Trigonometry
Difficulty: Medium

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Trigonometry
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 1

Time allowed: **102 minutes**

Score: **/89**

Percentage: **/100**

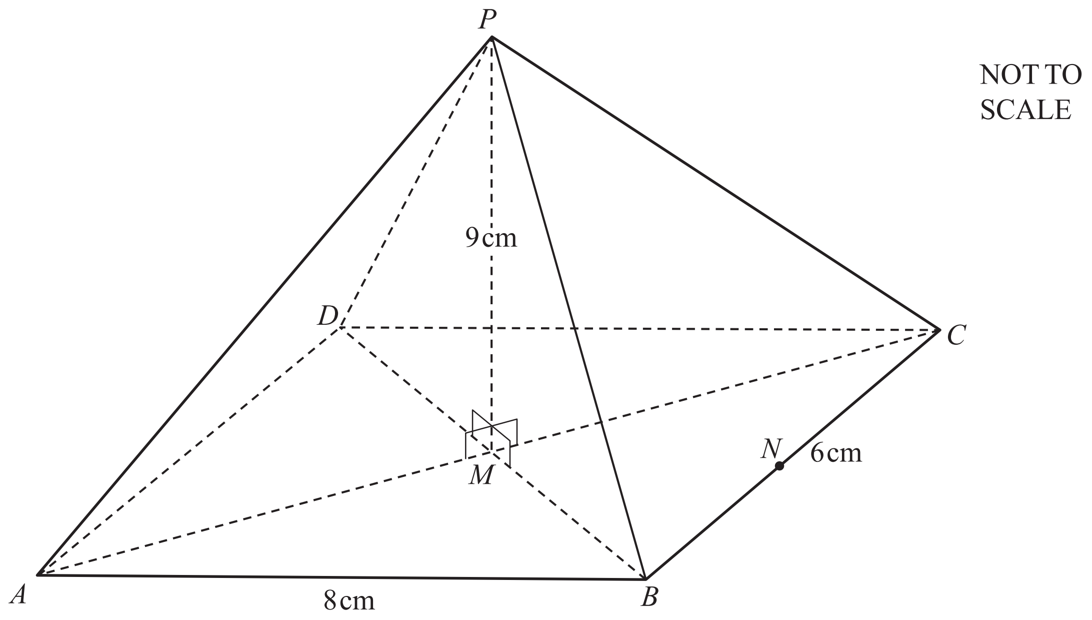
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980) Assembled by A/S

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



The diagram shows a pyramid on a rectangular base $ABCD$.
AC and BD intersect at M and P is vertically above M.
$AB = 8$ cm, $BC = 6$ cm and $PM = 9$ cm.

(a) N is the midpoint of BC.

Calculate angle PNM. [2]

We have that

$$NM = \frac{1}{2}AB$$

$$= 4$$

Angle PNM can be found using the trigonometric relation

$$\tan\theta = \frac{opp}{adj}$$

$$\rightarrow \tan PNM = \frac{9}{4}$$

$$\rightarrow \boldsymbol{PNM = 66.04}$$

(b) Show that $BM = 5$ cm. [1]

BM is half the length of BD.

BD can be found using Pythagoras'

$$BD^2 = 8^2 + 6^2$$

$$= 100$$

$$\rightarrow BD = 10$$

Hence

$$BM = \frac{1}{2}(10)$$

$$= \mathbf{5}$$

(c) Calculate the angle between the edge PB and the base $ABCD$. [2]

Again, using the trigonometric relation

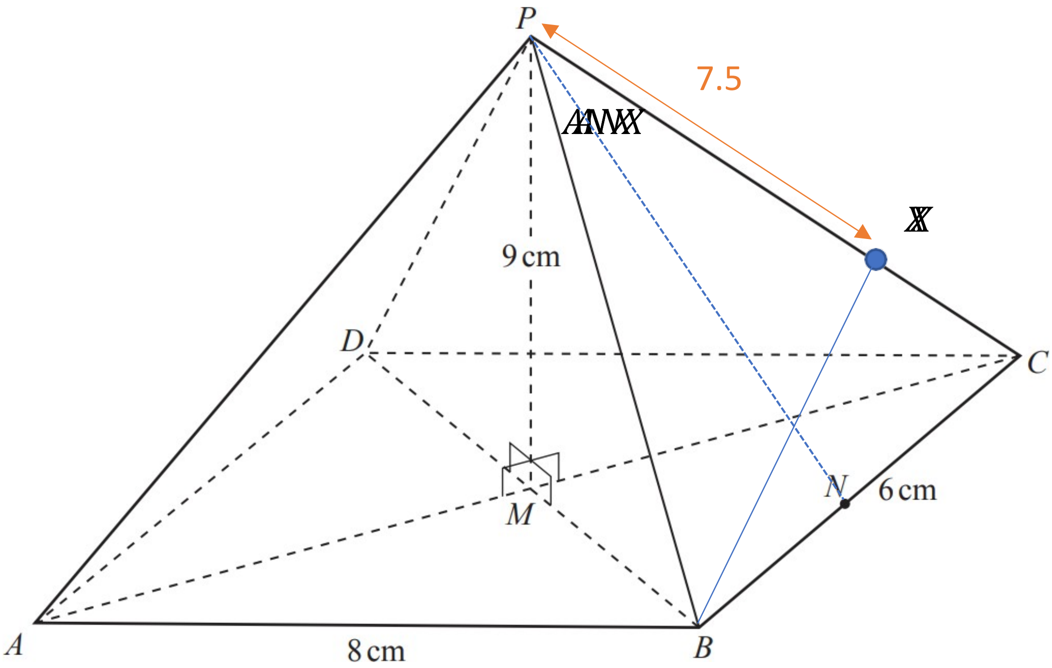
$$\tan\theta = \frac{opp}{adj}$$

$$\rightarrow \tan PBM = \frac{9}{5}$$

$$\rightarrow \boldsymbol{PBM = 60.95}$$

(d) A point X is on PC so that $PX = 7.5$ cm.

Calculate BX. [6]



If we find length BP and angle BPX then we can use the cosine rule to find BX as

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\rightarrow BX^2 = BP^2 + PX^2 - 2(BP)(PX)\cos BPX$$

BP can be found using Pythagoras'

$$BP^2 = 9^2 + 5^2$$

$$= 106$$

$$\rightarrow BP = \sqrt{106}$$

BPX can then be found by considering

$$BPX = 2 \times BPN$$

$$\sin BPN = \frac{opp}{hyp}$$

$$\rightarrow \sin BPN = \frac{3}{\sqrt{106}}$$

$$\rightarrow BPN = 16.94$$

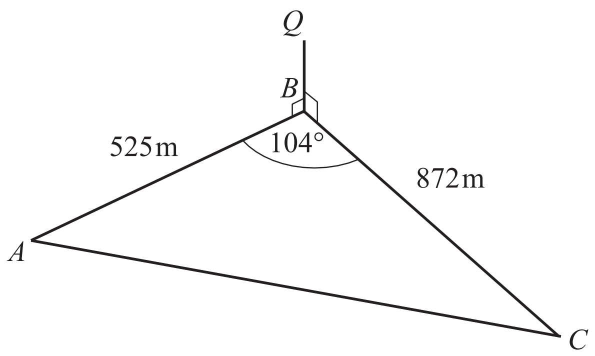
$$\rightarrow BPX = 33.88$$

Hence

$$BX^2 = 106 + 7.5^2 - 2\left(\sqrt{106}\right)(7.5)\cos 33.88$$

$$= 34.0374485$$

$$\rightarrow \boldsymbol{BX = 5.83}$$



NOT TO SCALE

ABC is a triangular field on horizontal ground.
There is a vertical pole BQ at B.
$AB = 525$ m, $BC = 872$ m and angle $ABC = 104°$.

(a) Use the cosine rule to calculate the distance AC. [4]

Cosine rule is

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\rightarrow AC^2 = 525^2 + 872^2 - 2(525)(872)\cos 104$$

$$= 1257512.688$$

$$\rightarrow \boldsymbol{AC = 1121}$$

(b) The angle of elevation of Q from C is $1.0°$.

Showing all your working, calculate the angle of elevation of Q from A. [4]

Use the trigonometric relation

$$\tan\theta = \frac{opp}{adj}$$

to write

$$\tan 1 = \frac{QB}{872}$$

$$\rightarrow QB = 872\tan 1$$

Hence

$$\tan QAB = \frac{QB}{525}$$

$$= \frac{872\tan 1}{525}$$

$$= 0.02899$$

$$\rightarrow \boldsymbol{QAB = 1.66}$$

(c) (i) Calculate the area of the field. [2]

Area of a triangle is

$$Area = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(525)(872)\sin 104$$

$$= \mathbf{222\ 100.7}$$

(ii) The field is drawn on a map with the scale 1 : 20 000.

Calculate the area of the field on the map in cm^2. [2]

Length scalar is 20 000, hence the area scalar is

$$20000^2$$

Convert field area into cm^2

$$222100.7 \times 100^2$$

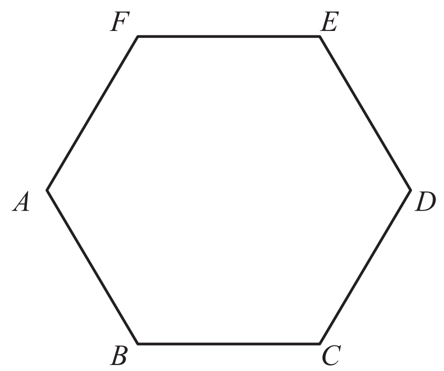
Then divide by the area scalar

$$222100.7 \times \frac{100^2}{20000^2}$$

$$= \mathbf{5.55}$$

Question 3

(a) The diagram shows a regular hexagon $ABCDEF$ of side 10cm.



NOT TO SCALE

(i) Show that angle $BAF = 120°$. [2]

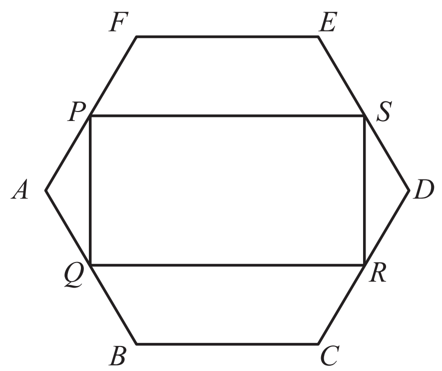
The total number of angles inside a regular polygon is: $(n-2) \times 180$

For a hexagon (6 sides), this means that the total number angles is $4 \times 180 = 720$

Each of the internal angles is the same, and there are 6 of them, the angle $BAF = \frac{720}{6} = 120°$

(ii) The vertices of a rectangle $PQRS$ touch the sides FA, AB, CD and DE.

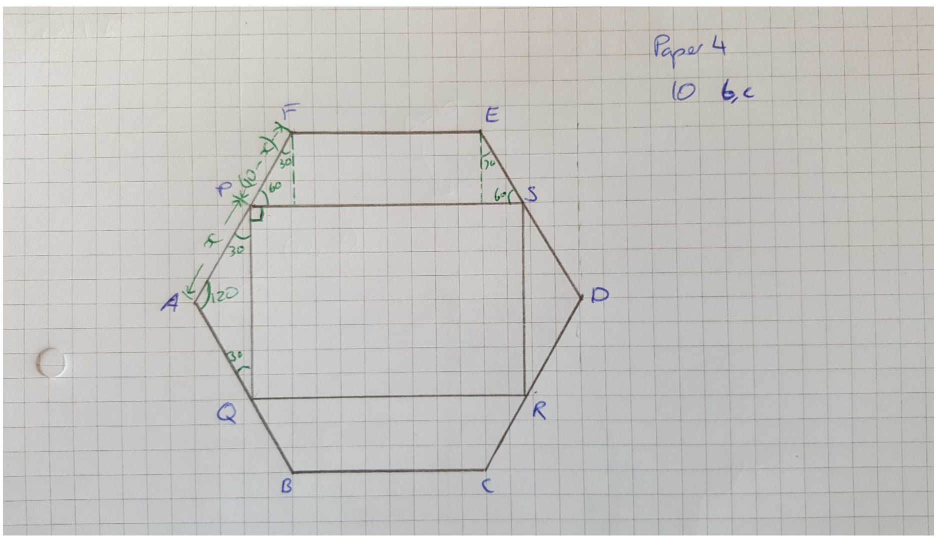
PS is parallel to FE and $AP = x$ cm.



NOT TO SCALE

Use trigonometry to find the length of PQ in terms of x. [3]

An annotated version of the hexagon looks like this:



To find PQ, we can use the COSINE rule:

$$a^2 = b^2 + c^2 - 2bccos(A)$$

Where:

$$a = PQ, b = x, c = x, A = 120°$$

So:

$$PQ^2 = x^2 + x^2 - 2x^2\,cos(120)$$

$$cos(120) = -\frac{1}{2}$$

$$PQ^2 = 3x^2$$

$$\boldsymbol{PQ = \sqrt{3}x}$$

(iii) $PF = (10 - x)$ cm.

Show that $PS = (20 - x)$ cm. [3]

We are looking for the angle FPS. We know that the angle inside the lower triangle drawn is 30°, the inside of the rectangle is a right angle, so the angle FPS is 60°.

Now we can say that:

$$PS = PFcos60 + FE + ES\,cos\,60$$

And

$$PF = ES$$

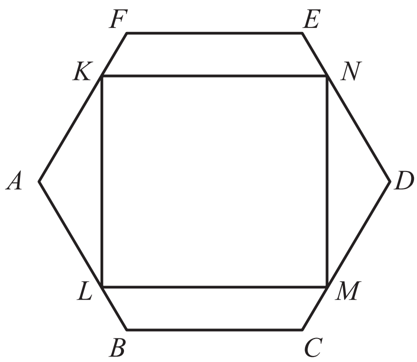
So

$$PS = \frac{1}{2}(10 - x) + \frac{1}{2}(10 - x) + 10$$

$$PS = 10 - x + 10$$

$$\boldsymbol{PS = 20 - x}$$

(b)



NOT TO SCALE

The diagram shows the vertices of a square *KLMN* touching the sides of the same hexagon *ABCDEF*, with *KN* parallel to *FE*.

Use your results from **part (a)(ii)** and **part (a)(iii)** to find the length of a side of the square.

[4]

Since this KLMN is a square, the sides KL and KN are the same length. These correspond to PQ and PS in the previous part of the question.

Therefore:

$$\sqrt{3}x = 20 - x$$

$$\sqrt{3}x + x = 20$$

$$x(\sqrt{3} + 1) = 20$$

$$x = \frac{20}{\sqrt{3} + 1}$$

$$x = 7.320508\ldots$$

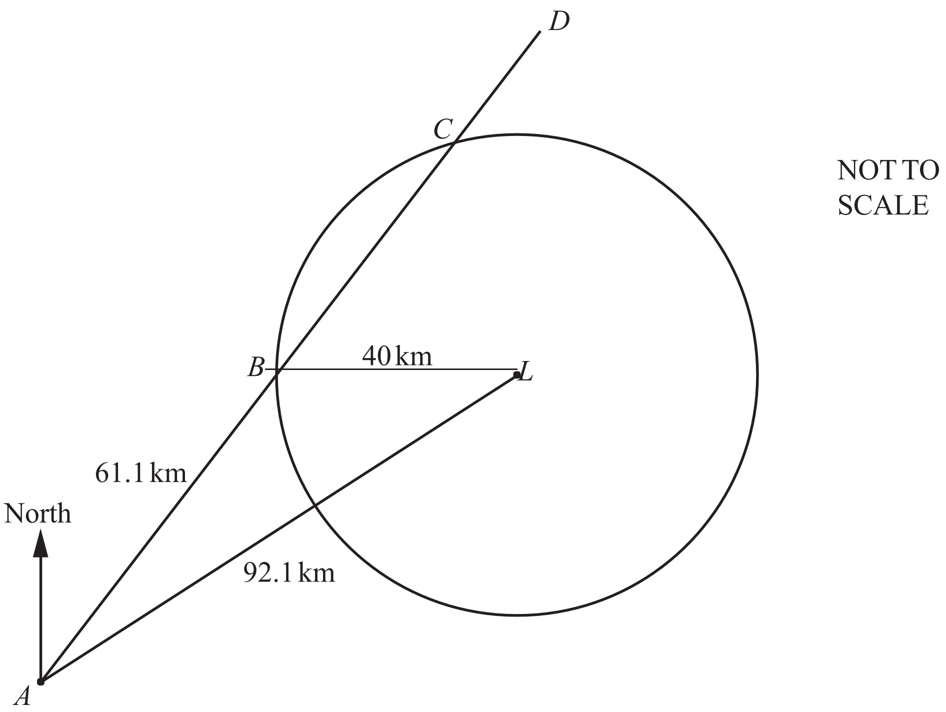
$$\boldsymbol{x = 7.32cm\ (3.s.f)}$$

Therefore the side is length:

$$KL = 20 - x$$

$$KL = 20 - \frac{20}{\sqrt{3} + 1}$$

$$\boldsymbol{KL = 12.7}$$



The diagram shows the position of a port, A, and a lighthouse, L.
The circle, centre L and radius 40 km, shows the region where the light from the lighthouse can be seen.
The straight line, $ABCD$, represents the course taken by a ship after leaving the port.
When the ship reaches position B it is due west of the lighthouse.

$AL = 92.1$ km, $AB = 61.1$ km and $BL = 40$ km.

(a) Use the cosine rule to show that angle $ABL = 130.1°$, correct to 1 decimal place.

[4]

Cosine Rule (with the correct letters):

$$\cos A\hat{B}L = \frac{a^2 + l^2 - b^2}{2al}$$

$$A\hat{B}L = \cos^{-1}\left(\frac{40^2 + 61.1^2 - 92.1^2}{2 \times 40 \times 61.1}\right)$$

$\boldsymbol{A\hat{B}L = 130.1°}$ (to 1dp)

(b) Calculate the bearing of the lighthouse, L, from the port, A. [4]

Find $B\hat{L}A$ using the Sine Rule:

$$\frac{\sin B\hat{L}A}{l} = \frac{\sin A\hat{B}L}{b}$$

$$\frac{\sin B\hat{L}A}{61.1} = \frac{\sin 130.1}{92.1}$$

$$\sin B\hat{L}A = 61.1 \times \frac{\sin 130.1}{92.1}$$

$$B\hat{L}A = \sin^{-1}\left(61.1 \times \frac{\sin 130.1}{92.1}\right)$$

$B\hat{L}A = 30.5°$ (to 1dp)

So (using a right-angled triangle):

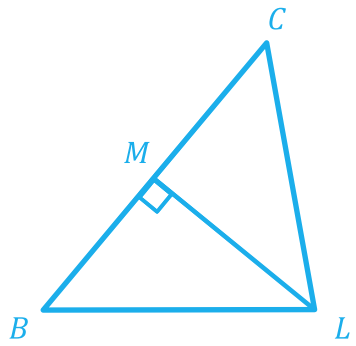
$$Bearing\ of\ L\ from\ A = 90 - 30.5$$

$$\boldsymbol{Bearing = 059.5°}$$

(Bearings always given as 3 figures before the decimal point)

(c) The ship sails at a speed of 28 km/h.

Calculate the length of time for which the light from the lighthouse can be seen from the ship. Give your answer correct to the nearest minute. [5]



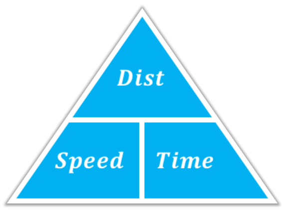
$$C\hat{B}L = 180 - A\hat{B}L = 180 - 130.1 = 49.9°$$

Find length BC using SOHCAHTOA:

$$BC = 2 \times BM = 2 \times BL \cos C\hat{B}L$$

$$BC = 2 \times 40 \times \cos 49.9$$

Find Time taken using:



$$Time = \frac{Dist}{Speed}$$

$$Time = \frac{BC}{28}$$

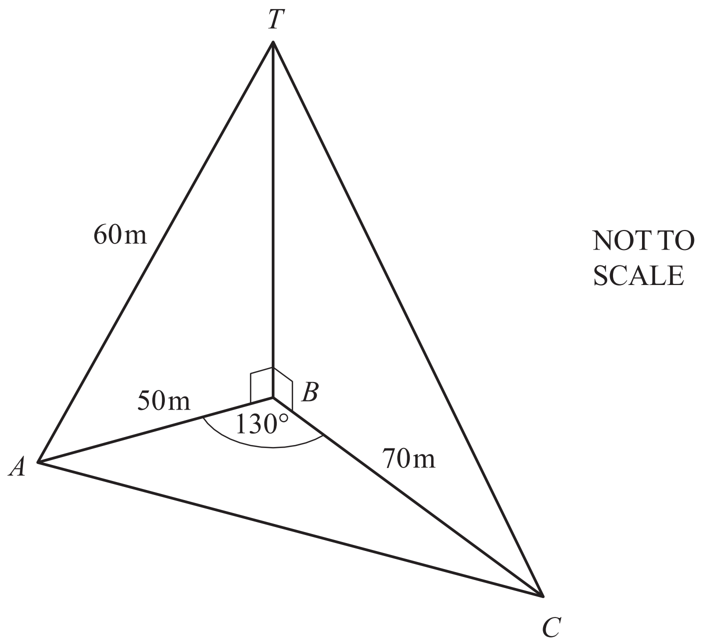
$$Time = \frac{2 \times 40 \times \cos 49.9}{28}$$

$Time = 1.84$ hours

Multiply the 0.84 by 60 to find the minutes part:

$\boldsymbol{Time}$ **= 1 hour 50 minutes**

(a)



A, B and C are points on horizontal ground.
BT is a vertical pole.
$AT = 60$ m, $AB = 50$ m, $BC = 70$ m and angle $ABC = 130°$.

(i) Calculate the angle of elevation of T from C. [5]

The angle of elevation of T from C is the angle BCT. The size of this angle can be calculated using trigonometry; however we need to do other calculations first.

We calculate the size of BT using Pythagoras' theorem (triangle ABT is a right angle triangle).

$$AT^2 = BT^2 + AB^2$$

Subtract AB^2 from both sides of the equation.

$$AT^2 - AB^2 = BT^2$$

Take square root of both sides and use the data given.

$$BT = \sqrt{AT^2 - AB^2}$$

$$BT = \sqrt{(60)^2 - (50)^2}$$

Use a calculator to find the length of BT.

$$BT = 33.17\ m$$

The angle BCT can now be calculated using trigonometry (triangle BCT is a right angle triangle).

$$\tan(BCT) = \frac{opposite}{adjacent} = \frac{BT}{BC}$$

Take arctan of both sides.

$$BCT = \arctan\left(\frac{BT}{BC}\right) = \arctan\left(\frac{33.17}{70}\right)$$

Use a calculator to work out the value of angle BCT.

$$BCT = \mathbf{25.4}°$$

[4]

(ii) Calculate the length AC.

$$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos(ABC)$$

$$AC^2 = (50)^2 + (70)^2 - 2 \times (50) \times (70) \times \cos(130°)$$

$$AC^2 = (2\,500 + 4\,900 + 4\,499.51)m^2$$

$$AC^2 = 11\,899.51\ m^2$$

Take square root of both sides to get the final answer:

$$AC = \mathbf{109\ m}\ (3sf)$$

(iii) Calculate the area of triangle ABC. [2]

The area of triangle ABC is given by:

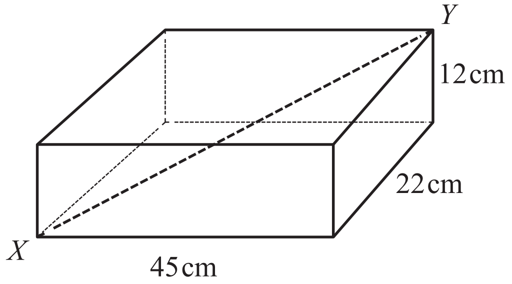
$$area = \frac{1}{2} AB \times BC \times \sin(ABC)$$

In our case AB=50m, BC=70m and ABC=130°.

$$area = \frac{1}{2}(50) \times (70) \times \sin(130°)$$

$$area = \mathbf{1340\ m^2}$$

(b)

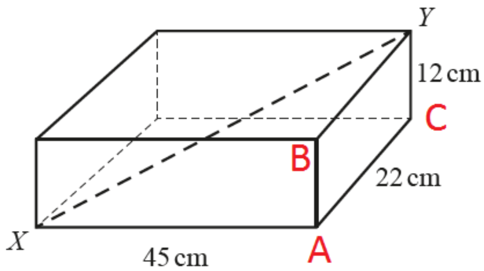


NOT TO SCALE

A cuboid has length 45 cm, width 22 cm and height 12 cm.

Calculate the length of the straight line XY. [4]

For the convenience, we mark a couple of other points on the diagram.



The length of XY can be found from Pythagoras' rule as:

$$XY^2 = XB^2 + BY^2$$

We do not yet know the length of XB, but this can also be found from Pythagoras' rule.

$$XB^2 = XA^2 + AB^2$$

We therefore have

$$XY^2 = XA^2 + AB^2 + BY^2$$

The shape is a cuboid, therefore AB=YC and BY=AC.

$$XY^2 = XA^2 + YC^2 + AC^2$$

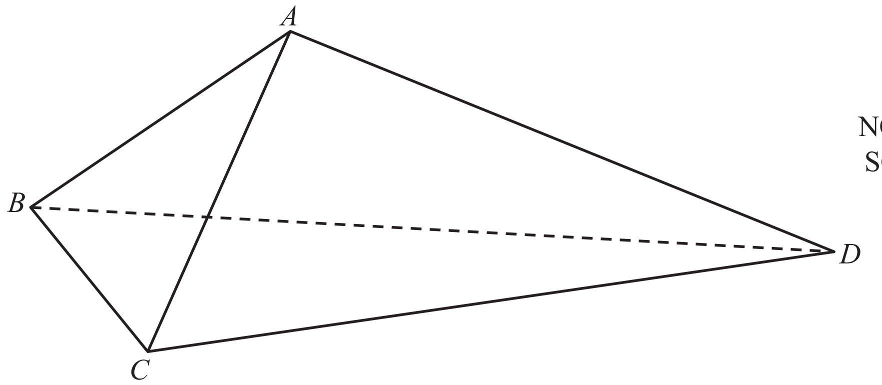
$$XY^2 = (45cm)^2 + (12cm)^2 + (22cm)^2$$

$$XY^2 = 2025 + 144 + 484$$

$$XY^2 = 2653$$

Take a square root.

Which gives us the final answer **YX = 51.5** (1 dp.)



The diagram shows a tent *ABCD*.
The front of the tent is an isosceles triangle *ABC*, with *AB* = *AC*.
The sides of the tent are congruent triangles *ABD* and *ACD*.

(a) *BC* = 1.2 m and angle *ABC* = 68°.

Find *AC*. [3]

As the triangle ABC is isosceles, we can divide it into two right-angle triangles by drawing a bisector of BC. Trigonometry can be used to calculate the length of AC.

(angle ABC=angle ACB)

$$\cos(ACB) = \frac{BC/2}{AC}$$

Multiply both sides by AC and divide both sides by cos(ACB).

$$AC = \frac{BC}{2 \times \cos(ACB)}$$

$$AC = \frac{1.2m}{2 \times \cos(68°)}$$

Use a calculator to get the value of AC.

$$\boldsymbol{AC = 1.6m}$$

(b) $CD = 2.3$ m and $AD = 1.9$ m.

Find angle ADC. [4]

Cosine rule can be used to find the value of ADC.

$$AC^2 = AD^2 + CD^2 - 2 \times AD \times CD \times \cos(ADC)$$

Subtract $AD^2 + CD^2$ from both sides of the equation.

$$AC^2 - AD^2 - CD^2 = -2 \times AD \times CD \times \cos(ADC)$$

Divide both sides by $-2 \times AD \times CD$

$$\cos(ADC) = \frac{AD^2 + CD^2 - AC^2}{2 \times AD \times CD}$$

We are given the following values: CD=2.3m, AD=1.9m and AC=1.6m

$$\cos(ADC) = \frac{(1.9m)^2 + (2.3m)^2 - (1.6m)^2}{2 \times (1.9m) \times (2.3m)}$$

$$\cos(ADC) = 0.725$$

Apply $\cos^{-1}$ on both sides of the equation. Use a calculator to find the value of angle ADC.

$$angle\ ADC = \mathbf{43.5°}$$

(c) The floor of the tent, triangle BCD, is also an isosceles triangle with $BD = CD$.

Calculate the area of the floor of the tent. [4]

Calculate the length of bisector of BC, which passes D. This is the height of the triangle BCD.

The length of this bisector can be calculated using Pythagoras' rule.

$$height = \sqrt{CD^2 - \left(\frac{BC}{2}\right)^2}$$

$$height = \sqrt{(2.3m)^2 - \left(\frac{1.2m}{2}\right)^2}$$

$$height = 2.22m$$

The area of a triangle is given as half the product of a length of a side and its height.

$$Area = \frac{1}{2} BC \times height$$

$$Area = \frac{1}{2}(1.2m) \times (2.22m)$$

$$Area = \mathbf{1.33\ m^2}$$

(d) When the tent is on horizontal ground, A is a vertical distance 1.25 m above the ground.

Calculate the angle between AD and the ground. [3]

The angle between AD (let this angle be x) and the ground (plane of BCD) can be calculated using trigonometry.

$$\sin(x) = \frac{height\ of\ A}{AD}$$

We know that the height of A above the ground is 1.25m and AD=1.9m.

$$\sin(x) = \frac{1.25m}{1.9m}$$

Apply $\sin^{-1}$ on both sides of the equation. Use a calculator to find the value of angle x.

$$x = \mathbf{41.1°}$$

(a) Andrei stands on level horizontal ground, 294 m from the foot of a vertical tower which is 55m high.

(i) Calculate the angle of elevation of the top of the tower. [2]

The angle of elevation can be calculated using trigonometry.

$$\tan(angle) = \frac{55}{294}$$

Take tan[-1] of both sides of the equation to calculate the angle of elevation.

$$angle = \arctan(\frac{55}{294})$$

$$\boldsymbol{angle = 10.6°}$$

(ii) Andrei walks a distance x metres directly towards the tower.
The angle of elevation of the top of the tower is now 24.8°.

Calculate the value of x. [4]

We use the same formula as before, but now we subtract x from Anderi's original distance from the tower (294m).

$$\tan(24.8°) = \frac{55}{294 - x}$$

Invert both fractions.

$$\frac{1}{\tan(24.8°)} = \frac{294 - x}{55}$$

Multiply both sides by 55m.

$$\frac{55\text{m}}{\tan(24.8°)} = 294 - x$$

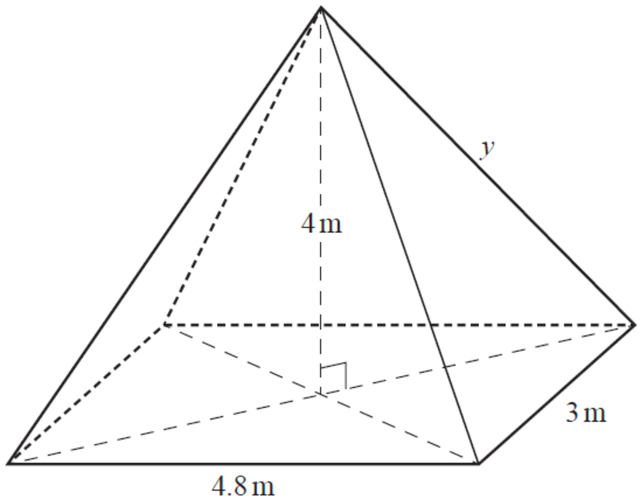
Subtract 294m from both sides of the equation.

$$x = 294m - \frac{55}{\tan(24.8°)}$$

Use a calculator to work out the value of x.

$$\boldsymbol{x = 175m}$$

(b) The diagram shows a pyramid with a horizontal rectangular base.



NOT TO
SCALE

The rectangular base has length 4.8 m and width 3 m and the height of the pyramid is 4 m.

Calculate

(i) y, the length of a sloping edge of the pyramid, [4]

The value of y can be calculated using the Pythagoras' rule.

$$y^2 = 4^2 + a^2$$

Where a is half the diagonal of the base.

The value of a can be also calculated using the Pythagoras' rule.

$$(2a)^2 = 4.8^2 + 3^2$$

Divide both sides by 4.

$$a^2 = \frac{4.8^2 + 3^2}{4}$$

Combine the two equations.

$$y^2 = 4^2 + \frac{4.8^2 + 3^2}{4}$$

$$y^2 = 24.01$$

Take square root of both sides of the equation and calculate the value of y.

$$\boldsymbol{y = 4.9m}$$

(ii) the angle between a sloping edge and the rectangular base of the pyramid. [2]

The angle between a sloping edge and the rectangular base of the pyramid can be calculated using trigonometry.

$$\sin(angle) = \frac{4}{y}$$

We have calculated the value of *y* in the previous equation.

$$\sin(angle) = \frac{4}{4.9}$$

Take $\sin^{-1}$ of both sides of the equation to calculate the angle between a sloping edge and the base.

$$angle = \arcsin(\frac{4}{4.9})$$

$$\boldsymbol{angle = 54.7°}$$

Trigonometry
Difficulty: Medium

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Trigonometry
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 2

Time allowed: **93 minutes**

Score: **/81**

Percentage: **/100**

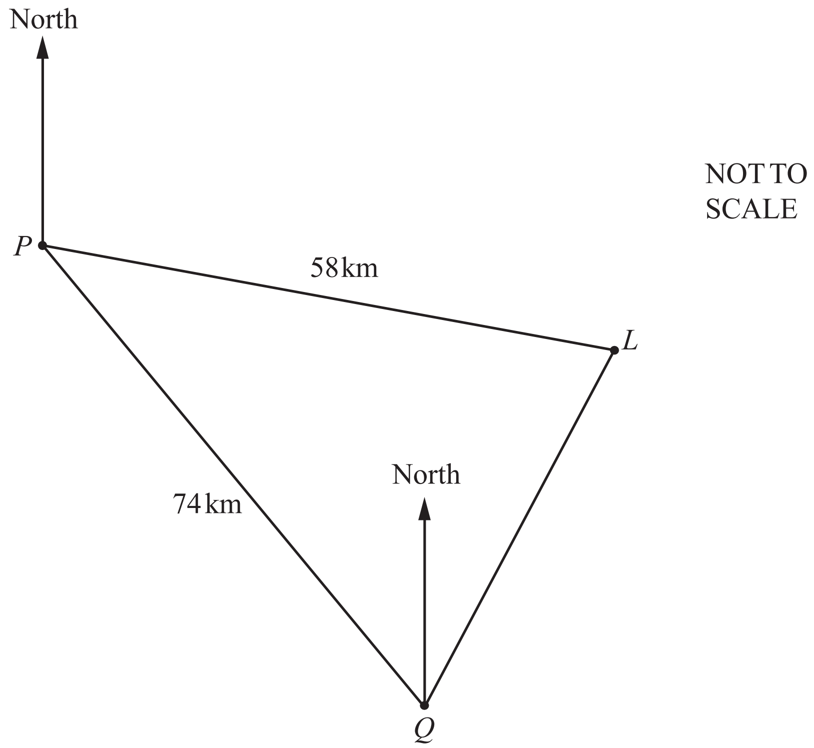
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



A ship sails from port P to port Q.
Q is 74 km from P on a bearing of 142°.
A lighthouse, L, is 58 km from P on a bearing of 110°.

(a) Show that the distance LQ is 39.5km correct to 1 decimal place. [5]

The angle QPL is:

$$QPL = 142 - 110$$

$$= 32$$

Now we can use the cosine rule to find QL:

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc \cos A}$$

$$\rightarrow QL^2 = 74^2 + 58^2 - 2(74)(58)\cos 32$$

$$= 1560.355$$

$$\rightarrow \boldsymbol{QL = 39.5}$$

(b) Use the sine rule to calculate angle PQL. [3]

The sine rule is

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\rightarrow \frac{\sin 32}{39.5} = \frac{\sin PQL}{58}$$

$$\rightarrow \sin PQL = \frac{58 \sin 32}{39.5}$$

$$= 0.778$$

$$\rightarrow PQL = 51.1$$

(c) Find the bearing of

(i) P from Q, [2]

Let this bearing be ϕ.

We know that:

$$\phi - 142 = 180$$

$$\rightarrow \phi = 180 + 142$$

$$= 322$$

(ii) L from Q. [1]

Let this bearing by ψ.

We have that:

$$\psi = PQL - (360 - \phi)$$

$$= 51.1 - (360 - 322)$$

$$= 13.1$$

$$= \mathbf{013.1}$$

(d) The ship takes 2 hours and 15 minutes to sail the 74 km from P to Q.

Calculate the average speed in knots. [3]
[1 knot = 1.85 km/h]

Speed distance time relation is

$$\boldsymbol{speed = \frac{distance}{time}}$$

$$\rightarrow speed = \frac{74km}{2.25\ hours}$$

$$= \frac{296}{9}\ kmh^{-1}$$

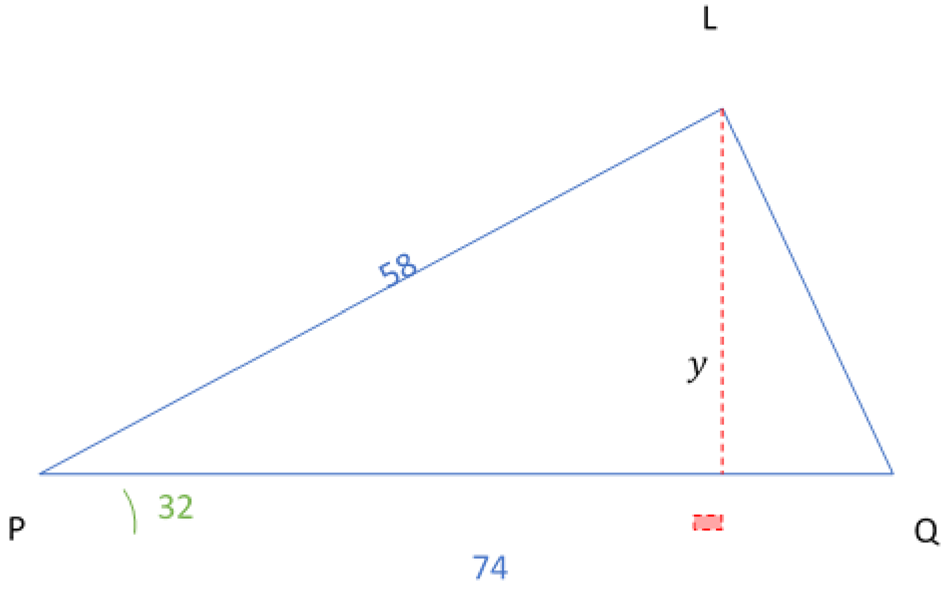
Divide by the conversion factor to get speed in knots:

$$\frac{296}{9} \div 1.85$$

$$= \mathbf{17.8}$$

(e) Calculate the shortest distance from the lighthouse to the path of the ship. [3]

The shortest distance, y, forms a right-angle triangle.



We can use the trigonometric relation:

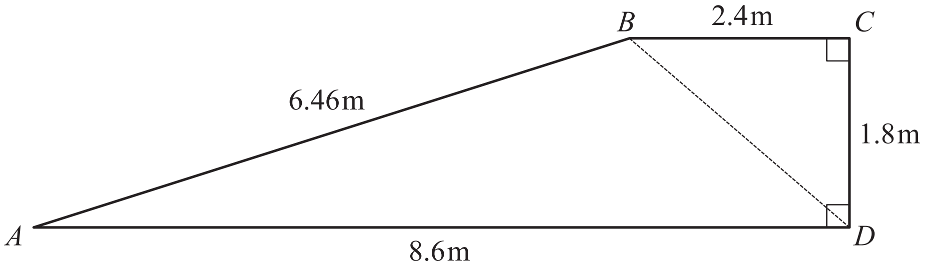
$$\sin \theta = \frac{opp}{hyp}$$

to write:

$$\sin 32 = \frac{y}{58}$$

$$\rightarrow y = 58 \sin 32$$

$$= \mathbf{30.7}$$

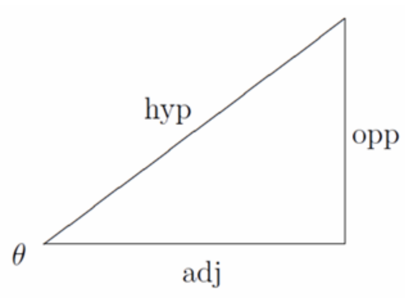


NOT TO SCALE

The diagram shows the cross section, $ABCD$, of a ramp.

(a) Calculate angle DBC. [2]

We cannot assume that angle BDC is 45 degrees.



$$\tan A = \frac{opp}{adj}$$

$$\tan(DBC) = \frac{1.8}{2.4}$$

$$\boldsymbol{DBC = 36.9°}$$

(b) (i) Show that BD is exactly 3 m. [2]

Using Pythagoras' Theorem,

$$BC^2 + CD^2 = BD^2$$

$$BD = \sqrt{BC^2 + CD^2}$$

$$= \sqrt{2.4^2 + 1.8^2}$$

$$\boldsymbol{= 3\ (shown)}$$

(ii) Use the cosine rule to calculate angle ABD. [4]

Substitute the values into the Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\ cos(A)$$

$$8.6^2 = 6.46^2 + 3^2 - 2(6.46)(3)\ cos(ABD)$$

$$cos(ABD) = -0.599$$

$$\boldsymbol{ABD = 126.8°}$$

(c) The ramp is a prism of width 4 m. [3]

Calculate the volume of this prism.

To find the volume, first find the area of the cross section, here two triangles, ABD and BCD and then multiplying the area by the width of the prism.

Width = 4m

$$Area\ ABD = \frac{1}{2} \times 6.46 \times 3 \times sin(126.8)$$

$$= 7.759m^2$$

$$Area\ BCD = \frac{1}{2} \times 2.4 \times 1.8 \times sin(90)$$

$$= 2.16m^2$$

$$Sum\ of\ areas = 7.759m^2 + 2.16m^2$$

$$= 9.919m^2$$

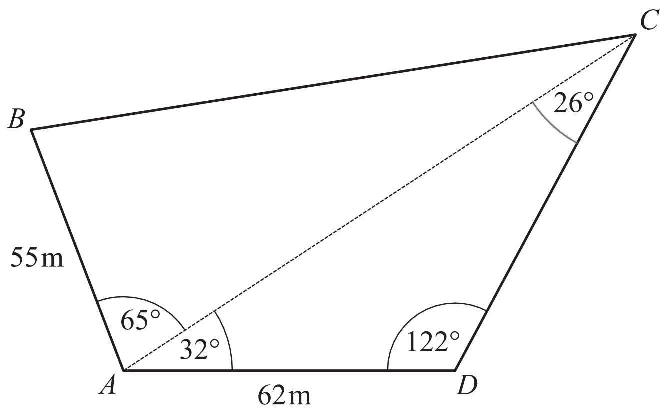
Therefore,

$$Volume\ of\ Prism = 9.919m^2 \times 4$$

$$\boldsymbol{= 39.7m^3}$$

Question 3

A field, $ABCD$, is in the shape of a quadrilateral.
A footpath crosses the field from A to C.



NOT TO SCALE

(a) Use the sine rule to calculate the distance AC and show that it rounds to 119.9 m, to 1 decimal place. [3]

Using Sine Rule,

$$\frac{AD}{\sin(26°)} = \frac{AC}{\sin(122°)}$$

$$\frac{62}{\sin(26°)} = \frac{AC}{\sin(122°)}$$

$$\boldsymbol{AC = 119.9m\ (1.d.p)(shown)}$$

(b) Calculate the length of BC. [4]

Using the Cosine Rule,

$$a^2 = b^2 + c^2 - 2(b)(c)\cos A$$

$$BC^2 = 55^2 + 119.9^2 - 2(55)(119.9)\cos 65°$$

$$\boldsymbol{BC = 108.8m}$$

(c) Calculate the area of triangle ACD. [2]

$$Area\ of\ triangle = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(62)(119.9)\sin 32°$$

$$= \mathbf{1970\ m^2}$$

(d) The field is for sale at \$4.50 per square metre.

Calculate the cost of the field. [3]

Calculate the area of the other triangle:

$$Area\ of\ triangle = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(55)(119.9)\sin 65°$$

$$= 2988.3\ m^2$$

Find total area of field:

$$Total\ Area = 1970\ m^2 + 2988.3\ m^2$$

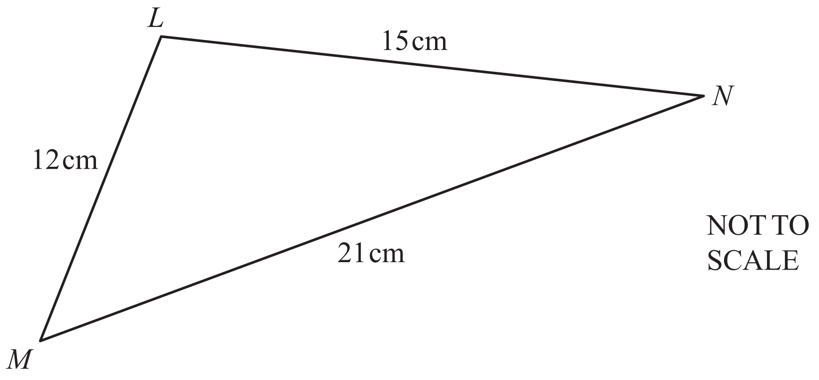
$$= 4958.3\ m^2$$

Now multiple by cost per square metre:

$$4958.3\ m^2 \times \frac{\$4.50}{m^2} = \$22312$$

$$\approx \mathbf{\$22300}$$

(a)



The diagram shows triangle LMN with $LM = 12$ cm, $LN = 15$ cm and $MN = 21$ cm.

(i) Calculate angle LMN.
Show that this rounds to 44.4°, correct to 1 decimal place. [4]

Use the cosine rule,

$$a^2 = b^2 + c^2 - 2bc \times \cos(A)$$

$$15^2 = 12^2 + 21^2 - 2(12)(21) \times \cos(A)$$

$$\cos A = \frac{5}{7}$$

$$A = 44.4° \ (1.d.p.)$$

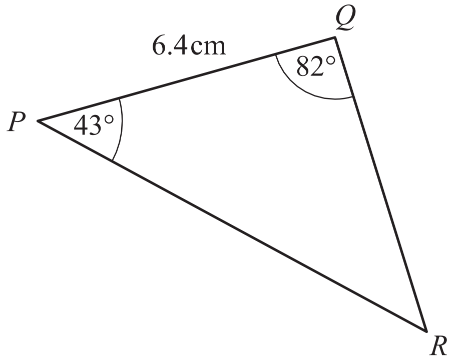
(ii) Calculate the area of triangle LMN. [2]

$$Area = \frac{1}{2}ab\sin(C)$$

$$= \frac{1}{2} \times (12) \times (21) \times \sin(44.4)$$

$$= 88.2\ cm^2$$

(b)



NOT TO SCALE

The diagram shows triangle PQR with $PQ = 6.4$cm, angle $PQR = 82°$ and angle $QPR = 43°$.

Calculate the length of PR. [4]

First, find angle QRP using angle sum of triangle:

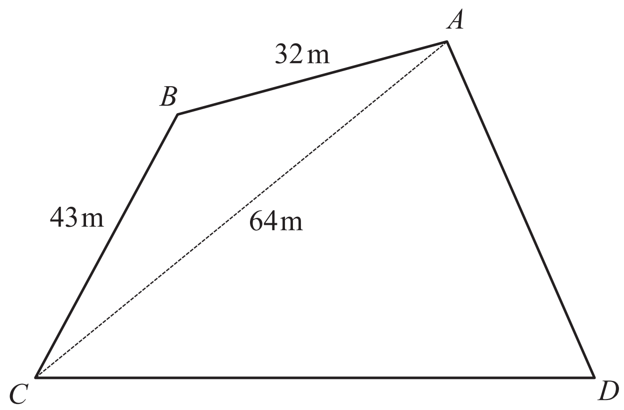
$$\mathbf{QRP = 180 - 82 - 43}$$

$$\mathbf{= 55°}$$

Use sine rule,

$$\frac{\sin(82)}{PR} = \frac{\sin(QRP)}{PQ} = \frac{\sin(55)}{6.4}$$

$$PR = 7.74\ cm$$



The diagram represents a field in the shape of a quadrilateral $ABCD$.
$AB = 32$ m, $BC = 43$ m and $AC = 64$ m.

(a) (i) Show clearly that angle $CAB = 37.0°$ correct to one decimal place. [4]

Use the cosine rule

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$

Where the side a is opposite angle A, and A represents angle CAB.

This gives us

$$43^2 = 32^2 + 64^2 - 2 \times 32 \times 64 \times \cos A$$

Rearranging for A

$$\rightarrow 43^2 - 32^2 - 64^2 = -4096\cos A$$

$$\rightarrow \cos A = \frac{-3271}{-4096}$$

$$\rightarrow \cos A = 0.799$$

$$\rightarrow A = \cos^{-1} 0.799$$

$$= 37.00490522$$

$$= \mathbf{37.0\ (1}\boldsymbol{dp}\mathbf{)}$$

(ii) Calculate the area of the triangle ABC. [2]

Area of a triangle is

$$\boldsymbol{Area = \frac{1}{2}bc\sin A}$$

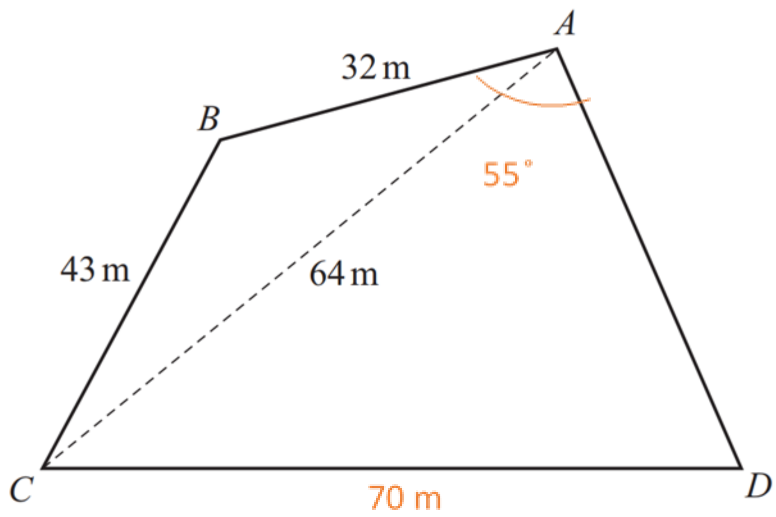
This gives us

$$Area = \frac{1}{2} \times 32 \times 64 \times \sin 37$$

$$\boldsymbol{= 616.3}$$

(b) $CD = 70$ m and angle $DAC = 55°$.

Calculate the perimeter of the whole field $ABCD$. [6]



Use the sine rule to find angle ADC

$$\frac{\sin \boldsymbol{A}}{\boldsymbol{a}} = \frac{\sin \boldsymbol{B}}{\boldsymbol{b}}$$

Where angles A and B are opposite sides a and b respectively.

$$\frac{\sin 55}{70} = \frac{\sin ADC}{64}$$

$$\rightarrow ADC = \sin^{-1}\left(\frac{64\sin 55}{70}\right)$$

$$= 48.5$$

All angles in a triangle add to 180 so we can find ACD

$$ACD = 180 - 55 - 48.5$$

$$= 76.5$$

We can find the length of AD by using the cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\rightarrow AD^2 = 70^2 + 64^2 - 2 \times 70 \times 64 \times \cos 76.5$$

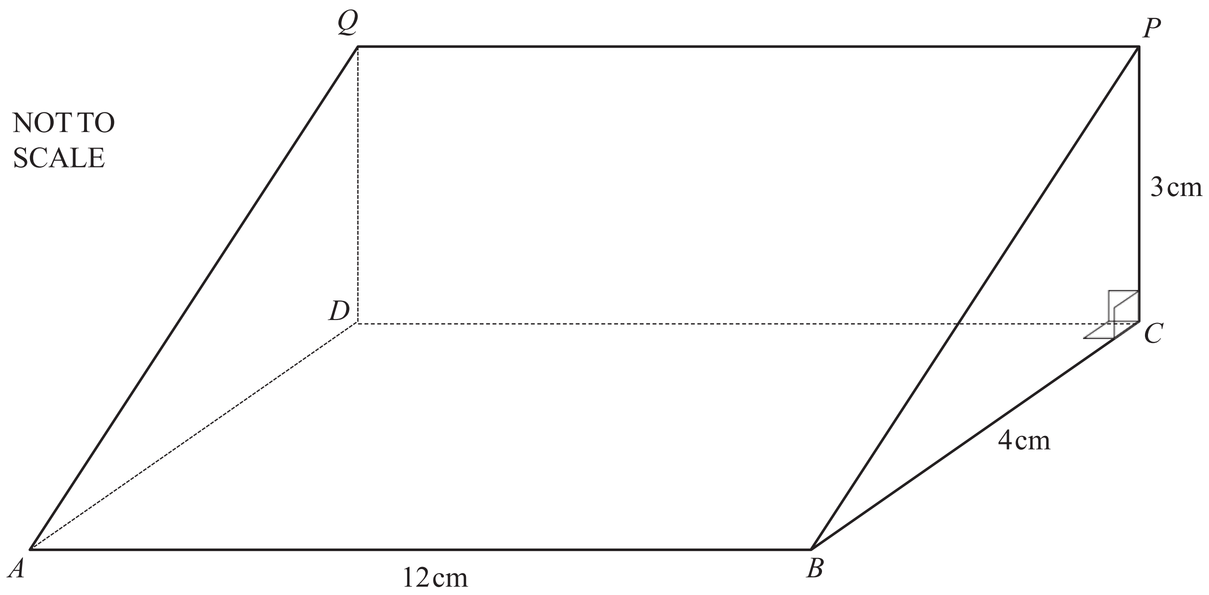
$$= 6904.32954$$

$$\rightarrow AD = 83.1$$

Hence the whole perimeter is

$$83.1 + 43 + 70 + 32$$

$$= \mathbf{228.1}$$



The diagram shows a triangular prism of length 12 cm.

The rectangle *ABCD* is horizontal and the rectangle *DCPQ* is vertical.

The cross-section is triangle *PBC* in which angle $BCP = 90°$, $BC = 4$ cm and $CP = 3$ cm.

(a) (i) Calculate the length of *AP*. [3]

PC is perpendicular on the plane of the rectangle ABCD.

AP represents the hypothenuse in the right-angled triangle, with PC perpendicular on AC.

We can work out the size of AC, the hypothenuse of the right-angled triangle ABC.

We can use Pythagoras' Theorem to work out the size of AC.

$4^2 + 12^2 = AC^2$

$AC^2 = 160$

We can use Pythagoras' Theorem to work out the size of AC.

$3^2 + 160 = AP^2$

$AP^2 = 169$

AP = 13 cm

(ii) Calculate the angle of elevation of P from A. [2]

The angle of elevation of P form A is equivalent to the angle PAC.

In the right-angle triangle PAC:

Tan PAC = $\frac{\text{PC}}{\text{AC}}$

Angle PAC = 13.32°

(b) (i) Calculate angle PBC. [2]

In the right-angle triangle PBC:

Tan PBC = $\frac{\text{PC}}{\text{BC}}$

Tan PBC = $\frac{3}{4}$

Angle PBC = 36.86°

(ii) X is on BP so that angle $BXC = 120°$.

Calculate the length of XC. [3]

In the triangle BXC we can use the sine rule to work out the size of XC.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Where A is an angle in the triangle and a is the opposite side to angle A.

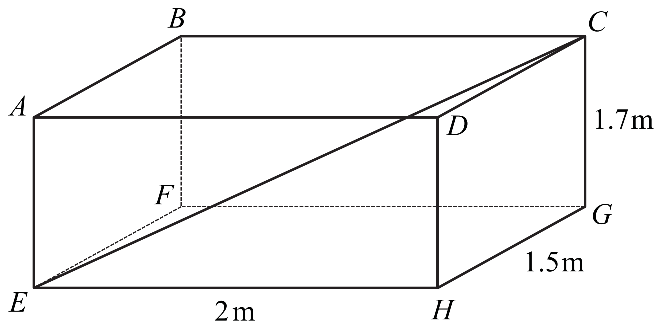
In triangle BXC:

$$\frac{\sin BXC}{BC} = \frac{\sin PBC}{XC}$$

$$\frac{\sin 120°}{4\ cm} = \frac{\sin 36.86°}{XC}$$

$$XC = \frac{4\text{ cm x }0.599}{0.866}$$

XC = 2.77



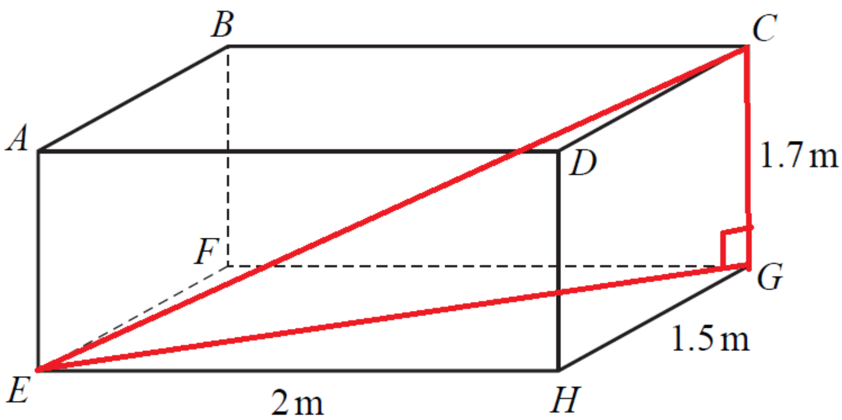
NOT TO SCALE

The diagram shows a box *ABCDEFGH* in the shape of a cuboid measuring 2 m by 1.5 m by 1.7 m.

(a) Calculate the length of the diagonal *EC*. [4]

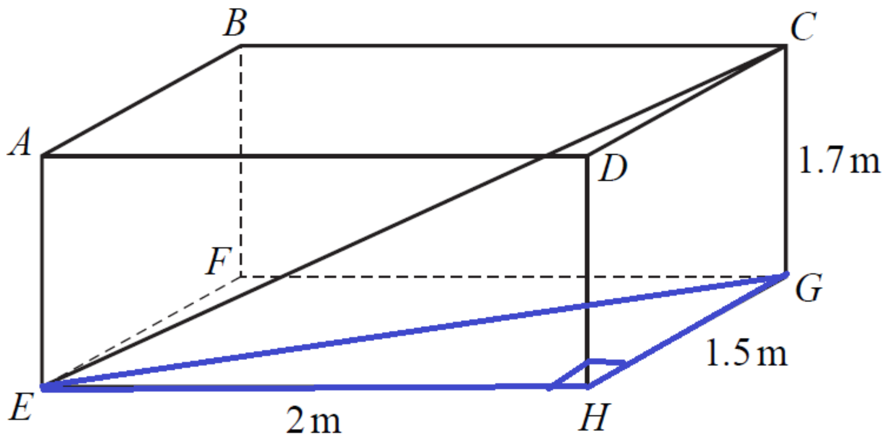
The length of a diagonal EC can be calculated using Pythagoras rule:

$$EC^2 = EG^2 + GC^2$$



We are not given the length of EG but we can calculate it using Pythagoras rule again:

$$EG^2 = EH^2 + HG^2$$



Hence we have a formula for the length of EC.

$$EC^2 = EH^2 + HG^2 + GC^2$$

Plug in given values:

$$EC^2 = (2m)^2+(1.5m)^2+(1.7m)^2$$

$$EC^2 = 4m^2 + 2.25m^2 + 2.89^2$$

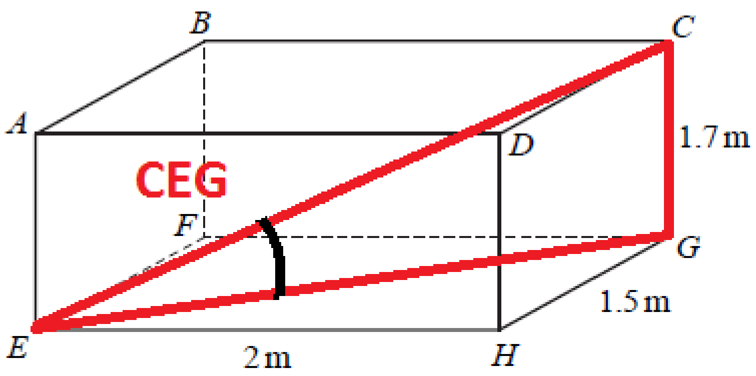
$$EC^2 = 9.14m^2$$

Take square root of both sides to get the length of the diagonal EC (3sf).

$$\mathbf{EC = 3.02m}$$

(b) Calculate the angle between EC and the base $EFGH$. [3]

The angle between EC and the base EFGH is essentially the angle CEG. We can calculate this angle using sine function (opposite over hypotenuse):



$$\sin(CEG) = \frac{CG}{EC}$$

Plug in known values:

$$\sin(CEG) = \frac{1.7m}{3.02m}$$

$$\sin(CEG) = 0.5629 \ldots$$

Apply inverse sin function to both sides of the equation to find the angle CEG.

$$\text{angle CE} \quad = \arcsin(0.5629 \ldots)$$

$$\textbf{angle CEG} = \mathbf{34.3°}$$

(c) (i) A rod has length 2.9 m, correct to 1 decimal place.

What is the upper bound for the length of the rod? [1]

The length of the rod is 2.9m correct to 1 decimal place. This means that it is correct to 0.1m.

To get the upper bound, we add half the precision to the given value.

$$\text{upper bound} = 2.9m + \frac{0.1m}{2}$$

$$\textbf{upper bound} = \mathbf{2.95}\boldsymbol{m}$$

(ii) Will the rod fit completely in the box?

Give a reason for your answer. [1]

We can fit the rod into the box if it is smaller (or just equal) the main diagonal of the box.

In part a), we have found that the diagonal is:

$$EC = 3.02m$$

This is greater that the upper bound of the rod length found in c)i), therefore:

yes the rod will fit.

Trigonometry
Difficulty: Medium

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Trigonometry
Paper	Paper 4
Difficulty	Medium
Booklet	Model Answers 3

Time allowed: **91 minutes**

Score: **/79**

Percentage: **/100**

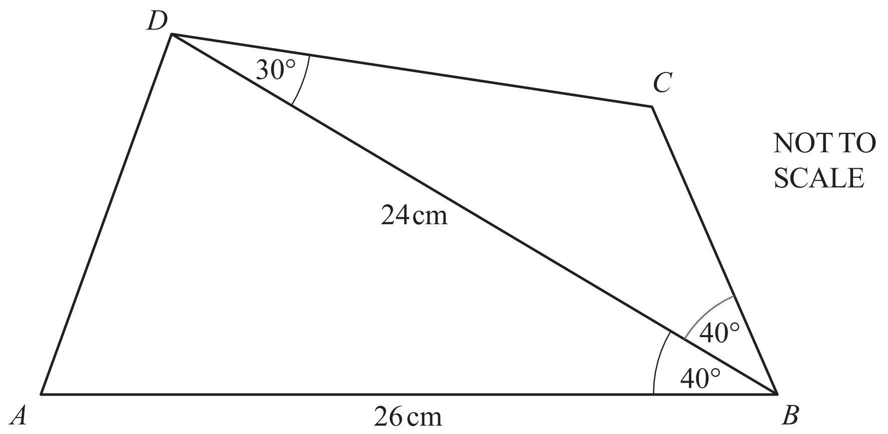
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



ABCD is a quadrilateral and *BD* is a diagonal.
AB = 26 cm, *BD* = 24 cm, angle *ABD* = 40°, angle *CBD* = 40° and angle *CDB* = 30°.

(a) Calculate the area of triangle *ABD*. [2]

The *Sine Rule* for Area of a general triangle is

$$A = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times 24 \times 26 \times \sin 40$$

$$= \mathbf{200.5}$$

(b) Calculate the length of *AD*. [4]

We use the Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Where side a is opposite angle A.

$$AD^2 = 24^2 + 26^2 - 2(24)(26)\cos 40$$

$$= 295.976535$$

$$\rightarrow \boldsymbol{AD = 17.2}$$

(c) Calculate the length of BC. [4]

Angles in a triangle add to 180, so

$$BCD = 180 - 30 - 40$$

$$= 110$$

Now using the Sine Rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

We have

$$\frac{\sin 30}{BC} = \frac{\sin 110}{24}$$

$$\rightarrow \boldsymbol{BC = \frac{24 \sin 30}{\sin 110} = 12.8}$$

(d) Calculate the shortest distance from the point C to the line BD. [2]

The shortest line from C to BD is perpendicular to BD, thus creating a right-angle triangle.

Let P be the point where the line from C intersects BD.

We can use the trigonometric relation

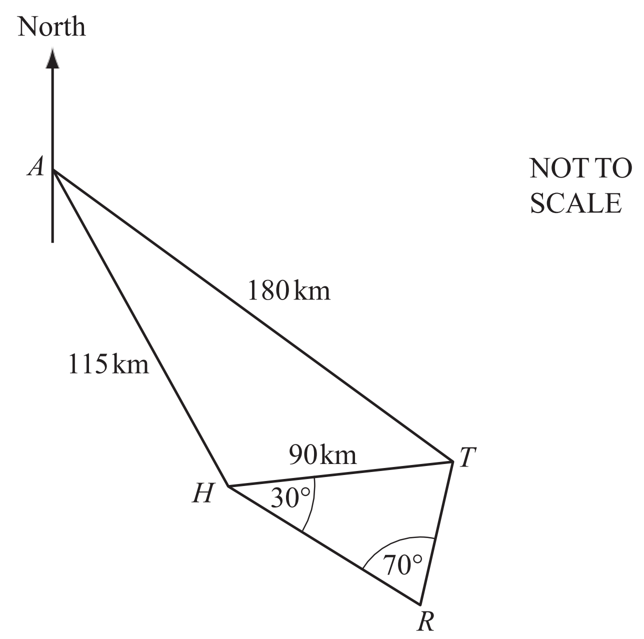
$$\sin\theta = \frac{opp}{hyp}$$

$$\rightarrow \sin 40 = \frac{CP}{BC}$$

$$\rightarrow CP = BC\sin 40$$

$$= 12.8\sin 40$$

$$= \mathbf{8.22}$$



The diagram shows some straight line distances between Auckland (A), Hamilton (H), Tauranga (T) and Rotorua (R).
$AT = 180$ km, $AH = 115$ km and $HT = 90$ km.

(a) Calculate angle HAT.
Show that this rounds to 25.0°, correct to 3 significant figures. [4]

We use the cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

where angle A is opposite side a.

This gives us

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\rightarrow \cos HAT = \frac{180^2 + 115^2 - 90^2}{2(115)(180)}$$

$$= 0.9064$$

$$\rightarrow HAT = 24.987$$

$$= \mathbf{25.0}$$

(b) The bearing of H from A is 150°.

Find the bearing of

(i) T from A, [1]

$150 - 25$

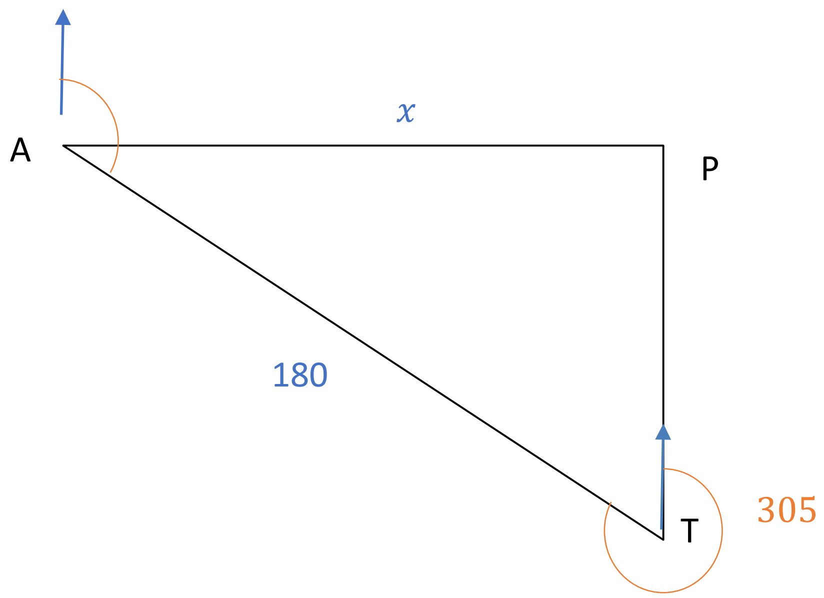
$= \mathbf{125}$

(ii) A from T. [1]

$180 + 125$

$= \mathbf{305}$

(c) Calculate how far T is east of A. [3]



We need to find the distance AP, which we have called x.

We can see that angle ATP is

$$360 - 305$$

$$= 55$$

Using trigonometric relations, we have

$$\sin ATP = \frac{opp}{hyp}$$

$$\rightarrow \sin 55 = \frac{x}{180}$$

$$\rightarrow x = 180 \times \sin 55$$

$$= \mathbf{147}$$

(d) Angle $THR = 30°$ and angle $HRT = 70°$.

Calculate the distance TR. [3]

We use sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

to get

$$\frac{\sin 70}{90} = \frac{\sin 30}{TR}$$

$$\rightarrow TR = \frac{90 \sin 30}{\sin 70}$$

$$= \mathbf{47.9}$$

(e) On a map the distance representing HT is 4.5cm.

The scale of the map is 1 : n.

Calculate the value of n. [2]

We have that

$$n \times 4.5\ cm = 90\ km$$

Scaling everything the same

$$90\ km = 90 \times 10^3\ m$$

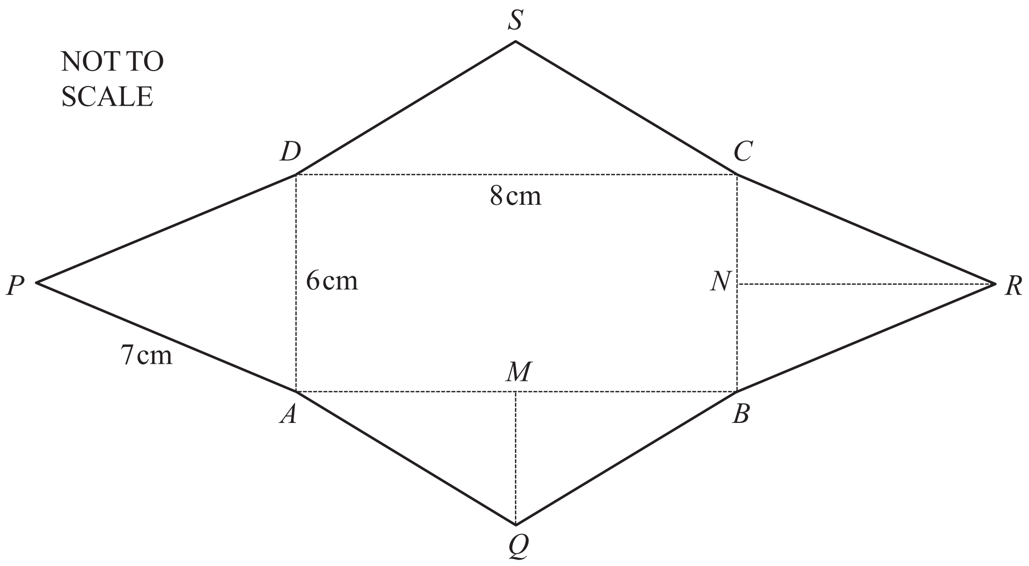
$$90 \times 10^3\ m = 90 \times 10^3 \times 100\ cm$$

$$= 9 \times 10^6$$

$$\rightarrow n \times 4.5 = 9 \times 10^6$$

$$\rightarrow n = 2 \times 10^6$$

$$= \mathbf{2\ 000\ 000}$$



The diagram above shows the net of a pyramid.

The base *ABCD* is a rectangle 8 cm by 6 cm.

All the sloping edges of the pyramid are of length 7 cm.

M is the mid-point of *AB* and *N* is the mid-point of *BC*.

(a) Calculate the length of

(i) *QM*, [2]

Using Pythagoras' Theorem

$$QM^2 = AQ^2 - AM^2$$

$$= 7^2 - 4^2$$

$$= 33$$

$$\rightarrow \boldsymbol{QM = 5.74}$$

(ii) RN. [1]

Again, using Pythagoras'

$$RN^2 + CN^2 = RC^2$$

$$\rightarrow RN^2 = 7^2 - 3^2$$

$$= 40$$

$$\rightarrow \boldsymbol{RN = 6.32}$$

(b) Calculate the surface area of thepyramid. [2]

Add the rectangle and the four triangles.

Area of a triangle is

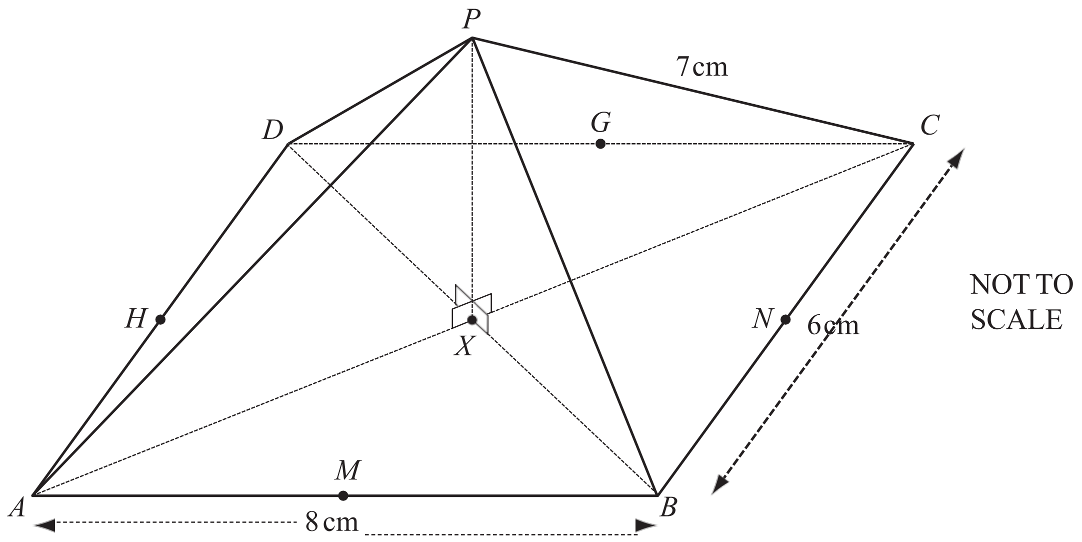
$$\boldsymbol{A = \frac{1}{2} base \times height}$$

Hence

$$A = 6 \times 8 + (6)(6.32) + (8)(5.74)$$

$$= \boldsymbol{131.84}$$

(c)



The net is made into a pyramid, with P, Q, R and S meeting at P.

The mid-point of CD is G and the mid-point of DA is H.

The diagonals of the rectangle $ABCD$ meet at X.

(i) Show that the height, PX, of the pyramid is 4.90 cm, correct to 2 decimal places. [2]

We have that

$$CX^2 = \left(\frac{1}{2}CB\right)^2 + \left(\frac{1}{2}AB\right)^2$$

$$= 3^2 + 4^2$$

$$= 25$$

$$\to CX = 5$$

Hence

$$CX^2 + PX^2 = 7^2$$

$$\to PX^2 = 49 - 25$$

$$= 24$$

$$\to \mathbf{PX = 4.90}$$

(ii) Calculate angle *PNX*. [2]

Using the trig ratio

$$\tan\theta = \frac{opp}{adj}$$

we have that

$$\tan PNX = \frac{PX}{NX}$$

$$\rightarrow \tan PNX = \frac{4.90}{4}$$

$$\rightarrow PNX = \tan^{-1}\left(\frac{4.90}{4}\right)$$

$$= \mathbf{50.8}$$

(iii) Calculate angle *HPN*. [2]

We have that

$$HPN = 2 \times XPN$$

and

$$XPN = 90 - 50.8$$

$$= 39.2$$

Hence

$$\boldsymbol{HPN = 78.4}$$

(iii) Calculate the angle between the edge PA and the base $ABCD$. [3]

Again, using

$$\tan\theta = \frac{opp}{adj}$$

$$\tan PAX = \frac{PX}{AX}$$

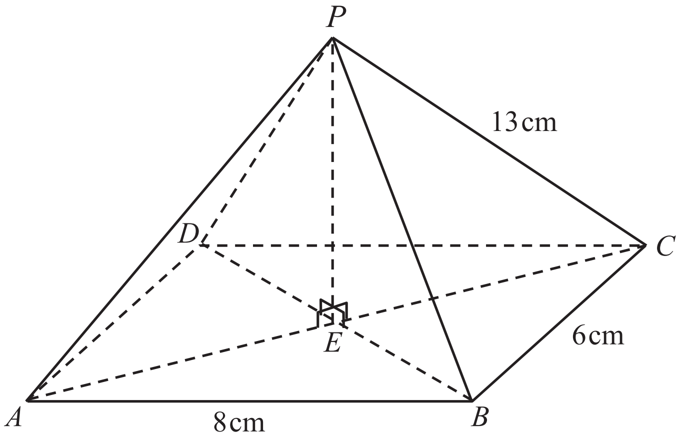
$$= \frac{4.9}{5}$$

$$\rightarrow PAX = \tan^{-1}\left(\frac{4.9}{5}\right)$$

$$= 44.4$$

(iii) Write down the vertices of a triangle which is a plane of symmetry of the pyramid. [1]

PHN or PGM



NOT TO SCALE

The diagram shows a pyramid on a horizontal rectangular base *ABCD*.
The diagonals of *ABCD* meet at *E*.
P is vertically above *E*.
$AB = 8$ cm, $BC = 6$ cm and $PC = 13$ cm.

(a) Calculate *PE*, the height of the pyramid. [3]

In the right-angled triangle ABC, we can use Pythagoras' Theorem to work out the size of AC.

$AC^2 = AB^2 + BC^2$

$AC^2 = 6^2 + 8^2$

AC = 10 cm

E is the middle of AC, so AE = 5 cm

We can use Pythagoras' Theorem in triangle PEC to work out the size of PE.

$PE^2 + 5^2 = 13^2$

PE = 12 cm

(b) Calculate the volume of the pyramid.

[The volume of a pyramid is given by $\frac{1}{3}$ × area of base × height.] [2]

area of the base = 6 cm x 8 cm = 48 cm^2

$V = \frac{1}{3} \times 48\ cm^2 \times 12\ cm$

V = 192 cm^3

(c) Calculate angle PCA. [2]

In triangle PEC:

Sin PCE = $\frac{\text{PE}}{\text{PC}}$

Sin PCE = $\frac{12}{13}$ = 0.923

Angle PCE = 67.38°

(d) M is the mid-point of AD and N is the mid-point of BC. Calculate angle MPN. [3]

EM = AB/2 = 4 cm

In the right-angled triangle PME:

Tan PME = $\frac{\text{PE}}{\text{ME}}$

Tan PME = $\frac{12\text{ cm}}{4\text{ cm}}$ = 3

Angle PME = 71.56°

PM = PN, therefore, the triangle MPN is isosceles, with the angles PME and PNE also equal.

Angle MPN = 180° - 2 x angle PME

Angel MPN = 180° - 143.1°

Angle MPN = 36.9°

(e) (i) Calculate angle PBC. [2]

In the right-angled triangle PMB with PM perpendicular on BC and M the mid-point of BC.

MB = MC = BC/2 = 3 cm

Cos PBC = $\frac{MB}{PB}$

Cos PBC = $\frac{3}{13}$ = 0.23

Angle PBC = 76.7°

(ii) K lies on PB so that $BK = 4$ cm.
Calculate the length of KC. [3]

In the triangle CKB, we can use the cosine rule to work out the length of KC.

Cosine rule:

$a^2 = b^2 + c^2 – 2bc \cos A$

where a, b and c are sides in the triangle and A is the angle opposite side a.

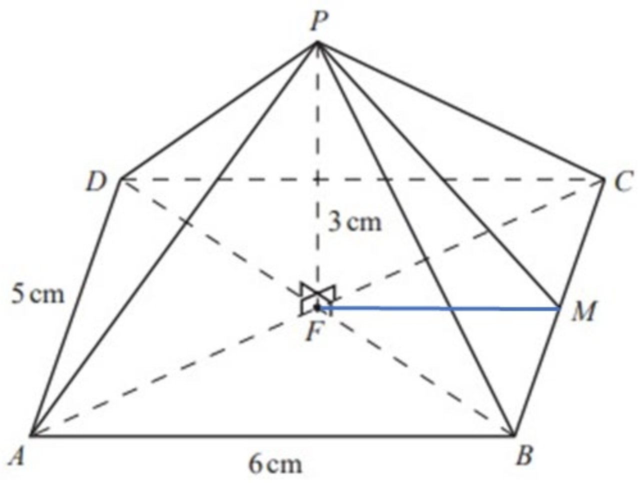
In our case:

$KC^2 = 4^2 + 6^2 – 2 \times 4 \times 6 \times \cos PBC$

Angle PBC = 76.7°

$KC^2 = 40.95$

KC = 6.40 cm



The diagram shows a pyramid on a rectangular base $ABCD$, with $AB = 6$ cm and $AD = 5$ cm.
The diagonals AC and BD intersect at F.
The vertical height $FP = 3$ cm.

(a) How many planes of symmetry does the pyramid have? [1]

A plane of symmetry is a plane which separates the pyramid in 2 figures which are the same.

One of the planes of symmetry is parallel with CB and DA and the other planes of symmetry is parallel with AB and DC.

The pyramid has 2 planes of symmetry.

(b) Calculate the volume of the pyramid.
[The volume of a pyramid is $\frac{1}{3}$ × area of base × height.] [2]

The formula for the volume of a pyramid is:

Volume = $\frac{1}{3}$ x area of the base x height

In our case, the area of the base is the area of the rectangle ABCD and the height is PF.

The area of a rectangle is:

A = length x width

The area of the rectangle ABCD is:

A = 5 cm x 6 cm

A = 30 cm^2

The volume of the pyramid is:

V = $\frac{1}{3}$ x 30 cm^2 x 3 cm

V = 30 cm^3

(c) The mid-point of BC is M.
Calculate the angle between PM and the base. [2]

The height PF is perpendicular on the plane of the rectangle ABCD.

FM is a line in the plane of the rectangle ABCD, therefore PF is perpendicular on FM.

The angle between PM and the base of the pyramid is the angle PMF.

In the right-angled triangle PFM:

tan PMF = $\frac{3\text{ cm}}{\text{FM}}$

FM is half the size of the side AB.

Therefore, FM = $\frac{6\,cm}{2}$

FM = 3 cm

tan PMF = $\frac{3\text{ cm}}{3\text{ cm}}$

tan PMF = 1

We know that tan 45° = 1

Therefore, angle PMF is 45°.

(d) Calculate the angle between PB and the base. [4]

FB is a line in the plane of the rectangle ABCD.

Therefore, the angle between PB and the base is the angle PBF.

FB is half the size of the diagonal DB.

In the right-angled triangle DAB, the diagonal DB is the hypothenuse.

Using Pythagoras' Theorem, we can work out the size of the diagonal DB.

$DA^2 + AB^2 = DB^2$

$DB^2 = 5^2 + 6^2\ cm^2$

$DB^2 = 25 + 36\ cm^2$

DB = 7.81 cm

$FB = \frac{DB}{2}$

FB = 3.9 cm

In the right-angled triangle PFB:

$\tan PBF = \frac{PF}{FB}$

$\tan PBF = \frac{3\ cm}{3.9\ cm}$

tan PBF = 0.769

angle PBF = 37.5°

(e) Calculate the length of PB. [2]

Using Pythagoras' Theorem in the right-angled triangle PBF, we can work out the size of the hypothenuse PB.

$FB^2 + PF^2 = PB^2$

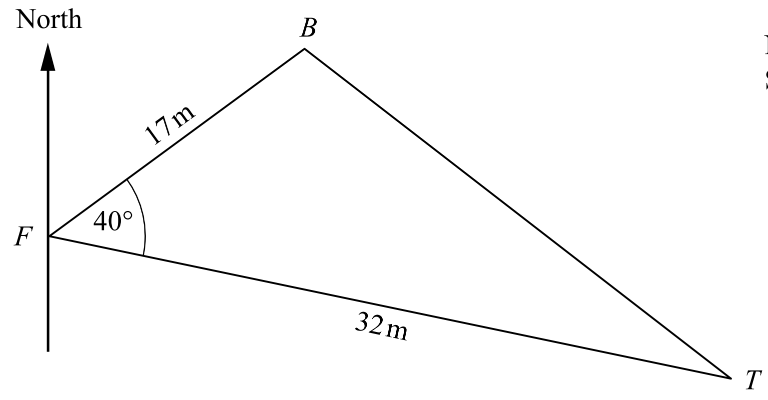
$PB^2 = 3^2 + 3.9^2 \text{ cm}^2$

$PB^2 = 9 + 15.21 \text{ cm}^2$

$PB^2 = 24.21 \text{ cm}^2$

PB = 4.92 cm

Question 6



NOT TO SCALE

Felipe (F) stands 17 metres from a bridge (B) and 32 metres from a tree (T). The points F, B and T are on level ground and angle BFT # $40°$.

(a) Calculate

(i) the distance BT, [4]

We can use the cosine rule to work out the size of BT.

Cosine rule:

$a^2 = b^2 + c^2 - 2bc \cos A$

where a, b and c are sides in the triangle and A is the angle opposite side a.

In our case:

$BT^2 = 17^2 + 32^2 - 2 \times 17 \times 32 \cos 40°$

BT = 21.91 m

(ii) the angle BTF. [3]

We can use the sine rule to work out the size of the angle BTF.

Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

where a and b are sides in the triangle, A is the angle opposite side a and B is the angle opposite b.

In our case:

$$\frac{\text{a}}{\sin \text{A}} = \frac{\mathbf{b}}{\mathbf{\sin B}}$$

$$\frac{21.9 \text{ m}}{\sin 40°} = \frac{\mathbf{17 \text{ m}}}{\mathbf{\sin BTF}}$$

sin BTF = 0.498

Angle BTF = 29.9°

(b) The bearing of B from F is 085°. Find the bearing of

(i) T from F, [1]

By looking at the diagram, we work out that:

The bearing of T from F is equal to the bearing of B from F

plus the angle BFT.

The bearing of T from F = 085° + 40° = 125°

(ii) F from T, [1]

The bearing of T from F is 125°.

The bearing needs to be measured from the line representing

North and clockwise.

In our case, the North lines from points F and T are parallel,

therefore, their corresponding angles will be equal.

Th bearing of F from T is: 180° + 125°

= 305°

(iii) B from T. [1]

The bearing of B from T is equal to the bearing of F from T plus the angle BTF.

The bearing of B from T = 29.9° + 305°

The bearing of B from T = 334.9°

(c) The top of the tree is 30 metres vertically above T.
Calculate the angle of elevation of the top of the tree from F. [2]

The tree will be perpendicular on the ground.

The angle of elevation can be worked out by calculating the tan of this angle in the right-angled triangle with the tree perpendicular FT.

tan angle = $\frac{30 \text{ m}}{32 \text{ m}}$

tan angle = 0.9375

angle = 43.2°

Trigonometry
Difficulty: Hard

Model Answers 1

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Trigonometry
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 1

Time allowed: **101 minutes**

Score: **/88**

Percentage: **/100**

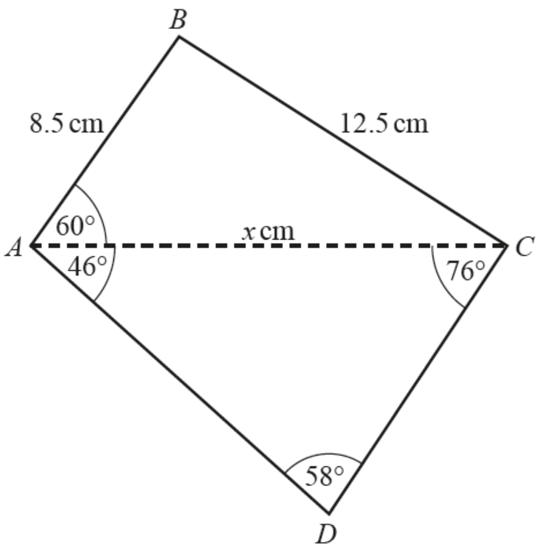
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



NOT TO SCALE

The diagram shows a quadrilateral *ABCD*.

(a) The length of *AC* is *x* cm.

Use the cosine rule in triangle *ABC* to show that $2x^2 - 17x - 168 = 0$. [4]

Cosine rule is

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\rightarrow 12.5^2 = 8.5^2 + x^2 - 2(8.5)x\cos 60$$

$$\rightarrow \frac{625}{4} = \frac{289}{4} + x^2 - 8.5x$$

$$\rightarrow x^2 - 8.5x - 84 = 0$$

$$\rightarrow 2x^2 - 17x - 168 = 0$$

(b) Solve the equation $2x^2 - 17x - 168 = 0$.
Show all your working and give your answers correct to 2 decimal places. [4]

Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\rightarrow x = \frac{(17 \pm \sqrt{1633})}{4}$$

$$\rightarrow x = 14.35, \qquad x = -5.85$$

(c) Use the sine rule to calculate the length of CD. [3]

Sine rule is

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\rightarrow \frac{x}{\sin 58} = \frac{CD}{\sin 46}$$

$$\rightarrow CD = \frac{14.35 \sin 46}{\sin 58}$$

$$= \mathbf{12.17}$$

(d) Calculate the area of the quadrilateral $ABCD$. [3]

Area of a triangle is

$$A = \frac{1}{2} ab \sin C$$

$$\rightarrow A_{ABC} = \frac{1}{2}(8.5)(14.35) \sin 60$$

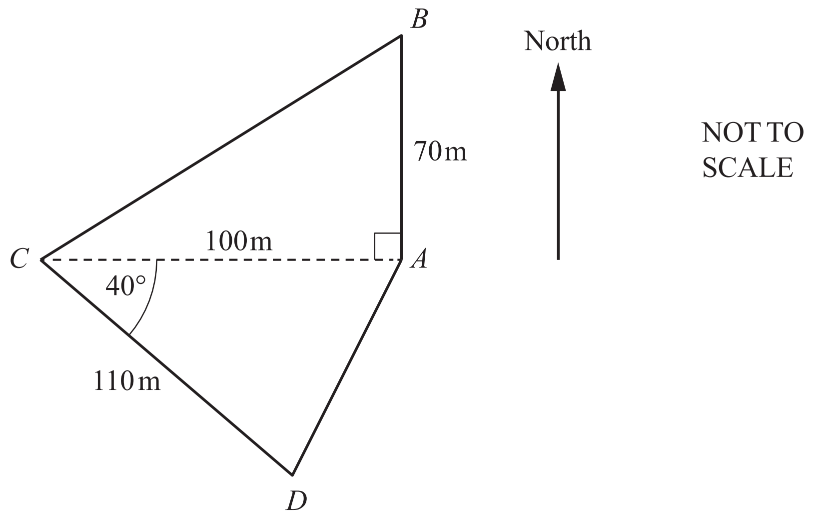
$$= 52.82$$

$$A_{ACD} = \frac{1}{2}(14.35)(12.17) \sin 76$$

$$= 84.73$$

$$\rightarrow A = 84.73 + 52.82$$

$$= \mathbf{137.5}$$



The diagram shows a field *ABCD*.

(a) Calculate the area of the field *ABCD*.

[3]

Area of Field = Area of Triangle ABC + Area of Triangle ACD

For ABC the base and the height are given so use Area= $\frac{1}{2} \times$ base $\times$ height

ACD is a non-Right-Angled Triangle so use Area = $\frac{1}{2} \times a \times b \times \sin C$

Area of Field = $\frac{1}{2} \times 100 \times 70 + \frac{1}{2} \times 100 \times 110 \times \sin 40$

Area of Field = 7035 m² (to nearest m^2)

(b) Calculate the perimeter of the field *ABCD*.

[5]

Perimeter = $AB + BC + CD + DA$

We know AB and CD. First, use Pythagoras to find BC:

$$BC^2 = 100^2 + 70^2$$

$$BC = \sqrt{100^2 + 70^2} = 122.065556\ldots$$

Now use the Cosine Rule to find DA:

$$DA^2 = 100^2 + 110^2 - 2 \times 100 \times 110 \times \cos 40$$

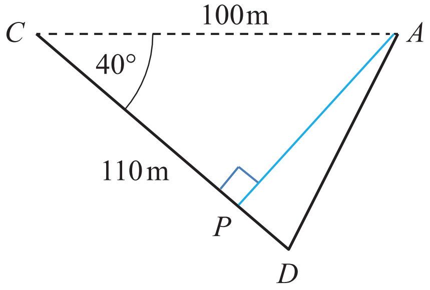
$$DA = \sqrt{100^2 + 110^2 - 2 \times 100 \times 110 \times \cos 40} = 72.4363324\ldots$$

So: Perimeter = $70 + 122.06\ldots + 110 + 72.43\ldots$

Perimeter = 375m (to nearest m)

(c) Calculate the shortest distance from A to CD.

[2]



Draw the length to be found (AP on the diagram above).

Remember the shortest distance is always at right-angles to the line.

Since APC is a right-angled triangle we can use SOHCAHTOA to find AP:

$$\text{OPP} = \text{HYP} \times \sin\theta$$

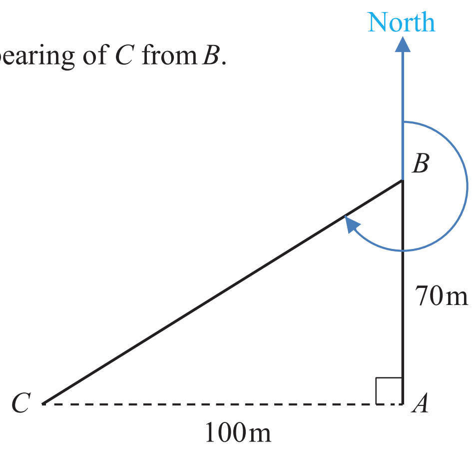
$$AP = 100 \times \sin 40$$

$$\boldsymbol{AP = 64.3\text{m}} \quad \text{(to nearest m)}$$

(d) B is due north of A.

Find the bearing of C from B.

[3]



Bearings measured Clockwise from the North line (as shown on the diagram).

Bearing = 180 + angle ABC

Use SOHCAHTOA to find angle ABC:

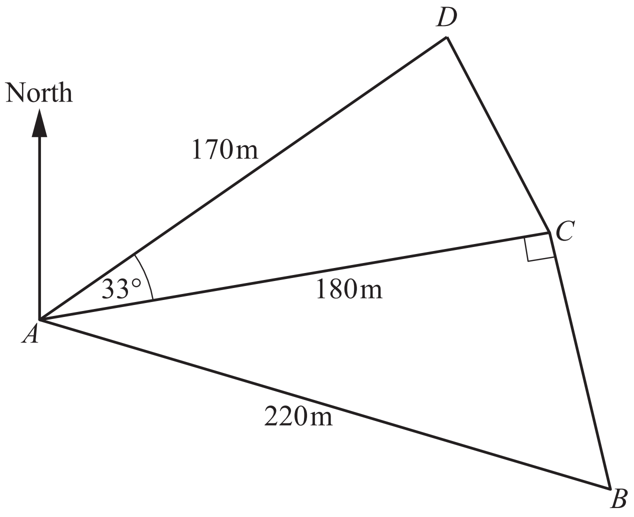
$$\tan\theta = \frac{OPP}{ADJ}$$

$$\tan ABC = \frac{100}{70}$$

$$ABC = \tan^{-1}\left(\frac{100}{70}\right) = 55°$$

$$\text{Bearing} = 180 + 55$$

$$\textbf{Bearing} = \boldsymbol{235°}$$



NOT TO SCALE

The diagram shows five straight footpaths in a park.
$AB = 220$ m, $AC = 180$ m and $AD = 170$ m.
Angle $ACB = 90°$ and angle $DAC = 33°$.

(a) Calculate BC. [3]

We know 2 sides of the right-angled triangle ABC and want to know the third so use Pythagoras Theorem:

$$BC^2 + AC^2 = AB^2$$

$$BC^2 + 180^2 = 220^2$$

$$BC = \sqrt{220^2 - 180^2}$$

$$\boldsymbol{BC = 126} \text{ (to 3sf)}$$

(b) Calculate CD. [4]

We know 2 sides and an angle of a non-right-angled triangle and want to know the third side so use the Cosine Rule:

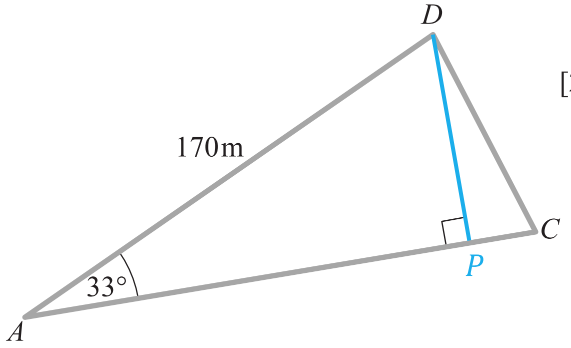
$$CD^2 = AC^2 + AD^2 - 2 \times AC \times AD \times \cos D\hat{A}C$$

$$CD = \sqrt{180^2 + 170^2 - 2 \times 180 \times 170 \times \cos 33}$$

$$BC = \sqrt{220^2 - 180^2}$$

$$\boldsymbol{CD = 99.9} \text{ (to 3sf)}$$

(c) Calculate the shortest distance from D to AC. [2]



Draw the perpendicular line from D to AC

Use SOHCAHTOA on the triangle ADP to find DP:

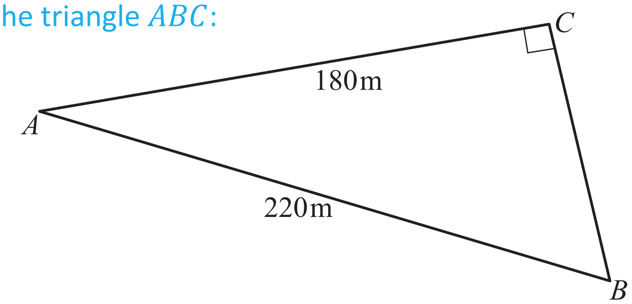
$$OPP = HYP \times \sin\theta$$

$$DP = 170 \times \sin 33$$

$$\mathbf{DP = 92.6m}$$

(d) The bearing of D from A is 047°.

Calculate the bearing of B from A. [3]



Need to find angle CAB so use SOHCAHTOA in the triangle ABC:

$$\cos\theta = \frac{ADJ}{HYP}$$

$$C\hat{A}B = \cos^{-1}\left(\frac{180}{220}\right)$$

$$C\hat{A}B = 35°$$

$$\mathbf{Bearing} = 47 + 33 + 35 = \mathbf{115°}$$

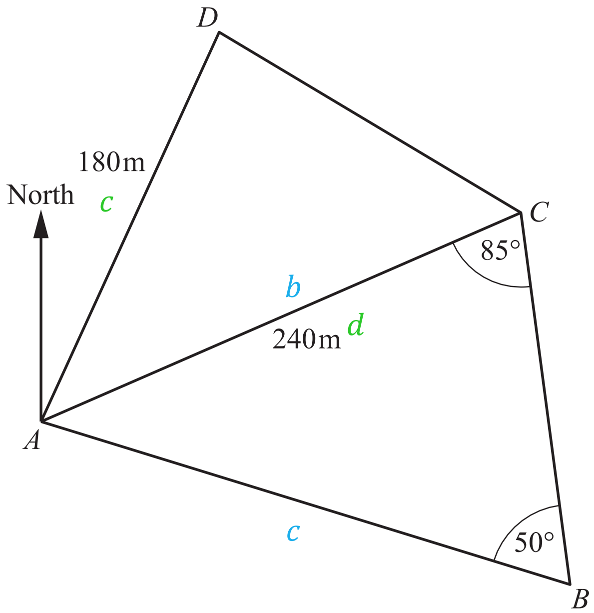
(e) Calculate the area of the quadrilateral $ABCD$. [3]

Use $Area = \frac{1}{2}bh$ for each triangle and add them together:

$$Area\ of\ ABCD = \frac{1}{2} \times AC \times DP + \frac{1}{2} \times AC \times CB$$

$$Area\ of\ ABCD = \frac{1}{2} \times 180 \times 92.6 + \frac{1}{2} \times 180 \times 126$$

$\mathbf{Area\ of\ ABCD = 19717}$ (to nearest whole number)



NOT TO SCALE

The diagram shows a field, $ABCD$.
$AD = 180$ m and $AC = 240$ m.
Angle $ABC = 50°$ and angle $ACB = 85°$.

(a) Use the sine rule to calculate AB. [3]

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{AB}{\sin 85} = \frac{240}{\sin 50}$$

$$AB = \frac{240}{\sin 50} \times \sin 85$$

$$\boldsymbol{AB = 312\text{m}} \text{ (to the nearest metre)}$$

(b) The area of triangle $ACD = 12\ 000\text{m}^2$.

Show that angle $CAD = 33.75°$, correct to 2 decimal places. [3]

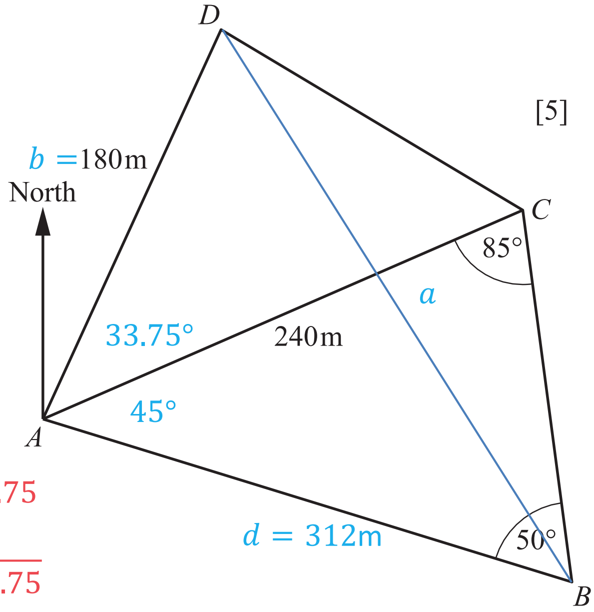
$$Area = \tfrac{1}{2} dc \sin A$$

$$12000 = \tfrac{1}{2} \times 240 \times 180 \times \sin C\hat{A}D$$

$$\sin C\hat{A}D = \frac{12000}{\frac{1}{2} \times 240 \times 180}$$

$$C\hat{A}D = \sin^{-1}\left(\frac{12000}{\frac{1}{2} \times 240 \times 180}\right)$$

$$\boldsymbol{C\hat{A}D = 33.75°}$$



(c) Calculate BD. [5]

First find $C\hat{A}B = 180 - 85 - 50 = 45°$

so $D\hat{A}B = 33.75 + 45 = 78.75°$

Now use the Cosine rule on triangle ADB:

$$a^2 = b^2 + d^2 - 2bd\cos A$$

$$BD^2 = 180^2 + 312^2 - 2 \times 180 \times 312 \cos 78.75$$

$$BD = \sqrt{180^2 + 312^2 - 2 \times 180 \times 312 \cos 78.75}$$

$\boldsymbol{BD} = \mathbf{328m}$ (to the nearest whole number)

(d) The bearing of D from A is 030°.

Find the bearing of

(i) B from A, [1]

The bearing of B from A is given by

$$Bearing = 30 + 33.75 + 45$$

$$\boldsymbol{Bearing = 108.75°}$$

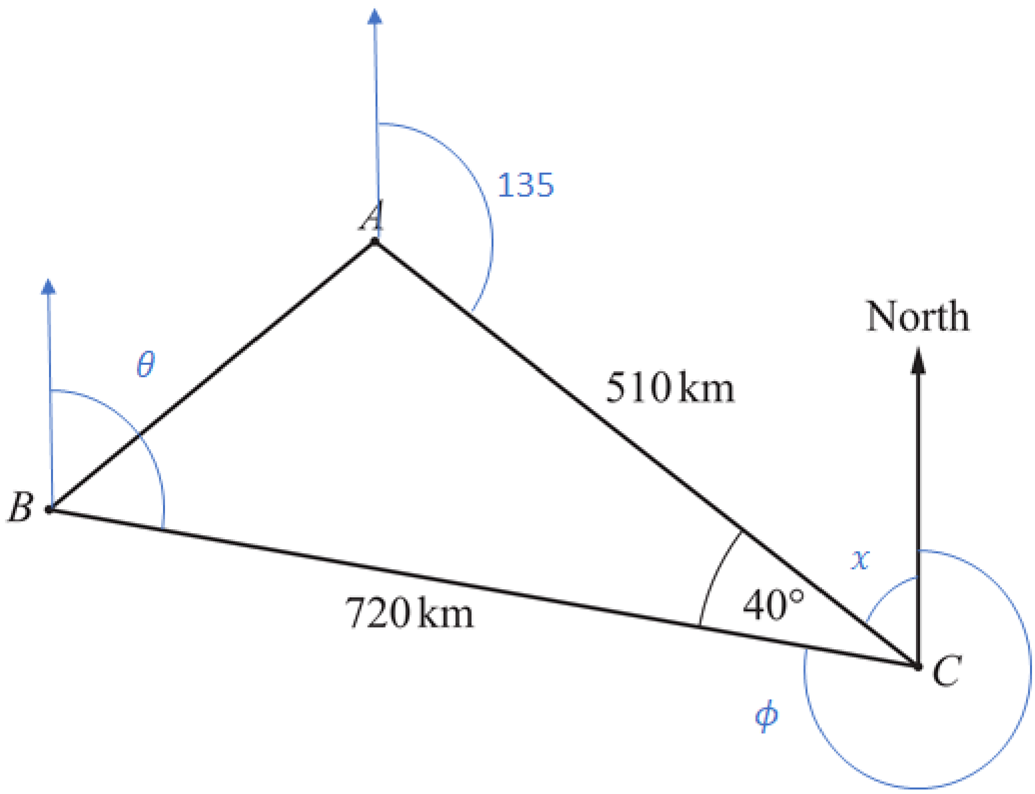
(ii) A from B. [2]

To take a reverse bearing add or subtract 180°

(keeping the answer between 0° and 360°)

$$Bearing = 108.75 + 180$$

$$\boldsymbol{Bearing = 288.75°}$$



The bearing of C from A is 135° and angle $ACB = 40°$.

(a) Find the bearing of

(i) B from C, [2]

Need to find the bearing ϕ. The angle x can be found as

$$135 + x = 180$$

$$\rightarrow x = 45$$

Hence

$$\phi + 40 + 45 = 360$$

$$\rightarrow \phi = 360 - 85$$

$$= \mathbf{275°}$$

(ii) C from B. [2]

Now need to find bearing θ.

$$\phi - \theta = 180$$

$$\rightarrow \theta = \phi - 180$$

$$= 275 - 180$$

$$= 095°$$

(b) Calculate AB and show that it rounds to 464.7 km, correct to 1 decimal place. [4]

Using the cosine rule, which can be written in this case as

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\rightarrow AB^2 = 510^2 + 720^2 - 2 \times 510 \times 720 \times \cos 40$$

$$= 215916.961$$

$$AB = 464.6686572$$

$$= \mathbf{464.7\ (1dp)}$$

(c) Calculate angle ABC. [3]

Using sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Where side b is opposite angle B.

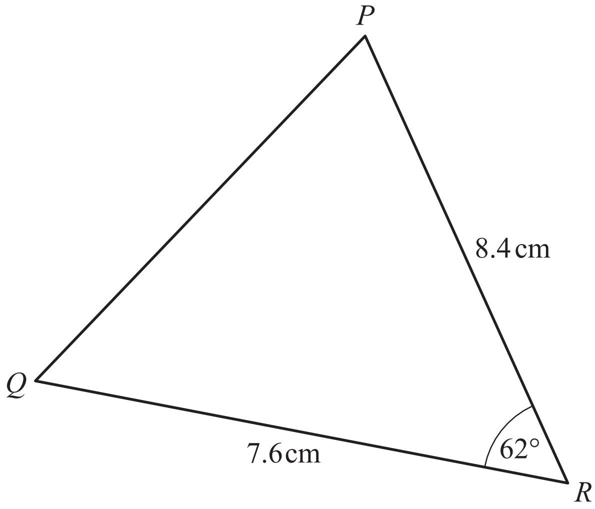
$$\frac{\sin ABC}{510} = \frac{\sin 40}{464.7}$$

$$\rightarrow \sin ABC = \frac{510 \sin 40}{464.7}$$

$$= 0.705$$

$$\rightarrow \boldsymbol{ABC = 44.9°}$$

(a)



NOT TO SCALE

In the triangle PQR, $QR = 7.6$ cm and $PR = 8.4$ cm.
Angle $QRP = 62°$.

Calculate

(i) PQ, [4]

The distance PQ can be found using the cosine rule.

$$PQ^2 = QR^2 + PR^2 - 2 \times QR \times PR \times \cos(PQR)$$

$$PQ^2 = (7.6cm)^2 + (8.4cm)^2 - 2 \times (7.6cm) \times (8.4cm) \times \cos(62°)$$

$$PQ^2 = 68.378\ cm^2$$

Take square root of both sides to get the final answer:

$$\boldsymbol{PQ = 8.27\ cm}\ (3sf)$$

(ii) the area of triangle PQR. [2]

The area of a triangle PQR is

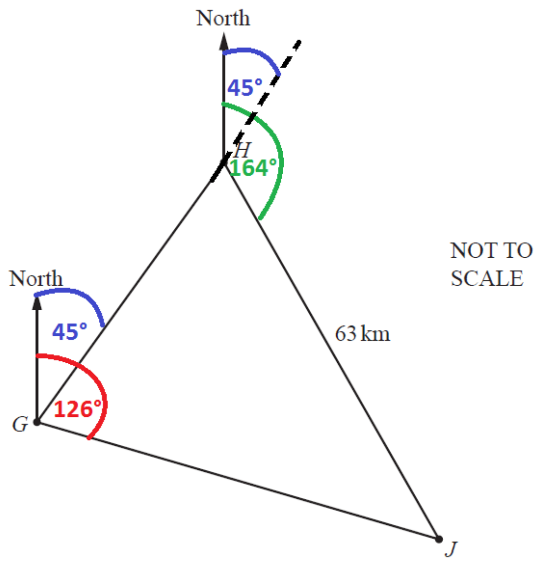
$$area = \frac{1}{2} QR \times PR \times \sin(QRP)$$

Use given values.

$$area = \frac{1}{2}(7.6cm) \times (8.4cm) \times \sin(62°)$$

$$area = \boldsymbol{28.2\ cm^2}$$

(b)



The diagram shows the positions of three small islands *G*, *H* and *J*.
The bearing of *H* from *G* is 045°.
The bearing of *J* from *G* is 126°.
The bearing of *J* from *H* is 164°.
The distance *HJ* is 63 km.

Calculate the distance *GJ*. [5]

We draw all the angles we are given on the diagram.

From the information we are given, we can calculate the size of angles HGJ and GHJ.

$$HGJ = NorthGJ - NorthGH = 126° - 45° = 81°$$

$$GHJ = 180° - (NorthHJ - NorthGJ) = 180° - (164° - 45°) = 61°$$

The size of GJ can be calculated using sine rule.

$$\frac{J}{\sin(GHJ)} = \frac{HJ}{\sin(HGJ)}$$

$$\frac{J}{\sin(61°)} = \frac{63}{\sin(81°)}$$

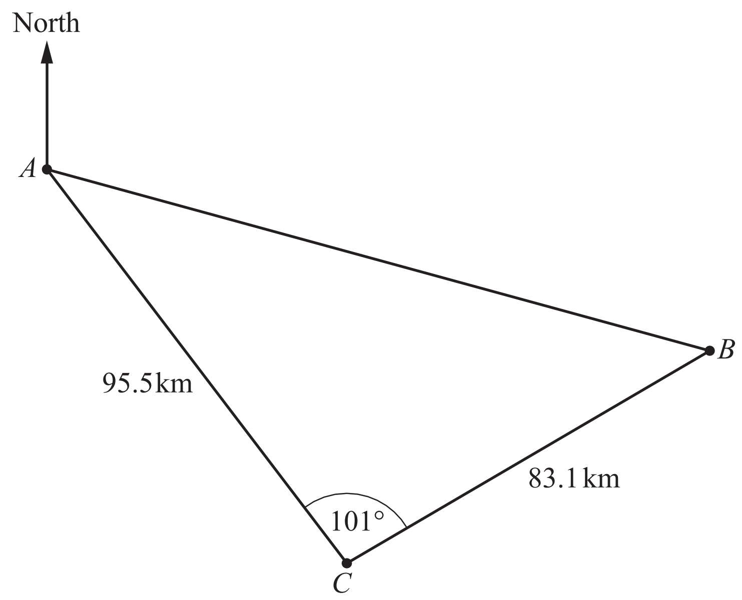
Multiply both sides of the equation bysin(61°)

$$GJ = \frac{63}{\sin(81°)} \times \sin(61°)$$

Use a calculator

$$J \quad = \mathbf{55.8km}$$

The diagram shows the positions of two ships, A and B, and a coastguard station, C.



NOT TO SCALE

(a) Calculate the distance, AB, between the two ships.
Show that it rounds to 138km, correct to the nearest kilometre.

[4]

The distance AB can be found using the cosine rule.

$$AB^2 = AC^2 + CB^2 - 2 \times AC \times CB \times \cos(ACB)$$

$$AB^2 = 95.5^2 + 83.1^2 - 2 \times (95.5) \times (83.1) \times \cos(101°)$$

$$AB^2 = 19054.40\ km^2$$

Take square root of both sides to get the final answer:

$$\boldsymbol{AB = 138\ km}\ (correct\ to\ the\ nearest\ km)$$

(b) The bearing of the coastguard station C from ship A is $146°$.

Calculate the bearing of ship B from ship A. [4]

Calculate the angle BAC using the sine rule.

$$\frac{\sin(ACB)}{AB} = \frac{\sin(BAC)}{AC}$$

Use the given values and the value of AB from part a)

$$\frac{\sin(101°)}{138} = \frac{\sin(BAC)}{83.1}$$

Multiply both sides by 83.1km.

$$\sin(BAC) = \frac{\sin(101°)}{138} \times 83.1$$

Take $\sin^{-1}$ of both sides.

$$BAC = \arcsin(0.5911)$$

Use your calculator to work out the value of ABC.

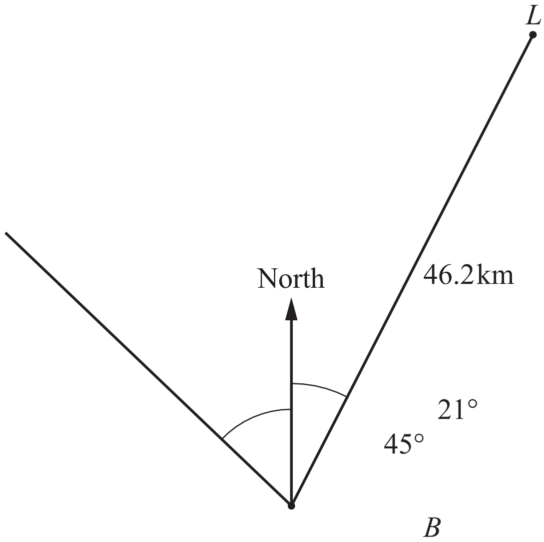
$$BAC = 36.2°$$

To get the bearing of ship B from ship A, subtract the size of angle BAC from the bearing of C from ship A.

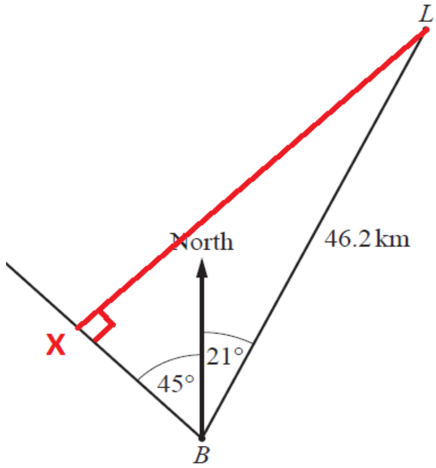
$$bearing\ B\ from\ A = 146° - 36.2°$$

$$= \mathbf{109.8°}$$

(c)



NOT TO SCALE



At noon, a lighthouse, *L*, is 46.2km from ship *B* on the bearing 021°.
Ship *B* sails north west.

Calculate the distance ship *B* must sail from its position at noon to be at its closest distance to the lighthouse.

[2]

The ship B is the closes to the lighthouse L when the line joining LB is perpendicular to the direction of the ship B. (Let X be the position of ship B at this time).

The triangle BLX forms a right angle triangle.

Angle XBL=45°+21°=66°

The distance ship B must sail (BX) can be calculated using trigonometry.

$$\cos(XBL) = \frac{BX}{BL}$$

Multiply both sides of the equation by BL and use given values.

$$BX = 46.2km \times \cos(66°)$$

$$\mathbf{BX = 18.8km}$$

Trigonometry
Difficulty: Hard

Model Answers 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Trigonometry
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 2

Time allowed: **99 minutes**

Score: **/86**

Percentage: **/100**

Grade Boundaries:

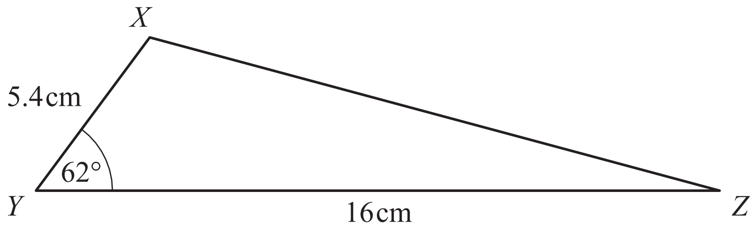
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

(a)



NOT TO SCALE

Show that the area of triangle XYZ is 38.1cm^2, correct to 1 decimal place. [2]

The area of triangle XYZ can be calculated using the formula:

$$Area = \frac{1}{2}XY \times YZ \times \sin(XYZ)$$

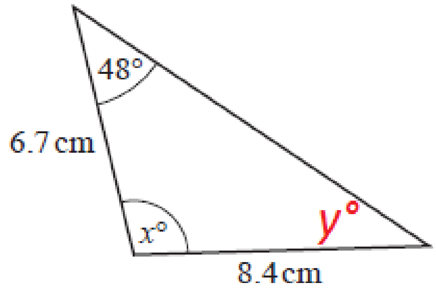
(The area of a triangle is given by half the product of its two sides and an angle between these two sides.)

$$Area = \frac{1}{2} \times (5.4cm) \times (16cm) \times \sin(62°)$$

Using a calculator:

$$\boldsymbol{Area = 38.14\ cm^2}$$

(b)



NOT TO SCALE

Calculate the value of x. [4]

Let the third angle of the triangle be y°.

The sum of all three interior angles of a triangle is 180°. Therefore:

$$180° = 48° + x° + y°$$

Subtract (48°+y°) from both sides.

$$132° - y° = x°$$

The size of angle y can be found using the sine rule.

$$\frac{\sin(y°)}{6.7cm} = \frac{\sin(48°)}{8.4cm}$$

Multiply both sides of the equation by 6.7

$$\sin(y°) = \frac{\sin(48°)}{8.4cm} \times 6.7cm$$

Use a calculator

$$\sin(y°) = 0.593$$

$$y° = \arcsin(0.593) = 36.4°$$

Use the previously found relation between x and y to work out x.

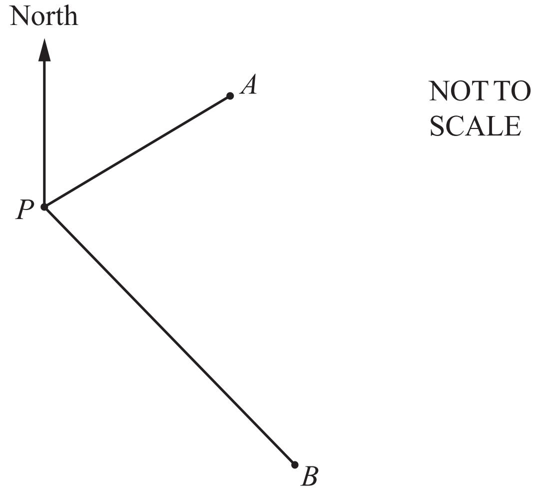
$$132° - y° = x°$$

$$132° - 36.4° = x°$$

We get the final answer:

$$\boldsymbol{x° = 95.6°}$$

(c)



Ship A is 180 kilometres from port P on a bearing of 063°.
Ship B is 245 kilometres from P on a bearing of 146°.

Calculate AB, the distance between the two ships. [5]

) The angle North-P-A is 63°. The angle North-P-B is 146°.

The difference between these two angles is the size of the angle APB.

$$APB = 146° - 63° = 83°$$

The distance AB between the two ships can be found using the cosine rule.

$$AB^2 = PA^2 + PB^2 - 2 \times PA \times PB \times \cos(APB)$$

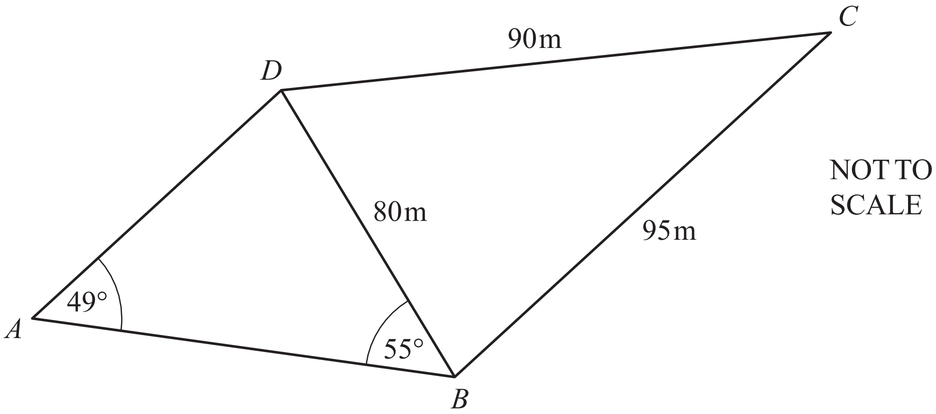
$$AB^2 = (180km)^2 + (245km)^2 - 2 \times (180km) \times (245km) \times \cos(83°)$$

$$AB^2 = (32\,400 + 60\,025 - 10\,748.88)km^2$$

$$AB^2 = 81\,676\ km^2$$

Take square root of both sides to get the final answer:

$$\boldsymbol{AB = 285.8\ km}$$



The diagram shows a quadrilateral *ABCD*.
Angle *BAD* = 49° and angle *ABD* = 55°.
BD = 80 m, *BC* = 95 m and *CD* = 90 m.

(a) Use the sine rule to calculate the length of *AD*. [3]

Sine rule is

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Hence

$$\frac{AD}{\sin 55} = \frac{80}{\sin 49}$$

$$\rightarrow AD = \frac{80 \sin 55}{\sin 49}$$

$$= \mathbf{86.83}$$

(b) Use the cosine rule to calculate angle *BCD*. [4]

Cosine rule is

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Hence

$$80^2 = 90^2 + 95^2 - 2(90)(95) \cos BCD$$

$$\rightarrow \cos BCD = \frac{80^2 - 90^2 - 95^2}{-2(90)(95)}$$

$$= \frac{143}{228}$$

$$\rightarrow \boldsymbol{BCD = 51.16}$$

(c) Calculate the area of the quadrilateral $ABCD$. [3]

Add the areas of the two triangles.

Area of a triangle is

$$\boldsymbol{A = \frac{1}{2}ab\sin C}$$

Need angle ADB

$$ADB = 180 - 49 - 55$$

$$= 76$$

$$\rightarrow A_1 = \tfrac{1}{2}(80)(86.83)\sin 76 = 3370$$

And

$$A_2 = \frac{1}{2}(90)(95)\sin 51.16$$

$$= 3330$$

Hence

$$A = 3330 + 3370$$

$$\boldsymbol{= 6700}$$

(d) The quadrilateral represents a field.
Corn seeds are sown across the whole field at a cost of \$3250 per hectare.

Calculate the cost of the corn seeds used.
1 hectare = 10 000 m^2 [3]

The area of the field is

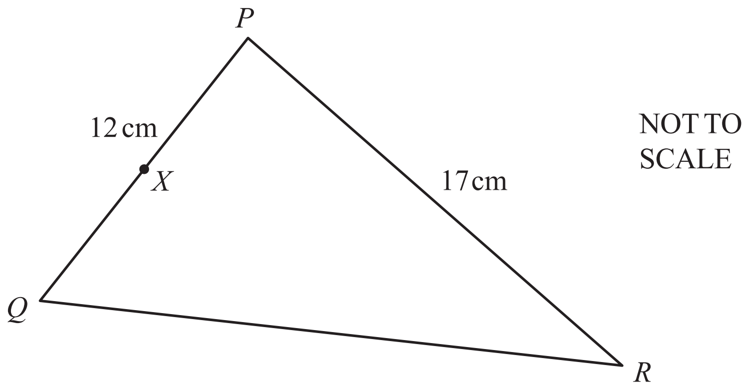
$$\frac{6700}{10000}\ hectares$$

Hence the cost is

$$\frac{67}{100} \times 3250$$

$$= \mathbf{2177.5}$$

(a)



The diagram shows triangle PQR with PQ = 12 cm and PR = 17 cm.
The area of triangle PQR is 97 cm^2 and angle QPR is acute.

(i) Calculate angle QPR. [3]

The area of a triangle is

$$A = \frac{1}{2} ab \sin C$$

Hence

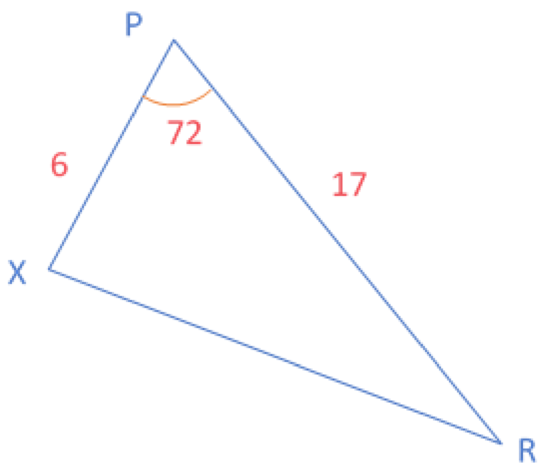
$$97 = \frac{1}{2}(12)(17) \sin QPR$$

$$\rightarrow \sin QPR = 0.951$$

$$\rightarrow QPR = 72.0$$

(ii) The midpoint of PQ is X.

Use the cosine rule to calculate the length of XR. [4]



The cosine rule is

$$a^2 = b^2 + c^2 - 2bc\cos A$$

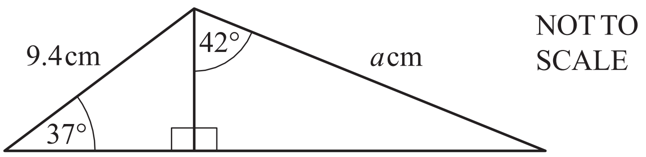
Hence

$$XR^2 = 6^2 + 17^2 - 2(6)(17)\cos 72$$

$$= 261.96$$

$$\rightarrow XR = 16.2$$

(b)



Calculate the value of a. [4]

The perpendicular height of the triangle is

$$h = 9.4\sin 37$$

And we have that

$$a\cos 42 = 9.4\sin 37$$

Hence

$$a = \frac{9.4\sin 37}{\cos 42}$$

$$= 7.61$$

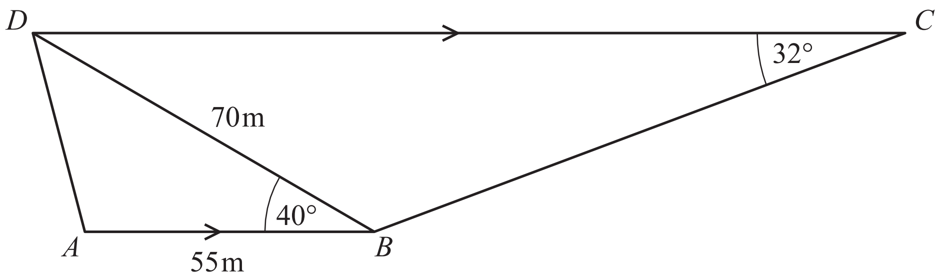
(c) $\sin x = \cos 40°, \quad 0° \leqslant x \leqslant 180°$

Find the two values of x. [2]

$x = \sin^{-1}(\cos 40)$

$= 50, \quad 180 - 50$

$= \mathbf{50}, \quad \mathbf{130}$



NOT TO SCALE

The diagram shows a school playground *ABCD*.
ABCD is a trapezium.
$AB = 55$ m, $BD = 70$ m, angle $ABD = 40°$ and angle $BCD = 32°$.

(a) Calculate *AD*. [4]

Using the Cosine Rule,

$$a^2 = b^2 + c^2 - 2bc \times cosA$$

$$= 55^2 + 70^2 - 2(55)(70) \times \cos(40°)$$

$$= 2026.5m$$

Therefore,

$$\boldsymbol{a = 45m}$$

(b) Calculate *BC*. [4]

Since bounded by 2 parallel lines, angle BDC is 40 degrees.

Using the Sine Rule,

$$\frac{70}{\sin(32°)} = \frac{BC}{\sin(40°)}$$

Therefore,

$$\boldsymbol{BC = 84.9m}$$

(c) (i) Calculate the area of the playground $ABCD$. [3]

To find total area, sum the areas of the 2 triangles.

For triangle ABD,

$$Area\ ABD = \frac{1}{2} \times 55 \times 70 \times \sin(40°)$$

$$= 1237.4m^2$$

Find angle DBC using sum of angles in triangle:

$$180° - 32° - 40° = 108°$$

For triangle BCD,

$$Area\ BCD = \frac{1}{2} \times 70 \times 84.9 \times \sin(108°)$$

$$= 2826m^2$$

Therefore the area is:

$$Total\ Area = 1237.4m^2 + 2826m^2$$

$$= \mathbf{4063m^2}$$

(ii) An accurate plan of the school playground is to be drawn to a scale of 1:200 .
Calculate the area of the school playground on the plan.
Give your answer in cm^2. [2]

Note that the scaling is done by length. This means that for every metre on the actual ground, 1/200 of the metre shows on the plan.

However, here we have an area we want to scale down, we adjust the scale accordingly to account for area:

1m is shown as (1/200)m on the plan,

So

$$1m^2 \text{ becomes} \left(\frac{1}{200}\right)^2 m^2$$

$$= 2.5 \times 10^{-5} m^2 \text{ on the plan}$$

Converting to cm^2,

$$2.5 \times 10^{-5} \times 100^2 = 0.25cm^2 \text{ on the plan}$$

Thus, for every m^2 on the actual playground, this translates to 0.25cm^2 area on the plan.

$$0.25 \times 4063m^2$$

$$= \mathbf{1016cm^2}$$

(d) A fence, *BD*, divides the playground into two areas.

Calculate the shortest distance from *A* to *BD*. [2]

We know the area of triangle to be 1237.4m^2

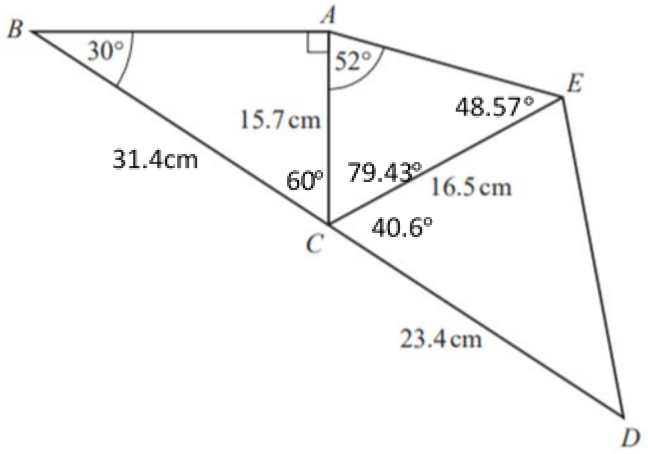
We know the equation for area of triangle to be ½ ×base×height

$$\frac{1}{2} \times 70 \times h = 1237.4m^2$$

$$where\ h\ is\ shortest\ dist.\ from\ A\ to\ BD$$

Therefore

$$\boldsymbol{h = 35.4m}$$



In the diagram, *BCD* is a straight line and *ABDE* is a quadrilateral.
Angle $BAC = 90°$, angle $ABC = 30°$ and angle $CAE = 52°$.
$AC = 15.7$ cm, $CE = 16.5$ cm and $CD = 23.4$ cm.

(a) Calculate *BC*. [3]

Since we know an angle and the length of the opposite side of the right angled triangle BAC, we can use sine to find BC.

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\sin 30 = \frac{15.7}{BC}$$

Multiplying through by BC, then dividing by sin(30).

$$BC = \frac{15.7}{\sin 30}$$

$$\boldsymbol{BC = 31.4cm}$$

(b) Use the sine rule to calculate angle *AEC*.
Show that it rounds to 48.57°, correct to 2 decimal places. [3]

The sine rule equation is given by:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Where A and B are angles and a and b are there opposite sides.

$$\frac{\sin 52}{16.5} = \frac{\sin AEC}{15.7}$$

Multiplying through by 15.7.

$\sin AEC = \frac{15.7 \sin 52}{16.5}$

$\sin AEC = 0.7498 \ldots$

Using inverse sine.

$AEC = \sin^{-1} 0.7498 \ldots$

$\boldsymbol{AEC = 48.57^o}$

(c) (i) Show that angle $ECD = 40.6°$, correct to 1 decimal place. [2]

To tackle this problem, it is best to work out all unknown angles until you arrive out the angle you want. Here we can start by finding angle ACB.

Angles in a triangle sum to 180°, therefore:

$30 + 90 + ACB = 180$

Subtracting 120 from both sides.

$ACB = 60^o$

We can use the same rule to find angle ACE as we found AEC in the previous part.

$52 + 48.57 + ACE = 180$

$100.57 + ACE = 180$

Subtracting 100.57 from both sides.

$ACE = 79.43^o$

Now we can find angle ECD as angles on a straight line sum to 180°.

$60 + 79.43 + ECD = 180$

$139.43 + ECD = 180$

Subtracting 139.43 from both sides.

$\boldsymbol{ECD = 40.6^o}$ **(1d.p)**

(ii) Calculate DE. [4]

Since we have a known angle between two known sides and we wish to find the opposite side, we can use the cosine rule.

$a^2 = b^2 + c^2 - 2bc \cos A$

Substituting values into equation.

$DE^2 = 16.5^2 + 23.4^2 - 2(16.5)(23.4) \cos 40.6$

Plugging numbers into a calculator.

$DE^2 = 233.5$

Square rooting both sides.

$\boldsymbol{DE = 15.3cm}$

(d) Calculate the area of the quadrilateral $ABDE$. [4]

To find the full area, the area of each triangle must be found individually.

To find the areas we can use the sine area rule as shown below.

$Area = \frac{1}{2}ab\sin C$

Triangle ACB:

$Area = \frac{1}{2}(15.7)(31.4)\sin 60$

$= 213.5\ cm^2$

Triangle AEC:

$Area = \frac{1}{2}(15.7)(16.5)\sin 79.43$

$= 127.3\ cm^2$

Triangle CED:

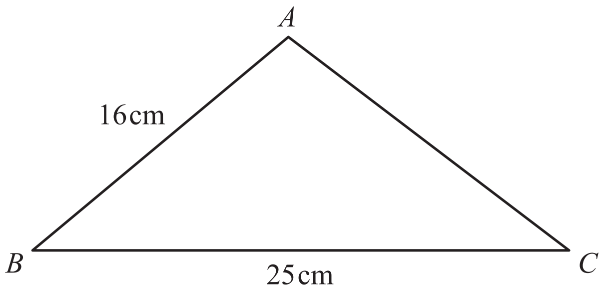
$Area = \frac{1}{2}(16.5)(23.4)\sin 40.6$

$= 125.6\ cm^2$

Summing all the areas to find total area.

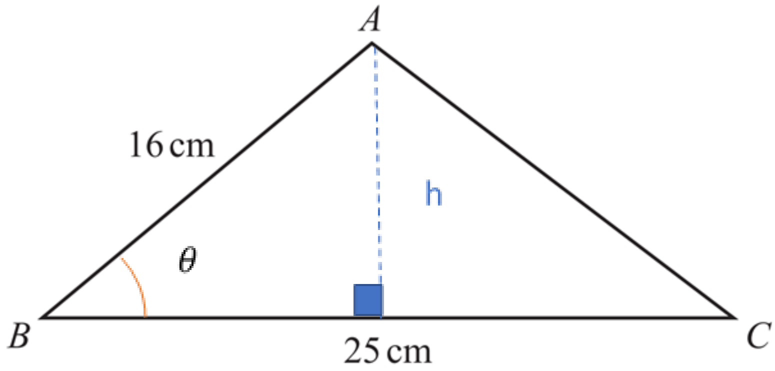
$Total\ Area = 213.5 + 127.3 + 125.5$

$\mathbf{= 466}$ **(3s.f)**



The area of triangle ABC is 130 cm^2.
$AB = 16$ cm and $BC = 25$ cm.

(a) Show clearly that angle $ABC = 40.5°$, correct to one decimal place. [3]



We have that

$$A = \frac{1}{2} \times base \times height$$

$$\rightarrow 130 = \frac{1}{2}(25)(h)$$

Rearrange for h

$$\rightarrow h = \frac{260}{25}$$

$$= 10.4$$

And we know, from trigonometry, that

$$\sin\theta = \frac{opp}{hyp}$$

$$\rightarrow \sin\theta = \frac{h}{16}$$

$$\rightarrow \theta = \sin^{-1}\frac{10.4}{16}$$

$$= \mathbf{40.5}$$

(b) Calculate the length of AC. [4]

Using the cosine rule

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$

Where side a and angle A are opposite on another

$$\rightarrow AC^2 = 16^2 + 25^2 - 2(16)(25)\cos 40.5$$

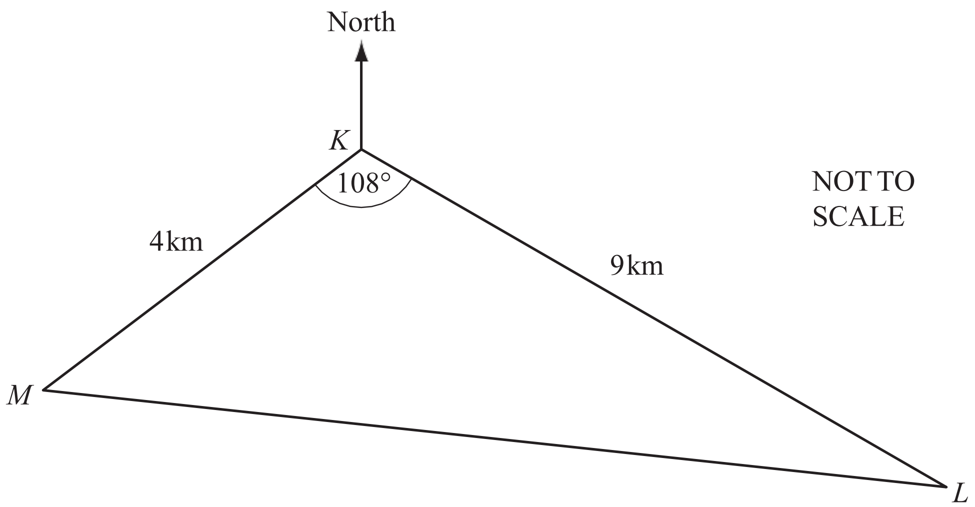
$$= 272.675$$

$$\rightarrow \boldsymbol{AC = 16.5}$$

(c) Calculate the shortest distance from A to BC. [2]

The shortest distance from A to BC is the height, which we have already found as

$$\boldsymbol{h = 10.4}$$



Three buoys K, L and M show the course of a boat race.
$MK = 4$ km, $KL = 9$ km and angle $MKL = 108°$.

(a) Calculate the distance ML. [4]

Use the cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

to get

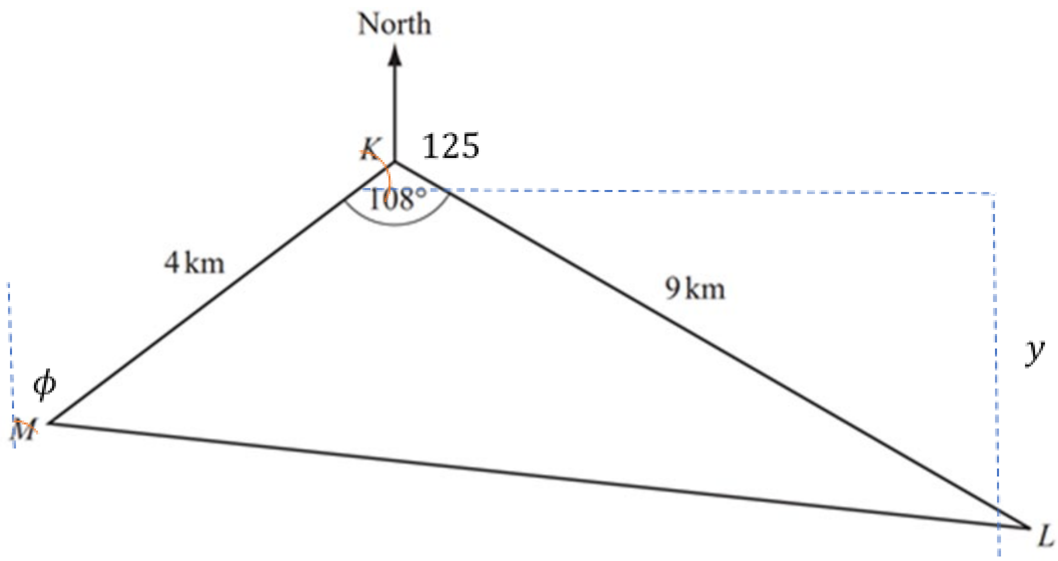
$$ML^2 = 4^2 + 9^2 - 2(4)(9)\cos 108$$

$$= 119.249$$

$$\rightarrow \boldsymbol{ML = 10.9}$$

(b) The bearing of L from K is 125°.

(i) Calculate how far L is south of K. [3]



We need to find the distance, y, as pictured on the diagram above.

We find this using the trigonometric relation

$$\sin\theta = \frac{opp}{hyp}$$

Where

$$\theta = 125 - 90$$

$$= 35$$

Hence

$$\sin 35 = \frac{y}{9}$$

$$\rightarrow y = 9\sin 35$$

$$\mathbf{= 5.16}$$

(ii) Find the three figure bearing of K from M. [2]

We need to find the angle ϕ, pictured above.

The bearing of M from K is

$$125 + 108$$

$$= 233$$

Hence the angle between K and M is

$$360 - 233$$

$$= 127$$

And we have that

$$\phi + 127 = 180$$

$$\rightarrow \boldsymbol{\phi} \mathbf{= 053}$$

Trigonometry
Difficulty: Hard

Model Answers 3

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Trigonometry
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 3

Time allowed: **99 minutes**

Score: **/86**

Percentage: **/100**

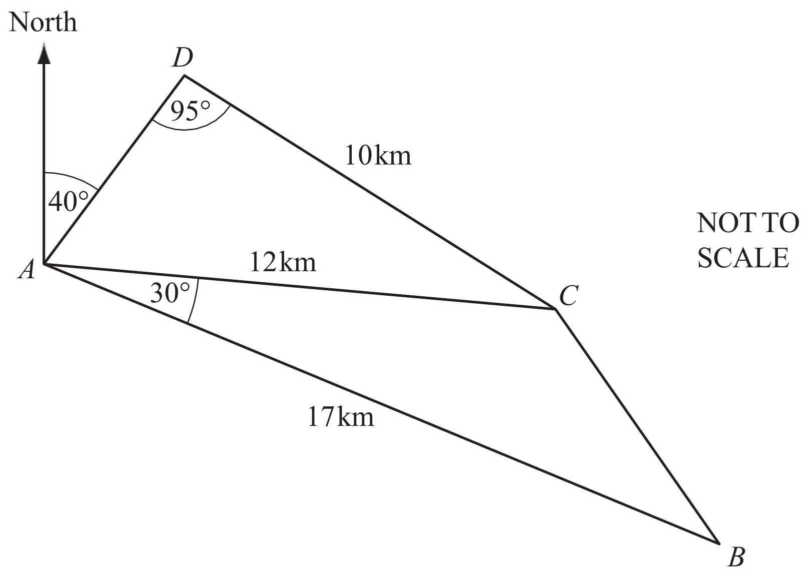
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



The diagram shows straight roads connecting the towns A, B, C and D.

$AB = 17$ km, $AC = 12$ km and $CD = 10$ km.

Angle $BAC = 30°$ and angle $ADC = 95°$.

(a) Calculate angle CAD. [3]

Using the sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

we have

$$\frac{\sin CAD}{10} = \frac{\sin 95}{12}$$

$$\rightarrow CAD = \sin^{-1}\left(\frac{10 \sin 95}{12}\right)$$

$$= \mathbf{56.1}$$

(b) Calculate the distance BC. [4]

Using the cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\rightarrow BC^2 = 12^2 + 17^2 - 2(12)(17)\cos 30$$

$$= 79.6616$$

$$\rightarrow \boldsymbol{BC = 8.93}$$

(c) The bearing of D from A is 040°.

Find the bearing of

(i) B from A, [1]

B from A is

$$40 + 56.1 + 30$$

$$\mathbf{= 126.1}$$

(ii) A from B. [1]

A from B is

$$360 - (180 - 126.1)$$

$$\mathbf{= 306.1}$$

(d) Angle ACB is obtuse.

Calculate angle BCD. [4]

Use sine rule to find BCA

$$\frac{\sin BCA}{17} = \frac{\sin 30}{8.93}$$

$$\rightarrow BCA = \sin^{-1}\left(\frac{17\sin 30}{8.93}\right)$$

$$= 72.1, \qquad 180 - 72.1$$

Because it's obtuse we know it must be the angle larger than 90˚

$$BCA = 107.9$$

All angles in a triangle add to 180, hence we have for triangle ADC

$$56.1 + 95 + ACD = 180$$

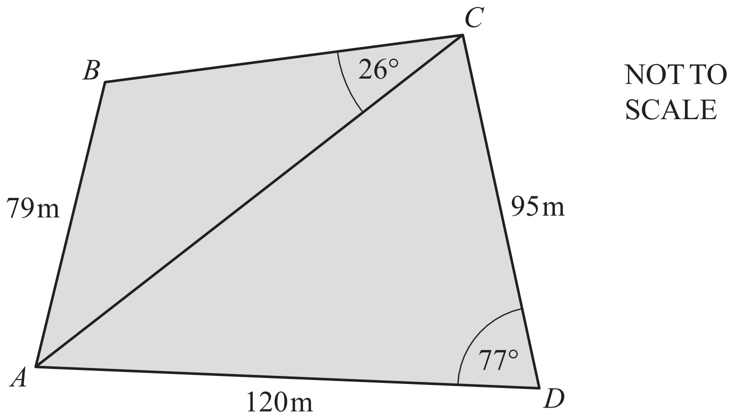
$$\rightarrow ACD = 180 - 151.1$$

$$= 28.9$$

Hence

$$BCD = 107.9 + 28.9$$

$$= \mathbf{136.8}$$



The quadrilateral *ABCD* represents an area of land.
There is a straight road from *A* to *C*.
$AB = 79$ m, $AD = 120$ m and $CD = 95$ m.
Angle $BCA = 26°$ and angle $CDA = 77°$.

(a) Show that the length of the road, *AC*, is 135 m correct to the nearest metre. [4]

We know the size of 2 sides in the triangle ADC and the angle between them,

therefore, we can use cosine rule to work out the length of the third side.

Cosine rule:

$a^2 = b^2 + c^2 - 2bc \times \cos A$

where a = AC, cos A = cos ADC, b = AD and c = CD.

We substitute the values that we know and work out a

$a^2 = 120^2 + 95^2 - 2 \times 95 \times 120 \times \cos 77°$

a = 135.26 m

(b) Calculate the size of the obtuse angle ABC. [4]

In the triangle ABC we can use the Sine rule to work out the size of the triangle ABC.

Sine rule:

$$\frac{\sin ABC}{AC} = \frac{\sin BCA}{AB}$$

$$\frac{\sin ABC}{135.26} = \frac{\sin 26°}{79}$$

Sin ABC = $\frac{135.26 \text{ x } \sin 26°}{79}$

Sin ABC = $\frac{135.26\ x\ 0.438}{79}$

Sin ABC = 0.7505

Angle ABC = 48.63°

The obtuse angle ABC will be 180° - 48.63°

= 131.37°

(c) A straight path is to be built from B to the nearest point on the road AC.

Calculate the length of this path. [3]

The nearest point from B on the line AC is represented by the perpendicular from B on AC, this perpendicular representing the path.

In the right-angled triangle formed with the path perpendicular on AC:

Sin BAC = $\frac{Path}{AB}$

AB = 79 m

We need to work out the size of the angle BAC.

In the triangle ABC, the sum of all 3 angles is 180°.

Angle BAC = 180° - 26° - 131.37°

Angle BAC = 22.63°

$\text{Sin } 22.63° = \frac{Path}{79\ m}$

Path = 79 m x 0.384

Path = 30.39 m

(d) Houses are to be built on the land in triangle *ACD*.
Each house needs at least 180 m^2 of land.

Calculate the maximum number of houses which can be built.
Show all of your working. [4]

To work out the maximum number of houses that can fit we need to work out the area of the triangle ACD and then divide it by the area needed by a house.

$\text{Area} = \frac{a \times b \times \sin C}{2}$

Where a and b represent 2 sides of the triangle and C is the angle between them

$\text{Area} = \frac{120\ m \times 95\ m \times \sin 77°}{2}$

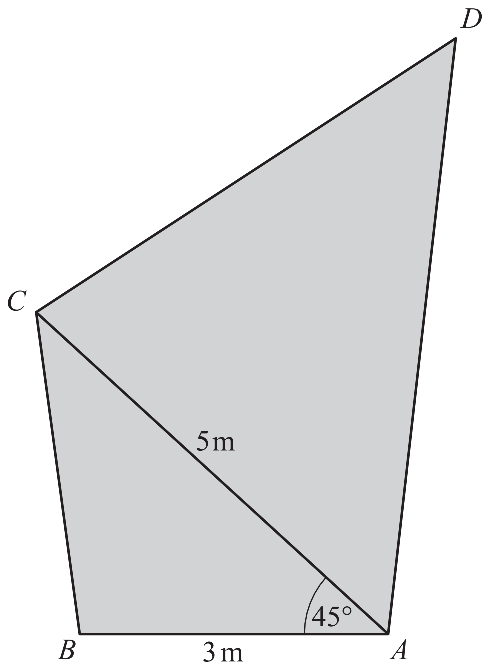
Area = 5553.9 m^2

The maximum number of houses is:

$$\frac{5553.9 \text{ m}^2}{180 \text{ m}^2} = 30.85$$

The number of houses needs to be a whole number therefore, the maximum number will be

30 houses.



Parvatti has a piece of canvas *ABCD* in the shape of an irregular quadrilateral.

AB = 3 m, *AC* = 5 m and angle *BAC* = 45°.

(a) (i) Calculate the length of *BC* and show that it rounds to 3.58 m, correct to 2 decimalplaces.

You must show all your working. [4]

We know the size of 2 sides in the triangle ABC and the angle between them,

therefore, we can use cosine rule to work out the length of the third side.

Cosine rule:

$$a^2 = b^2 + c^2 - 2bc \times \cos A$$

where a = BC, cos A = cos CAB, b = AC and c = AB.

We substitute the values that we know and work out a

$a^2 = 5^2 + 3^2 - 2 \times 5 \times 3 \times \cos 45°$

$a^2 = 25 + 9 - 30 \times \frac{\sqrt{2}}{2}$

a = 3.575 m

The third decimal place is 5, which is greater or equal than 5, so the second decimal, 7, rounds up to the next digit, 8.

a = 3.57 m

(ii) Calculate angle BCA. [3]

In the angle BCA we know at least 2 of the sides' lengths and one of the angles, therefore, we can apply the sine rule to work out the sin BCA and then the size of the angle BCA.

$$\frac{\boldsymbol{a}}{\textbf{sin}\ \boldsymbol{A}} = \frac{\textbf{b}}{\textbf{sin}\ \boldsymbol{B}}$$

Where sin A = sin BAC, sin B = sin BCA, a = BC and b = CA.

$$\frac{3.58\ \mathrm{m}}{\sin 45°} = \frac{3\ \mathrm{m}}{\sin BCA}$$

$$\text{sin BCA} = \frac{3\ \mathrm{m}\ \mathrm{x}\ \frac{\sqrt{2}}{2}}{3.58\ \mathrm{m}}$$

sin BCA = 0.593

angle BCA = 36.4°

(b) $AC = CD$ and angle $CDA = 52°$.

(i) Find angle DCA. [1]

If AC = CD, it means that the triangle CDA is an isosceles triangle.

This means the angles CDA and CAD will also be equal.

Angle CDA = 52°

The sum of all 3 angles in a triangle is 180°.

180° = 52° x 2 + angle DCA

180° = 104° + angle DCA

Angle DCA = 76°

(ii) Calculate the area of the canvas. [3]

The area of the canvas is represented by the area of the 2 triangles added up.

To work out the area of one of the triangles we can use the formula:

$A = \frac{1}{2}ab \sin C$

Where a and b are 2 sides of the triangle and angle C is the angle between them.

In triangle ABC, the area is:

$A_{ABC} = \frac{1}{2} \times 5\text{m} \times 3\text{m} \times \sin 45°$

$A_{ABC} = 5.3 \text{ m}^2$

In triangle ACD, the area is:

$A_{ACD} = \frac{1}{2}$ x DC x AC x sin DCA

AC = CD

$A_{ACD} = \frac{1}{2}$ x 5m x 5m x sin 76°

$A_{ACD} = 12.12\ m^2$

$A_{canvas} = 12.12\ m^2 + 5.3\ m^2$

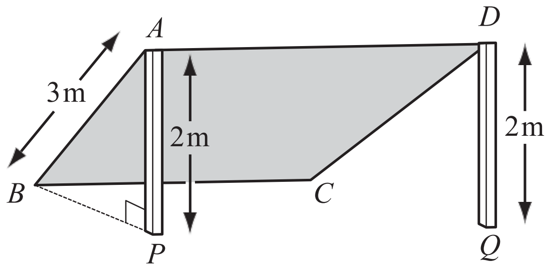
$A_{canvas} = 17.42\ m^2$

(c) Parvatti uses the canvas to give some shade.

She attaches corners *A* and *D* to the top of vertical poles, *AP* and *DQ*, each of height 2 m.

Corners *B* and *C* are pegged to the horizontal ground.

AB is a straight line and angle *BPA* = 90°.



NOT TO SCALE

Calculate angle *PAB*. [2]

In the right-angled triangle PAB with AB as hypothenuse:

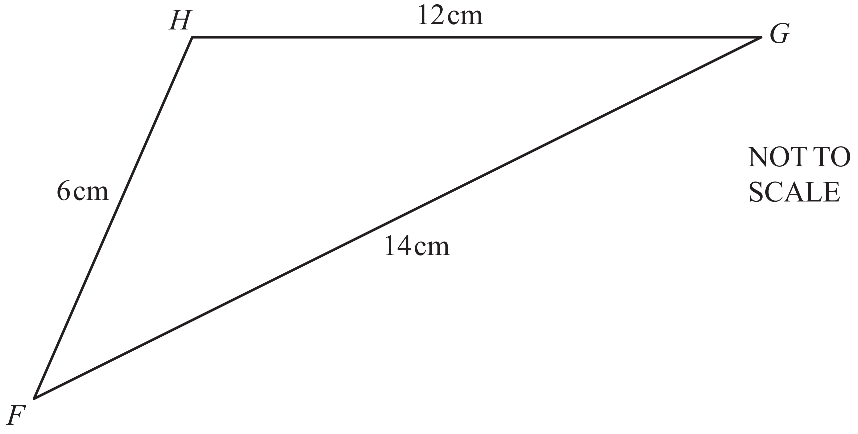
Cos PAB = $\frac{AP}{AB}$

Cos PAB = $\frac{2\ cm}{3\ cm}$

Cos PAB = 0.666

Angle PAB = 48.189°

(a)



The diagram shows triangle FGH, with $FG = 14$ cm, $GH = 12$ cm and $FH = 6$ cm.

(i) Calculate the size of angle HFG. [4]

We use the cosine rule to calculate the size of angle HFG:

$$\cos(HFG) = \frac{HF^2 + FG^2 - GH^2}{2 \times HF \times FG}$$

Plug in the given values:

$$\cos(HFG) = \frac{(6cm)^2 + (14cm)^2 - (12cm)^2}{2 \times (6cm) \times (14cm)}$$

$$\cos(HFG) = 0.5238..$$

Apply inverse cosine to both sides to find the size of the angle HFG

$$angle\ HFG = arccos(0.5238..)$$

Correct to three significant figures:

$$\boldsymbol{angle\ HFG = 58.4^\circ}$$

(ii) Calculate the area of triangle FGH. [2]

Use the sine formula for the area of the triangle.

$$area\ HFG = \frac{1}{2} \times HF \times FG \times \sin(HFG)$$

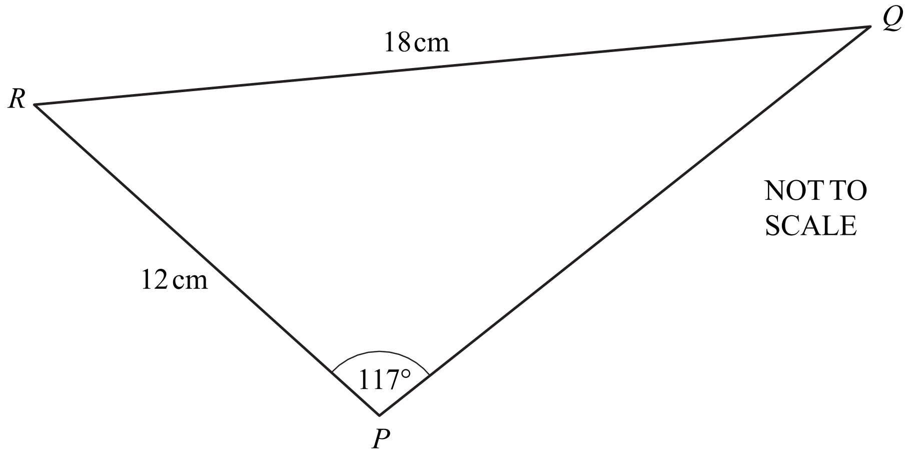
Use the angle from part a).

$$area\ HFG = \frac{1}{2} \times (6cm) \times (14cm) \times \sin(58.4°)$$

The area of HFG correct to three significant figures:

$$\boldsymbol{area\ HFG = 35.8cm^2}$$

(b)



The diagram shows triangle PQR, with RP = 12 cm, RQ = 18cm and angle RPQ = 117°.

Calculate the size of angle RQP. [3]

Use the sine rule to calculate the size of angle RQP.

$$\frac{\sin(RQP)}{RP} = \frac{\sin(RPQ)}{RQ}$$

Multiply both sides by the size of RP.

$$\sin(RQP) = \frac{\sin(RPQ) \times RP}{RQ}$$

Plug in known values:

$$\sin(RQP) = \frac{\sin(117°) \times 12cm}{18cm}$$

Plug in known values:

$$\sin(RQP) = 0.5940 \ldots$$

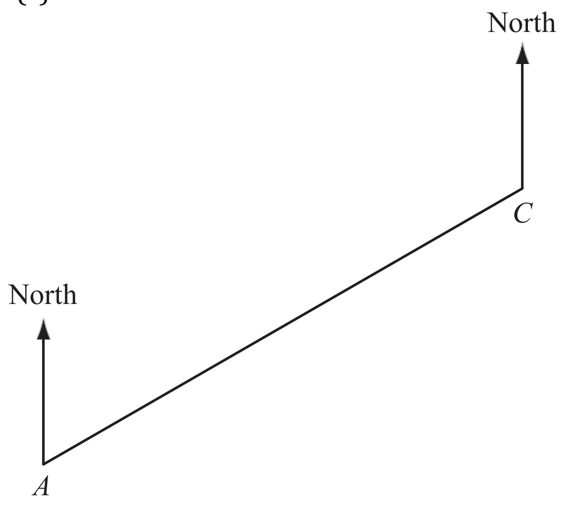
Apply inverse sine to both sides to find the size of the angle RQP

$$angle\ RQP = arcsin(0.5940 \ldots)$$

Correct to three significant figures:

$$\boldsymbol{angle\ RQP = 36.4°}$$

(a)



The scale drawing shows the positions of two towns A and C on a map.
On the map, 1 centimetre represents 20 kilometres.

(i) Find the distance in kilometres from town A to town C. [2]

We measure the length of the line from A to C as:

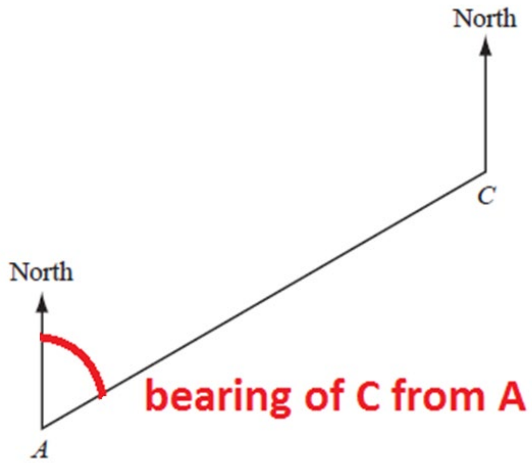
$$7.3cm$$

One centimetre on the map represents 20 kilometres so to get the actual distance in kilometres, we multiply by 20.

$$distance = 7.3cm \times 20\frac{km}{cm}$$

$$\boldsymbol{distance = 146km}$$

(ii) Measure and write down the bearing of town C from town A. [1]



We measure the bearing of town C from town A as (approximately):

$$\boldsymbol{bearing\ of\ C\ from\ A = 061°}$$

(iii) Town B is 140 km from town C on a bearing of 150°.

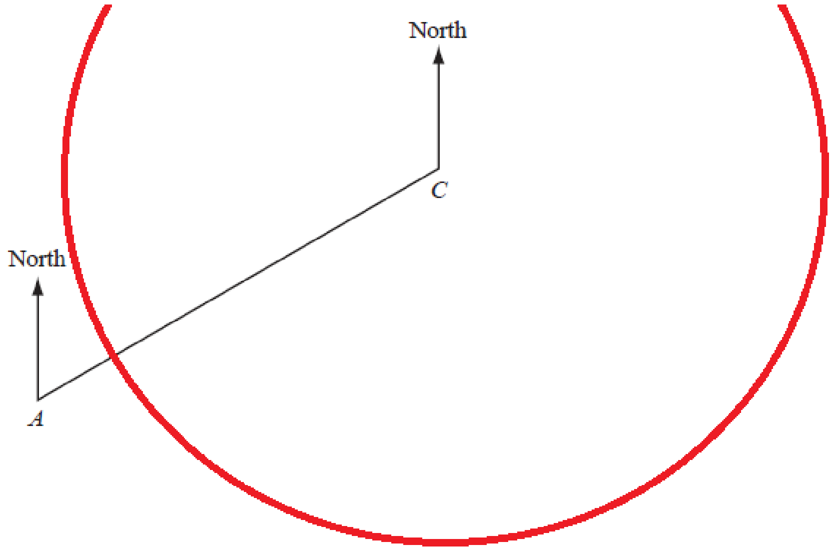
Mark accurately the position of town B on the scale drawing. [2]

First, we need to convert the distance 140km to the distance on the map. As one centimetre represents 20 km, we divide by 20 to get the distance on the map in centimetres.

$$distance\ on\ map = 140km \div 20\frac{km}{cm}$$

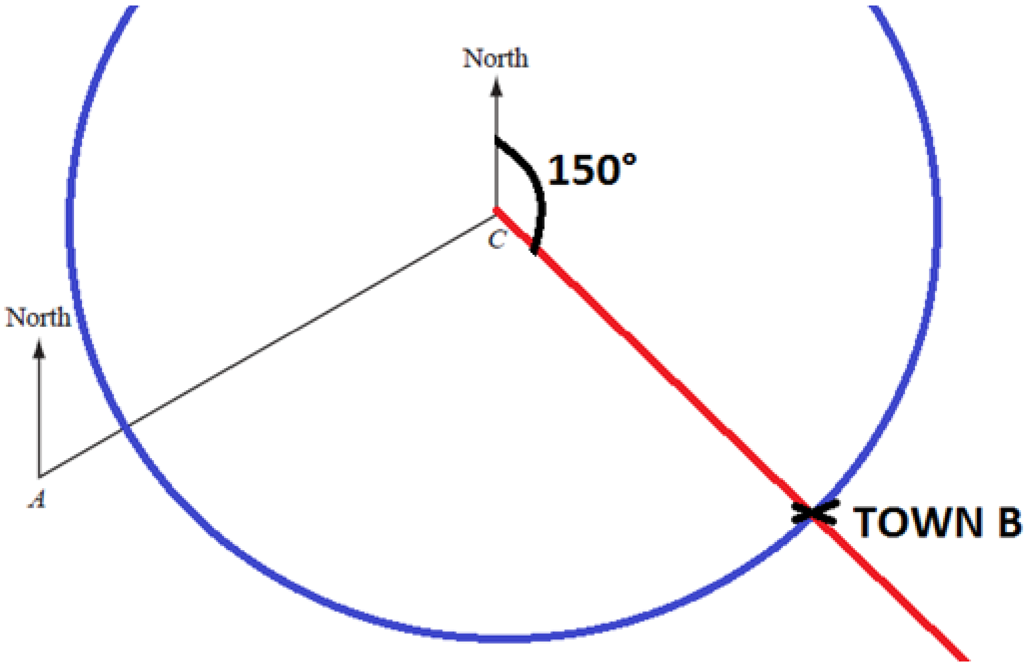
$$\boldsymbol{distance\ on\ map = 7\ cm}$$

We draw a circle with radius 7cm and with the point C as the centre.



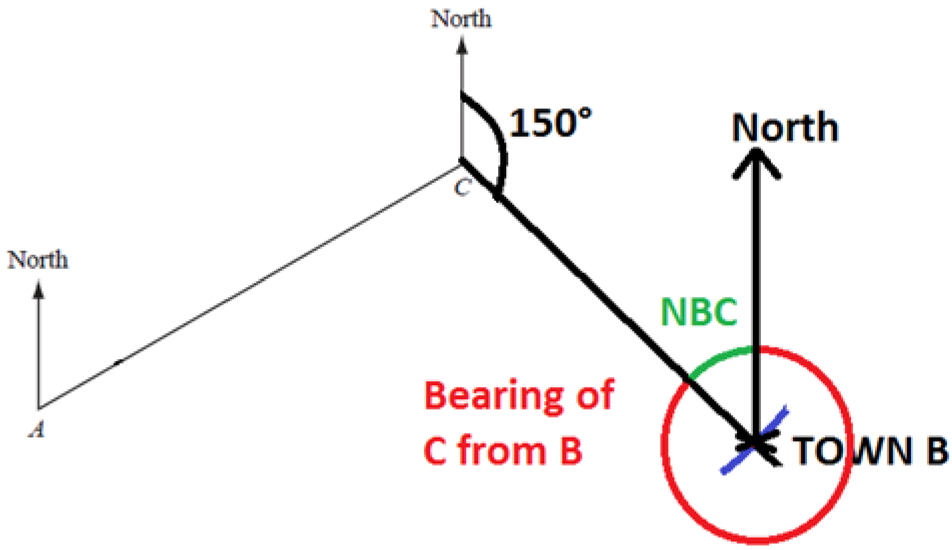
The town B is at a bearing of 150°, so we measure the angle 150° from C relative to the North.

The town B lies where the circle crosses the line which is at 150° from CN.



(iv) Find the bearing of town C from town B. [1]

We draw the line towards North from Town B.



The angles NCB and NBC are interior angles so their sum must be 180°.

$$180° = angle\ NCB + angle\ NBC$$

$$180° = 150° + angle\ NBC$$

Subtract 150° from both sides to get the size of angle NBC.

$$angle\ NBC = 30°$$

From the diagram, we can see that the bearing of town C from town B and angle NBCadd to 360°.

$$360° = bearing\ of\ C\ from\ B + angle\ NBC$$

$$360° = bearing\ of\ C\ from\ B + 30°$$

Subtract 30° from both sides to get the bearing of town C from B.

$$\boldsymbol{bearing\ of\ C\ from\ B = 330°}$$

(v) A lake on the map has an area of 0.15 cm^2.

Work out the actual area of the lake. [2]

To convert area, we need to apply the scale twice (because scale in only a linear factor).

$$actual\ area = area\ on\ the\ map \times (scale)^2$$

$$actual\ area = 0.15cm^2 \times (20\frac{km}{cm})^2$$

Multiply the numbers together.

$$\boldsymbol{actual\ area = 60\ km^2}$$

(b) A plane leaves town *C* at 11 57 and flies 1500 km to another town, landing at 14 12.

Calculate the average speed of the plane. [3]

If the plane leaves at 11 57 and lands as 14 12, then the flight is:

$$flight\ time = 14{:}12 - 11{:}57$$

$$flight\ time = 2\ hours\ 15\ minutes$$

Convert into hours, since the answer should be in kilometres per hour.

$$flight\ time = 2\ .25\ hours$$

The distance between the cities is 1500km, hence the formula for the average speed:

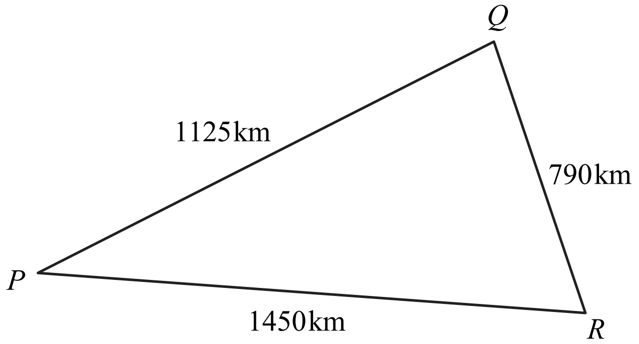
$$average\ distance = \frac{total\ distance}{flight\ time}$$

$$average\ distance = \frac{1500km}{2.25h}$$

Divide the numbers to find the average speed of the plane (correct to 3sf):

$$\boldsymbol{average\ distance = 667\ km/h}$$

(c)



The diagram shows the distances between three towns P, Q and R.

Calculate angle PQR. [4]

We use the cosine rule to calculate the size of angle PQR:

$$\cos(PQR) = \frac{PQ^2 + QR^2 - PR^2}{2 \times PQ \times QR}$$

Plug in the given values:

$$\cos(PQR) = \frac{(1125km)^2 + (790km)^2 - (1450km)^2}{2 \times (1125km) \times (790km)}$$

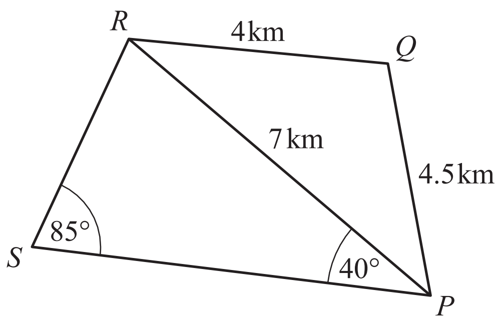
$$\cos(PQR) = -0.1197..$$

Apply inverse cosine to both sides to find the size of the angle PQR

$$angle\ PQR = arccos(-0.1197..)$$

Correct to three significant figures:

$$\boldsymbol{angle\ PQR = 96.9°}$$



NOT TO SCALE

The diagram shows five straight roads.
$PQ = 4.5$ km, $QR = 4$ km and $PR = 7$ km.
Angle $RPS = 40°$ and angle $PSR = 85°$.

(a) Calculate angle PQR and show that it rounds to 110.7°. [4]

Use the cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Where side a and angle A are opposite one another.

$$7^2 = 4^2 + 4.5^2 - 2(4)(4.5)\cos PQR$$

$$\rightarrow \cos PQR = -\frac{12.75}{36}$$

$$\rightarrow PQR = 110.74238$$

$$= \mathbf{110.7}$$

(b) Calculate the length of the road RS and show that it rounds to 4.52 km. [3]

Use the sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Where angle A is opposite side a and angle B is opposite side b.

$$\frac{\sin 85}{7} = \frac{\sin 40}{RS}$$

$$\rightarrow RS = \frac{7 \sin 40}{\sin 85}$$

$$= 4.5167$$

$$= \mathbf{4.52}$$

(c) Calculate the area of the quadrilateral *PQRS*.
[Use the value of 110.7° for angle *PQR* and the value of 4.52 km for *RS*.] [5]

Area of a triangle is

$$\boldsymbol{Area = \frac{1}{2} bc \sin A}$$

We need angle SRP.

All angles in a triangle add to 180 so

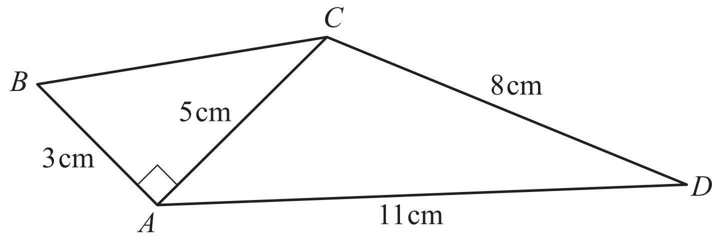
$$SRP = 180 - 85 - 40$$

$$= 55$$

Adding the two triangles together gives us

$$\frac{1}{2}(4)(4.5)\sin 110.7 + \frac{1}{2}(7)(4.52)\sin 55$$

$$= \mathbf{21.4}$$



NOT TO SCALE

In the quadrilateral $ABCD$, $AB = 3$ cm, $AD = 11$ cm and $DC = 8$ cm.
The diagonal $AC = 5$ cm and angle $BAC = 90°$.

Calculate

(a) the length of BC, [2]

Use Pythagoras' Theorem to get

$$3^2 + 5^2 = BC^2$$

$$\rightarrow BC^2 = 34$$

$$\rightarrow \boldsymbol{BC = 5.83}$$

(b) angle ACD, [4]

Use the cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Where side a and angle A are opposite one another.

$$\rightarrow 11^2 = 5^2 + 8^2 - 2(5)(8)\cos ACD$$

$$\rightarrow \cos ACD = \frac{5^2 + 8^2 - 11^2}{80}$$

$$= -\frac{2}{5}$$

$$\rightarrow \boldsymbol{ACD = 113.6}$$

(c) the area of the quadrilateral *ABCD*. [3]

Area of triangle ABC is found using

$$A = \frac{1}{2} base \times height$$

$$= \frac{1}{2}(3)(5)$$

$$= 7.5$$

Area of triangle ACD is found using

$$A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(5)(8)\sin 113.6$$

$$= 18.3$$

Adding these together gives us the area of ABCD

$$7.5 + 18.3$$

$$= \mathbf{25.8}$$

Trigonometry
Difficulty: Hard

Model Answers 4

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Trigonometry
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 4

Time allowed: **83 minutes**

Score: **/72**

Percentage: **/100**

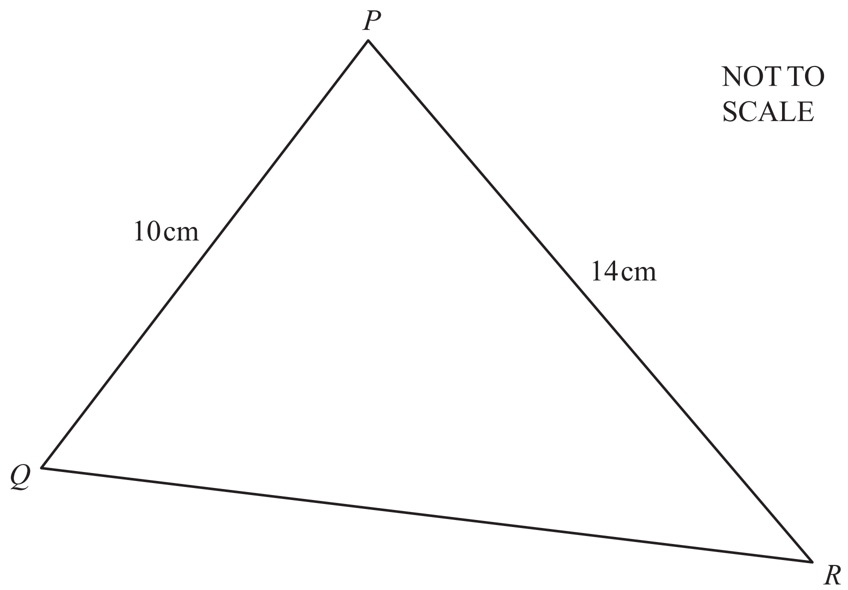
Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%



In triangle PQR, angle QPR is acute, $PQ = 10$ cm and $PR = 14$ cm.

(a) The area of triangle PQR is 48 cm^2.

Calculate angle QPR and show that it rounds to 43.3°, correct to 1 decimal place.
You must show all your working. [3]

The area of a triangle can be worked out as:

$A = \frac{a\ b\ \sin C}{2}$

Where a and b are sides in the triangle and C is the angle between them.

In our case:

$48\ cm^2 = \frac{10\ cm \times 14\ cm \times \sin QPR}{2}$

sin QPR = 0.685

Angle QPR = 43.29°

Correct to 1 decimal place = 43.3°

(b) Calculate the length of the side QR. [4]

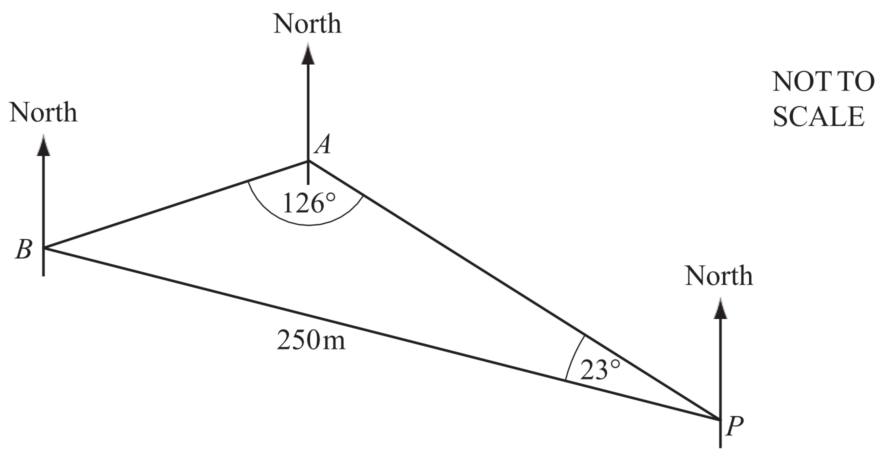
We can use the cosine rule in the triangle QPR:

$a^2 = b^2 + c^2 - 2bc \cos A$

where a, b and c are sides in the triangle and A is the angle opposite side a.

$QR^2 = 14^2 + 10^2 - 2 \times 14 \times 10 \times \cos 43.3°$

QR = 9.6 cm



The diagram shows three straight horizontal roads in a town, connecting points P, A and B.

PB =250 m, angle APB = 23° and angle BAP = 126°.

(a) Calculate the length of the road AB. [3]

In the triangle PAB we can use the sine rule to work out the size of AB.

Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Where a is a side in the triangle and A is the angle opposite side a.

In our case, we can write this as:

$$\frac{AB}{\sin 23°} = \frac{250\ m}{\sin 126°}$$

$$AB = \frac{250\ m\ x\ 0.39}{0.809}$$

AB = 121 m

(b) The bearing of A from P is 303°.

Find the bearing of

(i) B from P, [1]

Bearings are measured clockwise starting from North.

The bearing of B from P will be equal to the bearing of A from P minus the angle APB.

The bearing of B from P = 303° - 23°

= **280°**

(ii) A from B. [2]

The bearing of B from P is 280°.

Therefore, the bearing of P form B is:

280° - 180° = 100°

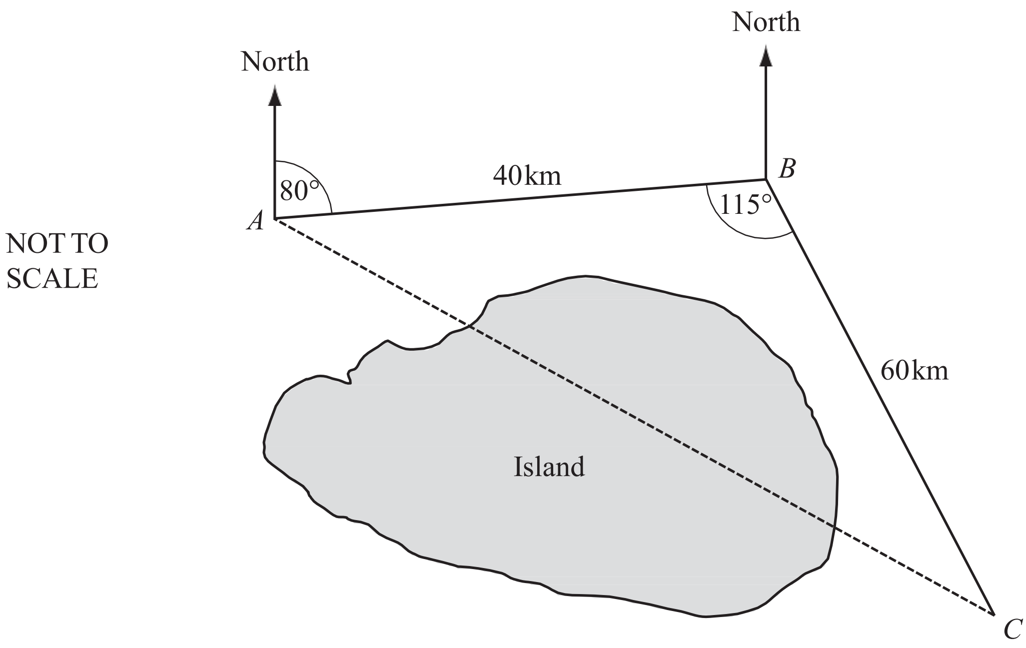
The bearing of A from B is equal to the bearing of P from B minus the angle ABP.

Angle ABP = 180° - 23° - 126°

Angle ABP = 31°

The bearing of A form B = 100° - 31°

The bearing of A form B = 69°



To avoid an island, a ship travels 40 kilometres from A to B and then 60 kilometres from B to C.

The bearing of B from A is 080° and angle ABC is 115°.

(a) The ship leaves A at 11 55.

It travels at an average speed of 35 km/h.

Calculate, to the nearest minute, the time it arrives at C. [3]

Total distance travelled is

$$40 + 60$$

$$= 100$$

Using

$$speed = \frac{distance}{time}$$

we have

$$time = \frac{distance}{speed}$$

$$= \frac{100}{35}$$

$$= 2\frac{6}{7}\ hours$$

$$= 2\ hours\ 51\ minutes$$

$$11\ 55 + 02\ 51$$

$$= 13\ 106$$

$$= \mathbf{14\ 46}$$

(b) Find the bearing of

(i) A from B, [1]

The bearing of A from B is

$$360 - (180 - 80)$$

$$= \mathbf{260}$$

(ii) C from B. [1]

$$360 - (180 - 80) - 115$$

$$= 260 - 115$$

$$= \mathbf{145}$$

(c) Calculate the straight line distance AC. [4]

Using cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

we have

$$AC^2 = 40^2 + 60^2 - 2(40)(60)\cos 115$$

$$= 7228.567656$$

$$\rightarrow AC = 85$$

(d) Calculate angle BAC. [3]

Using sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

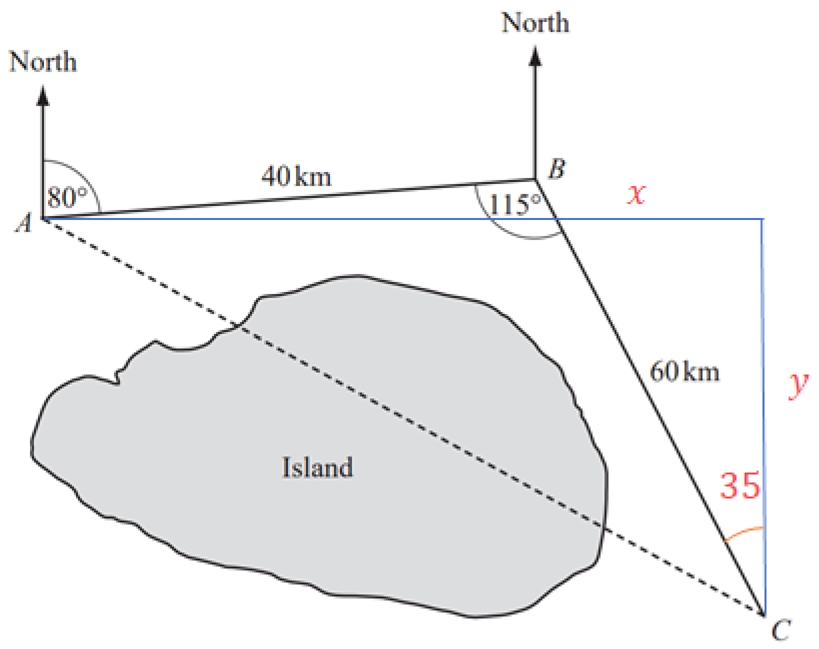
we have

$$\frac{\sin BAC}{60} = \frac{\sin 115}{85}$$

$$\rightarrow BAC = \sin^{-1}\left(\frac{60\sin 115}{85}\right)$$

$$= 39.8$$

(e) Calculate how far C is east of A. [3]



We need to find x.

The bearing of B from C (shown on diagram) is given by

$$180 - 145$$

$$= 035$$

Angle BCA is found using the fact that angles in a triangle sum to 180

$$BCA + 115 + 39.8 = 180$$

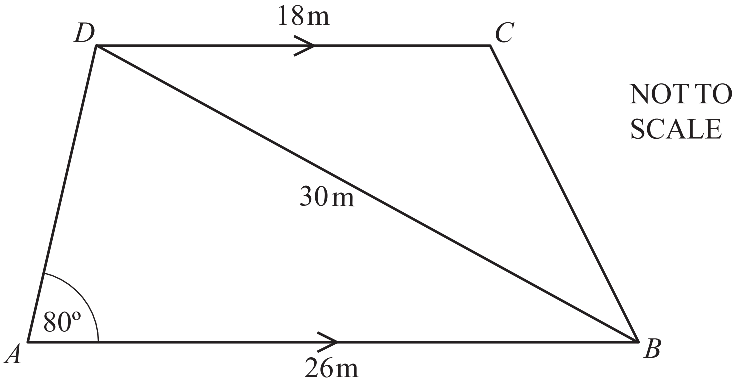
$$\rightarrow BCA = 25.2$$

Hence, we have

$$x = AC \sin(25.2 + 35)$$

$$= 85 \sin 60.2$$

$$= \mathbf{73.8}$$



The diagram shows the plan of a garden.

The garden is a trapezium with $AB = 26$ metres, $DC = 18$ metres and angle $DAB = 80°$.

A straight path from B to D has a length of 30 metres.

(a) **Use trigonometry**, showing all your working, to calculate

(i) angle ADB, [3]

Using sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\rightarrow \frac{\sin ADB}{26} = \frac{\sin 80}{30}$$

$$\rightarrow \sin ADB = \frac{26 \sin 80}{30}$$

$$= 0.8535$$

$$\rightarrow ADB = 58.6$$

(ii) the length of BC, [4]

We have that

$$BDC = ADC - 58.6$$

$$= 100 - 58.6$$

$$= 41.4$$

now using cosine rule

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$

we have that

$$BC^2 = 18^2 + 30^2 - 2(18)(30)\cos 41.4$$

$$= 413.88$$

$$\rightarrow \boldsymbol{BC = 20.3}$$

(iii) the area of the garden. [3]

Area of a triangle is

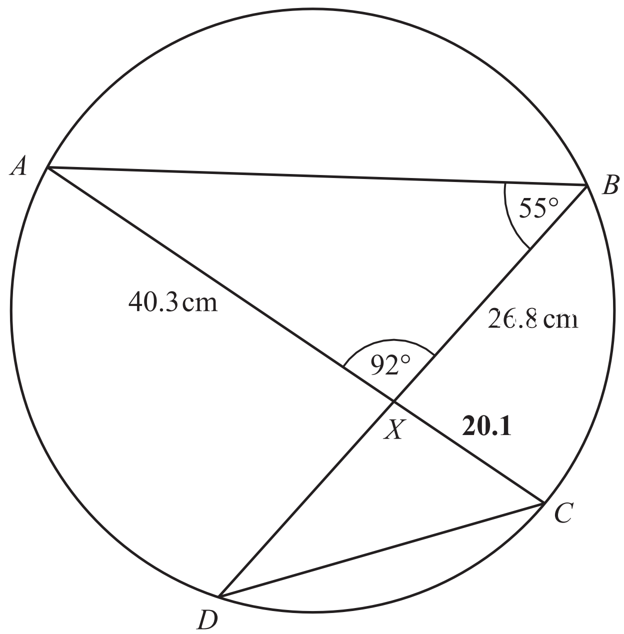
$$\boldsymbol{A = \frac{1}{2}ab\sin C}$$

Hence area of the garden is

$$\frac{1}{2}(18)(30)\sin 41.4 + \frac{1}{2}(30)(26)\sin 41.4$$

$$= \boldsymbol{436.5}$$

(a)



NOT TO SCALE

A, B, C and D lie on a circle.

AC and BD intersect at X.

Angle $ABX = 55°$ and angle $AXB = 92°$.

$BX = 26.8$ cm, $AX = 40.3$ cm and $XC = 20.1$ cm.

(i) Calculate the area of triangle AXB.
You must show your working. [2]

We can use the following formula for the area of a triangle:

$$A = \frac{ab \sin C}{2}$$

Where a and b are 2 sides of the triangle and angle C is the angle between them.

In our case:

$$A = \frac{40.3 \text{ cm} \times 26.8 \text{ cm} \times \sin 92°}{2}$$

A = 539.69 cm^2

(ii) Calculate the length of AB.
You must show your working. [3]

In the triangle AXB we ca use the sine rule to work out the size of AB.

Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Where a is a side in the triangle and A is the angle opposite side a.

In our case, we can write this as:

$$\frac{AB}{\sin 92°} = \frac{40.3\ cm}{\sin 55°}$$

$$AB = \frac{40.3\ cm\ x\ 0.999}{0.819}$$

AB = 49.167 cm

(iii) Write down the size of angle ACD. Give a reason for your answer. [2]

In a circle, the angles subtended at the circumference by the same arc are equal.

In our case, angles ABD and ACD are equal, 55°.

(iv) Find the size of angle BDC. [1]

In the triangle BAX, angle BAX is equal to:

Angle BAX = 180° - 92° - 55°

Angle BAX = 33°

In a circle, the angles subtended at the circumference by the same arc are equal.

angles BAC and BDC are equal, 33°.

(v) Write down the geometrical word which completes the statement [1]

"Triangle AXB is **similar.** to triangle DXC."

(vi) Calculate the length of XD. [2]
You must show your working.

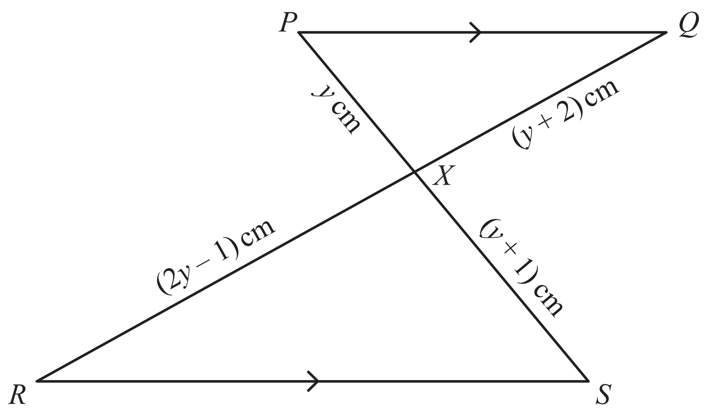
Since the 2 triangles ABX and CDX are similar, the ratios of their corresponding lengths will be equal.

$$\frac{26.8\ cm}{20.1\ cm} = \frac{40.3\ cm}{XD}$$

$$XD = \frac{40.3\ cm\ x\ 20.1\ cm}{26.8\ cm}$$

XD = 30.2 cm

(b)



NOT TO SCALE

In the diagram PQ is parallel to RS.

PS and QR intersect at X.

$PX = y$ cm, $QX = (y + 2)$ cm, $RX = (2y - 1)$ cm and $SX = (y + 1)$ cm.

(i) Show that $y^2 - 4y - 2 = 0$. [3]

Similar to point a)v), the 2 triangles are similar shapes, so we can write the following equality:

$$\frac{y}{y+2} = \frac{y+1}{2y-1}$$

We simplify the 2 fractions to obtain a second order equation.

$$y = \frac{(y+1)(y+2)}{2y-1}$$

$2y^2 - y = y^2 + y + 2y + 2$

We move all terms on one side.

$\mathbf{y^2 - 4y - 2 = 0}$

(ii) Solve the equation $y^2 - 4y - 2 = 0$.

Show all your working and give your answers correct to two decimal places. [4]

We solve the second order equation using the formula:

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our case, a = 1, b = -4 and c = -2

$$y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \; x \; 1 \; x \; (-2)}}{2 \; x \; 1}$$

$$y = \frac{4 \pm \sqrt{24}}{2}$$

y_1 = -0.45 and x_2 = 4.45

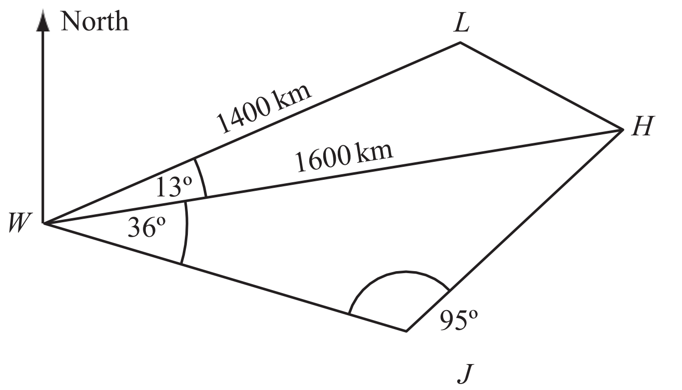
(iii) Write down the length of RX. [1]

RX = 2y – 1

We select the value y = 4.45 since the length of a triangle's side cannot be negative.

RX = 2 x 4.45 – 1

RX = 7.9 cm



The diagram shows the positions of four cities in Africa, Windhoek (W), Johannesburg (J), Harari (H) and Lusaka (L).
WL = 1400 km and WH = 1600 km.
Angle LWH = 13°, angle HWJ = 36° and angle WJH = 95°.

(a) Calculate the distance LH. [4]

We can use the cosine rule in the triangle LWH:

$a^2 = b^2 + c^2 - 2bc \cos A$

where a, b and c are sides in the triangle and A is the angle opposite side a.

$LH^2 = 1400^2 + 1600^2 - 2 \times 1400 \times 1600 \times \cos 13°$

LH = 393 km

(b) Calculate the distance *WJ*. [4]

In the triangle HJW, angle H is:

Angle H = 180° - 95° - 36°

Angle H = 49°

In the triangle HJW we ca use the sine rule to work out the size of WJ.

Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Where a is a side in the triangle and A is the angle opposite side a.

In our case, we can write this as:

$$\frac{WJ}{\sin 49°} = \frac{1600\ km}{\sin 95°}$$

$$WJ = \frac{1600\ km\ x\ 0.754}{0.996}$$

WJ = 1212 km

(c) Calculate the area of quadrilateral *WJHL*. [3]

The area of the quadrilateral is the sum of the areas of the 2 triangles.

We can use the following formula for the area of a triangle:

$$A = \frac{ab \sin C}{2}$$

Where a and b are 2 sides of the triangle and angle C is the angle between them.

In our case, for triangle LWH:

$$A = \frac{1400 \text{ x } 1600 \text{ x } \sin 13°}{2}$$

A = 251945.18 km^2

For triangle HWJ:

$$A = \frac{1211.2 \text{ x } 1600 \text{ x } \sin 36°}{2}$$

A = 569916.12 km^2

Area of the quadrilateral = 251945 km^2 + 569986 km^2

Area of the quadrilateral

= 821931.135 km^2

(d) The bearing of Lusaka from Windhoek is 060°.
Calculate the bearing of

(i) Harari from Windhoek, [1]

The bearing of H from W is equal to the bearing of L from W + the angle LWH.

The bearing of H from W = 60° + 13°

The bearing of H from W = 73°

(ii) Windhoek from Johannesburg. [1]

The bearing of J from W is equal to the bearing of H from W plus the angle HWJ.

The bearing of J from W = 73° + 36° = 109°

The bearing of W from J is equal to 360° minus the bearing of J from W.

The bearing of W from J = 360° - 109

° = 289°

(e) On a map the distance between Windhoek and Harari is 8 cm.
Calculate the scale of the map in the form 1:*n*. [2]

8 cm are represented by 1600 km, so 1 cm will be represented by:

1600 km = 1600 x 10^5 cm

$$n = \frac{1600 \times 10^5 \text{ cm} \times 1 \text{ cm}}{8 \text{ cm}}$$

n = 200 x 10^5 cm

= 20 000 000 cm

Trigonometry
Difficulty: Hard

Model Answers 5

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Trigonometry
Paper	Paper 4
Difficulty	Hard
Booklet	Model Answers 5

Time allowed: **84 minutes**

Score: **/73**

Percentage: **/100**

Grade Boundaries:

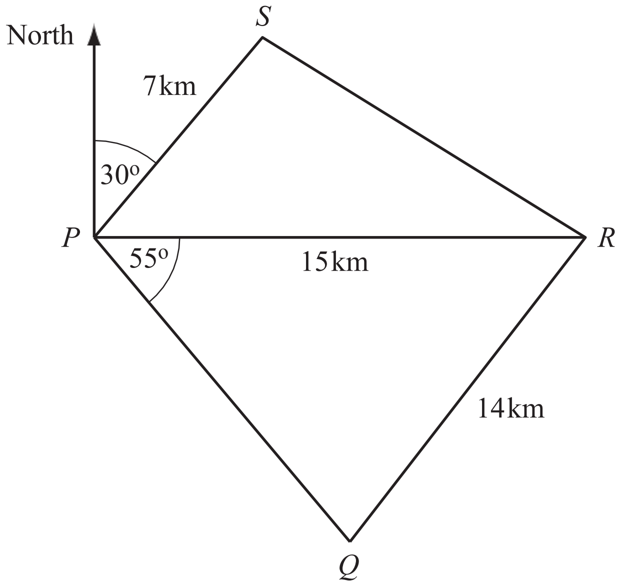
CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

NOT TO SCALE



The quadrilateral $PQRS$ shows the boundary of a forest.
A straight 15 kilometre road goes due East from P to R.

(a) The bearing of S from P is 030° and PS = 7 km.

(i) Write down the size of angle SPR. [1]

The arrow pointing North is perpendicular on the plane of the triangle SPR.

Therefore:

90° = angle SPR + 30°

Angle SPR = 60°

(ii) Calculate the length of RS. [4]

In the triangle SPR, we know the sides SP and PR and the size of the angle SPR.

We can use the cosine rule to work out the length of the side RS:

$a^2 = b^2 + c^2 - 2bc \cos A$

In this case, a = RS, b = SP = 7 km and c = PR = 15 km and the angle A is the angle

SPR = 60

$RS^2 = 7^2 + 15^2 - 2 \times 7 \times 15 \times \cos(60)$ km^2

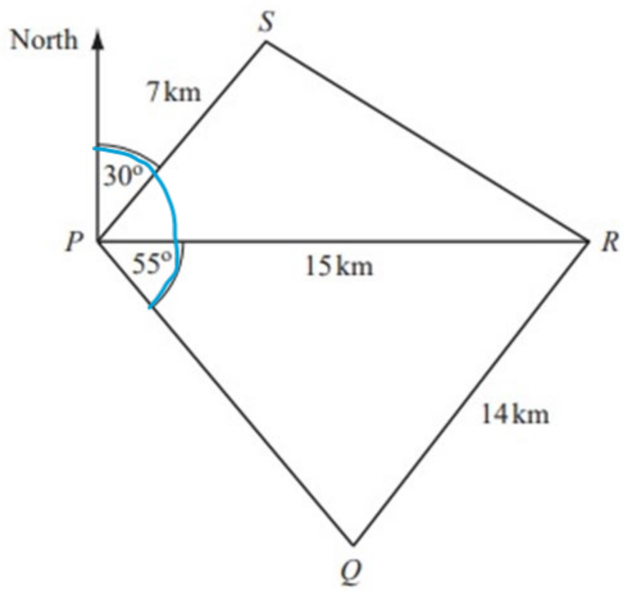
$RS^2 = 49 + 225 - 105$ km^2

$RS^2 = 169$ km^2

RS = 13 km

(b) Angle $RPQ = 55°$ and $QR = 14$ km.

(i) Write down the bearing of Q from P. [1]



The bearing of Q from P is marked in blue on the figure above.

From the figure, we can see that the bearing of Q from P is equal to the sum of the angle SPR, angle RPQ and the bearing of S from P.

The bearing of Q from P is:

30° + 55° + 60° = **145°**

(ii) Calculate the acute angle PQR. [3]

In the triangle PQR, we can use the sine rule to work out the size of the angle PQR.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

In our case, A = angle PQR, a = PR = 15 km, B = QPR = 55 and b = QR = 14 km

$$\frac{15\ km}{\sin PQR} = \frac{14\ km}{\sin 55°}$$

sin PQR = $\frac{15\ km\ x\ \sin 55°}{14\ km}$

Using a calculator, we work out that sin 55° = 0.819.

sin PQR = 0.877

angle PQR = 61.4°

(iii) Calculate the length of PQ. [3]

In the triangle PQR, the sum of the interior angles is 180°.

Angle PRQ = 180° - 55° - 61.4°

Angle PRQ = 63.6°

In the triangle PQR, we can use the sine rule to work out the size of the angle PQR.

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

In our case, A = angle PRQ, a = PQ, B = QPR = 55 and b = QR = 14 km

$\frac{PQ}{\sin PRQ} = \frac{14\ km}{\sin 55°}$

PQ = $\frac{14\ km\ x\ \sin PRQ}{\sin 55°}$

Using a calculator, we work out that sin 55° = 0.819 and sin 63.6° = 0.895.

PQ = 15.31 km

(c) Calculate the area of the forest, correct to the nearest square kilometre. [4]

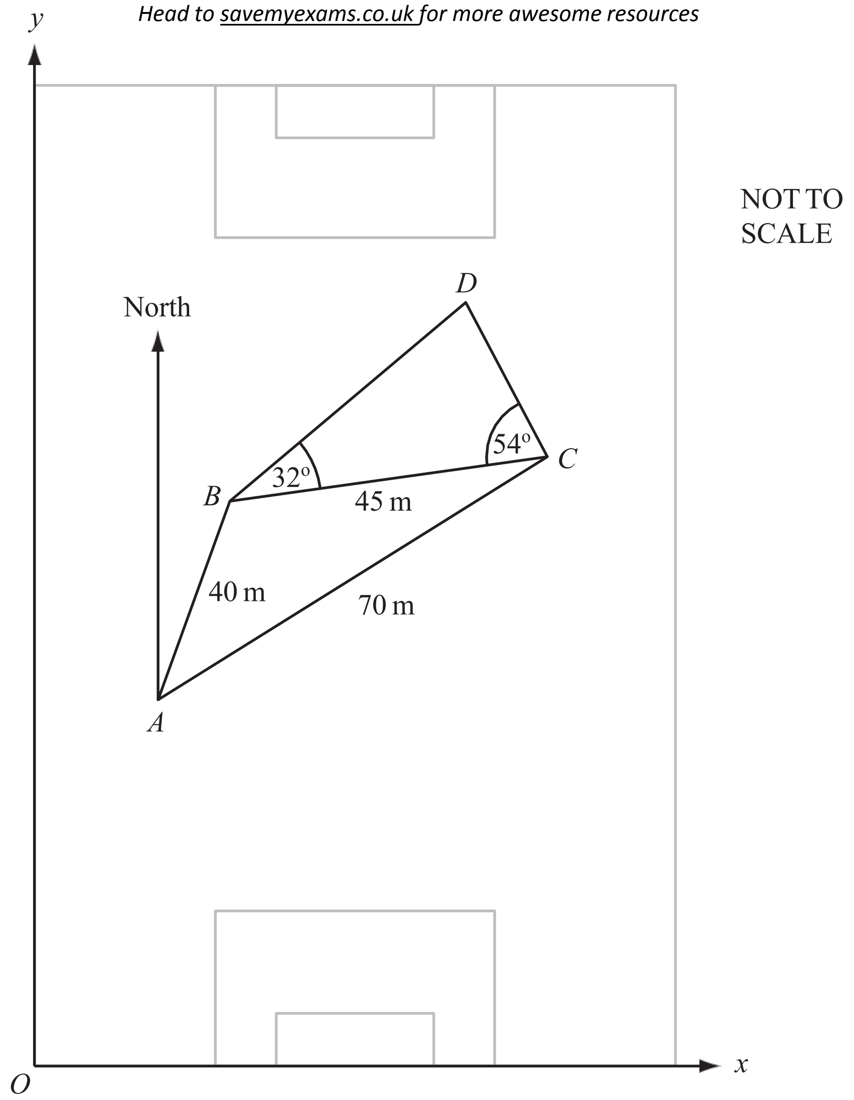
Use the formula Area $= \frac{1}{2} \times a \times b \times \sin C$ for each triangle and add them together.

Area $= \frac{1}{2} \times PS \times PR \times \sin SPR + \frac{1}{2} \times PR \times PQ \times \sin QPR$

Area $= \frac{1}{2} \times 7 \times 15 \times \sin 60 + \frac{1}{2} \times 15 \times 15.31 \times \sin 55$

Area = 140 km² (nearest square km)

Question 2



(a) During a soccer match a player runs from A to B and then from B to C as shown in the diagram.
$AB = 40$ m, $BC = 45$ m and $AC = 70$ m.

(i) Show by calculation that angle $BAC = 37°$, correct to the nearest degree. [3]

In the triangle BAC, we can use the cosine rule to work out the size of the angle BAC.

$a^2 = b^2 + c^2 - 2bc \cos A$

where A = angle BAC, a = BC = 45 m, b = AC = 70 m and c = AB = 40 m

$45^2 \text{ m}^2 = 70^2 \text{ m}^2 + 40^2 \text{ m}^2 - 2 \times 70 \text{ m} \times 40 \text{ m} \times \cos BAC$

$2025 \text{ m}^2 = 4900 \text{ m}^2 + 1600 \text{ m}^2 - 5600 \text{ m}^2 \times \cos BAC$

$\cos BAC = 0.799$

angle BAC = 37°

(ii) The bearing of C from A is 051°. Find the bearing of B from A. [1]

The bearing of B from A is equal to the bearing of C from A minus angle BAC.

51° - 37° = **14°**

(iii) Calculate the area of triangle ABC. [3]

We can calculate the area of a triangle using the formula:

Area = $\frac{1}{2}$ a b sin C

Where a and b are sides of the triangle and angle C is the angle between a and b.

Area = $\frac{1}{2}$ x 40 cm x 70 cm x sin 37°

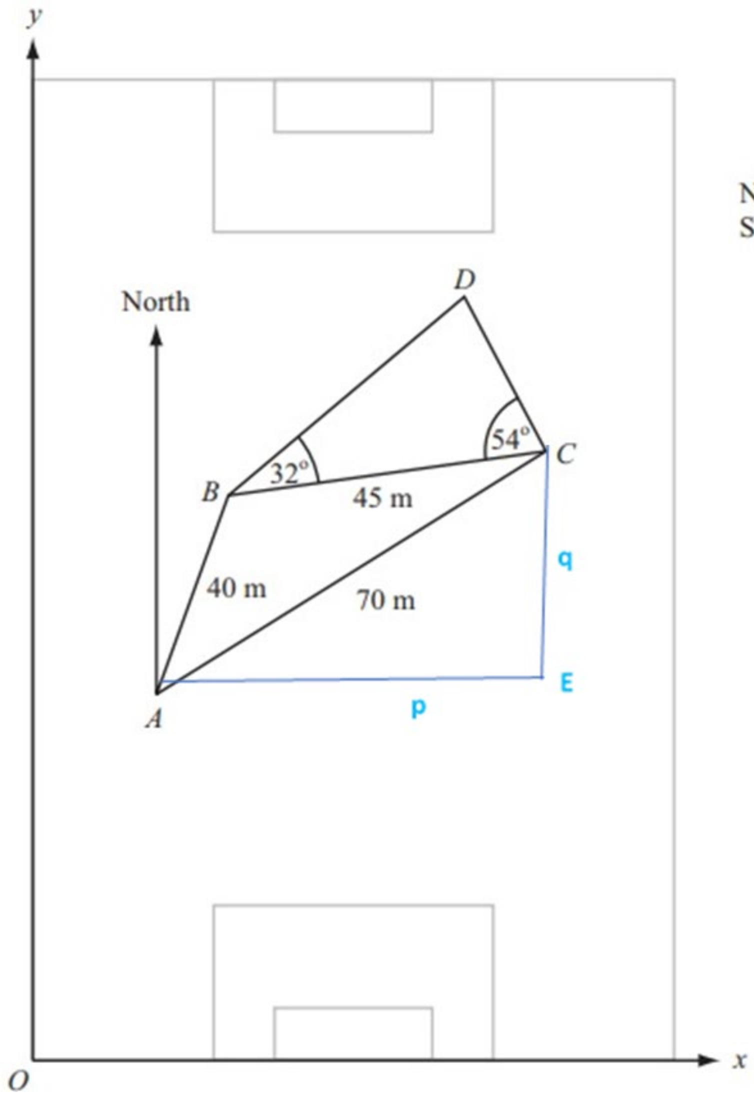
Area = 1400 cm^2 x 0.601

Area = 843 cm^2

(b) x- and y-axes are shown in the diagram.

$\overrightarrow{AC} = \begin{pmatrix} p \\ q \end{pmatrix}$, where p and q are measured in metres.

(i) Show that $p = 54.4$. [2]



We use positive numbers to describe a translation up or the right and negative numbers to describe a translation down or to the left.

In a column vector, the upper value represents a movement to the left or right while the lower value represents a movement either up or down.

In the right-angled triangle CEA, p represents the movement towards right while q represents the upwards movement.

CE is a parallel line to the arrow pointing north.

The bearing of C from A and the angle ACE represent alternate angles, therefore, they are congruent and equal to 51°.

In the right-angled triangle CEA, we can define:

$$\sin CEA = \frac{\text{opposite side}}{\text{hypothenuse}}$$

$$\cos CEA = \frac{\text{adjacent side}}{\text{hypothenuse}}$$

$$\sin 51° = \frac{p}{70 \text{ cm}}$$

p = 54.4 cm

(ii) Find the value of q. [2]

$$\cos 51° = \frac{q}{70}$$

q = 44.1 cm

(c) Another player is standing at D.
$BC = 45$ m, angle $BCD = 54°$ and angle $DBC = 32°$.
Calculate the length of BD. [4]

In the triangle DBC, the interior angles add up to 180°.

Angle BDC = 180° - 32° - 54°

Angle BDC = 94°

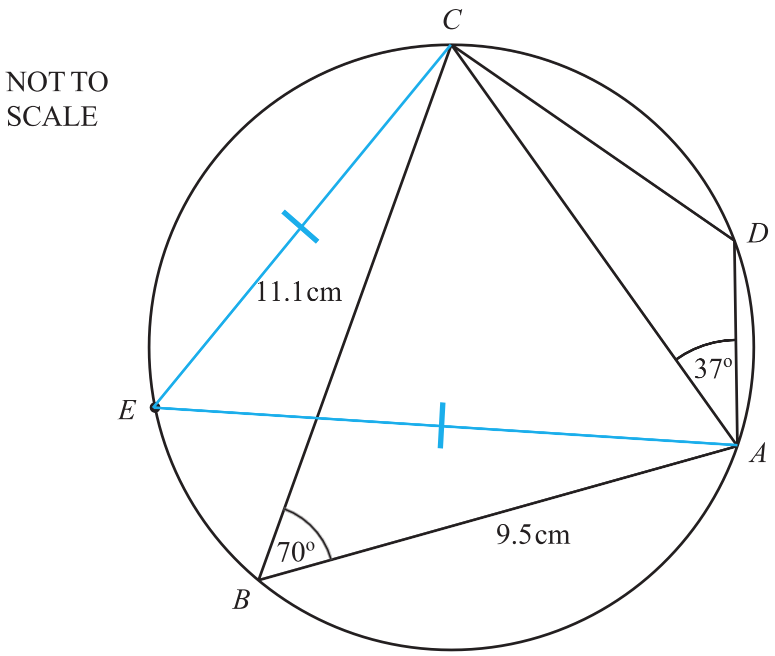
In triangle BDC we can use the sine rule to work out the size of BD.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

In our case, angle A = angle DCB, angle B = angle DBC and b = DC.

$$\frac{\sin 54°}{BD} = \frac{\sin 94°}{45 \text{ cm}}$$

BD = 36.5 cm



$ABCD$ is a cyclic quadrilateral.
$AB = 9.5$ cm, $BC = 11.1$ cm, angle $ABC = 70°$ and angle $CAD = 37°$.

(a) Calculate the length of AC. [4]

The first thing to do here is identify the type of problem.

ABC is a non-right-angled triangle and you are working with three sides and an angle.

This is therefore a Cosine Rule problem.

Using the labelling given (and a for the side opposite angle A etc.):

$$b^2 = a^2 + c^2 - 2ac\cos B$$

Plug in the numbers:

$$AC^2 = 11.1^2 + 9.5^2 - 2 \times 11.1 \times 9.5 \times \cos 70$$

Square Rooting gives:

$$AC = \sqrt{141.3279\ldots}$$

Put into calculator and round to 3sf:

$$\mathbf{AC = 11.9cm}$$

(b) Explain why angle $ADC = 110°$. [1]

Looking at the Circle Theorems we find that:

Opposite Angles in a Cyclic Quadrilateral add to 180°

$$ADC + 70 = 180°$$

$$\boldsymbol{ADC = 110°}$$

(c) Calculate the length of AD. [4]

Again identify the type of problem.

ADC is a non-right-angled triangle and you are working with two sides and two angles.

This is therefore a Sine Rule problem.

Using the labelling given (and a for the side opposite angle A etc.):

$$\frac{c}{\sin C} = \frac{d}{\sin D}$$

Find angle C using "Angles in a Triangle add to 180°":

$$C = 180 - 110 - 37 = 33°$$

Use the "Ans" button on your calculator instead of "11.9" for greater accuracy

Plug the numbers into the Sine Rule:

$$\frac{AD}{\sin 33} = \frac{11.9}{\sin 110}$$

$$AD = \frac{11.9 \times \sin 33}{\sin 110}$$

Put into calculator and round to 3sf:

$$\boldsymbol{AD = 6.89\text{cm}}$$

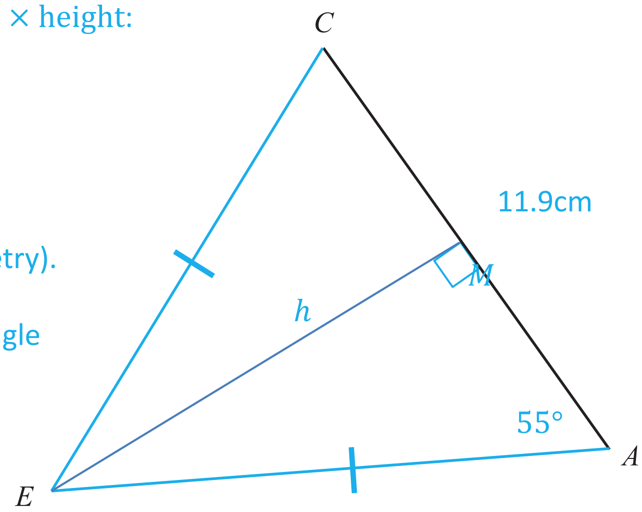
(d) A point E lies on the circle such that triangle ACE is isosceles, with $EA = EC$.

(i) Write down the size of angle AEC. [1]

$\boldsymbol{AEC = 70°}$ because angles in the same segment are equal

(ii) Calculate the area of triangle ACE. [3]

Using the general Area formula: Area $= \frac{1}{2} \times \text{base} \times \text{height}$:



Redraw the triangle ACE.

base$= AC = 11.9$ (from (a))

The height EM is such that $AM = MC$ (by symmetry).

Angle EAM is the base angle of an Isosceles Triangle

so $E\hat{A}M = \frac{1}{2}(180 - 70) = 55°$

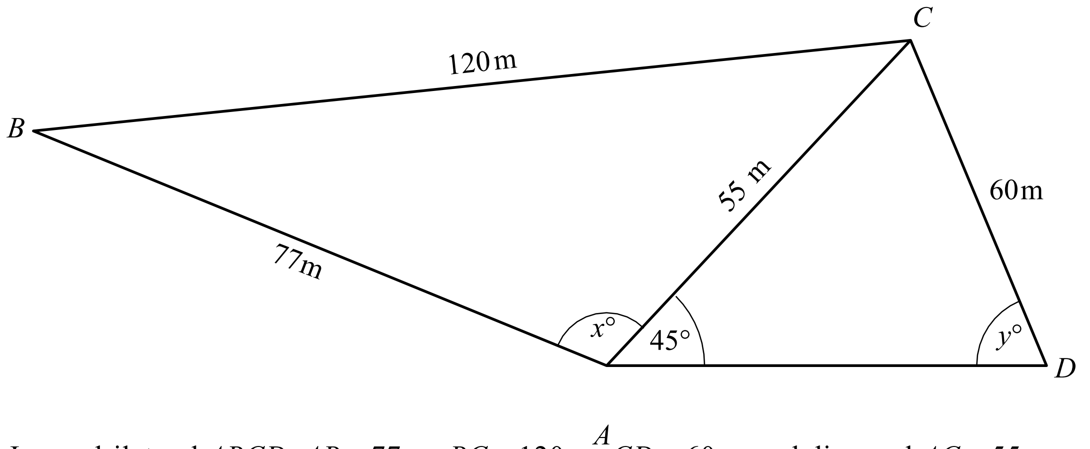
Find the height, h, using SOHCAHTOA:

height$= h = AM \times \tan 55 = \frac{1}{2} \times 11.9 \times \tan 55 = 8.497 \ldots$

And now plug the numbers into the Area formula above:

Area $= \frac{1}{2} \times 11.9 \times 8.497$

Area $= \mathbf{50.5cm^2}$ (to 3sf having used very accurate values from earlier)



In quadrilateral $ABCD$, $AB = 77$ m, $BC = 120$ m, $CD = 60$ m and diagonal $AC = 55$ m.
Angle $CAD = 45°$, angle $BAC = x°$ and angle $ADC = y°$.

(a) Calculate the value of x. [4]

We can use the cosine rule to work out the size of x

Cosine rule:

$a^2 = b^2 + c^2 - 2bc \cos A$

where a, b and c are sides in the triangle and A is the angle opposite side a.

In our case:

$120^2 = 77^2 + 55^2 - 2 \times 77 \times 55 \cos x°$

$x = 130°$

(b) Calculate the value of y. [4]

We can use the sine rule to work out the size of the angle y.

Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

where a and b are sides in the triangle, A is the angle opposite side a and

B is the angle opposite b.

In our case:

$$\frac{a}{\sin A} = \frac{\mathbf{b}}{\mathbf{\sin B}}$$

$$\frac{60\text{ m}}{\sin 45°} = \frac{\mathbf{55\text{ m}}}{\mathbf{\sin y}}$$

sin y = 0.648

y = 40.4°

(c) The bearing of D from A is 090°.
Find the bearing of

(i) A from C, [2]

The bearing of C from A is equal to the bearing of D from A minus the angle CAD.

The bearing of C from A = 90° - 45°

The bearing of C from A = 45°

The bearing needs to be measured from the line representing North and clockwise.

In our case, the North lines from points A and C are parallel, therefore, their corresponding angles will be equal.

Th bearing of A from C is: 180° + 45°

= **225°**

(ii) B from A. [2]

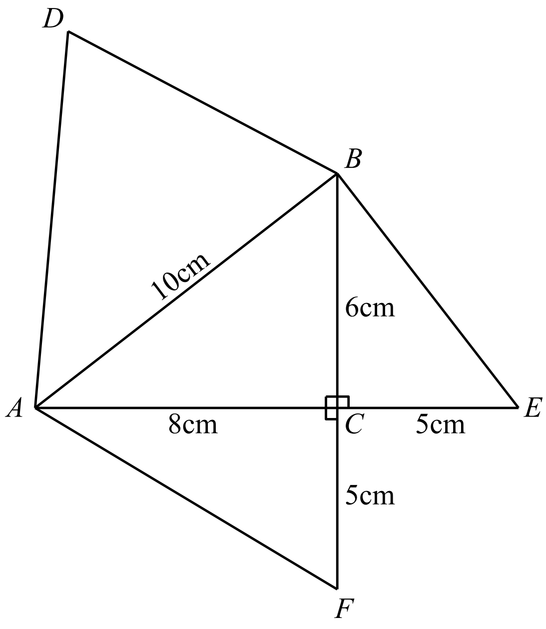
The angle the line BA makes with the extension of the horizontal line AD is equal to:

$180° - 45° - x = 180° - 45° - 130° = 5°$

Th bearing of B from A is equal to 180° plus the bearing of D from A plus the angle BA makes with the horizontal, 5°.

The bearing of B from A = 180° + 90° + 5°

= 275°



NOT TO SCALE

The diagram shows a sketch of the net of a solid tetrahedron (triangular prism).
The right-angled triangle ABC is its base.
$AC = 8$ cm, $BC = 6$ cm and $AB = 10$ cm. $FC = CE = 5$ cm.

(a) (i) Show that $BE = \sqrt{61}$ cm. [1]

In the right-angled triangle BCE we apply Pythagoras' Theorem:

$6^2 + 5^2 = BE^2$

$BE = \sqrt{61}$

(ii) Write down the length of DB. [1]

Since the net represents a triangular prism, the lengths DB and BE are equal.

$DB = \sqrt{61}$

(ii) Explain why $DA = \sqrt{89}$ cm. [2]

Similarly, AF and DA are equal.

In the right-angled triangle CAF we apply Pythagoras' Theorem:

$8^2 + 5^2 = AF^2$

$DA = AF = \sqrt{89}$

(b) **Calculate** the size of angle *DBA*. [4]

We can use the cosine rule to work out the size of the angle DBA.

Cosine rule:

$a^2 = b^2 + c^2 - 2bc \cos A$

where a, b and c are sides in the triangle and A is the angle opposite side a.

In our case:

$(\sqrt{89})^2 = 10^2 + (\sqrt{61})^2 - 2 \times 10 \times \sqrt{61} \cos DBA$

$\cos DBA = 0.46$

Angle DBA = 62.5°

(c) Calculate the area of triangle *DBA*. [3]

The formula for the area of a triangle is:

$A = \frac{ab \sin C}{2}$

Where a and b are sides in the triangle and angle C is the angle between them.

$A = \frac{DB \times BF \ \sin DBA}{2}$

$A = \frac{10 \times \sqrt{61} \ \sin 62.5°}{2}$

A = 34.6 cm²

(d) Find the total surface area of the solid. [3]

The surface area of the solid will be equal to the sum of the areas for the 4 triangles.

A = $\frac{\text{height x base}}{2}$

A BCE = $\frac{6 \text{ x } 5}{2}$

A BCE = 15 cm^2

A BCA = $\frac{6 \text{ x } 8}{2}$

A BCA = 24 cm^2

A CAF = $\frac{8 \text{ x } 5}{2}$

A CAF = 20 cm^2

Surface area = 15 cm^2 + 20 cm^2 + 24 cm^2 + 34.6 cm^2

Surface area = 93.6 cm^2

(e) Calculate the volume of the solid. [3]
[The volume of a tetrahedron is $\frac{1}{3}$ (area of the base) × perpendicular height.]

The height of the triangular prims will be DC = CE = CF = 5 cm.

A BCA = $\frac{6 \text{ x } 8}{2}$

A BCA = 24 cm^2

V = $\frac{1}{3}$ x 24 cm^2 x 5 cm

V = 40 cm^3