AoI-based Scheduling of Correlated Sources for Timely Inference

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***Abstract*—We consider a setting where multiple correlated sources send real-time observations over a wireless communication channel to a receiver. The receiver uses the delivered observations to infer multiple time-varying targets. Due to limited communication resources, these observations may not always be fresh. To quantify data timeliness, we utilize the *Age of Information* (AoI) metric. Our goal is to minimize real-time inference error by developing signal-agnostic scheduling policies that leverage AoI without requiring knowledge of the actual target values or the specific source observations. For the two-source case, we obtain an optimal cyclic policy with low computational complexity. For more than two-sources, we establish an information-theoretic lower bound on inference error. Building upon this lower bound, we approximate the scheduling problem and propose an approximate Whittle index policy that is asymptotically optimal as the number of sources increases and the correlation among sources decreases. Our scheduling policies hold for arbitrary target and source processes and loss functions. Finally, we conduct simulations of a network of cameras with overlapping field of views tracking multiple mobile objects to demonstrate the effectiveness of our policies.**

I. Introduction

Next-generation communications (Next-G) (e.g., 6G) are expected to support many intelligent applications such as environmental forecasting, surveillance, networked control of robot or UAV swarms, communication between connected vehicles, and massive sensing via the Internet of Things (IoTs). These applications often require timely inference of dynamic targets (e.g., positions of moving objects, changes in environmental conditions).

In this paper, we investigate timely inference of multiple time-varying targets based on observations collected from remote sources (e.g., sensors, cameras, IoT devices, UAVs). These observations are transmitted to a receiver over a capacity-limited communication channel. Furthermore, the source observations can exhibit some sort of correlation in their dynamics. For example, in environmental monitoring, temperature readings from geographically close sensors can be correlated. Similarly, in target tracking with UAV-mounted cameras, observations from cameras with overlapping fields of view are correlated. An important, yet difficult, task in wireless communication networks is to design an effective and low-complexity scheduling policy to monitor these correlated sources that minimizes inference errors, leading to improved real-time performance.

Due to limited communication resources, observations delivered from remote sources may not be fresh. *Age of Information* (AoI), introduced in [1], [2], provides a convenient measure of information freshness of the receiver. Specifically, consider packets sent from a source to a receiver: if $U(t)$ is the generation time of the most recently received packet by time t, then the AoI at time t is the difference between t and $U(t)$. Recent works on remote inference [3]–[6] have shown that the inference errors for different tasks can be expressed as functions of AoI. Additionally, AoI can be readily tracked, making it a promising metric for determining how to prioritize resource allocation. Recent works [7], [8] have also shown that AoI-based scheduling is sufficient to obtain near-optimal scheduling of correlated Gauss–Markov sources with linear time-invariant (LTI) system models. While this is promising, many real-world systems exhibit non-linear dynamics and complex correlation structures. Motivated by this, in this work, we pose the following research question:

> ***How can we develop AoI-based scheduling of correlated sources to minimize inference errors for arbitrary target processes?***

Summary of contributions. In answering the above question, we make the following contributions:

- To minimize time-averaged sum of inference errors for multiple targets, we formulate the problem of scheduling correlated sources over a capacity-limited channel. For the set of all causal and signal-agnostic scheduling policies, we show that the problem can be expressed as minimizing time-averaged sum of penalty functions of the AoI values (see Lemma 1 and (11)-(12)).
- For the two-source case, we obtain an optimal cyclic policy with low computational complexity (see Theorem 1). Prior work [9] showed the existence of an optimal cyclic policy but required solving a computationally difficult minimum average cost cycle problem over a large graph. Our result provides a significantly more efficient solution.
- For more than two sources, we establish an information-theoretic lower bound (see Lemma 2 and Lemma 3) on inference error and use it to approximate the problem as a restless multi-armed bandit problem. We then develop an approximated Whittle index-based policy (see Definition 2), with the approximation error vanishing as source correlation decreases. We show that our approximated Whittle index policy is asymptotically optimal for low correlation among sources (see Theorem 3). Our scheduling policies hold for arbitrary target and source processes and loss functions.

- Finally, we conduct simulations of a network of cameras with overlapping field of views tracking multiple mobile objects to demonstrate the effectiveness of our policies.

Related Works. Researchers have explored AoI-based scheduling policies to improve the performance of communication networks [10]–[12], control systems [13], [14], remote estimation [15]–[17], and remote inference [3], [4], [18]. The paper [3] is the first to analyze how AoI values of correlated source observations affect remote inference. Our work is mostly related to AoI-based scheduling of correlation sources [7], [8]. In particular, a simple probabilistic correlation model is considered in [7]. An LTI system model with Gauss-Markov source processes is considered in [8]. Moreover, the analysis in [8] is limited to quadratic loss function. Our work generalizes the AoI-based scheduling of correlated sources to arbitrary loss functions and target/source processes.

II. System Model

Consider M sources communicating over a wireless channel to a receiver (Fig. 1). At every time slot t, each source m observes a time-varying signal $X_{m,t} \in \mathcal{X}_m$, where $\mathcal{X}_m$ represents the set of possible observations from source m. A scheduler progressively schedules updates from different source to be sent to the receiver. At each time slot t, the receiver uses the received observations to infer M targets $(Y_{1,t}, Y_{2,t}, \ldots, Y_{m,t})$ of interest. Each target $Y_{m,t} \in \mathcal{Y}_m$ is time-varying and can be inferred from the corresponding source $X_{m,t}$. We consider that observations $(X_{1,t}, X_{2,t}, \ldots, X_{M,t})$ of all sources are correlated to each other.

A. Communication Model

Due to interference and bandwidth limitations, we assume that at most N out of M sources can be scheduled to transmit their observations at any time slot t. If source m is scheduled at time t, it transmits the current observation $X_{m,t}$ to the receiver. For simplicity, we assume reliable channels, i.e., observations sent at time slot t are delivered error-free at time slot $t+1$. Because of communication constraints, the receiver may not have fresh observations from all sources. Let $X_{m,t-\Delta_m(t)}$ be the most recently delivered signal observation from source m which was generated $\Delta_m(t)$ time-slots ago. We call $\Delta_m(t)$ the age of information (AoI) [2], [4] of source m. Let $U_m(t)$ be the generation time of the most recent delivered observation from source m. Then, the AoI can be formally defined as:

$$\Delta_m(t) = t - U_m(t). \tag{1}$$

Let $\pi_m(t) \in \{0, 1\}$ be the scheduling decision of source m. At time slot t, if $\pi_m(t) = 1$, the m-th source is scheduled to transmit its observation to the receiver; otherwise, if $\pi_m(t) = 0$, this transmission does not occur. If $\pi_m(t-1) = 0$, AoI $\Delta_m(t) = \Delta_m(t-1)+1$ grows by 1; otherwise, if $\pi_m(t-1) = 1$, AoI drops to $\Delta_m(t) = 1$.

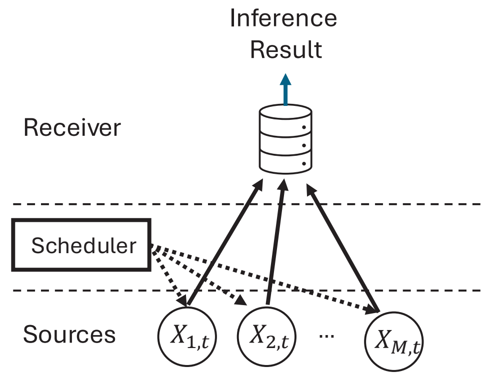


Fig. 1: A remote inference system with M correlated sources.

B. Inference Model

The receiver employs M predictors. The m-th predictor ϕ_m uses the most recent observations from all sources, $X_{n,t-\Delta_n(t)}$ for $n = 1, 2, \ldots, M$, along with their corresponding AoI values $\Delta_n(t)$, to produce an inference result $a_{m,t} \in \mathcal{A}_m$ for target $Y_{m,t}$. Specifically, given the most recently delivered source observations and their AoI values

$$(X_{n,t-\Delta_n(t)}, \Delta_n(t))_{n=1}^M = (x_n, \delta_n)_{n=1}^M,$$

the inference result

$$a_{m,t} = \phi_m((x_m, \delta_m)_{m=1}^M) \in \mathcal{A}_m$$

minimizes the expected loss function $\mathbb{E}[L(Y_{m,t}, a_m)]$ over all possible inference results $a_m \in \mathcal{A}_m$. $L(y_m, a_m)$ is the loss incurred when the actual target is $Y_{m,t} = y_m$ and the predicted output is $a_{m,t} = a_m$. The loss function $L(\cdot, \cdot)$ and the output space $\mathcal{A}_m$ can be designed according to the goal of the system. For example, $\mathcal{A}_m = \mathcal{Y}_m$ and quadratic loss $\|y - \hat{y}\|^2$ can be used for a regression task. In maximum likelihood estimation, we can use logarithmic loss function and the output space $\mathcal{A}_m$ can be $\mathcal{P}_{\mathcal{Y}_m}$, which is the set of all probability distributions on $\mathcal{Y}_m$. At time t, the expected inference error for target $Y_{m,t}$ is given by $\mathbb{E}\left[L(Y_{m,t}, a_{m,t})\right]$.

III. Problem Formulation

We focus on a class of *signal-agnostic* scheduling policies, where scheduling decisions are made without knowledge of the observed process's signal values, i.e., the centralized scheduler does not have access to $\{(Y_{m,t}, X_{m,t}), m = 1, \ldots, M, t = 0, 1, \ldots\}$. Moreover, we make the following assumption on the observed and the target processes:

Assumption 1. *The process*

$$\{(Y_{m,t}, X_{m,t}), m = 1, 2, \ldots, M, t = 0, 1, \ldots\}$$

is stationary, i.e., the joint distribution of

$$(Y_{m,t}, X_{m,t-k_m}, m = 1, 2, \ldots, M)$$

does not change over time t for all $k_1, k_2, \ldots, k_M \geq 0$.

Now, we are ready to formulate our problem. We denote the scheduling policy as

$$\pi = (\pi_m(0), \pi_m(1), \dots)_{m=1,2,\dots,M}.$$

We let Π denote the set of all signal-agnostic and causal scheduling policies π. Our goal is to find a policy $\pi \in \Pi$ that minimizes the time-averaged sum of inference errors:

$$\mathrm{L}_{\mathrm{opt}} = \inf_{\pi\in\Pi} \lim_{T\to\infty} \mathbb{E}_\pi \left[\frac{1}{T}\sum_{t=0}^{T-1}\sum_{m=1}^{M} L(Y_{m,t}, a_{m,t})\right], \quad (2)$$

$$\text{s.t.} \sum_{m=1}^{M} \pi_m(t) = N, t = 0, 1, \dots, \quad (3)$$

where $L(Y_{m,t}, a_{m,t})$ is the inference error for the m-th target at time t and $\mathrm{L}_{\mathrm{opt}}$ is the minimum average inference error.

A. AoI-Based Problem Formulation

Having formulated the multi-source scheduling problem in (2)-(3), we now demonstrate that it can be equivalently expressed as the minimization of a penalty function of the AoI values $(\Delta_1(t), \Delta_2(t), \dots, \Delta_M(t))$. To show that, we use the concept of generalized conditional entropy [19], [20] or specifically, the L-conditional entropy [4].

1) L-information Theoretic Metrics: For a random variable Y, the L-entropy is given by

$$H_L(Y) = \min_{a\in\mathcal{A}} \mathbb{E}_{Y\sim P_Y}[L(Y, a)]. \quad (4)$$

The L-conditional entropy of Y given $X = x$ is [4], [19], [20]

$$H_L(Y|X = x) = \min_{a\in\mathcal{A}} \mathbb{E}_{Y\sim P_{Y|X=x}}[L(Y, a)] \quad (5)$$

and the L-conditional entropy of Y given X is

$$H_L(Y|X) = \mathbb{E}[H_L(Y|X = x)]. \quad (6)$$

Moreover, an L-mutual information among two random variables Y and X is defined as [4], [19], [20]

$$I_L(Y; X) = H_L(Y) - H_L(Y|X). \quad (7)$$

The L-mutual information $I_L(Y; X)$ quantifies the reduction of expected loss in predicting Y by observing X. The L-conditional mutual information among two random variables Y and X given Z is defined as [4], [19], [20]

$$I_L(Y; X|Z) = H_L(Y|Z) - H_L(Y|X, Z). \quad (8)$$

Lemma 1 is first proved in [3]. We restate the result for the completeness of the paper.

Lemma 1. *The inference error for $Y_{m,t}$ can be expressed as*

$$\mathbb{E}_\pi [L(Y_{m,t}, a_{m,t})] = H_L\left(Y_{m,t}|(X_{n,t-\Delta_n(t)}, \Delta_n(t))_{n=1}^M\right), \quad (9)$$

where $\Delta_n(t)$ is the AoI of source n under the scheduling policy π. Moreover, if Assumption 1 holds, then the L-conditional entropy

$$H_L\left(Y_{m,t}|(X_{n,t-\Delta_n(t)}, \Delta_n(t))_{n=1}^M\right)$$

is a function of AoI values $(\Delta_1(t), \Delta_2(t), \dots, \Delta_M(t))$.

Lemma 1 implies that the inference error $\mathbb{E}_\pi [L(Y_{m,t}, a_{m,t})]$ can be represented as an L-conditional entropy of $Y_{m,t}$ given most recently delivered observations from all sources and their AoI values. Under Assumption 1, the L-conditional entropy is a function of AoI values $(\Delta_1(t), \Delta_2(t), \dots, \Delta_M(t))$. For the simplicity of presentation, we represent the function as

$$\begin{aligned} &g_m(\Delta_1(t), \Delta_2(t), \dots, \Delta_M(t)) \\ &= H_L\left(Y_{m,t}|(X_{n,t-\Delta_n(t)}, \Delta_n(t))_{n=1}^M\right). \end{aligned} \quad (10)$$

By using Lemma 1 and (10), we can express the problem (2)-(3) as a minimization time-averaged sum of penalty functions of the AoIs.

$$\mathrm{L}_{\mathrm{opt}} = \inf_{\pi\in\Pi} \lim_{T\to\infty} \mathbb{E}_\pi \left[\frac{1}{T}\sum_{t=0}^{T-1}\sum_{m=1}^{M} g_m\Big(\Delta_1(t), \Delta_2(t), \dots, \Delta_M(t)\Big)\right], \quad (11)$$

$$\text{s.t.} \sum_{m=1}^{M} \pi_m(t) = N, t = 0, 1, \dots. \quad (12)$$

Solving the problem (11)-(12) poses significant challenges because scheduling decisions are coupled due to constraint (12) and the penalty function $g_m(\Delta_1(t), \Delta_2(t), \dots, \Delta_M(t))$ depends on AoI of signal observations from all the sources.

IV. Design of Scheduling Policies

In this section, we introduce two policies. One policy is a stationary cyclic policy and show that it optimizes the scheduling problem (11)-(12) for $M = 2$ sources. Another policy is designed for more than two sources by approximating the main problem. A stationary cyclic policy is defined next.

Definition 1 (Stationary Cyclic Policy). *A stationary cyclic policy is a stationary policy that cycles through a finite subset of points in the state space, repeating a fixed sequence of actions in a particular order.*

Now, we provide an optimal stationary cyclic policy for $M = 2$ and $N = 1$ in Theorem 1.

Theorem 1. *For $M = 2$ source case and $N = 1$, there exists a stationary cyclic policy that is optimal for* (11)-(12)*, where the policy consists of a period $\tau_1^* + \tau_2^*$ and in each period, source 1 is scheduled for τ_1^* consecutive time slots, immediately followed by source 2 being scheduled for τ_2^* consecutive time slots. The period τ_1^* and τ_2^* minimizes*

$$\begin{aligned} \mathrm{L}_{\mathrm{opt}} = \min_{\substack{\tau_1=0,1,\dots\\ \tau_2=0,1,\dots}} \frac{1}{\tau_1+\tau_2}\Bigg(&\sum_{k=0}^{\tau_1-1}\sum_{m=1}^{2}\big(g_m(1, 2+k)\big) \\ &+ \sum_{j=0}^{\tau_2-1}\sum_{m=1}^{2}\big(g_m(2+j, 1)\big)\Bigg), \end{aligned} \quad (13)$$

where $\mathrm{L}_{\mathrm{opt}}$ is the optimal objective value of (11)-(12)*.*

Proof. See Appendix B. □

Remark 1. *Theorem 1 presents a low-complexity optimal cyclic scheduling policy for two correlated sources. To our knowledge, this is the first such result. Prior work [9] showed the existence of an optimal cyclic policy but required solving a computationally difficult minimum average cost cycle problem over a large graph. Our result provides a significantly more efficient solution for $M = 2$ sources.*

For $M > 2$, a low complexity optimal stationary policy to (11)-(12) is difficult to obtain. In this sequence, we approximate the penalty function $g_m(\delta_1, \delta_2, \ldots, \delta_M)$ by using

$$f_m(\delta_m) = H_L(Y_{m,t}|X_{m,t-\delta_m}, \mathbf{X}_{-m,t-1}), \tag{14}$$

where

$$\mathbf{X}_{-m,t-1} = [X_{1,t-1}, \ldots, X_{m-1,t-1}, X_{m+1,t-1}, \ldots, X_{M,t-1}]$$

includes signal observations generated one time slot ago from all sources excluding source m.

Lemma 2. *If the following holds for any $m \neq n$, δ_m, μ_n:*

$$I_L(Y_{m,t}; X_{n,t-\mu_n}|X_{1,t-\delta_1}, \ldots, X_{M,t-\delta_M}) \leq \epsilon^2_{m,n}, \tag{15}$$

then we have

$$g_m(\delta_1, \delta_2, \ldots, \delta_M) = f_m(\delta_m) + O(\max_{n\neq m} \epsilon^2_{m,n}). \tag{16}$$

where L-conditional mutual information $I_L()$ is defined in (8).

Proof. See Appendix C. □

The condition (15) tells that given the current observations, the reduction of inference error for target m by adding a new observation from source $n \neq m$, probably the freshest observation, is at most $\epsilon^2_{m,n}$. Lemma (2) implies that if $\epsilon_{m,n}$ tends to zero for all $n \neq m$, then the approximation error goes to zero, i.e., the function $f(\delta_m)$ is a good approximation of inference error function $g(\delta_1, \delta_2, \ldots, \delta_M)$ under low correlation among sources. Furthermore, the following lemma shows that the function $f(\delta_m)$ is a lower bound.

Lemma 3. *If the following Markov chain holds: $Y_{m,t} \leftrightarrow X_{n,t-1} \leftrightarrow X_{n,t-\delta_n}$ for all $k_n \geq 1$ and $n \neq m$, we have*

$$g_m(\delta_1, \delta_2, \ldots, \delta_M) \geq f_m(\delta_m). \tag{17}$$

Proof. See Appendix D. □

Our information-theoretic lower bound, $f_m(\delta_m)$, extends the bound in [8] to a significantly broader class of systems, including non-linear and non-Gaussian dynamics. As shown in Appendix A, this bound coincides with the bound in [8, Theorem 2] when specialized to LTI systems with Gaussian processes.

Now, we approximate the problem (11)-(12):

$$\mathrm{f}_{\mathrm{opt}} = \inf_{\pi\in\Pi} \lim_{T\to\infty} \mathbb{E}_\pi \left[\frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^{M} f_m(\Delta_m(t))\right], \tag{18}$$

$$\text{s.t.} \sum_{m=1}^{M} \pi_m(t) = 1, t = 0, 1, \ldots, \tag{19}$$

where $\mathrm{f}_{\mathrm{opt}}$ is the optimal objective value of (18)-(19).

The approximated problem (18)-(19) is a restless multi-armed bandit (RMAB) problem, in which $\Delta_m(t)$ is the state of the m-th bandit. The bandits are "restless" because the state $\Delta_m(t)$ undergoes changes even when the m-th bandit is passive [12], [21]. Whittle index policy [4], [12], [21] is known to be an efficient policy for RMAB. In [12] and [4], Whittle index policy has been developed for RMAB with penalty functions of AoI. In Theorem 2, we use [4, Theorem 10] to obtain Whittle index for the RMAB (18)-(19).

Theorem 2. *For problem* (18)-(19)*, the Whittle index $W_m(\delta)$ of source m is given by*

$$W_m(\delta) = \mu_m(\delta)\gamma_m(\delta) - \sum_{k=1}^{\mu_m(\delta)} f_m(k), \tag{20}$$

where $\gamma_m(\delta)$ and $\mu_m(\delta)$ are given by

$$\gamma_m(\delta) = \inf_{\tau=1,2,\ldots} \frac{1}{\tau} \sum_{k=1}^{\tau} f_m(\delta + k), \tag{21}$$

$$\mu_m(\delta) = \inf_{\tau=1,2,\ldots} \Big\{\tau : \gamma_m(\tau) \geq \gamma_m(\delta)\Big\}. \tag{22}$$

Now, we are ready to provide the scheduling policy for the main problem (11)-(12). Whittle index cannot be obtained for the main problem because (11)-(12) is not RMAB. Hence, we provide our "Approximated Whittle Index Policy".

Definition 2 (**Approximated Whittle Index Policy**)**.** *At every time slot t, N sources with maximum indices $W_m(\Delta_m(t))$ are scheduled. Let π_{AWI} denote our Approximated Whittle Index Policy.*

Definition 3 (**Asymptotically optimal**)**.** *Initially, we have N channels and M bandits. Let $\bar{p}^r_\pi$ represent the expected long-term average inference error under policy π, where both the number of channels rN and the number of bandits rM are scaled by r. The policy π_{AWI} will be asymptotically optimal if $\bar{p}^r_{\pi_{\mathrm{AWI}}} \leq \bar{p}^r_\pi$ for all $\pi \in \Pi$ as r approaches ∞, while maintaining a constant ratio $\alpha = \frac{rM}{rN}$.*

We denote by $\mathbf{V}^m_t(\delta)$ the fraction of class m bandits (bandits that share same penalty function f_m) with AoI $\Delta_m(t) = \delta$. We further define the vectors $\mathbf{V}^m_t$ to contain $V^m_t(\delta)$ for all $\delta = 1, 2, \ldots, \delta_{\mathrm{bound}}$. Here, we consider a truncated AoI space as a larger value of AoI is rarely visited. For a policy π, we can have a mapping Ψ_π of the state transitions, given by

$$\Psi_\pi((\mathbf{v}^m)^M_{m=1}) = \mathbb{E}_\pi[(\mathbf{V}^m_{t+1})^M_{m=1}|(\mathbf{V}^m_t)^M_{m=1} = (\mathbf{v}^m)^M_{m=1}]. \tag{23}$$

We define the t-th iteration of the maps $\Psi_{\pi,t\geq 0}(\cdot)$ as $\Psi_{\pi,0}(x) = x$ and $\Psi_{\pi,t+1}(x) = \Psi_\pi(\Psi_{\pi,t}(x))$.

Definition 4. *[22]* ***Uniform Global attractor.*** *Given an equilibrium point $(\mathbf{v}^{m*})^M_{m=1}$, if for all $\epsilon > 0$, there exists $T(\epsilon) > 0$*

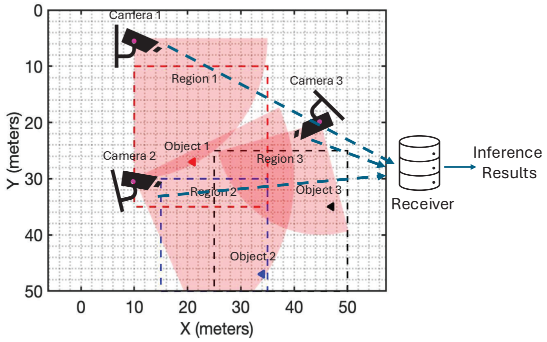


Fig. 2: Multiple mobile objects tracking system.

such that for all $t \geq T(\epsilon)$ and for all $(\mathbf{v}^m)_{m=1}^M$, one has $\|\Psi_{\pi,t}((\mathbf{v}^m)_{m=1}^M) - (\mathbf{v}^{m*})_{m=1}^M)\|_1 \leq \epsilon$.

Theorem 3. *If $\epsilon_{m,n}$ in (15) for all $m \neq n$, $m, n = 1, 2, \ldots, M$ decreases to zero and the global attractor condition [4] holds, then our Approximated Whittle Index Policy π_{AWI} is asymptotically optimal.*

Proof. See Appendix E. □

Remark 2. *While we leverage the Whittle index $W_m(\delta)$ derived in [4] for the approximated function $f_m(\delta_m)$, our work differs significantly. The prior work [4] does not consider correlation between sources, and their derived index is not applicable to our main penalty function $g_m(\delta_1, \delta_2, \ldots, \delta_M)$. If correlation among sources is low, i.e., $\epsilon_{m,n}$ is close to zero for all $m \neq n$, our policy is asymptotically optimal as M and N increases to ∞ while keeping their ratio fixed.*

V. Simulation Results

In this section, we evaluate the performance of our proposed scheduling policies. We consider the following four policies:

1. *Proposed Policy:* For $M = 2$ sources, the proposed policy is provided in Theorem 1. For $M > 2$ sources, proposed policy is in Definition 2.
2. *Maximum Age First (MAF) Policy:* At each time slot t, the MAF policy selects source m with the highest AoI.
3. *Random Policy:* At each time slot t, the random policy selects one source m from all sources following a uniform distribution.
4. *Naive Cyclic Policy:* The period of the naive cyclic policy is $\sum_{m=1}^M \tau$, where in each period, each source m is scheduled for τ consecutive time slots.

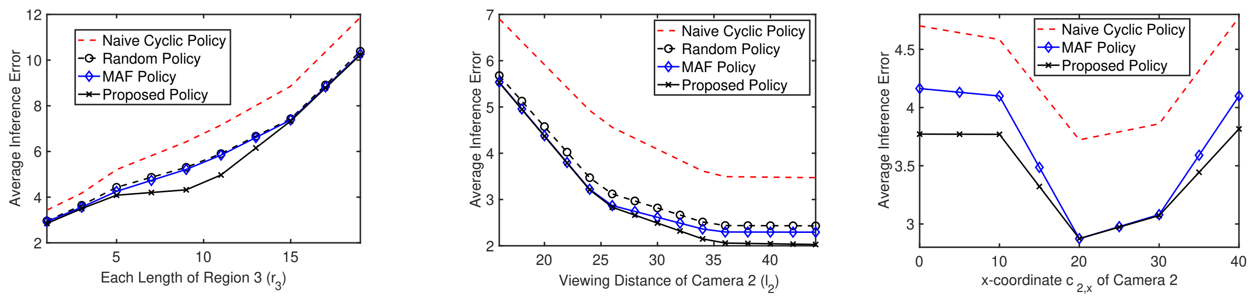
A. M Mobile Objects Tracking with M cameras and $N = 1$:

Setup: We simulated a multi-camera mobile object tracking scenario in MATLAB as illustrated in Fig. 2, where M cameras track M mobile objects within M overlapping square regions (r_m^2 area with top-left corner co-ordinate is denoted by z_m). Each camera monitors a specific region with defined viewing range (angular range $[-\pi/2, -\pi/2]$, angular direction θ_m, and viewing distance l_m). The objects move randomly within their respective regions. At each time slot, only one camera can transmit its observation to a central receiver due to a capacity-limited channel. The receiver then predicts the location of each object using a simple algorithm that prioritizes camera observations based on their AoI. The predictor starts with the camera observation with smallest AoI and checks if the target object is visible. If so, it extracts the object's location from that image. Otherwise, it discards the observation and repeats the process with the next freshest one. If no observations contain the object, its initial position is used as the prediction. We denote the image from camera m at time t as $X_{m,t}$, the true object location as $Y_{m,t}$, the predicted location as $\hat{Y}_{m,t}$. The inference error at time t is calculated by $\sum_{m=1}^M |Y_{m,t} - \hat{Y}_{m,t}|$. In this MATLAB simulation, we collected a dataset that includes $30,000$ realizations of cameras observations and objects locations. Out of them, first $10,000$ is used for getting the proposed policy. Then, we use rest of the dataset for performance evaluations.

Results: We used $M = 3$. Figure 3(b) shows the average inference error as a function of the side length (r_3) of the third square region, while keeping the side lengths of the first and second regions fixed at $r_1 = r_2 = 20$. The other parameters are $l_1 = 25, l_2 = 30, l_3 = 20$, $\theta_1 = 45, \theta_2 = 20, \theta_3 = 120$, $z_1 = (10, 10), z_2 = (15, 30)$, and $z_3 = (25, 25)$. As r_3 increases, the average inference error increases for all policies, reflecting the increased uncertainty in predicting the location of object 3 within a larger area. Our proposed policy consistently outperforms the other policies, particularly for r_3 ranging from 5 to 15. The naive cyclic policy, with a cycle length of 10, performs the worst, highlighting the need for careful analysis in designing scheduling policies for practical implementation. The MAF and random policies exhibit similar performance. This also shows that we need to carefully design AoI-based policy. Our proposed policy achieves significant gains when r_3 is between 5 and 15 because, in this range, region 3 is almost equally covered by cameras 2 and 3. This overlap allows our policy to strategically prioritize camera 2, leading to improved performance compared to the other policies. When r_3 is close to 1, the uncertainty in predicting object 3's location is minimal, resulting in similar performance across all policies. Fig. 3(c) plots the average inference error vs. viewing distance of 2nd camera l_2. The other parameters are $l_1 = 25, l_2 = 45, l_3 = 20$, $\theta_1 = 45, \theta_2 = -50, \theta_3 = 120$, $z_1 = (10, 10), z_2 = (15, 30)$, and $z_3 = (25, 25)$. As the viewing distance l_2 of camera 2, increases, camera 2 covers most of the part of other regions: region 1 and region 3. Our proposed policy prioritized the camera 2 in selection. This yields performance improvement compared to other policies, when l_2 increases. The simulation results Fig. 3(b)-(c) shows that our proposed policy utilized inherent correlation among the observations and targets to achieve better performance compared to MAF, random, and Naive cyclic policy, which do not utilize correlation.

B. K Mobile Objects Tracking with 2 cameras and $N = 1$:

Our proposed policy (Theorem 1) can be generalized to any number of targets K by simply by replacing the sum of inference error $\sum_{m=1}^2 g_m(\delta_1, \delta_2)$ in (13) with $\sum_{m=1}^K g_m(\delta_1, \delta_2)$.



(a) Average Inference Error vs. r_3 (b) Average Inference Error vs. l_2 (c) Average Inference Error vs. $c_{3,x}$

Fig. 3: Mobile objects tracking results: (a) Average inference error vs r_3 length of Squared Region 1 and Region 3, (b) Average inference error vs viewing distance l_2 of Camera 2, and (c) Average inference error vs x-coordinate $c_{2,x}$ of Camera 2.

To illustrate this, we consider a scenario with a single target $K = 1$ and two cameras $M = 2$. We modify the environment in Figure 2 by removing object 2, object 3, camera 3, region 2, and region 3, while retaining the remaining parameters from Figure 3(a) with l_2 set to 20. We then analyze the average inference error as a function of the x-coordinate ($c_{2,x}$) of camera 2. The performance gap between our proposed policy and the other policies increases when $c_{2,x}$ is 0, 10, or 40. This is because the field of view of camera 2 on region 1 changes as camera 2 moves along the x-axis. As $c_{2,x}$ increases from 0 to 20, camera 2's view of region 1 expands, and then it shrinks as $c_{2,x}$ increases further from 20 to 40. When camera 2's view of region 1 is limited ($c_{2,x} = 0, 10,$ or 40), the contribution of its observations to object tracking is reduced. Consequently, our proposed optimal policy strategically selects camera 2 less frequently ($\tau_2^* << \tau_1^*$). In contrast, the MAF and naive cyclic policies do not adapt to this varying visibility and thus exhibit inferior performance. This highlights the advantage of our policy in dynamically adjusting camera selection based on inherent correlation.

VI. Conclusion

In this paper, we studied AoI-based scheduling policies of correlated sources to minimize inference errors for multiple time-varying targets. An optimal policy was found for the two-source case. For more than two sources, we provided an approximate Whittle index policy by utilizing an information-theoretic lower bound. Our policies hold for general loss functions and general target and source processes. Simulation results for multiple objects tracking system validated the advantage of our policies.

Appendix A
Lower bound $f_m(\delta)$ for LTI and Gaussian Process

Let observation of m-th source $X_{m,t}$ and the target $Y_{m,t} = X_{m,t}$ evolves as follows:

$$X_{m,t} = a_m X_{m,t-1} + W_{m,t}, \tag{24}$$

where the noise vector $\mathbf{W}_t = [W_{1,t}, W_{2,t}, \ldots, W_{M,t}]$ is an i.i.d. multi-variate normal random variable across time, i.e., $\mathbf{W}_t \sim \mathcal{N}(0, Q)$ and Q is the covariance matrix. Let $q_{i,j}$ denote the (i,j)-th element of the covariance matrix Q. For this example, if $L(y, \hat{y}) = (y - \hat{y})^2$ is a quadratic loss, then we have

$$f_m(\delta) = \begin{cases} \bar{q}_{m,m}\delta, & \text{if } a_m = 1, \\ \bar{q}_{m,m}\frac{a_m^{2\delta}-1}{a_m^2-1}, & \text{otherwise,} \end{cases} \tag{25}$$

where

$$\bar{q}_{m,m} = q_{m,m} - \mathbf{q}_m Q_{-1}^{-1}\mathbf{q}_m^T$$

,

$$\mathbf{q}_m = [q_{m,1}, \ldots, q_{m,m-1}, q_{m,m+1}, \ldots, q_{m,M}]$$

is the covariance of the m-th noise process with other process, and Q_{-1}^{-1} is the noise covariance sub-matrix of the other processes excluding source m.

Appendix B
Proof of Theorem 1

Given $(\Delta_1(t), \Delta_2(t)) = (\delta_1, \delta_2)$, we can show that there exists a stationary deterministic policy that satisfies the following Bellman optimality equation [23]:

$$\begin{aligned} h(\delta_1, \delta_2) = & \min_{(\pi_1(t), \pi_2(t)) \in \mathcal{A}'} g(\delta_1, \delta_2) - \mathrm{L_{opt}} \\ & + \pi_1(t) h(1, \delta_2 + 1) + \pi_2(t) h(\delta_1 + 1, 1), \end{aligned} \tag{26}$$

where $g(\delta_1, \delta_2) = \sum_{m=1}^{2} g_m(\delta_1, \delta_2)$, $\mathcal{A}' = \{(0,1),(1,0)\}$, $h(\delta_1, \delta_2)$ is the relative value function for the state (δ_1, δ_2), $\mathrm{L_{opt}}$ is the average inference error under an optimal policy, $(1, \delta_2 + 1)$ is the next state if source 1 is scheduled, $(\delta_1 + 1, 1)$ is the next state if source 2 is scheduled.

We can further express the Bellman equation (26) for state $(\delta, 1)$ at time t as follows:

$$h(\delta, 1) = \min_{\tau_2 \in \{0,1,\ldots\}} \sum_{k=0}^{\tau_2} \Big(g(\delta + k, 1) - \mathrm{L_{opt}} \Big) + h(1, 2), \tag{27}$$

where $\tau_2 = 0, 1, \ldots$ is the time to keep scheduling source 2 after time t. By solving (27), we get that the optimal $\tau_2(\delta)$ satisfies

$$\tau_2(\delta) = \inf \Big\{ \tau \in \mathbb{Z}^+ : \gamma_1(\delta + \tau) \geq \mathrm{L_{opt}} \Big\}, \tag{28}$$

where $\gamma_1(\delta+\tau)$ is defined as

$$\gamma_1(\delta) = \inf_{k=1,2,\ldots} \frac{1}{k}\sum_{j=0}^{k-1} g(\delta+1+j,1). \quad (29)$$

Similarly, we can express the Bellman equation (26) for state $(1,\delta)$ as follows:

$$h(1,\delta)= \min_{\tau_1\in\{0,1,\ldots\}} \sum_{k=0}^{\tau_1}\left(g(1,\delta+k)-\mathrm{L}_{\mathrm{opt}}\right)+h(2,1), \quad (30)$$

where $\tau_1 = 0,1,\ldots$ is the time to keep scheduling source 1 after time t. By solving (30), we get that the optimal $\tau_1(\delta)$ satisfies

$$\tau_1(\delta) = \inf\left\{\tau\in\mathbb{Z}^+ : \gamma_2(\delta+\tau)\geq \mathrm{L}_{\mathrm{opt}}\right\}, \quad (31)$$

where $\gamma_2(\delta+\tau)$ is defined as

$$\gamma_2(\delta) = \inf_{k=1,2,\ldots} \frac{1}{k}\sum_{j=0}^{k-1} g(1,\delta+1+j). \quad (32)$$

Moreover, by using (27) and (30), we get

$$\begin{aligned} h(1,2) &= \sum_{k=0}^{\tau_1(2)}\left(g(1,2+k)-\mathrm{L}_{\mathrm{opt}}\right)+h(2,1)\\ &= \sum_{k=0}^{\tau_1(2)}\left(g(1,2+k)-\mathrm{L}_{\mathrm{opt}}\right)\\ &\quad+\sum_{k=0}^{\tau_2(2)}\left(g(2+k,1)-\mathrm{L}_{\mathrm{opt}}\right)+h(1,2), \end{aligned} \quad (33)$$

which yields

$$\begin{aligned} &\mathrm{L}_{\mathrm{opt}}\\ &=\frac{\sum_{k=0}^{\tau_1(2)}\left(g(1,2+k)\right)+\sum_{k=0}^{\tau_2(2)}\left(g(2+k,1)\right)}{\tau_1(2)+\tau_2(2)}\\ &=\min_{\substack{\tau_1=0,1,\ldots\\ \tau_2=0,1,\ldots}}\frac{\sum_{k=0}^{\tau_1}\left(g(1,2+k)\right)+\sum_{k=0}^{\tau_2}\left(g(2+k,1)\right)}{\tau_1+\tau_2}. \end{aligned} \quad (34)$$

Notice that the optimal objective $\mathrm{L}_{\mathrm{opt}}$ is same as if source 1 is scheduled for τ_1^* consecutive time slots, followed by source 2 being scheduled for τ_2^* consecutive time slots.

Appendix C
Proof of Lemma 2

For the simplicity of presentation, we prove the lemma for $M=3$ and $m=1$. In this case, We need to show

$$\begin{aligned} &H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-\delta_2},X_{3,t-\delta_3})\\ &=H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-1},X_{3,t-1})+O(\max_{n=2,3}\epsilon_{1,n}^2) \end{aligned} \quad (35)$$

If condition (15) holds for $n=2$, then by definition (8)

$$\begin{aligned} &H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-\delta_2},X_{3,t-\delta_3})\\ &-H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-\delta_2},X_{3,t-\delta_3},X_{2,t-\mu_2})\leq\epsilon_{1,2}^2. \end{aligned} \quad (36)$$

By substituting $\mu_2=1$ in (36), we get

$$\begin{aligned} &H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-\delta_2},X_{3,t-\delta_3})\\ &-H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-\delta_2},X_{3,t-\delta_3},X_{2,t-1})\leq\epsilon_{1,2}^2. \end{aligned} \quad (37)$$

As the following inequality holds [4]

$$H_L(Y|X,Z)\leq H_L(Y|X), \quad (38)$$

by removing $X_{2,t-\delta_2}$ from

$$(X_{1,t-\delta_1},X_{2,t-\delta_2},X_{3,t-\delta_3},X_{2,t-1})$$

in (37), we get

$$\begin{aligned} &H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-\delta_2},X_{3,t-\delta_3})\\ &-H_L(Y_{1,t}|X_{1,t-\delta_1},X_{3,t-\delta_3},X_{2,t-1})\leq\epsilon_{1,2}^2. \end{aligned} \quad (39)$$

Similarly, we can write for $n=3$

$$\begin{aligned} &H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-1},X_{3,t-\delta_3})\\ &-H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-1},X_{3,t-1})\leq\epsilon_{1,3}^2. \end{aligned} \quad (40)$$

By adding (41) and (40), we get

$$\begin{aligned} &H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-\delta_2},X_{3,t-\delta_3})\\ &-H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-1},X_{3,t-1})\leq\sum_{n=2}^{3}\epsilon_{1,n}^2\\ &=O(\sum_{n=2}^{3}\epsilon_{1,n}^2)=O(\max_{n=2,3}\epsilon_{1,n}^2). \end{aligned} \quad (41)$$

This completes the proof for $M=3$ and $m=1$. Following the same steps, we can prove it for any values of M and m.

Appendix D
Proof of Lemma 3

For the simplicity of presentation, we prove the lemma for $M=3$ and $m=1$. Since $Y_{1,t}\leftrightarrow X_{n,t-1}\leftrightarrow X_{n,t-k_n}$ for all $\delta_n\geq 1$ and $n=2,3$, we have

$$\begin{aligned} &H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-\delta_2},X_{3,t-\delta_3})\\ &\geq H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-1},X_{3,t-\delta_3})\\ &\geq H_L(Y_{1,t}|X_{1,t-\delta_1},X_{2,t-1},X_{3,t-1}). \end{aligned} \quad (42)$$

This completes the proof for $M=3$ and $m=1$. Following the same steps, we can prove it for any values of M and m.

APPENDIX E
PROOF OF THEOREM 3

By using [4, Theorem 11], we get that our policy π_{AWI} is asymptotically optimal to the problem (18)-(19). For this reason, it is sufficient to show that if $\epsilon_{m,n}$ is close to zero for all $m \neq n$ and $m, n = 1, 2, \ldots, M$, then the problem (11)-(12) and (18)-(19) are equivalent.

For any policy $\pi \in \Pi$, we have

$$\begin{aligned}
&\mathbb{E}_\pi\left[\frac{1}{T}\sum_{t=0}^{T-1}\sum_{m=1}^{M} g_m\bigg(\Delta_1(t), \Delta_2(t), \ldots, \Delta_M(t)\bigg)\right] \\
=&\mathbb{E}_\pi\left[\frac{1}{T}\sum_{t=0}^{T-1}\sum_{m=1}^{M} f_m(\Delta_m(t)) + O\bigg(\max_{m\neq n} \epsilon_{m,n}^2\bigg)\right] \\
=&\mathbb{E}_\pi\left[\frac{1}{T}\sum_{t=0}^{T-1}\sum_{m=1}^{M} f_m(\Delta_m(t))\right] + \sum_{m=1}^{M} O\bigg(\max_{m\neq n} \epsilon_{m,n}^2\bigg).
\end{aligned} \tag{43}$$

As $\epsilon_{m,n}$ becomes close to zero for all $m \neq n$ and $m, n = 1, 2, \ldots, M$, the term

$$\sum_{m=1}^{M} O\bigg(\max_{m\neq n} \epsilon_{m,n}^2\bigg)$$

reduces to zero. This completes the proof.

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