Lab 5: The Calculus of Images

Primary Goal: Learn how to approximate the curvature of an image using finite differences.
Secondary Goal: Practice writing Matlab functions.

Your goal is to compute the curvature matrix of a grayscale image. For a function u(x,y), the curvature κ measures the rate at which the unit gradient vector is changing:

$$\kappa = \nabla \cdot \left(\frac{\nabla u}{\|\nabla \mathrm{u}\|} \right) = \frac{u_{xx} {u_y}^2 - 2u_x u_y u_{xy} + u_{yy} {u_x}^2}{\left({u_x}^2 + {u_y}^2 \right)^{3/2}}$$

We can approximate the derivatives using the finite difference approximations shown in Figure 1. To avoid division by zero, we introduce a small "fudge factor" a into the denominator. You can experiment with different values of a, but I recommend $a = 0.01$.

$$\kappa = \frac{u_{xx} {u_y}^2 - 2u_x u_y u_{xy} + u_{yy} {u_x}^2}{\left({u_x}^2 + {u_y}^2 \right)^{3/2} + a}$$

$$u_x \approx \frac{u(x+1,y) - u(x-1,y)}{2} \qquad u_y \approx \frac{u(x,y+1) - u(x,y-1)}{2}$$

$$u_{xx} \approx u(x+1,y) - 2u(x,y) + u(x-1,y) \qquad u_{yy} \approx u(x,y+1) - 2u(x,y) + u(x,y-1)$$

$$u_{xy} \approx \frac{u(x+1,y+1) + u(x-1,y-1) - u(x-1,y+1) - u(x+1,y-1)}{4}$$

Figure 1. Finite difference approximations of first and second partial derivatives.

Write a Matlab function that computes the curvature matrix of an input grayscale image. First compute each of the derivatives shown in Figure 1 and then combine them carefully to calculate the curvature κ. When you combine your derivative matrices in the formula for κ, be sure to use the Matlab point-wise arithmetic operators: .* ./ .^ . Copy and paste your function code into a figure in your report.

Download the test image for Lab 5 from our course website. Run your curvature function on the image **test.bmp** and display the curvature image. We expect the curvature to be zero in flat regions and along straight edges. The curvature should be non-zero along the rounded edges of the circles. If you look closely at your curvature image, you should see points of negative curvature at the 4 corners of the rectangle.

What to Include in Your Report

1. *[5 points]* Write a short paragraph describing what you did. Your text should reference your figures.
2. *[10 points]* A figure showing your Matlab function for computing curvature. Include descriptive comments in your code. Include a caption on your figure.
3. *[5 points]* A figure showing the **test.bmp** image and its curvature matrix side-by-side in an appropriate subplot. Include a caption on your figure.