Macroeconomic Analysis of the Child Benefit: Fertility, Demographics, and Welfare*

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Abstract

This study examines the macroeconomic and welfare effects of the child benefit, using a general equilibrium overlapping generations model incorporating fertility choices. The model is calibrated to the Japanese economy and produces the benefit elasticity of fertility in line with the empirical estimates. Expanding the per-child payment would lead to welfare gains for future generations in stationary equilibrium. Notably, the long-run gains extend to individuals who are childless throughout their lives and do not receive child benefits. Higher fertility rates in the new equilibrium and the resulting demographic change account for the results via several channels. However, the accrual of welfare gains is gradual and spans approximately 100 years. This is because the gains are contingent on the demographic shift, necessitating sufficient transition periods.

Keywords: child benefit, overlapping generations, fertility, welfare.

JEL Classification: E60, J13, J18

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1 Introduction

Total fertility rates have dramatically fallen over the last decades across the globe, and they are now below the population replacement level in many advanced economies. Sustained low fertility, without sufficient increases in mortality and younger immigrants, leads to demographic aging, raising several macroeconomic concerns.[1] Importantly, for those countries with social security programs such as public pensions and healthcare, this demographic change will result in reduced benefits or significant tax burdens for households to sustain the payments (e.g., Kitao, 2015); both threaten the households' welfare and living standards.

In response, those countries have introduced pro-natal policies that aim to remove obstacles to fertility and help people bear and raise children. A widely used instrument is the child benefit (CB), cash transfers to households with children.[2] In 2018, average OECD countries paid CB to households with children that amounted to nearly 5% of the average labor earnings in the country.[3]

Although empirical studies show that the CB has positive effects on fertility, as I discuss in detail below, its macroeconomic and welfare implications remain unclear despite their importance in designing the policy well. These implications are out of the scope of those empirical studies, and previous macroeconomic studies on CB abstract fertility choices to focus on its effects on maternal labor supply in detail.

This study constructs a general equilibrium overlapping generations (GE-OLG) model with fertility choices calibrated to the Japanese economy, and re-examines the macroeconomic effects and welfare implications of the CB. It demonstrates the effects of CB policies in an aging economy, considering that Japan is a leading country in both the magnitude of demographic aging and low fertility. This study thus provides insights, regarding the effectiveness of pro-natal transfers, into other countries expected to face more severe demographic aging in the future, akin to Japan's present-day situation.

In this model, individuals have some innate characteristics regarding gender, skill, and marital status (single or married). Married households (couples) choose the number of children they have, whereas unmarried households (singles) cannot. The demographic structure, captured by age distribution, is endogenously determined by couples' fertility choices.

[1] Bloom et al. (2023) discuss the macroeconomic concerns of low fertility in advanced economies, including those related to the labor force shortage and social security systems.

[2] Its eligibility is typically independent of childcare expenditures and parents' labor market status, which makes it different from other family-related transfers, such as childcare subsidies and paid parental leave.

[3] See PF1.3 of the database https://www.oecd.org/els/family/database.htm.

The government runs two sets of programs in the economy: child benefit and social security programs. This model approximates Japan's CB system, as closely as possible, to what is conducted in reality. In the model, the government pays $5,000-11,000$ yen per child-month to households with children under 15 years old, depending on the household income.[4] The monthly per-child payment amounted to roughly 3% of the monthly salary of males aged 20-54 in 2020.[5]

In this model, the social security programs provided by the government consist of the pay-as-you-go (PAYG) pension and health insurance, which are self-financed by a proportional tax on labor earnings. Households' medical needs arise exogenously and deterministically. It depends only on age and evolves with aging, as observed in the data. The government runs the health insurance program, subsidizing a fraction of the medical expenditure. PAYG pension and health insurance programs imply that a larger old-age population share leads to a significant tax burden for the working-age population, holding these payment schemes fixed.[6]

The model is calibrated to the 2000s Japanese economy. An important targeted moment is the benefit elasticity of fertility, and the calibrated model produces an elasticity value that is in line with empirical estimates. I then solve the stationary equilibrium and transition dynamics associated with payment expansion or reduction. While other policy variables are fixed as in the benchmark (e.g., the income replacement rate for the PAYG pension and the copayment rate for public health insurance), two tax rates are adjusted to balance the government budget: (1) the labor income tax rate for self-financing the social security programs, and (2) the consumption tax rate to balance the other government budget.[7] In some experiments, I also solve the model under *exogenous fertility*, where the number of children in each type of household is fixed as in the benchmark. This

[4] Appendix B provides an overview of the current payment scheme.

[5] The average monthly salary is computed from the Basic Survey on Wage Structure released by the Ministry of Labour, Health, and Welfare (MHLW).

[6] In Japan, public pension and health insurance expenditures constitute a sizable portion of the overall expenditures on social securities and other welfare programs such as family-related transfers. According to the National Institute of Population and Social Security Research (IPSS), the total expenditures on these two programs amounted to 17.3% of the GDP in 2019.

[7] This construction that the consumption tax rate is adjusted to balance the government budget follows previous studies on social security reforms in Japan (e.g., Braun and Joines, 2015; Kitao, 2015). They consider that situation partly because this seems to be a more realistic way of reform as the consumption tax rate is relatively lower in Japan compared with other countries and thus, there is relatively more room for increasing the tax rate: in 2021, the consumption tax rate in Japan was 10%, while many European countries set the value added tax to around 20%. For the last decade, the Japanese government has actively discussed increasing the tax rate and actually increased it: the rate rose to 8% from 5% in 2014 and 10% in 2019.

exogenous fertility setup aligns with previous macroeconomic studies and reveals how considering fertility choices makes differences in the macroeconomic and welfare implications of payment expansion.

The main findings are summarized as follows: First, considering fertility choices significantly changes the macroeconomic implications of payment expansion. For example, the equilibrium labor income tax rate declines in the payment scale under endogenous fertility because the higher fertility rate causes the working-age population to expand relative to the old-age, leading to a lower per-capita expenditure on social security benefits. On the contrary, under exogenous fertility, the demographic structure is fixed by construction and the labor income tax rate does not change.

The equilibrium output increases in the payment scale under endogenous fertility. When tripling the per-child payment, for example, the per-capita output is 2.3% higher than the benchmark. However, it does not change significantly under exogenous fertility. The difference comes from changes in the demographic structure; the working-age population share expands in the long run and the output increases under endogenous fertility, whereas the exogenous fertility version of the model does not capture the demographic shift and the output stays almost constant.

Although the consumption tax rates under both fertility settings increase in payment scale, the tax rate under endogenous fertility is lower than that under exogenous fertility for each expansion scenario. This is because the per-capita output is greater under endogenous fertility than under exogenous fertility, and a greater output means a larger tax base, making the equilibrium tax rate lower holding the expenditure constant. Notice that the expenditure on the CB is more significant under endogenous fertility because the number of children in the economy increases with the expansion, which is not captured by the model with exogenous fertility. Despite the larger expenditure, the tax base increase is sufficiently significant so that the equilibrium tax rate under endogenous fertility is lower than that under exogenous fertility.

Second, the endogenous fertility version of the model shows that expanding the per-child payment leads to welfare gains for future generations in terms of the consumption equivalent variation. Notably, the long-run gains extend to individuals who are childless throughout their lives and do not receive child benefits. For each type of household, welfare gains are more significant under endogenous fertility than under exogenous fertility. Importantly, the gains for non-recipients do not accrue under the exogenous fertility version of the model, and they are worse off by the expansion; the welfare implications of the expansion can differ, even qualitatively, between the two different fertility settings.

To study the source of the welfare effects, I decompose the gains for each household into several objects varying between the initial and final steady states, such as tax rates, factor prices, and the scale of the per-child payment. This decomposition demonstrates that different types of households differently benefit from the expansion. For example, less-skilled couples benefit most from the larger payment while, interestingly, high-skilled couples benefit most from the lower labor income tax rate. In other words, for high-skilled couples, the gains from this channel are more significant than the payment increase, unlike for less-skilled couples; households with higher earnings-potential benefit more from this channel. Lower labor income tax also accounts for most of the gains for non-recipients. Recall that the decline of the labor income tax comes from the demographic shift triggered by the higher fertility rate due to a larger payment, and it does not change under exogenous fertility. These results indicate that the welfare gains from payment expansion accrue not only from its redistributive nature (i.e., net beneficiaries are better off due to the redistribution) but also from equilibrium feedback, primarily via tax rates, originated by demographic change. As discussed in Appendix D, I find that the results obtained in the steady state analysis are robust to (1) the value for fertility elasticity and (2) the assumption on human capital accumulation of mothers.

Lastly, the long-run effects take nearly 100 years to accrue because the effects are mainly driven by the change in demographic structure, necessitating a long time. In addition, during the transition, the tax burden for financing the expansion is much larger than its long-run level. Recall that a larger output and tax base mitigate the tax rate increase for financing expansion in the long run, and the higher output is also contingent on demographic change. However, the demographic shift requires sufficient transition periods, and the tax base expansion takes a long time. Consequently, current and some future generations are worse off because of the higher tax burden during the initial transition periods, with poor realization of the gains.

The remainder of the paper is organized as follows: Section 2 reviews several strands of literature related to this study. Section 3 describes the model, which is calibrated in Section 4. Counterfactual policy experiments are implemented in Section 5, and Section 6 concludes the paper.

2 Related Literature

This study is mostly related to the growing literature on the quantitative analysis of family related policies using lifecycle models (e.g., Erosa et al., 2010; Domeij and Klein, 2013; Oguro et al., 2013; Bick, 2016; Guner et al., 2020; Okamoto, 2020; Cavalcanti et al., 2021; Hannusch, 2022; Jakobsen et al., 2022; Ortigueira and Siassi, 2022; Zhou, 2022). This study contributes to the literature by investigating the macroeconomic and welfare implications of CB building on an equilibrium framework with fertility choices and richer heterogeneity across households. Although some studies have embedded fertility choices in their models, they focus on the impacts of other policies, such as parental leave and childcare subsidies (e.g., Erosa et al., 2010; Bick, 2016). And previous works considering the CB, with some notable exceptions, abstract fertility choices to investigate the impacts of a broader set of child-related transfers on female labor supply (e.g., Guner et al., 2020; Hannusch, 2022). This study shows that a CB expansion would benefit even non-recipients in the long run because of its effects on the demographic structure, although the accrual of the gains requires sufficient periods. In addition, it turns out that considering fertility responses to CB policy reforms results in significant differences in their macroeconomic and welfare implications.

One of the closely related papers is a seminal work by Guner et al. (2020) using an OLG model with exogenous fertility to study the effects of child-related transfers in the US on maternal labor supply, female human capital accumulation, and welfare. My study differs from theirs in considering fertility choices and endogenous demographic structure, while they consider a comprehensive set of child-related transfers not limited to unconditional cash transfers such as CB.[8] Another key difference is that my study draws the transition path of macroeconomic variables between the initial and new equilibrium associated with policy reforms. This enables us to understand their short-run effects, how long the economy takes to reach the new equilibrium, and the resulting welfare effects on each cohort. Given that the long-run effects accrue primarily from demographic change, which will require sufficient periods, solving the transition dynamics is a vital part of this study because many cohorts living in the transition periods could not benefit from the reforms, and whether this is the case is critical for policy discussion.

There are some notable exceptions in recent studies in that examining the macroeconomic effects of pro-natal cash transfers using a model with fertility choices. Zhou (2022) constructs a heterogeneous-agent OLG model equipped with the quantity-quality trade-off paradigm to study the effects of family policies such as the baby bonus and childcare subsidies. My study differs from his in that my model considers the heterogeneity in family structure, which helps us examine the welfare implications of the CB policies for non-recipients, while his model considers a broader set of policy instruments on the quantity-quality trade-off framework. Kim et al. (2023) construct a two-period OLG model with quantity-quality trade-off to shed light on the role of the *status externalities* on the low fertility rate in South Korea. They also simulate the expansion of pro-natal

[8]At the same time, I abstract a few components considered in Guner et al. (2020). A critical one is the process of female human capital accumulation: it evolves endogenously in their model whereas it is exogenously given in my model. I consider the robustness of my results to the construction of female human capital accumulation in Appendix D, by a simple extension of the model.

transfers and observe its effects on fertility rates and aggregate human capital. My study differs from theirs in using a lifecycle model with finer periods and capturing the impacts of the CB on the demographic structure.[9]

Finally, this study is also related to the empirical literature on the effects of cash benefits to households with children, especially CB and baby bonuses. Previous studies investigate their impacts on parental labor supply (e.g., for studies in Japan, Bessho, 2018; Asakawa and Sasaki, 2022), on fertility (e.g., Milligan, 2005; Cohen et al., 2013; Riphahn and Wiynck, 2017), and on both (Azmat and González, 2010; González, 2013; Laroque and Salanié, 2014; Yamaguchi, 2019).

Regarding the labor supply effects, the consensus is that generous cash transfers would depress parents' labor supply, especially that of mothers, in the short run (e.g., Asakawa and Sasaki, 2022). At the same time, they show that considering the labor supply response is important for evaluating potential options for policy reform. For example, Bessho (2018) uses a discrete choice model of labor supply to show that Japan's CB expansion conducted in 2010-11 depressed parents' labor supply, and that funds required for the reform are underestimated by 22% if the labor supply responses are disregarded. Recently, Corinth et al. (2022) suggest that replacing the Child Tax Credit with the child allowance in the US is not as effective in reducing child poverty because poorer parents with young children would reduce their labor supply due to the introduction of the child allowance. Building on these insights, I incorporate the labor supply choices to capture CB's effects on maternal labor supply.

For the fertility effects, previous works show that the cash benefits such as the CB and baby bonus have a significant impact on fertility.[10] Milligan (2005) exploits a reform of Quebec's baby bonus and argues that an extra 1,000 Canadian dollars benefit would increase fertility by 16.9%, implying a benefit elasticity of 0.107. Azmat and González (2010) study the Spanish income tax reform, including an increase in yearly supplements per child from 300 to 1,200 euros, to identify its fertility effects. They show that the reform increased the fertility rate by 5%, and the benefit elasticity of fertility was 0.022.

[9]Some other related studies also use the OLG model with fertility choices (Oguro et al., 2013; Okamoto, 2020). The primary difference from those studies is twofold. First, this study disciplines the model to replicate the benefit elasticity of fertility in line with empirical estimates. Given that fertility responses against CB policies determine their effects on the demographic structure and, thereby, macroeconomic variables and welfare, fertility elasticity is a critical moment in evaluating the policy more accurately. Second, this study considers richer heterogeneity across households, especially in skill and marital status, allowing us to evaluate the heterogeneous welfare implications of CB policies.

[10]From the viewpoint of institutional similarity, this paper relies on empirical studies on the CB and baby bonus, and I do not include papers studying a reform of the EITC on the list (e.g., Baughman and Dickert-Conlin, 2009).

Cohen et al. (2013) uses a variation in the child subsidy payment conducted around 2003 in Israel, providing a larger subsidy for third or higher births. They conclude that the benefit elasticity of fertility was 0.176. González (2013) adopts the regression discontinuity design to study fertility effects implied by Spain's reform in 2007 introducing a one-time payment of 2,500 euros (about 3,800 USD) for births, almost 4.5 times the monthly minimum wage for full-time workers. It finds a statistically significant impact on fertility, increasing conceptions by approximately $5 - 6\%$. Laroque and Salanié (2014) estimate a discrete choice model of fertility and participation using the French Labor Force Survey. They simulate the introduction of an unconditional transfer of 150 euros per month for households with children, which would require funds amounting to 0.3% of GDP. This introduction results in a nearly 20% higher fertility rate.[11]

Overall, the literature reports a significant effect of CB or baby bonuses on fertility. However, the results regarding the dependence of elasticity on household income are mixed; some find that the elasticity is larger for households with higher income(e.g., Milligan, 2005; Riphahn and Wiynck, 2017),[12] while others report the opposite (e.g., Azmat and González, 2010; Cohen et al., 2013).

This study complements the literature by examining the macroeconomic and welfare implications of expanding the cash transfer, building on the GE-OLG framework. In doing that, the model is calibrated so that the benefit elasticity of fertility implied by the model is consistent with the empirical estimates. The benefit elasticity of fertility is a key moment for constructing a reasonable quantitative model, as we are interested in the equilibrium outcome considering fertility responses to potential CB reforms. Although there are few empirical studies on the fertility effects in Japan,[13] we can rely on Yamaguchi (2019), which implies an elasticity value of 0.025 as a targeted value for calibrating the model. Section 4 and Appendix B provide a detailed discussion of this point, and Appendix D checks the robustness of my main results to the fertility elasticity in the model.

[11] González (2013) and Laroque and Salanié (2014) also examine the effect of the cash transfer on maternal employment and show that the generous transfer reduces maternal employment.

[12] Riphahn and Wiynck (2017) study Germany's child benefit reform in 1996. They show that the fertility effects on low-income couples are not statistically significant. They also find some support that the fertility effect is significant for higher-income couples deciding on a second birth.

[13] One of the primary reasons for this is that, in Japan, the central government designs the CB system, and there is no room for local governments to establish their original system. Previous works (e.g., Milligan, 2005) indicate that using the regional difference in the benefit system can be a powerful device for identifying the fertility effect.

3 Model

This section describes a stationary OLG economy populated by heterogeneous households.

3.1 Primitives

Demographics: Let $j \in \mathcal{J} \equiv \{1, ..., J\}$ denote the age of an individual. Individuals live for a maximum of J periods, and a fraction of each cohort dies in each period. The government collects accidental bequests left by those who die before age J and makes a lump-sum transfer to each household, denoted by a_b. Let $s_{j,j+1}$ be the conditional probability of being alive at $j+1$, given that the person is alive at age j. The unconditional probability of being alive at age $k \in \{2, ..., J\}$ is denoted by $S_k = \Pi_{j=1}^{k-1} s_{j,j+1}$ with $S_1 = 1$. Every period, the mass of new generation in which each individual has a_1 units of initial assets enters the economy. The cohort size at age $j = 1$ grows at a rate n across cohorts, which is endogenously determined by households' fertility choices in this model. The growth rates of cohort size and the survival probabilities characterize the measure of individuals aged j, denoted by μ_j, which is normalized to add up to one (i.e., $\Sigma_j \mu_j = 1$).[14]

Endowment: Individuals entering the economy are endowed with gender, marital status, and skill. Let $g \in \mathcal{G} \equiv \{M, F\}$ denote gender, where M stands for male and F stands for female. Next, let $z \in \mathcal{Z}$ denote the skill of individuals. A fraction of males and females in a cohort are matched exogenously at the start of their lives, forming a family. Let $m \in \mathcal{M} \equiv \{C, S\}$ denote marital status where C stands for couple and S stands for single. These three factors are time-invariant; couples do not divorce, and single households do not marry anyone throughout their lives. Let $\mu_{g,z,m}$ represent the fraction of individuals with time-invariant states (g, z, m). For couples, let superscripts h and w denote the variables for the husband and wife, respectively. In addition, among couples, let π_{z^h,z^w} represent the fraction of those comprised of a husband with skill z^h and a wife with skill z^w. Finally, let $\boldsymbol{\theta}_C = (z^h, z^w) \in \boldsymbol{\Theta}_C$ and $\boldsymbol{\theta}_S = (g, z) \in \boldsymbol{\Theta}_S$ represent vectors of the characteristics of married and single households, respectively. I refer to $\boldsymbol{\theta}_C$ and $\boldsymbol{\theta}_S$ as the *type* of household.

Preference: Every period, each household draws utility from consumption and disutility from labor supply. Individuals can participate in the labor market after entering the economy and are forced to retire at the end of age $j = J_R$. In this model, all husbands

[14]In other words, the population mass is fixed to one, and the cohort growth rate matters only to the age distribution, not the population size.

supply one unit of labor exogenously and inelastically,[15] and wives and singles choose their working hours. Let c and l denote the consumption and working hours.

Couples also draw utility from having children. Upon marriage, couples choose the number of children they have, while singles do not make the fertility decisions and have no children throughout their lives. If couples decide to have children, they have all births at once, at age $j = J_b$, which is common across households.

Let $u^C(c, l; b)$ and $u^S(c, l)$ denote the flow utility for couples and singles, where $b \in \mathcal{B}$ represents the number of children they have. The utility from having children is captured by a function $v(b)$. The lifetime utility of couples and singles, denoted as U^C and U^S, are formulated as follows:

$$U^C = v(b) + \sum_{j=1}^{J} S_j \cdot \beta^{j-1} \cdot u^C(c_j, l_j; b),$$

$$U^S = \sum_{j=1}^{J} S_j \cdot \beta^{j-1} \cdot u^S(c_j, l_j),$$

where β is the subjective discount factor.

Labor earnings: Three factors determine individuals' labor earnings: the wage rate w determined in the competitive labor market, labor productivity, and hours worked l. The labor productivity of an individual with (j, g, z) is represented by a function $\bar{\omega}(j, g, z)$, capturing the wage differences across different ages, gender, and skill. Lastly, let $y = w \times \bar{\omega}(j, g, z) \times l$ represent an individual's flow labor earnings.[16]

Costs of children: Two types of costs arise upon having children: monetary and time costs. First, households with children incur child-related expenditures up to a certain period, depending on the type of parents and the number of children. A function $CE(\boldsymbol{x}_{CE})$ captures child-related expenditures for couples, where $\boldsymbol{x}_{CE} = (j, b, z^h, z^w)$. In addition to the monetary cost, households with young children must spend a fraction of their disposable time per child, denoted by $\eta > 0$.

Child benefit: Couples receive the CB payment according to a function $CB(\boldsymbol{x}_{CB}; X)$, where $\boldsymbol{x}_{CB} = (j, b, y^h, y^w)$. y^h and y^w denote the annual labor earnings of husband and wife, and $X \geq 0$ is a scale parameter for the per-child payment where $X = 1$ in the benchmark. Hereafter, I use terms "the scale parameter for the per-child payment" (or

[15] According to the Employment Status Survey (ESS) in 2017 published by the Administration of Internal Affairs and Communications (MIC), about 98% of prime-age married males are in the labor market, and more than 95% of them work full-time.

[16] As in Guner et al. (2020), there is no uncertainty for labor productivity.

just "the scale parameter") and "(gross) expansion rate" interchangeably to refer to the parameter X. Section 4 represents the functional form, constructed to approximate the actual payment system in Japan. In Section 5, I then change X to simulate an expansion of the CB to study the effects of CB expansion.

Medical needs and social security transfers: Every period, medical needs, captured by m_j, arise for each individual, depending on their age. The needs are exogenous and deterministic. The government runs the public health insurance program and subsidizes ω_j fraction of the medical expenditure, depending on age j. The government also runs the PAYG pension program, providing pension benefits to individuals aged $j > J_R$. The per-person payment denoted by ss is assumed to be constant across retirees with an average-income replacement rate ρ; that is, $ss = \rho\bar{y}$ where $\bar{y}$ denotes the average labor earnings of workers.

Firm: A representative firm rents capital (K) and effective labor (L) in competitive factor markets at rates of $r + \delta$ and w, where δ denotes the depreciation cost of capital that the firm has to incur. It chooses the inputs to maximize the profit obtained by selling final goods in the competitive market, where the goods are produced according to a production function $F(K, L)$.

Government: The government raises revenue from taxing consumption, labor earnings, and capital income, with linear tax rates of τ_c, τ_l, and τ_a, respectively. The labor income tax rate τ_l comprises a part necessary for financing the social security benefits (τ_{ss}) and another (τ_{-ss}); that is, $\tau_l = \tau_{ss} + \tau_{-ss}$. Government expenditures are broken down into three parts: expenditures on (1) the child benefit (CB), (2) social security programs, and (3) others (G). Social security expenditures, comprising those for the public pension and health insurance programs, are self-financed by the proportional tax on labor earnings τ_{ss}:

$$\tau_{ss} w L = \underbrace{ss \sum_{j=J_R+1}^{J} \mu_j}_{\text{pension}} + \underbrace{\sum_{j=1}^{J} \omega_j m_j \mu_j}_{\text{health insurance}} . \tag{1}$$

The other tax rates are set so that the following budget constraint is satisfied:

$$\tau_c C + \tau_{-ss} w L + \tau_a r K = CB + G, \tag{2}$$

where C denotes the aggregate consumption.

3.2 Households' Problems

Given the economic environment described thus far, I formulate household decision problems.

Couples: Hereafter, I abstract the subscript j from the variables and denote a' to represent the asset position at the beginning of the next period. Couples choose the number of children at the start of their lives. Given the number of children and their household type, they make choices on consumption, wife's labor supply, and savings, throughout their lives. They can borrow or lend at interest rate r, but there is a borrowing limit ϕ. The value function for couples with type $\boldsymbol{\theta}_C$ and initial asset a_1 is as follows:

$$W^C(a_1, \boldsymbol{\theta}_C) = \max_{b \in \mathcal{B}} \{v(b) + V^C(a_1, 1, b, \boldsymbol{\theta}_C)\} \tag{3}$$

where

$$V^C(a, j, b, \boldsymbol{\theta}_C) = \begin{cases} \max_{\{c,a',l^w\}} \{u^C(c, l^w; b) + s_{j,j+1}\beta V^C(a', j+1, b, \boldsymbol{\theta}_C)\} & \text{if } j \in \{1, ..., J_R\} \\ \max_{\{c,a'\}} \{u^C(c, 0; b) + s_{j,j+1}\beta V^C(a', j+1, b, \boldsymbol{\theta}_C)\} & \text{if } j \in \{J_R+1, ..., J-1\} \\ \max_{\{c\}} u^C(c, 0; b) & \text{if } j = J \end{cases} \tag{4}$$

s.t.

$$(1+\tau_c)c = \begin{cases} (1+(1-\tau_a)r)(a+2a_b) - a' + (1-\tau_l)(y^h + y^w) & \\ -CE(\boldsymbol{x}_{CE}) + CB(\boldsymbol{x}_{CB}; X) - 2(1-\omega_j)m_j & \text{if } j \in \{1, ..., J_R\} \\ (1+(1-\tau_a)r)(a+2a_b) - a' + 2[ss - (1-\omega_j)m_j] & \text{if } j \in \{J_R+1, ..., J-1\} \\ (1+(1-\tau_a)r)(a+2a_b) + 2[ss - (1-\omega_j)m_j] & \text{if } j = J \end{cases} \tag{5}$$

$$c \geq 0, a' \geq -\phi, l^w \in [0, 1 - b \cdot \eta]. \tag{6}$$

Singles: Singles make choices regarding consumption, labor supply, and savings, throughout their lives. The value function for singles of type $\boldsymbol{\theta}_S$ with asset a is as follows:

$$V^S(a,j,\boldsymbol{\theta}_S)=\begin{cases}\max_{\{c,a',l\}}\{u^S(c,l)+s_{j,j+1}\beta V^S(a',j+1,\boldsymbol{\theta}_S)\} & \text{if } j\in\{1,...,J_R\}\\ \max_{\{c,a'\}}\{u^S(c,0)+s_{j,j+1}\beta V^S(a',j+1,\boldsymbol{\theta}_S)\} & \text{if } j\in\{J_R+1,...,J-1\}\\ \max_{\{c\}} u^S(c,0) & \text{if } j=J\end{cases} \tag{7}$$

s.t.

$$(1+\tau_c)c=\begin{cases}(1+(1-\tau_a)r)(a+a_b)-a'+(1-\tau_l)y-(1-\omega_j)m_j & \text{if } j\in\{1,...,J_R\}\\ (1+(1-\tau_a)r)(a+a_b)-a'+ss-(1-\omega_j)m_j & \text{if } j\in\{J_R+1,...,J-1\}\\ (1+(1-\tau_a)r)(a+a_b)+ss-(1-\omega_j)m_j & \text{if } j=J\end{cases} \tag{8}$$

$$c\geq 0, a'\geq -\phi, l\in[0,1]. \tag{9}$$

3.3 Equilibrium

I solve the stationary equilibrium of the economy. In equilibrium, households choose consumption, savings, and labor supply, and couples choose the number of children, to maximize their lifetime utility. The representative firm maximizes its profit by choosing labor and capital inputs. Prices clear factor markets. The government budget constraints are satisfied each period. Importantly, households' fertility choices endogenously determine the demographic structure, which is represented by μ_j. Stationarity implies that μ_j is time-invariant in equilibrium. Appendix A provides a detailed definition of equilibrium.

4 Calibration

This section provides an overview of the calibration of the model, and Appendix B provides a detailed discussion. The calibration procedure involves three steps as follows:

1. **Preliminary**:
   - Specify functional forms for preference and technology.
   - Set some parameter values and distributions, following the literature or based on Japan's aggregate data.
   - Make an initial guess for a parameter value governing the curvature of utility from having children, γ, in a function (12) below.

2. **Inner loop**: Except for γ, calibrate the rest of the parameters to be determined inside the model such that the model-implied values for each target moment are sufficiently close to their targets.

3. **Outer loop**: Conduct a counterfactual experiment increasing the per-child payment and compute the benefit elasticity of fertility to the payment expansion, holding prices and tax rates constant. Until the implied elasticity is sufficiently close to its target value, update the guess for γ and return to Step 2.

Demographics: The length of one period in the model is five years. Individuals enter the economy when they are 25 years old, retire at the end of age 64 (i.e., $J_R = 8$), and live to the end of age 104 at a maximum (i.e., $J = 16$). I set $J_b = 2$ so that households have births at the age of 30. The survival probability is set based on the Vital Statistics (2019).[17] Individuals enter the economy with no assets (i.e., $a_1 = 0$), except for the lump-sum transfer of the accidental bequests. Measures of individuals with time-invariant states $\mu_{g,z,m}$ and couples in each education pair π_{z^h,z^w} are set based on the ESS (2017).

Productivity: I set the productivity function $\bar{\omega}$ based on the Basic Survey on Wage Structure (2020) of the MHLW, considering the skill space as follows:

$$\mathcal{Z} = \{<HS, HS, SC, COL, COL+\},$$

where each entry represents the educational background. $<HS, HS, SC, COL$, and $COL+$ stand for less than high school (elementary school or junior high school graduate), high school, some college, college, and graduate school, respectively.

Child Benefit: The following function for the monthly payment of the CB, $CB(\boldsymbol{x}_{CB}; X)$, is conducted to approximate the current CB system in Japan:

$$CB(\boldsymbol{x}_{CB}; X) = \begin{cases} \mathbb{I}_{j_c \in [0,14]}[b \times \yen 11,000] \times X & \text{if } \max\{y^h, y^w\} \leq 9.6 \text{ million yen}, \\ \mathbb{I}_{j_c \in [0,14]}[b \times \yen 5,000] \times X & \text{otherwise}, \end{cases}$$

where $j_c (= j - J_b)$ denotes the children's age, and $\mathbb{I}_{j_c \in [0,14]}$ is an indicator function taking 1 if $j_c \in [0, 14]$ and 0 otherwise. Recall that $X = 1$ for the benchmark. The current system provides 5,000 yen per child-month, for example, for households with three dependents if the highest earner's annual earnings in the household are more than 9.6 million yen. For those who earn less than the value, they receive on average 11,000 yen. According to the

[17] See https://www.mhlw.go.jp/english/database/db-hw/outline/index.html.

ESS (2017),[18] 7.9% of husbands with children under 18 earned more than 10 million yen, implying that the majority receive 11,000 yen per child-month.[19] Note that the ESS says that the median annual earnings of husbands with children under 18 lie between 2 and 2.49 million yen, which means that the per-child payment covers nearly $5-6\%$ of the median earnings.

Preference: Periodic utility functions for couples and singles are given as follows:

$$u^C(c, l^w; b) = 2 \cdot \frac{(c/\Lambda(b))^{1-\sigma} - 1}{1-\sigma} - \varphi \frac{(l^w + b \cdot \eta)^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}}, \tag{10}$$

and

$$u^S(c, l) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \varphi \frac{l^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}}, \tag{11}$$

where $\varphi > 0$ affects the disutility from additional working hours and μ represents the Frisch elasticity. $\Lambda(b)$ indicates the equivalence scale, and I adopt the OECD modified equivalence scale where $\Lambda(b) = 1.5 + 0.3b$. This specification applies until their children become independent (i.e., when the children turn 25 years old). I set $\varphi = 1.26$ so that the average hours worked in the model matches the data.[20] The time cost of childcare η is set to 0.339 to replicate the ratio between the working hours of mothers with young children and their childcare time.[21] In addition to the time costs, this model also captures education expenditures with a function CE, set according to the White Paper on Education, Culture, Sports, Science and Technology (2009) of the Ministry of Education. The subjective discount factor β is set to 1.056 so that the initial steady state approximates a capital-to-output ratio of 2.8.

For utility from consumption, I consider a case in which $\sigma \to 1$, the logarithmic function. The Frisch elasticity μ is set to 0.85 based on Japan's empirical estimate.[22] The borrowing limit ϕ is set to zero (i.e., households cannot borrow).

[18] See Table 250-2 in https://www.e-stat.go.jp/en/stat-search/files?page=1&layout=datalist&toukei=00200532&tstat=000001107875&cycle=0&tclass1=000001107876&tclass2=000001107878&tclass3val=0.

[19] The precise level of payment varies according, in addition to income level, to the number of children and children's age. For more detail about the current system, see Appendix B.

[20] The Survey on Time Use and Leisure Activities (2016, STL) conducted by the MIC reports the average hours worked for each marital status (couple or single) and gender. Normalizing the hours worked by married men as 1, I set the target for calibrating φ to 0.77, which is the average hours worked for singles and wives. See https://www.stat.go.jp/english/data/shakai/2016/gaiyo.html.

[21] According to the STL (2016), mothers with a child under 6 spend 142 minutes on market work and 197 minutes on childcare per day, implying that the ratio is given as 0.721.

[22] Since Kuroda and Yamamoto (2008) reports that the elasticity considering both intensive and extensive margin ranges between 0.7 and 1.0, I take the intermediate value.

The utility from having children is captured by the following function:

$$v(b) = \kappa \cdot \frac{(1+b)^{1-\gamma} - 1}{1-\gamma}, \tag{12}$$

where κ is set to 367.8, targeting the annual growth rate of the cohort size, -1.15%. The curvature parameter γ is determined in Step 3. The targeted moment is the benefit elasticity of fertility to the payment increase, indicating the percent change in the completed fertility rate in response to a 1%-increase in the per-child payment. I rely on Yamaguchi (2019) and set the target value to 0.025.[23] To check if the model-implied elasticity is sufficiently close to the target, I conduct counterfactual experiments changing the scale parameter for per-child payment, X. I consider seven cases where X takes any of the values in a set $\{2, 2.5, 3, 3.5, 4, 4.5, 5\}$, which are similar to the scales of the expansion examined in empirical studies and my experiments in Section 5. I perform the simulation holding prices and tax rates constant,[24] and compute the completed fertility rate of women with each expansion case. The elasticity is then calculated for each expansion rate using the implied fertility rates, and the mean of the seven elasticity values is computed. It turns out that setting $\gamma = 3.5$ leads to an average elasticity of 0.024 with that range of expansion rates,[25] which is sufficiently close to the target value.

Social Security Programs and Taxes: Based on empirical estimates in Japan, I set $\tau_a = 0.35$ and $\tau_l = 0.35$ in the benchmark case.[26] The consumption tax rate τ_c is set to 0.1, as Japan currently has. The income replacement rate ρ for the PAYG pension is set to $\rho = 0.122$ so that the total spending on public pension corresponds to 10% of GDP. Age-dependent medical needs and copayment rates are computed based on data prepared by the MLHW.[27] The benchmark social security tax rate τ_{ss} is determined so that the constraint (1) is satisfied, which pins down another part of the labor income tax rate as $\tau_{-ss} = \tau_l - \tau_{ss}$. These values then pin down the other government expenditures in the benchmark, G, to equate both sides of the government budget constraint (2).

Technology: The representative firm produces the final good using the Cobb-Douglas

[23] Appendix B provides a detailed discussion of this point, and Appendix D checks the robustness of my main results to the fertility elasticity in the baseline model.

[24] This setting is the same as in Yamaguchi (2019) and in other studies using the dynamic discrete choice framework in the empirical literature.

[25] Table 8 in Appendix B reports the computed elasticity for each expansion rate with $\gamma = 3.5$.

[26] Hansen and İmrohoroğlu (2016) estimate the 2010's capital income tax rate in Japan, and Gunji and Miyazaki (2011) estimate the average marginal tax rate, including social security taxes, on labor income.

[27] The data is available from `https://www.mhlw.go.jp/wp/hakusyo/kousei/17/backdata/xls/1/03-01-09.xls` (in Japanese).

production function, $F(K,L) = K^{\alpha}L^{1-\alpha}$. Capital share α is 0.36, and annual depreciation is 0.089, following Kitao (2015).

Table 1 summarizes the parameter values calibrated inside the model (i.e., in Steps 2 and 3).

Parameter	Value	Target (description)	Target	Model
β	1.056	Capital/Output	2.8	2.6
η	0.339	Working/Childcare	0.721	0.720
φ	1.26	Average hours worked	0.77	0.77
κ	367.8	Annual growth rate of birth numbers	-1.15%	-1.16%
γ	3.5	Benefit elasticity of fertility	0.025	0.024
ρ	0.122	Pension expenditure to GDP	10%	10%

Table 1: Parameters calibrated inside the model.

Non-targeted moment: This model replicates fertility differentials across education categories, which is a non-targeted moment. To observe this, I use the National Fertility Survey (NFS),[28] reporting completed fertility rates by educational background of married women.

According to the report, in 2015, the completed fertility rate for college graduated women was 1.89 and for those graduating high school was 1.99, implying that more educated women tend to have fewer children than less educated women. This also implies that the ratio between the fertility rates of high school and college graduates was 0.95.

In the benchmark model, the fertility rate is 2.27 with $z^w = HS$ and 2.12 with $z^w = COL$, yielding a ratio of 0.93. The model replicates the observation that high-skilled women are less likely to have children than low-skilled women, with a similar degree to the data. Table 2 summarizes the results.

	Data		**Model**	
Education	HS	COL	HS	COL
# of births	1.99	1.89	2.28	2.12
COL/HS	0.95		0.93	

Table 2: Fertility in different education categories. *Note*: The data part comes from the NFS. A column "Education" indicates the wife's educational background, where HS refers to high school graduates, and COL refers to college graduates. For the model part, I report the completed fertility of married women with each education status in the model.

[28] See https://www.ipss.go.jp/ps-doukou/e/doukou15/Nfs15_gaiyoEng.html.

A primary factor behind this gap is the difference in opportunity costs across different education categories. Recall that, in the model, having children takes a fraction of the time cost until the children are grown. As the average wage of more educated women is higher, the opportunity cost of having children is also higher, making them have fewer children.

5 Numerical Analysis

5.1 Methodology

Using the model, this section examines the effects of increasing/decreasing the per-child payments. Specifically, I solve the stationary equilibrium defined in Section 3 and Appendix A, with different levels of the per-child payment. In each exercise, I change the scale parameter for CB (X), holding other government variables, such as G, ρ, and $\{\omega_j\}_j$, fixed as in the benchmark.

Upon reforms, the government adjusts the consumption tax rate to balance the budget (2). In addition, the social security tax rate (τ_{ss}) is adjusted to satisfy the budget constraint for self-financing social security (1), which changes the total tax rate on labor earnings ($\tau_l = \tau_{ss} + \tau_{-ss}$). Even though parameters, such as the income replacement rate for the PAYG pension, are fixed as in the benchmark, the social security expenditures can differ from the benchmark, especially in the case when the demographic structure changes due to the expansion. Second, I solve the transition dynamics associated with the change in per-child payment, describing how the economy will reach a new equilibrium and how the policy change affects the welfare of cohorts living in the transition periods.

Hereafter, I use the term "non-recipients" and "unmarred (or single) households" interchangeably. I find that all types of couples in the benchmark have a positive number of children. In addition, all households with children under 15 years old are eligible for the benefit in this model, as explained in Section 4, meaning that all couples qualify as program recipients at certain periods in their lives. Thus, only unmarried individuals are ineligible for the CB throughout their lives; in this sense, they are only non-recipients.

Welfare Measure: In each exercise, I compute the consumption equivalent variation (CEV) of each household type. The CEV for a household indicates how much of its lifetime consumption vector in the benchmark needs to be scaled up for the household to be indifferent between living in the benchmark and the new equilibrium.[29] To formulate

[29]As Jones et al. (2007) explain, defining the Pareto efficiency is not straightforward in models with

the CEV with my model, let $W(a, \boldsymbol{\theta}_C; X)$ ($V^S(a, j, \boldsymbol{\theta}_S; X)$) be the value function for couples (singles) with type $\boldsymbol{\theta}_C(\boldsymbol{\theta}_S)$, where the scale parameter for per-child payment is given by X. Then, the CEV for those couples, where the scale parameter is given by X', is defined as a real number λ satisfying the following equation:[30]

$$W(a, \boldsymbol{\theta}_C; X') = \sum_{j=1}^{J} S_j \cdot \beta^{j-1} u^C(\bar{c}_j(1+\lambda), \bar{l}_j; \bar{b}) + v(\bar{b}),$$

where $\{\bar{c}_j\}_{j=1}^{J}, \{\bar{l}_j\}_{j=1}^{J_R}$, and $\bar{b}$ are solutions for the couple's problem in the benchmark where the CB payment function is given by $CB(\boldsymbol{x}_{CB}; X = 1)$.

Similarly, the CEV for singles of type $\boldsymbol{\theta}_S$ is defined as a real number λ satisfying the following equation:

$$V^S(a, 1, \boldsymbol{\theta}_S; X') = \sum_{j=1}^{J} S_j \cdot \beta^{j-1} u^S(\bar{c}_j(1+\lambda), \bar{l}_j).$$

Note that this formulation implies that a positive value of CEV indicates welfare improvement for the household owing to the reform.

5.2 Steady States

I solve the stationary equilibrium with four expansion rates, $X = 0, 2, 3$ and 4. The case of $X = 0$ corresponds to the CB removal. Setting an upper bound of $X = 4$ seems reasonable because that scale, at least in the short run, corresponds roughly to the current maximum amount of per-child payments among OECD countries, which covers approximately 15% of the average labor income.[31]

5.2.1 Aggregate Variables

First, the fertility rate increases with expansions. For example, with an expansion rate of three, the equilibrium fertility rate is 1.48, while the benchmark fertility rate is 1.41. Consequently, the working-age population share increases by 1.5 percentage points (p.p.).

This demographic change affects other macroeconomic variables. First, the long-run labor income tax rate declines in the expansion rate. For example, with an expansion rate of three, the income tax rate declines by 1.5 p.p. The aggregate labor supply (i.e., total labor supply in efficiency unit, over the population) increases due to a higher working-age population share.

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endogenous fertility.

[30] Suppose that the amount of initial asset endowment for each type is the same between the benchmark economy and any another one.

[31] For more detail, see PF1.3 of the database https://www.oecd.org/els/family/database.htm.

A greater labor force with a lower tax rate on labor earnings will increase disposable income and savings, resulting in greater per-capita capital, output, and accidental bequests. In particular, expanding the system will increase the output by $1.4 - 5.0\%$ in the long run. The consumption tax rate should increase to finance the additional payment for the CB by $0.4 - 1.7$ p.p. with those expansion scenarios. Owing to a more substantial increase in the total labor supply compared with capital, the wage rate slightly declines, and the interest rate increases in each expansion scenario. Due to the labor income tax decline, however, the after-tax wage rate, $(1 - \tau_l)w$, increases with CB expansion.

The removal has minor impacts on the entire economy. The equilibrium fertility rate drops to 1.39 with the removal. As the demographic structure does not change much, the aggregate quantity, such as per-capita output, does not change significantly. Due to a lower working-age share, the labor income tax rate financing the social security programs slightly increases (by 0.1 p.p.). On the contrary, the consumption tax rate declines by 0.3 p.p. due to the removal.

I also solve equilibria with those expansion scenarios under *exogenous fertility*, where the number of children in each type of household is exogenous and fixed as in the benchmark. The long-run labor income tax rate does not change with expansion or removal, mainly because the change in the demographic structure is not considered. Next, the per-capita output mostly remains the same in each scenario, as the demographic structure is fixed.

There are two notable points about comparing the results under endogenous fertility with those under exogenous fertility. First, although the equilibrium consumption tax rate increases with expansion rate X, it is higher under exogenous fertility than under endogenous fertility for each scenario, as visualized in Fig 1(a). The result might be surprising since the total expenditure on the CB is larger under endogenous fertility, as Fig 1(c) shows, because the number of children in the economy increases under endogenous fertility, in addition to the per-child payment. However, recall that the per-capita output increases by $1.4 - 5.0\%$ under endogenous fertility, while there is no significant change under exogenous counterpart. As Fig 1(d) indicates, the output increase entails a consumption increase, expanding the tax base and mitigating the tax burden for financing the expansion. This explains how the tax rate for funding the expansion is lower under endogenous fertility even though the total expenditure is more significant.

Next, the labor income tax rate is stable with expansion under exogenous fertility, whereas it declines under endogenous fertility (see Fig 1(b)). As Fig 1(e) indicates, the equilibrium age distribution changes with expansion under exogenous fertility, making the working-age (old-age) share larger (smaller). Then, the total expenditure of the social security benefits decreases in the expansion rate under endogenous fertility (see Fig 1(f)).

	Expansion Rate (X)				
	0	1	2	3	4
Demographic Structure					
Completed fertility	1.39	1.41	1.45	1.48	1.53
Working-age share (Δ p.p.)	-0.34	–	0.94	1.50	2.60
Annual population growth (%)	-1.20	-1.16	-1.06	-1.00	-0.88
Quantity (per-capita, Δ%)					
Output	0.0(-0.1)	–	1.4(0.0)	2.3(0.2)	5.0(0.8)
Capital	-0.1(-0.5)	–	0.8(0.0)	2.0(0.7)	4.2(2.2)
Labor (in efficiency unit)	0.0(0.1)	–	1.7(0.0)	2.5(0.0)	5.4(0.0)
Bequests	0.7(0.0)	–	0.8(1.2)	0.9(1.6)	0.3(2.6)
Prices					
Interest (%)	5.19(5.23)	5.18	5.27(5.18)	5.23(5.12)	5.28(4.99)
Wage (Δ%)	0.0(-0.2)	–	-0.3(0.0)	-0.2(0.2)	-0.4(0.8)
After-tax ($[1-\tau_l]w, \Delta\%$)	-0.2(-0.2)	–	1.1(0.0)	2.0(0.2)	4.0(0.8)
Taxes (%)					
Consumption	9.7 (9.6)	10	10.4 (10.6)	11.1 (11.2)	11.7 (11.9)
Labor income	35.1 (35)	35	34.1 (35)	33.5 (35)	32.2 (35)

Table 3: Changes in aggregate variables in the long run with different per-child payments. *Note*: Values in the parenthesis in each cell are those obtained under exogenous fertility. Note that, by construction, the demographic structure is invariant with expansion under exogenous fertility. Thus, I omit values under exogenous fertility for that part. Cases of $X = 0$ and $X = 1$ correspond to the CB removal and benchmark, respectively.

On the contrary, age distribution is invariant under exogenous fertility by construction. Then, the social security expenditure is stable (or slightly increasing owing to fewer hours worked, as described below). Consequently, the required tax rate for balancing the budget is lower than the benchmark under endogenous fertility, whereas it does not change under exogenous fertility.

The results suggest that considering fertility responses non-trivially changes the macroeconomic implications of CB reforms. Each cell in Table 3 reports changes in aggregate variables, and values in parentheses indicate those under exogenous fertility.

Labor Supply: Changes in aggregate labor supply in the efficiency unit reported in

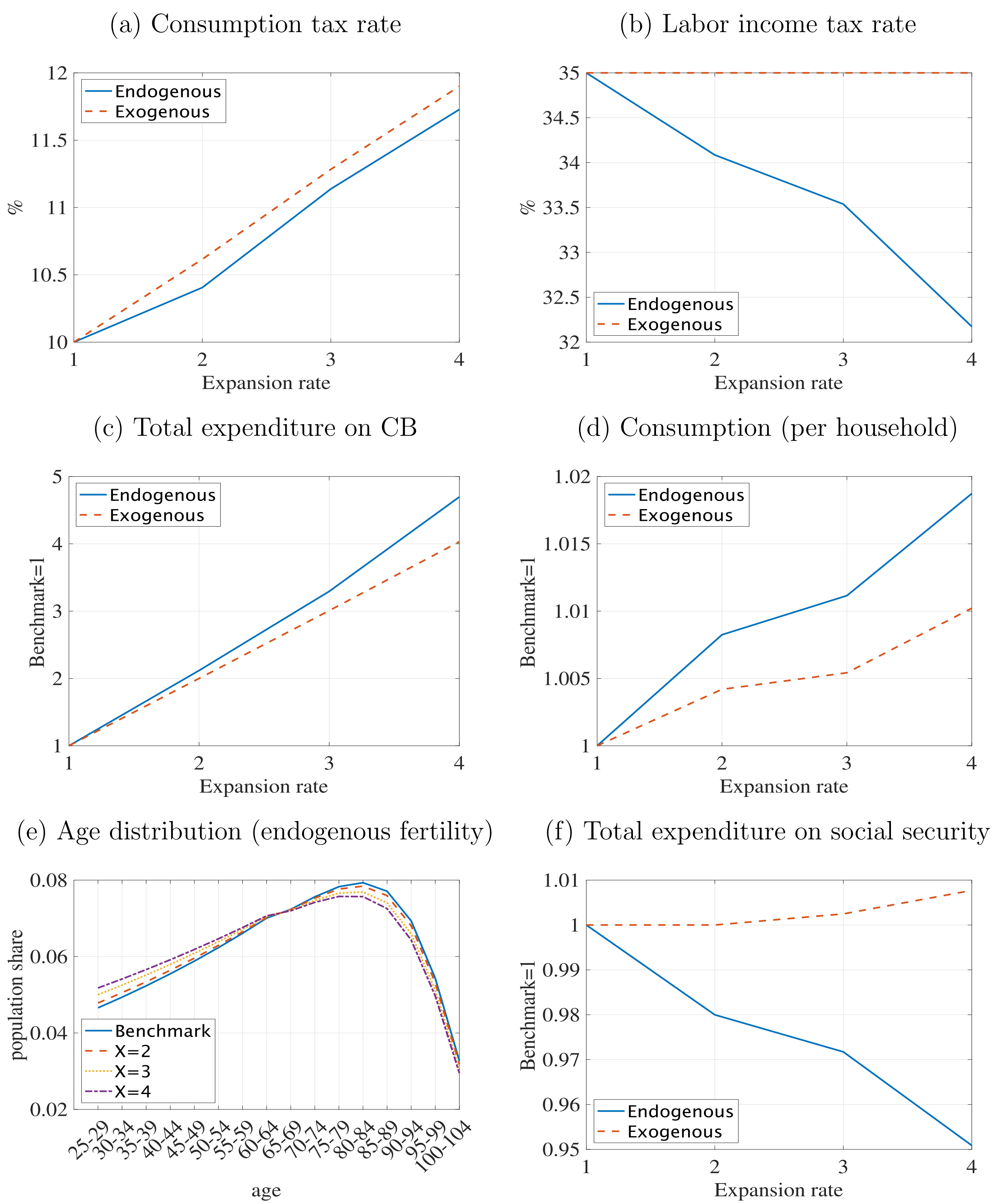

Fig 1: Payment expansion, government expenditures, and tax rates. *Note*: In each figure, "Endogenous" in the legend plots the results under endogenous fertility, while "Exogenous" plots the results under exogenous fertility where the number of children for each household is fixed as in the benchmark.

Table 3 reflect changes in the demographic structure and hours worked by each household type. Given that the total labor supply and working-age population share increase by expansions in the long run, it is unclear whether the aggregate hours worked increase or decrease.

In particular, it is worth investigating how wives' hours worked respond to the CB expansion, similar to previous works with exogenous fertility showing that the expansion depresses the maternal labor supply due to an income effect. Under endogenous fertility, the expansion can further depress labor supply due to increased time costs for childcare.[32] At the same time, note that the labor income tax rate declines and the after-tax wage rate increases in the long run, which can either motivate them to work more due to a substitution effect or reduce working incentives due to an income effect.

I compute the hours worked by wives and singles under both endogenous and exogenous fertility. The results are summarized in Table 4. I find that, first, the payment expansion depresses maternal labor supply under exogenous fertility, in line with previous studies: with the expansion rate of 2, 3, and 4, hours worked by wives aged 25-54 decline by 0.7, 1.9, and 1.9 %, respectively. Wives also reduce working hours under endogenous fertility, but there is no clear result of under which fertility setting the degree of reduction is more significant: under endogenous fertility with $X = 2$, 3, and 4, wives reduce their working hours by 1.1, 1.8, and 2.5 %.

On the contrary, singles work more under endogenous fertility, whereas they do less under exogenous fertility. With the expansion rate of 2, 3, and 4, hours worked by singles aged 25-54 increase by 0.0, 0.3, and 1.3 %, under endogenous fertility. Under exogenous fertility, on the other hand, they work less by 0.2, 0.3, and 0.5 % with expansion rates of 2, 3, and 4. The result suggests that singles work more under endogenous fertility as a substitution effect originating form a higher after-tax wage rate surpasses the income effect, while they work less under exogenous fertility due to a substitution effect caused by a relatively higher price of consumption goods, coming from a higher consumption tax rate with almost unchanged after-tax wage rate. Aggregating changes in hours worked by wives and singles,[33] I find that the expansions reduce total hours worked by $0.2 - 0.4\%$ under endogenous fertility and $0.3 - 0.7\%$ under exogenous fertility.

[32]The opposite would also be the case (i.e., mothers may want to work more to finance monetary costs for the new child).

[33]Recall that the labor supply by married men is inelastic.

	Expansion Rate (X)			
	0	2	3	4
Wives	0.3(0.4)	−1.1(−0.7)	−1.8(−1.9)	−2.5(−1.9)
Singles	−0.1(0.0)	0.0(−0.2)	0.3(−0.3)	1.3(−0.5)
Average	0.0(0.1)	−0.3(−0.3)	−0.4(−0.6)	−0.2(−0.7)

Table 4: Changes in hours worked in the long run with different per-child payments. *Note*: Cases of $X = 0$ and $X = 1$ correspond to the CB removal and benchmark, respectively. Values in the parenthesis in each cell are those obtained under exogenous fertility.

5.2.2 Welfare

What are the welfare implications of payment expansion and reduction? Table 5 summarizes the changes in welfare for each household type and expansion rate under endogenous and exogenous fertility.[34] As with other tables, values in parentheses represent results under exogenous fertility. Here, a positive value of CEV means welfare improvement for the household and vice versa. Here, "High skilled" households are defined as those in which the husband and wife complete at least a college education, and "Low skilled" as those in which the husband and wife complete at most a high school education.

One of the most striking results is that all types of households, including non-recipients who do not have children throughout their lives, are better off by expansion in the long run under endogenous fertility. Expanding the per-child payment leads to welfare gains for couples by $7.2 - 20.2\%$ and for singles by $1.1 - 1.9\%$.[35]

Another remarkable point is that welfare gains are more significant under endogenous fertility than under exogenous fertility, and for singles, the results differ even qualitatively. For example, tripling the payment leads to a 7.2% welfare gain for couples under endogenous fertility, whereas it leads to only 3.1% gain under exogenous fertility. Singles are better off by 1.1% with tripling the payment, whereas they are worse off by 0.7% under exogenous fertility.

[34]Table 9 and 10 in Appendix C report the welfare result for more detailed categories of households in the case of $X = 3$.

[35]Every type of single (non-recipient) is better off by the expansion regardless of skill and gender under endogenous fertility. Table 10 in Appendix C reports the CEV for more detailed categories of singles in the case of $X = 3$ as an example.

	Expansion Rate (X)			
	0	2	3	4
Couples	$-2.7(-2.7)$	7.2(3.1)	11.4(5.5)	20.2(8.7)
High skilled	$-2.3(-2.3)$	6.6(2.5)	10.2(4.5)	18.8(7.2)
Low skilled	$-3.7(-3.9)$	8.2(4.5)	13.7(8.2)	23.5(12.7)
Singles	0.7(0.5)	$1.3(-0.1)$	$1.1(-0.7)$	$1.9(-0.9)$

Table 5: Changes in CEV (%) in the long run with different per-child payments. *Note*: The parenthesis in each cell reports the CEV of the agent with an expansion scenario under exogenous fertility. Here, a positive value of CEV means welfare improvement for the household, and vice versa. "High skilled" households are defined as those in which the husband and wife complete at least a college education, and "Low skilled" as those in which the husband and wife complete at most a high school education.

What derives these results? With an expansion, the values of some objects in household problems change in equilibrium: tax rates (on consumption and labor income), factor prices (interest rate and wage rate), lump-sum transfers of accidental bequests, and the per-child payment for couples with children. These are taken as given for households and affect their utility-possibility frontier and behavior.

To evaluate the relative importance of changes in the factors on household welfare, I conduct a decomposition analysis as follows. First, I solve a new equilibrium with an expansion rate and obtain the prices, tax rates, and lump-sum transfers in long-run equilibrium under the new payment scale. Second, I solve each household's maximization problem by replacing any of the objects (i.e., consumption tax, labor income tax, factor prices, per-child payment, or lump-sum transfer of accidental bequests) in the benchmark with that of the new equilibrium, holding the other objects fixed. Finally, I compute the CEV for each household type. Note that this methodology does not imply that each effect adds up to the overall effect.[36]

Table 6 summarizes the results in the case of $X = 3$,[37] where the first five columns report the CEV of each household type when replacing the labor income tax rate τ_l, consumption tax rate τ_c, factor prices, lump-sum transfers of accidental bequests, and the payment scale in the benchmark, with the new equilibrium counterpart. Some important results are presented there.

[36]We may perform the decomposition by changing each object step by step so that each effect adds up to the overall effect. This "step-by-step" method, however, is sensitive to the order of changing variables if there is a critical interaction among them.

[37]The other cases (i.e., $X = 2$ and 4) imply similar results to those in this case.

	τ_l	τ_c	(w, r)	X	a_b	Overall
Couples	6.4	-2.8	-0.1	7.6	0.8	11.4
High skill	6.7	-2.8	-0.1	6.4	0.7	10.2
Low skill	5.9	-2.8	0.0	10.2	0.9	13.7
Singles	2.0	-1.4	0.0	0.0	0.5	1.9

Table 6: Welfare decomposition under the expansion rate of three ($X = 3$). *Note*: Fours columns report the CEV of each household type, replacing labor income tax rate (τ_l), consumption tax rate (τ_c), factor prices (w, r), lump-sum transfers of accidental bequests (a_b), and the payment (X) in benchmark with that in the new equilibrium. Note that the decomposition method adopted here does not imply that each effect sums to the overall effect.

First, the gains from a reduction in labor income tax τ_l are so significant that they cancel out and surpass the welfare losses from the higher consumption tax rate, while the price changes do not significantly affect welfare. Gains from the lower τ_l amount to 6.4% for couples and 2.0% for singles, which are greater than losses from a higher consumption tax rate; couples are worse off by 2.8% due to the higher consumption tax rate, and singles are also worse off by 1.4%.

In addition, gains from the larger payment are more significant for low-skilled couples whose marginal utility from consumption is relatively high, which aligns with the result of Guner et al. (2020). Through this channel, low-skilled couples are better off by 10.2%, while high-skilled couples are better off by 6.4%. Interestingly, for high-skilled couples, the gains from a lower labor income tax rate are greater than that from the larger payment. On the contrary, for low-skilled couples, the gains from a larger payment are about 1.7 times larger than that from the lower labor income tax rate. As high-skilled couples have a higher earnings potential, they benefit relatively more from the lower tax rate than low-skilled ones. This decomposition analysis shows that the welfare effects of the CB arise not only from its redistributive nature (i.e., net beneficiaries are better off due to the redistribution) but also from equilibrium feedback originating from fertility rate increase and the resulting demographic change when considering fertility choices, which makes even non-recipients better off.

Recall that the lump-sum transfer a_b increases, for example, by 0.9% in the long run with an expansion rate of three as indicated in Table 3. As a higher a_b implies a larger budget set for households, most household types are better off via this channel, although the effects are relatively modest.

To summarize, this decomposition suggests that the impact of changes in labor income tax is a critical source of the welfare effects for recipients and non-recipients. For recipients, gains from the lower tax rate can be more significant than those due to more generous payments, particularly for high-skilled couples. The decomposition also provides an answer to why the welfare implications for non-recipients differ between the two fertility settings. Under exogenous fertility, non-recipients are worse off by expansions because they just have to incur an additional tax burden, whereas they are better off under endogenous fertility in the long run, primarily because of the lower tax rate on labor earnings.

5.3 Transition Dynamics

The main result of the steady state analysis is that expanding the per-child payment will benefit future generations, including non-recipients. Then, relevant questions arise: how long do the effects take time to accrue? What are the welfare implications for cohorts living in transitional periods?

To answer these questions, I solve the transition dynamics of each expansion scenario.[38] I assume a reform occurs at the beginning of 2025, which is unexpected for households. For computational simplicity, I also assume that payment expansion applies only to households with children born after expansion. Thus, for example, although households with children born one period before the expansion are eligible for the CB, their per-child payments are the same as those before the reform. Parents in a cohort born in 1991-95 are the first beneficiaries of the reform, which I refer the 1995 cohort hereafter.

Fig 2 represents the transition path of the macroeconomic variables. Fig 2(a) plots the completed fertility of each birth cohort, and the vertical line in the figure indicates the first cohort benefiting from the reform (i.e., the 1995 cohort). Fig 2(b), 2(c), and 2(d) represent the transition paths of the per-capita GDP, labor income tax rate, and consumption tax rate.

First, although the fertility rate jumps upon expansion as Fig 2(a) shows, the labor income tax rate takes a long time to decline because the demographic structure can only change slowly; even if the fertility rate increases, children born today cannot participate immediately in economic activities, and they will be in the labor force after around 20 years have passed.

Next, the tax burden for financing the CB (i.e., τ_c) increases upon expansion as the fertility rate and government expenditures on CB increase. During the transition periods, the consumption tax rate exceeds the long-run level in the case of $X = 3$ and $X = 4$, reaching the new levels after sufficient periods have passed. This is because although the aggregate output increases in the long run, it takes a long time to reach the new

[38] Note that, in addition to prices, tax rates, and quantities, the age distribution changes during the transition.

level, as Fig 2(b) shows, because of the same reason as that for the slow decline of labor income tax; the long-run output increases owing to the larger share of the working-age population, but the demographic change requires sufficient periods. Thus, a higher tax rate is required to balance the government budget before reaching a new steady state. All factors will reach new levels at around 2130, implying that the transition will take approximately 100 years.

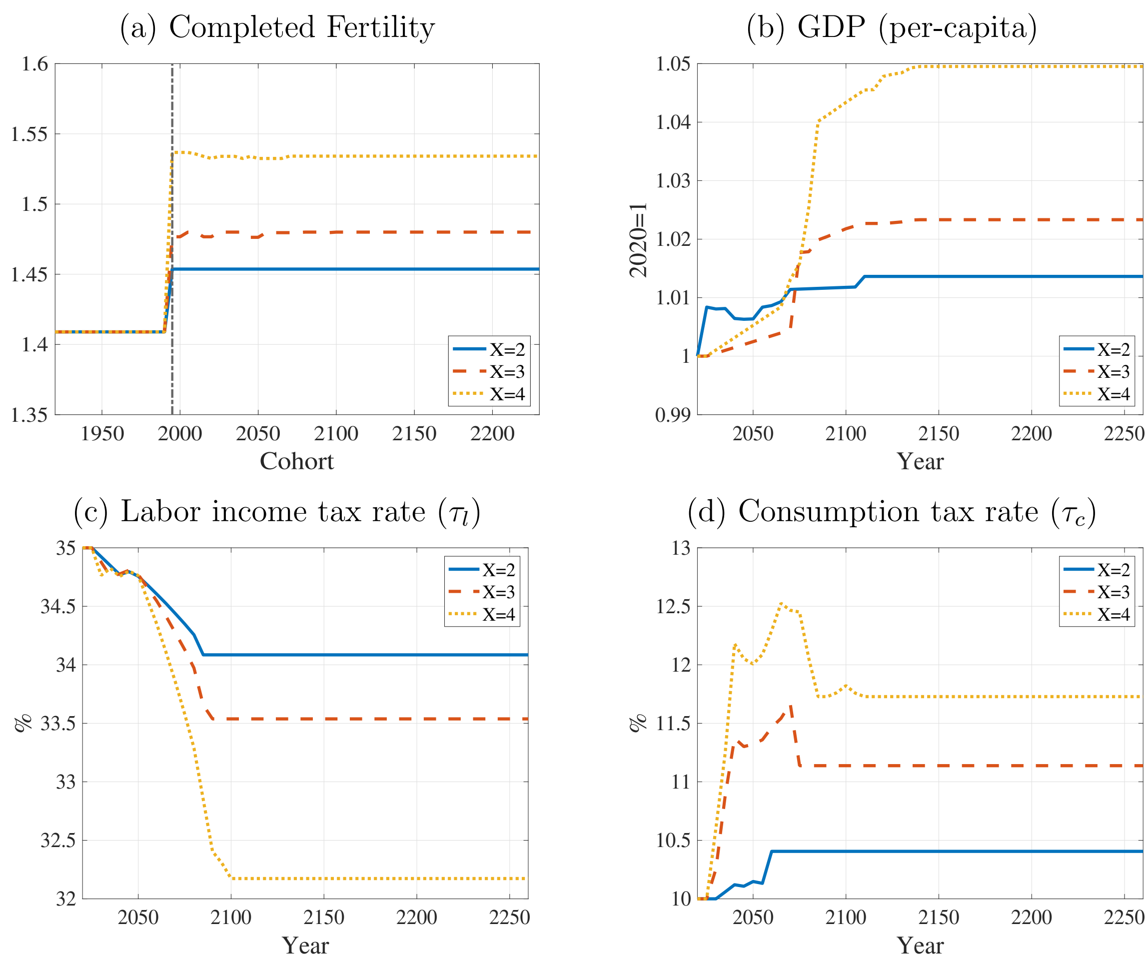

Fig 2: Aggregate variables during the transition. *Note*: The per-capita GDP is represented by normalizing its value in 2020 as one.

Due to the long time required for the transition and the larger consumption tax burden in the medium run, welfare gains also take a long time to accrue. Fig 3 shows the CEV for each type and birth cohort in each expansion scenario. As the figures show, current generations, defined as cohorts older than the 1995 cohort, and some future generations (i.e., those of non-recipients), defined as those younger than the 1995 cohort, are worse off in each scenario.

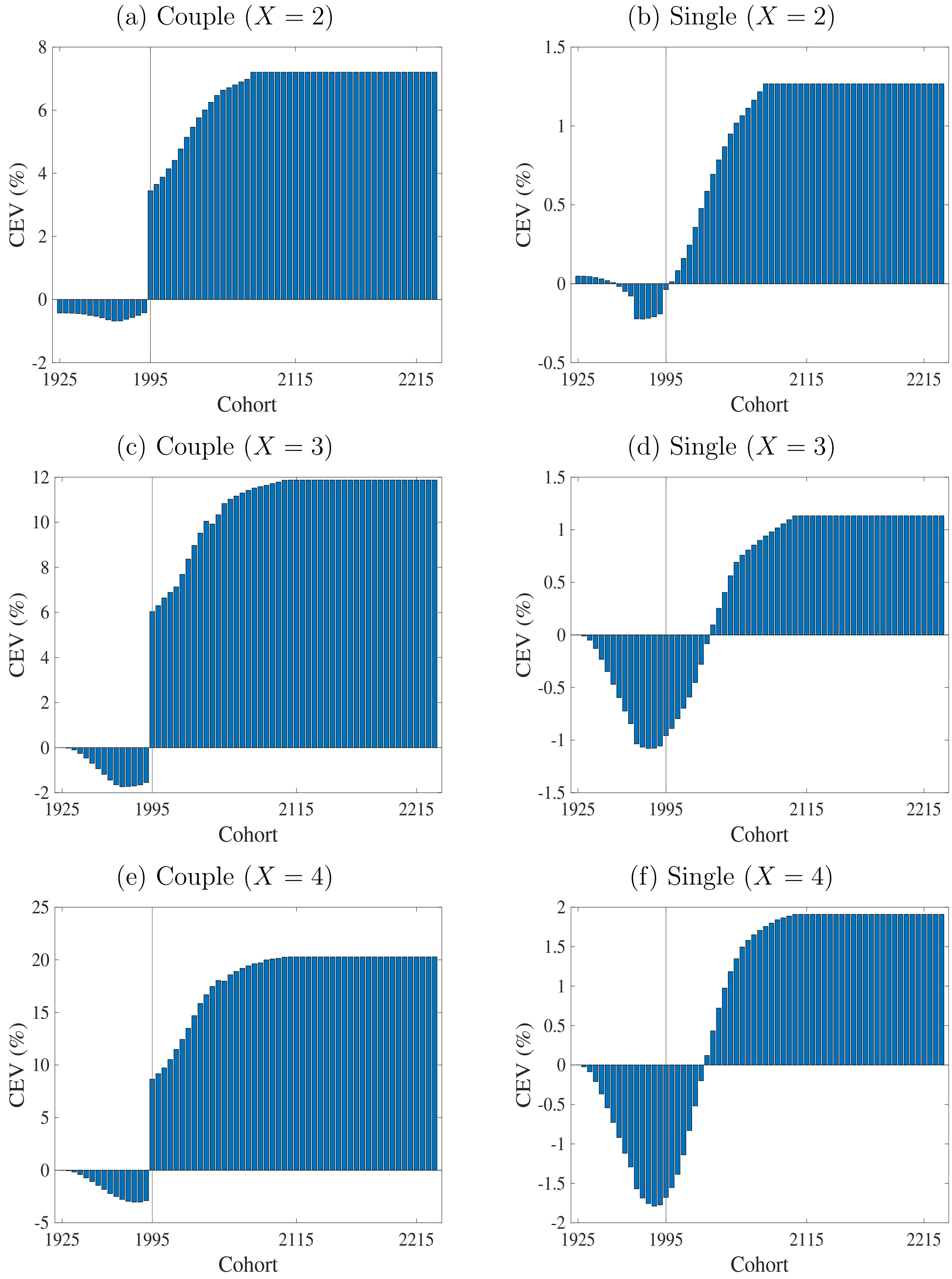

Fig 3: Welfare effects of the payment expansion during the transition. *Note*: The horizontal axis in each figure represents the birth cohort. "1995" in the figure stands for the 1991-95 cohort, and "1925" for the 1921-25 cohort, whose age is 100-104 when the expansion occurs. The vertical line indicates the first cohort benefiting from the reform (i.e., the 1995 cohort).

6 Concluding Remarks

This study examines the macroeconomic and welfare effects of CB policies, embedding fertility choices in the GE-OLG model. The model is calibrated to the 2000s Japanese economy and produces the benefit elasticity of fertility in line with the empirical estimates. Expanding the per-child payment will lead to welfare gains for future generations in stationary equilibrium, including non-recipients who do not have children throughout their lives. Decomposition analysis suggests that the equilibrium feedback especially through tax rates, driven by behavioral changes in fertility and the corresponding demographic change, accounts for the result. However, it takes a long time to reach the new equilibrium, and during the transition, the tax rate for financing the expansion is higher than its long-run level. Consequently, the welfare gains take approximately 100 years to accrue, and current and some future generations are worse off.

This article concludes by discussing promising directions for future research. First, this study abstracts marriage decisions by exogenously assigning marital status to individuals. Given that the CB policies are relevant to fertility decisions and only couples can procreate, the CB policies are also expected to affect marriage decisions by changing relative values of marriage and being single. Considering marriage choices would lead to more significant effects of the CB expansion on the TFR if the policy increases the relative value of marriage and increases marriage rates. The current version of this study with exogenous marriage is a first step to understanding the effects of CB policies, which makes it easier to understand their welfare effects on non-beneficiaries, and incorporating marriage decisions would be a promising direction for future research. Second, this study abstracts the quality investments for children; monetary costs of children are exogenously given for each type of household, and these expenditures do not affect children's human capital. Daruich (2022) shows that large-scale early childhood policies, captured as an increase in monetary investments in children, yield substantial welfare gains using a GE-OLG model with exogenous fertility. Thus, the CB can be evaluated as a more welfare-improving policy by capturing its positive effects on aggregate human capital. In contrast, previous studies using OLG models with fertility choices show that pro-natal transfers would lower aggregate human capital because they make parents shift from the "quality" toward "quantity" of children (e.g., Zhou, 2022; Kim et al., 2023). Based on those considerations, it is unclear whether and how considering the quality investments for children affects my results, especially regarding the welfare effects. Although embedding this element in my model adds complexity and computational challenges to the analysis, this would also be an essential step to understanding the macroeconomic consequences of pro-natal policies.

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Appendix A Definition of Equilibrium

Definition 1 (Stationary Equilibrium). *Given an initial asset endowment $\bar{a}$ common for each household type, and policy rules (a payment function for the CB, an income replacement rate of PAYG pension benefit ρ, copayment rates of the public health insurance ω, government expenditures G, and a capital income tax rate τ_a), a stationary equilibrium consists of*

- *value functions for couples $W^C(a, \boldsymbol{\theta}_C)$ and $V^C(a, j, b, \boldsymbol{\theta}_C)$, and the value function for singles $V^S(a, j, \theta_S)$,*
- *policy functions for the number of children for couples $b^C(\boldsymbol{\theta}_C)$, policy functions for consumption ($c^C(j, \boldsymbol{\theta}_C)$ and $c^S(j, \boldsymbol{\theta}_S)$), for savings ($s^C(j, \boldsymbol{\theta}_C)$ and $s^S(j, \boldsymbol{\theta}_S)$), and for labor supply ($l^C(j, \boldsymbol{\theta}_C)$ and $l^S(j, \boldsymbol{\theta}_S)$),*
- *an age distribution over the population μ_j, and wealth distribution μ_a,*
- *quantities (aggregate capital $\hat{K}$ and aggregate labor in efficiency unit, $\hat{L}$),*
- *prices (an interest rate $\hat{r}$ and wage rate $\hat{w}$),*
- *policy rules (the consumption tax rate τ_c and labor income tax rate τ_l),*

such that

1. *each household maximizes their lifetime utility: $W^C(a, \boldsymbol{\theta}_C)$ is defined as (3), and V^m solves for the functional equations (4) and (7), subject to (5), (6), (8), and (9), for $m \in \{C, S\}$. And $b^C(\boldsymbol{\theta}_C)$, $c^C(j, \boldsymbol{\theta}_C)$, $c^S(j, \boldsymbol{\theta}_S)$,$s^C(j, \boldsymbol{\theta}_C)$, $s^S(j, \boldsymbol{\theta}_S)$,$l^C(j, \boldsymbol{\theta}_C)$ and $l^S(j, \boldsymbol{\theta}_S)$ are corresponding policy functions.*
2. *$\hat{r}$ and $\hat{w}$ are determined competitively.*
3. *factor markets clear:*

$$\hat{K} = \sum_{j \in \mathcal{J}} \left[\phi \sum_{z^h, z^w} s^C(j, \boldsymbol{\theta}_C) \times \Psi^C(j, \boldsymbol{\theta}_C) \right.$$
$$\left. + (1 - \phi) \sum_{g,z} s^S(j, \boldsymbol{\theta}_S) \times \Psi^S(j, \boldsymbol{\theta}_S) \right],$$
$$\hat{L} = \sum_{j \le J_R} \left[\phi \sum_{z^h, z_w} \left[\bar{\omega}(j, M, z^h) + l^C(j, \boldsymbol{\theta}_C) \times \bar{\omega}(j, F, z^w) \right] \times \Psi^C(j, \boldsymbol{\theta}_C) \right.$$
$$\left. + (1 - \phi) \sum_{g,z} l^S(j, \boldsymbol{\theta}_S) \times \Psi^S(j, \boldsymbol{\theta}_S) \right],$$

where ϕ is the share of married households. $\Psi^C(j, \boldsymbol{\theta}_C)$ and $\Psi^S(j, \boldsymbol{\theta}_S)$ are distributions over type and age for couples and singles, which are consistent with the measure of individuals $\mu_{g,z,m}$ and age distribution μ_j.

4. *The age distribution μ_j is characterized by couples' policy function for the number of children $b^C(\boldsymbol{\theta}_C)$ and the wealth distribution μ_a is characterized by policy functions for savings. More specifically, the age distribution μ_j is characterized by the equilibrium cohort growth rate g_n and survival probabilities conditional on age S_j as follows:*

$$
\begin{aligned}
N_j &= (1 + g_n) \cdot N_{j+1} \quad \forall j, \\
\mu_j &= N_j \cdot S_j \quad \forall j, \\
\sum_j \mu_j &= 1,
\end{aligned}
$$

where N_j denotes the population mass of the cohort when they enter the economy for that aged j (i.e., the cohort's birth number).

5. *The government budget constraints ((1) and (2)) are balanced.*

Appendix B More on calibration

Preference: Assuming that the birth number grew at a constant rate in 1940-2020, the growth rate is computed as -1.15%, based on the Vital Statistics (2020) of the MHLW. Note that the oldest age in this economy corresponds to $J = 16$, and the one period is five years, meaning that the difference between the age of the youngest agents and the oldest ones is nearly 80. Thus, I choose the period between 1940 ($= 2020 - 80$) and 2020. As Fig 4 shows, the birth number or cohort size shows a declining trend during the period, while there are some baby boom and bust phases.

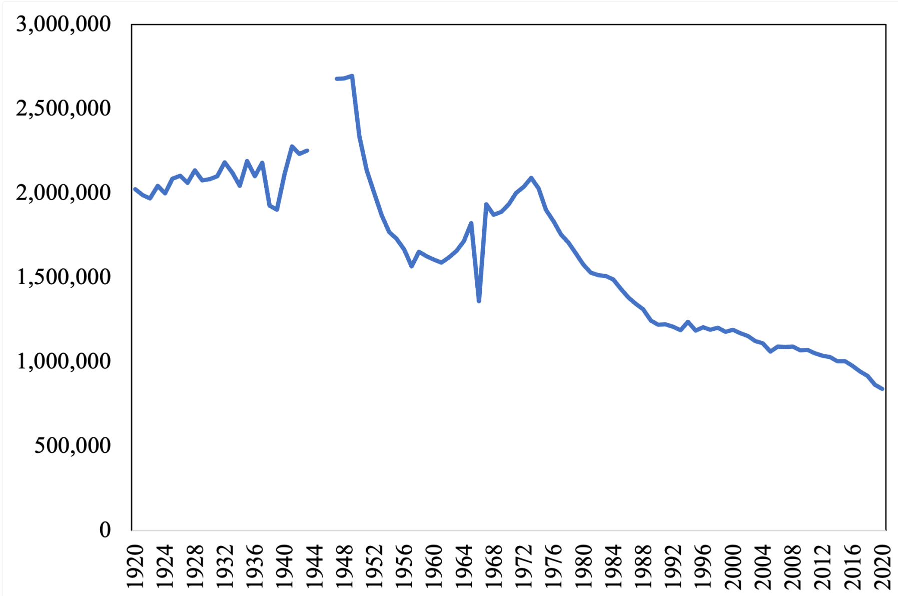

Fig 4: The transition of birth numbers in Japan (1920-2020). *Note*: The data comes from the Vital Statistics (2020) of the MHLW. Values in 1944-46 are missing. The vertical axis represents the birth number, and the horizontal axis represents the year.

To pin down the curvature parameter value for utility from having children, I set the target value of 0.025 for the benefit elasticity of fertility, relying on Yamaguchi (2019). He estimates a dynamic discrete choice model of female employment and fertility decisions to evaluate some options for parental leave policies in Japan, using the Japanese panel data. His main focus is on the parental leave policy, but he also conducts counterfactual experiments to increase the baby bonus and observe its effects on fertility and female labor supply. Although he does not specify the fertility elasticity explicitly, I compute the (aggregate) elasticity as follows. Note that he simulates introducing the baby bonus of 1, 3, and 5 million yen, with fixing the parental leave system. A proxy of completed fertility rate in each scenario is reported as 2.23, 2.34, and 2.46 for each. Considering the expansion of the baby bonus from 1 to 3 million yen and 5 million yen, increases in reported fertility rates from 2.23 to 2.34 and 2.46 imply the benefit elasticity of 0.25 and 0.26, respectively; they belong to the lower class of elasticity values reported in studies outside Japan, which are reviewed in Section 1. Thus, I set the target as 0.025.

Although this is the elasticity for the baby bonus, using this value as the target in calibration should be innocuous for the following reasons. First, the baby bonus is the most similar benefit to the CB in Japan's context: it is a cash transfer, most households are eligible, and the eligibility is almost independent of parents' labor market status and their earnings.[39] Further, the scale of expanding the baby bonus in Yamaguchi (2019) is

[39]As elaborated in the main text, although there is an income test for eligibility of the CB, the income limit is not so severe, and the majority is eligible.

similar to the CB expansion scenarios considered in Section 5.[40]

Child-related Expenditures: As the Declining Birthrate White Paper (2010) of the Cabinet Office[41] implies, we may classify two broad categories of child-related expenditures by households: (i) living expenses, such as expenditures for clothes, food, and housing, and (ii) education expenditures. Note that the model captures the additional living expenses induced by having a child, deflating household consumption by the equivalence scale.

This model also captures education expenditures with function CE. I set the function assuming the child attains the same education level as the parents. Data for average tuition costs in each education category comes from the White Paper on Education, Culture, Sports, Science and Technology (2009) of the Ministry of Education.[42]

Child Benefit: A function representing payment of the CB is conducted to approximate the actual transfer systems described below. Households with children before junior high school completion may receive the payment,[43] and the per-child payment varies according to the household's income, age, and birth order.

The program defines household income as the maximum income earned by a household member in the previous year. Consider a household, for example, consisting of a husband, wife, and newborn babies. This household is potentially eligible to receive CB. Let y^h and y^w denote the annual earnings of husband and wife in the previous year. In this case, the household income represented by I defined for the program is given by $I = \max\{y^h, y^w\}$.

The program has income tests determining eligibility status and payment in two steps. First, if a household's income is above a threshold, the household is not eligible. The threshold value varies according to the number of dependents. As a typical example, for households consisting of a husband and three dependents (two young children and a wife who does not work in the market), the threshold is 12 million yen. Next, even though household income is not above it, the per-child payment depends on income. Considering the example of the household with three dependents, if the income is above 9.6 million yen, they receive 5,000 yen per child-month, regardless of the child's age or birth order.

[40]The majority receives approximately 2 million yen per child until the child reaches 16 years old, under the current CB system. Then, doubling or tripling the per-child payment as examined in Section 5 implies that the per-child payment increases by about $2 \sim 4$ million yen, which is similar to Yamaguchi (2019)'s experiment.

[41]See https://www8.cao.go.jp/shoushi/shoushika/whitepaper/measures/english/w-2010/pdf/2_p41_56.pdf.

[42]See https://warp.ndl.go.jp/info:ndljp/pid/11293659/www.mext.go.jp/b_menu/hakusho/html/hpab200901/1305844.htm.

[43]In principle, one completes junior high school education at 15 years old in Japan.

Otherwise, they can receive 10,000-15,000 yen per child-month, depending on the child's age and birth order. When the child's age is between 0 and 2, the monthly payment for each child amounts to 15,000 yen. In addition, for children whose birth order is higher than third, the payment for the child is 15,000 yen until primary school completion. When the child is in junior high school, the per-child payment is 10,000 yen regardless of the birth order. Table 7 summarizes the transfer rules.

Earnings (y, million yen)	Child's age	Payment (per child-month, yen)
$y \geq 12$		No payment
$9.6 \leq y < 12$		
	$0-2$	5,000
	$3-$ primary school	5,000
	J.H. school	5,000
$y < 9.6$		
	$0-2$	15,000
	$3-$ primary school	10,000 (1st and 2nd child)
		15,000 (3rd child or more)
	J.H. school	10,000

Table 7: The per-child payment in Japan's child benefit. *Note*: This is the case for a household consisting of a worker with three dependents (typically, a wife who does not participate in the labor market and two children).

	Expansion Rate (X)								
	2	2.5	3	3.5	4	4.5	5	Mean	SD
Elasticity	0.030	0.021	0.020	0.019	0.028	0.025	0.025	**0.024**	0.0041

Table 8: The benefit elasticity of fertility with $\gamma = 3.5$. *Note*: I compute the completed fertility rate with each expansion case. The elasticity is then calculated for each expansion rate using the implied fertility rates. "Mean" reports the mean of the seven elasticity values. "SD" stands for the standard deviation of the seven values.

Appendix C More on welfare analysis

$z^w \backslash z^h$	$< HS$	HS	SC	COL	$COL+$
$< HS$	15.1 (9.2)	14.5 (8.6)	14.1 (7.5)	12.6 (6.4)	11.4 (5.1)
HS	14.7 (8.7)	14.2 (8.2)	13.8 (7.2)	12.3 (6.2)	11.2 (5.0)
SC	13.5 (7.1)	13.0 (6.6)	12.5 (6.5)	12.0 (5.0)	10.6 (4.1)
COL	12.2 (6.3)	11.9 (5.9)	11.9 (5.0)	10.7 (4.5)	10.2 (3.7)
$COL+$	10.9 (4.0)	10.6 (3.8)	10.2 (3.8)	10.0 (3.5)	9.7 (2.9)

Table 9: Couples welfare with $X = 3$ simulated in Section 5.2. *Note*: Values in parentheses in each cell report the CEV of each household with each expansion scenario under exogenous fertility. Here, a positive value of CEV means welfare improvement for the household and vice versa.

$g \backslash z$	$< HS$	HS	SC	COL	$COL+$
Female	0.9 (-0.3)	1.0 (-0.3)	1.0 (-0.5)	1.2 (-0.7)	1.2 (-0.8)
Male	1.1 (-0.6)	1.1 (-0.6)	1.2 (-0.7)	1.2 (-0.8)	1.3 (-0.9)

Table 10: Singles welfare with $X = 3$ simulated in Section 5.2. *Note*: Values in parentheses in each cell report the CEV of each household with each expansion scenario under exogenous fertility. Here, a positive value of CEV means welfare improvement for the household and vice versa.

Appendix D Robustness

a) Child penalty

In the analysis conducted in Section 5, individuals' lifecycle profiles of productivity are exogenously given. I examine whether and how the results change if the endogenous accumulation of mothers' human capital is considered with a simple extension of the model. Empirical literature shows that women experience a persistent wage decline after births (e.g., Adda et al., 2017; Lundborg et al., 2017; Kleven et al., 2019, etc), whereas men seldom do that, which is often called the *child penalty*.Capturing this point might affect my main results since otherwise, the long-run effects of the CB on the aggregate output and consumption, which are relevant to taxation effects, could be overestimated. To check if this is the case, I reconstruct the productivity function to capture the depreciation due to having births. Using the original version of the function $\bar{\omega}$, I formulate the productivity function, $\tilde{\omega}(j, g, z, b)$, as follows:

$$\tilde{\omega}(j, g, z, b) = (1 - \mathbb{I}_{j \geq J_b}[b \cdot \delta_h]) \times \bar{\omega}(j, g, z) \quad \text{if } g = F.$$

Here, the number of children b enters the function only for females. If a woman has a child, her wage decreases at rate δ_h after having birth, compared with the wage she would have if she did not have birth. To the best of my knowledge, no published papers estimate the child penalty in Japan. As a remedy, I rely on the seminal work by Kleven et al. (2019), estimating the impact of children on gender inequality in terms of labor supply, wage rate, and earnings, using the Danish administrative data. They define the child penalty as the percentage by which women are falling behind men due to children and show that the child penalty averaged over 20 years after having a birth, in terms of wage rates, amounted to 14.3%. Thus, I set $\delta_h = 0.15$ and assume that men do not incur the depreciation so that the model produces that magnitude of the wage dispersion after having a birth.

Considering the child penalty, I simulate four scenarios of the per-child payment expansion as in Section 5, and the results are reported in Table 11 and 12. In Table 11, I report percentage changes in the average maternal human capital (denoted by H), which is defined as follows:

$$H = \frac{1}{J_R} \sum_{j=1}^{J_R} \sum_{z^h, z^w} \tilde{\omega}(j, F, z^w, b(z^h, z^w)) \times \pi_{z^h, z^w},$$

where $b(z^h, z^w)$ denotes the optimal choice for the birth number for couples with education pair of (z^h, z^w). The average maternal human capital declines at most 4.7% under endogenous fertility. However, it is insignificant for other macroeconomic variables and welfare, and the arguments presented in Section 5 still apply: the expansion leads to a higher output and lower labor income tax rate, and even non-recipients are better off in the long run.

	Expansion Rate (X)				
	0	1	2	3	4
Demographic Structure					
Completed fertility	1.39	1.41	1.45	1.48	1.53
Quantity (per-capita, Δ%)					
Output	0.0(−0.1)	–	1.6(0.5)	3.1(1.1)	5.6(1.6)
Maternal human capital (Δ%)	0.7 (0)	–	−1.0 (0)	−2.0 (0)	−4.7 (0)
Taxes (%)					
Consumption	9.5 (9.1)	10	10.2 (10.3)	10.8 (11.0)	11.7 (11.6)
Labor income	35.3 (35)	35	34.2 (35)	33.4 (35)	31.7 (35)

Table 11: Main aggregate variables with payment expansion with child penalty. *Note*: Values in the parenthesis in each cell are those obtained under exogenous fertility. Note that, by construction, the demographic structure is invariant with expansion under exogenous fertility. Thus, I omit values under exogenous fertility for that part. Cases of $X = 0$ and $X = 1$ correspond to the CB removal and benchmark, respectively.

	Expansion Rate (X)			
	0	2	3	4
Household Type				
Couples	−4.0(−2.0)	6.5(3.2)	12.4(6.6)	19.8(10.0)
High skilled	−3.6(−1.5)	5.9(2.6)	11.1(5.5)	18.4(8.3)
Low skilled	−5.1(−3.0)	7.9(4.6)	15.3(9.3)	26.7(13.8)
Singles	1.1(0.5)	0.7(−0.3)	0.8(−0.8)	0.7(−1.2)

Table 12: CEV (%) with payment expansion with child penalty. *Note*: Values in the parenthesis in each cell report the CEV of each household with each expansion scenario under exogenous fertility. Here, a positive value of CEV means welfare improvement for the household and vice versa. Also, “High skilled” households are defined as those in which the husband and wife complete at least a college education, and “Low skilled” as those in which the husband and wife complete at most a high school education.

	Expansion Rate (X)			
	0	2	3	4
Wives	1.0(0.9)	−1.3(0.0)	−2.7(−1.2)	−4.4(−1.1)
Singles	−0.1(−0.1)	0.0(0.0)	0.4(−0.1)	1.5(−0.2)
Average	0.2(0.2)	−0.3(0.0)	−0.5(−0.3)	−0.5(−0.3)

Table 13: Changes in equilibrium hours worked with payment expansion with child penalty. *Note*: Cases of $X = 0$ and $X = 1$ correspond to the CB removal and benchmark, respectively. Values in the parenthesis in each cell are those obtained under exogenous fertility.

b) The benefit elasticity of fertility

Recall that, in the benchmark economy, the benefit elasticity of fertility is 0.024 with the curvature parameter $\gamma = 3.5$. I check the robustness of the results obtained in the steady state analysis in Section 5 to different values of the fertility elasticity, as it is the most critical moment in determining the effects of CB expansion through demographic change. More specifically, I compute two additional equilibria, one with a more modest elasticity and another with a higher elasticity. In the former case, I set $\gamma = 4$ and re-calibrate the model, implying the benefit elasticity of 0.21. In contrast, γ is set to 3 in the latter case, leading to the elasticity of 0.30. The results under both economies are reported in Table 14-17. Most qualitative results discussed in Section 5 hold in both economies, and quantitative results do not change significantly.

	Expansion Rate (X)				
	0	1	2	3	4
Demographic Structure					
Completed fertility	1.37	1.41	1.45	1.48	1.56
Quantity (per-capita, Δ%)					
Output	0.0(-0.1)	–	1.8(0.0)	2.3(0.2)	5.2(0.8)
Taxes (%)					
Consumption	9.4 (9.6)	10	10.4 (10.6)	11.2 (11.3)	11.7 (11.9)
Labor income	35.5 (35)	35	33.8 (35)	33.5 (35)	31.9 (35)

Table 14: Changes in main aggregate variables in the long run with different per-child payments with $\gamma = 3$. Values in the parenthesis in each cell are those obtained under exogenous fertility. Note that, by construction, the demographic structure is invariant with expansion under exogenous fertility. Thus, I omit values under exogenous fertility for that part. *Note*: Cases of $X = 0$ and $X = 1$ correspond to the CB removal and benchmark, respectively.

	Expansion Rate (X)			
	0	2	3	4
Household Type				
Couples	-2.5(-2.7)	8.4(2.9)	11.5(5.5)	20.2(8.8)
High skilled	-2.2(-2.3)	7.9(2.3)	10.3(4.4)	18.6(7.3)
Low skilled	-3.2(-3.9)	9.4(4.2)	14.0(8.2)	23.6(12.9)
Singles	1.3(0.5)	1.6(-0.2)	0.9(-0.7)	1.5(-0.8)

Table 15: Changes in CEV (%) in the long run with different per-child payments with $\gamma = 3$. *Note*: Values in the parenthesis in each cell report the CEV of each household with each expansion scenario under exogenous fertility. Here, a positive value of CEV means welfare improvement for the household and vice versa. Also, "High skilled" households are defined as those in which the husband and wife complete at least a college education, and "Low skilled" as those in which the husband and wife complete at most a high school education.

	Expansion Rate (X)				
	0	1	2	3	4
Demographic Structure					
Completed fertility	1.39	1.41	1.45	1.45	1.52
Quantity (per-capita, Δ%)					
Output	0.0(−0.1)	–	1.4(0.0)	2.3(0.2)	5.0(0.8)
Taxes (%)					
Consumption	9.7 (9.6)	10	10.4 (10.6)	11.1 (11.3)	11.9 (11.9)
Labor income	35.1 (35)	35	34.1 (35)	34.1 (35)	32.6 (35)

Table 16: Changes in main aggregate variables in the long run with different per-child payments with $\gamma = 4$. Values in the parenthesis in each cell are those obtained under exogenous fertility. Note that, by construction, the demographic structure is invariant with expansion under exogenous fertility. Thus, I omit values under exogenous fertility for that part. *Note*: Cases of $X = 0$ and $X = 1$ correspond to the CB removal and benchmark, respectively.

	Expansion Rate (X)			
	0	2	3	4
Household Type				
Couples	−2.5(−2.7)	7.2(3.1)	9.5(5.5)	17.2(8.7)
High skilled	−2.2(−2.3)	6.7(2.5)	8.3(4.5)	15.8(7.2)
Low skilled	−3.3(−3.9)	8.4(4.5)	11.8(8.2)	20.3(12.7)
Singles	0.8(0.5)	1.3(−0.1)	0.5(−0.7)	0.9(−0.9)

Table 17: Changes in CEV (%) in the long run with different per-child payments with $\gamma = 4$. *Note*: Values in the parenthesis in each cell report the CEV of each household with each expansion scenario under exogenous fertility. Here, a positive value of CEV means welfare improvement for the household and vice versa. Also, "High skilled" households are defined as those in which the husband and wife complete at least a college education, and "Low skilled" as those in which the husband and wife complete at most a high school education.