Continuous Optimization: Assignment 1

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Hiroyasu Akada **Honglu Ma**

Exercise 1

(a)

Claim: $lim_{k\to\infty} x^{(k)} = \frac{1}{30}$
Proof:
Let $\epsilon > 0$ be given. Choose $N = \max\{\frac{1}{90\epsilon}, 8\}$. Assume $n > N$. We have

$$n > N \Rightarrow n > 9 > \sqrt[3]{600} \Rightarrow n^3 > 600 \Rightarrow 5n^3 > 3000 \Rightarrow 10n^3 - 5n^3 > 3000 \Rightarrow 3000 + 5n^3 < 10n^3$$

and obviously

$$900n^4 > 150n^3 + 900n^4$$

To check the validity of the limit we need to show $\left|x^{(n)} - x^\star\right| < \epsilon$ where $x^\star = \frac{1}{30}$.

$$\begin{aligned}
\left|\frac{n^4 - 100}{5n^3 + 30k^4} - \frac{1}{30}\right| &= \left|\frac{30n^4 - 3000 - 5n^3 - 30k^4}{150n^3 + 900n^4}\right| \\
&= \left|\frac{-3000 - 5n^3}{150n^3 + 900n^4}\right| \\
&= \frac{3000 + 5n^3}{150n^3 + 900n^4} \\
&< \frac{10n^3}{900n^4} = \frac{1}{90n} && \text{(by the inequalities above)} \\
&< \frac{1}{90N} \\
&< \frac{1}{\frac{90}{90\epsilon}} = \epsilon
\end{aligned}$$

(b)

We have the sequence as such:

$$x^{(1)} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}, \quad x^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad x^{(3)} = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}, \quad x^{(4)} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$x^{(5)} = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix}, \quad x^{(6)} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \quad x^{(7)} = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix}, \quad x^{(8)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \ldots$$

We can prove by showing that $x^{(k)}$ is not a cauchy sequence thus does not converges (Proposition A.5, Lecture Script) i.e. $\exists \epsilon > 0$ such that $\forall N \in \mathbb{N}$, $\exists n, m > N$ such that $\left\|x^{(n)} - x^{(m)}\right\| \geq \epsilon$.
Proof:
Let $\epsilon = 1$, for all $N \in \mathbb{N}$, choose $n = 8N$ and $m = 8N + 4$. We have

$$\left\|x^{(8N)} - x^{(8N+4)}\right\| = \left\|\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right\| = 2 \geq 1 = \epsilon$$

We need to prove by induction that the sequence is periodic. We can determine cluster point by constructing subsequence but how to prove that we have found all of them?