Communication Systems

Assignment 4

Master’s Degree in Communication engineering

3198

3213

3282

January 20, 2024

Politecnico di Torino

Table of Contents

Mirroring (1/2)

In this exercise, we aim to generate the transmitted OFDM signal using IFFT. We set $\Delta = 1$ and $N = 128$. To prevent power leakage outside our available spectrum due to the side lobes containing much power, we turned off $N_G = 10$ tones on both the left and right ends.
Given the vector $\underline{X}$ containing the N QAM symbols:

$$\underline{X} = (X_0, X_1, \ldots, X_i, \ldots, X_{N-1}),$$

we introduce a new vector $\underline{X'}$ with $N' = 2N$ elements obtained by mirroring:

$$\underline{X'} = (X_0, X_1, \ldots, X_{N-1}, 0, X^*_{N-1}, \ldots, X^*_i, \ldots, X^*_1).$$

Mirroring (2/2)

We know that:

$$s_m = \Re\left\{\sum_{i=0}^{N'-1} X'_i e^{j\frac{2\pi}{N'}im}\right\}.$$

Therefore, we applied the IFFT to the vector $\underline{X'}$ to obtain the $2N$ real samples of $\underline{s}$:

$$\underline{s} = \left(s_0, \ldots, s_m, \ldots, s_{N'-1}\right),$$

representing the samples of the transmitted OFDM signal.

Transmitting One Symbol

1 Exercise 1 - OFDM transmitted signal

All tones were turned off except for transmitting the symbol $X_{10} = -j$. The plot below shows that the corresponding transmitted signal is a sinusoidal wave with 10 periods.

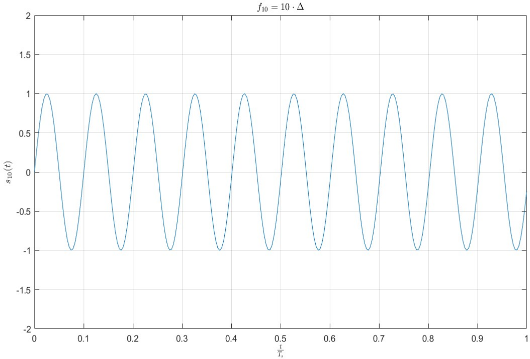


Figure: Transmitted signal for $X_{10} = -j$ showing a sinusoidal pattern with 10 periods.

Transmitting Over All Tones

1 Exercise 1 - OFDM transmitted signal

All tones were activated, excluding the external tones, with a 4-QAM symbol transmitted over each tone. The figure below shows the transmitted signal $s(t)$.

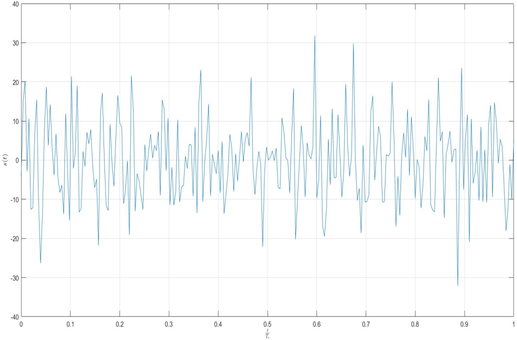


Figure: Transmitted OFDM signal $s(t)$

Histogram of The Amplitudes

The histogram of the amplitudes of the transmitted OFDM signal is plotted below. The histogram indicates that the transmitted OFDM signal approximates a Gaussian probability density function (pdf) with zero mean. This aligns with the characteristic that the transmitted OFDM signal resembles Gaussian noise with zero mean.

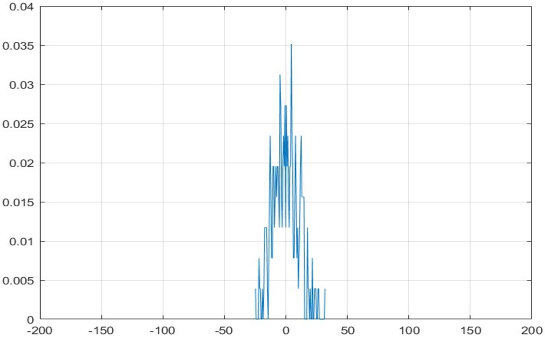


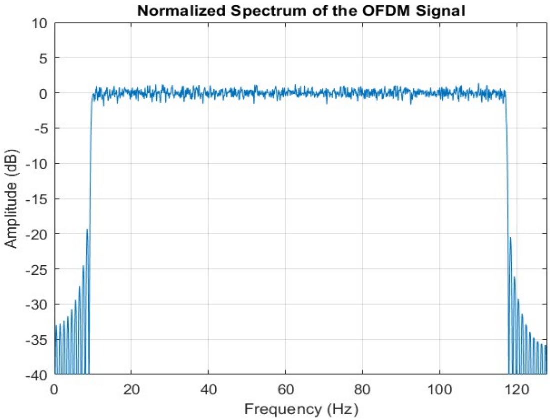
Table of Contents

Estimating OFDM Spectrum

In this exercise, we estimated the spectrum of the OFDM transmitted signal with a frequency resolution of $f_{\text{res}} = \frac{\Delta}{10} = 0.1$. Given that the sampling frequency is $f_{\text{sam}} = 2N\Delta = 256$, we considered windows of $\frac{f_{\text{sam}}}{f_{\text{res}}} = 2560$ samples. To achieve this, we generated a very long sequence containing many samples. The spectrum was estimated using the Welch method, where the signal is divided into segments of length equal to the window size, where no overlapping were used. The spectrum is computed over each window, and then the results are averaged over all segments.

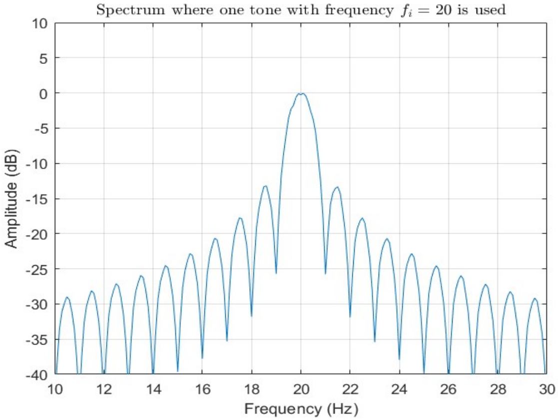
OFDM Spectrum - all tones

The Spectrum when all tones were used, excluding the external tones, was plotted in dB, normalized to have 0 dB maximum.



OFDM Spectrum - single tone

The Spectrum when only single tone with frequency $f_i = 20$ was used, was plotted in dB, also normalized to have 0 dB maximum.



In this exercise, we consider an OFDM system with $\Delta = 1$, $N = 128$, and carrier frequencies $f_i = i\Delta$.

We consider a channel with the following impulse response:

$$h(t) = a_1\delta(t - D_1) + a_2\delta(t - D_2) + a_3\delta(t - D_3) + a_4\delta(t - D_4),$$

where the delays are given by:

$$D_1 = 1, \quad D_2 = 1.01, \quad D_3 = 1.015, \quad D_4 = 1.02,$$

and the amplitudes are complex values with absolute magnitudes:

$$|a_1| = 1, \quad |a_2| = 0.5, \quad |a_3| = 0.9, \quad |a_4| = 0.3.$$

The phase of these amplitudes is random, uniformly distributed between 0 and 2π.

Hartogs-Hughes Algorithm (1/2)

We have randomly generated four phases. Given the impulse response $h(t)$, we computed the corresponding transfer function $H(f)$ using the Fourier transform, expressed as $H(f) = \mathcal{F}\{h(t)\}$:

$$
\begin{aligned}
H(f) &= \int h(t) e^{-j2\pi f t}\, dt && (1)\\
&= a_1 \cdot e^{-j2\pi f D_1} + a_2 \cdot e^{-j2\pi f D_2} \\
&\quad + a_3 \cdot e^{-j2\pi f D_3} + a_4 \cdot e^{-j2\pi f D_4} && (2)
\end{aligned}
$$

where $a_1 = |a_1| e^{j\phi_1}, \quad a_2 = |a_2| e^{j\phi_2}, \quad a_3 = |a_3| e^{j\phi_3}, \quad a_4 = |a_4| e^{j\phi_4}$

Hartogs-Hughes Algorithm (2/2)

Continuing from the previous slide, we evaluated $H_i = H(f = f_i)$ for $f_i = i\Delta$. The attenuation values were computed as $A_i = \frac{1}{|H_i|^2}$.

To apply the Hartogs-Hughes algorithm, we assume the normalized attenuations are $\alpha_i = A_i$, with the total transmission power $P_{\text{TX}} = \mathrm{median}(A_i) \times N$. The Hartogs-Hughes algorithm employs the bit loading formula:

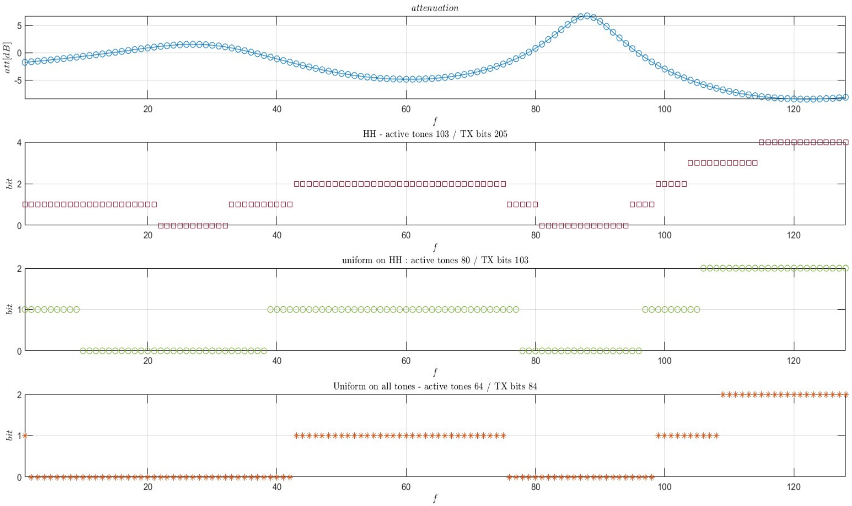
$$m_i \leq \left\lfloor \log_2 \left(1 + \frac{P_i}{\alpha_i} \right) \right\rfloor$$

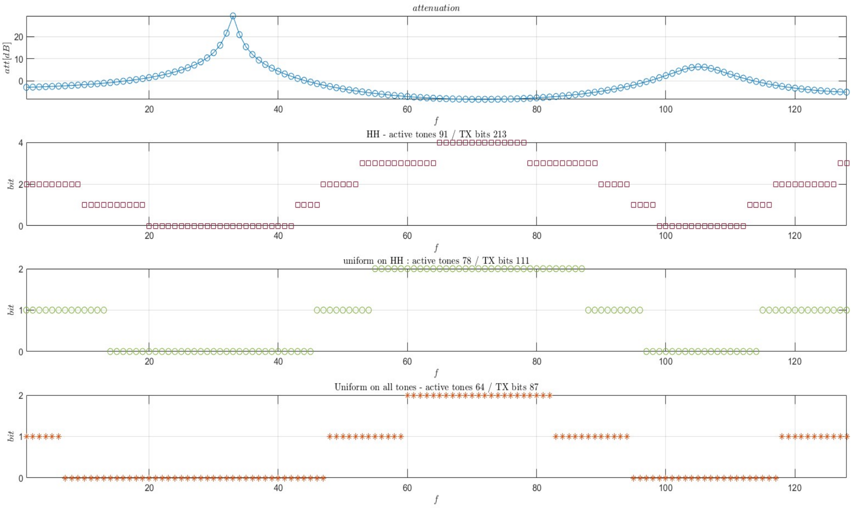
in order to determines the cost in terms of power for sending a bit on each channel, and the cost for each additional bit. Power is then allocated iteratively, selecting the channel with the minimum cost for bit transmission until all P_{TX} is allocated, or when the remaining power is less than the cost of the channel with the lowest additional bit cost. At the end of this process, we had determined the number of bits allocated for each channel, and correspondingly, the specific constellation that should be used on each channel.

Visualization of OFDM Bit Loading

The following figures illustrates the impact of (H-H) algorithm on power allocation and bit transmission. the random phases were generated twice to provide two figures where each is showing:

- Attenuation Plot in dB, showing the attenuation values across different tones.
- Bits per Tone - This plot shows the number of bits transmitted in each tone as determined by the H-H algorithm. The plot's title includes the total number of active tones and the cumulative number of transmitted bits.
- Uniform Power Distribution over Active Tones - Here, the total power P_{TX} is uniformly distributed over the tones identified as active by the H-H algorithm. The plot details the number of bits transmitted per active tone, with the title indicating the number of active tones and the total transmitted bits.
- Uniform Power Distribution over All Tones - In this plot, P_{TX} is uniformly distributed over all N tones, showing the number of bits transmitted per tone. The title again specifies the number of active tones and the total number of transmitted bits.


attenuation
att[dB]
5
0
-5
20
40
60
80
100
120
f
HH - active tones 103 / TX bits 205
bit
4
2
0
uniform on HH : active tones 80 / TX bits 103
bit
2
1
0
Uniform on all tones - active tones 64 / TX bits 84
bit
2
1
0



attenuation
att[dB]
20
10
0
20
40
60
80
100
120
f
HH - active tones 91 / TX bits 213
bit
4
2
0
uniform on HH : active tones 78 / TX bits 111
bit
2
1
0
Uniform on all tones - active tones 64 / TX bits 87
bit
2
1
0

Comments on Results (1/2)

The implications of applying the Hughes-Hartogs algorithm to an OFDM system with multiple tones include:

- **Effective Tone Utilization:** The Hughes-Hartogs algorithm effectively identifies a subset of active tones from the total available, focusing on those with higher Signal-to-Noise Ratios. This selective activation illustrates the algorithm's efficiency in utilizing the available bandwidth.
- **Uniform vs. Optimized Power Distribution:** Comparing uniform power distribution across all tones with the selective allocation by the Hughes-Hartogs algorithm reveals notable differences in performance. The algorithm's selective approach typically led to more efficient transmission, as opposed to the uniform distribution strategy.

Comments on Results (2/2)

- **Algorithm's Adaptability:** The Hughes-Hartogs algorithm shoewd its adaptability by effectively balancing power distribution. The contrast between uniform distribution and optimized allocation underlines the algorithm's capability to adjust according to channel conditions.
- **Strategic Power Allocation:** The results highlighted the importance of strategic power allocation in communication systems. By optimizing power distribution, the Hughes-Hartogs algorithm enhanced system performance, maximizing data throughput within the constraints of power budget.

Overall, the Hughes-Hartogs algorithm significantly enhances OFDM system performance by intelligently allocating power to optimize data transmission efficiency.