Financial Costs of Children, Education Subsidies, and Parental Choices in Equilibrium*

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Abstract

The sustained low fertility has driven pro-natal policies in many countries. In Japan, increasing grants for college students has garnered much attention because the low fertility could largely be attributed to the financial costs of having children, especially college students. However, whether it works as expected and the resulting macroeconomic effects are obscure and may depend on its target. I construct an incomplete market, general equilibrium overlapping generations model embedding parental choices on fertility and transfers to their children, calibrated using Japan's household panel survey. Children's college enrollment requires parents to incur significant financial costs, as observed in data, and whether their children enroll in college or not is uncertain before having a birth. I find that introducing the grants would increase the fertility and college enrollment rates to some extent, leading to a greater labor supply in efficiency units. Ability-tested transfers would bring the largest gains in terms of output, while income-tested ones would do the least owing to their impacts on the education sorting by ability, and these heterogeneous effects are amplified by fertility responses. Expanding unconditional or income-tested programs would be more effective in increasing fertility than expanding the ability-tested ones because the insurance effects of the grants on fertility choices work stronger for the former cases.

Keywords: Education subsidy, fertility, intergenerational linkages.

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1 Introduction

The sustained low fertility has driven the recent rise in pro-natal policies aiming at increasing birthrates in many countries:[1] low fertility is a root cause of demographic aging and poses challenges for economic growth and fiscal sustainability. In line with this trend, the Japanese government, in which one of the lowest-fertility countries and facing significant demographic aging, has introduced education subsidies for college students in 2020 and is now discussing its expansion.[2] It aims at accelerating human capital accumulation to deal with those challenges related to demographic aging, and also aims at rising fertility rates by relaxing the financial constraints of having children. Although it has been uncommon to consider education subsidies for college students as a pro-natalist measure, there are suggestive observations indicating that financial burden to get children have college education may one of the critical factors behind the low fertility in Japan.

First, people are more likely to give up having their desired number of children "because child-rearing and education take too much financial costs." The National Fertility Survey (NFS, 2015)[3] shows that there are some discrepancy between "desired" and "planned" number of children for wives[4] and ask them whose planned number is lower than the desired one the reasons why. The result shows that "because child-rearing and education take too much financial costs" is most frequently chosen among other choices, such as "because it would interfere with my job." For example, 81.1 % of wives aged $30-34$ whose planned number is lower than the desired one chose the financial reason, while 24.2% of them chose the "job" reason.

As similar to most advanced economies, having a child who enrolls in college is far expensive than having a child not enrolling in college in Japan. Average households make educational spending of about 8 million yen per child until their child graduates high school,[5] and they have to make further spending of 4.8 million per child to cover tuition fees and living expenses.[6] This great increase in expenditure upon child's college

[1]Those countries include Hungary, Poland, Greece, Japan, South Korea, etc. See, for example, articles reporting Poland's and Greece's reforms (European Commission, 2018; The Guardian, 2020).

[2]See, for example, the following document by the Cabinet Office https://www5.cao.go.jp/keizai-shimon/kaigi/cabinet/honebuto/2023/2023_basicpolicies_ja.pdf (available only in Japanese).

[3]This survey is conducted by the National Institute of Population and Social Security Research (IPSS). See, https://www.ipss.go.jp/site-ad/index_english/survey-e.asp.

[4]For example, for couples married for $5-9$ years in 2015, the desired number of children was 2.33 on average, while the planned number was 2.03.

[5]See the Survey of Children's Learning Expenses conducted by the Ministry of Education, Culture, Sports, Science and Technology (MEXT) https://www.mext.go.jp/b_menu/toukei/chousa03/gakushuuhi/1268091.htm.

[6]See the Survey on Student Life (2018) conducted by the MEXT https://www.jasso.go.jp/

enrollment may partly reflect the fact that Japan is one of the least countries in terms of public spending on tertiary education.[7]

Lastly, the college enrollment rate in Japan now exceeds 50% and continues to increase over time. Correspondingly, a significant fraction of parents have desired the higher level of education for their children.[8] Then, introducing and expanding education subsidies for college students are expected to affect fertility behavior of a significant fraction of parents in Japan by reducing the financial costs of having children.

Whether it works as expected is obscure, however: it might just crowd out household savings and distort their decisions, and changes in equilibrium objects such as prices and distribution might lead to some unintended consequences. Further, the answer may depend on the target/eligibility of the subsidy (e.g., merit-base, need-base, unconditional, etc.). Without considering those points (i.e., behavioral changes of households, equilibrium feedback, heterogeneous implications of different types of subsidies), that policy intervention would be misleading and come at a further cost.

Motivated by those considerations, I investigate the macroeconomic implications of education subsidies for college students by developing the incomplete market, general equilibrium (GE) overlapping generations (OLG) model incorporating choices on college enrollment, fertility, and inter-vivo transfers (IVT). The key ingredients are fertility choices and internerational linkages. Households draw utility from having children but it is costly in terms of time and money. In particular, if their children enroll in college, parents have to make some IVT to support them, which comes at significant financial costs. Whether they enroll in college is uncertain for parents before having births, but the probability depends on parents' characteristics such as human capital and school taste. In this economy, the education subsidy for college students would reduce the expected costs of children or provide the insurance against the expenditure uncertainty related to the children's college enrollment.

I calibrate the model to the Japanese economy using the Japanese Panel Survey of Consumers (JPSC). I use the 60s cohort of married females and their spouses to consider their completed fertility rate, educational spending on their children, and their relationship to education and lifetime income. The model replicates important moments, such as the intergenerational persistence of education level, average educational spending made by parents, the distribution of the number of children, and fertility differentials across education levels. The benchmark model captures the government-provided student loans

statistics/gakusei_chosa/__icsFiles/afieldfile/2021/03/09/data18_all.pdf (in Japanese).

[7]In 2018, it spent 0.44% of GDP on tertiary education, the second lowest after Luxembourg among OECD countries and less than half of the OECD average (0.95%). See, https://data.oecd.org/.

[8]Accoding to the NFS (2015), about 68% of parents desire that their children complete college education, while it was 54.1 % in 1992.

but does not include the grants such as that introduced in 2020 to examine the potential impacts of introducing several types of subsidies.

I use the model to investigate the macroeconomic effects of introducing and expanding various types of education subsidies. More specifically, I consider three types of the subsidies and conduct counterfactual experiments introducing and expanding each of them. First, as in Abbott et al. (2019), I introduce (1) the unconditional transfers of ¥$120,000$ per college student per year. Starting from this unconditional transfers, I further considers those (2) income-tested, and (3) ability-tested so that the short-run expenditure in each reform is the same.

Main findings are summarized as follows. First, each program increases the college enrollment rate by at most 1.7 percent points (p.p.) and the total fertility rate (TFR) by $1.6 - 2.6\%$ in the long-run equilibrium, which leads to a greater labor supply in efficiency units due to higher share of skilled workers and working-age population. At the same time, the most cases reduce aggregate savings as they reduce saving incentives and crowd out the IVT. In addition, the higher TFR implies a larger share of younger generations who have relatively less assets, leading to lower aggregate savings. Its positive impacts on aggregate labor surpass the negative impacts on aggregate savings with unconditional and ability-tested transfers, implying a greater aggregate output. Importantly, the ability-tested program would achieve the greatest output and fiscal gains: it would lead to 3.3% output growth, which increases the tax base and reduce the labor income tax rate by 0.9%, which is adjusted to balance the government budget. On the contrary, introducing income-tested program lowers output by 0.5%. The human capital level of students newly enrolling, thanks to the subsidy, is relatively lower than in the case of introducing ability-tested one, given the intergenerational correlation of human capital. Then, the marginal effects of the subsidy on labor efficiency are lower, and so are the effects on output. The variance of income is reduced the most in the income-tested program by targeting students in lower income households.

To understand the roles of fertility responses, I solve the exogenous fertility version of this model where the number of children in each state of household are fixed as in the benchmark. I find that the heterogeneous effects of those subsidies on aggregate output are attenuated toward origin under exogenous fertility: output gains of ability-tested and unconditional programs become smaller and output losses of income-tested one also become smaller. The attenuation under exogenous fertility suggests that fertility responses upon the policy intervention amplify their heterogeneous implications for aggregate output. Note that, in principle, households whose fertility behavior is affected are those targeted by the policy (e.g., those with lower human capital in the case of income-tested transfers). If we consider the fertility responses, the targeted households may increase fertility, and their children are likely to have similar characteristics to their parents via intergenerational linkage of human capital and school tastes. As a result, the share of those households increases through this fertility channel, strengthening the heterogeneous effects on aggregate output. As a result of the smaller output gains, the equilibrium tax rate is no longer smaller than in the benchmark with any programs.

Lastly, expanding payments for unconditional and income-tested transfers have almost no effects on college enrollment but would lead to a higher TFR and output, and to a lower tax rate even for income-tested transfers. On the other hand, the marginal effects of increasing the ability-tested transfer are limited: the college enrollment rate and the TFR are almost the same when introducing the subsidy even when the payments are tripled. Because high-ability students are more likely to have high-ability parents implied by the intergenerational persistence of human capital, and those high-ability parents earn higher income and will have sufficient amount of assets. Then, the income effects and insurance effects of education subsidy are weaker for those richer parents, and expanding education subsidies would have less impacts on fertility choices of those households. Still, the output growth and the decline of tax rate are more significant than income-tested transfers in any scale.

Those results may draw some policy implications. First, ability-tested programs would bring the greatest gains in output and tax base to some extent, but the marginal gains would decrease faster because many of their parents are rich enough and the income effects and insurance effects of education subsidies on their fertility behavior are limited. On the contrary, expanding programs covering lower and broader income classes, such as unconditional or income-tested ones, would be more effective in achieving the pro-natalist goal of increasing fertility: it reduces the financial costs of children or providing insurance against the expenditure risks related to children, and the effects are more significant for lower income households. However, we must keep in mind that the ability-tested program would still lead to a higher output and lower tax rate than the income-tested program despite the smaller impacts on the TFR and demographic structure.

This study contributes to the vast literature on the macroeconomic analysis of education subsidies.[9] Most studies build on the incomplete market GE-OLG model embedding college enrollment choices and IVT. A seminal work of Abbott et al. (2019) incorporate students' labor supply decision into the framework so that the model captures the crowding out of education subsidies through the labor supply channel, and investigate

[9]See, for example, Benabou (2002); De la Croix and Doepke (2004); Akyol and Athreya (2005); Kulikova (2015); Krueger and Ludwig (2016); Kotera and Seshadri (2017); Lawson (2017); Abbott et al. (2019); Lee and Seshadri (2019); Abbott (2022); Daruich (2022); Zheng and Graham (2022); Matsuda and Mazur (2022); Matsuda (2022).

several types of education subsidies for college students, such as loans and grants, with and without some tests for eligibility in terms of income and ability. Without some notable exceptions,[10] previous studies adopt models without fertility choices. I contribute to the literature by incorporating fertility choices into the model otherwise standard in the literature, considering college enrollment choices and IVT on the incomplete market GE-OLG framework. I show that the fertility responses amplify the aggregate impacts of the education subsidies.

This study methodologically relates to the macroeconomic studies based on the quantity-quality trade-off framework.[11] This literature shows that considering the quantity-quality trade-off and the resulting differentials in those choices can have critical implications for macroeconomic variables such as aggregate output and inequality. Although they do not investigate the effects of education policies to focus on how the fertility differentials matter to the variables of their interest, their results suggest that cross-sectional heterogeneity in quantity-quality choices is vital in considering the macroeconomic effects of education subsidies and their design. Building on those insights, I contribute to the literature by extending the framework in some respects and investigating the macroeconomic effects of education subsidies. De La Croix and Doepke (2003) use a two-period OLG model capturing fertility differentials and suggest that the higher inequality implies a lower aggregate output through the fertility differentials. This study is different from theirs in constructing a full-fledged lifecycle model on the incomplete market framework and elaborating on college enrollment choices. Daruich and Kozlowski (2020) construct a partial equilibrium lifecycle model with inter-vivo transfers and education and fertility choices. They suggest that the negative income-fertility relationship is the critical factor determining the low intergenerational mobility in the US. This study is different from theirs in adopting the general equilibrium framework to capture the GE and taxation effects due to introducing education subsidies, which are shown to be critical in the literature of education policies (e.g., Krueger and Ludwig, 2016).

The rest of the paper is organized as follows. Section 2 describes the model, which is calibrated in Section 3. Section 4 conducts numerical analysis using the model and Section 5 concludes.

[10]For example, De la Croix and Doepke (2004) compare the macroeconomic implications of a public and private schooling regime based on a two-period OLG model with fertility choices. My study differs from it in constructing a full-fledged lifecycle model on the incomplete market framework and elaborating on college enrollment choices.

[11]See, for example, De La Croix and Doepke (2003); Daruich and Kozlowski (2020); Kim et al. (2023); Zhou (2022).

2 Model

This section describes the GE-OLG model incorporating choices on college enrollment, inter-vivo transfers, and fertility. I embed fertility choices into a framework otherwise standard in the literature (e.g., Krueger and Ludwig, 2016; Abbott et al., 2019; Matsuda and Mazur, 2022). In this economy, agents choose to go to college or enter the labor market after graduating high school. College enrollment increases their skill and potential lifetime earnings but takes some costs in terms of money and disutility. As standard in the literature, the model captures several channels of intergenerational linkage critical to the enrollment choices. First, the disutility of education ("psychic costs") depends on their parents' educational background. Second, parents choose the amount of inter-vivo transfers for their children. Third, the human capital endowment of children correlates to that of their parents, which is critical to education return. Students can fund their education costs through the inter-vivo transfers, labor earnings through part-time jobs, student loans with cheaper interest payments, and grants. The subsidized loans and grants are income- and ability-tested and provided by the government. After enrolling in college, they enter the labor market and choose how much to consume, save, and work, subject to idiosyncratic productivity shocks. At a point in time, they choose how many children they have; they draw the utility from the "quantity and quality" of their children. After their children graduate high school, they make inter-vivo transfers to their children if they enroll in college. Wealthier parents can afford to make more generous transfers, which helps their children go to college, while poorer parents cannot.

2.1 Preliminaries

Production: A representative firm chooses labor and capital inputs in competitive factor markets to produce final goods. There are two types of labor in this economy: one is the skilled labor L_h who graduates college, and the rest is referred to as the unskilled labor, denoted by L_l. I allow them to be imperfect substitutes by considering the total labor efficiency unit L is given as:

$$L = \left[(1-\omega_h)(L_l)^{\chi} + \omega_h(L_h)^{\chi}\right]^{1/\chi},$$

where ω_h governs the relative productivity of the skilled workers.

The representative firm operates with a Cobb-Douglas production function with aggregate capital K and labor L to produce the aggregate output Y:

$$Y = K^{\alpha}L^{1-\alpha}.$$

Let r, w_l, and w_h denote the rental rate of capital and wage rates for skilled and unskilled labor. Capital depreciates at δ, and the firm has to incur the capital depreciation cost.

Demographics: In this model, one period corresponds to two years. Agents live with their parents until graduating high school and do not make any decisions at this stage. Upon high school graduation, they make a college enrollment decision. If they decide to go to college, it takes two periods (four years) to complete.[12] After graduation, they enter the labor market and can work until a retirement age of $j = J_R$. After retirement, they face uncertainty regarding their survival in the next period. Let $\zeta_{j,j+1}$ denote the probability of surviving at age $j+1$ conditional on surviving at age j for each $j \in \{J_R + 1, ..., J\}$ with $\zeta_{J,J+1} = 0$. The cohort size grows at rate g_n, which is endogenously determined in this model based on households' choices on fertility. Let μ_j denote the age distribution of the economy, determined by the cohort population growth rate and survival probabilities. The mass of the economy is normalized to 1 (i.e., $\sum_j \mu_j = 1$) as in previous studies (e.g., Guner et al., 2020; Zhou, 2022).

Intergenerational Linkages and Initial Endowments: Three factors shape college enrollment choices and generate their heterogeneity: inter-vivo transfers from their parents, disutility (psychic costs) of college enrollment, and human capital determining future earnings and education returns. Inter-vivo transfers that potential enrollees receive are heterogeneous across agents due to wealth heterogeneity in parent generations. Second, college enrollment takes some psychic costs, denoted by ϕ. They are stochastic and drawn from a distribution $g^{\phi}_{e_p}$, depending on parents' education e_p. Likewise, the human capital endowment, denoted by h, is drawn from a distribution $g^{h}_{h_p}$ varying with the parents' human capital level h_p.

Preferences: Throughout their lifetime, they draw utility from consumption c and leisure l according to a utility function $u(c,l)$. They discount future utility by β. At age $j = J_F$, they choose the number of children they have, denoted by n. They can draw additional utility from having children in several ways. First, they draw utility from the "quality" of the children by making monetary investments q per child until the children graduate high school, captured by a utility function $v(q)$. The utility is discounted by a function $b(n)$. Further, the model captures the variation of paternalistic motive adopted in Abbott et al. (2019) so that parents draw utility by transferring assets to children if they attend college. More specifically, parents consider children's expected lifetime utility in their education choices with a discount rate λ_a if they attend college and make some inter-vivo transfers to support them financially.

[12] This model does not consider the possibility of dropping out from college because the drop out rate is insignificant in Japan. In 2021, for example, the dropout rate was 2.5%. See, `https://www.mext.go.jp/content/20220603-mxt_kouhou01-000004520_03.pdf` (in Japanese).

Costs of Children: Having children is costly in terms of money and time. First, q units of the per-child investment require $n \cdot q$ units of money, and if they go to college and parents decide to make inter-vivo transfers a_{CL} per child, they have to make an additional expenditure $n \cdot a_{CL}$. In addition to the monetary costs, having children requires κ units of time until children become independent after graduating high school.

Labor Earnings : Households choose hours worked h to earn income. The labor earnings of a household are given by $w_e \eta_{j,z,e,h}(T - l)$; it is a product of the market wage w_e, labor efficiency $\eta_{j,z,e,h}$, and hours worked $(T - l)$. Here, T denotes the disposable time that can be devoted to work or leisure where $T = 1 - \kappa$ if they have children before graduating high school and $T = 1$ otherwise. Labor efficiency depends on age j, idiosyncratic productivity shock component z, education level e, and time-invariant human capital h.

Financial Markets: Financial markets are incomplete due to the lack of state-contingent claims. Households can self-insure against risks, accruing interest payments at a rate of r. Following the literature, I assume that households can borrow only in the periods of living with children at rate $r^- = r + \iota$ where $\iota > 0$ (i.e., borrowers incur the overseeing costs ι) up to a borrowing limit $\underline{A}$. In addition, eligible students have access to student loans subsidized by the government, which entails the interest rate of $r^s = r + \iota_s$. This loan is income- and ability-tested, and eligible students can borrow up to a limit $\underline{A}_s$. The rest of the population cannot borrow.

Government: The government raises the revenue by levying three types of taxes: consumption, labor income, and capital income taxes, where the tax rates are represented as τ_c, τ_w, and τ_a. In addition, the government collects accidental bequests and devotes them to cover expenditures. They use this revenue to fund (1) the public pension benefit, which gives p unit of money to retired households each period, (2) subsidized loans for college students, (3) grants for eligible college students, where the per eligible student is given by $g(h, h_p)$, (4) lamp-sum transfers ψ that is introduced for replicating the progressivity of labor income tax schedule in a simple way following the literature, (5) cash benefits for households with children under 18 with per-child payment of b, and (6) the other expenditure G. The grants can depend on students' and parents' human capital. I regard the former as a proxy of the "ability" of the student and the latter as household income, so the function allows us to consider several types of grants, such as merit-base and need-base ones. I do not consider any grants in the benchmark to replicate the economy before grants are introduced in 2020 and set $g(h, h_p) = 0$ for each (h, h_p). The government budget constraint is given as follows:

$$\tau_c \cdot C + \tau_w \cdot (L_l + L_h) + \tau_a \cdot K + B = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + S + \psi + b \cdot \mu_{j \le 18} + G, \quad (1)$$

where C, B, μ_{old}, $\mu_{j\le 18}$, K_s, and S represent the total consumption, total accidental bequests, population mass of retired households, that of children under age 18, the total amount of borrowing by college students, and the total grant payments.

2.2 Household Problems

This section describes problems agents face throughout their lifecycle. The first is about college enrollment choice, choosing whether to go to college or enter the labor market after graduating high school.

Education Stage: After graduating high school, they draw the time-invariant human capital h from the distribution $g^h_{h_p}$ and psychic cost ϕ from the distribution $g^\phi_{e_p}$. They also receive inter-vivo transfers from their parents $a_{CL} \ge 0$. Some of them can access subsidized loans to fund expenditures that arise during the college education stage, which is income- and ability-tested. I regard the student's human capital h as a proxy of the ability and the parent's ability as the parent's income that is referred to when judging if the student is eligible to borrow the subsidized loans. Thus, the state variables for the students are comprised of asset a_{CL}, the human capital h, psychic costs ϕ, and their parent's human capital h_p. They compare the expected value for entering the labor market as a high school graduate, $\mathbb{E}V^w$, with the value for enrolling in college, V_{g1} net of the psychic cost ϕ, and they choose college enrollment if the latter is greater than the former; otherwise they enter the labor market. The decision problem is formulated as follows:

$$\begin{aligned} V_{g0}(a_{CL}, \phi, h, h_p) = \max_{e \in \{0,1\}} \Big\{ &(1-e) \cdot \mathbb{E}_{z_0}[V^w(a=0, j=19, z_0; e=0, h)], \\ & e \cdot V_{g1}(a_{CL}; h, h_p) - \phi \Big\}, \end{aligned} \quad (2)$$

where $e \in \{0,1\}$ indicates the education choice where $e = 1$ means college enrollment and $e = 0$ does entering the labor market as a high school graduate. V^w denotes the value function for workers, which I formulate in the next subsection. The initial draw of z, z_0, is uncertain and is according to the invariant distribution of z, $\bar{\pi}_z$, so the expectation operator is put next to the V^w. The value for college enrollment, V_{g1}, is defined as follows:

$$\begin{aligned} V_{g1}(a_{CL}; h, h_p) &= \max_{c,l,a'} \{u(c,l) + \beta V_{g2}(a'; h, h_p)\}, \\ V_{g2}(a; h, h_p) &= \max_{c,l,a'} \{u(c,l) + \beta \mathbb{E}_{z_0}[V^w(a^s(a'), j=23, z_0; e=1, h)]\}. \end{aligned} \quad (3)$$

The budget constraints differ according to eligibility to the student loans. The budget constraints for eligible students are give as follows:

$$(1+\tau_c)c + p_{CL} + a' - (1-\tau_w)w_l(1-\bar{t}-l) - \psi - g(h,h_p) = \begin{cases} (1+(1-\tau_a)r)a & \text{if } \; a \geq 0, \\ (1+r^s)a & \text{otherwise}, \end{cases} \tag{4}$$

$$a' \geq -\underline{A}_s,$$

The rest of the students cannot access to the student loans, which implies that their budget constraints are given as follows:

$$(1+\tau_c)c + p_{CL} + a' = (1+(1-\tau_a)r)a + (1-\tau_w)w_l(1-\bar{t}-l) + \psi,$$
$$a' \geq 0.$$

College students draw utility from consumption and leisure and must pay tuition fees p_{CL}. Normalizing their total disposable time as 1, students must spend a $\bar{t}$ fraction of time on study. Thus, they choose the time allocation between leisure and working over the disposable time $1-\bar{t}$. One unit of labor supply gives college students w_l unit of wages. They can fund the consumption and tuition fees through (1) transfers from their parents a_{CL}, (2) borrowing through student loans if eligible, and (3) labor earnings by themselves. Following the literature, I assume that fixed payments are made for 20 years (10 periods) following college graduation and transform college loans into regular bonds according to the following formula:

$$a^s(a') = a' \times \frac{r^s}{1-(1+r^s)^{-10}} \times \frac{1-(1+r^-)^{-10}}{r^-}.$$

Working Stage without Children: The remaining component in (2) and (3), V^w, represents the value function for working stage for age $j \in \{J_E, ..., J_F - 1, J_{IVT}+1, ..., J_R\}$, where J_E denotes the age of education period. That is, I define another value function for those with children and deciding inter-vivo transfers (i.e., those aged $j \in \{J_F, ..., J_{IVT}\}$). The state variables for this stage consists of asset a, age j, idiosyncratic component of labor productivity z, education level e, and human capital h. Except a period before the inter-vivo transfer choice, the uncertainty in this stage is about only next period's productivity, which is denoted by z' following a Markov chain $\pi_z(z', z)$. Households choose consumption, leisure, and savings given the stage variables. The value function is formulated as follows:

$$V^w(a,j,z;e,h) = \max_{c,l,a'}\{u(c,l) + \beta\mathbb{E}[V^w(a',j+1,z';e,h)]\}$$

s.t.

$$(1+\tau_c)c + a' = (1-\tau_w)w_e\eta_{j,z,e,h}(1-l) + \psi + (1+(1-\tau_a)r)a,$$
$$z' \sim \pi(z',z), \;\; a' \geq 0.$$

Recall that they are not allowed to borrow in this stage.

Fertility Choices and Working Stage with Children: When aged $j = J_F$, they choose whether and how many to have children. As in the previous life stages, they draw utility from consumption and leisure and decide on consumption, time allocation, and savings. In addition to that, they also have to choose how much to invest in their children, which draws utility for parents.[13] The utility depends on the per-child investment q and the number of children n. Then, the value function for households aged $j = J_F$, V^f, is formulated as follows:

$$V^f(a, z, e, h) = \max_{n \in \{0,1,..,N\}} \left\{ V^{wf}(a, j = J_F, z; e, h, n) \right\}$$

where

$$\begin{aligned} V^{wf}(a, j, z; e, h, n) = &\max_{c,l,q,a'} \{u(c, l) + b(n) \cdot v(q) \\ &+ \left\{ \begin{array}{cc} \beta \mathbb{E}_{z'|z}[V^{wf}(a', j+1, z'; e, h, n)] & \text{if } j \in \{J_F + 1, ..., J_{IVT} - 2\} \\ \beta \mathbb{E}_{z'|z,\phi_k|e,h_k|h}[V^{IVT}] & \text{if } j = J_{IVT} - 1 \end{array} \right\}, \end{aligned}$$

subject to

$$(1 + \tau_c)(c + nq) + a' = Y_{wf},$$

where

$$\begin{aligned} Y_{wf} = &(1 - \tau_w) w_e \eta_{j,z,e,h} (1 - l - \kappa) \\ &+ b + \psi + \left\{ \begin{array}{cc} (1 + (1 - \tau_a) r) a & \text{if} \quad a \geq 0, \\ (1 + r^-) a & \text{otherwise}, \end{array} \right. \\ a' \geq &-\underline{A}. \end{aligned}$$

Here, ϕ_k and h_k denote the psychic costs and human capital for their children. The total quality investment to children, nq, enters in the budget constraint, and utility from the quality and quantity of children, $b(n) \cdot v(q)$, enters in the objective function in the periods with children. V^{IVT} represents the value function for the stage of making inter-vivo transfers at $j = J_{IVT}$, which is described in the following subsection.

In addition to the next period's productivity z', they are uncertain about their children's human capital h_k and psychic costs ϕ_k at age $j = J_{IVT} - 1$ (i.e., one period before making the inter-vivo transfer decision). Note that those three components (z', h_k, and ϕ_k) are independent. Recall that the children's human capital correlates to the parent's,

[13] This study does not consider children's endogenous human capital accumulation through parental investments and assumes parents invest in children's quality just because it draws utility. Incorporating the endogenous human capital accumulation is left for future research.

and the distribution of their psychic costs depends on the parent's education level. Uncertainty about h_k and ϕ_k can be translated into uncertainty about expenditure toward children as inter-vivo transfers, given that those two components shape children's education decisions and the transfers will take place if they enroll in college.

Inter-vivo Transfers: At age $j = J_{IVT}$, which corresponds to a period when their children graduate high school and face the college enrollment choice, they decide on how much to transfer to their children. Some of the children's characteristics have been realized at this point, such as their psychic cost ϕ_k and human capital h_k. Further, they know children's policy function for education $e_k(a_{CL}, \phi_k, h_k, h)$. Then, they choose the amount of per-child transfer a_{CL} based on the state variables and policy function, which is formulated as follows:

$$V^{IVT}(a, j = J^{IVT}, z; \phi_k, h_k, e, h, n) = \max_{c,l,a',a_{CL}} \Big\{ V^w(a - \tilde{a}_{CL}, j = J_{IVT}, z; e, h) \\ + e_k \cdot b(n) \cdot \lambda_a \cdot V_{g0}(a_{CL}, \phi_k, h_k, h) \Big\},$$

where $\tilde{a}_{CL} = \frac{n \cdot a_{CL}}{1+(1-\tau_a)r}$ and $e_k = e_k(a_{CL}, \phi_k, h_k, h)$ represents children's policy function for education that returns 1 if they enroll in college and 0 otherwise. The budget constraint is given as follows:

$$(1 + \tau_c)c + a' + n a_{CL} = Y_{IVT}, \\ a' \geq 0,$$

where

$$Y_{IVT} = (1 - \tau_w) w_e \eta_{j,z,e,h} (1 - l) + \psi + (1 + (1 - \tau_a) r) a.$$

Retirement Stage: After retirement age J_R, they are forced to retire from the labor market. They spend all their time on leisure and make consumption-saving decisions. Two points differ from previous choice problems: they receive the pension benefit p from the government and face uncertainty about the next period's survival. The value function for the retirement stage is formulated as follows:

$$V^r(a, j; e) = \max_{c,a'} u(c, 1) + \beta \xi_{j,j+1} V^r(a', j + 1; e)$$

s.t.

$$(1 + \tau_c)c + a' = p + (1 + (1 - \tau_a) r) a + \psi, \\ a' \geq 0 \ (a' = 0 \text{ when } j = J).$$

2.3 Stationary Equilibrium

I solve the stationary equilibrium. In equilibrium, households make every choice to maximize their expected utility, the firm maximizes its profit, and the government budget constraint is balanced. The stationarity implies that the distribution over state variables is invariant. Importantly, the age distribution is determined endogenously according to households' fertility choices.

3 Calibration

To calibrate the model, I mainly rely on the Japanese Panel Survey of Consumers (JPSC), a panel survey of Japanese women and their household members. It started in 1993 with a representative sample of 1,500 women aged $24-34$ and contains information about, for example, their income, educational background, marriage, fertility, and expenditures to detailed categories, including those on children's education. I focus on the cohort born in 1959-69, the first cohort of this survey, especially to compute the completed fertility and intergenerational mobility of education. I keep only married households as in previous works (e.g., Daruich and Kozlowski, 2020) because the model focuses on choices made within married households such as fertility and educational investments. Unless otherwise mentioned, target moments for internally determined parameters described below are computed based on the JPSC's 1959-69 cohort data.

Demographics: I consider that households make fertility choices at the start of age $J_F = 30$, and correspondingly, inter-vivo transfers choices at the start of age $J_{IVT} = 48$. They retire at the end of age $J_R = 64$ and can live at most for $J = 104$ years. The survival probability $\zeta_{j,j+1}$ is set based on the Vital Statistics (2019).[14]

Prices: Annual college tuition fees p_{CL} are set to 1.05 million yen. κ is set to 0.044 so that they spend 13.3% of their working hours on childcare.[15]

Preferences: Instantaneous utility for students and adults are given as follows:

$$u(c,l) = \frac{(c^{\mu} l^{1-\mu})^{1-\gamma}}{1-\gamma}.$$

μ is internally determined as 0.28 so that the households spend one-third of the total disposable time on market work. The utility function must always be positive or negative

[14] See https://www.mhlw.go.jp/english/database/db-hw/outline/index.html.

[15] See, Kitao and Nakakuni (2023).

in models of altruism with endogenous fertility, and I set $\gamma = 0.5$ following the literature. Instantaneous utility from the quality of children is given as:

$$v(q) = \lambda_q \frac{q^{1-\gamma}}{1-\gamma},$$

where $\lambda_q = 0.62$ so that the annual educational expenditure per child amounts to 7% of average income at age 28. Similarly, the altruism discount parameter λ_a is set to 1.03 so that the annual average inter-vivo transfers for college students amounts to 0.24% of average income at age 28. Following Kim et al. (2023), I let discount function of the number of children $b(n)$ be non-parametric, and assume that $b(n) = b_n$ for each $n \in \{0, 1, 2, 3, 4\}$ with $b(0) = 0$. Those parameters are determined so that the model replicates the distribution of the number of children. The time for studying $\bar{t}$ is set to 0.8 so that the income share of labor earnings for college students in the model is close to 20%. I assume that $\beta = 0.98$ as in Zhou (2022).

Financial Markets: I set the borrowing limits $\underline{A} = 20$ million yen and $\underline{A}_s = 2.88$ million yen.[16] The borrowing wadges are set $\iota = 0.07$ and $\iota_s = 0.055$ so that the model approximates the share of working households with a negative net worth (54%) and the share of borrowing students (44%).

School Taste: Following Daruich and Kozlowski (2020), I assume that the psychic cost of college enrollment follows the following CDF:

$$G^{\phi}_{e_p} = \begin{cases} \phi^{\omega} & \text{if } e^p = 0 \\ 1 - (1-\phi)^{\omega} & \text{if } e^p = 1 \end{cases}$$

Similar to Daruich and Kozlowski (2020), I define a mobility index of education as $(2 - \text{trace}(P))/2$, where P is the intergenerational transition matrix of education where $(i, j)-$th entry indicates the probability that children acquire skill i given that their parents' skill is j. ω is set so that the model matches the mobility index of 0.31 as closely as possible. I also assume that ϕ is given as $\phi = \psi_{CL} \cdot \tilde{\phi}$, where $\tilde{\phi}$ is distributed on an interval $[0, 1]$ and ψ_{CL} governs the scale of psychic cost and thus college enrollment rate. ψ_{SL} is set so that the model approximates the college enrollment rate of 37.7%.

[16] The former is based on the Family Income and Expenditure Survey by the Ministry of Interna Affairs and Communications and the latter is based on the Student Life Survey by the Ministry of Education, Culture, Sports, Science and Technology.

Intergenerational Transmission of Human Capital: The intergenerational transmission of human capital is according to the following formula:

$$\log(h) = \rho_h \left[\log(h_p) - \log(\bar{h}_p)\right] + \varepsilon_h,$$
$$\varepsilon_h \sim N(0, \sigma_h),$$

where $\bar{h}_p$ represents the mean value of parents' human capital. I assume $\rho_h = 0.19$ based on Daruich and Kozlowski (2020) while internally determining σ_h as 0.71 so that the variance of log income at age 28 in this model is close to 0.27.

Income Process and Education Return: The efficiency labor of agents aged j, education e, human capital h, and productivity z, $\eta_{j,z,e,h}$, is given as follows:

$$\log \eta_{j,z,e,h} = f^e(h) + \gamma_{j,e} + z,$$
$$f^e(h) = h + e \cdot (\alpha_{CL} h^{\beta_{CL}}),$$
$$z' = \rho_z z + \zeta, \quad \zeta \sim N(0, \sigma_z).$$

To set $\gamma_{j,e}$, I estimate the second-order polynomial of hourly wages on age using JPSC. As reported in Table 1, the age gradient is larger for college graduates than the rest of the workers, but the degree is modest compared with the US case (e.g., Abbott et al., 2019). Due to data limitations, we cannot estimate the AR(1) process for z using any data source in Japan. Then, I assume $\rho_z = 0.95$ and $\sigma_z = 0.02$, reasonable values frequently used in the literature. The function $f^e(h)$ indicates that education return depends on human capital h: people with higher human capital can obtain greater returns through college education. α_{CL} and β_{CL} are determined so that the model replicates the ratio of log wage between college graduates and the rest of the population at age $j = 28$ and the log wage variance for college graduates at age $j = 28$.

	CL≤	CL
Age	0.041	0.048
Age^2 $\times 10,000$	-4.551	-5.364

Table 1: Wage age-profile. *Note*: CL indicates the college-graduate households where the husband or wife is a college graduate. CL≤ represents the rest of the population.

Production: I set the capital share $\alpha = 0.33$ and $\delta = 0.07$ following Kitao (2015). χ is set to 0.39 following Abbott et al. (2019). ω_h is internally determined to 0.45 so that the average wage ratio between college graduates and the rest in the model amounts to 1.36. Z is determined so that the wage rate for low-skilled labor is normalized to one.

Government: Tax rates are set to $\tau_c = 0.1$, $\tau_w = 0.35$, and $\tau_a = 0.35$ in the benchmark. The lump-sum transfer is set to $\psi = 0.01$ to match the ratio between the variance of log net income and that of log gross income (0.6). The pension benefit p is set so that the government provides ¥160, 000 per household per month. The cash transfer b is given as ¥10, 000 per child per month. The other expenditure G is set so that the government budget constraint is balanced in the benchmark and fixed throughout the counterfactual experiments. Table 2 and Table 3 summarise the parameters externally and internally determined.

Parameter	Value	Description
$\underline{A}_s$	20 million yen	Borrowing limit for students
$\underline{A}$	2.88 million yen	Borrowing limit
p_{CL}	1.05 million yen/year	Tuition fees
κ	0.044	Time costs
$\xi_{j,j+1}$	–	survival prob.
τ_c	0.10	Consumption tax
τ_a	0.35	Capital income tax
τ_w	0.35	Labor income tax
p	¥160, 000/month	Pension benefits
b	¥10, 000/month	Cash transfers
α	0.33	Capital share
δ	0.07	Depreciation rate
χ	0.39	Elasticity of substitution
ρ_z	0.95	Persistence
σ_z	0.02	Transitory
γ	0.5	Curvature
β	0.98	Discount factor
ρ_h	0.19	Transmission of h

Table 2: Parameters externally determined.

Parameter	Value	Moment	Data	Model
μ	0.23	Work hours	0.33	0.33
$\bar{t}$	0.8	Income share of labor earnings	0.20	0.17
ι_s	0.055	Share of students using loans	0.44	0.32
ι	0.07	Household share with negative net worth	0.54	0.58
ω_h	0.45	CL−CL≤ income ratio	1.36	1.40
ψ	0.01	Var(log disposable income)/Var(log gross income)	0.60	0.68
λ_q	0.62	Average transfer / Average income at age 28	0.07	0.07
λ_a	1.03	Average transfer / Average income at age 28	0.27	0.25
ω	1.51	Intergenerational mobility of education	0.31	0.37
σ_h	0.71	Variance of log(income) at age 28	0.27	0.32
ψ_{CL}	18.7	College enrollment rate	0.377	0.369
α_{CL}	0.1	Log wage ratio (CL−CL≤) at age 28	0.34	0.38
β_{CL}	0.1	Var log wage for CL at age 28	0.14	0.24
b_1	0.54	Share of one child	0.16	0.18
b_2	0.64	Share of two children	0.55	0.60
b_3	0.67	Share of three children	0.22	0.21
b_4	0.68	Share of four or more children	0.02	0.00
Z	1.99	Low skill wage	1.0	1.0

Table 3: Parameters internally determined.

3.1 Non-targeted Moments

The benchmark model performs well in some respects that are non-targeted. First, the benchmark model captures the fertility differentials across education levels observed in the data. The benchmark model implies that the fertility ratio between college graduates and the rest is given as 0.96, which is close to the data counterpart of 0.91. Second, the benchmark model replicates the intergenerational transition matrix of education well, which are not directly targeted. $(i, j)-$th entry of the matrix in Table 4 indicates the probability that children acquire skill j given that their parent's skill is i in the benchmark model, and values in parenthesis represent the data counterparts.

Parents/Children	<CL	CL
<CL	0.725 (0.798)	0.275 (0.202)
CL	0.470 (0.423)	0.530 (0.577)

Table 4: Intergenerational transition matrix of education. *Note*: $(i, j)-$th entry of the matrix indicates the probability that children acquire skill j given that their parent's skill is i in the benchmark model, and values in parenthesis represent the data counterparts.

In addition to the educational mobility, the benchmark model well approximates the composition of student income, consisting of inter-vivo transfers made by their parents (IVT), subsidized loans, and labor income through part-time jobs, as reported in Table 5.

	IVT	Loan	Labor
Model	0.72	0.17	0.11
Data	0.63	0.20	0.17

Table 5: Composition of Students' Revenue.

4 Numerical Analysis

This section investigates the education subsidies for college students in the model with fertility choices calibrated in the previous section. To this end, I consider five types of subsidies and conduct counterfactual experiments introducing each. First, as in Abbott et al. (2019), I introduce (1) the unconditional transfers of ¥120,000 per college student per year. Starting from these unconditional transfers, I further consider those (2) income-tested, and (3) ability-tested, where the short-run expenditure in each reform is the same. More specifically, I set a grant function $g(h, h_p)$ that may return the payments for students, although it always returns zero in the benchmark, depending on students' and parents' human capital as proxies of ability and household income, respectively. For the conditional transfer scheme (2) and (3), I arbitrarily set the limits for eligibility: the bottom 40% of students in terms of household income are eligible for the income-tested transfers, and the top 40% of them in terms of ability are eligible for the ability-tested ones. Again, the payment per eligible student is determined so that the short-run expenditure is the same as the introduction of unconditional transfers. Table 6 summarizes the share of eligible students and payment per eligible student. Following the literature, the introduction is funded by adjusting the labor income tax rate.

	Eligibility		
	(1) Uncond.	(2) Income	(3) Ability
Share	1.00	0.40	0.40
Payment	0.12	0.30	0.30

Table 6: Subsidy Scheme. *Note*: The row "Share" represents the share of eligible students over all college students, while "Payment" represents the annual payment per eligible student in each scheme (in a million yen).

4.1 Introducing Education Subsidies

Table 7 summarizes the main results. The introduction of grants behaves similarly in each form in terms of college enrollment rate and TFR: the college enrollment rate increases by at most 1.69 p.p., the TFR increases by $0.007 - 0.054$ points. The per-student IVT is crowded out at most 7.77%.

The increases in college graduates and TFR can be translated into increases in the quantity and quality of workers in the long run, leading to greater labor in efficiency units for each program. On the other hand, the aggregate capital level is lower in most cases. First, the greater subsidies reduce the saving incentives for households. In addition, the higher TFR implies that the population share of younger generations increases, who tend to have fewer assets: this composition change also contributes to lower aggregate savings.

To summarize, the education subsidies increase the aggregate labor while decreasing the aggregate capital, which makes it indeterminate whether the aggregate output increases or decreases. The results indicate that, with unconditional and ability-tested transfers, the positive impacts on labor supply surpass the negative ones on aggregate savings, resulting in greater output in equilibrium. In particular, the ability-tested program leads to the greatest output growth of 3.3%. This result is consistent with Abbott et al. (2019): the average education return is higher, and the distortion of saving and IVT decisions arising from introducing grants are smaller.

Although the introduction of grants requires the government to raise additional revenue, the labor income tax rate decreases in the long-run equilibrium because of the greater output and larger tax base. The ability-tested and unconditional transfer would lead to 0.9 and 0.3 p.p. declines in tax rate.

Contrary to the ability-tested and unconditional transfers, the income-tested ones would lower output because the adverse effects on aggregate savings are more significant than the positive effects on aggregate labor. An explanation behind these qualitative differences in output changes is that, with income-tested transfers, the human capital level of students newly enrolling, thanks to the subsidy, is relatively lower, given the intergenerational correlation of human capital. Then, the marginal effects of the subsidy on labor efficiency are lower with the income-test, and so are the effects on output.

	(1) Uncond.	(2) Income	(3) Ability
CL share (40.91)	41.14	40.97	42.60
TFR (2.094)	2.145	2.148	2.127
IVT (Δ%)	-2.46	-7.77	-0.21
Output (Δ%)	0.7	-0.5	3.3
Capital (Δ%)	-2.4	-3.5	3.5
Labor (Δ%)	1.5	0.7	2.1
Tax (35.0)	34.7	35.0	34.1

Table 7: Main results. *Note*: The values in parenthesis in the first column indicate the benchmark results. Rows where the first column has (Δ%) indicate the %-changes compared to the benchmark.

4.2 Inspecting the Mechanism

Given the results presented in the previous section, this section investigates their mechanisms. First, I conduct a decomposition analysis in Section 4.2.1 to discuss and understand the mechanism behind increases in the TFR and college enrollment rates upon introducing the subsidies. Next, in Section 4.2.2, I consider the roles of fertility responses in understanding the results by solving the exogenous fertility version of the model.

4.2.1 Decomposition: behavioral and composition effects

What causes the increases in the TFR and college enrollment rates? Those increases can be broken down into behavioral effects and composition effects, where the behavioral effects can be further broken down into the direct effects driven by the expansion of household budget constraint due to the introduction of the subsidy and the indirect effects driven by changes in market prices and tax rates (i.e., GE effects and taxation effects). The composition effects capture changes driven by changes of distribution over state variables such as education, age, and human capital. To isolate those effects, I conduct a decomposition as follows. I first solve an equilibrium of each of the five policies examined above (Say, "policy A"). Then, I solve an equilibrium by replacing one of the four objectives (Subsidies, prices, tax rate, and the state distribution) in the benchmark with that computed in the policy A. Table 8 summarises the decomposition results. Rows

"Grant", "Prices", "Tax", and "Distribution" report the results when only grant, prices, labor income tax rate, and distribution change, respectively.

College Enrollment: If the grant is introduced while other variables are fixed as in the benchmark (e.g, prices, tax rate, and distribution), the college enrollment rate increases closely to the long-run level for each policy. For example, the ability-tested transfers would lead to a 1.7 p.p. higher college enrollment rate in the long-run equilibrium, and if we do not consider any changes in prices, tax rate, and distribution but consider the grant increase, the college enrollment rate increases by 1.47 p.p.

Next, the GE effects do not affect the college enrollment rate in this case, given that the changes in the college enrollment rate and the TFR are subtle, and so are price changes. Taxation effects also do not affect college enrollment choices, even in cases with the ability-tested transfers where the equilibrium tax rate is 0.9% lower than in the benchmark.

The composition effects, obtained by changing only the state distribution, imply a higher college enrollment rate for each policy. The most relevant factor is the change in the skill distribution of parents (i.e., the share of college graduates for parents). The increase in the share of college graduates means that the share of those more likely to have children enrolling in college because of the intergenerational persistence of education.

Fertility: If the grant is introduced while other variables are fixed as in the benchmark, the TFR significantly increases for each policy. For example, the unconditional transfers for college students lead to a 2.5% higher TFR in the long-run equilibrium, and if we do not consider any changes in prices, tax rate, and distribution but consider the grant increase, the college enrollment rate increases exactly 2.5%.

It is worth digging deeper into the impacts of education subsidies on fertility behavior. In this model, having children takes monetary costs, and some of them are subject to uncertainty: the IVT takes place only if children enroll in college, and whether they do is uncertain until they graduate high school, where children's characteristics such as taste of education and human capital realize. In this sense, having children entails some expenditure risks. Under the incomplete market framework, these expenditure risks would lower fertility rates.[17] In that case, education subsidies would provide insurance against the risks, and for those who expect that their children are most likely to enroll in college, they reduce the fixed costs of having children.

The GE effects do not affect the TFR in this case, given that the changes in the college

[17] Santos and Weiss (2016) and Sommer (2016) show the recent rise in income volatility in the US would lead to lower fertility given that having children involves a consumption commitment for the parents.

enrollment rate and the TFR are subtle, and so are price changes. Taxation effects also do not affect college enrollment choices, as in the cases of college enrollment rate.

	Grant	Prices	Tax	Composition	All
CL share					
(1) Uncond.	41.15	40.94	40.94	40.97	41.14
(2) Income	40.94	40.94	40.94	41.00	40.97
(3) Ability	42.37	40.94	40.94	41.15	42.60
TFR					
(1) Uncond.	2.147	2.094	2.094	2.091	2.145
(2) Income	2.131	2.094	2.094	2.108	2.148
(3) Ability	2.200	2.094	2.093	2.041	2.127

Table 8: Decomposition results. *Note*: Rows "Grant", "Prices", "Tax", and "Distribution" report the results when only grant, prices, labor income tax rate, and distribution change, respectively.

4.2.2 Roles of Fertility Responses

As discussed in Section 1, this study differs from the previous studies in the literature on education subsidies for college students in that modeling fertility choices and allowing households' fertility choices to respond against policy changes in education subsidies. Thus, it is worth examining the roles of fertility responses in accounting for the impacts of the subsidies on macroeconomic variables such as output. To this end, I next solve the exogenous fertility version of the model, a standard setup used in the literature on financial aid for college students. More specifically, I conduct the same procedure in the previous section, except that the policy functions for fertility are fixed as in the benchmark.

Table 9 reports the main results under exogenous fertility. Values in parenthesis in each cell represent the result under endogenous fertility reported in Table 7. First, the introduction of grants results in a lower college enrollment rate than under endogenous fertility for most cases. For example, introducing the income-tested grants results in a college enrollment rate 40.97 under endogenous fertility, whereas it is 40.91 under exogenous fertility. Under exogenous fertility, the grants make some children enroll in college who otherwise cannot. Under endogenous fertility, in addition to that effect, the college enrollment rate can further increase through the fertility increase for those likely to have children willing to go to college.

From the second row onward, the changes in aggregate variables are reported. The increases in aggregate output, capital, and efficiency labor under exogenous fertility are modest compared with those under endogenous fertility in each case. Importantly, changes in output are attenuated toward the origin. That is, the output growth with unconditional and ability-tested transfers and the output loss with income-tested ones are both modest under exogenous fertility.

The attenuation suggests that fertility responses upon the policy intervention amplify the heterogeneous implications for aggregate output. First, note that, in principle, households whose fertility behavior is affected are those targeted by the policy (e.g., those with lower human capital in the case of income-tested transfers). Next, recall the contrast between the ability-tested program and income-tested one under endogenous fertility in terms of their effects on aggregate output: The ability-tested program would lead to the greatest output growth by strengthening educational sorting by ability, while income-tested one would lead to lower gain in terms of output because students newly enrolling possess relatively poor human capital and their education return is lower. If we consider the fertility responses, the targeted households may increase fertility, and their children are likely to have similar characteristics to their parents via intergenerational linkage of human capital and school tastes. As a result, the share of those households increases through this fertility channel, strengthening the heterogeneous effects on aggregate output.

Finally, the equilibrium tax rate is higher under exogenous fertility than endogenous fertility. Although the grant introduction decreases the equilibrium tax rate by $1.2 - 2.2$ p.p. under endogenous fertility, the degree of the tax rate decline is more modest, or the tax rate is higher than in the benchmark under exogenous fertility. This is mainly because the aggregate labor in efficiency units is greater under endogenous fertility due to a higher college enrollment rate and a larger share of the working-age population.

	(1) Uncond.	(2) Income	(3) Ability
CL share (40.91)	41.13 (41.14)	40.91 (40.97)	42.68 (42.60)
Output (Δ%)	0.4 (0.7)	-0.2 (-0.5)	1.5 (3.3)
Capital (Δ%)	0.9 (-2.4)	0.1 (-3.5)	3.0 (3.5)
Labor (Δ%)	0.1 (1.5)	-0.2 (0.7)	0.6 (2.1)
Tax (35.0)	35.0 (34.7)	35.0 (35.0)	35.0 (34.1)

Table 9: Results under exogenous fertility. *Note*: Values in parenthesis in each cell represent the result under endogenous fertility reported in Table 7. The values in parenthesis in the first column indicate the benchmark results. Rows where the first column has (Δ%) indicate the %-changes compared to the benchmark.

4.3 Expanding the Education Subsidies

Finally, I consider the effects of expanding those subsidies examined so far. More specifically, I keep considering the same thresholds for each targeted program but multiply their payments by $X > 1$. Table 10 summarizes the share of eligible students and payment per eligible student.

	(1) Uncond.	(2) Income	(3) Ability
Share	1.00	0.40	0.40
Payment	$X\times0.12$	$X\times0.30$	$X\times0.30$

Table 10: Subsidy scheme with expansion. *Note*: The row "Share" represents the share of eligible students over all college students, while "Payment" represents the annual payment per eligible student in each scheme (million yen).

Table 11 summarizes the results and there are several takeaways there. First, expanding payments for unconditional and income-tested transfers would lead to higher college enrollment and fertility rates, leading to a higher output and lower tax rate even for income-tested transfers.

Next, the marginal effects of increasing the ability-tested transfer are limited: the college enrollment rates are almost the same as in the case of $X = 1$ (i.e., when introducing the subsidy), and the TFR increases only $0.001 - 0.005$. This result should be straightforward to understand because high-ability students are more likely to have high-ability parents implied by the intergenerational persistence of human capital, and those high-ability parents earn higher income and will have sufficient amount of assets. Then, the income and insurance effects of education subsidy discussed in Section 4.2.1 are weaker for those richer parents, and expanding education subsidies would have less impacts on fertility choices of those households. While this limited increases in factors contributing to a greater labor supply, the larger transfers distort households decisions. As a result, the output growth becomes lower as the payment increases, and correspondingly, the tax rate decline becomes modest. Still, the output growth and the decline of tax rate are more significant than any scale of income-tested transfers.

	$X = 1$	$X = 2$	$X = 3$
(1) Unconditional			
CL share (40.91)	41.14	42.61	42.63
TFR (2.094)	2.145	2.165	2.188
Output (Δ%)	0.7	4.3	3.9
Tax rate (35.0)	34.7	34.1	34.1
(2) Income-tested			
CL share (40.91)	40.97	41.11	41.12
TFR (2.094)	2.148	2.149	2.161
Output (Δ%)	-0.5	2.1	1.9
Tax rate (35.0)	35.0	34.3	34.4
(3) Ability-tested			
CL share (40.91)	42.60	42.60	42.60
TFR (2.094)	2.127	2.132	2.133
Output (Δ%)	3.3	3.2	3.1
Tax rate (35.0)	34.1	34.2	34.3

Table 11: Results of expanding payments. *Note*: The values in parenthesis in the first column indicate the benchmark results. Rows where the first column has (Δ%) indicate the %-changes compared to the benchmark. $X = 1$ corresponds to the results discussed in Section 4.1. $X = 2$ and $X = 3$ indicate the results obtained by multiplying the payments per eligible student by 2 and 3, respectively.

We may draw some policy implications from the results. First, expanding programs covering lower and broader income classes, such as unconditional or income-tested ones, would be more effective in achieving the pro-natalist goal of increasing fertility by compensating the costs of having children or providing insurance against expenditure risks arising from having children. On the contrary, the marginal effects of expanding payments for the ability-tested program would decrease faster than other programs because fewer margins would be affected by the expansion. However, we must keep in mind that the ability-tested program would still lead to a higher output and lower tax rate than the income-tested program despite the smaller impacts on the TFR.

5 Concluding Remarks

This study constructs a GE-OLG model embedding choices on college enrollment, inter-vivo transfers, and fertility, to investigate the macroeconomic implications of education subsidies for college students. The model is calibrated to the Japanese economy using panel data and the benchmark model replicates the intergenerational mobility of education, financial costs of having children including inter-vivo transfers made if children go to college, and the distribution of birth numbers.

I show that introducing each program increases the college enrollment rate by at most 1.7 percent points (p.p.) and the total fertility rate (TFR) by $1.6-2.6\%$ in the long-run equilibrium, which leads to a greater labor supply in efficiency units due to higher share of skilled workers and working-age population. The ability-tested program would achieve the greatest output and fiscal gains, while introducing income-tested program would lower output. These results are attributed to the human capital level of students newly enrolling, and the fertility responses amplify the heterogeneous effects.

Although ability-tested programs would bring the greatest gains in output and tax base to some extent, the marginal gains would decrease faster because many of their parents are rich enough and the income effects and insurance effects of education subsidies on their fertility behavior are limited. On the contrary, expanding programs covering lower and broader income classes, such as unconditional or income-tested ones, would be more effective in achieving the pro-natalist goal of increasing fertility: it reduces the financial costs of children or providing insurance against the expenditure risks related to children, and the effects are more significant for lower income households.

I would like to conclude by listing several points that have not been considered in this preliminary version and to be examined further for revision. First, while the current version focuses on the positive aspects of the education subsidies, examining their welfare effects should be relevant especially to draw some policy implications. In doing so, I build on the work by Golosov et al. (2007) and adopt an efficiency criteria that can be applied to models with endogenous fertility. Second, using the model to answer questions that have actively being discussed would also be a promising direction. In Japan, for example, the government discusses the expansion of the income threshold for income-tested grants for college students conditional on the number of children as a pro-natal policy. The model can be used to examine the effects of this potential reform because the model captures the discreteness of fertility choices. Finally, solving the transition dynamics and investigating the time horizon of the policy effects will provide another picture of the effects of education subsidies with fertility choices.

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