The Calculus of Images

Imaging Lab 5 - May, 2017

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Curvature Matrix of an Image

In this assignment, we are writing a Matlab function to compute the curvature matrix of an input grayscale image. The function takes a grayscale image as an input argument, computes first and second derivative of image and returns a the curvature matrix.
The curvature k (and the discredited approximation) shown below measures the rate at which the unit gradient vector is changing:

$$k = \nabla \cdot \left(\frac{\nabla u}{\|\nabla u\|} \right) \approx \frac{u_{xx}u_y^2 - 2u_x u_y u_{xy} + u_{yy}u_x^2}{(u_x^2 + u_y^2)^{3/2}}$$

Listing 1 shows the Curvature function's implementation in Matlab:

```
function k = Curvature(img)
    % Computes the curvature matrix of an input grayscale image

    a = 0.01;                % fudge factor
    [m,n] = size(img);       % image size
    img = double(img);       % convert input image to double

    % u_x = (u(x+1,y) - u(x-1,y)) / 2
    u_x = (img(:,[2:n,n]) - img(:,[1,1:n-1])) / 2;

    % u_y = (u(x,y+1) - u(x,y+1)) / 2
    u_y = (img([2:m,m],:) - img([1,1:m-1],:)) / 2;

    % u_xx = u(x+1,y) - 2u(x,y) + u(x-1,y)
    u_xx = img(:,[2:n,n]) - 2 * img + img(:,[1,1:n-1]);

    % u_yy = u(x,y+1) - 2u(x,y) + u(x,y-1)
    u_yy = img([2:m,m],:) - 2 * img + img([1,1:m-1],:);

    % u_xy = (u(x+1,y+1) + u(x-1,y-1) - u(x-1,y+1) - u(x+1,y-1))/4
    u_xy = (img([2:m,m],[2:n,n]) + img([1,1:m-1],[1,1:n-1]) -
       img([2:m,m],[1,1:n-1]) - img([1,1:m-1],[2:n,n])) / 4;

    % multiplications and powers are all componentwised
    k_num = (u_xx.*u_y.^2) - 2*(u_x.*u_y.*u_xy) + (u_yy.*u_x.^2);
    k_denom = (u_x.^2 + u_y.^2).^(3/2) + a;

    % componentwise division
    k = k_num ./ k_denom;
end
```

Listing 1: Matlab Curvature Function

Figure 1 shows a test image and its curvature matrix side-by-side. The curvature is zero in flat regions and along straight edges; however it is visible along the perimeter of the circles and the edges of the square:

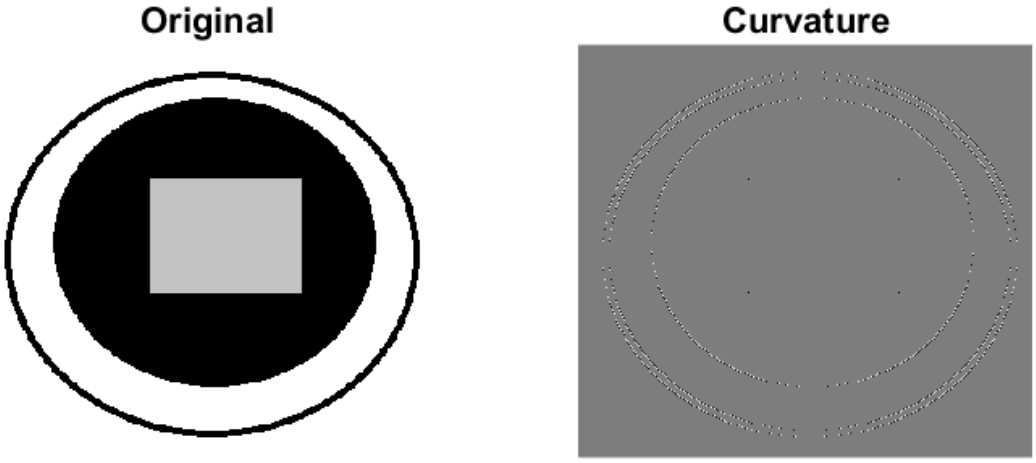

Figure 1: Curvature image of a grayscale test image