Homework 2

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Answer to the Question Number 1

Given, $p(x) = sin(x)$ [where $x \in (0, \frac{\pi}{2})$]

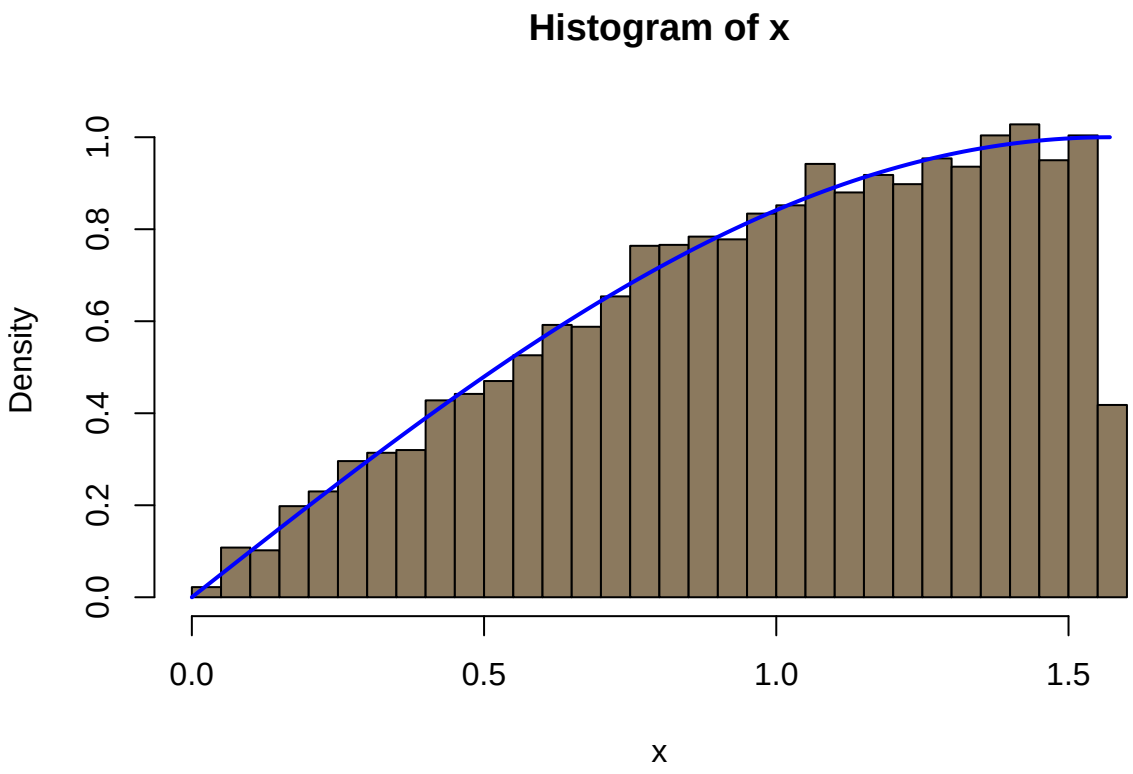
Now the cdf of $p(x)$ will be $P(x) = \int_0^x sin(x)dx = -cos(x)\Big|_0^x = 1 - cos(x)$

$\Rightarrow u = 1 - cos(x)$ [where $u \sim Uinf(0, 1)$]

$\Rightarrow cos(x) = 1 - u$

$\Rightarrow x = arccos(1 - u)$

```
set.seed(7)
n = 10000
u = runif(n)
x = acos(u)
hist(x, freq = F, breaks = 30, col = "navajowhite4")
curve(sin(x), 0, pi/2, col = "blue", add = T, lw = 2)
```

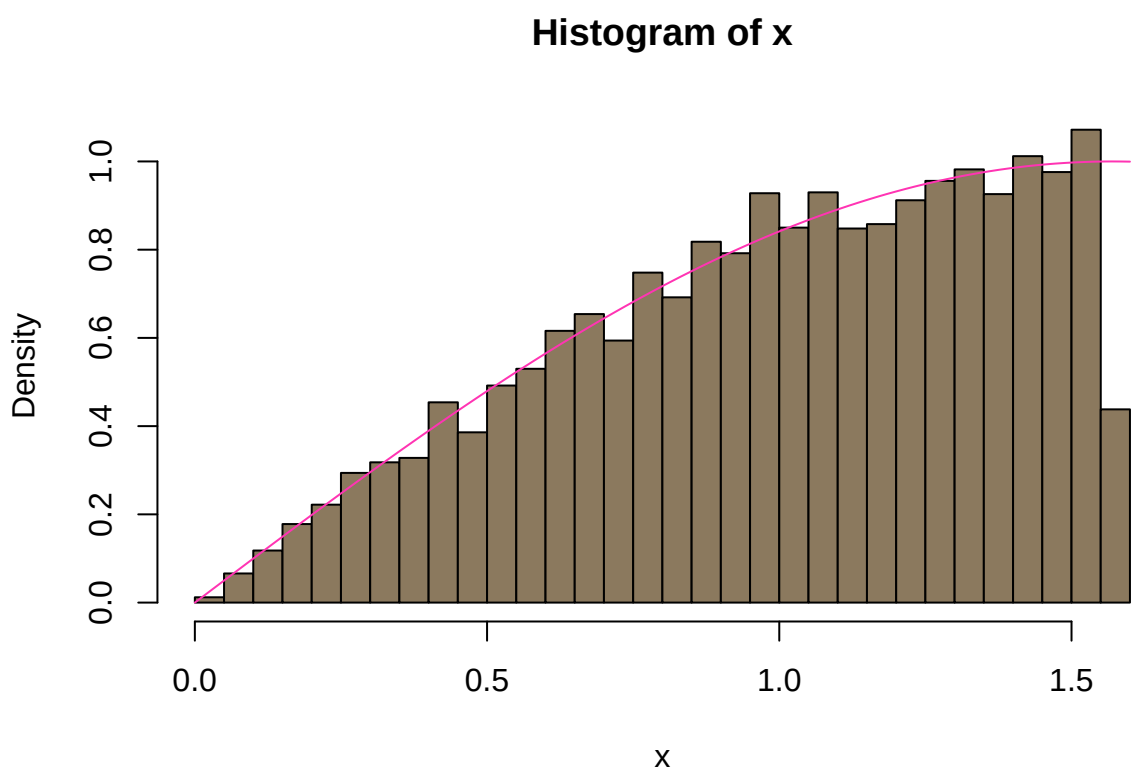



```
set.seed(7)
arsample <- function(dist, minsup, maxsup, maxdist, n)
{
  rv = rep(0,n)
  for (i in 1:n)
  {
    while(1)
    {
      x = runif(1, minsup, maxsup)
      y = runif(1, 0, maxdist)
      if (y <= dist(x))
      {
        rv[i] = x
        break
      }
    }
  }
  return(rv)
}

sinfun <- function(x)
{
  return(sin(x))
}

x = arsample(sinfun, 0, pi/2, 1, 10000)
```

```r
hist (x, freq = F, breaks = 30, col ="navajowhite4")
curve (sinfun, add = T, col = "maroon1")
```



Answer to the Question Number 2

Given, $p(x) = \lambda x^{-\lambda-1}$ where $x \in [1, \infty)$ and $\lambda \geq 2$

So, the cdf will be

$P(x) = \int_1^x \lambda x^{-\lambda-1} dx = -x^{-\lambda}\Big|_1^x = 1 - x^{-\lambda}$

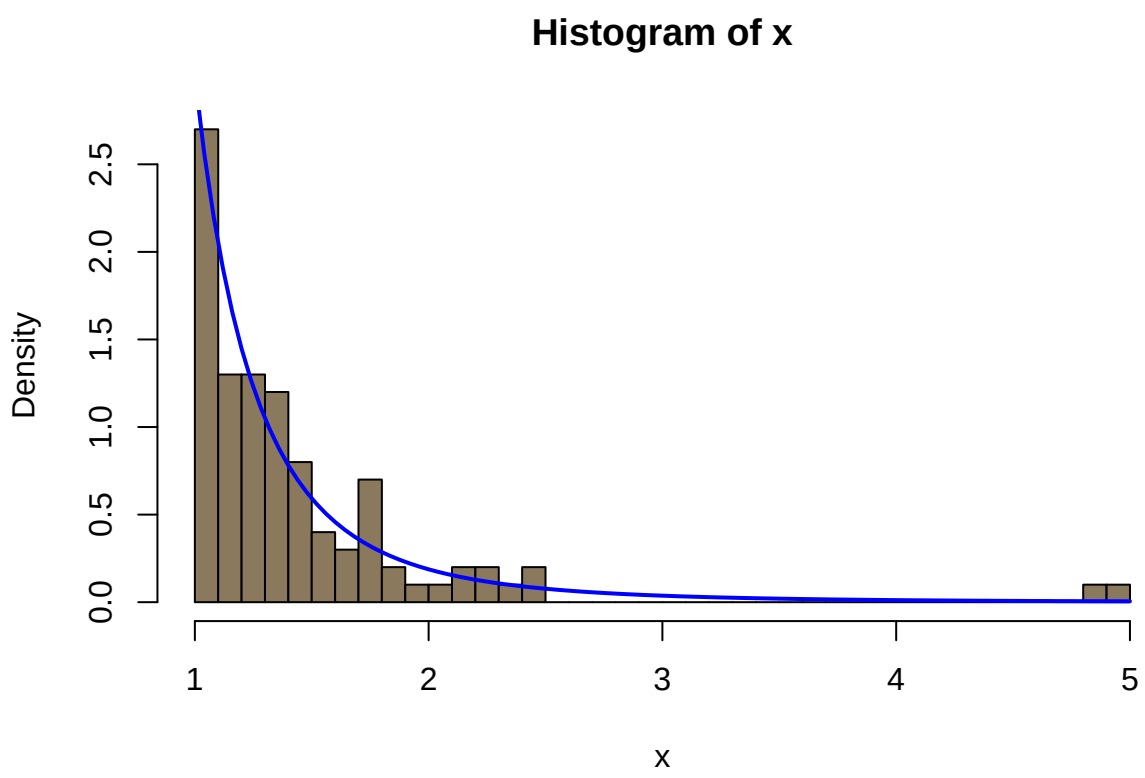
Let, $u = 1 - x^{-\lambda}$ [where $u \sim Uinf(0, 1)$]

$\Rightarrow x^{-\lambda} = 1 - u$

$\Rightarrow x = (1 - u)^{-1/\lambda}$

```r
set.seed(7)
pareto <- function(lambda)
{
  u = runif(n)
  x = 1/u^(1/lambda)
}
n = 100
x = pareto(3)
hist(x, breaks = 30, freq = F, col = "navajowhite4", xlim = c(1,5))
```

```r
fy <- function(y,lambda) lambda*(y^(-lambda-1))
curve(fy(x,3), add = T, col = "blue", lw = 2)
```



Mean

$$E(X^{2.736}) = \int_1^\infty x^{2.736}\lambda x^{-\lambda-1}dx = \lambda\int_1^\infty x^{-\lambda+1.736}dx = \lambda\frac{x^{-\lambda+2.736}}{-\lambda+2.736}\Big|_1^\infty = 3\frac{x^{-3+2.736}}{-3+2.736}\Big|_1^\infty = 3\frac{1}{-0.264x^{0.264}}\Big|_1^\infty =$$

$$-0 + \frac{3}{0.264} = 11.36$$

```r
set.seed(12345)
lambda = 3
mean(pareto(lambda)^2.736)
```

```
## [1] 10.12916
```