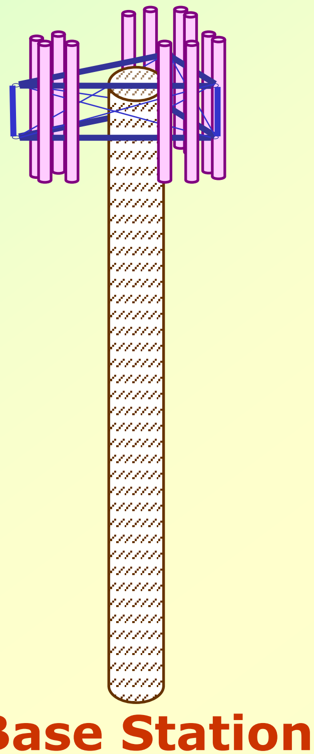
EEE4121F-A
Mobile and Wireless Networks

Olabisi E. Falowo
olabisi.falowo@uct.ac.za

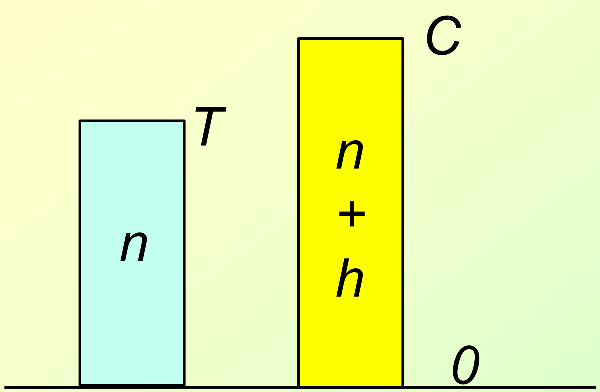
A Model of Mobile Networks

Previous Example

- In the previous lecture, we have considered the following example of heterogeneous wireless networks

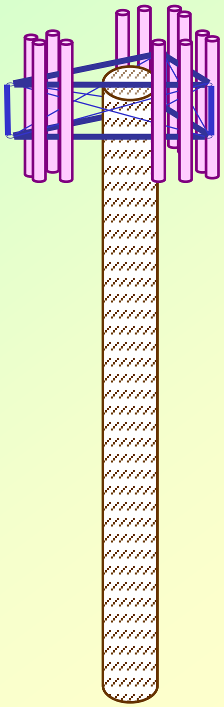


A cell of a GSM network has 8 timeslots for supporting new and handoff calls. Assuming that all states of the cell are equally probable, calculate the blocking probability for new calls and dropping probabilities for handoff calls if the cell uses a threshold-based bandwidth reservation scheme, with threshold, T=6 **(i.e. a maximum of 6 new calls can be admitted into the network)**



A Model of Mobile Networks

Previous Example



Base Station

- Given: b=1, C=8, H=6, where b is the bandwidth required for a new or handoff call.
- The current state of the heterogeneous system is represented as follows:

$$\Omega = (n, h)$$

- The non-negative integer n and h denote the number of ongoing new calls and ongoing handoff calls in the cell of the GSM network, respectively.
- The state S of all admissible states is given as:

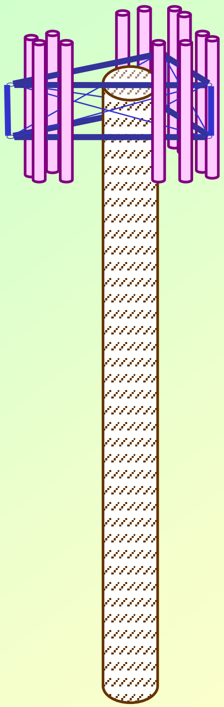
$$S = \{\Omega = (n, h): \ ((n + h) * b \le C) \ \wedge \ (n * b \le T) \}$$

- The blocking states for new calls are $s \subset S$ for which (b+ (n+h)*b > C) or (b+ n*b > T)

- The dropping states for handoff calls are $s \subset S$ for which (b+ (n+h)*b > C)

A Model of Mobile Networks

$$S = \{\Omega = (\mathrm{n},\mathrm{h}):\ ((n+h)*\mathrm{b} \leq \mathrm{C}\ )\ \ \wedge\ (n*b\ \leq T)\ \ \}$$



Base Station

Admissible states for [n, h]

[00], [01], [02], [03], [04], [05], [06], [07], [08]
[10], [11], [12], [13], [14], [15], [16], [17]
[20], [21], [22], [23], [24], [25], [26]
[30], [31], [32], [33], [34], [35]
[40], [41], [42], [43], [44]
[50], [51], [52], [53]
[60], [61], [62]

Limitation of the model

- *We assume that all states are equally probable, which is not realistic.*

Blocking states for [n, h]

[00], [01], [02], [03], [04], [05], [06], [07], [08]
[10], [11], [12], [13], [14], [15], [16], [17]
[20], [21], [22], [23], [24], [25], [26]
[30], [31], [32], [33], [34], [35]
[40], [41], [42], [43], [44]
[50], [51], [52], [53]
[60], [61], [62]

$$P_b = 9/42 = 0.2143$$

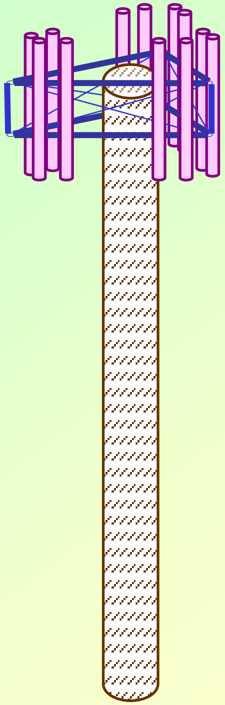
Dropping states for [n, h]

[00], [01], [02], [03], [04], [05], [06], [07], [08]
[10], [11], [12], [13], [14], [15], [16], [17]
[20], [21], [22], [23], [24], [25], [26]
[30], [31], [32], [33], [34], [35]
[40], [41], [42], [43], [44]
[50], [51], [52], [53]
[60], [61], [62]

$$P_d = 7/42 = 0.1667$$

Realistic Models of Mobile Networks

$$S = \{\Omega = (n,h):\ ((n+h)*b \le C)\ \wedge\ (n*b \le T)\ \}$$



Base Station

Admissible states for [n, h]

[00], [01], [02], [03], [04], [05], [06], [07], [08]
[10], [11], [12], [13], [14], [15], [16], [17]
[20], [21], [22], [23], [24], [25], [26]
[30], [31], [32], [33], [34], [35]
[40], [41], [42], [43], [44]
[50], [51], [52], [53]
[60], [61], [62]

In a realistic model

- *All states are not equally probable.*

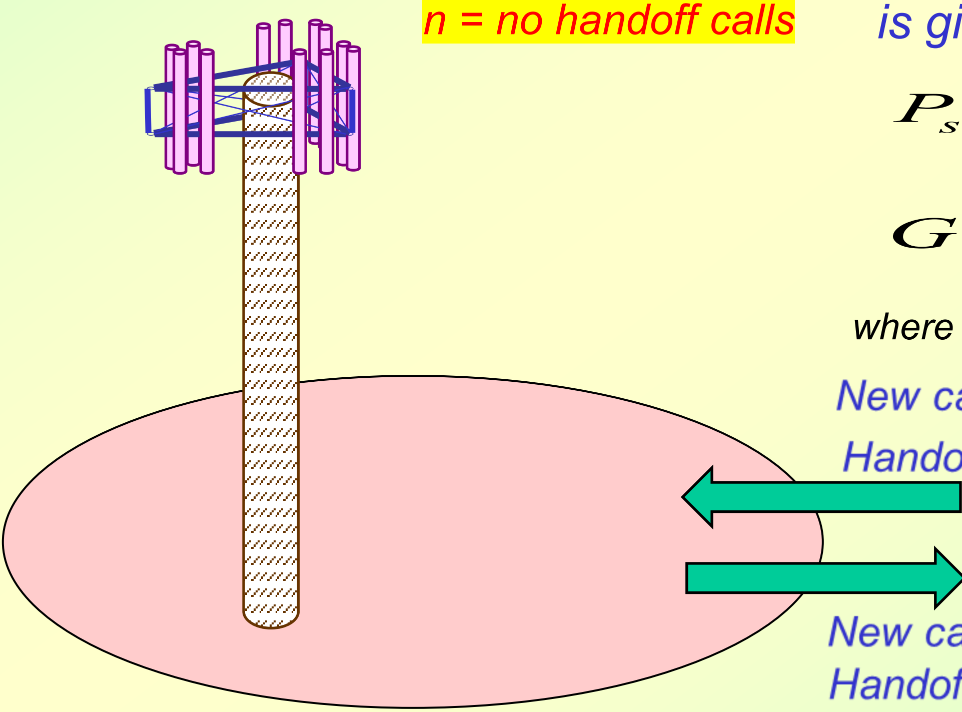
- *In a real network, the probability of the network being in a particular state depends on the arrival rates and departure rates of different classes/types of calls in the network*
- ***All states are not equally probable***
- *For example, if the network is in a business district, at mid-night, there will be few people using the network whereas during the busy hours of the day, there will be many people using the network*

Realistic Models of Mobile Networks

Admissible states for [m, n]

[00], [01], [02], [03], [04], [05], [06], [07], [08]
[10], [11], [12], [13], [14], [15], [16], [17]
[20], [21], [22], [23], [24], [25], [26]
[30], [31], [32], [33], [34], [35]
[40], [41], [42], [43], [44]
[50], [51], [52], [53]
[60], [61], [62]

m = no of new calls
n = no handoff calls



- *We will consider the network load in this model*

Network load as a result of new calls

$$\rho_m = (\lambda_m / \mu_m)$$

Network load as a result of handoff calls,

$$\rho_n = (\lambda_n / \mu_n)$$

The probability P_s of being in any state $(s \in S)$ is given as follows:

$$P_s = \left(\frac{(\rho_m)^m}{m!} \frac{(\rho_n)^n}{n!} \right) / G$$

$$G = \sum_{s \in S} \left(\frac{(\rho_m)^m}{m!} \frac{(\rho_n)^n}{n!} \right)$$

where G is the normalization constant

New call arrival rate (λ_m)
Handoff call arrival rate (λ_n)

New call departure rate (μ_m)
Handoff departure arrival rate (μ_n)

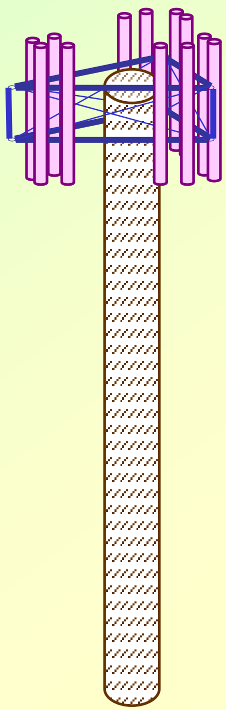
Realistic Models of Mobile Networks

Example 1

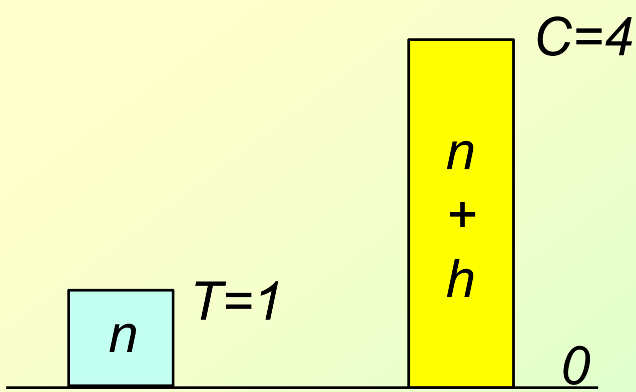
A homogenous cellular network providing voice service has a capacity of 4 basic bandwidth units (bbu). In the network, new calls are rejected when the current bbu being used is up to 1 whereas handoff calls are rejected only when all the available bbu are being used. Assume that the arrival rate of new calls is 1 call per minute, the arrival rate of handoff calls is 0.5 call per minute, the departure rate of new calls is 0.5 call per minute, and the departure rate of handoff calls is 0.5 call per minute. Moreover, a call requires 1 bbu.

(i) Evaluate the probability of blocking a new call.

(ii) Evaluate the probability of dropping a handoff call.



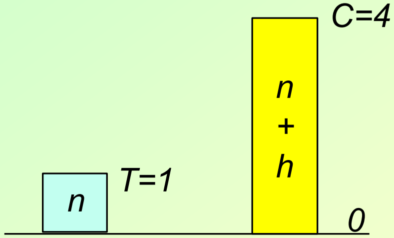
Base Station



$C=4$
$T=1$
$b=1$

Realistic Models of Mobile Networks

Solution 1



The current state of the cellular network is represented as follows:

$$\Omega = (m, n)$$

The non-negative integer m and n denote the number of ongoing new calls and ongoing handoff calls in a cell of the network, respectively.

Set S of all admissible states is given as:

$$S = \{\Omega = (m,n) : ((m+n)b \le C) \wedge ((m)\ b \le T)\ \}$$

$$S = \{\Omega = (m,n) : ((m+n)b \le 4) \wedge ((m)\ b \le 1)\ \}$$

Admissible states for [m, n]

[00], [01], [02], [03], [04]

[10], [11], [12], [13]

Network load as a result of new calls, $\rho_m = (\lambda_m / \mu_m)$

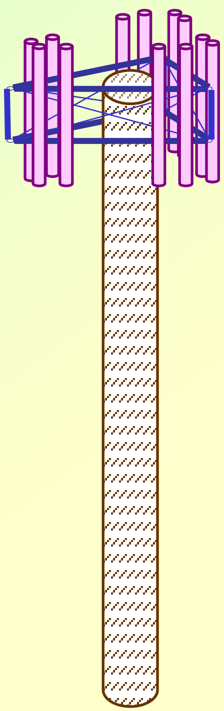
Network load as a result of handoff calls, $\rho_n = (\lambda_n / \mu_n)$

$$\rho_m = (1 / 0.5)$$

$$\rho_m = 2$$

$$\rho_n = (0.5 / 0.5)$$

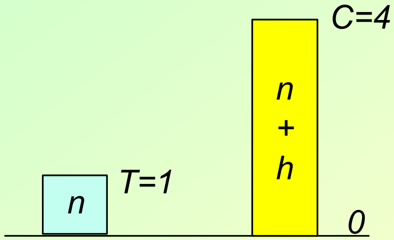
$$\rho_n = 1$$

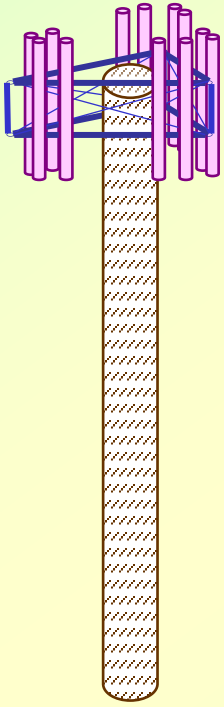


Base Station

Realistic Models of Mobile Networks

Solution 1





Base Station

Admissible states for [m, n]

[00], [01], [02], [03], [04]

[10], [11], [12], [13]

The probability P_s of being in any state $(s \in S)$ is given as follows:

$$P_s = \left(\frac{(\rho_m)^m}{m!} \frac{(\rho_n)^n}{n!} \right) / G$$

$$G = \sum_{s \in S} \left(\frac{(\rho_m)^m}{m!} \frac{(\rho_n)^n}{n!} \right)$$

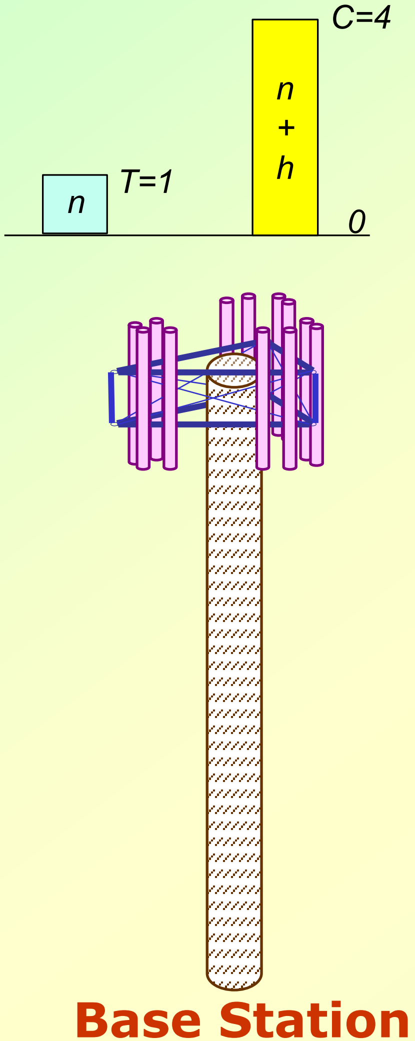
$$P_{(0,0)} = \left(\frac{(2)^0}{0!} \frac{(1)^0}{0!} \right) / G \quad = 1/G$$

$$P_{(0,1)} = \left(\frac{(2)^0}{0!} \frac{(1)^1}{1!} \right) / G \quad = 1/G$$

$$P_{(0,2)} = \left(\frac{(2)^0}{0!} \frac{(1)^2}{2!} \right) / G \quad = 0.5/G$$

Realistic Models of Mobile Networks

Solution 1



Admissible states for [m, n]

[00], [01], [02], [03], [04]

[10], [11], [12], [13]

$$P_{(0,3)} = \left(\frac{(2)^0 (1)^3}{0! \quad 3!} \right) / G \quad = 0.167 / G$$

$$P_{(0,4)} = \left(\frac{(2)^0 (1)^4}{0! \quad 4!} \right) / G \quad = 0.0416 / G$$

$$P_{(1,0)} = \left(\frac{(2)^1 (1)^0}{1! \quad 0!} \right) / G \quad = 2 / G$$

$$P_{(1,1)} = \left(\frac{(2)^1 (1)^1}{1! \quad 1!} \right) / G \quad = 2 / G$$

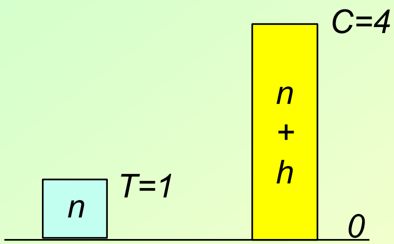
$$P_{(1,2)} = \left(\frac{(2)^1 (1)^2}{1! \quad 2!} \right) / G \quad = 1 / G$$

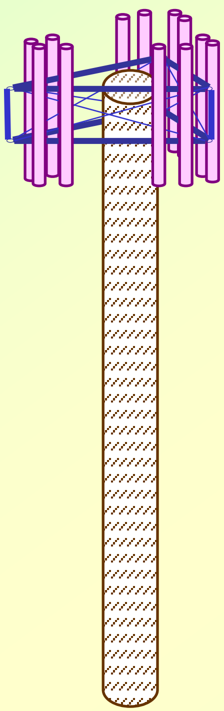
$$P_{(1,3)} = \left(\frac{(2)^1 (1)^3}{1! \quad 3!} \right) / G \quad = 0.333 / G$$

Realistic Models of Mobile Networks

Solution 1

Calculate the value of G





Base Station

$$P_s = \left(\frac{(\rho_m)^m}{m!} \frac{(\rho_n)^n}{n!} \right) / G$$

$$G = \sum_{s \in S} \left(\frac{(\rho_m)^m}{m!} \frac{(\rho_n)^n}{n!} \right)$$

G = (1 + 1 + 0.5+0.167+0.0416+2+2+1+0.333)
G = 8.04

$$P_{(0,0)} = \left(\frac{(2)^0}{0!} \frac{(1)^0}{0!} \right) / G \quad = 1/G$$

$$P_{(0,1)} = \left(\frac{(2)^0}{0!} \frac{(1)^1}{1!} \right) / G \quad = 1/G$$

$$P_{(0,2)} = \left(\frac{(2)^0}{0!} \frac{(1)^2}{2!} \right) / G \quad = 0.5/G$$

$$P_{(0,3)} = \left(\frac{(2)^0}{0!} \frac{(1)^3}{3!} \right) / G \quad = 0.167/G$$

$$P_{(0,4)} = \left(\frac{(2)^0}{0!} \frac{(1)^4}{4!} \right) / G \quad = 0.0416/G$$

$$P_{(1,0)} = \left(\frac{(2)^1}{1!} \frac{(1)^0}{0!} \right) / G \quad = 2/G$$

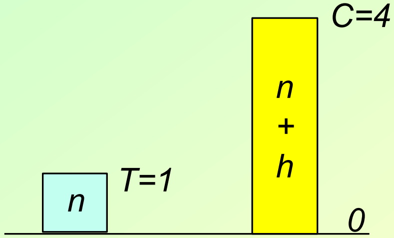
$$P_{(1,1)} = \left(\frac{(2)^1}{1!} \frac{(1)^1}{1!} \right) / G \quad = 2/G$$

$$P_{(1,2)} = \left(\frac{(2)^1}{1!} \frac{(1)^2}{2!} \right) / G \quad = 1/G$$

$$P_{(1,3)} = \left(\frac{(2)^1}{1!} \frac{(1)^3}{3!} \right) / G \quad = 0.333/G$$

Realistic Models of Mobile Networks

Solution 1 (i) Calculate new call blocking probability (P_b)



Find the set of blocking states (S_b) for [m, n]

*(b+ (m+n)*b > C) or (b+ (m)*b > T)*

[00], [01], [02], [03], [04]

[10], [11], [12], [13]

The probability of blocking a new call (P_b) is given as:

$$P_b = \sum_{s \in S_b} P(s)$$

Where S_b is the set of blocking states

$$P_b = P_{(0,4)} + P_{(1,0)} + P_{(1,1)} + P_{(1,2)} + P_{(1,3)}$$

$$P_b = (0.0416 + 2 + 2 + 1 + 0.333)/8.04$$

$$P_b = 0.668$$

$$P_{(0,0)} = \left(\frac{(2)^0}{0!}\frac{(1)^0}{0!}\right)/G \quad = 1/G$$

$$P_{(0,1)} = \left(\frac{(2)^0}{0!}\frac{(1)^1}{1!}\right)/G \quad = 1/G$$

$$P_{(0,2)} = \left(\frac{(2)^0}{0!}\frac{(1)^2}{2!}\right)/G \quad = 0.5/G$$

$$P_{(0,3)} = \left(\frac{(2)^0}{0!}\frac{(1)^3}{3!}\right)/G \quad = 0.167/G$$

$$P_{(0,4)} = \left(\frac{(2)^0}{0!}\frac{(1)^4}{4!}\right)/G \quad = 0.0416/G$$

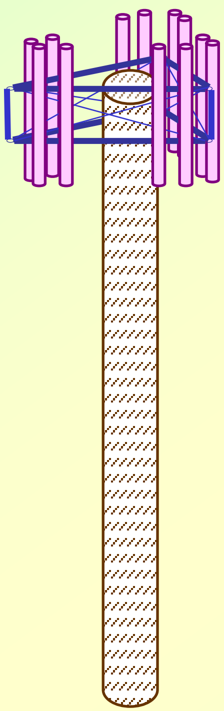
$$P_{(1,0)} = \left(\frac{(2)^1}{1!}\frac{(1)^0}{0!}\right)/G \quad = 2/G$$

$$P_{(1,1)} = \left(\frac{(2)^1}{1!}\frac{(1)^1}{1!}\right)/G \quad = 2/G$$

$$P_{(1,2)} = \left(\frac{(2)^1}{1!}\frac{(1)^2}{2!}\right)/G \quad = 1/G$$

$$P_{(1,3)} = \left(\frac{(2)^1}{1!}\frac{(1)^3}{3!}\right)/G \quad = 0.333/G$$

$$G = 8.04$$

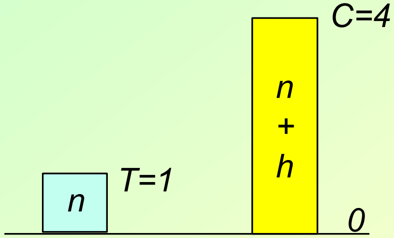


Base Station

Realistic Models of Mobile Networks

Solution 1 (ii) Calculate handoff call dropping probability (P_d)

Find the set of dropping states (S_d) for [m, n]



*(b+ (m+n)*b > C)*

[00], [01], [02], [03], [04]

[10], [11], [12], [13]

$$P_{(0,0)} = \left(\frac{(2)^0 (1)^0}{0! \quad 0!}\right) / G = 1/G$$

$$P_{(0,1)} = \left(\frac{(2)^0 (1)^1}{0! \quad 1!}\right) / G = 1/G$$

$$P_{(0,2)} = \left(\frac{(2)^0 (1)^2}{0! \quad 2!}\right) / G = 0.5/G$$

$$P_{(0,3)} = \left(\frac{(2)^0 (1)^3}{0! \quad 3!}\right) / G = 0.167/G$$

$$P_{(0,4)} = \left(\frac{(2)^0 (1)^4}{0! \quad 4!}\right) / G = 0.0416/G$$

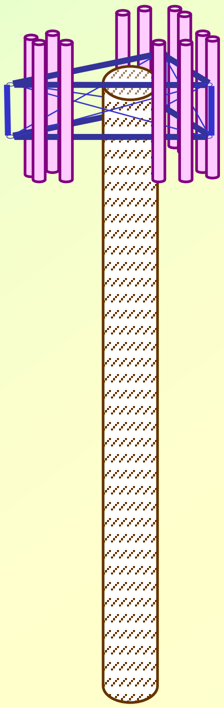
$$P_{(1,0)} = \left(\frac{(2)^1 (1)^0}{1! \quad 0!}\right) / G = 2/G$$

$$P_{(1,1)} = \left(\frac{(2)^1 (1)^1}{1! \quad 1!}\right) / G = 2/G$$

$$P_{(1,2)} = \left(\frac{(2)^1 (1)^2}{1! \quad 2!}\right) / G = 1/G$$

$$P_{(1,3)} = \left(\frac{(2)^1 (1)^3}{1! \quad 3!}\right) / G = 0.333/G$$

$$G = 8.04$$



Base Station

The probability of dropping a handoff call (P_d) is given by:

$$P_d = \sum_{s \in S_d} P(s)$$

Where S_d is the set of dropping states

$$P_{(0,4)} + P_{(1,3)}$$

$$P_d = (0.0416 + 0.333) / 8.04$$

P_d= 0.0466

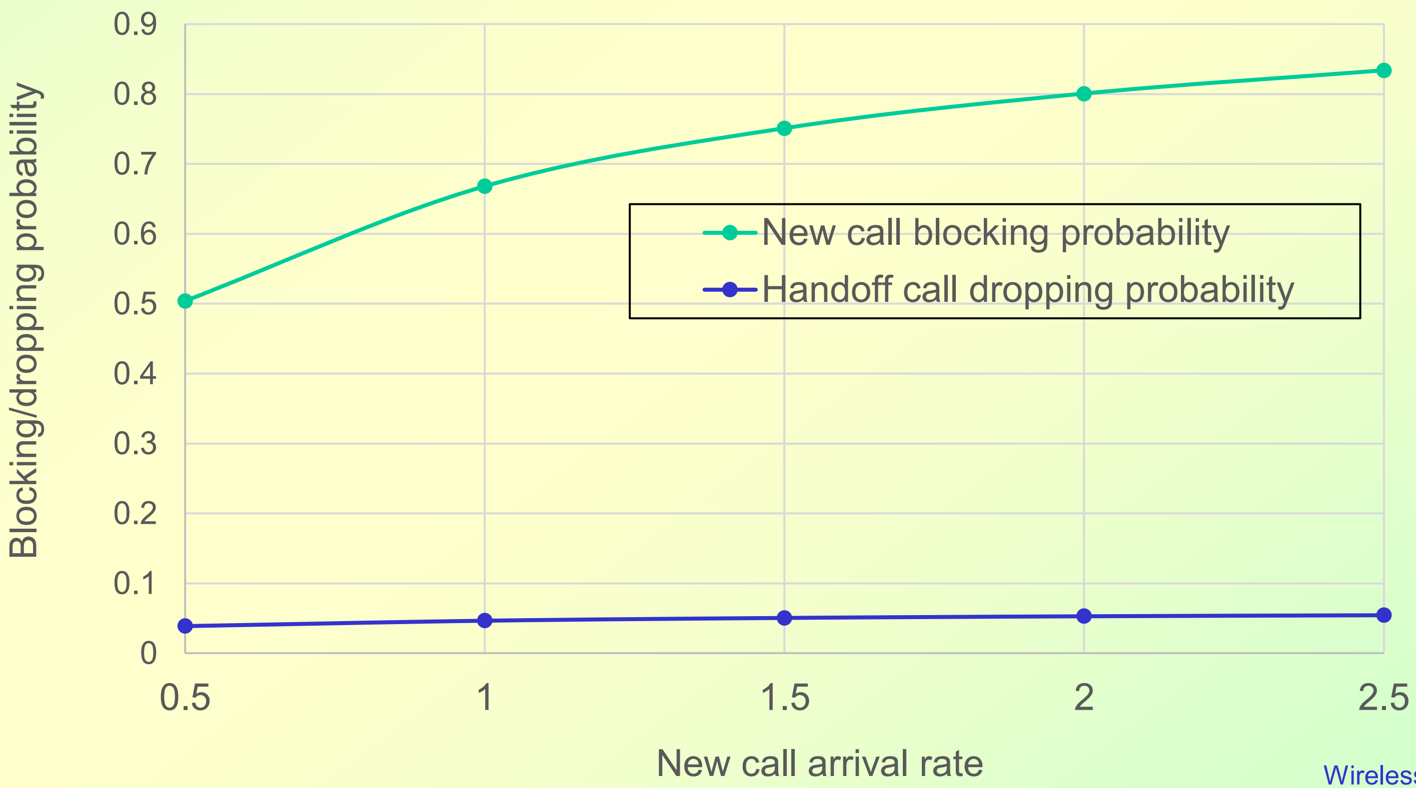
Sample MATLAB CODE

```
clc;
c1=4; t1=1; b1=1; xm=1; xn=0.5; um=0.5; un=0.5;
lm=xm/um;
ln=xn/un;
%xm=new call arrival rate; %xn=handoff call arrival rate;
%um=new call departure rate; %un=handoff call departure rate;
%m1=no of new calls; %n1=no-handoff calls;
%SB=summation of the probabilities of blocking states;
%SD= summation of the probabilities of dropping states;
%ST=normalization constant;
s1=0; s2=0; T=0; ST=0; SB=0; SD=0;
P =zeros((t1+1),(c1+1));
    for m1=0:t1
    for n1=0:c1
if ((b1*(m1+n1)<=c1) & (b1*m1<=t1))
    P((m1+1),(n1+1))= ((lm^m1)*(ln^n1))/(factorial(m1) * factorial(n1));
    ST=ST+ P((m1+1), (n1+1));
%Summation of the probabilities for blocking states
    if (b1+ (b1*(m1+n1)) > c1) | ((b1+ (b1*m1)) > t1)
     SB=SB+P((m1+1), (n1+1));
    end
%Summation of the probabilities for dropping states
    if b1+ (b1*(m1+n1)) > c1
     SD=SD+P((m1+1), (n1+1));
    end
    states=[(m1),(n1)]
end
end
end
Block_new_calls =SB/ST
Block_handoff_calls =SD/ST
```

Effect of New Call Arrival Rate on Pb and Pd

λ_m	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**
P_b	0.5039	0.668	0.7510	0.8006	0.8338
P_d	0.0388	0.0466	0.0506	0.0530	0.0545

All other parameters have the same values as in the previous example



Exercise 1

Determine the effect of threshold value (T) on P_b and P_d using the following values.

$C=4$, $b=1$, $\lambda_m=1$, $\lambda_n=0.5$, $\mu_m=0.5$, $\mu_n=0.5$

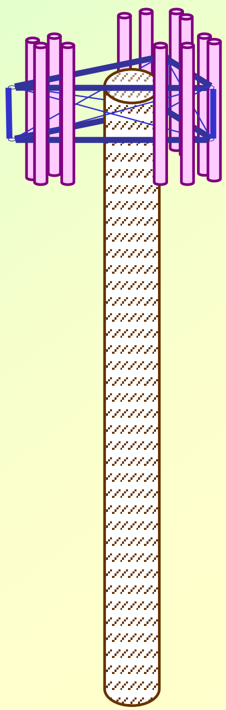
Plot the graphs to illustrate the effect.

T	**0**	**1**	**2**	**3**	**4**
P_b					
P_d					

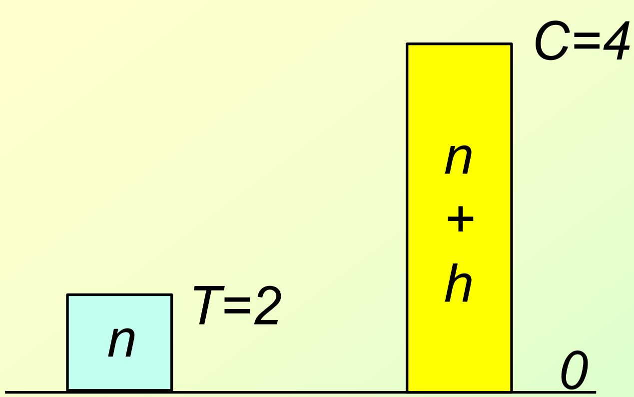
Realistic Models of Mobile Networks

Example 2

A homogenous cellular network providing voice service has a capacity of 4 basic bandwidth units (bbu). In the network, new calls are rejected when the current bbu being used is up to 2 whereas handoff calls are rejected only when all the available bbu are being used. Assume that the arrival rate of new calls is 1 call per minute, the arrival rate of handoff calls is 0.5 call per minute, the departure rate of new calls is 0.5 call per minute, and the departure rate of handoff calls is 0.5 call per minute. Moreover, a **call requires 2 bbu**. (i) Evaluate the probability of blocking a new call.
(ii) Evaluate the probability of dropping a handoff call.



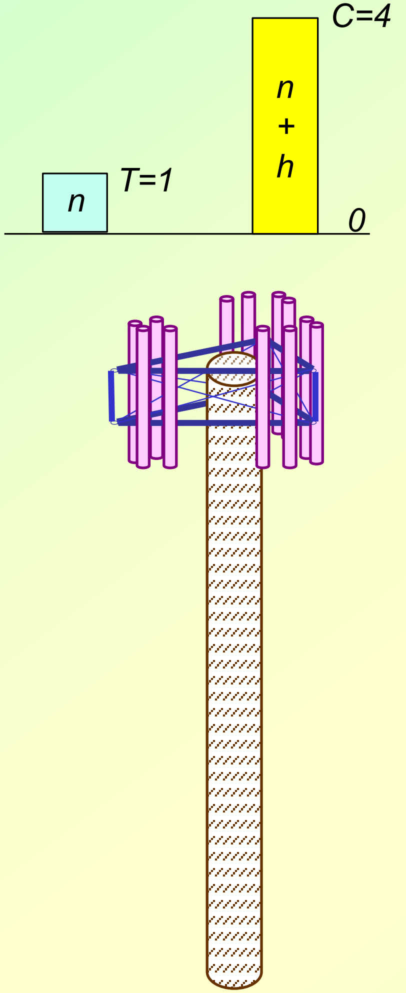
Base Station



$C=4$
$T=2$
$b=2$

Realistic Models of Mobile Networks

Solution 2



Base Station

The current state of the cellular network is represented as follows:

$$\Omega = (m, n)$$

The non-negative integer m and n denote the number of ongoing new calls and ongoing handoff calls in a cell of the network, respectively.

Set S of all admissible states is given as:

$$S = \{\Omega = (m, n) : ((m+n)b \le C) \wedge ((m)\ b \le T)\ \}$$

$$S = \{\Omega = (m, n) : ((m+n)2 \le 4) \wedge ((m)2 \le 2)\ \}$$

Admissible states for [m, n]

[00], [01], [02]

[10], [11]

Network load as a result of new calls, $\rho_m = (\lambda_m / \mu_m)$

Network load as a result of handoff calls, $\rho_n = (\lambda_n / \mu_n)$

$$\rho_m = (1 / 0.5)$$

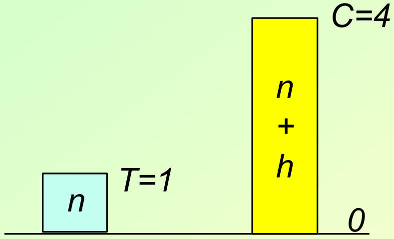
$$\rho_m = 2$$

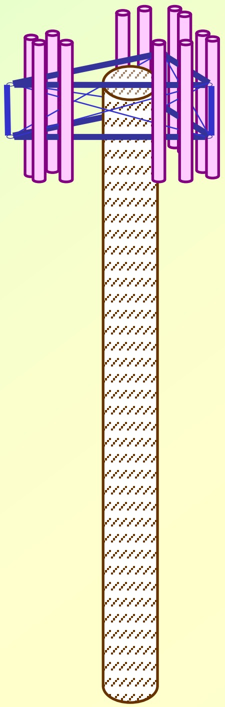
$$\rho_n = (0.5 / 0.5)$$

$$\rho_n = 1$$

Realistic Models of Mobile Networks

Solution 2





Base Station

Admissible states for [m, n]

[00], [01], [02],

[10], [11],

The probability P_s of being in any state ($s \in S$) is given as follows:

$$P_s = \left(\frac{(\rho_m)^m (\rho_n)^n}{m! \quad n!} \right) / G$$

$$G = \sum_{s \in S} \left(\frac{(\rho_m)^m (\rho_n)^n}{m! \quad n!} \right)$$

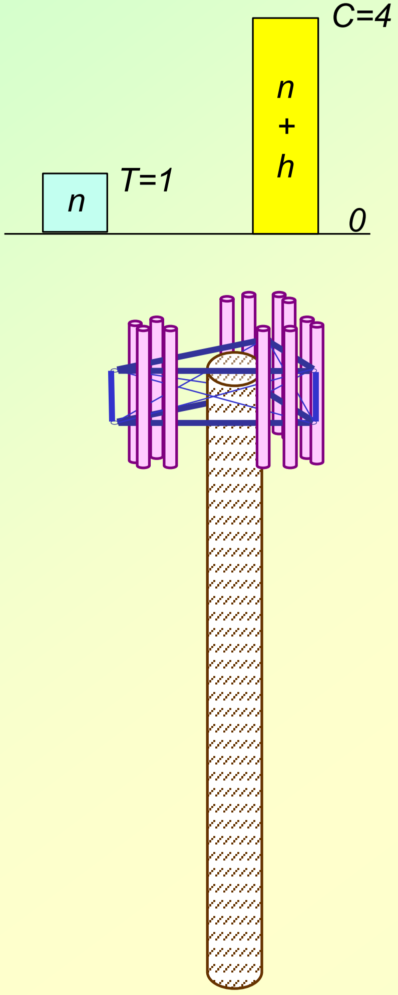
$$P_{(0,0)} = \left(\frac{(2)^0 (1)^0}{0! \quad 0!} \right) / G \quad = 1/G$$

$$P_{(0,1)} = \left(\frac{(2)^0 (1)^1}{0! \quad 1!} \right) / G \quad = 1/G$$

$$P_{(0,2)} = \left(\frac{(2)^0 (1)^2}{0! \quad 2!} \right) / G \quad = 0.5/G$$

Realistic Models of Mobile Networks

Solution 2



Base Station

Admissible states for [m, n]

[00], [01], [02],

[10], [11],

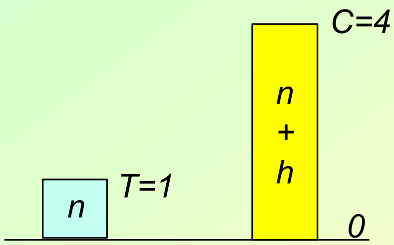
$$P_{(1,0)} = \left(\frac{(2)^1 (1)^0}{1! \quad 0!} \right) / G \quad = 2/G$$

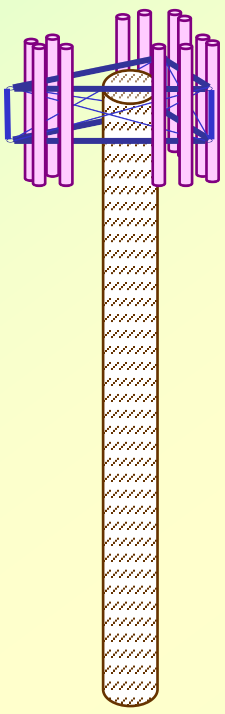
$$P_{(1,1)} = \left(\frac{(2)^1 (1)^1}{1! \quad 1!} \right) / G \quad = 2/G$$

Realistic Models of Mobile Networks

Solution 2

Calculate the value of G





Base Station

$$P_s = \left(\frac{(\rho_m)^m}{m!} \frac{(\rho_n)^n}{n!} \right) / G$$

$$G = \sum_{s \in S} \left(\frac{(\rho_m)^m}{m!} \frac{(\rho_n)^n}{n!} \right)$$

$$P_{(0,0)} = \left(\frac{(2)^0}{0!} \frac{(1)^0}{0!} \right) / G \quad = 1/G$$

$$P_{(0,1)} = \left(\frac{(2)^0}{0!} \frac{(1)^1}{1!} \right) / G \quad = 1/G$$

$$P_{(0,2)} = \left(\frac{(2)^0}{0!} \frac{(1)^2}{2!} \right) / G \quad = 0.5/G$$

$$P_{(1,0)} = \left(\frac{(2)^1}{1!} \frac{(1)^0}{0!} \right) / G \quad = 2/G$$

$$P_{(1,1)} = \left(\frac{(2)^1}{1!} \frac{(1)^1}{1!} \right) / G \quad = 2/G$$

$G = (1 + 1 + 0.5 + 2 + 2)$

$G = 6.5$

Solution 2 (i) Calculate new call blocking probability (P_b)

Find the set of blocking states (S_b) for [m, n]

*(b+ (m+n)*b > C) or (b+ (m)*b > T)*

*(2+ (m+n)*2 > 4) or (2+ (m)*2 > 2)*

$$P_{(0,0)} = \left(\frac{(2)^0(1)^0}{0!\quad 0!}\right)/G \quad = 1/G$$

$$P_{(0,1)} = \left(\frac{(2)^0(1)^1}{0!\quad 1!}\right)/G \quad = 1/G$$

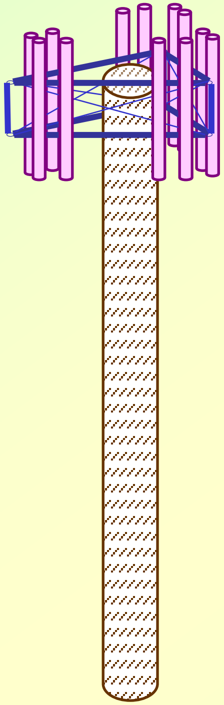
$$P_{(0,2)} = \left(\frac{(2)^0(1)^2}{0!\quad 2!}\right)/G \quad = 0.5/G$$

$$P_{(1,0)} = \left(\frac{(2)^1(1)^0}{1!\quad 0!}\right)/G \quad = 2/G$$

$$P_{(1,1)} = \left(\frac{(2)^1(1)^1}{1!\quad 1!}\right)/G \quad = 2/G$$

[00], [01], [02],

[10], [11],



The probability of blocking a new call (P_b) is given as:

$$P_b = \sum_{s \in S_b} P(s)$$

Where S_b is the set of blocking states

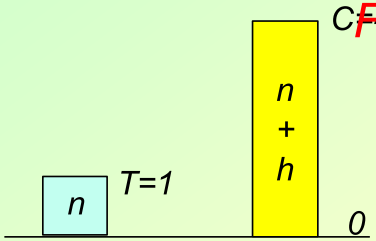
$P_b = P_{(0,2)} + P_{(1,0)} + P_{(1,1)}$

$P_b = (0.5 + 2 + 2) /6.5$

$P_b = 0.692$

Realistic Models of Mobile Networks

Solution 2 (ii) Calculate handoff call dropping probability (P_d)



Find the set of dropping states (S_d) for [m, n]

*(b+ (m+n)*b > C)*

[00], [01], [02],

[10], [11],

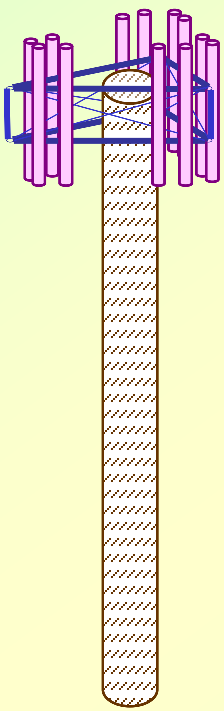
$$P_{(0,0)} = \left(\frac{(2)^0 (1)^0}{0! \quad 0!} \right) / G \quad = 1/G$$

$$P_{(0,1)} = \left(\frac{(2)^0 (1)^1}{0! \quad 1!} \right) / G \quad = 1/G$$

$$P_{(0,2)} = \left(\frac{(2)^0 (1)^2}{0! \quad 2!} \right) / G \quad = 0.5/G$$

$$P_{(1,0)} = \left(\frac{(2)^1 (1)^0}{1! \quad 0!} \right) / G \quad = 2/G$$

$$P_{(1,1)} = \left(\frac{(2)^1 (1)^1}{1! \quad 1!} \right) / G \quad = 2/G$$



Base Station

The probability of dropping a handoff call (P_d) is given by:

$$P_d = \sum_{s \in S_d} P(s)$$

Where S_d is the set of dropping states

$P_d = P_{(0,2)} + P_{(1,1)}$

$P_d = (0.5 + 2) / 6.5$

$P_d = 0.385$

Exercise 2

Through simulation, determine the effect of b (bbu) on P_b and P_d using the following values. $C=20$, $T=10$, $\lambda_m=1$, $\lambda_n=0.5$, $\mu_m=0.5$, $\mu_n=0.5$

Plot the graphs to illustrate the effect of b (bbu) on P_b and P_d

b	**1**	**2**	**3**	**4**	
P_b					
P_d					

Exercise 3

Through simulation, determine the effect of new call departure rate (μ_m) on P_b and P_d using the following values.

$C=20$, $T=10$, $b=1$, $\lambda_m=1$, $\lambda_n=0.5$, $\mu_n=0.5$

Plot the graphs to illustrate the effect of μ_m on P_b and P_d

μ_m	0.25	0.50	0.75	1.00	
P_b					
P_d					

EEE4121F-A

"**Success** is not final; failure is not fatal: It is the courage to continue that counts.“

Never Give Up!