Sequences
Difficulty: Medium

Question Paper 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Medium
Booklet	Question Paper 1

Time allowed: **63 minutes**

Score: **/55**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) Assembled by A-S

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a) The nth term of a sequence is $8n - 3$.

(i) Write down the first two terms of this sequence. [1]

(ii) Show that the number 203 is not in this sequence. [2]

(b) Find the nth term of these sequences.

(i) 13, 19, 25, 31, … [2]

(ii) 4, 8, 14, 22, … [2]

(c) … , 20, 50, …

The second term of this sequence is 20 and the third term is 50.
The rule for finding the next term in this sequence is subtract y then multiply by 5.

Find the value of y and work out the first term of this sequence. [4]

(a) Complete the table for the four sequences A, B, C and D.

	Sequence				Next term	*n*th term
A	2	5	8	11		
B	20	14	8	2		
C	1	4	9	16		
D	0	2	6	12		

[10]

(b) The sum of the first n terms of a sequence is $\frac{n(3n+1)}{2}$.

(i) When the sum of the first n terms is 155, show that $3n^2 + n - 310 = 0$. [2]

(ii) Solve $3n^2 + n - 310 = 0$. [3]

(iii) Complete the statement.

The sum of the first ..... terms of this sequence is 155. [1]

The first three diagrams in a sequence are shown below.
The diagrams are made by drawing lines of length 1 cm.

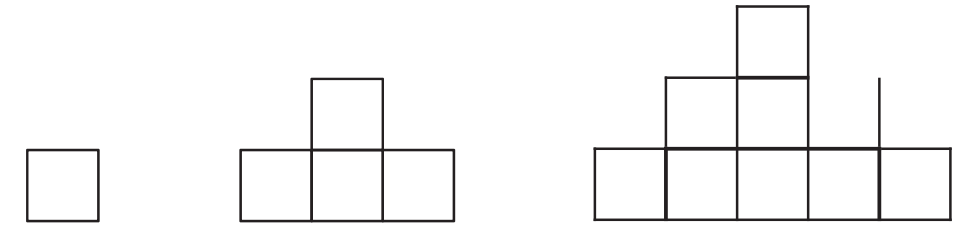

Diagram 1 Diagram 2 Diagram 3

(a) The areas of each of the first three diagrams are shown in this table.

Diagram	1	2	3
Area (cm^2)	1	4	9

(i) Find the area of Diagram 4. [1]

(ii) Find, in terms of n, the area of Diagram n. [1]

(b) The numbers of 1 cm lines needed to draw each of the first three diagrams are shown in this table.

Diagram	1	2	3
Number of 1 cm lines	4	13	26

(i) Find the number of 1 cm lines needed to draw Diagram 4. [1]

(ii) In which diagram are 118 lines of length 1 cm needed? [1]

(c) The **total** number of 1 cm lines needed to draw both Diagram 1 and Diagram 2 is 17.
The **total** number of 1 cm lines needed to draw all of the first n diagrams is

$$\frac{2}{3}n^3 + an^2 + bn.$$

Find the value of a and the value of b.
Show all your working. [6]

Complete the table for each sequence.

Sequence	1st term	2nd term	3rd term	4th term	5th term	6th term		*n*th term
A	15	8	1	–6				
B	$\frac{5}{18}$	$\frac{6}{19}$	$\frac{7}{20}$	$\frac{8}{21}$				
C	2	5	10	17				
D	2	6	18	54				

[11]

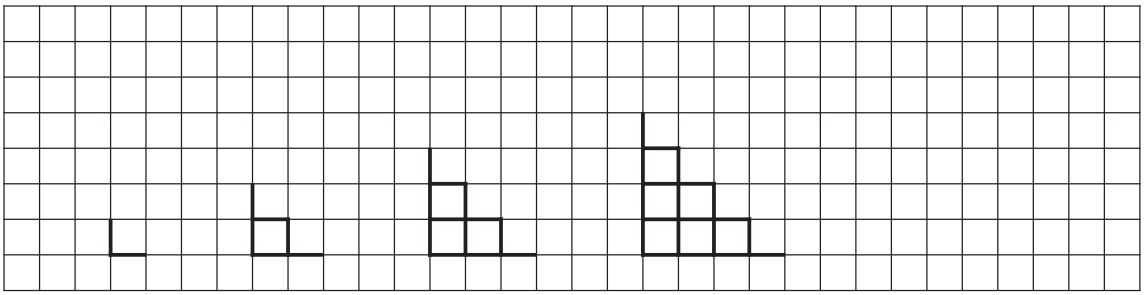

Diagram 1 Diagram 2 Diagram 3 Diagram 4 Diagram 5

Diagram 1 shows two lines of length 1 unit at right angles forming an L.

Two Ls are added to Diagram 1 to make Diagram 2. This forms one small square.

Three Ls are added to Diagram 2 to make Diagram 3. This forms three small squares.
The sequence of Diagrams continues.

(a) Draw Diagram 5. [1]

(b) Complete the table.

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5
Number of lines of length 1 unit	2	6	12	20	
Number of small squares	0	1	3	6	

[2]

(c) Find an expression, in terms of n, for the number of lines of length 1 unit in Diagram n. [2]

(d) Find an expression, in terms of n, for the number of small squares in Diagram n. [2]

Sequences
Difficulty: Medium

Question Paper 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Medium
Booklet	Question Paper 2

Time allowed: **66 minutes**

Score: **/57**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

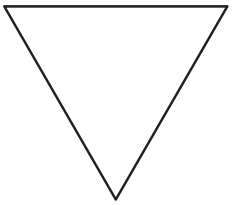

Diagram 1

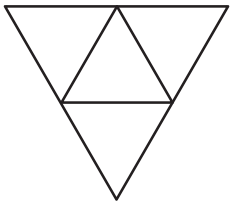

Diagram 2

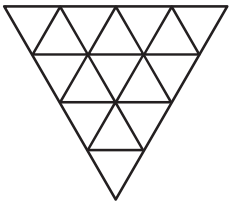

Diagram 3

The first three diagrams in a sequence are shown above.
Diagram 1 shows an equilateral triangle with sides of length 1 unit.

In Diagram 2, there are 4 triangles with sides of length $\frac{1}{2}$ unit.

In Diagram 3, there are 16 triangles with sides of length $\frac{1}{4}$ unit.

(a) Complete this table for Diagrams 4, 5, 6 and n. [6]

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5	Diagram 6		Diagram n
Length of side	1	$\frac{1}{2}$	$\frac{1}{4}$					
Length of side as a power of 2	2^0	2^{-1}	2^{-2}					

(b) (i) Complete this table for the number of the smallest triangles in Diagrams 4, 5 and 6. [2]

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5	Diagram 6
Number of smallest triangles	1	4	16			
Number of smallest triangles as a power of 2	2^0	2^2	2^4			

(ii) Find the number of the smallest triangles in Diagram n, giving your answer as a power of 2. [1]

(c) Calculate the number of the smallest triangles in the diagram where the smallest triangles have sides of Length $\frac{1}{128}$ unit. [2]

Complete the table for the following sequences.
The first row has been completed for you.

	Sequence				Next two terms		*n*th term	
	1	5	9	13	17	21	$4n - 3$	
(a)	12	21	30	39				[3]
(b)	80	74	68	62				[3]
(c)	1	8	27	64				[2]
(d)	2	10	30	68				[2]

(a) The nth term of a sequence is $n(n+1)$.

(i) Write the two missing terms in the spaces. 2, 6, ..., 20, ... [2]

(ii) Write down an expression in terms of n for the $(n + 1)$th term. [1]

(iii) The difference between the nth term and the $(n + 1)$th term is $pn + q$.

Find the values of p and q. [2]

(iv) Find the positions of the two consecutive terms which have a difference of 140. [2]

(b) A sequence $u_1, u_2, u_3, u_4, \ldots\ldots\ldots\ldots$ is given by the following rules.

$u_1 = 2$, $u_2 = 3$ and $u_n = 2u_{n-2} + u_{n-1}$ for $n \geqslant 3$.

For example, the third term is u_3 and $u_3 = 2u_1 + u_2 = 2 \times 2 + 3 = 7$.
So, the sequence is 2, 3, 7, u_4, u_5, …..

(i) Show that $u_4 = 13$. [1]

(ii) Find the value of u_5. [1]

(iii) Two consecutive terms of the sequence are 3413 and 6827 .

Find the term before and the term after these two given terms. [2]

(a) The first five terms P_1, P_2, P_3, P_4 and P_5 of a sequence are given below.

$1 = 1 = P_1$

$1 + 2 = 3 = P_2$

$1 + 2 + 3 = 6 = P_3$

$1 + 2 + 3 + 4 = 10 = P_4$

$1 + 2 + 3 + 4 + 5 = 15 = P_5$

(i) Write down the next term, P_6, in the sequence 1, 3, 6, 10, 15… [1]

(ii) The formula for the nth term of this sequence is

$$P_n = \frac{1}{2} n(n + 1).$$

Show this formula is true when $n = 6$. [1]

(iii) Use the formula to find P_{50}, the 50th term of this sequence. [1]

(iv) Use your answer to **part (iii)** to find $3 + 6 + 9 + 12 + 15 + \ldots\ldots\ldots\ldots + 150$. [1]

(v) Find $1 + 2 + 3 + 4 + 5 + \ldots\ldots\ldots\ldots + 150$. [1]

(vi) Use your answers to **parts (iv)** and (v) to find the sum of the numbers less than 150 which are **not** multiples of 3. [1]

(b) The first five terms, S_1, S_2, S_3, S_4 and S_5 of a different sequence are given below.

(1×1) $= 1 = S_1$

$(1 \times 2) + (2 \times 1)$ $= 4 = S_2$

$(1 \times 3) + (2 \times 2) + (3 \times 1)$ $= 10 = S_3$

$(1 \times 4) + (2 \times 3) + (3 \times 2) + (4 \times 1)$ $= 20 = S_4$

$(1 \times 5) + (2 \times 4) + (3 \times 3) + (4 \times 2) + (5 \times 1)$ $= 35 = S_5$

(i) Work out the next term, S_6, in the sequence 1, 4, 10, 20, 35… [2]

(ii) The formula for the nth term of this sequence is

$$S_n = \frac{1}{6} n(n + 1)(n + 2).$$

Show this formula is true for $n = 6$. [1]

(iii) Find $(1 \times 20) + (2 \times 19) + (3 \times 18)$ …… $+ (20 \times 1)$. [1]

(c) Show that $S_6 - S_5 = P_6$, where P_6 is your answer to **part (a)(i).** [1]

(d) Show by algebra that $S_n - S_{n-1} = P_n$. $\left[P_n = \frac{1}{2} n(n + 1)\right]$ [3]

In all the following sequences, after the first two terms, the rule is to add the previous two terms to find the next term.

(a) Write down the next two terms in this sequence.

1 1 2 3 5 8 13 .... [1]

(b) Write down the first two terms of this sequence.

.... 3 11 14 [2]

(c) (i) Find the value of d and the value of e.

2 d e 10 [3]

(ii) Find the value of x, the value of y and the value of z.

-33 x y z 18 [5]

Sequences
Difficulty: Medium

Question Paper 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Medium
Booklet	Question Paper 3

Time allowed: **59 minutes**

Score: **/51**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

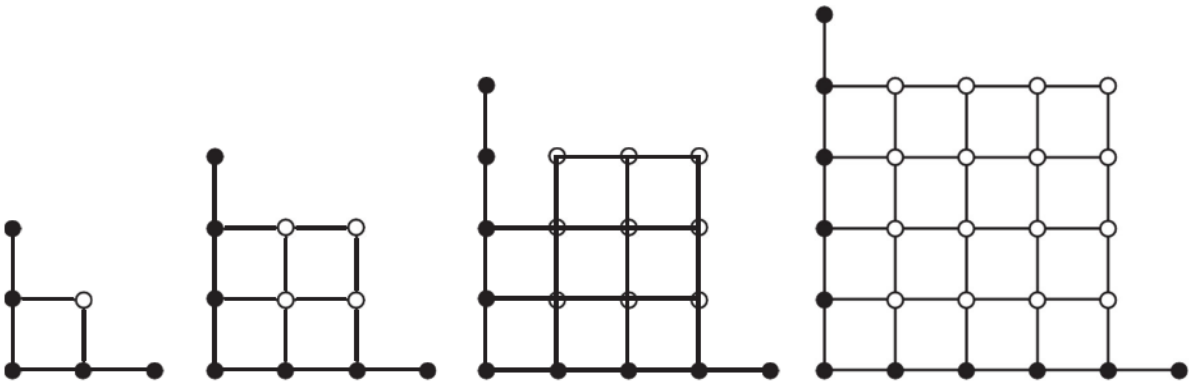

Diagram 1	Diagram 2	Diagram 3	Diagram 4
1 white dot	4 white dots	9 white dots	16 white dots
5 black dots	7 black dots	9 black dots	11 black dots
6 lines	14 lines	26 lines	42 lines

The four diagrams above are the first four of a pattern.

(a) Diagram 5 has been started below.
Complete this diagram and write down the information about the numbers of dots and lines.

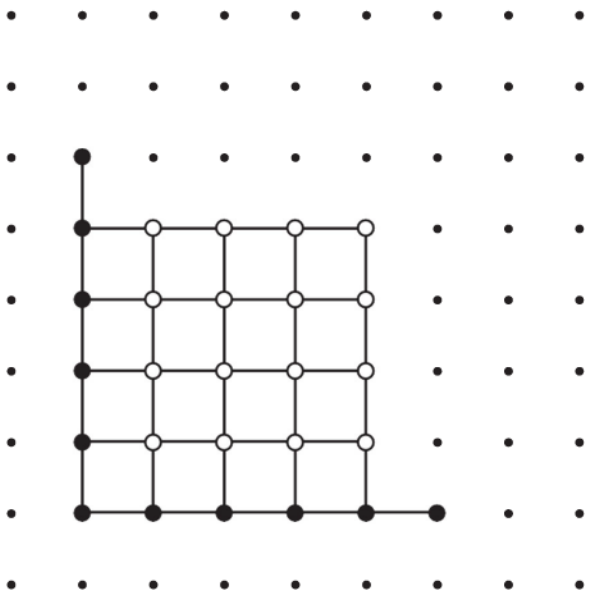

Diagram 5

..... white dots

..... black dots

..... lines

[4]

(b) Complete the information about the number of dots and lines in Diagram 8. [3]

(c) Complete the information about the number of dots in Diagram n.
Give your answers in terms of n. [2]

(d) The number of lines in diagram n is $k(n^2 + n + 1)$.

Find

(i) the value of k, [1]

(ii) the number of lines in Diagram 100. [1]

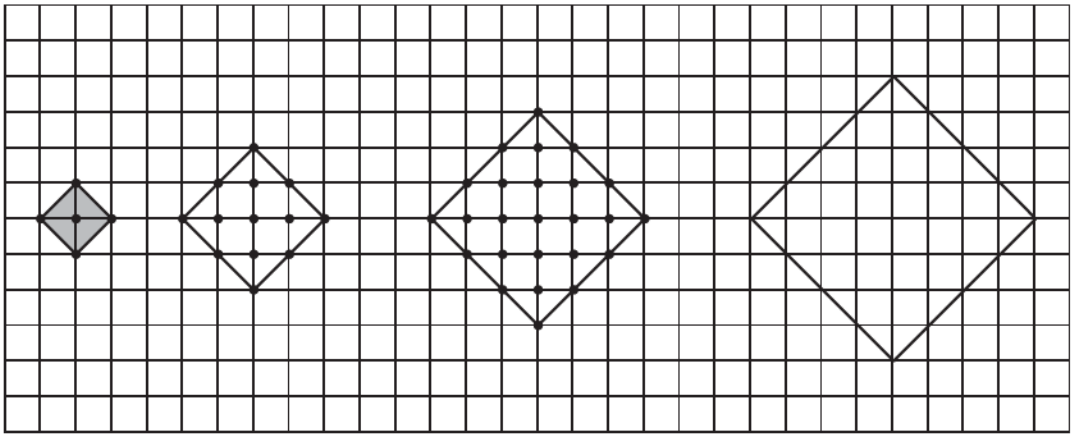

Diagram 1 Diagram 2 Diagram 3 Diagram 4

The diagrams show squares and dots on a grid.

Some dots are on the sides of each square and other dots are inside each square.

The area of the square (shaded) in Diagram 1 is 1 unit^2.

(a) Complete Diagram 4 by marking all the dots. [1]

(b) Complete the columns in the table below for Diagrams 4, 5 and n.

Diagram	1	2	3	4	5	-------	n
Number of units of area	1	4	9			-------	
Number of dots inside the square	1	5	13			-------	$(n-1)^2+n^2$
Number of dots on the sides of the square	4	8	12			-------	
Total number of dots	5	13	25			-------	

[7]

(c) For Diagram 200, find the number of dots

(i) inside the square, [1]

(ii) on the sides of the square. [1]

(d) Which diagram has 265 dots inside the square? [1]

												Total
Row 1								1	=	1
Row 2						3	+	5	=	8
Row 3			7	+	9	+	11	=	27
Row 4	13	+	15	+	17	+	19	=	64
Row 5										
Row 6										

The rows above show sets of consecutive odd numbers and their totals.

(a) Complete Row 5 and Row 6. [2]

(b) What is the special name given to the numbers 1, 8, 27, 64…? [1]

(c) Write down in terms of n,

(i) how many consecutive odd numbers there are in Row n, [1]

(ii) the total of these numbers. [1]

(d) The first number in Row n is given by $n^2 - n + 1$.

Show that this formula is true for Row 4. [1]

(e) The total of Row 3 is 27. This can be calculated by $(3 \times 7) + 2 + 4$.

The total of Row 4 is 64. This can be calculated by $(4 \times 13) + 2 + 4 + 6$. The total of Row 7 is 343. Show how this can be calculated in the same way. [1]

(f) The total of the first n **even** numbers is $n(n + 1)$.

Write down a formula for the total of the first $(n - 1)$ **even** numbers. [1]

(g) Use the results of **parts (d), (e)** and **(f)** to show clearly that the total of the numbers in Row n gives your answer to **part (c)(ii).** [2]

$$1 + 2 + 3 + 4 + 5 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}$$

(a) (i) Show that this formula is true for the sum of the first 8 natural numbers. [2]

(ii) Find the sum of the first 400 natural numbers. [1]

(b) (i) Show that $2 + 4 + 6 + 8 + \ldots\ldots\ldots\ldots\ldots + 2n = n(n + 1)$. [1]

(ii) Find the sum of the first 200 even numbers. [1]

(iii) Find the sum of the first 200 odd numbers. [1]

(c) (i) Use the formula at the beginning of the question to find the sum of the first $2n$ natural numbers. [1]

(ii) Find a formula, in its simplest form, for

$$1 + 3 + 5 + 7 + 9 + \ldots\ldots\ldots\ldots\ldots\ldots + (2n - 1).$$

Show your working. [2]

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(a) Write down the 10th term and the nth term of the following sequences.

(i) 1, 2, 3, 4, 5 …, …, [1]

(ii) 7, 8, 9, 10, 11 …, …, [1]

(iii) 8, 10, 12, 14, 16 …, … . [3]

(b) Consider the sequence

$1(8-7), 2(10-8), 3(12-9), 4(14-10),$ ……, …… .

(i) Write down the next term and the 10th term of this sequence in the form $a(b-c)$ where a, b and c are integers.

[3]

(ii) Write down the nth term in the form $a(b-c)$ and then simplify your answer. [2]

Sequences
Difficulty: Hard

Question Paper 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Hard
Booklet	Question Paper 1

Time allowed: **91 minutes**

Score: **/79**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

The table shows the first four terms in sequences A, B, C and D.

Complete the table.

Sequence	1st term	2nd term	3rd term	4th term	5th term		nth term
A	16	25	36	49			
B	5	8	11	14			
C	11	17	25	35			
D	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$			

[12]

The table shows the first five terms of sequences A, B and C.

Sequence	1st term	2nd term	3rd term	4th term	5th term	6th term
A	3	4	5	6	7	
B	0	1	4	9	16	
C	–3	–3	–1	3	9	

(a) Complete the table for the 6th term of each sequence. [2]

(b) Write down the nth term of sequence A. [1]

(c) (i) Find the nth term of sequence B. [2]

(ii) Find the value of n when the nth term of sequence B is 8281. [2]

(d) (i) Find the nth term of sequence C in its simplest form. [2]

(ii) Find the 8th term of sequence C. [1]

(e) The nth term of another sequence D is $\left(-\frac{1}{2}\right)^{n-1}$.

Complete the table for the first four terms of sequence D.

Sequence	1st term	2nd term	3rd term	4th term
D				

[3]

The first four terms of sequences A, B, C and D are shown in the table.

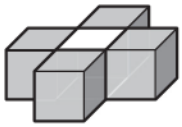
Sequence	1st term	2nd term	3rd term	4th term	5th term		nth term
A	$\frac{1}{3}$	$\frac{2}{4}$	$\frac{3}{5}$	$\frac{4}{6}$			
B	3	4	5	6			
C	–1	0	1	2			
D	–3	0	5	12			

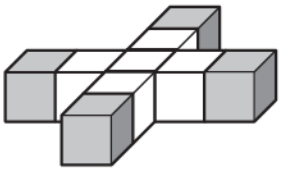
(a) Complete the table. [8]

(b) Which term in sequence A is equal to $\frac{36}{37}$? [2]

(c) Which term in sequence D is equal to 725? [2]

Layer 1 

Layer 2 

Layer 3 

The diagrams show layers of white and grey cubes.
Khadega places these layers on top of each other to make a tower.

(a) Complete the table for towers with 5 and 6 layers. [4]

Number of layers	1	2	3	4	5	6
Total number of **white** cubes	0	1	6	15		
Total number of **grey** cubes	1	5	9	13		
Total number of cubes	1	6	15	28		

(b) (i) Find, in terms of n, the **total** number of **grey** cubes in a tower with n layers. [2]

(ii) Find the total number of grey cubes in a tower with 60 layers. [1]

(iii) Khadega has plenty of white cubes but only 200 grey cubes.
How many layers are there in the highest tower that she can build? [2]

(c) The expression for the **total** number of **white** cubes in a tower with n layers is $pn^2 + qn + 3$.

Find the value of p and the value of q.
Show all your working. [5]

(d) Find an expression, in terms of n, for the **total** number of cubes in a tower with n layers.
Give your answer in its simplest form. [2]

(a)

1	$= 1$
$1 + 2$	$= 3$
$1 + 2 + 3$	$= 6$
$1 + 2 + 3 + 4$	$= 10$

(i) Write down the next line of this pattern. [1]

(ii) The sum of the first n integers is $\frac{n}{k}(n + 1)$.

Show that $k = 2$.

[2]

(iii) Find the sum of the first 60 integers. [1]

(iv) Find n when the sum of the first n integers is 465. [2]

(v) $1 + 2 + 3 + 4 + \ldots\ldots + x = \frac{(n - 8)(n - 7)}{2}$

Write x in terms of n. [1]

(b)

$1^3 = 1$

$1^3 + 2^3 = 9$

$1^3 + 2^3 + 3^3 = 36$

$1^3 + 2^3 + 3^3 + 4^3 = 100$

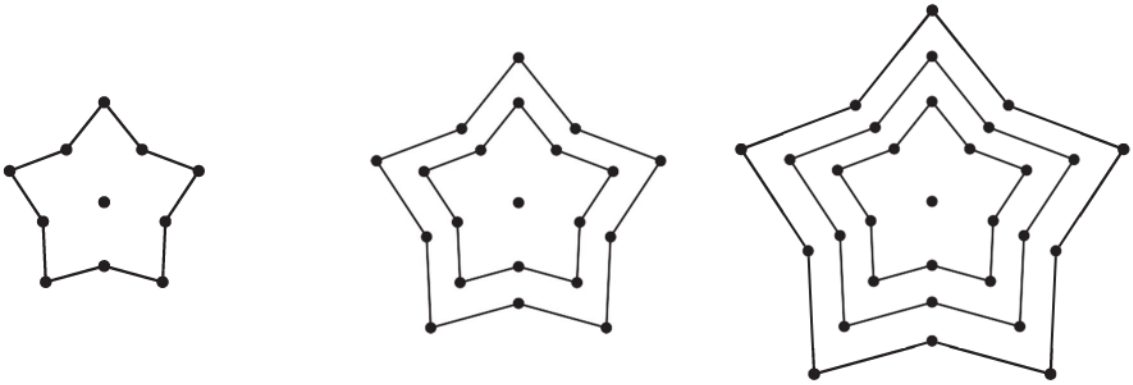
(i) Complete the statement.

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = \ldots\ldots = (\ldots\ldots)^2$ [2]

(ii) The sum of the first n integers is $\frac{n}{2}(n+1)$.

Find an expression, in terms of n, for the sum of the first n cubes. [1]

(iii) Find the sum of the first 19 cubes. [2]



Star 1 Star 2 Star 3

The diagrams show a sequence of stars made of lines and dots.

(a) Complete the table for Star 5, Star 7 and Star n.

	Star 1	Star 2	Star 3	Star 4	Star 5		Star 7		Star n
Number of lines	10	20	30	40					
Number of dots	11	21	31	41					

[4]

(b) The sums of the number of dots in two consecutive stars are shown in the table.

Star 1 and Star 2	Star 2 and Star 3	Star 3 and Star 4
32	52	72

Find the sum of the number of dots in

(i) Star 10 and Star 11, [1]

(ii) Star n and Star $(n + 1)$, [1]

(iii) Star $(n + 7)$ and Star $(n + 8)$. [1]

(c) The **total number of dots** in the first n stars is given by the expression $5n^2 + 6n$.

(i) Show that this expression is correct when $n = 3$. [2]

(ii) Find the total number of dots in the first 10 stars. [1]

(d) The total number of dots in the first n stars is $5n^2 + 6n$.
The number of dots in the $(n + 1)$th star is $10(n + 1) + 1$.

Add these two expressions to show that the total number of dots in the first $(n + 1)$ stars is

$$5(n + 1)^2 + 6(n + 1) .$$

You must show each step of your working. [4]

Sequences
Difficulty: Hard

Question Paper 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Hard
Booklet	Question Paper 2

Time allowed: **94 minutes**

Score: **/82**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

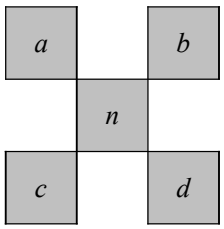
CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Consecutive integers are set out in rows in a grid.

(a) This grid has 5 columns.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35



The shape drawn encloses five numbers 7, 9, 13, 17 and 19. This is the $n = 13$ shape.

In this shape, $a = 7$, $b = 9$, $c = 17$ and $d = 19$.

(i) Calculate $bc - ad$ for the $n = 13$ shape. [1]

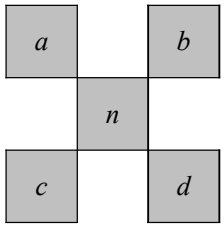
(ii) For the 5 column grid, $a = n - 6$.

Write down b, c and d in terms of n for this grid. [2]

(iii) Write down $bc - ad$ in terms of n.
Show clearly that it simplifies to 20. [2]

(b) This grid has 6 columns. The shape is drawn for $n = 10$.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

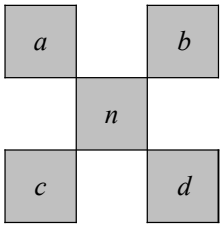


(i) Calculate the value of $bc - ad$ for $n = 10$. [1]

(ii) Without simplifying, write down $bc - ad$ in terms of n for this grid. [2]

(c) This grid has 7 columns.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

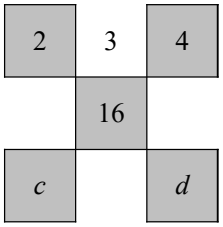


Show clearly that $bc - ad = 28$ for $n = 17$.

Answer(c) [1]

(d) Write down the value of $bc - ad$ when there are t columns in the grid. [1]

(e) Find the values of c, d and $bc - ad$ for this shape.



[2]

(a) Complete the table for the 6 th term and the nth term in each sequence.

	Sequence	6 th term		n th term
A	11, 9, 7, 5, 3			
B	1, 4, 9, 16, 25			
C	2, 6, 12, 20, 30			
D	3, 9, 27, 81, 243			
E	1, 3, 15, 61, 213			

[12]

(b) Find the value of the 100 th term in

(i) Sequence A, [1]

(ii) Sequence C. [1]

(c) Find the value of n in Sequence D when the n th term is equal to 6561. [1]

(d) Find the value of the 10 th term in Sequence E. [1]

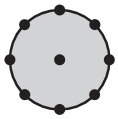


Diagram 1

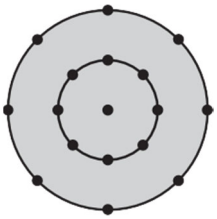


Diagram 2

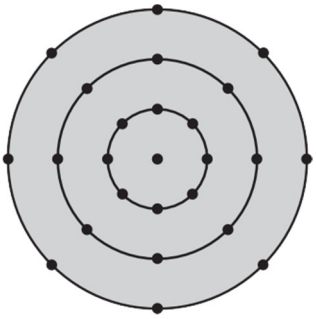


Diagram 3

The diagrams show a sequence of dots and circles.
Each diagram has one dot at the centre and 8 dots on each circle.
The radius of the first circle is 1 unit.
The radius of each new circle is 1 unit greater than the radius of the previous circle.

(a) Complete the table for diagrams 4 and 5.

Diagram	1	2	3	4	5
Number of dots	9	17	25		
Area of the largest circle	π	4π	9π		
Total length of the circumferences of the circles	2π	6π	12π		

[4]

(b) (i) Write down, in terms of n, the number of dots in diagram n. [2]

(ii) Find n, when the number of dots in diagram n is 1097. [2]

(c) Write down, in terms of n and π, the area of the largest circle in

(i) diagram n, [1]

(ii) diagram $3n$. [1]

(d) Find, in terms of n and π, the total length of the circumferences of the circles in diagram n. [2]

The first and the nth terms of sequences A, B and C are shown in the table below.

(a) Complete the table for each sequence. [5]

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence A	1					n^3
Sequence B	4					$4n$
Sequence C	4					$(n+1)^2$

(b) Find

(i) the 8th term of sequence A, [1]

(ii) the 12th term of sequence C. [1]

(c) (i) Which term in sequence A is equal to 15625? [1]

(ii) Which term in sequence C is equal to 10000? [1]

(d) The first four terms of sequences D and E are shown in the table below.

Use the results from **part (a)** to find the 5th and the nth terms of the sequences D and E. [4]

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence D	5	16	39	80		
Sequence E	0	1	4	9		

(a) (i) Work out the first 3 terms of the sequence whose nth term is $n(n + 2)$. [2]

(ii) Which term in this sequence is equal to 168? [3]

(b) Find a formula for the nth term of the following sequences.

(i) 5 8 11 14 17 …… [2]

(ii) 1 2 4 8 16 …… [2]

(c)

Diagram 1 Diagram 2 Diagram 3

A sequence of diagrams is formed by drawing equilateral triangles each of side one centimetre.
Diagram 1 has 3 one centimetre lines.
Diagram 2 has 9 one centimetre lines.

The formula for the **total** number of one centimetre lines needed to draw all of the first n **diagrams** is

$$an^3 + bn^2 + n.$$ [6]

Find the values of a and b.

(a) (i) The first three positive integers 1, 2 and 3 have a sum of 6.

Write down the sum of the first 4 positive integers. [1]

(ii) The formula for the sum of the first n integers is $\frac{n(n+1)}{2}$.

Show the formula is correct when $n = 3$. [1]

(iii) Find the sum of the first 120 positive integers. [1]

(iv) Find the sum of the integers

$121 + 122 + 123 + 124 +$ ..... $+ 199 + 200.$ [2]

(v) Find the sum of the even numbers

$2 + 4 + 6 +$ ..... $+ 800.$ [2]

(b) (i) Complete the following statements about the sums of cubes and the sums of integers. [2]

$1^3 = 1$	$1 = 1$
$1^3 + 2^3 = 9$	$1 + 2 = 3$
$1^3 + 2^3 + 3^3 =$	$1 + 2 + 3 =$ .....
$1^3 + 2^3 + 3^3 + 4^3 =$	$1 + 2 + 3 + 4 =$ .....

(ii) The sum of the first 14 integers is 105.

Find the sum of the first 14 cubes. [1]

(iii) Use the formula in part(a)(ii) to write down a formula for the sum of the first n cubes. [1]

(iv) Find the sum of the first 60 cubes. [1]

(v) Find n when the sum of the first n cubes is 278784. [2]

Sequences
Difficulty: Hard

Question Paper 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Hard
Booklet	Question Paper 3

Time allowed: **93 minutes**

Score: **/81**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

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CIE IGCSE Maths (0980)

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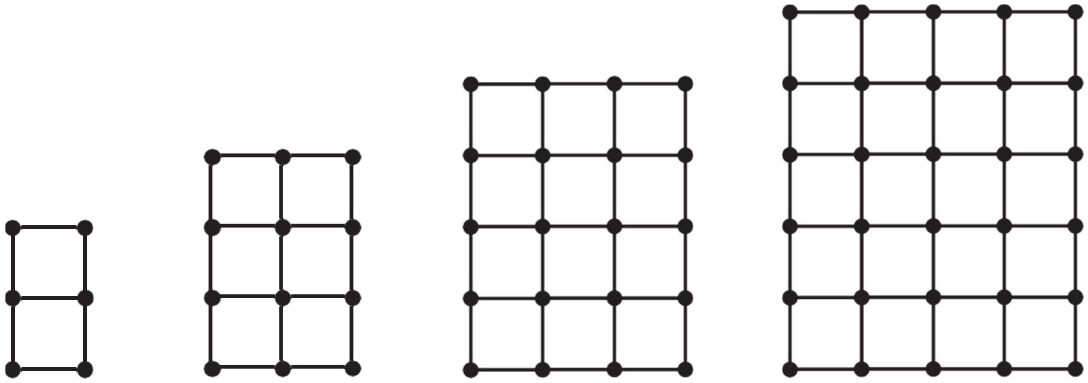


Diagram 1 Diagram 2 Diagram 3 Diagram 4

The first four Diagrams in a sequence are shown above.
Each Diagram is made from dots and one centimetre lines.
The area of each small square is 1 cm^2.

(a) Complete the table for Diagrams 5 and 6.

Diagram	1	2	3	4	5	6
Area (cm^2)	2	6	12	20		
Number of dots	6	12	20	30		
Number of one centimetre lines	7	17	31	49		

[4]

(b) The area of Diagram n is $n(n + 1)\,cm^2$.

(i) Find the area of Diagram 50. [1]

(ii) Which Diagram has an **area** of 930 cm^2? [1]

(c) Find, in terms of n, the number of **dots** in Diagram n. [1]

(d) The number of one centimetre lines in Diagram n is $2n^2 + pn + 1$.

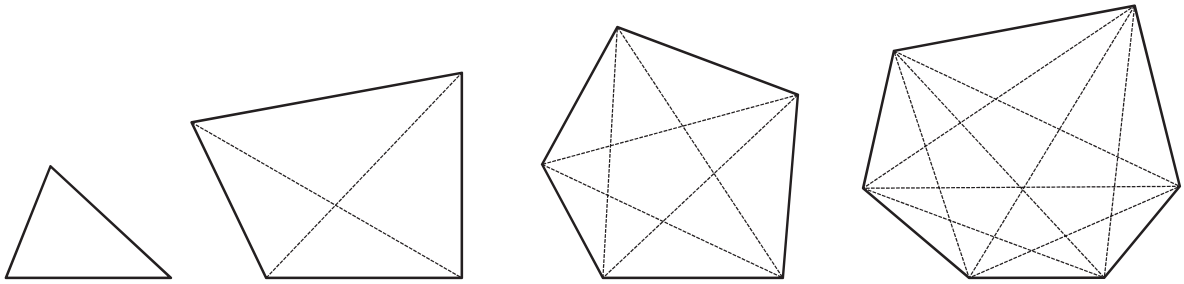
(i) Show that $p = 4$. [2]

(ii) Find the number of one centimetre lines in Diagram 10. [1]

(iii) Which Diagram has 337 one centimetre lines? [3]

(e) For **each** Diagram, the number of squares of area 1 cm^2 is A, the number of dots is D and the number of one centimetre lines is L.

Find a connection between A, D and L that is true for each Diagram. [1]



The diagrams show some polygons and their diagonals.

(a) Complete the table.

Number of sides	Name of polygon	Total number of diagonals
3	triangle	0
4	quadrilateral	2
5		5
6	hexagon	9
7	heptagon	14
8		

[3]

(b) Write down the total number of diagonals in

(i) a decagon (a 10-sided polygon), [1]

(ii) a 12-sided polygon. [1]

(c) A polygon with n sides has a total of $\frac{1}{p}n(n-q)$ diagonals, where p and q are integers.

(i) Find the values of p and q. [3]

(ii) Find the total number of diagonals in a polygon with 100 sides. [1]

(iii) Find the number of sides of a polygon which has a total of 170 diagonals. [2]

(d) A polygon with $n + 1$ sides has 30 more diagonals than a polygon with n sides.

Find n. [1]

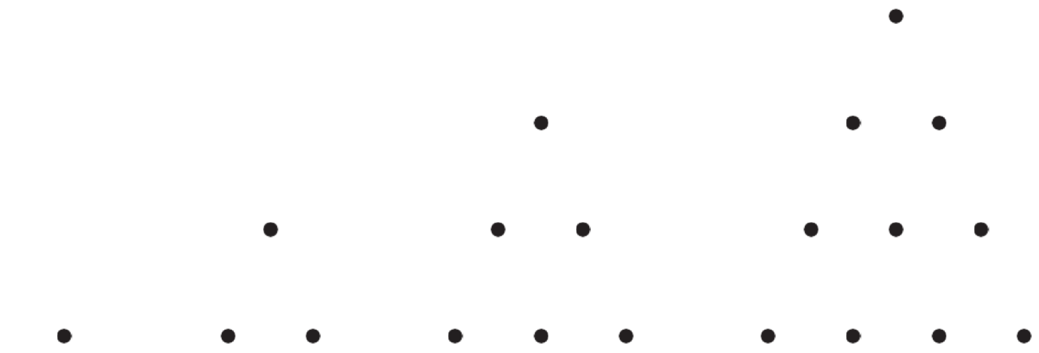


Diagram 1 Diagram 2 Diagram 3 Diagram 4

The first four terms in a sequence are 1, 3, 6 and 10.
They are shown by the number of dots in the four diagrams above.

(a) Write down the next four terms in the sequence. [2]

(b) (i) The sum of the two consecutive terms 3 and 6 is 9. The sum of the two consecutive terms 6 and 10 is 16.

Complete the following statements using different pairs of terms.

The sum of the two consecutive terms ..... and ..... is .....

The sum of the two consecutive terms ..... and ..... is ..... [1]

(ii) What special name is given to these sums? [1]

(c) (i) The formula for the nth term in the sequence 1, 3, 6, 10… is $\frac{n(n+1)}{k}$,

where k is an integer.

Find the value of k. [1]

(ii) Test your formula when $n = 4$, **showing your working.** [1]

(iii) Find the value of the 180th term in the sequence. [1]

(d) (i) Show clearly that the sum of the nth and the $(n + 1)$th terms is $(n + 1)^2$. [3]

(ii) Find the values of the two consecutive terms which have a sum of 3481. [2]

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

A 3 by 3 square

x	b	c
d	e	f
g	h	i

can be chosen from the 6 by 6 grid above.

(a) One of these squares is

8	9	10
14	15	16
20	21	22

.

In this square, $x = 8$, $c = 10$, $g = 20$ and $i = 22$.

For this square, calculate the value of

(i) $(i - x) - (g - c)$, [1]

(ii) $cg - xi$. [1]

(b)

x	b	c
d	e	f
g	h	i

(i) $c = x + 2$. Write down g and i in terms of x. [2]

(ii) Use your answers to **part(b)(i)** to show that $(i - x) - (g - c)$ is constant. [1]

(iii) Use your answers to **part(b)(i)** to show that $cg - xi$ is constant. [2]

(c) The 6 by 6 grid is replaced by a 5 by 5 grid as shown.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

A 3 by 3 square

x	b	c
d	e	f
g	h	i

can be chosen from the 5 by 5 grid.

For any 3 by 3 square chosen from this 5 by 5 grid, calculate the **value** of

(i) $(i - x) - (g - c)$, [1]

(ii) $cg - xi$. [1]

(d) A 3 by 3 square is chosen from an n by n grid.

(i) Write down the value of $(i - x) - (g - c)$. [1]

(ii) Find g and i in terms of x and n. [2]

(iii) Find $cg - xi$ in its simplest form. [1]

The table shows some terms of several sequences.

Term	1	2	3	4		8	
Sequence P	7	5	3	1		p	
Sequence Q	1	8	27	64		q	
Sequence R	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$		r	
Sequence S	4	9	16	25		s	
Sequence T	1	3	9	27		t	
Sequence U	3	6	7	−2		u	

(a) Find the values of p, q, r, s, t and u. [6]

(b) Find the nth term of sequence

(i) **P,** [1]

(ii) **Q,** [1]

(iii) **R,** [1]

(iv) **S,** [1]

(v) **T,** [1]

(vi) **U.** [1]

(c) Which term in sequence **P** is equal to −777? [2]

(d) Which term in sequence **T** is equal to 177 147? [2]

Diagram 1 Diagram 2 Diagram 3

The first three diagrams in a sequence are shown above.

The diagrams are made up of dots and lines. Each line is one centimetre long.

(a) Make a sketch of the next diagram in the sequence. [1]

(b) The table below shows some information about the diagrams.

Diagram	1	2	3	4	---------	n
Area	1	4	9	16	---------	x
Number of dots	4	9	16	p	---------	y
Number of one centimetre lines	4	12	24	q	---------	z

(i) Write down the values of p and q. [2]

(ii) Write down each of x, y and z in terms of n. [4]

(c) The **total** number of one centimetre lines in the first n diagrams is given by the expression

$$\frac{2}{3}n^3 + fn^2 + gn.$$

(i) Use $n = 1$ in this expression to show that $f + g = \frac{10}{3}$. [1]

(ii) Use $n = 2$ in this expression to show that $4f + 2g = \frac{32}{3}$. [2]

(iii) Find the values of f and g. [3]

(iv) Find the total number of one centimetre lines in the first 10 diagrams. [1]