Continuous Optimization: Assignment 10

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Exercise 1

We can reformulate the problem as follows:

$$\min_{x\in\mathbb{R}^n} \langle c, x\rangle \quad \text{s.t.} \quad \forall i \in \{1,\dots,m\} : b_i - \langle a_i, x\rangle = 0 \quad \text{and} \quad \forall j \in \{1,\dots,n\} : -x_j \leq 0$$

where a_i is the i-th row of A, b_i is the i-th element of b and x_j is the j-th element of x.
We can name the objective function as $f(x) = \langle c, x\rangle$, the equality constraints as $f_i(x) = b_i - \langle a_i, x\rangle$, $\forall i \in \{1,\dots,m\}$ and the inequality constraints as $g_j(x) = -x_j$, $\forall j \in \{1,\dots,n\}$ such that we have a problem that fits the general form provided in Corollary 15.19, namely:

$$\min_{x\in\mathbb{R}^n} f(x) \quad \text{s.t.} \quad f_i(x) = 0, i \in \mathcal{E} \quad \text{and} \quad g_j(x) \leq 0, j \in \mathcal{I}$$

By Corollary 15.19, we know that at optimal $x^\star$, we have

$$\begin{aligned}
\nabla f(x^\star) + \sum_{i\in\mathcal{E}} \lambda_i \nabla f_i(x^\star) + \sum_{j\in\mathcal{A}(x^\star)} \mu_j \nabla g_j(x^\star) &= 0 \\
c - \sum_{i=1}^{m} \lambda_i a_i - \sum_{j\in\mathcal{A}(x^\star)} \mu_j e_j &= 0 \\
c - A^\top \lambda - \mu &= 0
\end{aligned}$$

where

$$\mu = \begin{cases} 0 & \text{if } x_j^\star > 0 \\ \mu_j > 0 & \text{if } x_j^\star = 0 \end{cases}$$

observe that $\mu \geq 0$ which is the fouth KKT condition and $\mu_j x_j^\star = 0$, $\forall j \in \{1,\dots,n\}$ which is the fifth KKT condition a.k.a the complementary condition. Also by reformulate the equation we derivate at optimal, we have the first KKT condition:

$$c = A^\top \lambda + \mu$$