# Continuous Optimization: Assignment 7

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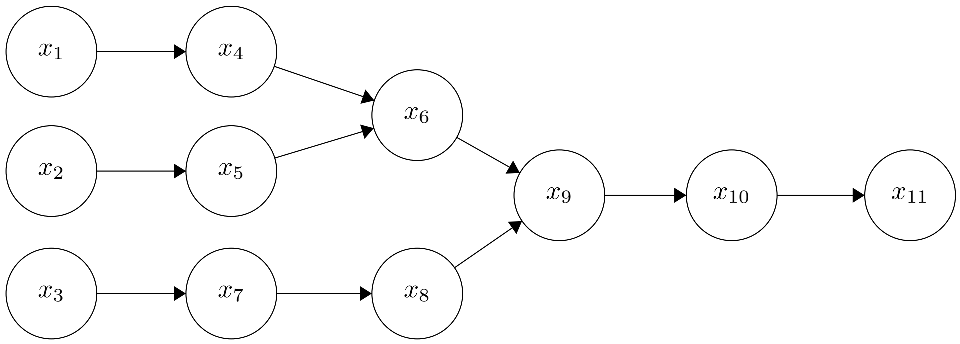
# Exercise 1

The computational graph for function $f$ is expressed as such




where

$$\begin{aligned}
x_4 &= x_1^2 \\
x_5 &= x_2^3 \\
x_7 &= x_3^4 \\
x_6 &= x_4 \cdot x_5 \\
x_8 &= \sin(x_7) \\
x_9 &= x_6 + x_8 \\
x_{10} &= \exp(x_9) \\
x_{11} &= x_{10}^2
\end{aligned}$$

and we have the following derivatives

$$\begin{aligned}
\frac{\partial x_4}{\partial x_1} &= 2x_1 \\
\frac{\partial x_5}{\partial x_2} &= 3x_2^2 \\
\frac{\partial x_7}{\partial x_3} &= 4x_3^3 \\
\frac{\partial x_6}{\partial x_4} &= x_5 \\
\frac{\partial x_6}{\partial x_5} &= x_4 \\
\frac{\partial x_8}{\partial x_7} &= \cos(x_7) \\
\frac{\partial x_9}{\partial x_6} &= 1 \\
\frac{\partial x_9}{\partial x_8} &= 1 \\
\frac{\partial x_{10}}{\partial x_9} &= \exp(x_9) \\
\frac{\partial x_{11}}{\partial x_{10}} &= 2x_{10}
\end{aligned}$$

Exercise 1 continued on next page. . .

## Forward Mode

We propogate the tangents through the computational graph to compute the derivative of $f$ with respect to $x_1$, $x_2$ and $x_3$. Normally, we would have to propogate all the bases i.e. $\hat{i}$, $\hat{j}$ and $\hat{k}$ one after another in order to all the partial derivatives but here each operation of the computational graph is a scalar operation and the starting node $x_1$, $x_2$ and $x_3$ are also scalas so we can simply set $\dot{x}_1 = 1$ when calculating $\frac{\partial f}{\partial x_1}$ and so on. To calculate the partial derivatives at point $(x_1, x_2, x_3) = (\bar{x}_1, \bar{x}_2, \bar{x}_3)$, we have

(i) $\frac{\partial f}{x_1}$ Set $\dot{x}_1 = 1, \dot{x}_2 = 0, \dot{x}_3 = 0$

$$\begin{aligned}
x_1 &= \bar{x}_1 & \dot{x}_1 &= 1 \\
x_2 &= \bar{x}_2 & \dot{x}_2 &= 0 \\
x_3 &= \bar{x}_3 & \dot{x}_3 &= 0 \\
x_4 &= x_1^2 = \bar{x}_1^2 & \dot{x}_4 &= 2x_1\dot{x}_1 = 2\bar{x}_1\dot{x}_1 = 2\bar{x}_1 \\
x_5 &= x_2^3 = \bar{x}_2^3 & \dot{x}_5 &= 3x_2^2\dot{x}_2 = 0 \\
x_6 &= x_4 \cdot x_5 = \bar{x}_1^2 \cdot \bar{x}_2^3 & \dot{x}_6 &= x_5\dot{x}_4 + x_4\dot{x}_5 = 2\bar{x}_1\bar{x}_2^3 \\
x_7 &= x_3^4 = \bar{x}_3^4 & \dot{x}_7 &= 4x_3^3\dot{x}_3 = 0 \\
x_8 &= \sin(x_7) = \sin(\bar{x}_3^4) & \dot{x}_8 &= \cos(x_7)\dot{x}_7 = 0 \\
x_9 &= x_6 + x_8 = \bar{x}_1^2 \cdot \bar{x}_2^3 + \sin(\bar{x}_3^4) & \dot{x}_9 &= \dot{x}_6 + \dot{x}_8 = 2\bar{x}_1\bar{x}_2^3 \\
x_{10} &= \exp(x_9) = \exp(\bar{x}_1^2 \cdot \bar{x}_2^3 + \sin(\bar{x}_3^4)) & \dot{x}_{10} &= \exp(x_9)\dot{x}_9 = 2\exp(\bar{x}_1^2 \cdot \bar{x}_2^3 + \sin(\bar{x}_3^4))\bar{x}_1\bar{x}_2^3 \\
x_{11} &= x_{10}^2 = \exp(2(\bar{x}_1^2 \cdot \bar{x}_2^3 + \sin(\bar{x}_3^4))) & \dot{x}_{11} &= 2x_{10}\dot{x}_{10} = 4\exp(2(\bar{x}_1^2 \cdot \bar{x}_2^3 + \sin(\bar{x}_3^4)))\bar{x}_1\bar{x}_2^3
\end{aligned}$$

(ii) $\frac{\partial f}{x_2}$ Set $\dot{x}_1 = 0, \dot{x}_2 = 1, \dot{x}_3 = 0$. Repeat the same process as above we get

$$\frac{\partial f}{\partial x_2} = 6\bar{x}_1^2\bar{x}_2^2 \exp(2(\bar{x}_1^2 \cdot \bar{x}_2^3 + \sin(\bar{x}_3^4)))$$

(iii) $\frac{\partial f}{x_3}$ Set $\dot{x}_1 = 0, \dot{x}_2 = 0, \dot{x}_3 = 1$. Repeat the same process as above we get

$$\frac{\partial f}{\partial x_3} = 8\bar{x}_3^3 \cos(\bar{x}_3^4) \exp(2(\bar{x}_1^2 \cdot \bar{x}_2^3 + \sin(\bar{x}_3^4)))$$