Exercises

Continuous Optimization

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www.mop.uni-saarland.de/teaching/OPT24

— Summer Term 2024 —

— Assignment 6 —

Exercise 1. [5 points]
Show that strong Wolfe condition implies the curvature condition (11.4).

Exercise 2. [5 points]
Let the curvature condition (11.4) be satisfied. Argue that there always exists a symmetric positive definite matrix B_{k+1} that also satisfies the secant equation (11.3). However the solution is not unique.

Exercise 3. [8 points]
Show that the curvature condition is automatically satisfied if f is strongly convex.

Exercise 4. [22 points]
Consider a set of points $\{(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)\}$ where $x_i \in \mathbb{R}^n$ and $y_i \in \{0, 1\}$ for all $i = 1, \dots, m$. For $w \in \mathbb{R}^n$, denote the following

$$h_w(x) = \frac{1}{1 + e^{-\langle w, x \rangle}} ,$$

an

$$f(w) = -\frac{1}{m} \sum_{i=1}^{m} \left(y_i \log(h_w(x_i)) + (1 - y_i) \log(1 - h_w(x_i)) \right) .$$

Your tasks are the following.

(a) Calculate the gradient and the Hessian for the function $f(w)$. Submit this part along with other theoretical exercises. (2 + 2 = 4 points)

(b) Prove that the function $f(w)$ is convex. (2 points)

(c) Complete the function `grad_func` in `my_functions.py`, to compute the gradient of f. (2 point)

(d) Implement Newton's method to solve $f(w)$, by filling in the TODOs in `NewtonMethod.py`. (2 points)

(e) Implement BFGS method to solve $f(w)$, by filling in the TODOs in `BFGS.py`. (6 points)

(f) Implement L-BFGS method to solve $f(w)$, by filling in the TODOs in `L_BFGS.py`. (6 points)

(g) You can compare all the algorithms with the command `python3 ex05_03_compare_algorithms.py`.

Submission Instructions: This assignment sheet comprises the theoretical and programming parts.

- **Theoretical Part:** Write down your solutions clearly on a paper, scan them and convert them into a file named *theory(Name).pdf* where Name denotes the name of the student submitting on behalf of the group.. Take good care of the ordering of the pages in this file. You are also welcome to submit the solutions prepared with LaTeX or some digital handwriting tool. You must write the full names of all the students in your group on the top of first page.
- **Programming part:** Submit your solution for the programming exercise with the filename `python3 ex05_03_compare_algorithms.py` where Name is the name of the student who submits the assignment on behalf of the group. You can only use python3.
- **Submission Folder:** Create a folder with the name *MatA_MatB_MatC* where MatA, MatB and MatC are the matriculation number (Matrikelnummer) of all the students in your group; depending on the number of people in the group. For example, if there are three students in a group with matriculation numbers 123456, 789012 and 345678 respectively, then the folder should be named: *123456_789012_345678*.
- **Submission:** Add all the relevant files to your submission folder and compress the folder into *123456_789012_345678.zip* file and upload it on the link provided on Moodle.
- **Deadline:** The submission deadline is 04.06.2024, 2:00 p.m. (always Tuesday 2 p.m.) via Moodle.