Financial Costs of Children, Education Subsidies, and Parental Choices in Equilibrium

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Low Fertility and Pro-natal Policies

Low fertility has driven the recent rise of pro-natal policies in many countries.

- Japan is one of the lowest-fertility countries.
  - Total fertility rate: 1.26, far below the replacement level ($\simeq 2$).
- Suggestive observations (detailed in next section):
  - Financial costs, esp. those for college education, are a key obstacle in JPN.
- Grant-type scholarships for college students have garnered attention and were recently introduced as a pro-natal measure.
- It is unconventional as a pro-natal one, little is known about its consequences.

  Previous: cash benefits (Kim et al., 2023), in-kind (Bick, 2016), both (Zhou, 2022), parental leave (Erosa et al., 2010).

Research Questions

1. Does it increase fertility?
    - In the short run, the answer is probably "yes."
        - Suggestive evidence from empirical studies:
          reducing the financial costs of children increases fertility.
          e.g.) Milligan (2005, REStat), Cohen et al. (2013, REStat), Malkova (2018, REStat), etc.
    - In the long run, it may not be the case.
        - Composition effects: Educated parents tend to have fewer children.
        - Taxation effects: That policy requires an additional tax burden.

Research Questions

1. Does it increase fertility?

2. What are the impacts on the other macro variables? (e.g., education, output)
   - Can be "two birds with one stone" (fertility $\uparrow$ + human capital $\uparrow$)

Research Questions

1. Does it increase fertility?
2. What are the impacts on the other macro variables? (e.g., education, output)
3. Do the macro implications depend on its target? How different are they?
   - Income-tested/Ability-tested/Unconditional
   - Differences from typical pro-natal transfers (e.g., baby bonuses, child benefits)

Research Questions

1. Does it increase fertility?
2. What are the impacts on the other macro variables? (e.g., education, output)
3. Do the macro implications depend on its target? How different are they?

$\Rightarrow$ Need for a structural (equilibrium) model with fertility and education choices.

This Paper

- Constructs an incomplete market GE-OLG model:
  - + Fertility + College enrollment + Inter-vivo transfers (IVT) choices.
- Calibrates the model to the Japanese economy using household panel data.
- Validates if the model implies reasonable fertility behavior.
  - The benefit elasticity of fertility (comparison with empirical estimates).
- Examines the macro effects of the grants for college students.

Main Results

1. Does it increase fertility (esp. in the long run)?
    - Yes. The introduced grants increase the TFR by 3% via **insurance effects**.
    - But its eligibility expansion does not increase the TFR further as it **increases the expected costs of children** for some households due to the higher probability of children attending college.

2. What are the impacts on the other macro variables?
    - College enrollment rate: 4 p.p. $\uparrow$. Output: 0.7% $\uparrow$.
    - Fertility margins **amplify** the policy impacts on these variables in the long run.

3. How different are the macro effects, depending on its target?
    - Income-tested ones lead to higher fertility (insurance effects are key).
    - Targeting college students can lead to higher fertility and greater output.

Financial costs are a key obstacle to fertility

Non-negligible gap between the "ideal" and "planned" numbers of children. Why?
⇒ "Financial" reason: "**Child-raising and education are too expensive**."

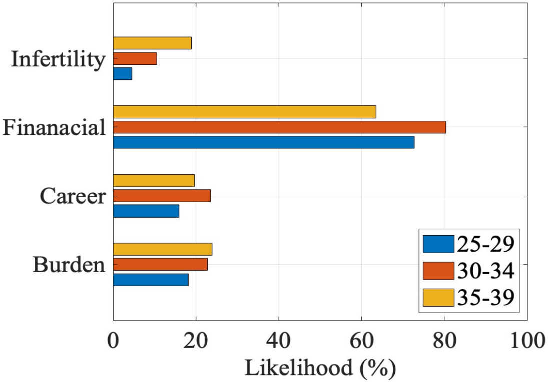


Figure: Reasons for the gap between the ideal and planned numbers of children by wife's age (NFS, 2015).

Japan's fewer subsidization in tertiary education

- Japan subsidizes a sizable fraction of schooling costs up to secondary education,
- but not for tertiary education, the 2nd lowest in OECD.

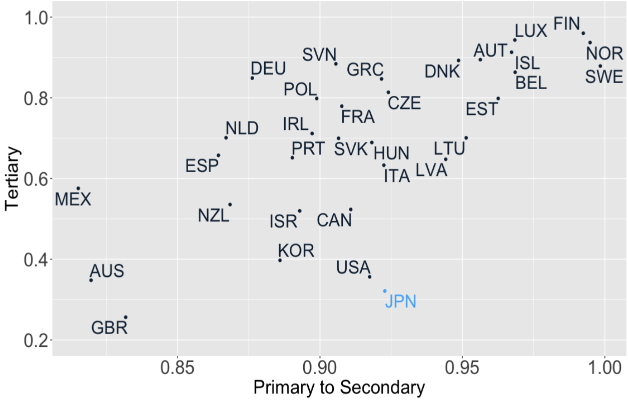


Figure: Subsidization rate for each education category (OECD, 2018).

A significant cost gap (HS grad. vs. CL grad.)

Sending a child to college makes a significant jump in total education costs.

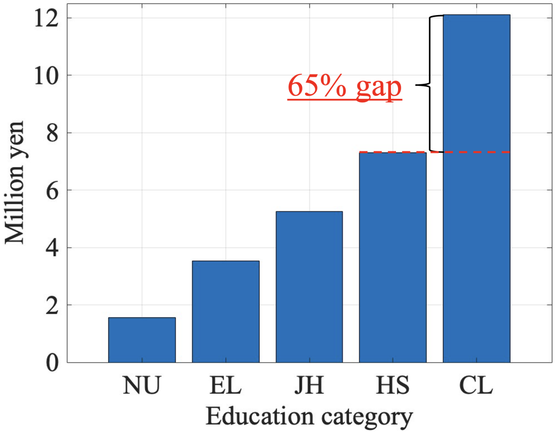


Figure: Cumulative education-expenditures. Sources: the SCLE (2021) and the SLS (2018) by the MEXT.

Desired education for children

More than 75% of parents desire a college education for their children.
– College education costs are relevant for most of the parents.

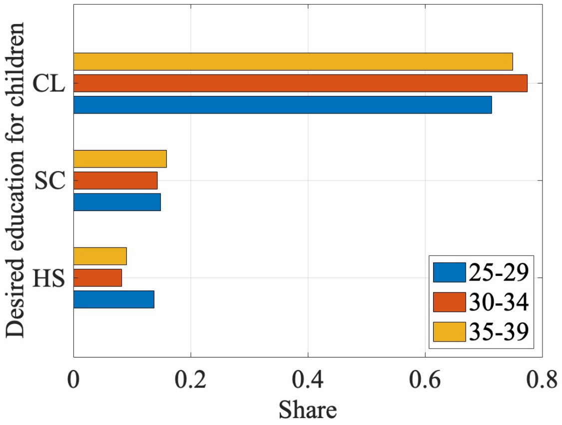


Figure: Wives' intention for children's education attainment (NFS, 2015).

Background: Summary

- Suggestive observations:
  - Financial costs, esp. those for college education, are critical.
- Grant-type scholarship for college students has garnered significant attention.
  - Grants were introduced in 2020 with some income tests.
  - Its eligibility expansion has been actively discussed recently.

Model

Model with fertility choices otherwise standard in (Macro) education literature.
Krueger and Ludwig (2016, JME), Abbott et al. (2019, JPE), Matsuda and Mazur (2022, JME), etc.

- Incomplete market GE-OLG framework.
- Production: allows for the imperfect substitutability b/w skilled and unskilled.
- Government subsidizes CL enrollment according to a grant function $g(h, I)$.
- Standard lifecycle $\underbrace{+\text{ Education } + \text{ IVT}}_{\text{Literature standard}}$ + Fertility

Production Technology

A representative firm operates with the Cobb-Douglus production technology:

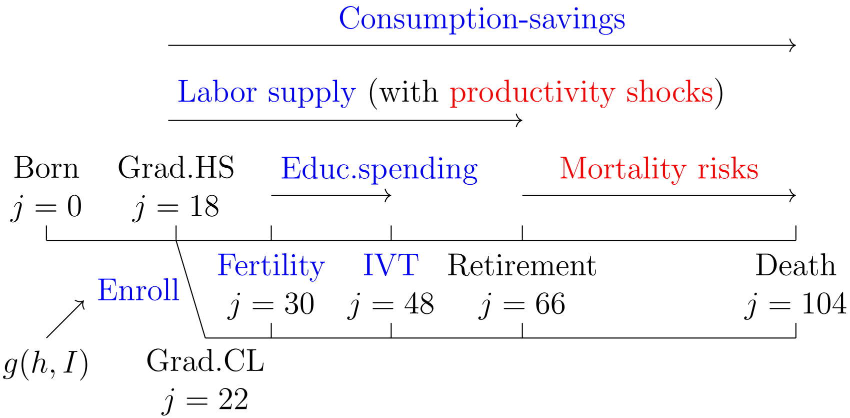
$$Y = ZK^{\alpha}L^{1-\alpha},$$

where

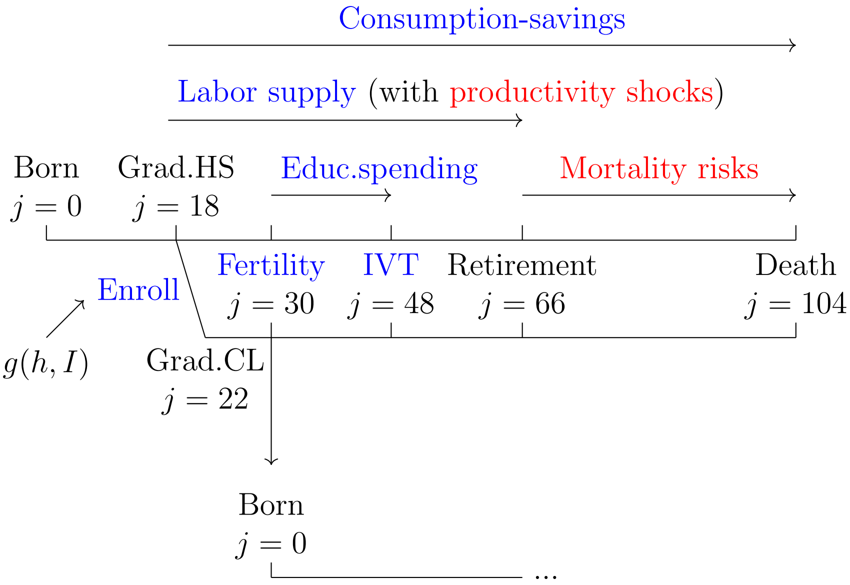
$$L = \left[\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}\right]^{1/\chi}.$$

- L_e: total efficiency labor with skill $e \in \{HS, CL\}$.

Lifecycle


Consumption-savings
Labor supply (with productivity shocks)
Born
$j = 0$
Grad.HS
$j = 18$
Educ.spending
Mortality risks
Enroll
Fertility
$j = 30$
IVT
$j = 48$
Retirement
$j = 66$
Death
$j = 104$
$g(h, I)$
Grad.CL
$j = 22$

Lifecycle



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Retirement
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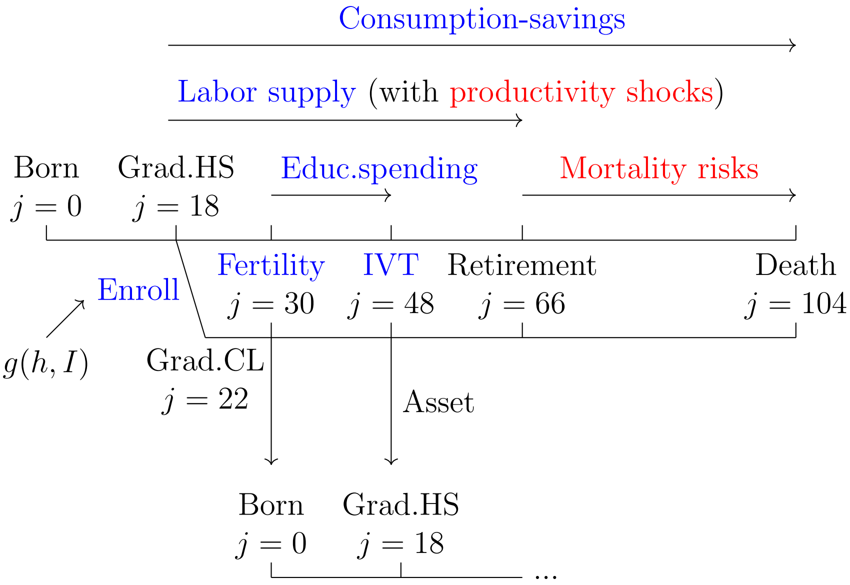
Born
$j = 0$
...

Lifecycle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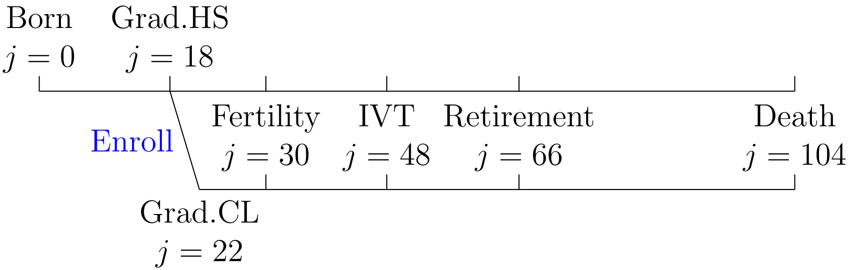
Mortality risks
Enroll
$g(h, I)$
Fertility
$j = 30$
IVT
$j = 48$
Retirement
$j = 66$
Death
$j = 104$
Grad.CL
$j = 22$
Asset
Born
$j = 0$
Grad.HS
$j = 18$
...

Education choices



Three factors (IG linkages) influence their college enrollment decisions:

1. IVTs made by their parents (a_{CL}).
2. Psychic costs of education ($\phi \sim g^{\phi}_{h,e_p}$).
3. Their human capital ($h \sim g^{h}_{h_p}$).

Budget constraints: college students

Expenditures:

- Tuition fees p_{CL} (exogenous)
- Living expenses c (endogenous/choice)

Revenue:

- Transfers from their parents a_{CL} (parent's choice).
- Labor earnings by themselves (endogenous/choice).
- Loans/Grants provided by the government, depending on h and I.

Education choices

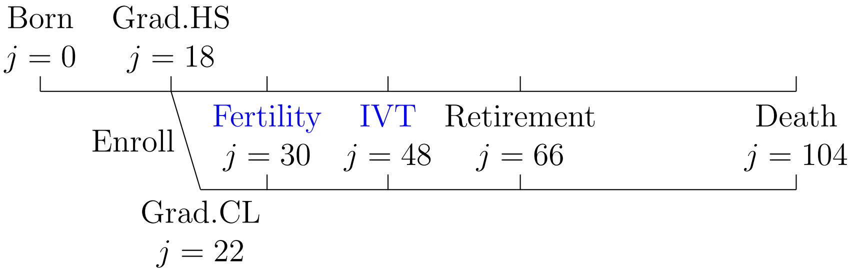
$$V_{g0}(a_{IVT}, \phi, h, I) = \max_{e \in \{0,1\}} \Big\{ (1-e) \cdot \mathbb{E}_{z_0}[\underbrace{V^w(a_{IVT}, j=18, z_0; e=0, h)}_{\text{Value for high school graduates}}]$$

$$+ e \cdot [\underbrace{V_{g1}(a_{IVT}; h, I) - \phi}_{\text{Net value for college graduates}}] \Big\}, \tag{1}$$

where $e = 1$ represents enrolling in college.

▸ Value functions

Fertility and IVT choices



Inter-vivo transfers

$$V^{IVT}(a, z; \phi_k, h_k, e, h, n) = \max_{c,l,a',a_{IVT}} \Big\{ V^w(a - \tilde{a}_{IVT}, j = J_{IVT}, z; e, h)$$

$$+ b(n) \cdot \lambda_a \cdot V_{g0}(a_{IVT}, \phi_k, h_k, I) \Big\}$$

$$\text{s.t.} \quad (1 + \tau_c)c + a' + n \cdot a_{IVT} = Y_{IVT},$$

$$a' \geq -\underline{A},$$

where $\tilde{a}_{CL} = \frac{n \cdot a_{CL}}{1+(1-\tau_a)r}$, $I = I(J_{IVT}, z, e, h)$, and Y_{IVT} is the disposable income.

Budget constraint details

Fertility choices and working stage with children I

The value function at age $j = J_f$:

$$V^f(a, z, e, h) = \max_{n \in \{0,1,..,N\}} \left\{ V^{wf}(a, j = J_F, z; e, h, n) \right\}$$

where, for $j = J_F, ..., J_{IVT} - 1$,

$$V^{wf}(a, j, z; e, h, n) = \max_{c,l,q,a'} \{ u(c/\Lambda(n), l) + b(n) \cdot v(q)$$

$$+ \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^{wf}(a', j+1, z'; e, h, n)] & \text{if } j < J_{IVT} - 1 \\ \beta \mathbb{E}_{z', \phi_k, h_k}[V^{IVT}(a', z'; \phi_k, h_k, e, h, n)] & \text{if } j = J_{IVT} - 1 \end{array} \right\}$$

s.t.

$$(1 + \tau_c)(c + n \cdot q) + a' = Y_{wf},$$

$$a' \geq -\underline{A},$$

Fertility choices and working stage with children II

where

$$Y_{wf} \equiv (1-\tau_w) w_e \eta_{j,z,e,h}(1-l-\kappa\cdot n) \\ + n\cdot B + \psi + \begin{cases} (1+(1-\tau_a)r)a & \text{if} \quad a \geq 0, \\ (1+r^-)a & \text{otherwise.} \end{cases}$$

1. Introduction
2. Background
3. Model
4. Calibration
5. Numerical Analysis
6. Literature
7. Conclusion
8. Appendix

Data

The Japanese Panel Survey of Consumers (JPSC)

- Panel survey of women and their household members.
- Starts in 1993 with 1,500 women aged 24-34.

Sample selection:

- Birth cohort: 1959-69
- Married (1993-2020)

Grants for college students

- As before 2020, I set $g(h, I) = 0$ for any (h, I) in the benchmark,
- and examine the effects of the introduction using the calibrated model.

Preferences

Instantaneous utility for students and adults:

$$u(c,l) = \frac{(c^{\mu} l^{1-\mu})^{1-\gamma}}{1-\gamma}$$

Instantaneous utility from education spending on children:

$$v(q) = \lambda_q \frac{q^{1-\gamma_q}}{1-\gamma_q}$$

The discount function $b(n)$ takes a non-parametric form (i.e., $b(n) = b_n$).

School taste

- Psychic costs ϕ are given as $\phi = \psi_{CL} \cdot \exp(-h) \cdot \tilde{\phi}$.
  - ψ_{CL} governs the college enrollment rate at the initial steady state.
  - $\tilde{\phi}$ is distributed on $[0, 1]$.
- As in Daruich and Kozlowski (2020), the CDF for $\tilde{\phi}$ is given as

$$G_{e^p}^{\tilde{\phi}} = \begin{cases} \tilde{\phi}^{\omega} & \text{if } e^p = 0 \\ 1 - (1 - \tilde{\phi})^{\omega} & \text{if } e^p = 1 \end{cases}$$

- Target for ψ_{CL}: the college enrollment rate (37.7%).
- Target for ω: intergenerational transition matrix of education.

Internally determined

Parameter	Value	Moment	Data	Model
μ	0.23	Work hours	0.33	0.30
$\bar{t}$	0.8	Income share of labor earnings	0.20	0.17
ι_s	0.055	Share of students using loans	0.44	0.34
ι	0.054	Household share with negative net worth	0.54	0.45
ω_h	0.52	CL−HS wage ratio	1.36	1.48
ψ	0.01	Var(log disposable income)/Var(log gross income)	0.60	0.68
λ_q	0.62	Average transfer / Average income at age 28	0.07	0.07
λ_a	1.03	Average transfer / Average income at age 28	0.27	0.27
ω	1.71	Intergenerational mobility of education	See Table 1	
σ_h	0.65	Variance of log(income) at age 28	0.27	0.24
ψ_{CL}	20.8	College enrollment rate	0.377	0.376
α_{CL}	0.1	Log wage ratio (CL−HS) at age 28	0.34	0.38
β_{CL}	0.1	Var log wage for CL at age 28	0.14	0.24
b_1	0.49	Share of one child	0.16	0.15
b_2	0.53	Share of two children	0.55	0.61
b_3	0.55	Share of three children	0.22	0.24
b_4	0.56	Share of four or more children	0.02	0.00
Z	1.99	Low skill wage	1.0	1.0

- Avg. expenditure before HS grad. ($\simeq$ 7 M yen) and after CL enrollment ($\simeq$ 5 M yen).

Education Mobility

Education status is persistent between parents and children.

Parents/Children	HS	CL
HS	0.725 (0.798)	0.275 (0.202)
CL	0.412 (0.423)	0.588 (0.577)

Table: Intergenerational transition matrix of education in the benchmark. *Note*: $(i, j)-$th entry of the matrix indicates the probability that children acquire skill j given that their parent's skill is i. Values in parentheses represent the data counterparts (JPSC).

Validation and Non-targeted Moments

- The benefit elasticity of fertility.
  - 1% increase in cash benefits increases fertility rates by $0.1 - 0.2\%$.
    e.g.) Milligan (2005), Cohen et al. (2013), etc.
  - The benchmark model implies the elasticity of 0.13. Detail
- Fertility differential across education (next page).
- Composition of students' revenue.
  (IVT:Labor:Loan $\simeq$ 0.6:0.2:0.2)

Fertility differential across education

More (less) educated parents have fewer (more) children.

	Data	Model
HS	2.12	2.25
CL	1.92	1.79

Table: Fertility differential across education in the benchmark.

Road Map

- Examining the effects of the existing income-tested grants.
- Setting the higher income threshold for eligibility.
- Different targets:
  - Income-tested vs. Ability-tested vs. Unconditional
  - Targeting college students vs. Typical pro-natal transfers
- Inspecting the mechanism:
  - Behavioral and distributional effects
  - Roles of endogenous fertility

Introducing Subsidies: What I Do

Consider a grant scheme $g(h, I)$ and solve the equilibrium:

$$g(h, I) = \begin{cases} g & \text{if } I \leq \bar{I} \\ 0 & \text{otherwise} \end{cases}$$

- Threshold $\bar{I} \simeq$ the 15 percentile of income dist.
- Amount $g \simeq 2/3$ of the students' average expenses in the benchmark.
- Government budget balanced by adjusting the labor income tax rate.

Results: Education and Fertility

Education:

- College enrollment rate: 4 p.p. $\uparrow$
  - Skill premium (w_{CL}/w_{HS}): 0.02 points $\downarrow$
- Educational mobility (HS$\rightarrow$CL): 2.5 p.p. $\uparrow$

Fertility:

- TFR: 3% $\uparrow$
- Largely driven by fertility increases among college graduates.

Misc.: Output: 0.7% $\uparrow$ (via greater share of working-age and skilled pop.)

Inspecting the mechanism

Changes in the TFR and college enrollment can be driven by changes in:

1. Grant function $g(h, I)$
   "**Direct effects**" (short-run effects)
2. Prices (w_{CL}, w_{HS}, r)
   "**GE (Price) effects**"
3. Tax rate τ_l
   "**Taxation effects**"
4. Distributions (e.g., college graduates share)
   "**Distributional effects**"

(in principle, those effects do not add up to the overall effects)

Decomposition: Fertility

	Direct	GE	Tax	Dist.	All
TFR	+2.3%	+0.9%	0.0	−0.4%	+3.0%
HS	+1.0%	0.0	0.0	0.0	+0.4%
CL	+4.5%	+2.5%	0.0	0.0	+10.7%

Table: Decomposing the effects on fertility.

- GE: CL share $\uparrow \Rightarrow w_{CL} \downarrow \Rightarrow$ Opportunity costs $\downarrow \Rightarrow$ Fertility $\uparrow$
- Dist.: CL share $\uparrow \Rightarrow$ Fertility $\downarrow$ as skilled parents tend to have fewer children.
- What is a direct effect? Is it equivalent to an income effect? Not necessarily.

Education

Decomposition: Fertility

- Note: HHs do not know whether eligible when making fertility choices.
    - A key source of uncertainty: income shocks (z).
- Some skilled parents, whose children are ex-post ineligible, increase fertility.

  $\Rightarrow$ The grants serve as partial insurance against the risks of having a child.

Grants As Insurance For Parents

- **Children as "consumption commitments"**: Santos and Weiss (2016), Sommer (2016).
  – Irreversible and force parents to spend required amounts of resources.
  ⇒ Income volatility makes parents tend to have fewer children.

- Children of skilled parents are more likely to (be willing to) attend college.
  – Expected costs of children are higher for skilled parents.
  ⇒ They benefit more from the insurance, and their fertility responds strongly.

Expansion: What I Do

Raise the threshold for the grant eligibility to cover students in middle-income HHs.

- **Threshold** $\simeq$ the $x(> 15)$ percentile of income dist.

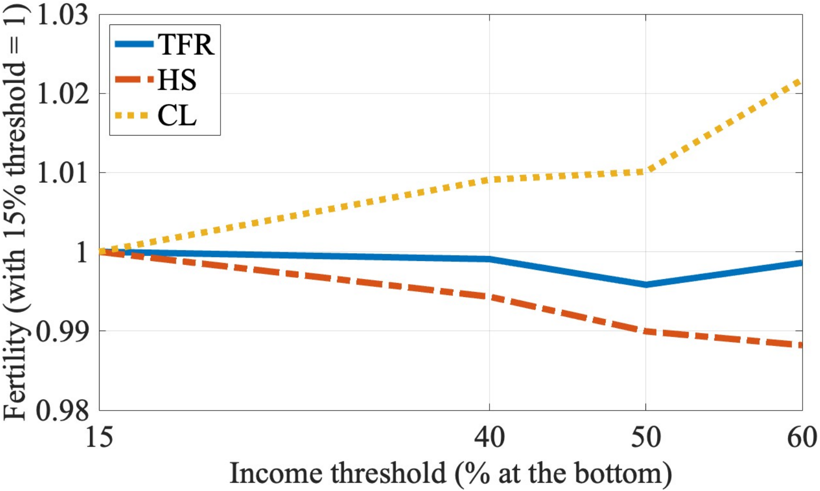
Expansion: Results

	Threshold			
	15%	40%	50%	60%
CL share (Δ p.p.)	+3.9	+4.7	+5.6	+6.2
Output (Δ%)	+0.70	+0.15	+1.07	+1.53
Tax(Δ p.p.)	+0.04	+0.17	+0.23	+0.30

Table: Main results of higher income thresholds.

- College enrollment rate, output, and tax rate all increase with the expansion.
- But this is not the case for the average fertility (next page).

Expansion: Fertility



Fertility Decomposition: Case of Threshold = 60%

High-school-graduate parents: Direct and GE effects explain the fertility decline.

- Higher education mobility $\Rightarrow$ Higher expected costs of children $\Rightarrow$ Fertility $\downarrow$
- CL share $\uparrow \Rightarrow w_{HS} \uparrow \Rightarrow$ Opportunity costs $\uparrow \Rightarrow$ Fertility $\downarrow$

Table

Roles of endogenous fertility

So far, we have focused on the policy effects on fertility.

- What are the implications of the fertility responses for other macro variables?

Procedure:

- Solve the equilibrium with each program under exogenous fertility.
- Policy functions for fertility are fixed as in the benchmark.

Exogenous fertility

Considering fertility responses implies:

- Higher college enrollment rates (via hetero. effects on fertility + IG linkages)
- Greater output (via greater share of working-age and skilled pop.)

  $\Rightarrow$ Fertility margins amplify the effects in the long run.
- Lower tax rate (via greater tax base)

Moments/Threshold	15%	40%	50%	60%
CL share (Δp.p.)	+3.4 (+3.9)	+3.9 (+4.7)	+4.7 (+5.6)	+5.2 (+6.2)
Output (Δ%)	+0.65 (+0.70)	+0.55 (+0.15)	+0.85 (+1.07)	+1.23 (+1.53)
Tax (Δp.p.)	+0.14 (+0.04)	+0.36 (+0.17)	+0.37 (+0.23)	+0.40 (+0.30)

Table: Values in parentheses represent the results under endogenous fertility.

Different macro effects with different target

Consider the following schemes with an expenditure-neutral way:

- Income-tested (the existing scheme).
- +Ability test: for those with $I \leq \bar{I}$ and $h \geq \underline{h}$:
  $\underline{h}$ is arbitrarily set to the median of the students' ability dist. in the benchmark.
- Unconditional ($\simeq 1/10$ of students' average expenses)
- Increasing cash benefits to households with children under 18 ($6.5\% \uparrow$).

Results

	Income	+Ability	Uncond.	Cash
CL share (Δ p.p.)	+3.9	+2.6	+5.1	+0.0
TFR (Δ%)	+3.0	+2.7	+0.4	+1.0

Table: Main results with several schemes with different targets. *Note*: values in each cell indicate changes from the benchmark value.

- Income-tested ones lead to higher fertility (insurance effects are a key).
- Targeting college students can lead to higher fertility and greater output.

Literature

(1) Fertility choices in incomplete market models:

Schoonbroodt and Tertilt (2014, JET), Santos and Weiss (2016, IER), Sommer (2016, JME), etc.

- This paper: Income volatility + Uncertainty about children's characteristics.
- The education subsidies would provide insurance against those multiple sources of uncertainty and increase fertility.

(2) Education subsidies for college students in macro models:

Benabou (2002, Ecta), Krueger and Ludwig (2016, JME), Abbott et al. (2019, JPE), Matsuda and Mazur (2022, JME).

- This paper: + Fertility choices
- Fertility margins amplify the effects on other macro variables.

Literature

(3) Macro effects of pro-natal policies:

Erosa et al. (2010, RED), Hagiwara (2021), Zhou (2022), Kim et al. (2023), Nakakuni (2023), etc.

- This paper: + Education choice (its discreteness matters to fertility choices),
- and investigates the effects of grants for college students on fertility.

(4) Macro models with fertility choices:

De la Croix and Doepke (2003, AER), De la Croix and Doepke (2004, JPubE), Daruich and Kozlowski (2020, RED), etc.

- This paper: + Full lifecycle + GE + uninsurable shocks + IG linkages,
- all crucial to studying the effects of grants for college students.

Conclusion

1. Does it increase fertility?
   - Yes. The introduced grants increase the TFR by 3% via insurance effects.
   - But its eligibility expansion does not increase the TFR as it increases the expected costs of children for some households due to the higher probability of children attending college.
2. What are the impacts on the other macro variables?
   - College enrollment rate: 4 p.p. $\uparrow$. Output: 0.7% $\uparrow$.
   - Fertility margins amplify the policy impacts on these variables.
3. How different are the macro effects, depending on its target?
   - Income-tested ones lead to higher fertility (insurance effects are key).
   - Targeting college students can lead to higher fertility and greater output.

Data

The National Fertility Survey:

- Conducted almost every five years by the IPSS.
- Survey questions regarding marriage, childbirth, and child-raising.
- married couples and singles.

Sample selection:

- Year: 2015 (latest among available)
- Married couples.
- Age (wife): $25 - 39$.

Ideal and planned number of children

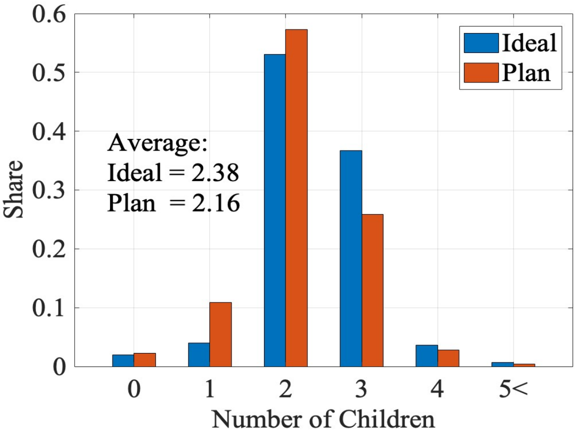


Figure: Distribution of ideal and planned number of children.

Preliminaries

- Technology
- Demographics
- Preferences
- Labor income
- Financial markets
- Government

Demographics

- The size of new cohort grows at rate g_n.
  - g_n is determined enodgenously.
- Mortality risks after retirement.
  - $\zeta_{j,j+1}$: Survival probability at age $j+1$ conditional on surviving until age j.
- g_n and $\{\zeta_{j,j+1}\}_{j \geq J_R}$ pin down the age distribution μ_j.

Preferences

- Households draw utility from consumption c and leisure l according to $u(c, l)$.
- If they have children under 18 (before completing HS), they further draw utility from the "quantity and quality" of children according to $b(n) \cdot v(q)$:
  - n: the number of children.
  - $b(n)$: increasing in $n \in \{0, 1, ..., N\}$ and concave. i.e., marginal utility gains from having additional child diminishes as n increases.
  - q: spending on children's quality.
    - Caveat: the spending does not affect children's human capital.

Preference: IVT

Utility from the IVT for households with n children is given as follows:

$$\underbrace{b(n) \cdot \lambda_a}_{\text{Discounting}} \cdot \underbrace{V_{g0}(a_{CL}, \phi_k, h_k, I)}_{\text{Value function for children}} .$$

Labor income

- Gross labor income is determined by:
    1. equilibrium wage rates w_e varying with skill (education levels) e,
    2. productivity $\eta_{j,z,e,h}$,
    3. hours worked.

- The productivity $\eta_{j,z,e,h}$ depends on:
    1. age j,
    2. skill e,
    3. human capital h,
    4. a stochastic component $z \sim \pi(z' \mid z)$.

Financial Markets

- Incomplete market: households can trade only claims for risk-free bonds.
- Households face debt limits that vary over the life-cycle:
  - (1) eligible college students and (2) households aged $j < (J_R - 1)$ can borrow.
    - $r^s = r + \iota_s$ for (1).
    - $r^- = r + \iota$ for (2).
  - Retired households cannot.

Government: revenue

- Consumption tax: τ_c
- Capital income tax: τ_a
- Labor income tax: τ_w
- Accidental bequests: Q

Government: expenditures

- Public pension: p per household
- Cash benefits for households with children under 17: B per child
- Lump-sum transfers ψ to generate the progressivity for τ_w.
- Education subsidies/loans for college students.
- The other expenditures: S

Financial aid for college students

Grants $g(h, I)$:

- Eligibility and payments can depend on:
  1. student' human capital (a proxy of "ability"),
  2. household income (I).
- No grants in the benchmark.

Loans:

- Eligible students can access to the subsidized loans with interest rate $r^s \leq r^-$.
- Eligibility is determined by (h, I).
- Government incurs the costs implied by the wedge b/w r^- and r^s.

Government Budget Constraint

$$\tau_c \cdot C + \tau_w \cdot (L_{HS} + L_{CL}) + \tau_a \cdot K + Q = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \leq 17} + S, \quad (2)$$

- C: total consumption,
- Q: total accidental bequests,
- μ_{old}: population mass of retired households,
- $\mu_{j \leq 17}$: population mass of children under age 17,
- K_s: total amount of borrowing by college students,
- μ_{es}: mass of students eligible for the grants.

Budget constraint for IVT choices

$$
\begin{aligned}
(1+\tau_c)c + a' + na_{IVT} = Y_{IVT},\\
a' \geq -\underline{A},
\end{aligned}
$$

where

$$
Y_{IVT} \equiv (1-\tau_w)w_e\eta_{j,z,e,h}(1-l) + \psi + \begin{cases} (1+(1-\tau_a)r)a & \text{if} \quad a \geq 0, \\ (1+r^-)a & \text{otherwise.} \end{cases}
$$

▸ Return

Value functions for college students

$$
\begin{aligned}
V_{g1}(a_{IVT};h,I) &= \max_{c,l,a'}\{u(c,l)+\beta V_{g2}(a';h,I)\},\\
V_{g2}(a;h,I) &= \max_{c,l,a'}\{u(c,l)+\beta \mathbb{E}_{z_0}[V^w(a^s(a'),j=22,z_0;e=1,h)]\}.
\end{aligned}
$$

Budget constraints for college students

$$a^s(a') = a' \times \frac{r^s}{1-(1+r^s)^{-10}} \times \frac{1-(1+r^-)^{-10}}{r^-}.$$

Eligible to loans:

$$\begin{aligned} &(1+\tau_c)c + p_{CL} + a' \qquad (3) \\ &-(1-\tau_w)w_{HS}(1-\bar{t}-l) - \psi - g(h,I) = \begin{cases} (1+(1-\tau_a)r)a & \text{if } \ a \geq 0, \\ (1+r^s)a & \text{otherwise.} \end{cases} \qquad (4) \\ &a' \geq -\underline{A}_s. \end{aligned}$$

The rest:

$$\begin{aligned} (1+\tau_c)c + p_{CL} + a' &= (1+(1-\tau_a)r)a + (1-\tau_w)w_{HS}(1-\bar{t}-l) + \psi + g(h,I), \\ a' &\geq 0. \end{aligned}$$

Working stage without children

$$
\begin{aligned}
V^w(a,j,z;e,h) = \max_{c,l,a'} \{ & u(c,l) \\
& + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^f(a',z',e,h)] & \text{if } \; j = J_F - 1 \\ \beta[V^r(a',j+1)] & \text{if } \; j = J_R - 1 \\ \beta \mathbb{E}_{z'}[V^w(a',j+1,z';e,h)] & \text{otherwise} \end{array} \right\} \\
\text{s.t.} & \\
& (1+\tau_c)c + a' = (1-\tau_w) w_e \eta_{j,z,e,h}(1-l) + \psi + (1+(1-\tau_a)r)a, \\
& z' \sim \pi(z',z), \\
& a' \geq \left\{ \begin{array}{ll} 0 & \text{if } \; j = J_R - 1, \\ -\underline{A} & \text{otherwise.} \end{array} \right.
\end{aligned}
\tag{5}
$$

Retirement stage

$$V^r(a,j;e) = \max_{c,a'} u(c,1) + \beta\xi_{j,j+1}V^r(a',j+1;e)$$

$$\text{s.t.}$$

$$(1+\tau_c)c + a' = p + (1+(1-\tau_a)r)a + \psi,$$

$$a' \geq 0 \ (a' = 0 \text{ when } j = J).$$

Return

Financial markets

Targets for

- ι: share of negative net worth,
- ι_s: share of students borrowing

Borrowing limits are set outside the model:

- $\underline{A}_s$: 2.88 million yen.
- $\underline{A}$: 20 million yen.

Intergenerational transmission of human capital

- The initial draw of human capital:

$$\log(h) = \rho_h \log(h_p) + \varepsilon_h,$$
$$\varepsilon_h \sim N(0, \sigma_h).$$

- $\rho_h = 0.19$ following Daruich and Kozlowski (2020).
- Target for σ_h: Variance of log(income) at age 28-29.

Income Process

- Labor productivity $\eta_{j,z,e,h}$:

$$\begin{aligned} \log \eta_{j,z,e,h} &= \log f^e(h) + \gamma_{j,e} + z \\ z' &= \rho_z z + \zeta, \quad \zeta \sim N(0, \sigma_z). \end{aligned}$$

- $\gamma_{j,e}$: estimate the second-order polynomial of hourly wages on age.

Education returns

- Consider the following human-capital production function of the non-linear form:

$$f^e(h) = h + e \cdot (\alpha_{CL} h^{\beta_{CL}})$$

- Target for α_{CL}: Log(wage) ratio (CL$-$ $\leq$CL) at age 28-29 (0.34).
- Target for β_{CL}: Variance of log(wage) for college grad. workers (0.14).

Production

- $\chi = 0.39$ following Matsuda and Mazur (2022).
- $\omega_h = 0.52$: to replicate the wage ratio between CL and the rest.
- $Z = 1.99$: s.t. low skill wage $= 1$.
- $\alpha = 0.33$.
- $\delta = 0.07$ (annual).

Externally determined

Parameter	Value	Description
$\underline{A}_s$	2.88 million yen	Borrowing limit for students
$\underline{A}$	20 million yen	Borrowing limit
p_{CL}	1.05 million yen/year	Tuition fees
κ	0.044	Time costs
$\xi_{j,j+1}$	–	survival prob.
τ_c	0.10	Consumption tax
τ_a	0.35	Capital income tax
τ_w	0.35	Labor income tax
p	¥$160,000$/month	Pension benefits
b	¥$10,000$/month	Cash transfers
α	0.33	Capital share
δ	0.07	Depreciation rate
χ	0.39	Elasticity of substitution
ρ_z	0.95	Persistence
σ_z	0.02	Transitory
ν	1	Education sorting by ability
γ	0.5	Curvature
β	0.98	Discount factor
ρ_h	0.19	Transmission of h

Wage age-profile

	College graduates	The rest
Age	0.048	0.041
$\text{Age}^2 \times 10,000$	-5.364	-4.551

Table: Wage age-profile

Validation: Fertility elasticity

- Set the per-child payment $B = B_0 \cdot X$ where $X \in \{1.1, 1.2, ..., 3.0\}$.
- Solve the household decisions with particular X and compute the TFR.
- Compute the implied elasticity for each X, denoted by ξ_X.
- After having $\{\xi_X\}_X$, compute the average elasticity $\bar{\xi}_X$.

Return

Introducing Subsidies: Aggregate Quantity

- Efficiency labor (per-capita): 1.3% $\uparrow$
  - Higher CL share $\Rightarrow$ Larger share of skilled workers.
  - Higher TFR $\Rightarrow$ Larger working-age population share.
- Capital (per-capita): 1.8% $\downarrow$
  - Reduce saving incentives/Crowd out IVTs.
  - Higher TFR $\Rightarrow$ Larger share of younger generations, who hold fewer assets.
- Output (per-capita): 0.7% $\uparrow$

Return

Decomposition: Education

	Direct	Prices	Tax	Dist.	All
CL share (Δ p.p.)	+2.6	0.0	0.0	+1.9	+4.0

Table: Decomposing the effects on education.

- Direct $\Rightarrow$ The subsidy relaxes the financial constraint (short/long run).
- Dist. $\Rightarrow$ The effects are amplified in the long run via IG linkages.
  CL share among parents $\uparrow$ $\Rightarrow$ share of children favoring college $\uparrow$ $\Rightarrow$ CL share $\uparrow$
- Fertility margins also play roles:
  College graduates have more children, who are likely to be college graduates.

Return

Fertility Decomposition: Case of Threshold = 60%

	Direct	Prices	Tax	Dist.	All
HS	−0.7%	−1.4%	0.0	0.0	−0.8%
CL	+9.1%	+4.3%	+2.4%	+0.6%	+13.2%

Table: Decomposing the effects on fertility when income threshold = 60%.

Return