Exercises

Continuous Optimization

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www.mop.uni-saarland.de/teaching/OPT24

— Summer Term 2024 —

— Assignment 12 —

Exercise 1. [10 points]

1. Which of the following functions are convex ?

   (i) x^p on $(-\infty, \infty)$ when $p > 1$.

   (ii) $-\log(x)$ on $(0, \infty)$

   (iii) $\exp(ax)$ on $(-\infty, \infty)$, $a \in \mathbb{R}$

   (iv) $x\log(x) + (1-x)\log(1-x)$ is convex on $(0, 1)$.

2. Which of the following sets are convex ?

   (i) $C = \{x \in \mathbb{R}^2 : ||x||_2 \leq 1\}$

   (ii) $C = \{x \in \mathbb{R}^2 : ||x||_2 = 1\}$

   (iii) $C = \{x \in \mathbb{R}^2 : ||x||_\infty = 1\}$

   (iv) $C = \{x \in \mathbb{R}^2 : ||x||_1 \leq 1\}$

3. Show that the function $f(x, y) = x^4 + y^4 - x - y$ is coercive while $f(x, y) = ax + by + c$ is not.

4. For the set $C = \{x \in \mathbb{R}^2 : ||x||_\infty \leq 1\}$, derive expressions for tangent and normal cones at $(1, 1)^\top$.

Exercise 2. [10 points]

(A) Show that Newton's method can be derived from the steepest descent method with appropriate change of variables.

(B) For the quadratic function $f(x) = \frac{1}{2}x^\top Qx + b^\top x$, with $Q = \begin{bmatrix} 4 & 3 \\ 3 & 6 \end{bmatrix}$ and $b = (1, -1)^\top$ find the minimizer by applying conjugate gradient method with initialization of your choice. You may use exact line search. Compute at most two steps.

Exercise 3. [10 points]

Consider the following function f with three input variables x_1, x_2, x_3:

$$f(x_1\,, x_2,\, x_3) = x_1 sin(x_2) + x_3 e^{x_1 x_2}\,,$$

find the computational graph and compute derivatives using the forward and backward mode.

Exercise 4. [10 points]

(A) Consider the following constrained optimization problem where Q is a symmetric positive definite matrix. Write the optimality conditions for this problem,

$$\min_{x\in\mathbb{R}^n} \sum_{i=1}^{n} -\log(\alpha_i + x_i), \quad s.t.\ x \geq 0\,, 1^\top x = 1\,.$$

(B) For the given constrained optimization problem, derive an expression for an iteration of the projected gradient algorithm.

$$\min_{x\in\mathbb{R}^n} f(x)\,, \quad s.t.\, \alpha^\top x = \beta.$$

where $\alpha \in \mathbb{R}^n$ and $\beta \in \mathbb{R}$.

Submission Instructions: This assignment sheet comprises the theoretical and programming parts.

- **Theoretical Part:** Write down your solutions clearly on a paper, scan them and convert them into a file named *theory(Name).pdf* where Name indicates the name of student submitting the assignment. Take good care of the ordering of the pages in this file. You are also welcome to submit the solutions prepared with LaTeX or some digital handwriting tool. You must write the full names of all the students in your group on the top of first page.
- **Submission Folder:** Create a folder with the name *MatA_MatB_MatC* where MatA, MatB and MatC are the matriculation number (Matrikelnummer) of all the students in your group; depending on the number of people in the group. For example, if there are three students in a group with matriculation numbers 123456, 789012 and 345678 respectively, then the folder should be named: *123456_789012_345678*.
- **Submission:** Add all the relevant files to your submission folder and compress the folder into *123456_789012_345678.zip* file and upload it on the link provided on Moodle.
- **Deadline:** The submission deadline is 16.07.2024, 2:00 p.m. (always Tuesday 2 p.m.) via Moodle.