DTProblog Example

The example below uses the decision theoretic problog program to execute the umbrella example.

In [1]:
```
from problog.tasks.dtproblog import dtproblog
from problog.program import PrologString
```

In [2]:
```
model = """
% probabilistic facts
0.3::rain.
0.5::wind.

% decision facts
?::umbrella.
?::raincoat.

broken_umbrella :- umbrella, rain, wind.
dry :- rain, raincoat.
dry :- rain, umbrella, not broken_umbrella.
dry :- not rain.

% utilities
utility(broken_umbrella, -40).
utility(raincoat, -20).
utility(umbrella, -2).
utility(dry, 60).
"""
```

In [3]:
```
program = PrologString(model)
decisions, score, statistics = dtproblog(program)
```

In [4]:
```
for name, value in decisions.items():
    print('{}: {}'.format(name, value))

print(decisions)
```

```
umbrella: 1
raincoat: 0
{umbrella: 1, raincoat: 0}
```

In [5]:
```
print(score)
```

```
43.00000000000001
```

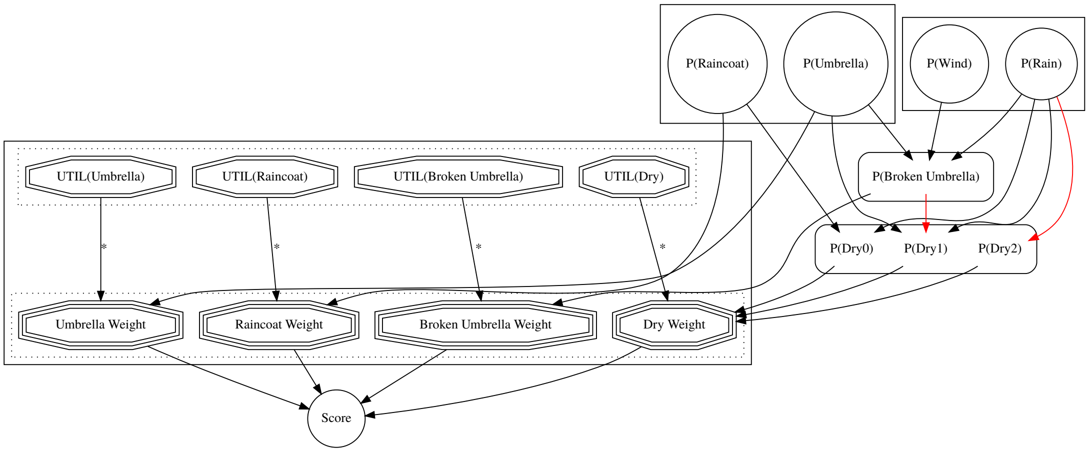
In [6]:
```
print(statistics)
```

```
{'eval': 4}
```

Bayesian Network

In [7]:
```
from graphviz import Digraph

g = Digraph('Bayesian Network', filename='fsm.gv')

g.attr(rank='same', size='10,10')
g.attr('node', shape='circle')

with g.subgraph(name='clusterA') as c:
    c.node('P(Rain)')
    c.node('P(Wind)')

with g.subgraph(name='clusterB') as c:
    c.node('P(Umbrella)')
    c.node('P(Raincoat)')

with g.subgraph(name='clusterC') as c:
    c.attr(style='rounded')
    c.attr('node', shape='none')
    c.node('P(Broken Umbrella)')

with g.subgraph(name='clusterH') as h:
    with h.subgraph(name='clusterD') as c:
        c.attr(style='dotted')
        c.attr('node', shape='tripleoctagon')
        c.node('Dry Weight')
        c.node('Broken Umbrella Weight')
        c.node('Umbrella Weight')
        c.node('Raincoat Weight')

    with h.subgraph(name='clusterE') as c:
        c.attr(style='dotted')
        c.attr('node', shape='doubleoctagon')
        c.node('UTIL(Broken Umbrella)')
        c.node('UTIL(Dry)')
        c.node('UTIL(Umbrella)')
        c.node('UTIL(Raincoat)')

with g.subgraph(name='clusterF') as c:
    c.attr(style='rounded')
    c.attr('node', shape='none')
    c.edge('P(Rain)', 'P(Dry0)')
    c.edge('P(Raincoat)', 'P(Dry0)')

    c.edge('P(Rain)', 'P(Dry1)')
    c.edge('P(Umbrella)', 'P(Dry1)')
    c.edge('P(Broken Umbrella)', 'P(Dry1)', color='red')

    c.edge('P(Rain)', 'P(Dry2)', color='red')

with g.subgraph(name='clusterG') as c:
    c.edge('P(Umbrella)', 'P(Broken Umbrella)')
    c.edge('P(Rain)', 'P(Broken Umbrella)')
    c.edge('P(Wind)', 'P(Broken Umbrella)')

g.edge('P(Dry0)', 'Dry Weight')
g.edge('P(Dry1)', 'Dry Weight')
g.edge('P(Dry2)', 'Dry Weight')
g.edge('UTIL(Dry)', 'Dry Weight', label='*')

g.edge('P(Broken Umbrella)', 'Broken Umbrella Weight')
g.edge('UTIL(Broken Umbrella)', 'Broken Umbrella Weight', label='*')

g.edge('P(Umbrella)', 'Umbrella Weight')
```

Out[7]:


P(Raincoat)
P(Umbrella)
P(Wind)
P(Rain)
UTIL(Umbrella)
UTIL(Raincoat)
UTIL(Broken Umbrella)
UTIL(Dry)
P(Broken Umbrella)
P(Dry0)
P(Dry1)
P(Dry2)
Umbrella Weight
Raincoat Weight
Broken Umbrella Weight
Dry Weight
Score

The Math

From code to equations + reductions using Algebra of Sets:

```
% probabilistic facts
0.3::rain.
0.5::wind.

% decision facts
?::umbrella.
?::raincoat.

broken_umbrella :- umbrella, rain, wind.
dry :- rain, raincoat.
dry :- rain, umbrella, not broken_umbrella.
dry :- not rain.

% utilities
utility(broken_umbrella, -40).
utility(raincoat, -20).
utility(umbrella, -2).
utility(dry, 60).
```

DT-Problog Utilities

Taken from: *DTProbLog: A Decision-Theoretic Probabilistic Prolog*

A single utility is defined as:
$$ \text{Util}(a_i|\sigma,\mathcal{DT}) = r_i \cdot P(u_i|\sigma(\mathcal{DT})) $$

- u_i is a literal
- r_i is the reward/consequence for achieving u_i
- $\mathcal{DT}$ Problog program
- σ strategy

Total Utility:
$$ \text{Util}(\sigma, \mathcal{DT}) = \sum_{a_i \in U} \text{Util}(a_i|\sigma(\mathcal{DT})) = \text{Util}(\sigma(\mathcal{DT})) $$

When faced with a decision problem, one is interested in computing the optimal strategy, which is, the maximum expected utility principle:
$$ \sigma^* = \text{argmax}_\sigma \text{Util}(\sigma(\mathcal{DT})) $$

This is the *solution* to the decision problem.

Equations

We are provided the following probabilistic facts about the weather:
$$ \begin{align} P(\text{rain}) &= 0.3\\ P(\text{wind}) &= 0.5 \end{align} $$

We decide to choose from the following options, which are shown as probabilities to fit in with the math:
$$ \begin{align} P(\text{umbrella}) &= \text{?}\\ P(\text{raincoat}) &= \text{?} \end{align} $$

Python Implementation

The code below uses regular python to achieve the same result as the umbrella example shown above.

In [8]:

```python
# compute total utility given that decisions are mutually exclusive

# define facts
pRain = 0.3
pWind = 0.5

# define utility
broken_umbrella_utility = -40
raincoat_utility = -20
umbrella_utility = -2
dry_utility = 60

def total_utility(pUmbrella, pRainCoat):
    pDry = (1-pRain) + pRain*pUmbrella*(1-pWind) + pRain*pRainCoat
    pBrokenUmbrella = pUmbrella*pRain*pWind

    print('Dry Weight: {}'.format(pDry * dry_utility))
    print('Broken Umbrella Weight: {}'.format(pBrokenUmbrella * broken_umbrella_utility))
    print('Umbrella Weight: {}'.format(pRainCoat * raincoat_utility))
    print('Raincoat Weight: {}'.format(pUmbrella * umbrella_utility))

    # compute and return the score
    return (pDry * dry_utility +
            pBrokenUmbrella * broken_umbrella_utility +
            pRainCoat * raincoat_utility +
            pUmbrella * umbrella_utility)

print('-------------------------------------------')
print('Umbrella Option:')
umbrella_score = total_utility(1., 0.)
print('-------------------------------------------')


print('\n-------------------------------------------')
print('Raincoat Option:')
raincoat_score = total_utility(0., 1.)
print('-------------------------------------------\n')

print('Umbrella Score: {}'.format(umbrella_score))
print('Raincoat Score: {}'.format(raincoat_score))
```

```
-------------------------------------------
Umbrella Option:
Dry Weight: 51.0
Broken Umbrella Weight: -6.0
Umbrella Weight: -0.0
Raincoat Weight: -2.0
-------------------------------------------

-------------------------------------------
Raincoat Option:
Dry Weight: 60.0
Broken Umbrella Weight: -0.0
Umbrella Weight: -20.0
Raincoat Weight: -0.0
-------------------------------------------

Umbrella Score: 43.0
Raincoat Score: 40.0
```

Nengo DTProblog Implementation

The code below uses nengo to achieve the same result as the umbrella example shown above.

Within Nengo we directly compute, using ensembles and pre-built networks, the following equations:

$$ \begin{align} \text{Util}(\sigma(\mathcal{DT})) &= \sum_{a_i \in U} \text{Util}(a_i|\sigma(\mathcal{DT}))\\ &= \text{Util}(\text{broken umbrella}| \sigma, \mathcal{DT}) + \text{Util}(\text{raincoat}| \sigma, \mathcal{DT}) + \text{Util}(\text{umbrella}| \sigma, \mathcal{DT}) + \text{Util}(\text{dry}| \sigma, \mathcal{DT})\\ &= r_\text{broken umbrella} \cdot P(\text{broken umbrella}) + r_\text{raincoat} \cdot P(\text{raincoat}) + r_\text{umbrella} \cdot P(\text{umbrella}) + r_\text{dry} \cdot P(\text{dry}) \end{align} $$

The pre-build networks of the Basal Ganglia and Thalamus can be used to compute the equation which operate as a WTA and a selector:

$$ \sigma^* = \text{argmax}_{\sigma \in \{P(\text{umbrella}) = 1, P(\text{raincoat}) = 1,\}} \text{Util}(\sigma(\mathcal{DT})) $$

The main pre-built networks used are:

- `nengo.networks.Product()`
- `nengo.networks.BasalGanglia()`
- `nengo.networks.Thalamus()`

The dimensions of the BasalGanglia and the Thalamus are 2, essentially one dimension per decision. The first dimension is the choice of `Umbrella` and the second is `Raincoat`. Ensembles are used to hold the probability for intermediate calculations, which store a value between 0 and 1 (thus we can define the radius).

Scaling is performed on the rewards such that the rewards are between 0 and 1 to fit within the ensemble radius. This allows for compression of the ensemble's dynamic range.

Sizes of the ensembles (`n_neurons`) are chosen arbitary large enough to avoid error. This can be reduced for efficiency.

Decision Changes

From the equations shown above, note that we can observe a decision change by manipulating the equations. Here we can see that `Umbrella` is chosen initially as it has a score of 43 which is greater than the score of when we chose `Raincoat` which is 40.

We can manipulate the equations such that we can choose `Raincoat`. One way of doing this is modifying the reward/cost of choosing `Raincoat` from -20 to -16. As a result, the score from choosing `Umbrella` would remain 43, but the score from choosing `Raincoat` would be 44, causing `Raincoat` to be chosen.

Implementing this to partly verify that Nengo is working as expected, we adjust the cost of `Raincoat` as time progresses by making it a function of time.

```python
def raincoat_cost(t):
    return -20. + 4. * t
```

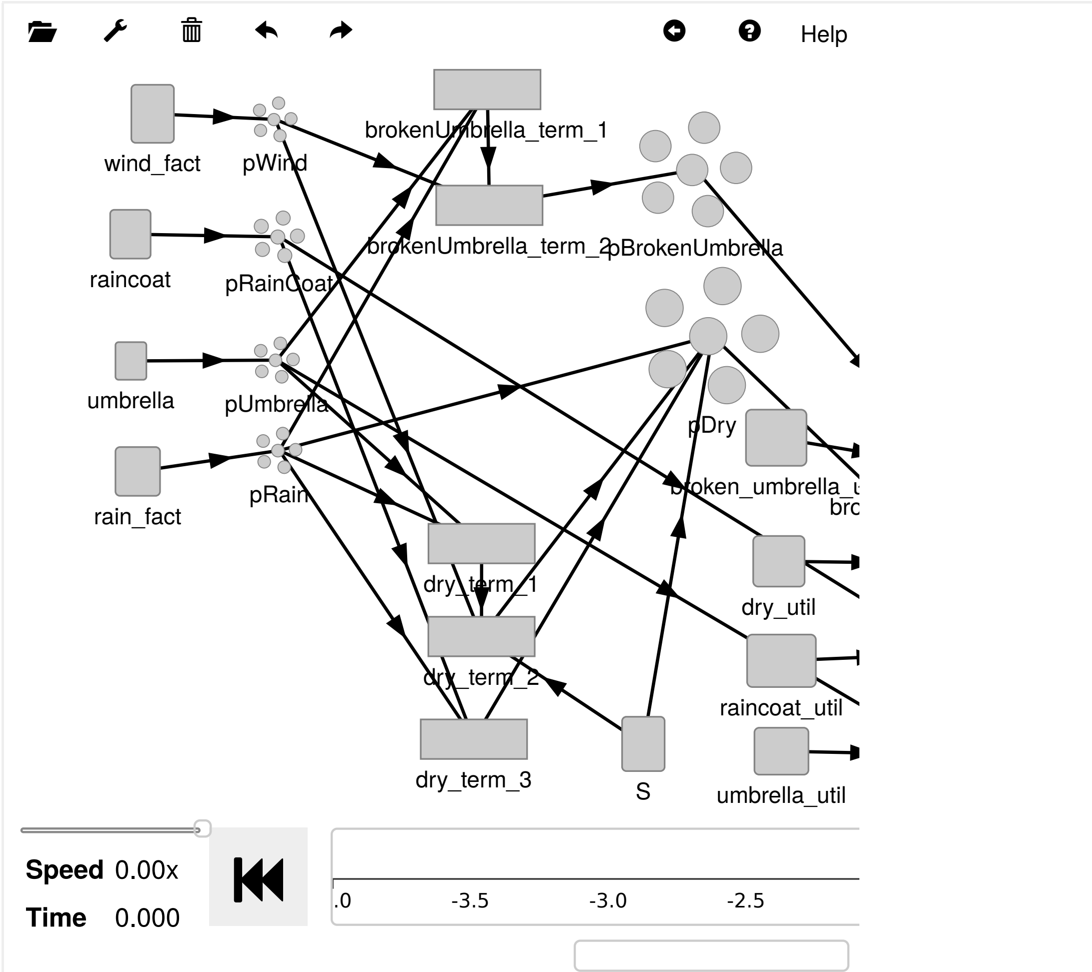
By adding this adjustment, we should observe a change in decision as time progresses. More specifically, around $t=1$, the decision should change.

In [11]:

```
import nengo

def raincoat_cost(t):
    return -20. + 4.*(t%5.)
    #return -20. + 4.*t

rain = 0.3
wind = 0.5
model = nengo.Network('Umbrella Example', seed=8)

n_neurons = 1000
with model:
    # define facts
    rain_fact = nengo.Node([rain])
    wind_fact = nengo.Node([wind])

    # define rewards
    broken_umbrella_util = nengo.Node([-40])
    #raincoat_util = nengo.Node([-20])
    raincoat_util = nengo.Node(raincoat_cost)
    umbrella_util = nengo.Node([-2])
    dry_util = nengo.Node([60])

    # world set (for NOT calculations)
    S = nengo.Node([1])

    # terms for computing total utility and constructing bayesian network
    pDry = nengo.Ensemble(n_neurons, 2)
    pRain = nengo.Ensemble(n_neurons, 1)
    pWind = nengo.Ensemble(n_neurons, 1)
    pUmbrella = nengo.Ensemble(n_neurons, 2)
    pRainCoat = nengo.Ensemble(n_neurons, 2)
    pBrokenUmbrella = nengo.Ensemble(n_neurons, 1)

    # define umbrella and raincoat
    umbrella = nengo.Node([1, 0])
    raincoat = nengo.Node([0, 1])
    nengo.Connection(umbrella, pUmbrella)
    nengo.Connection(raincoat, pRainCoat)

    # construct pRain
    nengo.Connection(rain_fact, pRain)

    # construct pWind
    nengo.Connection(wind_fact, pWind)

    # construct pDry, size 2 (umbrella, raincoat)
    # (1-pRain)
    nengo.Connection(S, pDry, transform=[[1], [1]])
    nengo.Connection(pRain, pDry, transform=[[-1], [-1]])

    # pRain*pUmbrella
    dry_term_1 = nengo.networks.Product(n_neurons, 1, label='dry_term_1')
    nengo.Connection(pRain, dry_term_1.input_a)
    nengo.Connection(pUmbrella[0], dry_term_1.input_b)

    # pRain*pUmbrella*(1-pWind)
    dry_term_2 = nengo.networks.Product(n_neurons, 1, label='dry_term_2')
    nengo.Connection(dry_term_1.output, dry_term_2.input_a)
    nengo.Connection(S, dry_term_2.input_b)
    nengo.Connection(pWind, dry_term_2.input_b, transform=-1)
    nengo.Connection(dry_term_2.output, pDry[0])
```

```
Model uses 20100 neurons
```

In [12]:
```python
# View Model
from nengo_gui.ipython import IPythonViz
IPythonViz(model)
```



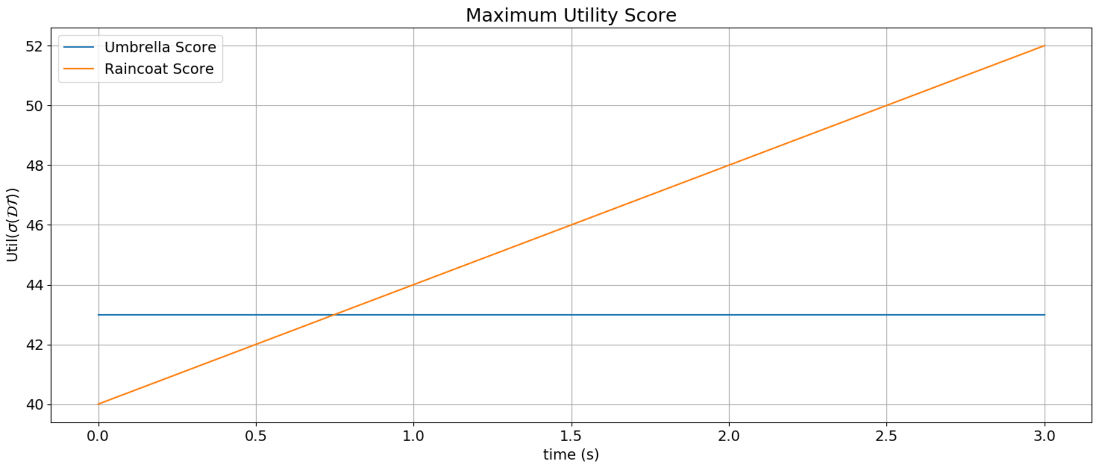
In [13]:
```python
with nengo.Simulator(model) as sim:
    sim.run(3)
```

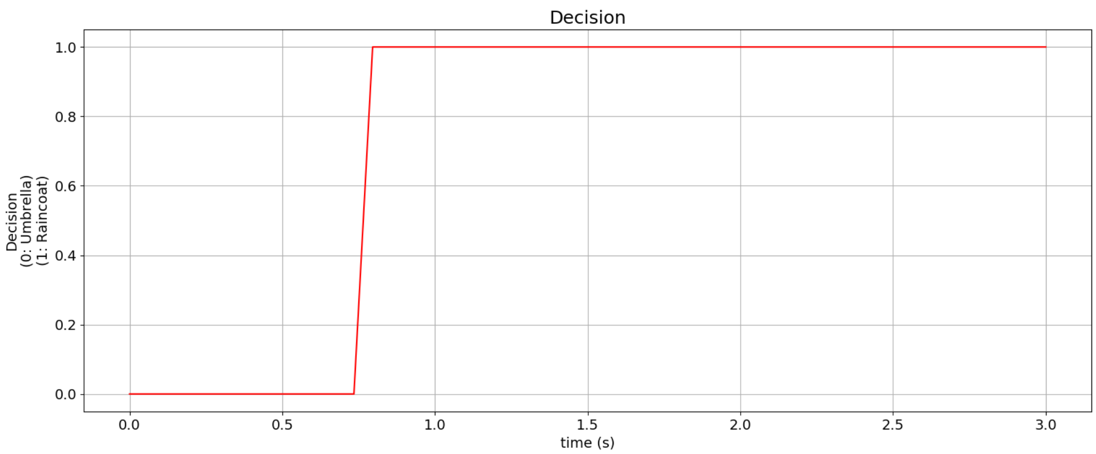
Build finished in 0:00:06.

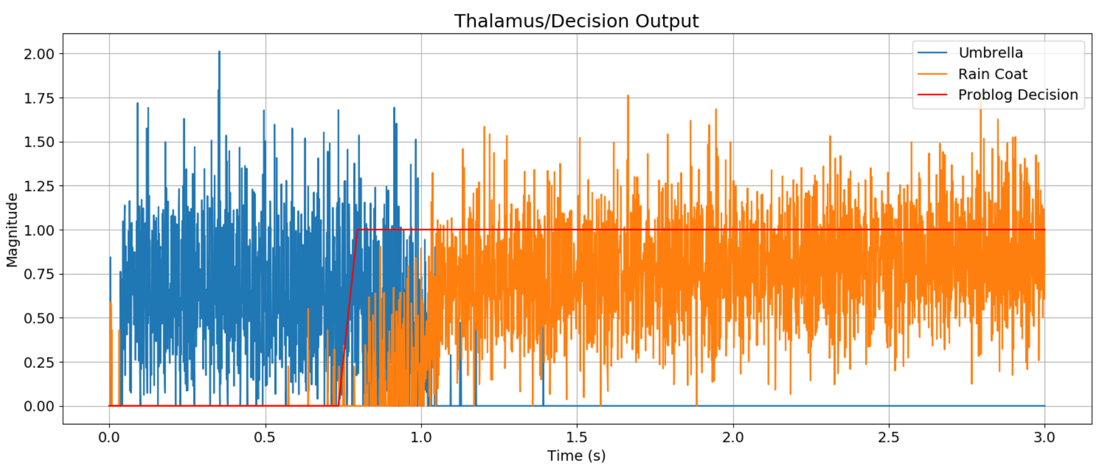
Simulation finished in 0:00:19.

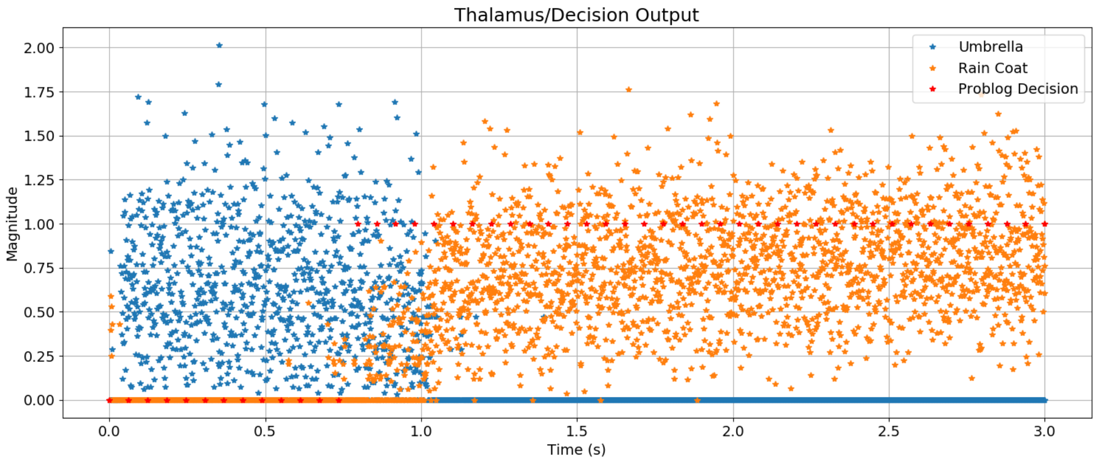
In [14]:
```
import matplotlib.pyplot as plt

plt.rcParams['figure.figsize'] = [18, 16]
plt.rcParams['figure.dpi'] = 100
#plt.rcParams['figure.subplot.top'] = 1
plt.rcParams['figure.titlesize'] = 18
plt.rcParams['axes.titlesize'] = 18
plt.rcParams['font.size'] = 14

#plt.figure(figsize=(18, 16), dpi=80, facecolor='w', edgecolor='k')
plt.figure()

plt.subplot(2, 1, 1)
plt.plot(sim.trange(), sim.data[p_dry_weight], '*')
plt.plot(sim.trange(), sim.data[p_brokenUmbrella_weight], '*')
plt.plot(sim.trange(), sim.data[p_raincoat_weight], '*')
plt.plot(sim.trange(), sim.data[p_umbrella_weight], '*')
plt.legend(('Dry', 'Broken Umbrella', 'Raincoat', 'Umbrella'), loc='best')
plt.title('Weighted Utilities')
plt.xlabel('Time (s)')
plt.ylabel('Magnitude')
plt.grid()

plt.subplot(2, 1, 2)
plt.plot(sim.trange(), sim.data[p_BG_in], '*')
plt.legend(('BG_in_0', 'BG_in_1',), loc='best')
plt.title('BG Input')
plt.xlabel('Time (s)')
plt.ylabel('Magnitude')
plt.grid()

plt.figure()

plt.subplot(2, 1, 1)
plt.plot(sim.trange(), sim.data[p_BG_out], '*')
plt.legend(('BG_out_0', 'BG_out_1',), loc='best')
plt.title('BG Output')
plt.xlabel('Time (s)')
plt.ylabel('Magnitude')
plt.grid()

plt.subplot(2, 1, 2)
plt.plot(sim.trange(), sim.data[p_TH_out], '*')
plt.legend(('Umbrella', 'Rain Coat',), loc='best')
plt.title('TH Output')
plt.xlabel('Time (s)')
plt.ylabel('Magnitude')
plt.grid()
```

In [16]:
```
# redefine the score computation with parameterized utilities
import numpy as np

# compute total utility given that decisions are mutually exclusive
def total_utility(pUmbrella, pRainCoat,

    # define facts
    pRain = 0.3,
    pWind = 0.5,

    # define utility
    broken_umbrella_utility = -40,
    raincoat_utility = -20,
    umbrella_utility = -2,
    dry_utility = 60):

    pDry = (1-pRain) + pRain*pUmbrella*(1-pWind) + pRain*pRainCoat
    pBrokenUmbrella = pUmbrella*pRain*pWind

    #print('Dry Weight: {}'.format(pDry * dry_utility))
    #print('Broken Umbrella Weight: {}'.format(pBrokenUmbrella * broken_umbrell
a_utility))
    #print('Umbrella Weight: {}'.format(pRainCoat * raincoat_utility))
    #print('Raincoat Weight: {}'.format(pUmbrella * umbrella_utility))

    # compute and return the score
    return (pDry * dry_utility +
            pBrokenUmbrella * broken_umbrella_utility +
            pRainCoat * raincoat_utility +
            pUmbrella * umbrella_utility)

# sample time
t = np.linspace(0, 3)

#print('--------------------------------------------')
#print('Umbrella Scores Over Time:')
umbrella_score = np.squeeze([total_utility(1., 0., raincoat_utility=rc_util) fo
r rc_util in [-20 + 4*t]])
#print(umbrella_score)
#print('--------------------------------------------')


#print('\n--------------------------------------------')
#print('Raincoat Scores Over Time:')
raincoat_score = np.squeeze([total_utility(0., 1., raincoat_utility=rc_util) fo
r rc_util in [-20 + 4*t]])
#print(raincoat_score)
#print('--------------------------------------------\n')

# compute maximum expected utility principle, set umbrella at index 0, raincoat
at index 1
decision = np.argmax((umbrella_score, raincoat_score), axis=0)


# plot
plt.rcParams['figure.figsize'] = [18, 7]
plt.rcParams['figure.dpi'] = 100
#plt.rcParams['figure.subplot.top'] = 1
plt.rcParams['figure.titlesize'] = 18
plt.rcParams['axes.titlesize'] = 18
plt.rcParams['font.size'] = 14

plt.figure()
```


Maximum Utility Score
Umbrella Score
Raincoat Score
Util(σ(𝒟𝒯))
time (s)


Decision
Decision
(0: Umbrella)
(1: Raincoat)
time (s)


Thalamus/Decision Output
Umbrella
Rain Coat
Problog Decision
Magnitude
Time (s)

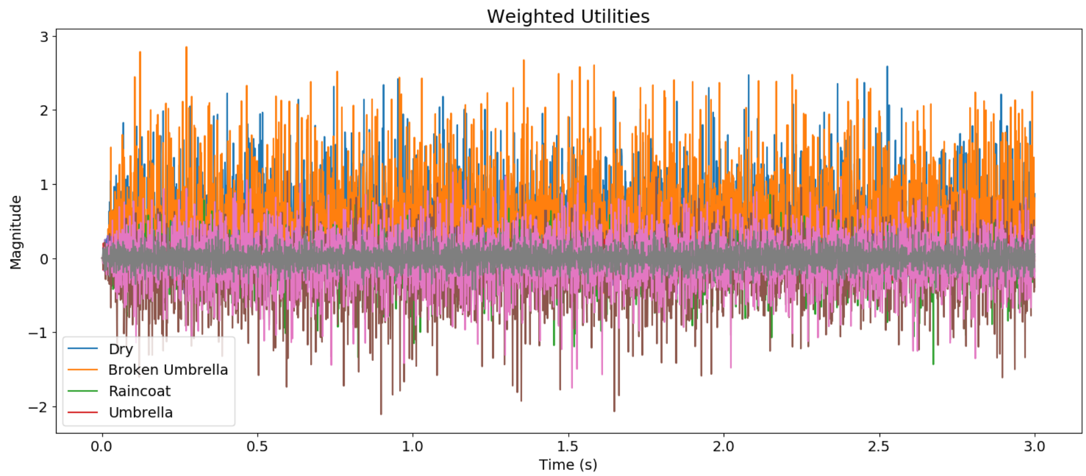


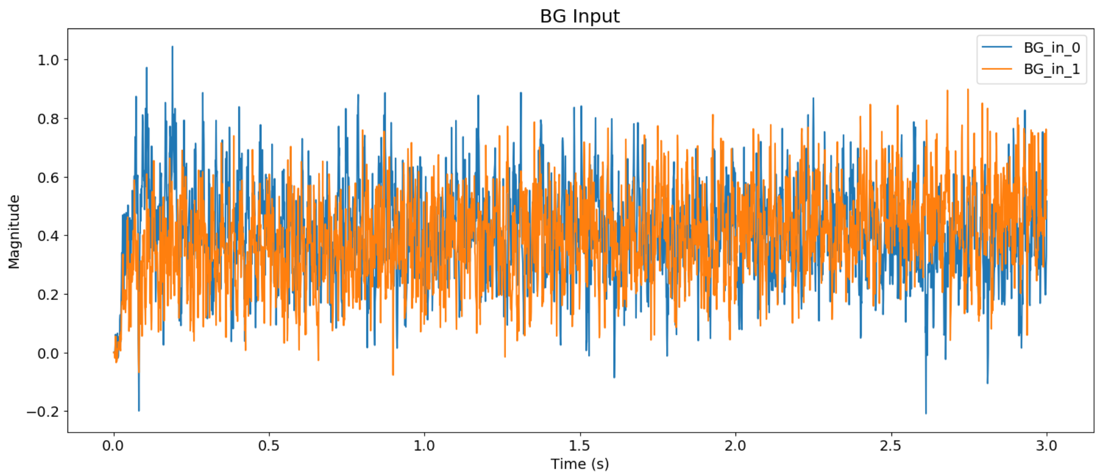
In [24]:
```
import nengo_loihi as loihi
loihi.set_defaults()
with loihi.Simulator(model) as sim:
    sim.run(3)
```

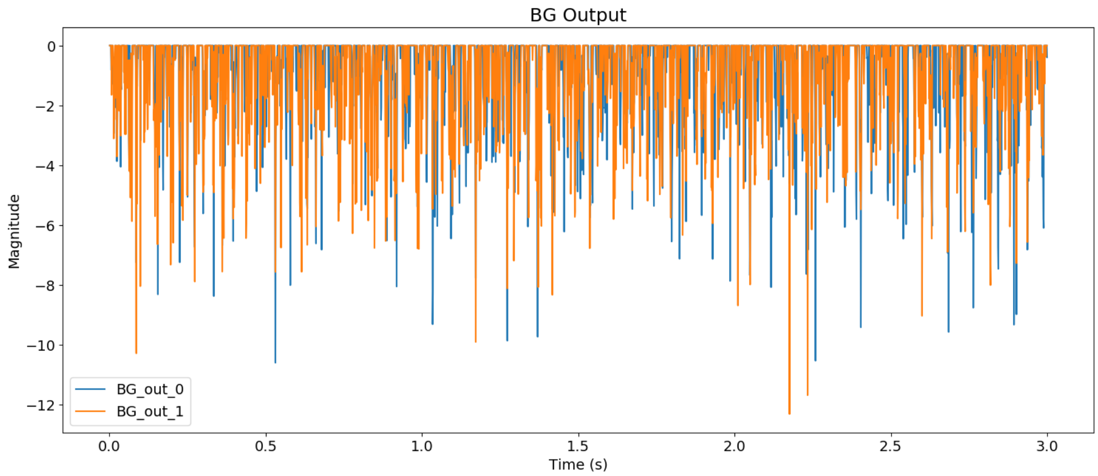
```
/home/turtlebot/.virtualenvs/turtlebot_py3/lib/python3.6/site-packages/nengo_lo
ihi/builder/ensemble.py:35: UserWarning: Intercepts are larger than intercept l
imit (0.95). High intercept values cause issues when discretizing the model for
running on Loihi.
  "the model for running on Loihi." % intercept_limit)
/home/turtlebot/.virtualenvs/turtlebot_py3/lib/python3.6/site-packages/nengo_lo
ihi/discretize.py:396: UserWarning: Lost 1 extra bits in weight rounding
  warnings.warn("Lost %d extra bits in weight rounding" % (-s2,))
/home/turtlebot/.virtualenvs/turtlebot_py3/lib/python3.6/site-packages/nengo_lo
ihi/discretize.py:396: UserWarning: Lost 2 extra bits in weight rounding
  warnings.warn("Lost %d extra bits in weight rounding" % (-s2,))
```

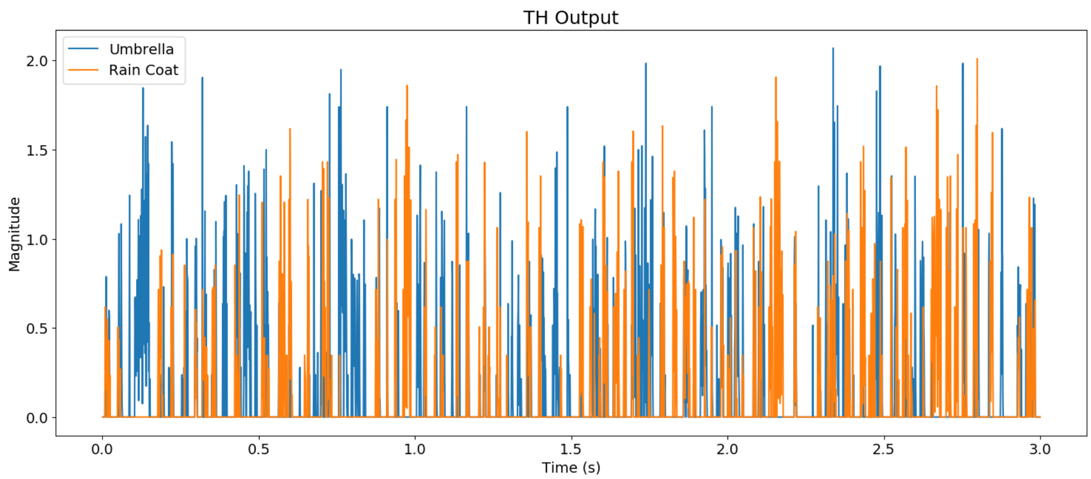
In [25]:
```
plt.rcParams['figure.figsize'] = [18, 16]
plt.rcParams['figure.dpi'] = 100
#plt.rcParams['figure.subplot.top'] = 1
plt.rcParams['figure.titlesize'] = 18
plt.rcParams['axes.titlesize'] = 18
plt.rcParams['font.size'] = 14

plt.figure()

plt.subplot(2, 1, 1)
plt.plot(sim.trange(), sim.data[p_dry_weight])
plt.plot(sim.trange(), sim.data[p_brokenUmbrella_weight])
plt.plot(sim.trange(), sim.data[p_raincoat_weight])
plt.plot(sim.trange(), sim.data[p_umbrella_weight])
plt.legend(('Dry', 'Broken Umbrella', 'Raincoat', 'Umbrella'), loc='best')
plt.title('Weighted Utilities')
plt.xlabel('Time (s)')
plt.ylabel('Magnitude')

plt.subplot(2, 1, 2)
plt.plot(sim.trange(), sim.data[p_BG_in])
plt.legend(('BG_in_0', 'BG_in_1',), loc='best')
plt.title('BG Input')
plt.xlabel('Time (s)')
plt.ylabel('Magnitude')

plt.figure()

plt.subplot(2, 1, 1)
plt.plot(sim.trange(), sim.data[p_BG_out])
plt.legend(('BG_out_0', 'BG_out_1',), loc='best')
plt.title('BG Output')
plt.xlabel('Time (s)')
plt.ylabel('Magnitude')

plt.subplot(2, 1, 2)
plt.plot(sim.trange(), sim.data[p_TH_out])
plt.legend(('Umbrella', 'Rain Coat',), loc='best')
plt.title('TH Output')
plt.xlabel('Time (s)')
plt.ylabel('Magnitude')
```

Out[25]: Text(0, 0.5, 'Magnitude')









\begin{thebibliography}{1} \bibitem{dtProblog} Van den Broeck, Guy et al. "DTProbLog: A Decision-Theoretic Probabilistic Prolog." Twenty-Fourth AAAI Conference on Artificial Intelligence. 2010. \end{thebibliography}