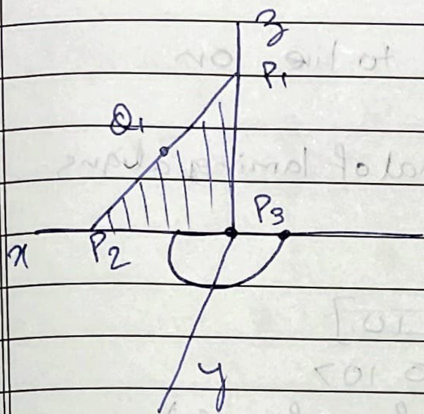


$P_1 = \langle 0\ 0\ 1 \rangle$

$P_2 = \langle 1\ 0\ 0 \rangle$

$P_3 = \langle 0\ 0\ 0 \rangle$

$Q_1 = \langle 0.5\ 0\ 0.5 \rangle$

Step 1:

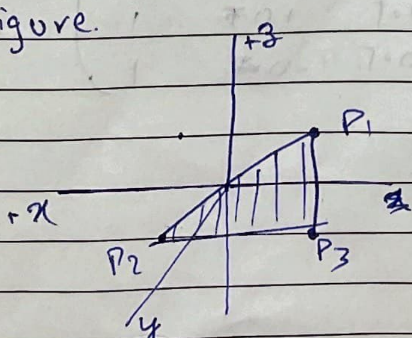
Translate Lamina to origin

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-1}{2} & 0 & \frac{-1}{2} & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-1}{2} & 0 & \frac{-1}{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} = \begin{bmatrix} -0.5 & 0 & 0.5 & 1 \\ 0.5 & 0 & -0.5 & 1 \\ -0.5 & 0 & -0.5 & 1 \end{bmatrix}$$

Figure:



Step 2: Rotate Lamina to lie on xy plane

→ or rotate s.t. normal of lamina aligns with z axis

$n(\text{lamina}) = \langle 0\ 1\ 0 \rangle$

(As can be seen from figure)

$\therefore$ to align with z axis, rotate about x axis by $\pi/2$

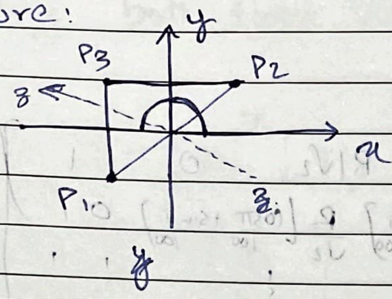
$$= R = \begin{bmatrix} \cancel{\cos\pi/2}\ 1 & \cancel{\sin\pi/2}\ 0 & 0 & 0 \\ \cancel{-\sin\pi/2}\ 0 & \cos\pi/2 & \sin\pi/2 & 0 \\ 0 & -\sin\pi/2 & \cos\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_1'' \\ P_2'' \\ P_3'' \end{bmatrix} = \begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} [R]$$

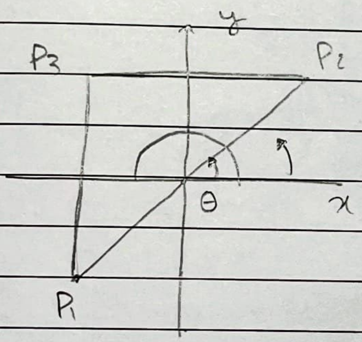
$$= \begin{bmatrix} -0.5 & +0.5 & 0 & 1 \\ 0.5 & +0.5 & +0 & 1 \\ -0.5 & +0.5 & -1.0 & 1 \end{bmatrix}$$

Figure:



Step 3:

Rotate Circle to align with Lamina about z axis



$\Rightarrow \tan\theta = \frac{0.5}{0.5}$

$\tan\theta = 1 \Rightarrow \theta = \pi/4$

$\therefore \cos\theta = 1/\sqrt{2}$

$\sin\theta = 1/\sqrt{2}$

$$\therefore R_z = \begin{bmatrix} \cancel{z}\cos\pi/4 & \sin\pi/4 & 0 & 0 \\ -\sin\pi/4 & \cos\pi/4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

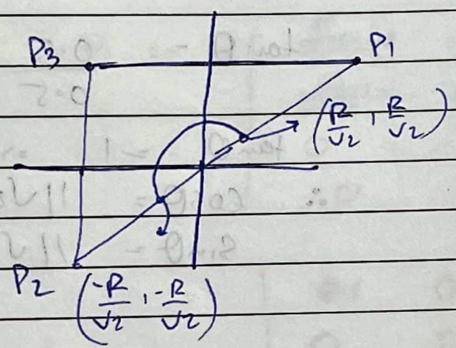
Let Circle be depicted by ~~pt~~ matrix C

$$C = \begin{bmatrix} R\cos(0) & R\sin(0) & 0 & 1 \\ R\cos(\pi/100) & R\sin(\pi/100) & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ R\cos(\pi) & R\sin(\pi) & 0 & 1 \end{bmatrix}$$

$$\therefore C' = [C][R]$$

$$= \begin{bmatrix} \cancel{R\cos}\cdot R/\sqrt{2} & R/\sqrt{2} & 0 & 1 \\ R/\sqrt{2} \times \left[\cos\frac{\pi}{100} - \sin\frac{\pi}{100}\right] & \frac{R}{\sqrt{2}}\left[\cos\frac{\pi}{100} + \sin\frac{\pi}{100}\right] & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{R}{\sqrt{2}}[-1] & \frac{R}{\sqrt{2}}[-1] & 0 & 1 \end{bmatrix}$$

FIGURE



Step 4 Undo rotations & translations

$$\Rightarrow R' \Rightarrow R^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.5 & 0 & 0.5 & 1 \end{bmatrix}.$$

Apply both these rotations to Circle & laming

$$\therefore C'' = C' [R]^T [T'].$$

$$\begin{bmatrix} (P_1)_f \\ (P_2)_f \\ (P_3)_f \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = R [L''] [R]^T [T'].$$

$$\therefore C'' = \begin{bmatrix} R/\sqrt{2} + 0.5 & 0 & -R/\sqrt{2} + 0.5 & 1 \\ \frac{R}{\sqrt{2}}\left(\cos\frac{\pi}{100} - \sin\frac{\pi}{100}\right) + 0.5 & 0 & \frac{-R}{\sqrt{2}}\left(\sin\frac{\pi}{100} + \cos\frac{\pi}{100}\right) + 0.1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{-R}{\sqrt{2}} + 0.5 & 0 & +\frac{R}{\sqrt{2}} + 0.5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_1(f) \\ P_2(f) \\ P_3(f) \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

FIGURE