Exercice 3

$N : \mathbb{R}^2 \longrightarrow \mathbb{R}$

$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \longmapsto \sqrt{4u_1^2 + 9u_2^2}$

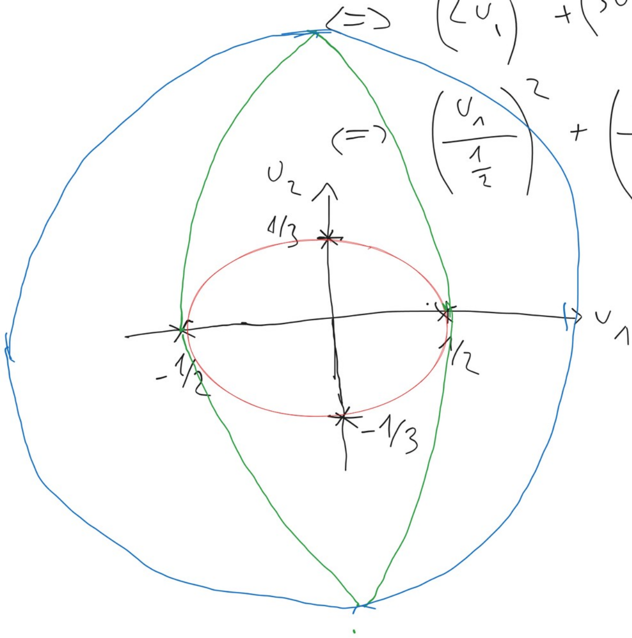
$C_1 = \{ u \in \mathbb{R}^2, N(u) = 1 \}$

$N(u) = 1 \iff \sqrt{4u_1^2 + 9u_2^2} = 1$

Donc : $4u_1^2 + 9u_2^2 = 1$

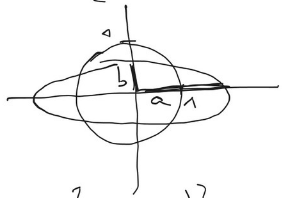
$\iff (2u_1)^2 + (3u_2)^2 = 1$

$\iff \left(\frac{u_1}{\frac{1}{2}}\right)^2 + \left(\frac{u_2}{\frac{1}{3}}\right)^2 = 1$



$x^2 + y^2 = 1$

$\|u\|_2^2 = 1$



$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

Si $u_2 = 0$: $\left(\frac{u_1}{\frac{1}{2}}\right)^2 = 1$

Si $u_1 = 0$ $\left(\frac{u_2}{\frac{1}{3}}\right)^2 = 1$

Exercice 4

$\cdot\ \|u\|_\infty = \max\limits_{1\le i\le n} |u_i|$

$\cdot\ \|u\|_2 = \sqrt{\sum\limits_{i=1}^{n} |u_i|^2} = \sqrt{|u_1|^2 + \underset{\uparrow_k}{\dots} + |u_n|^2}$

Soit $1\le k\le n$ tel que : $|u_k| = \max\limits_{1\le i\le n} |u_i|$

Si $|u_k| = 0$: $u = 0_{\mathbb{R}^2}$. $\|u\|_\infty \le \|u\|_2$.

Sinon :

$$\|u\|_2 = \sqrt{|u_1|^2 + \dots + |u_k|^2 + \dots + |u_n|^2} = \sqrt{|u_k|^2\left(\left|\frac{u_1}{u_k}\right|^2 + \dots + 1 + \dots \left|\frac{u_n}{u_k}\right|^2\right)}$$

$$= |u_k| \sqrt{\left|\frac{u_1}{u_k}\right|^2 + \dots + 1 + \dots \left|\frac{u_n}{u_k}\right|^2}$$

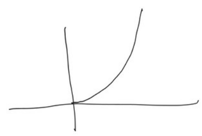
$$= |u_k| \times \sqrt{1+\varepsilon}$$

$$\varepsilon = \left|\frac{u_1}{u_k}\right|^2 + \dots + \left|\frac{u_n}{u_k}\right|^2 \ge 0$$

$\sqrt{1+\varepsilon} \ge 1$

Donc $\|u\|_2 = \|u\|_\infty \times \sqrt{1+\varepsilon} \ge \|u\|_\infty \times 1$

Donc $\|u\|_\infty \le \|u\|_2$.



$\cdot\ \|u\|_2 \le \|u\|_1$? $\quad \|u\|_1 = \sum\limits_{i=1}^{n} |u_i| = |u_1| + \dots + |u_n|$

$$\sqrt{|u_1|^2 + \dots + |u_n|^2} \le |u_1| + \dots + |u_n| \iff \boxed{|u_1|^2 + \dots + |u_n|^2} \le (|u_1| + \dots + |u_n|)^2$$

$$(|u_1| + \dots + |u_n|)^2 = (|u_1| + \dots + |u_n|)(|u_1| + \dots + |u_n|) = |u_1|^2 + |u_1| \times (\dots) + |u_2|^2 + |u_2|(\dots) + \dots$$

$$= \boxed{|u_1|^2 + \dots + |u_n|^2} + \underset{\ge 0}{\varepsilon}$$

Donc $\|u\|_2 \le \|u\|_1$

$\cdot\ \|u\|_1 \le \|u\|_\infty$?

$|u_1| + \dots + |u_n| \le n \cdot \max\limits_{1\le i\le n} |u_i|$?

$\cdot\ \forall\, 1\le i\le n$: $|u_i| \le \max\limits_{1\le i\le n} |u_i| = \|u\|_\infty$

Donc

$\|u\|_1 \le \|u\|_\infty + \dots + \|u\|_\infty = n\|u\|_\infty$.

Orthogonalité

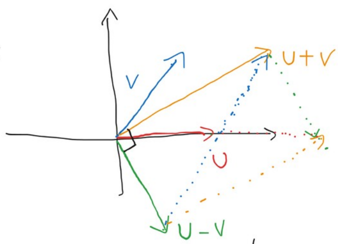
Exercice 1

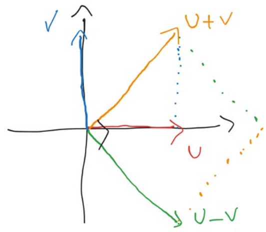
$(a+b)(c+d)$ . .

1) Soient $u,v \in E$. $\|u\| = \sqrt{\langle u,u\rangle}$.

$$\langle u+v, u-v\rangle = 0 \iff \langle u,u-v\rangle + \langle v,u-v\rangle = 0$$
$$\iff \langle u,u\rangle - \langle u,v\rangle + \langle v,u\rangle - \langle v,v\rangle = 0$$
$$\iff \|u\|^2 - \|v\|^2 = 0$$
$$\iff \|u\|^2 = \|v\|^2 \iff \|u\| = \|v\|$$

3) Dans $\mathbb{R}^2$:





2) Si $\|u\| = \|v\|$, $u+v$ et $u-v$ forment les côtés d'un rectangle dont les deux demi-diagonales sont représentées par u et v.

Exercice 2

$$\langle u,v\rangle = u_1v_1 - (u_1v_2 + u_2v_1) + 2u_2v_2$$

1) Soient $u,v,w \in \mathbb{R}^2$ et $\lambda \in \mathbb{R}$:

- Bilinéarité :

- $\langle u+\lambda v, w\rangle \overset{?}{=} \langle u,w\rangle + \lambda\langle v,w\rangle$

$$\langle u+\lambda v, w\rangle = (u_1+\lambda v_1)w_1 - \left((u_1+\lambda v_1)w_2 + (u_2+\lambda v_2)w_1\right) + 2\times(u_2+\lambda v_2)w_2$$
$$= u_1w_1 + \lambda v_1w_1 - (u_1w_2 + \lambda v_1w_2 + u_2w_1 + \lambda v_2w_1) + 2u_2w_2 + 2\lambda v_2w_2$$
$$= \underbrace{u_1w_1 - (u_1w_2 + u_2w_1) + 2u_2w_2}_{\langle u,w\rangle} + \lambda\Big(\underbrace{v_1w_1 - (v_1w_2 + v_2w_1) + 2v_2w_2}_{\langle v,w\rangle}\Big)$$
$$= \langle u,w\rangle + \lambda\langle v,w\rangle$$

- $\langle u, v+\lambda w\rangle = u_1(v_1+\lambda w_1) - \left(u_1(v_2+\lambda w_2) + u_2(v_1+\lambda w_1)\right) + 2u_2(v_2+\lambda w_2)$

$$= u_1v_1 + \lambda u_1w_1 - (u_1v_2 + \lambda u_1w_2 + u_2v_1 + \lambda u_2w_1) + 2u_2v_2 + \lambda\cdot 2u_2w_2$$
$$= \underbrace{u_1v_1 - (u_1v_2 + u_2v_1) + 2u_2v_2}_{\langle u,v\rangle} + \lambda\times\Big(\underbrace{u_1w_1 - (u_1w_2 + u_2w_1) + 2u_2w_2}_{\langle u,w\rangle}\Big)$$
$$= \langle u,v\rangle + \lambda\langle u,w\rangle$$

. Symétrie : $\langle u,v\rangle \overset{?}{=} \langle v,u\rangle$

$\langle u,v\rangle = u_1v_1 - (u_1v_2+u_2v_1) + 2u_2v_2$

$= v_1u_1 - (v_2u_1 + v_1u_2) + 2v_2u_2$

$= \langle v,u\rangle$

. Défini positif : $\forall u \in \mathbb{R}^2 \quad u \neq 0_{\mathbb{R}^2}, \langle u,u\rangle > 0$

Soit $u \neq 0_{\mathbb{R}^2}$:

$$\langle u,u\rangle = u_1u_1 - (u_1u_2 + u_2u_1) + 2u_2u_2$$

$$= u_1^2 - (u_1u_2 + u_2u_1) + 2u_2^2$$

$$= \underbrace{u_1^2 - 2u_1u_2 + u_2^2} + u_2^2$$

$$= (u_1 - u_2)^2 + u_2^2 > 0 \quad (\text{car } u \neq 0_{\mathbb{R}^2})$$

Finalement, $\langle \cdot,\cdot\rangle$ est un produit scalaire sur $\mathbb{R}^2$.

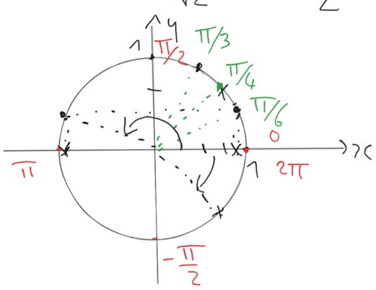
2) $u = \begin{pmatrix}1\\1\end{pmatrix}$

$\|u\| = \sqrt{\langle u,u\rangle} = \sqrt{1\times1 - (1\times1 + 1\times1) + 2\times1\times1} = 1$

3) $\|u\|_2 = \sqrt{1^2+1^2} = \sqrt{2}$

4) $\cos\theta = \dfrac{u\cdot v}{\|u\|_2 \cdot \|v\|_2} = \dfrac{-1\times(-1) + 0\times(-1)}{\sqrt{(-1)^2+0^2}\ \sqrt{(-1)^2+(-1)^2}}$ $\quad u = \begin{pmatrix}-1\\0\end{pmatrix} \quad v = \begin{pmatrix}-1\\-1\end{pmatrix}$

$\cos\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$



$x^2+y^2 = 1$

$x = \cos\theta$

$y = \sin\theta$

	x	y
	$\cos\theta$	$\sin\theta$
$\theta = 0$:	1	0
$\theta = \frac{\pi}{6}$:	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\theta = \frac{\pi}{4}$:	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\theta = \frac{\pi}{3}$:	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\theta = \frac{\pi}{2}$:	0	1

$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

Donc $\theta = \frac{\pi}{4}$ avec le produit scalaire euclidien.

5) $\cos(\theta) = \dfrac{\langle u,v\rangle}{\|u\|\cdot\|v\|}$

$\langle u,v\rangle = (-1)\times(-1) - ((-1)\times(-1) + 0\times(-1)) + 2\times0\times(-1)$

$= 1 - 1 = 0 \qquad u \perp v$ avec $\langle \cdot,\cdot\rangle$

$\cos(\theta) = 0$ donc $\theta = \frac{\pi}{2}$ avec $\langle \cdot,\cdot\rangle$.

6) $u = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad v = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$u.v = 2 \times -1 \; + 1 \times 2 \; = 0$

Donc $u \perp v$ pour le produit scalaire euclidien.

7) $\langle u, v \rangle = 2 \times (-1) - (2 \times 2 + 1 \times -1) + 2 \times 1 \times 2$

$= -2 - 3 + 4 = -1 \neq 0$

8) Trouver $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{R})$ telle que :

$\forall u, v \in \mathbb{R}^2 \quad \langle u, v \rangle = u^T \underbrace{A v}$

$Av = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} av_1 + bv_2 \\ cv_1 + dv_2 \end{pmatrix}$

$\Leftrightarrow \; u_1 v_1 - (u_2 v_1 + u_1 v_2) + 2 u_2 v_2 = (u_1 \; u_2) \begin{pmatrix} av_1 + bv_2 \\ cv_1 + dv_2 \end{pmatrix}$

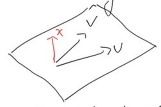
$= u_1 (av_1 + bv_2) + u_2 (cv_1 + dv_2)$

$u_1 v_1 - u_1 v_2 - u_2 v_1 + 2 u_2 v_2 = a u_1 v_1 + b u_1 v_2 + c u_2 v_1 + d u_2 v_2$

Par identification : $A = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$

Exercice 3

F et G orthogonaux $\Leftrightarrow \forall x \in F, \forall y \in G, \; x \cdot y = 0$

F 

$x \in F, \; \exists \lambda_1, \lambda_2 \in \mathbb{R}$ tels que : $x = \lambda_1 u + \lambda_2 v$

$y \in G, \; \exists \lambda_3, \lambda_4 \in \mathbb{R}$ tels que : $y = \lambda_3 w + \lambda_4 t$

$x \cdot y = (\lambda_1 u + \lambda_2 v) \cdot (\lambda_3 w + \lambda_4 t)$

$= \lambda_1 \lambda_3 u \cdot w + \lambda_1 \lambda_4 u \cdot t + \lambda_2 \lambda_3 v \cdot w + \lambda_2 \lambda_4 v \cdot t$

$u.w = \begin{pmatrix} -1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$

$= 1 - 1 - 1 + 1$

$= 0$

$v.t = \begin{pmatrix} 1 \\ -1 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$

$= 1 - 1 - 1 + 1$

$= 0$

$u.t = \begin{pmatrix} -1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$

$= -1 - 1 + 1 + 1$

$= 0$

$v.w = \begin{pmatrix} 1 \\ -1 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$

$= -1 - 1 + 1 + 1$

$= 0$

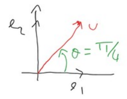
Donc $F \perp G$.

Exercice 4

1) Dans $\mathbb{R}^2$ $\quad e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\cos(\theta_1) = \frac{u.e_1}{\|u\|_2 \|e_1\|_2} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \theta_1 = \frac{\pi}{4}.$

$\cos(\theta_2) = \frac{u.e_2}{\|u\|_2 \|e_2\|_2} = \frac{1}{\sqrt{2}} \qquad \theta_2 = \frac{\pi}{4}.$

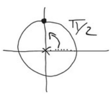


2) Dans $\mathbb{R}^3$: $\qquad u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\cos(\theta_1) = \frac{u.e_1}{\|u\|_2 \|e_1\|_2} = \frac{1}{\sqrt{3}}$

3) Dans $\mathbb{R}^n$: $\quad u = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \qquad \|u\|_2 = \sqrt{n}$

$\cos(\theta_1) = \frac{1}{\sqrt{n}}$

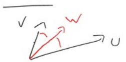


4) $\cos(\theta_1) = \frac{1}{\sqrt{n}} \xrightarrow[n \to +\infty]{} 0$

À l'infini : $\theta_1 = \frac{\pi}{2}$.

5) À l'infini, la droite diagonale à toutes les droites engendrées par les vecteurs de la base canonique est orthogonale à celles-ci.

Exercice 5 :



Applications linéaires.

Exercice 1

$$\mathcal{D} = \mathrm{Vect}(u) \qquad u = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}. \qquad \|u\|_2^2 = 2$$

$$p_{\mathcal{D}} : \mathbb{R}^3 \to \mathbb{R}^3$$
$$v \mapsto \frac{v \cdot u}{\|u\|_2^2}\, u = \frac{1}{2} \times (v \cdot u)\, u$$

1) $$M_B(p_D) = \begin{pmatrix} p_{\mathcal{D}}(e_1) & p_{\mathcal{D}}(e_2) & p_D(e_3) \end{pmatrix}$$

$$p_{\mathcal{D}}(e_1) = \frac{1}{2}(e_1 \cdot u)\, u = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$p_{\mathcal{D}}(e_2) = \frac{1}{2}(e_2 \cdot u)\, u = \frac{1}{2} \times 0 . u = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

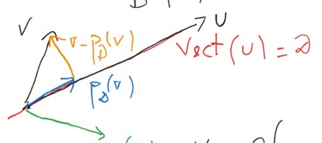
$$p_{\mathcal{D}}(e_3) = \frac{1}{2}(e_3 \cdot u)\, u = -\frac{1}{2} u = -\frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$M_B(p_D) = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

2) $$M_B^2(p_D) = \frac{1}{4}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 2 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$= M_B(p_D)$$

3)



$$s_{\mathcal{D}}(v) = v - 2(v - p_{\mathcal{D}}(v)) = v - 2v + 2p_D(v) = 2p_{\mathcal{D}}(v) - v$$

$$\boxed{s_D = 2p_D - \mathrm{id}}$$

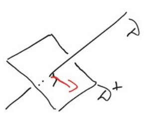
$$s_{\mathcal{D}} : \mathbb{R}^3 \to \mathbb{R}^3$$
$$v \mapsto 2p_D(v) - v$$

$$M_B(s_{\mathcal{D}}) = 2M_B(p_{\mathcal{D}}) - I_3 = 2.\frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix} = -1\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

4) $$M_B^2(s_{\mathcal{D}}) = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

5) $\mathcal{D}^\perp$



$$\mathcal{D}^\perp = \left\{ v \in \mathbb{R}^3,\ \underset{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}}{u \cdot v = 0} \right\} = \left\{ v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3,\ x - z = 0 \right\}$$

L'équation de $\mathcal{D}^\perp$ est $x - z = 0$

7) Bonus: $$\mathrm{Ker}(p_D) = \left\{ v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3\ \underbrace{\frac{1}{2}(u \cdot v)\, u = 0}_{} \right\}$$

$$\frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} u = \frac{1}{2}(x - z)\, u = 0$$
$$\Rightarrow x = z$$

$$\mathrm{Ker}(p_D) = \mathcal{D}^\perp$$

Exercice 2

$$u = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \quad v = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

1) Soit p_u la projection sur $\mathrm{Vect}(u)$

— p_v — $\mathrm{Vect}(v)$

Alors $p_P = p_u + p_v$

$$p_u(w) = \frac{u \cdot w}{\|u\|^2} u = \frac{1}{3}(u \cdot w) u \qquad \Bigg| \qquad p_v(w) = \frac{v \cdot w}{\|v\|^2} v = \frac{1}{2}(v \cdot w) v$$

$$p_u(e_1) = -\frac{1}{3}u \quad p_u(e_2) = \frac{1}{3}u \quad p_u(e_3) = -\frac{1}{3}u \qquad \Bigg| \qquad p_v(e_1) = \frac{1}{2}v \quad p_v(e_2) = 0_{\mathbb{R}^3} \quad p_v(e_3) = -\frac{1}{2}v$$

$$p_P(e_1) = p_u(e_1) + p_v(e_1) = -\frac{1}{3}\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \frac{1}{6}\left(\begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}\right)$$

$$= \frac{1}{6}\begin{pmatrix} 5 \\ -2 \\ -1 \end{pmatrix}$$

$$p_P(e_2) = \frac{1}{3}u = \frac{1}{6}\begin{pmatrix} -2 \\ 2 \\ -2 \end{pmatrix}$$

$$p_P(e_3) = -\frac{1}{3}u - \frac{1}{2}v = \frac{1}{6}\left(\begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}\right) = \frac{1}{6}\begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}$$

$$M_B(p_P) = \frac{1}{6}\begin{pmatrix} 5 & -2 & -1 \\ -2 & 2 & -2 \\ -1 & -2 & 5 \end{pmatrix}$$

2) $M_B^2(p_P) = M_B(p_P)$

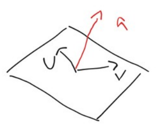
3) $s_P = 2p_P - id \qquad M(s_P) = 2M(p_P) - I_3$

$$= \frac{1}{3}\begin{pmatrix} 2 & -2 & -1 \\ -2 & -1 & -2 \\ -1 & -2 & 2 \end{pmatrix}$$

4) $M^2(s_P) = I_3$

5)

6) P



on cherche $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = a$: $\begin{cases} u \cdot a = 0 \\ v \cdot a = 0 \end{cases}$

$$\Leftrightarrow \begin{cases} -x + y - z = 0 \\ x - z = 0 \end{cases} \Leftrightarrow \begin{cases} y = 2z \\ x = z \end{cases}$$

$a = \begin{pmatrix} x \\ 2x \\ x \end{pmatrix}$ avec $x \in \mathbb{R}$

$a = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ convient

Equation de P : $x + 2y + z = 0$

Astuce :

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \wedge \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \times 5 - 3 \times 3 \\ 3 \times (-1) - 1 \times 5 \\ 1 \times 3 - 2 \times (-1) \end{pmatrix}$$

$\begin{matrix} 1 & & -1 \end{matrix}$