Continuous Optimization: Assignment 9

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Exercise 2

The minimization problem is given by

$$\min_{x\in\mathbb{R}^n} ||x||^2 \quad \text{s.t.} \quad Ax = b$$

It is equivalent to

$$\min_{x\in\mathbb{R}^n} \frac{1}{2}||x||^2 \quad \text{s.t.} \quad Ax = b$$

Let $f(x) = \frac{1}{2}||x||^2$ and $c_i(x) = a_i^\top x$ where a_i is the i-th row of A.
The constraint $Ax = b$ can be rewritten as m smaller constraints: $c_i(x) = b_i$ for $i = 1, \ldots, m$.
Using the Lagrange multiplier method, we compose such equation:

$$\begin{aligned} \nabla f(x) &= \sum_{i=1}^{m} \lambda_i \nabla c_i(x) \\ x &= \sum_{i=1}^{m} \lambda_i a_i \\ x &= A^\top \lambda \end{aligned}$$

where λ_i is the Lagrange multiplier for the i-th constraint and λ is a column vector consists of all multipliers.
We also have the constraint level sets:

$$\begin{aligned} Ax &= b \\ AA^\top \lambda &= b \\ \lambda &= (AA^\top)^{-1} b \\ \Rightarrow x &= A^\top (AA^\top)^{-1} b \end{aligned}$$