

$$A \rightarrow PLU$$

$$PA = LU$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 & 0 \\ l_{31} & & 1 & 0 & 0 \\ & & & 1 & 0 \\ l_{51} & l_{53} & & & 1 \end{bmatrix}$$

$$Ax = b$$

$$PAx = Pb$$

$$\underbrace{LU}x = Pb$$

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & 0 & u_{34} \\ 0 & 0 & 0 & u_{44} \end{bmatrix}$$

$$A = \mathbb{R}^{m \times n}$$

$$P \in \mathbb{R}^{m \times m}$$

$$L \in \mathbb{R}^{m \times m}$$

$$U \in \mathbb{R}^{m \times n}$$

$$m < n$$

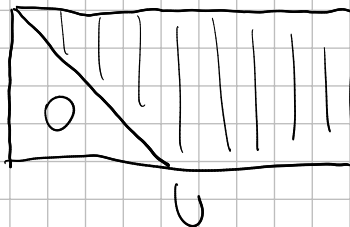
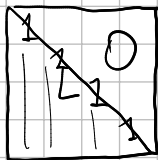
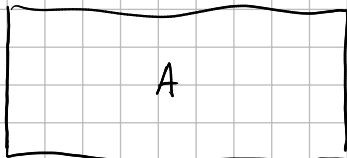
$$P \in \mathbb{R}^{m \times m}$$

$$L \in \mathbb{R}^{m \times n}$$

$$U \in \mathbb{R}^{n \times n}$$

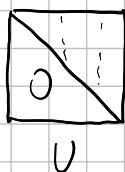
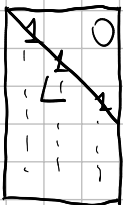
$$m > n$$

$$m < n$$



U

$$m > n$$



U

$$Ax = b$$

$$PAx = Pb$$

$$LUx = Pb$$

$$y = Ux$$

$$\boxed{Ly = Pb}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} Pb(1) \\ Pb(2) \\ Pb(3) \end{bmatrix}$$

$$y_1 = Pb(1)$$

$$y_2 = Pb(2) - l_{21} y_1$$

$$y_3 = Pb(3) - l_{31} y_1 - l_{32} y_2$$

$$Ux = y$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$x_3 = y_3 / u_{33}$$

$$x_2 = (y_2 - u_{23} x_3) / u_{22}$$

$$x_1 = (y_1 - u_{12} x_2 - u_{13} x_3) / u_{11}$$

$$\det(A) = ?$$

$$\det(PA) = \det(P) \det(A) = \det(LU) = \det(L) \det(U)$$

$$\begin{array}{cccc} \det(P) & \det(A) & \det(L) & \det(U) \\ \neq 0 & ? & 1 & \prod_{i=1}^n u_{ii} \\ & 0 & 1 & 0 \end{array}$$

$$\begin{bmatrix} 3 & 2 & -4 \\ 0 & \frac{1}{3} & -\frac{5}{3} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$$

$\cup$ 
 $x = y$

$$\frac{1}{3}y - \frac{5}{3}z = 2$$

$$y = \left(2 + \frac{5z}{3}\right) \cdot 3 = 6 + 5z$$

$$3x + 2(6 + 5z) - 4z = 6$$

$$3x + 12 + 10z - 4z = 6$$

$$3x + 6z = -6$$

$$x = (-6 - 6z)/3 = 2(1 + z)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 + 2z \\ 6 + 5z \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix} + z \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$$

$$1 \cdot x + 2 \cdot y + 0 \cdot z + 1w = 5$$

$$-1y + 5z + 1w = 2$$

$$-2z + 4w = 6$$

$$z = (6 - 4w) / (-2) = -3 + 2w$$

$$-y + 5(-3 + 2w) + w = 2$$

$$y = -17 + 11w$$

$$x + 2(-17 + 11w) + 1w = 5$$

$$x = 39 - 23w$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 39 - 23w \\ -17 + 11w \\ -3 + 2w \\ w \end{bmatrix} = \begin{bmatrix} 39 \\ -17 \\ -3 \\ 0 \end{bmatrix} + w \begin{bmatrix} -23 \\ 11 \\ 2 \\ 1 \end{bmatrix}$$