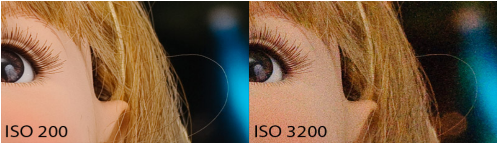
Lecture 4:
Nonlinear Filters

Image Noise

- Physical effects (dust, raindrops)
- Electrical noise
  - Caused by fluctuations in electric circuit.
  - Thermal noise
  - Impulse noise
- Quantization noise
- Poor illumination

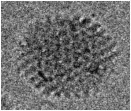


Source: http://photographylife.com/what-is-iso-in-photography

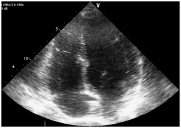
Applications

Astronomy

Microscopy



Medical Imaging



Computer Animation

Noise vs. Blur

- Noise adds static by randomly corrupting individual pixels.
- Blur smooths the edges by averaging over multiple pixels.

$g = f + RANDOM$

$g = f * w$

- Mathematically speaking, noise is generally an additive process, while blur is generally multiplicative.

Artificial Noise

- We can add artificial noise to images using the Matlab imnoise command.
- We have to specify what type of noise we want and how much.
- The most common type of noise is Gaussian (white noise).

```
A = imread('cameraman.tif');
B = imnoise(A, 'gaussian', 0, 0.3);
```

Distribution of noise

Mean

Variance

Additive Noise

Gaussian Noise *(MRI)*

$$f = g + N(0, \sigma)$$


```
A=imread('cameraman.tif');
B=imnoise(uint8(A),'gaussian',0,0.02);
imagesc(B);
```

Poisson Noise *(PET)*

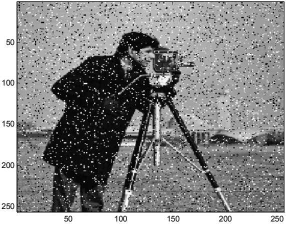
$$f = Poisson(g)$$


```
A=imread('cameraman.tif');
B=imnoise(uint8(A),'poisson');
imagesc(B);
```

Non-additive Noise

Salt & Pepper Noise

(Reset random pixels)



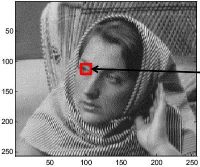

```
A=imread('cameraman.tif');
B=imnoise(uint8(A),'salt & pepper',0.1);
imagesc(B);
```

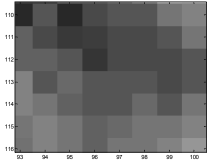
Speckle Noise (Ultrasound)


```
A=imread('cameraman.tif');
B=imnoise(uint8(A),'speckle',0.05);
imagesc(B);
```

Filters

- A <u>filter</u> is a process that removes or enhances some feature of an image.
- Commonly, the word "filter" describes an operation on the *neighborhood* of an image.





Linear vs. Nonlinear

- A linear filter can be expressed as a convolution (weighted average over a neighborhood):

$$g = f * w$$

- Any filter that cannot be written as a convolution is called a nonlinear filter.

- Linear filters can be computed very quickly using Fourier Transforms by The Convolution Theorem.
- Nonlinear filters will generally be much slower.

Median Filter

- The median filter computes the median of the neighborhood centered over the pixel.
- Compared to the mean filter, the median filter preserves edges better.

255	255	0
255	100	0
255	255	0

Mean

$$= \frac{255+255+0+255+100+0+255+255+0}{9}$$

$= 152.8$

Median

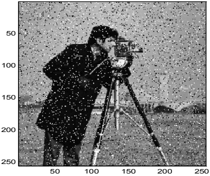
=0, 0, 0 100, $\boxed{255}$, 255, 255, 255, 255

= 255

Median Filter

- The median filter and its variants are particularly well-suited for salt & pepper noise.
- The Matlab command `nlfilter` calculates nonlinear functions on neighborhoods. It is much slower than imfilter.

```
B = nlfilter(A, [3,3], @(x) median(x(:)) );
```



- Can also use the command medfilt2.

Visualizing Noise

- A common way to test denoising algorithms is to subtract the denoised image from the noisy image.
- This subtracted image is called the <u>residual</u>.
- If the algorithm did a good job denoising, then we should only see random dots and no structure.

Quantifying Noise

- There are two major statistics used to determine the amount of noise in an image.
- Both statistics compare the noisy image f to an ideal noise-free image f_{ideal}.
- Both statistics are easier to write using the Frobenius norm:

$$\boxed{\|A\|_F = \sqrt{\sum_i \sum_j f(i,j)^2}}$$

1. Root Mean Square Error (RMSE)

$$RMSE = \frac{1}{N} \|f - f_{ideal}\|_F$$

where N=#pixels in image

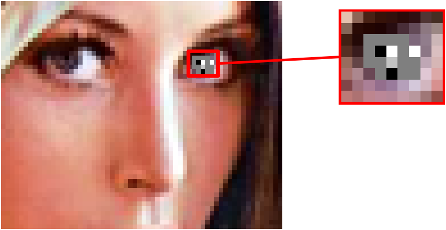
2. Signal-to-Noise Ratio (SNR)

$$\boxed{SNR = 20 \log \frac{\|f_{ideal}\|}{\|f - f_{ideal}\|}}$$

Larger SNR = Less Noise

Beyond Local

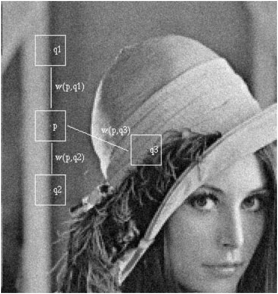
- Suppose Lena's left eye was obscured by noise.
- A neighborhood around the eye will not tell us the eye's true color.



- *Solution*: Look at the other eye!

Image Similarity

- Natural images tend to have similar neighborhoods.



Nonlocal-Means

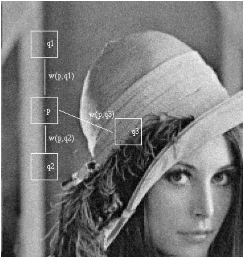
- *(Buades-Coll-Morel, 2005)* proposed a denoising method that averages over all pixels in the image that have a similar neighborhood.

- They call their method *Nonlocal (NL) Means*.

$$\boxed{NL(x) = \frac{\sum_q w(x,q) f(q)}{\sum_q w(x,q)}}$$

where w is a weight that judges the similarity between the neighborhoods.

Similar ⟶ w=1,   Different ⟶ w=0



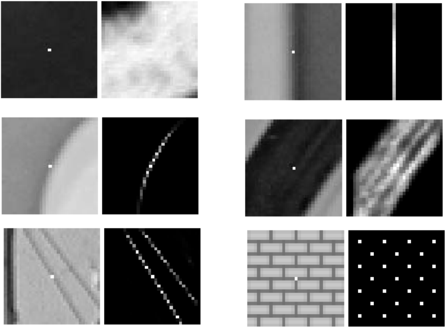
Nonlocal-Means

- Let N(x) denote the neighborhood around pixel x.
- We want w(x,q)=1 if the neighborhoods N(x) and N(q) are similar.
- And we want w(x,q)=0 if the neighborhoods N(x) and N(q) are completely different.

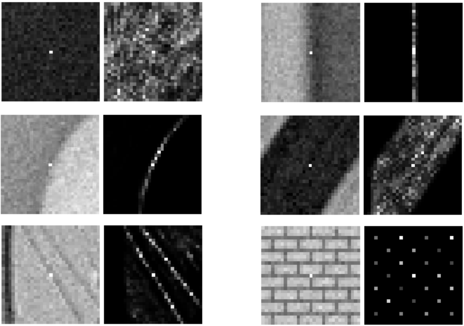
$$\boxed{w(x,q) = exp\left(-\frac{\|N(x)-N(q)\|_F{}^2}{h^2}\right)}$$

- The parameter h controls the similarity.
- Generally choose square neighborhood, e.g. 5x5.
- Can add Gaussian weights to norm to emphasize center.

Weights from Clean Images

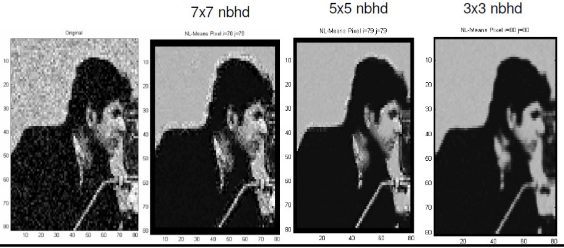


Weights from Noisy Images



NL-Means

- Smaller neighborhoods remove noise better, but may over-smooth the image.



NL-Means

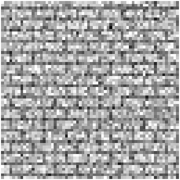
- Many consider NL-Means and its variants to be the best denoising method for natural images.

Noisy

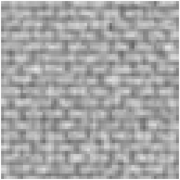
NL-Means

NL-Means

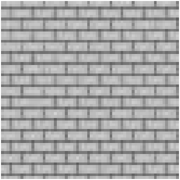
- NL-Means performs best on images with repeating patterns.



Noisy



Gauss Filtered



NL-Means

Implementation

- NL-Means is painfully slow. There are 2 tricks to speed it up:

1. *Search Somewhat Local* -- Restrict the neighborhood comparisons to a window around the current pixel, rather than the whole image.

2. *Save Pixel Comparisons* -- Store all pixel-to-pixel comparisons in an efficient data structure to avoid making unnecessary passes through the image

Color Images

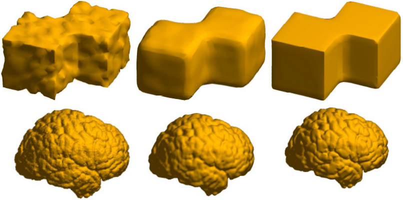
- We can extend NL-Means to color images by comparing 3D neighborhood cubes.
- NL-Means actually achieves better results on color images than on grayscale. We can shrink the neighborhood size because of the extra information.

Noisy

NL-Means

3D NL-Means

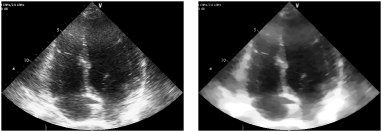
- NL-Means extends to 3D for surface denoising *(Dong-Ye-Osher-Dinov, 2007)*.
- NL-Means recovers corners and flattens smooth regions better than many denoising methods.



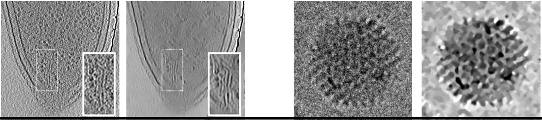
Noisy Mean Curvature Smoothing NL-Means

Ultrasound

- Nonlocal methods are good at removing the speckle noise in Ultrasound images *(Sawatzky, 2011)*.

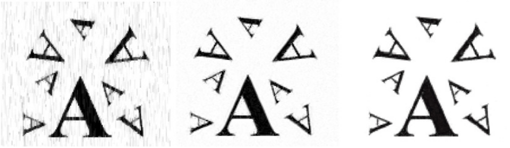


- NL-Means is particularly good for microscopy *(Darbon-Cunha-Chan-Osher-Jensen, 2008)*.



More Neighborhoods

- Standard NL-Means only compares neighborhoods under translation.
- We could also consider neighborhoods under scaling and rotation transformations *(Lou-Favaro-Soatto-Bertozzi, 2008)*.
- But this will slow down the computation.



Noisy NL-Means Extended NL-Means

Use with Caution

- Nonlocal methods are powerful denoisers, but they have to be used carefully.
- Exploiting image similarity could erase small-scale features that only occur in one part of the image.
- But these small features are often the most important part in some applications. For example, in medical imaging we look for tumors, hemorrhages, muscle tears, bone fractures, etc.