DIGITAL SIGNAL PROCESSING LAB

(EL-302)

LABORATORY MANUAL

ENGR. MUHAMMAD IBRAR KHAN

Discrete Fourir Transform(DFT)

(LAB # 07)

Student Name: ______________________________________

Roll No: ______________ Section: ___

Date performed: ___________, 2018

NATIONAL UNIVERSITY OF COMPUTER AND EMERGING SCIENCES, ISLAMABAD

Prepared by: Engr. Muhammad Asim
Last Edited by: Engr. Muhammad Asim, Feb 20, 2016
Verified by: Dr. Shahzad Saleem
Updated: Spring 2016

Lab # 07: Discrete Fourier Transform

Learning Objectives

1) Introduction to DFT
2) DTFT vs DFT
3) FFT and IFFT
4) Observing relationship between k and f in Hz
5) Properties of DFT
6) Linear Convolution of two sequences using DFT

Equipment Required

1. PC
2. MATLAB

1. Introduction

Discrete Fourier Transform (DFT) is used for performing frequency analysis of discrete time signals. DFT gives a discrete frequency domain representation whereas the other transforms are continuous in frequency domain. The N point DFT of discrete time signal x[n] is given by the equation

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kn}{N}} \quad ; \qquad k = 0,1,2, \dots \dots \dots \dots \dots \dots \dots \dots \dots N-1$$

Where N is chosen such that $N \geq L$, where L=length of $x[n]$. The inverse DFT allows us to recover the sequence x[n] from the frequency samples.

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{\frac{j2\pi kn}{N}} ; \; n = 0,1,2 \dots \dots . . N-1$$

Also the DFT operator $W_N = e^{-\frac{j2\pi}{N}}$ is referred as twiddle factor.

X(k) is a complex number (remember $e^{-j\theta} = cos\theta - jsin\theta$). It has both magnitude and phase which are plotted versus k. These plots are magnitude and phase spectrum of $x[n]$. The 'k' gives us the frequency information. Here k=N in the frequency domain corresponds to sampling frequency (fs). Increasing N, increases the frequency resolution, i.e. it improves the spectral characteristics of the sequence. For example if f_S = 8kHz and N=8 point DFT, then in the resulting spectrum, k=1 corresponds to 1kHz frequency. For the same f_S and x[n], if N=80 point DFT is computed, then in the resulting spectrum, k=1 corresponds to 100Hz frequency. Hence, the resolution in frequency is increased. Since N ≥ L , increasing N from 8 to 80 for the same x[n] implies x[n] is still the same sequence (<8), the rest of x[n] is padded with zeros. This implies that there is no further information in time domain, but the resulting spectrum has higher frequency resolution. This spectrum is known as 'high density spectrum' (resulting from zero padding x[n]). Instead of zero padding, for higher N, if more number of points of x[n] are taken (more data in time domain), then the resulting spectrum is called a 'high resolution spectrum'.

The discrete Fourier transform (DFT) X[k] of a finite-length sequence x[n] can be easily computed in MATLAB using the function "fft" command. There are two versions of this function:

1) $fft(x)$ Computes the DFT X[k] of the sequence x[n] where the length of X[k] is the same as that

of x[n] which means N=L.

2) $fft(x, L)$ Computes the N-point DFT of a sequence x[n] of length L where N ≥ L. If N>L, x[n] is zero-padded with N−L trailing zero-valued samples before the DFT is computed.

The inverse discrete Fourier transform (IDFT) x[n] of a DFT sequence X[k] can likewise be computed using the function ifft, which also has two versions.

DTFT vs DFT

Task#01:

1) Generate a DT multi-tone sinusoidal sequence $x[n]$ with f_1=200Hz; f_2=600Hz; f_3=1300Hz and sampling frequency and Fs=3000Hz.

$$x[n] = \sin\left(\frac{2\pi f_1 n}{Fs}\right) + \sin\left(\frac{2\pi f_2 n}{Fs}\right) + \sin\left(\frac{2\pi f_3 n}{Fs}\right) \quad 0 \le n \le 31$$

2) Write a MATLAB program to compute the N-point DFT X[k] of sequence x[n] above

$$X = D_N x \qquad \text{While}$$

$$\mathbf{D}_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^1 & W_N^2 & \cdots & W_N^{(N-1)} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{(N-1)} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)^2} \end{bmatrix}$$

3) Plot (Stem) the magnitude and phase spectrum of X[k] with respect to $2\pi k/N$.
4) Compute the DTFT of $x[n]$ by using $[X_{DTFT}, w] = freqz(x)$
5) Plot the X_{DTFT} with respect to w on the same above figure of DFT using “line” command instead of “plot” because plot and stem cannot work together.
6) Observe and comment on the sampling of DTFT.
7) Write a MATLAB program to compute and plot the N-point IDFT of X[k] by using following formula $x = D_N{}^{-1} X \;\; while \;\; D_N{}^{-1} = \frac{1}{N} D_N{}^*$

   Hint: You can use the “conj” command to compute $D_N{}^*$.
8) Plot the sequence x[n] obtained after taking IDFT..

FFT and IFFT

Task#02:

1) Write a matlab function to compute and plot the the N-point DFT and N-point IDFT of a sequence using fft and ifft commands. The function should be of following type $Function[] = DFT(n, x, N)$ Where n shows indexes of x.
2) Run the program for different values of the DFT length N.

Observing relationship between k and f in Hz

Task#03:

1) Generate a DT sinusoidal sequence $x[n]$ with f=900Hz, Fs=2000Hz and $0 \leq n \leq 200$.
2) Compute its DFT with N=201 using "fft" and plot its magnitude and phase spectrum with respect to k.
3) Check from the graph that which value of k is related to f=900Hz.
4) Compute k for f=900Hz using $k = \frac{fN}{F_s}$.

Verifying Circular Time Shift and Frequency Shifting Property of DFT

Task#04:

1) Generate a sinusoidal sequence $g[n] = sin(2\pi * 0.5 * n), \quad n = 0{:}7;$
2) Write a matlab code to verify the circular time shift property of DFT of $\boldsymbol{g}[\boldsymbol{n}]$ with shift n_o=5.

$$g[< n - n_0 >_N] \xrightarrow{DFT} G[k]W^{kn_0}$$

3) Write a matlab code to verify the frequency shift property of DFT of $\boldsymbol{g}[\boldsymbol{n}]$ with shift k_o = 5.

$$W_N{}^{-k_o n} g[n] \overset{DFT}{\longleftrightarrow} G[< k - k_o >_N]$$

Verifying N-point Circular convolution Property of DFT

Task#05:

By multiplying two N-point DFTs in the frequency domain, we get the circular convolution in the time domain.

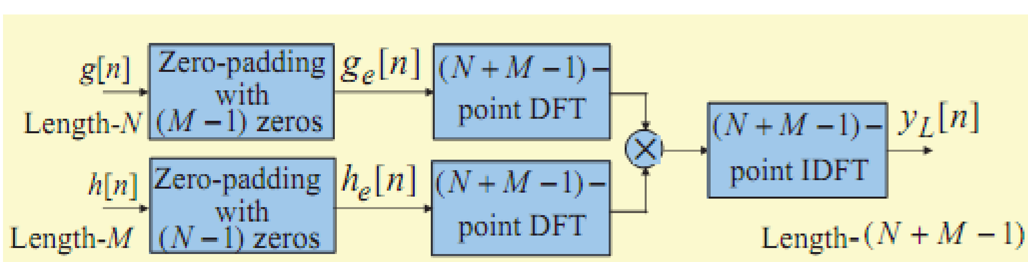
1) Generate two sinusoidal sequences
$g[n] = sin(2\pi * 0.5 * n1), \quad n1 = 0{:}6; \qquad h[n] = sin(2\pi * 0.3 * n2), \quad n2 = 0{:}6;$
2) Write a matlab code to verify the circular convolution property of DFT

$$y[n] = x[n] \textcircled{4} h[n] \xleftrightarrow{N-DFT} Y(k) = X(k) \times H(k)$$

$$\boldsymbol{y}[\boldsymbol{n}] = \boldsymbol{x}[\boldsymbol{n}]$$

Linear Convolution of two finite length sequences using DFT

Let g[n] and h[n] be two finite-length sequences of length N and M, respectively. We want to compute the linear convolution of these sequences with L=N+M-1 using DFT then the corresponding implementation scheme is illustrated below



Task#06:

Task#06:

1) Generate two sinusoidal sequences
   $g[n] = sin(2\pi * 0.5 * n),\ \ n = 0{:}\,6;\ \ h[n] = cos(2\pi * 0.3 * n),\ \ \ n = 0{:}\,9;$
2) Compute and plot the linear convolution of above two signals using “conv”.
3) Now use the above mentioned scheme to compute and plot the linear convoltuion of above two sequences.
4) Compare the results of above two methods and comment on these results.

Student's feedback: Purpose of feedback is to know the strengths and weaknesses of the system for future improvements. This feedback is for the 'current lab session'. Circle your choice:

[-3 = Extremely Poor, -2 = Very Poor, -1 = Poor, 0 = Average, 1 = Good, 2 = Very Good, 3 = Excellent]:

The following table should describe your experience with:

S#	**Field**	**Rating**	**Describe in words if required**
1	Overall Session	-3 | -2 | -1 | 0 | 1 | 2 | 3	
2	Lab Instructor	-3 | -2 | -1 | 0 | 1 | 2 | 3	
3	Lab Staff	-3 | -2 | -1 | 0 | 1 | 2 | 3	
4	Equipment	-3 | -2 | -1 | 0 | 1 | 2 | 3	
5	Atmosphere	-3 | -2 | -1 | 0 | 1 | 2 | 3	

Any other valuable feedback: ____

____

____

____

Student's Signature: ______________________________

MARKS AWARDED	**Attitude**	**Neatness**	**Correctness of results**	**Initiative**	**Originality**	**Conclusion**	**TOTAL**
TOTAL	**10**	**10**	**10**	**20**	**20**	**30**	**100**
EARNED							

Lab Instructor's Comments:____

____

____

____

Lab Instructor's Signature: ______________________________