Equilibrium Effects of the Child Benefit: Fertility, Labor Supply, and Welfare*

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Abstract

This article examines the macroeconomic and welfare effects of the child benefit, incorporating fertility choices into the general equilibrium overlapping generations model with two-earner households. The model is calibrated to the Japanese economy and produces the benefit elasticity of fertility in line with empirical estimates. Expanding the per-child payment will lead to welfare gains for future generations in stationary equilibrium, including non-recipients who are without children throughout their lives. Equilibrium feedback via factor prices and tax rates, mainly driven by changes in fertility rates and demographic structure, accounts for the results. Although the expansion tends to depress the maternal labor supply due to an income effect, as well as to increased time costs for childcare owing to higher fertility, the equilibrium feedback mitigates the reduction of labor supply in the long run.

Keywords: child benefit, fertility, labor supply, welfare.

JEL Classification: E60, J13, J18

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1 Introduction

The child benefit (CB), or child allowance, prevails across countries. Although the detail of the transfer system hinges on each administrator, it typically means periodic cash transfers to households with children up to a certain age, unconditional on the labor market status of parents or expenditures to childcare services.[1] In 2018, average OECD countries paid CB and tax credits, covering nearly 5% of the average labor earnings in the country, to eligible households with children.[2]

Advanced economies have either considered or implemented the expansion of the CB (or baby bonus, providing a one-shot cash transfer for births) based on pro-natal motives in the last few years (e.g., Hungary, Poland, Greece, Japan, and so on).[3] The background is that most have experienced declining birthrates over the last several decades, a root cause of demographic aging, which is costly in fiscal and welfare terms.

Empirical studies might support this action because they report that cash transfers have a statistically significant effect on fertility (e.g., Milligan, 2005; Azmat and González, 2010; Cohen et al., 2013; Laroque and Salanié, 2014).[4] In the long run, the rise in fertility rates changes the demographic structure, which could have critical importance for macroeconomic variables and welfare. However, the macroeconomic studies on CB abstract fertility choices to elaborate on its effects on maternal labor supply (Guner et al., 2020; Hannusch, 2022).

This study incorporates fertility choices into the general equilibrium overlapping generations (GE-OLG) model, and re-examines the macroeconomic effects and welfare implications of the CB for heterogeneous households. *Ex-ante*, individuals differ in their gender, skill (or educational background), and marital status (single or married). In other words, the model also builds on the two-earner household framework, as in previous works, enabling us to consider secondary earners' decisions on labor supply in married households. Married households (couples) choose the number of children they have, while unmarried (single) households cannot. The demographic structure, or age distribution, is endogenously determined by aggregating couples' fertility choices. The model is calibrated to the 2000s Japanese economy. An important targeted moment is the benefit

[1]The payment amount typically varies with the child's age or birth order. In some cases, it takes the form of a refundable tax credit like the US child allowance temporarily conducted since 2021.

[2]See, PF1.3 of the database `https://www.oecd.org/els/family/database.htm`.

[3]See, for example, articles reporting Poland's (European Commission, 2018) and Greece's reforms (The Guardian, 2020).

[4]Theoretical studies would also provide a rationale for the cash transfers because of the positive externality of children (Folbre, 1994), market incompleteness (Schoonbroodt and Tertilt, 2014), and that having children involves a consumption commitment for the child (Baudin et al., 2015; Santos and Weiss, 2016; Sommer, 2016).

elasticity of fertility, and the calibrated model produces an elasticity value in line with the empirical estimates. The government runs two sets of programs in the economy: the child benefit and social security programs.

I model Japan's CB system as closely as possible to what is conducted in reality. In the model, the government pays 5,000 to 15,000 yen per child per month to households with children under 15 years old,[5] depending on household income, age and birth order. The monthly per-child payment amounted to roughly 3% of the monthly salary of males aged 20-54 in 2020.[6]

The social security programs consist of the pay-as-you-go (PAYG) public pension and public health insurance, financed by labor income tax proportional to labor earnings in this model. Households' medical needs arise exogenously and deterministically. It depends only on their age and evolves with aging, as observed in the data. The government runs the health insurance program, incurring a fraction of the medical expenditure. The PAYG pension and public health insurance programs imply a larger share of older people and consequently, either a significant tax burden for the working-age population, or smaller social security payments.

I solve the stationary equilibrium and transition dynamics associated with payment expansion or reduction. Tax rates are adjusted to balance the government budget, and other policy variables are fixed as in the benchmark (e.g., the income replacement rate for PAYG pension and the copayment rate for public health insurance). Note that the social security expenditure can vary across the equilibria, given that the demographic structure is endogenous. The labor income tax rate is adjusted in response to those expenditure changes. In addition, the consumption tax rate is adjusted to balance the government budget and finance an additional expenditure to the CB. This construction reflects a current policy situation in Japan where expansions of child-related policies are more likely to be funded by raising the consumption tax rate.

The main findings are summarized as follows. First, in the long run, expanding the per-child payment benefits recipients and non-recipients who do not have children through their lives. This result is different from that of Guner et al. (2020), who use a model with exogenous fertility and report that non-recipients incur some welfare costs upon payment expansion. I demonstrate that the welfare gains for non-recipients do not accrue in most expansion cases under the exogenous fertility version of the model. A decomposition analysis suggests that a long-run decline of the labor income tax rate, stemming from a larger share of the working-age population and lower tax burden for funding social

[5] 1 US dollars (USD) corresponds to around 130 yen.

[6] The average monthly salary is computed from the Basic Survey on Wage Structure (BSWS) released by the Ministry of Labour, Health, and Welfare (MHLW).

security benefits, is a vital force accounting for the result. In addition, in the long run, the expansion substantially benefits high-skilled households with children, even though the payment size relative to earnings is smaller; they gain from the reduction of labor income tax, as their pre-tax earnings are higher.

Next, although the expansion tends to depress the maternal labor supply, the side effect is mitigated in the long run. On the one hand, as previous studies show, the expansion will reduce their labor supply due to an income effect. In addition, the model captures that having a young child costs a fraction of mother's disposable time. Then, the expansion can further depress their labor supply due to an increased childcare time owing to higher fertility. On the other hand, long-run changes in prices and tax rates would induce them to work more. In particular, given that the expansion raises an equilibrium fertility rate, the demographic structure changes in the long run, and the workers-to-retiree population ratio rises, reducing the labor income tax rate in this model. The lower tax rate increases the opportunity cost of leisure, inducing them to work more if a substitution effect of the tax reduction on labor supply is sufficiently large. Under the exogenous fertility setting, tripling the benchmark CB payment reduces the working hours of wives aged 25-64 by 1.64% and wives with children under 15 by 3.90%. Under endogenous fertility, each reduction is mitigated as 0.67% and 0.29%, respectively, regardless of a higher fertility and larger time costs for childcare. These results suggest that, in the long run, the equilibrium feedback driven by demographic change mitigates the reduction of maternal labor supply due to the CB expansion.

Lastly, the long-run effects take nearly 100 years to accrue, because the effects are mainly driven by the change in demographic structure and labor income tax rate, which takes a long time to change. In the medium run, the tax burden for financing the expansion is much larger than its long-run level because the aggregate output (or tax base) also takes a long time to reach the new level. As a result, current and some future generations are worse off because of the higher tax burden during the initial transition period, with a poor realization of the long-run gains.

Examining the effects of CB on the Japanese economy, where the magnitude of demographic aging with declining birthrates is unprecedented, demonstrates the effectiveness of the policies in an aging economy. This should be a valuable case study for other countries that are expected to face further demographic aging in the future, as in today's Japan.

Related Literature: This study is related to the macroeconomic literature using models with two-earner households. It departs from the standard model comprised of only single-earner households and explicitly models choices of secondary earners. Previous works highlight the importance of considering their behavior to study aggregate implications of the rise in wage volatility (Heathcote et al., 2010), the self-insurance mechanism of households (Blundell et al., 2016), and tax and social security transfers (Guner et al., 2012a,b; Braun et al., 2016; Bick and Fuchs-Schündeln, 2018; Borella et al., 2021; Kitao and Mikoshiba, 2022; Ortigueira and Siassi, 2022).

This study is mostly related to the growing literature on the macroeconomic analysis of family-related policies using quantitative models with heterogeneous agents (Erosa et al., 2010; Domeij and Klein, 2013; Guner et al., 2020; Cavalcanti et al., 2021; Hannusch, 2022). The literature extend the two-earner household framework to capture the policy effects on maternal labor supply and their aggregate implications. Erosa et al. (2010) study the aggregate implications of parental leave policies by developing a general equilibrium, labor market matching framework embedded fertility and employment choices. They show that generous policies provide substantial gains to females at the expense of male welfare by affecting bargaining power between the two. Domeij and Klein (2013) develop the lifecycle model with two-earner households to examine whether the government should subsidize preschool day care. They show that subsidizing day care at a rate of 50% maximizes social welfare and doubles the labor supply of mothers with young children. Recently, Hannusch (2022) examines what explains the cross-country difference in the gap between the employment rates of mothers with and without preschool children. She shows that the difference in child-related transfer systems accounts for most of the cross-country difference. Although Cavalcanti et al. (2021) use the standard single-earner model, they incorporate endogenous unwanted fertility into the GE-OLG framework to study the role of family planning policies in Kenya. They show that legalizing and subsidizing abortion is a cost-effective measure for improving living standards from a long-term perspective.

My main contribution to the two strands of literature is to examine the macroeconomic and welfare effects of the CB, incorporating fertility choices into the model with two-earner households. Although some studies have embedded fertility choices, they focus on the policy impacts other than those of the CB (e.g., Erosa et al., 2010; Cavalcanti et al., 2021). At the same time, previous studies on the CB abstract fertility choices to elaborate on its impact on female labor supply using models with heterogeneity in two-earner households (e.g., Guner et al., 2020; Hannusch, 2022).

This study is most closely related to Guner et al. (2020), using an OLG model with exogenous fertility. They study the effects of a wide range of child-related transfers in the US, such as childcare subsidies and tax credits, on maternal labor supply, female human capital accumulation, and welfare. The transfers they study include a tax credit for households with children, which is non-refundable and unconditional on the labor market status of parents. As one of the experiments, they also simulate the tax credit expansion, approximating the actual reform under the Tax Cut and Job Act of 2017.

The reform makes it refundable, similar to the CB system adopted in other countries, and the per-child payment is doubled. They show that the expansion leads to substantial welfare gains for recipients, especially households with children at the bottom of the skill distribution, at the expense of non-recipient welfare, such as households who never have children. Average welfare improves by 1.71% in terms of consumption equivalence, largely due to the redistributive nature of the reform.[7] At the same time, such a generous transfer depresses the parental labor supply due to an income effect and results in a lower aggregate output.

My study differs from theirs in that I incorporate fertility choices into the model and elaborate on the equilibrium effects of the CB, taking policy impacts on fertility and demographic structure into account. I show that results under the exogenous fertility version of my model are qualitatively in line with those of Guner et al. (2020), while the endogenous fertility version has different implications for the aggregate variables, including female labor supply, and welfare.

Finally, this study is also related to the empirical literature on the effects of cash benefits to households with children, especially CB and baby bonus. They investigate its effects on parental labor supply (e.g., for Japanese studies, see, Bessho, 2018; Asakawa and Sasaki, 2022), on fertility (e.g., Milligan, 2005; Cohen et al., 2013; Riphahn and Wiynck, 2017), and on both (Azmat and González, 2010; González, 2013; Laroque and Salanié, 2014; Yamaguchi, 2019).

Regarding the labor supply effects, the consensus is that generous cash transfers would depress parents' labor supply, especially that of mothers, in the short run (e.g., Asakawa and Sasaki, 2022). At the same time, they show that considering the labor supply response is important for evaluating potential options for policy reform. For example, Bessho (2018) uses a discrete choice model of labor supply to show that Japan's CB expansion conducted in 2010-11 depressed parents' labor supply, and that funds required for the reform are underestimated by 22% if the labor supply responses were disregarded. Recently, Corinth et al. (2022) suggest that replacing the Child Tax Credit with the child allowance in the US is not so effective in reducing child poverty because poorer parents with young children would reduce their labor supply due to the introduction of the child allowance. Note that these empirical works typically study the short-run effects. I show that the side effects are mitigated in the long run, considering equilibrium effects stemming from changes in fertility rates and demographic structure.

For the fertility effects, previous works show that the cash benefits such as the CB

[7] They compute another measure of average welfare by weighting discounted value of households' utility by the inverse of shadow value associated with one dollar transfer at birth, to disregard the distributional effects.

and baby bonus have a significant impact on fertility.[8] Milligan (2005) exploits a reform of Quebec's baby bonus and argues that an extra 1,000 Canadian dollars benefit would increase fertility by 16.9%, implying a benefit elasticity of 0.107. Azmat and González (2010) study the Spanish income tax reform, including an increase in yearly supplements per child from 300 to 1,200 euros, to identify its fertility effects. They show that the reform increased the fertility rate by 5%, and the benefit elasticity of fertility was 0.022. Cohen et al. (2013) uses a variation in the child subsidy payment conducted around 2003 in Israel, providing a larger subsidy for third or higher births. They conclude that the benefit elasticity of fertility was 0.176. González (2013) adopt the regression discontinuity design to study fertility effects implied by Spain's reform in 2007 introducing a one-time payment of 2,500 euros (about 3,800 USD) for births, almost 4.5 times the monthly minimum wage for full-time workers. She finds that it has a statistically significant impact on fertility and increases conceptions by approximately $5-6$ percent. Laroque and Salanié (2014) estimates a discrete choice model of fertility and participation using the French Labor Force Survey. They simulate the introduction of an unconditional transfer of 150 euros per month for households with children, which would require funds amounting to 0.3% of GDP. This introduction results in a nearly 20% higher fertility rate.[9]

Overall, the literature reports a significant effect of CB or baby bonus on fertility. However, there is no consensus on how the elasticity depends on household income: some find that the elasticity is larger for households with higher income(e.g., Milligan, 2005; Riphahn and Wiynck, 2017),[10] while others report the opposite (e.g., Azmat and González, 2010; Cohen et al., 2013).

Building on their findings, this study disciplines the model so that the benefit elasticity of fertility implied by the model is consistent with the empirical estimates. The benefit elasticity of fertility is a key moment for constructing a reasonable quantitative model, as we are interested in the equilibrium outcome considering fertility responses with potential CB reforms. Although there are few empirical studies on the fertility effects in Japan,[11]

[8] From the viewpoint of institutional similarity, this paper relies on the empirical studies on the CB and baby bonus, and I do not include papers studying a reform of the EITC on the list (e.g., Baughman and Dickert-Conlin, 2009)

[9] González (2013) and Laroque and Salanié (2014) also examine the effect of the cash transfer on maternal employment and show that the generous transfer reduces maternal employment.

[10] Riphahn and Wiynck (2017) study Germany's child benefit reform in 1996. They show that the fertility effects on low-income couples are not statistically significant. At the same time, they find some support that the fertility effect is significant for higher-income couples deciding on a second birth.

[11] One of the primary reasons for this is that, in Japan, the central government designs the CB system, and there is no room for local governments to establish their original system. Previous works (e.g., Milligan, 2005) indicate that using the regional difference in the benefit system is a powerful device for identifying the fertility effect.

we can use results implied by Yamaguchi (2019) to calibrate the model. He estimates a dynamic discrete choice model of female employment and fertility decisions to evaluate some options for parental leave policies in Japan, using the Japanese panel data. Although his main focus is on the parental leave policy, he also conducts counterfactual experiments to increase the baby bonus and observe its effects on fertility and female labor supply. Although he does not explicitly specify the fertility elasticity, we can compute it as 0.026 based on the results, belonging to the lower class of the elasticity values reported in studies outside Japan. I use the value as a target in calibrating the model. Section 3 provides a detailed discussion of this point.

The rest of the paper is organized as follows: Section 2 describes the model, which is calibrated in Section 3. Then, counterfactual policy experiments are implemented in Section 4, and Section 5 concludes the paper.

2 Model

This section describes a stationary overlapping generations economy consisting of heterogeneous households.

2.1 Primitives

Demographics: Let $j \in \mathcal{J} \equiv \{1, ..., J\}$ denote the age of an individual. Individuals live for a maximum of J periods, and a fraction of individuals in each cohort die in each period. The government collects accidental bequests left by those who die before reaching age J and makes a lump-sum transfer to each household, denoted by a_b. Let $s_{j,j+1} \in \mathbb{R}_+$ be the conditional probability of being alive at $j+1$, given that the person is alive at age j. The unconditional probability of being alive at age $k \in \{2, ..., J\}$ is denoted by $S_k = \Pi_{j=1}^{k-1} s_{j,j+1}$ and $S_{j=1} = 1$. Every period, the mass of new generation in which each individual has a_1 units of initial assets enters the economy. The cohort size at age $j = 1$ grows at a rate of n across cohorts, which is endogenously determined by households' fertility choices in this model. The growth rates of cohort size and the survival probabilities characterize the measure of individuals aged j, denoted by μ_j, which is normalized to add up to one (i.e., $\Sigma_j \mu_j = 1$).

Endowment: Each individual entering the economy has several characteristics (i.e., individuals are heterogeneous *ex-ante*): gender, marital status, and educational attainment. First, $g \in \mathcal{G} \equiv \{M, F\}$ denotes gender, where M stands for male and F stands for female.

Second, they differ in educational attainment or skill, denoted by z, where $z \in \mathcal{Z} \subset \mathbb{R}_+$. Third, a fraction of males and females in a cohort are matched exogenously at the start of their lives, forming a family. Let $m \in \{C, S\}$ denote marital status where C stands for couple and S stands for single. Note that the three factors are time-invariant, and thus, couples do not divorce, and single households do not marry anyone throughout their lives. Let $\mu_{g,z,m}$ represent the fraction of individuals with a time-invariant state (g, z, m). For couples, let the superscripts h and w denote the variables for the husband and wife, respectively. In addition, among couples, let π_{z^h,z^w} represent the fraction of those comprised of a husband with skill z^h and a wife with skill z^w. Finally, let $\boldsymbol{\theta}_C \equiv \{z^h, z^w\} \in \boldsymbol{\Theta}_C$ and $\boldsymbol{\theta}_S \equiv \{g, z\} \in \boldsymbol{\Theta}_S$ represent vectors of the married and single households' characteristics, respectively. I refer to a specific factor on $\boldsymbol{\Theta}_C$ or $\boldsymbol{\Theta}_S$ as the *type* of the household.

Preference: Every period, each household draws utility from consumption and disutility from labor supply. Individuals can participate in the labor market after entering the economy and retire at the end of age $j = J_R$. In this model, all husbands supply one unit of labor exogenously and inelastically,[12] and wives and single individuals choose their working hours. Let c and l denote consumption and working hours, and $u^C(c, l; b)$ and $u^S(c, l)$ denote the flow utility for married and unmarried households, where $b \in \mathcal{B}$ represents the number of children they have.

Households also draw utility from having children, captured by $v(b)$. Upon marriage, couples choose the number of children they have. If they decide to have children, they have all births at once, at age $j = J_b$, which is common across households. By contrast, unmarried individuals do not make the fertility decisions. The lifetime utility of married and unmarried households are formulated as follows:

$$v(b) + \sum_{j=1}^{J} S_j \cdot \beta^{j-1} \cdot u^C(c_j, l_j; b),$$

$$\sum_{j=1}^{J} S_j \cdot \beta^{j-1} \cdot u^S(c_j, l_j),$$

where β is the subjective discount factor.

Labor earnings: Three factors determine labor earnings: wage rate w determined in the competitive labor market, labor productivity, and hours worked l. Labor productivity is represented as $\bar{\omega}(j, g, z)$, capturing the wage difference across different ages, gender, and

[12] According to the Employment Status Survey (ESS) in 2017 published by the Administration of Internal Affairs and Communications (MIC), about 98% of prime-age married males are in the labor market, and more than 95% work full-time.

education. Lastly, let $y = w \times \bar{\omega}(j, g, z) \times l$ represent an individual's flow labor earnings.[13]

Costs of children: There are two costs of having children: monetary and time costs. First, households with children incur child-related expenditures up to a certain period, depending on the type of parent and the number of children. A function $CE(j, b, \boldsymbol{\theta}_C)$ captures child-related expenditures for couples. In addition to the monetary cost, households with young children have to spend a fraction of their disposable time per child, $\eta > 0$.

Child Benefits: For married households, the government provides the CB payments according to functinos $CB(j, b, y^h, y^w)$. The payment function is constructed to approximate the actual payment system in Japan. I elaborate the CB function in Section 3.

Medical needs and social security transfers: Every period, medical needs, captured by $m(j)$, arise for each individual, depending on their age. The needs are exogenous and deterministic. The government runs the public health insurance program and subsidizes ω_j fraction of the medical expenditure, depending on age j. The government also runs the PAYG pension program, providing pension benefits to individuals aged $j > J_R$. The per-person payment denoted by ss is assumed to be constant across retirees with an average income replacement rate ρ. The lifecycle structure of this model is illustrated in Fig 1.

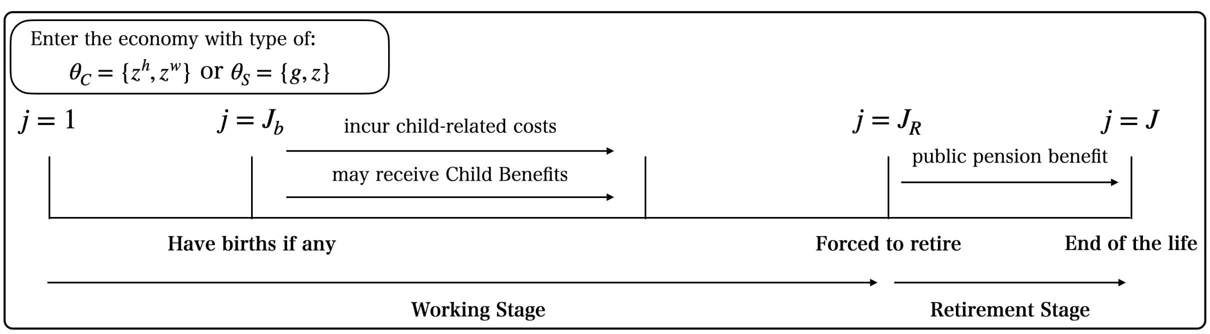

Fig 1: Model's lifecycle.

Technology: A representative firm operates with the constant-return-to-scale (CRS) production function $F(K, L)$, with capital inputs K and labor input L in efficiency unit. The aggregate capital depreciates at a rate of $\delta \geq 0$ per period, and the firm must incur depreciation costs.

Government: The government raises revenue from taxing consumption, labor earnings, and capital income. Let τ_c, τ_l, and τ_a represent the tax rates of each. The government

[13] As in Guner et al. (2020), there is no uncertainty for labor productivity.

uses revenue for CB, PAYG pension, public health insurance, and other government expenditures G. The government funds the PAYG pension and health insurance with the labor income tax.

2.2 Decision Problems

Given the economic environment and primitives described thus far, I formulate decision problems for households and the firm.

Married households: Hereafter, I abstract the subscript j from the variables and denote a' to represent the asset position at the beginning of the next period. They choose the number of children at the start of their lives. Given the number of children and their household type, they make choices on consumption, wife's labor supply, and savings, throughout their lives. They can borrow or lend at an interest rate r, but there is a borrowing limit ϕ. The value function for couples with type $\boldsymbol{\theta}_C$ and an initial asset a is as follows:

$$W^C(a, \boldsymbol{\theta}_C) = \max_{b \in \mathcal{B}} \{v(b) + V^C(a, j = 1, b, \boldsymbol{\theta}_C)\}$$

where

$$V^C(a, j, b, \boldsymbol{\theta}_C) = \begin{cases} \max_{\{c,a',l^w\}} \{u^C(c, l^w; b) + s_{j,j+1}\beta V^C(a', j+1, b, \boldsymbol{\theta}_C)\} & \text{if } j \in \{1, ..., J_R\} \\ \max_{\{c,a'\}} \{u^C(c, 0; b) + s_{j,j+1}\beta V^C(a', j+1, b, \boldsymbol{\theta}_C)\} & \text{if } j \in \{J_R + 1, ..., J - 1\} \\ \max_{\{c\}} u^C(c, 0; b) & \text{if } j = J \end{cases} \tag{1}$$

s.t.

$$(1 + \tau_c)c = \begin{cases} (1 + (1 - \tau_a)r)(a + 2a_b) - a' + (1 - \tau_l)(y^h + y^w) & \\ -CE(j, b, \boldsymbol{\theta}_C) + CB(j, b, y^h, y^w) - 2(1 - \omega_j)m(j) & \text{if } j \in \{1, ..., J_R\} \\ (1 + (1 - \tau_a)r)(a + 2a_b) - a' + 2[ss - (1 - \omega_j)m(j)] & \text{if } j \in \{J_R + 1, ..., J - 1\} \\ (1 + (1 - \tau_a)r)(a + 2a_b) + 2[ss - (1 - \omega_j)m(j)] & \text{if } j = J \end{cases} \tag{2}$$

$$c \geq 0, a' \geq -\phi, l \in [0, 1 - b \cdot \eta]. \tag{3}$$

Single households: Single households make choices regarding consumption, labor supply, and savings, throughout their lives. The value function for single households of type

$\boldsymbol{\theta}_S$ and an initial asset a is as follows:

$$V^S(a, j, \boldsymbol{\theta}_S) = \begin{cases} \max_{\{c,a',l\}}\{u^S(c,l) + s_{j,j+1}\beta V^S(a', j+1, \boldsymbol{\theta}_S)\} & \text{if } j \in \{1, ..., J_R\} \\ \max_{\{c,a'\}}\{u^S(c,0) + s_{j,j+1}\beta V^S(a', j+1, \boldsymbol{\theta}_S)\} & \text{if } j \in \{J_R+1, ..., J-1\} \\ \max_{\{c\}} u^S(c,0) & \text{if } j = J \end{cases} \tag{4}$$

s.t.

$$(1+\tau_c)c = \begin{cases} (1+(1-\tau_a)r)(a+a_b) - a' + (1-\tau_l)y - (1-\omega_j)m(j) & \text{if } j \in \{1, ..., J_R\} \\ (1+(1-\tau_a)r)(a+a_b) - a' + ss - (1-\omega_j)m(j) & \text{if } j \in \{J_R+1, ..., J-1\} \\ (1+(1-\tau_a)r)(a+a_b) + ss - (1-\omega_j)m(j) & \text{if } j = J \end{cases} \tag{5}$$

$$c \geq 0, a' \geq -\phi, l \in [0,1]. \tag{6}$$

Firm: The representative firm rents capital and effective labor in competitive factor markets, produces final goods according to the production function, and sells the goods in the competitive final good market. The user cost of capital is given by $(r+\delta)$, as the firm must incur depreciation costs. Given the user cost $(r+\delta)$ and wage rate w, the firm solves the standard static problem for each period:

$$\max_{(K,L)\geq 0} F(K,L) - (r+\delta)K - wL. \tag{7}$$

Government budget: Each period, the following government budget constraint must be satisfied:

$$T_c + T_l + T_a = CB + G + \underbrace{\bar{y}\rho \sum_{j=J_R+1}^{J} \mu_j}_{\text{pension expenditure}} + \underbrace{\sum_{j=1}^{J} \omega_j m(j)\mu_j}_{\text{health insurance expenditure}}.$$

On the revenue side, T_c, T_l, and T_a denote tax revenues from consumption, labor income, and capital income. For the expenditure side, CB represents total expenditures on CB, and $\bar{y}$ denotes the average labor earnings in the period.

2.3 Equilibrium

I solve the stationary equilibrium of the economy. In equilibrium, households choose consumption, savings, and labor supply, and couples choose the number of children, to maximize their lifetime utility. The representative firm maximizes its profits by choosing labor and capital inputs. Prices clear factor markets. The government budget constraint is satisfied period by period. Particularly important is that households' fertility choices endogenously determine the demographic structure, represented by μ_j. Stationarity implies that μ_j is time-invariant in equilibrium. Appendix A provides a detailed definition of equilibrium.

3 Calibration

The calibration procedure involves three steps as follows:

1. **Preliminary**:
   - Specify functional forms for preference and technology.
   - Set some parameter values and distributions, following the literature or based on Japan's aggregate data.
   - Make an initial guess for a parameter value governing the curvature of utility from having children, γ in equation (10) below.
2. **Inner loop**: Except for γ, calibrate the rest of the parameters to be determined inside the model such that the model-implied value in equilibrium and target in each targeted moment are sufficiently close.
3. **Outer loop**: Conduct a counterfactual experiment increasing the per-child payment and compute the benefit elasticity of fertility to the payment expansion, holding prices and tax rates constant. Until the implied elasticity is sufficiently close to the target value, update the guess for γ and return to Step 2 to run the inner loop with the new guess for γ.

Demographics: The length of one period in the model is five years. Individuals enter the economy when they are 25 years old, retire at the end of age 64 (i.e., $J_R = 8$), and live to the end of age 104 at a maximum (i.e., $j = J = 16$). I set $j_b = 2$ so that households have births at age 30. The survival probability on age is set based on the Vital Statistics (2019).[14] Individuals enter the economy with no assets, except for the lump-sum transfer of the accidental bequests (i.e., $a_1 = 0$). Measures of individuals with time-invariant

[14] See, https://www.mhlw.go.jp/english/database/db-hw/outline/index.html.

states $\mu_{g,z,m}$ and couples in each education pair π_{z^h,z^w} are set based on the ESS (2017).

Productivity: I set the productivity function $\bar{\omega}$ based on the Basic Survey on Wage Structure (2020) by the MHLW, discretizing the space for educational attainment as follows:

$$\mathcal{Z} = \{< HS, HS, SC, COL, COL+\}.$$

$< HS, HS, SC, COL$, and $COL+$ stand for less than high school (elementary school- or junior high school-graduate), high school, some college, college, and graduate school, respectively.

Preference: Periodic utility functions for couples and singles are given as:

$$u^C(c, l^w; b) = 2 \cdot \frac{(c/\Lambda(b))^{1-\sigma} - 1}{1-\sigma} - \varphi \frac{(l^w + b \cdot \eta)^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}}, \tag{8}$$

and

$$u^S(c, l) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \varphi \frac{l^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}}, \tag{9}$$

where $\varphi > 0$ affects the disutility from additional working hours and μ represents the Frisch elasticity. $\Lambda(b)$ indicates the equivalent scale, and I adopt the OECD modified equivalent scale where $\Lambda(b) = 1.5 + 0.3b$. This specification applies until their children become independent (i.e., when the children turn 25 years old and form their own household). Thereafter, we have $\Lambda(b) = 1.5$ for any $b \in \mathcal{B}$. For the utility from consumption, I consider a case in which $\sigma \to 1$, the logarithmic function. The Frisch elasticity μ is set to 0.85 based on Japan's empirical estimate.[15] The borrowing limit ϕ is set to 0 (i.e., households cannot borrow).

The utility from having children is captured by the following function:

$$v(b) = \kappa \cdot \frac{(1+b)^{1-\gamma} - 1}{1-\gamma}, \tag{10}$$

where κ is set to 573.1, targeting the annual growth rate of the cohort size, -1.15%.[16] The curvature parameter γ is determined in Step 3. The targeted moment is the benefit

[15] Since Kuroda and Yamamoto (2008) reports that the elasticity considering both intensive and extensive margin ranges between 0.7 and 1.0, I take the intermediate value.

[16] Assuming that the birth number grew at a constant rate in 1940-2020, the constant rate is computed as -0.0115, based on the Vital Statistics (2020) of the MHLW. Note that the oldest age in this economy is given as $J = 16$, and the one period is five years, meaning that the difference between the age of the youngest agents and the oldest ones is nearly 80. Thus, I choose the period between 1940 ($= 2020 - 80$) and 2020. As Fig 4 on Appendix B shows, the birth number or cohort size shows a declining trend during the period, while there are some baby boom and baby bust phases.

elasticity of fertility to the payment increase, indicating at what percent the completed fertility rate changes in response to a 1% increase in the per-child payment. I rely on Yamaguchi (2019), and set the target value as 0.026.[17]

Although this is the elasticity for the baby bonus, using this value as the target in calibration is not problematic for the following reasons. First, the baby bonus is the most similar benefit to the CB in Japan's context in that it is the cash transfer, most households are eligible, and the eligibility is almost independent of the labor market status of parents and their earnings.[18] Further, the scale of expanding the baby bonus in Yamaguchi (2019) is similar to the CB expansion scenarios considered in Section 4.[19]

To check if the model implied elasticity is sufficiently close to the target, I conduct counterfactual experiments increasing the per-child payment by $(1 + x)$, holding prices and tax rates constant. x can take seven values from $\{1, 1.5, 2, 2.5, 3, 3.5, 4\}$, which are similar to the scales of the expansion examined in empirical studies and Section 4. I compute the completed fertility rate of women with each case of the payment increase.[20] The elasticity is then calculated for each expansion rate using the implied fertility rates. Next, I compute the mean of the seven elasticity values, to be close to the target value of 0.026. Finally, γ is set to 4.5, resulting in a mean elasticity of 0.0288. Table 1 reports the computed elasticity for each expansion rate with $\gamma = 4.5$.

	Expansion Rate $(1 + x)$								
	2	2.5	3	3.5	4	4.5	5	Mean	SD
Elasticity	0.020	0.025	0.028	0.044	0.025	0.028	0.030	**0.0288**	0.0077

Table 1: The benefit elasticity of fertility with $\gamma = 4.5$. *Note*: I compute the completed fertility rate of women with each case of the payment increase. The elasticity is then calculated for each expansion rate using the implied fertility rates. "Mean" reports the mean of the seven elasticity values, to be close to the target value of 0.026. "SD" stands for the standard deviation of the seven values.

[17]Although he does not specify the fertility elasticity explicitly, I compute the elasticity as follows. Note that he simulates introducing the baby bonus of 1, 3, and 5 million yen, with fixing the parental leave system. A proxy of completed fertility rate in each scenario is reported as 2.23, 2.34, and 2.46 for each. Considering the expansion of the baby bonus from 1 to 3 million yen and to 5 million yen, increases in the fertility rate from 2.23 to 2.34 and to 2.46 imply the benefit elasticity of 0.25 and 0.26, respectively. Thus, I set the target as 0.026.

[18]As elaborated above, although there is an income test for eligibility of the CB, the income limit is not so severe so that the majority is eligible.

[19]As listed above, the majority receives about 2 million yen per child in total (without discounting), under the current CB system. Then, doubling or tripling the per-child payment as examined in Section 4 implies that the expenditure would increase about 2 $\sim$ 4 million yen, which are similar to Yamaguchi (2019)'s experiment.

[20]This setting is the same as Yamaguchi (2019) and other studies using the dynamic discrete choice framework in the empirical literature.

I set $\varphi = 1.55$ so that the average hours worked conditional on working in the model matches the data.[21] The time cost of children η is set to 0.069 so that the ratio between childcare time and working hours of mothers with young children is in line with data from the Survey on Time Use and Leisure Activities (2016) by the MIC.[22] The subjective discount factor β is set to 1.033 so that the initial steady state produces the capital to output ratio of 2.8.

Child-related Expenditures: As the Declining Birthrate White Paper (2010) by the Cabinet Office[23] implies, we may classify two broad categories of child-related expenditures: (i) living expenses, such as expenditures for clothes, foods, and housing; and (ii) education expenditures. Note that the model captures the additional living expenses induced by having a child, deflating household consumption by the equivalence scale.

This model also captures education expenditures with function CE. I set the function assuming that the child attains the same education level as the parents. Data for average tuition costs in each education category come from the White Paper on Education, Culture, Sports, Science and Technology (2009) by the Ministry of Education.[24]

Child Benefit: A function representing payment of the CB is conducted to approximate the actual transfer systems described below. Households with children before junior high school completion may receive the payment,[25] and the per-child payment varies according to the household's income, age, and birth order.

The program defines household income as the maximum income earned by a household member in the previous year. Consider a household, for example, consisting of a husband, wife, and newborn babies. This household is potentially eligible to receive CB. Let y^h and y^w denote the annual earnings of husband and wife in the previous year. In this case, the household income represented by I defined for the program is given by $I = \max\{y^h, y^w\}$.

The program has income tests determining eligibility status and payment, in two steps. First, if a household's income is above a threshold, the household is not eligible. The threshold value varies according to the number of dependents. As a typical example, for households consisting of a husband and three dependents (two young children and a

[21] I use weekly hours worked in ESS (2017) for this purpose. I focus on people who worked more than 200 days in the year and less than 48 hours per week as a sample. Then, the upper bound for working hours corresponds to 48 hours per week in this model.

[22] See, https://www.stat.go.jp/english/data/shakai/2016/gaiyo.html.

[23] See, https://www8.cao.go.jp/shoushi/shoushika/whitepaper/measures/english/w-2010/pdf/2_p41_56.pdf.

[24] See, https://warp.ndl.go.jp/info:ndljp/pid/11293659/www.mext.go.jp/b_menu/hakusho/html/hpab200901/1305844.htm.

[25] In principle, one completes junior high school education at the age of 15 in Japan.

wife who does not work in the market), the threshold is 12 million yen. Next, even though household income is not above it, the per-child payment depends on income. Considering the example household with three dependents, if the income is above 9.6 million yen, they receive 5,000 yen for each child per month, regardless of the child's age or birth order. Otherwise, they can receive 10,000-15,000 yen per child-month, depending on the child's age and birth order. When the child's age is between 0 and 2, the monthly payment for each child amounts to 15,000 yen. In addition, for children whose birth order is higher than third, the payment for the child is 15,000 yen until primary school completion. When the child is in junior high school, the per-child payment is 10,000 yen regardless of the birth order. Table 2 summarizes the transfer rules.

Earnings (y, million yen)	Child's age	Payment (per child-month, yen)
$y \geq 12$		No payment
$9.6 \leq y < 12$		
	$0-2$	5,000
	3− primary school	5,000
	J.H. school	5,000
$y < 9.6$		
	$0-2$	15,000
	3− primary school	10,000 (1st and 2nd child)
		15,000 (3rd child or more)
	J.H. school	10,000

Table 2: The per-child payment in Japan's child benefit. *Note*: This is the case for a household consisting of a worker with three dependents (typically consisting of a wife that does not participate in the labor market and two children).

According to the ESS (2017),[26] 3% of husbands with children under 18 earned more than 12.5 million yen, while 4.9% of them earned more than 10 but less than 12.5 million yen. Thus, the income test in Japan's CB is not so severe that the majority of households with children can receive payments. Thus, I assume that all households with children before junior high school graduation can receive a positive benefit for simplicity. More specifically, using the threshold values 9.6 and 12 million yen, the model assumes that

[26] See Table 250-2 in https://www.e-stat.go.jp/en/stat-search/files?page=1&layout=datalist&toukei=00200532&tstat=000001107875&cycle=0&tclass1=000001107876&tclass2=000001107878&tclass3val=0.

those earning more than 12 million yen receive 5,000 yen, and that other households receive the payment as described in Table 2.

Lastly, the ESS says that the median of husbands' annual income with children under 18 lies between 2 and 2.49 million yen, which means that the per-child payment covers 5-7% of the median income.

Social Security Programs and Taxes: Based on empirical estimates in Japan, I set $\tau_a = 0.35$ and $\tau_l = 0.35$ in the benchmark case.[27] The consumption tax rate τ_c is set as 0.1 as Japan currently has. The income replacement rate ρ for the PAYG pension is set to $\rho = 0.114$, so that the total spending for public pension corresponds to 9.5% of GDP. The medical needs and copayment rates depending on age are computed based on data prepared by the MLHW.[28] These values then pin down the other government expenditure in the benchmark, G, to equate the both sides of the government budget constraint (8).

Technology: The representative firm produces the final good using the Cobb-Douglas production function, $F(K, L) = K^{\alpha} L^{1-\alpha}$. Capital share α is 0.36, and the annual depreciation is 0.089, following Kitao (2015).

Table 3 summarizes the parameter values internally calibrated in the model (i.e., in Steps 2 and 3).

Parameter	Value	Target (description)	Target	Model
β	1.033	Capital/Output	2.8	2.8
η	0.069	Child-rearing / Market work	0.236	0.236
φ	1.55	Average hours worked (per week)	38.64 h	38.49 h
κ	573.1	Annual growth rate of birth numbers	-1.15%	-1.13%
γ	4.50	Benefit elasticity of fertility	0.026	0.0288
ρ	0.114	Pension expenditure to GDP	9.5%	9.5%

Table 3: Parameters calibrated inside the model.

Non-targeted moment: The model successfully replicates the fertility differential across education categories, not targeted in the calibration. To observe this, I use the National Fertility Survey (NFS),[29] reporting fertility rates for each educational background among

[27] Hansen and İmrohoroğlu (2016) estimate the 2010's capital income tax rate in Japan, and Gunji and Miyazaki (2011) estimate average marginal tax rate, including social security taxes, on labor income.

[28] The data is available from https://www.mhlw.go.jp/wp/hakusyo/kousei/17/backdata/xls/1/03-01-09.xls (in Japanese).

[29] See, https://www.ipss.go.jp/ps-doukou/e/doukou15/Nfs15_gaiyoEng.html.

married women whose marriage duration is 15-19 years.

According to the report, in 2015, the completed fertility rate for college-graduated women was 1.89, and it was 1.99 for women graduating high school, which means that more educated women tend to have fewer children than less educated women. The ratio between the fertility rates of high school and college graduates was 0.9497.

In the benchmark, the fertility rate is 2.09 with $z = HS$, and is 2.00 with $z = COL$, implying that the ratio is 0.9569. The model replicates the basic observation that high-skilled women are less likely to have children than low-skilled women are, and the ratio is relatively close to the data. Table 4 summarizes the results.

	Data		**Model**	
Education	HS	COL	HS	COL
# of births	1.99	1.89	2.09	2.00
COL/HS	0.950		0.957	

Table 4: Fertility in different education categories. *Note*: The data part comes from the NFS. The column "Education" indicates the wife's educational attainment, where HS refers to high school graduates, and COL refers to college graduates. The survey asks sample couples whose marriage duration is 15-19 years the birth number. For the model part, I report the completed fertility of married women with each education status in the model.

A primary factor behind this differential is the difference in opportunity costs across different education categories. Recall that, in the model, having children takes a fraction of the time cost until the children are sufficiently grown. Because the average wage of more educated women is higher, the opportunity cost of having children is also higher, leading them to fewer children.

4 Numerical Analysis

4.1 Methodology

Using the model, this section examines the effects of increasing the per-child payments and removing the program. Specifically, I first solve the stationary equilibrium defined in Section 2 and Appendix A, with different levels of the per-child payment. In each exercise, I change only the CB payment, holding other government variables, such as G, ρ, and ω_j, fixed as in the benchmark. Let $CB^{initial}_{j,y}$ be the per-child payment for households with children aged j and the income vector of parent y in the benchmark. Then, the per-child payment after an expansion, denoted by $CB^{new}_{j,y}$, is expressed as $CB^{new}_{j,y} = (1+x) \times CB^{initial}_{j,y}$

for each (j, y), for some $x \geq -1$. The case in which $x = -1$ is equivalent to removing the system. Upon reforms, the government adjusts the consumption tax rate to balance the budget. In addition, the labor income tax rate is adjusted in response to changes in spending on social security programs. Even though parameters, such as the income replacement rate for the PAYG pension and the copayment rate for the health insurance program, are fixed as in the benchmark, changes in demographic structure change the social security expenditure and social security tax rate on labor earnings. Second, I solve the transition dynamics associated with the change in per-child payment, describing how the economy will reach a new equilibrium and how the policy change affects the welfare of cohorts living in the transition periods.

Hereafter, I use the term "non-recipients" and "unmarred (or single) households" interchangeably. I find that all types of couples in the benchmark have a positive number of children. In addition, all households with children before junior high school completion (i.e., those under 15 years old) are eligible for the benefit in this model, as explained in Section 3, meaning that all couples qualify as program recipients at certain periods in their lives. Thus, only unmarried individuals are not eligible for the CB throughout their entire life, and in this sense, they are only non-recipients.

Welfare Measure: In each exercise, I compute the Consumption Equivalent Variation (CEV) of each household type. Here, the CEV for a household indicates how much its lifetime consumption vector in the benchmark needs to be scaled up for the household to be indifferent between living in the benchmark and the new equilibrium.[30] Thus, note that a positive value of CEV indicates welfare improvement for the household, and vice versa.

Welfare Decomposition: Note that the values of some objects differ across equilibria: factor prices (interest rate and wage rate), tax rates (on consumption and labor income), and the per-child payment. These are taken as given for households and different values of them have different implications for their behavior and welfare.

To evaluate the relative importance of changes in the factors to the changes in household welfare, I conduct a decomposition analysis as follows. First, I solve a new equilibrium with an expansion rate, and obtain the prices and tax rates in long-run equilibrium under the new payment scale. Second, I solve each household's maximization problem by replacing any of the four objects (i.e., the consumption tax, labor income tax, prices, or per-child payment), with that of the new equilibrium, holding the other three objects

[30] As Jones et al. (2007) explain, defining the Pareto efficiency is not straightforward in models with endogenous fertility.

fixed as the benchmark. Finally, I compute the CEV for each household type. Note that, this methodology does not imply that each effect adds up to the overall effect.[31]

4.2 Steady States

I solve the stationary equilibrium with five different expansion rates, $1 + x = 0, 2, 3$, and 4. The case of $1 + x = 0$ corresponds to CB removal. Setting the upper bound around 3 seems to be reasonable because the scale of $1+x = 3$, at least in the short run, corresponds roughly to the current maximum amount of per-child payments among OECD countries, which covers about 15% of the average labor income.[32]

4.2.1 Aggregate Variables

Table 5 reports changes in aggregate variables. First, the completed fertility rate in equilibrium drops to 1.38 by the removal from the benchmark fertility rate of 1.42. Contrary, the fertility rate increases with expansions. For example, with an expansion rate of 3, the equilibrium fertility rate is 1.47. As a result, the working-age population share increases by 0.97 percentage point (p.p.).

This demographic change triggers equilibrium feedback via several channels. First, the labor income tax rate τ_l declines in the long run as the expansion rate increases. For example, with an expansion rate of 3, the income tax rate declines by 1.3 p.p. Also, aggregate labor supply in per-capita term (i.e., total labor supply in efficiency unit, over the population over 25 years old) increases for two reasons: a larger working-age population due to the change in demographic structure and a greater number of hours worked. I later show that aggregate hours worked increase by the expansion and examine the mechanism.

A greater labor supply per capita with a lower tax rate on labor income will increase income and savings, resulting in a greater per-capita output. While the removal will reduce the output by 2.9%, expanding the system will increase the output by $1.4 \sim 3.7\%$ in the long run. The output increase is equivalent to an increase in the tax base, mitigating the tax burden for financing the expansion, represented as the consumption tax rate τ_c. Due to a more substantial increase in per-capita labor compared to capital, the wage rate slightly declines, and the interest rate increases in each expansion scenario. Due to the labor income tax decline, however, the after-tax wage rate, $(1 - \tau_l)w$, increases with CB expansion.

[31] We may do the decomposition by changing each of the four objects step by step so that each effect sums to one. This "step-by-step" method, however, is sensible to the order of the variable changed, if there is a critical interaction among the variables.

[32] For more detail, see, PF1.3 of the database https://www.oecd.org/els/family/database.htm.

Under *exogenous fertility*, where the policy function for fertility is fixed as the benchmark, I find that the results change even qualitatively. The results are summarized in Table 9 in Appendix B. First, the long-run labor income tax rate does not change with expansion or removal, mainly because the change in demographic structure is not considered. Next, the per-capita output mostly stays the same in each scenario, as the demographic structure is fixed. The consumption tax rate in each expansion scenario is almost the same as those under endogenous fertility, even though more significant tax revenue for funding the CB is required under endogenous fertility owing to the greater number of children. Notably, while the consumption tax rate declines by the removal under exogenous fertility, it increases under endogenous fertility. The difference in output changes under the two settings is a key to understanding the consumption tax rate responses. Under endogenous fertility, in the long run, the removal (expansion) lowers (rises) the equilibrium fertility rate, meaning less (more) labor endowment in the economy, less (more) output produced, and a smaller (larger) tax base. The results obtained under exogenous fertility suggest that considering fertility responses non-trivially changes the macroeconomic implications of CB reforms.

Hours worked: The labor supply per capita reported in Table 5 reflects changes in the demographic structure and hours worked by each household type. Given that the per-capita labor supply and the working-age population share increase by expansions in the long run, we do not know whether the aggregate hours worked to increase or decrease.

In particular, it is worth investigating how the female labor supply responds to the CB expansion, recalling that previous works with exogenous fertility show that the expansion depresses the maternal labor supply due to an income effect. Under endogenous fertility, the expansion further depresses the labor supply due to increased time costs for childcare.[33] At the same time, note that the labor income tax rate declines and the after-tax wage rate increases in the long run, which would motivate them to work more due to a substitution effect.

In the case of tripling the benchmark CB payment,[34] the working hours of wives aged 25-64 are reduced by 0.67% and those of wives with children under 15 by 0.29%. Contrary, hours worked by unmarried individuals increase by 3%, and the aggregate hours worked increase by 0.81%.

I also compute the hours worked by each type under exogenous fertility, a setting adopted in previous studies. Hours worked by all subcategories are less than those under

[33] The opposite direction would also be the case (i.e., women may want to work more to finance monetary costs for the new child).

[34] The other cases show the similar result as this one does.

	Expansion Rate $(1+x)$				
	0	1	2	3	4
Demographic Structure					
Completed fertility	1.38	1.42	1.45	1.47	1.52
Working-age share (Δ p.p.)	-0.86	–	$+0.24$	$+0.97$	$+2.00$
Annual population growth (%)	-1.22	-1.13	-1.10	-1.02	-0.91
Quantity (per-capita, Δ%)					
Output	-2.9	–	$+1.4$	$+2.4$	$+3.7$
Capital	-2.6	–	$+0.8$	$+1.7$	$+2.3$
Labor (in efficiency unit)	-3.1	–	$+1.7$	$+3.1$	$+4.5$
Prices					
Interest (%)	3.80	3.85	3.92	4.00	4.02
Wage (Δ%)	$+0.19$	–	-0.32	-0.67	-0.76
After-tax ($[1-\tau_l]w, \Delta\%$)	-1.48	–	$+1.11$	$+1.16$	$+2.26$
Taxes (%)					
Consumption	10.3	10.0	10.5	11.4	11.8
Labor income	36.1	35.0	34.1	33.7	33.0

Table 5: Changes in aggregate variables in the long run with different per-child payments. *Note*: Cases of $1+x=0$ and $1+x=1$ correspond to the CB removal and benchmark, respectively.

endogenous fertility. Tripling the benchmark payment, wives decrease working hours by 1.64% and those with children under 15 by 3.90%. This result is in line with previous studies with exogenous fertility, showing that cash transfers to households with children depress maternal labor supply (Bessho, 2018; Guner et al., 2020), and the reduction is more significant than that under endogenous fertility. Unmarried individuals increase working hours by 1.67%, resulting in a 0.11 % increase in aggregate, both are lower than those under endogenous fertility. This exercise suggests that changes in factor prices and tax rates, driven by changes in the demographic structure, mitigate the reduction of female labor supply due to the CB expansion in the long run. The results are summarized in Table 6.

	Fertility set-up	
	Endogenous	Exogenous
Wives (aged 25-64)	-0.67	-1.64
with children under 15	-0.29	-3.90
Singles	$+3.00$	$+1.67$
Average	$+0.81$	$+0.11$

Table 6: Percentage changes in hours worked with expansion rate of three. *Note*: Recall that, every period, married men aged under 64 supply one unit of labor inelastically.

4.2.2 Welfare

What are the welfare implications of payment expansion and reduction? Table 7 summarizes the changes in welfare for each household type and expansion rate. Here, a positive value of CEV means welfare improvement for the household, and vice versa. The most important point is that all types of households, including non-recipients who do not have children throughout their lives, are worse off by the removal and better off by expansions in the long run. Removing the CB leads to a 1.2 % welfare loss for unmarried individuals in terms of CEV, while long-run gains by expansions amount to $0.5 \sim 1.8\%$. Among couples, high- and low-skilled couples are substantially better off by expansions. The result is notable because the payment is lump-sum fashion or even regressive, meaning that it covers a small fraction of their income for high-skilled households. Here, high-skilled household is defined as that husband and wife complete at least a college education, and low-skilled as that husband and wife complete at most high school education.

The CEV implied under exogenous fertility is also reported in the parenthesis in each cell of Table 7. There are two remarks: gains by the expansion and losses by the removal under endogenous fertility are more significant than those under exogenous fertility for each household and expansion rate, and for non-recipients, the sign of welfare effects is different between the two settings of fertility with most scenarios. For example, the removal makes non-recipients better off by 1.4% under exogenous fertility, while it makes them worse off by 1.2% under endogenous fertility. Further, tripling the per-child payment makes them worse off by 0.4% under exogenous fertility, while it makes them better off by 0.5% under endogenous fertility.

	Expansion Rate $(1+x)$			
	0	2	3	4
Household Type				
Couples	−7.8(−2.4)	+4.4(+1.7)	+6.2(+3.5)	+8.5(+2.7)
High skilled	−7.7(−2.1)	+4.2(+1.3)	+5.6(+2.9)	+7.3(+2.2)
Low skilled	−8.6(−2.8)	+5.6(+2.6)	+7.6(+5.6)	+10.1(+5.2)
Singles	−1.2(+1.4)	+1.8(+0.3)	+0.5(−0.4)	+0.7(−1.6)
Average	−5.4(−1.0)	+3.5(+1.2)	+4.1(+2.1)	+5.7(+1.1)

Table 7: Changes in CEV (%) in the long run with different per-child payments. *Note*: The parenthesis in each cell report the CEV of the agent with an expansion scenario under exogenous fertility. Here, a positive value of CEV means welfare improvement for the household, and vice versa. Also, "High skilled" household is defined as that husband and wife complete at least a college education, and "Low skilled" as that husband and wife complete at most high school education. Each type of couples has at least one child in equilibrium, even though I allow them not to have children.

Why are these the cases? To consider, note that, in equilibrium, the values of some objects in household problems change with different scales of per-child payment: tax rates (on consumption and labor income) and factor prices (interest rate and wage rate).[35] These are taken as given for households and different values of them have different welfare implications.

To evaluate the relative importance of changes in the factors, I conduct a decomposition analysis as discussed in Section 4.1, under endogenous fertility. I choose the scenario of tripling the per-child payment (i.e., an expansion rate of 3).[36] Four columns report the CEV of each household type, when replacing the labor income tax rate τ_l, consumption tax rate τ_c, factor prices, and the payment in the benchmark, with the new equilibrium counterpart.

[35]Note that this study does not consider the progress in production technology. An only thing that matters to factor prices is the ratio between supplied capital and labor, and different allocations imply different prices in relative term.

[36]The other cases show the similar result as this one does.

Table 8 summarizes the results, and some important results are presented there. First, the gains from a reduction in labor income tax τ_l are so significant that they cancel out the welfare losses from the higher consumption tax rate, while the price changes do not affect their welfare much. For couples, the gain from a reduction in τ_l is larger for high skilled ones. They benefit from the expansion by 2.9% through the labor income tax channel, while low skilled couples do by 2.6% from the same channel. This is because high-skilled workers have more taxable earnings, and a 1% decline in τ_l gives them a relatively larger amount of resources than it does low-skilled workers. That is why high-skilled households are also substantially better off by the expansion, as reported in Table 7. In addition, gains from the payment increase itself are larger for low-skilled households whose marginal utility from consumption is relatively high, which is in line with the result of Guner et al. (2020). Through this channel, low-skilled couples are better off by 10.1%, while high-skilled couples are so by 7.3%. This decomposition suggests that the impact of changes in the labor income tax is the critical source for the welfare effects.

The decomposition also suggests an answer to the question of why the welfare implications for non-recipients differ between the two fertility settings. Under exogenous fertility, non-recipients are better off in the long run by the removal because of no tax burden for the CB and a lower consumption tax rate, as indicated in Table 9 on Appendix B. Under endogenous fertility, they are worse off regardless of no tax burden for CB because the reduced tax base resulting from the removal rather increases the total tax burden in terms of consumption and labor income taxes. A similar argument applies to the expansion cases. They are worse off by expansions under exogenous fertility because they have to incur an additional tax burden, while they are better off under endogenous fertility because the larger tax base lowers the total tax burden in the long run.

	τ_l	τ_c	Prices	Payment	Overall
Household Type					
Couples	+2.8	−2.9	−0.1	+5.1	+8.5
High skill	+2.9	−2.9	−0.2	+4.3	+7.3
Low skill	+2.6	−2.9	−0.1	+7.0	+10.1
Singles	+1.0	−1.4	0.0	–	+0.7
Average	+2.1	−2.4	−0.1	+3.3	+5.7

Table 8: Welfare decomposition under the expansion rate of three. *Note*: Fours columns report the CEV of each household type, replacing labor income tax rate (τ_l), consumption tax rate (τ_c), factor prices, and the payment in benchmark with that in the new equilibrium. Note that the decomposition method adopted here does not imply that each effect sums to the overall effect.

4.3 Transition Dynamics

The main result of the steady state analysis is that expanding the per-child payment will benefit future generations, including non-recipients and high-skilled couples. Then, relevant questions arise: How long do the effects take to accrue? What are the welfare implications for cohorts living in transition periods?

To answer these questions, I solve the transition dynamics of an expansion scenario.[37] I arbitrarily choose the equilibrium with an expansion rate of three as the final steady state, which corresponds roughly to the largest payment amount among today's OECD countries.[38] I assume that the reform occurs at the beginning of 2025. For computational simplicity, I assume that the payment expansion applies to households with children born after the expansion. Thus, for example, although those with children born one period before the expansion are eligible for the CB, their per-child payment is the same as before the reform. Parents in a cohort born in 1991-95 are the first beneficiaries of the reform.

Fig 2 represents the transition path of macroeconomic variables. The horizontal axis of the top figure represents the birth cohort. "1995" stands for the 1991-95 cohort, and "1925" for the 1921-25 cohort, whose age is 100-104 when the expansion occurs. The vertical line indicates that the first cohort benefiting from the reform (i.e., the 1995 cohort). GDP, capital, and labor, all in per-capita terms, are represented by normalizing their values in 2020 as one.

First, the birth number jumps upon expansion, and correspondingly, the tax burden for financing the CB also jumps. In the medium run, the consumption tax rate exceeds the long-run level and reaches around 11.7%. This is because, although the aggregate output increases in the long run, it takes a long time to reach the new level. Then, a higher tax rate is required to balance the government budget before reaching the new steady state. As the bottom right of Fig 2 shows, the aggregate capital and output decline in the short run, gradually recovering and reaching the new levels. One reason for the short-run decline is that parents dissave to incur an additional monetary cost for additional children. The labor income tax rate and after-tax wage rate also take a long time to change because demographic change triggered by the expansion drives the tax rate decline, and the demographic structure can change only slowly. Even if the fertility rate jumps, children cannot participate immediately in economic activities, and they will be in the labor force after around 20 years have passed. All factors have reached the new levels around the year 2100, meaning that the transition will take nearly 100 years.

[37] Note that, in addition to prices, tax rates, and quantities, the age distribution changes during the transition.

[38] See, PF1.3 of the database https://www.oecd.org/els/family/database.htm.

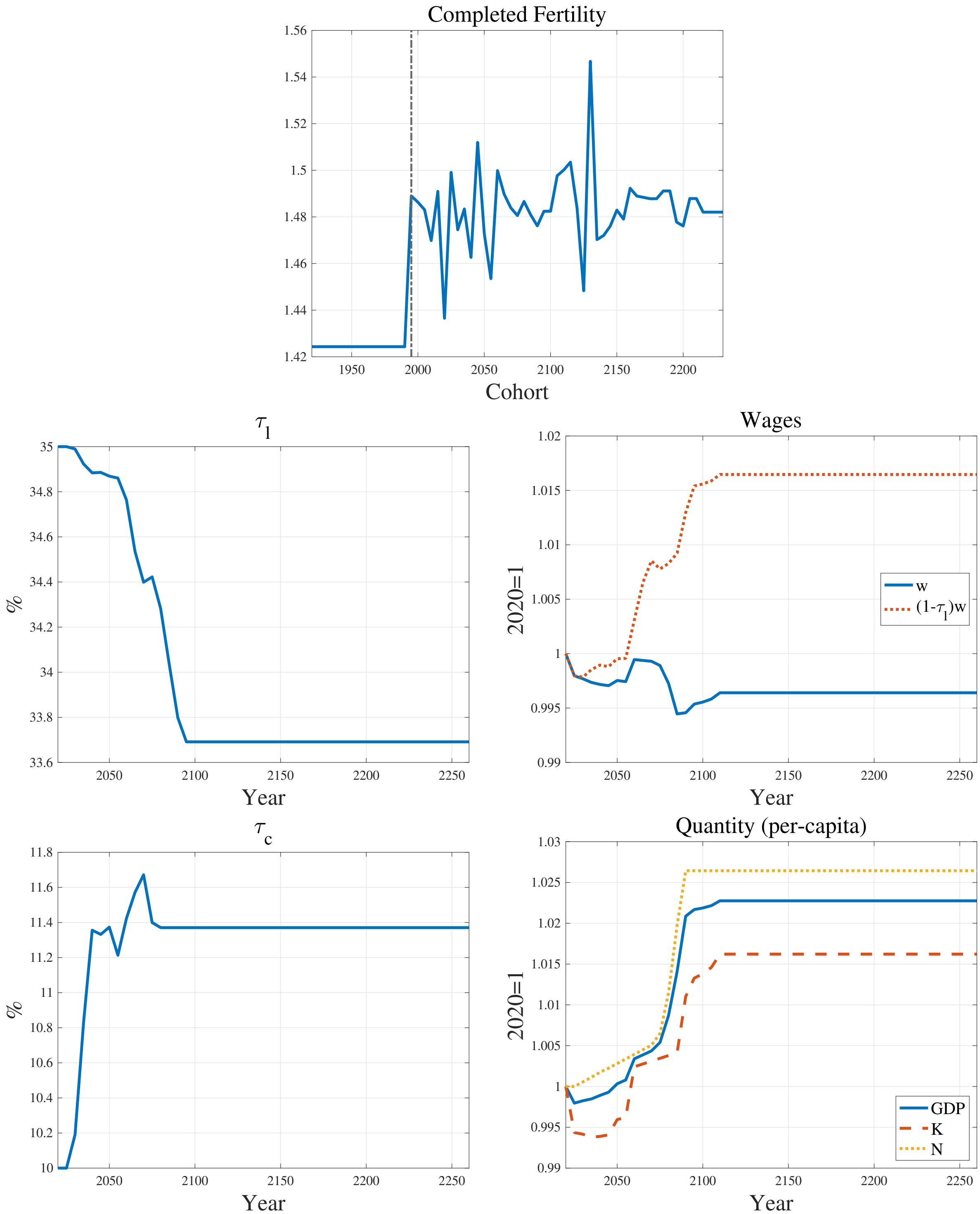

Fig 2: Aggregate variables during the transition. *Note*: The aggregate output, capital, and labor, all in per capita terms, are represented by normalizing their value in 2020 as one.

Due to the long time required for the transition and the larger consumption tax burden in the medium run, the welfare gains also take a long time to accrue. Fig 3 draws the CEV of each type and birth cohort. As the figures show, some current generations, defined as cohorts born before the expansion, and some future generations of non-recipients, those born after the expansion, are worse off. On average, current generations are worse off by $2 \sim 3\%$, but all cohorts younger than 1995 are better off (top center of Fig 3). This argument applies to couples' welfare (bottom left of Fig 3), while this is not the case for non-recipients: some future generations of non-recipients, as well as their current generations, are worse off (bottom right of Fig 3).

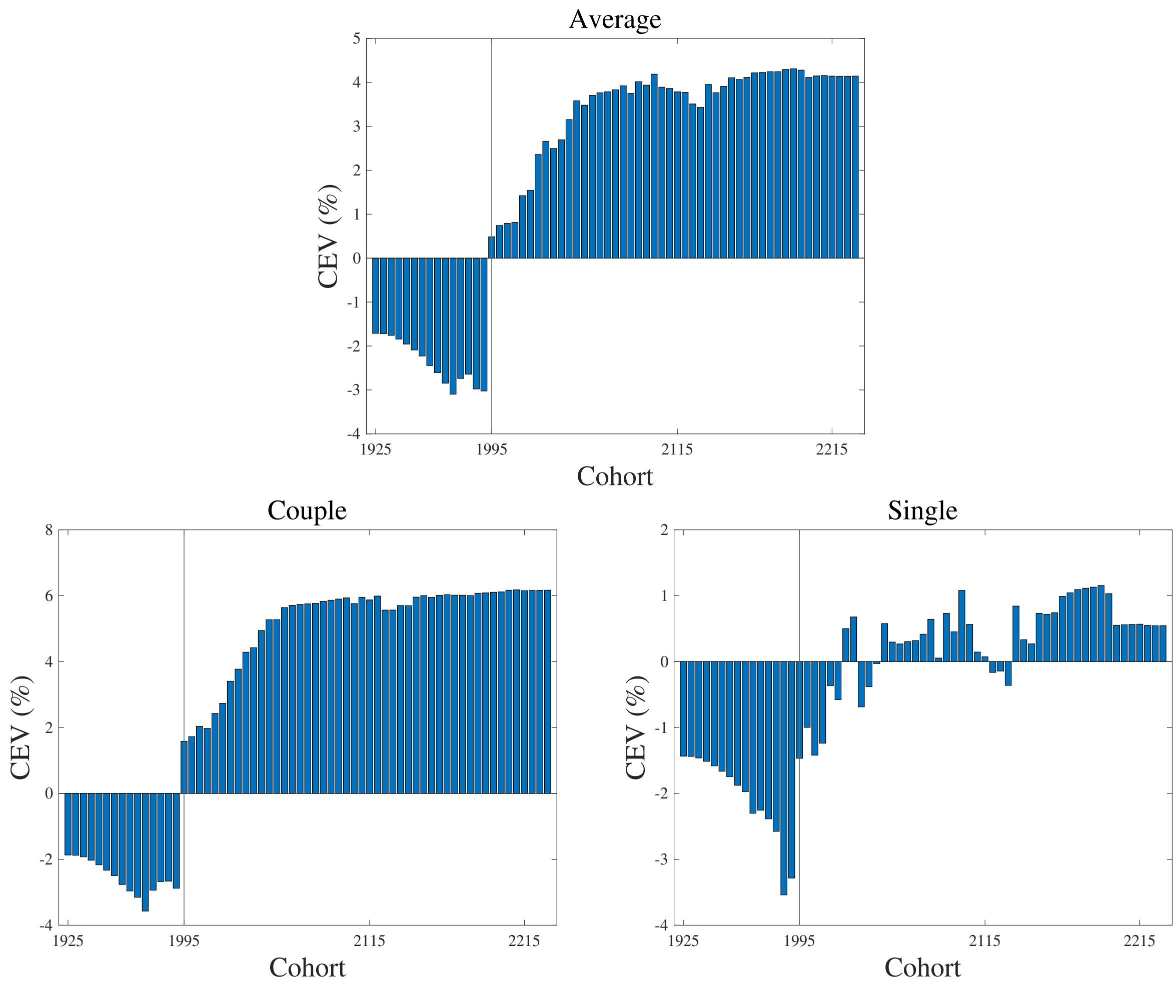

Fig 3: Welfare effects of the payment expansion along the transition. *Note*: The horizontal axis in each figure represents the birth cohort. "1995" in the figure stands for the 1991-95 cohort, and "1925" for the 1921-25 cohort, whose age is 100-104 when the expansion occurs. The vertical line indicates the first cohort benefiting from the reform (i.e., the 1995 cohort). The dotted line on each figure indicates the CEV of each type in the new equilibrium.

5 Concluding Remarks

This article examines the macroeconomic and welfare effects of CB policies, embedding fertility choices in the GE-OLG model with two-earner households. The model is calibrated to the 2000s Japanese economy and produces the benefit elasticity of fertility in line with empirical estimates. Expanding the per-child payment will lead to welfare gains for future generations in stationary equilibrium, including non-recipients who do not have children throughout their lives. Decomposition analysis suggests that the equilibrium feedback via factor prices and tax rates, driven mainly by behavioral changes in fertility and the corresponding demographic change, accounts for the result. Although the expansion tends to depress maternal labor supply, the equilibrium feedback mitigates the side effects in the long run and even results in more hours worked in the aggregate. The short-run tax rates associated with the expansion are substantially higher than their long-run level. Welfare gains take more than 100 years to accrue, making current generations and some future ones worse off.

This study abstracts the endogenous human capital accumulation of mothers and children; females' education status and age exogenously determine their labor efficiency, regardless of their experience in the labor market, and parental investment does not affect children's future earnings. On the one hand, it is unclear whether and how considering children's human capital accumulation changes the results obtained in this study. Recently, Daruich (2022) shows that large-scale early childhood policies, captured as an increase in monetary investment in children in his model, yield substantial welfare gains, using a GE-OLG model with exogenous fertility. In contrast, Kim et al. (2022) build on the quantity-quality trade-off framework incorporating the *status externality* of children's human capital and show that pro-natalist cash transfers would lead to welfare losses by reducing the aggregate (average) human capital. This is especially true when the transfers improve the fertility rates of low-income (and low human capital) parents, as in Kim et al. (2022). On the other hand, without considering female human capital accumulation, we might have overestimated the effects of the CB expansion. However, the equilibrium feedback in long-run equilibrium mitigates the side effect of expanding CB on labor supply. Thus, the policy impact on female human capital accumulation might be insignificant. Although embedding all the factors in one model is challenging, it might be a promising direction for future research.

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Appendix A Definition of Equilibrium

Definition 1 (A Stationary Equilibrium)**.** *Given an initial distribution of asset over households aged $j = 1$, $\mu_{a_1}(\cdot)$, and policy rules (a payment function for the CB, an income replacement rate of PAYG pension benefit ρ, the copayment rate of the public health insurance ω, a government expenditure G, and capital income tax rate τ_a), a stationary equilibrium consists of*

- *Value functions for couples $W^C(a, \boldsymbol{\theta}_C)$ and $V^C(a, j, b, \boldsymbol{\theta}_C)$, and the value function for single households $V^S(a, j; b, \theta_S)$,*
- *policy functions for number of children for couples $b^C(a, \boldsymbol{\theta}_C)$, policy functions for consumption ($c^C(a, j, b, \boldsymbol{\theta}_C)$ and $c^S(a, j, \boldsymbol{\theta}_S)$), for savings ($s^C(a, j, b, \boldsymbol{\theta}_C)$ and $s^S(a, j, \boldsymbol{\theta}_S)$), and for labor supply ($l^C(a, j, b, \boldsymbol{\theta}_C)$ and $l^S(a, j, \boldsymbol{\theta}_S)$),*
- *an age distribution over the population $\mu_j(\cdot)$, and unconditional wealth distribution $\mu_a(\cdot)$,*
- *quantities (aggregate capital $\hat{K}$ and aggregate labor in efficiency unit, $\hat{N}$),*
- *prices (an interest rate $\hat{r}$ and wage rate $\hat{w}$),*
- *policy rules (the consumption tax rate τ_c and labor income tax rate τ_l),*

such that

1. *Each household maximizes their lifetime utility: $W^C(a, \boldsymbol{\theta}_C)$ is defined as (2.2), where V^m solves for the functional equations (1) and (4), subject to (2), (3), (5), and (6), for $m \in \{C, S\}$. And $b^C(a, \boldsymbol{\theta}_C)$, $c^C(a, j, b, \boldsymbol{\theta}_C)$, $c^S(a, j, \boldsymbol{\theta}_S)$,$s^C(a, j, b, \boldsymbol{\theta}_C)$, $s^S(a, j, \boldsymbol{\theta}_S)$,$l^C(a, j, b, \boldsymbol{\theta}_C)$ and $l^S(a, j, \boldsymbol{\theta}_S)$ are corresponding policy functions.*
2. *$\hat{r}$ and $\hat{w}$ are determined competitively.*
3. *Markets clear:*

$$
\begin{aligned}
\hat{K} = \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}} \int_a \Bigg[\int_{\boldsymbol{\theta}_C} s^C(a,j,\boldsymbol{\theta}_C) \times \Psi^C(a,j,b,\boldsymbol{\theta}_C) d\boldsymbol{\theta}_C \\
+ \sum_{g \in \{M,F\}} \int_z s^S(a,j,\boldsymbol{\theta}_S) \times \Psi^S(a,j,\boldsymbol{\theta}_S) dz \Bigg] da, \\
\hat{N} = \sum_{j \leq J_R} \sum_{b \in \mathcal{B}} \int_a \Bigg[\int_{\boldsymbol{\theta}_C} \Big[\bar{\omega}(j, g=M, z^h) + l^C(a,j,b,\boldsymbol{\theta}_C) \times \bar{\omega}(j, g=F, z^w) \Big] \\
\times \Psi^C(a,j,b,\boldsymbol{\theta}_C) d\boldsymbol{\theta}_C \\
+ \sum_{g \in \{M,F\}} \int_z \Big[l^S(a,j,\boldsymbol{\theta}_S) \times \Psi^S(a,j,\boldsymbol{\theta}_S) \Big] dz \Bigg] da
\end{aligned}
$$

where $\Psi^C(a,j,b,\boldsymbol{\theta}_C)$ *and* $\Psi^S(a,j,\boldsymbol{\theta}_S)$ *are joint distributions over states for couples and singles.*

4. *The government budget* (8) *is balanced, given the policy rules.*
5. *The age distribution* $\mu_j(\cdot)$ *is characterized by couples' policy function for number of children* $b^C(a,\boldsymbol{\theta}_C)$ *and the wealth distribution* $\mu_a(\cdot)$ *is characterized by policy functions for savings.*

Appendix B Calibration and Numerical Results

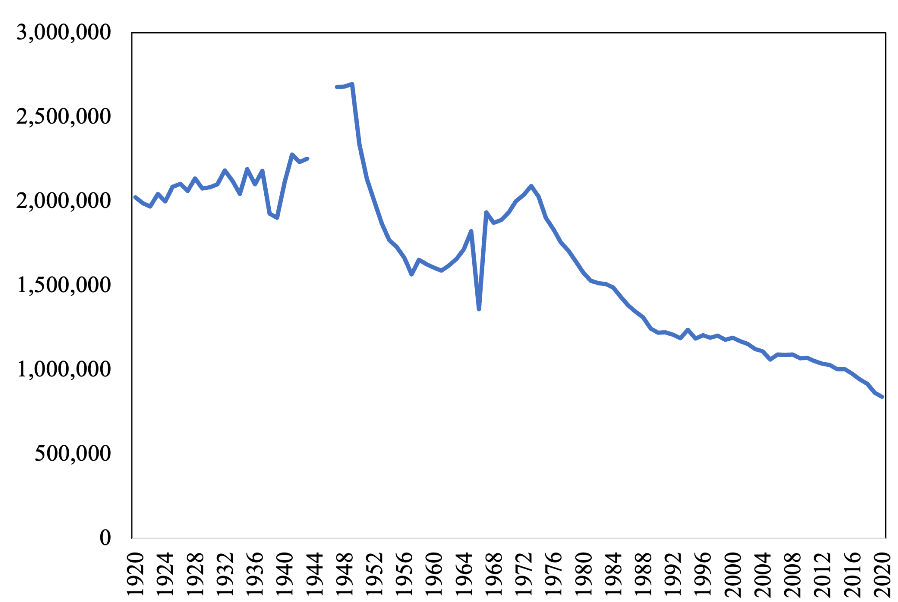

Fig 4: The transition of birth numbers in Japan (1920-2020). *Note*: The data come from the Vital Statistics (2020) of the MHLW. Values in 1944-46 are missing. The vertical axis represents the birth number, and the horizontal axis represents the year.

	Expansion Rate $(1+x)$				
	0 (Removal)	1	2	3	4
Demographic Structure	Same as benchmark				
Quantity (per-capita, Δ%)					
Output	+0.2	–	+0.3	+0.8	+0.2
Capital	−1.1	–	−0.6	+0.7	−0.2
Labor (in efficiency unit)	+1.4	–	+0.9	+0.9	+0.4
Prices (Δ%)					
Interest	4.11	3.85	3.97	3.86	3.89
Wage	−1.2	–	−0.6	0.0	−0.2
Taxes					
Consumption (10%)	9.4	10.0	10.2	11.0	11.9
Labor income (35%)	35	35	35	35	35

Table 9: Changes in aggregate variables in the long run with different per-child payments, under exogenous fertility. *Note*: Cases of $1+x=0$ and $1+x=1$ correspond to the CB removal and benchmark, respectively.