SYDE 552-750 Assignment: LNP Neurons

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March 22, 2016

```
In [20]: %pylab inline
         import numpy as np
         import matplotlib.pyplot as plt
         import scipy.signal
         from scipy.optimize import curve_fit
         plt.rcParams['lines.linewidth'] = 4
         plt.rcParams['font.size'] = 20
```

```
Populating the interactive namespace from numpy and matplotlib
```

1 Spike Statistics

1.1 Generate 50 10-second trials of Poisson spikes at 25 spikes/s with a 5ms absolute refractory period. To do this, draw samples from an appropriate ISI distribution.

```
In [21]: def generate_poisson_spikes(T,dt,trials,rate,rng):

                 #I'll implement the 5ms refractory period as using dt=5ms.
                 spike_raster=[]
                 spike_times=[]
                 for trial in range(trials):
                         spike_raster_trial=[]
                         spike_times_trial=[]
                         for t in range(int(T/dt)):
                                 spike_here=(rng.rand()<rate*dt)
                                 spike_raster_trial.append(1*spike_here)
                                 if spike_here:
                                         spike_times_trial.append(t*dt)
                         spike_raster.append(spike_raster_trial)
                         spike_times.append(spike_times_trial)
                 return np.array(spike_raster),np.array(spike_times)

         def get_ISI(spike_times):

                 #calculate across all trials
                 ISI=[]
                 for trial in range(len(spike_times)):
                         for t in range(len(spike_times[trial])-1):
                                 ISI.append(spike_times[trial][t+1]-spike_times[trial][t])
                 return ISI

         def get_CV(ISI):
```

```
        return np.std(ISI)/np.average(ISI)

def get_fano_factor(spike_raster,t_range):

        count=[np.sum(raster[:t_range]) for raster in spike_raster]
        avg=np.average(count)
        if avg != 0:
                var=np.var(count)
        else:
                var=0
        return var/avg

def get_fano_factor_list(T,dt,trials,rate,rng,n_FFs,t_range):

        FF_list=[]
        for i in range(n_FFs):
                spike_raster, spike_times = generate_poisson_spikes(T,dt,trials,rate,rng)
                FF_i=get_fano_factor(spike_raster,t_range)
                FF_list.append(FF_i)
        return FF_list
```

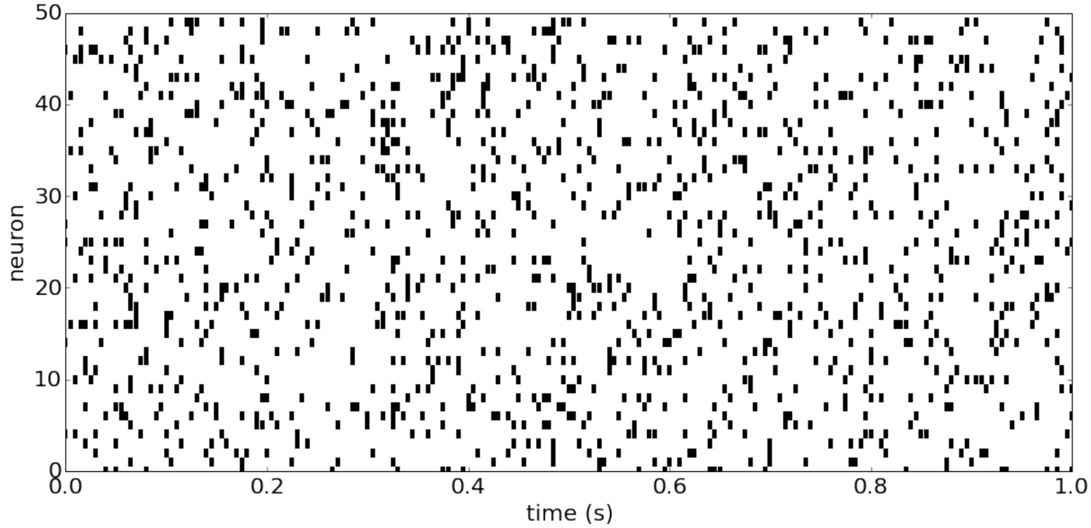
1.2 Plot the spike raster (1s) and the ISI histogram (10ms bins from 0-500ms

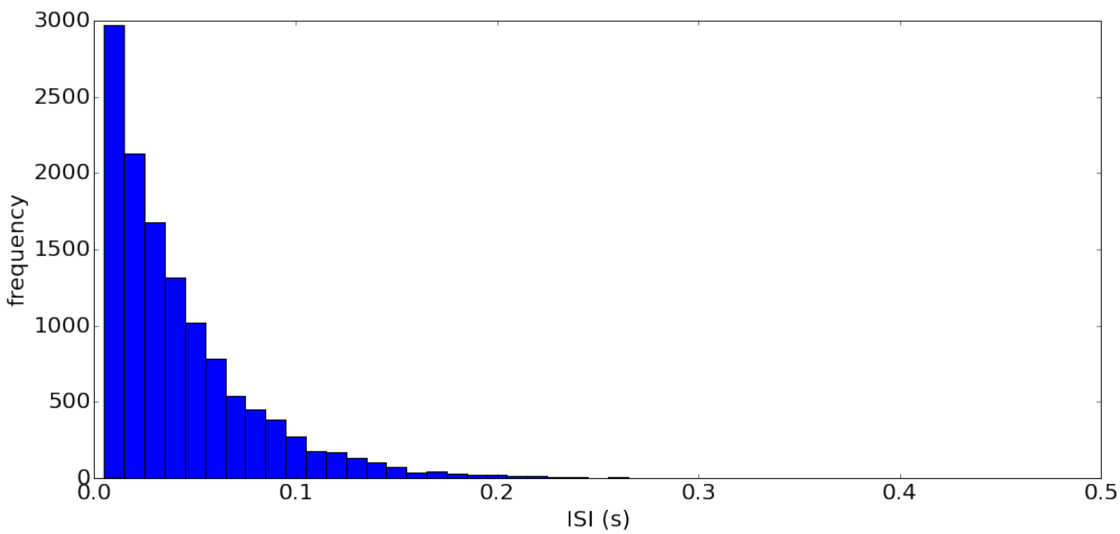
1.3 Calculate the coefficient of variation of the spike rate over all trials.

1.4 Calculate the Fano factor for the first 100ms. Comment on the consistency of the Fano factor over multiple runs. Comment on the difference in Fano factors with 5ms vs 1ms refractory period.

```
In [22]: def one():

                T=10 #seconds
                dt=0.005
                trials=50
                rate=25 #Hz
                seed=3
                t=np.arange(0,T,dt)
                rng=np.random.RandomState(seed=seed)

                spike_raster, spike_times = generate_poisson_spikes(T,dt,trials,rate,rng)

                ISI = get_ISI(spike_times)
                n_bins=int(np.max(ISI)/(2*dt))

                #Plot the spike raster for first 1.0 seconds
                fig=plt.figure(figsize=(16,16))
                ax=fig.add_subplot(211)
                ax.eventplot(spike_times,colors=[[0,0,0]])
                ax.set_xlim(0,1.0)
                ax.set_ylim(0,trials)
                ax.set_xlabel('time (s)')
                ax.set_ylabel('neuron')
```

```
    #plot ISI histogram
    ax=fig.add_subplot(212)
    ax.hist(ISI,n_bins)
    ax.set_xlim(0,dt*100)
    ax.set_xlabel('ISI (s)')
    ax.set_ylabel('frequency')
    plt.show()

    CV = get_CV(ISI)
    print "The coefficient of variation is", CV

    t_range=int(0.100/dt)
    n_FFs = 50

    FF_list_1 = get_fano_factor_list(T,dt,trials,rate,rng,n_FFs,t_range)
    print "Fano Factor for $t_{ref}=%s$, %s trials:" %(dt, n_FFs)
    print "mean: %s" %np.average(FF_list_1), "std: %s" %np.std(FF_list_1)

    dt=0.001
    FF_list_2 = get_fano_factor_list(T,dt,trials,rate,rng,n_FFs,t_range)
    print "Fano Factor for $t_{ref}=%s$, %s trials:" %(dt, n_FFs)
    print "mean: %s" %np.average(FF_list_2), "std: %s" %np.std(FF_list_2)

one()
```






```
The coefficient of variation is 0.919684065222
Fano Factor for $t_{ref}=0.005$, 50 trials:
mean: 0.839471102108 std: 0.176345519687
Fano Factor for $t_{ref}=0.001$, 50 trials:
mean: 0.942815010008 std: 0.191469341661
```

The Fano factor is significantly below 1 with a 5ms refractory period (mean=0.83), but will sometimes reach the expected value of 1.0 due to its high standard deviation across trials (std=0.18).

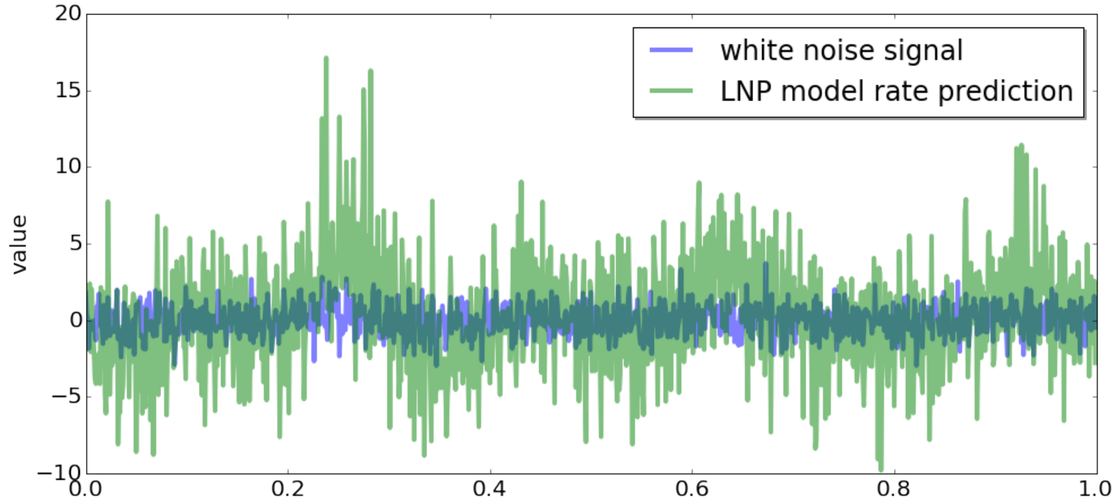
A smaller refractory period of 1ms brings the Fano factor significantly closer to 1 (mean=0.94), but the standard deviation across trials remains as high (or higher) as with the long refractory period (std=0.19).

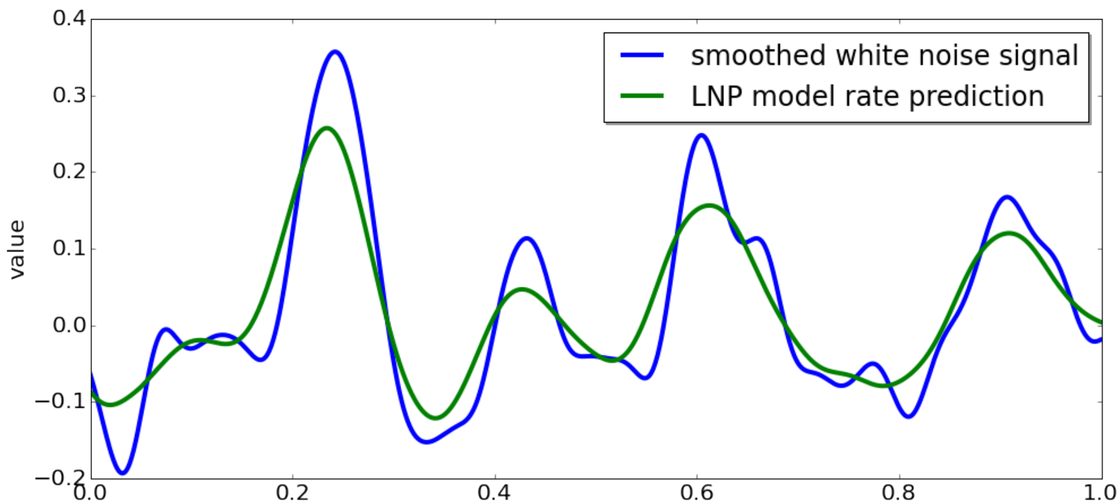
2 LNP Models

2.1 Create a linear poisson model of the synthetic neuron given in the previous assignment (Neuron Responses). To do this, convolve white noise (dt=0.001s, mean=0.0, std=1.0) with a kernel calculated using the spike-triggered average of that signal (use the STA method from the previous assignment). Plot 1s of the white noise stimulus alongside the firing rate prediction from the linear poisson model.

```
In [23]: def white_noise(mean=0.0,std=1.0,T=100,dt=0.001,rng=np.random.RandomState()):
                return rng.normal(mean,std,T/dt)

         def synthetic_neuron(drive,rng):
                """
                Simulates a mock neuron with a time step of 1ms.
                Arguments:
                drive - input to the neuron (expect zero mean; SD=1)
                Returns:
                rho - response function (0=non-spike and 1=spike at each time step)
                """

                dt = 0.001
                T = dt*len(drive)
                time = np.arange(0, T, dt)
                lagSteps = 0.02/dt
                drive = np.concatenate((np.zeros(lagSteps), drive[lagSteps:]))
                system = scipy.signal.lti([1], [0.03**2, 2*0.03, 1])
                _, L, _ = scipy.signal.lsim(system, drive[:,np.newaxis], time)
                rate = np.divide(30, 1 + np.exp(50*(0.05-L)))
                spikeProb = rate*dt
                return rng.rand(len(spikeProb)) < spikeProb

         def spike_trig_avg(stim,spikes,dt,window_width):

                window = np.arange(0,int(window_width / dt),1)
                #truncate spikes in first window timesteps
                spike_indices=np.where(spikes[len(window):]==1)[0].flatten()
                spike_triggered_avg=[]
                for t in window:
                        stim_sum_i=[]
                        for i in spike_indices:
                                #undo truncation when indexing from stimulus
                                stim_sum_i.append(stim[(i+len(window))-t])
                        spike_triggered_avg.append(np.average(stim_sum_i))

                spike_triggered_avg=np.array(spike_triggered_avg).flatten()/len(spike_indices)

                return -1.0*window*dt, spike_triggered_avg

In [24]: def two_a():


                T=1.0
                dt=0.001
                mean=0.0
```

```
        std=1.0
        seed=3

        #generate noisy signal with gaussian sampled numbers
        rng=np.random.RandomState(seed=seed)
        noise=white_noise(mean,std,T,dt,rng)
        t=np.arange(0,T,dt)

        #generate colored noise by convolving the noise signal with a gaussian
        sigma=0.020
        G = np.exp(-(t-np.average(t))**2/(2*sigma**2))
        G = G / sum(G)
        colored_noise=np.convolve(noise,G,'same')

        #feed colored noise into Bryan's spike generator
        spikes=synthetic_neuron(noise,rng)
        smooth_spikes=synthetic_neuron(colored_noise,rng)
        rate=spikes.sum()/T
        smooth_rate=smooth_spikes.sum()/T

        #calculate the spike-triggered average
        window_width=0.100
        window, sta = spike_trig_avg(noise,spikes,dt,window_width)
        smooth_window, smooth_sta = spike_trig_avg(colored_noise,smooth_spikes,dt,window_width

        #the math says to scale with <r> = rate, but this overshoots for smoothed noise
        kernel = rate * sta / std**2
        smooth_kernel = smooth_sta / std**2

        LNP = np.convolve(noise, kernel, mode='same')
        smooth_LNP = np.convolve(colored_noise, smooth_kernel, mode='same')

        #Plot the white noise signal together with the LNP rate prediction
        fig=plt.figure(figsize=(16,16))
        ax=fig.add_subplot(211)
        ax.plot(t,noise,label='white noise signal',alpha=0.5)
        ax.plot(t,LNP,label='LNP model rate prediction',alpha=0.5)
        # ax.set_xlim(0,T)
        legend=ax.legend(loc='best',shadow=True)
        # ax.set_xlabel('time (seconds)')
        ax.set_ylabel('value')
        ax=fig.add_subplot(212)
        ax.plot(t,colored_noise,label='smoothed white noise signal')
        ax.plot(t,smooth_LNP,label='LNP model rate prediction')
        # ax.set_xlim(0,T)
        legend=ax.legend(loc='best',shadow=True)
        # ax.set_xlabel('time (seconds)')
        ax.set_ylabel('value')

two_a()
```





I was unsure whether the top plot demonstrated that the model was correctly estimating the white noise signal, due to the high variability. I therefore smoothed the white noise signal and fed this stimulus into the synthetic neuron to produce spikes and a new rate estimate. The bottom plot shows that this estimate matches the original signal quite closely, validating my methodology. The only issue appears to be the magnitude of the first estimtae, but since I multiplied the spike triggered average by $< r > /\sigma_s^2$, I believe the scaling is as it should be.

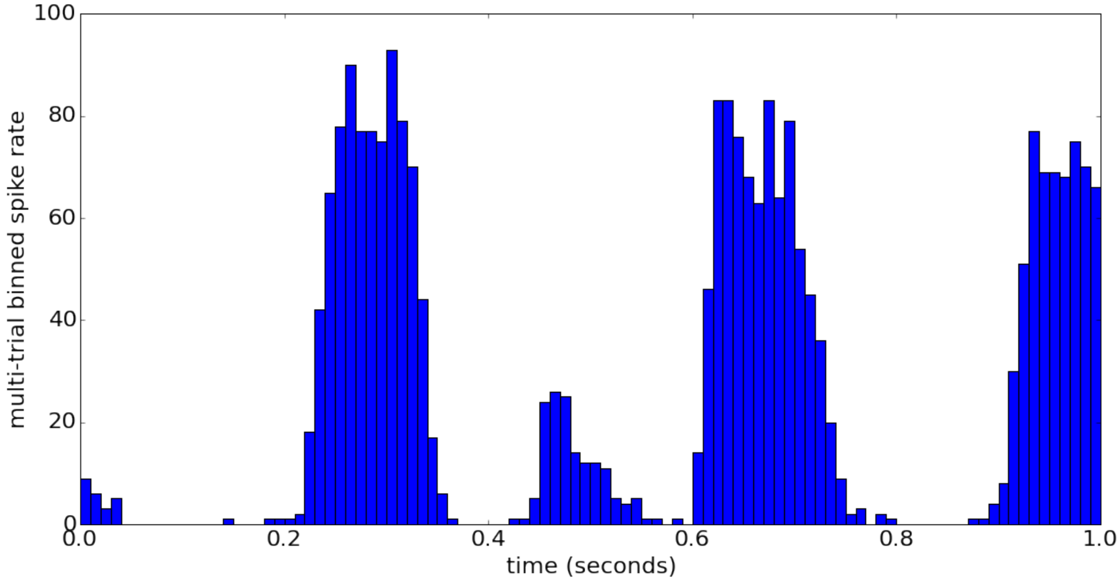
2.2 Calculate and plot the multi-trial firing rate of the synthetic neuron (using methods from the previous assignment) over 250 repeated trials with the same white noise stimulus. Overlay the prediction from the linear poisson model.

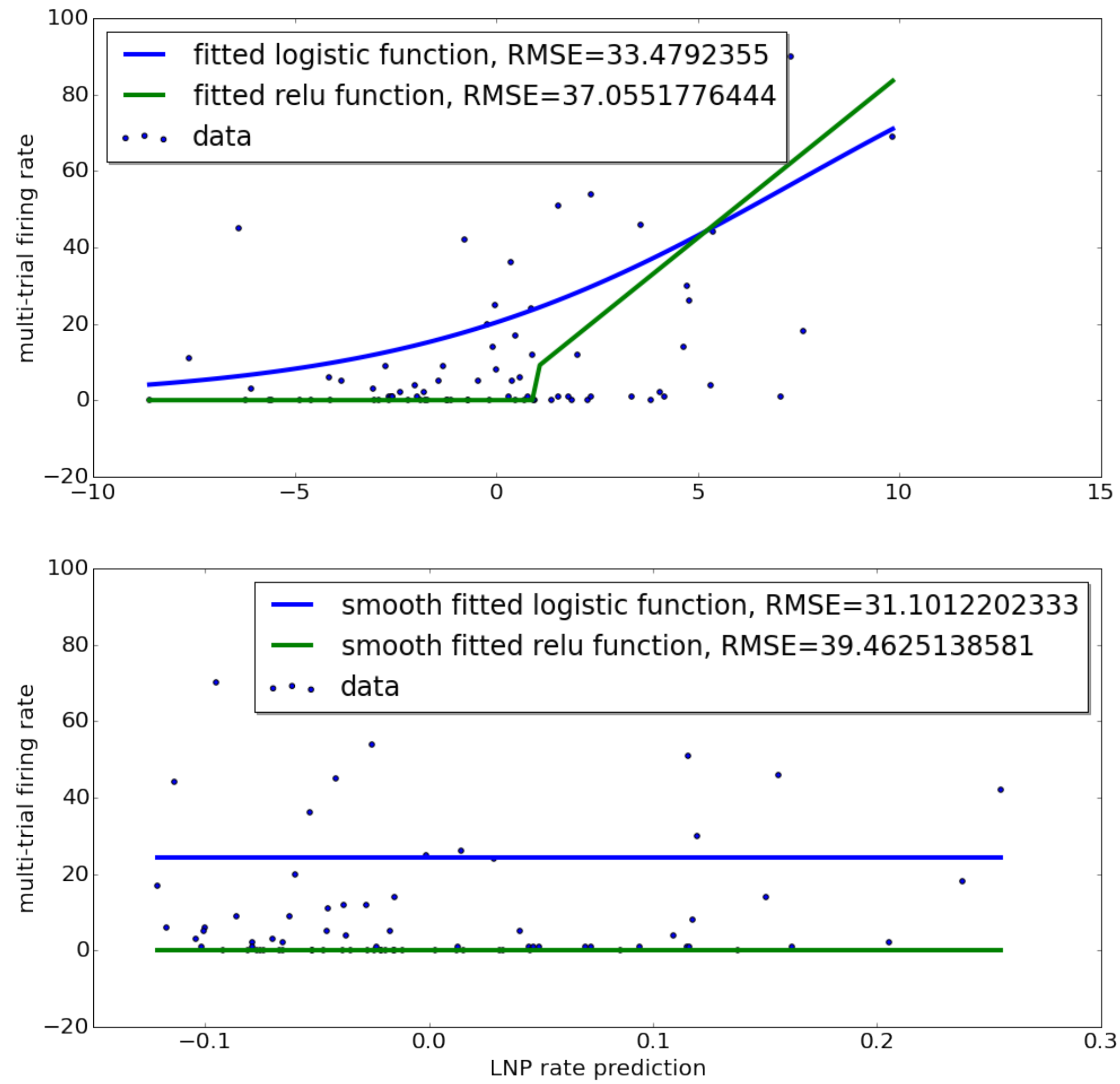
2.3 Scatterplot the multi-trial firing rate vs. the linear poisson model prediction. Fit the neural nonlinearity using `scipy.optimize.curve_fit()`.

```
In [25]: def two_b_thru_c():

                 T=1.0
```

```
dt=0.001
mean=0.0
std=1.0
seed=3

#generate noisy signal with gaussian sampled numbers
rng=np.random.RandomState(seed=seed)
noise=white_noise(mean,std,T,dt,rng)
t=np.arange(0,T,dt)

#generate colored noise by convolving the noise signal with a gaussian
sigma=0.020
G = np.exp(-(t-np.average(t))**2/(2*sigma**2))
G = G / sum(G)
colored_noise=np.convolve(noise,G,'same')

#feed colored noise into Bryan's spike generator
spikes=synthetic_neuron(noise,rng)
smooth_spikes=synthetic_neuron(colored_noise,rng)
rate=spikes.sum()/T
smooth_rate=smooth_spikes.sum()/T

#calculate the spike-triggered average
window_width=0.100
window, sta = spike_trig_avg(noise,spikes,dt,window_width)
smooth_window, smooth_sta = spike_trig_avg(colored_noise,smooth_spikes,dt,window_width

#the math says to scale with <r> = rate, but this overshoots for smoothed noise
kernel = rate * sta / std**2
smooth_kernel = smooth_sta / std**2

LNP = np.convolve(noise, kernel, mode='same')
smooth_LNP = np.convolve(colored_noise, smooth_kernel, mode='same')

#Calculate the 'multi-trial' (or 'time-varying') firing rate
#by counting the number of spikes in a small time window
#accross all trials
bin_width=0.010
n_trials=250
multitrial_binned_rate=[]
for i in range(int(T/bin_width)):
        bin_i=0
        for j in range(n_trials):
                spikes=synthetic_neuron(noise,rng)
                spike_times=np.where(spikes==True)[0]*dt
                for t in spike_times:
                        bin_i+=(i*bin_width<=t<(i+1)*bin_width)
        multitrial_binned_rate.append(bin_i)

#plot the multitrial firing rate
fig=plt.figure(figsize=(16,8))
ax=fig.add_subplot(111)
ax.bar(np.arange(0,T,bin_width),multitrial_binned_rate,width=bin_width)
# ax.set_xlim(0,T)
```

```
        ax.set_xlabel('time (seconds)')
        ax.set_ylabel('multi-trial binned spike rate')
        plt.show()

        #curve fit the nonlinearity to a logistic (sigmoid) and rectified linear functions
        def logistic(t, a, b, c):
            return a / (1 + np.exp(b*(c-t)))
        def relu(t, threshold, m):
            return m * t * (t > threshold)
        LNP_subset = LNP[::bin_width/dt]
        popt, pcov = curve_fit(logistic, LNP_subset, multitrial_binned_rate)
        popt2, pcov2 = curve_fit(relu, LNP_subset, multitrial_binned_rate)
        t_odd=np.linspace(np.min(LNP_subset),np.max(LNP_subset),100)
        fitted_logistic=logistic(t_odd,*popt)
        fitted_relu=relu(t_odd,*popt2)
        #curve fit to smoothed LNP prediction
        LNP_smooth_subset = smooth_LNP[::bin_width/dt]
        popt_smooth, pcov_smooth = curve_fit(logistic, LNP_smooth_subset, multitrial_binned_ra
        popt2_smooth, pcov2_smooth = curve_fit(relu, LNP_smooth_subset, multitrial_binned_rate
        t_odd_smooth=np.linspace(np.min(LNP_smooth_subset),np.max(LNP_smooth_subset),100)
        fitted_logistic_smooth=logistic(t_odd_smooth,*popt_smooth)
        fitted_relu_smooth=relu(t_odd_smooth,*popt2_smooth)

        #plot LNP rate prediction vs multitrial firing rate
        fig=plt.figure(figsize=(16,16))
        ax=fig.add_subplot(211)
        ax.scatter(LNP_subset,multitrial_binned_rate,label="data")
        ax.plot(t_odd,fitted_logistic,label='fitted logistic function, RMSE=%s'
                                %np.sqrt(np.average((fitted_logistic-multitrial_binned_rate)**
        ax.plot(t_odd,fitted_relu,label='fitted relu function, RMSE=%s'
                                %np.sqrt(np.average((fitted_relu-multitrial_binned_rate)**2)))
        legend=ax.legend(loc='best',shadow=True)
        # ax.set_xlabel('LNP rate prediction')
        ax.set_ylabel('multi-trial firing rate')
        ax=fig.add_subplot(212)
        ax.scatter(LNP_smooth_subset,multitrial_binned_rate,label="data")
        ax.plot(t_odd_smooth,fitted_logistic_smooth,label='smooth fitted logistic function, RM
                                %np.sqrt(np.average((fitted_logistic_smooth-multitrial_binned_
        ax.plot(t_odd_smooth,fitted_relu_smooth,label='smooth fitted relu function, RMSE=%s'
                                %np.sqrt(np.average((fitted_relu_smooth-multitrial_binned_rate
        legend=ax.legend(loc='best',shadow=True)
        ax.set_xlabel('LNP rate prediction')
        ax.set_ylabel('multi-trial firing rate')
        plt.show()

two_b_thru_c()
```


multi-trial binned spike rate
time (seconds)
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100



This plots should show that the actual firing rate is systematically different from the LNP rate prediction, specifically that they follow a sigmoid curve. If this were the case, running the curve fit on this plot would produce a nice estimate of the neural nonlinearity. I tried several procedures for generate more data, smoothing the data, and binning the result, but none of them significantly improved the fit.

2.4 Describe how you could generate spikes with a linear-nonlinear model of the synthetic neuron using the above results.

To estimate the firing rate of the synthetic neuron using an LNP model, I would generate a LP rate prediction as above (calculate the STA of the stimulus, use it to find the kernel, and convolve the stimulus with the kernel), then feed this estimate into a nonlinear activation function. This activation function could be optimized by using a curve fitting algorithm to find parameters which minimized the error between a trial dataset of firing rates and the LNP predictions, assuming a standard activation function such as a sigmoid, tanh, or rectified linear. Depending on the resolution of the data, it may be necessary to "bin" the linear poisson rate prediction before doing the curve-fitting.