DIGITAL SIGNAL PROCESSING LAB

(EL-302)

LABORATORY MANUAL

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FIR Filter Design

(LAB # 13)

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Lab # 13: FIR Filter Design

Learning Objectives

a. To design FIR digital filter using different window functions

Equipment Required

1. Matlab

1. Introduction

The difference equation of FIR filter is

$$y(n) = b_0 x(n) + b_1 x(n-1) + \cdots + b_{M-1} x(n-M+1)$$
$$= \sum_{k=0}^{M-1} b_k x(n-k)$$

Advantages of FIR filter in design and implementation:

1) The phase response can be exactly linear.
2) They are relatively easy to design since there is no stability problem
3) They are efficient to implement.
4) The DFT can be used in their implementation.

Advantages of a linear-phase response are:

1) Design problem contains only real arithmetic and not complex arithmetic.
2) Linear-phase filters provide no delay distortion and only a 6x4 amount of delay
3) For the filter of length M (or order N=M - 1) the number of operations are of the order of M/2 as in the linear-phase filter implementation.

The only disadvantage in using FIR filter is that order of an FIR filter, in most cases, is considerably higher than the order of an equivalent IIR filter meeting the sane specifications, and FIR filter has thus higher computational complexity.

2. Properties of Linear-Phase FIR filters

The impulse response of filter h(n), $0 \le n \le M-1$ of length M. The system function of the filter is

$$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n} = z^{-(M-1)} \sum_{n=0}^{M-1} h(n) z^{M-1-n}$$

Which has M-1 poles at origin z=0 and M-1 zeros at anywhere in the z-plane. The frequency response of the filter is

$$H(e^{j\omega}) = \sum_{n=0}^{M-1} h(n) e^{-j\omega n}, \quad -\pi < \omega \le \pi$$

$$\angle H(e^{j\omega}) = -\alpha\omega, \quad -\pi < \omega \le \pi$$

The condition for linear-phase

$$h(n) = h(M-1-n), \quad 0 \le n \le (M-1) \text{ with } \alpha = \frac{M-1}{2}$$

Where α is a constant phase delay and h(n) is symmetric about α.

Another type,

$$\angle H(e^{j\omega}) = \beta - \alpha\omega$$

$$h(n) = -h(M-1-n), \quad 0 \le n \le (M-1);\ \alpha = \frac{M-1}{2},\ \beta = \pm\frac{\pi}{2}$$

3. Linear-phase FIR filter design using Windowing techniques

The basic idea behind the window design is to choose a proper ideal frequency-selective filter (which always has a non-causal, infinite-duration impulse response) and then truncate (or window) its impulse response to obtain a linear-phase and causal FIR filter.

Ideal frequency selective filter by $H_d(e^{jw})$ has unity magnitude gain and linear-phase characteristics over its pass band, and zero response over its stop band. An ideal LPF of bandwidth $\omega c < \pi$ is given by

$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\alpha\omega}, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

Where ωc is cut-off frequency, and α is called the sample delay. The impulse response of this filter is of infinite duration and is given by

$$h_d(n) = \mathcal{F}^{-1}\left[H_d(e^{j\omega})\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1 \cdot e^{-j\alpha\omega}e^{j\omega n}d\omega$$

$$= \frac{\sin\left[\omega_c(n-\alpha)\right]}{\pi(n-\alpha)}$$

$h_d(n)$ is symmetric with respect to α but not causal. For that truncation (windowing) operation is used. To obtain a causal and linear-phase FIR filter h(n) of length M, we must have

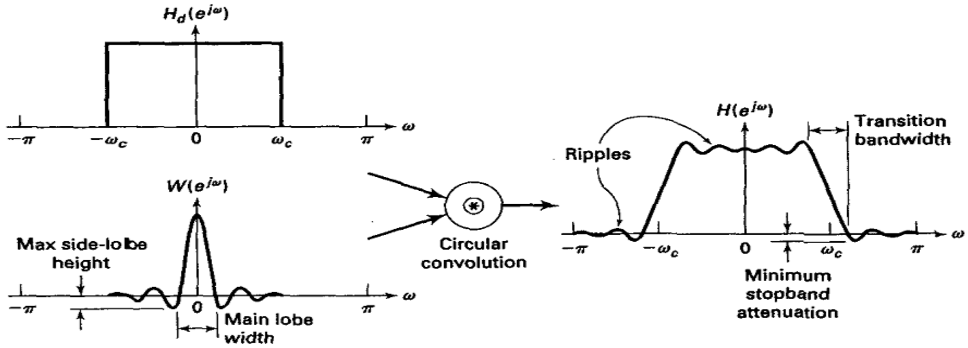
$$h(n) = \begin{cases} h_d(n), & 0 \le n \le M-1 \\ 0, & \text{elsewhere} \end{cases} \qquad \text{and} \qquad \alpha = \frac{M-1}{2}$$

$$h(n) = h_d(n)w(n)$$

In general where w(n) is called window function In the frequency domain the causal FIR filter response $H_d(e^{j\omega})$ of is given by the periodic convolution of $H(e^{j\omega})$ and the window response *W(e^jω)* , that is

$$H(e^{j\omega}) = H_d(e^{j\omega}) \circledast W(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} W\left(e^{j\lambda}\right) H_d\left(e^{j(\omega-\lambda)}\right) d\lambda$$

The pictorial representation of the windowing in frequency domain is shown below.



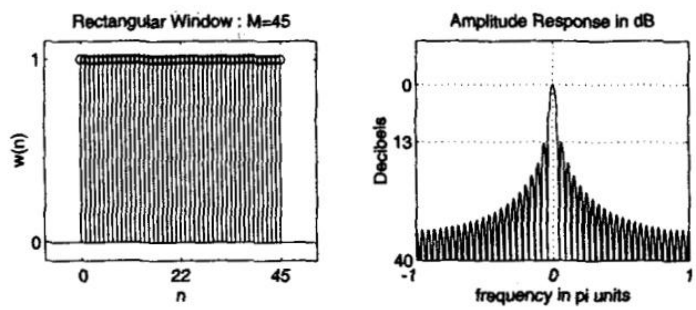
Observations for the above operation are:

1. Since the window w(n) has a finite length equal to M, its response has a peaky main lobe whose width is proportional to and has side lobes of smaller heights.
2. The periodic convolution produces a smeared version of the ideal response .
3. The main lobe produces a transition band in whose width is responsible for the transition width.. The wider the main lobe, the wider will be the transition width.
4. The side lobes produce ripples that have similar shapes in both the pass band and stop band.

FIR filter design through windows:

a) Rectangular window

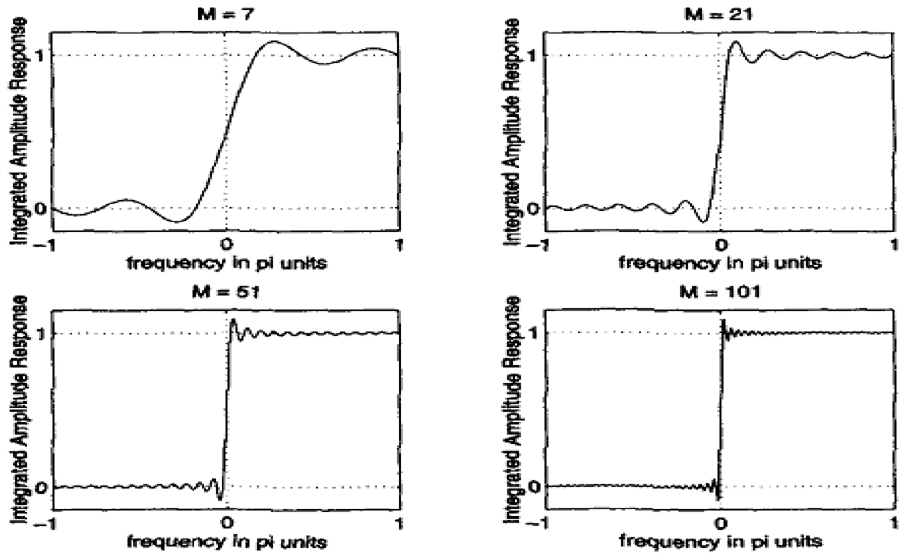
$$w(n) = \begin{cases} 1, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$



There are problems in rectangular windowing method:
i) Minimum stop band attenuation is 21dB
ii) Due to direct truncation, desired impulse response suffers from

Gibb's phenomenon. That is if we increase M, the width of each side lobe will decrease, but the area under each lobe will remain constant. Therefore the relative amplitudes of side lobes will remain constant, and the minimum stop band attenuation will remain at 21 dB.
This implies that all ripples will bunch up near the band edges.

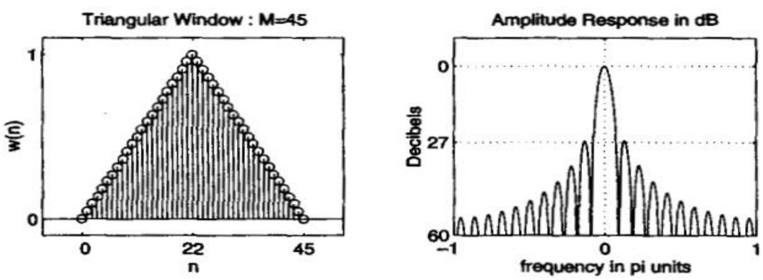
Gibb's phenomenon is shown below.



The other type's windows are used to reduce this problems.

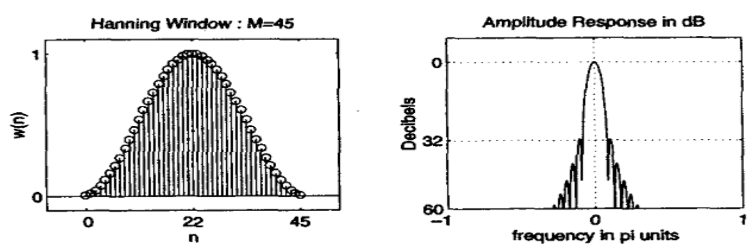
b) Bartlett (triangular) window

$$w(n) = \begin{cases} \dfrac{2n}{M-1}, & 0 \le n \le \dfrac{M-1}{2} \\ 2 - \dfrac{2n}{M-1}, & \dfrac{M-1}{2} \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$



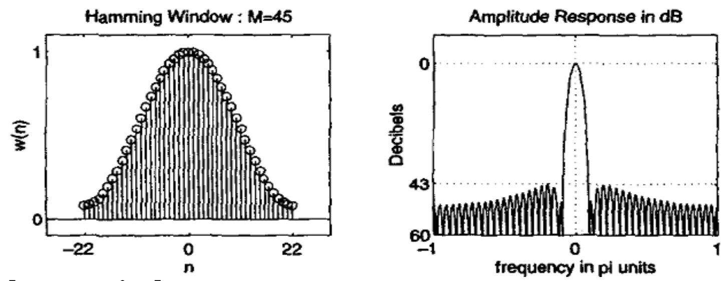
c) Hanning window

$$w(n) = \begin{cases} 0.5\left[1 - \cos\left(\frac{2\pi n}{M-1}\right)\right], & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$



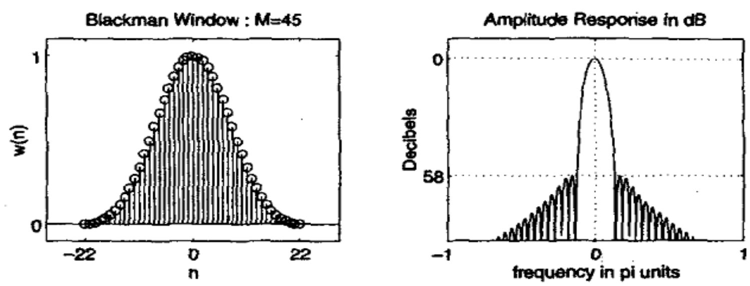
d) Hamming window

$$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right), & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$



e) Blackman window

$$w(n) = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{M-1}\right) + 0.08\cos\left(\frac{4\pi n}{M-1}\right), & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$



f) Kaiser window

$$w(n) = \frac{I_0\left[\beta\sqrt{1-\left(1-\frac{2n}{M-1}\right)^2}\right]}{I_0[\beta]}, \quad 0 \le n \le M-1$$

Where Io[.] is the modified zero order Bessel function and β is a parameter that depends on M and that can be chosen according to the required transition width and optimum stop band attenuation.

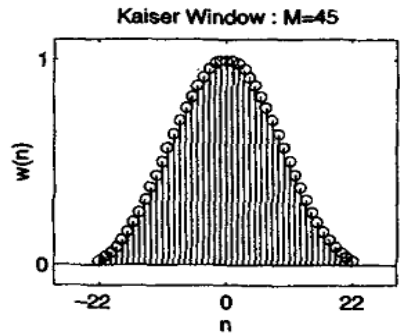
Design equations for Kaiser Window are:

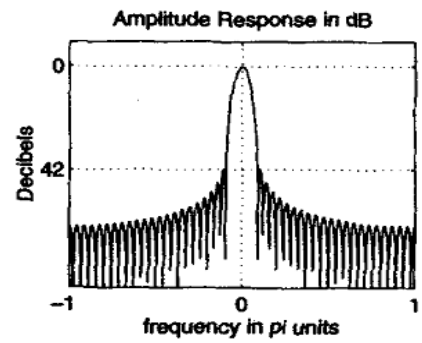
Given ω_p, ω_s, R_p, and A_s

$$\text{Norm. transition width} = \Delta f \triangleq \frac{\omega_s - \omega_p}{2\pi}$$

$$\text{Filter order } M \simeq \frac{A_s - 7.95}{14.36\Delta f} + 1$$

$$\text{Parameter } \beta = \begin{cases} 0.1102\,(A_s - 8.7), & A_s \ge 50 \\ 0.5842\,(A_s - 21)^{0.4} + 0.07886\,(A_s - 21), & 21 < A_s < 50 \\ 0 & \text{Otherwise} \end{cases}$$





MATLAB functions:

1) RECTWIN *Rectangular window.*
W = RECTWIN(N) *returns the N-point rectangular window.*
2) BARTLETT **Bartlett window.**
W = BARTLETT(N) *returns the N-point Bartlett window.*
3) HANN *Hann window.*
HANN(N) *returns the N-point symmetric Hann window in a column vector.*
4) HAMMING *Hamming window.*
HAMMING(N) *returns the N-point symmetric Hamming window in a column vector.*
5) BLACKMAN *Blackman window.*
BLACKMAN(N) *returns the N-point symmetric Blackman window in a column vector.*
6) KAISER *Kaiser window.*
W = KAISER(N) *returns an N-point Kaiser window in the column vector W.*
W = KAISER(N,BTA) *returns the BETA-valued N-point Kaiser window. If omitted, BTA is set to 0.500.*

4. Procedure

i. Steps to design FIR filter using windows:

1. Calculate order of the filter N using given specifications pass band ripple Rp in dB, stop band ripple As in dB, pass band edge frequency in Hz, stop band edge frequency in Hz and sampling rate in samples/sec.
Or
Specify the order, sampling rate and cut-off frequencies of the filter
2. Choose the window function and compute its coefficients
3. Compute the ideal filter coefficients according to the order of the filter
4. Compute FIR filter coefficients according to the obtained window function and ideal filter coefficients.
5. If the resulting filter has too wide or too narrow transition region, it is necessary to change the filter order by increasing or decreasing it according to needs.

ii. FIR filter using common windows

1. Start
2. Enter cut-off frequencies, sampling rate and order of the filter *(or pass bandedge frequency, stop band edge frequency, pass band ripple, stop band ripple and sampling rate. The compute cut-off frequency by $\omega c= (\omega p + \omega s)/2$, order of the filter by where c is the constant and value depends on the width of main lobe of windows)*

3. Compute the length of the filter, M
4. Find coefficients of window sequence w(n) by the functions *rectwin( ), bartlett( ), hann( ), hamming()* and *blackman( )*
5. Find desired filter coefficients h(n) by the function *fir1( )*
6. Find filter response by the function *freqz( )*
7. Plot the window, magnitude response and phase response
8. Stop

iii. MATLAB

```
%FIR filter using windows
clc;
clear all;
close all;
disp('FIR filter design using windows');
%Inputs
ftype=input('Enter filter type (LPF, HPF, BPF or BSF): ');
switch ftype
case 'LPF'
fc=input('Enter cut-off frequency fc in Hz ');
%Error message
if fc<=0
error('Cut-off frequency must be larger than zero');
end
case 'HPF'
fc=input('Enter cut-off frequency fc in Hz ');
%Error message
if fc<=0
error('Cut-off frequency must be larger than zero');
end
case 'BPF'
fc=input('Enter cut-off frequencies fc1 and fc2 (fc2 >fc1) in Hz ');
if (fc(1) || fc(2))<=0
error('Cut-off frequencies must be larger than 0');
end
if fc(1)<=fc(2)
error('Lower cut-off frequency must be smaller than upper cut-off frequency');
end
case 'BSF'
fc=input('Enter cut-off frequencies fc1 and fc2 (fc2 >fc1) in Hz ');
if (fc(1) || fc(2))<=0
error('Cut-off frequencies must be larger than 0');
end
if fc(1)<=fc(2)
error('Lower cut-off frequency must be smaller than upper cut-off frequency');
end
end
Fs=input('Enter sampling frequency F in samples/sec ');
N=input('Enter the order of the filter ');
%length of filter
M=N+1;
%conversion and normalization of frequencies
%pi radians/second

wc=2*fc/Fs;
```

```
%input to window type
wintype=input('Enter the window (Rectangular,Bartlett,Hann,Hamming,Blackman,
Kaiser): ');
switch wintype
case 'Rectangular'
%Rectangular window
w=rectwin(M);
case 'Bartlett'
%Bartlett window
w=bartlett(M);
case 'Hann'
%Hanning window
w=hann(M);
case 'Hamming'
%Hamming window
w=hamming(M);
case 'Blackman'
%Blackmann window
w=blackman(M);
case 'Kaiser'
%Kaiser window
beta=input('Enter shape parameter beta');
w=kaiser(M,beta);
end
%Digital filter response
switch ftype
case 'LPF'
bz=fir1(N,wc,w);
case 'HPF'
bz=fir1(N,wc,'high',w);
case 'BPF'
bz=fir1(N,wc,'bandpass',w);
case 'BSF'
bz=fir1(N,wc,'stop',w);
end
%Frequency response
[H,f]=freqz(bz,1,512,Fs);
%display H(z)
disp('Filter coefficients are:');
disp('b');
disp(bz);
%Plotting windows and responses
n=0:1:M-1;
subplot(3,1,1);

plot (n,w)

xlabel('n');
ylabel('w(n)');
title([wintype,' window for FIR ',ftype]);
subplot(3,1,2);
plot(f,20*log10(abs(H)));
xlabel('f in Hz');
ylabel('H(w)in dB');
title(['Magnitude response of FIR ',ftype,' filter with ',wintype,' window']);
grid on;
```

```
subplot(3,1,3);
plot(f,angle(H));
xlabel('f in Hz');
ylabel('Phase of H(w)');
title(['Phase response of FIR ',ftype,' filter with ',wintype,' window']);
grid on;
```

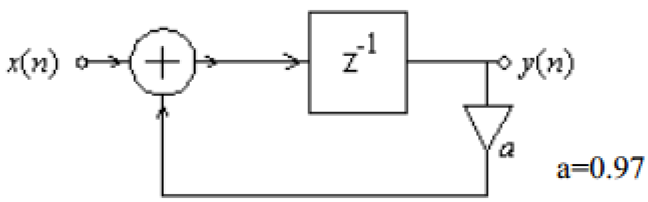
5) Exercise:

Task#1: Window-Based FIR filter design:
Consider the low-pass specification for the design of Kaiser window

$$1-\delta_s \le |H(e^{jw})| \le 1+\delta_s \quad , \quad |w| \le 0.3\pi,$$
$$|H(e^{jw})| \le \delta_s \quad , \quad 0.4\pi \le |w| \le \pi$$

Where δ_s =0.003162. Plot using MATLAB the gain response of kasier window.

Task#2: Generate the code for the given FIR filter, having the coefficient b0=1, b1= -1 and b2=1. Taking sinusoid as x(n) having 80 samples, frequency =1/8 and phase= pi/6.



$$y(n) = x(n\text{-}1) + ay(n\text{-}1)$$

Student's feedback: Purpose of feedback is to know the strengths and weaknesses of the system for future improvements. This feedback is for the 'current lab session'. Circle your choice:

[-3 = Extremely Poor, -2 = Very Poor, -1 = Poor, 0 = Average, 1 = Good, 2 = Very Good, 3 = Excellent]:

The following table should describe your experience with:

S#	Field	Rating	Describe your experience in words
1	Overall Session	-3 | -2 | -1 | 0 | 1 | 2 | 3	
2	Lab Instructor	-3 | -2 | -1 | 0 | 1 | 2 | 3	
3	Lab Staff	-3 | -2 | -1 | 0 | 1 | 2 | 3	
4	Equipment	-3 | -2 | -1 | 0 | 1 | 2 | 3	
5	Atmosphere	-3 | -2 | -1 | 0 | 1 | 2 | 3	

Any other valuable feedback:

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Student's Signature: ______________________________

MARKS AWARDED	Attitude	Neatness	Correctness of results	Initiative	Originality	Conclusion	TOTAL
TOTAL	**10**	**10**	**10**	**20**	**20**	**30**	**100**
EARNED							

Lab Instructor's Comments:______________________________________

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Lab Instructor's Signature: ______________________