Education Policies, Fertility Differentials, and Macroeconomic Outcomes*

Kanato Nakakuni†

April 16, 2024

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Abstract

In most high-income countries, college graduates have fewer children and invest more in their children's education than non-college graduates. Considering this fertility differential, what are the macroeconomic consequences of college financial aid policy? I construct a heterogeneous-agent general-equilibrium overlapping-generations model with choices regarding fertility, college enrollment, and inter vivos transfers. Fertility choices are irreversible, and households make the choice in the presence of uncertainty regarding their future income and children's characteristics. The calibrated and validated model demonstrates that income-tested college subsidy provides partial insurance against costly states associated with having a child, especially for college graduates, stimulating the fertility of ex-post-ineligible parents whose children do not receive the subsidy. This fertility response reduces the differential and amplifies the policy effects on college enrollment, output, and welfare, operating through several channels such as intergenerational linkages, distribution changes, and general equilibrium feedback.

Keywords: Fertility, education, intergenerational linkages
JEL codes: C68, I28, J13, J24

*I am deeply indebted to my advisors, Makoto Nirei and Sagiri Kitao, for their continuous guidance and support. I am also grateful to Minsu Chang, James Graham, Fergal Hanks, Chihiro Inoue, Akira Ishide, Yusuke Ishihata, Ryo Jinnai, Akihisa Kato, Colin McKenzie, Kazushige Matsuda, Yena Park, Ponpoje Porapakkarm, Xavier Ragot, Yuta Takahashi, Satoshi Tanaka, Kazuhiro Teramoto, Makoto Watanabe, Minchul Yum, and seminar participants at the Annual Macro Conference at Osaka University, Hitotsubashi University, OzMac workshop at Monash University, and Waseda University for their many helpful comments. This study is financially supported by the Japan Society for the Promotion of Science (23KJ0374). This paper was originally circulated under the title "Finanicial costs of children, education subsidies, and fertility choices in equilibrium."

†University of Tokyo, Japan. Email address: nakakunik@gmail.com.

1 Introduction

In most high-income countries, college graduates have fewer children and invest more in their children's education than non-college graduates.[1] Parents' joint decisions on fertility and educational investments determine future household distributions on age, income, and human capital. Therefore, this fertility differential and education distribution matter to the long-run household distributions and thus broad macroeconomic factors, such as inequality, aggregate labor productivity, and national income.[2]

Given these considerations, this paper examines the macroeconomic consequences of college financial aid policy by constructing a new general-equilibrium (GE) model that features fertility and college enrollment choices. My central contribution is to highlight the critical roles of the fertility margins in evaluating the macroeconomic performance of the policy. More specifically, an income-tested college subsidy reduces the fertility differential by providing partial insurance against costly states associated with having children for college graduates. Further, this fertility response amplifies the policy effects on college enrollment, output, and welfare, operating through several channels such as intergenerational linkages, distribution changes, and general equilibrium feedback.

In this model, agents choose whether to attend college after graduating from high school. In college, students must finance their tuition fees and consumption, which can be done via inter vivos transfers (IVTs) from their parents, their own labor earnings, and, if eligible, government-provided financial aid in the form of grants and loans. At a certain point in life, the agents make fertility choices regarding how many children to have. They derive utility from the number of children, education investments, and the IVTs made for them upon their high school graduations. When they make fertility choices, they are uncertain about (1) their future income because of idiosyncratic shocks, (2) potential children's human capital, which determines education return, and (3) their school tastes. The latter two, (2) and (3), are correlated with the parents' characteristics.

I use the Japanese Panel Survey of Consumers (JPSC) to calibrate the model to Japan, where the government recently introduced grants for college students as parents have incurred a substantial fraction of college education costs. The model replicates key

[1]For example, in the 1960s cohort in Japan, the complete fertility of college graduate females is approximately 10% lower than that of high school graduates, as elaborated in Section 4.2.2. This fertility differential between college and high school graduates is also observed in other countries, such as the US, France, Spain, and Germany (See, Figures 13 and 15 in Doepke et al. (2023)). Appendix A. provides some theoretical explanations for why and when a negative fertility-income relationship arises using simple models, which is informative for understanding the fertility differential across education groups. See Jones et al. (2010) for more details on the theoretical treatment in this respect.

[2]Previous studies highlight the implications of fertility differentials across income distribution for macroeconomic outcomes, such as economic growth (De La Croix and Doepke, 2003) and social mobility (Daruich and Kozlowski, 2020).

moments, such as the (1) average amount of parental asset transfers for college students, (2) intergenerational persistence of education levels, and (3) fertility differential across education levels. I also validate the model's fertility behavior using empirical estimates for the cash-benefit elasticity of fertility, indicating the extent to which fertility rates increase in response to cash transfers. The benchmark model captures government-provided student loans, which are income- and ability-tested, but does not include the existing grants introduced in 2020. Thus, no grants are available to college students in the benchmark model, given that no grants were available to them until their introduction in 2020 in Japan.[3] Existing grants are income-tested, and only households in approximately the bottom 15% of the income distribution are eligible. The payments cover approximately two-thirds of students' average expenses.

The main findings are summarized as follows. First, the introduction of income-tested grants increases the fertility of college graduates by 7.4%, while it does not significantly affect high school graduates. The reduction of skill premium in the long run, due to a 3.9 percentage point (p.p.) higher college enrollment rate, reduces the opportunity costs of having children for college graduates relative to high school graduates (i.e., a GE effect), which explains one-fourth of the fertility increase. Additionally, the significant increase in college graduates' fertility is mainly explained by the fertility increase of ex-post ineligible ones whose children do not receive grants. Decomposition analysis suggests that, in addition to the GE effect, an insurance effect accounts for a substantial part of the fertility increase; the income-tested grants provide partial insurance against costly states associated with having a child (or another child), especially for college graduates.

To understand this result, note that the intergenerational persistence of education levels implies that when college graduates make fertility choices, they expect their (potential) children to attend college. Thus, they also expect sizable IVTs they have to make to support their college enrollment. However, the future realization of negative income shocks would make it infeasible for parents to financially support their children's enrollment, or they may end up with low consumption in exchange for supporting the enrollment: either situation is costly for parents. Thus, income volatility with higher expected IVT causes hesitation among some college graduates regarding having a child. In this situation, the grants provide partial insurance against such a costly state, make college graduates feel more comfortable having a child (or another child), and increase their fertility.

Second, these fertility responses amplify the policy effects on college enrollment rate, per-capita output, and welfare in the long run through intergenerational linkages and

[3] This model does not allow students to access financial aid (grants or loans) provided by institutions besides the government (e.g., colleges or other private entities). In 2018, among students in four-year universities who used any financial aid, 83.8% used only government-provided aid, 8.6% used only non-government-provided aid, and 7.6% used both, according to the Student Life Survey (SLS, 2018).

distribution changes. As college graduates' fertility increases, those "marginal" children born are also likely to attend college due to the intergenerational transmission of human capital, education tastes, and assets (IVT). Thus, the fertility responses lead to a higher share of college graduates in the long run, thereby contributing to a lower skill premium and a greater per-capita that results from the increase in the average productivity of workers. The higher aggregate fertility implies a more significant share of the working-age population, contributing to the greater per-capita output. The more significant output implies greater tax bases, reducing the equilibrium tax rate. Those factors (i.e., a higher probability of attending college, lower inequality, and lower taxes) bring more significant welfare gains for newborn agents under the veil of ignorance. The roles of these fertility margins are highlighted by the results under an exogenous fertility version of the model in which household decision rules regarding fertility are fixed to those in the benchmark. Compared to the exogenous fertility setup, the grant introduction under endogenous fertility leads to a 2.5% higher college enrollment rate, a 2.5 times greater increase in per-capita output, and 46% more significant welfare gains.

The fertility differential decreases further with the marginal expansion of the policy, setting a higher income threshold. This is because the college graduates' fertility increases further with expansion, primarily due to the insurance effect, while the high school (HS) graduates' fertility decreases with eligibility expansion, primarily for the following reasons. The expansion increases educational mobility in the sense that the children of HS graduates are more likely to attend college. Higher educational mobility makes HS graduates expect their (potential) children to be more likely to attend college when making fertility decisions. The higher probability of children's college enrollment increases the expected costs of having children, given that the grants are not sufficiently generous to cover 100% of the costs of sending a child to college; the higher expected costs then decrease fertility among HS graduates. The reduced fertility differential contributes to a higher college enrollment via intergenerational linkages. However, the positive effects on output in expansion diminish as the broader coverage significantly crowds out savings among households with children, reducing the capital stock of the economy.

2 Related Literature

This paper contributes to several strands of literature. The first is the macroeconomic analysis of college financial aid policies.[4] I incorporate fertility choices into a framework

[4]See, for example, Benabou (2002); Akyol and Athreya (2005); Krueger and Ludwig (2016); Lawson (2017); Abbott, Gallipoli, Meghir, and Violante (2019); Lee and Seshadri (2019); Matsuda and Mazur (2022); Matsuda (2022).

that is otherwise standard in the literature (i.e., the heterogeneous-agent GE-OLG model with education and IVT choices). As a result, this model allows for the fertility differential across education groups, as observed in data, which starkly contrasts with models in the literature in which all households have the same number of children. My extended framework with fertility choices and differentials highlights the roles of fertility margins in understanding the macroeconomic performance of college financial aid policies.

Second, this study is closely related to macroeconomic studies based on the quantity-quality trade-off framework.[5] My primary contribution to the literature is constructing a new model with college enrollment choices in the GE framework, which enables us to examine college financial aid policies.[6] From a modeling viewpoint, the closest work is Daruich and Kozlowski (2020). They construct a partial equilibrium lifecycle model with college enrollment, fertility, and IVT choices to investigate the role of fertility choices and family transfers in explaining intergenerational mobility in the US. This study differs from theirs in adopting the GE framework and examining the macroeconomic consequences of college financial aid policies.[7]

Third, this study is closely related to the literature on fertility choices in incomplete market models.[8] Previous studies demonstrate that having a child can be considered making "consumption commitments," and that income volatility thus makes households hesitate to have children.[9] This study contributes to the literature by examining policies that can potentially reduce the risks associated with having a child. This study then highlights the insurance effects of income-tested college subsidies on fertility choices, especially for college graduates whose children are likely to attend college.

Fourth, this study also relates to the literature on the macroeconomic and fiscal implications of demographic aging and the associated analyses of, for example, tax reforms,

[5]See, for example, De La Croix and Doepke (2003); Manuelli and Seshadri (2009); Cordoba, Liu, and Ripoll (2019); Daruich and Kozlowski (2020); Zhou (2022); Kim, Tertilt, and Yum (2023).

[6]Several papers are closely related to this study. The critical difference is that this study incorporates college enrollment choices into a full-lifecycle model to examine college education policies. Kim, Tertilt, and Yum (2023) build a two-period OLG model with the quantity-quality trade-off to examine the role of the status externalities in education on the low fertility in South Korea. Zhou (2022) builds a multi-period GE-OLG model capturing the quantity-quality trade-off and endogenous human capital accumulation for children to study the macroeconomic consequences of family policies. De la Croix and Doepke (2004) construct a two-period OLG model with fertility choices to compare the macroeconomic implications of public and private schooling regimes.

[7]A higher college enrollment rate facilitated by college subsidies reduces the skill premium in the long run. The change in skill premium implies the change in income distribution (e.g., Krueger and Ludwig, 2016) and also affects the incentive of college enrollment (e.g., Abbott, Gallipoli, Meghir, and Violante, 2019). My results presented in Section 5 also highlight the roles of GE effects in explaining the long-run effects of education subsidies on fertility.

[8]For example, Schoonbroodt and Tertilt (2014); Santos and Weiss (2016); Sommer (2016).

[9]This is the case, especially in the early stages of their lives, thus delaying marriage and fertility (Santos and Weiss, 2016). Delaying fertility leads to low fertility, given that the ability to reproduce declines with age (Sommer, 2016).

social security reforms, and immigration policies.[10] This study adds an education policy to the list, a countermeasure that enhances aggregate human capital in a country facing a declining labor force. I show that some education subsidies (e.g., unconditional grants for college students) unintentionally decrease the labor force in the long run due to a composition effect; college financial aid increases the population share of college graduates with fewer children.

Finally, this study contributes to the macroeconomic literature on pro-natal policies[11] by adding information about the effects of college financial aid, which is a novel pro-natal policy implemented in Japan. As discussed in Appendix B., the college financial aid policies are also considered pro-natal policies in Japan, given facts suggesting that the financial costs for parents to support their children's college enrollment are a significant impediment to fertility decisions. Compared with typical pro-natal transfers such as baby bonuses, the notable feature of the financial aid is that it increases the average human capital by promoting skill acquisition.[12] By examining this novel policy in Japan, this study provides insights into countries considering countermeasures against macroeconomic concerns of low fertility.

3 Model

I incorporate fertility choices into a model otherwise standard in the macroeconomic literature of college financial aid policies: the heterogeneous-agent GE-OLG model with IVT and college enrollment choices. Section 3.1 provides an overview of the lifecycle of this economy. Section 3.2 elaborates on preliminaries of the model and then Section 3.3 formulates households' decision problems. Section 3.4 discusses the stationary equilibrium of this economy.

3.1 Overview of the lifecycle

Fig 1 represents the households' lifecycle in this model. Time is discrete, and one period corresponds to two years in this model. Let j denote the agents' age. They live with

[10] See, for example, Nishiyama and Smetters (2007); Imrohoroğlu and Kitao (2012); Braun and Joines (2015); İmrohoroğlu, Kitao, and Yamada (2017); Hsu and Yamada (2019).

[11] Previous studies examine the effects of childcare subsidization (e.g., Bick, 2016), cash transfers (e.g., Kim, Tertilt, and Yum, 2023; Nakakuni, 2023), both of them (e.g., Hagiwara, 2021; Zhou, 2022), parental leave policies (e.g., Erosa, Fuster, and Restuccia, 2010; Yamaguchi, 2019; Kim and Yum, 2023), and tax reform (e.g., Jakobsen, Jørgensen, and Low, 2022).

[12] Previous studies show that pro-natal transfers would lower aggregate human capital because they make parents shift from the "quality" toward "quantity" of children (e.g., Zhou, 2022; Kim et al., 2023). Actually, Appendix H. shows that the income-tested grants would lead to greater aggregate human capital and output than pro-natal transfers conducted in an expenditure-neutral way.

their parents until they graduate from high school at the beginning of age $j = J_E (:= 18)$; until then, they make no decisions. After graduating high school, they choose whether to attend college or enter the labor market after graduating high school, represented as a node "Grad.HS" in the figure. If they do not enroll in college, they enter the labor market as a high school graduate. If they choose to attend college, it takes four years (two model periods) to complete, and they enter the labor market as a college graduate after graduation, represented as a node "Grad.CL" in the figure.[13]

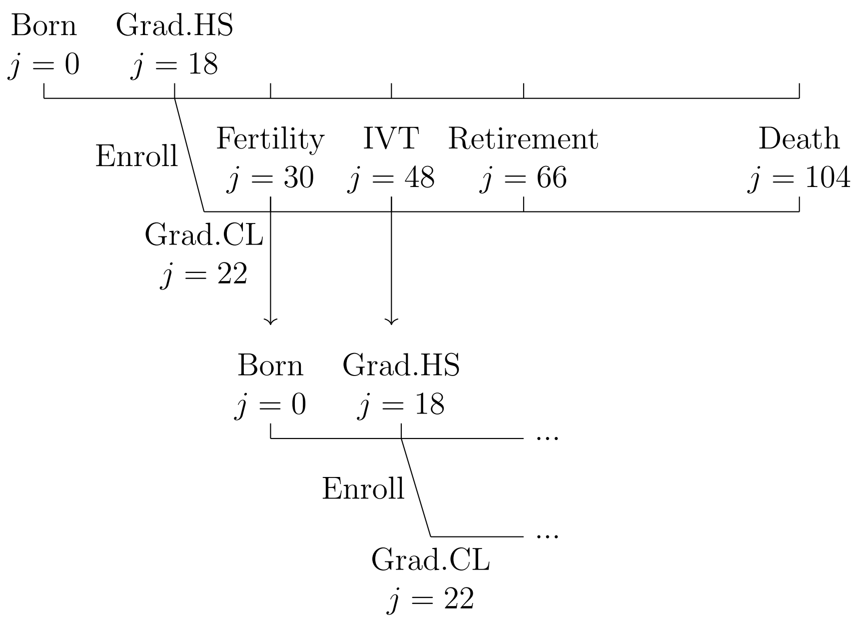

Fig 1: Model's lifecycle.

After completing their education, they enter the labor market. At the beginning of age $j = J_F (:= 30)$, they make fertility choices by choosing how many children they have. The timing of the fertility choice is common for high school and college graduates. The lifecycle of a new cohort starts at this point (provided that the fertility rate is positive), represented in the bottom half of Fig 1. After their children graduate from high school, corresponding to the beginning of the age $j = J_{IVT} (:= 48)$ for parents, they decide how much money to transfer to their children. This IVT decision affects the children's college enrollment choices at the node "Grad.HS." Households retire from the labor market at the beginning of age $j = J_R (:= 66)$. After that period, they face mortality risks; every period, a certain fraction of them is hit by exogenous mortality shocks and exits from the economy. They can live for J $(:= 104)$ years at the longest, and they exit the economy after age J.

Dividing the model's lifecycle into several stages, summarized in Table 1, helps us

[13]This model does not consider the possibility of dropping out from college because, as I mentioned, the dropout rate is insignificant in Japan.

understand its structure; according to these stages, I formulate household problems in Section 3.3. Also, to provide a whole picture of the model's lifecycle, I provide lists of choices and state variables in Tables 2 and 3 in advance, including their relevant stages. Each variable is elaborated on in the following.

Stage	Corresponding age
Education stage	$18(-21)$
Working stage without children (1)	$18(22)-29$
Fertility stage	30
Working stage with children	$30-47$
Inter vivos transfers stage	48
Working stage without children (2)	$48-65$
Retirement stage	$66-$

Table 1: The lifecycle stages and the corresponding age. *Note*: Numbers in parentheses indicate the corresponding age for those who choose to attend college.

Notation	Age/Period	Description
Throughout the entire stages		
c	$18-$	Consumption
l	$18-$	Leisure
a'	$18-$	Saving
Education stage		
e	18	College enrollment
Working stage with children & IVT stage		
n	30	The number of children (fertility)
q	$30-47$	Education spending
a_{IVT}	48	Inter vivos transfers

Table 2: Summary of the choice variables over the life stages.

Notation	Age/Period	Description
Throughout the entire stages		
j	18–	Age
a	18–	Asset
Until retirement		
z	$18(22)-65$	Labor productivity $\sim$ AR(1)
e	$18(22)-65$	Education (HS or CL)
h	$18-65$	Human capital
Education stage		
ϕ	18	Psychic costs of college enrollment
I	$18(-21)$	Household income
Working stage with children		
n	$30-48$	The number of children
Inter vivos transfers stage		
h_k	48	Children's human capital endowment
ϕ_k	48	Children's psychic costs of college education

Table 3: Summary of the stage variables over the life stages. *Note*: Numbers in parentheses indicate the corresponding age for those who choose to attend college. "HS" and "CL" stand for high school and college.

3.2 Preliminaries

Production: A representative firm chooses labor and capital inputs in competitive factor markets to produce final goods. There are two types of labor inputs in this economy; the college graduates (skilled) and high school graduates (unskilled). Their total labor supply in efficiency units are represented as L_{CL} and L_{HS}, respectively. I allow them to be imperfect substitutes by considering the aggregate labor in efficiency units, L, is given as:

$$L = \left[\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}\right]^{1/\chi},$$

where $\omega_{HS} \equiv 1 - \omega_{CL}$ and $\omega_{CL} \in [0, 1]$ governs the relative productivity of the skilled workers. The representative firm operates with a Cobb-Douglas production function with aggregate capital K and labor L to produce the output Y:

$$Y = ZK^{\alpha}L^{1-\alpha},$$

where Z represents the factor neutral productivity. Let r, w_{HS}, and w_{CL} denote the rental rate of capital and wage rates for unskilled and skilled labor. Capital depreciates at δ, and the firm has to incur the capital depreciation costs.

Demographics: Every period, a mass of new cohorts enters the economy. The size of the new cohort is endogenously determined by aggregating fertility choices. The age distribution of this economy is thus determined by (1) households' fertility choices, which are endogenous, and (2) mortality risks after retirement, which are exogenous. Let $\zeta_{j,j+1}$ denote the probability of surviving at age $j+1$ conditional on surviving at age j for each $j \in \{J_R, ..., J\}$ with $\zeta_{J,J+1} = 0$.

Intergenerational Linkages and Initial Endowments: After graduating high school (at the beginning of age 18), agents are endowed with a triple (a_{IVT}, h, ϕ): (1) assets transferred from their parents (a_{IVT}), (2) human capital (h), which governs education returns in future earnings, and (3) psychic costs of college education (ϕ). They draw the human capital from a distribution $g^h_{h_p}$, varying with the parents' human capital level h_p. They also draw the psychic costs from a distribution g^{ϕ}_{h,e_p}, varying with the student's human capital h and parent's education e_p.

Preferences: Throughout their lifetime, they draw utility from consumption c and leisure l according to a utility function $u(c, l)$. They discount future utility by β. At age $j = J_F$, they choose the number of children they have, denoted by $n \in \{0, 1, ..., N\}$. They draw additional utility from having children in several ways. First, they draw utility from education spending q per child until the children graduate from high school, captured by a utility function $v(q)$. The utility $v(q)$ is discounted by a function $b(n)$, increasing and concave in the number of children n. Further, based on altruistic motives, parents draw utility by transferring assets to their children after the children graduate from high school. More specifically, parents consider children's expected lifetime utility in their education stage, with a discount rate $\lambda_a \cdot b(n)$, where λ_a represents the altruistic discount factor.

Costs of Children: Having children is costly in money and time. First, q units of the per-child education spending require $n \cdot q$ units of money, and they will make an additional expenditure $n \cdot a_{IVT}$ upon high school graduation of their children.[14] In addition to the monetary costs, having a child requires κ units of time until the child graduates from high school and becomes independent.

[14]Children within a household are homogeneous, as assumed in the literature (e.g., Abbott, Gallipoli, Meghir, and Violante, 2019). Thus, education spending and IVT do not vary among children within the household.

Labor Earnings: Households choose hours worked to earn income. The labor earnings of a household are given by $w_e \eta_{j,z,e,h}(T-l)$; it is a product of the market wage w_e, which depends on education level $e \in \{HS, CL\}$, labor efficiency or productivity $\eta_{j,z,e,h}$, and hours worked $(T-l)$. Here, T denotes the disposable time that can be devoted to work or leisure where $T = 1 - \kappa \cdot n$ if they have n children who have not graduated high school and $T = 1$ otherwise. Labor efficiency depends on age j, idiosyncratic productivity shock component z, education level e, and human capital h.

Financial Markets: Financial markets are incomplete due to the lack of state-contingent claims. Households can self-insure against risks by savings, accruing interest payments at a rate of r. Households in the working stages can borrow at rate $r^- = r + \iota$ where $\iota > 0$ (i.e., borrowers incur the overseeing costs ι) up to a borrowing limit $\underline{A}$. In contrast, retired households are not allowed to borrow, as in the literature. In addition, eligible students have access to student loans subsidized by the government, which entails the interest rate of $r^s = r + \iota_s$. This loan is income- and ability-tested, and eligible students can borrow up to a limit $\underline{A}_s$.

Government: The government raises the revenue by levying three types of taxes: consumption, labor income, and capital income taxes, where each tax rate is represented as τ_c, τ_w, and τ_a. In addition, the government collects accidental bequests and devotes them to cover expenditures. They use this revenue to fund (1) the public pension benefits, which gives p units of money to retired households each period, (2) subsidized loans for college students, (3) grants for eligible college students, where the payment per eligible student is represented by $g(h, I)$ where $I \geq 0$ denotes the household income when the student faces education choice problem (i.e., their parent's age is J_{IVT}), (4) lamp-sum transfers ψ that is introduced for replicating the progressivity of labor income tax schedule in a simple way following the literature, (5) cash benefits for households with children under 17 with per-child payment of B, and (6) other expenditures S. I do not consider grants in the benchmark to replicate the economy before grants are introduced in 2020 and set $g(h, I) = 0$ for each (h, I). The government budget constraint is given as follows:

$$\tau_c \cdot C + \tau_w \cdot (L_{HS} + L_{CL}) + \tau_a \cdot K + Q = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \leq 17} + S, \quad (1)$$

where C, Q, μ_{old}, $\mu_{j \leq 17}$, K_s, and G represent the total consumption, total accidental bequests, population mass of retired households, that of children under age 17, the total amount of borrowing by college students, and the total grant payments.

3.3 Household Problems

This section formulates household problems in each stage defined in Table 1. The first is the education stage in which they choose whether to attend college or enter the labor market after graduating high school.

Education Stage: After graduating high school, they draw the human capital h from the distribution $g^h_{h_p}$ and psychic cost ϕ from the distribution g^{ϕ}_{h,e_p}. They also receive IVT from their parents $a_{IVT} \geq 0$. Some of them can access subsidized loans to fund expenditures that arise during the college education stage, which is income- and ability-tested. Thus, the state variables for the students are comprised of asset a_{IVT}, the human capital h, psychic costs ϕ, and their parent's income I. They compare the expected value for entering the labor market as a high school graduate, $\mathbb{E}V^w$, with the value for enrolling in college, V_{g1} net of the psychic cost ϕ. They choose college enrollment if the latter is greater than the former; otherwise, they enter the labor market. The decision problem is formulated as follows:

$$\begin{aligned} V_{g0}(a_{IVT}, \phi, h, I) = \max_{e \in \{0,1\}} \Big\{ &(1-e) \cdot \mathbb{E}_{z_0}[V^w(a_{IVT}, j=18, z_0; e=0, h)] \\ &+ e \cdot [V_{g1}(a_{IVT}; h, I) - \phi] \Big\}, \end{aligned} \tag{2}$$

where $e \in \{0,1\}$ indicates the education choice where $e=1$ means college enrollment and $e=0$ does entering the labor market as a high school graduate. V^w denotes the value function for workers, which I formulate in the next subsection. The initial draw of z, z_0, is uncertain and is according to the invariant distribution of z, $\bar{\pi}_z$, so the expectation operator is put next to the V^w. The value for college enrollment, V_{g1}, is defined as follows:

$$\begin{aligned} V_{g1}(a_{IVT}; h, I) &= \max_{c,l,a'} \{u(c,l) + \beta V_{g2}(a'; h, I)\}, \\ V_{g2}(a; h, I) &= \max_{c,l,a'} \{u(c,l) + \beta \mathbb{E}_{z_0}[V^w(a^s(a'), j=22, z_0; e=1, h)]\}. \end{aligned} \tag{3}$$

The budget constraints differ according to eligibility to the student loans. The budget constraints for eligible students are give as follows:

$$\begin{aligned}(1+\tau_c)c + p_{CL} + a' &= (1-\tau_w)w_{HS}(1-\bar{t}-l) + \psi + g(h,I) \\ &\quad + \begin{cases} (1+(1-\tau_a)r)a & \text{if } \ a \geq 0 \\ (1+r^s)a & \text{otherwise} \end{cases} \\ a' &\geq -\underline{A}_s. \end{aligned} \tag{4}$$

The rest of the students cannot access to the student loans, which implies that their budget constraints are given as follows:

$$\begin{aligned}(1+\tau_c)c + p_{CL} + a' &= (1-\tau_w)w_{HS}(1-\bar{t}-l) + \psi + g(h,I) + (1+(1-\tau_a)r)a, \\ a' &\geq 0. \end{aligned}$$

College students draw utility from consumption and leisure and must pay tuition fees p_{CL}. They can fund the consumption and tuition fees through (1) transfers from their parents a_{IVT}, (2) borrowing through student loans if eligible, (3) government-provided grants if eligible, and (4) labor earnings by themselves. Students must spend a $\bar{t}$ fraction of their time on study while in college. Thus, they choose the time allocation between leisure and working over the disposable time $1-\bar{t}$, where the total disposable time is normalized as 1. One unit of labor supply gives college students w_{HS} units of wages.[15] Following the literature, I assume that fixed payments are made for 20 years (10 periods) following college graduation and transform college loans into regular bonds according to the following formula:

$$a^s(a') = a' \times \frac{r^s}{1-(1+r^s)^{-10}} \times \frac{1-(1+r^-)^{-10}}{r^-}.$$

Working Stage without Children: The remaining component in (2) and (3), V^w, represents the value function for working households without children (i.e., for those aged $j \in \{J_E, ..., J_F - 1, J_{IVT}+1, ..., J_R - 1\}$). The state variables for this stage consists of asset a, age j, idiosyncratic component of labor productivity z, education level e, and human capital h. The uncertainty in this stage is only about the next period's productivity, which is denoted by z' following a Markov process $\pi_z(z', z)$. Households choose consumption,

[15] I assume that, while in college, there is no heterogeneity in labor efficiency and no uncertainty regarding the next period's productivity, and one unit of hours worked brings one unit of labor efficiency.

leisure, and savings given the state variables. The value function is formulated as follows:

$$
\begin{aligned}
V^w(a,j,z;e,h) = \max_{c,l,a'} \{ & u(c,l) \\
& + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^f(a',z',e,h)] & \text{if } \ j = J_F - 1 \\ \beta [V^r(a', j+1)] & \text{if } \ j = J_R - 1 \\ \beta \mathbb{E}_{z'}[V^w(a', j+1, z'; e, h)] & \text{otherwise} \end{array} \right\}
\end{aligned} \tag{5}
$$

s.t.

$$
\begin{aligned}
& (1+\tau_c)c + a' = (1-\tau_w) w_e \eta_{j,z,e,h}(1-l) + \psi + (1 + (1-\tau_a) r) a, \\
& z' \sim \pi(z', z), \\
& a' \geq \left\{ \begin{array}{ll} 0 & \text{if } \ j = J_R - 1, \\ -\underline{A} & \text{otherwise.} \end{array} \right.
\end{aligned}
$$

Value functions V^f and V^r represent those for the fertility stage and retirement stage, which are formulated in the following subsections.

Fertility Stage and Working Stage with Children: When they reach the age of $j = J_F$, they choose how many children they have. The problem is formulated as follows.

$$
V^f(a,z,e,h) = \max_{n \in \{0,1,..,N\}} \left\{ V^{wf}(a, j = J_F, z; e, h, n) \right\},
$$

where $V^{wf}(a,j,z;e,h,n)$ in the parenthesis represents the value function for the working stage with children (i.e., for households aged $j \in \{J_F, ..., J_{IVT} - 1\}$). As in the previous life stages, they draw utility from consumption and leisure and decide on consumption, time allocation, and savings. In addition, they draw utility from the number of children n and education spending q for each child during the stage.[16] The value function V^{wf}, for $j \in \{J_F, ..., J_{IVT} - 1\}$, is then formulated as follows:

$$
\begin{aligned}
V^{wf}(a,j,z;e,h,n) = \max_{c,l,q,a'} \{ & u(c/\Lambda(n), l) + b(n) \cdot v(q) \\
& + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^{wf}(a', j+1, z'; e, h, n)] & \text{if } j \in \{J_F, ..., J_{IVT} - 2\} \\ \beta \mathbb{E}_{z',\phi_k,h_k}[V^{IVT}(a', z'; \phi_k, h_k, e, h, n)] & \text{if } j = J_{IVT} - 1 \end{array} \right\}
\end{aligned}
$$

s.t.

$$
\begin{aligned}
& (1+\tau_c)(c + nq) + a' = Y_{wf}, \\
& a' \geq -\underline{A}, z' \sim \pi(z', z), h_k \sim g_h^{h_k}, \phi_k \sim g_{e,h_k}^{\phi_k},
\end{aligned}
$$

[16] This study does not consider children's endogenous human capital accumulation through parental investments and assumes parents make educational spending on children just because it draws utility. Incorporating the endogenous human capital accumulation is left for future research.

where

$$Y_{wf} \equiv (1-\tau_w) w_e \eta_{j,z,e,h}(1-l-\kappa \cdot n) \\ + n \cdot B + \psi + \begin{cases} (1+(1-\tau_a)r)a & \text{if} \quad a \geq 0, \\ (1+r^-)a & \text{otherwise.} \end{cases}$$

Here, ϕ_k and h_k denote the psychic costs and human capital for their children. The household consumption is deflated by the equivalence scale $\Lambda(n)$, depending on the number of children n. The total education spending on children, nq, enters the budget constraint, and utility from children, $b(n) \cdot v(q)$, enters the objective function in the periods with children. V^{IVT} represents the value function for the IVT stage at $j = J_{IVT}$, which is described in the following subsection. Households are uncertain about their children's human capital h_k and psychic costs ϕ_k until the beginning of age $j = J_{IVT}$, which can be interpreted as uncertainty about the expenditures on children in the form of IVT, as will be evident in the following subsection.

Inter Vivos Transfers Stage: At age $j = J_{IVT}$, households decide how much to transfer to their children. This timing corresponds to when the children graduate from high school and face the college enrollment choice. Two characteristics of their children are realized at this point: their psychic costs ϕ_k and human capital h_k. After observing the realized characteristics, households choose the optimal amount of per-child transfer a_{IVT}, formulated as follows:

$$V^{IVT}(a, z; \phi_k, h_k, e, h, n) = \max_{a_{IVT} \geq 0} \Big\{ V^w(a - \tilde{a}_{IVT}, j = J_{IVT}, z; e, h) \\ + b(n) \cdot \lambda_a \cdot V_{g0}(a_{IVT}, \phi_k, h_k, I) \Big\},$$

where $\tilde{a}_{IVT} = \frac{n \cdot a_{IVT}}{1+(1-\tau_a)r}$ and the parent's state vector pins down household income I. They care about the lifetime utility of each child, $V_{g0}(a_{IVT}, \phi_k, h_k, I)$, based on altruism, discounted by an altruistic discount factor λ_a.

Note that the children's characteristics (h_k, ϕ_k) govern education returns and the willingness to attend college, which in turn governs the marginal gains from IVT for parents,[17]

$$b(n) \cdot \lambda_a \cdot \frac{\partial V_{g0}(a_{IVT}, \phi_k, h_k, I)}{\partial a_{IVT}}.$$

Thus, uncertainty about their children's characteristics, which households face one period

[17] For better clarity, this representation implicitly assumes that the value function V_{g0} is differentiable. However, in principle, this is not the case because of the discrete nature of the college enrollment choice.

ahead of the IVT stage, is interpreted as uncertainty about the expenditures on children in the form of IVT.

Retirement Stage: At the beginning of age J_R, households are forced to retire from the labor market. After that, they spend all their time on leisure and make consumption-saving decisions. Two points differ from previous choice problems; they receive the pension benefit p from the government and face uncertainty about the next period's survival. The value function for the retirement stage is formulated as follows:

$$
\begin{aligned}
V^r(a, j) = \max_{c, a'} \; & u(c, 1) + \beta \xi_{j,j+1} V^r(a', j+1) \\
\text{s.t.} \quad & \\
& (1+\tau_c)c + a' = p + (1 + (1-\tau_a)r)a + \psi, \\
& a' \geq 0.
\end{aligned}
$$

3.4 Stationary Equilibrium

I solve the stationary equilibrium of this economy. In equilibrium, households make every choice to maximize their expected utility, the firm maximizes its profit, and the government budget is balanced. Stationarity implies that the distribution over state variables is invariant. Importantly, the age distribution is determined endogenously according to households' fertility choices. See Appendix D. for the detailed definition of equilibrium and Appendix E. for the computational algorithm for solving the equilibrium.

4 Calibration

To calibrate the model, I use the Japanese Panel Survey of Consumers (JPSC), a panel survey of Japanese women and their household members. It started in 1993 with a representative sample of 1,500 women aged $24-34$ and contains information about, for example, their income, educational background, marital status, fertility, and expenditures to detailed categories, including those on children's education. I focus on the cohort born in 1959-69, the oldest cohort of this survey, especially to compute the completed fertility and intergenerational mobility of education. I keep only married households as in previous works (e.g., Daruich and Kozlowski, 2020) because the model focuses on choices made within married households such as fertility and educational investments.[18]

[18]Hence, an agent or household in this model refers to households with two individuals. Then, "children" in this model can also be interpreted as a household unit. That is, having n children can be interpreted as reproducing $n/2$ units of households. For example, if a household has two children, it means reproducing one household unit with two individuals.

Unless otherwise mentioned, targeted moments for the parameters internally determined, described below, are computed based on the JPSC's 1959-69 cohort data.

4.1 Targeted Moments

Preferences: Instantaneous utility for households are given as follows:

$$u(c,l) = \frac{(c^{\mu} l^{1-\mu})^{1-\gamma}}{1-\gamma}.$$

μ is internally determined as 0.23 so that the households spend one-third of the total disposable time on market work. Instantaneous utility from education spending on a child is given as:

$$v(q) = \lambda_q \frac{q^{1-\gamma}}{1-\gamma},$$

where $\lambda_q = 0.62$ so that the annual educational expenditure per child amounts to 7% of average income at age 28. The utility must be always positive (or always negative) in models of altruism with endogenous fertility, and I set $\gamma = 0.5$ following the literature (e.g., Daruich and Kozlowski, 2020). The altruistic discount factor λ_a is set to 1.03 so that the annual average IVT for college students amounts to 27% of average income at age 28.[19] Following Kim, Tertilt, and Yum (2023), I let the discount function of the number of children $b(n)$ be non-parametric, and assume that $b(n) = b_n$ for each $n \in \{0, 1, 2, 3, 4\}$ with $b(0) = b_0 = 0$. Those parameters are determined so that the model replicates the distribution of the completed fertility. The time for studying $\bar{t}$ is set to 0.8 so that the income share of labor earnings for college students in the model is close to 21% (SLS, 2018). I assume that $\beta = 0.98$ as in Zhou (2022).

Financial Markets: I set the borrowing limits $\underline{A} = 20$ million yen and $\underline{A}_s = 2.88$ million yen.[20] The borrowing wedges are set $\iota = 0.055$ and $\iota_s = 0.054$ so that the model approximates the share of working households with a negative net worth (54%) and the share of students borrowing from the government-provided student loans (44%).

School Taste: I assume that the psychic costs ϕ are given as $\phi = \psi_{CL} \cdot \exp(-\nu \cdot h) \cdot \tilde{\phi}$. First, ψ_{CL} governs the scale of psychic costs and thus college enrollment rate. The second term $\exp(-\nu \cdot h)$ allows high ability students to have smaller psychic costs, as standard in

[19] Although some parents whose children do not enroll in college make IVT in the model, the amount is negligibly small. One reason is that the marginal gains from IVT are more significant if their children attend college, as students are financially constrained, primarily because of their limited earning ability.

[20] The former is based on the Family Income and Expenditure Survey by the Ministry of Interna Affairs and Communications and the latter is based on the SLS (2018).

the literature, and ν governs the education sorting by ability. Finally, $\tilde{\phi}$ is stochastic, and the distribution depends on their parent's education. Following Daruich and Kozlowski (2020), I assume that $\tilde{\phi}$ is distributed on an interval $[0,1]$ and follows the following CDF:

$$G^{\tilde{\phi}}_{e_p} = \begin{cases} \tilde{\phi}^{\omega} & \text{if } e^p = 0 \\ 1-(1-\tilde{\phi})^{\omega} & \text{if } e^p = 1 \end{cases}$$

Here, ω governs the intergenerational transmission of school tastes. ψ_{CL} is set to 20.8 so that the model approximates the college enrollment rate of 37.7%. ν is set to 1 in the benchmark, and ω is set to 1.71 so that the intergenerational transition matrix of education in the model matches the data counterpart as closely as possible. Table 4 reports the transition matrix in the model and data. $(i,j)-$th entry of the matrix indicates the probability that children acquire skill (education) $j \in \{HS, CL\}$ given that their parent's skill is $i \in \{HS, CL\}$ in the benchmark model, and values in parentheses represent the data counterparts. The table indicates that the education level is persistent across generations: children of high school graduates attend college with a probability less than 0.3, whereas children of college graduates do with a probability approximately 0.6.

Parents/Children	HS	CL
HS	0.725 (0.798)	0.275 (0.202)
CL	0.412 (0.423)	0.588 (0.577)

Table 4: Intergenerational transition matrix of education. *Note*: $(i,j)-$th entry of the matrix indicates the probability that children acquire skill (education) $j \in \{HS, CL\}$ given that their parent's skill is $i \in \{HS, CL\}$ in the benchmark model, and values in parentheses represent the data counterparts.

Intergenerational Transmission of Human Capital: The intergenerational transmission of human capital is according to the following formula:

$$\log(h) = \rho_h \cdot \log(h_p) + \varepsilon_h,$$
$$\varepsilon_h \sim N(0, \sigma_h).$$

I set ρ_h to 0.30 to approximate the intergenerational income elasticity (0.3) and also set σ_h to 0.65 so that the variance of log income at age 28 in this model is close to 0.27.

Income Process and Education Return: The efficiency labor of an agent aged j, education e, human capital h, and productivity z, $\eta_{j,z,e,h}$, is given as follows:

$$\log \eta_{j,z,e,h} = \log[f^e(h)] + \gamma_{j,e} + z,$$
$$f^e(h) = h + e \cdot (\alpha_{CL} h^{\beta_{CL}}),$$
$$z' = \rho_z z + \zeta, \quad \zeta \sim N(0, \sigma_z).$$

To set $\gamma_{j,e}$, I estimate the second-order polynomial of hourly wages on age using JPSC. As reported in Table 5, the income gradient on age is larger for college graduates than the rest of the workers, but the degree is modest compared with the US case (e.g., Abbott, Gallipoli, Meghir, and Violante, 2019). I assume $\rho_z = 0.95$ and $\sigma_z = 0.02$, values in the ranges over those frequently used in the literature.[21] The function $f^e(h)$ indicates that the education return depends on human capital h: people with higher human capital can obtain greater returns through college education. Following Daruich and Kozlowski (2020), α_{CL} and β_{CL} are determined so that the model replicates the ratio of log wage between college graduates and the rest of the population at age $j = 28$ and the log wage variance for college graduates at age $j = 28$.

	HS	CL
Age	$+0.041$	$+0.048$
$\text{Age}^2 \times 10,000$	-4.551	-5.364

Table 5: Parameters governing the age profile of wages. *Note*: CL indicates the college graduate households where the husband or wife is a college graduate. HS represents the rest of the population.

Production: I set the capital share $\alpha = 0.33$ and $\delta = 0.07$ following Kitao (2015). χ is set to 0.39 following Matsuda and Mazur (2022). ω_{CL} is internally determined to 0.52 so that the average wage ratio between college graduates and the rest in the model amounts to 1.36. Z is determined so that the wage rate for high school graduates is normalized to one in the benchmark.

Government: Tax rates are set to $\tau_c = 0.1$, $\tau_w = 0.35$, and $\tau_a = 0.35$ in the benchmark. The lump-sum transfer is set to $\psi = 0.01$ to match the ratio between the variance of log net income and that of log gross income (0.6). The pension benefit p is set so that the government provides ¥$160,000$ per household per month. The cash transfer B, which we refer to "child benefit" or "typical pro-natal transfers" hereafter, is given as ¥$10,000$ per child per month, approximating the actual payment. The other expenditure S is set so

[21] Due to data limitations, it is hard to accurately estimate the AR(1) process for z using any data source in Japan.

that the government budget constraint is balanced in the benchmark and fixed throughout the counterfactual experiments.

Miscellaneous: The survival probability $\zeta_{j,j+1}$ is set based on the Vital Statistics (2019).[22] Annual college tuition fees p_{CL} are set to 1.05 million yen. κ is set to 0.044 so that they spend 13.3% of their working hours on childcare.[23] Table 6 and Table 7 summarise the parameters externally and internally determined.

Parameter	Value	Description
$\underline{A}_s$	2.88 million yen	Borrowing limit for students
$\underline{A}$	20 million yen	Borrowing limit
p_{CL}	1.05 million yen/year	Tuition fees
κ	0.044	Time costs
$\xi_{j,j+1}$	$-$	survival prob.
τ_c	0.10	Consumption tax
τ_a	0.35	Capital income tax
τ_w	0.35	Labor income tax
p	¥$160,000$/month	Pension benefits
b	¥$10,000$/month	Cash transfers
α	0.33	Capital share
δ	0.07	Depreciation rate
χ	0.39	Elasticity of substitution
ρ_z	0.95	Persistence
σ_z	0.02	Transitory
ν	1.0	Education sorting by ability
γ	0.5	Curvature
β	0.98	Discount factor

Table 6: Parameters externally determined.

[22]See, `https://www.mhlw.go.jp/english/database/db-hw/outline/index.html`.
[23]See, Kitao and Nakakuni (2023).

Parameter	Value	Moment	Data	Model
μ	0.23	Work hours	0.33	0.30
$\bar{t}$	0.8	Income share of labor earnings	0.21	0.20
ι_s	0.054	Share of students using loans	0.44	0.34
ι	0.055	Household share with negative net worth	0.54	0.45
ω_{CL}	0.52	CL−HS wage ratio	1.36	1.48
ψ	0.01	Var(log disposable income)/Var(log gross income)	0.60	0.68
λ_q	0.62	Average transfer / Average income at age 28	0.07	0.07
λ_a	1.03	Average transfer / Average income at age 28	0.27	0.27
ω	1.71	Intergenerational mobility of education	See Table 4	
ρ_h	0.30	Intergenerational income elasticity	0.30	0.27
σ_h	0.65	Variance of log(income) at age 28	0.27	0.24
ψ_{CL}	20.8	College enrollment rate	0.377	0.376
α_{CL}	0.1	Log wage ratio (CL−HS) at age 28	0.34	0.38
β_{CL}	0.1	Var log wage for CL at age 28	0.14	0.24
b_1	0.49	Share of one child	0.16	0.15
b_2	0.53	Share of two children	0.55	0.61
b_3	0.55	Share of three children	0.22	0.24
b_4	0.56	Share of four or more children	0.02	0.00
Z	1.99	Low skill wage	1.0	1.0

Table 7: Parameters internally determined.

4.2 Non-targeted Moments and Validation

This subsection checks the validity of the calibrated model. First, I check if the benchmark model generates a reasonable value of the benefit elasticity of fertility, which is non-targeted in calibration. Second, I check if the model performs well on other critical non-targeted moments: fertility differential across education levels and the revenue breakdown for college students.

4.2.1 The benefit elasticity of fertility

Previous works show that cash benefits such as the child benefit and baby bonus have a significant impact on fertility. Many of them report that the benefit elasticity of fertility, the percentage increase in fertility rate against the one percent increase in the cash transfer, is about $0.1-0.2$. For example, Milligan (2005) studies a reform of Quebec's baby bonus and shows that an extra 1,000 Canadian dollars benefit would increase fertility by 16.9%, which implies a benefit elasticity of 0.107. Cohen, Dehejia, and Romanov (2013) uses a variation in the child subsidy payment observed around 2003 in Israel, providing a larger subsidy for third or higher births. They show that the benefit elasticity of fertility was 0.176. González (2013) adopts the regression discontinuity design to study the effects of Spain's reform in 2007, introducing a one-time payment of 2,500 euros (about 3,800

USD) for births, almost 4.5 times the monthly minimum wage for full-time workers. It finds a statistically significant impact on fertility, increasing conceptions by $5-6\%$.

To examine how this model performs in this respect, I conduct the following exercise. Let B_0 denote the per-child cash transfers for households with children under 17 in the benchmark. I solve the household problem, holding prices, tax rate, and distribution fixed, with several levels of the per-child payment $B = B_0 \cdot (1+x)$ for some $x \in X$, where X is a set of positive real numbers. This procedure brings the implied fertility rate, and with the expansion rate x, we can compute the implied benefit elasticity of fertility for the case of the expansion rate x. I set $X = \{0.1, 0.2, ..., 1.9, 2.0\}$, which is a reasonable range in the context of the expansion examined in the empirical studies, and compute the implied elasticity for each x. Then, I take the average of those 20 values. I find that the average elasticity is 0.138 (with a standard deviation of 0.025), which is consistent with the empirical estimates.

4.2.2 The fertility differential across education levels

More educated parents have fewer children than less educated ones. According to my sample of the JPSC, college graduates' completed fertility was 1.92, which is lower than the rest's, 2.12. This is observed in another data source where we can check the completed fertility rate by different education levels: the National Fertility Survey (NFS), which is provided by the National Institute of Population and Social Security Research (IPSS).[24] According to the NFS (2015), college graduate wives' completed fertility has been lower than less educated ones almost every survey year since 1977. The latest survey in 2015 reports that the completed fertility of wives with a college degree was 1.89, and that of high school graduate wives was 1.98. In this benchmark model, the completed fertility of college graduate wives is 1.79, which is lower than the high school graduate wives', 2.28. The benchmark model captures the qualitative feature of the fertility differential across education levels, although the gap is somewhat significant compared with data counterparts. Table 8 summarizes the fertility differential across education levels in the model and data.

	Model	JPSC	NFS
HS	2.28	2.12	1.98
CL	1.79	1.92	1.89

Table 8: Fertility differential across education in the benchmark model and data. *Note*: "NFS" stands for the National Fertility Survey conducted by the ISPP, and the table reports the values from the 2015 survey. It reports the completed fertility of wives with different educational backgrounds.

A factor generating the gap in this model is the differences in opportunity costs, given

[24] See, https://www.ipss.go.jp/site-ad/index_english/survey-e.asp.

that having children requires a fixed fraction of time. More educated parents face the higher opportunity costs of having children as their potential earnings are higher. In this model, the intergenerational persistence of education can also generate this differential because children of educated parents are more likely to attend college, which raises financial costs in the form of asset transfers upon children's college enrollment.

4.2.3 Composition of students' revenue

Capturing the composition of students' revenue –how college students finance their living expenses and tuition fees– is also important as it is critical not only to students' education choices but also to parents' IVT choices and, thereby, fertility choices. According to the SLS (2018), the students' revenue consists of three parts. First, the greatest part, 61% of their revenue, is accounted for by asset transfers from their parents. Second, students' labor earnings account for 21%, and lastly, the rest (18%) is financed by student loans. Although the revenue share of labor earnings (21%) is a targeted moment, the rest is not targeted. As Table 9 shows, the model captures the overall revenue composition as well. The IVT and loans account for 66% and 14% of their revenue, close to the data counterparts.

	IVT	Loan	Labor
Data	0.61	0.18	0.21
Model	0.66	0.14	0.20

Table 9: Composition of Students' Revenue.

5 Numerical Analysis

This section investigates the effects of grants for college students in stationary equilibrium, using the model with fertility choices calibrated in the previous section. Section 5.1 simulates the introduction of the existing income-tested grants started in 2020 in Japan. Section 5.2 then examines the mechanism through which the macroeconomic effects of the introduction are realized. Lastly, Section 5.3 investigates the effects of raising the income threshold so that students in broader income classes of households are eligible. Following the literature, I adjust the labor income tax to balance the government budget upon the introduction and expansion.

Welfare Measure: Our variables of primary interest are fertility, college enrollment, and per-capita output. In addition to these variables, I also examine the welfare effects of the policy in the following exercises. However, the welfare analysis using models with endogenous fertility is not straightforward theoretically and philosophically. One of the well-known difficulties in the economics or theoretical context is that the standard concept of Pareto efficiency is not applicable to models with endogenous fertility (Golosov, Jones, and Tertilt, 2007).[25] Given that the literature on the normative analysis of endogenous fertility models is still developing and there is no one "correct" method to tackle this issue (at least for now), this study captures the welfare effects of the policy by the consumption equivalence under the veil of ignorance under the benchmark economy relative to the new steady state.[26]

To formalize the measure, let $P \in \{0, 1, 2, ...\}$ denote education subsidy schemes or policies with $P = 0$ representing the benchmark economy without grants. Next, let $V^P(\lambda)$ be the expected lifetime utility in stationary equilibrium for newborn agents with a consumption scaling parameter λ and policy P:

$$V^P(\lambda) = \int_{\boldsymbol{x}_1} V^P_{j=1}(\boldsymbol{x}_1; \lambda) d\mu(\boldsymbol{x}_1). \tag{6}$$

$\mu(\boldsymbol{x}_j)$ is the measure over the age-specific state space where $j \in \{1, ..., J\}$ and $V^P_{j=1}(\boldsymbol{x}_1; \lambda)$ represents the expected utility for an agent aged $j = 1$ with a state vector $\boldsymbol{x}_1$:

$$\begin{aligned} V^P_{j=1}(\boldsymbol{x}_1; \lambda) = \mathbb{E}\Bigg\{ & \sum_{j=1}^{J} \beta^{j-1} \cdot u(c_j \cdot (1+\lambda), l_j) \\ & + \sum_{j=J_F}^{J_{IVT-1}} \beta^{j-1} b(n) \cdot v(q_j) + \beta^{J_{IVT}-1} \lambda_a \cdot b(n) \cdot V_{g0}(\boldsymbol{x}_{J_{IVT}}) \Bigg\} \end{aligned}$$

Here, $\{c_j, l_j, q_j, n, a_{IVT}\}$ are optimal choices with the policy P and each state $\boldsymbol{x}_j$. Finally, the consumption equivalence with a policy P is given as a scalar λ satisfying the following equation:

$$V^0(\lambda) = V^P(0).$$

That is, the consumption equivalence λ makes the newborn agents indifferent between the new economy and the benchmark by scaling up the benchmark consumption by λ.

[25]This is because models with endogenous fertility require a welfare comparison between two different sets of individuals. Golosov, Jones, and Tertilt (2007) then propose two efficiency concepts that can apply to models with endogenous fertility. For example, the $\mathcal{A}-$efficiency proposed by them focuses on individuals being alive in both economies. This concept is particularly beneficial if we examine the optimal policy as they show that the unique solution for the planning problem considering the utility of the existing agents (i.e., those who are alive just before the policy is introduced) corresponds to the $\mathcal{A}-$efficiency.

[26]The related work Zhou (2022) adopts the same welfare criteria in its steady-state analysis. In its transition analysis, Zhou (2022) also computes the average welfare of the existing households already alive before the policy changes to evaluate the $\mathcal{A}-$efficiency of the policy.

5.1 Introducing Education Subsidies

In 2020, the Japanese government introduced income-tested subsidies for college students in low-income households; until then, the government has provided only student loans. Households are eligible if their last year's annual labor income is less than a threshold value $\bar{I}$, and students in those households receive g amount of money each year while in college. The grant function $g(h, I)$ representing the existing grants is formulated as follows:

$$g(h, I) = \begin{cases} g & \text{if } I < \bar{I} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Note that the benchmark case can be interpreted as $g(h, I) = 0$ for any (h, I). Although the income threshold $\bar{I}$ and payment g in the actual system vary with some characteristics of households and students, such as family structure and whether the student commutes to college from home or not, $\bar{I}$ approximately corresponds to the 15 percentile of the household income distribution, and the payment approximately amounts to two-thirds of the average expenses of college students, including tuition fees and life expenses. I set $\bar{I}$ and g based on this information and income distribution and students' expenditure in the benchmark model (initial steady state). I then solve the stationary equilibrium by introducing this new grant function.

The main numerical results are as follows. First, introducing the means-tested grants increases the college enrollment rate by 3.9 p.p. in the long run. Educational mobility increases in the sense that children of high school graduates are 2.5 p.p. more likely to attend college.[27] Because of the higher college enrollment rate, implying a greater supply of college graduate labor, the skill premium decreases by 0.02 points. Introducing the grants increases the TFR by 3% or 0.064 points. Importantly, this increase is primarily driven by a 7.4% increase in college graduates' fertility, whereas the fertility rate of high school graduates is almost stable.

Those changes in demographic structure and skill distribution affect other aggregate variables. In the long run, the higher college enrollment rate implies a higher share of skilled labor, and the higher TFR implies a greater share of the working-age population. As a result, per-capita labor supply in efficiency units increases by 1.3% in the long run. On the contrary, per-capita capital decreases by 0.9% for some reasons. First, introducing the grants reduces saving incentives for a substantial fraction of households and crowd-out IVT.[28] In addition, the higher TFR implies a greater share of younger

[27] This mobility concept is a variant of an income mobility measure adopted in Zheng and Graham (2022).

[28] Note that some households can increase savings upon the grant introduction. For example, households

generations, who hold fewer assets than older ones. Despite its negative effects on capital accumulation, the positive impacts on the labor force and productivity are sufficiently greater so that the per-capita output increases by 1.0% in the long run. The standard deviation of wages, $w_e \cdot \eta_{j,z,e,h}$, is reduced by 1.3 % compared with the benchmark because of the greater educational mobility and the pecuniary externalities of college enrollment (i.e., the shrinking skill premium due to the greater supply of college graduates). Also, the intergenerational income elasticity decreases by 6.0% (i.e., intergenerational mobility of income increases) because the income-tested grants help students in poor households enroll in college. A GE effect through the reduced skill premium also contributes to higher mobility. The welfare improves by 5.1% in consumption equivalence under the veil of ignorance, and its sources are discussed in the following section.

5.2 Inspecting the Mechanism

Previous results imply that the grant introduction significantly affects fertility and college enrollment rates, which leads to greater output. The introduction also brings welfare gains in the long run. However, the mechanism behind these changes is not so obvious because several objects aside from the grant function, such as prices, tax rate, and distribution, vary between the benchmark and new equilibrium, and each can play important roles in accounting for the overall effects on the TFR, college enrollment rate, and welfare. In addition, the roles of fertility margins are also worth investigating, that is, how the results differ under exogenous fertility in which the fertility behavior is fixed to that in the benchmark. This is because the fertility setup would matter to the IVT and, thereby, children's education choices, given that fertility and IVT choices are joint decisions. Fertility margins can also have distributional implications through intergenerational linkages if fertility responses are heterogeneous across household characteristics.

I conduct a decomposition analysis in Section 5.2.1 to discuss and understand the mechanism behind the changes in the TFR, college enrollment rates, and welfare upon introducing the grants. Next, in Section 5.2.2, I consider the roles of fertility responses in understanding the results by solving the exogenous fertility version of the model.

5.2.1 Decomposition

What causes the increases in fertility and college enrollment rates? Those increases can be broken down into behavioral effects and distributional (or composition) effects, where the behavioral effects can be further broken down into the direct effects, driven only

whose children do not attend college in the benchmark but attend college when the grant is introduced may need more assets in the new equilibrium to make some IVT upon children's college enrollment. This is because the grants are not sufficiently generous to cover 100% of the expenditures for students.

by a change in the subsidization scheme (i.e., grant function $g(h, I)$), and the indirect effects, driven by changes in factor prices (i.e., GE effects) or tax rates (i.e., Taxation effects). Note that the direct effects can also be interpreted as the short-run effects of the introduction, where prices, tax, and distribution are fixed. Finally, the distributional effects capture changes driven by distribution changes over the state variables such as education, age, and human capital. To isolate each effect, I conduct a decomposition exercise as follows. I first solve an equilibrium by introducing the new grant function. Then, I solve household problems by replacing one of the four objects (grant function, prices, tax rate, and household distribution) in the benchmark with that in the new equilibrium. Note that this method does not guarantee that each implied effect adds up to the overall effects because all factors except grant function are endogenous and interconnected; however, my results indicate that each effect adds up roughly to the overall effects. Table 10 summarises the decomposition results.

	Direct	GE	Tax	Dist.	All
CL share (Δ p.p.)	+2.6	−0.2	0.0	+1.9	+3.9
TFR (Δ %)	+2.3	+0.9	0.0	−0.4	+3.0
HS (Δ %)	+1.0	0.0	0.0	0.0	+0.4
CL (Δ %)	+4.5	+2.5	0.0	0.0	+7.4
Output (Δ %)	−0.7	−0.8	−0.1	+0.9	+1.0
Welfare (%)	+2.6	−0.7	−0.2	+2.3	+5.1

Table 10: Decomposition results. *Note*: Columns "Direct," "GE," "Tax," and "Dist." report the results when only grant function, prices, labor income tax rate, and distribution change, respectively. A column "All" indicates the results in the benchmark and the long-run equilibrium with the grants. Rows "CL share" represent the change in college enrollment rate in p.p. and rows "HS" and "CL" represent the percentage changes in fertility rates for high school and college graduates. Welfare gains are represented in terms of consumption equivalence.

College Enrollment: For the long-run increase in the college enrollment rate, critical forces are direct and distributional effects. If the grant is introduced while other variables (i.e., prices, tax rate, and distribution) are fixed as in the benchmark, the college enrollment rate increases by 2.6 p.p., more than half of the overall increase. These direct effects capture the effects of relaxing financial constraints on college enrollment decisions. GE effects put downward pressure on college enrollment because the lower college premium reduces the incentive for college enrollment.

Note that these direct effects do not explain the overall effects, so we need other forces accounting for the increase in college enrollment rate; that turns out to be the distributional effects. The implied changes in the distribution in the long run, holding grant function, prices, and tax rate fixed as in the benchmark, lead to a 1.9 p.p. higher college enrollment rate. The most relevant factor is the change in the skill distribution of parents (i.e., the share of college graduates). The increase in the share of college graduates implies a higher share of those more likely to have children enrolling in college due to the intergenerational persistence of education. Then, the distributional change amplifies the short-run increase in college enrollment rate facilitated by the direct effects.

Fertility: For the long-run increase in the TFR, vital forces are direct and GE effects. If the grant is introduced while other variables are fixed as in the benchmark, the TFR increases by 2.3% in the long run (direct effects), which corresponds to three-fourths of the overall effects. As in explaining the overall effects, that 2.3% increase due to the direct effects is largely driven by the fertility increase of college graduates; college graduates increase fertility by 4.5%, whereas high school graduates do by 1%.

Further, among each education group, ex-post ineligible households increase fertility. The average fertility rates of ex-post eligible high school and college graduates increase by 1.6% and 3.9%, as indicated in Fig 2a. Given that the ineligible households are the majority of the cohort as indicated by Fig 2b, these fertility increases among ex-post eligible ones largely explain the direct effects; they explain a 0.7 p.p and 1.2 p.p. of the 2.3% direct effect, respectively (Fig 3). In other words, more than 80% of the direct effect is explained by the fertility increases of ineligible households.

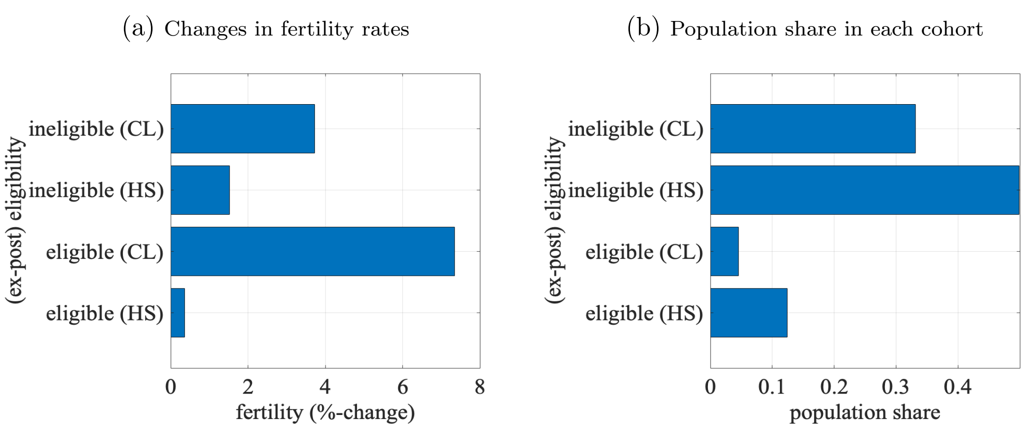

Fig 2: %-changes in fertility rates of each eligibility-education status (2a) and population share of each eligibility-education status (2b). *Note*: "HS" and "CL" stand for high school and college graduates.

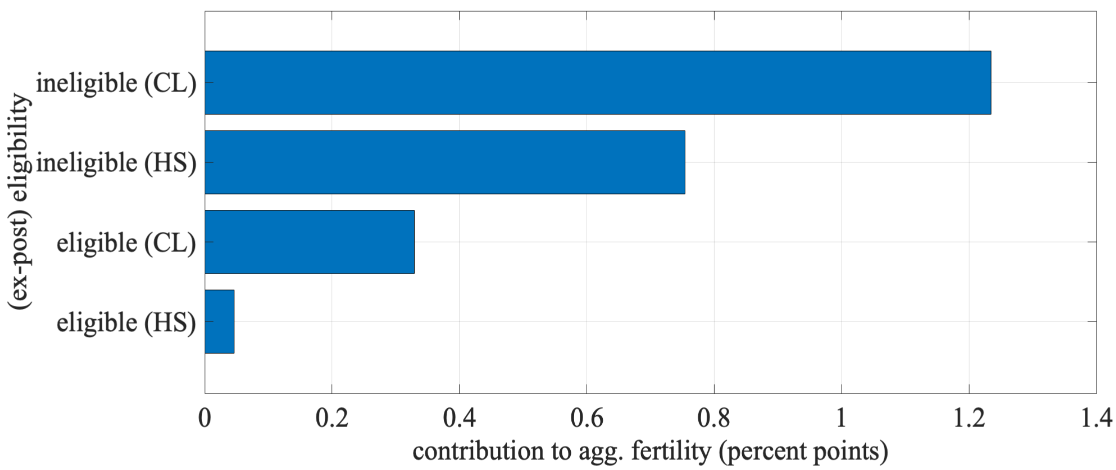

Fig 3: Contribution to the change in aggregate fertility by education and ex-post eligibility status. *Note*: "HS" and "CL" stand for high school and college graduates.

Why can the grant introduction increase fertility, even among ex-post ineligible households? The short answer is due to an insurance effect of the income-tested grants. To understand this statement, recall that the eligibility is uncertain when they make fertility choices; they are beneficiaries if (1) their children enroll in college and (2) their earnings when their children make the education choices, are sufficiently low. Those are subject to uncertainty regarding their productivity and children's characteristics (school tastes and human capital), which are not realized when they make fertility choices. If their children have characteristics favoring college enrollment (i.e., higher human capital endowment and/or lower psychic costs of education), they would like to make more IVTs to support them, and vice versa. Thus, they face expenditure uncertainty until their children's education choices terminate, in addition to income risks.

Consider that those "shocks" are realized simultaneously; their children have characteristics willing to attend college, but they are poor due to the realization of negative income shocks. Without the income-tested grants, the realization of negative income shocks will make it infeasible for those "unlucky" parents to financially support their children's enrollment, or they may end up with low consumption in exchange for supporting the enrollment: either situation is costly for parents. Thus, in fertility decision-making, they may choose fewer children to avoid such a costly situation they might encounter in the future. The income-tested grants provide partial insurance against those risks and make some parents comfortable having a child (or another child). Because college graduates are more likely to have children with characteristics favoring college enrollment, they benefit more from this insurance, and their fertility behavior responds strongly.

Next, the decomposition result shows that the GE effect also plays a role in accounting for the TFR increase. If the prices are set to the long-run equilibrium levels while other objects are fixed as in the benchmark, the college graduates' fertility increases by 2.5%, and the TFR then increases by 0.9%. The key is the decline of the wage rate for college graduate workers, w_{CL}. In the long-run equilibrium with the grants, w_{CL} decreases by 2.2%. First, the greater supply of college graduate workers depress w_{CL} relative to w_{HS}. In addition, greater aggregate labor supply in efficiency units and lower capital accumulation discussed in Section 5.1 imply the lower marginal productivity of labor, decreasing w_{CL}. The lower w_{CL} implies the lower opportunity costs of having children for college graduates, making some have more children. Also, for college graduate parents, the lower w_{CL} reduces the (expected) utility of the IVTs, given that their children are likely to attend college, as discussed above. The lower utility of the IVTs also contributes to higher fertility because the marginal utility gains from investing in children's "quality" (by making IVTs to send a child to college) are relatively lower than those from increasing its "quantity."[29]

Lastly, the distributional effect implies a 0.4% decline in the average fertility. The introduction of the grants increases the college enrollment rate, meaning that the share of college graduates increases in the long run. Given that college graduates have fewer children than high school graduates as discussed in Section 4.2.2, the greater share of college graduates means a greater share of those who tend to have fewer children, which can lead to a lower average fertility.[30] However, direct and GE effects are significant and lead to the higher TFR.

Output: The distributional effect is critical in accounting for the output increase. In the long run, the working-age population increases due to a higher equilibrium TFR, and the share of skilled workers increases. These forces increase the labor supply in efficiency units, increasing output in the long run.

In contrast, the direct and GE effects put downward pressure on output. As indicated in Table 10, the GE effects lower the college enrollment rate due to the reduced skill premium. The resulting lower share of skilled workers leads to a lower output.

Next, recall that the direct effect can be considered the short-run effect of the introduction, where other macroeconomic variables, such as prices, tax rates, skill distribution, and age distribution, are fixed as in the benchmark. In the short run, aggregate savings and labor supply decrease due to higher fertility; having a child requires parents to spend a fraction of their time and some additional money, reducing their working hours and savings. Thus, the aggregate output decreases in the short run.

[29] This fertility response of college graduates to the change in skill premium aligns with an empirical observation (Lehr, 2003).

[30] This effect is comparable with a composition effect highlighted by Zhou (2022) in understanding the effects of public education subsidies on fertility. In his model, in the long run, public education subsidies increase the share of parents with higher human capital, who tend to have fewer children.

Welfare: The direct effect explains more than half of the welfare gains. Importantly, introducing the grants makes agents attend college with smaller costs and enables someone who could not attend college in the benchmark to do that, bringing them a higher lifetime income. The rest of the gains are explained by the distributional effect. The education subsidy increases the share of college graduates in the long run, and the lifetime utility for college graduates is, in principle, greater than that for high school graduates, particularly because of their higher income due to the college education returns. Thus, this distributional change increases the expected lifetime utility. Note that the direct effect is about the change in $V^P_{j=1}$ in equation (6), while the distributional effect is about the change in μ there.

5.2.2 Roles of Fertility Responses

I next solve the exogenous fertility version of the model. More specifically, I follow the same procedure in the previous section 5.1, except that the policy functions for fertility are fixed as in the benchmark. Therefore, the TFR under exogenous fertility is not necessarily the same as in the benchmark because the household distribution changes.

	Endogenous	Exogenous
CL share (Δ p.p.)	+3.9	+2.9
TFR (Δ %)	+3.0	−0.8
Output (Δ %)	+1.0	+0.4
Capital (Δ %)	−0.8	+0.6
Labor (Δ %)	+1.3	+0.2
Tax (Δ p.p.)	+0.02	+0.71
STD(wage) (Δ %)	−1.3	−1.1
Welfare (%)	+5.1	+3.5

Table 11: Results under exogenous fertility. *Note*: Results for output, labor, and capital represent percentage changes compared with the benchmark. Those for college enrollment and labor income tax rates represent changes in percentage points compared with the benchmark. "STD(wage)" represents the standard deviation of wages, where wages for an agent with (j, z, e, h) is captured as $w_e \cdot \eta_{j,z,e,h}$. Welfare gains are represented in terms of consumption equivalence.

Table 11 reports the main results under exogenous fertility. First, the grant introduction results in a 2.9 p.p. increase in the college enrollment rate under exogenous fertility, which is lower than a 3.9 p.p. increase under endogenous fertility. Under both exogenous and endogenous fertility, the grants make some children who otherwise cannot enroll in college. Under endogenous fertility, in addition to that effect, the college enrollment rate can further increase through fertility margins. As I discussed in Section 5.1, college graduates increase fertility than high-school graduate parents. Their children will likely attend college due to the intergenerational transmission of school tastes and human capital. And when those children become parents in the future, their children, if they have, are also likely to have similar characteristics to theirs, favoring college enrollment. Through this mechanism, the long-run share of college graduates increases, implying that the fertility margins amplify the effects on college enrollment.

From the second row onward, the changes in other aggregate variables are reported. Under exogenous fertility, per-capita labor in efficiency units increases by 0.2 % compared with the benchmark. The degree of this increase is modest compared with a 1.3% increase under endogenous fertility for two reasons. First, the share of skilled labor is lower under exogenous fertility than endogenous fertility, as discussed above. Second, the working-age population share is slightly lower under exogenous fertility than in the benchmark, an opposite result to the endogenous fertility setup: the introduction under exogenous fertility leads to a 0.8% lower fertility rate due to a composition effect. The lower labor supply in efficiency units leads to lower output gains under exogenous fertility: the exogenous setup implies a 0.4% increase in per-capita output, while the endogenous fertility setup implies a 1% increase.

Even though the college enrollment rate and TFR under exogenous fertility are lower than under endogenous fertility, implying a lower government expenditure on the grants, the required tax increase is 0.69 p.p. higher under exogenous fertility. This is because the greater labor supply in efficiency units under endogenous fertility increases the tax revenue in the long run. The reduction of the standard deviation in wages is more significant under endogenous fertility, mainly because the higher college enrollment rate under endogenous fertility leads to a lower skill premium. The welfare gain is 46% greater under endogenous fertility because of those channels (i.e., lower tax and lower inequality). Also, the higher college enrollment rate in the economy increases the welfare under the veil of ignorance via the direct and distributional effects.

5.3 Expansion

Finally, I consider the effects of raising the income threshold $\bar{I}$ so that students in households of broader income classes can be eligible. Recall that I set the threshold $\bar{I}$ so that the existing grants target the students in households at the bottom 15 % of the income distribution. In this experiment, I increase $\bar{I}$ so that it corresponds to the 40, 50, and 60 percentiles of the income distribution and solve the stationary equilibrium in each case. For students in the household at the bottom 15 %, the payment is still given by g, which amounts to two-thirds of the average expenses of students in the benchmark. For students in households higher than the 15 percentile but less than x percentile of the income distribution, where x takes either 40, 50, or 60, the payment is given by $g/2$, which amounts to one-third of the average expenses of students. In other words, the payment tapers off in income. Letting $\bar{I}_{15\%}$ and $\bar{I}_{x\%}$ denote the income level of 15 and $x \in \{40, 50, 60\}$ percentiles of the income distribution, the grant function $g(h, I; x)$ with a threshold $\bar{I}_{x\%}$ can now be formulated as:

$$g(h, I; x) = \begin{cases} g & \text{if } I < \bar{I}_{15\%} \\ g/2 & \text{if } I \in [\bar{I}_{15\%}, \bar{I}_{x\%}) \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Table 12 summarizes the results, and there are several takeaways there. First, the equilibrium college enrollment increases as the income threshold is higher. Setting the threshold at the 40, 50, and 60 percentiles of the income distribution, the enrollment rate increases by 4.7 p.p., 5.6 p.p., and 6.2 p.p., respectively, in stationary equilibrium. This is also the case for educational mobility and labor income tax rates. The expansion requires additional revenue, so the equilibrium tax rate should also increase by 0.3 p.p. for the case of the 60% threshold.

However, the expansion would not significantly increase the TFR. Recall that the introduction leads to a 3% increase in TFR, from the benchmark level of 2.096 to 2.160 in the long-run equilibrium with the grants. With grant functions $g(h, I; 40)$, $g(h, I; 50)$, and $g(h, I; 60)$, the equilibrium TFR would be 2.158, 2.151, and 2.157, respectively; the TFR even decreases locally with expansion. Fertility rates for each skill help us understand this situation. First, college graduates continue to increase fertility. With grant functions $g(h, I; 40)$, $g(h, I; 50)$, and $g(h, I; 60)$, their equilibrium fertility rate would be 1.996, 1.998, and 2.021, all higher than the fertility rate with the existing grants, 1.978. This result is straightforward to understand given that the insurance and GE effects contribute to the fertility increase of college graduates when the grants are introduced, discussed in Section 5.2. On the contrary, fertility among high school graduates decreases with expansion, making the TFR remain almost constant even though fertility among college graduates increases. Fig 4 depicts the changes in the TFR and fertility rates for high school and college graduates, where the equilibrium TFR with the existing program (with the 15 percentile threshold) is normalized to one.

	Threshold			
	15%	40%	50%	60%
CL share (Δ p.p.)	+3.9	+4.7	+5.6	+6.2
HS→CL (Δ p.p.)	+2.5	+2.6	+3.1	+3.3
Tax (Δ p.p.)	+0.02	+0.17	+0.23	+0.30
Output (Δ%)	+1.0	+1.3	+0.2	−0.5
Welfare (%)	+5.1	+6.5	+6.3	+5.5

Table 12: Main results of higher income thresholds. *Note*: Rows "CL share" and "HS→CL" represent the changes in college enrollment rate and educational mobility in the sense of probability that children of high school graduates attend college. Output changes are expressed as percentage changes. Changes in college enrollment rate, educational mobility, and tax rate are represented as changes in percentage points. Welfare gains are represented in terms of consumption equivalence.

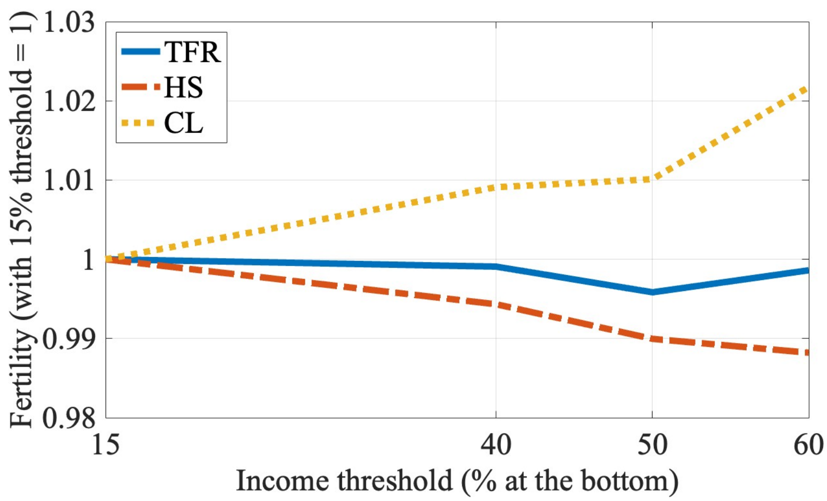

Fig 4: Changes in fertility rates with expansion. *Note*: The equilibrium TFR with the existing program (with the 15% threshold) is normalized to one. "HS" and "CL" represent fertility rates for high school and college graduates.

Why do high school graduates decrease fertility as the income threshold is higher? To understand this, I conduct the decomposition when the grant function is given by $g(h, I; 60)$, and the results are summarized in Table 13. Two effects are critical to explain the fertility decreases of high school graduates: the direct and GE effects.

In this case with the grant function $g(h, I; 60)$, the wage rate for high school graduates, w_{HS}, is 0.6% higher in the long run than in the benchmark, especially because of the significant supply of college graduates. This higher wage rate makes the opportunity costs of having children for high school graduates higher, which puts downward pressure on their fertility rates.

Next, the direct effects imply a lower fertility rate for high school graduates. This result is somewhat confusing given that we discuss the insurance effects of the grants on fertility, which is critical to account for the fertility increases of college graduates. Why do the direct effects lead to lower fertility for high school graduates? In the benchmark without grants, children of high school graduates are unlikely to attend college. If the grants are introduced and their target expands, educational mobility increases in the sense that children of high school graduates are more likely to attend college than in the benchmark, as represented in Table 12. Given that the grants are not generous enough to cover 100% of the costs to send their children to college, this higher probability of children going to college implies that those parents are more likely to have to make additional transfers upon children's college enrollment. The expansion thus increases the expected costs of children for high school graduates, which lowers their fertility.

	Direct	GE	Tax	Dist.	All
HS (Δ%)	-0.7	-1.4	0.0	0.0	-0.8
CL (Δ%)	$+9.1$	$+4.3$	$+2.4$	$+0.6$	$+13.2$

Table 13: Decomposing the effects on fertility with $g(h, I, 60)$. *Note*: Rows "HS" and "CL" represent the percentage changes in fertility rates for high school and college graduates.

Last, the output increases in expansion up to the case of the 40 percentile threshold, but they start declining after that point, as Table 12 and Fig 5 indicate. If the income threshold becomes sufficiently high, households with children have less incentive to save to support their college enrollment because they will likely be eligible for the grants. This higher probability of eligibility also causes an income effect on labor supply. Thus, the aggregate capital and labor supply decrease when the income threshold becomes sufficiently high, reducing the output. This concavity of output gains with expansion applies to the welfare gains. The marginal utility gains of the grants decrease in the values of the income threshold while the tax rate increases with expansion. Then, the net gains start declining at some point.

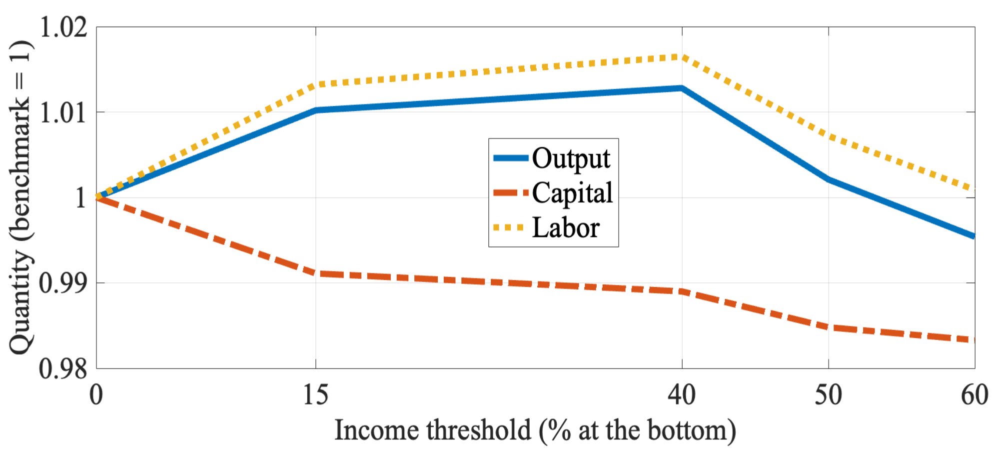

Fig 5: Changes in aggregate output, capital, and labor supply in efficiency unit with expansion. *Note*:

6 Concluding Remarks

This paper examines the macroeconomic consequences of college financial aid policy by constructing a new GE-OLG model that features fertility and college enrollment choices. My central contribution is to highlight the critical roles of the fertility margins in evaluating the macroeconomic performance of the policy. I show that income-tested college subsidy reduces the fertility differential by providing partial insurance against costly states associated with having children for college graduates, which amplifies the policy effects on college enrollment and output.

My model is tailored to examine the financial aid policies by capturing relevant ingredients, such as students' labor supply and minute lifecycle (age) structure. Instead, this paper abstracts the children's skill formation through parental investments before college education, as previous studies in college financial aid policies do not. These ingredients are critical in considering broader policies contributing to children's skill formation, including family policies and early childhood education policies.[31] Given that skill formation in earlier stages affects returns in later education stages, examining the policies relevant to the earlier stages will provide a more comprehensive understanding of the macroeconomic effects of education policies. This study is the first step for this ambitious task, and incorporating the skill formation throughout childhood into the current model is left for future research.

[31] Examples of recent works in this respect are Lee and Seshadri (2019); Abbott (2022); Daruich (2022); Zhou (2022); Yum (2023); Moschini and Tran-Xuan (2023); Gu and Zhang (2024).

References

B. Abbott. Incomplete markets and parental investments in children. *Review of Economic Dynamics*, 44:104–124, 2022.

B. Abbott, G. Gallipoli, C. Meghir, and G. L. Violante. Education policy and intergenerational transfers in equilibrium. *Journal of Political Economy*, 127(6):2569–2624, 2019.

A. Akyol and K. Athreya. Risky higher education and subsidies. *Journal of Economic Dynamics and Control*, 29(6):979–1023, 2005.

R. Benabou. Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica*, 70(2):481–517, 2002.

A. Bick. The quantitative role of child care for female labor force participation and fertility. *Journal of the European Economic Association*, 14(3):639–668, 2016.

R. A. Braun and D. H. Joines. The implications of a graying japan for government policy. *Journal of Economic Dynamics and Control*, 57:1–23, 2015.

A. Cohen, R. Dehejia, and D. Romanov. Financial incentives and fertility. *The Review of Economics and Statistics*, 95(1):21, 2013.

J. C. Cordoba, X. Liu, and M. Ripoll. Accounting for the international quantity-quality trade-off. Working Paper, 2019.

D. Daruich. The macroeconomic consequences of early childhood development policies. Working paper, 2022.

D. Daruich and J. Kozlowski. Explaining intergenerational mobility: The role of fertility and family transfers. *Review of Economic Dynamics*, 36:220–245, 2020.

D. De La Croix and M. Doepke. Inequality and growth: why differential fertility matters. *American Economic Review*, 93(4):1091–1113, 2003.

D. De la Croix and M. Doepke. Public versus private education when differential fertility matters. *Journal of Development Economics*, 73(2):607–629, 2004.

M. Doepke, A. Hannusch, F. Kindermann, and M. Tertilt. The economics of fertility: A new era. In *Handbook of the Economics of the Family*. Elsevier, 2023.

A. Erosa, L. Fuster, and D. Restuccia. A general equilibrium analysis of parental leave policies. *Review of Economic Dynamics*, 13(4):742–758, 2010.

M. Golosov, L. E. Jones, and M. Tertilt. Efficiency with endogenous population growth. *Econometrica*, 75(4):1039–1071, 2007.

L. González. The effect of a universal child benefit on conceptions, abortions, and early maternal labor supply. *American Economic Journal: Economic Policy*, 5(3):160–88, 2013.

S. Gu and L. Zhang. The macroeconomic consequences of parental investment competition for college admissions. *Working Paper*, 2024.

R. Hagiwara. Macroeconomic and welfare effects of the fiscal and social security reforms (in japanese). *Unpublished Manuscript*, 2021. https://hermes-ir.lib.hit-u.ac.jp/hermes/ir/re/71565/eco020202000503.pdf.

M. Hsu and T. Yamada. Population aging, health care, and fiscal policy reform: The challenges for japan. *The Scandinavian Journal of Economics*, 121(2):547–577, 2019.

S. Imrohoroğlu and S. Kitao. Social security reforms: Benefit claiming, labor force participation, and long-run sustainability. *American Economic Journal: Macroeconomics*, 4(3):96–127, 2012.

S. İmrohoroğlu, S. Kitao, and T. Yamada. Can guest workers solve japan's fiscal problems? *Economic Inquiry*, 55(3):1287–1307, 2017.

K. M. Jakobsen, T. Jørgensen, and H. Low. Fertility and family labor supply. *CESifo Working Paper*, 2022.

L. E. Jones, A. Schoonbroodt, and M. Tertilt. Fertility theories: Can they explain the negative fertility-income relationship? In J. Shoven, editor, *Demography and the Economy*, chapter 2, pages 43–100. University of Chicago Press, Chicago, 2010.

D. Kim and M. Yum. Parental leave policies, fertility, and labor supply. *Working Paper*, 2023.

S. Kim, M. Tertilt, and M. Yum. Status externalities and low birth rates in korea. *Working paper*, 2023.

S. Kitao. Fiscal cost of demographic transition in japan. *Journal of Economic Dynamics and Control*, 54:37–58, 2015.

S. Kitao and K. Nakakuni. On the trends of technology, family formation, and women's time allocations. *Working Paper*, 2023.

D. Krueger and A. Ludwig. On the optimal provision of social insurance: Progressive taxation versus education subsidies in general equilibrium. *Journal of Monetary Economics*, 77:72–98, 2016.

N. Lawson. Liquidity constraints, fiscal externalities and optimal tuition subsidies. *American Economic Journal: Economic Policy*, 9(4):313–343, 2017.

S. Y. Lee and A. Seshadri. On the intergenerational transmission of economic status. *Journal of Political Economy*, 127(2):855–921, 2019.

C. S. Lehr. Fertility and education premiums. *Journal of Population Economics*, 16: 555–578, 2003.

R. E. Manuelli and A. Seshadri. Explaining international fertility differences. *The Quarterly Journal of Economics*, 124(2):771–807, 2009.

K. Matsuda. Progressive taxation versus college subsidies with college dropout. *Journal of Money, Credit and Banking*, 2022.

K. Matsuda and K. Mazur. College education and income contingent loans in equilibrium. *Journal of Monetary Economics*, 132:100–117, 2022.

K. Milligan. Subsidizing the Stork: New Evidence on Tax Incentives and Fertility. *The Review of Economics and Statistics*, 3(87):18, 2005.

E. Moschini and M. Tran-Xuan. Family policies and child skill accumulation. *Available at SSRN 4518632*, 2023.

K. Nakakuni. Macroeconomic analysis of the child benefit: Fertility, demographic structure, and welfare. *Working Paper*, 2023.

S. Nishiyama and K. Smetters. Does social security privatization produce efficiency gains? *The Quarterly Journal of Economics*, 122(4):1677–1719, 2007.

C. Santos and D. Weiss. "why not settle down already?" a quantitative analysis of the delay in marriage. *International Economic Review*, 57(2):425–452, 2016.

A. Schoonbroodt and M. Tertilt. Property rights and efficiency in olg models with endogenous fertility. *Journal of Economic Theory*, 150:551–582, 2014.

K. Sommer. Fertility choice in a life cycle model with idiosyncratic uninsurable earnings risk. *Journal of Monetary Economics*, 83:27–38, 2016.

S. Yamaguchi. Effects of parental leave policies on female career and fertility choices. *Quantitative Economics*, 10(3):1195–1232, 2019.

M. Yum. Parental time investment and intergenerational mobility. *International Economic Review*, 64(1):187–223, 2023.

A. Zheng and J. Graham. Public education inequality and intergenerational mobility. *American Economic Journal: Macroeconomics*, 14(3):250–282, 2022.

A. Zhou. The macroeconomic consequences of family policies. *Available at SSRN*, 3931927, 2022.

A. Illustrative Examples for Fertility Differentials

A. (i) The Price of Time Theory

Following Jones, Schoonbroodt, and Tertilt (2010), I consider the following problem:

$$\max_{c,n}\left\{\frac{c^{1-\sigma}}{1-\sigma}+\theta\frac{n^{1-\sigma}}{1-\sigma}\right\}$$

s.t.

$$c = w\cdot(1-n\cdot\chi)$$

Here, c, n, w, and χ denote consumption, fertility, an exogenous wage, and time costs of children. The optimal fertility decision n^* is given as:

$$n^*=\frac{1}{w^{\frac{1-\sigma}{\sigma}}\cdot(\chi/\theta)^{\frac{1}{\theta}}+\chi},$$

meaning that the negative income-fertility relationship holds if $\sigma > 1$ (i.e., the substitutability between consumption and children are high enough). Because having children is time-consuming, its (opportunity) cost is higher for high-income households. At the same time, high-income households, by definition, have higher incomes and afford to have more children. If the substitutability between consumption and children is sufficiently high, the substitution effect dominates the income effect, implying the negative income-fertility relationship.

A. (ii) The Quantity-Quality Trade-off

Following De La Croix and Doepke (2003), I consider the following problem:

$$\max_{c,n,e}\{\ln(c)+\theta\ln(nq)\}$$

s.t.

$$c+b\cdot n\cdot e = w\cdot(1-n\cdot\chi),$$

$$q=(\eta+e)^{\gamma},$$

$$c,n>0, e\geq 0.$$

Here, the choice variables c, n, and e represent the consumption, fertility, and per-child education investment. The investment e is transformed into the "quality" of children, q, according to a human capital production function $q = (\eta + e)^\gamma$. The parameter $\gamma \in (0, 1)$ governs the marginal gains of investment, and $\eta > 0$ implies that the quality of children takes a positive value even if parents make no investments. Parameters χ and b represent the time cost of having a child and a unit (monetary) cost of investment.

The optimal solutions for the fertility and education investment are given as

$$e^* = \frac{\gamma w \chi / b - \eta}{1 - \gamma}, \tag{9}$$

$$n^* = \frac{1}{1 + \theta} \cdot \frac{1 - \gamma}{\chi - \eta b / w}. \tag{10}$$

There are two critical observations in the optimal choices (9) and (10): (1) the optimal number of children is decreasing in wage level w, and (2) the optimal number of children is increasing in the unit education cost b whereas the optimal investment is decreasing in b. The second observation means that if education becomes cheaper, the households in this setup substitute the "quantity" with the "quality" of children.

B. Education Costs and Fertility Choices in Japan

Japan is a leading country in demographic aging,[32] leading to the shrinking labor force, output, and tax base, while the public expenditures on social security benefits are increasing. As a countermeasure against this demographic issue, the government introduced grants for college students in 2020. Two key underlying premises are: (1) it will increase the "quality" of the labor force in the long run, and (2) it will increase the fertility rate, which increase the "quantity" of the labor force in the long run. This pro-natal motive is explicitly described in an act for introducing the grants.[33]

> The aim is ... to foster an environment where people can bear and raise their children with a sense of ease by alleviating the economic burden associated with higher education, thereby contributing to addressing the rapid decline in the birthrate in our country.
> (Act on Support for Higher Education Studies, enacted on May 17, 2019)

Although that expectation for education policies as a pro-natal policy is unconventional, there are facts suggesting that the financial costs for parents to support their

[32] Japan's current fertility rate has been around 1.3, far below the population replacement level. In addition, the old-age dependency ratio is more than 50% and is projected to reach 80% in 2050, which is by far the highest among the OECD countries (See, https://data.oecd.org/).

[33] See, https://www.mext.go.jp/a_menu/koutou/hutankeigen/detail/__icsFiles/afieldfile/2019/05/17/1417025_02_1.pdf (available only in Japanese).

children's college enrollment are a significant impediment to fertility decisions in Japan. I first list the four facts below and then elaborate on each one by one:

1. Couples are most likely to abandon having an ideal number of children because of financial costs.

2. A significant financial cost gap exists between those who have children enrolled in college and those who do not.

3. A substantial fraction of parents desire a college education for their children.

4. Japan is one of the least in subsidizing tertiary education.

Fact 1. *Couples are most likely to abandon having an ideal number of children because of financial costs.*

This fact is drawn from the National Fertility Survey (NFS), which is provided by the National Institute of Population and Social Security Research (IPSS).[34] The NFS is a cross-sectional household survey that asks respondents about their preferences or intentions regarding fertility, marriage, child-raising, and education, as well as their basic information, including their education, age, and income. It is conducted nearly every five years, and the latest survey available is in 2015, which collected 5,334 couples in which the wife is aged 18 to 49. Hereafter, I present the results focusing on married couples in which the wife is aged 25 to 39 years, leaving 2,420 couples.[35] According to the NFS, a non-negligible gap exists between the ideal and planned numbers of children. In the 2015 survey, the ideal number of children for wives aged 25 to 39 was on average 2.38, whereas the planned number was 2.16. Fig 6 represents the distribution of the ideal and planned numbers of children, where blue (red) bars indicate the share of wives who desire (plan) to have each number of children from zero to more than five. This figure suggests that the gap originates from the downward revision of the ideal at the intensive margin. There is no significant share gap between those whose ideal number is zero and those whose planned number is zero, and a substantial fraction of wives who desire three children end up with one or two children.

[34] See, https://www.ipss.go.jp/site-ad/index_english/survey-e.asp.

[35] Its 53.1% of the sample consists of couples in which the wife is over age 40, so the sample size shrinks if we target the younger couples. I focus on wives under age 39 because here I am interested in the fertility intention of those in the stage of fertility decision. Note that the cohort fertility rate is stable after age 40 for any cohort in Japan. See, for example, p7 of https://www.mhlw.go.jp/toukei/saikin/hw/jinkou/tokusyu/syussyo07/dl/gaikyou.pdf (in Japanese). Excluding those aged 18 to 24 does not affect the result significantly because they consist only of 1.5% of the observations for wives aged 18 to 49.

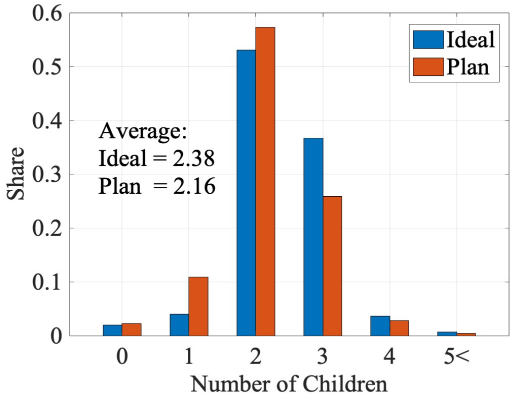

Fig 6: Distribution of ideal and planned numbers of children.

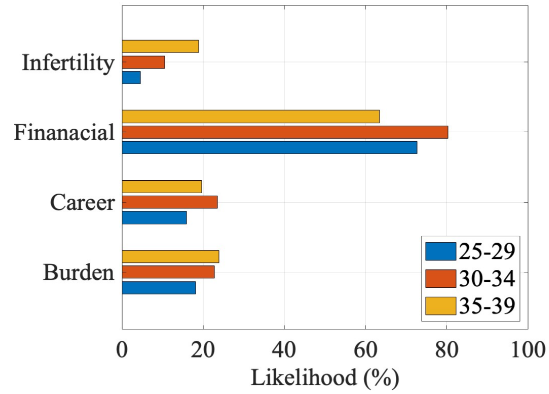

Fig 7: Reasons for the gap between the ideal and planned number of children.

Why do people abandon their ideal number of children? The NFS asks them to pick their reasons among several options, and the result indicates that they are most likely to choose the financial reason: "raising children and education are too expensive." Fig 7 represents the likelihood[36] that wives of each age group abandon their ideal number of children for a particular reason, and "Financial" represents the financial reason. Aside from this, "Career" represents "it would interfere with my job," and "Burden" represents "I would not handle the psychological and physical burden" (arising from achieving the ideal number). On average, more than 75% of them chose the financial reason, which dominates other reasons, such as "Career" and "Burden," chosen by only 20% of them.

To sum up, there is a non-negligible gap between the ideal and planned numbers of children for couples, and a substantial fraction of them answer the reason as "raising children and education are too expensive." These results establish the first fact: couples are most likely to abandon having an ideal number of children because of financial costs.

Fact 2. *A significant financial cost gap exists between those who have children enrolled in college and those who do not.*

Fact 1 indicates that the financial cost of having children is a critical constraint on fertility choices in Japan. However, it is silent on how and in which cases having children is so expensive; Fact 2 addresses them from the viewpoint of education costs. To do this, I use two data sets: (1) the Survey of Children's Learning Expenses (SCLE, 2021) and (2) the Student Life Survey (SLS, 2018), both cross-sectional household surveys and conducted by the Ministry of Education, Culture, Sports, Science and Technology (MEXT).[37]

[36] I use the term "likelihood" instead of "share" because it allows respondents to choose multiple options.

[37] See, https://www.mext.go.jp/b_menu/toukei/chousa03/gakushuuhi/1268091.htm for the SCLE and https://www.jasso.go.jp/statistics/gakusei_chosa/__icsFiles/afieldfile/2021/03/09/data18_all.pdf (in Japanese) for the SLS.

The SCLE covers more than 53,000 students from preschool to high school and reports the per-student average education expenditure for each expenditure category and education stage (i.e., preschool, elementary, junior high, and high school). The expenditure category includes not only school-related ones (e.g., tuition fees and textbooks) but also extracurricular activities (e.g., cram school, music, arts, and sports). In addition to that, I use the SLS for parents' expenditures when their children enroll in college.[38]

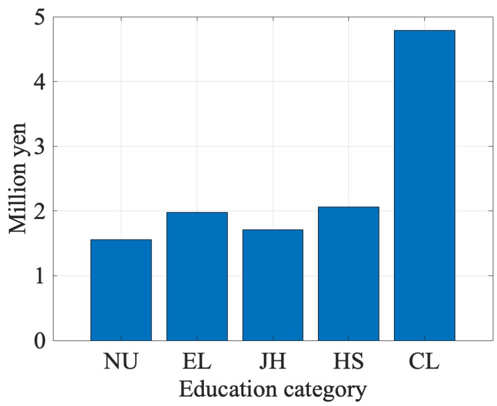

Fig 8: Average education expenditures.

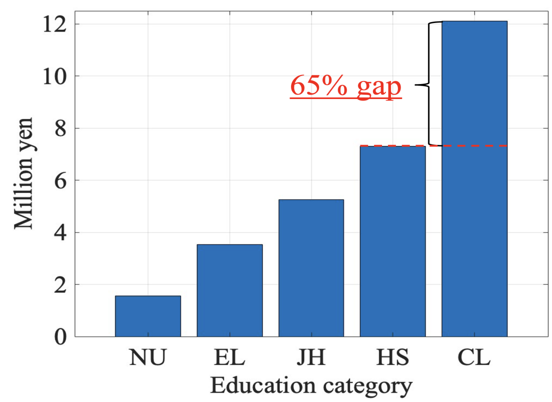

Fig 9: Cumulative education-expenditures.

Fig 8 shows the average per-child expenditure for each education category until completion. "NU," "EL," "JH," "HS," and "CL" each stand for nursery school or preschool, elementary school, junior high school, high school, and college, only consisting of a four-year college. All expenditures are conditional on enrollment in the education stage. Fig 9 represents the cumulative education expenditures, showing that the expenditure jumps up when children attend college. Given that the high school graduation rate is approximately 100% in Japan, the figure tells us that raising one child on average takes the education costs of 7.31 million yen, which amounts to 4.5% of the individuals' average lifetime labor earnings.[39] If their children attend college, they have to spend another 4.8 million yen, meaning there is more than a 60% increase in education costs if parents send their children to college. This expensiveness of college education can, at least partly, be attributed to the fact that Japan is one of the least in subsidizing tertiary education while subsidizing a sizable portion of schooling costs up to secondary education. These observations establish Fact 2: A significant financial cost gap exists between those who have children enrolled in college and those who do not.

[38]In 2018, the SLS correct answers from 43,394 students attending tertiary education, including college, some college, and graduate school.

[39]I construct a proxy of the average individual's lifetime earnings based on the 2022 Basic Survey on Wage Structure (BSWS) by the Ministry of Health, Labour, and Welfare (MHLW). First, I compute the average monthly earnings of ordinary workers for each age unconditional on any other characteristics such as sex and education. Then, I sum up the average earnings for each age, which amounts to about 160 million yen, and regard it as a proxy of the individuals' average lifetime labor earnings.

Fact 3. *A substantial fraction of parents desire a college education for their children.*

Facts 1 and 2 suggest that the financial costs for children enrolled in college are a critical obstacle to fertility in Japan. However, one might not still be convinced because Japan's college enrollment rate is approximately 55%, far below 100%; thus, college education costs seem relevant only to half of the population.[40] The following fact answers "No, they should not." to this argument by showing that far more fraction of parents desire a college education for their children than the current college enrollment rate.

We return to the NFS (2015), which asked respondents about the desired education level for their children. Fig 10 summarizes the results by wife's age. Here, "SC" stands for some college. The figure shows that approximately 75% of the parents desire a college education for their children, which is significantly higher than the college enrollment rate observed in any period in Japan. This observation suggests that, although college enrollment in Japan has been far below 100%, education costs for a college education can be relevant not only for a specific part of the population because many parents would like to send their children to college.

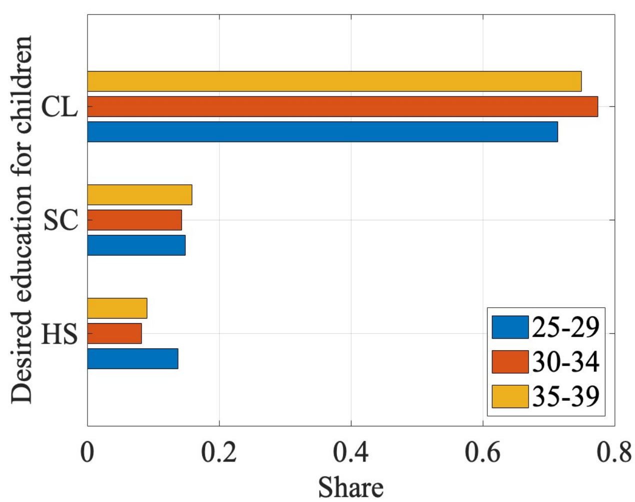

Fig 10: Wives' intention for children's education attainment.

Fact 4. *Japan is one of the least in subsidizing tertiary education.*

Japan is one of the least in subsidizing tertiary education, which roughly corresponds to four-year college education in Japan, given that the enrollment rates for tertiary education

[40] Dropout rates are insignificant in Japan, so the enrollment rate is almost equivalent to the graduation rate. For example, the dropout rate was 2.5% in 2021. See, `https://www.mext.go.jp/content/20220603-mxt_kouhou01-000004520_03.pdf` (in Japanese).

other than the four-year college are significantly lower than four-year college enrollment rate.[41]

To see this, I use the cross-country data provided by the OECD[42] and define the *subsidization rate* for a specific education category e as follows:

$$s_e = \frac{y_e^{pub}}{y_e^{pri} + y_e^{pub}},$$

where y_e^{pri} and y_e^{pub} represent the private and public spending on education e, both represented as a share of the GDP. Public spending includes expenditures on educational institutions and educational-related subsidies for households or students. Private spending refers to expenditures financed by households and other private entities. Private spending includes expenditures on school but excludes those outside educational institutions (e.g., textbooks purchased by households, private tutoring, and student living costs).[43] In other words, the subsidization rate indicates what fraction of (potential) school-related costs are funded by the government.

Japan subsidies more than 90% of the costs for primary-to-secondary education, which is higher than the OECD average. On the contrary, the subsidization rate for tertiary education was only 32%, which is the second lowest among OECD countries and less than half of the OECD average of 70%. This fact shows plenty of room for increasing subsidies for college students, which also has driven recent policy discussions on introducing and expanding education subsidies for college students.

C. Empirical Analysis

This section provides suggestive evidence for the positive fertility effects of college subsidy presented in Section 5. More specifically, I show that parents cease reproduction because having and educating children takes too much financial cost, more likely when their children enroll in college.

I use the 60s cohort data in the JPSC, as in Section 4. A unique feature of the JPSC is that it asks respondents about their preferences or intentions regarding fertility, such as "Do you want more children in the future?" If they answer "No," it then asks the reasons from fourteen options, which they can choose multiple. The main options are: (1) having and educating children takes too much financial cost; (2) I would rather spend

[41] According to the Basic School Survey by the MEXT, the enrollment rate for some college was 4% in 2022 and is declining over the past thirty years.

[42] I use 2018's data given that it is the latest year in which data on a significant number of countries is available.

[43] For more details, see the OECD data (https://data.oecd.org/).

more time with my husband or for myself; (3) I want to continue my job; and (4) I cannot expect my husband's effort in child-rearing. Fig 11 reports the likelihood for each of the four reasons, where I refer to (1), (2), (3), and (4) as *Finanial*, *Time*, *Job*, and *Husband*, respectively. This figure indicates that more than 40% of parents cease reproduction due to the financial reason, which is the most important among other reasons.

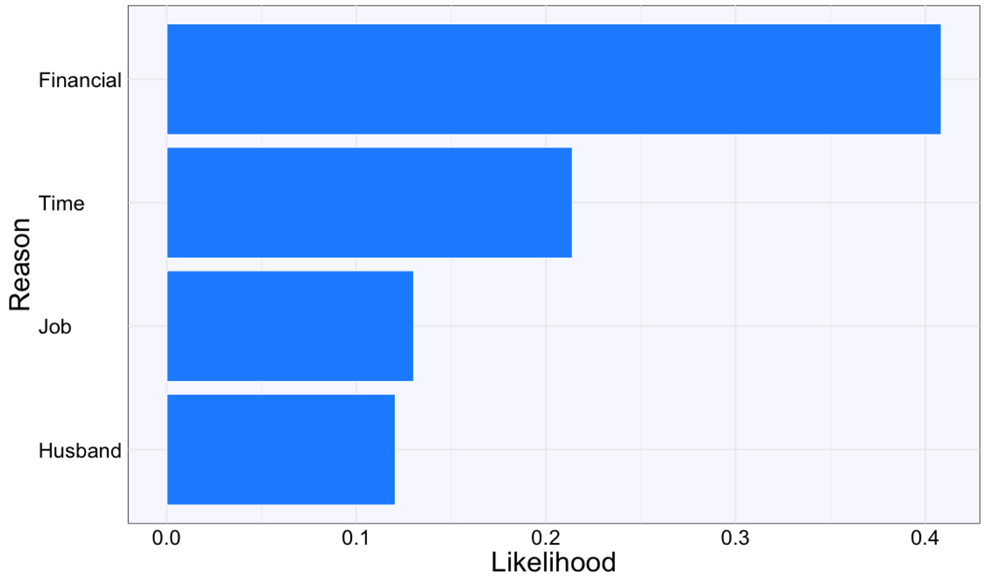

Fig 11: Likelihood for reasons why parents cease reproduction. *Note*: Each of the four reasons, *Finanial*, *Time*, *Job*, and *Husband*, represent: (1) having and educating children takes too much financial cost; (2) I would rather spend more time with my husband or for myself; (3) I want to continue my job; and (4) I cannot expect my husband's effort in child-rearing.

Next, I examine how the likelihood of ceasing reproduction for financial reasons is related to their children's education level based on the following regression:

$$Financial_i = \alpha + \beta \cdot Child\ CL_i + \varepsilon_i \tag{11}$$

Here, $Financial_i$ is a dummy variable, taking one if they cease reproduction for financial reasons and zero otherwise. Also, $Child\ CL_i$ is a dummy variable, taking one if at least one of their children attends a four-year college. I conduct the regression analysis based on the two models (the linear probability model and the logit model) and two categories of sample (full sample and high education). Here, I define "high education" households if the wife or husband is a college graduate.

Table 14 summarizes the results. Based on the full sample, the estimates for β are positive and statistically significant for both specifications. Based on the high education sample, the estimates are still positive and statistically significant for both specifications, and the values are more significant than those based on the full sample.

There are two possibilities for how their children's education status would be related to fertility choices. First, some parents expect potential or existing children younger than college enrollment age to attend college, which will impose a significant financial burden on the parents in the future. Second, children have already attended college and started paying financial costs to support their enrollment. Either way, the results suggest that the presence (or expectation) of children attending college affects fertility decisions due to financial burdens.

	(1)	(2)	(3)	(4)
Child CL_i	0.2009***	0.35128***	1.4158***	2.8973**
	(0.0506)	(0.0902)	(0.3867)	(1.0284)
Sample	Full	High	Full	High
Model	OLS	OLS	Logit	Logit
Observations	484	212	484	212

Table 14: Estimation results. *Note*: Values in parentheses represent standard errors. ** and *** denote statistical significance at the 5 and 1 percent levels, respectively. "High" represents the high education households where the wife or husband is a college graduate.

	Income	Fertility	*Child CL*	*Financial*
Low Educ.	581.4	2.05	0.197	0.419
High Educ.	887.8	1.86	0.417	0.385
Average	699.8	1.97	0.285	0.401

Table 15: Descriptive statistics. *Note*: I define "high education" households if the wife or husband is a college graduate; the rest are defined as "low education." Income refers to the average household income when the wife is aged 40 to 43 following Kim et al. (2023). Fertility refers to the completed fertility rate. *Financial* represents the share of households who cease reproduction for financial reasons. *Child CL* represents the share of households where at least one of their children is a college graduate.

D. Equilibrium Definition

Let $\boldsymbol{x}_j^e$ be an age-specific state vector for agents with education level $e \in \{HS, CL\}$ and $\mu_j^e(\boldsymbol{x}_j^e)$ be the measure of agents with state vector $\boldsymbol{x}_j^e$. Let $I_l(h, I)$ and $I_g(h, I)$ be indicator functions for loans and grants, respectively, returning 1 if students with (h, I) are eligible and 0 otherwise.

Given exogenous parameters and policy rules $\{\tau_a, \tau_c, \iota, \iota_s, B, S, \psi, p, I_l(h, I), I_g(h, I)\}$, a *stationary recursive competitive equilibrium* consists of

- value functions $\{V_{g0}, V_{g1}, V_{g2}, V^w, V^f, V^{wf}, V^{IVT}, V^r\}$,
- policy functions for consumption, savings, leisure $\{c_j^e(\boldsymbol{x}_j^e), a_j^e(\boldsymbol{x}_j^e), l_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J}$, working hours $\{h_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J_R}$, fertility $\{n_{J_F}^e(\boldsymbol{x}_{J_F}^e)\}$, IVT $\{a_{J_{IVT}^e}(\boldsymbol{x}_{J_{IVT}}^e)\}$, and college enrollment $\{e_{J_E}(\boldsymbol{x}_{J_E})\}$,

- prices (r, w_{HS}, w_{CL}),
- labor income tax rate τ_w,
- aggregate quantities (K, L_{HS}, L_{CL}),
- measures for households $\{\mu_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J}$,

such that:

1. The decision rules of students, workers, and retired households solve their problems, and $\{V_{g0}, V_{g1}, V_{g2}, V^w, V^f, V^{wf}, V^{IVT}, V^r\}$ are the associated value functions.
2. The representative firm maximizes its profit and optimally chooses capital and labor inputs:

$$r + \delta = \alpha \cdot Z \cdot \left(\frac{K}{L}\right)^{\alpha - 1}, \tag{12}$$

$$w_{HS} = \tilde{Z} \cdot \omega_{HS} \cdot L_{HS}^{\chi - 1}, \tag{13}$$

$$w_{CL} = \tilde{Z} \cdot \omega_{CL} \cdot L_{CL}^{\chi - 1}, \tag{14}$$

   where

$$L = [\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}]^{1/\chi},$$

$$\tilde{Z} = (1 - \alpha) \cdot Z \cdot \left(\frac{K}{L}\right)^{\alpha} \cdot [\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}]^{1/\chi - 1}.$$

3. The labor market for each skill $e \in \{HS, CL\}$ clears:

$$L_e = \sum_{j=J_E}^{J_R} \int_{\boldsymbol{x}_j^e} \eta_j^e(\boldsymbol{x}_j^e) \cdot h_j^e(\boldsymbol{x}_j^e) \; d\mu_j^e(\boldsymbol{x}_j^e), \tag{15}$$

   where $\eta_j^e(\boldsymbol{x}_j^e)$ represents the labor efficiency of agents with a state vector $\boldsymbol{x}_j^e$.

4. The capital market clears:

$$K = \sum_{j=J_E}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} a_j^e(\boldsymbol{x}_j^e) \; d\mu_j^e(\boldsymbol{x}_j^e). \tag{16}$$

5. The government budget is balanced:

$$\tau_c \cdot C + \tau_w \cdot (L_{HS} + L_{CL}) + \tau_a \cdot K + Q = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \leq 17} + S,$$

where

$$C = \sum_{j=J_E}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} c_j(\boldsymbol{x}_j^e) \ d\mu_j^e(\boldsymbol{x}_j^e),$$

$$Q = \sum_{j=J_R+1}^{J} \frac{1-\zeta_{j-1,j}}{\zeta_{j-1,j}} \sum_{e} \int_{\boldsymbol{x}_j^e} a_j^e(\boldsymbol{x}_j^e) \ d\mu_j(\boldsymbol{x}_j^e),$$

$$p \cdot \mu_{old} = \sum_{j=J_R}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} p \ d\mu_j^e(\boldsymbol{x}_j^e),$$

$$K_s = \int_{\boldsymbol{x}^s} \max\{0, -a^s(\boldsymbol{x}^e)\} \cdot I_l(h, I) \ d\boldsymbol{x}^s$$

$$G = \int_{\boldsymbol{x}^s} g(h, I) \cdot I_g(h, I) \ d\boldsymbol{x}^s$$

$$B \cdot \mu_{j \leq 17} = \sum_{j=J_F}^{J_{IVT}-1} \sum_{e} \int_{\boldsymbol{x}_j^e} B \cdot n \ d\mu_j^e(\boldsymbol{x}_j^e),$$

where $\boldsymbol{x}^s$, $\mu^s(\boldsymbol{x}^s)$, and $\{a^s(\boldsymbol{x}^s)\}$ represent a state vector for college students, measure of college students, and students' policy function for saving, respectively.

6. Distributions (measures) and households' behavior are consistent.

E. Computational Algorithm

E. (i) Stationary Equilibrium

For any variable or distribution x, let $\tilde{x}$ and $\hat{x}$ represent its guessed and model-implied values. Also, let μ_j represent the distribution over state variables for age j. Dividing the computation process into three blocks makes it easier to understand. The first block is the outer loop, searching for equilibrium prices. The other two blocks are inner loops; one is the *optimization block*, solving household problems given prices, and another is the *distribution block*, searching for the stationary distributions given prices and policy functions obtained in the optimization block. The computational algorithm proceeds as follows:

1. Guess prices $\tilde{\boldsymbol{p}} = (\tilde{r}, \tilde{w}_{HS}, \tilde{w}_{CL})$.

2. *Optimization block*:

   - Guess the value function for agents at the beginning of age $j = 18$ ($\tilde{V}_{g0}$).
   - Given $\tilde{V}_{g0}$, solve backward from the period of IVT choice to that of the education choice, which gives the model-implied value function for V_{g0}, $\hat{V}_{g0}$.

- Check if
$$d(\tilde{V}_{g0}, \hat{V}_{g0}) < \varepsilon, \tag{17}$$
where $d(\cdot)$ and $\varepsilon > 0$ represent an arbitrary metric function and error tolerance. If (17) is not satisfied, update $\tilde{V}_{g0}$ and follow the same procedure until convergence. The correct V_{g0} pins down all value functions and policy functions given a set of prices $\tilde{\boldsymbol{p}}$.

3. *Distribution block*:
   - Guess the distribution for age J_{IVT}. This $\tilde{\mu}_{J_{IVT}}$ and policy functions for IVT derive the implied distribution for agents aged 18, $\hat{\mu}_{18}$. Given $\hat{\mu}_{18}$ and policy functions, compute the implied distributions for age $j = 19, ..., J_{IVT}$, and obtain $\hat{\mu}_{J_{IVT}}$.
   - Check if
   $$d(\tilde{\mu}_{J_{IVT}}, \hat{\mu}_{J_{IVT}}) < \varepsilon. \tag{18}$$
   If (18) is not satisfied, update $\tilde{\mu}_{J_{IVT}}$ and follow the same procedure until convergence.
   - After obtaing the correct distributions for age $j = 18, ..., J_{IVT}$, compute the distribution for age $j_{IVT}+1$ onward, using those distributions and policy functions.

4. After computing value functions, policy functions, and distributions, compute the implied quantities, $\hat{L}$ and $\hat{K}$ based on (15) and (16), which gives the implied prices $\hat{\boldsymbol{p}}$ based on $\hat{L}$, $\hat{K}$, (12), (13), and (14).

5. Check if
$$d(\tilde{\boldsymbol{p}}, \hat{\boldsymbol{p}}) < \varepsilon. \tag{19}$$
If (19) is not satisfied, update $\tilde{\boldsymbol{p}}$, return to the optimization block, and follow the same procedure until convergence.

E. (ii) Transition Dynamics

For any variable or distribution x, let $\tilde{x}$ and $\hat{x}$ represent its guessed and model-implied values. Also, let $\boldsymbol{q}_t = (K_t, L_t)$ be a vector of aggregate quantities in a year t, and $\boldsymbol{q} = (\boldsymbol{q}_t)_{t=1,...,T}$ for some T. The computational algorithm for transition dynamics proceeds as follows:

1. *Preliminaries*: Set an arbitrarily large number for transition periods, T. Given aggregate quantities in the initial and final steady states, $\boldsymbol{q}_1$ and $\boldsymbol{q}_T$, guess a sequence

of aggregate quantities, $\{\tilde{\boldsymbol{q}}_t\}_{t=1,\ldots,T}$, where $\tilde{\boldsymbol{q}}_1 = \boldsymbol{q}_1$ and $\tilde{\boldsymbol{q}}_T = \boldsymbol{q}_T$, which pins down guesses for prices $\{\tilde{r}_t, \tilde{w}_t\}_{t=1,\ldots,T}$. Also, guess a sequence of the labor income tax rate $\{\tilde{\tau}_{l,t}\}_{t=1,\ldots,T}$, where $\tilde{\tau}_{l,1}$ and $\tilde{\tau}_{l,T}$ are the tax rates at the initial and final steady states.

2. *Optimization*: Given $\{\tilde{r}_t, \tilde{w}_t\}_{t=1,\ldots,T}$ and $\{\tilde{\tau}_{l,t}\}_{t=1,\ldots,T}$, solve the optimization problem for each cohort born before period T.

3. *Distribution*: Given decision rules for each cohort obtained in the above optimization block, construct the implied distributions:

   - $\tilde{\mu}_a(j;t)$: (aggregate) savings distribution across age in year t.
   - $\tilde{\mu}_n(j;t)$: (aggregate) labor supply distribution across working-age in year t.
   - $\tilde{\mu}_{j|t}$: age distribution in year t.

4. Given the decision rules and the implied distributions, compute the model-implied aggregate quantities, $\hat{\boldsymbol{q}}$. Also, compute the implied government revenue and expenditures, denoted by $\hat{\boldsymbol{G}}_r = (\hat{\boldsymbol{G}}_{r,t})_{t=1,\ldots,T}$ and $\hat{\boldsymbol{G}}_e = (\hat{\boldsymbol{G}}_{e,t})_{t=1,\ldots,T}$.

5. Check if

$$d(\tilde{\boldsymbol{q}}, \hat{\boldsymbol{q}}) < \varepsilon$$

   and

$$d(\hat{\boldsymbol{G}}_r, \hat{\boldsymbol{G}}_e) < \varepsilon.$$

   If the first (second) condition is not satisfied, update $\tilde{\boldsymbol{q}}$ ($\{\tilde{\tau}_{l,t}\}_{t=1,\ldots,T}$) and follow the same procedure until convergence.

F. Transition Dynamics

[In progress. TBA]

G. Reallocating Resources to Different Programs

This section examines the performance of the potential alternative programs to the existing income-tested grants. To this end, I consider another two scenarios, in addition to the introduction of income-tested grants to the benchmark model, examined in Section 5.1. The first scenario is to introduce grants for college students with income and ability tests so that "high-ability" students in poor households are eligible. More specifically, I keep the income threshold adopted in the existing scheme but arbitrarily set the lower bound for students' human capital for the eligibility, $\underline{h}$, to its median. The payment function $g(h, I)$ for this scheme is defined as follows:

$$g(h, I) = \begin{cases} g_1 & \text{if } I < \bar{I} \text{ \& } h > \underline{h} \\ 0 & \text{otherwise} \end{cases}$$

Here, the payment g_1 is set so that the short-run expenditure upon the introduction is the same as that of the existing income-tested grants. As a result, g_1 covers approximately 100% of the average student's expenditure in the benchmark, which is greater than the payment in the existing scheme with only income tests because the number of eligible students is fewer.

The second scenario introduces unconditional grants for college students regardless of ability and income. The payment function is given as $g(h, I) = g_2$ for any students with (h, I), where g_2 is set so that the short-run government expenditure is the same as the existing program. As a result, g_2 covers approximately 10% of the average students' expenditure in the benchmark, which is less significant than in the existing income-tested grants because this alternative program covers a broader range of students.

I solve the stationary equilibrium with each scenario, and the results regarding fertility and enrollment rates are summarized in Table 16. A highlight is that the existing scheme with income tests would lead to the highest equilibrium TFR among other scenarios. Notably, the unconditional grants lead to a 1.2 p.p. higher college enrollment rate than the income-tested ones and a -0.3% lower TFR because of the composition effect. These results highlight the insurance effect of the income-tested grants on fertility and the downward pressure the grants exert on aggregate fertility through composition changes.

	Income	+Ability	Uncond.
CL share (Δ p.p.)	+3.9	+2.6	+5.1
TFR (Δ%)	+3.0	+2.7	−0.3
HS	+0.4	−3.5	+4.0
CL	+7.4	+8.4	+3.9
Output (Δ%)	+1.0	−0.1	−0.7
STD (wage) (Δ%)	−1.3	−0.9	−0.1
Welfare (%)	+4.8	+2.6	+1.6

Table 16: Main results with several schemes with different targets. *Note*: values in each cell indicate changes from the benchmark value. Rows "HS" and "CL" indicate the percentage changes in the fertility of high school and college graduates.

H. Comparison With Pro-natal Transfers

This section examines the differences in the macroeconomic implications between the education subsidy and child benefit (i.e., typical pro-natal cash transfers).[44] To this end, I simulate the expansion of the unconditional cash transfers to households with children B from its benchmark value while setting $g(h, I) = 0$ for any (h, I) as in the benchmark. The new level of the per-child payment B is set so that the short-run expenditure upon the introduction is the same as that of the existing income-tested grants, which leads to a 6.5% increase in the per-child payment.

I solve the stationary equilibrium with the expansion of the per-child payment B, and the main results are summarized in Table 17. Columns "Education" and "Typical" represent the results of the introduction of income-tested grants and the expansion of child benefit ("typical" pro-natal transfers).

The child benefit expansion leads to a 2.5% increase of the TFR in the long-run equilibrium, comparable to introducing the income-tested grants but slightly lower than that. The previous decomposition analysis suggests that targeting students in unlucky households in which negative income shocks are realized would be effective, at least marginally, in increasing the TFR. A notable observation is that the characteristics of households who respond to the policy differ between the two cases; the education subsidy induces skilled households to have more children, while the child benefit has a similar degree of impact on skilled and unskilled households regarding fertility. This is because the former is more beneficial for households whose potential children are likely to attend college, and college graduates tend to be those households due to the intergenerational persistence of education.

The college enrollment rate does not change in response to the child benefit expansion. The expansion also does not affect the average human capital, defined as the workers' average of labor efficiency $\eta_{j,z,e,h}$,[45] while the education subsidy increases the average human capital by 1.0% primarily because of the higher college enrollment rate. The difference in the average human capital also leads to the difference in the per-capita output. While the education subsidy increases the per-capita output by 1.0% in the long run, the child benefit expansion would rather lead to a 0.3% lower per-capita output. As in the case of introducing the education subsidy, the child benefit expansion also leads to lower (physical) capital accumulation due to crowding out and demographic change,

[44] In Section G., I also consider reallocating resources into different targets to implement the education subsidy by considering unconditional transfers to college students and introducing ability tests.

[45] Formally, I define the average human capital as $\sum_j \int_{z,h} [(1-s) \cdot \eta_{j,z,e=0,h} + s \cdot \eta_{j,z,e=1,h}] dF(z,h)\mu_j$, where s represents the college enrollment rate, $F(z,h)$ represents the stationary distribution over (z,h), and μ_j is the stationary distribution of age.

as discussed in Section 5.1. The education subsidy has sufficiently large positive effects on aggregate labor supply in efficiency units, which surpasses the negative impacts on physical capital accumulation and increases output. The child benefit expansion has more modest impacts on aggregate labor efficiency than the education subsidy, leading to the gap in output gains between the two policies.

The child benefit expansion does not affect the standard deviation of wages, whereas the education subsidy leads to a lower standard deviation by enabling some students in poor households to attend college and acquire skills. Lastly, the expansion leads to a 0.3% of the welfare gain, which is substantially lower than a 4.8% gain with the education subsidy. As discussed in Section 5.2.1, the welfare gains of the education subsidy come mainly from the higher expected lifetime income in each state and the higher probability of being college graduates and enjoying higher earnings facilitated by the higher college enrollment rate and educational mobility, as opposed to the child benefit expansion do. Thus, the education subsidy leads to greater welfare gains than the child benefit expansion under the veil of ignorance.

A caveat is that this model does not capture the endogenous human capital accumulation before high school graduation and the dynamic complementarity of human capital. Households may increase their investments in their children upon the child benefit expansion, which contributes to greater human capital; however, that channel is not considered in this current analysis. Those ingredients can be critical in comparing the effects of these different programs, especially on college enrollment rates, aggregate human capital, and output. The exercise in this framework is a first step, and incorporating those ingredients – fertility, college enrollment, dynamic complementarity of human capital– in one framework is left for future research.

	Education	Typical
TFR (Δ%)	+3.0	+2.5
HS	+0.4	+2.5
CL	+7.4	+2.4
CL share (Δ p.p.)	+3.9	0.0
HS→CL	+2.5	0.0
Avg. HC (Δ%)	+1.0	0.0
Output (Δ%)	+1.0	−0.3
STD (wage) (Δ%)	−1.3	0.0
Welfare (%)	+4.8	+0.3

Table 17: Main results with several schemes with different targets. *Note*: Columns "Education" and "Typical" represent the results with the introduction of the education subsidy and expansion of child benefit ("typical" pro-natal transfers), respectively. Values in each cell indicate changes from the benchmark value. Rows "HS" and "CL" indicate the percentage changes in the fertility of high school and college graduates. "HS→CL" represent the changes in college enrollment rate and educational mobility in the sense of probability that children of high school graduates attend college. "Ave. HC" stands for the average human capital.