Education Policies, Fertility Differentials, and Macroeconomic Outcomes*

Kanato Nakakuni†

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Abstract

This paper examines the macroeconomic consequences of college financial aid policies, considering their effects through fertility margins. I construct a heterogeneous agent general-equilibrium overlapping-generations model with choices on fertility, college enrollment, and inter-vivo transfers, calibrated to Japan. Income-tested grants provide partial insurance against the risks of having a child arising through multiple sources of uncertainty, especially for college graduates, and increase the fertility of ex-post ineligible ones. These fertility margins significantly amplify their positive effects on college enrollment and per-capita output via intergenerational linkages and changes in demographic structure. While the eligibility expansion or unconditional grants lead to higher college enrollment rates, they put significant downward pressure on aggregate fertility and the labor force in the long run through several channels, resulting in fewer output gains.

Keywords: Fertility, education, intergenerational linkages
JEL codes: C68, I28, J13, J24.

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†University of Tokyo, Japan. Email address: nakakunik@gmail.com.

1 Introduction

Education subsidy is a fundamental policy for achieving higher aggregate productivity in the presence of credit market imperfections and uncertain education returns. It plays a more critical role in countries experiencing low fertility or demographic aging, where the working-age population share is decreasing, per-capita output and tax base are thus shrinking, whereas social security expenditures are increasing. The policy will enhance the labor force's "quality," mitigating the issues of shrinking its "quantity."

However, the subsidy, especially for college education (i.e., college financial aid policies), would have unintended consequences on the "quantity": it increases the share of educated people who typically have fewer children,[1] lowering the aggregate fertility (a composition effect). Also, cheaper education may shift parents from the "quantity" to the "quality" of children (a behavioral effect). Either effect can unintentionally accelerate the shrinking labor force's "quantity" in the long run, while both will contribute to higher "quality." What are the macroeconomic consequences of college financial aid policies, considering their impacts on the "quantity" and "quality" of the labor force?

To answer the question, I construct a heterogeneous-agent general-equilibrium (GE) overlapping-generations (OLG) model with choices on fertility, college enrollment, and inter-vivo transfers (IVT). A central contribution of this study is to build a new GE model that incorporates fertility and college enrollment choices, which sheds light on the critical roles of fertility margins in evaluating the macroeconomic performance of college financial aid policies.

In this model, agents choose whether to attend college after graduating high school. While in college, students finance tuition fees and their consumption by the IVT from their parents, labor earnings, and government financial aid if eligible. At a certain point in life, they make fertility choices, choosing how many children they have. They draw utility from the number of children, education spending until their children graduate high school, and IVT made for them upon their high school graduation based on altruism. They also make labor supply decisions until retirement age, and their labor productivity is subject to idiosyncratic risks.

The model is calibrated to Japan, a leading country in low fertility and demographic aging, using the Japanese Panel Survey of Consumers (JPSC). The model replicates key moments, such as (1) the average parental asset transfers for college students, (2) the intergenerational persistence of education levels, and (3) fertility differentials across education levels. I also validate the model's fertility behavior using empirical estimates for

[1] Appendix A. provides some theoretical explanations for why and when a negative fertility-income relationship arises using simple models, which is informative for understanding the fertility differential across education groups. For more details, see Jones, Schoonbroodt, and Tertilt (2010).

the cash-benefit elasticity of fertility, indicating the extent to which fertility rates increase in response to cash transfers. The benchmark model captures government-provided student loans, which are income- and ability-tested, but does not include the existing grants introduced in 2020. The existing grants are income-tested, and approximately the bottom 15% of the income distribution of households are eligible. The payments cover approximately two-thirds of the students' average expenses. No grants have been available to college students until the introduction in 2020, meaning that no grants are available to them in the benchmark model.

The main findings are summarized as follows. First, the introduction of income-tested grants leads to a 3.9 percentage point (p.p.) higher college enrollment rate, putting downward pressure on aggregate fertility and the labor force in the long run, given that college graduates have fewer children. However, their overall effects on aggregate fertility are positive (approximately 3% compared with the benchmark) despite the composition effect because they significantly increase the fertility of college graduates, notably including ex-post ineligible ones; approximately 40% of the increase (i.e., 1.2 p.p. of the 3 percent increase) is explained by the fertility increase of ex-post ineligible college graduates. A decomposition analysis suggests that the income-tested grants provide partial insurance against income uncertainty and the risks of having a child.

To understand this result, note that the intergenerational persistence of education levels implies that college graduates expect their (potential) children to attend college when they make fertility choices. Thus, they also expect sizable IVT they have to make to support their college enrollment. However, the future realization of negative income shocks would make it infeasible for parents to financially support their children's enrollment, or they may end up with low consumption in exchange for supporting the enrollment: either situation is costly for parents. Thus, income volatility with higher expected IVT makes college graduates hesitate to have a child. In this situation, the grants provide partial insurance against such a costly state, make college graduates comfortable having a child, and increase the fertility of college graduates, including ex-post ineligible ones.

These fertility margins amplify the policy effects on college enrollment rate and per-capita output in the long run through intergenerational linkages and changes in demographic structure. First, as college graduates' fertility increases, those "marginal" children born are likely to attend college due to intergenerational transmission of human capital, education tastes, and assets (IVT). This fertility margin contributes to a higher share of college graduates in the long run, which in turn contributes to greater per-capita output. Second, the higher aggregate fertility implies a more significant share of the working-age population, which increases per-capita output. These roles of fertility margins are highlighted by results in an exogenous fertility version of the model in which household decision rules regarding fertility are fixed as in the benchmark. For example, the grant introduction leads to a 1% increase in per-capita output under endogenous fertility, which is 2.5 times larger than under exogenous fertility.

Aggregate fertility and per-capita output increase further by a marginal expansion setting a higher income threshold. However, the marginal effects diminish in expansion and eventually become negative. First, the effects on aggregate fertility diminish in expansion because the fertility of high school (HS) graduates decreases in the eligibility expansion primarily for the following reasons. The expansion increases education mobility in the sense that children of HS graduates are more likely to attend college. The higher education mobility makes HS graduates expect their (potential) children to be more likely to attend college. The higher probability of children's college enrollment increases the expected costs of having children, given that the grants are not generous enough to cover 100% of the costs of sending their children to college, and the higher expected costs decrease the HS graduates' fertility. Although the college graduates' fertility increases due to the insurance effect, the lower fertility of HS graduates leads to limited marginal effects on aggregate fertility. The reduced fertility differential across the two education groups contributes to a higher college enrollment rate via intergenerational linkages. However, the positive effects on output in expansion diminish as the broader coverage leads to lower savings among households with children, reducing the capital stock of the economy.

Last, I examine the performance of the potential alternative programs, such as unconditional grants for college students regardless of income, in an expenditure-neutral way. A highlight is that unconditional grants lead to the highest college enrollment rate among other programs and a 0.3% lower fertility rate than in the benchmark because of the composition effect. This result highlights the insurance effect of the income-tested grants on fertility and the downward pressure on aggregate fertility via composition effects.

The rest of the paper is organized as follows. Section 2 reviews several strands of literature related to this study. Section 3 describes the model, which is calibrated in Section 4. Section 5 conducts numerical analysis and Section 6 concludes.

2 Related Literature

This paper contributes to several strands of literature. The first is the literature on macroeconomic analysis of college financial aid policies.[2] I incorporate fertility choices into a framework otherwise standard in the literature (i.e., the heterogeneous-agent GE-

[2]See, for example, Benabou (2002); Akyol and Athreya (2005); Kulikova (2015); Krueger and Ludwig (2016); Lawson (2017); Abbott, Gallipoli, Meghir, and Violante (2019); Lee and Seshadri (2019); Abbott (2022); Daruich (2022); Matsuda and Mazur (2022); Matsuda (2022).

OLG model with education and IVT choices). As a result, this model allows the fertility differential between education groups as observed in data, which starkly contrasts models in this literature in which all households have the same number of children. My extended framework with fertility choices and fertility differential shed light on the roles of fertility margins in understanding the macroeconomic performance of the policy.

Second, this study closely relates to macroeconomic studies based on the quantity-quality trade-off framework.[3] My primary contribution to the literature is constructing a new model with college enrollment choices in the GE framework, enabling us to examine college financial aid policies.[4] From the modeling viewpoint, the closest work is Daruich and Kozlowski (2020). They construct a partial equilibrium lifecycle model with college enrollment, fertility, and IVT choices to investigate the roles of fertility choices and family transfers in explaining intergenerational mobility in the US. This study differs from theirs in adopting the GE framework, which is critical to examine the macroeconomic consequences of college education policies.[5]

Third, it is closely related to the literature on fertility choices in incomplete market models.[6] Previous studies point out that having a child can be considered making "consumption commitments," and income volatility thus makes households hesitate to have children.[7] This study contributes to the literature by examining policies that can potentially reduce the "risks" of having a child. It highlights the insurance effects of income-tested college subsidies on fertility choices, especially for college graduates whose children are likely to attend college. These findings build on a new model with college enrollment choices and intergenerational linkages.

Last, this study contributes to the literature on pro-natal policies using lifecycle models with fertility choices[8] by studying the effects of college financial aid, a novel pro-natal

[3]See, for example, De La Croix and Doepke (2003); Manuelli and Seshadri (2009); Cordoba, Liu, and Ripoll (2019); Daruich and Kozlowski (2020); Zhou (2022); Kim, Tertilt, and Yum (2023).

[4]Several papers are closely related to this study. The critical difference is that this study incorporates college enrollment choices into a full-lifecycle model to examine college education policies. Kim, Tertilt, and Yum (2023) build a two-period OLG model with the quantity-quality trade-off to examine the role of the status externalities in education on the low fertility in South Korea. Zhou (2022) builds a multi-period GE-OLG model capturing the quantity-quality trade-off and endogenous human capital accumulation for children to study the macroeconomic consequences of family policies. De la Croix and Doepke (2004) construct a two-period OLG model with fertility choices to compare the macroeconomic implications of public and private schooling regimes.

[5]A higher college enrollment rate facilitated by college subsidies reduces the skill premium in the long run. The change in skill premium implies the change in income distribution (e.g., Krueger and Ludwig, 2016) and also affects the incentive of college enrollment (e.g., Abbott, Gallipoli, Meghir, and Violante, 2019). My results presented in Section 5 also highlight the roles of GE effects in explaining the long-run effects of education subsidies on fertility.

[6]For example, Schoonbroodt and Tertilt (2014); Santos and Weiss (2016); Sommer (2016).

[7]This is the case, especially in the early stages of their lives, thus delaying marriage and fertility (Santos and Weiss, 2016). Delaying fertility leads to low fertility, given that the ability to reproduce declines with age (Sommer, 2016).

[8]Previous studies examine the effects of childcare subsidization (e.g., Bick, 2016), cash transfers (e.g.,

policy conducted in Japan. As discussed in Appendix B., the college financial aid policies are also considered a pro-natal policy in Japan, given facts suggesting that the financial costs for parents to support their children's college enrollment are a significant impediment to fertility decisions. Compared with typical pro-natal transfers such as baby bonuses, its notable feature is that it will increase the average human capital by promoting skill acquisition.[9] By examining this novel policy in Japan, this study provides insights into countries considering countermeasures against macroeconomic concerns of low fertility.

3 Model

This section describes the incomplete market GE-OLG model incorporating choices on college enrollment, IVT, and fertility. I embed fertility choices into a framework otherwise standard in the macroeconomic literature of college subsidy (e.g., Krueger and Ludwig, 2016; Abbott, Gallipoli, Meghir, and Violante, 2019; Matsuda and Mazur, 2022). Section 3.1 provides an overview of the lifecycle of this economy. Section 3.2 elaborates on preliminaries of the model and then Section 3.3 formulates households' decision problems. Section 3.4 discusses the stationary equilibrium of this economy.

3.1 Overview of the lifecycle

Fig 1 represents the households' lifecycle in this model. In this model, one period corresponds to two years. Letting j denote age, agents live with their parents until they graduate from high school at the beginning of age $j = J_E (:= 18)$ and then choose whether to attend college or enter the labor market after graduating high school, represented as a node "Grad.HS" in the figure. If they do not enroll in college, they enter the labor market as a high school graduate. If they choose to attend college, it takes four years (two model-periods) to complete, and they enter the labor market as a college graduate after graduation, represented as a node "Grad.CL" in the figure.[10]

Kim, Tertilt, and Yum, 2023; Nakakuni, 2023), both of them (e.g., Hagiwara, 2021; Zhou, 2022), parental leave policies (e.g., Erosa, Fuster, and Restuccia, 2010; Yamaguchi, 2019; Kim and Yum, 2023), and tax reform (e.g., Jakobsen, Jørgensen, and Low, 2022).

[9]Previous studies show that pro-natal transfers would lower aggregate human capital because they make parents shift from the "quality" toward "quantity" of children (e.g., Zhou, 2022; Kim et al., 2023). Actually, Appendix E. shows that the income-tested grants would lead to greater aggregate human capital and output than pro-natal transfers conducted in an expenditure-neutral way.

[10]This model does not consider the possibility of dropping out from college because, as I mentioned, the dropout rate is insignificant in Japan.

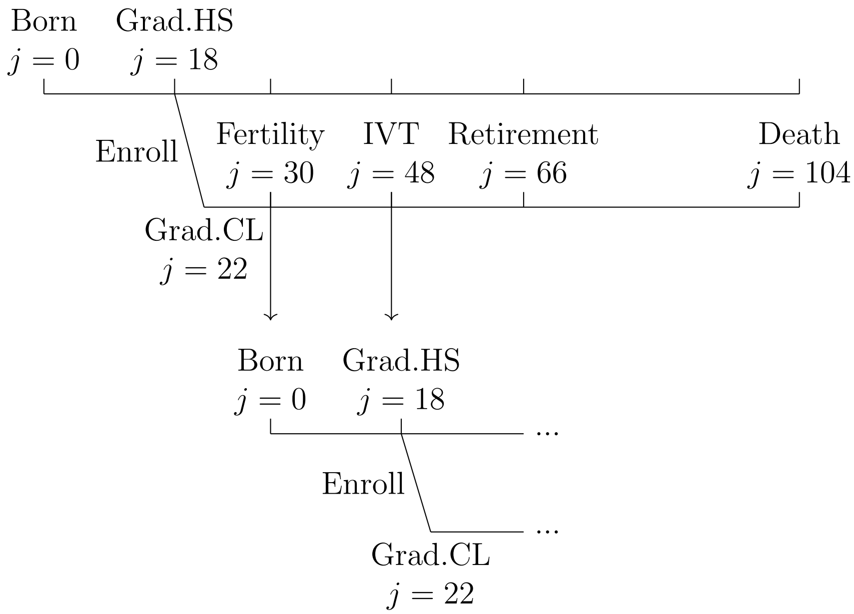

Fig 1: Model's lifecycle.

College enrollment increases their skill and potential lifetime earnings but takes some costs in terms of money and disutility. As standard in the literature, the model captures several channels of intergenerational linkage critical to the enrollment choices. First, the disutility of education ("psychic costs") depends on their parents' educational background. Second, parents choose the amount of IVT for their children. Third, the human capital endowment of children correlates to that of their parents, which is critical to education return. Students can fund their education costs through the IVT, labor earnings through part-time jobs, student loans with cheaper interest payments, and grants. The subsidized loans and grants can be income- or ability-tested and provided by the government.

After completing their education and entering the labor market, they choose how much to consume, save, and work, subject to idiosyncratic productivity shocks. At the beginning of age $j = J_F (:= 30)$, they choose how many children they have, which is common for high school and college graduates. They draw utility from the number of children and education spending for each child, but having children requires some costs in terms of money and time. The lifecycle of a new cohort starts at this point, represented in the bottom half of the Fig 1. After their children graduate from high school, corresponding to the beginning of the age $j = J_{IVT} (:= 48)$ for parents, they decide how much money to transfer to their children. This IVT decision affects the children's college enrollment choice at the node "Grad.HS." They are forced to retire at the beginning of age $j = J_R (:= 66)$. After that period, they face mortality risks; every period, a certain fraction of them is hit by exogenous mortality shocks and exits from the economy. They can live for J $(:= 104)$ years at the longest, but they exit the economy after age J.

3.2 Preliminaries

Production: A representative firm chooses labor and capital inputs in competitive factor markets to produce final goods. There are two types of labor inputs in this economy; the college graduates (skilled) and high school graduates (unskilled). Their total labor supply in efficiency units are represented as L_{CL} and L_{HS}, respectively. I allow them to be imperfect substitutes by considering the aggregate labor in efficiency units, L, is given as:

$$L = \left[\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}\right]^{1/\chi},$$

where $\omega_{HS} \equiv 1 - \omega_{CL}$ and ω_{CL} governs the relative productivity of the skilled workers.

The representative firm operates with a Cobb-Douglas production function with aggregate capital K and labor L to produce the output Y:

$$Y = ZK^{\alpha}L^{1-\alpha},$$

where Z represents the factor neutral productivity. Let r, w_{HS}, and w_{CL} denote the rental rate of capital and wage rates for unskilled and skilled labor. Capital depreciates at δ, and the firm has to incur the capital depreciation cost.

Demographics: As I explained above, after retirement, they face uncertainty regarding their survival in the next period. Let $\zeta_{j,j+1}$ denote the probability of surviving at age $j+1$ conditional on surviving at age j for each $j \in \{J_R, ..., J\}$ with $\zeta_{J,J+1} = 0$. In equilibrium, the cohort size (i.e., the number of births) grows at a constant rate g_n, which is endogenously determined in this model based on households' choices on fertility. Let μ_j denote the age distribution of the economy, determined by the cohort population growth rate and survival probabilities. The population mass of the economy is normalized to 1 (i.e., $\sum_j \mu_j = 1$) as in previous studies (e.g., Guner, Kaygusuz, and Ventura, 2020).[11]

Intergenerational Linkages and Initial Endowments: After graduating high school (at the beginning of age 18), agents draw their human capital from a distribution $g^h_{h_p}$ varying with the parents' human capital level h_p. They also draw the psychic costs of education from a distribution g^{ϕ}_{h,e_p}, depending on the student's human capital and parent's education e_p. Lastly, they may receive some assets (IVT) from their parents. These three sources of intergenerational linkages shape their college enrollment choices and generate their heterogeneity.

[11] Thus, the fertility rate matters to the age distribution, not population mass.

Preferences: Throughout their lifetime, they draw utility from consumption c and leisure l according to a utility function $u(c,l)$. They discount future utility by β. At age $j = J_F$, they choose the number of children they have, denoted by $n \in \{0, 1, ..., N\}$. They can draw additional utility from having children in several ways. First, they draw utility from education spending q per child until the children graduate from high school, captured by a utility function $v(q)$. The utility is discounted by a function $b(n)$, increasing and concave in the number of children n. Further, based on altruistic motives, parents draw utility by transferring assets to their children after high school graduation. More specifically, parents consider children's expected lifetime utility in their education choice stage, with a discount rate $\lambda_a \cdot b(n)$, where λ_a represents the altruistic discount factor.

Costs of Children: Having children is costly in terms of money and time. First, q units of the per-child investment require $n \cdot q$ units of money, and they will make an additional expenditure $n \cdot a_{IVT}$ upon high school graduation of their children. In addition to the monetary costs, having children requires κ units of time until children graduate from high school and become independent.

Labor Earnings: Households choose hours worked to earn income. The labor earnings of a household are given by $w_e \eta_{j,z,e,h}(T - l)$; it is a product of the market wage w_e, labor efficiency or productivity $\eta_{j,z,e,h}$, and hours worked $(T-l)$. Here, T denotes the disposable time that can be devoted to work or leisure where $T = 1 - \kappa \cdot n$ if they have n children before graduating high school and $T = 1$ otherwise. Labor efficiency depends on age j, idiosyncratic productivity shock component z, education level e, and time-invariant human capital h.

Financial Markets: Financial markets are incomplete due to the lack of state-contingent claims. Households can self-insure against risks by savings, accruing interest payments at a rate of r. Households aged $j < J_R - 1$ except college students can borrow at rate $r^- = r + \iota$ where $\iota > 0$ (i.e., borrowers incur the overseeing costs ι) up to a borrowing limit $\underline{A}$, while we do not allow retired households' net worth to be negative. In addition, eligible students have access to student loans subsidized by the government, which entails the interest rate of $r^s = r + \iota_s$. This loan is income- and ability-tested, and eligible students can borrow up to a limit $\underline{A}_s$.

Government: The government raises the revenue by levying three types of taxes: consumption, labor income, and capital income taxes, where these tax rates are represented as τ_c, τ_w, and τ_a. In addition, the government collects accidental bequests and devotes them to cover expenditures. They use this revenue to fund (1) the public pension benefits, which gives p units of money to retired households each period, (2) subsidized loans for college students, (3) grants for eligible college students, where the payment per eligible student is represented by $g(h, I)$ where $I \geq 0$ denotes the household income when the student faces education choice problem (i.e., their parent's age is J_{IVT}), (4) lamp-sum transfers ψ that is introduced for replicating the progressivity of labor income tax schedule in a simple way following the literature, (5) cash benefits for households with children under 17 with per-child payment of B, and (6) other expenditures S. I do not consider grants in the benchmark to replicate the economy before grants are introduced in 2020 and set $g(h, I) = 0$ for each (h, I). The government budget constraint is given as follows:

$$\tau_c \cdot C + \tau_w \cdot (L_{HS} + L_{CL}) + \tau_a \cdot K + Q = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \leq 17} + S, \quad (1)$$

where C, Q, μ_{old}, $\mu_{j \leq 17}$, K_s, and G represent the total consumption, total accidental bequests, population mass of retired households, that of children under age 17, the total amount of borrowing by college students, and the total grant payments.

3.3 Household Problems

This section describes problems agents face throughout their lifecycle. The first is about college enrollment choice, choosing whether to attend college or enter the labor market after graduating high school.

Education Stage: After graduating high school, they draw the time-invariant human capital h from the distribution $g^h_{h_p}$ and psychic cost ϕ from the distribution g^{ϕ}_{h,e_p}. They also receive IVT from their parents $a_{IVT} \geq 0$. Some of them can access subsidized loans to fund expenditures that arise during the college education stage, which is income- and ability-tested. Thus, the state variables for the students are comprised of asset a_{IVT}, the human capital h, psychic costs ϕ, and their parent's income I. They compare the expected value for entering the labor market as a high school graduate, $\mathbb{E}V^w$, with the value for enrolling in college, V_{g1} net of the psychic cost ϕ. They choose college enrollment if the latter is greater than the former; otherwise, they enter the labor market. The decision problem is formulated as follows:

$$\begin{aligned} V_{g0}(a_{IVT}, \phi, h, I) = \max_{e \in \{0,1\}} \Big\{ &(1 - e) \cdot \mathbb{E}_{z_0}[V^w(a_{IVT}, j = 18, z_0; e = 0, h)] \\ &+ e \cdot [V_{g1}(a_{IVT}; h, I) - \phi] \Big\}, \end{aligned} \quad (2)$$

where $e \in \{0,1\}$ indicates the education choice where $e = 1$ means college enrollment and $e = 0$ does entering the labor market as a high school graduate. V^w denotes the value function for workers, which I formulate in the next subsection. The initial draw of z, z_0, is uncertain and is according to the invariant distribution of z, $\bar{\pi}_z$, so the expectation operator is put next to the V^w. The value for college enrollment, V_{g1}, is defined as follows:

$$\begin{aligned} V_{g1}(a_{IVT}; h, I) &= \max_{c,l,a'}\{u(c,l) + \beta V_{g2}(a'; h, I)\}, \\ V_{g2}(a; h, I) &= \max_{c,l,a'}\{u(c,l) + \beta \mathbb{E}_{z_0}[V^w(a^s(a'), j = 22, z_0; e = 1, h)]\}. \end{aligned} \tag{3}$$

The budget constraints differ according to eligibility to the student loans. The budget constraints for eligible students are give as follows:

$$\begin{aligned} &(1+\tau_c)c + p_{CL} + a' - (1-\tau_w)w_{HS}(1-\bar{t}-l) \\ &\qquad\qquad - \psi - g(h,I) = \begin{cases} (1+(1-\tau_a)r)a & \text{if } a \geq 0, \\ (1+r^s)a & \text{otherwise,} \end{cases} \\ &\qquad a' \geq -\underline{A}_s. \end{aligned} \tag{4}$$

The rest of the students cannot access to the student loans, which implies that their budget constraints are given as follows:

$$\begin{aligned} (1+\tau_c)c + p_{CL} + a' &= (1+(1-\tau_a)r)a + (1-\tau_w)w_{HS}(1-\bar{t}-l) + \psi + g(h,I), \\ a' &\geq 0. \end{aligned}$$

College students draw utility from consumption and leisure and must pay tuition fees p_{CL}. Normalizing their total disposable time as 1, students must spend a $\bar{t}$ fraction of time on study. Thus, they choose the time allocation between leisure and working over the disposable time $1 - \bar{t}$. One unit of labor supply gives college students w_{HS} units of wages.[12] They can fund the consumption and tuition fees through (1) transfers from their parents a_{IVT}, (2) borrowing through student loans if eligible, (3) government-provided grants if eligible, and (4) labor earnings by themselves. Following the literature, I assume that fixed payments are made for 20 years (10 periods) following college graduation and transform college loans into regular bonds according to the following formula:

$$a^s(a') = a' \times \frac{r^s}{1-(1+r^s)^{-10}} \times \frac{1-(1+r^-)^{-10}}{r^-}.$$

[12] I assume that, while in college, there is no heterogeneity in labor efficiency and no uncertainty regarding the next period's productivity, and one unit of hours worked brings one unit of labor efficiency.

Working Stage without Children: The remaining component in (2) and (3), V^w, represents the value function for working stage for age $j \in \{J_E, ..., J_F - 1, J_{IVT}+1, ..., J_R - 1\}$. The state variables for this stage consists of asset a, age j, idiosyncratic component of labor productivity z, education level e, and human capital h. The uncertainty in this stage is only about the next period's productivity, which is denoted by z' following a Markov process $\pi_z(z', z)$. Households choose consumption, leisure, and savings given the state variables. The value function is formulated as follows:

$$
\begin{aligned}
V^w(a, j, z; e, h) = \max_{c,l,a'} \{ & u(c, l) \\
& + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^f(a', z', e, h)] & \text{if } \ j = J_F - 1 \\ \beta [V^r(a', j+1)] & \text{if } \ j = J_R - 1 \\ \beta \mathbb{E}_{z'}[V^w(a', j+1, z'; e, h)] & \text{otherwise} \end{array} \right\} \\
\text{s.t.} & \\
& (1+\tau_c)c + a' = (1-\tau_w) w_e \eta_{j,z,e,h}(1-l) + \psi + (1 + (1-\tau_a) r) a, \\
& z' \sim \pi(z', z), \\
& a' \geq \left\{ \begin{array}{ll} 0 & \text{if } \ j = J_R - 1, \\ -\underline{A} & \text{otherwise.} \end{array} \right.
\end{aligned}
\tag{5}
$$

Value functions V^f and V^r represent those for the fertility choice stage and retirement stage, respectively, which are formulated in the following subsections.

Fertility Choices and Working Stage with Children: When aged $j = J_F$, they choose whether and how many to have children. As in the previous life stages, they draw utility from consumption and leisure and decide on consumption, time allocation, and savings. In addition to that, as I mentioned, they also draw utility from the number of children n and education spending q for each child.[13] Then, the value function for households aged $j = J_F$, V^f, is formulated as follows:

$$
V^f(a, z, e, h) = \max_{n \in \{0,1,..,N\}} \left\{ V^{wf}(a, j = J_F, z; e, h, n) \right\}
$$

[13]This study does not consider children's endogenous human capital accumulation through parental investments and assumes parents make educational spending on children just because it draws utility. Incorporating the endogenous human capital accumulation is left for future research.

where, for $j = J_F, ..., J_{IVT} - 1$,

$$V^{wf}(a, j, z; e, h, n) = \max_{c,l,q,a'} \{u(c/\Lambda(n), l) + b(n) \cdot v(q) + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^{wf}(a', j+1, z'; e, h, n)] & \text{if } j \in \{J_F, ..., J_{IVT} - 2\} \\ \beta \mathbb{E}_{z', \phi_k, h_k}[V^{IVT}(a', z'; \phi_k, h_k, e, h, n)] & \text{if } j = J_{IVT} - 1 \end{array} \right\}$$

s.t.

$$(1 + \tau_c)(c + nq) + a' = Y_{wf},$$

$$a' \geq -\underline{A},$$

where

$$Y_{wf} \equiv (1 - \tau_w) w_e \eta_{j,z,e,h} (1 - l - \kappa \cdot n) + n \cdot B + \psi + \left\{ \begin{array}{ll} (1 + (1 - \tau_a) r) a & \text{if} \quad a \geq 0, \\ (1 + r^-) a & \text{otherwise.} \end{array} \right.$$

Here, ϕ_k and h_k denote the psychic costs and human capital for their children. The household consumption is deflated by the equivalence scale $\Lambda(n)$, depending on the number of children n. The total education spending on children, nq, enters the budget constraint, and utility from children, $b(n) \cdot v(q)$, enters the objective function in the periods with children. V^{IVT} represents the value function for the stage of making IVT at $j = J_{IVT}$, which is described in the following subsection.

In addition to the next period's productivity z', they are uncertain about their children's human capital h_k and psychic costs ϕ_k until the beginning of age $j = J_{IVT}$. Recall that the children's human capital correlates to the parent's, and the distribution of their psychic costs depends on the parent's education level and student's own human capital. Uncertainty about h_k and ϕ_k can be translated into uncertainty about expenditures on children in the form of IVT, given that those two components govern the marginal gains from IVT for parents,

$$b(n) \cdot \lambda_a \cdot \frac{\partial V_{g0}(\boldsymbol{x})}{\partial a_{IVT}},$$

where $\boldsymbol{x} = (a_{IVT}, \phi_k, h_k, I)$.[14]

Inter-vivo Transfers: At age $j = J_{IVT}$, which corresponds to a period when their children graduate from high school and face the college enrollment choice, they decide on how much to transfer to their children. Some of the children's characteristics are realized

[14] This representation implicitly assumes that the value function V_{g0} is differentiable for better clarity. However, in principle, this is not the case because of the discrete nature of the college enrollment choice.

at this point, such as their psychic cost ϕ_k and human capital h_k. Then, they choose the amount of per-child transfer a_{IVT} given the state variables, which is formulated as follows:

$$\begin{aligned} V^{IVT}(a, z; \phi_k, h_k, e, h, n) = \max_{c,l,a',a_{IVT}} \Big\{ & V^w(a - \tilde{a}_{IVT}, j = J_{IVT}, z; e, h) \\ & + b(n) \cdot \lambda_a \cdot V_{g0}(a_{IVT}, \phi_k, h_k, I) \Big\}, \end{aligned}$$

where $\tilde{a}_{IVT} = \frac{n \cdot a_{IVT}}{1+(1-\tau_a)r}$ and the parent's state vector pins down household income I. The budget constraint is given as follows:

$$\begin{aligned} (1+\tau_c)c + a' + na_{IVT} &= Y_{IVT}, \\ a' &\geq -\underline{A}, \end{aligned}$$

where

$$Y_{IVT} \equiv (1-\tau_w)w_e\eta_{j,z,e,h}(1-l) + \psi + \begin{cases} (1+(1-\tau_a)r)a & \text{if} \quad a \geq 0, \\ (1+r^-)a & \text{otherwise.} \end{cases}$$

Retirement Stage: At the beginning of age J_R, they are forced to retire from the labor market. After that, they spend all their time on leisure and make consumption-saving decisions. Two points differ from previous choice problems; they receive the pension benefit p from the government and face uncertainty about the next period's survival. The value function for the retirement stage is formulated as follows:

$$\begin{aligned} V^r(a, j) &= \max_{c,a'} u(c, 1) + \beta\xi_{j,j+1}V^r(a', j+1) \\ \text{s.t.} & \\ & (1+\tau_c)c + a' = p + (1+(1-\tau_a)r)a + \psi, \\ & a' \geq 0. \end{aligned}$$

3.4 Stationary Equilibrium

I solve the stationary equilibrium of this economy. In equilibrium, households make every choice to maximize their expected utility, the firm maximizes its profit, and the government budget is balanced. The stationarity implies that the distribution over state variables is invariant. Importantly, the age distribution is determined endogenously according to households' fertility choices. See Appendix C. for the detailed definition of equilibrium and Appendix D. for the computational algorithm for solving the equilibrium.

4 Calibration

To calibrate the model, I mainly rely on the Japanese Panel Survey of Consumers (JPSC), a panel survey of Japanese women and their household members. It started in 1993 with a representative sample of 1,500 women aged $24-34$ and contains information about, for example, their income, educational background, marriage, fertility, and expenditures to detailed categories, including those on children's education. I focus on the cohort born in 1959-69, the oldest cohort of this survey, especially to compute the completed fertility and intergenerational mobility of education. I keep only married households as in previous works (e.g., Daruich and Kozlowski, 2020) because the model focuses on choices made within married households such as fertility and educational investments.[15] Unless otherwise mentioned, target moments for internally determined parameters described below are computed based on the JPSC's 1959-69 cohort data.

4.1 Targeted Moments

Preferences: Instantaneous utility for agents are given as follows:

$$u(c,l) = \frac{(c^{\mu} l^{1-\mu})^{1-\gamma}}{1-\gamma}.$$

μ is internally determined as 0.23 so that the households spend one-third of the total disposable time on market work. Instantaneous utility from education spending on a child is given as:

$$v(q) = \lambda_q \frac{q^{1-\gamma}}{1-\gamma},$$

where $\lambda_q = 0.62$ so that the annual educational expenditure per child amounts to 7% of average income at age 28. The utility must be always positive (or always negative) in models of altruism with endogenous fertility, and I set $\gamma = 0.5$ following the literature (e.g., Daruich and Kozlowski, 2020). The altruistic discount factor λ_a is set to 1.03 so that the annual average IVT for college students amounts to 27% of average income at age 28.[16] Following Kim, Tertilt, and Yum (2023), I let the discount function of number of children $b(n)$ be non-parametric, and assume that $b(n) = b_n$ for each $n \in \{0,1,2,3,4\}$ with $b(0) = b_0 = 0$. Those parameters are determined so that the model replicates the

[15] Hence, an agent or household in this model refers to households with two individuals. Then, "children" in this model can also be interpreted as a household unit. That is, having n children can be interpreted as reproducing $n/2$ units of households. For example, if a household has two children, it means reproducing one household unit with two individuals. If a household has one child, it means a reproduction of 0.5 units of households with two individuals.

[16] Although some parents whose children do not enroll in college make IVT in the model, the amount is negligibly small. One reason is that the marginal gains from IVT are more significant if their children attend college, as students are financially constrained, primarily because of their limited earning ability.

distribution of the completed fertility. The time for studying $\bar{t}$ is set to 0.8 so that the income share of labor earnings for college students in the model is close to 21% (SLS, 2018). I assume that $\beta = 0.98$ as in Zhou (2022).

Financial Markets: I set the borrowing limits $\underline{A} = 20$ million yen and $\underline{A}_s = 2.88$ million yen.[17] The borrowing wedges are set $\iota = 0.055$ and $\iota_s = 0.054$ so that the model approximates the share of working households with a negative net worth (54%) and the share of students borrowing from the government-provided student loans (44%).

School Taste: I assume that the psychic costs ϕ are given as $\phi = \psi_{CL} \cdot \exp(-\nu \cdot h) \cdot \tilde{\phi}$. First, ψ_{CL} governs the scale of psychic costs and thus college enrollment rate. The second term $\exp(-\nu \cdot h)$ allows high ability students to have smaller psychic costs, as standard in the literature, and ν governs the education sorting by ability. Finally, $\tilde{\phi}$ is stochastic and the distribution depends on their parent's education. Following Daruich and Kozlowski (2020), I assume that $\tilde{\phi}$ is distributed on an interval $[0, 1]$ and follows the following CDF:

$$G^{\tilde{\phi}}_{e_p} = \begin{cases} \tilde{\phi}^{\omega} & \text{if } e^p = 0 \\ 1 - (1 - \tilde{\phi})^{\omega} & \text{if } e^p = 1 \end{cases}$$

Here, ω governs the intergenerational transmission of school tastes. ψ_{CL} is set to 20.8 so that the model approximates the college enrollment rate of 37.7%. ν is set to 1 in the benchmark, and ω is set to 1.71 so that the intergenerational transition matrix of education in the model matches the data counterpart as closely as possible. Table 1 reports the transition matrix in the model and data. $(i, j)-$th entry of the matrix indicates the probability that children acquire skill j given that their parent's skill is i in the benchmark model, and values in parentheses represent the data counterparts. The table indicates that the education level is persistent across generations; children of high school graduates attend college with a probability less than 0.3, whereas children of college graduates do with a probability approximately 0.6.

Parents/Children	HS	CL
HS	0.725 (0.798)	0.275 (0.202)
CL	0.412 (0.423)	0.588 (0.577)

Table 1: Intergenerational transition matrix of education. *Note*: $(i, j)-$th entry of the matrix indicates the probability that children acquire skill j given that their parent's skill is i in the benchmark model, and values in parentheses represent the data counterparts.

[17] The former is based on the Family Income and Expenditure Survey by the Ministry of Interna Affairs and Communications and the latter is based on the SLS (2018).

Intergenerational Transmission of Human Capital: The intergenerational transmission of human capital is according to the following formula:

$$\log(h) = \rho_h \cdot \log(h_p) + \varepsilon_h,$$
$$\varepsilon_h \sim N(0, \sigma_h).$$

I assume $\rho_h = 0.19$ based on Daruich and Kozlowski (2020) while internally determining σ_h as 0.65 so that the variance of log income at age 28 in this model is close to 0.27.

Income Process and Education Return: The efficiency labor of an agent aged j, education e, human capital h, and productivity z, $\eta_{j,z,e,h}$, is given as follows:

$$\log \eta_{j,z,e,h} = \log[f^e(h)] + \gamma_{j,e} + z,$$
$$f^e(h) = h + e \cdot (\alpha_{CL} h^{\beta_{CL}}),$$
$$z' = \rho_z z + \zeta, \quad \zeta \sim N(0, \sigma_z).$$

To set $\gamma_{j,e}$, I estimate the second-order polynomial of hourly wages on age using JPSC. As reported in Table 2, the income gradient on age is larger for college graduates than the rest of the workers, but the degree is modest compared with the US case (e.g., Abbott, Gallipoli, Meghir, and Violante, 2019). I assume $\rho_z = 0.95$ and $\sigma_z = 0.02$, values in the ranges over those frequently used in the literature.[18] The function $f^e(h)$ indicates that the education return depends on human capital h: people with higher human capital can obtain greater returns through college education. Following Daruich and Kozlowski (2020), α_{CL} and β_{CL} are determined so that the model replicates the ratio of log wage between college graduates and the rest of the population at age $j = 28$ and the log wage variance for college graduates at age $j = 28$.

	HS	CL
Age	+0.041	+0.048
Age^2 $\times$ 10,000	−4.551	−5.364

Table 2: Parameters governing the age profile of wages. *Note*: CL indicates the college graduate households where the husband or wife is a college graduate. HS represents the rest of the population.

Production: I set the capital share $\alpha = 0.33$ and $\delta = 0.07$ following Kitao (2015). χ is set to 0.39 following Matsuda and Mazur (2022). ω_{CL} is internally determined to 0.52 so that the average wage ratio between college graduates and the rest in the model amounts to 1.36. Z is determined so that the wage rate for unskilled labor is normalized to one.

[18] Due to data limitations, it is hard to accurately estimate the AR(1) process for z using any data source in Japan.

Government: Tax rates are set to $\tau_c = 0.1$, $\tau_w = 0.35$, and $\tau_a = 0.35$ in the benchmark. The lump-sum transfer is set to $\psi = 0.01$ to match the ratio between the variance of log net income and that of log gross income (0.6). The pension benefit p is set so that the government provides ¥160,000 per household per month. The cash transfer B, which we refer to "child benefit" or "typical pro-natal transfers" hereafter, is given as ¥10,000 per child per month, approximating the actual payment. The other expenditure S is set so that the government budget constraint is balanced in the benchmark and fixed throughout the counterfactual experiments.

Miscellaneous: The survival probability $\zeta_{j,j+1}$ is set based on the Vital Statistics (2019).[19] Annual college tuition fees p_{CL} are set to 1.05 million yen. κ is set to 0.044 so that they spend 13.3% of their working hours on childcare.[20] Table 3 and Table 4 summarise the parameters externally and internally determined.

Parameter	Value	Description
$\underline{A}_s$	2.88 million yen	Borrowing limit for students
$\underline{A}$	20 million yen	Borrowing limit
p_{CL}	1.05 million yen/year	Tuition fees
κ	0.044	Time costs
$\xi_{j,j+1}$	$-$	survival prob.
τ_c	0.10	Consumption tax
τ_a	0.35	Capital income tax
τ_w	0.35	Labor income tax
p	¥160,000/month	Pension benefits
b	¥10,000/month	Cash transfers
α	0.33	Capital share
δ	0.07	Depreciation rate
χ	0.39	Elasticity of substitution
ρ_z	0.95	Persistence
σ_z	0.02	Transitory
ν	1.0	Education sorting by ability
γ	0.5	Curvature
β	0.98	Discount factor
ρ_h	0.19	Transmission of h

Table 3: Parameters externally determined.

[19]See, https://www.mhlw.go.jp/english/database/db-hw/outline/index.html.
[20]See, Kitao and Nakakuni (2023).

Parameter	Value	Moment	Data	Model
μ	0.23	Work hours	0.33	0.30
$\bar{t}$	0.8	Income share of labor earnings	0.21	0.20
ι_s	0.054	Share of students using loans	0.44	0.34
ι	0.055	Household share with negative net worth	0.54	0.45
ω_{CL}	0.52	CL−HS wage ratio	1.36	1.48
ψ	0.01	Var(log disposable income)/Var(log gross income)	0.60	0.68
λ_q	0.62	Average transfer / Average income at age 28	0.07	0.07
λ_a	1.03	Average transfer / Average income at age 28	0.27	0.27
ω	1.71	Intergenerational mobility of education	See Table 1	
σ_h	0.65	Variance of log(income) at age 28	0.27	0.24
ψ_{CL}	20.8	College enrollment rate	0.377	0.376
α_{CL}	0.1	Log wage ratio (CL−HS) at age 28	0.34	0.38
β_{CL}	0.1	Var log wage for CL at age 28	0.14	0.24
b_1	0.49	Share of one child	0.16	0.15
b_2	0.53	Share of two children	0.55	0.61
b_3	0.55	Share of three children	0.22	0.24
b_4	0.56	Share of four or more children	0.02	0.00
Z	1.99	Low skill wage	1.0	1.0

Table 4: Parameters internally determined.

4.2 Non-targeted Moments and Validation

This subsection checks the validity of the calibrated model. First, I check if the benchmark model generates a reasonable value of the benefit elasticity of fertility, which is non-targeted in calibration. Second, I check if the model performs well on other critical non-targeted moments, fertility differential across education levels and the revenue breakdown for college students.

4.2.1 The benefit elasticity of fertility

Previous works show that cash benefits such as the CB and baby bonus have a significant impact on fertility. Many of them report that the benefit elasticity of fertility, the percentage increase in fertility rate against the one percent increase in the cash transfer, is about $0.1 - 0.2$. For example, Milligan (2005) studies a reform of Quebec's baby bonus and shows that an extra 1,000 Canadian dollars benefit would increase fertility by 16.9%, which implies a benefit elasticity of 0.107. Cohen, Dehejia, and Romanov (2013) uses a variation in the child subsidy payment observed around 2003 in Israel, providing a larger subsidy for third or higher births. They show that the benefit elasticity of fertility was 0.176. González (2013) adopts the regression discontinuity design to study the effects of Spain's reform in 2007, introducing a one-time payment of 2,500 euros (about 3,800 USD) for births, almost 4.5 times the monthly minimum wage for full-time workers. It finds a statistically significant impact on fertility, increasing conceptions by $5-6\%$.

To examine how this model performs in this respect, I conduct the following exercise. Let B_0 denote the per-child cash transfers for households with children under 17 in the benchmark. I solve the household problem, holding prices, tax rate, and distribution fixed, with several levels of the per-child payment $B = B_0 \cdot (1 + x)$ for some $x \in X$, where X is a set of positive real numbers. This procedure brings the implied fertility rate, and with the expansion rate x, we can compute the implied benefit elasticity of fertility for the case of the expansion rate x. I set $X = \{0.1, 0.2, ..., 1.9, 2.0\}$, which is a reasonable range in the context of the expansion examined in the empirical studies, and compute the implied elasticity for each x. Then, I take the average of those 20 values. I find that the average elasticity is 0.138 (with a standard deviation of 0.025), which is consistent with the empirical estimates.

4.2.2 The fertility differential across education levels

More educated parents have fewer children than less educated ones. According to my sample of the JPSC, college graduates' completed fertility was 1.92, which is lower than the rest's, 2.12. This is observed in another data source that we can check the completed fertility by different education levels, the NFS. According to the NFS, college graduate wives' completed fertility has been lower than less educated ones almost every survey year since 1977. The latest survey in 2015 reports that the completed fertility of wives with a college degree was 1.89, and that of high school graduate wives was 1.98. In this benchmark model, the completed fertility of college graduate wives is 1.79, which is lower than the high school graduate wives', 2.28. The benchmark model captures the qualitative feature of the fertility differential across education levels, although the gap is somewhat significant compared with data counterparts. Table 5 summarizes the fertility differential across education levels in the model and data.

	Model	JPSC	NFS
HS	2.28	2.12	1.98
CL	1.79	1.92	1.89

Table 5: Fertility differential across education in the benchmark model and data. *Note*: "NFS" stands for the National Fertility Survey conducted by the ISPP, and the table reports the values from the 2015 survey. It reports the completed fertility of wives with different educational backgrounds.

A factor generating the gap in this model is the differences in opportunity costs, given that having children requires a fixed fraction of time. More educated parents face the higher opportunity costs of having children as their potential earnings are higher. In this model, the intergenerational persistence of education can also generate this differential because children of educated parents are more likely to attend college, which raises financial costs in the form of asset transfers upon children's college enrollment.

4.2.3 Composition of students' revenue

Capturing the composition of students' revenue – how college students finance their living expenses and tuition fees – is also important as it is critical not only to students' education choices but also to parents' IVT choices and, thereby, fertility choices. According to the SLS (2018), the students' revenue consists of three parts. First, the greatest part, 61% of their revenue, is accounted for by asset transfers from their parents. Second, students' labor earnings account for 21%, and lastly, the rest (18%) is financed by student loans. Although the revenue share of labor earnings (21%) is a targeted moment, the rest is not targeted. As Table 6 shows, the model captures the overall revenue composition as well; the IVT and loans account for 66% and 14% of their revenue, close to the data counterparts.

	IVT	Loan	Labor
Data	0.61	0.18	0.21
Model	0.66	0.14	0.20

Table 6: Composition of Students' Revenue.

5 Numerical Analysis

This section investigates the effects of education subsidies for college students in the model with fertility choices calibrated in the previous section in stationary equilibrium. Section 5.1 simulates the introduction of the existing income-tested subsidy started in 2020. Section 5.2 then examines the mechanism through which the macroeconomic effects of the introduction are realized. Section 5.3 examines the performance of the potential alternative programs to the existing income-tested grants. Lastly, Section 5.4 investigates the effects of raising the income threshold so that students in broader income classes of households are eligible. Following the literature, I adjust the labor income tax to balance the government budget upon the introduction and expansion.

Welfare Measure: Our primary focus is fertility, college enrollment, and output. In addition to these variables, I also examine the welfare effects of the policy in the following exercises. However, the welfare analysis using models with endogenous fertility is not straightforward theoretically and philosophically. One of the well-known difficulties in the economics or theoretical context is an issue Golosov, Jones, and Tertilt (2007) address: the standard notion of the Pareto efficiency is not well-defined under endogenous fertility because it ends up with a comparison between two different sets of individuals.[21] Given that the literature on the normative analysis of endogenous fertility models is still developing and there is no one "correct" method to tackle this issue at least for now, this study captures the welfare effects of the policy by the consumption equivalence under the veil of ignorance under the benchmark economy relative to the new steady state.[22]

To formalize the measure, let $P \in \{0, 1, 2, ...\}$ denote education subsidy schemes or policies with $P = 0$ representing the benchmark economy without grants. Next, let $V^P(\lambda)$ be the expected lifetime value for newborn agents with a consumption scaling parameter λ and policy P:

$$V^P(\lambda) = \int_{\boldsymbol{x}_1} V^P_{j=1}(\boldsymbol{x}_1; \lambda) d\mu(\boldsymbol{x}_1). \tag{6}$$

$\mu(\boldsymbol{x}_j)$ is the measure over the age-specific state space where $j \in \{1, ..., J\}$ and $V^P_{j=1}(\boldsymbol{x}_1; \lambda)$ represents the expected value for an agent aged $j = 1$ with a state vector $\boldsymbol{x}_1$:

$$\begin{aligned} V^P_{j=1}(\boldsymbol{x}_1; \lambda) = E\Bigg\{ & \sum_{j=1}^{J} \beta^{j-1} \cdot u(c_j \cdot (1+\lambda), l_j) \\ & + \sum_{j=J_F}^{J_{IVT-1}} \beta^{j-1} b(n) \cdot v(q_j) + \beta^{j-1} \lambda_a \cdot b(n) \cdot V_{g0}(\boldsymbol{x}_{J_{IVT}}) \Bigg\} \end{aligned}$$

Here, $\{c_j, l_j, q_j, n, a_{IVT}\}$ are optimal choices with the policy P and each state $\boldsymbol{x}_j$. Finally, the consumption equivalence with a policy P is given as a scalar λ satisfying the following equation:

$$V^0(\lambda) = V^P(0).$$

That is, the consumption equivalence λ makes the newborn agent indifferent between the new economy and the benchmark by scaling up the benchmark consumption by λ.

5.1 Introducing Education Subsidies

In 2020, the Japanese government introduced income-tested subsidies for college students in low-income households; until then, the government has provided only student loans.

[21] Golosov, Jones, and Tertilt (2007) then propose two efficiency concepts that can apply to models with endogenous fertility. For example, the $\mathcal{A}-$efficiency proposed by them focuses on individuals being alive in both economies to be compared. This notion is particularly beneficial if we examine the optimal policy as they show that the unique solution for the planning problem considering the utility of the existing agents (i.e., those who are alive just before the policy is introduced) corresponds to the $\mathcal{A}-$ efficiency.

[22] The related work Zhou (2022) adopts the same welfare criteria in its steady-state analysis. In its transition analysis, Zhou (2022) also computes the average welfare of the existing households already alive before the policy changes to refer to the $\mathcal{A}-$efficiency discussed above.

Households are eligible if their last year's annual labor income is less than a threshold value $\bar{I}$, and students in those households receive g amount of money each year while in college. The grant function $g(h, I)$ can be formulated as follows:

$$g(h, I) = \begin{cases} g & \text{if } I < \bar{I} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Note that the benchmark case can be interpreted as $g(h, I) = 0$ for any (h, I). Although the income threshold $\bar{I}$ and payment g in the actual system vary with some characteristics of households and students, such as family structure and whether the student commutes to college from home or not, $\bar{I}$ approximately corresponds to the 15 percentile of the household income distribution, and the payment approximately amounts to two-thirds of the average expenses of college students. I set $\bar{I}$ and g based on this information, and income distribution and students' expenditure in the benchmark model (initial steady state). I then solve the stationary equilibrium by introducing this new grant function.

The main numerical results are as follows. First, introducing the means-tested subsidy would increase the college enrollment rate by 3.9 p.p. in the long run. Education mobility increases in the sense that children of high school graduates are 2.5 p.p. more likely to attend college.[23] Because of the higher college enrollment rate, implying a greater supply of college graduate labor, the skill premium decreases by 0.02 points. Introducing the subsidy would increase the TFR by 3% or 0.064 points. Importantly, this increase is primarily driven by a 7.4% increase in college graduates' fertility, whereas the fertility rate of high school graduates is almost stable.

Those changes in demographic structure and skill distribution affect other aggregate variables. In the long run, the higher college enrollment rate implies a higher share of skilled labor, and the higher TFR implies a greater share of the working-age population. As a result, per-capita labor supply in efficiency units increases by 1.3% in the long run. On the contrary, per-capita capital decreases by 0.9% for some reasons. First, introducing the subsidy reduces saving incentives for a substantial fraction of households and crowd-out IVT.[24] In addition, the higher TFR implies a greater share of younger generations, who hold fewer assets than older ones. Despite its negative effects on capital accumulation, the positive impacts on the labor force and productivity are sufficiently greater so that the per-capita output increases by 1.0% in the long run. The standard deviation of wages,

[23] This mobility concept is a variant of an income mobility measure adopted in Zheng and Graham (2022).

[24] Note that some households, characterized on the state space, can increase savings due to the subsidy introduction. For example, households whose children do not attend college in the benchmark but attend college when the subsidy is introduced may need more assets in the new equilibrium to make some IVT upon children's college enrollment. This is because the grants are not sufficiently significant to cover 100% of the expenditures for students.

$w_e \cdot \eta_{j,z,e,h}$, is reduced by 1.12 % compared with the benchmark because of the greater education mobility and the pecuniary externalities of college enrollment (i.e., shrinking skill premium due to the greater supply of college graduates). The welfare improves by 4.8% in consumption equivalence under the veil of ignorance, and its source is discussed in the following section.

5.2 Inspecting the Mechanism

Previous results imply that the subsidy introduction significantly affects the college enrollment rate and TFR, which leads to a greater output. The introduction also brings welfare gains in the long run. However, the mechanism behind these changes is not so obvious because several objects aside from the grant function, such as prices, tax rate, and distribution, vary between the benchmark and new equilibrium, and each can play important roles in accounting for the overall effects on the TFR, college enrollment rate, and welfare. In addition, the roles of fertility margins are also worth investigating, that is, how the results differ under exogenous fertility where the fertility choice is fixed. This is because the fertility setup would matter to the IVT and, thereby, children's education choices, given that fertility and IVT choices are joint decisions. Fertility margins can also have distributional implications through intergenerational linkages if fertility responses are heterogeneous across household characteristics. In this section, I conduct a decomposition analysis in Section 5.2.1 to discuss and understand the mechanism behind the changes in the TFR, college enrollment rates, and welfare upon introducing the subsidies. Next, in Section 5.2.2, I consider the roles of fertility responses in understanding the results by solving the exogenous fertility version of the model.

5.2.1 Decomposition

What causes the increases in the TFR and college enrollment rates? Those increases can be broken down into behavioral effects and distributional (or composition) effects, where the behavioral effects can be further broken down into the direct effects, driven only by a change in the subsidization scheme (i.e., grant function $g(h, I)$), and the indirect effects, driven by changes in factor prices (i.e., GE effects) or tax rates (i.e., Taxation effects). Note that the direct effects can also be interpreted as the short-run effects of the introduction, where prices, tax, and distribution are fixed. Finally, the distributional effects capture changes driven by distribution changes over state variables such as education, age, and human capital. To isolate each effect, I conduct a decomposition exercise as follows. I first solve an equilibrium by introducing the new grant function. Then, I solve household problems by replacing one of the four objects (grant function, prices, tax rate, and state distribution) in the benchmark with that in the new equilibrium. Note that this method does not guarantee that each implied effect adds up to the overall effects because all factors except grant function are endogenous and interconnected. Table 7 summarises the decomposition results.

	Direct	GE	Tax	Dist.	All
CL share (Δ p.p.)	+2.6	−0.2	0.0	+1.9	+3.9
TFR (Δ %)	+2.3	+0.9	0.0	−0.4	+3.0
HS (Δ %)	+1.0	0.0	0.0	0.0	+0.4
CL (Δ %)	+4.5	+2.5	0.0	0.0	+7.4
Output (Δ %)	−0.7	−0.8	−0.1	+0.9	+1.0
Welfare (%)	+2.9	−0.7	−0.2	+1.9	+4.8

Table 7: Decomposition results. *Note*: Columns "Direct," "GE," "Tax," and "Dist." report the results when only grant function, prices, labor income tax rate, and distribution change, respectively. A column "All" indicates the results in the benchmark and the long-run equilibrium with the grants. Rows "CL share" represent the change in college enrollment rate in p.p. and rows "HS" and "CL" represent the percentage changes in fertility rates for high school and college graduates. Welfare gains are represented in terms of consumption equivalence.

College Enrollment: For the long-run increase in the college enrollment rate, critical forces are direct and distributional effects. If the grant is introduced while other variables (i.e., prices, tax rate, and distribution) are fixed as in the benchmark, the college enrollment rate increases by 2.6 p.p., more than half of the overall increase. These direct effects capture the effects of relaxing financial constraints on college enrollment decisions. Note that these direct effects do not explain the overall effects, so we need other forces accounting for the increase in college enrollment rate; that turns out to be the distributional effects. GE effects put downward pressure on college enrollment because the lower college premium reduces the incentive for college enrollment.

The implied changes in the distribution in the long run, holding grant function, prices, and tax rate fixed as in the benchmark, lead to a 1.9 p.p. higher college enrollment rate. The most relevant factor is the change in the skill distribution of parents (i.e., the share of college graduates). The increase in the share of college graduates implies a higher share of those more likely to have children enrolling in college due to the intergenerational persistence of education. Then, the distributional change amplifies the short-run increase in college enrollment rate due to the direct effects.

Fertility: For the long-run increase in the TFR, vital forces are direct and GE effects. If the grant is introduced while other variables are fixed as in the benchmark, the TFR increases by 2.3% in the long run (direct effects), which corresponds to three-fourths of the overall effects. As in explaining the overall effects, that 2.3% increase due to the direct effects is largely driven by the fertility increase of college graduates; college graduates increase fertility by 4.5%, whereas high school graduates do by 1%.

Further, among both education groups, ex-post ineligible households increase fertility; the average fertility rates of ex-post eligible high school and college graduates increase by 1.6% and 3.9%, as indicated in Fig 2a. Given that the ineligible households are the majority of the cohort as indicated by Fig 2b, these fertility increases of ex-post eligible households largely explain the direct effects; the fertility increases of ex-post ineligible high school and college graduates explain a 0.7 p.p and 1.2 p.p. of the 2.3% direct effect, respectively (Fig 3). In other words, more than 80% of the direct effect is explained by the fertility increases of ineligible households.

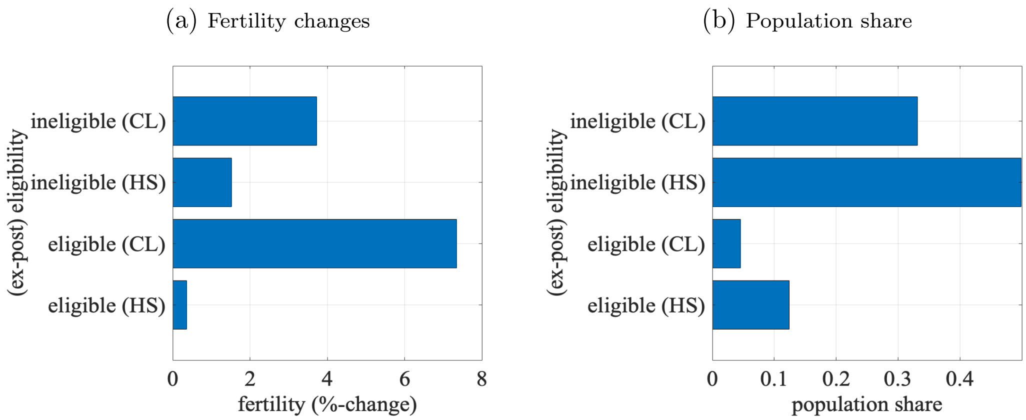

Fig 2: Share of each eligibility-education status and changes in fertility. *Note*: "HS" and "CL" stand for high school and college graduates.

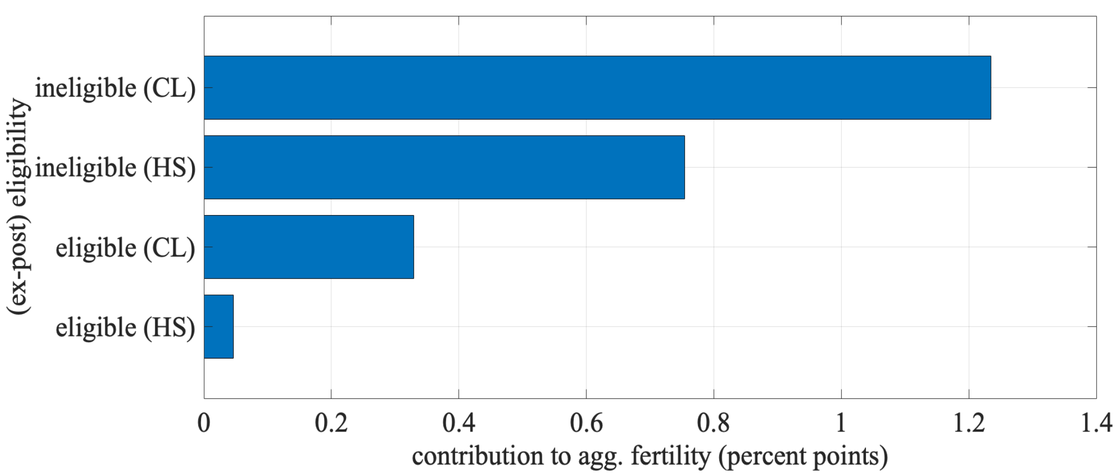

Fig 3: Contribution to the aggregate fertility change by ex-post eligibility status and education. *Note*: "HS" and "CL" stand for high school and college graduates.

Why can the subsidy introduction increase fertility, even among ex-post ineligible households? The short answer is due to an insurance effect of the income-tested subsidy.

To understand this argument, recall that the eligibility is uncertain when they make fertility choices; they are beneficiaries if (1) their children enroll in college and (2) their earnings when their children face the education choices, are sufficiently low. Those are subject to uncertainty regarding their productivity and children's characteristics (school tastes and human capital), which are not realized when they make fertility choices. If their children have characteristics favoring college enrollment (i.e., higher human capital endowment and/or lower psychic costs of education), they would like to make more IVT to support them, and vice versa. Thus, they face expenditure uncertainty until their children's education choices terminate, in addition to income risks.

Consider that those "shocks" are realized simultaneously; their children have characteristics willing to attend college, and they are also willing to make a sufficient amount of IVT, but they are poor due to the realization of negative income shocks. Without the income-tested grants, the realization of negative income shocks will make it infeasible for those "unlucky" parents to financially support their children's enrollment, or they may substantially reduce their consumption in exchange for supporting their college enrollment; either situation is costly. Thus, in fertility decision-making, they may choose fewer children to avoid such a costly situation in the future. The income-tested grants provide partial insurance against those risks and can make parents comfortable having another child. Because college graduates are more likely to have children with characteristics favoring college enrollment, they benefit more from this insurance, and their fertility behavior responds strongly.

Next, the decomposition result shows that the GE effect also plays a role in accounting for the TFR increase. If the prices are set to the long-run equilibrium levels while other objects are fixed as in the benchmark, the college graduates' fertility increases by 2.5%, and the TFR then increases by 0.9%. The key is the decline of the wage rate for college graduate workers, w_{CL}. In the long-run equilibrium with the grants, w_{CL} decreases by 2.2%. First, the greater supply of college graduate workers depress w_{CL} relative to w_{HS}. In addition, greater aggregate labor supply in efficiency units and lower capital accumulation discussed in Section 5.1 imply the lower marginal productivity of labor, decreasing w_{CL}. The lower w_{CL} implies the lower opportunity costs of having children for college graduates, making some have more children.

Lastly, the distributional effect implies a 0.4% decline in the average fertility. The introduction of the grants increases the college enrollment rate, meaning that the share of college graduates increases, especially in the long run. Given that college graduates have fewer children than high school graduates as discussed in Section 4.2.2, the greater share of college graduates means a greater share of those who tend to have fewer children, which can lead to a lower average fertility.[25] However, direct and GE effects are significant and lead to the higher TFR.

Output: The distributional effect is critical in accounting for the output increase. In the long run, the working-age population increases due to a higher equilibrium TFR, and the share of skilled workers increases. These forces increase the labor supply in efficiency units, increasing output in the long run.

In contrast, the direct and GE effects put downward pressure. As indicated in Table 7, the GE effects lower the college enrollment rate due to the reduced skill premium. The resulting lower share of skilled workers leads to a lower output.

Next, recall that the direct effect can be considered the short-run effect of the introduction, where other macroeconomic variables, such as prices, tax rates, skill distribution, and age distribution, are fixed as in the benchmark. In the short run, both aggregate savings and labor supply due to the higher fertility; having a child requires parents to spend a fraction of time and some additional money, reducing savings. Thus, the aggregate output decreases in the short run.

Welfare: The direct effect explains approximately 60% of the welfare gain. Importantly, introducing the subsidy makes agents attend college with smaller costs and enables someone who could not attend college in the benchmark to do that, bringing them a higher lifetime income. The rest of the gains are explained by the distributional effect. The education subsidy increases the share of college graduates in the long run, and the lifetime values of college graduates are, in principle, greater than those of high school graduates, particularly because of their higher income due to the college education return. Thus, this distributional change increases the expected lifetime utility. Note that the direct effect is about the change in $V_{j=1}^P$ in equation (6), while the distributional effect is about the change in μ there.

5.2.2 Roles of Fertility Responses

I next solve the exogenous fertility version of the model. More specifically, I follow the same procedure in the previous section 5.1, except that the policy functions for fertility are fixed as in the benchmark. Therefore, the TFR under exogenous fertility is not necessarily the same as in the benchmark because the household distribution changes.

[25] This effect is comparable with a composition effect highlighted by Zhou (2022) in understanding the effects of public education subsidies on fertility. In his model, public education subsidies increase parents with higher human capital, and they tend to have fewer children.

	Endogenous	Exogenous
CL share (Δ p.p.)	+3.9	+2.9
TFR (Δ %)	+3.0	−0.8
Output (Δ %)	+1.0	+0.4
Capital (Δ %)	−0.8	+0.6
Labor (Δ %)	+1.3	+0.2
Tax (Δ p.p.)	+0.04	+0.13
STD(wage) (Δ %)	−1.3	−1.1
Welfare (%)	+4.8	+4.7

Table 8: Results under exogenous fertility. *Note*: Results for output, labor, and capital represent percentage changes compared with the benchmark. Those for college enrollment and labor income tax rates represent changes in percentage points compared with the benchmark. "STD(wage)" represents the standard deviation of wages, where wages for an agent with (j, z, e, h) is captured as $w_e \cdot \eta_{j,z,e,h}$. Welfare gains are represented in terms of consumption equivalence.

Table 8 reports the main results under exogenous fertility. First, the introduction of grants results in a 2.9 p.p. increase in the college enrollment rate under exogenous fertility, which is lower than a 3.9 p.p. increase under endogenous fertility. Under both exogenous and endogenous fertility, the grants make some children who otherwise cannot enroll in college. Under endogenous fertility, in addition to that effect, the college enrollment rate can further increase through fertility margins. As I discussed in Section 5.1, college graduates increase fertility than high-school graduate parents. Their children will likely attend college due to the intergenerational transmission of school tastes and human capital. And when those children become parents in the future, their children, if they have, are also likely to have similar characteristics to theirs, favoring college enrollment. Through this mechanism, the long-run share of college graduates increases, implying that the fertility margins amplify the effects on college enrollment.

From the second row onward, the changes in aggregate variables are reported. Under exogenous fertility, per-capita labor in efficiency units increases by 0.2 % compared with the benchmark mainly because of the higher share of skilled labor. However, the extent of this increase is modest compared with a 1.3% increase under endogenous fertility because the college enrollment rate is lower. Notably, the introduction under exogenous fertility leads to a 0.8% lower fertility rate due to a composition effect; the share of college graduates increases who have fewer children. Thus, the working-age population share is slightly lower than in the benchmark, opposite to the endogenous fertility setup. The greater labor supply in efficiency units leads to a greater aggregate output; the endogenous fertility setup implies a 1% increase in per-capita output, while the exogenous setup does a 0.4% increase.

Even though the college enrollment rate under exogenous fertility is lower than under endogenous fertility, implying a lower government expenditure on the grants, the required tax increase is 0.09 p.p. higher under exogenous fertility. This is because the greater labor supply in efficiency units under endogenous fertility increases the tax revenue in the long run. The reduction of the standard deviation in wages is more significant under endogenous fertility, mainly because the higher college enrollment rate under endogenous fertility leads to a lower skill premium. The difference in the welfare effect is quantitatively subtle, but the gain is slightly greater under endogenous fertility. As I discussed above, the higher college enrollment rate in the economy increases the welfare under the veil of ignorance via the direct and distributional effects. Then, endogenous fertility can lead to higher welfare gains due to the higher college enrollment rate facilitated by fertility margins.

5.3 Reallocating Resources to Different Programs

This section examines the performance of the potential alternative programs to the existing income-tested grants. To this end, I consider another two scenarios, in addition to the introduction of income-tested grants to the benchmark model, examined in Section 5.1. The first scenario is to introduce grants for college students with income and ability tests so that "high-ability" students in poor households are eligible. More specifically, I keep the income threshold adopted in the existing scheme but arbitrarily set the lower bound for students' human capital for the eligibility, $\underline{h}$, to its median. The payment function $g(h, I)$ for this scheme is defined as follows:

$$g(h, I) = \begin{cases} g_1 & \text{if } I \leq \bar{I} \ \& \ h \geq \underline{h} \\ 0 & \text{otherwise} \end{cases}$$

Here, the payment g_1 is set so that the short-run expenditure upon the introduction is the same as that of the existing income-tested grants. As a result, g_1 covers approximately 100% of the average student's expenditure in the benchmark, which is greater than the payment in the existing scheme with only income-test because the number of eligible students is fewer.

The second scenario introduces unconditional grants for college students regardless of ability and income. The payment function is given as $g(h, I) = g_2$ for any students with (h, I), where g_2 is set so that the short-run government expenditure is the same as the existing program. As a result, g_2 covers approximately 10% of the average students' expenditure in the benchmark, which is less significant than in the existing income-tested grants because this alternative program covers a broader range of students.

I solve the stationary equilibrium with each scenario, and the results regarding fertility and enrollment rates are summarized in Table 9. A key highlight is that the existing scheme with income tests would lead to the highest equilibrium TFR among other scenarios. Notably, the unconditional grants lead to a 1.2 p.p. higher college enrollment rate than the income-tested ones and a -0.3% lower TFR because of the composition effect. This result highlights the insurance effect of the income-tested grants on fertility and the downward pressure on aggregate fertility through composition changes.

	Income	+Ability	Uncond.
CL share (Δ p.p.)	+3.9	+2.6	+5.1
TFR (Δ%)	+3.0	+2.7	−0.3
HS	+0.4	−3.5	+4.0
CL	+7.4	+8.4	+3.9
Output (Δ%)	+1.0	−1.2	−0.7
STD (wage) (Δ%)	−1.3	−0.9	−0.1
Welfare (%)	+4.8	+2.6	+1.6

Table 9: Main results with several schemes with different targets. *Note*: values in each cell indicate changes from the benchmark value. Rows "HS" and "CL" indicate the percentage changes in the fertility of high school and college graduates.

5.4 Expansion

Lastly, I consider the effects of raising the income threshold $\bar{I}$ so that students in households of broader income classes can be eligible. Recall that I set $\bar{I}$ so that the existing subsidy targets the students in households at the bottom 15 % of the income distribution. In this experiment, I increase $\bar{I}$ to correspond to the 40, 50, and 60 percentile of the income distribution and solve the stationary equilibrium in each case. For students in the household at the bottom 15 %, the payment is still given by g, which amounts to two-thirds of the average expenses of students in the benchmark. For students in households higher than the 15 percentile but less than x percentile of the income distribution, where x takes either 40, 50, or 60, the payment is given by $g/2$, which amounts to one-third of the average expenses of students. In other words, the payment tapers off in income. Letting $\bar{I}_{15\%}$ and $\bar{I}_{x\%}$ denote the income level of 15 and $x \in \{40, 50, 60\}$ percentiles of the income distribution, the grant function $g(h, I; x)$ with a threshold $\bar{I}_{x\%}$ can now be formulated as:

$$g(h, I; x) = \begin{cases} g & \text{if } I < \bar{I}_{15\%} \\ g/2 & \text{if } I \in [\bar{I}_{15\%}, \bar{I}_{x\%}) \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Table 10 summarizes the results, and there are several takeaways there. First, the equilibrium college enrollment increases as the income threshold is higher. Setting the threshold at the 40, 50, and 60 percentiles of the income distribution, the enrollment rate increases by 4.7 p.p., 5.6 p.p., and 6.2 p.p., respectively, in stationary equilibrium. This is also the case for education mobility and labor income tax rates. The expansion requires additional revenue, so the equilibrium tax rate should also increase by 0.3 p.p. for the case of the 60% threshold.

However, the expansion would not significantly increase the TFR. Recall that the introduction leads to a 3% increase in TFR, from the benchmark level of 2.096 to 2.160 in the long-run equilibrium with the grants. With grant functions $g(h, I; 40)$, $g(h, I; 50)$, and $g(h, I; 60)$, the equilibrium TFR would be 2.158, 2.151, and 2.157, respectively; the TFR even decreases locally by expansions. Fertility rates for each skill help us understand this situation. First, college graduates continue to increase fertility. With grant functions $g(h, I; 40)$, $g(h, I; 50)$, and $g(h, I; 60)$, their equilibrium fertility rate would be 1.996, 1.998, and 2.021, all higher than the fertility rate with the existing subsidy, 1.978. This result is straightforward to understand given that the insurance and GE effects contribute to the fertility increase of college graduates when the grants are introduced, discussed in Section 5.2. On the contrary, high school graduates reduce fertility instead, making the TFR remain almost constant even though the income threshold is higher. Fig 4 depicts the changes in the TFR and fertility rates for high school and college graduates, where the equilibrium TFR with the existing program (with the 15 percentile threshold) is normalized to one.

	Threshold			
	15%	40%	50%	60%
CL share (Δ p.p.)	+3.9	+4.7	+5.6	+6.2
HS→CL (Δ p.p.)	+2.5	+2.6	+3.1	+3.3
Tax (Δ p.p.)	+0.04	+0.17	+0.23	+0.30
Output (Δ%)	+1.0	+1.3	+0.2	−0.5
Welfare (%)	+4.8	+6.5	+6.3	+5.5

Table 10: Main results of higher income thresholds. *Note*: Rows "CL share" and "HS→CL" represent the changes in college enrollment rate and education mobility in the sense of probability that children of high school graduates attend college. Output changes are expressed as percentage changes. Changes in college enrollment rate, education mobility, and tax rate are represented as changes in percentage points. Welfare gains are represented in terms of consumption equivalence.

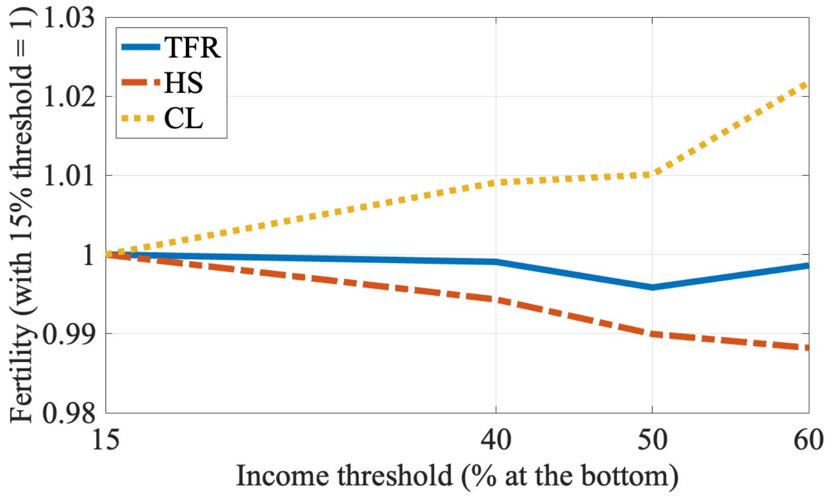

Fig 4: Changes in fertility rates with expansion. *Note*: The equilibrium TFR with the existing program (with the 15% threshold) is normalized to one. "HS" and "CL" represent fertility rates for high school and college graduates.

Why do high school graduates decrease fertility as the income threshold is higher? To understand this, I conduct the decomposition when the grant function is given by $g(h, I; 60)$, and the results are summarized in Table 11. Two effects are critical to explain the fertility decreases of high school graduates: the direct and GE effects.

In this case with the grant function $g(h, I; 60)$, the wage rate for high school graduates, w_{HS}, is 0.6% higher in the long run than in the benchmark, especially because of the significant supply of college graduates. This higher wage rate makes the opportunity costs of having children for high school graduates higher, which puts downward pressure on their fertility rates.

Next, the direct effects imply a lower fertility rate for high school graduates. This result is somewhat confusing given that we discuss the insurance effects of the subsidy on fertility, which is critical to account for the fertility increases of college graduates. Why do the direct effects lead to lower fertility for high school graduates? In the benchmark without grants, children of high school graduates are unlikely to attend college. If the grants are introduced and their target expands, education mobility increases in the sense that children of high school graduates are more likely to attend college than in the benchmark, as represented in Table 10. Given that the grants are not generous enough to cover 100% of the costs to send their children to college, this higher probability of children going to college implies that those parents are more likely to have to make additional transfers upon children's college enrollment. The expansion thus increases the expected costs of children for high school graduates, which would lower their fertility.

	Direct	GE	Tax	Dist.	All
HS (Δ%)	-0.7	-1.4	0.0	0.0	-0.8
CL (Δ%)	$+9.1$	$+4.3$	$+2.4$	$+0.6$	$+13.2$

Table 11: Decomposing the effects on fertility with $g(h, I, 60)$. *Note*: Rows "HS" and "CL" represent the percentage changes in fertility rates for high school and college graduates.

Last, the output increases in expansion up to the 40 percentile threshold, but they start declining after that point, as Table 10 and Fig 5 indicate. If the income threshold becomes sufficiently high, households with children have less incentive to save because they are likely to be eligible for the grants even if their children attend college. This higher probability of eligibility also causes an income effect on labor supply. Thus, the aggregate capital and labor supply decrease when the income threshold becomes sufficiently high, reducing the output. This concavity of gains in expansion applies to the welfare gain. The marginal utility gains of the subsidy decrease in values of the income threshold, while the tax rate increases in expansion. Then, the net gains start declining at some point.

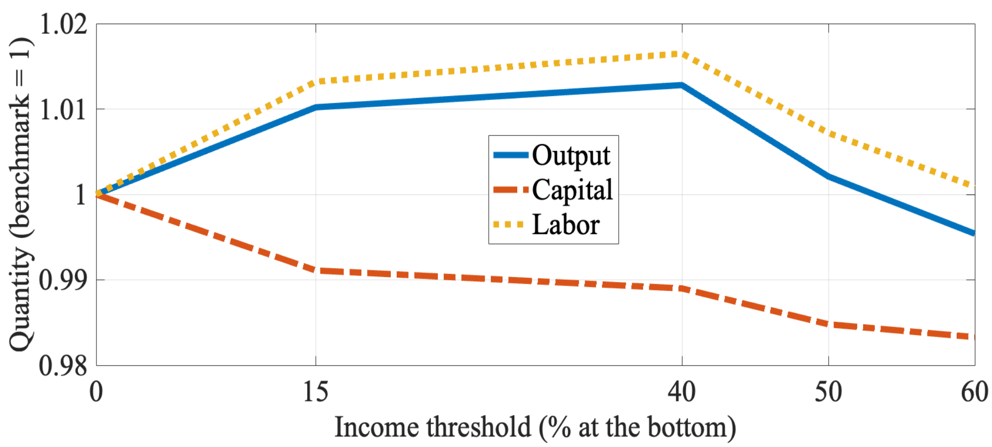

Fig 5: Changes in aggregate output, capital, and labor supply in efficiency unit with expansion. *Note*:

6 Concluding Remarks

What are the macroeconomic consequences of college financial aid policies, considering their impacts on the "quantity" and "quality" of the labor force? To answer the question, I construct a heterogeneous-agent GE-OLG model that incorporates choices regarding college enrollment, fertility, and inter-vivo transfers (IVT). The model is calibrated to Japan and replicates key moments, such as (1) the average parental asset transfers for college students, (2) the intergenerational persistence of education levels, and (3) fertility differentials across education levels. I validate the model's fertility behavior using empirical estimates for the cash-benefit elasticity of fertility, indicating the extent to which fertility rates increase in response to cash transfers.

Using this quantitative model, I show that income-tested grants provide partial insurance against the risks of having a child arising through multiple sources of uncertainty, especially for college graduates, and increase the fertility of ex-post ineligible ones. These fertility margins significantly amplify their positive effects on college enrollment and per-capita output via intergenerational linkages and changes in demographic structure. While the eligibility expansion or unconditional grants lead to higher college enrollment rates, they put significant downward pressure on aggregate fertility and the labor force in the long run through several channels, resulting in fewer output gains. A central contribution of this study is to build a new GE model that incorporates fertility and college enrollment choices, which sheds light on the critical roles of fertility margins in evaluating the macroeconomic performance of college financial aid policies.

My model is tailored to examine the financial aid policies by capturing relevant ingredients, such as students' labor supply and minute lifecycle (age) structure. Instead, this paper abstracts the endogenous human capital accumulation for children before college enrollment and the dynamic complementarity of human capital, as previous studies in college financial aid policies do not. These ingredients are critical in considering broader education policies, including early childhood education policies. Given that education in earlier stages affects education returns in later stages, examining the policies targeting several stages will provide a more comprehensive understanding of the macroeconomic effects of education policies. This study is a first step for this ambitious task, and incorporating those ingredients – fertility, college enrollment, dynamic complementarity of human capital– in one framework is left as an important avenue for future research.

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A. Illustrative Examples for Fertility Differentials

i) The Price of Time Theory

Following Jones, Schoonbroodt, and Tertilt (2010), I consider the following problem:

$$\max_{c,n} \frac{c^{1-\sigma}}{1-\sigma} + \theta \frac{n^{1-\sigma}}{1-\sigma}$$

s.t.

$$c = w \cdot (1 - n \cdot \chi)$$

Here, c, n, w, and χ denote consumption, fertility, an exogenous wage, and time costs of children. The optimal fertility decision n^* is given as:

$$n^* = \frac{1}{w^{\frac{1-\sigma}{\sigma}} \cdot (\chi/\theta)^{\frac{1}{\theta}} + \chi},$$

meaning that the negative income-fertility relationship holds if $\sigma > 1$ (i.e., the substitutability between consumption and children are high enough). Because having children is time-consuming, its (opportunity) cost is higher for high-income households. At the same time, high-income households, by definition, have higher incomes and afford to have more children. If the substitutability between consumption and children is sufficiently high, the substitution effect dominates the income effect, implying the negative income-fertility relationship.

B. Education Costs and Fertility Choices in Japan

Japan is a leading country in demographic aging,[26] leading to the shrinking labor force, output, and tax base, while the public expenditures on social security benefits are increasing. As a countermeasure against this demographic issue, the government introduced grants for college students in 2020. Two key underlying premises are: (1) it will increase the "quality" of the labor force in the long run, and (2) it will increase the fertility rate, which increase the "quantity" of the labor force in the long run. This pro-natal motive is explicitly described in an act for introducing the grants.[27]

[26] Japan's current fertility rate has been around 1.3, far below the population replacement level. In addition, the old-age dependency ratio is more than 50% and is projected to reach 80% in 2050, which is by far the highest among the OECD countries (See, https://data.oecd.org/).

[27] See, https://www.mext.go.jp/a_menu/koutou/hutankeigen/detail/__icsFiles/afieldfile/2019/05/17/1417025_02_1.pdf (available only in Japanese).

> The aim is ... to foster an environment where people can bear and raise their children with a sense of ease by alleviating the economic burden associated with higher education, thereby contributing to addressing the rapid decline in the birthrate in our country.
> (Act on Support for Higher Education Studies, enacted on May 17, 2019)

Although that expectation for education policies as a pro-natal policy is unconventional, there are facts suggesting that the financial costs for parents to support their children's college enrollment are a significant impediment to fertility decisions in Japan. I first list the four facts below and then elaborate on each one by one:

1. Couples are most likely to abandon having an ideal number of children because of financial costs.

2. A significant financial cost gap exists between those who have children enrolled in college and those who do not.

3. A substantial fraction of parents desire a college education for their children.

4. Japan is one of the least in subsidizing tertiary education.

Fact 1. *Couples are most likely to abandon having an ideal number of children because of financial costs.*

This fact is drawn from the National Fertility Survey (NFS), which is provided by the National Institute of Population and Social Security Research (IPSS).[28] The NFS is a cross-sectional household survey that asks respondents about their preferences or intentions regarding fertility, marriage, child-raising, and education, as well as their basic information, including their education, age, and income. It is conducted nearly every five years, and the latest survey available is in 2015, which collected 5,334 couples in which the wife is aged 18 to 49. Hereafter, I present the results focusing on married couples in which the wife is aged 25 to 39 years, leaving 2,420 couples.[29] According to the NFS, a non-negligible gap exists between the ideal and planned numbers of children. In the 2015 survey, the ideal number of children for wives aged 25 to 39 was on average 2.38, whereas the planned number was 2.16. Fig 6 represents the distribution of the ideal and planned

[28] See, https://www.ipss.go.jp/site-ad/index_english/survey-e.asp.

[29] Its 53.1% of the sample consists of couples in which the wife is over age 40, so the sample size shrinks if we target the younger couples. I focus on wives under age 39 because here I am interested in the fertility intention of those in the stage of fertility decision. Note that the cohort fertility rate is stable after age 40 for any cohort in Japan. See, for example, p7 of https://www.mhlw.go.jp/toukei/saikin/hw/jinkou/tokusyu/syussyo07/dl/gaikyou.pdf (in Japanese). Excluding those aged 18 to 24 does not affect the result significantly because they consist only of 1.5% of the observations for wives aged 18 to 49.

numbers of children, where blue (red) bars indicate the share of wives who desire (plan) to have each number of children from zero to more than five. This figure suggests that the gap originates from the downward revision of the ideal at the intensive margin. There is no significant share gap between those whose ideal number is zero and those whose planned number is zero, and a substantial fraction of wives who desire three children end up with one or two children.

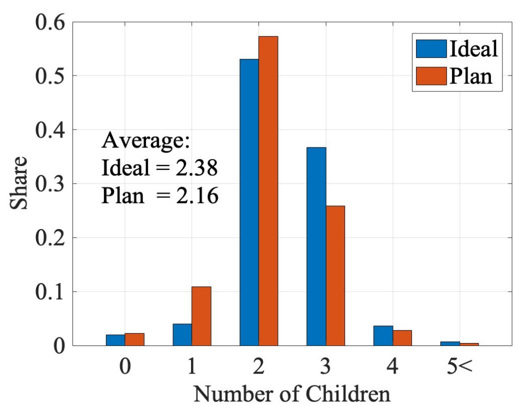

Fig 6: Distribution of ideal and planned numbers of children.

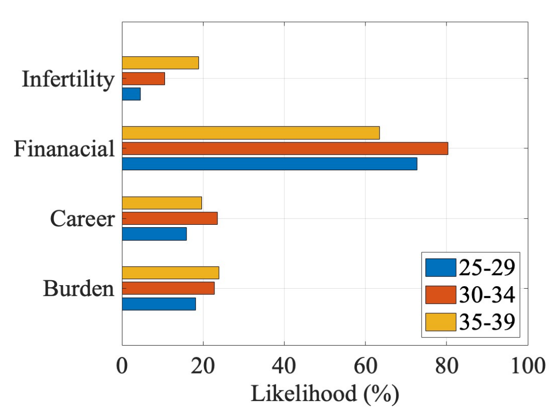

Fig 7: Reasons for the gap between the ideal and planned number of children.

Why do people abandon their ideal number of children? The NFS asks them to pick their reasons among several options, and the result indicates that they are most likely to choose the financial reason: "raising children and education are too expensive." Fig 7 represents the likelihood[30] that wives of each age group abandon their ideal number of children for a particular reason, and "Financial" represents the financial reason. Aside from this, "Career" represents "it would interfere with my job," and "Burden" represents "I would not handle the psychological and physical burden" (arising from achieving the ideal number). On average, more than 75% of them chose the financial reason, which dominates other reasons, such as "Career" and "Burden," chosen by only 20% of them.

To sum up, there is a non-negligible gap between the ideal and planned numbers of children for couples, and a substantial fraction of them answer the reason as "raising children and education are too expensive." These results establish the first fact: couples are most likely to abandon having an ideal number of children because of financial costs.

Fact 2. *A significant financial cost gap exists between those who have children enrolled in college and those who do not.*

Fact 1 indicates that the financial cost of having children is a critical constraint on

[30] I use the term "likelihood" instead of "share" because it allows respondents to choose multiple options.

fertility choices in Japan. However, it is silent on how and in which cases having children is so expensive; Fact 2 addresses them from the viewpoint of education costs. To do this, I use two data sets: (1) the Survey of Children's Learning Expenses (SCLE, 2021) and (2) the Student Life Survey (SLS, 2018), both cross-sectional household surveys and conducted by the Ministry of Education, Culture, Sports, Science and Technology (MEXT).[31] The SCLE covers more than 53,000 students from preschool to high school and reports the per-student average education expenditure for each expenditure category and education stage (i.e., preschool, elementary, junior high, and high school). The expenditure category includes not only school-related ones (e.g., tuition fees and textbooks) but also extracurricular activities (e.g., cram school, music, arts, and sports). In addition to that, I use the SLS for parents' expenditures when their children enroll in college.[32]

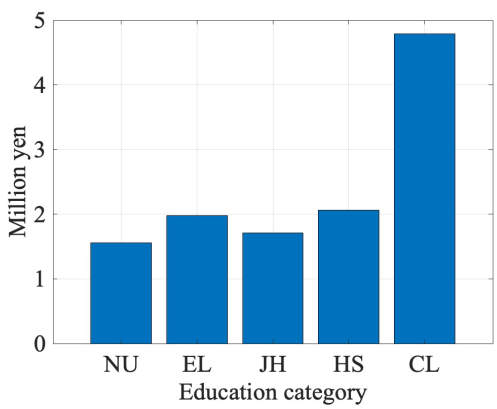

Fig 8: Average education expenditures.

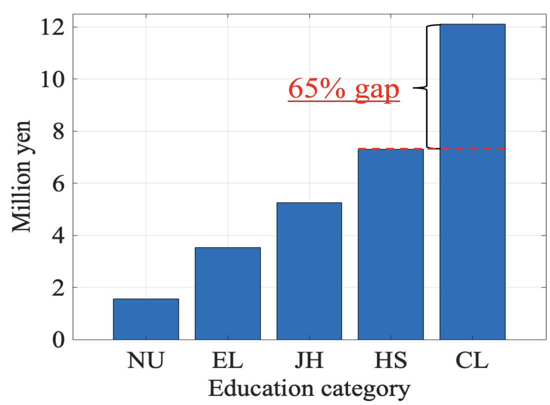

Fig 9: Cumulative education-expenditures.

Fig 8 shows the average per-child expenditure for each education category until completion. "NU," "EL," "JH," "HS," and "CL" each stand for nursery school or preschool, elementary school, junior high school, high school, and college, only consisting of a four-year college. All expenditures are conditional on enrollment in the education stage. Fig 9 represents the cumulative education expenditures, showing that the expenditure jumps up when children attend college. Given that the high school graduation rate is approximately 100% in Japan, the figure tells us that raising one child on average takes the education costs of 7.31 million yen, which amounts to 4.5% of the individuals' average lifetime labor earnings.[33] If their children attend college, they have to spend another 4.8 million yen, meaning there is more than a 60% increase in education costs if parents

[31] See, https://www.mext.go.jp/b_menu/toukei/chousa03/gakushuuhi/1268091.htm for the SCLE and https://www.jasso.go.jp/statistics/gakusei_chosa/__icsFiles/afieldfile/2021/03/09/data18_all.pdf (in Japanese) for the SLS.

[32] In 2018, the SLS correct answers from 43,394 students attending tertiary education, including college, some college, and graduate school.

[33] I construct a proxy of the average individual's lifetime earnings based on the 2022 Basic Survey on Wage Structure (BSWS) by the Ministry of Health, Labour, and Welfare (MHLW). First, I compute the average monthly earnings of ordinary workers for each age unconditional on any other characteristics

send their children to college. This expensiveness of college education can, at least partly, be attributed to the fact that Japan is one of the least in subsidizing tertiary education while subsidizing a sizable portion of schooling costs up to secondary education. These observations establish Fact 2: A significant financial cost gap exists between those who have children enrolled in college and those who do not.

Fact 3. *A substantial fraction of parents desire a college education for their children.*

Facts 1 and 2 suggest that the financial costs for children enrolled in college are a critical obstacle to fertility in Japan. However, one might not still be convinced because Japan's college enrollment rate is approximately 55%, far below 100%; thus, college education costs seem relevant only to half of the population.[34] The following fact answers "No, they should not." to this argument by showing that far more fraction of parents desire a college education for their children than the current college enrollment rate.

We return to the NFS (2015), which asked respondents about the desired education level for their children. Fig 10 summarizes the results by wife's age. Here, "SC" stands for some college. The figure shows that approximately 75% of the parents desire a college education for their children, which is significantly higher than the college enrollment rate observed in any period in Japan. This observation suggests that, although college enrollment in Japan has been far below 100%, education costs for a college education can be relevant not only for a specific part of the population because many parents would like to send their children to college.

such as sex and education. Then, I sum up the average earnings for each age, which amounts to about 160 million yen, and regard it as a proxy of the individuals' average lifetime labor earnings.

[34] Dropout rates are insignificant in Japan, so the enrollment rate is almost equivalent to the graduation rate. For example, the dropout rate was 2.5% in 2021. See, https://www.mext.go.jp/content/20220603-mxt_kouhou01-000004520_03.pdf (in Japanese).

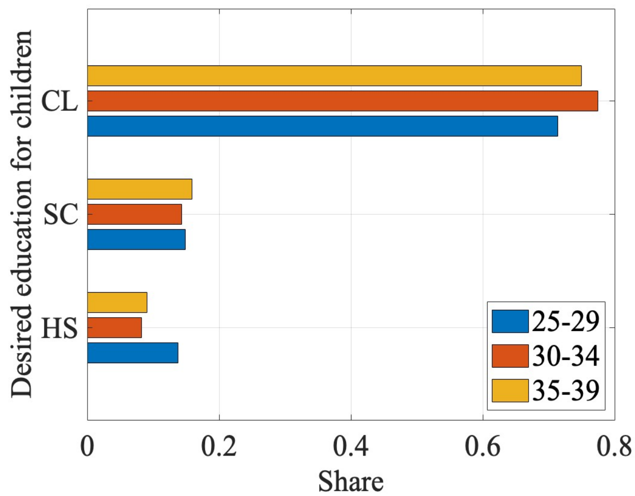

Fig 10: Wives' intention for children's education attainment.

Fact 4. *Japan is one of the least in subsidizing tertiary education.*

Japan is one of the least in subsidizing tertiary education, which roughly corresponds to four-year college education in Japan, given that the enrollment rates for tertiary education other than the four-year college are significantly lower than four-year college enrollment rate.[35]

To see this, I use the cross-country data provided by the OECD[36] and define the *subsidization rate* for a specific education category e as follows:

$$s_e = \frac{y_e^{pub}}{y_e^{pri} + y_e^{pub}},$$

where y_e^{pri} and y_e^{pub} represent the private and public spending on education e, both represented as a share of the GDP. Public spending includes expenditures on educational institutions and educational-related subsidies for households or students. Private spending refers to expenditures financed by households and other private entities. Private spending includes expenditures on school but excludes those outside educational institutions (e.g., textbooks purchased by households, private tutoring, and student living costs).[37] In other words, the subsidization rate indicates what fraction of (potential) school-related costs are funded by the government.

Japan subsidies more than 90% of the costs for primary-to-secondary education, which is higher than the OECD average. On the contrary, the subsidization rate for tertiary

[35] According to the Basic School Survey by the MEXT, the enrollment rate for some college was 4% in 2022 and is declining over the past thirty years.

[36] I use 2018's data given that it is the latest year in which data on a significant number of countries is available.

[37] For more details, see the OECD data (https://data.oecd.org/).

education was only 32%, which is the second lowest among OECD countries and less than half of the OECD average of 70%. This fact shows plenty of room for increasing subsidies for college students, which also has driven recent policy discussions on introducing and expanding education subsidies for college students.

C. Equilibrium Definition

Let $\boldsymbol{x}_j^e$ be an age-specific state vector for agents with education level $e \in \{HS, CL\}$ and $\mu_j^e(\boldsymbol{x}_j^e)$ be the measure of agents with state vector $\boldsymbol{x}_j^e$. Let $I_l(h, I)$ and $I_g(h, I)$ be indicator functions for loans and grants, respectively, returning 1 if students with (h, I) are eligible and 0 otherwise.

Given exogenous parameters and policy rules $\{\tau_a, \tau_c, \iota, \iota_s, B, S, \psi, p, I_l(h, I), I_g(h, I)\}$, a *stationary recursive competitive equilibrium* consists of

- value functions $\{V_{g0}, V_{g1}, V_{g2}, V^w, V^f, V^{wf}, V^{IVT}, V^r\}$,
- policy functions for consumption, savings, leisure $\{c_j^e(\boldsymbol{x}_j^e), a_j^e(\boldsymbol{x}_j^e), l_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J}$, working hours $\{h_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J_R}$, fertility $\{n_{J_F}^e(\boldsymbol{x}_{J_F}^e)\}$, IVT $\{a_{J_{IVT}^e}(\boldsymbol{x}_{J_{IVT}}^e)\}$, and college enrollment $\{e_{J_E}(\boldsymbol{x}_{J_E})\}$,
- prices (r, w_{HS}, w_{CL}),
- labor income tax rate τ_w,
- aggregate quantities (K, L_{HS}, L_{CL}),
- measures for households $\{\mu_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J}$,

such that:

1. The decision rules of students, workers, and retired households solve their problems, and $\{V_{g0}, V_{g1}, V_{g2}, V^w, V^f, V^{wf}, V^{IVT}, V^r\}$ are the associated value functions.
2. The representative firm maximizes its profit and optimally chooses capital and labor inputs:

$$r + \delta = \alpha \cdot Z \cdot \left(\frac{K}{L}\right)^{\alpha-1}, \tag{9}$$

$$w_{HS} = \tilde{Z} \cdot \omega_{HS} \cdot L_{HS}^{\chi-1}, \tag{10}$$

$$w_{CL} = \tilde{Z} \cdot \omega_{CL} \cdot L_{CL}^{\chi-1}, \tag{11}$$

where

$$L = [\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}]^{1/\chi},$$
$$\tilde{Z} = (1-\alpha) \cdot Z \cdot \left(\frac{K}{L}\right)^{\alpha} \cdot [\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}]^{1/\chi - 1}.$$

3. The labor market for each skill $e \in \{HS, CL\}$ clears:

$$L_e = \sum_{j=J_E}^{J_R} \int_{\boldsymbol{x}_j^e} \eta_j^e(\boldsymbol{x}_j^e) \cdot h_j^e(\boldsymbol{x}_j^e)\ d\mu_j^e(\boldsymbol{x}_j^e), \tag{12}$$

where $\eta_j^e(\boldsymbol{x}_j^e)$ represents the labor efficiency of agents with a state vector $\boldsymbol{x}_j^e$.

4. The capital market clears:

$$K = \sum_{j=J_E}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} a_j^e(\boldsymbol{x}_j^e)\ d\mu_j^e(\boldsymbol{x}_j^e). \tag{13}$$

5. The government budget is balanced:

$$\tau_c \cdot C + \tau_w \cdot (L_{HS} + L_{CL}) + \tau_a \cdot K + Q = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \leq 17} + S,$$

where

$$C = \sum_{j=J_E}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} c_j(\boldsymbol{x}_j^e)\ d\mu_j^e(\boldsymbol{x}_j^e),$$
$$Q = \sum_{j=J_R+1}^{J} \frac{1-\zeta_{j-1,j}}{\zeta_{j-1,j}} \sum_{e} \int_{\boldsymbol{x}_j^e} a_j^e(\boldsymbol{x}_j^e)\ d\mu_j(\boldsymbol{x}_j^e),$$
$$p \cdot \mu_{old} = \sum_{j=J_R}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} p\ d\mu_j^e(\boldsymbol{x}_j^e),$$
$$K_s = \int_{\boldsymbol{x}^s} \max\{0, -a^s(\boldsymbol{x}^e)\} \cdot I_l(h, I)\ d\boldsymbol{x}^s$$
$$G = \int_{\boldsymbol{x}^s} g(h, I) \cdot I_g(h, I)\ d\boldsymbol{x}^s$$
$$B \cdot \mu_{j \leq 17} = \sum_{j=J_F}^{J_{IVT}-1} \sum_{e} \int_{\boldsymbol{x}_j^e} B \cdot n\ d\mu_j^e(\boldsymbol{x}_j^e),$$

where $\boldsymbol{x}^s$, $\mu^s(\boldsymbol{x}^s)$, and $\{a^s(\boldsymbol{x}^s)\}$ represent a state vector for college students, measure of college students, and students' policy function for saving, respectively.

6. Distributions (measures) and households' behavior are consistent.

D. Computational Algorithm: Stationary Equilibrium

For any variable or distribution x, let $\tilde{x}$ and $\hat{x}$ represent its guessed and model-implied values. Also, let μ_j represent the distribution over state variables for age j. Dividing the computation process into three blocks makes it easier to understand. The first block is the outer loop, searching for equilibrium prices. The other two blocks are inner loops; one is the *optimization block*, solving household problems given prices, and another is the *distribution block*, searching for the stationary distributions given prices and policy functions obtained in the optimization block. The computational algorithm proceeds as follows:

1. Guess prices $\tilde{\boldsymbol{p}} = (\tilde{r}, \tilde{w}_{HS}, \tilde{w}_{CL})$.

2. *Optimization block*:

   - Guess the value function for agents at the beginning of age $j = 18$ ($\tilde{V}_{g0}$).
   - Given $\tilde{V}_{g0}$, solve backward from the period of IVT choice to that of the education choice, which gives the model-implied value function for V_{g0}, $\hat{V}_{g0}$.
   - Check if
   $$d(\tilde{V}_{g0}, \hat{V}_{g0}) < \varepsilon, \tag{14}$$
   where $d(\cdot)$ and $\varepsilon > 0$ represent an arbitrary metric function and error tolerance. If (14) is not satisfied, update $\tilde{V}_{g0}$ and follow the same procedure until convergence. The correct V_{g0} pins down all value functions and policy functions given a set of prices $\tilde{\boldsymbol{p}}$.

3. *Distribution block*:

   - Guess the distribution for age J_{IVT}. This $\tilde{\mu}_{J_{IVT}}$ and policy functions for IVT derive the implied distribution for agents aged 18, $\hat{\mu}_{18}$. Given $\hat{\mu}_{18}$ and policy functions, compute the implied distributions for age $j = 19, ..., J_{IVT}$, and obtain $\hat{\mu}_{J_{IVT}}$.
   - Check if
   $$d(\tilde{\mu}_{J_{IVT}}, \hat{\mu}_{J_{IVT}}) < \varepsilon. \tag{15}$$
   If (15) is not satisfied, update $\tilde{\mu}_{J_{IVT}}$ and follow the same procedure until convergence.

- After obtaing the correct distributions for age $j = 18, ..., J_{IVT}$, compute the distribution for age $j_{IVT}+1$ onward, using those distributions and policy functions.

4. After computing value functions, policy functions, and distributions, compute the implied quantities, $\hat{L}$ and $\hat{K}$ based on (12) and (13), which gives the implied prices $\hat{\boldsymbol{p}}$ based on $\hat{L}$, $\hat{K}$, (9), (10), and (11).

5. Check if

$$d(\tilde{\boldsymbol{p}}, \hat{\boldsymbol{p}}) < \varepsilon. \tag{16}$$

If (16) is not satisfied, update $\tilde{\boldsymbol{p}}$, return to the optimization block, and follow the same procedure until convergence.

E. Comparison With Pro-natal Transfers

This section examines the differences in the macroeconomic implications between the education subsidy and child benefit (i.e., typical pro-natal cash transfers).[38] To this end, I simulate the expansion of the unconditional cash transfers to households with children B from its benchmark value while setting $g(h, I) = 0$ for any (h, I) as in the benchmark. The new level of the per-child payment B is set so that the short-run expenditure upon the introduction is the same as that of the existing income-tested grants, which leads to a 6.5% increase in the per-child payment.

I solve the stationary equilibrium with the expansion of the per-child payment B, and the main results are summarized in Table 12. Columns "Education" and "Typical" represent the results of the introduction of income-tested grants and the expansion of child benefit ("typical" pro-natal transfers).

The child benefit expansion leads to a 2.5% increase of the TFR in the long-run equilibrium, comparable to introducing the income-tested education subsidy but slightly lower than that. The previous decomposition analysis suggests that targeting students in unlucky households in which negative income shocks are realized would be effective, at least marginally, in increasing the TFR. A notable observation is that the characteristics of households who respond to the policy differ between the two cases; the education subsidy induces skilled households to have more children, while the child benefit has a similar degree of impact on skilled and unskilled households regarding fertility. This is because the former is more beneficial for households whose potential children are likely to attend

[38] In Section 5.3, I also consider reallocating resources into different targets to implement the education subsidy by considering unconditional transfers to college students and introducing ability tests.

college, and college graduates tend to be those households due to the intergenerational persistence of education.

The college enrollment rate does not change in response to the child benefit expansion. The expansion also does not affect the average human capital, defined as the workers' average of labor efficiency $\eta_{j,z,e,h}$,[39], while the education subsidy increases the average human capital by 1.0% primarily because of the higher college enrollment rate. The difference in the average human capital also leads to the difference in the per-capita output. While the education subsidy increases the per-capita output by 1.0% in the long run, the child benefit expansion would rather lead to a 0.3% lower per-capita output. As in the case of introducing the education subsidy, the child benefit expansion also leads to lower (physical) capital accumulation due to crowding out and demographic change, as discussed in Section 5.1. The education subsidy has sufficiently large positive effects on aggregate labor supply in efficiency units, which surpasses the negative impacts on physical capital accumulation and increases output. The child benefit expansion has more modest impacts on aggregate labor efficiency than the education subsidy, leading to the gap in output gains between the two policies.

The child benefit expansion does not affect the standard deviation of wages, whereas the education subsidy leads to a lower standard deviation by enabling some students in poor households to attend college and acquire skills. Lastly, the expansion leads to a 0.3% of the welfare gain, which is substantially lower than a 4.8% gain with the education subsidy. As discussed in Section 5.2.1, the welfare gains of the education subsidy come mainly from the higher expected lifetime income in each state and the higher probability of being college graduates and enjoying higher earnings facilitated by the higher college enrollment rate and education mobility, as opposed to the child benefit expansion do. Thus, the education subsidy leads to greater welfare gains than the child benefit expansion under the veil of ignorance.

A caveat is that this model does not capture the endogenous human capital accumulation before high school graduation and the dynamic complementarity of human capital. Households may increase their investments in their children upon the child benefit expansion, which contributes to greater human capital; however, that channel is not considered in this current analysis. Those ingredients can be critical in comparing the effects of these different programs, especially on college enrollment rates, aggregate human capital, and output. The exercise in this framework is a first step, and incorporating those ingredients – fertility, college enrollment, dynamic complementarity of human capital– in one

[39]Formally, I define the average human capital as $\sum_j \int_{z,h} [(1-s)\cdot\eta_{j,z,e=0,h} + s\cdot\eta_{j,z,e=1,h}]dF(z,h)\mu_j$, where s represents the college enrollment rate, $F(z,h)$ represents the stationary distribution over (z,h), and μ_j is the stationary distribution of age.

framework is left for future research.

	Education	Typical
TFR (Δ%)	+3.0	+2.5
HS	+0.4	+2.5
CL	+7.4	+2.4
CL share (Δ p.p.)	+3.9	0.0
HS→CL	+2.5	0.0
Avg. HC (Δ%)	+1.0	0.0
Output (Δ%)	+1.0	−0.3
STD (wage) (Δ%)	−1.3	0.0
Welfare (%)	+4.8	+0.3

Table 12: Main results with several schemes with different targets. *Note*: Columns "Education" and "Typical" represent the results with the introduction of the education subsidy and expansion of child benefit ("typical" pro-natal transfers), respectively. Values in each cell indicate changes from the benchmark value. Rows "HS" and "CL" indicate the percentage changes in the fertility of high school and college graduates. "HS→CL" represent the changes in college enrollment rate and education mobility in the sense of probability that children of high school graduates attend college. "Ave. HC" stands for the average human capital.