PINN Loss Function Components for Heat Equation

1. Data Loss ($L_{\mathbf{data}}$)

$$L_{\text{data}} = \text{MSE}(u_{\text{pred}}, u_{\text{true}}) \tag{1}$$

Explanation: This term measures how well the neural network's predictions (u_{pred}) match the known data points (u_{true}). It ensures the model fits the available data.

- MSE stands for Mean Squared Error.
- Minimizing this loss improves the accuracy of the model's predictions at known data points.
- This is similar to loss functions in traditional machine learning.

2. PDE Loss ($L_{\mathbf{pde}}$)

$$L_{\text{pde}} = \text{MSE}\left(\frac{\partial u}{\partial t} - \alpha \frac{\partial^2 u}{\partial x^2}, 0\right) \tag{2}$$

Explanation: This term enforces the heat equation ($\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$) throughout the domain.

- $\frac{\partial u}{\partial t}$ is the partial derivative of u with respect to time.
- $\frac{\partial^2 u}{\partial x^2}$ is the second partial derivative of u with respect to space.
- α is the thermal diffusivity coefficient.
- This loss encourages the neural network to learn solutions that satisfy the heat equation.
- It allows the model to generalize to areas where no data is available.

3. Boundary Condition Loss ($L_{\mathbf{bc}}$)

$$L_{\text{bc}} = \text{MSE}(u_{\text{pred}}(x = 0, t) - u_{\text{left}}, 0) + \text{MSE}(u_{\text{pred}}(x = L, t) - u_{\text{right}}, 0) \tag{3}$$

Explanation: This term ensures that the solution satisfies the boundary conditions of the problem.

- $u_{\text{pred}}(x = 0, t)$ is the predicted value at the left boundary.
- $u_{\text{pred}}(x = L, t)$ is the predicted value at the right boundary.
- u_{left} and u_{right} are the specified boundary conditions.
- This loss ensures that the neural network learns to respect the physical constraints at the boundaries.

4. Total Loss (L_{total})

$$L_{\text{total}} = \lambda_1 L_{\text{data}} + \lambda_2 L_{\text{pde}} + \lambda_3 L_{\text{bc}} \tag{4}$$

Explanation: This is the overall loss function that the neural network aims to minimize.

- λ_1, λ_2, and λ_3 are weighting factors for each loss component.
- These weights balance the importance of fitting data, satisfying the PDE, and meeting boundary conditions.
- Adjusting these weights can prioritize different aspects of the solution based on the specific problem and available data.