Algorithms: CSE 202 — Homework I

Problem 1: Next greater element

Given an array, print the Next Greater Element (NGE) for every element. The Next Greater Element of an item x is the first greater element to its right in the array.

Problem 2: Sorted matrix search

Given an $m \times n$ matrix in which each row and column is sorted in ascending order, design an algorithm to find an element.

Problem 3: Maximum overlap of two intervals

Design an algorithm that takes as input a list of intervals $[a_i, b_i]$ for $1 \leq i \leq n$ and outputs the length of the maximum overlap of two distinct intervals in the list.

Problem 4: 132 pattern

Given a sequence of n distinct positive integers $a_1, \ldots, a_n$, a 132-pattern is a subsequence a_i, a_j, a_k such that $i < j < k$ and $a_i < a_k < a_j$. For example: the sequence $31, 24, 15, 22, 33, 4, 18, 5, 3, 26$ has several 132-patterns including 15, 33, 18 among others. Design an algorithm that takes as input a list of n numbers and checks whether there is a 132-pattern in the list.

Problem 5: Toeplitz matrices

A *Toeplitz matrix* is an $n \times n$ matrix $A = (a_{ij})$ such that $a_{ij} = a_{i-1,j-1}$ for $i = 2, 3, \ldots, n$ and $j = 2, 3, \ldots, n$.

1. Is the sum of two Toeplitz matrices necessarily Toeplitz? What about the product?
2. Describe how to represent a Toeplitz matrix so that two $n \times n$ Toeplitz matrices can be added in $O(n)$ time.
3. Give an $O(n \lg n)$-time algorithm for multiplying an $n \times n$ Toeplitz matrix by a vector of length n. Use your representation from part (b).
4. Give an efficient algorithm for multiplying two $n \times n$ Toeplitz matrices. Analyze its running time.