Continuous Optimization: Assignment 8

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Exercise 1

The constraint set C is defined as

$$C := \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; x_1,\, x_2 \geq 0,\; x_1 x_2 = 0 \right\}$$

which are the points lie on the axes of the first quadrant shown in Figure 1.

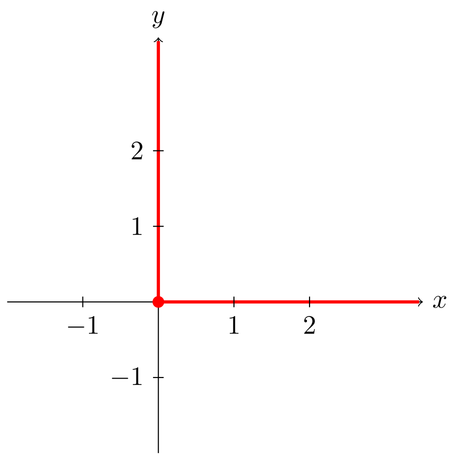

Figure 1: Illustration of the constraint set C marked in red.

Tangent Cone

The tangent cone of a set C at a point $\bar{x} \in C$ is the closure of the set of all feasible directions of C at $\bar{x}$. Apparently, the vectors on the axes pointing at positive directions are the feasible directions of C at $\bar{x}$ and also the closure of such set is itself.

Thus, the tangent cone to C at $\bar{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is

$$T_C(\bar{x}) = \left\{ \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; t_1,\, t_2 \geq 0,\; t_1 t_2 = 0 \right\}$$

as shown in Figure 2.

Normal Cone

The normal cone of a set C at a point $\bar{x} \in C$ is the set of all vectors v s.t. $\langle v, x - \bar{x} \rangle \leq 0, \forall x \in C$. Another way to visualize this is that v must form acute angles between all feasible directions of C at $\bar{x}$.

Apparently, the normal cone to C at $\bar{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is

$$N_C(\bar{x}) = \left\{ \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; n_1,\, n_2 \leq 0 \right\}$$

as shown in Figure 3.

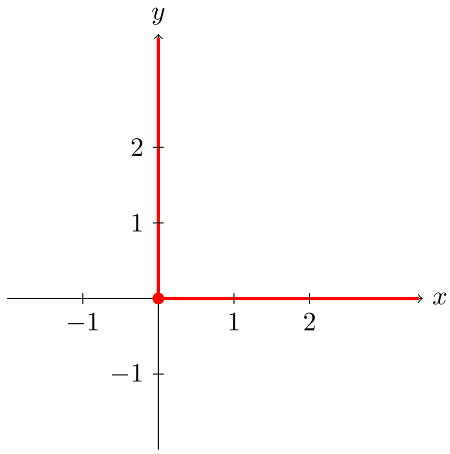

Figure 2: Illustration of the tangent cone of C at $\bar{x}$ marked in red.

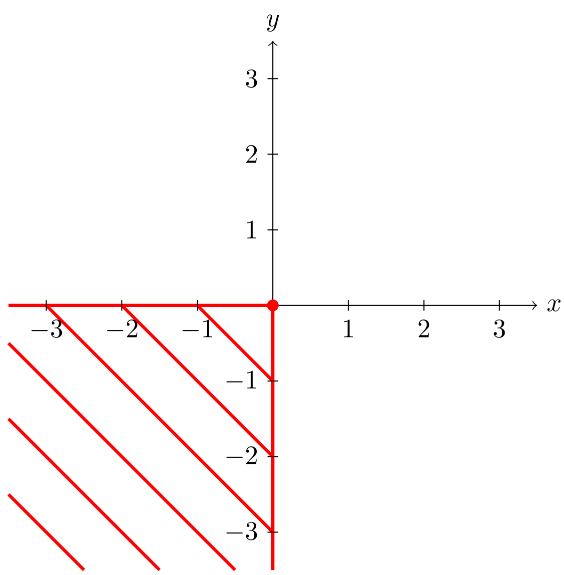

Figure 3: Illustration of the normal cone of C at $\bar{x}$ marked in red.