Continuous Optimization: Assignment 1

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Exercise 1

(a)

Claim: $lim_{k\to\infty}x^{(k)} = \frac{1}{30}$
Proof:
Let $\epsilon > 0$ be given. Choose $N = \max\{\frac{1}{90\epsilon}, 8\}$. Assume $n > N$. We have

$$n > N \Rightarrow n > 9 > \sqrt[3]{600} \Rightarrow n^3 > 600 \Rightarrow 5n^3 > 3000 \Rightarrow 10n^3 - 5n^3 > 3000 \Rightarrow 3000 + 5n^3 < 10n^3$$

and obviously

$$900n^4 > 150n^3 + 900n^4$$

To check the validity of the limit we need to show $\left|x^{(n)} - x^\star\right| < \epsilon$ where $x^\star = \frac{1}{30}$.

$$\begin{aligned}
\left|\frac{n^4 - 100}{5n^3 + 30k^4} - \frac{1}{30}\right| &= \left|\frac{30n^4 - 3000 - 5n^3 - 30k^4}{150n^3 + 900n^4}\right| \\
&= \left|\frac{-3000 - 5n^3}{150n^3 + 900n^4}\right| \\
&= \frac{3000 + 5n^3}{150n^3 + 900n^4} \\
&< \frac{10n^3}{900n^4} = \frac{1}{90n} && \text{(by the inequalities above)} \\
&< \frac{1}{90N} \\
&< \frac{1}{\frac{90}{90\epsilon}} = \epsilon
\end{aligned}$$