 \times \frac{1}{2} \times 12 \times 5$$

$$\boldsymbol{Area\ of\ Shaded\ Segment} = \mathbf{392cm^2}\ \text{(to 3sf)}$$

(iii) The length of the cylinder is 40 cm.

Calculate the volume of water in the cylinder. [1]

For a Prism:

$Volume = Area\ of\ Cross\text{-}section \times Length$

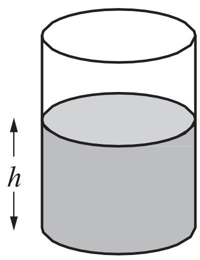
$$Volume = Area\ of\ Cross\text{-}section \times Length$$

$$Volume\ of\ water = 392 \times 40$$

$$\boldsymbol{Volume\ of\ water} = \mathbf{15687cm^2}\ \text{(to nearest whole number)}$$

(d) The cylinder is turned so that it stands on one of its circular ends.
In this position, the depth of the water is h. [2]

Find h.



NOT TO SCALE

A cylinder is a Prism:

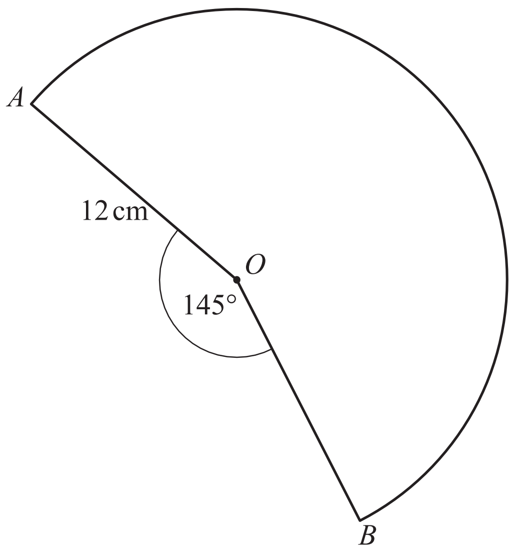
$$Volume = Area\ of\ Cross\text{-}section \times Length$$

$$Volume = \pi r^2 h$$

$$15687 = \pi \times 13^2 \times h$$

$$h = \frac{15687}{\pi \times 13^2}$$

$$\boldsymbol{h} = \mathbf{29.5cm}\ \text{(to 3sf)}$$



NOT TO SCALE

The diagram shows a sector, centre O, and radius 12 cm.

(a) Calculate the area of the sector. [3]

Area of a sector is

$$\frac{\theta}{360} \times \pi r^2$$

Where θ is the angle of the sector and r is the radius.

$$\theta = 360 - 145$$

$$= 215$$

Hence

$$A = \frac{215}{360} \times 12^2 \times \pi$$

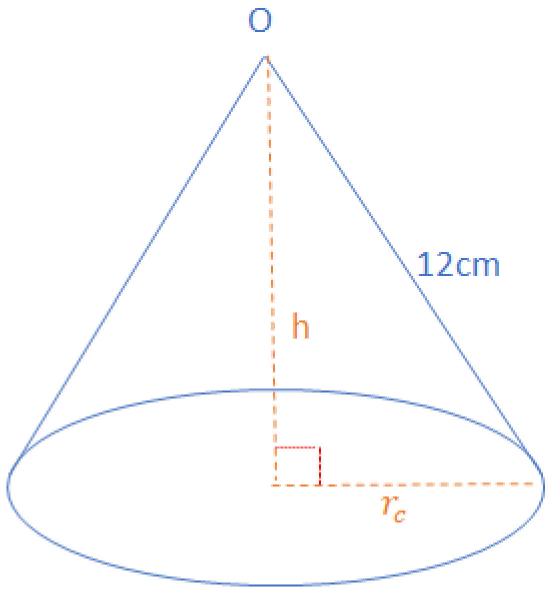
$$= \mathbf{270.18}$$

(b) The sector is made into a cone by joining OA to OB.

Calculate the volume of the cone.

[The volume, V, of a cone with base radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [6]

The length of the sector and hence the circumference of the bottom of the cone is



$$C = \frac{\theta}{180}\pi \times r$$

$$= \frac{215}{180}\pi \times 12$$

$$= 45.03$$

This is also equal to

$$2\pi r_c = 45.03$$

$$\rightarrow r_c = 7.17$$

The height can be found using Pythagoras' Theorem

$$r_c^2 + h^2 = 12^2$$

$$\rightarrow h^2 = 144 - 7.17^2$$

$$\rightarrow h = 9.62$$

The volume is then

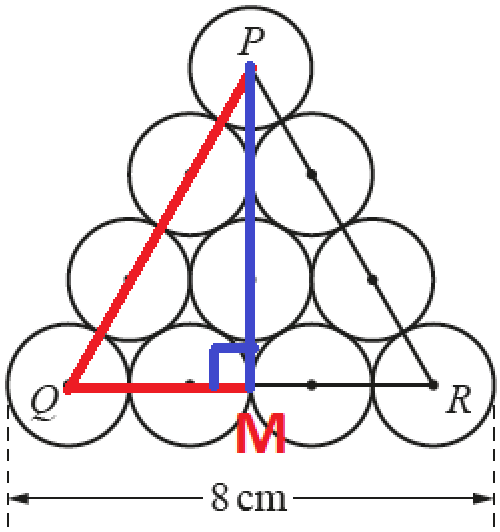
$$V = \frac{1}{3}\pi r_c^2 h$$

$$= \frac{1}{3}\pi(7.17)^2(9.62)$$

$$\mathbf{= 518}$$

(a) The ten circles in the diagram each have radius 1cm.
The centre of each circle is marked with a dot.

Calculate the height of triangle PQR. [3]



By inspecting the length of QR, we conclude that it is equal to 6 radii of the circles.

Hence:

$$QR = 6cm$$

Triangle PQR is an equilateral triangle.

$$QP = QR$$

We can use Pythagoras theorem to calculate the height. Let M be the midpoint of QR.

$$QM = 3cm$$

From Pythagoras theorem:

$$QP^2 = height^2 + QM^2$$

$$height^2 = QP^2 - QM^2$$

Take square root of both sides:

$$height = \sqrt{QP^2 - QM^2}$$

Substitute known values:

$$height = \sqrt{(6cm)^2 - (3cm)^2}$$

Hence the height of triangle PQR:

$$\boldsymbol{height = 5.20\ cm\ (3sf)}$$

(b) Mr Patel uses whiteboard pens that are cylinders of radius 1 cm.

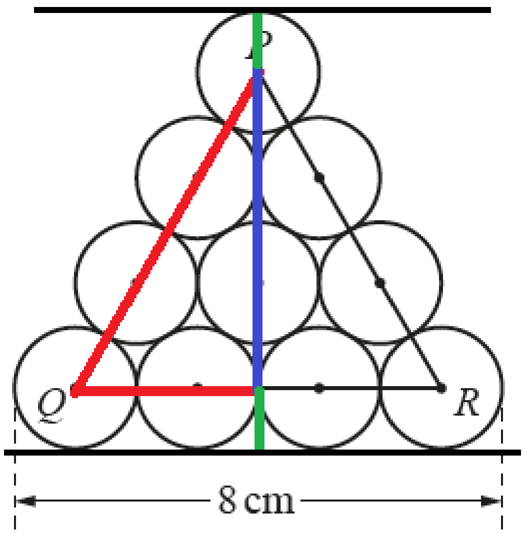
[1]

(i) The diagram shows 10 pens stacked in a tray.
The tray is 8 cm wide.
The point A is the highest point in the stack.

Find the height of A above the base, BC, of the tray.

We have almost found the answer in part a), however we still need to add the length of two radii to our answers, because of the radius of the pen on top and the line at the bottom.

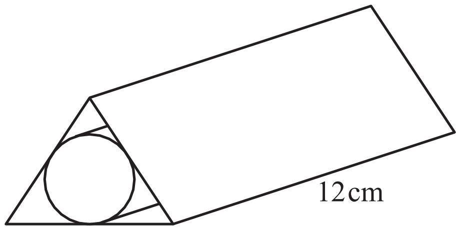
The height from part a) is marked blue and the two radii we need to add are green.



Add 2 cm to the answer from part a).

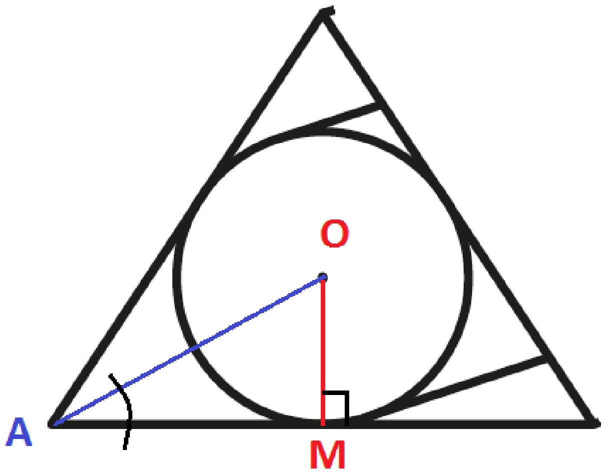
$$\textbf{height of A above the base} = \mathbf{7.20\ cm\ (3sf)}$$

(ii) The diagram shows a box that holds one pen.
The box is a prism of length 12 cm.
The cross section of the prism is an equilateral triangle.
The pen touches each of the three rectangular faces of the box.



Calculate the volume of this box. [5]

The length of the box is known. In order to calculate the volume, we need to know the cross-section area.



The red line is the radius of the pen which is 1centimetre.

$$OM = 1cm$$

The triangle is an equilateral triangle, so all internal angles are 60°. The blue line connecting point A with the centre O is a bisector of one of these angles, so a newly formed angle is 30°.

$$angle\ OAM = 30°$$

Use trigonometric identity:

$$\tan(angle\ OAM) = \frac{OM}{AM}$$

$$\mathrm{AM} = \frac{OM}{\tan(angle\ OAM)}$$

Substitute known values:

$$\mathrm{AM} = \frac{1cm}{\tan(30°)}$$

$$\mathrm{AM} = \sqrt{3}cm$$

The size of AM is half the size of the base.

$$triangle\ base = 2\sqrt{3}cm$$

The area of an equilateral triangle is given by:

$$area = \frac{1}{2}\,(base)^2 \times \sin(60°)$$

In our case:

$$area = \frac{1}{2}\,(2\sqrt{3}cm)^2 \times \sin(60°)$$

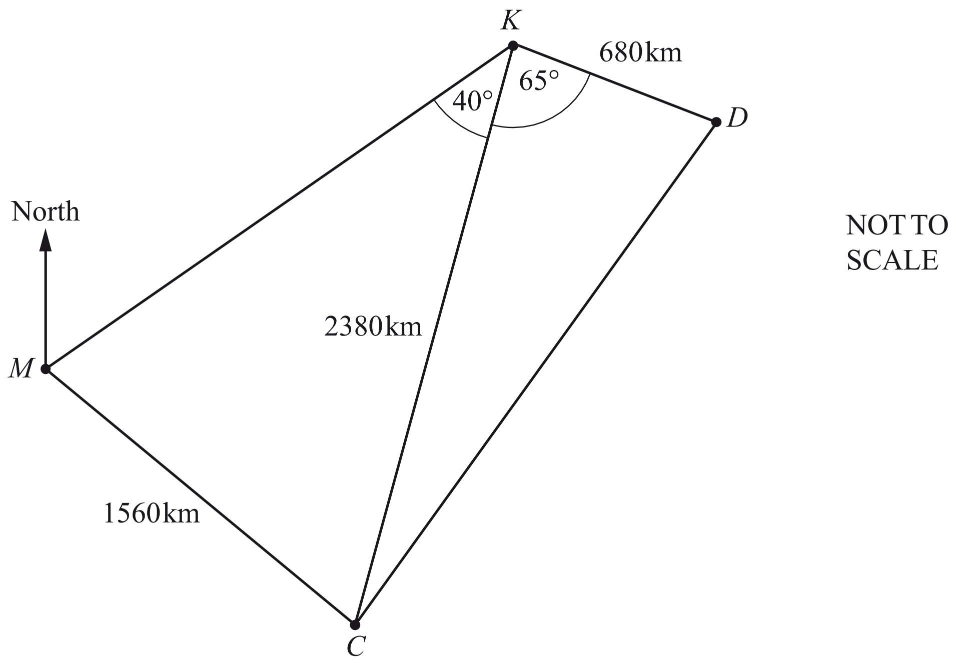
$$area = 3\sqrt{3}cm^2$$

Multiply the cross-section area of the triangle by the length of the box to obtain the volume.

$$volume = area \times length$$

$$volume = 3\sqrt{3}cm^2 \times 12cm$$

$$\boldsymbol{volume = 62.4\ cm^3\ (3sf)}$$



The diagram shows some distances between Mumbai (M), Kathmandu (K), Dhaka (D) and Colombo (C).

(a) Angle $CKD = 65°$.

Use the cosine rule to calculate the distance CD.

[4]

The distance CD between Colombo and Dhaka can be found using the cosine rule.

$$CD^2 = KD^2 + KC^2 - 2 \times KD \times KC \times \cos(CKD)$$

$$CD^2 = (680km)^2 + (2380km)^2 - 2 \times (680km) \times (2380km) \times \cos(65°)$$

$$CD^2 = 4\,758\,869.21\ km^2$$

Take square root of both sides to get the final answer:

$$\boldsymbol{CD = 2\,181\ km}\ (3sf)$$

(b) Angle $MKC = 40°$.

Use the sine rule to calculate the acute angle KMC.

Using sine rule and triangle MKC.

$$\frac{\sin(KMC)}{KC} = \frac{\sin(MKC)}{MC}$$

$$\frac{\sin(KMC)}{2380km} = \frac{\sin(40°)}{1560km}$$

Multiply both sides by 2380km.

$$\sin(KMC) = \frac{\sin(40°)}{1560km} \times 2380km$$

$$\sin(KMC) = 0.981$$

Take $\sin^{-1}$ of both sides of the equation.

$$\boldsymbol{KMC = 78.7°}$$ [3]

(c) The bearing of K from M is 050°.

Find the bearing of M from C.

[2]

The angle North-M-K is 50°. We have calculated the angle KMC in part b).

It is also needed to add extra 180° as the bearing is of M from C.

Add all these angles together.

$$50° + 78.1° + 180° = 308.7°$$

Therefore the bearing of M from C is **308.7°**.

(d) A plane from Colombo to Mumbai leaves at 2115 and the journey takes 2 hours 24 minutes.

(i) Find the time the plane arrives at Mumbai.

[1]

Add 2 hours and 24 hours to 21.15.

$$21\ h\ 15\ \text{min} + 2\ h\ 24\ min = 23h\ 39\ min$$

The plane arrives at Mumbai at **23.39**.

(ii) Calculate the average speed of the plane. [2]

The average speed is defined as the ratio of total distance covered and total time taken.

We convert 2 hours and 24 minutes into hours (by dividing 24 minutes by 60).

$$2h\ 24\ min = 2h + \frac{24min}{60\ \text{min}\ per\ hour} = 2.4h$$

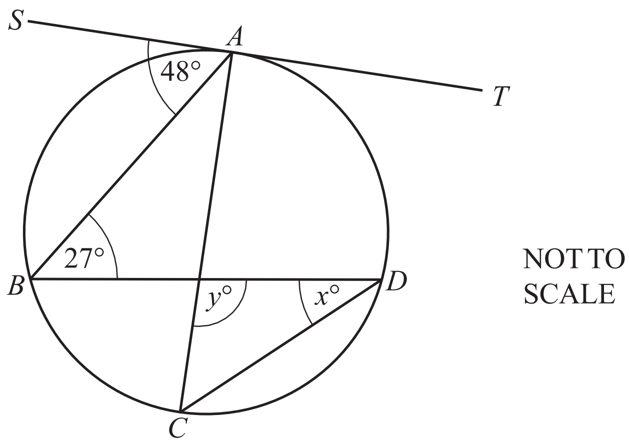
The total distance covered (CM) is 1560km.

Time taken to cover this distance is 2.4h.

$$average\ speed = \frac{1560km}{2.4h}$$

$$= \mathbf{650\ km/h}$$

(a) The points A, B, C and D lie on a circle.
AC is a diameter of the circle.
ST is the tangent to the circle at A.



Find the value of

(i) x, [2]

Angle SAC is a right angle triangle, because ST is the tangent to the circle at A.

$$SAC = SAB + BAC$$

$$90° = 48° + BAC$$

Subtract 48° from both sides of the equation.

$$BAC = 42°$$

As AC is a diameter of the circle, the triangles are mathematically similar.

Angle BAC must have the same size as BDC. This means that:

$$\boldsymbol{x = 42°}$$

(ii) y. [2]

The angle ACD is subtended from the same points (A and D) as the angle ABD, so they must have the same size. (Angles in the same segment are equal)

Therefore angle ACD=27°.

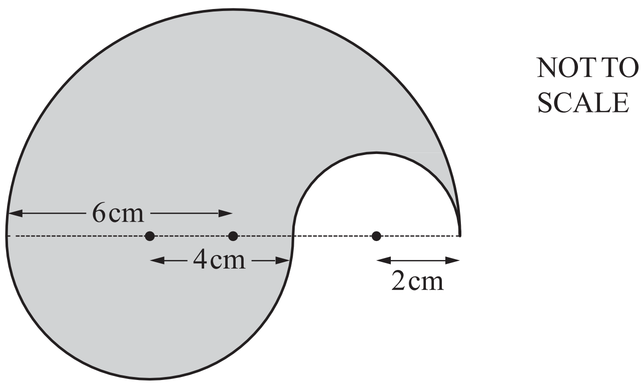
The sum of all interior angles of a triangle is 180°.

$$180° = y + 27° + 42°$$

Subtract 69° from both sides of the equation to work out the value of y.

$$\boldsymbol{y = 111°}$$

(b) The diagram shows a shaded shape formed by three semi-circular arcs.
The radius of each semi-circle is shown in the diagram.



(i) Calculate the perimeter of the shadedshape. [2]

The perimeter is made of three half-circumferences of circles with radii: 2cm, 4cm and 6cm.

The circumference of a circle is $2\pi r$ where r is the radius of the circle.

Therefore the half-circumference is πr.

$$Perimeter = \pi(2cm) + \pi(4cm) + \pi(6cm)$$

$$Perimeter = \mathbf{37.7cm}$$

(ii) The shaded shape is made from metal 1.6mm thick.

Calculate the volume of metal used to make this shape.
Give your answer in cubic millimetres. [5]

The area of a circle with radius *r* is $Area = \pi r^2$

The total area of the shaded region is half the area of a circle with radius 4cm plus half the area of a circle with radius 6cm, which is missing half the area of a circle with radius 2cm.

$$Total\ area = \frac{1}{2}area(r = 4cm) + \frac{1}{2}area(r = 6cm) - \frac{1}{2}area(r = 2cm)$$

Use the formula mentioned earlier.

$$Toal\ area = \frac{1}{2}\pi(4cm)^2 + \frac{1}{2}\pi(6cm)^2 - \frac{1}{2}\pi(2cm)^2$$

Use a calculator to work out the total area.

$$Total\ area = 25.13cm^2 + 56.55cm^2 - 6.28cm^2$$

$$Total\ area = 75.4cm^2$$

Convert the area into square millimetres (times by 100).

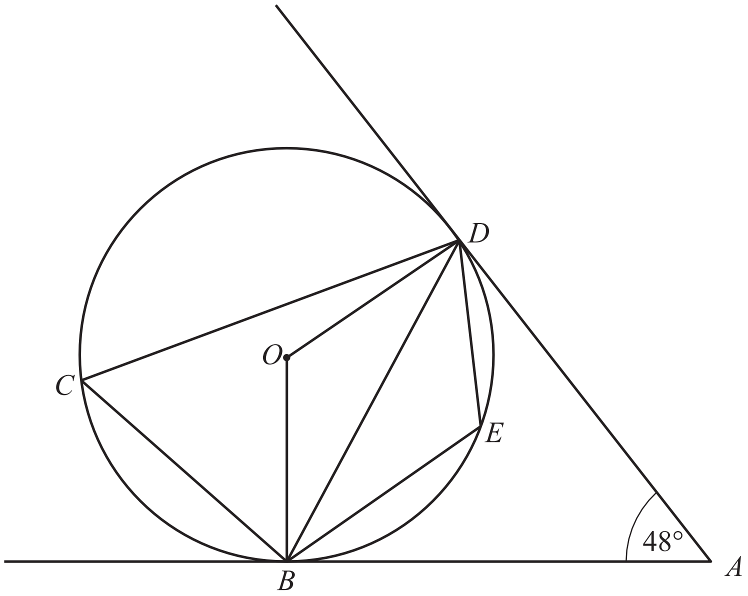
$$Toal\ area = 7540mm^2$$

To get the volume of the shape, multiply the area by the thickness.

$$Volume = area \times thickness$$

$$Volume = 7540mm^2 \times 1.6mm$$

$$\boldsymbol{Volume = 12\ 064mm^3}$$



NOT TO SCALE

In the diagram, B, C, D and E lie on the circle, centre O.
AB and AD are tangents to the circle.
Angle $BAD = 48°$.

(a) Find

(i) angle ABD,

[1]

Both AB and AD are tangents to the circle, therefore the angle BDA is an isosceles triangle.

Angle ABD and ADB are the same size.

The sum of all interior angles of a triangle is 180°.

$$180° = 48° + ABD + ADB = 48° + 2ABD$$

Subtract 48° from both sides of the equation and divide both sides by 2.

$$\boldsymbol{ABD = 66°}$$

(ii) angle OBD,

[1]

As O is the centre of the circle and AB is the tangent, the angle OBA must be 90°.

$$OBA = OBD + DBA$$

$$90° = OBD + 66°$$

Subtract 66° from both sides of the equation.

$$\boldsymbol{OBD = 24°}$$

(iii) angle BCD, [2]

Opposite angles of a cyclic quadrilateral add up to 180°.

In our case, the opposite angles are DAB and DOB.

$$180° = DAB + DOB$$

$$180° = DOB + 48°$$

$$DOB = 132°$$

The value of BCD will be half the value of the angle BOD (the angle subtended at the centre of a circle is double the size of the angle subtended at the edge from the same two points).

The two common points for these angles are B and D.

Therefore we have ***BCD* = 66°.**

(iv) angle BED. [1]

Opposite angles of a cyclic quadrilateral add up to 180°.

In our case, the opposite angles are DCB and DEB.

$$180° = BCD + BED$$

$$180° = 66° + BED$$

Subtract 66° from both sides of the equation.

$$\boldsymbol{BED = 114°}$$

(b) The radius of the circle is 15 cm.

Calculate the area of triangle BOD. [2]

The area of a triangle BOD is

$$area = \frac{1}{2} OB \times OD \times \sin(BOD)$$

Use given values (both OB and OD are equal to the radius of the circle).

$$area = \frac{1}{2}(15cm) \times (15cm) \times \sin(132°)$$

$$area = \mathbf{83.6\ cm^2}$$

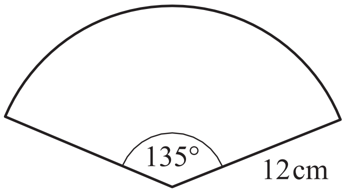
(c) Give a reason why $ABOD$ is a cyclic quadrilateral. [1]

Cyclic because opposite angles of ABOD add up to 180°.

(a) A sector of a circle has radius 12 cm and an angle of 135°.

(i) Calculate the length of the arc of this sector.
Give your answer as a multiple of π.

[2]



NOT TO SCALE

The length of the arc is given by the following formula:

$$Arc\ length\ of\ sector = \frac{interior\ angle}{360°} \times 2\pi \times radius\ of\ the\ sector$$

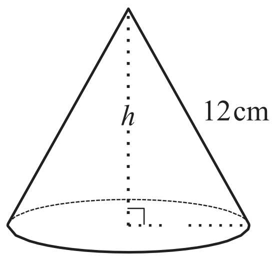
In our case, the interior angle is 135° and the radius of the sector is 12cm.

$$= \frac{135°}{360°} \times 2\pi \times 12cm$$

$$Arc\ length\ of\ sector = \mathbf{9\pi}\ cm$$

(ii) The sector is used to make a cone.

(a) Calculate the base radius, r.



NOT TO SCALE

[2]

The circumference of the cone will be $2\pi \times r$ and this will be identical to the arc length of the sector.

$$2\pi \times r = 9\pi\ cm$$

We divide both sides by 2π to get the radius of the cone

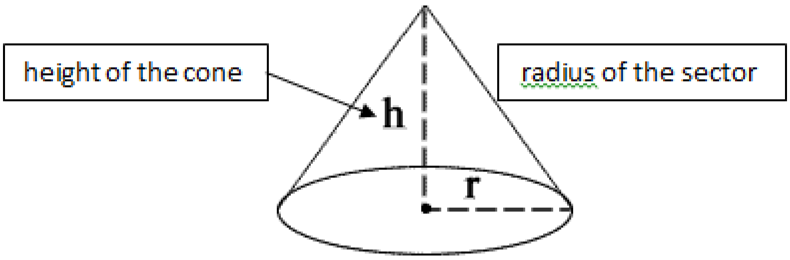
$$\mathbf{r = 4.5cm}$$

r

(b) Calculate the height of the cone, *h*. [3]

The height *h* of the cone can be found from Pythagoras triangle

$$12^2 = h^2 + r^2$$



Subtracting r^2 from both sides

$$h^2 = 12^2 - r^2$$

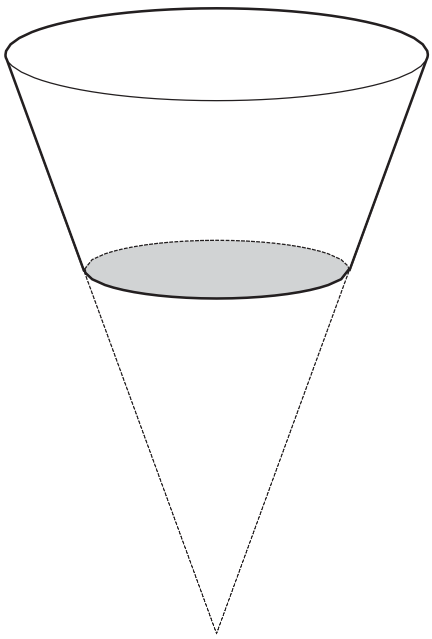
Use the value of *r* from the previous part of the question.

$$h^2 = 12^2 - 4.5^2$$

Taking square root of both sides, we get the final answer:

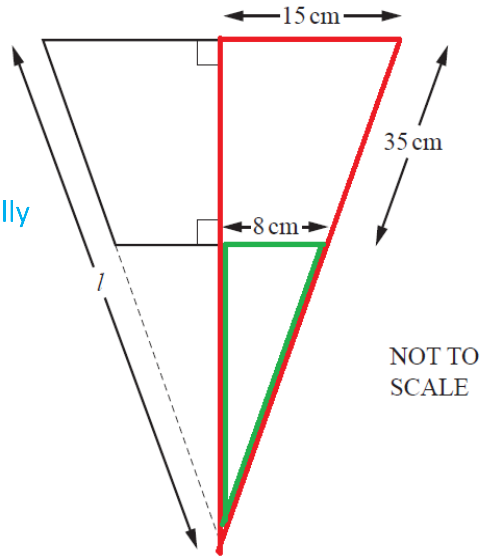
$$\boldsymbol{h = 11.1\ cm}$$

(b) The diagram shows a plant pot.
It is made by removing a small cone from a larger cone and adding a circular base.



NOT TO SCALE

This is the cross section of the plant pot.



[3]

(i) Find l.

The red and the green triangles are mathematically similar.

This means that the ratio of their sides is equal.

$$\frac{8}{15} = \frac{l - 35}{l}$$

Multiply both sides by (15cm)x*l*.

$$8l = (l - 35) \times 15$$

Multiply out the bracket.

$$8l = 15l - 525$$

$$525 = 7l$$

Divide both sides by 7 to find the value of *l*.

$$\boldsymbol{l = 75cm}$$

(ii) Calculate the total surface area of the outside of the plant pot.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.] [3]

To get the curved surface area of the outside of the plant pot, we subtract the curved area of the smaller cone from the curved area of the larger cone.

$$Curved\ area = Larger\ cone - Smaller\ cone$$

The curved surface area of a cone with radius *r* and slant height *l* is *A*=π*rl*.

Use the given and calculated values.

$$Curved\ area = \pi \times (15cm) \times (75cm) - \pi \times (8cm) \times (75cm - 35cm)$$

Find the answer using a calculator:

$$Curved\ area = 1125\pi\ cm^2 - 320\pi\ cm^2 = 805\pi\ cm^2$$

To get the total surface area, we need to add the area of the circular base (radius=8cm).

$$Total\ area = curved\ area + base$$

$$Total\ area = 805\pi\ cm^2 + \pi(8cm)^2$$

Using a calculator:

$$\mathbf{Total\ area = 2730\ cm^2}$$

(c) Some cones are mathematically similar.
For these cones, the mass, M grams, is proportional to the cube of the base radius, r cm.
One of the cones has mass 1458 grams and base radius 4.5 cm. [2]

(i) Find an expression for M in terms of r.

The mass M is proportional to the cube of the base radius r.

$$M = kr^3$$

Where k is the proportionality constant.

$$1458 = k \times 4.5^3 = k \times 91.125$$

Divide both sides of the equation by 91.125.

$$k = 16$$

Therefore we can express M in terms of r.

$$\boldsymbol{M = 16r^3}$$

(ii) Two of the cones have radii in the ratio 2 : 3.

Write down the ratio of their masses. [1]

As mentioned earlier, the mass is proportional to the cube of the base radius.

To get the ratio of the masses from the ratio of the radii, we simply cube both sides of the ratio.

$$Ratio\ of\ masses = 2^3 : 3^3$$

$$\boldsymbol{Ratio\ of\ masses = 8 : 27}$$

Perimeters, Area and Volumes
Difficulty: Hard

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Perimeters, Area and Volumes
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 3

Time allowed: **117 minutes**

Score: **/102**

Percentage: **/100**

Grade Boundaries:

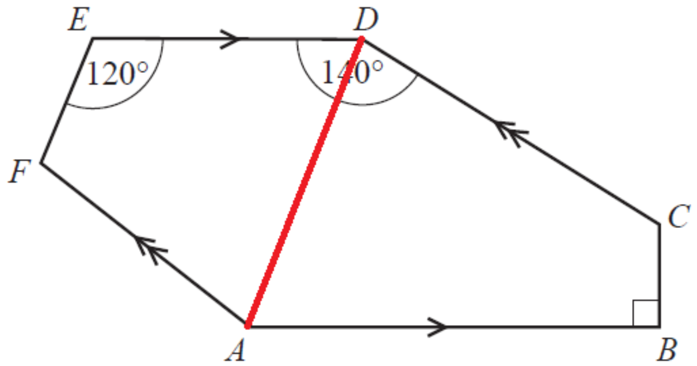
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a)



NOT TO SCALE

In the hexagon $ABCDEF$, AB is parallel to ED and AF is parallel to CD.
Angle $ABC = 90°$, angle $CDE = 140°$ and angle $DEF = 120°$.

Calculate angle EFA. [4]

As ED is parallel to AB and DC is parallel to FA, we know that the angles EDC and FAB have the same size. $FAB = EDC = 140°$

We can therefore draw a line between D and A which will divide the hexagon into two.

The sum of the two angles must stay the same, therefore

$$EDA + FAD = 140°$$

The sum of the interior angles of quadrilateral FEDA is 360° (as are all quadrilaterals).

We can write this as:

$$360° = DEF + EDA + FAD + EFA$$

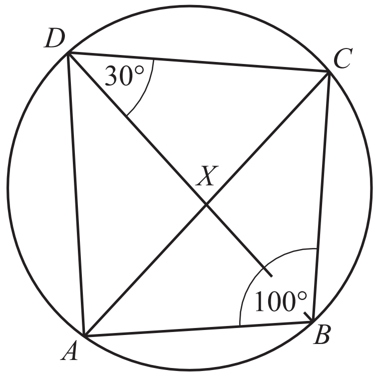
We know all angles apart from EFA.

$$360° = 120° + 140° + FAD$$

Subtract 260° from both sides of the equation.

$$\boldsymbol{angle\ FAD = 100°}$$

(b)



NOT TO SCALE

In the cyclic quadrilateral $ABCD$, angle $ABC = 100°$ and angle $BDC = 30°$.
The diagonals intersect at X.

(i) Calculate angle ACB. [2]

Since ABCD is a cyclic quadrilateral, the sum of opposite sides must be 180°.

$$180° = ABC + ADC = ABC + ADB + BDC$$

Using this, we can calculate the size of angle ADB.

$$180° = 100° + ADB + 30°$$

Subtract 130° from both sides of the equation.

$$ADB = 50°$$

Angles ADE and ACB have the same size (triangles AXD and BXC are similar). Therefore:

$$\mathbf{ACB = 50°}$$

(ii) Angle $BXC = 89°$.

Calculate angle CAD. [2]

Point X is the point where the diagonals intersect. Because of this, the angles BXC and AXD have the same size. $BXC = AXD = 89°$

The sum of all internal angles of a triangle AXD is 180°.

$$180° = AXD + XDA + DAX$$

We know the size of ADX from part b)i), because this is the same angle as ADB

$$180° = AXD + ADX + DAX$$

$$180° = 89° + 50° + DAX$$

Subtract 139° from both sides of the equation.

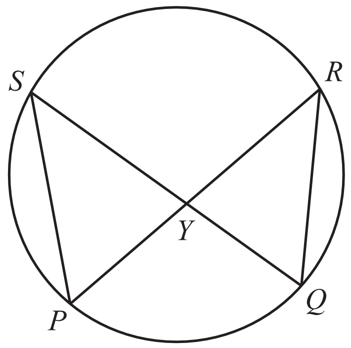
$$\mathbf{DAX = 41°}$$

(iii) Complete the statement. [1]

Triangle AXD and BXC are **similar.**

The sizes of their internal angles are the same.

(c)



P, Q, R and S lie on a circle.
PR and QS intersect at Y.
$PS = 11$ cm, $QR = 10$ cm and the area of triangle $QRY = 23$ cm^2.

Calculate the area of triangle PYS. [2]

The angles are similar; therefore the ratio of their areas must be the same as the ratio of the squares of their corresponding sides (squares, because areas are two dimensional, but length only one dimensional).

$$\frac{PS^2}{QR^2} = \frac{area\ PYS}{area\ QRY}$$

Use data given.

$$\frac{11^2}{10^2} = \frac{area\ PYS}{23\ cm^2}$$

Divide both sides by 23 cm^2.

$$area\ PYS = 23\ cm^2 \times \frac{11^2}{10^2}$$

Use calculator to work out the area of triangle PYS.

$$\boldsymbol{area\ PYS = 27.8cm^2}$$

(d) A regular polygon has n sides.
Each exterior angle is equal to $\frac{n}{10}$ degrees.

(i) Find the value of n. [3]

The sum of an exterior angle and the corresponding interior angle is 180° (straight line).

$$180° = \frac{n}{10} + interior\ angle$$

Subtract n/10 from both sides.

$$interior\ angle = \frac{1800 - n}{10}°$$

The sum of interior angles of shape with n vertices is given by $(n - 2) \times 180°$.

The shape is a regular polygon, the sum must also be $n \times (interior\ angle)$, as all interior angles have the same size. We know that this interior angle is 156°.

$$n \times \frac{1800 - n}{10} = (n - 2) \times 180°$$

$$n \times \frac{(1800 - n)}{10} = n \times 180° - 360°$$

Multiply both sides by 10.

$$n \times (1800 - n)° = n \times 1800° - 3600°$$

Multiply out the bracket.

$$1800n° - n^{2}° = n \times 1800° - 3600°$$

Subtract $n \times 1800°$ from both sides.

$$n^2 = 3600°$$

Take the positive square root of 3600 (negative angle does not make sense).

$$n = 60$$

We get the final answer: $n = 60$.

This regular polygon has **60 sides**.

(ii) Find the size of an interior angle of this polygon. [2]

As stated in the previous part, the interior angle is:

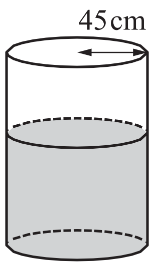
$$interior\ angle = \frac{1800 - n}{10}°$$

Use n=60.

$$\boldsymbol{interior\ angle = 174°}$$

(a) A cylindrical tank contains 180000 cm^3 of water.
The radius of the tank is 45 cm.

Calculate the height of water in the tank.



NOT TO SCALE

[2]

The volume of a cylindrical tank is given as:

$$V = h\pi r^2$$

Where *h* is the height and *r* is the radius of the tank.

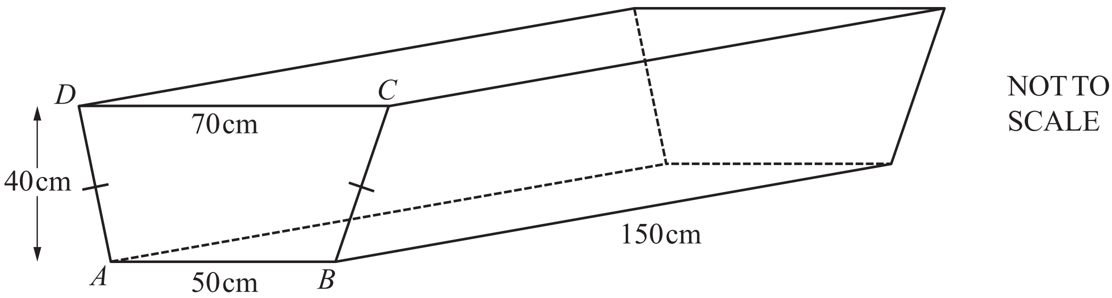
Use knows values.

$$180\,000\ cm^3 = h\pi\ x\ 45^2$$

Divide both sides by $\pi(45cm)^2$ to calculate the height of water.

$$\boldsymbol{h = 28.3\ cm}$$

(b)



The diagram shows an empty tank in the shape of a horizontal prism of length 150cm.
The cross section of the prism is an isosceles trapezium *ABCD*.
AB = 50 cm, *CD* = 70 cm and the vertical height of the trapezium is 40 cm.

(i) Calculate the volume of the tank. [3]

The volume of the tank is given as the product of the area of the trapezium face and the length of the prism.

The area of a trapezium with parallel sides 70 cm and 50 cm and height 40 cm is given as:

$$Trapezium = \frac{(70+50)}{2} \times 40$$

$$Trapezium = 2400\ cm^2$$

The volume of the tank:

$$Volume = trapezium \times length$$

$$Volume = 2400\ \times 150$$

$$Volume = \mathbf{360\ 000\ cm^3}$$

(ii) Write your answer to **part (b)(i)** in litres. [1]

One cubic centimetre is equal to one millilitre.

$$Volume = 360\ 000\ cm^3 = 360\ 000\ ml$$

There are 1000 ml in one litre (divide the number by 1000 to get the volume in litres).

$$\boldsymbol{Volume = 360\ l}$$

(c) The 180 000 cm^3 of water flows from the tank in **part (a)** into the tank in **part (b)** at a rate of 15 cm^3/s.
Calculate the time this takes.
Give your answer in hours and minutes. [3]

We divide the volume by the rate to get the time needed to empty the tank.

$$\frac{180\ 000\ cm^3}{15\ cm^3\ per\ second} = 12\ 000\ hours$$

Divide the number of seconds to empty the tank by 60 to get the time in min (60 second in a minute).

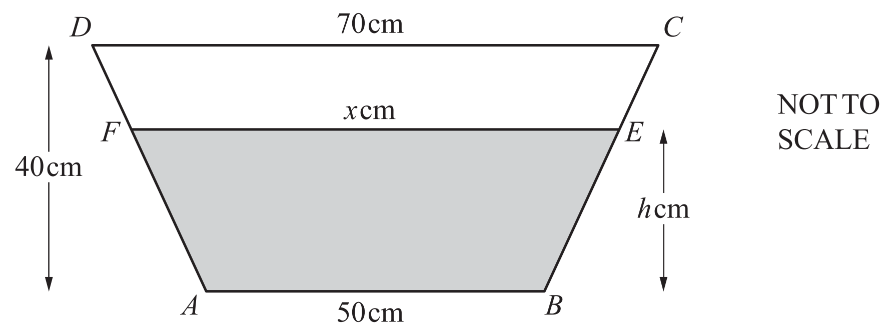
$$\frac{12\ 000\ seconds}{60\ seconds\ in\ min} = 200\ min$$

This time is equivalent to **3 hours and 20 minutes.**

$$3 \times 60min\ in\ hour + 20min = 180min + 20min$$

$$= \boldsymbol{200min}$$

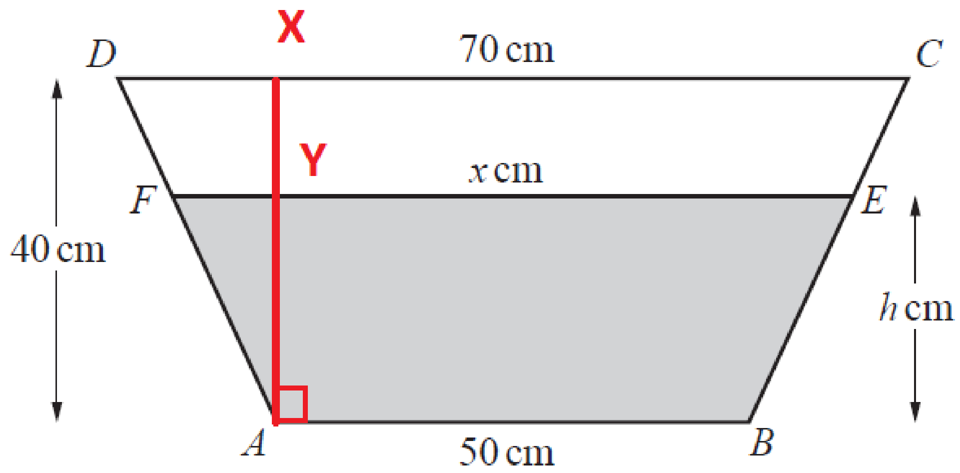
(d)



The 180 000 cm^3 of water reaches the level EF as shown above.
$EF = x$ cm and the height of the water is h cm.

(i) Using the properties of similar triangles, show that $h = 2(x - 50)$. [2]

Triangles AYF and AXD are similar.



The ratio of the parallel sides FY and DX must be the same as the ratio of heights AY and AX.

$$\frac{FY}{DX} = \frac{AY}{AX}$$

Use given values.

$$\frac{(x-50)/2}{(70-50)/2} = \frac{h}{40}$$

$$\frac{(x-50)/2}{10} = \frac{h}{40}$$

Multiply both sides by 40cm.

$$4 \times \frac{x-50}{2} cm = hcm$$

We get the final form of the equation.

$$\mathbf{2(x-50) = h}$$

(ii) Using $h = 2(x - 50)$, show that the shaded area, in cm^2, is $x^2 - 2500$. [1]

The area of a trapezium with parallel sides x cm and 50 cm and height h cm is given as:

$$Trapezium = \frac{(x+50)}{2} \times h$$

Use the value of h from previous part of the question.

$$area = \frac{(x+50)}{2} \times 2(x-50)$$

Multiply the brackets together.

$$\boldsymbol{area = (x^2 - 2500)cm^2}$$

(iii) Find the value of x. [2]

We know that the volume of water in the tank is 180 000cm^3 and that the length is 150cm.

This means that the shaded area is

$$area = \frac{180\,000}{150} = 1200\ cm^2$$

Use the formula from part d)ii).

$$1200cm^2 = (x^2 - 2500)cm^2$$

Add 2500 to both sides of the equation.

$$3700 = x^2$$

Take positive square root of the answer to find the value of x (negative length would not make sense).

$$\boldsymbol{x = 60.83}$$

(iv) Find the value of h. [1]

Use the formula from part d)i)

$$2(x-50)=h$$

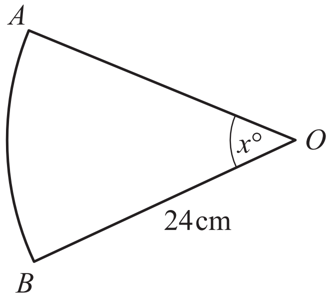
Plug in the value of x.

$$2(60.83-50)=h$$

Use a calculator to find the value of h.

$$\boldsymbol{h=21.7}$$

(a) The diagram shows a sector of a circle with centre O and radius 24cm.



(i) The total perimeter of the sector is 68 cm.

Calculate the value of x. [3]

Total perimeter of the sector is the sum of the two radii and the arc between point A and B.

$$perimeter = 2 \times radius + AB$$

$$68\ cm = 2 \times 24cm + AB$$

Subtract 48cm from both sides of the equation to get the length of arc AB.

$$20\ cm = AB$$

The length of arc AB is given as:

$$AB = \frac{x°}{360°} \times 2\pi \times radius$$

$$20cm = \frac{x°}{360°} \times 2\pi \times 24cm$$

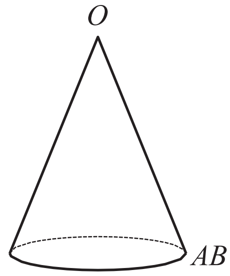
By rearranging:

$$x° = \frac{20cm \times 360°}{2\pi \times 24cm}$$

Use a calculator to get the value of *x* (interior angle of the sector)

$$\boldsymbol{x° = 47.7°}$$

(ii) The points A and B of the sector are joined together to make a hollow cone.
The arc AB becomes the circumference of the base of the cone.



NOT TO SCALE

Calculate the volume of the cone.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [6]

The volume of cone is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height.

The circumference of the cone will be $2\pi \times r$ and this will be identical to the arc length of the sector.

$$Arc\ length\ of\ sector\ AB = 20cm$$

As mentioned earlier, this equals to $2\pi \times r$. (AB becomes the circumference of the circular base)

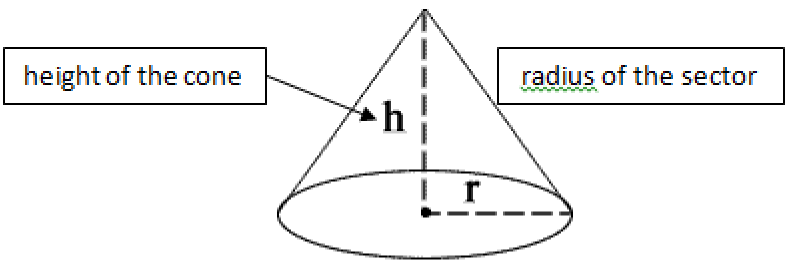
$$2\pi \times r = 45.03cm$$

We divide both sides by 2π to get the radius of the cone

$$r = 3.18\ cm$$

The height h of the cone can be found from Pythagoras triangle

$$radius\ of\ the\ sector^2 = h^2 + r^2$$



Subtracting r^2 from both sides

$$h^2 = radius\ of\ the\ sector^2 - r^2$$

$$h^2 = (24cm)^2 - (3.18cm)^2$$

Taking square root of both sides, we get the final answer:

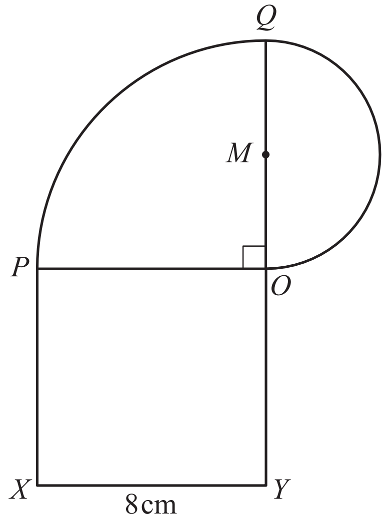
$$h = 23.8\ cm$$

Now, we can calculate the volume:

$$V = \frac{1}{3}\pi \times (3.18cm)^2 \times 23.8cm$$

$$\boldsymbol{Volume = 252\ cm^3}$$

(b)



NOT TO SCALE

The diagram shows a shape made from a square, a quarter circle and a semi-circle.
OPXY is a square of side 8 cm.
OPQ is a quarter circle, centre *O*.
The line *OMQ* is the diameter of the semi-circle.

Calculate the area of the shape. [5]

The total area of the shape is the sum of the areas of its parts:

$$total\ area = square\ OPXY + quarter\ circle\ OPQ + semicircle$$

- Area of square OPXY:

The area of a square is the square of its side.

$$Area\ square\ OPXY = XY^2$$

$$Area\ square\ OPXY = (8cm)^2$$

$$Area\ square\ OPXY = 64\ cm^2$$

- Area of quarter circle OPQ

The radius of the circle is identical to the length of a side of the square OPXY (*r*=8cm).

Area of a quarter circle is a quarter of area of a circle with the same radius:

$$Area\ quarter\ circle = \frac{1}{4}\pi\, r^2$$

$$Area\ quarter\ circle = \frac{1}{4}\pi\, (8cm)^2$$

$$Area\ quarter\ circle = 50.27\ cm^2$$

- Area of semicircle

The diameter of the semicircle is identical to XY. Therefore the radius is a half of this length (*r*=4cm).

Area of a semicircle is half the area of a circle with the same radius:

$$Area\ semicircle = \frac{1}{2}\pi\, r^2$$

$$Area\ semicircle = \frac{1}{2}\pi\, (4cm)^2$$

$$Area\ semicircle = 25.13\ cm^2$$

Sum these areas to get the area of the shape:

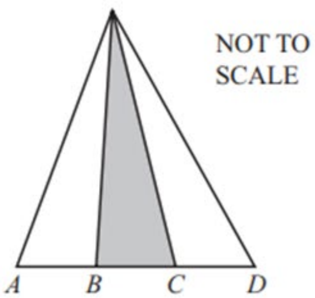
$$total\ area = square\ OPXY + quarter\ circle\ OPQ + semicircle$$

$$total\ area = 64\ cm^2 + 50.27\ cm^2 + 25.13\ cm^2$$

$$\boldsymbol{total\ area = 139.4\ cm^2}$$

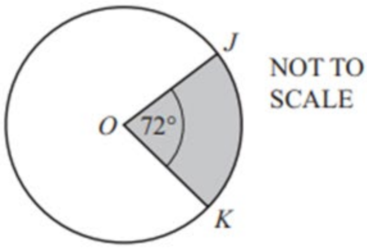
The total area of each of the following shapes is X.
The area of the shaded part of each shape is kX.

For each shape, find the value of k and write your answer below each diagram. [10]



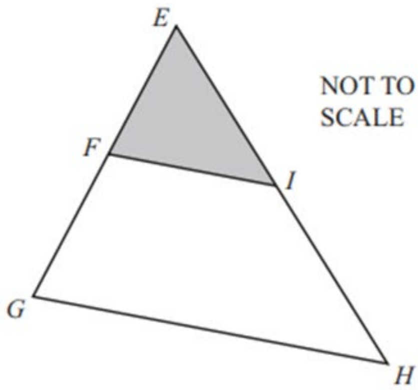
$AB = BC = CD$

$k = \frac{1}{3}$



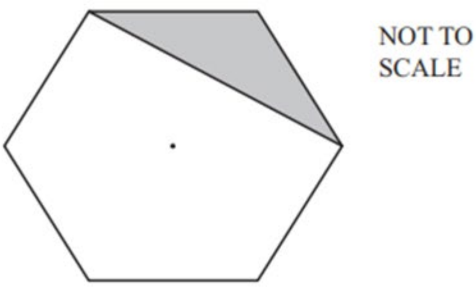
Angle $JOK = 72°$

$k = \frac{72}{360}$



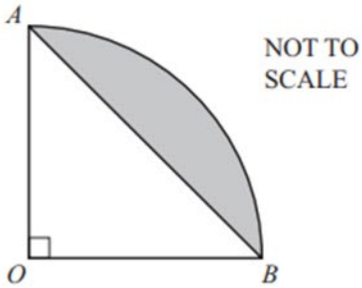
$EF = FG$ and $EI = IH$

$k = \frac{1}{4}$



The shape is a regular hexagon.

$k = \frac{1}{6}$



The diagram shows a sector of a circle centre O.
Angle $AOB = 90°$

$k = \frac{\pi - 2}{\pi}$

For the bottom rightmost shape

$$A_{shaded} = A_{sector} - A_{triangle}$$

$$= \frac{90}{360}\pi r^2 - \frac{1}{2}r^2$$

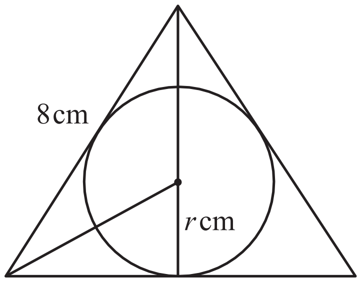
$$= \frac{1}{4}r^2(\pi - 2)$$

As a fraction this is

$$\frac{1}{4}r^2(\pi - 2) \div \frac{1}{4}\pi r^2$$

$$= \boldsymbol{\frac{\pi - 2}{\pi}}$$

(a)



NOT TO SCALE

The three sides of an equilateral triangle are tangents to a circle of radius r cm.
The sides of the triangle are 8 cm long.

Calculate the value of r.
Show that it rounds to 2.3, correct to 1 decimal place. [3]

Each angle in the triangle is 60.

Use

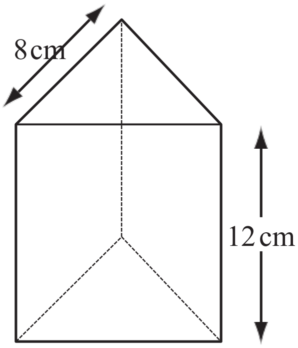
$$\tan\theta = \frac{opp}{adj}$$

$$\rightarrow \tan 30 = \frac{r}{4}$$

$$\rightarrow r = 4\tan 30$$

$$= \mathbf{2.3}$$

(b)



NOT TO SCALE

The diagram shows a box in the shape of a triangular prism of height 12cm.
The cross section is an equilateral triangle of side 8 cm.

Calculate the volume of the box. [4]

Area of the triangle is

$$\boldsymbol{A = \frac{1}{2}ab\sin C}$$

$$= \frac{1}{2}(8^2)\sin 60$$

$$= 16\sqrt{3}$$

Hence the volume is

$$16\sqrt{3} \times 12$$

$$= \mathbf{332.6}$$

(c) The box contains biscuits.
Each biscuit is a cylinder of radius 2.3 centimetres and height 4 millimetres.

Calculate

(i) the largest number of biscuits that can be placed in the box, [3]

Biscuits have same radius as the circle in the triangle problem.

i.e. they are placed in a neat, cylindrical order, and the limiting factor is their height, 0.4 cm.

$$12 \div 0.4$$

$$= \mathbf{30}$$

(ii) the volume of one biscuit in cubic centimetres, [2]

$$V = \pi r^2 \times h$$

$$= 2.3^2\pi \times 0.4$$

$$= \mathbf{6.65}$$

(iii) the percentage of the volume of the box **not** filled with biscuits. [3]

Volume filled with biscuits is

$$30 \times 6.65$$

$$= 199.5$$

Volume not filled is

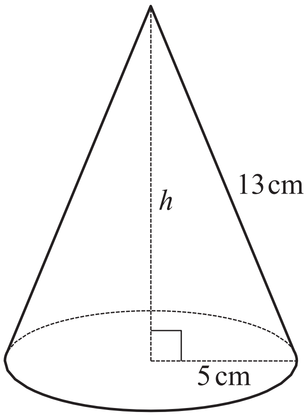
$$332.6 - 199.5$$

$$= 133.1$$

As a percentage this is

$$\frac{133.1}{332.6} \times 100$$

$$= \mathbf{40.02}$$



NOT TO SCALE

(a) The diagram shows a cone of radius 5 cm and slant height 13 cm.

(i) Calculate the curved surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.] [2]

Curved surface area:

$$\pi \times 5cm \times 13cm = \mathbf{204.2cm^2}$$

Note here that as per the question, the curved surface area is calculated using slant height, which is not to be confused with perpendicular height. The slant height is given, at 13cm.

(ii) Calculate the perpendicular height, h, of the cone. [3]

The perpendicular height can easily be calculated using Pythagoras' Theorem,

$$5^2 + h^2 = 13^2$$

Rearranging:

$$h^2 = 13^2 - 5^2$$

$$h = \sqrt{13^2 - 5^2}$$

$$= \mathbf{12cm}$$

(iii) Calculate the volume of the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [2]

Remember for the volume to use the perpendicular height, and

not the slant height!

$$Volume = \frac{1}{3} \times \pi \times 5^2 \times 12$$

$$= \mathbf{314.2cm^3}$$

(iv) Write your answer to **part (a)(iii)** in cubic metres.
Give your answer in standard form. [2]

To convert to cubic metres, note that for every 100cm makes 1m.

Additionally, because this is a volume, the conversion has to be cubed.

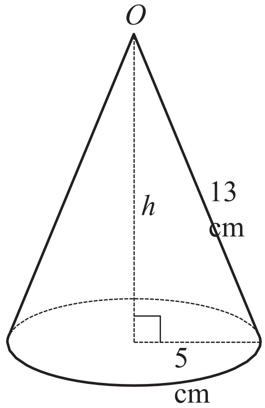
Since a metre is 100cm, we can expect the value for the volume to become

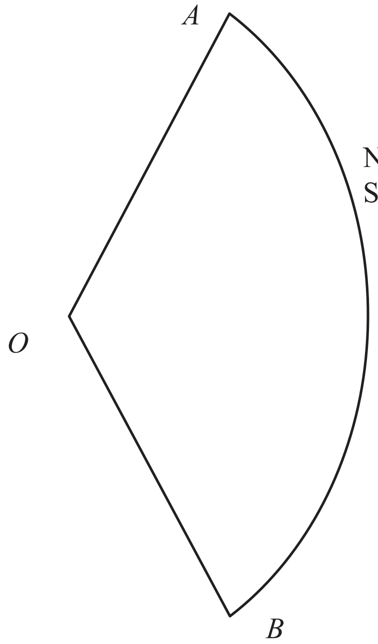
smaller when converted to metres, so we divide:

$$\frac{314.2cm^3}{100^3}$$

$$= \mathbf{3.142 \times 10^{-4}}$$

(b)





The cone is now cut along a slant height and it opens out to make the sector AOB of a circle.

Calculate angle AOB. [4]

Imagine the cone being cut up and opened out. It can be seen that OA and OB equate to the slant height of the cone.

The question states that this is a sector of a circle, which makes OA and OB the radius of said circle. OA is equal to OB.

The length of curved line AB is also equal to the circumference of the base of the cone.

First, calculate the length of curved line AB:

$$Circumference\ of\ cone\ base = 2 \times \pi \times r$$

$$= 2 \times \pi \times 5cm$$

$$= 31.42cm$$

Secondly, calculate the circumference of the larger circle with radius OB

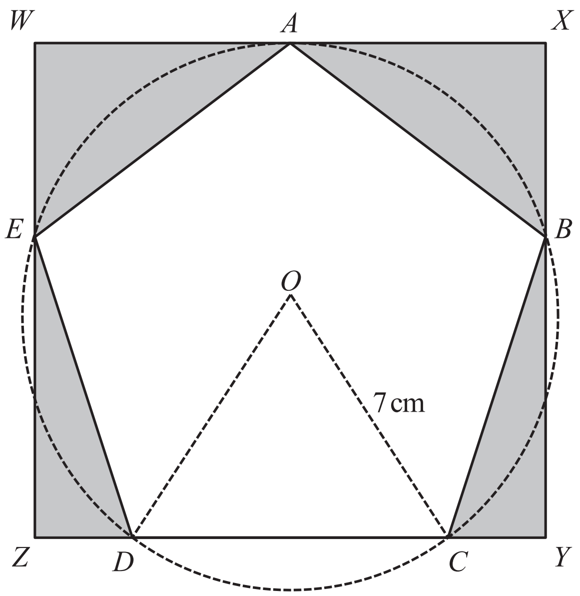
$$Circumference\ of\ large\ circle = 2 \times \pi \times slant\ height\ of\ cone$$

$$= 2 \times \pi \times 13cm$$

$$= 81.68cm$$

Lastly, we can now calculate the area of the sector as:

$$\frac{31.42}{81.68} \times 360° = \mathbf{138.5°}$$



The vertices A, B, C, D and E of a regular pentagon lie on the circumference of a circle, centre O, radius 7 cm.
They also lie on the sides of a rectangle $WXYZ$.

(a) Show that

(i) angle $DOC = 72°$, [1]

It is an even fifth of the full circle. Hence

$$\frac{360}{5}$$

$$= \mathbf{72}$$

(ii) angle $DCB = 108°$, [2]

It is a regular pentagon, so all its angles are equal, and

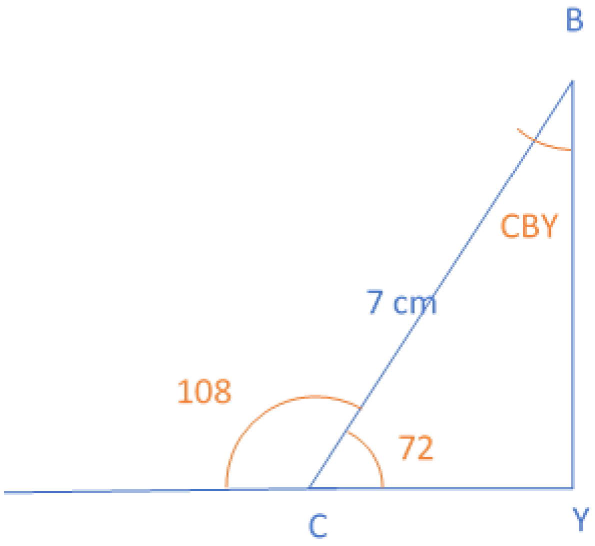
they must add to make 540.

$$\frac{540}{5}$$

$$= \mathbf{108}$$

(iii) angle $CBY = 18°$. [1]

Consider the following shape



Angle BCY is found by using

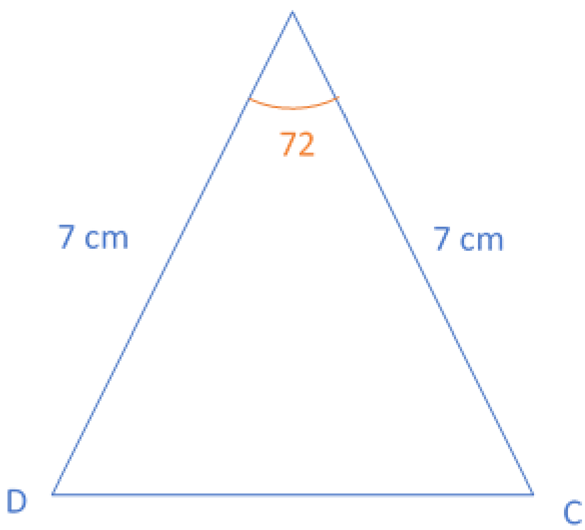
$$180 - 108 = 72$$

All angles in a triangle add to 180, hence

$$CBY = 180 - 72 - 90$$

$$= \mathbf{18}$$

(b) Show that the length CD of one side of the pentagon is 8.23 cm correct to three significant figures. [3]



We use the cosine rule to find side DC, as so

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$DC^2 = 7^2 + 7^2 - 2 \times 7 \times 7 \times \cos(72)$$

$$= 67.72$$

$$DC = \sqrt{67.72}$$

$$= \mathbf{8.23}$$

(c) Calculate

(i) the area of the triangle *DOC*, [2]

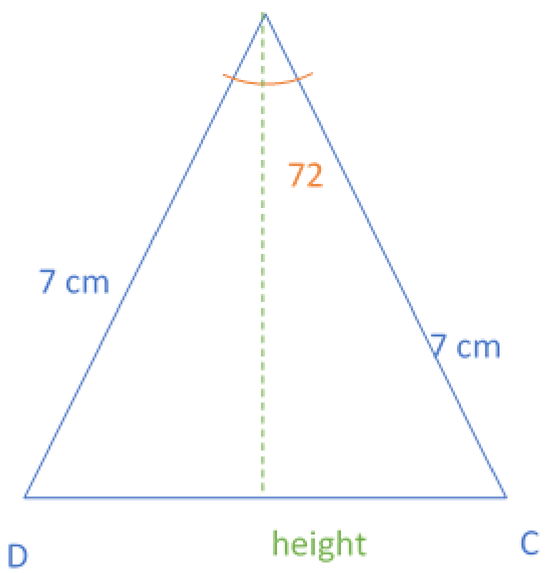
The area of a triangle is

$$A = \frac{1}{2} \times base \times height$$

For the triangle DOC (above) this is

$$A = \frac{1}{2} \times 8.23 \times height$$

We find the height using trigonometry. Consider the following shape



We use the relationship

$$\cos\theta = \frac{adj}{hyp}$$

$$\rightarrow \cos\frac{72}{2} = \frac{height}{7}$$

$$\rightarrow height = 7 \times \cos 36$$

$$= 5.66$$

Hence

$$A = \frac{1}{2} \times 8.23 \times 5.66$$

$$= \mathbf{23.3}$$

(ii) the area of the pentagon *ABCDE*, [1]

The pentagon is 5 equal triangles all with the area equal to that found in part c.i.

$$A = 5 \times 23.3$$

$$= \mathbf{116.5}$$

(iii) the area of the sector *ODC*, [2]

The area of a sector is

$$\boldsymbol{A = \frac{1}{2}r^2\theta}$$

If $\theta$is measured in radians. We must convert the angle of our sector to radians like so

$$\frac{72}{180} \times \pi$$

Then

$$A = \frac{1}{2} \times 7^2 \times \frac{72}{180}\pi$$

$$= \mathbf{30.8}$$

(iv) the length XY. [2]

The length XY is the height of the rectangle. We therefore only need to add the radius of the circle (distance OA) to the height of triangle DOC

$$XY = 7 + 7\cos 36$$

$$= \mathbf{12.66}$$

(d) Calculate the ratio

area of the pentagon $ABCDE$: area of the rectangle $WXYZ$.

Give your answer in the form 1 : n. [5]

To find the area of the rectangle we need to find its width (we have the height courtesy of part c.iv.).

The width is

$$W = ZD + DC + CY$$

From symmetry we know that

$$ZD = CY$$

and we already know DC, hence

$$W = 8.23 + 2 \times CY$$

The length CB is the same as DC so the length of CY is

$$8.23 \sin 18$$

$$= 2.54$$

$$\rightarrow W = 8.23 + 2 \times 2.54$$

$$= 13.31$$

The area of the rectangle is then

$$13.31 \times 12.66$$

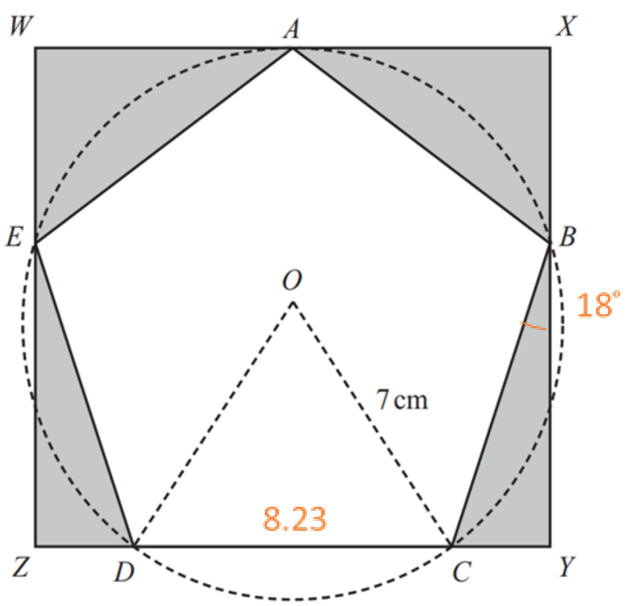
$$= 168.5$$

The ratio of the areas is

$$1 : \frac{168.5}{116.5}$$

$$= \mathbf{1 : 1.45}$$

Note that the relevant diagram is found below.



Perimeters, Area and Volumes
Difficulty: Hard

Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Perimeters, Area and Volumes
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 4

Time allowed: **106 minutes**

Score: **/92**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Boris has a recipe which makes 16 biscuits.

The ingredients are

160 g flour,

160g sugar,

240 g butter,

200 g oatmeal.

(a) Boris has only 350 grams of oatmeal but plenty of the other ingredients.

(i) How many biscuits can he make? [2]

If he can make 16 biscuits with 200g oatmeal, then with 350g he will make:

$$\text{Number of biscuits} = \frac{16 \text{ biscuits x } 350\text{g}}{200\text{g}}$$

Number of biscuits = 28

(ii) How many grams of butter does he need to make this number of biscuits? [2]

We know he needs 240g of butter for 16 biscuits.

Therefore, for 28 biscuits he will need:

$$\text{Amount of butter} = \frac{28 \text{ biscuits x } 240\text{g}}{16 \text{ biscuits}}$$

Amount of butter = 420g

(b) The ingredients are mixed together to make dough.

This dough is made into a sphere of volume 1080 cm^3.

Calculate the radius of this sphere.

[The volume, V, of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.] [3]

$V = \frac{4}{3}\pi r^3$

V = 1080 cm^3

We solve the equality making r the subject.

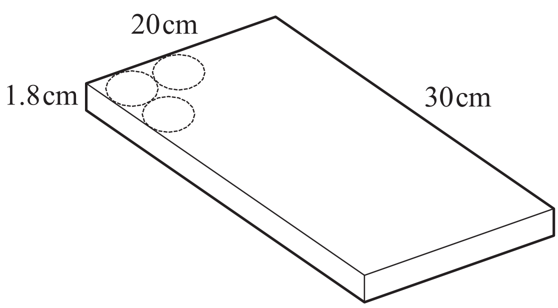
$\frac{4}{3}\pi r^3$ = 1080 cm^3

πr^3 = 810 cm^3

r^3 = 257.83 cm^3

r = 6.364 cm

(c)



NOT TO SCALE

The 1080 cm^3 of dough is then rolled out to form a cuboid 20 cm × 30 cm × 1.8 cm.

Boris cuts out circular biscuits of diameter 5 cm.

(i) How many whole biscuits can he cut from this cuboid? [1]

To work out the number of biscuits which can be cut from the cuboid we need to work out the surface area of the face of the cuboid and then divide the area by the 5cm diameter for each of the biscuits.

Number of biscuits = $\frac{20\text{cm}}{5\text{ cm}} \times \frac{30cm}{5cm}$

Number of biscuits = 24

(ii) Calculate the volume of dough left over. [3]

The total volume of the dough taken up by the 120 biscuits will be 120 times the volume of one of these cylinders with the diameter 5 cm and the height 1.8 cm.

To work out the volume of one cookie we need to use the formula for the volume of a cylinder.

V = πr^2h

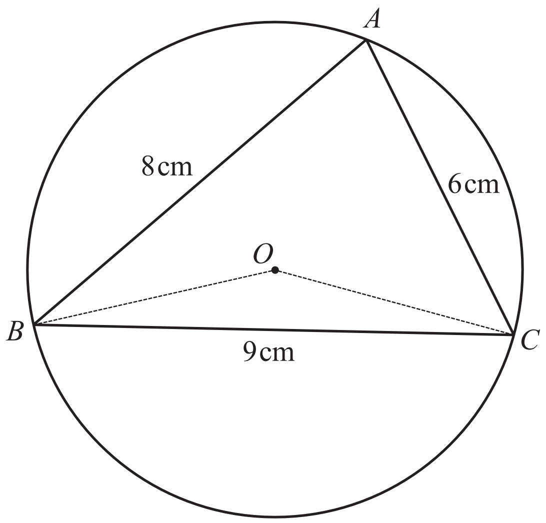
V = $\pi 2.5^2$ x 1.8

V cookie = 35.34 cm^3

The volume of dough left will be:

V = 1080 cm^3 – 35.34 cm^3 x 24

V = 232 cm^3



NOT TO SCALE

The circle, centre O, passes through the points A, B and C.

In the triangle ABC, $AB = 8$ cm, $BC = 9$ cm and $CA = 6$ cm.

(a) Calculate angle BAC and show that it rounds to 78.6°, correct to 1 decimal place. [4]

We use the cosine rule to calculate the size of angle BAC:

$$\cos(BAC) = \frac{AB^2 + CA^2 - BC^2}{2 \times AB \times CA}$$

Plug in the given values:

$$\cos(BAC) = \frac{(8cm)^2+(6cm)^2-(9cm)^2}{2 \times (8cm) \times (6cm)}$$

$$\cos(BAC) = 0.1979..$$

Apply inverse cosine to both sides to find the size of the angle BAC

$$angle\ BAC = arccos(0.1979..)$$

$$angle\ BAC = 78.58\ \ldots°$$

Correct to one decimal place:

$$\boldsymbol{angle\ BAC = 78.6°}$$

(b) M is the midpoint of BC.

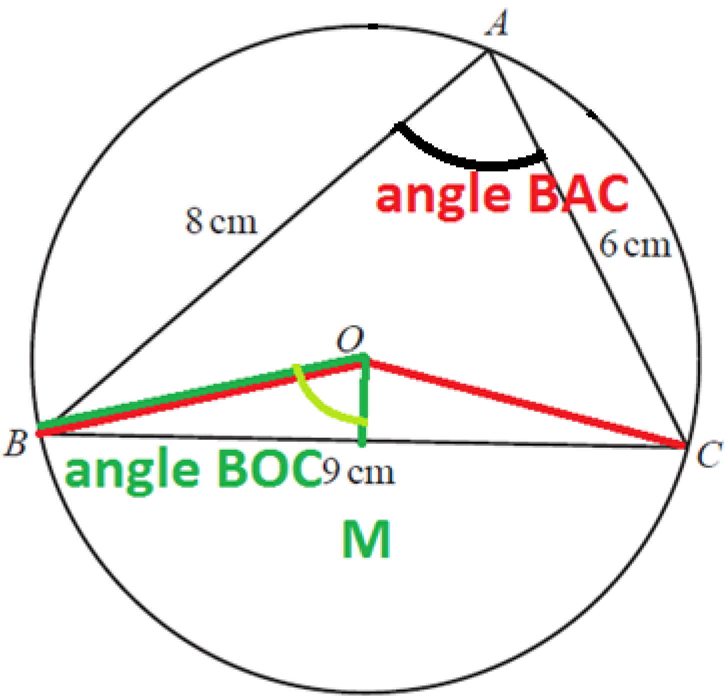
(i) Find angle BOM. [1]

Since M is the midpoint of BC, the size of angle BOM will be half the size of the angle BOC.

$$angle\ BOM = \frac{1}{2} angleBOC$$

The angle subtended at the centre of a circle (BOC) is twice the angle subtended at the edge (BAC) from the same chord. In our case, the chord is the line BC. Hence:

$$angle\ BAC = \frac{1}{2} angle\ BOC$$



Combining these two equations, we have:

$$angle\ BOM = angle\ BAC$$

$$\boldsymbol{angle\ BOM = 78.6°}$$

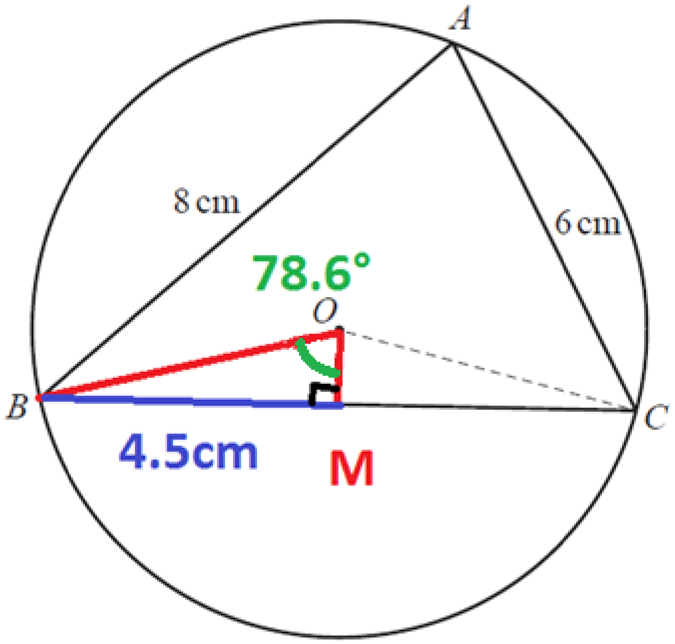
(ii) Calculate the radius of the circle and show that it rounds to 4.59 cm, correct to 3 significant figures. [3]

The size of BM is half the size of BC (M is the midpoint of BC).

$$BM = \frac{1}{2}BC$$

$$BM = 4.5cm$$

We want to find the radius of the circle (BO). To do this, we use trigonometry and the fact that the triangle BOM is a right-angle triangle.



The sine of the angle BOM is equal to the ratio of the opposite side relative to the hypotenuse.

$$\sin(BOM) = \frac{opposite}{hypotenuse}$$

$$\sin(BOM) = \frac{BM}{BO}$$

Multiply both sides by the length of BO (radius).

$$\mathrm{BO} \times \sin(BOM) = BM$$

Divide both sides by the value of sine of BOM.

$$\mathrm{BO} = \frac{BM}{\sin(BOM)}$$

Plug in the values and work out the size of radius BO.

$$\mathrm{BO} = \frac{4.5cm}{\sin(78.6°)}$$

$$\mathrm{BO} = 4.5906\ldots cm$$

Correct to 3 significant figures:

$$\mathbf{BO = 4.59}\boldsymbol{cm}$$

(c) Calculate the area of the triangle ABC as a percentage of the area of the circle. [4]

The area of a triangle ABC as a percentage of the area of

the circle can be obtained from the ratio of their areas.

$$percentage\ area = \frac{triangle\ area}{circle\ area} \times 100\%$$

The area of a circle with radius *r* is given by:

$$circle\ area = \pi r^2$$

In our case:

$$radius = r = OB$$

The area of the triangle with sides *a*, *b* which form an angle C is given by:

$$triangle\ area = \frac{1}{2}ab \times \sin(C)$$

In our case, we pick these sides as AB and CA, and the angle they form is BAC.

$$triangle\ area = \frac{1}{2}AB \times CA \times \sin(BAC)$$

Hence we have the formula for the area as a percentage.

$$percentage\ area = \frac{\frac{1}{2}AB \times CA \times \sin(BAC)}{\pi \times OB^2} \times 100\%$$

Substitute the known values and calculate the area as a percentage:

$$percentage\ area = \frac{\frac{1}{2}(8cm) \times (6cm) \times \sin(78.6°)}{\pi \times (4.59cm)^2} \times 100\%$$

$$\boldsymbol{percentage\ area = 35.5\%}$$

(a) Calculate the volume of a cylinder of radius 31 **centimetres** and length 15 **metres**.
Give your answer in cubic metres. [3]

$V = \pi r^2 \times l$

$= \pi(0.31)^2 \times 15$

$= 1.4415\pi$

$= \mathbf{4.53}$

(b) A tree trunk has a circular cross-section of radius 31 cm and length 15 m.
One cubic metre of the wood has a mass of 800 kg.
Calculate the mass of the tree trunk, giving your answer in tonnes. [2]

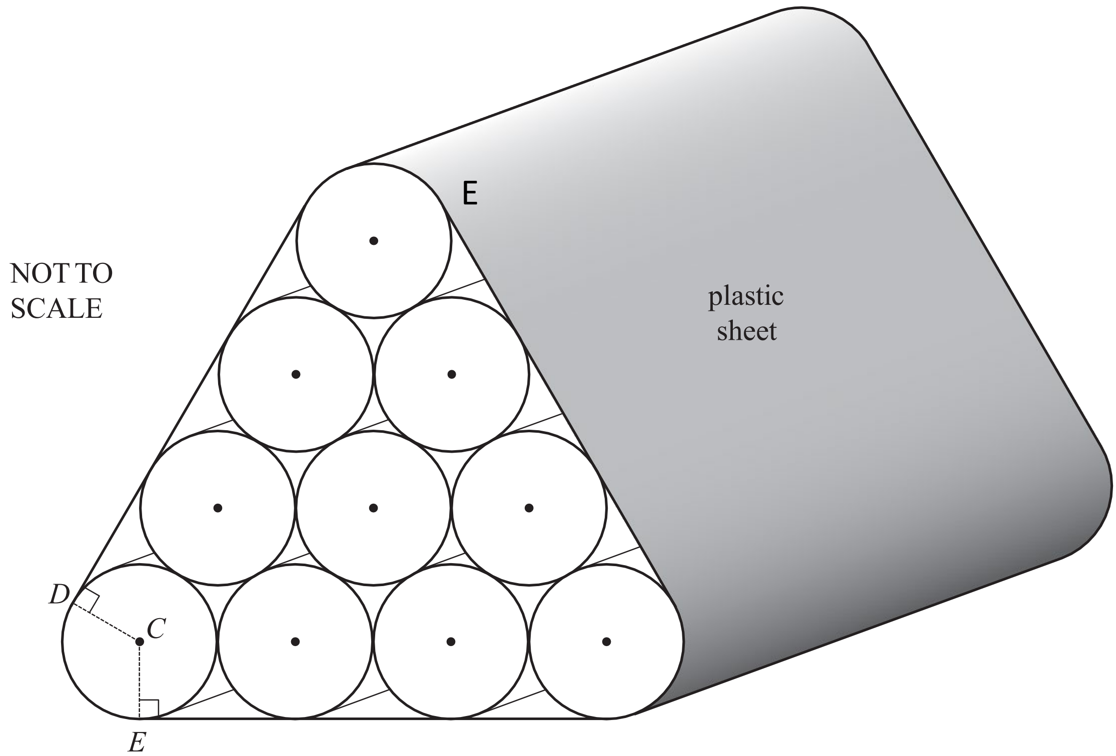
Multiply volume and density for mass

4.53×800

$= 3624\ kg$

$= \mathbf{3.624\ tonnes}$

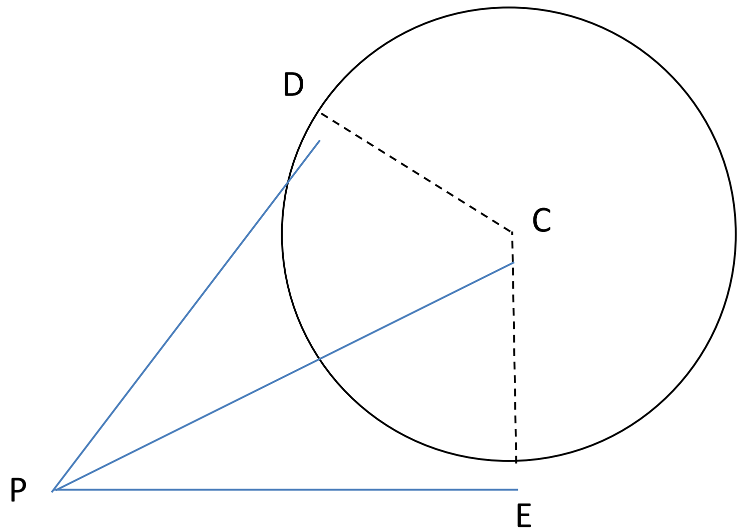
(c)



The diagram shows a pile of 10 tree trunks.
Each tree trunk has a circular cross-section of radius 31 cm and length 15 m.
A plastic sheet is wrapped around the pile.

C is the centre of one of the circles.
CE and *CD* are perpendicular to the straight edges, as shown.

(i) Show that angle *ECD* = 120°. [2]



The line PC bisects the angle ECD.

Angle DPE is half of ECD.

Hence, angle DPC is a quarter of ECD.

All angles in a triangle add to 180, and using triangle DPC

we have

$$90 + \frac{1}{2}ECD + \frac{1}{4}ECD = 180$$

$$\rightarrow \frac{3}{4}ECD = 90$$

$$\rightarrow \boldsymbol{ECD = 120}$$

(ii) Calculate the length of the arc *DE*, giving your answer in metres. [2]

Length of an arc is

$$l = r\theta$$

If θ is measured in radians.

Converting ECD to radians gives us

$$DE = 0.31 \times \frac{120}{180}\pi$$

$$= \boldsymbol{0.649}$$

(iii) The edge of the plastic sheet forms the perimeter of the cross-section of the pile.
The perimeter consists of three straight lines and three arcs.
Calculate this perimeter, giving your answer in metres. [3]

The straight lines are 6 times the radius.

The arcs are what was just calculated in (c)(ii).

$$3 \times 6 \times 0.31 + 3 \times 0.649$$

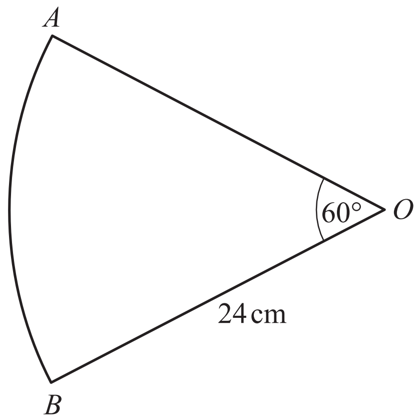
$$= \mathbf{7.527}$$

(iv) The plastic sheet does not cover the two ends of the pile.
Calculate the area of the plastic sheet. [1]

Just multiply by the length

$$7.527 \times 15$$

$$= \mathbf{112.9}$$



(a) The sector of a circle, centre O, radius 24 cm, has angle $AOB = 60°$.

Calculate

(i) the length of the arc AB, [2]

Use $(2 \times \pi \times r \times \theta)/ 360$

$= (2 \times \pi \times 24 \times 60) / 360$

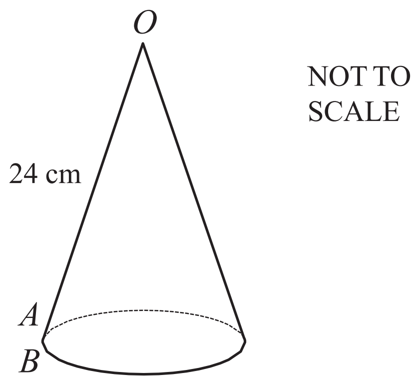
$= 8\pi = 25.1$

(ii) the area of the sector OAB. [2]

$(60 \times \pi \times 24^2)/ 360$

$= 96\pi = 301$

(b) The points A and B of the sector are joined together to make a hollow cone as shown in the diagram. The arc AB of the sector becomes the circumference of the base of the cone.



Calculate

(i) the radius of the base of the cone, [2]

circumference = πd

π x diameter = 8π

diameter = 8

r = 8/2 = 4

(ii) the height of the cone, [2]

Right angle triangle

$a^2 + b^2 = c^2$

$h^2 + r^2 = l^2$

$h^2 + 4^2 = 24^2$

$h = \sqrt{(24^2 - 4^2)}$

$h = 4\sqrt{35}$

= 23.7

(iii) the volume of the cone.
[The volume, V, of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [2]

$V = \pi r^2 h/3$

$V = (\pi \times 4^2 \times 23.7) / 3$

= 396.4

(c) A different cone, with radius x and height y, has a volume W.

Find, **in terms of *W***, the volume of

(i) a similar cone, with both radius and height 3 times larger, [1]

$W = (\pi \times x^2 \times y) / 3$

$V = (\pi \times (3x)^2 \times 3y) / 3 = (27\pi \times x^2 \times y) / 3 = 27 \times W$

27W

(ii) a cone of radius $2x$ and height y. [1]

$V = (\pi \times (2x)^2 \times y) / 3 = (4\pi \times x^2 \times y) / 3 = 4 \times W$

4W

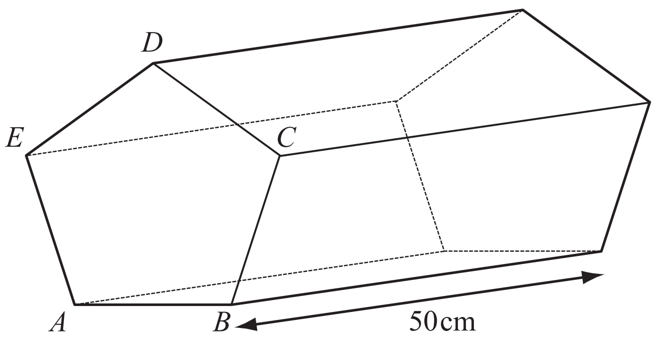


Diagram 1

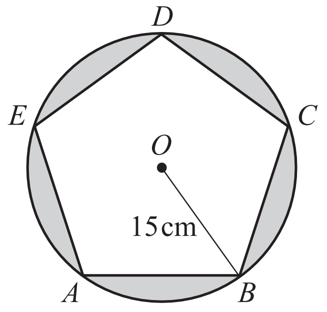


Diagram 2

Diagram 1 shows a solid wooden prism of length 50 cm.

The cross-section of the prism is a regular pentagon $ABCDE$.

The prism is made by removing 5 identical pieces of wood from a solid wooden cylinder.

Diagram 2 shows the cross-section of the cylinder, centre O, radius 15 cm.

(a) Find the angle AOB. [1]

We have that

$$5 \times AOB = 360$$

$$\rightarrow \boldsymbol{AOB = 72}$$

(b) Calculate

(i) the area of triangle AOB, [2]

Area of a triangle is

$$\boldsymbol{A = \frac{1}{2}ab\sin C}$$

Hence

$$A_{AOB} = \frac{1}{2}15^2 \sin 72$$

$$= \boldsymbol{107}$$

(ii) the area of the pentagon $ABCDE$, [1]

The area of the pentagon is just 5 of the previously found identical triangles

$$5 \times 107$$

$$= \mathbf{535}$$

(ii) the volume of wood removed from the cylinder. [4]

The area of the circle is

$$A_c = \pi(15)^2$$

$$= 225\pi$$

Hence the volume of the original cylinder is

$$50 \times 225\pi$$

$$= 11250\pi$$

and the volume of the prism is

$$535 \times 50$$

$$= 26750$$

Hence the volume removed is

$$11250\pi - 26750$$

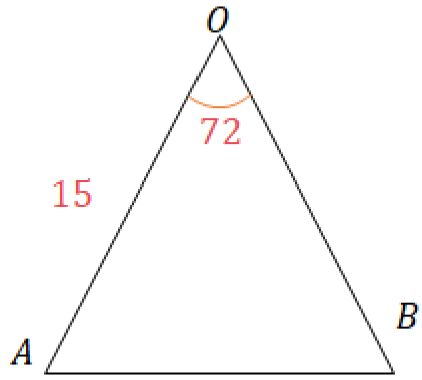
$$= \mathbf{8590}$$

(c) Calculate the total surface area of the prism. [4]

The total surface area is the two pentagonal faces plus 5 long rectangular sides.

$$A = 2 \times 535 + 5 \times AB \times 50$$

The length of AB is found using



$$\sin\theta = \frac{opp}{hyp}$$

$$\rightarrow \sin\left(\frac{72}{2}\right) = \frac{1}{2}\frac{AB}{15}$$

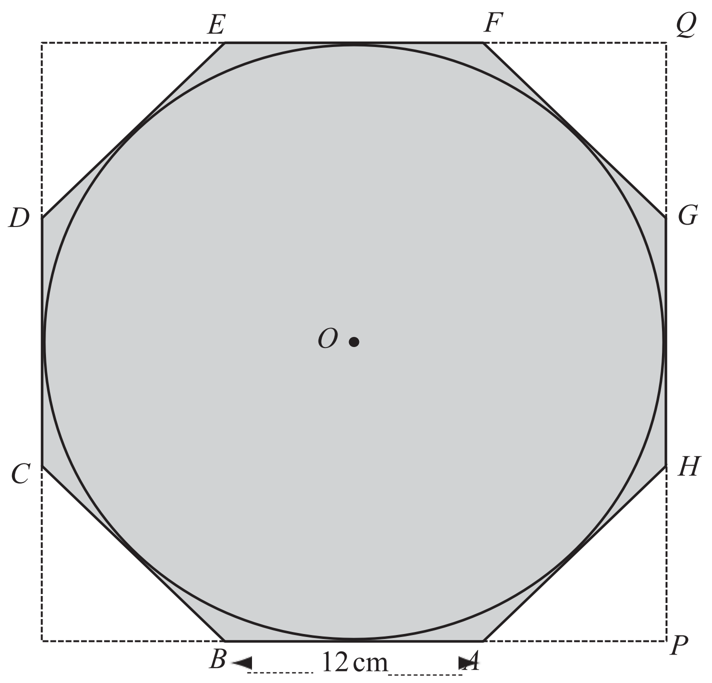
$$\rightarrow AB = 30\sin 36$$

$$= 17.63$$

Hence, we have

$$A = 1070 + 250 \times 17.63$$

$$= \mathbf{5477.5}$$



A circle, centre O, touches all the sides of the regular octagon $ABCDEFGH$ shaded in the diagram.

The sides of the octagon are of length 12 cm.

BA and GH are extended to meet at P. HG and EF are extended to meet at Q.

(a) (i) Show that angle BAH is 135°. [2]

All the angles in a polygon sum to

$$S = (n-2) \times 180$$

Hence

$$S = 6 \times 180$$

$$= 1080$$

It is a regular shape, so

$$BAH = \frac{1080}{8}$$

$$= \mathbf{135}$$

(ii) Show that angle APH is $90°$. [1]

We have that

$$PAH = 180 - 135$$

$$= 45$$

and

$$PHA = 180 - 135$$

$$= 45$$

And since angles in a triangle sum to 180 we have that

$$APH + 45 + 45 = 180$$

$$\rightarrow \boldsymbol{APH = 90}$$

(b) Calculate

(i) the length of *PH*, [2]

Using the trig relation

$$\sin\theta = \frac{opp}{hyp}$$

we have

$$\sin PAH = \frac{PH}{AH}$$

$$\rightarrow \sin 45 = \frac{PH}{12}$$

$$\rightarrow PH = 12 \times \sin 45$$

$$= 6\sqrt{2}$$

$$= \mathbf{8.49}$$

(ii) the length of *PQ*, [2]

$$PQ = PH + HG + GQ$$

$$= 12 + 2 \times 8.49$$

$$= \mathbf{29.0}$$

(iii) the area of triangle *APH*, [2]

Area of a triangle is

$$A = \frac{1}{2} base \times height$$

$$\rightarrow A = \frac{1}{2} 6\sqrt{2} \times 6\sqrt{2}$$

$$= \mathbf{36}$$

(iv) the area of the octagon. [3]

Area of the square minus the area of the four triangles.

$$A = 29 \times 29 - 4 \times 36$$

$$= \mathbf{697}$$

(c) Calculate

(i) the radius of the circle, [2]

The diameter of the circle is the same length as a side of the square, hence

$$2r = 29.0$$

$$\rightarrow \boldsymbol{r = 14.5}$$

(ii) the area of the circle as a percentage of the area of the octagon. [3]

Area of the circle

$$A = \pi r^2$$

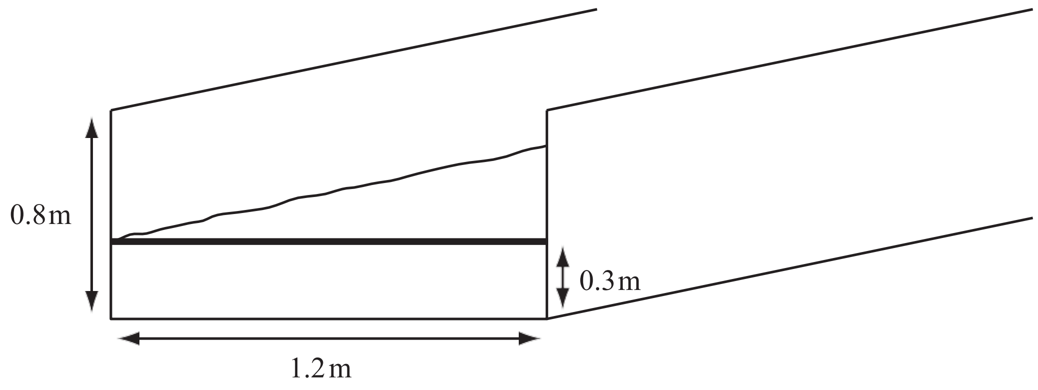
$$= \pi(14.5)^2$$

$$= 660.5$$

As a percentage, this is

$$\frac{660.5}{697} \times 100\%$$

$$= \mathbf{94.8\%}$$



NOT TO SCALE

The diagram shows water in a channel.

This channel has a rectangular cross-section, 1.2 metres by 0.8 metres.

(a) When the depth of water is 0.3 metres, the water flows along the channel at 3 metres/minute.

Calculate the number of cubic metres which flow along the channel in one hour. [3]

The volume of the water that goes through per minute will be:

3 m/minute x 0.3 m depth x 1.2 m width = 1.08 m^3/minute

For 60 minutes, the volume will be: 60 minutes x 1.08 m^3/minute

= 64.8 m^3

(b) When the depth of water in the channel increases to 0.8 metres, the water flows at 15 metres/minute.

Calculate the percentage increase in the number of cubic metres which flow along the channel in one hour.

[4]

We need to work out the volume of water which goes through at the new rate and depth.

Then, we can calculate the percentage increase relative to the amount obtained at a).

15 m/minutes x 0.8 m depth x 1.2 m width = 14.4 m^3/minute

For 60 minutes, the volume will be: 60 minutes x 14.4 m^3/minute = 864 m^3

We represent the percentage with the unknown x.

$64.8\ m^3 + 64.8\ m^3 \times \frac{x}{100} = 864\ m^3$

We solve the equality for x.

$64.8x\ m^3 = 79920\ m^3$

x = 1233.3%

(c) The water comes from a cylindrical tank.

When 2 cubic metres of water leave the tank, the level of water in the tank goes down by 1.3 **millimetres**.

Calculate the radius of the tank, in **metres**, correct to one decimal place. [4]

The formula for the volume of a cylinder is:

V = $\pi r^2 h$

For h – 1.3 millimetres the volume becomes V – 2 m^3.

Therefore, 2 m^3 is the volume of a cylinder with the height 1.3 millimetres.

We convert mm in m since the result needs to be in m.

1.3 mm = 0.0013 m

2 m^3 = $\pi r^2 x\ 0.0013m$

πr^2 = 1538.4 m^2

r = 22.1 m

(d) When the channel is empty, its **interior** surface is repaired.

This costs \$0.12 per square metre. The total cost is \$50.40.

Calculate the length, in metres, of the channel. [4]

The channel has the shape of a rectangle with the top open, so the perimeter will be:

0.8 m x 2 + 1.2 m = 2.8 m

The perimeter multiplied by the length of the channel gives the surface area.

The area is also equal to:

$\frac{\$50.4}{\$0.12/\mathrm{m}^2}$ = 420 m^2

2.8 m x length = 420 m^2

Length = 150 m

Perimeters, Area and Volumes
Difficulty: Hard

Model Answers 5

Level	IGCSE
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Time allowed: **85 minutes**

Score: **/74**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a) Calculate the volume of a cylinder with radius 30 cm and height 50 cm. [2]

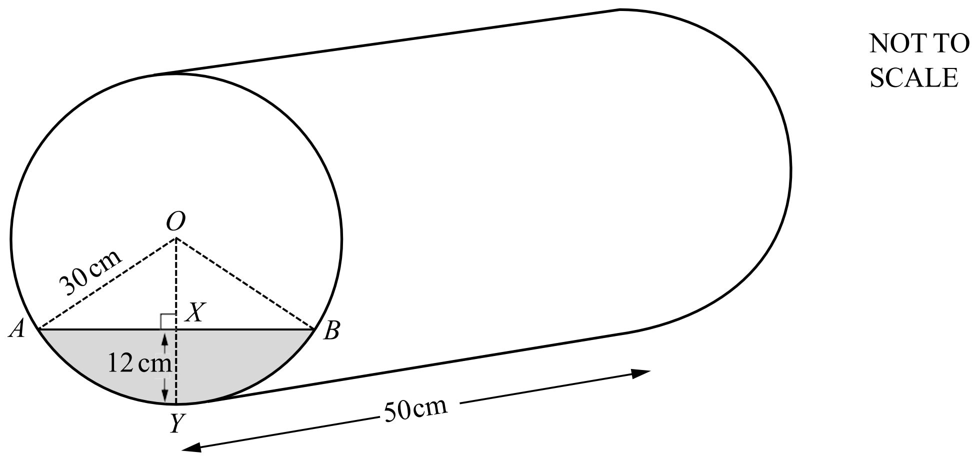
The formula for the volume of a cylinder is:

V = $\pi r^2 h$

V = $\pi 30^2 x\ 50$

V = 141 000 cm^3

(b)



A cylindrical tank, radius 30 cm and length 50 cm, lies on its side. It is partially filled with water.
The shaded segment $AXBY$ in the diagram shows the cross-section of the water. The greatest depth, XY, is 12 cm.
$OA = OB = 30$ cm.

(i) Write down the length of OX. [1]

The radius OY = 30 cm.

OX = OY – XY

OX = 30 cm – 12 cm

OX = 18 cm

(ii) Calculate the angle AOB correct to two decimal places, showing all your working. [3]

OA = AB = 30 cm

Therefore, the triangle AOB is isosceles.

The angle AOB will be twice the size of the angle AOX.

In the right-angled triangle AOX:

cos AOX = $\frac{\text{OX}}{\text{OA}}$

cos AOX = $\frac{18\text{ cm}}{30\text{ cm}}$

cos AOX = $\frac{3}{5}$

Angle AOX = 53.13°

Angle AOB = 2 x Angle AOX

Angle AOB = 106.26°

(c) Using angle $AOB = 106.3°$, find

(i) the area of the sector $AOBY$, [3]

The formula for the area of a sector is:

A = $\pi r^2 \frac{\text{Angle AOB}}{360°}$

A = $\pi 30^2 \frac{106.3°}{360°}$

A = 835 cm²

(ii) the area of triangle AOB, [2]

The formula for the area of a triangle is:

$A = \frac{\text{AB x OX}}{2}$

AB will be twice the size of the side AX, since the 2

triangles AOX and BOX are congruent.

In the right-angled triangle AOX:

$\sin \text{AOX} = \frac{\text{AX}}{\text{AO}}$

$\sin 53.13° = \frac{\text{AX}}{30 \text{ cm}}$

AX = 23.99 cm

AB =2 x AX

AB = 2 x 23.99 cm

AB = 48 cm

$A = \frac{48 \text{ cm x } 18 \text{ cm}}{2}$

A = 432 cm^2

(iii) the area of the shaded segment $AXBY$. [1]

The area AXBY is equal to the area of the sector minus the

area of the triangle AOB.

Area AXBY = 835 cm^2 – 432 cm^2

Area AXBY = 403 cm^2

(d) Calculate the volume of water in the cylinder, giving your answer

(i) in cubic centimetres, [2]

The volume of water can be worked out by multiplying the area of the AXBY region by the height, 50 cm. AXBY represents the base of the cylinder containing water.

V = 403 cm^2 x 50 cm

V = 20 150 cm^3

(ii) in litres. [1]

1 L = 1 dm^3

We need to convert cm^3 in dm^3 by dividing the amount by 1000.

V = 20 1500 cm^3/1000

V = 20.1 dm^3 = 20.1 L

(e) How many more litres must be added to make the tank half full? [2]

The amount which needs to be added is represented by the difference between the volume of the cylinder half full, 70 500 cm^3 (from a)/2), and the volume of water already in the tank, 20.1 L.

We convert the total volume in L.

V = 70 500 cm^3 = 70.5 L

The volume of water which needs to be added is:

V = 70.5 L – 20.1 L

V = 50.4 L

In each of the diagrams below, triangle ABC is an isosceles right-angled triangle.
AB # AC # 6 cm.
A straight line or a circular arc divides the triangle into two parts, one of which is shaded.

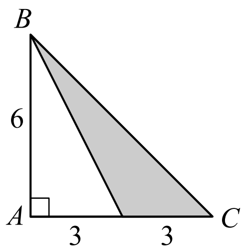


Diagram 1

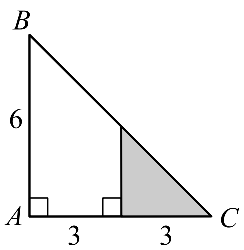


Diagram 2

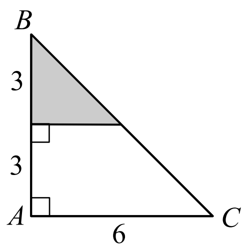


Diagram 3

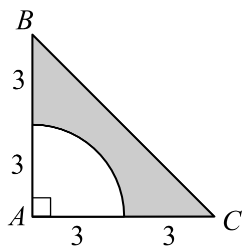


Diagram 4

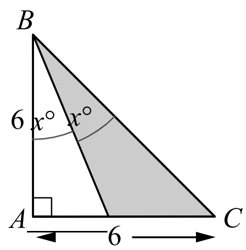


Diagram 5

(a) Which diagram has a shaded region showing **all** the points in the triangle which are

(i) closer to BC than to BA, [1]

The locus of points that are closer to BC than to BA are at the right of the bisector for the angle ABC, separating it into 2 congruent angles, x.

Diagram 5.

(ii) more than 3 cm from A, [1]

The locus of points which are exactly 3 cm away from point A are the outline of a circle with the centre in A and radius = 3cm. To be more than 3 cm away, the shaded region would be outside this circle. **Diagram 4.**

(iii) closer to C than to A? [1]

The locus of points that are closer to C than to A are at the right of a perpendicular bisector on the side AC. **Diagram 2.**

(b) For **each** of the five diagrams, calculate the shaded area. [11]

Diagram 1:

The area of a right-angled triangle is calculated by the formula:

$A = \frac{a \times h}{2}$

Where a is the base and h is the side perpendicular on the base.

In our case, the area is:

$A = \frac{6 \times 6}{2} = 18\ cm^2$

The shaded area represents half of the total area of the triangle.

Shaded area = 9 cm^2

Diagram 2:

The shaded area represents a right-angled triangle with the side 3 cm perpendicular on the base which is half the size of the side of the triangle 6 cm.

$A = \frac{3 \times 3}{2}$

= 4.5 cm^2

Diagram 3:

The shaded area in this case has the same area as the one in Diagram 2,

4.5 cm^2.

Diagram 4:

The shaded area in this case represents the area of the triangle minus the area of a quarter of a circle with the radius 3 cm.

Area of a circle $= \pi r^2$

Area of a circle $= 9\pi$ cm^2

Area of a quarter of a circle = $9\pi/4$ = 7.06 cm^2

Shaded area = 18 cm^2 – 7.06 cm^2

Shaded area = 10.94 cm^2

Diagram 5:

The triangle is isosceles with one right angle.

The angle ABC and ACB are equal and therefore the angle x can be calculated by:

x = (180° - 90°)/4 = 22.5°

tan 22.5° = side of the unshaded triangle/6cm

The side of the unshaded triangle is = 2.485 cm

The area of the shaded region is the area of the triangle minus the area of the unshaded triangle.

Shaded area = 18 cm^2 – (6 cm x 2.485 cm)/2

Shaded area = 10.5 cm^2

(a) Calculate the area of an equilateral triangle with sides 10 cm. [2]

The area of a quadrilateral triangle is:

$$A = \frac{a \times b \times \sin C}{2}$$

Where a and b are sides in the triangle and C is the angle between them

$$A = \frac{10 \times 10 \times \sin 60^\circ}{2}$$

A = 43.3 cm²

(b) Calculate the radius of a circle with circumference 10 cm. [2]

The circumference of a circle has the formula:

$C = 2\pi r$

$10 \text{ cm} = 2\pi r$

r = 1.59 cm

(c)

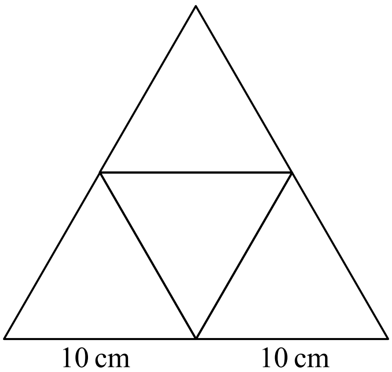


Diagram 1

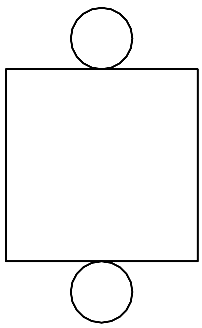


Diagram 2

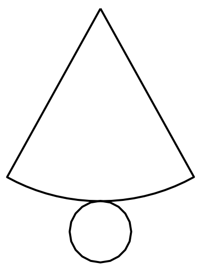


Diagram 3

The diagrams represent the nets of 3 solids. Each straight line is 10 cm long. Each circle has circumference 10 cm. The arc length in Diagram 3 is 10 cm.

(i) Name the solid whose net is Diagram 1. Calculate its surface area. [3]

The solid is a tetrahedron.

The surface area will be 4 times the area of an equilateral triangle with

the side 10 cm.

A = 4 x 43.3 cm^2

A = 173.2 cm^2

(ii) Name the solid whose net is Diagram 2. Calculate its volume. [4]

The solid is a cylinder.

The formula for the volume of a cylinder is:

A = 10 cm x $\pi\ x\ 1.59^2$

A = 79.55 cm^2

(iii) Name the solid whose net is Diagram 3. Calculate its perpendicular height. [4]

The solid is a cone.

A right-angled triangle is formed by the height

perpendicular on the cone's base.

In this triangle with a hypothenuse 10 cm, the height can

be worked out by using Pythagoras' Theorem.

$10^2 = h^2 + r^2$

h = $\sqrt{10^2 + 1.59^2}$

h = 9.87

(a)

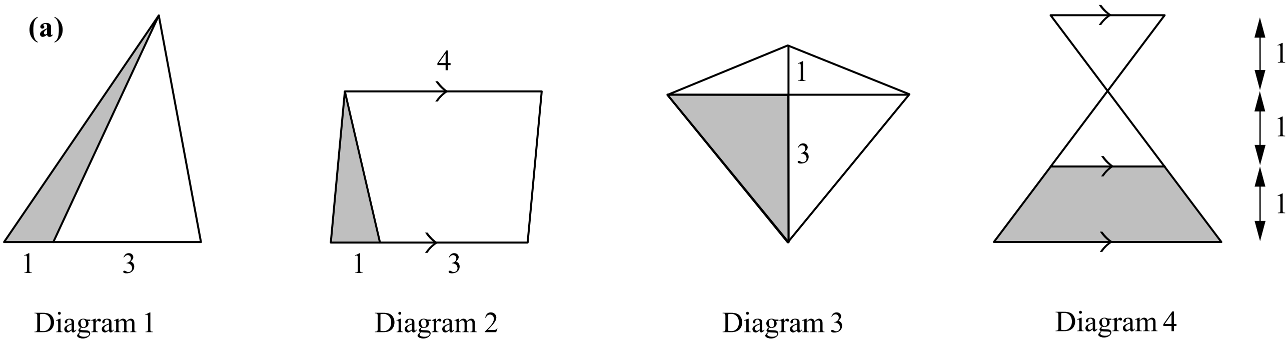


Diagram 1 shows a triangle with its base divided in the ratio 1 : 3.

Diagram 2 shows a parallelogram with its base divided in the ratio 1 : 3.

Diagram 3 shows a kite with a diagonal divided in the ratio 1 : 3.

Diagram 4 shows two congruent triangles and a trapezium each of height 1 unit.

For each of the four diagrams, write down the **percentage** of the total area which is shaded. [7]

Diagram 1 is a triangle divided into a region with base 1 and a region with base 3. This separates the area into a quarter (the shaded region) and 3 quarters (the unshaded region).

Therefore, the shaded region represents 25% of the triangle area.

Diagram 2 is a trapezium with the base separated into a 1 region and a 3 region. This as well separates the trapezium into a quarter of the total area and three quarters of the total area.

The shaded area is a triangle which represents half of a small parallelogram representing q quarter of the area of the big parallelogram.

The shaded area represents 25/2

= **12.5%**

Diagram 3 is a kite with its diagonal divided into 1 and 3.

The area of the kite is therefore divided into a quarter, and 3 quarters.

The shaded area is half of the 3 quarters of the total area of the kite.

A = $\frac{1}{2}$ x 75%

= **37.5%**

In Diagram 4, the shaded area represents approximately 3 times the area of a small triangle from the same diagram.

Therefore, the shaded area represents $\frac{3}{5}$ of the total area of the shape.

For 100% total area, the shaded area is 60%.

(b)

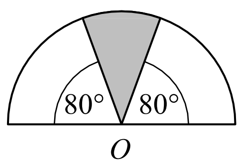


Diagram 5

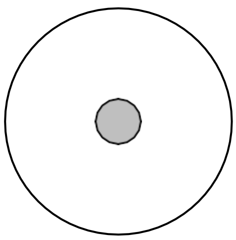


Diagram 6

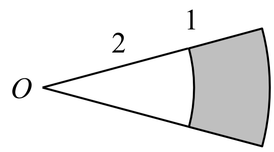


Diagram 7

Diagram 5 shows a semicircle, centre O.

Diagram 6 shows two circles with radii 1 unit and 5 units.

Diagram 7 shows two sectors, centre O, with radii 2 units and 3 units.

For each of diagrams 5, 6 and 7, write down the **fraction** of the total area which is shaded. [6]

Diagram 5:

Area of a sector = $\pi r^2 \frac{\text{sector angle}}{360°}$

Area of the shaded sector = $\pi r^2 \frac{180°-160°}{360°}$ = $\pi r^2 \frac{20°}{360°}$

The ratio of the sectors

$$= \frac{\text{Area shaded sector}}{\text{Area semicricle}} = \frac{\pi r^2 \frac{20°}{360°}}{\pi r^2 \frac{180°}{360°}} = \mathbf{\frac{1}{9}}$$

Diagram 6:

$A = \pi r^2$

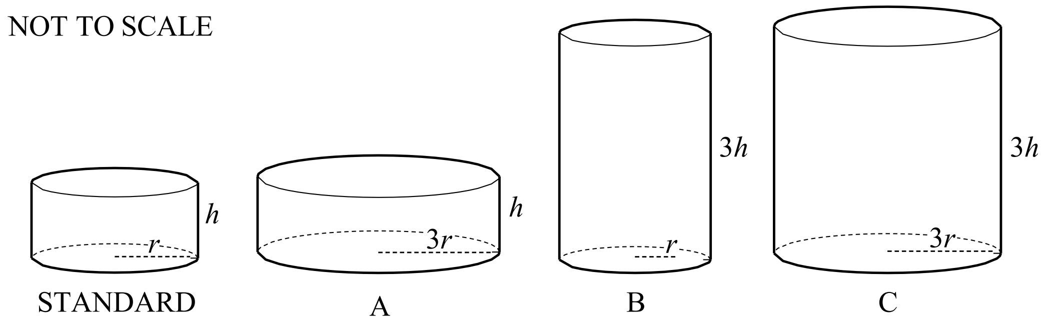
The ratio of the circles

$$= \frac{\text{Area shaded circle}}{\text{Area total circle}} = \frac{\pi 1^2}{\pi 5^2} = \mathbf{\frac{1}{25}}$$

Diagram 7:

The ratio of the sectors = $\frac{\text{Area unshaded sector}}{\text{Area total sector}} = \frac{\pi 2^2 \frac{x°}{360°}}{\pi 3^2 \frac{x°}{360°}} = \frac{4}{9}$

The fraction of the shaded are will be: 1 - $\frac{4}{9}$ = $\frac{5}{9}$



Sarah investigates cylindrical plant pots.
The standard pot has base radius r cm and height h cm.
Pot A has radius $3r$ and height h. Pot B has radius r and height $3h$. Pot C has radius $3r$ and height $3h$.

(a) **(i)** Write down the volumes of pots A, B and C in terms of π , r and h. [3]

The formula for the volume of a cylinder is:

V = $\pi r^2 h$

For Pot A:

V = $\pi(3r)^2 h$

V = $9\pi r^2 h$

For Pot B:

V = $\pi r^2 3h$

V = $3\pi r^2 h$

For Pot C:

V = $\pi(3r)^2 3h$

V = $27\pi r^2 h$

(ii) Find in its lowest terms the ratio of the volumes of A : B : C. [2]

A : B : C = $9\pi r^2 h : 3\pi r^2 h : 27\pi r^2 h$

A : B : C = 3 : 1 : 9

(iii) Which one of the pots A, B or C is mathematically similar to the standard pot? Explain your answer. [2]

Pot C is the one similar to the standard cylinder.

This is because similar shapes will have the ratio of their corresponding lengths equal.

In case of Pot C:

$$\frac{r}{3r} = \frac{h}{3h} = \frac{1}{3}$$

(iv) The surface area of the standard pot is S cm^2. Write down in terms of S the surface area of the similar pot. [2]

Since the 2 are similar shapes, the square of the ratio of its lengths will be equal to the ratio of their corresponding areas.

$$\left(\frac{r}{3r}\right)^2 = \frac{\text{S cm}^2}{\text{Area of a similar pot}}$$

$$\left(\frac{1}{3}\right)^2 = \frac{\text{S cm}^2}{\text{Area of a similar pot}}$$

Area of a similar pot = 9S cm²

(b) Sarah buys a cylindrical plant pot with radius 15 cm and height 20 cm. She wants to paint its outside surface (base and curved surface area).

(i) Calculate the area she wants to paint. [2]

The area of the plant pot is equal to the area of the base, a circle, plus the area of the curved cylinder surface.

$$A = \pi r^2 + 2\pi r h$$

For r = 15 cm and h = 20 cm.

$$A = \pi 15^2 + 2\pi \times 15 \times 20$$

A = 2590 cm²

(ii) Sarah buys a tin of paint which will cover 30 m^2.
How many plant pots of this size could be painted on their outside surfaces **completely** using this tin of paint? [4]

We convert the surface area in m^2.

$A = 2590\ cm^2 = 0.259\ m^2$

The number of pots is: $\frac{30\ m^2}{0.259\ m^2}$

Number of pots = 115.83

Therefore, 115 pots can be painted completely.