Education Subsidy as Insurance for Fertility Choices*

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Abstract

This paper studies the macroeconomic roles of college education subsidies and highlights a new critical margin to take into account: fertility choices. I develop a heterogeneous-agent general-equilibrium (GE) overlapping-generations model with fertility and college enrollment choices. Using a Japanese household panel survey, I calibrate the model to Japan, where the government subsidizes college education much less than in other countries and parents bear sizable costs. Counterfactual experiments demonstrate that an income-tested college subsidy provides partial insurance against costly states associated with having children under income uncertainty; because of the irreversible nature of fertility choices, having a (or another) child can lead to significantly lower consumption per household member—or inability to finance children's education—in the event of a negative income shock, resulting in substantial welfare losses. The resulting fertility responses due to this insurance amplify the policy's effects on inequality reduction, welfare improvement, and increasing income levels in the long run through the GE effects and intergenerational linkages. I also provide suggestive evidence for those insurance mechanisms using the survey data.

Keywords: Fertility, education, overlapping-generations, college subsidy
JEL codes: C68, I28, J13, J24

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1 Introduction

Government subsidies for college education are prevalent across countries. While they are often viewed as an effective tool for mitigating income inequality, promoting social mobility, fostering economic growth, and enhancing welfare, it is crucial to understand through which margin these policies can achieve the goals, as well as their potential side effects, which are vital to designing effective policies. In this spirit, previous studies highlight critical aspects or margins to the costs and benefits of college subsidies: these include the redistributional effects of the policy through reduced skill premiums due to a greater supply of college graduate workers (e.g., Krueger and Ludwig, 2016), crowding-out of parental transfers that can dampen the effects of policy (Abbott et al., 2019), adverse selection of students with low ability (e.g., Matsuda and Mazur, 2022), and the complementarity with policies promoting pre-college human capital accumulation that matters to success in the completion of college education(Krueger et al., 2024).

The central contribution of this paper is to find a new key margin in considering college subsidies: fertility choices. This contribution has three main blocks. The first one is to find that an income-tested college subsidy serves as partial insurance for fertility choices; because of the irreversible nature of fertility choices, having (more) children can lead to, *ex post*, significantly lower consumption per household member—or inability to finance children's education—in the event of a negative income shock, resulting in substantial welfare losses. The income-tested subsidy provides partial insurance against such costly states, which is particularly beneficial for more educated parents; they are more willing to spend on their children's education, expecting their children's high returns from college given the intergenerational persistence of abilities and tastes. The second is to show that the resulting fertility responses induced by this insurance amplify the policy's effects on inequality reduction, welfare improvement, and raising education and income levels, operating through the GE effects and intergenerational linkages. Third, these findings build on another contribution from a modeling perspective: this study is the first to construct a GE model encompassing fertility and college enrollment choices. Each component—GE framework, fertility, and college enrollment—turns out to be indispensable to highlight the notable implications of college subsidies through fertility margins and the associated GE effects.

The model builds on the heterogeneous-agent GE overlapping-generations (OLG) framework in which an individual's labor income is subject to idiosyncratic risks. In addition to the standard consumption-savings and leisure-labor choices, I incorporate three other choices into this framework. The first is the college enrollment choices: agents choose whether to attend college after graduating high school. In college, students must finance their tuition fees and consumption, which can be done via inter vivos transfers (IVTs) from their parents, their own labor earnings, and, if eligible, government-provided financial aid in the form of grants and loans. The second is the fertility choices: at a certain point in life, the agents make fertility choices regarding how many children to have. The third is the IVT choices made by parents: when their children graduate from high school, they choose how much assets to transfer to them based on altruism. Children differ in their abilities and tastes, which are realized at this IVT stage and correlated with their (parents') characteristics. The ability and taste matter to return and disutility from a college education. These two factors then determine the (altruistic) parents' gains from IVTs and their willingness to spend on their children's education.

The model is calibrated to Japan using the Japanese Panel Survey of Consumers (JPSC). Japan is a valuable laboratory in studying the macroeconomic roles of college education subsidies because (1) the government has subsidized higher education much less than in other countries, (2) thus parents bear sizable financial costs to support their children's education, and (3) children's college enrollments are associated with a higher probability of their parents deciding to stop having an additional child.[1]

The calibrated model replicates key moments, such as the (1) average amount of parental asset transfers for college students, which amounts to about 3% of average lifetime income, (2) intergenerational persistence of education levels, which is a proxy of a differential in educational investments across education levels, and (3) fertility differential across education levels. I also validate the model's fertility behavior using empirical estimates for the cash-benefit elasticity of fertility, indicating the extent to which fertility rates increase in response to cash transfers.

The benchmark model captures government-provided student loans, which are income- and ability-tested, but does not include the existing grants introduced in 2020.[2] Existing grants are income-tested, and only households in approximately the bottom 15% of the income distribution are eligible. The payments cover approximately two-thirds of students' average expenses.

The main findings are summarized as follows. First, the introduction of income-tested grants leads to a 7.4% higher fertility for college graduates in the long-run equilibrium, while it does not significantly affect high school graduates' fertility. Notably, almost a fraction of this increase is explained by fertility increase of *ineligible* households whose income is higher than the threshold and thus whose children are ineligible for the grants.

[1] Appendix C. discusses those facts in Japan. I discuss the point (2) in detail in Section 4 as well.

[2] This model does not allow students to access financial aid (grants or loans) provided by institutions besides the government (e.g., colleges or other private entities). In 2018, among students in four-year universities who used any financial aid, 83.8% used only government-provided aid, 8.6% used only non-government-provided aid, and 7.6% used both, according to the Student Life Survey (SLS, 2018).

A decomposition analysis suggests that two forces are critical to explaining the higher fertility of ineligible households. The first is a GE effect, which explains one-fourth of the increase. The grants increase the college enrollment rate by 3.9 percentage points (p.p.) in the long run and, thereby, suppress the wage rate for college graduates, reducing the opportunity costs of having children. The second—and most important—force is an insurance effect, which explains about half of the fertility increase.

In what sense do the grants provide insurance? To understand this point, note that fertility choices are irreversible and thus risky under income uncertainty; having (more) children can lead, *ex post*, to significantly lower consumption per household member—or to inability to finance children's education—in the event of a negative income shocks, resulting in substantial welfare costs. The income-tested grants provide partial insurance against such costly states, which is particularly beneficial for more educated parents as they are more willing to spend on their children's education, expecting their high returns from college given the intergenerational persistence of abilities and tastes. As a result, some college graduate parents have another child as they benefit *ex ante* from this insurance, even though they may be ineligible *ex post*.

Second, these fertility responses amplify the policy's effects on inequality reduction, welfare improvement, and raising education and output levels in the long run, highlighted by the comparison with policy's effects under *exogenous* fertility setup in which household decision rules regarding fertility are fixed to those in the benchmark.

As college graduates' fertility increases under endogenous fertility, those "marginal" children newly born are also likely to attend college due to the intergenerational transmissions of abilities, tastes, and assets (IVTs). Thus, the fertility responses lead to a higher share of college graduates in the long run through these intergenerational linkages, thereby contributing to a lower skill premium. The higher enrollment rate also leads to a greater per-capita output because of the higher average productivity of workers, and the more significant output implies greater tax bases, mitigating an increase in the tax rate required to finance the policy. These effects (i.e., a higher probability of attending college, lower inequality, and lower taxes), amplified through fertility margins and the associated GE effects, bring more significant welfare gains for newborn agents under the veil of ignorance.

Solving the transitional dynamics reveals that these GE effects are not realized in the short run. The accrual of welfare gains is thus gradual, making some existing cohorts who are alive when the policy is implemented worse off during the transition periods.

The marginal expansion of the policy by setting a higher income threshold increases college graduates' fertility further, primarily due to the insurance effect, and this fertility response contributes to a higher college enrollment rate via intergenerational linkages.

However, the positive effects on output in expansion diminish as the broader coverage significantly crowds out labor supply and savings among households with children, hindering (physical) capital accumulation.

Lastly, I conduct an empirical analysis using the JPSC to provide suggestive evidence for the insurance roles. More specifically, I conduct a regression analysis and show that (1) households facing more income uncertainty—which is captured by their contract type in the workplace following Guner, Kaya, and Sánchez-Marcos (2024)—leads to significantly lower birth probabilities; and (2) this negative effect of income uncertainty on birth probability is more significant for more educated couples.

2 Related Literature

This paper contributes to several strands of literature. The first is the macroeconomic analysis of college education subsidies.[3] I incorporate fertility choices into a framework that is otherwise standard in the literature (i.e., the heterogeneous-agent GE-OLG model with IVT and college enrollment choices). Based on this extended framework, this study contributes to the literature by finding the critical roles of fertility margins in understanding the macroeconomic roles of college education subsidies.

Second, this study is closely related to macroeconomic studies based on the quantity-quality trade-off framework.[4] The primary difference from the literature is incorporating college enrollment choices into the GE framework. Modeling college enrollment choices and considering the resulting heterogeneity in workers' skills enable us to capture a GE effect (i.e., a redistributional effect due to the change in skill premium), which is critical in evaluating the education policies (Krueger and Ludwig, 2016). My results presented in Section 5 indeed highlight the roles of GE effects driven by the change in skill distribution in explaining the long-run effects of education subsidies.

Third, this study is closely related to the literature on fertility choices in incomplete market models.[5] Previous studies demonstrate that having a child can be considered making "consumption commitments," and that income volatility thus makes households hesitate to have children.[6] This study contributes to the literature by examining policies

[3] See, for example, Benabou (2002); Akyol and Athreya (2005); Krueger and Ludwig (2016); Lawson (2017); Abbott, Gallipoli, Meghir, and Violante (2019); Lee and Seshadri (2019); Matsuda and Mazur (2022); Matsuda (2022); Krueger, Ludwig, and Popova (2024).

[4] See, for example, De La Croix and Doepke (2003); Manuelli and Seshadri (2009); Cordoba, Liu, and Ripoll (2019); Daruich and Kozlowski (2020); Zhou (2022); Kim, Tertilt, and Yum (2023).

[5] For example, Schoonbroodt and Tertilt (2014); Santos and Weiss (2016); Sommer (2016); Coskun and Dalgic (2024); Guner, Kaya, and Sánchez-Marcos (2024).

[6] This is the case, especially in the early stages of their lives, thus delaying marriage and fertility (Santos and Weiss, 2016). Delaying fertility leads to low fertility, given that the ability to reproduce declines with age (Sommer, 2016).

that can potentially reduce or insure the risks associated with having a child. This study then highlights the insurance effects of income-tested college subsidies on fertility choices, especially for college graduates whose children are likely to attend college.

Finally, this study contributes to the literature on pro-natal policies using structural models[7] by adding information about the effects of college subsidies, a novel pro-natal policy implemented in Japan. As discussed in Appendix C., the college education subsidies are also considered pro-natal policies in Japan, given facts suggesting that the financial costs for parents to support their children's college enrollment are a significant impediment to fertility decisions. Compared with typical pro-natal transfers such as baby bonuses, the notable feature of the education subsidy is that it increases the average human capital by promoting skill acquisition.[8] By examining this novel policy in Japan, this study provides insights into countries considering countermeasures against macroeconomic concerns of low fertility.

3 Model

I incorporate fertility choices into a model otherwise standard in the macroeconomic literature of college financial aid policies: the heterogeneous-agent GE-OLG model with IVT and college enrollment choices. Section 3.1 provides an overview of the lifecycle of this economy. Section 3.2 elaborates on preliminaries of the model and then Section 3.3 formulates households' decision problems. Section 3.4 discusses the stationary equilibrium of this economy.

3.1 Overview of the lifecycle

Fig 1 represents the households' lifecycle in this model. Time is discrete, and one period corresponds to two years in this model. Let j denote the agents' age. They live with their parents until they graduate from high school at the beginning of age $j = J_E (:= 18)$; until then, they make no decisions. After graduating high school, they choose whether to attend college or enter the labor market after graduating high school, represented as a node "Grad.HS" in the figure. If they do not enroll in college, they enter the labor market as a high school graduate. If they choose to attend college, it takes four years

[7] Previous studies examine the effects of childcare subsidization (e.g., Bick, 2016), cash transfers (e.g., Kim, Tertilt, and Yum, 2023; Nakakuni, 2024), both of them (e.g., Hagiwara, 2021; Zhou, 2022), parental leave policies (e.g., Erosa, Fuster, and Restuccia, 2010; Yamaguchi, 2019; Wang, 2022; Kim and Yum, 2023), and tax reform (e.g., Jakobsen, Jørgensen, and Low, 2022).

[8] Previous studies show that pro-natal transfers would lower aggregate human capital because they make parents shift from the "quality" toward "quantity" of children (e.g., Zhou, 2022; Kim et al., 2023). Actually, Appendix H. shows that the income-tested grants would lead to greater aggregate human capital and output than pro-natal transfers conducted in an expenditure-neutral way.

(two model periods) to complete, and they enter the labor market as a college graduate after graduation, represented as a node "Grad.CL" in the figure.[9]

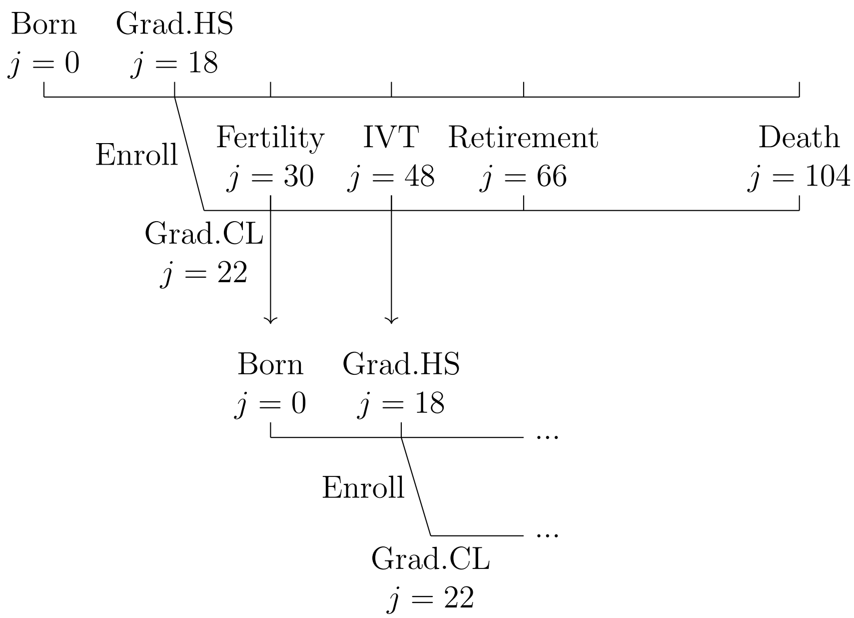

Fig 1: Model's lifecycle.

After completing their education, they enter the labor market. At the beginning of age $j = J_F (:= 30)$, they make fertility choices by choosing how many children they have. The timing of the fertility choice is common for high school and college graduates. The lifecycle of a new cohort starts at this point (provided that the fertility rate is positive), represented in the bottom half of Fig 1. After their children graduate from high school, corresponding to the beginning of the age $j = J_{IVT} (:= 48)$ for parents, they decide how much money to transfer to their children. This IVT decision affects the children's college enrollment choices at the node "Grad.HS." Households retire from the labor market at the beginning of age $j = J_R (:= 66)$. After that period, they face mortality risks; every period, a certain fraction of them is hit by exogenous mortality shocks and exits from the economy. They can live for J $(:= 104)$ years at the longest, and they exit the economy after age J.

Dividing the model's lifecycle into several stages, summarized in Table 1, helps us understand its structure; according to these stages, I formulate household problems in Section 3.3. Also, to provide a whole picture of the model's lifecycle, I provide lists of choices and state variables in Tables 2 and 3 in advance, including their relevant stages. Each variable is elaborated on in the following.

[9]This model does not consider the possibility of dropping out from college because, as I mentioned, the dropout rate is insignificant in Japan.

Stage	Corresponding age
Education stage	$18(-21)$
Working stage without children (1)	$18(22) - 29$
Fertility stage	30
Working stage with children	$30 - 47$
Inter vivos transfers stage	48
Working stage without children (2)	$48 - 65$
Retirement stage	$66-$

Table 1: The lifecycle stages and the corresponding age. *Note*: Numbers in parentheses indicate the corresponding age for those who choose to attend college.

Notation	Age/Period	Description
Throughout the entire stages		
c	$18-$	Consumption
l	$18-$	Leisure
a'	$18-$	Saving
Education stage		
e	18	College enrollment
Working stage with children & IVT stage		
n	30	The number of children (fertility)
q	$30 - 47$	Education spending
a_{IVT}	48	Inter vivos transfers

Table 2: Summary of the choice variables over the life stages.

Notation	Age/Period	Description
Throughout the entire stages		
j	18−	Age
a	18−	Asset
Until retirement		
z	$18(22) - 65$	Labor productivity $\sim$ AR(1)
e	$18(22) - 65$	Education (HS or CL)
h	$18 - 65$	Human capital
Education stage		
ϕ	18	Psychic costs of college enrollment
I	$18(-21)$	Household income
Working stage with children		
n	$30 - 48$	The number of children
Inter vivos transfers stage		
h_k	48	Children's human capital endowment
ϕ_k	48	Children's psychic costs of college education

Table 3: Summary of the stage variables over the life stages. *Note*: Numbers in parentheses indicate the corresponding age for those who choose to attend college. "HS" and "CL" stand for high school and college.

3.2 Preliminaries

Production: A representative firm chooses labor and capital inputs in competitive factor markets to produce final goods. There are two types of labor inputs in this economy; the college graduates (skilled) and high school graduates (unskilled). Their total labor supply in efficiency units are represented as L_{CL} and L_{HS}, respectively. I allow them to be imperfect substitutes by considering the aggregate labor in efficiency units, L, is given as:

$$L = [\omega_{HS} \cdot (L_{HS})^\chi + \omega_{CL} \cdot (L_{CL})^\chi]^{1/\chi},$$

where $\omega_{HS} \equiv 1 - \omega_{CL}$ and $\omega_{CL} \in [0, 1]$ governs the relative productivity of the skilled workers, and χ governs the elasticity of substitution between the skilled and unskilled workers. The representative firm operates with a Cobb-Douglas production function with aggregate capital K and labor L to produce the output Y:

$$Y = ZK^\alpha L^{1-\alpha},$$

where Z represents the factor neutral productivity. Let r, w_{HS}, and w_{CL} denote the rental rate of capital and wage rates for unskilled and skilled labor. Capital depreciates at δ, and the firm has to incur the capital depreciation costs.

Demographics: Every period, a mass of new cohorts enters the economy. The size of the new cohort is endogenously determined by aggregating fertility choices. The age distribution of this economy is thus determined by (1) households' fertility choices, which are endogenous, and (2) mortality risks after retirement, which are exogenous. Let $\zeta_{j,j+1}$ denote the probability of surviving at age $j+1$ conditional on surviving at age j for each $j \in \{J_R, ..., J\}$ with $\zeta_{J,J+1} = 0$.

Intergenerational Linkages and Initial Endowments: After graduating high school (at the beginning of age 18), agents are endowed with a triple (a_{IVT}, h, ϕ): (1) assets transferred from their parents (a_{IVT}), (2) human capital (h), which governs education returns in future earnings, and (3) psychic costs of college education (ϕ). They draw the human capital from a distribution $g^h_{h_p}$, varying with the parents' human capital level h_p. They also draw the psychic costs from a distribution g^ϕ_{h,e_p}, varying with the student's human capital h and parent's education e_p.

Preferences: Throughout their lifetime, they draw utility from consumption c and leisure l according to a utility function $u(c, l)$. They discount future utility by β. At age $j = J_F$, they choose the number of children they have, denoted by $n \in \{0, 1, ..., N\}$. They draw additional utility from having children in several ways. First, they draw utility from education spending q per child until the children graduate from high school, captured by a utility function $v(q)$. The utility $v(q)$ is discounted by a function $b(n)$, increasing and concave in the number of children n. Further, based on altruistic motives, parents draw utility by transferring assets to their children after the children graduate from high school. More specifically, parents consider children's expected lifetime utility in their education stage, with a discount rate $\lambda_a \cdot b(n)$, where λ_a represents the altruistic discount factor.

Costs of Children: Having children is costly in money and time. First, q units of the per-child education spending require $n \cdot q$ units of money, and they will make an additional expenditure $n \cdot a_{IVT}$ upon high school graduation of their children.[10] In addition to the monetary costs, having a child requires κ units of time until the child graduates from high school and becomes independent.

[10] Children within a household are homogeneous, as assumed in the literature (e.g., Abbott, Gallipoli, Meghir, and Violante, 2019). Thus, education spending and IVT do not vary among children within the household.

Labor Earnings: Households choose hours worked to earn income. The labor earnings of a household are given by $w_e \eta_{j,z,e,h}(T-l)$; it is a product of the market wage w_e, which depends on education level $e \in \{HS, CL\}$, labor efficiency or productivity $\eta_{j,z,e,h}$, and hours worked $(T-l)$. Here, T denotes the disposable time that can be devoted to work or leisure where $T = 1 - \kappa \cdot n$ if they have n children who have not graduated high school and $T = 1$ otherwise. Labor efficiency depends on age j, idiosyncratic productivity shock component z, education level e, and human capital h.

Financial Markets: Financial markets are incomplete due to the lack of state-contingent claims. Households can self-insure against risks by savings, accruing interest payments at a rate of r. Households in the working stages can borrow at rate $r^- = r + \iota$ where $\iota > 0$ (i.e., borrowers incur the overseeing costs ι) up to a borrowing limit $\underline{A}$. In contrast, retired households are not allowed to borrow, as in the literature. In addition, eligible students have access to student loans subsidized by the government, which entails the interest rate of $r^s = r + \iota_s$. This loan is income- and ability-tested, and eligible students can borrow up to a limit $\underline{A}_s$.

Government: The government raises the revenue by levying three types of taxes: consumption, labor income, and capital income taxes, where each tax rate is represented as τ_c, τ_w, and τ_a. In addition, the government collects accidental bequests and devotes them to cover expenditures. They use this revenue to fund (1) the public pension benefits, which gives p units of money to retired households each period, (2) subsidized loans for college students, (3) grants for eligible college students, where the payment per eligible student is represented by $g(I)$ where I represents household income when the student faces education choice problem (i.e., their parent's age is J_{IVT}), (4) lamp-sum transfers ψ that is introduced for replicating the progressivity of labor income tax schedule in a simple way following the literature, (5) cash benefits for households with children under 17 with per-child payment of B, and (6) other expenditures S. I do not consider grants in the benchmark to replicate the economy before grants are introduced in 2020 and set $g(I) = 0$ for any I. The government budget constraint is given as follows:

$$\begin{aligned} \tau_c \cdot C + \tau_w \cdot (w_{HS} L_{HS} + w_{CL} L_{CL}) + \tau_a \cdot r \int_{\boldsymbol{x}} \mathbb{I}_{a(\boldsymbol{x})>0} a(\boldsymbol{x}) dF(\boldsymbol{x}) + Q \\ = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \leq 17} + S, \quad (1) \end{aligned}$$

where C, Q, μ_{old}, $\mu_{j \leq 17}$, K_s, and G represent the total consumption, total accidental bequests, population mass of retired households, that of children under age 17, the total amount of borrowing by college students, and the total grant payments. The term $\tau_a \cdot$

$r \int_{\boldsymbol{x}} \mathbb{I}_{a(\boldsymbol{x})>0} a(\boldsymbol{x}) dF(\boldsymbol{x})$ represents revenue from capital income taxation, where $\boldsymbol{x}$, $a(\boldsymbol{x})$, $\mathbb{I}_{a(\boldsymbol{x})>0}$, and $F(\boldsymbol{x})$ represent the household state space, household's policy function for savings, an indicator function returning one if $a(\boldsymbol{x})$ is positive, and distribution over the state space $\boldsymbol{x}$.

3.3 Household Problems

This section formulates household problems in each stage defined in Table 1. The first is the education stage in which they choose whether to attend college or enter the labor market after graduating high school.

Education Stage: After graduating high school, they draw the human capital h from the distribution $g^h_{h_p}$ and psychic cost ϕ from the distribution g^ϕ_{h,e_p}. They also receive IVT from their parents $a_{IVT} \geq 0$. Some of them can access subsidized loans to fund expenditures that arise during the college education stage, which is income- and ability-tested. Thus, the state variables for the students are comprised of asset a_{IVT}, the human capital h, psychic costs ϕ, and their parent's income I. They compare the expected value for entering the labor market as a high school graduate, $\mathbb{E}V^w$, with the value for enrolling in college, V_{g1} net of the psychic cost ϕ. They choose college enrollment if the latter is greater than the former; otherwise, they enter the labor market. The decision problem is formulated as follows:

$$V_{g0}(a_{IVT}, \phi, h, I) = \max_{e\in\{0,1\}} \Big\{ (1-e) \cdot \mathbb{E}_{z_0}[V^w(a_{IVT}, j=18, z_0; e=0, h)] + e \cdot [V_{g1}(a_{IVT}; h, I) - \phi] \Big\}, \tag{2}$$

where $e \in \{0,1\}$ indicates the education choice where $e=1$ means college enrollment and $e=0$ does entering the labor market as a high school graduate. V^w denotes the value function for workers, which I formulate in the next subsection. The initial draw of z, z_0, is uncertain and is according to the invariant distribution of z, $\bar{\pi}_z$, so the expectation operator is put next to the V^w. The value for college enrollment, V_{g1}, is defined as follows:

$$\begin{aligned} V_{g1}(a_{IVT}; h, I) &= \max_{c,l,a'} \{u(c,l) + \beta V_{g2}(a'; h, I)\}, \\ V_{g2}(a; h, I) &= \max_{c,l,a'} \{u(c,l) + \beta \mathbb{E}_{z_0}[V^w(a^s(a'), j=22, z_0; e=1, h)]\}. \end{aligned} \tag{3}$$

The budget constraints differ according to eligibility to the student loans. The budget constraints for eligible students are give as follows:

$$
\begin{aligned}
(1+\tau_c)c + p_{CL} + a' = (1-\tau_w)w_{HS}(1-\bar{t}-l) + \psi + g(I) \\
+ \begin{cases} (1+(1-\tau_a)r)a & \text{if } \; a \geq 0 \\ (1+r^s)a & \text{otherwise} \end{cases} \\
a' \geq -\underline{A}_s.
\end{aligned} \tag{4}
$$

The rest of the students cannot access to the student loans, which implies that their budget constraints are given as follows:

$$
\begin{aligned}
(1+\tau_c)c + p_{CL} + a' &= (1-\tau_w)w_{HS}(1-\bar{t}-l) + \psi + g(I) + (1+(1-\tau_a)r)a, \\
a' &\geq 0.
\end{aligned}
$$

College students draw utility from consumption and leisure and must pay tuition fees p_{CL}. They can fund the consumption and tuition fees through (1) transfers from their parents a_{IVT}, (2) borrowing through student loans if eligible, (3) government-provided grants if eligible, and (4) labor earnings by themselves. Students must spend a $\bar{t}$ fraction of their time on study while in college. Thus, they choose the time allocation between leisure and working over the disposable time $1-\bar{t}$, where the total disposable time is normalized as 1. One unit of labor supply gives college students w_{HS} units of wages.[11] Following the literature, I assume that fixed payments are made for 20 years (10 periods) following college graduation and transform college loans into regular bonds according to the following formula:

$$
a^s(a') = a' \times \frac{r^s}{1-(1+r^s)^{-10}} \times \frac{1-(1+r^-)^{-10}}{r^-}.
$$

Working Stage without Children: The remaining component in (2) and (3), V^w, represents the value function for working households without children (i.e., for those aged $j \in \{J_E, ..., J_F - 1, J_{IVT}, ..., J_R - 1\}$). The state variables for this stage consists of asset a, age j, idiosyncratic component of labor productivity z, education level e, and human capital h. The uncertainty in this stage is only about the next period's productivity, which is denoted by z' following a Markov process $\pi_z(z', z)$. Households choose consumption,

[11] I assume that, while in college, there is no heterogeneity in labor efficiency and no uncertainty regarding the next period's productivity, and one unit of hours worked brings one unit of labor efficiency.

leisure, and savings given the state variables. The value function is formulated as follows:

$$
\begin{aligned}
V^w(a,j,z;e,h) = \max_{c,l,a'} \{ & u(c,l) \\
& + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^f(a',z',e,h)] & \text{if } \ j = J_F - 1 \\ \beta [V^r(a', j+1)] & \text{if } \ j = J_R - 1 \\ \beta \mathbb{E}_{z'}[V^w(a', j+1, z'; e, h)] & \text{otherwise} \end{array} \right\} \\
\text{s.t.} & \\
& (1+\tau_c)c + a' = (1-\tau_w) w_e \eta_{j,z,e,h}(1-l) + \psi + (1+(1-\tau_a)r)a, \\
& z' \sim \pi(z', z), \\
& a' \geq \left\{ \begin{array}{ll} 0 & \text{if } \ j = J_R - 1, \\ -\underline{A} & \text{otherwise.} \end{array} \right.
\end{aligned}
\tag{5}
$$

Value functions V^f and V^r represent those for the fertility stage and retirement stage, which are formulated in the following subsections.

Fertility Stage and Working Stage with Children: When they reach the age of $j = J_F$, they choose how many children they have. The problem is formulated as follows.

$$
V^f(a,z,e,h) = \max_{n \in \{0,1,..,N\}} \left\{ V^{wf}(a, j = J_F, z; e, h, n) \right\},
$$

where $V^{wf}(a,j,z;e,h,n)$ in the parenthesis represents the value function for the working stage with children (i.e., for households aged $j \in \{J_F, ..., J_{IVT} - 1\}$). As in the previous life stages, they draw utility from consumption and leisure and decide on consumption, time allocation, and savings. In addition, they draw utility from the number of children n and education spending q for each child during the stage.[12] The value function V^{wf}, for $j \in \{J_F, ..., J_{IVT} - 1\}$, is then formulated as follows:

$$
\begin{aligned}
V^{wf}(a,j,z;e,h,n) = \max_{c,l,q,a'} \{ & u(c/\Lambda(n), l) + b(n) \cdot v(q) \\
& + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^{wf}(a', j+1, z'; e, h, n)] & \text{if } j \in \{J_F, ..., J_{IVT} - 2\} \\ \beta \mathbb{E}_{z', \phi_k, h_k}[V^{IVT}(a', z'; \phi_k, h_k, e, h, n)] & \text{if } j = J_{IVT} - 1 \end{array} \right\} \\
\text{s.t.} & \\
& (1+\tau_c)(c + nq) + a' = Y_{wf}, \\
& a' \geq -\underline{A}, z' \sim \pi(z', z), h_k \sim g_h^{h_k}, \phi_k \sim g_{e,h_k}^{\phi_k},
\end{aligned}
$$

[12]This study does not consider children's endogenous human capital accumulation through parental investments and assumes parents make educational spending on children just because it draws utility. Incorporating the endogenous human capital accumulation is left for future research.

where

$$
\begin{aligned}
Y_{wf} \equiv & (1-\tau_w) w_e \eta_{j,z,e,h}(1-l-\kappa \cdot n) \\
& + n \cdot B + \psi + \begin{cases} (1+(1-\tau_a)r)a & \text{if} \quad a \geq 0, \\ (1+r^-)a & \text{otherwise.} \end{cases}
\end{aligned}
$$

Here, ϕ_k and h_k denote the psychic costs and human capital for their children. The household consumption is deflated by the equivalence scale $\Lambda(n)$, depending on the number of children n. The total education spending on children, nq, enters the budget constraint, and utility from children, $b(n) \cdot v(q)$, enters the objective function in the periods with children. V^{IVT} represents the value function for the IVT stage at $j = J_{IVT}$, which is described in the following subsection. Households are uncertain about their children's human capital h_k and psychic costs ϕ_k until the beginning of age $j = J_{IVT}$, which can be interpreted as uncertainty about the expenditures on children in the form of IVT, as will be evident in the following subsection.

Inter Vivos Transfers Stage: At the beginning of age $j = J_{IVT}$, households decide how much to transfer to their children. This timing corresponds to when the children graduate from high school and face the college enrollment choice. Two characteristics of their children are realized at this point: their psychic costs ϕ_k and human capital h_k. After observing the realized characteristics, households choose the optimal amount of per-child transfer a_{IVT}, formulated as follows:

$$
\begin{aligned}
V^{IVT}(a, z; \phi_k, h_k, e, h, n) = \max_{a_{IVT} \geq 0} \Big\{ & V^w(a - \tilde{a}_{IVT}, j = J_{IVT}, z; e, h) \\
& + b(n) \cdot \lambda_a \cdot V_{g0}(a_{IVT}, \phi_k, h_k, I) \Big\},
\end{aligned}
$$

where $\tilde{a}_{IVT} = \frac{n \cdot a_{IVT}}{1+(1-\tau_a)r}$ and the parent's state vector pins down household income I. They care about the lifetime utility of each child, $V_{g0}(a_{IVT}, \phi_k, h_k, I)$, based on altruism, discounted by an altruistic discount factor λ_a.

Note that the children's characteristics (h_k, ϕ_k) govern education returns and the willingness to attend college, which in turn governs the marginal gains from IVT for parents,[13]

$$
b(n) \cdot \lambda_a \cdot \frac{\partial V_{g0}(a_{IVT}, \phi_k, h_k, I)}{\partial a_{IVT}}.
$$

Thus, uncertainty about their children's characteristics, which households face one period

[13]For better clarity, this representation implicitly assumes that the value function V_{g0} is differentiable. However, in principle, this is not the case because of the discrete nature of the college enrollment choice.

ahead of the IVT stage, is interpreted as uncertainty about the expenditures on children in the form of IVT.

Retirement Stage: At the beginning of age J_R, households are forced to retire from the labor market. After that, they spend all their time on leisure and make consumption-saving decisions. Two points differ from previous choice problems; they receive the pension benefit p from the government and face uncertainty about the next period's survival. The value function for the retirement stage is formulated as follows:

$$
\begin{aligned}
V^r(a,j) = &\max_{c,a'} u(c,1) + \beta\xi_{j,j+1}V^r(a',j+1) \\
&\text{s.t.} \\
&\quad (1+\tau_c)c + a' = p + (1+(1-\tau_a)r)a + \psi, \\
&\quad a' \geq 0.
\end{aligned}
$$

3.4 Stationary Equilibrium

I solve the stationary equilibrium of this economy. In equilibrium, households make every choice to maximize their expected utility, the firm maximizes its profit, and the government budget is balanced. Stationarity implies that the distribution over state variables is invariant. Importantly, the age distribution is determined endogenously according to households' fertility choices. See Appendix D. for the detailed definition of equilibrium and Appendix E. for the computational algorithm for solving the equilibrium.

4 Calibration

To calibrate the model, I use the Japanese Panel Survey of Consumers (JPSC), a panel survey of Japanese women and their household members. It started in 1993 with a representative sample of 1,500 women aged $24-34$ and contains information about, for example, their income, educational background, marital status, fertility, and expenditures to detailed categories, including those on children's education. I focus on the cohort born in 1959-69, the oldest cohort of this survey, especially to compute the completed fertility and intergenerational mobility of education. I keep only married households as in previous works (e.g., Daruich and Kozlowski, 2020) because the model focuses on choices made within married households such as fertility and educational investments.[14]

[14]Hence, an agent or household in this model refers to households with two individuals. Then, "children" in this model can also be interpreted as a household unit. That is, having n children can be interpreted as reproducing $n/2$ units of households. For example, if a household has two children, it means reproducing one household unit with two individuals.

Unless otherwise mentioned, targeted moments for the parameters internally determined, described below, are computed based on the JPSC's 1959-69 cohort data.

4.1 Targeted Moments

Preferences: Instantaneous utility for households are given as follows:

$$u(c,l) = \frac{(c^{\mu} l^{1-\mu})^{1-\gamma}}{1-\gamma}.$$

μ is internally determined as 0.23 so that the households spend one-third of the total disposable time on market work. Instantaneous utility from education spending on a child is given as:

$$v(q) = \lambda_q \frac{q^{1-\gamma}}{1-\gamma},$$

where $\lambda_q = 0.62$ so that the annual educational expenditure per child amounts to 7% of average income at age 28. The utility must be always positive (or always negative) in models of altruism with endogenous fertility, and I set $\gamma = 0.5$ following the literature (e.g., Daruich and Kozlowski, 2020). The altruistic discount factor λ_a is set to 1.03 so that the annual average IVT for college students amounts to 27% of average income at age 28.[15] Following Kim, Tertilt, and Yum (2023), I let the discount function of the number of children $b(n)$ be non-parametric, and assume that $b(n) = b_n$ for each $n \in \{0, 1, 2, 3, 4\}$ with $b(0) = b_0 = 0$. Those parameters are determined so that the model replicates the distribution of the completed fertility. The time for studying $\bar{t}$ is set to 0.8 so that the income share of labor earnings for college students in the model is close to 21% (SLS, 2018). I assume that $\beta = 0.98$ as in Zhou (2022).

Financial Markets: I set the borrowing limits $\underline{A} = 20$ million yen and $\underline{A}_s = 2.88$ million yen.[16] The borrowing wedges are set $\iota = 0.055$ and $\iota_s = 0.054$ so that the model approximates the share of working households with a negative net worth (54%) and the share of students borrowing from the government-provided student loans (44%).

School Taste: I assume that the psychic costs ϕ are given as $\phi = \psi_{CL} \cdot \exp(-\nu \cdot h) \cdot \tilde{\phi}$. First, ψ_{CL} governs the scale of psychic costs and thus college enrollment rate. The second term $\exp(-\nu \cdot h)$ allows high ability students to have smaller psychic costs, as standard in

[15] Although some parents whose children do not enroll in college make IVT in the model, the amount is negligibly small. One reason is that the marginal gains from IVT are more significant if their children attend college, as students are financially constrained, primarily because of their limited earning ability.

[16] The former is based on the Family Income and Expenditure Survey by the Ministry of Interna Affairs and Communications and the latter is based on the SLS (2018).

the literature, and ν governs the education sorting by ability. Finally, $\tilde{\phi}$ is stochastic, and the distribution depends on their parent's education. Following Daruich and Kozlowski (2020), I assume that $\tilde{\phi}$ is distributed on an interval $[0,1]$ and follows the following CDF:

$$G_{e_p}^{\tilde{\phi}} = \begin{cases} \tilde{\phi}^{\omega} & \text{if } e^p = 0 \\ 1-(1-\tilde{\phi})^{\omega} & \text{if } e^p = 1 \end{cases}$$

Here, ω governs the intergenerational transmission of school tastes. ψ_{CL} is set to 20.8 so that the model approximates the college enrollment rate of 37.7%. ν is set to 1 in the benchmark, and ω is set to 1.71 so that the intergenerational transition matrix of education in the model matches the data counterpart as closely as possible. Table 4 reports the transition matrix in the model and data. $(i,j)-$th entry of the matrix indicates the probability that children acquire skill (education) $j \in \{HS, CL\}$ given that their parent's skill is $i \in \{HS, CL\}$ in the benchmark model, and values in parentheses represent the data counterparts. The table indicates that the education level is persistent across generations: children of high school graduates attend college with a probability less than 0.3, whereas children of college graduates do with a probability approximately 0.6.

Parents/Children	HS	CL
HS	0.725 (0.798)	0.275 (0.202)
CL	0.412 (0.423)	0.588 (0.577)

Table 4: Intergenerational transition matrix of education. *Note*: $(i,j)-$th entry of the matrix indicates the probability that children acquire skill (education) $j \in \{HS, CL\}$ given that their parent's skill is $i \in \{HS, CL\}$ in the benchmark model, and values in parentheses represent the data counterparts.

Intergenerational Transmission of Human Capital: The intergenerational transmission of human capital is according to the following formula:

$$\log(h) = \rho_h \cdot \log(h_p) + \varepsilon_h,$$
$$\varepsilon_h \sim N(0, \sigma_h).$$

I set ρ_h to 0.30 to approximate the intergenerational income elasticity (0.3) and also set σ_h to 0.65 so that the variance of log income at age 28 in this model is close to 0.27.

Income Process and Education Return: The efficiency labor of an agent aged j, education e, human capital h, and productivity z, $\eta_{j,z,e,h}$, is given as follows:

$$\begin{aligned}\log \eta_{j,z,e,h} &= \log[f^e(h)] + \gamma_{j,e} + z,\\ f^e(h) &= h + e \cdot (\alpha_{CL} h^{\beta_{CL}}),\\ z' &= \rho_e^z z + \zeta, \quad \zeta \sim N(0, \sigma_e^z).\end{aligned}$$

To set $\gamma_{j,e}$, I estimate the second-order polynomial of hourly wages on age using JPSC. As reported in Table 5, the income gradient on age is larger for college graduates than the rest of the workers, but the degree is modest compared with the US case (e.g., Abbott, Gallipoli, Meghir, and Violante, 2019).

Following Daruich and Kozlowski (2020), I construct a series of two-year total household income (i.e., the sum of husband's and wife's labor earnings) for each couple and estimate the process for stochastic components z for each education group. The estimation results are summarized in Table 5; the persistence parameters for high school and college graduates are 0.98 and 0.97, and parameters governing income volatility for high school and college graduates are 0.02 and 0.03, which are similar to the process estimated in Daruich and Kozlowski (2020) using the US data in their levels. The function $f^e(h)$ indicates that the education return depends on human capital h: people with higher human capital can obtain greater returns through college education. Following Daruich and Kozlowski (2020), α_{CL} and β_{CL} are determined so that the model replicates the ratio of log wage between college graduates and the rest of the population at age $j = 28$ and the log wage variance for college graduates at age $j = 28$.

	HS	CL
Age	+0.041	+0.048
Age^2 $\times$ 10,000	−4.551	−5.364
ρ_e^z (persistence)	0.98	0.97
σ_e^z (volatility)	0.02	0.03

Table 5: Parameters governing the age profile and stochastic process for wages. *Note*: CL indicates the college graduate households where the husband or wife is a college graduate. HS represents the rest of the population.

Production: I set the capital share $\alpha = 0.33$ and $\delta = 0.07$ following Kitao (2015). χ is set to 0.39 following Matsuda and Mazur (2022). ω_{CL} is internally determined to 0.52 so that the average wage ratio between college graduates and the rest in the model amounts to 1.36. Z is determined so that the wage rate for high school graduates is normalized to one in the benchmark.

Government: Tax rates are set to $\tau_c = 0.1$, $\tau_w = 0.35$, and $\tau_a = 0.35$ in the benchmark. The lump-sum transfer is set to $\psi = 0.01$ to match the ratio between the variance of log net income and that of log gross income (0.6). The pension benefit p is set so that the government provides ¥160, 000 per household per month. The cash transfer B, which we refer to "child benefit" or "typical pro-natal transfers" hereafter, is given as ¥10, 000 per child per month, approximating the actual payment. The other expenditure S is set so that the government budget constraint is balanced in the benchmark and fixed throughout the counterfactual experiments.

Miscellaneous: The survival probability $\zeta_{j,j+1}$ is set based on the Vital Statistics (2019).[17] Annual college tuition fees p_{CL} are set to 1.05 million yen. κ is set to 0.044 so that they spend 13.3% of their working hours on childcare.[18] Table 6 and Table 7 summarise the parameters externally and internally determined.

Parameter	Value	Description
$\underline{A}_s$	2.88 million yen	Borrowing limit for students
$\underline{A}$	20 million yen	Borrowing limit
p_{CL}	1.05 million yen/year	Tuition fees
κ	0.044	Time costs
$\xi_{j,j+1}$	–	survival prob.
τ_c	0.10	Consumption tax
τ_a	0.35	Capital income tax
τ_w	0.35	Labor income tax
p	¥160, 000/month	Pension benefits
b	¥10, 000/month	Cash transfers
α	0.33	Capital share
δ	0.07	Depreciation rate
χ	0.39	Elasticity of substitution
ρ_e^z	See Table 5	Persistence
σ_e^z	See Table 5	Transitory
$\{\gamma_{j,e}\}_{j,e}$	See Table 5	Wage growth in age
ν	1.0	Education sorting by ability
γ	0.5	Curvature
β	0.98	Discount factor

Table 6: Parameters externally determined.

[17] See, https://www.mhlw.go.jp/english/database/db-hw/outline/index.html.
[18] See, Kitao and Nakakuni (2023).

Parameter	Value	Moment	Data	Model
μ	0.23	Work hours	0.33	0.30
$\bar{t}$	0.8	Income share of labor earnings	0.21	0.20
ι_s	0.054	Share of students using loans	0.44	0.34
ι	0.055	Household share with negative net worth	0.54	0.45
ω_{CL}	0.52	CL−HS wage ratio	1.36	1.48
ψ	0.01	Var(log disposable income)/Var(log gross income)	0.60	0.68
λ_q	0.62	Average transfer / Average income at age 28	0.07	0.07
λ_a	1.03	Average transfer / Average income at age 28	0.27	0.27
ω	1.71	Intergenerational mobility of education	See Table 4	
ρ_h	0.30	Intergenerational income elasticity	0.30	0.27
σ_h	0.65	Variance of log(income) at age 28	0.27	0.24
ψ_{CL}	20.8	College enrollment rate	0.377	0.376
α_{CL}	0.1	Log wage ratio (CL−HS) at age 28	0.34	0.38
β_{CL}	0.1	Var log wage for CL at age 28	0.14	0.24
b_1	0.49	Share of one child	0.16	0.15
b_2	0.53	Share of two children	0.55	0.61
b_3	0.55	Share of three children	0.22	0.24
b_4	0.56	Share of four or more children	0.02	0.00
Z	1.99	Low skill wage	1.0	1.0

Table 7: Parameters internally determined.

4.2 Non-targeted Moments and Validation

This subsection checks the validity of the calibrated model. First, I check if the model generates the fertility differential across education levels observed in the data. Second, I check if the benchmark model generates a reasonable value of the benefit elasticity of fertility, which is non-targeted in calibration. Lastly, I check if the benchmark model generates a realistic composition of revenues for college students over the IVTs, labor earnings, and student loans.

4.2.1 The fertility differential across education levels

More educated parents have fewer children than less educated ones. According to my sample of the JPSC, college graduates' completed fertility was 1.92, which is lower than the rest's, 2.12. This is observed in the National Fertility Survey (NFS) provided by the National Institute of Population and Social Security Research (IPSS), another data source where we can check the completed fertility rate by different education levels.[19] According to the NFS (2015), college graduate wives' completed fertility has been lower than less educated ones almost every survey year since 1977. The latest survey in 2015 reports that the completed fertility of wives with a college degree was 1.89, and that of

[19] See, `https://www.ipss.go.jp/site-ad/index_english/survey-e.asp`.

high school graduate wives was 1.98. In this benchmark model, the completed fertility of college graduate wives is 1.79, which is lower than the high school graduate wives', 2.28. The benchmark model captures the fertility differential across education levels with a similar degree, as summarized in Table 8.

	Model	JPSC	NFS
HS	2.28	2.12	1.98
CL	1.79	1.92	1.89

Table 8: Fertility differential across education in the benchmark model and data. *Note*: "NFS" stands for the National Fertility Survey conducted by the ISPP, and the table reports the values from the 2015 survey. It reports the completed fertility of wives with different educational backgrounds.

Why does this fertility differential result in this model? First, the opportunity costs of having children are more significant for college graduates, making child-bearing more costly for them. This is because having a child requires parents to incur a fixed amount of time, and college graduates' wage rates are higher than those of high school graduates. Second, college graduate parents are more willing to pay for the IVT, making a child more costly. This is because the children's human capital and psychic costs of education, (h_k, ϕ_k), govern education returns and the willingness to attend college, which in turn governs the marginal gains from IVT for parents. In particular, the psychic costs of children correlate with those of parents, so college graduate parents derive higher utility from the IVTs than high school graduate parents.

Appendix A. presents a simple static model that highlights these mechanisms in a close form. These two mechanisms align with the major explanations proposed by the theoretical literature on the fertility differentials across income distribution:[20] (1) having children is more costly for higher-income households, provided that having children is time-intensive, and (2) higher-income households have higher demands for child quality or an advantage in parental investments.

4.2.2 The benefit elasticity of fertility

Previous works show that cash benefits such as the child benefit and baby bonus have a significant impact on fertility. Many of them report that the benefit elasticity of fertility, the percentage increase in fertility rate against the one percent increase in the cash transfer, is about $0.1 - 0.2$. For example, Milligan (2005) studies a reform of Quebec's baby bonus and shows that an extra 1,000 Canadian dollars benefit would increase fertility by 16.9%, which implies a benefit elasticity of 0.107. Cohen, Dehejia, and Romanov (2013) uses a variation in the child subsidy payment observed around 2003 in Israel, providing a

[20] See Jones et al. (2010) for review on the literature.

larger subsidy for third or higher births. They show that the benefit elasticity of fertility was 0.176. González (2013) adopts the regression discontinuity design to study the effects of Spain's reform in 2007, introducing a one-time payment of 2,500 euros (about 3,800 USD) for births, almost 4.5 times the monthly minimum wage for full-time workers. It finds a statistically significant impact on fertility, increasing conceptions by $5-6\%$.

To examine how this model performs in this respect, I conduct the following exercise. Let B_0 denote the per-child cash transfers for households with children under 17 in the benchmark. I solve the household problem, holding prices, tax rate, and distribution fixed, with several levels of the per-child payment $B = B_0 \cdot (1+x)$ for some $x \in X$, where X is a set of positive real numbers. This procedure brings the implied fertility rate, and with the expansion rate x, we can compute the implied benefit elasticity of fertility for the case of the expansion rate x. I set $X = \{0.1, 0.2, ..., 1.9, 2.0\}$, which is a reasonable range in the context of the expansion examined in the empirical studies, and compute the implied elasticity for each x. Then, I take the average of those 20 values. I find that the average elasticity is 0.138 (with a standard deviation of 0.025), which is consistent with the empirical estimates.

4.2.3 Composition of students' revenue

Capturing the composition of students' revenue –how college students finance their living expenses and tuition fees– is also important as it is critical not only to students' education choices but also to parents' IVT choices and, thereby, fertility choices. According to the SLS (2018), the students' revenue consists of three parts. First, the greatest part, 61% of their revenue, is accounted for by asset transfers from their parents. Second, students' labor earnings account for 21%, and lastly, the rest (18%) is financed by student loans. Although the revenue share of labor earnings (21%) is a targeted moment, the rest is not targeted. As Table 9 shows, the model captures the overall revenue composition as well. The IVT and loans account for 66% and 14% of their revenue, close to the data counterparts.

	IVT	Loan	Labor
Data	0.61	0.18	0.21
Model	0.66	0.14	0.20

Table 9: Composition of Students' Revenue.

5 Numerical Analysis

This section investigates the effects of grants for college students using the calibrated model. Section 5.1 examines the long-run implications of introducing the existing income-tested grants started in Japan in 2020. Section 5.2 then examines the mechanism through which the macroeconomic effects of the introduction are realized. Section 5.3 investigates the long-run effects of raising the income threshold so that students in broader income classes of households are eligible. Following the literature, I adjust the labor income tax to balance the government budget upon the introduction and expansion of these (steady-state) analyses. Lastly, Section 5.4 studies the transition dynamics upon an introduction of the income-tested grants.

Welfare Measure: In addition to macroeconomic variables such as fertility rates, college enrollment rates, and output, I also examine the welfare effects of the policy in the following exercises. However, the welfare analysis using models with endogenous fertility is not straightforward theoretically and philosophically. One of the well-known difficulties in the economics or theoretical context is that the standard concept of Pareto efficiency is not applicable to models with endogenous fertility (Golosov, Jones, and Tertilt, 2007).[21] Given that the literature on the normative analysis of endogenous fertility models is still developing and there is no one "correct" method to tackle this issue (at least for now), this study captures the welfare effects of the policy by the consumption equivalence under the veil of ignorance under the benchmark economy relative to the new steady state.[22]

To formalize the measure, let $P \in \{0, 1, 2, ...\}$ denote education subsidy schemes or policies with $P = 0$ representing the benchmark economy without grants. Next, let $V^P(\lambda)$ be the expected lifetime utility in stationary equilibrium for newborn agents with a consumption scaling parameter λ and policy P:

$$V^P(\lambda) = \int_{\boldsymbol{x}_1} V^P_{j=1}(\boldsymbol{x}_1; \lambda) d\mu(\boldsymbol{x}_1). \tag{6}$$

$\mu(\boldsymbol{x}_j)$ is the measure over the age-specific state space where $j \in \{1, ..., J\}$ and $V^P_{j=1}(\boldsymbol{x}_1; \lambda)$ represents the expected utility for an agent aged $j = 1$ with a state vector $\boldsymbol{x}_1$:

[21]This is because models with endogenous fertility require a welfare comparison between two different sets of individuals. Golosov, Jones, and Tertilt (2007) then propose two efficiency concepts that can apply to models with endogenous fertility. For example, the $\mathcal{A}-$efficiency proposed by them focuses on individuals being alive in both economies. This concept is particularly beneficial if we examine the optimal policy as they show that the unique solution for the planning problem considering the utility of the existing agents (i.e., those who are alive just before the policy is introduced) corresponds to the $\mathcal{A}-$efficiency.

[22]The related work Zhou (2022) adopts the same welfare criteria in its steady-state analysis. In its transition analysis, Zhou (2022) also computes the average welfare of the existing households already alive before the policy changes to evaluate the $\mathcal{A}-$efficiency of the policy.

$$
\begin{aligned}
V^P_{j=1}(\boldsymbol{x}_1;\lambda) = \mathbb{E}\Bigg\{ & \sum_{j=1}^{J} \beta^{j-1} \cdot u(c_j \cdot (1+\lambda), l_j) \\
& + \sum_{j=J_F}^{J_{IVT-1}} \beta^{j-1} b(n) \cdot v(q_j) + \beta^{J_{IVT}-1} \lambda_a \cdot b(n) \cdot V_{g0}(\boldsymbol{x}_{J_{IVT}}) \Bigg\}
\end{aligned}
$$

Here, $\{c_j, l_j, q_j, n, a_{IVT}\}$ are optimal choices with the policy P and each state $\boldsymbol{x}_j$. Finally, the consumption equivalence with a policy P is given as a scalar λ satisfying the following equation:

$$
V^0(\lambda) = V^P(0).
$$

That is, the consumption equivalence λ makes the newborn agents indifferent between the new economy and the benchmark by scaling up the benchmark consumption by λ.

5.1 Introducing Education Subsidies

In 2020, the Japanese government introduced income-tested subsidies for college students in low-income households; until then, the government has provided only student loans. Households are eligible if their last year's annual labor income is less than a threshold value $\bar{I}$, and students in those households receive g amount of money each year while in college. The grant function $g(I)$ representing the existing grants is formulated as follows:

$$
g(I) = \begin{cases} g & \text{if } I < \bar{I} \\ 0 & \text{otherwise} \end{cases} \tag{7}
$$

Note that the benchmark case can be interpreted as $g(I) = 0$ for any I. Although the income threshold $\bar{I}$ and payment g in the actual system vary with some characteristics of households and students, such as family structure and whether the student commutes to college from home or not, $\bar{I}$ approximately corresponds to the 15 percentile of the household income distribution, and the payment approximately amounts to two-thirds of the average expenses of college students, including tuition fees and life expenses. I set $\bar{I}$ and g based on this information and income distribution and students' expenditure in the benchmark model (initial steady state). I then solve the stationary equilibrium by introducing this new grant function.

The main numerical results are as follows. First, introducing the means-tested grants increases the college enrollment rate by 3.9 p.p. in the long run. Educational mobility increases in the sense that children of high school graduates are 2.5 p.p. more likely to attend college.[23] Because of the higher college enrollment rate, implying a greater supply of college graduate labor, the skill premium decreases by 0.02 points. Introducing the grants increases the TFR by 3% or 0.064 points. Importantly, this increase is primarily driven by a 7.4% increase in college graduates' fertility, whereas the fertility rate of high school graduates is almost stable.

Those changes in demographic structure and skill distribution affect other aggregate variables. In the long run, the higher college enrollment rate implies a higher share of skilled labor, and the higher TFR implies a greater share of the working-age population. As a result, per-capita labor supply in efficiency units increases by 1.3% in the long run. On the contrary, per-capita capital decreases by 0.9% for several reasons. First, introducing the grants reduces saving incentives for a substantial fraction of households and crowd-out IVT.[24] In addition, the higher TFR implies a greater share of younger generations, who hold fewer assets than older ones. Despite its negative effects on capital accumulation, the positive impacts on the labor force and productivity are sufficiently greater so that the per-capita output increases by 1.0% in the long run. The standard deviation of wages, $w_e \cdot \eta_{j,z,e,h}$, is reduced by 1.3 % compared with the benchmark because of the greater educational mobility and the pecuniary externalities of college enrollment (i.e., the shrinking skill premium due to the greater supply of college graduates). Also, the intergenerational income elasticity decreases by 6.0% (i.e., intergenerational mobility of income increases) because the income-tested grants help students in poor households enroll in college. A GE effect through the reduced skill premium also contributes to higher mobility. The welfare improves by 5.1% in consumption equivalence under the veil of ignorance, and its sources are discussed in the following section.

5.2 Inspecting the Mechanism

Previous results imply that the grant introduction significantly affects fertility and college enrollment rates, which leads to greater output. The introduction also brings welfare gains in the long run. However, the mechanism behind these changes is not so obvious because several objects aside from the grant function, such as prices, tax rate, and distribution, vary between the benchmark and new equilibrium, and each can play important roles in accounting for the overall effects on fertility, college enrollment, and welfare. In addition, the roles of fertility margins are also worth investigating, that is, how the re-

[23] This mobility concept is a variant of an income mobility measure adopted in Zheng and Graham (2022).

[24] Note that some households can increase savings upon the grant introduction. For example, households whose children do not attend college in the benchmark but attend college when the grant is introduced may need more assets in the new equilibrium to make some IVT upon children's college enrollment. This is because the grants are not sufficiently generous to cover 100% of the expenditures for students.

sults differ under exogenous fertility in which the fertility behavior is fixed to that in the benchmark. This is because the fertility setup would matter to the IVT and, thereby, children's education choices, given that fertility and IVT choices are joint decisions. Fertility margins can also have distributional implications through intergenerational linkages if fertility responses are heterogeneous across household characteristics.

I conduct a decomposition analysis in Section 5.2.1 to discuss and understand the mechanism behind the changes in the TFR, college enrollment rates, and welfare upon introducing the grants. Next, in Section 5.2.2, I consider the roles of fertility responses in understanding the results by solving the exogenous fertility version of the model.

5.2.1 Decomposition

What causes the increases in fertility and college enrollment rates? Those increases can be broken down into behavioral effects and distributional (or composition) effects, where the behavioral effects can be further broken down into the direct effects, driven only by a change in the subsidization scheme (i.e., grant function $g(I)$), and the indirect effects, driven by changes in factor prices (i.e., GE effects) or tax rates (i.e., Taxation effects). Note that the direct effects can also be interpreted as the short-run effects of the introduction, where prices, tax, and distribution are fixed. Finally, the distribution effects capture changes driven by distribution changes over the state variables such as education, age, and human capital. To isolate each effect, I conduct a decomposition exercise as follows. I first solve an equilibrium by introducing the new grant function. Then, I solve household problems by replacing one of the four objects (grant function, prices, tax rate, and household distribution) in the benchmark with that in the new equilibrium. Note that this method does not guarantee that each implied effect adds up to the overall effects because all factors except grant function are endogenous and interconnected; however, my results indicate that each effect adds up roughly to the overall effects. Table 10 summarises the decomposition results.

	Direct	GE	Tax	Dist.	All
CL share (Δ p.p.)	+2.6	−0.2	0.0	+1.9	+3.9
TFR (Δ %)	+2.3	+1.4	0.0	−0.4	+3.0
HS (Δ %)	+1.0	0.0	0.0	0.0	+0.4
CL (Δ %)	+4.5	+2.5	0.0	0.0	+7.4
Output (Δ %)	−0.7	−0.8	−0.1	+0.9	+1.0
Welfare (%)	+2.6	−0.7	−0.2	+2.3	+5.1

Table 10: Decomposition results. *Note*: Columns "Direct," "GE," "Tax," and "Dist." report the results when only grant function, prices, labor income tax rate, and distribution change, respectively. A column "All" indicates the results in the benchmark and the long-run equilibrium with the grants. Rows "CL share" represent the change in college enrollment rate in p.p. and rows "HS" and "CL" represent the percentage changes in fertility rates for high school and college graduates. Welfare gains are represented in terms of consumption equivalence.

College Enrollment: For the long-run increase in the college enrollment rate, critical forces are direct and distribution effects. If the grant is introduced while other variables (i.e., prices, tax rate, and distribution) are fixed as in the benchmark, the college enrollment rate increases by 2.6 p.p., more than half of the overall increase. These direct effects capture the effects of relaxing financial constraints on college enrollment decisions. GE effects put downward pressure on college enrollment because the lower college premium reduces the incentive for college enrollment.

Note that these direct effects do not explain the overall effects, so we need other forces accounting for the increase in college enrollment rate; that turns out to be the distribution effects. The implied changes in the distribution in the long run, holding grant function, prices, and tax rate fixed as in the benchmark, lead to a 1.9 p.p. higher college enrollment rate. The most relevant factor is the change in the skill distribution of parents (i.e., the share of college graduates). The increase in the share of college graduates implies a higher share of those more likely to have children enrolling in college due to the intergenerational persistence of education. Then, the distributional change amplifies the short-run increase in college enrollment rate facilitated by the direct effects.

Fertility: For the long-run increase in the TFR, vital forces are direct and GE effects. If the grant is introduced while other variables are fixed as in the benchmark, the TFR increases by 2.3% in the long run (direct effects), which corresponds to three-fourths of the overall effects. As in explaining the overall effects, that 2.3% increase due to the direct effects is largely driven by the fertility increase of college graduates; college graduates increase fertility by 4.5%, whereas high school graduates do by 1%.

Further, among each education group, ex-post ineligible households increase fertility. The average fertility rates of ex-post eligible high school and college graduates increase by 1.6% and 3.9%, as indicated in Fig 2a. Given that the ineligible households are the majority of the cohort as indicated by Fig 2b, these fertility increases among ex-post eligible ones largely explain the direct effects; they explain a 0.7 p.p and 1.2 p.p. of the 2.3% direct effect, respectively (Fig 3). In other words, more than 80% of the direct effect is explained by the fertility increases of ineligible households.

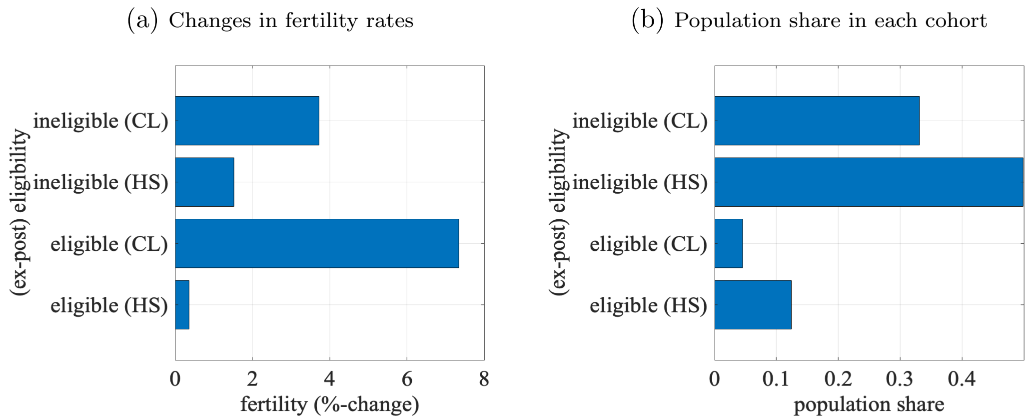

Fig 2: %-changes in fertility rates of each eligibility-education status (2a) and population share of each eligibility-education status (2b). *Note*: "HS" and "CL" stand for high school and college graduates.

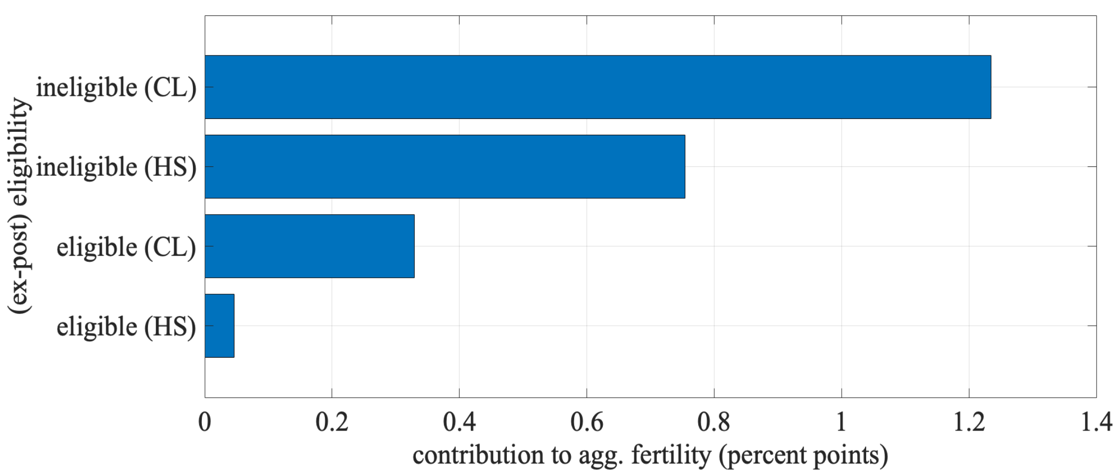

Fig 3: Contribution to the change in aggregate fertility by education and ex-post eligibility status. *Note*: "HS" and "CL" stand for high school and college graduates.

Why can the grant introduction increase fertility, even among ex-post ineligible households? The short answer is due to an insurance effect of the income-tested grants. To understand this statement, recall that the eligibility is uncertain when they make fertility choices; they are beneficiaries if (1) their children enroll in college and (2) their earnings when their children make the education choices, are sufficiently low. Those are subject to uncertainty regarding their productivity and children's characteristics (school tastes and human capital), which are not realized when they make fertility choices. If their children have characteristics favoring college enrollment (i.e., higher human capital endowment and/or lower psychic costs of education), they would like to make more IVTs to support them, and vice versa. Thus, they face expenditure uncertainty until their children's education choices terminate, in addition to income risks.

Consider that those "shocks" are realized simultaneously; their children have characteristics willing to attend college, but they are poor due to the realization of negative income shocks. Without the income-tested grants, the realization of negative income shocks will make it infeasible for those "unlucky" parents to financially support their children's enrollment, or they may end up with low consumption in exchange for supporting the enrollment: either situation is costly for parents. Thus, in fertility decision-making, they may choose fewer children to avoid such a costly situation they might encounter in the future. The income-tested grants provide partial insurance against those risks and make some parents comfortable having a child (or another child). Because college graduates are more likely to have children with characteristics favoring college enrollment, they benefit more from this insurance, and their fertility behavior responds strongly.

Next, the decomposition result shows that the GE effect also plays a role in accounting for the TFR increase. If the prices are set to the long-run equilibrium levels while other objects are fixed as in the benchmark, the college graduates' fertility increases by 2.5%, and the TFR then increases by 1.4%. The key is the decline of the wage rate for college graduate workers, w_{CL}. In the long-run equilibrium with the grants, w_{CL} decreases by 2.2%. First, the greater supply of college graduate workers depress w_{CL} relative to w_{HS}. In addition, greater aggregate labor supply in efficiency units and lower capital accumulation discussed in Section 5.1 imply the lower marginal productivity of labor, decreasing w_{CL}. The lower w_{CL} implies the lower opportunity costs of having children for college graduates, making some have more children. Also, for college graduate parents, the lower w_{CL} reduces the (expected) utility of the IVTs, given that their children are likely to attend college, as discussed above. The lower utility of the IVTs also contributes to higher fertility because the marginal utility gains from investing in children's "quality" (by making IVTs to send a child to college) are relatively lower than those from increasing its "quantity."[25]

Lastly, the distribution effect implies a 0.4% decline in the average fertility. The introduction of the grants increases the college enrollment rate, meaning that the share of college graduates increases in the long run. Given that college graduates have fewer children than high school graduates as discussed in Section 4.2.1, the greater share of college graduates means a greater share of those who tend to have fewer children, which can lead to a lower average fertility.[26] However, direct and GE effects are significant and

[25]This fertility response of college graduates to the change in skill premium aligns with an empirical observation (Lehr, 2003).

[26]This effect is comparable with a composition effect highlighted by Zhou (2022) in understanding the effects of public education subsidies on fertility. In his model, in the long run, public education subsidies

lead to the higher TFR.

Output: The distribution effect is critical in accounting for the output increase. In the long run, the working-age population increases due to a higher equilibrium TFR, and the share of skilled workers increases. These forces increase the labor supply in efficiency units, increasing output in the long run.

In contrast, the direct and GE effects put downward pressure on output. As indicated in Table 10, the GE effects lower the college enrollment rate due to the reduced skill premium. The resulting lower share of skilled workers leads to a lower output.

Next, recall that the direct effect can be considered the short-run effect of the introduction, where other macroeconomic variables, such as prices, tax rates, skill distribution, and age distribution, are fixed as in the benchmark. In the short run, aggregate savings and labor supply decrease due to higher fertility; having a child requires parents to spend a fraction of their time and some additional money, reducing their working hours and savings. Thus, the aggregate output decreases in the short run.

Welfare: The direct effect explains more than half of the welfare gains. Importantly, introducing the grants makes agents attend college with smaller costs and enables someone who could not attend college in the benchmark to do that, bringing them a higher lifetime income. The rest of the gains are explained by the distribution effect. The education subsidy increases the share of college graduates in the long run, and the lifetime utility for college graduates is, in principle, greater than that for high school graduates, particularly because of their higher income due to the college education returns. Thus, this distributional change increases the expected lifetime utility. Note that the direct effect is about the change in $V^P_{j=1}$ in equation (6), while the distribution effect is about the change in μ there.

5.2.2 Roles of Fertility Responses

I next solve the exogenous fertility version of the model. More specifically, I follow the same procedure in the previous section 5.1, except that the policy functions for fertility are fixed as in the benchmark. Therefore, the TFR under exogenous fertility is not necessarily the same as in the benchmark because the household distribution changes.

increase the share of parents with higher human capital, who tend to have fewer children.

	Endogenous	Exogenous
CL share (Δ p.p.)	+3.9	+2.9
TFR (Δ %)	+3.0	−0.8
Output (Δ %)	+1.0	+0.4
Capital (Δ %)	−0.8	+0.6
Labor (Δ %)	+1.3	+0.2
Tax (Δ p.p.)	+0.02	+0.71
STD(wage) (Δ %)	−1.3	−1.1
Welfare (%)	+5.1	+3.5

Table 11: Results under exogenous fertility. *Note*: Results for output, labor, and capital represent percentage changes compared with the benchmark. Those for college enrollment and labor income tax rates represent changes in percentage points compared with the benchmark. "STD(wage)" represents the standard deviation of wages, where wages for an agent with (j, z, e, h) is captured as $w_e \cdot \eta_{j,z,e,h}$. Welfare gains are represented in terms of consumption equivalence.

Table 11 reports the main results under exogenous fertility. First, the grant introduction results in a 2.9 p.p. increase in the college enrollment rate under exogenous fertility, which is lower than a 3.9 p.p. increase under endogenous fertility. Under both exogenous and endogenous fertility, the grants make some children who otherwise cannot enroll in college. Under endogenous fertility, in addition to that effect, the college enrollment rate can further increase through fertility margins. As I discussed in Section 5.1, college graduates increase fertility than high-school graduate parents. Their children will likely attend college due to the intergenerational transmission of school tastes and human capital. And when those children become parents in the future, their children, if they have, are also likely to have similar characteristics to theirs, favoring college enrollment. Through this mechanism, the long-run share of college graduates increases, implying that the fertility margins amplify the effects on college enrollment.

From the second row onward, the changes in other aggregate variables are reported. Under exogenous fertility, per-capita labor in efficiency units increases by 0.2 % compared with the benchmark. The degree of this increase is modest compared with a 1.3% increase under endogenous fertility for two reasons. First, the share of skilled labor is lower under exogenous fertility than endogenous fertility, as discussed above. Second, the working-age population share is slightly lower under exogenous fertility than in the benchmark, an opposite result to the endogenous fertility setup: the introduction under exogenous fertility leads to a 0.8% lower fertility rate due to a composition effect. The lower labor supply in efficiency units leads to lower output gains under exogenous fertility: the exogenous setup implies a 0.4% increase in per-capita output, while the endogenous fertility setup implies a 1% increase.

Even though the college enrollment rate and TFR under exogenous fertility are lower than under endogenous fertility, implying a lower government expenditure on the grants, the required tax increase is 0.69 p.p. higher under exogenous fertility. This is because the greater labor supply in efficiency units under endogenous fertility increases the tax revenue in the long run. The reduction of the standard deviation in wages is more significant under endogenous fertility, mainly because the higher college enrollment rate under endogenous fertility leads to a lower skill premium. The welfare gain is 46% greater under endogenous fertility because of those channels (i.e., lower tax and lower inequality). Also, the higher college enrollment rate in the economy increases the welfare under the veil of ignorance via the direct and distribution effects.

5.3 Expansion

In this section, I consider the effects of raising the income threshold $\bar{I}$ so that students in households of broader income classes can be eligible. Recall that I set the threshold $\bar{I}$ so that the existing grants target the students in households at the bottom 15 % of the income distribution. In this experiment, I increase $\bar{I}$ so that it corresponds to the 40, 50, and 60 percentiles of the income distribution and solve the stationary equilibrium in each case. For students in the household at the bottom 15 %, the payment is still given by g, which amounts to two-thirds of the average expenses of students in the benchmark. For students in households higher than the 15 percentile but less than x percentile of the income distribution, where x takes either 40, 50, or 60, the payment is given by $g/2$, which amounts to one-third of the average expenses of students. In other words, the payment tapers off in income. Letting $\bar{I}_{15\%}$ and $\bar{I}_{x\%}$ denote the income level of 15 and $x \in \{40, 50, 60\}$ percentiles of the income distribution, the grant function $g(h, I; x)$ with a threshold $\bar{I}_{x\%}$ can now be formulated as:

$$g(h, I; x) = \begin{cases} g & \text{if } I < \bar{I}_{15\%} \\ g/2 & \text{if } I \in [\bar{I}_{15\%}, \bar{I}_{x\%}) \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Table 12 summarizes the results, and there are several takeaways there. First, the equilibrium college enrollment increases as the income threshold is higher. Setting the threshold at the 40, 50, and 60 percentiles of the income distribution, the enrollment rate increases by 4.7 p.p., 5.6 p.p., and 6.2 p.p., respectively, in stationary equilibrium. This is also the case for educational mobility and labor income tax rates. The expansion requires additional revenue, so the equilibrium tax rate should also increase by 0.3 p.p. for the case of the 60% threshold.

However, the expansion would not significantly increase the TFR. Recall that the introduction leads to a 3% increase in TFR, from the benchmark level of 2.096 to 2.160 in the long-run equilibrium with the grants. With grant functions $g(h, I; 40)$, $g(h, I; 50)$, and $g(h, I; 60)$, the equilibrium TFR would be 2.158, 2.151, and 2.157, respectively; the TFR even decreases locally with expansion. Fertility rates for each skill help us understand this situation. First, college graduates continue to increase fertility. With grant functions $g(h, I; 40)$, $g(h, I; 50)$, and $g(h, I; 60)$, their equilibrium fertility rate would be 1.996, 1.998, and 2.021, all higher than the fertility rate with the existing grants, 1.978. This result is straightforward to understand given that the insurance and GE effects contribute to the fertility increase of college graduates when the grants are introduced, discussed in Section 5.2. On the contrary, fertility among high school graduates decreases with expansion, making the TFR remain almost constant even though fertility among college graduates increases. Fig 4 depicts the changes in the TFR and fertility rates for high school and college graduates, where the equilibrium TFR with the existing program (with the 15 percentile threshold) is normalized to one.

	Threshold			
	15%	40%	50%	60%
CL share (Δ p.p.)	+3.9	+4.7	+5.6	+6.2
HS$\to$CL (Δ p.p.)	+2.5	+2.6	+3.1	+3.3
Tax (Δ p.p.)	+0.02	+0.17	+0.23	+0.30
Output (Δ%)	+1.0	+1.3	+0.2	−0.5
Welfare (%)	+5.1	+6.5	+6.3	+5.5

Table 12: Main results of higher income thresholds. *Note*: Rows "CL share" and "HS→CL" represent the changes in college enrollment rate and educational mobility in the sense of probability that children of high school graduates attend college. Output changes are expressed as percentage changes. Changes in college enrollment rate, educational mobility, and tax rate are represented as changes in percentage points. Welfare gains are represented in terms of consumption equivalence.

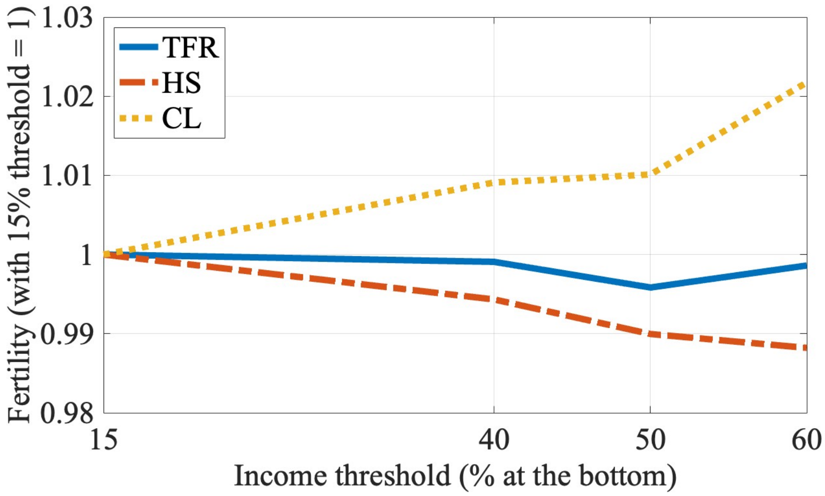

Fig 4: Changes in fertility rates with expansion. *Note*: The equilibrium TFR with the existing program (with the 15% threshold) is normalized to one. "HS" and "CL" represent fertility rates for high school and college graduates.

Why do high school graduates decrease fertility as the income threshold is higher? To understand this, I conduct the decomposition when the grant function is given by $g(h, I; 60)$, and the results are summarized in Table 13. Two effects are critical to explain the fertility decreases of high school graduates: the direct and GE effects.

In this case with the grant function $g(h, I; 60)$, the wage rate for high school graduates, w_{HS}, is 0.6% higher in the long run than in the benchmark, especially because of the significant supply of college graduates. This higher wage rate makes the opportunity costs of having children for high school graduates higher, which puts downward pressure on their fertility rates.

Next, the direct effects imply a lower fertility rate for high school graduates. This result is somewhat confusing given that we discuss the insurance effects of the grants on fertility, which is critical to account for the fertility increases of college graduates. Why do the direct effects lead to lower fertility for high school graduates? In the benchmark without grants, children of high school graduates are unlikely to attend college. If the grants are introduced and their target expands, educational mobility increases in the sense that children of high school graduates are more likely to attend college than in the benchmark, as represented in Table 12. Given that the grants are not generous enough to cover 100% of the costs to send their children to college, this higher probability of children going to college implies that those parents are more likely to have to make additional transfers upon children's college enrollment. The expansion thus increases the expected costs of children for high school graduates, which lowers their fertility.

	Direct	GE	Tax	Dist.	All
HS ($\Delta\%$)	-0.7	-1.4	0.0	0.0	-0.8
CL ($\Delta\%$)	$+9.1$	$+4.3$	$+2.4$	$+0.6$	$+13.2$

Table 13: Decomposing the effects on fertility with $g(h, I, 60)$. *Note*: Rows "HS" and "CL" represent the percentage changes in fertility rates for high school and college graduates.

Last, the output increases in expansion up to the case of the 40 percentile threshold, but they start declining after that point, as Table 12 and Fig 5 indicate. If the income threshold becomes sufficiently high, households with children have less incentive to save to support their college enrollment because they will likely be eligible for the grants. This higher probability of eligibility also causes an income effect on labor supply. Thus, the aggregate capital and labor supply decrease when the income threshold becomes sufficiently high, reducing the output. This concavity of output gains with expansion applies to the welfare gains. The marginal utility gains of the grants decrease in the values of the income threshold while the tax rate increases with expansion. Then, the net gains start declining at some point.

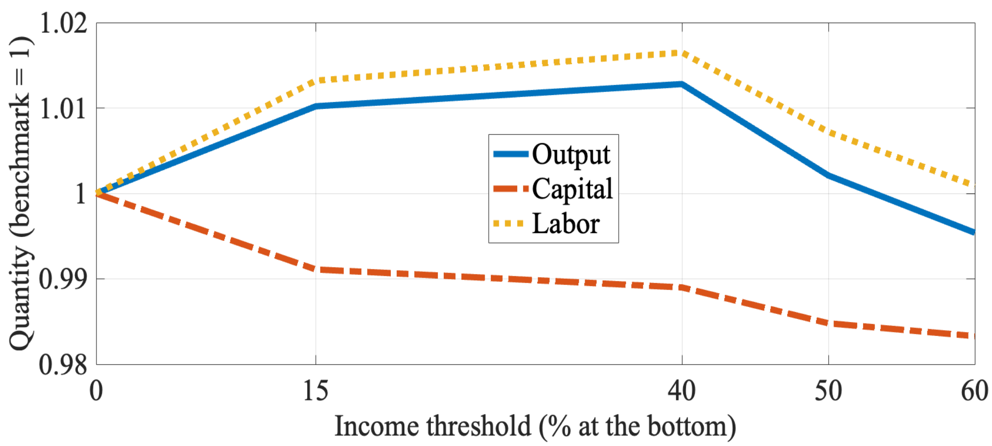

Fig 5: Changes in aggregate output, capital, and labor supply in efficiency unit with expansion. *Note*:

5.4 Transitional Dynamics

Finally, this section investigates the transition dynamics of the economy upon the introduction of the existing income-tested grants. The introduction occurs unexpectedly for households in the initial equilibrium (benchmark economy). Households have perfect foresight about future prices and tax rates and make subsequent choices to maximize utility. For computational reasons, I assume that the government collects the lump-sum taxes to balance the budget during the transition and that the labor income tax rate immediately reaches its final steady-state value, $\tau_l = 0.3502$. [27]

[27] Some previous studies also adopt this strategy for computational reasons (e.g., Daruich, 2022).

First, Fig 6 represents changes in college enrollment rates, labor supply of college and high school graduates, and wage structure. As the top-left in Fig 6 indicates, college enrollment rates (within cohorts) increase upon the introduction but do not immediately reach the new equilibrium value and gradually converge to it. This is because, as highlighted in Section 5.2, a substantial part of the long-run increase in college enrollment rate is explained by the distribution (composition) effects: students who benefit from the policy become parents in the future, whose children will be more likely to attend college due to intergenerational persistence of educational attainment. Also, the fertility responses strengthen the distribution effects: college graduates increase fertility, and those "marginal" children born are more likely to be college graduates, whose children will be more likely to attend college. Along with the increase in college enrollment, the labor supply of college graduates (in efficiency units) gradually increases, as shown in the top-right. The increase in college graduates' labor supply leads to the decline in the wage rates for college graduates and the increase in those for high school graduates (see the bottom-left figure), reducing the skill premium (see the bottom-right).

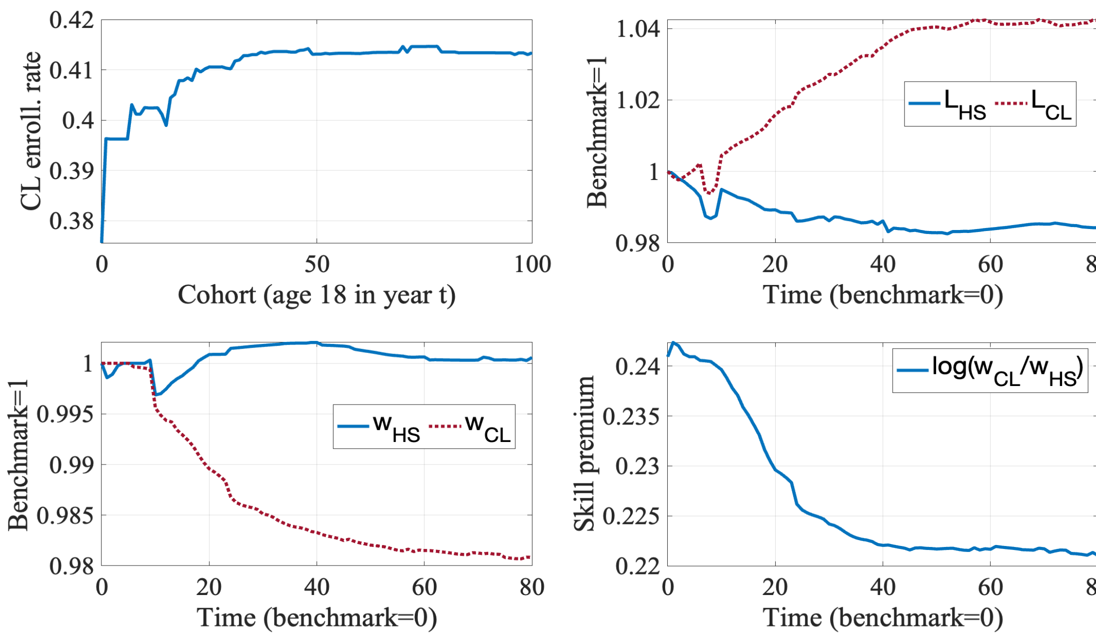

Fig 6: College enrollment rates, labor supply, and wage structure during the transition. *Note*: In the top-left figure about college enrollment rates, the horizontal axis represents the cohort, where an x-th cohort refers to a cohort aged 18 (the age of enrollment choice) in year $t = x$ where the policy is implemented in $t = 1$. In the rest of the figures, the horizontal axis represents the time t where the benchmark economy corresponds to $t = 0$. One time period refers to 2 years in this model.

Next, the left figure in Fig 7 plots the transition path for per-capita output. Per-capita output decreases in the short and medium runs mainly for two reasons. First, the grants crowd out some households' savings, leading to lower capital stock in the economy.

Second, fertility rates increase upon the introduction, leading to a lower labor supply because having children requires time costs. However, as discussed in Section 5.2, the introduction leads to higher outputs in the long run through distribution changes.

The right figure in Fig 7 also highlights the sources of fertility increases discussed in Section 5.2. College graduates' fertility increases due to the insurance effects, operating even in the short run. However, they do not immediately reach the new equilibrium, and the GE effects gradually push the fertility rates, taking approximately twenty model periods. Contrary to college graduates, high school graduates' fertility in the short run exceeds the long-run level, mainly due to the insurance effect. However, the GE effects exert downward pressure on their fertility because higher wage rates during the transition (see the bottom-left in Fig 6) increase the opportunity costs of having children. Therefore, their fertility rates gradually decrease and reach the long-run equilibrium level.

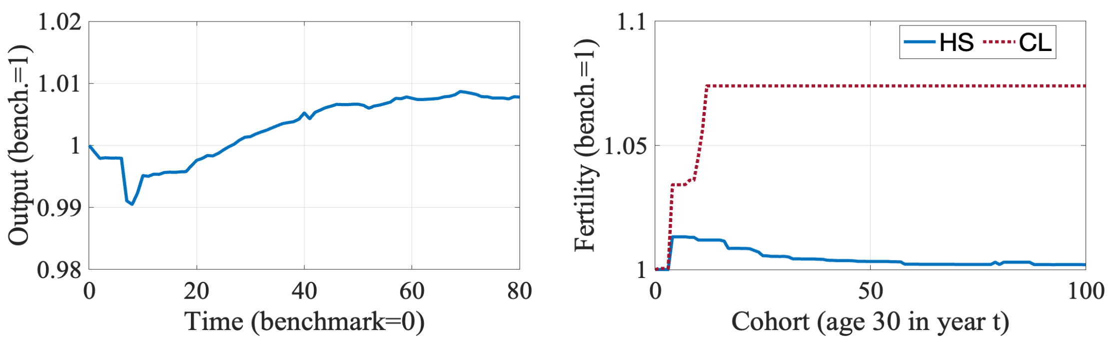

Fig 7: Per-capita output, capital, and fertility rates during transition. *Note*: In the right figure about fertility rates, the horizontal axis represents the cohort, where an x-th cohort refers to a cohort aged 30 (the age of fertility choices) where the policy is implemented in $t = 1$. "HS" and "CL" represent fertility rates for high school and college graduates in the cohort. In the right figure about output and capital, the horizontal axis represents the time t where the benchmark economy corresponds to $t = 0$. One time period refers to 2 years in this model.

Finally, Fig 8 represents the lump-sum taxes during the transition and welfare implications for each cohort. Despite the immediate increase of the labor income tax upon the introduction, the government needs to collect additional lump-sum taxes to finance the grants because tax bases do not sufficiently expand in the short run. As a result, the tax burdens are more significant in the medium run, and most of the existing cohorts, who have been alive just before the introduction, especially older generations, will be worse off. Specifically, net-beneficiaries (i.e., cohorts younger than those aged 50 in year $t = 1$) are better off, whereas the others are not. As time passes and tax bases expand, the required lump-sum tax returns to zeros as in the benchmark economy (recall that the labor income tax rate is immediately set to the final steady-state value upon the grant introduction).

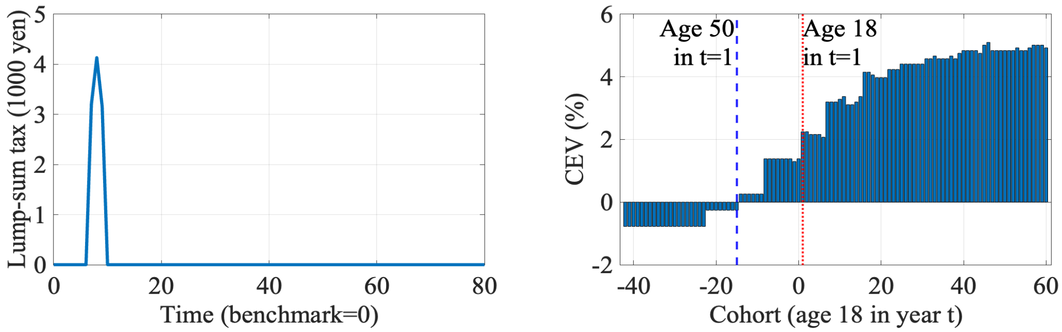

Fig 8: Tax burdens during transition and welfare effects for each cohort. *Note*: In the left figure, the horizontal axis represents the time t where the benchmark economy corresponds to $t = 0$. One time period refers to 2 years in this model. In the right figure about welfare, an x-th cohort refers to a cohort aged 18 (the age of enrollment choice) in year $t = x$ where the policy is implemented in $t = 1$.

6 Empirical Analysis

In Section 5, I show that income-tested grants provide partial insurance against the risks associated with fertility choices under income uncertainty, and more educated couples benefit more from this insurance. This section aims to provide evidence supporting those observations obtained in the model. More specifically, this section shows that (1) income uncertainty, captured by temporary contracts in the labor market, leads to lower birth probabilities and that (2) this negative effect of income uncertainty on birth probability is more significant for more educated couples.

In doing so, I exploit the characteristics of the Japanese labor market, which is the coexistence of two different types of labor contracts: regular (permanent) and contingent (temporary). This idea follows previous studies exploiting dual labor market structure in Spain or Italy.[28] In Japan, as in those countries, most regular (permanent) jobs are characterized as, as the name suggests, permanent contracts, and they are more stable and less susceptible to aggregate fluctuation. On the contrary, contingent (temporary) jobs are characterized by temporary contracts and higher separation rates.

Table 14 summarizes some descriptive statistics regarding the difference between permanent and temporary contracts in the 1960s cohort. The share of wives with permanent (temporary) contracts was about 75% (25%). There is a significant difference between them in their separation rate: annually, only 0.86% of permanent workers experience job separation, whereas 3.87% of temporary workers do. Also, the annual probability of giving birth differs significantly between the two types of workers: permanent workers give birth with an annual probability of 0.124, whereas contract workers do only with a probability of 0.012. In addition, among temporary workers, college graduates have significantly

[28]For example, Modena, Rondinelli, and Sabatini (2014); Guner, Kaya, and Sánchez-Marcos (2024).

lower birth probability (0.005) than non-college graduates do (0.015).

	Share	Separation	Pr(birth)	Pr(Child=CL)
Permanent	0.754	0.0086	0.124	0.303
College >	0.454	0.0091	0.122	0.236
College	0.300	0.0000	0.128	0.415
Temporary	0.246	0.0387	0.012	0.321
College >	0.183	0.0353	0.015	0.234
College	0.063	0.0909	0.005	0.489

Table 14: Separation rates and birth probabilities.

To assess the effects of temporary contract—a proxy of income uncertainty—on fertility, I estimate the following model:

$$\Pr(y_{i,t} = 1) = L(\alpha + \beta T_{i,t-1} + \boldsymbol{x}_{i,t}\boldsymbol{\theta} + \boldsymbol{z}_i\boldsymbol{\eta} + \phi_t + \varepsilon_{i,t}). \quad (9)$$

Here, $y_{i,t}$ takes one if the wife in a couple i gives birth in year t, and $L(\cdot)$ is the standard Logistic function. $T_{i,t-1}$ is a dummy variable taking one if the wife worked under a temporary contract in year $t-1$, and β is the coefficient of our interest. The vector $\boldsymbol{x}_{i,t}$ includes other (time-variant) couples' characteristics, such as household income and wife's age. The vector $\boldsymbol{z}_i$ includes the time-invariant couple's characteristics, such as the educational background of the wife and husband. I also control for the year fixed-effects, ϕ_t.

Table 15 reports the log odds-ratio estimates. The first column presents the estimates where I control only for the temporary contract indicator. In the second and third columns, I gradually add couple's characteristics and year fixed-effects. Each column reports that the log odds-ratio for giving birth decreases by 2.2 points for wives with temporary contracts. It is statistically significant and implies about an 89 percent decrease in birth probability. Next, I consider whether the degree of the decline depends on the couple's education level by introducing an interaction term, $T_{i,t-1} \cdot College_i$ where $College_i$ takes one if either of wife or husband is a college graduate. The fourth column reports the result. The coefficient for $T_{i,t}$ is -2.0 and is still significant. The coefficient for the interaction term is -1.08. The result is close to the 10 percent significance level, suggesting that the birth probabilities of college graduates are lower than those of non-college graduates among temporary workers.

	(1)	(2)	(3)	(4)
$T_{t,i-1}$	-2.358^{***}	-2.202^{***}	-2.200^{***}	-2.001^{***}
	(0.194)	(0.196)	(0.196)	(0.209)
$T_{t,i-1} \cdot College_i$				-1.082^{*}
				(0.620)
Personal characteristics	No	Yes	Yes	Yes
Year fixed effects	No	No	Yes	Yes
Interaction ($T_{t,i-1} \cdot College_i$)	–	No	No	Yes
Observations	9143			

Table 15: Employment type and birth probabilities of women aged 24-39. *Note:* Values in parentheses represent standard errors. ***, **, and * denote statistical significance at the 1, 5 and 10 percent levels, respectively.

7 Concluding Remarks

This paper examines the macroeconomic consequences of college financial aid policy by constructing a new GE-OLG model that features fertility and college enrollment choices. My central contribution is to highlight the critical roles of the fertility margins in evaluating the macroeconomic performance of the policy. My calibrated model demonstrates that income-tested college subsidy reduces the fertility differential by providing partial insurance against costly states associated with having children for college graduates, which amplifies the policy effects on college enrollment and output.

My model is tailored to examine the college education subsidies by capturing relevant ingredients, such as students' labor supply and minute lifecycle (age) structure. Instead, this paper abstracts the children's skill formation through parental investments before college education, as previous studies in college financial aid policies do not. These ingredients are critical in considering broader policies contributing to children's skill formation, including family policies and early childhood education policies.[29] In addition, this paper abstracts competition for college admission and externalities of parental investments, which are also important to take into account when we evaluate the broader education policies, such as education taxes and changing enrollment capacity.[30] Incorporating these ingredients into the current model and considering broader education policies are left for future research.

[29]Examples of recent works in this respect are Lee and Seshadri (2019); Abbott (2022); Daruich (2022); Zhou (2022); Yum (2023); Moschini and Tran-Xuan (2023); Gu and Zhang (2024).

[30]See, for example, Kim et al. (2023); Kang (2024); Gu and Zhang (2024).

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A. Illustrative Examples for Fertility Differentials

A. (i) The Price of Time Theory

Following Jones, Schoonbroodt, and Tertilt (2010), I consider the following problem:

$$\max_{c,n} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \theta \frac{n^{1-\sigma}}{1-\sigma} \right\}$$

s.t.

$$c = w \cdot (1 - n \cdot \chi)$$

Here, c, n, w, and χ denote consumption, fertility, an exogenous wage, and time costs of children. The optimal fertility decision n^* is given as:

$$n^* = \frac{1}{w^{\frac{1-\sigma}{\sigma}} \cdot (\chi/\theta)^{\frac{1}{\theta}} + \chi},$$

meaning that the negative income-fertility relationship holds if $\sigma > 1$ (i.e., the substitutability between consumption and children are high enough). Because having children is time-consuming, its (opportunity) cost is higher for high-income households. At the same time, high-income households, by definition, have higher incomes and afford to have more children. If the substitutability between consumption and children is sufficiently high, the substitution effect dominates the income effect, implying the negative income-fertility relationship.

A. (ii) The Quantity-Quality Trade-off

Following De La Croix and Doepke (2003), I consider the following problem:

$$\max_{c,n,e} \{\ln(c) + \theta \ln(nq)\}$$

s.t.

$$c + b \cdot n \cdot e = w \cdot (1 - n \cdot \chi),$$

$$q = (\eta + e)^{\gamma},$$

$$c, n > 0, e \geq 0.$$

Here, the choice variables c, n, and e represent the consumption, fertility, and per-child education investment. The investment e is transformed into the "quality" of children, q, according to a human capital production function $q = (\eta + e)^{\gamma}$. The parameter $\gamma \in (0,1)$ governs the marginal gains of investment, and $\eta > 0$ implies that the quality of children takes a positive value even if parents make no investments. Parameters χ and b represent the time cost of having a child and a unit (monetary) cost of investment.

The optimal solutions for the fertility and education investment are given as

$$e^* = \frac{\gamma w \chi / b - \eta}{1 - \gamma}, \tag{10}$$

$$n^* = \frac{1}{1+\theta} \cdot \frac{1-\gamma}{\chi - \eta b / w}. \tag{11}$$

There are two critical observations in the optimal choices (10) and (11): (1) the optimal number of children is decreasing in wage level w, and (2) the optimal number of children is decreasing in the investment efficiency γ whereas the optimal investment is increasing in γ. These two observations capture the main theoretical explanations for the negative relationship between income and fertility: (1) having children is more costly for higher-income households, provided that having children is time-intensive, and (2) higher-income households have higher demands for child quality or an advantage in parental investments.[31]

My full quantitative model captures these two channels, contributing to replicating the fertility differentials between college and high school graduates observed in the data. First, the opportunity costs of having children are more significant for college graduates, making child-bearing more costly for them. This is because having a child requires parents to incur a fixed amount of time, and college graduates' wage rates are higher than those of high school graduates. Second, college graduate parents are more willing to pay for the IVT, making a child more costly. This is because the children's human capital and psychic costs of education, (h_k, ϕ_k), govern education returns and the willingness to attend college, which in turn governs the marginal gains from IVT for parents. In particular, the psychic costs of children correlate with those of parents, so college graduate parents derive higher utility from the IVTs than high school graduate parents.

B. Fertility Choices with Income Risk

This section illustrates a simple model of fertility choices under income uncertainty and shows that the higher income uncertainty leads to lower fertility.

[31] For more details, see Jones et al. (2010).

Setup: I consider a household problem where a household chooses the number of children, n, and consumption, c, to maximize its expected utility while facing income uncertainty. Households first choose the number of children, n, before the realization of the income level; the income level can take on either $y - \varepsilon$ or $y + \varepsilon$, with $\varepsilon \in (0, y)$ for some $y > 0$, with equal probabilities. Let q be the unit cost of having children,[32] and for simplicity, the number of children chosen must be positive. After the income level is realized, they choose the consumption level, c. The household problem is formulated as follows:

$$\max_{n>0,c>0} \; v(n) + \mathbb{E}_y U(c)$$

s.t.

$$c + q \cdot n = \begin{cases} y + \varepsilon & w.p. \; 1/2 \\ y - \varepsilon & w.p. \; 1/2 \end{cases}$$

Now, based on standard assumptions on utility functions, namely that (1) $v''(n) < 0$, (2) $U''(c) < 0$, and (3) $U'''(c) > 0$, it holds that

$$\frac{dn}{d\varepsilon} < 0, \tag{12}$$

meaning that the higher income uncertainty leads to lower fertility.

Derivation: I first take the first-order condition with respect to n as follows:

$$\begin{aligned} q^{-1} \cdot v'(n) &= \mathbb{E}_y U'(c) \\ &= \frac{1}{2}[U'(y + \varepsilon - n \cdot q) + U'(y - \varepsilon - n \cdot q)] \end{aligned}$$

To capture the relationship between n and ε, I then express n as a function of ε and take the first-order derivative of ε with respect to n as follows:

[32] q takes a positive value and is assumed to be lower than the worst income level, $y - \varepsilon$, to ensure positive consumption and fertility.

$$
\begin{aligned}
2q^{-1}v''(n(\varepsilon))\frac{dn}{d\varepsilon} &= U''(y+\varepsilon-q\cdot n(\varepsilon)) - U''(y-\varepsilon-q\cdot n(\varepsilon)) \\
&\quad - q\cdot\frac{dn}{d\varepsilon}[U''(y+\varepsilon-q\cdot n(\varepsilon)) + U''(y-\varepsilon-q\cdot n(\varepsilon))] \\
\Leftrightarrow 2q^{-1}v''(n(\varepsilon))\frac{dn}{d\varepsilon} &+ q\cdot\frac{dn}{d\varepsilon}[U''(y+\varepsilon-q\cdot n(\varepsilon)) + U''(y-\varepsilon-q\cdot n(\varepsilon))] \\
&= U''(y+\varepsilon-q\cdot n(\varepsilon)) - U''(y-\varepsilon-q\cdot n(\varepsilon)).
\end{aligned}
$$

Finally, we have the stated result (12) as follows:

$$
\frac{dn}{d\varepsilon} = \frac{\overbrace{U''(y+\varepsilon-q\cdot n(\varepsilon)) - U''(y-\varepsilon-q\cdot n(\varepsilon))}^{>0\ (\because\ U'''>0)}}{\underbrace{2q^{-1}v''(n(\varepsilon)) + q\cdot[U''(y+\varepsilon-q\cdot n(\varepsilon)) + U''(y-\varepsilon-q\cdot n(\varepsilon))])}_{<0\ (\because\ v''<0\ \&\ U''<0)}} < 0
$$

C. Education Costs and Fertility Choices in Japan

Japan is a leading country in demographic aging,[33] leading to the shrinking labor force, output, and tax base, while the public expenditures on social security benefits are increasing. As a countermeasure against this demographic issue, the government introduced grants for college students in 2020. Two key underlying premises are: (1) it will increase the "quality" of the labor force in the long run, and (2) it will increase the fertility rate, which increase the "quantity" of the labor force in the long run. This pro-natal motive is explicitly described in an act for introducing the grants.[34]

> The aim is ... to foster an environment where people can bear and raise their children with a sense of ease by alleviating the economic burden associated with higher education, thereby contributing to addressing the rapid decline in the birthrate in our country.
> (Act on Support for Higher Education Studies, enacted on May 17, 2019)

Although that expectation for education policies as a pro-natal policy is unconventional, there are facts suggesting that the financial costs for parents to support their children's college enrollment are a significant impediment to fertility decisions in Japan. I first list the five facts below and then elaborate on each one by one:

[33] Japan's current fertility rate has been around 1.3, far below the population replacement level. In addition, the old-age dependency ratio is more than 50% and is projected to reach 80% in 2050, which is by far the highest among the OECD countries (See, https://data.oecd.org/).

[34] See, https://www.mext.go.jp/a_menu/koutou/hutankeigen/detail/__icsFiles/afieldfile/2019/05/17/1417025_02_1.pdf (available only in Japanese).

1. Couples are most likely to abandon having an ideal number of children because of financial costs.
2. A significant financial cost gap exists between those who have children enrolled in college and those who do not.
3. A substantial fraction of parents desire a college education for their children.
4. Japan is one of the least in subsidizing tertiary education.
5. Children's college enrollments are associated with their parents' lower fertility and higher probabilities of deciding to stop having an additional child

Fact 1. *Couples are most likely to abandon having an ideal number of children because of financial costs.*

This fact is drawn from the National Fertility Survey (NFS), which is provided by the National Institute of Population and Social Security Research (IPSS).[35] The NFS is a cross-sectional household survey that asks respondents about their preferences or intentions regarding fertility, marriage, child-raising, and education, as well as their basic information, including their education, age, and income. It is conducted nearly every five years, and the latest survey available is in 2015, which collected 5,334 couples in which the wife is aged 18 to 49. Hereafter, I present the results focusing on married couples in which the wife is aged 25 to 39 years, leaving 2,420 couples.[36] According to the NFS, a non-negligible gap exists between the ideal and planned numbers of children. In the 2015 survey, the ideal number of children for wives aged 25 to 39 was on average 2.38, whereas the planned number was 2.16. Fig 9 represents the distribution of the ideal and planned numbers of children, where blue (red) bars indicate the share of wives who desire (plan) to have each number of children from zero to more than five. This figure suggests that the gap originates from the downward revision of the ideal at the intensive margin. There is no significant share gap between those whose ideal number is zero and those whose planned number is zero, and a substantial fraction of wives who desire three children end up with one or two children.

[35] See, https://www.ipss.go.jp/site-ad/index_english/survey-e.asp.

[36] Its 53.1% of the sample consists of couples in which the wife is over age 40, so the sample size shrinks if we target the younger couples. I focus on wives under age 39 because here I am interested in the fertility intention of those in the stage of fertility decision. Note that the cohort fertility rate is stable after age 40 for any cohort in Japan. See, for example, p7 of https://www.mhlw.go.jp/toukei/saikin/hw/jinkou/tokusyu/syussyo07/dl/gaikyou.pdf (in Japanese). Excluding those aged 18 to 24 does not affect the result significantly because they consist only of 1.5% of the observations for wives aged 18 to 49.

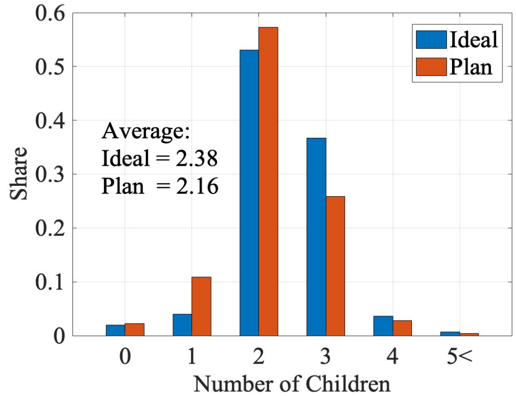

Fig 9: Distribution of ideal and planned numbers of children.

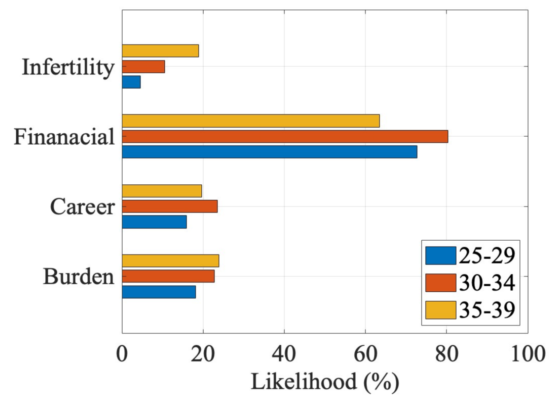

Fig 10: Reasons for the gap between the ideal and planned number of children.

Why do people abandon their ideal number of children? The NFS asks them to pick their reasons among several options, and the result indicates that they are most likely to choose the financial reason: "raising children and education are too expensive." Fig 10 represents the likelihood[37] that wives of each age group abandon their ideal number of children for a particular reason, and "Financial" represents the financial reason. Aside from this, "Career" represents "it would interfere with my job," and "Burden" represents "I would not handle the psychological and physical burden" (arising from achieving the ideal number). On average, more than 75% of them chose the financial reason, which dominates other reasons, such as "Career" and "Burden," chosen by only 20% of them.

We observe the same even using the JPSC, which asks respondents about their preferences or intentions regarding fertility, such as "Do you want more children in the future?" If they answer "No," it then asks the reasons from fourteen options, which they can choose multiple. The main options are: (1) having and educating children takes too much financial cost; (2) I would rather spend more time with my husband or for myself; (3) I want to continue my job; and (4) I cannot expect my husband's effort in child-rearing. Fig 11 reports the likelihood for each of the four reasons, where I refer to (1), (2), (3), and (4) as *Finanial*, *Time*, *Job*, and *Husband*, respectively. This figure indicates that more than 40% of parents cease reproduction due to the financial reason, which is the most important among other reasons.

[37] I use the term "likelihood" instead of "share" because it allows respondents to choose multiple options.

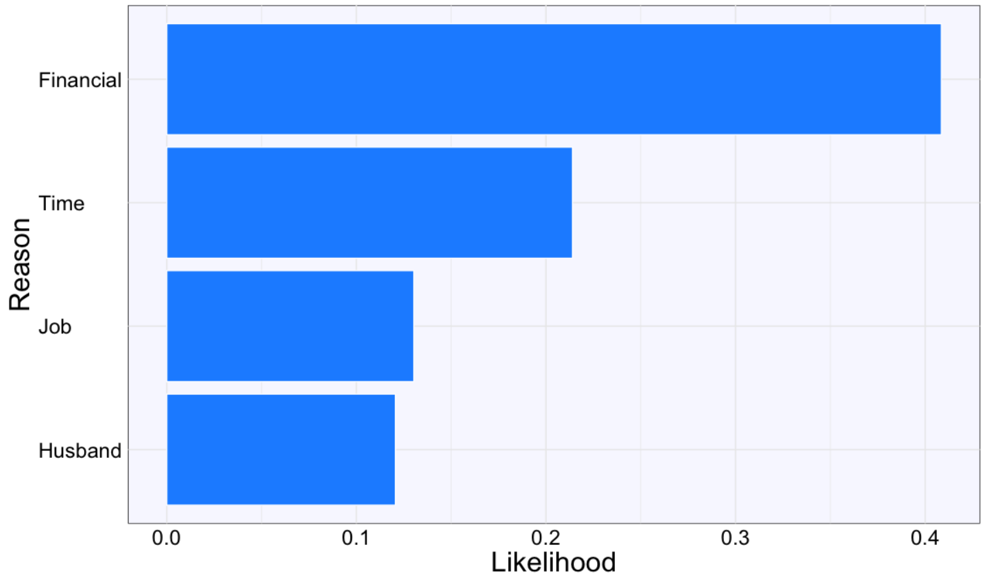

Fig 11: Likelihood for reasons why parents cease reproduction. *Note*: Each of the four reasons, *Finanial*, *Time*, *Job*, and *Husband*, represent: (1) having and educating children takes too much financial cost; (2) I would rather spend more time with my husband or for myself; (3) I want to continue my job; and (4) I cannot expect my husband's effort in child-rearing.

To sum up, there is a non-negligible gap between the ideal and planned numbers of children for couples, and a substantial fraction of them answer the reason as "raising children and education are too expensive." These results establish the first fact: couples are most likely to abandon having an ideal number of children because of financial costs.

Fact 2. *A significant financial cost gap exists between those who have children enrolled in college and those who do not.*

Fact 1 indicates that the financial cost of having children is a critical constraint on fertility choices in Japan. However, it is silent on how and in which cases having children is so expensive; Fact 2 addresses them from the viewpoint of education costs. To do this, I use two data sets: (1) the Survey of Children's Learning Expenses (SCLE, 2021) and (2) the Student Life Survey (SLS, 2018), both cross-sectional household surveys and conducted by the Ministry of Education, Culture, Sports, Science and Technology (MEXT).[38] The SCLE covers more than 53,000 students from preschool to high school and reports the per-student average education expenditure for each expenditure category and education stage (i.e., preschool, elementary, junior high, and high school). The expenditure category includes not only school-related ones (e.g., tuition fees and textbooks) but also

[38]See, https://www.mext.go.jp/b_menu/toukei/chousa03/gakushuuhi/1268091.htm for the SCLE and https://www.jasso.go.jp/statistics/gakusei_chosa/__icsFiles/afieldfile/2021/03/09/data18_all.pdf (in Japanese) for the SLS.

extracurricular activities (e.g., cram school, music, arts, and sports). In addition to that, I use the SLS for parents' expenditures when their children enroll in college.[39]

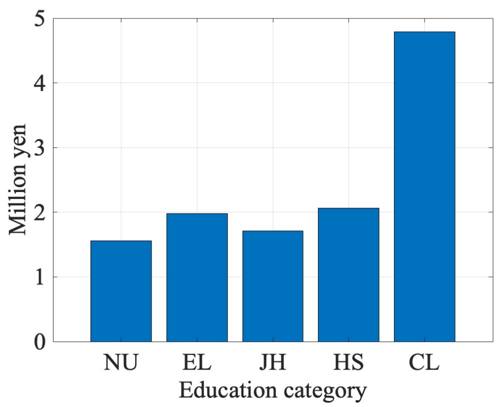

Fig 12: Average education expenditures.

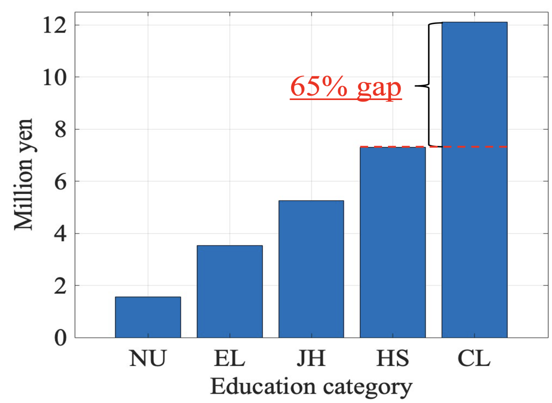

Fig 13: Cumulative education-expenditures.

Fig 12 shows the average per-child expenditure for each education category until completion. "NU," "EL," "JH," "HS," and "CL" each stand for nursery school or preschool, elementary school, junior high school, high school, and college, only consisting of a four-year college. All expenditures are conditional on enrollment in the education stage. Fig 13 represents the cumulative education expenditures, showing that the expenditure jumps up when children attend college. Given that the high school graduation rate is approximately 100% in Japan, the figure tells us that raising one child on average takes the education costs of 7.31 million yen, which amounts to 4.5% of the individuals' average lifetime labor earnings.[40] If their children attend college, they have to spend another 4.8 million yen, meaning there is more than a 60% increase in education costs if parents send their children to college. This expensiveness of college education can, at least partly, be attributed to the fact that Japan is one of the least in subsidizing tertiary education while subsidizing a sizable portion of schooling costs up to secondary education. These observations establish Fact 2: A significant financial cost gap exists between those who have children enrolled in college and those who do not.

Fact 3. *A substantial fraction of parents desire a college education for their children.*

Facts 1 and 2 suggest that the financial costs for children enrolled in college are a

[39]In 2018, the SLS correct answers from 43,394 students attending tertiary education, including college, some college, and graduate school.

[40]I construct a proxy of the average individual's lifetime earnings based on the 2022 Basic Survey on Wage Structure (BSWS) by the Ministry of Health, Labour, and Welfare (MHLW). First, I compute the average monthly earnings of ordinary workers for each age unconditional on any other characteristics such as sex and education. Then, I sum up the average earnings for each age, which amounts to about 160 million yen, and regard it as a proxy of the individuals' average lifetime labor earnings.

critical obstacle to fertility in Japan. However, one might not still be convinced because Japan's college enrollment rate is approximately 55%, far below 100%; thus, college education costs seem relevant only to half of the population.[41] The following fact answers "No, they should not." to this argument by showing that far more fraction of parents desire a college education for their children than the current college enrollment rate.

We return to the NFS (2015), which asked respondents about the desired education level for their children. Fig 14 summarizes the results by wife's age. Here, "SC" stands for some college. The figure shows that approximately 75% of the parents desire a college education for their children, which is significantly higher than the college enrollment rate observed in any period in Japan. This observation suggests that, although college enrollment in Japan has been far below 100%, education costs for a college education can be relevant not only for a specific part of the population because many parents would like to send their children to college.

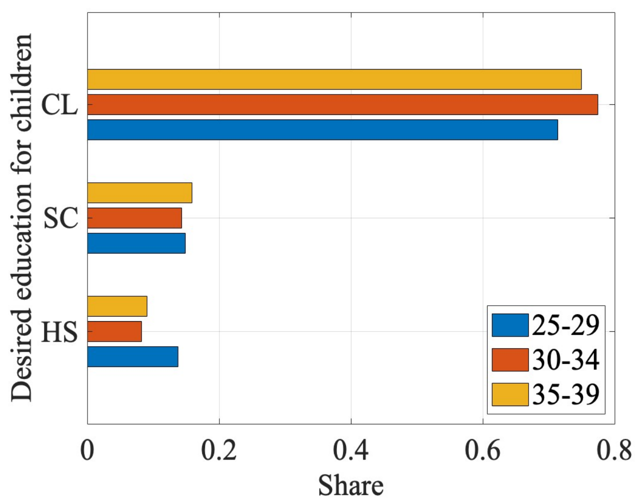

Fig 14: Wives' intention for children's education attainment.

Fact 4. *Japan is one of the least in subsidizing tertiary education.*

Japan is one of the least in subsidizing tertiary education, which roughly corresponds to four-year college education in Japan, given that the enrollment rates for tertiary education other than the four-year college are significantly lower than four-year college enrollment rate.[42]

[41] Dropout rates are insignificant in Japan, so the enrollment rate is almost equivalent to the graduation rate. For example, the dropout rate was 2.5% in 2021. See, https://www.mext.go.jp/content/20220603-mxt_kouhou01-000004520_03.pdf (in Japanese).

[42] According to the Basic School Survey by the MEXT, the enrollment rate for some college was 4% in 2022 and is declining over the past thirty years.

To see this, I use the cross-country data provided by the OECD[43] and define the *subsidization rate* for a specific education category e as follows:

$$s_e = \frac{y_e^{pub}}{y_e^{pri} + y_e^{pub}},$$

where y_e^{pri} and y_e^{pub} represent the private and public spending on education e, both represented as a share of the GDP. Public spending includes expenditures on educational institutions and educational-related subsidies for households or students. Private spending refers to expenditures financed by households and other private entities. Private spending includes expenditures on school but excludes those outside educational institutions (e.g., textbooks purchased by households, private tutoring, and student living costs).[44] In other words, the subsidization rate indicates what fraction of (potential) school-related costs are funded by the government.

As Table 15 shows, Japan subsidizes more than 90% of the costs for primary-to-secondary education, which is higher than the OECD average. On the contrary, the subsidization rate for tertiary education was only 32%, which is the second lowest among OECD countries and less than half of the OECD average of 70%. This fact shows plenty of room for increasing subsidies for college students, which also has driven recent policy discussions on introducing and expanding education subsidies for college students.

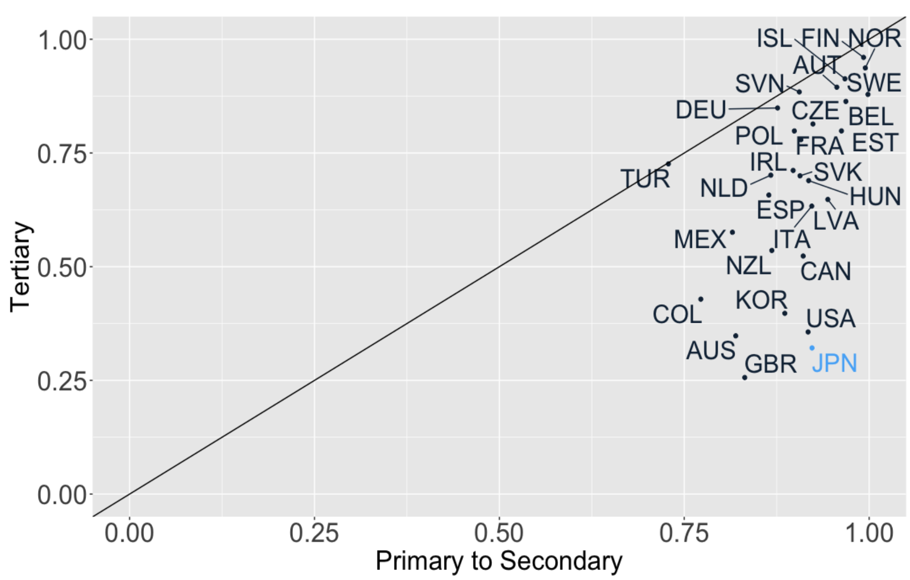

Fig 15: Subsidization rate for each education category (OECD, 2018).

Fig 16 plots the ratio of per-student government expenditures on tertiary education

[43] I use 2018's data given that it is the latest year in which data on a significant number of countries is available.

[44] For more details, see the OECD data (https://data.oecd.org/).

to per-capita GDP. On average, the government expenditures for each student enrolling in tertiary education amount to 33% of per-capita GDP, but the Japanese government only does to 21%.

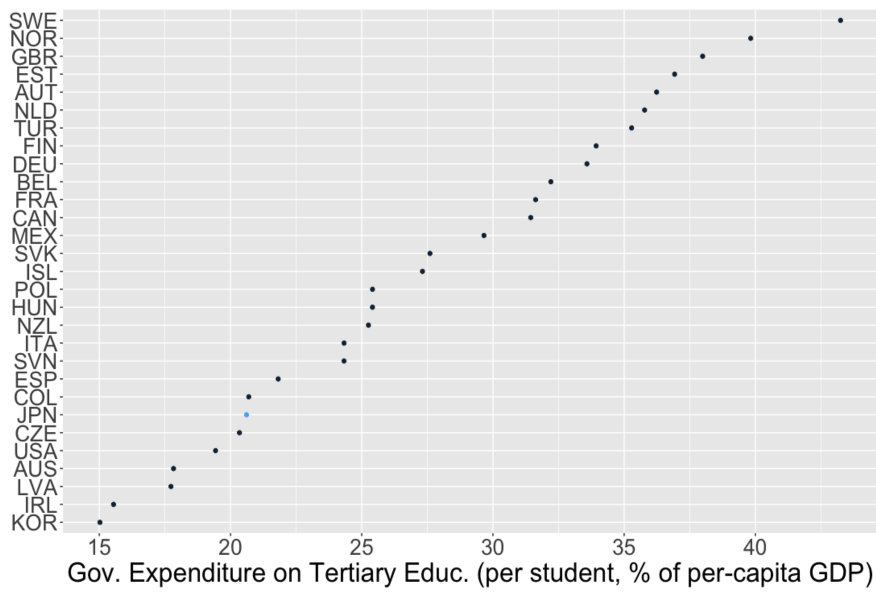

Fig 16: Per-student government spending on tertiary education to per-capita GDP (OECD, 2016-17).

Fig 17 represents the cross-country relationship between log fertility and log government spending on tertiary education (the latter is captured by the ratio of per-student government expenditures on tertiary education to per-capita GDP). Although not statistically significant, we can see the positive relationship between the government expenditures on tertiary education and fertility, which is not surprising given that the expected costs of having children are associated with lower fertility (e.g., Malkova, 2018).

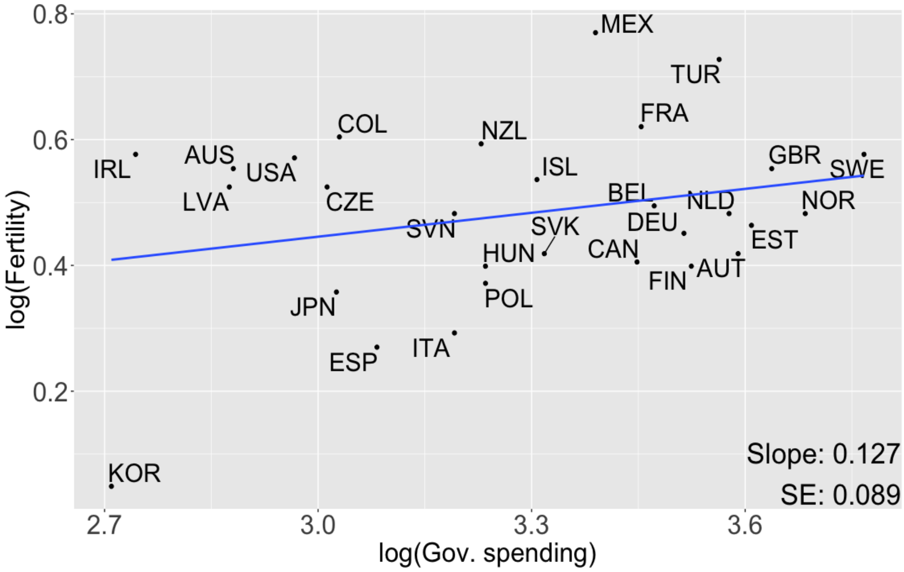

Fig 17: Relationship between total fertility and government spending on tertiary education (OECD, 2016-17).

Fact 5. *Children's college enrollments are ex post associated with higher probabilities of their parents deciding to stop having an additional child.*

This fact links the above four facts and suggests that college education costs are an important determinant of fertility choices in Japan. To see this, I conduct the following regression to examine the relationship between fertility *intentions* and children's college enrollment:

$$\Pr(y_i = 1) = L(\alpha + \beta\, ChildCL_i + \boldsymbol{x}_i\boldsymbol{\theta} + \varepsilon_i),$$

Here, y_i is a dummy variable taking one if a couple i decides to stop having an additional child due to the financial reason. The results are summarized in Table 16. The first column

	(1)	(2)
$ChildCL_i$	0.515***	0.870***
	(0.186)	(0.222)
Personal characteristics	No	Yes
Observations	692	

Table 16: Children's college enrollment and fertility intentions of women aged 24-39. *Note:* Values in parentheses represent standard errors. *** denotes statistical significance at the 1 percent level.

The first column presents the estimates where I control only for the children's college enrollment indicator. In the second column, I add couples' characteristics. The first and second columns report that the log odds ratio for couples choosing the financial reason to stop having children increases by 0.515 and 0.870 points for couples whose children will enroll in college. Both are statistically significant, and the latter implies a 2.4 times higher odds ratio for stopping having an additional child due to financial reasons.

D. Equilibrium Definition

Let $\boldsymbol{x}_j^e$ be an age-specific state vector for agents with education level $e \in \{HS, CL\}$ and $\mu_j^e(\boldsymbol{x}_j^e)$ be the measure of agents with state vector $\boldsymbol{x}_j^e$. Let $I_l(h, I)$ and $I_g(h, I)$ be indicator functions for loans and grants, respectively, returning 1 if students with (h, I) are eligible and 0 otherwise.

Given exogenous parameters and policy rules $\{\tau_a, \tau_c, \iota, \iota_s, B, S, \psi, p, I_l(h, I), I_g(h, I)\}$, a *stationary recursive competitive equilibrium* consists of

- value functions $\{V_{g0}, V_{g1}, V_{g2}, V^w, V^f, V^{wf}, V^{IVT}, V^r\}$,
- policy functions for consumption, savings, leisure $\{c_j^e(\boldsymbol{x}_j^e), a_j^e(\boldsymbol{x}_j^e), l_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J}$, working hours $\{h_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J_R}$, fertility $\{n_{J_F}^e(\boldsymbol{x}_{J_F}^e)\}$, IVT $\{a_{J_{IVT}^e}(\boldsymbol{x}_{J_{IVT}}^e)\}$, and college enrollment $\{e_{J_E}(\boldsymbol{x}_{J_E})\}$,
- prices (r, w_{HS}, w_{CL}),
- labor income tax rate τ_w,
- aggregate quantities (K, L_{HS}, L_{CL}),
- measures for households $\{\mu_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J}$,

such that:

1. The decision rules of students, workers, and retired households solve their problems, and $\{V_{g0}, V_{g1}, V_{g2}, V^w, V^f, V^{wf}, V^{IVT}, V^r\}$ are the associated value functions.

2. The representative firm maximizes its profit and optimally chooses capital and labor inputs:

$$r + \delta = \alpha \cdot Z \cdot \left(\frac{K}{L}\right)^{\alpha-1}, \tag{13}$$

$$w_{HS} = \tilde{Z} \cdot \omega_{HS} \cdot L_{HS}^{\chi-1}, \tag{14}$$

$$w_{CL} = \tilde{Z} \cdot \omega_{CL} \cdot L_{CL}^{\chi-1}, \tag{15}$$

where

$$
\begin{aligned}
L &= \left[\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}\right]^{1/\chi}, \\
\tilde{Z} &= (1-\alpha) \cdot Z \cdot \left(\frac{K}{L}\right)^{\alpha} \cdot \left[\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}\right]^{1/\chi - 1}.
\end{aligned}
$$

3. The labor market for each skill $e \in \{HS, CL\}$ clears:

$$
L_e = \sum_{j=J_E}^{J_R} \int_{\boldsymbol{x}_j^e} \eta_j^e(\boldsymbol{x}_j^e) \cdot h_j^e(\boldsymbol{x}_j^e)\; d\mu_j^e(\boldsymbol{x}_j^e), \tag{16}
$$

where $\eta_j^e(\boldsymbol{x}_j^e)$ represents the labor efficiency of agents with a state vector $\boldsymbol{x}_j^e$.

4. The capital market clears:

$$
K = \sum_{j=J_E}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} a_j^e(\boldsymbol{x}_j^e)\; d\mu_j^e(\boldsymbol{x}_j^e). \tag{17}
$$

5. The government budget is balanced:

$$
\tau_c \cdot C + \tau_w \cdot (L_{HS} + L_{CL}) + \tau_a \cdot K + Q = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \le 17} + S,
$$

where

$$
\begin{aligned}
C &= \sum_{j=J_E}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} c_j(\boldsymbol{x}_j^e)\; d\mu_j^e(\boldsymbol{x}_j^e), \\
Q &= \sum_{j=J_R+1}^{J} \frac{1-\zeta_{j-1,j}}{\zeta_{j-1,j}} \sum_{e} \int_{\boldsymbol{x}_j^e} a_j^e(\boldsymbol{x}_j^e)\; d\mu_j(\boldsymbol{x}_j^e), \\
p \cdot \mu_{old} &= \sum_{j=J_R}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} p\; d\mu_j^e(\boldsymbol{x}_j^e), \\
K_s &= \int_{\boldsymbol{x}^s} \max\{0, -a^s(\boldsymbol{x}^e)\} \cdot I_l(h, I)\; d\boldsymbol{x}^s \\
G &= \int_{\boldsymbol{x}^s} g(h, I) \cdot I_g(h, I)\; d\boldsymbol{x}^s \\
B \cdot \mu_{j \le 17} &= \sum_{j=J_F}^{J_{IVT}-1} \sum_{e} \int_{\boldsymbol{x}_j^e} B \cdot n\; d\mu_j^e(\boldsymbol{x}_j^e),
\end{aligned}
$$

where $\boldsymbol{x}^s$, $\mu^s(\boldsymbol{x}^s)$, and $\{a^s(\boldsymbol{x}^s)\}$ represent a state vector for college students, measure of college students, and students' policy function for saving, respectively.

6. Distributions (measures) and households' behavior are consistent.

E. Computational Algorithm

E. (i) Stationary Equilibrium

For any variable or distribution x, let $\tilde{x}$ and $\hat{x}$ represent its guessed and model-implied values. Also, let μ_j represent the distribution over state variables for age j. Dividing the computation process into three blocks makes it easier to understand. The first block is the outer loop, searching for equilibrium prices. The other two blocks are inner loops; one is the *optimization block*, solving household problems given prices, and another is the *distribution block*, searching for the stationary distributions given prices and policy functions obtained in the optimization block. The computational algorithm proceeds as follows:

1. Guess prices $\tilde{\boldsymbol{p}} = (\tilde{r}, \tilde{w}_{HS}, \tilde{w}_{CL})$.

2. *Optimization block*:

   - Guess the value function for agents at the beginning of age $j = 18$ ($\tilde{V}_{g0}$).
   - Given $\tilde{V}_{g0}$, solve backward from the period of IVT choice to that of the education choice, which gives the model-implied value function for V_{g0}, $\hat{V}_{g0}$.
   - Check if

   $$d(\tilde{V}_{g0}, \hat{V}_{g0}) < \varepsilon, \tag{18}$$

   where $d(\cdot)$ and $\varepsilon > 0$ represent an arbitrary metric function and error tolerance. If (18) is not satisfied, update $\tilde{V}_{g0}$ and follow the same procedure until convergence. The correct V_{g0} pins down all value functions and policy functions given a set of prices $\tilde{\boldsymbol{p}}$.

3. *Distribution block*:

   - Guess the distribution for age J_{IVT}. This $\tilde{\mu}_{J_{IVT}}$ and policy functions for IVT derive the implied distribution for agents aged 18, $\hat{\mu}_{18}$. Given $\hat{\mu}_{18}$ and policy functions, compute the implied distributions for age $j = 19, ..., J_{IVT}$, and obtain $\hat{\mu}_{J_{IVT}}$.
   - Check if

   $$d(\tilde{\mu}_{J_{IVT}}, \hat{\mu}_{J_{IVT}}) < \varepsilon. \tag{19}$$

   If (19) is not satisfied, update $\tilde{\mu}_{J_{IVT}}$ and follow the same procedure until convergence.

- After obtaing the correct distributions for age $j = 18, ..., J_{IVT}$, compute the distribution for age $j_{IVT}+1$ onward, using those distributions and policy functions.

4. After computing value functions, policy functions, and distributions, compute the implied quantities, $\hat{L}$ and $\hat{K}$ based on (16) and (17), which gives the implied prices $\hat{\boldsymbol{p}}$ based on $\hat{L}$, $\hat{K}$, (13), (14), and (15).

5. Check if

$$d(\tilde{\boldsymbol{p}}, \hat{\boldsymbol{p}}) < \varepsilon. \tag{20}$$

If (20) is not satisfied, update $\tilde{\boldsymbol{p}}$, return to the optimization block, and follow the same procedure until convergence.

E. (ii) Transition Dynamics

For any variable or distribution x, let $\tilde{x}$ and $\hat{x}$ represent its guessed and model-implied value (or distribution). Also, let $\boldsymbol{q}_t = (K_t, L_t)$ be a vector of aggregate quantities in a year t, and $\boldsymbol{q} = (\boldsymbol{q}_t)_{t=1,...,T}$ for some T. The computational algorithm for transition dynamics proceeds as follows:

1. *Preliminaries*: Set an arbitrarily large number for transition periods, T. Given aggregate quantities in the initial and final steady states, $\boldsymbol{q}_1$ and $\boldsymbol{q}_T$, guess a sequence of aggregate quantities, $\{\tilde{\boldsymbol{q}}_t\}_{t=1,...,T}$, where $\tilde{\boldsymbol{q}}_1 = \boldsymbol{q}_1$ and $\tilde{\boldsymbol{q}}_T = \boldsymbol{q}_T$, which pins down guesses for prices $\{\tilde{r}_t, \tilde{w}_t\}_{t=1,...,T}$. Also, guess a sequence of the labor income tax rate $\{\tilde{\tau}_{l,t}\}_{t=1,...,T}$, where $\tilde{\tau}_{l,1}$ and $\tilde{\tau}_{l,T}$ are the tax rates at the initial and final steady states.

2. *Optimization*: Given $\{\tilde{r}_t, \tilde{w}_t\}_{t=1,...,T}$ and $\{\tilde{\tau}_{l,t}\}_{t=1,...,T}$, solve the optimization problem for each cohort born before period T.

3. *Distribution*: Given decision rules for each cohort obtained in the above optimization block, construct the implied distributions:

   - $\tilde{\mu}_a(j;t)$: (aggregate) savings distribution across age in year t.
   - $\tilde{\mu}_n(j;t)$: (aggregate) labor supply distribution across working-age in year t.
   - $\tilde{\mu}_{j|t}$: age distribution in year t.

4. Given the decision rules and the implied distributions, compute the model-implied aggregate quantities, $\hat{\boldsymbol{q}}$. Also, compute the implied government revenue and expenditures, denoted by $\hat{\boldsymbol{G}}_r = (\hat{\boldsymbol{G}}_{r,t})_{t=1,...,T}$ and $\hat{\boldsymbol{G}}_e = (\hat{\boldsymbol{G}}_{e,t})_{t=1,...,T}$.

5. Check if

$$d(\tilde{\boldsymbol{q}}, \hat{\boldsymbol{q}}) < \varepsilon$$

and

$$d(\hat{\boldsymbol{G}}_r, \hat{\boldsymbol{G}}_e) < \varepsilon.$$

If the first (second) condition is not satisfied, update $\tilde{\boldsymbol{q}}$ ($\{\tilde{\tau}_{l,t}\}_{t=1,\ldots,T}$) and follow the same procedure until convergence.

F. More on Calibration

F. (i) Income process

TBA

G. Reallocating Resources to Different Programs

This section examines the performance of the potential alternative programs to the existing income-tested grants. To this end, I consider another two scenarios, in addition to the introduction of income-tested grants to the benchmark model, examined in Section 5.1. The first scenario is to introduce grants for college students with income and ability tests so that "high-ability" students in poor households are eligible. More specifically, I keep the income threshold adopted in the existing scheme but arbitrarily set the lower bound for students' human capital for the eligibility, $\underline{h}$, to its median. The payment function $g(h, I)$ for this scheme is defined as follows:

$$g(h, I) = \begin{cases} g_1 & \text{if } I < \bar{I} \ \& \ h > \underline{h} \\ 0 & \text{otherwise} \end{cases}$$

Here, the payment g_1 is set so that the short-run expenditure upon the introduction is the same as that of the existing income-tested grants. As a result, g_1 covers approximately 100% of the average student's expenditure in the benchmark, which is greater than the payment in the existing scheme with only income tests because the number of eligible students is fewer.

The second scenario introduces unconditional grants for college students regardless of ability and income. The payment function is given as $g(h, I) = g_2$ for any students with (h, I), where g_2 is set so that the short-run government expenditure is the same as the existing program. As a result, g_2 covers approximately 10% of the average students' expenditure in the benchmark, which is less significant than in the existing income-tested grants because this alternative program covers a broader range of students.

I solve the stationary equilibrium with each scenario, and the results regarding fertility and enrollment rates are summarized in Table 17. A highlight is that the existing scheme with income tests would lead to the highest equilibrium TFR among other scenarios. Notably, the unconditional grants lead to a 1.2 p.p. higher college enrollment rate than the income-tested ones and a -0.3% lower TFR because of the composition effect. These results highlight the insurance effect of the income-tested grants on fertility and the downward pressure the grants exert on aggregate fertility through composition changes.

	Income	+Ability	Uncond.
CL share (Δ p.p.)	+3.9	+2.6	+5.1
TFR (Δ%)	+3.0	+2.7	−0.3
HS	+0.4	−3.5	+4.0
CL	+7.4	+8.4	+3.9
Output (Δ%)	+1.0	−0.1	−0.7
STD (wage) (Δ%)	−1.3	−0.9	−0.1
Welfare (%)	+5.1	+2.6	+1.6

Table 17: Main results with several schemes with different targets. *Note*: values in each cell indicate changes from the benchmark value. Rows "HS" and "CL" indicate the percentage changes in the fertility of high school and college graduates.

H. Comparison With Pro-natal Transfers: Macroeconomic Effects

This section examines the differences in the macroeconomic implications between the education subsidy and child benefit (i.e., typical pro-natal cash transfers).[45] To this end, I simulate the expansion of the unconditional cash transfers to households with children B from its benchmark value while setting $g(I) = 0$ for any I as in the benchmark. The new level of the per-child payment B is set so that the short-run expenditure upon the introduction is the same as that of the existing income-tested grants, which leads to a 6.5% increase in the per-child payment.

I solve the stationary equilibrium with the expansion of the per-child payment B, and the main results are summarized in Table 18. Columns "Education" and "Typical" represent the results of the introduction of income-tested grants and the expansion of child benefit ("typical" pro-natal transfers).

[45] In Section G., I also consider reallocating resources into different targets to implement the education subsidy by considering unconditional transfers to college students and introducing ability tests.

The child benefit expansion leads to a 2.5% increase of the TFR in the long-run equilibrium, comparable to introducing the income-tested grants but slightly lower than that. The previous decomposition analysis suggests that targeting students in unlucky households in which negative income shocks are realized would be effective, at least marginally, in increasing the TFR. A notable observation is that the characteristics of households who respond to the policy differ between the two cases; the education subsidy induces skilled households to have more children, while the child benefit has a similar degree of impact on skilled and unskilled households regarding fertility. This is because the former is more beneficial for households whose potential children are likely to attend college, and college graduates tend to be those households due to the intergenerational persistence of education.

The college enrollment rate does not change in response to the child benefit expansion. The expansion also does not affect the average human capital, defined as the workers' average of labor efficiency $\eta_{j,z,e,h}$,[46], while the education subsidy increases the average human capital by 1.0% primarily because of the higher college enrollment rate. The difference in the average human capital also leads to the difference in the per-capita output. While the education subsidy increases the per-capita output by 1.0% in the long run, the child benefit expansion would rather lead to a 0.3% lower per-capita output. As in the case of introducing the education subsidy, the child benefit expansion also leads to lower (physical) capital accumulation due to crowding out and demographic change, as discussed in Section 5.1. The education subsidy has sufficiently large positive effects on aggregate labor supply in efficiency units, which surpasses the negative impacts on physical capital accumulation and increases output. The child benefit expansion has more modest impacts on aggregate labor efficiency than the education subsidy, leading to the gap in output gains between the two policies.

The child benefit expansion does not affect the standard deviation of wages, whereas the education subsidy leads to a lower standard deviation by enabling some students in poor households to attend college and acquire skills. Lastly, the expansion leads to a 0.3% of the welfare gain, which is substantially lower than a 4.8% gain with the education subsidy. As discussed in Section 5.2.1, the welfare gains of the education subsidy come mainly from the higher expected lifetime income in each state and the higher probability of being college graduates and enjoying higher earnings facilitated by the higher college enrollment rate and educational mobility, as opposed to the child benefit expansion do. Thus, the education subsidy leads to greater welfare gains than the child benefit expansion

[46]Formally, I define the average human capital as $\sum_j \int_{z,h} [(1-s)\cdot\eta_{j,z,e=0,h} + s\cdot\eta_{j,z,e=1,h}]dF(z,h)\mu_j$, where s represents the college enrollment rate, $F(z,h)$ represents the stationary distribution over (z,h), and μ_j is the stationary distribution of age.

under the veil of ignorance.

A caveat is that this model does not capture the endogenous human capital accumulation before high school graduation and the dynamic complementarity of human capital. Households may increase their investments in their children upon the child benefit expansion, which contributes to greater human capital; however, that channel is not considered in this current analysis. Those ingredients can be critical in comparing the effects of these different programs, especially on college enrollment rates, aggregate human capital, and output. The exercise in this framework is a first step, and incorporating those ingredients – fertility, college enrollment, dynamic complementarity of human capital– in one framework is left for future research.

	Education	Typical
TFR (Δ%)	+3.0	+2.5
HS	+0.4	+2.5
CL	+7.4	+2.4
CL share (Δ p.p.)	+3.9	0.0
HS→CL	+2.5	0.0
Avg. HC (Δ%)	+1.0	0.0
Output (Δ%)	+1.0	−0.3
STD (wage) (Δ%)	−1.3	0.0
Welfare (%)	+5.1	+0.3

Table 18: Main results with several schemes with different targets. *Note*: Columns "Education" and "Typical" represent the results with the introduction of the education subsidy and expansion of child benefit ("typical" pro-natal transfers), respectively. Values in each cell indicate changes from the benchmark value. Rows "HS" and "CL" indicate the percentage changes in the fertility of high school and college graduates. "HS→CL" represent the changes in college enrollment rate and educational mobility in the sense of probability that children of high school graduates attend college. "Ave. HC" stands for the average human capital.

I. Comparison With Other Transfers in Insuring the Risks

This section studies four other transfers that potentially reduce or insure the risks associated with fertility choices under income uncertainty, other than the income-tested subsidy examined in Section 5: (1) child allowance (i.e., unconditional transfers for households with children), (2) unconditional education subsidy, (3) wage insurance that provides cash benefits with households with a negative income shock, and (4) fertility-dependent wage insurance that provides wage insurance benefits with households depending on the number of children they have. I implement the introduction of these policies in an expenditure neutral way. More specifically, I feed the income-tested education subsidy into the model as I do in Section 5, and then set policy parameters for each cash transfer so that the short-run government expenditures of each policy is the same as those of the income-tested education subsidy.

Uniform education subsidy: For the uniform education subsidy, the payment function is given as $g(I) = \tilde{g}$ for any students with (I), where g is set in the expenditure neural manner and then covers approximately 10% of the average students' expenditure in the benchmark.

Child allowance: For child allowance, I simulate the expansion of the unconditional cash transfers to households with children B from its benchmark value. The new level of the per-child payment B is 6.5% higher than the benchmark.

Wage insurance: Wage insurance provides workers facing negative income shocks (i.e., $z < 0$) with cash transfers. In the uniform wage insurance, I introduce the following payment function into the budget constraint for workers:

$$w_I(z) = \begin{cases} w_I & \text{if } z < 0 \\ 0 & \text{otherwise} \end{cases}$$

For the fertility-dependent wage insurance, I introduce the following payment function into the budget constraint for workers:

$$\tilde{w}_I(z, n, j) = \begin{cases} n \cdot \tilde{w}_I & \text{if } z < 0 \& j < J_{IVT} \\ 0 & \text{otherwise} \end{cases}$$

Under the fertility-dependent wage insurance, workers facing a negative income shock receive wage insurance benefits proportional to the number of children younger than 18 that they have.

.(i) Effects on fertility with and without income uncertainty

I simulate each policy in two different setups and examine the fertility responses. The one setup is "with income risk," which is the calibrated model in which the parameter governing income volatility takes a positive value (i.e., $\sigma_z = 0.02$). The other setup is "without income risk," in which there is no stochastic component in household's income. In other words, I set $\rho_z = 1$ and $\sigma_z = 0$ to consider an economy without income uncertainty. To capture the direct effect of the policy on fertility choices, I first conduct this simulation under partial equilibrium setting in which prices, taxes, and distributions are held fixed.

The results are summarised in Table 19. First, introducing fertility-dependent wage insurance increases fertility by 0.9%, whereas introducing uniform wage insurance has no effects on fertility. Next, as the illustrative model suggests, the increase in fertility rates due to the policy is more significant under income uncertainty, compared with a model without income uncertainty. For example, the expansion of child allowance leads to a 1.7% increase in fertility rates under income uncertainty, whereas, without income uncertainty, the increase is only a 0.2%. This result suggests that policies compensating costs of having children play more critical roles under income uncertainty.

	W/risk	No risk
Child allowance	+1.7%	+0.2%
Universal. Edu.	+3.2%	+1.6%
Income-test Edu.	+2.2%	+0.2%
Wage ins. (uniform)	+0.0%	–
Wage ins. (fertility)	+0.9%	–

Table 19: Changes in fertility with and without income uncertainty.

The difference in the fertility responses between with and without income uncertainty is more significant for income-tested education subsidy: it increases fertility by 0.2% without income risk but 2.2% with income risk.