

DIGITAL SIGNAL PROCESSING LAB

(EL-302)

LABORATORY MANUAL

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Design of Simple FIR and IIR Filters

(LAB # 10)

Student Name: _____

Roll No: _____ Section: ____

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Lab # 08: Design of Simple FIR and IIR Filters

Learning Objectives

- FIR Lowpass Filter Design
- FIR Highpass Filter Design
- IIR Lowpass Filter Design
- IIR Highpass Filter Design
- All Pass Transfer Function cascaded with IIR Transfer Function

Equipment Required

1. PC
2. Matlab

1. Introduction:

One of the main applications of digital signal processing is in filtering discrete-time signals to remove undesirable components. The frequency responses of the four types of ideal filters are shown in Figure below. These filters have doubly infinite impulse responses and are not realizable. In many applications, fairly simple realizable approximations to these filters are quite adequate.

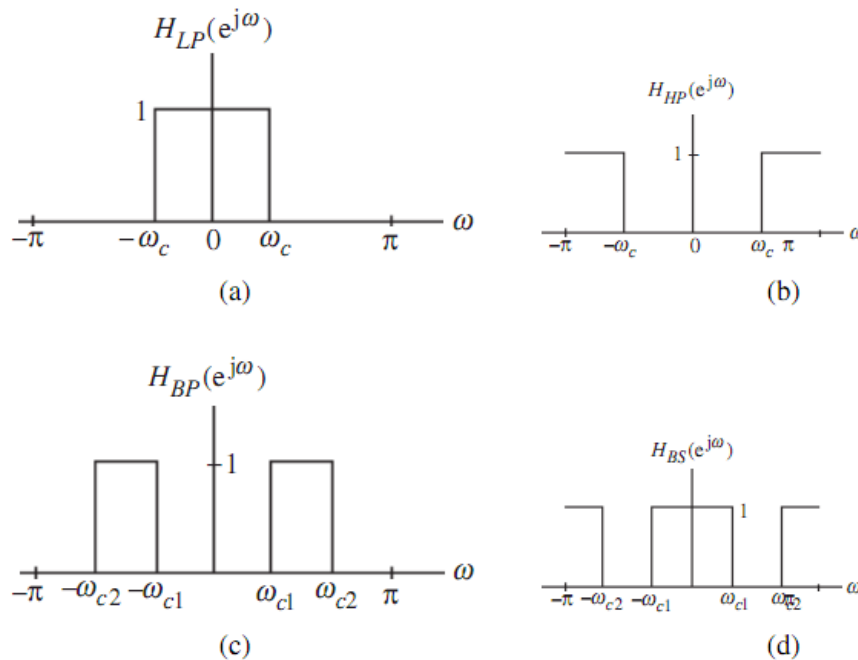


Figure Frequency responses of ideal filters: (a) lowpass filter, (b) highpass filter, (c) bandpass filter, and (d) bandstop filter.

The impulse response $h_{LP}[n]$ of the ideal low pass filter of above Figure is given by

$$h_{LP}[n] = \frac{\sin(w_c n)}{\pi n}$$

FIR Filters:

It is always possible to design an FIR transfer function with an exact linear phase response. Such a transfer function corresponds either to a symmetric impulse response defined by

$$h[n] = h[N - n] \quad 0 \leq n \leq N$$

Or an antisymmetric impulse response defined by

$$h[n] = -h[N - n] \quad 0 \leq n \leq N$$

Where N is the order of the transfer function and the length of $h[n]$ is $N+1$. There are four types of such transfer functions:

Type 1: Symmetric Impulse Response with Odd Length.

Type 2: Symmetric Impulse Response with Even Length.

Type 3: Antisymmetric Impulse Response with Odd Length.

Type 4: Antisymmetric Impulse Response with Even Length.

A Type 2 FIR transfer function must have a zero at $z = -1$, and as a result, it cannot be used to design a high pass filter. A Type 3 FIR transfer function must have a zero at $z = 1$ and $z = -1$ and, therefore, cannot be used to design either a low pass, a high pass, or a band stop filter. A Type 4 FIR transfer function is not appropriate for designing a low pass filter due to the presence of a zero at $z = 1$. The Type 1 FIR filter has no such restrictions and can be used to design almost any type of filter.

The simplest low pass FIR filter of length 2 (Moving Average Filter with $M=2$) has a transfer function given by

$$H_0(z) = \frac{1}{2} (1 + z^{-1})$$

It can be shown that this filter has a 3-dB cutoff frequency $w_c = \frac{\pi}{2}$. By cascading a number of these simple FIR low pass filters, a low pass filter with a sharper magnitude response can be obtained.

A cascade of K sections of $H_0(z)$ has a Transfer function

$$G_0(z) = \left(\frac{1}{2} (1 + z^{-1}) \right)^K$$

And has 3-dB cutoff frequency at

$$w_c = 2 \cos^{-1} \left(2^{-\frac{1}{2K}} \right)$$

A slight modification of the difference equation of Moving Average Filter yields a high pass filter whose transfer function is given by

$$H_1(z) = \frac{1}{M} \sum_{n=0}^{M-1} (-1)^n z^{-n}$$

Its 3-dB cutoff frequency is given by

$$\omega_c = \frac{\pi}{2}$$

Same as with cascaded low pass filters, we can cascade the simple high pass filters to obtain sharper magnitude response.

IIR Filters:

A first-order low pass IIR transfer function $H_{LP}(z)$ is given by

$$H_{LP}(z) = \frac{(1 - \alpha)}{2} \cdot \frac{(1 + z^{-1})}{1 - \alpha z^{-1}} \quad 0 < |\alpha| < 1$$

Where $|\alpha| < 1$ for stability. The frequency ω_c where the gain is 3dB below its maximum value at dc ($\omega=0$), called the 3-dB cutoff frequency, is related to the parameter α through

$$\alpha = \frac{1 - \sin \omega_c}{\cos \omega_c}$$

A first-order high pass IIR transfer function $H_{HP}(z)$ is given by

$$H_{HP}(z) = \frac{(1 + \alpha)}{2} \cdot \frac{(1 - z^{-1})}{1 - \alpha z^{-1}} \quad 0 < |\alpha| < 1$$

Where $|\alpha| < 1$ for stability. The frequency ω_c where the gain is 3dB below its maximum value at dc ($\omega=0$), called the 3-dB cutoff frequency, is related to the parameter α through

$$\alpha = \frac{1 - \sin \omega_c}{\cos \omega_c}$$

By cascading the simple digital filters described above, digital filters with sharper magnitude responses can be implemented. For example, for a cascade of K first-order low pass sections, the overall structure has a transfer function $G_{LP}(z)$ given by

$$G_{LP}(z) = \left(\frac{(1 - \alpha)}{2} \cdot \frac{(1 + z^{-1})}{1 - \alpha z^{-1}} \right)^k$$

Similarly transfer function $G_{HP}(z)$ for a cascade of K first-order High pass sections is given by

$$G_{HP}(z) = \left(\frac{(1 + \alpha)}{2} \cdot \frac{(1 - z^{-1})}{1 - \alpha z^{-1}} \right)^k$$

Task#01:

- Implement the above given transfer function $H_0(z)$ of FIR Lowpass filter using “tf” function in matlab.
- Find poles and zeros of above computed transfer function and plot these using “zplane” command. Write down type of the filter.
- Find frequency response using “freqz” function and plot its magnitude and phase response.
- Observe the phase response whether it is linear or not?
- Plot the gain function of filter and find its 3dB cutoff frequency. Compare it with mathematically computed ω_c .

Task#02:

- Implement the above given transfer function $G_0(z)$ of cascaded FIR Lowpass filter using “tf” function in matlab with $K=3$.
- Find poles and zeros of above computed transfer function and plot these using “zplane” command. Write down type of the filter.
- Find frequency response using “freqz” function and plot its magnitude and phase response.
- Compare this frequency response with frequency response of a single low pass FIR filter.
- Observe the phase response whether it is linear or not?
- Plot the gain function of filter and find its 3dB cutoff frequency. Compare it with mathematically computed ω_c .

Task#03:

- Implement the above given transfer function $H_1(z)$ of FIR Highpass filter using “tf” function in matlab.
- Find poles and zeros of above computed transfer function and plot these using “zplane” command. Write down type of the filter.
- Find frequency response using “freqz” function and plot its magnitude and phase response.
- Observe the phase response whether it is linear or not?
- Plot the gain function of filter and find its 3dB cutoff frequency. Compare it with mathematically computed ω_c .

Task#04:

- Implement the transfer function $G_1(z)$ of cascaded FIR Highpass filter using “tf” function in matlab with $K=3$.
- Find poles and zeros of above computed transfer function and plot these using “zplane” command. Write down type of the filter.
- Find frequency response using “freqz” function and plot its magnitude and phase response.
- Compare this frequency response with frequency response of a single high pass FIR filter.
- Observe the phase response whether it is linear or not?
- Plot the gain function of filter and find its 3dB cutoff frequency. Compare it with mathematically computed ω_c .

Task#05:

- Implement the above given transfer function $H_{LP}(z)$ of IIR Lowpass filter using “tf” function in matlab.
- Find poles and zeros of above computed transfer function and plot these using “zplane” command.
- Find frequency response using “freqz” function and plot its magnitude and phase response.
- Observe the phase response whether it is linear or not?
- Plot the gain function of filter and find its 3dB cutoff frequency. Compare it with mathematically computed w_c .
- Change the value of α and observe how w_c changes with α .

Task#06:

- Implement the above given transfer function $H_{HP}(z)$ of IIR Highpass filter using “tf” function in matlab.
- Find poles and zeros of above computed transfer function and plot these using “zplane” command.
- Find frequency response using “freqz” function and plot its magnitude and phase response.
- Observe the phase response whether it is linear or not?
- Plot the gain function of filter and find its 3dB cutoff frequency. Compare it with mathematically computed w_c .
- Change the value of α and observe how w_c changes with α .

Task#07:

- Implement the above given transfer function $G_{LP}(z)$ of cascaded Lowpass IIR filter system using “tf” function in matlab with $K=3$.
- Find poles and zeros of above computed transfer function and plot these using “zplane” command.
- Find frequency response using “freqz” function and plot its magnitude and phase response.
- Compare this frequency response with frequency response of a single low pass IIR filter.
- Observe the phase response whether it is linear or not?
- Plot the gain function of filter and find its 3dB cutoff frequency. Compare it with mathematically computed w_c .

Task#08:

- Implement the above given transfer function $G_{HP}(z)$ of cascaded Highpass IIR filter system using “tf” function in matlab with $K=3$.
- Find poles and zeros of above computed transfer function and plot these using “zplane” command.
- Find frequency response using “freqz” function and plot its magnitude and phase response.
- Compare this frequency response with frequency response of a single high pass IIR filter.
- Observe the phase response whether it is linear or not?

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- f) Plot the gain function of filter and find its 3dB cutoff frequency. Compare it with mathematically computed ω_c

Task#09:

- a) Using MATLAB compute and plot the group delay of the causal LTI discrete-time system with a transfer function given by

$$H(z) = \frac{z^{-1} - 1.2z^{-2} + z^{-3}}{1 - 1.3z^{-1} + 1.04z^{-2} - 0.222z^{-3}}$$

HINT: Use the “grpdelay” function.

- b) Find allpass transfer function for above system by using “iirgrpdelay” function.
c) Cascade the $H(z)$ with above computed allpass T.F to generate $H_1(z)$.
d) Find group delay of $H_1(z)$ and compare it with above computed group delay.

Student's feedback: Purpose of feedback is to know the strengths and weaknesses of the system for future improvements. This feedback is for the 'current lab session'. Circle your choice:

[-3 = Extremely Poor, -2 = Very Poor, -1 = Poor, 0 = Average, 1 = Good, 2 = Very Good, 3 = Excellent]:

The following table should describe your experience with:

| S# | Field | Rating | | | | | | | Describe in words if required |
|----|-----------------|--------|----|----|---|---|---|---|-------------------------------|
| 1 | Overall Session | -3 | -2 | -1 | 0 | 1 | 2 | 3 | |
| 2 | Lab Instructor | -3 | -2 | -1 | 0 | 1 | 2 | 3 | |
| 3 | Lab Staff | -3 | -2 | -1 | 0 | 1 | 2 | 3 | |
| 4 | Equipment | -3 | -2 | -1 | 0 | 1 | 2 | 3 | |
| 5 | Atmosphere | -3 | -2 | -1 | 0 | 1 | 2 | 3 | |

Any other valuable feedback: _____

Student's Signature: _____

| MARKS AWARDED | Attitude | Neatness | Correctness of results | Initiative | Originality | Conclusion | TOTAL |
|------------------|----------|----------|---------------------------|------------|-------------|------------|-------|
| TOTAL | 10 | 10 | 10 | 20 | 20 | 30 | 100 |
| EARNED | | | | | | | |

Lab Instructor's Comments: _____

Lab Instructor's Signature: _____