

# **DIGITAL SIGNAL PROCESSING LAB**

## **(EL-302)**

### **LABORATORY MANUAL**

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#### **Discrete Time Fourier Transform(DTFT)**

**(LAB # 05)**

Student Name: \_\_\_\_\_

Roll No: \_\_\_\_\_ Section: \_\_\_\_

Date performed: \_\_\_\_\_, 2019



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## Lab # 05: Discrete Time Fourier Transform

### Learning Objectives

To study various properties of discrete time Fourier transform and verify these properties on various signals in Matlab.

### Equipment Required

1. PC
2. MATLAB

### 1. Introduction

If  $x[n]$  is absolutely summable, that is

$$\sum_{-\infty}^{\infty} |x[n]| < \infty$$

Then its discrete-time Fourier transform is given by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

The inverse discrete-time Fourier transform (IDTFT) of  $X(e^{j\omega})$  is given by

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

The discrete-time Fourier transform (DTFT)  $X(e^{j\omega})$  of a sequence  $x[n]$  is a continuous function of  $\omega$ . Since the data in MATLAB is in vector form,  $X(e^{j\omega})$  can only be evaluated at a prescribed set of discrete frequencies. Moreover, only a class of the DTFT that is expressed as a rational function in  $e^{-j\omega}$  in the form

$$X(e^{j\omega}) = \frac{p_0 + p_1 e^{-j\omega} + \dots + p_m e^{-j\omega m}}{d_0 + d_1 e^{-j\omega} + \dots + d_n e^{-j\omega n}} \quad 1)$$

can be evaluated.

### 2. DTFT Computation

The DTFT  $X(e^{j\omega})$  of a sequence  $x[n]$  of the form of Eq. (1) can be computed easily at a prescribed set of  $L$  discrete frequency points  $\omega = \omega_l$  using the MATLAB function `freqz`. Since  $X(e^{j\omega})$  is a continuous function of  $\omega$ , it is necessary to make  $L$  as large as possible so that the plot generated using the command `plot` provides a reasonable replica of the actual plot of the DTFT. In MATLAB, `freqz` computes the  $L$ -point DFT of the sequences  $\{p_0 \ p_1 \ \dots \ p_M\}$  and  $\{d_0 \ d_1 \ \dots \ d_M\}$ , and then forms their ratio to arrive at  $X(e^{j\omega_l})$ .

$l = 1, 2, \dots, L$ . For faster computation,  $L$  should be chosen as a power of 2, such as 256 or 512.

### Task 01

- 1) Evaluate the DTFT of following in the range  $0 \leq \omega \leq \pi$  (512 points)

$$U(e^{j\omega}) = \frac{0.7 - 0.5e^{-j\omega} + 0.3e^{-2j\omega} + e^{-3j\omega}}{1 + 0.3e^{-j\omega} - 0.5e^{-2j\omega} + 0.7e^{-3j\omega}}$$

- 2) Compute and Plot the magnitude response, phase response, real part and imaginary part of  $U(e^{j\omega})$  w.r.t.  $\frac{\omega}{\pi}$ . Label your plots properly.

Hint : Commands used; “freqz”, “angle”, ”abs”, “imag”, “real”

### Task 02

Is the DTFT a periodic function of  $\omega$ ? If it is, what is the period?

### Project 4.1

```
% Compute the frequency samples of the DTFT
w = -4*pi:8*pi/511:4*pi;
num = [2 1];den = [1 -0.6];
h = freqz(num, den, w);
% Plot the DTFT
subplot(2,1,1)
plot(w/pi,real(h));grid
title('Real part of H(e^{j\omega})')
xlabel('\omega /\pi');
ylabel('Amplitude');
subplot(2,1,2)
plot(w/pi,imag(h));grid
title('Imaginary part of H(e^{j\omega})')
xlabel('\omega /\pi');
ylabel('Amplitude');
figure;
subplot(2,1,1)
plot(w/pi,abs(h));grid
title('Magnitude Spectrum |H(e^{j\omega})|')
xlabel('\omega /\pi');
ylabel('Amplitude');
subplot(2,1,2)
plot(w/pi,angle(h));grid
title('Phase Spectrum arg[H(e^{j\omega})]')
xlabel('\omega /\pi');
ylabel('Phase, radians');
```

### Task 03:

- 1) What is the expression of the DTFT being evaluated in the above Program and on what range this DTFT is computed in the program?
- 2) Comment on your results. Can you explain the jump in the phase spectrum ? The jump can be removed using the MATLAB command “unwrap”. Evaluate the phase spectrum with the jump removed.
- 3) Is the DTFT a periodic function of  $\omega$ ? If it is, what is the period?

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**Task 04:**

- 1) Write matlab Program to evaluate and plot the magnitude and phase response of DTFT of the following finite-length sequence in the range  $0 \leq \omega \leq \pi$  (512 points)

$$g[n] = [1 \ 3 \ 5 \ 7 \ 9 \ 11 \ 13 \ 15 \ 17]$$

- 2) Modify the your code to plot the phase response in degrees.
- 3) Explain the jumps in your phase spectrum?

**DTFT Properties:**

The DTFT satisfies a number of useful properties that are often utilized in a number of applications. A detailed listing of these properties and their analytical proofs can be found in any text on digital signal processing. These properties can also be verified using MATLAB. We list below a few selected properties that will be encountered later in this exercise.

**Time-Shifting Property** – If  $G(e^{j\omega})$  denotes the DTFT of a sequence  $g[n]$ , then the DTFT of the time-shifted sequence  $g[n - n_0]$  is given by  $e^{-j\omega n_0} G(e^{j\omega})$ .

**Frequency-Shifting Property** – If  $G(e^{j\omega})$  denotes the DTFT of a sequence  $g[n]$ , then the DTFT of the sequence  $e^{j\omega_0 n} g[n]$  is given by  $G(e^{j(\omega - \omega_0)})$ .

**Convolution Property** – If  $G(e^{j\omega})$  and  $H(e^{j\omega})$  denote the DTFTs of the sequences  $g[n]$  and  $h[n]$ , respectively, then the DTFT of the sequence  $g[n] * h[n]$  is given by  $G(e^{j\omega}) H(e^{j\omega})$ .

**Time-Reversal Property** – If  $G(e^{j\omega})$  denotes the DTFT of a sequence  $g[n]$ , then the DTFT of the time-reversed sequence  $g[-n]$  is given by  $G(e^{-j\omega})$ .

**Task 05:**

- 1) Write matlab Program to evaluate and plot the magnitude and phase response of DTFT of the following finite-length sequence in the range  $-\pi \leq \omega \leq \pi$  (256 points)

$$g[n] = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9]$$

- 2) Verify the time shifting property of DTFT using  $n_0=10$ . Plot the magnitude and phase response of the DTFT of time shifted sequence and then also plot the magnitude and phase response of  $e^{-j\omega n_0} G(e^{j\omega})$ .

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- 3) Verify the frequency shifting property of DTFT using  $w_0 = 0.5\pi$ . Plot the magnitude and phase response of the DTFT of frequency shifted sequence and then also plot the magnitude and phase response of  $G(e^{j(\omega-w_0)})$ .

**Task 06:**

- 1) Write matlab Program to evaluate and plot the magnitude and phase response of DTFT of the following finite-length sequences in the range  $-\pi \leq w \leq \pi$  (256 points)

$$g[n] = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9], h[n] = [1 \ -2 \ 3 \ -2 \ 1]$$

- 2) Verify the Convolution property of DTFT by performing convolution on both of above sequences. Plot the magnitude and phase response of the DTFT of convolved sequence and then also plot the magnitude and phase response of  $G(e^{j\omega}) * H(e^{j\omega})$ .

**Student's feedback:** Purpose of feedback is to know the strengths and weaknesses of the system for future improvements. This feedback is for the 'current lab session'. Circle your choice:

[-3 = Extremely Poor, -2 = Very Poor, -1 = Poor, 0 = Average, 1 = Good, 2 = Very Good, 3 = Excellent]:

The following table should describe your experience with:

S#	Field	Rating							Describe in words if required
1	Overall Session	-3	-2	-1	0	1	2	3	
2	Lab Instructor	-3	-2	-1	0	1	2	3	
3	Lab Staff	-3	-2	-1	0	1	2	3	
4	Equipment	-3	-2	-1	0	1	2	3	
5	Atmosphere	-3	-2	-1	0	1	2	3	

Any other valuable feedback: \_\_\_\_\_

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Student's Signature: \_\_\_\_\_

MARKS AWARDED	Attitude	Neatness	Correctness of results	Initiative	Originality	Conclusion	TOTAL
TOTAL	10	10	10	20	20	30	100
EARNED							

Lab Instructor's Comments: \_\_\_\_\_

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Lab Instructor's Signature: \_\_\_\_\_