Financial Costs of Children, Education Subsidies, and Parental Choices in Equilibrium*

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Abstract

Sustained low fertility rates and the resulting demographic aging have driven the development of pro-natal policies in many countries. In this context, education subsidies for college students have garnered significant attention in Japan, a leading country in this demographic change. In this study, I first present a set of facts suggesting that low fertility in Japan could largely be attributed to the financial costs for parents to support their children's college enrollment. Then, I construct an equilibrium lifecycle model with education, fertility, and asset transfer choices. The existing means-tested grants introduced in 2020 would increase the fertility of high-skilled parents, including ex-post ineligible parents, by providing insurance against multiple sources of uncertainty. They would also increase the college enrollment rate, amplified by fertility margins and distributional effects in the long run. Setting a higher income threshold increases the fertility of high-skilled parents further. However, the low-skilled parents would rather decrease because, contrary to high-skilled parents, their expected costs of children would significantly increase because of the higher probability of their children going to college due to the greater education mobility facilitated by eligibility expansions.

Keywords: Education subsidy, fertility, intergenerational linkages

JEL codes: C68, I28, J13, J24.

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1 Introduction

In many countries, sustained low fertility has driven the recent rise in pro-natal policies, aiming to increase birth rates;[1] low fertility is a root cause of demographic aging and poses challenges to economic growth and fiscal sustainability. A typical measure is cash transfers to households with children, and empirical studies show that they have significant effects on fertility (e.g., Milligan, 2005; Cohen et al., 2013; González, 2013), and that reducing financial costs also has significant impacts (Malkova, 2018).

In this context, the education subsidies for college students have garnered significant attention in Japan,[2] a leading country in this demographic change. They are considered a "two birds with one stone" policy addressing concerns related to the demographic changes. First, they would bring about greater average human capital by promoting skill acquisition, mitigating the negative effects of low fertility on aggregate output and the tax base. Second, they are also expected to increase the fertility rate because, as discussed in Section 2, several observations suggest that the financial costs arising from children's college enrollment should be a critical factor in low fertility in Japan: (1) couples are most likely to abandon having an ideal number of children because of financial costs; (2) a significant financial cost gap exists between those who have children enrolled in college and those who do not; (3) a substantial fraction of parents desire a college education for their children; and (4) Japan is one of the least in subsidizing tertiary education.

However, whether the policy works as expected to increase fertility, college enrollment, and aggregate output is unclear. This might distort agents' decisions and just crowd out savings. Changes in equilibrium objects such as prices and distribution may also lead to unintended effects, resulting in economic and fiscal costs.

I investigate the macroeconomic implications of education subsidies for college students by developing an incomplete market general equilibrium (GE) overlapping generations (OLG) model that incorporates choices regarding college enrollment, fertility, and inter-vivo transfers (IVT). The key ingredients are fertility choices and intergenerational linkages. Households draw utility from having children, but it is costly in terms of time and money. In particular, parents make IVT, based on altruistic motives, for their children after they graduate from high school and choose whether to attend college. The optimal IVT depends on children's characteristics and the resulting policy function for education. Children differ in school tastes − to what degree they draw disutility from

[1] Those countries include Hungary, Poland, Greece, Japan, South Korea, etc. See, for example, articles reporting Poland's and Greece's reforms (European Commission, 2018; The Guardian, 2020).

[2] See, for example, the following document published by the Cabinet Office `https://www5.cao.go.jp/keizai-shimon/kaigi/cabinet/honebuto/2023/2023_basicpolicies_ja.pdf` (available only in Japanese).

college enrollment − and human capital that is informative about their future earnings and education return. These characteristics correlate with parents' and are realized for children and parents when the children make the education choices.

The model is calibrated to the Japanese economy using the Japanese Panel Survey of Consumers (JPSC). I use the 60s cohort of married females and their family members, and the model replicates key moments such as the intergenerational persistence of education level and the scale of parental asset transfers for college students. I then check the model's validity and show that it replicates some important non-targeted moments, such as the benefit elasticity of fertility and fertility differentials across education levels.

The benchmark model captures government-provided student loans but does not include grants introduced in 2020. The subsidy is income-tested, and approximately the bottom 15% of the income distribution of households is eligible. The payments cover approximately two-thirds of the students' average expenses. I examine the effects of this existing subsidization scheme by introducing it into the benchmark model and solving the equilibrium. I also simulate an increase in the income threshold for the income-tested program.

The main findings are summarized as follows. First, introducing the existing program would increase the total fertility rate (TFR) by 3% in the long run, primarily driven by the fertility increase of college graduate (high-skilled) parents. Notably, the fertility increase of ex-post ineligible parents explains the higher TFR. Note that whether they are beneficiaries of the grants is uncertain when the parents make fertility decisions due to several sources of uncertainty: labor productivity shocks and children's characteristics (i.e., their school tastes and human capital, which are critical to education choices but are not realized before birth). A decomposition analysis suggests that the education subsidy provides insurance against the uncertainty arising from children's college enrollment choices and income volatility, which can increase fertility. Because high-skilled parents are more likely to have children with characteristics favoring college enrollment due to intergenerational linkages, they benefit more from this insurance, at least in the ex-ante sense, and their fertility behavior responds more strongly.

Second, introducing the existing program would increase the college enrollment rate by four percentage points (p.p.) in the long run. To understand the source of this increase, note that the following four components differ between the benchmark and long-run equilibrium, creating differences in college enrollment rates: the subsidization scheme, prices, labor tax rate adjusted to balance the government budget, and household distribution (e.g., skill distribution). I decompose the long-run effects into four effects and find that a *direct effect* driven only by the change in the subsidy scheme explains two-thirds of the long-run increase in college enrollment; the subsidy relaxes the financial constraints on students' enrollment decisions. However, this direct effect is not sufficiently large to account for the overall effects, and I find that a *distributional effect* driven only by changes in household distribution is the missing piece. In the short run, the college enrollment rate increases because of the direct effect, and these cohorts will become parent generations in the future. In the long run, the skill distribution then changes among the parents' generations (i.e., the share of college graduates increases). Because college graduate parents are more likely to have children with characteristics favoring college enrollment owing to intergenerational linkages, the change in skill distribution of parent generations contributes to the higher college enrollment rate in the long run.

In addition to this decomposition, I simulate the exogenous fertility version of this model, in which policy functions for fertility for each household are fixed as in the benchmark. I find that the college enrollment rate is significantly higher under endogenous fertility. Since high-skilled parents' fertility increases more and their children are more likely to attend college, considering fertility responses leads to a higher college enrollment rate through these fertility margins.

Higher TFR and college enrollment rates would lead to a greater per-capita labor supply in efficiency units, which is 1.3% greater than the benchmark; a higher TFR implies a greater share of the working-age population, and a higher college enrollment rate implies a greater share of skilled workers. However, introducing the subsidy leads to a 1.8% reduction in per-capita capital in the long run. The subsidy reduces savings incentives for many households and crowds out IVT. In addition, a higher TFR implies a greater share of younger generations holding fewer assets than older ones. Despite its negative effects on capital accumulation, the positive impacts on labor supply are sufficiently greater so that the per-capita output increases by 0.7% in the long run.

Finally, I simulate the program's expansion by subsidizing students in broader income classes, which is actively discussed in Japan. The long-run college enrollment rate and per-capita output increase as the income threshold increases. However, its positive effect on the TFR is limited and can be locally negative. For example, in the first experiment introducing a subsidy, the long-run TFR is 2.160. If the income threshold is set to the 60 percentile instead of the 15 percentile of the income distribution, the TFR would be 2.157, higher than the benchmark but lower than the first experiment of introducing subsidies. Although the fertility of high-skilled parents continues to increase with the expansion, that of low-skilled parents rather decreases. Then, the TFR is almost stable despite the broader coverage of income classes because the fertility decrease of low-skilled parents cancels out the fertility increase of high-skilled parents.

Why do low-skilled parents reduce fertility? The decomposition analysis suggests that the direct effects are essential to account for the fertility decline. This starkly contrasts the previous results that the direct effects increase the fertility of high-skilled parents by providing insurance against multiple sources of uncertainty. In the benchmark without grants, children of low-skilled parents are less likely to attend college; therefore, parents will likely not have to make the significant IVT required to support their children's college enrollment. If grants are introduced and their targets expand, education mobility increases in the sense that children of low-skilled parents are more likely to attend college than in the benchmark. Given that the grants are not generous enough to cover 100% of the costs of sending their children to college, this higher probability of children going to college implies that those parents are more likely to make more significant transfers upon their children's college enrollment, increasing the expected costs of children.

Related Literature: This study relates to several strands of literature. First, it is closely related to the literature on fertility choices in incomplete market models.[3] Some studies show that market incompleteness matters to household fertility choices, highlighting the nature of fertility choices and having children; they are discrete and irreversible choices and require at least a certain amount of resources over long periods. Then, rising income volatility makes households hesitate to have children, especially in the early stages of their lives, thus delaying marriage and fertility (Santos and Weiss, 2016). Delaying fertility leads to low fertility, given that the ability to reproduce declines with age (Sommer, 2016). This study makes two major contributions to the literature. First, besides the income risks considered in the literature, this model captures expenditure uncertainty stemming from the uncertainty about children's characteristics and education choices. Additionally, I show that the education subsidies for college students would provide insurance against those multiple sources of uncertainty. However, the effects on fertility can be qualitatively different between skilled and unskilled households.

Second, this study relates to macroeconomic studies based on the quantity-quality trade-off framework.[4] Kim et al. (2023) build an OLG model with quantity-quality trade-off to examine the role of the status externalities in education on the low fertility in South Korea. This study is related to theirs in that both focus on parental choices regarding education and fertility when examining the behavior of aggregate fertility. Unlike Kim et al. (2023), my model does not capture status externalities but is tailored to study the effects of education subsidies for college students by building a full lifecycle model with college enrollment and IVT choices.

Other papers in this literature show that considering the quantity-quality trade-off and

[3]For example, Schoonbroodt and Tertilt (2014); Santos and Weiss (2016); Sommer (2016).

[4]See, for example, De La Croix and Doepke (2003); Manuelli and Seshadri (2009); Cordoba et al. (2019); Daruich and Kozlowski (2020); Kim et al. (2023); Zhou (2022).

differentials in those choices can have critical implications for macroeconomic variables such as aggregate output and inequality. De La Croix and Doepke (2003) use a two-period OLG model capturing fertility differentials and show that the higher inequality implies a lower aggregate output through the fertility differentials. Daruich and Kozlowski (2020) construct a partial equilibrium lifecycle model with IVT, education, and fertility choices to investigate the roles of negative income-fertility relationships in shaping the low intergenerational mobility in the US.

Although they do not investigate the effects of education policies, as they are not of primary interest, their results suggest that cross-sectional heterogeneity in quantity-quality choices is vital in considering the macroeconomic effects of education subsidies and their design. Building on these insights, I contribute to the literature by extending the framework in several respects[5] and investigating the macroeconomic effects of education subsidies. I show that education subsidies for college students would have heterogeneous implications for fertility behavior across different skills, which would matter to other macroeconomic variables such as college enrollment rate.

Third, this study is closely related to the literature on family-related policies using lifecycle models with fertility choices.[6] Previous studies examine the effects of childcare subsidization (e.g., Bick, 2016), cash transfers (e.g., Nakakuni, 2023), both of them (e.g., Zhou, 2022), parental leave policies (e.g., Erosa et al., 2010), and tax reform (e.g., Jakobsen et al., 2022). In this context, education subsidies for college students can be interpreted as cash transfers targeting a specific type of household. This study contributes to the literature by examining the effects of these specific types of cash transfers on fertility, demographic structure, and other macroeconomic variables. In doing so, the model is tailored to capture the college enrollment and IVT decisions, which are critical for studying education subsidies. I believe this study has valuable implications for other countries, especially East Asian countries, where the education costs of having children are a critical factor for the recent low fertility rates.[7]

In this literature, the closest study to this is Zhou (2022). It builds an OLG model capturing the quantity-quality trade-off and endogenous human capital accumulation for

[5] This study differs from De La Croix and Doepke (2003) in constructing a full-fledged lifecycle model on the incomplete market framework and elaborating on college enrollment choices. This study is also different from Daruich and Kozlowski (2020) in adopting the general equilibrium framework to capture the GE and taxation effects due to introducing education subsidies, which are shown to be critical in the literature of education policies (e.g., Krueger and Ludwig, 2016).

[6] For example, Erosa et al. (2010); Bick (2016); Yamaguchi (2019); Cavalcanti et al. (2021); Hagiwara (2021); Jakobsen et al. (2022); Zhou (2022); Nakakuni (2023).

[7] Gauthier (2016) argues that in Japan, South Korea, and Taiwan, the financial burden of having children is a key obstacle to fertility, showing survey data for each country.

children to study the effects of family policies such as the baby bonus, childcare subsidies, as well as public education subsidies. Unlike its study, I focus on the effects of education subsidies for college students; thus, the model is tailored to incorporate college enrollment choices. In contrast, my model exogenizes the parental time investment and the accumulation of children's human capital before college education, which are critical in considering a broader set of family policies as Zhou (2022) does.

Finally, this study contributes to the literature on macroeconomic analysis of education subsidies, especially for college students.[8] This study makes two major contributions to the literature. First, it is the first to incorporate fertility choices into the standard framework adopted in the literature − the incomplete market GE-OLG model with education and IVT choices.[9] This allows us to quantify the effects of the subsidies on fertility and demographic structure, and the resulting feedback to other macroeconomic variables. Building on this new framework, the second contribution is to show that fertility responses amplify the policy impacts on macroeconomic variables such as the college enrollment rate and aggregate output.

The rest of the paper is organized as follows. Section 2 presents a set of facts suggesting that financial costs for parents to support children's college enrollment could largely be attributed to low fertility in Japan. Section 3 describes the model, which is calibrated in Section 4. Section 5 conducts numerical analysis and Section 6 concludes.

2 Background: Financial Costs of Children and Fertility Choices in Japan

This section presents facts suggesting that the financial costs for parents to support children's college enrollment could largely be attributed to low fertility in Japan. This explains why education subsidies for college students are drawing attention in Japan as a pro-natal measure and motivates my quantitative analysis. First, I list the four facts below and then elaborate on each one by one:

1. Couples are most likely to abandon having an ideal number of children because of financial costs.

[8] See, for example, Benabou (2002); Akyol and Athreya (2005); Kulikova (2015); Krueger and Ludwig (2016); Kotera and Seshadri (2017); Lawson (2017); Abbott et al. (2019); Lee and Seshadri (2019); Abbott (2022); Daruich (2022); Zheng and Graham (2022); Matsuda and Mazur (2022); Matsuda (2022).

[9] De la Croix and Doepke (2004) construct a two-period OLG model with fertility choices to compare the macroeconomic implications of a public and private schooling regime. My study differs from it in constructing a full-fledged lifecycle model on the incomplete market framework incorporating college enrollment and IVT choices.

2. A significant financial cost gap exists between those who have children enrolled in college and those who do not.

3. A substantial fraction of parents desire a college education for their children.

4. Japan is one of the least in subsidizing tertiary education.

Fact 1. Couples are most likely to abandon having an ideal number of children because of financial costs.

This fact is drawn from the National Fertility Survey (NFS), which is provided by the National Institute of Population and Social Security Research (IPSS).[10] The NFS is a cross-sectional household survey that asks respondents about their preferences or intentions regarding fertility, marriage, child-raising, and education, as well as their basic information, including their education, age, and income. It is conducted nearly every five years, and the latest survey available is in 2015, which collected 5,334 couples in which the wife is aged 18 to 49. Hereafter, I present the results focusing on married couples in which the wife is aged 25 to 39 years, leaving 2,420 couples.[11] According to the NFS, a non-negligible gap exists between the ideal and planned numbers of children. In the 2015 survey, the ideal number of children for wives aged 25 to 39 was on average 2.38, whereas the planned number was 2.16. Fig 1 represents the distribution of the ideal and planned number of children, where blue (red) bars indicate the share of wives who desire (plan) to have each number of children from zero to more than five. This figure suggests that the gap originates from the downward revision of the ideal at the intensive margin. There is no significant share gap between those whose ideal number is zero and those whose planned number is zero, and a substantial fraction of wives who desire three children end up with one or two children.

[10] See, https://www.ipss.go.jp/site-ad/index_english/survey-e.asp.

[11] Its 53.1% of the sample consists of couples in which the wife is over age 40, so the sample size shrinks if we target the younger couples. I focus on wives under age 39 because here I am interested in the fertility intention of those in the stage of fertility decision. Note that the cohort fertility rate is stable after age 40 for any cohort in Japan. See, for example, p7 of https://www.mhlw.go.jp/toukei/saikin/hw/jinkou/tokusyu/syussyo07/dl/gaikyou.pdf (in Japanese). Excluding those aged 18 to 24 does not affect the result significantly because they consist only of 1.5% of the observations for wives aged 18 to 49.

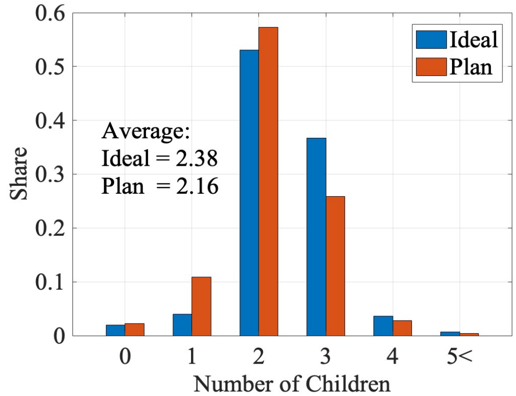

Fig 1: Distribution of ideal and planned number of children.

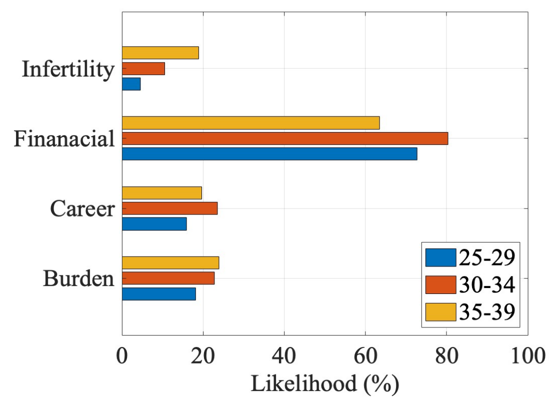

Fig 2: Reasons for the gap between the ideal and planned number of children.

Why do people abandon their ideal number of children? The NFS asks them to pick their reasons among several options, and the result indicates that they are most likely to choose the financial reason: "raising children and education is too expensive." Fig 2 represents the likelihood[12] that wives of each age group abandon their ideal number of children for a particular reason, and "Financial" represents the financial reason. Aside from this, "Career" represents "it would interfere with my job," and "Burden" represents "I would not handle the psychological and physical burden." On average, more than 75% of them chose the financial reason, which dominates other reasons, such as "Career" and "Burden," chosen by only 20% of them.

To sum up, there is a non-negligible gap between the ideal and planned numbers of children for couples, and a substantial fraction of them answer the reason as "raising children and education is too expensive." These results establish the first fact: couples are most likely to abandon having an ideal number of children because of financial costs.

Fact 2. A significant financial cost gap exists between those who have children enrolled in college and those who do not.

Fact 1 indicates that the financial cost of having children is a critical constraint on the number of children. However, it is silent on how and in which cases having children is so expensive; Fact 2 addresses them in education costs. To do this, I use two data sets: (1) the Survey of Children's Learning Expenses (SCLE, 2021) and (2) the Student Life Survey (SLS, 2018), both cross-sectional household surveys and conducted by the Ministry of Education, Culture, Sports, Science and Technology (MEXT).[13] The SCLE covers

[12] I use the term "likelihood" instead of "share" because it allows respondents to choose multiple options.

[13] See, https://www.mext.go.jp/b_menu/toukei/chousa03/gakushuuhi/1268091.htm for the SCLE and https://www.jasso.go.jp/statistics/gakusei_chosa/__icsFiles/afieldfile/2021/03/09/

more than 53,000 students from preschool to high school and reports the per-student average education expenditure for each expenditure category and education stage (i.e., preschool, elementary, junior high, and high school). The expenditure category includes not only school-related ones (e.g., tuition fees and textbooks) but also extracurricular activities (e.g., cram school, music, arts, and sports). In addition to that, I use the SLS for expenditures arising when their children enroll in college.[14]

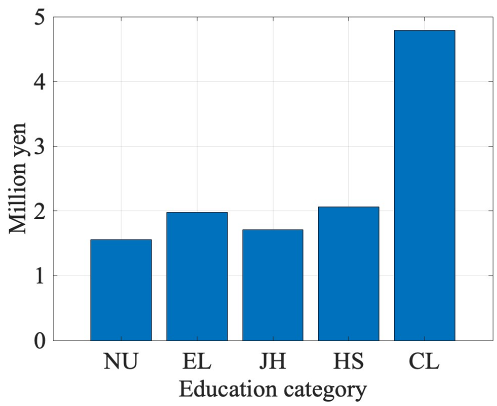

Fig 3: Average education expenditures.

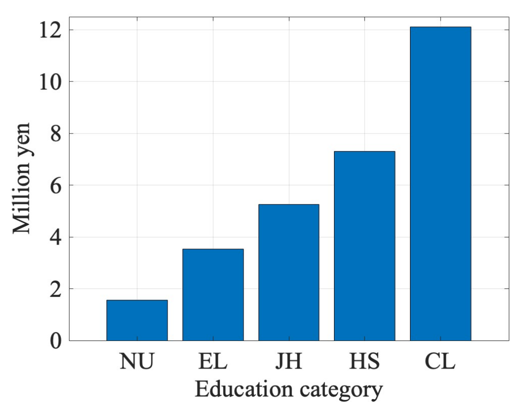

Fig 4: Cumulative education-expenditures.

Fig 3 shows the average per-child expenditure for each education category until completion. "NU," "EL," "JH," "HS," and "CL" each stand for nursery school or preschool, elementary school, junior high school, high school, and college, only consisting of a four-year college. All expenditures are conditional on enrollment in the education stage. In Japan, the high school graduation rate is approximately 100%, whereas the college enrollment rate was about 55% in 2022 and has increased over the last decades. Fig 4 represents the cumulative education expenditures, showing that the expenditure jumps up when children attend college. The figure tells us that raising one child on average takes the education costs of 7.31 million yen, which amounts to 4.5% of the individuals' average lifetime labor earnings.[15] If their children attend college, they have to spend another 4.8 million yen, meaning there is more than a 50% increase in education costs if parents send their children to college. These observations establish Fact 2: A significant financial cost gap exists between those who have children enrolled in college and those who do not.

`data18_all.pdf` (in Japanese) for the SLS.

[14] In 2018, the SLS correct answers from 43,394 students attending tertiary education, including college, some college, and graduate school.

[15] I construct a proxy of the average individual's lifetime earnings based on the 2022 Basic Survey on Wage Structure (BSWS) by the Ministry of Health, Labour, and Welfare (MHLW). First, I compute the average monthly earnings of ordinary workers for each age unconditional on any other characteristics such as sex and education. Then, I sum up the average earnings for each age, which amounts to about 160 million yen, and regard it as a proxy of the individuals' average lifetime labor earnings.

Fact 3. A substantial fraction of parents desire a college education for their children.

Facts 1 and 2 suggest that the financial costs for children enrolled in college are a critical obstacle to fertility in Japan. However, one might not still be convinced because the college enrollment rate is approximately 55%; therefore, the education costs for college students should be relevant only for half of the population.[16] The following fact answers "No, they should not." to this argument: far more fraction of parents desire a college education for their children than the current college enrollment rate.

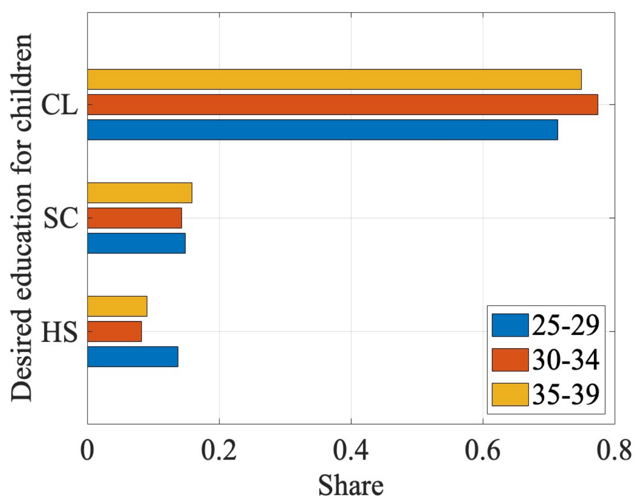

Fig 5: Wives' intention for children's education attainment.

We return to the NFS (2015), which asked respondents about the desired education level for their children. Fig 5 summarizes the results by the wife's age. Here, "SC" stands for some college. The figure shows that more than 70% of the parents desire a college education for their children, which is significantly higher than the college enrollment rate observed in any period in Japan. This observation suggests that, although college enrollment in Japan has been far below 100%, education costs for a college education can be relevant not only for a specific part of the population because many parents would like to send their children to college.

Fact 4. Japan is one of the least in subsidizing tertiary education.

These three facts presented suggest that the education costs for college students are critical to fertility decisions in Japan. A final fact (Fact 4) complements the previous ones by

[16] Dropout rates are insignificant in Japan, so the enrollment rate is almost equivalent to the graduation rate. For example, the dropout rate was 2.5% in 2021. See, https://www.mext.go.jp/content/20220603-mxt_kouhou01-000004520_03.pdf (in Japanese).

showing that Japan is one of the least in subsidizing tertiary education, which roughly corresponds to four-year college education in Japan, given that the enrollment rates for tertiary education other than the four-year college are significantly lower than four-year college enrollment rate.[17] I use the cross-country data provided by the OECD to present Fact 4.

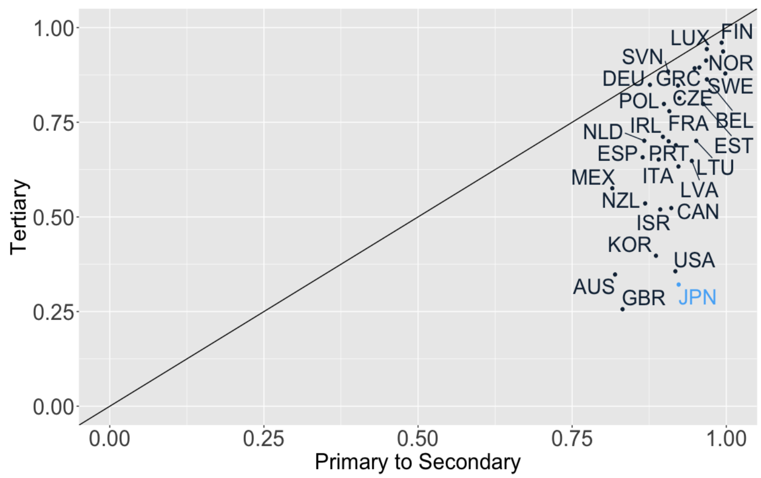

Fig 6: Subsidization rate for each education category (OECD, 2018).

Fig 6 shows the subsidization rates for tertiary education and primary-to-secondary education across OECD countries in 2018.[18] The subsidization rate s_e for a specific education category e is defined as

$$s_e = \frac{y_e^{pub}}{y_e^{pri} + y_e^{pub}},$$

where y_e^{pri} and y_e^{pub} represent the private and public spending on education e, both represented as a share of the GDP. Public spending includes expenditures on educational institutions and educational-related subsidies for households or students. Private spending refers to expenditures financed by households and other private entities. Private spending includes expenditures on school but excludes those outside educational institutions (e.g., textbooks purchased by households, private tutoring, and student living costs).[19]

The horizontal axis in Fig 6 represents the subsidization rate for primary-to-secondary education, whereas the vertical axis represents the subsidization rate for tertiary education. As the figure shows, the subsidization rate for primary-to-secondary education was more than 75% in all countries. On the contrary, there is a significant variation in tertiary

[17] According to the Basic School Survey by the MEXT, the enrollment rate for some college was 4% in 2022 and is declining over the past thirty years.

[18] This is the latest one that many countries' data are available.

[19] For more detail, see the OECD data (https://data.oecd.org/).

education subsidization; while some countries, such as Finland, Luxembourg, and Germany, subsidize tertiary education to a similar degree as subsidizing primary-to-secondary education, other countries, including Japan, the US, and the UK spend significantly less on tertiary education than they do on primary-to-secondary education. In particular, Japan's subsidization rate for tertiary education was 32%, the second lowest after the UK among OECD countries and less than half of the OECD average of 70%. This lower degree of subsidization for tertiary education can partly contribute to the situation in which the education costs of college education are critical to fertility decisions, as suggested by Facts 1 and 2. In addition, Fact 4 shows plenty of room for increasing subsidies for college students, which has driven recent policy discussions on introducing and expanding education subsidies for college students.

3 Model

This section describes the incomplete market GE-OLG model incorporating choices on college enrollment, IVT, and fertility. I embed fertility choices into a framework otherwise standard in the literature (e.g., Krueger and Ludwig, 2016; Abbott et al., 2019; Matsuda and Mazur, 2022). First, Section 3.1 provides an overview of the lifecycle of this economy. Section 3.2 elaborates on preliminaries of the model and then Section 3.3 formulates households' decision problems. Finally, Section 3.4 discusses the stationary equilibrium of this economy.

3.1 Overview of the lifecycle

Fig 7 represents the households' lifecycle in this model. In this model, one period corresponds to two years. Letting j denote age, agents live with their parents until they graduate high school at the beginning of age $j = J_E (:= 18)$ and then choose whether to attend college or enter the labor market after graduating high school, represented as a node "Grad.HS" in the figure. If they do not enroll in college, they enter the labor market as a high school graduate. If they choose to attend college, it takes four years (two model-periods) to complete, and they enter the labor market as a college graduate after graduation, represented as a node "Grad.CL" in the figure.[20]

[20] This model does not consider the possibility of dropping out from college because, as I mentioned, the dropout rate is insignificant in Japan.

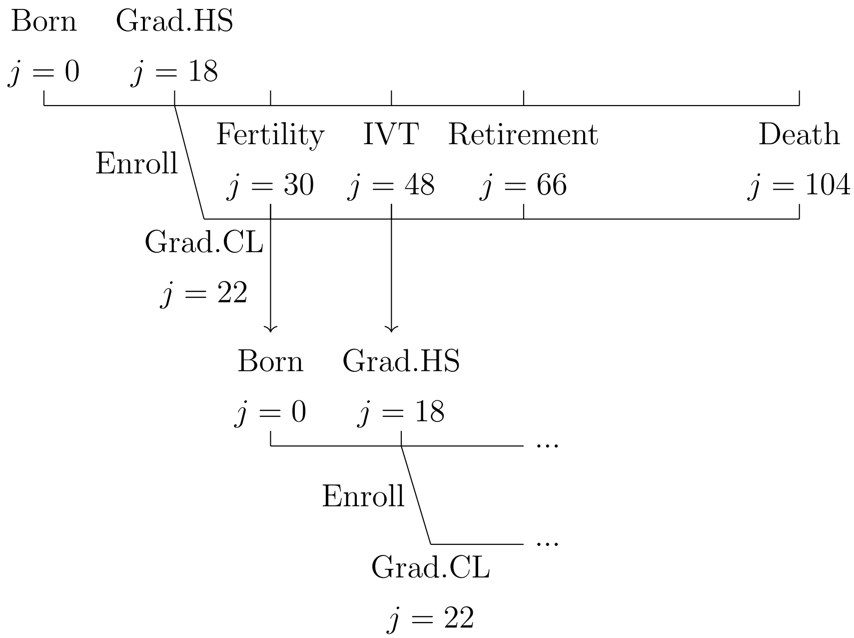

Fig 7: Model's lifecycle.

College enrollment increases their skill and potential lifetime earnings but takes some costs in terms of money and disutility. As standard in the literature, the model captures several channels of intergenerational linkage critical to the enrollment choices. First, the disutility of education ("psychic costs") depends on their parents' educational background. Second, parents choose the amount of IVT for their children. Third, the human capital endowment of children correlates to that of their parents, which is critical to education return. Students can fund their education costs through the IVT, labor earnings through part-time jobs, student loans with cheaper interest payments, and grants. The subsidized loans and grants are income- and ability-tested and provided by the government.

After completing their education and entering the labor market, they choose how much to consume, save, and work, subject to idiosyncratic productivity shocks. At the beginning of age $j = J_F (:= 30)$, they choose how many children they have, which is common for high school and college graduates. They draw the utility from the "quantity and quality" of their children, but having children requires some costs in terms of money and time. The lifecycle of a new cohort starts at this point, represented in the bottom half of the Fig 7. After their children graduate high school, corresponding to the beginning of the age $j = J_{IVT} (:= 48)$ for parents, they decide how much money to transfer to their children. This IVT decision affects the children's college enrollment choice at the node "Grad.HS." They are forced to retire at the beginning of age $j = J_R (:= 66)$. After that period, they face mortality risks; every period, a certain fraction of them is hit by exogenous mortality shocks and exits from the economy. They can live for J $(:= 104)$ years at the longest, but they exit the economy after age J.

3.2 Preliminaries

Production: A representative firm chooses labor and capital inputs in competitive factor markets to produce final goods. There are two types of labor inputs in this economy; the college graduates (skilled) and high school graduates (unskilled). Their total labor supply in efficiency units are represented as L_{CL} and L_{HS}, respectively. I allow them to be imperfect substitutes by considering the aggregate labor in efficiency units, L, is given as:

$$L = \left[\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}\right]^{1/\chi},$$

where $\omega_{HS} \equiv 1 - \omega_{CL}$ and ω_{CL} governs the relative productivity of the skilled workers.

The representative firm operates with a Cobb-Douglas production function with aggregate capital K and labor L to produce the output Y:

$$Y = ZK^{\alpha}L^{1-\alpha},$$

where Z represents the factor neutral productivity. Let r, w_{HS}, and w_{CL} denote the rental rate of capital and wage rates for skilled and unskilled labor. Capital depreciates at δ, and the firm has to incur the capital depreciation cost.

Demographics: As I explained above, after retirement, they face uncertainty regarding their survival in the next period. Let $\zeta_{j,j+1}$ denote the probability of surviving at age $j+1$ conditional on surviving at age j for each $j \in \{J_R, ..., J\}$ with $\zeta_{J,J+1} = 0$. The cohort size grows at rate g_n, which is endogenously determined in this model based on households' choices on fertility. Let μ_j denote the age distribution of the economy, determined by the cohort population growth rate and survival probabilities. The mass of the economy is normalized to 1 (i.e., $\sum_j \mu_j = 1$) as in previous studies (e.g., Guner et al., 2020; Zhou, 2022).

Intergenerational Linkages and Initial Endowments: After graduating high school (at the beginning of age 18), agents draw the psychic costs from a distribution g^{ϕ}_{h,e_p}, depending on the student's ability and parent's education e_p. They also draw their human capital from a distribution $g^{h}_{h_p}$ varying with the parents' human capital level h_p. Lastly, they may receive some assets (IVT) from their parents. These three sources of intergenerational linkages shape their college enrollment choices and generate their heterogeneity.

Preferences: Throughout their lifetime, they draw utility from consumption c and leisure l according to a utility function $u(c, l)$. They discount future utility by β. At age $j = J_F$, they choose the number of children they have, denoted by $n \in \{0, 1, ..., N\}$. They can draw additional utility from having children in several ways. First, they draw utility from the "quality" of the children by making monetary investments q per child until the children graduate high school, captured by a utility function $v(q)$. The utility is discounted by a function $b(n)$, increasing and concave in the number of children n. Further, based on altruistic motives, parents draw utility by transferring assets to their children after high school graduation. More specifically, parents consider children's expected lifetime utility in their education choice stage, with a discount rate $\lambda_a \cdot b(n)$, where λ_a represents the altruistic discount factor.

Costs of Children: Having children is costly in terms of money and time. First, q units of the per-child investment require $n \cdot q$ units of money, and they will make an additional expenditure $n \cdot a_{IVT}$ upon high school graduation of their children. In addition to the monetary costs, having children requires κ units of time until children graduate high school and become independent.

Labor Earnings: Households choose hours worked h to earn income. The labor earnings of a household are given by $w_e \eta_{j,z,e,h}(T-l)$; it is a product of the market wage w_e, labor efficiency or productivity $\eta_{j,z,e,h}$, and hours worked $(T-l)$. Here, T denotes the disposable time that can be devoted to work or leisure where $T = 1 - \kappa \cdot n$ if they have n children before graduating high school and $T = 1$ otherwise. Labor efficiency depends on age j, idiosyncratic productivity shock component z, education level e, and time-invariant human capital h.

Financial Markets: Financial markets are incomplete due to the lack of state-contingent claims. Households can self-insure against risks by savings, accruing interest payments at a rate of r. Households aged $j < J_R - 1$ except college students can borrow at rate $r^- = r + \iota$ where $\iota > 0$ (i.e., borrowers incur the overseeing costs ι) up to a borrowing limit $\underline{A}$, while we do not allow retired households' net worth to be negative. In addition, eligible students have access to student loans subsidized by the government, which entails the interest rate of $r^s = r + \iota_s$. This loan is income- and ability-tested, and eligible students can borrow up to a limit $\underline{A}_s$.

Government: The government raises the revenue by levying three types of taxes: consumption, labor income, and capital income taxes, where these tax rates are represented as τ_c, τ_w, and τ_a. In addition, the government collects accidental bequests and devotes them to cover expenditures. They use this revenue to fund (1) the public pension benefit, which gives p units of money to retired households each period, (2) subsidized loans for college students, (3) grants for eligible college students, where the payment per eligible student is represented by $g(h, I)$ where $I \geq 0$ denotes the household income when the student faces education choice problem (i.e., their parent's age is J_{IVT}), (4) lamp-sum transfers ψ that is introduced for replicating the progressivity of labor income tax schedule in a simple way following the literature, (5) cash benefits for households with children under 17 with per-child payment of B, and (6) the other expenditure S. I do not consider grants in the benchmark to replicate the economy before grants are introduced in 2020 and set $g(h, I) = 0$ for each (h, I). The government budget constraint is given as follows:

$$\tau_c \cdot C + \tau_w \cdot (L_{HS} + L_{CL}) + \tau_a \cdot K + Q = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \leq 17} + S, \quad (1)$$

where C, Q, μ_{old}, $\mu_{j\leq 17}$, K_s, and G represent the total consumption, total accidental bequests, population mass of retired households, that of children under age 17, the total amount of borrowing by college students, and the total grant payments.

3.3 Household Problems

This section describes problems agents face throughout their lifecycle. The first is about college enrollment choice, choosing whether to attend college or enter the labor market after graduating high school.

Education Stage: After graduating high school, they draw the time-invariant human capital h from the distribution $g^h_{h_p}$ and psychic cost ϕ from the distribution g^{ϕ}_{h,e_p}. They also receive IVT from their parents $a_{IVT} \geq 0$. Some of them can access subsidized loans to fund expenditures that arise during the college education stage, which is income- and ability-tested. Thus, the state variables for the students are comprised of asset a_{IVT}, the human capital h, psychic costs ϕ, and their parent's income I. They compare the expected value for entering the labor market as a high school graduate, $\mathbb{E}V^w$, with the value for enrolling in college, V_{g1} net of the psychic cost ϕ. They choose college enrollment if the latter is greater than the former; otherwise, they enter the labor market. The decision problem is formulated as follows:

$$V_{g0}(a_{IVT}, \phi, h, I) = \max_{e \in \{0,1\}} \Big\{ (1-e) \cdot \mathbb{E}_{z_0}[V^w(a_{IVT}, j = 18, z_0; e = 0, h)] + e \cdot [V_{g1}(a_{IVT}; h, I) - \phi] \Big\}, \quad (2)$$

where $e \in \{0, 1\}$ indicates the education choice where $e = 1$ means college enrollment and $e = 0$ does entering the labor market as a high school graduate. V^w denotes the value function for workers, which I formulate in the next subsection. The initial draw of z, z_0, is uncertain and is according to the invariant distribution of z, $\bar{\pi}_z$, so the expectation operator is put next to the V^w. The value for college enrollment, V_{g1}, is defined as follows:

$$
\begin{aligned}
V_{g1}(a_{IVT}; h, I) &= \max_{c,l,a'} \{u(c,l) + \beta V_{g2}(a'; h, I)\}, \\
V_{g2}(a; h, I) &= \max_{c,l,a'} \{u(c,l) + \beta \mathbb{E}_{z_0}[V^w(a^s(a'), j = 22, z_0; e = 1, h)]\}.
\end{aligned} \tag{3}
$$

The budget constraints differ according to eligibility to the student loans. The budget constraints for eligible students are give as follows:

$$
\begin{aligned}
&(1+\tau_c)c + p_{CL} + a' - (1-\tau_w)w_{HS}(1-\bar{t}-l) \\
&\qquad - \psi - g(h, I) = \begin{cases} (1+(1-\tau_a)r)a & \text{if } \ a \geq 0, \\ (1+r^s)a & \text{otherwise,} \end{cases} \\
&\qquad a' \geq -\underline{A}_s.
\end{aligned} \tag{4}
$$

The rest of the students cannot access to the student loans, which implies that their budget constraints are given as follows:

$$
\begin{aligned}
&(1+\tau_c)c + p_{CL} + a' = (1+(1-\tau_a)r)a + (1-\tau_w)w_{HS}(1-\bar{t}-l) + \psi + g(h, I), \\
&\qquad a' \geq 0.
\end{aligned}
$$

College students draw utility from consumption and leisure and must pay tuition fees p_{CL}. Normalizing their total disposable time as 1, students must spend a $\bar{t}$ fraction of time on study. Thus, they choose the time allocation between leisure and working over the disposable time $1-\bar{t}$. One unit of labor supply gives college students w_{HS} units of wages.[21] They can fund the consumption and tuition fees through (1) transfers from their parents a_{IVT}, (2) borrowing through student loans if eligible, (3) government-provided grants if eligible, and (4) labor earnings by themselves. Following the literature, I assume that fixed payments are made for 20 years (10 periods) following college graduation and transform college loans into regular bonds according to the following formula:

$$
a^s(a') = a' \times \frac{r^s}{1-(1+r^s)^{-10}} \times \frac{1-(1+r^-)^{-10}}{r^-}.
$$

Working Stage without Children: The remaining component in (2) and (3), V^w, represents the value function for working stage for age $j \in \{J_E, ..., J_F - 1, J_{IVT}+1, ..., J_R - 1\}$. The state variables for this stage consists of asset a, age j, idiosyncratic component of labor productivity z, education level e, and human capital h. The uncertainty in this stage is only about the next period's productivity, which is denoted by z' following a

[21] I assume that, while in college, there is no heterogeneity in labor efficiency and no uncertainty regarding the next period's productivity, and one unit of hours worked brings one unit of labor efficiency.

Markov process $\pi_z(z', z)$. Households choose consumption, leisure, and savings given the state variables. The value function is formulated as follows:

$$
\begin{aligned}
V^w(a, j, z; e, h) = \max_{c,l,a'} \{ & u(c, l) \\
& + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^f(a', z', e, h)] & \text{if } \ j = J_F - 1 \\ \beta [V^r(a', j+1)] & \text{if } \ j = J_R - 1 \\ \beta \mathbb{E}_{z'}[V^w(a', j+1, z'; e, h)] & \text{otherwise} \end{array} \right\} \\
\text{s.t.} \quad & \\
& (1+\tau_c)c + a' = (1-\tau_w) w_e \eta_{j,z,e,h}(1-l) + \psi + (1 + (1-\tau_a) r) a, \\
& z' \sim \pi(z', z), \\
& a' \geq \left\{ \begin{array}{ll} 0 & \text{if } \ j = J_R - 1, \\ -\underline{A} & \text{otherwise.} \end{array} \right.
\end{aligned}
\tag{5}
$$

Value functions V^f and V^r represent those for the fertility choice stage and retirement stage, respectively, which are formulated in the following subsections.

Fertility Choices and Working Stage with Children: When aged $j = J_F$, they choose whether and how many to have children. As in the previous life stages, they draw utility from consumption and leisure and decide on consumption, time allocation, and savings. In addition to that, as I mentioned, they also draw utility from the number of children n and the quality of children q, the latter is captured by the per-child educational spending.[22] The utility depends on the per-child investment q and the number of children n. Then, the value function for households aged $j = J_F$, V^f, is formulated as follows:

$$
V^f(a, z, e, h) = \max_{n \in \{0,1,..,N\}} \left\{ V^{wf}(a, j = J_F, z; e, h, n) \right\}
$$

where, for $j = J_F, ..., J_{IVT} - 1$,

$$
\begin{aligned}
V^{fw}(a, j, z; e, h, n) = \max_{c,l,q,a'} \{ & u(c/\Lambda(n), l) + b(n) \cdot v(q) \\
& + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^{wf}(a', j+1, z'; e, h, n)] & \text{if } j \in \{J_F, ..., J_{IVT} - 2\} \\ \beta \mathbb{E}_{z', \phi_k, h_k}[V^{IVT}(a', z'; \phi_k, h_k, e, h, n)] & \text{if } j = J_{IVT} - 1 \end{array} \right\} \\
\text{s.t.} \quad & \\
& (1+\tau_c)(c + nq) + a' = Y_{fw}, \\
& a' \geq -\underline{A},
\end{aligned}
$$

[22] This study does not consider children's endogenous human capital accumulation through parental investments and assumes parents make educational spending on children just because it draws utility. Incorporating the endogenous human capital accumulation is left for future research.

where

$$Y_{wf} \equiv (1-\tau_w) w_e \eta_{j,z,e,h}(1-l-\kappa \cdot n) + n \cdot B + \psi + \begin{cases} (1+(1-\tau_a)r)a & \text{if} \quad a \geq 0, \\ (1+r^-)a & \text{otherwise.} \end{cases}$$

Here, ϕ_k and h_k denote the psychic costs and human capital for their children. The household consumption is deflated by the equivalence scale $\Lambda(n)$, depending on the number of children n. The total education spending on children, nq, enters the budget constraint, and utility from the quantity and quality of children, $b(n) \cdot v(q)$, enters the objective function in the periods with children. V^{IVT} represents the value function for the stage of making IVT at $j = J_{IVT}$, which is described in the following subsection.

In addition to the next period's productivity z', they are uncertain about their children's human capital h_k and psychic costs ϕ_k until the beginning of age $j = J_{IVT}$. Recall that the children's human capital correlates to the parent's, and the distribution of their psychic costs depends on the parent's education level and student's own human capital. Uncertainty about h_k and ϕ_k can be translated into uncertainty about expenditures on children in the form of IVT, given that those two components govern the marginal gains from IVT for parents,

$$b(n) \cdot \lambda_a \cdot \frac{\partial V_{g0}(\boldsymbol{x})}{\partial a_{IVT}},$$

where $\boldsymbol{x} = (a_{IVT}, \phi_k, h_k, I)$.[23]

Inter-vivo Transfers: At age $j = J_{IVT}$, which corresponds to a period when their children graduate high school and face the college enrollment choice, they decide on how much to transfer to their children. Some of the children's characteristics are realized at this point, such as their psychic cost ϕ_k and human capital h_k. Then, they choose the amount of per-child transfer a_{IVT} given the state variables, which is formulated as follows:

$$V^{IVT}(a, z; \phi_k, h_k, e, h, n) = \max_{c,l,a',a_{IVT}} \Big\{ V^w(a - \tilde{a}_{IVT}, j = J_{IVT}, z; e, h) + b(n) \cdot \lambda_a \cdot V_{g0}(a_{IVT}, \phi_k, h_k, I) \Big\},$$

where $\tilde{a}_{IVT} = \frac{n \cdot a_{IVT}}{1+(1-\tau_a)r}$ and the parent's state vector pins down household income I. The budget constraint is given as follows:

$$(1+\tau_c)c + a' + n a_{IVT} = Y_{IVT},$$
$$a' \geq -\underline{A},$$

[23] To help our understanding, this representation implicitly assumes that the value function V_{g0} is differentiable, but in principle, this is not the case because of the discreteness of the education choices.

where

$$Y_{IVT} \equiv (1-\tau_w)w_e\eta_{j,z,e,h}(1-l) + \psi + \begin{cases} (1+(1-\tau_a)r)a & \text{if} \quad a \geq 0, \\ (1+r^-)a & \text{otherwise.} \end{cases}$$

Retirement Stage: At the beginning of age J_R, they are forced to retire from the labor market. After that, they spend all their time on leisure and make consumption-saving decisions. Two points differ from previous choice problems: they receive the pension benefit p from the government and face uncertainty about the next period's survival. The value function for the retirement stage is formulated as follows:

$$V^r(a,j) = \max_{c,a'} u(c,1) + \beta\xi_{j,j+1}V^r(a',j+1)$$

$$\text{s.t.}$$

$$(1+\tau_c)c + a' = p + (1+(1-\tau_a)r)a + \psi,$$

$$a' \geq 0.$$

3.4 Stationary Equilibrium

I solve the stationary equilibrium. In equilibrium, households make every choice to maximize their expected utility, the firm maximizes its profit, and the government budget is balanced. The stationarity implies that the distribution over state variables is invariant. Importantly, the age distribution is determined endogenously according to households' fertility choices. See Appendix A for the detailed definition of equilibrium and Appendix B for the computational algorithm for solving the equilibrium.

4 Calibration

To calibrate the model, I mainly rely on the Japanese Panel Survey of Consumers (JPSC), a panel survey of Japanese women and their household members. It started in 1993 with a representative sample of 1,500 women aged $24-34$ and contains information about, for example, their income, educational background, marriage, fertility, and expenditures to detailed categories, including those on children's education. I focus on the cohort born in 1959-69, the oldest cohort of this survey, especially to compute the completed fertility and intergenerational mobility of education. I keep only married households as in previous works (e.g., Daruich and Kozlowski, 2020) because the model focuses on choices made within married households such as fertility and educational investments.[24] Unless oth-

[24]Hence, an agent or household in this model refers to households with two individuals. Then, "children" in this model can also be interpreted as a household unit. That is, having n children can be

erwise mentioned, target moments for internally determined parameters described below are computed based on the JPSC's 1959-69 cohort data.

4.1 Targeted Moments

Preferences: Instantaneous utility for agents are given as follows:

$$u(c,l) = \frac{(c^{\mu} l^{1-\mu})^{1-\gamma}}{1-\gamma}.$$

μ is internally determined as 0.23 so that the households spend one-third of the total disposable time on market work. Instantaneous utility from the quality of children is given as:

$$v(q) = \lambda_q \frac{q^{1-\gamma}}{1-\gamma},$$

where $\lambda_q = 0.62$ so that the annual educational expenditure per child amounts to 7% of average income at age 28. The utility must be always positive (or always negative) in models of altruism with endogenous fertility, and I set $\gamma = 0.5$ following the literature (e.g., Daruich and Kozlowski, 2020). The altruistic discount factor λ_a is set to 1.03 so that the annual average IVT for college students amounts to 27% of average income at age 28.[25] Following Kim et al. (2023), I let the discount function of number of children $b(n)$ be non-parametric, and assume that $b(n) = b_n$ for each $n \in \{0, 1, 2, 3, 4\}$ with $b(0) = b_0 = 0$. Those parameters are determined so that the model replicates the distribution of the completed fertility. The time for studying $\bar{t}$ is set to 0.8 so that the income share of labor earnings for college students in the model is close to 20% (SLS, 2018). I assume that $\beta = 0.98$ as in Zhou (2022).

Financial Markets: I set the borrowing limits $\underline{A} = 20$ million yen and $\underline{A}_s = 2.88$ million yen.[26] The borrowing wadges are set $\iota = 0.055$ and $\iota_s = 0.054$ so that the model approximates the share of working households with a negative net worth (54%) and the share of borrowing students (44%).

interpreted as reproducing $n/2$ units of households. For example, if a household has two children, it means reproducing one household unit with two individuals. If a household has one child, it means a reproduction of 0.5 units of households with two individuals.

[25] Although some parents whose children do not enroll in college make IVT, the amount is negligibly small. One reason is that the marginal gains from IVT are more significant if their children enroll in college, as college students are financially constrained, primarily because of their limited earning ability.

[26] The former is based on the Family Income and Expenditure Survey by the Ministry of Interna Affairs and Communications and the latter is based on the SLS (2018).

School Taste: I assume that the psychic costs ϕ are given as $\phi = \psi_{CL} \cdot \exp(-\nu \cdot h) \cdot \tilde{\phi}$. First, ψ_{CL} governs the scale of psychic costs and thus college enrollment rate. The second term $\exp(-\nu \cdot h)$ allows high ability students to have smaller psychic costs, as standard in the literature, and ν governs the education sorting by ability. Finally, $\tilde{\phi}$ is stochastic and the distribution depends on their parent's education. Following Daruich and Kozlowski (2020), I assume that $\tilde{\phi}$ is distributed on an interval $[0, 1]$ and follows the following CDF:

$$G_{e_p}^{\tilde{\phi}} = \begin{cases} \tilde{\phi}^{\omega} & \text{if } e^p = 0 \\ 1 - (1 - \tilde{\phi})^{\omega} & \text{if } e^p = 1 \end{cases}$$

Here, ω governs the intergenerational transmission of school tastes. ψ_{CL} is set to 20.8 so that the model approximates the college enrollment rate of 37.7%. ν is set to 1 in the benchmark, and ω is set to 1.71 so that the intergenerational transition matrix of education in the model matches the data counterpart as close as possible. Table 1 reports the transition matrix in the model and data. $(i, j)-$th entry of the matrix indicates the probability that children acquire skill j given that their parent's skill is i in the benchmark model, and values in parenthesis represent the data counterparts. The table indicates that the education level is persistent across generations; children of high school graduate parents attend college with a probability less than 0.3, whereas children of college graduate parents do with a probability approximately 0.6.

Parents/Children	HS	CL
HS	0.725 (0.798)	0.275 (0.202)
CL	0.412 (0.423)	0.588 (0.577)

Table 1: Intergenerational transition matrix of education. *Note*: $(i, j)-$th entry of the matrix indicates the probability that children acquire skill j given that their parent's skill is i in the benchmark model, and values in parenthesis represent the data counterparts.

Intergenerational Transmission of Human Capital: The intergenerational transmission of human capital is according to the following formula:

$$\log(h) = \rho_h \cdot \log(h_p) + \varepsilon_h,$$
$$\varepsilon_h \sim N(0, \sigma_h).$$

I assume $\rho_h = 0.19$ based on Daruich and Kozlowski (2020) while internally determining σ_h as 0.65 so that the variance of log income at age 28 in this model is close to 0.27.

Income Process and Education Return: The efficiency labor of an agent aged j, education e, human capital h, and productivity z, $\eta_{j,z,e,h}$, is given as follows:

$$\log \eta_{j,z,e,h} = \log[f^e(h)] + \gamma_{j,e} + z,$$
$$f^e(h) = h + e \cdot (\alpha_{CL} h^{\beta_{CL}}),$$
$$z' = \rho_z z + \zeta, \quad \zeta \sim N(0, \sigma_z).$$

To set $\gamma_{j,e}$, I estimate the second-order polynomial of hourly wages on age using JPSC. As reported in Table 2, the income gradient on age is larger for college graduates than the rest of the workers, but the degree is modest compared with the US case (e.g., Abbott et al., 2019). I assume $\rho_z = 0.95$ and $\sigma_z = 0.02$, values in the ranges over those frequently used in the literature.[27] The function $f^e(h)$ indicates that the education return depends on human capital h: people with higher human capital can obtain greater returns through college education. Following Daruich and Kozlowski (2020), α_{CL} and β_{CL} are determined so that the model replicates the ratio of log wage between college graduates and the rest of the population at age $j = 28$ and the log wage variance for college graduates at age $j = 28$.

	HS	CL
Age	0.041	0.048
$\text{Age}^2 \times 10,000$	-4.551	-5.364

Table 2: Wage age-profile. *Note*: CL indicates the college graduate households where the husband or wife is a college graduate. HS represents the rest of the population.

Production: I set the capital share $\alpha = 0.33$ and $\delta = 0.07$ following Kitao (2015). χ is set to 0.39 following Matsuda and Mazur (2022). ω_{CL} is internally determined to 0.52 so that the average wage ratio between college graduates and the rest in the model amounts to 1.36. Z is determined so that the wage rate for low-skilled labor is normalized to one.

Government: Tax rates are set to $\tau_c = 0.1$, $\tau_w = 0.35$, and $\tau_a = 0.35$ in the benchmark. The lump-sum transfer is set to $\psi = 0.01$ to match the ratio between the variance of log net income and that of log gross income (0.6). The pension benefit p is set so that the government provides ¥160,000 per household per month. The cash transfer B is given as ¥10,000 per child per month, which approximates the actual payment. The other expenditure S is set so that the government budget constraint is balanced in the benchmark and fixed throughout the counterfactual experiments.

[27] Due to data limitations, it is hard to accurately estimate the AR(1) process for z using any data source in Japan.

Miscellaneous: The survival probability $\zeta_{j,j+1}$ is set based on the Vital Statistics (2019).[28] Annual college tuition fees p_{CL} are set to 1.05 million yen. κ is set to 0.044 so that they spend 13.3% of their working hours on childcare.[29] Table 3 and Table 4 summarise the parameters externally and internally determined.

Parameter	Value	Description
$\underline{A}_s$	2.88 million yen	Borrowing limit for students
$\underline{A}$	20 million yen	Borrowing limit
p_{CL}	1.05 million yen/year	Tuition fees
κ	0.044	Time costs
$\xi_{j,j+1}$	–	survival prob.
τ_c	0.10	Consumption tax
τ_a	0.35	Capital income tax
τ_w	0.35	Labor income tax
p	¥$160,000$/month	Pension benefits
b	¥$10,000$/month	Cash transfers
α	0.33	Capital share
δ	0.07	Depreciation rate
χ	0.39	Elasticity of substitution
ρ_z	0.95	Persistence
σ_z	0.02	Transitory
ν	1.0	Education sorting by ability
γ	0.5	Curvature
β	0.98	Discount factor
ρ_h	0.19	Transmission of h

Table 3: Parameters externally determined.

[28]See, https://www.mhlw.go.jp/english/database/db-hw/outline/index.html.

[29]See, Kitao and Nakakuni (2023).

Parameter	Value	Moment	Data	Model
μ	0.23	Work hours	0.33	0.30
$\bar{t}$	0.8	Income share of labor earnings	0.20	0.17
ι_s	0.054	Share of students using loans	0.44	0.34
ι	0.055	Household share with negative net worth	0.54	0.45
ω_{CL}	0.52	CL−HS wage ratio	1.36	1.48
ψ	0.01	Var(log disposable income)/Var(log gross income)	0.60	0.68
λ_q	0.62	Average transfer / Average income at age 28	0.07	0.07
λ_a	1.03	Average transfer / Average income at age 28	0.27	0.27
ω	1.71	Intergenerational mobility of education	See Table 1	
σ_h	0.65	Variance of log(income) at age 28	0.27	0.24
ψ_{CL}	20.8	College enrollment rate	0.377	0.376
α_{CL}	0.1	Log wage ratio (CL−HS) at age 28	0.34	0.38
β_{CL}	0.1	Var log wage for CL at age 28	0.14	0.24
b_1	0.49	Share of one child	0.16	0.15
b_2	0.53	Share of two children	0.55	0.61
b_3	0.55	Share of three children	0.22	0.24
b_4	0.56	Share of four or more children	0.02	0.00
Z	1.99	Low skill wage	1.0	1.0

Table 4: Parameters internally determined.

4.2 Non-targeted Moments and Validation

This subsection checks the validity of the calibrated model. First, I validate if the benchmark model generates a reasonable value of the benefit elasticity of fertility, which is non-targeted in calibration. Second, I check if the model performs well on other critical non-targeted moments, fertility differential across education levels and the revenue breakdown for college students.

4.2.1 The benefit elasticity of fertility

Previous works show that cash benefits such as the CB and baby bonus have a significant impact on fertility. Many of them report that the benefit elasticity of fertility, the percentage increase in fertility rate against the one percent increase in the cash transfer, is about $0.1-0.2$. For example, Milligan (2005) studies a reform of Quebec's baby bonus and shows that an extra 1,000 Canadian dollars benefit would increase fertility by 16.9%, which implies a benefit elasticity of 0.107. Cohen et al. (2013) uses a variation in the child subsidy payment observed around 2003 in Israel, providing a larger subsidy for third or higher births. They show that the benefit elasticity of fertility was 0.176. González (2013) adopts the regression discontinuity design to study the effects of Spain's reform in 2007, introducing a one-time payment of 2,500 euros (about 3,800 USD) for births, almost 4.5 times the monthly minimum wage for full-time workers. It finds a statistically significant impact on fertility, increasing conceptions by $5-6\%$.

To examine how this model performs in this respect, I conduct the following exercise. Let B_0 denote the per-child cash transfers for households with children under 17 in the benchmark. I solve the household problem, holding prices, tax rate, and distribution fixed, with several levels of the per-child payment $B = B_0 \cdot (1+x)$ for some $x \in X$, where X is a set of positive real numbers. This procedure brings the implied fertility rate, and with the expansion rate x, we can compute the implied benefit elasticity of fertility for the case of the expansion rate x. I set $X = \{0.1, 0.2, ..., 1.9, 2.0\}$, which is a reasonable range in the context of the expansion examined in the empirical studies, and compute the implied elasticity for each x. Then, I take the average of those 20 values. I find that the average elasticity is 0.138 (with a standard deviation of 0.025), which is consistent with the empirical estimates.

4.2.2 The fertility differential across education levels

More educated parents have fewer children than less educated ones. According to my sample of the JPSC, college graduate parents' completed fertility was 1.92, which is lower than the rest's, 2.12. This is observed in another data source that we can check the completed fertility by different education levels, the NFS. According to the NFS, college graduate wives' completed fertility has been lower than less educated ones almost every survey year since 1977. The latest survey in 2015 reports that the completed fertility of wives with a college degree was 1.89, and that of high school graduate wives was 1.89. In this benchmark model, the completed fertility of college graduate wives is 1.79, which is lower than the high school graduate wives', 2.28. The benchmark model captures the qualitative feature of the fertility differential across education levels, although the gap is somewhat significant compared with data counterparts. Table 5 summarizes the fertility differential across education levels in the model and data.

	Model	JPSC	NFS
HS	2.28	2.12	1.98
CL	1.79	1.92	1.89

Table 5: Fertility differential across education in the benchmark. *Note*: "NFS" stands for the National Fertility Survey conducted by the ISPP, and the table reports the values from the 2015 survey. It reports the completed fertility of wives with different educational backgrounds.

A primary factor behind this gap is the differences in opportunity costs, given that having children requires a fixed fraction of time. More educated parents face the higher opportunity costs of having children as their potential earnings are higher. In this model, the intergenerational persistence of education can also generate this differential because more educated parents' children, if any, are more likely to attend college, which raises financial costs in the form of more asset transfers upon children's college enrollment.

4.2.3 Composition of students' revenue

Capturing the composition of students' revenue − how college students finance their living expenses and tuition fees − is also important as it is critical not only to students' education choices but also to parents' IVT choices and, thereby, fertility choices. According to the SLS (2018), the students' revenue consists of three parts. First, the greatest part, 61% of their revenue, is accounted for by asset transfers from their parents. Second, students' labor earnings account for 21%, and lastly, the rest (18%) is financed by student loans. Although the revenue share of labor earnings (21%) is a targeted moment, the rest is not targeted. As Table 6 shows, the model captures the overall revenue composition as well; the IVT and loans account for 66% and 14% of their revenue, which is close to the data counterparts.

	IVT	Loan	Labor
Data	0.61	0.18	0.21
Model	0.66	0.14	0.20

Table 6: Composition of Students' Revenue.

5 Numerical Analysis

This section investigates the effects of education subsidies for college students in the model with fertility choices calibrated in the previous section. Section 5.1 simulates the introduction of the existing income-tested subsidy started in 2020. Section 5.2 then examines the mechanism through which the macroeconomic effects of the introduction are realized. Finally, Section 5.3 investigates the effects of raising the income threshold so that students in broader income classes of households are eligible. Following the literature, I adjust the labor income tax to balance the government budget upon the introduction and expansion.

5.1 Introducing Education Subsidies

In 2020, the Japanese government introduced income-tested subsidies for college students in low-income households; until then, the government has provided only student loans. Households are eligible if their last year's annual labor income is less than a non-negative value $\bar{I}$, and students in those households receive g amount of money each year while in college. The grant function $g(h, I)$ can be formulated as follows:

$$g(h, I) = \begin{cases} g & \text{if } I \leq \bar{I} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Note that the benchmark case can be interpreted as $g = 0$ for any $\bar{I}$. Although the income threshold $\bar{I}$ and payment g in the actual system vary with some characteristics of households and students, such as family structure and whether the student commutes to college from home or not, $\bar{I}$ approximately corresponds to the 15 percentile of the household income distribution, and the payment approximately amounts to two-thirds of the average expenses of college students. I set $\bar{I}$ and g based on this information, and income distribution and students' expenditure in the benchmark model (initial steady state). I then solve the stationary equilibrium by introducing this new grant function.

The main numerical results are as follows. First, introducing the means-tested subsidy would increase the college enrollment rate by almost 4 p.p. in the long run. Education mobility increases in the sense that children of high school graduate parents are 2.5 p.p. more likely to attend college.[30] Because of the higher college enrollment rate, implying a greater supply of college graduate labor, the skill premium decreases by 0.02 points. Introducing the subsidy would increase the TFR by 3%, largely driven by the fertility increase of college graduate parents.

Those changes in demographic structure and skill distribution affect other aggregate variables. In the long run, the higher college enrollment rate implies a higher share of skilled labor, and the higher TFR implies a greater share of the working-age population. As a result, per-capita labor supply in efficiency units increases by 1.3% in the long run.

[30] This mobility concept is a variant of an income mobility measure adopted in Zheng and Graham (2022), "economic opportunity."

On the contrary, per-capita capital decreases by 1.8% for some reasons. First, introducing the subsidy reduces saving incentives for a substantial fraction of households and crowd-out IVT.[31] In addition, the higher TFR implies a greater share of younger generations, who hold fewer assets than older ones. Despite its negative effects on capital accumulation, the positive impacts on the labor force and productivity are sufficiently greater so that the per-capita output increases by 0.7% in the long run.

5.2 Inspecting the Mechanism

Previous results imply that the subsidy introduction significantly affects the college enrollment rate and TFR, which leads to a greater output. However, the mechanism behind these changes is not so obvious because several objects aside from the grant function, such as prices, tax rate, and distribution, vary between the benchmark and new equilibrium, and each can play important roles in accounting for the overall effects on the TFR and college enrollment rate. In addition, the roles of fertility margins are also worth investigating, that is, how the results differ under exogenous fertility where the fertility choice is fixed. This is because the fertility setup would matter to the IVT and, thereby, children's education choices, given that fertility and IVT choices are joint decisions. Fertility margins can also have distributional implications through intergenerational linkages if fertility responses are heterogeneous across household characteristics. In this section, I conduct a decomposition analysis in Section 5.2.1 to discuss and understand the mechanism behind increases in the TFR and college enrollment rates upon introducing the subsidies. Next, in Section 5.2.2, I consider the roles of fertility responses in understanding the results by solving the exogenous fertility version of the model.

5.2.1 Decomposition: behavioral and composition effects

What causes the increases in the TFR and college enrollment rates? Those increases can be broken down into behavioral effects and distributional (or composition) effects, where the behavioral effects can be further broken down into the direct effects, driven only by a change in the subsidization scheme (i.e., grant function $g(h, I)$), and the indirect effects, driven by changes in market prices (i.e., GE effects) or tax rates (i.e., Taxation effects). Note that the direct effects can also be interpreted as the short-run effects of the introduction, where prices, tax, and distribution are fixed. Finally, the distributional effects

[31] Note that some households, characterized on the state space, can increase savings due to the subsidy introduction. For example, households whose children do not attend college in the benchmark but attend college when the subsidy is introduced may need more assets in the new equilibrium to make some IVT upon children's college enrollment. This is because the grants are not sufficiently significant to cover 100% of the expenditures for students.

capture changes driven by distribution changes over state variables such as education, age, and human capital. To isolate those effects, I conduct a decomposition exercise as follows. I first solve an equilibrium by introducing the new grant function. Then, I solve household problems by replacing one of the four objectives (grant function, prices, tax rate, and state distribution) in the benchmark with that in the new equilibrium. Note that this method does not guarantee that each implied effect adds up to one because all factors except grant function are endogenous and interconnected. Table 7 summarises the decomposition results.

College Enrollment: For the long-run increase in the college enrollment rate, critical forces are direct and distributional effects, whereas GE and taxation effects do not play significant roles, at least in the case of introduction. If the grant is introduced while other variables are fixed as in the benchmark (e.g., prices, tax rate, and distribution), the college enrollment rate increases by 2.6 p.p., more than half of the overall increase in the long run. These direct effects capture the effects of relaxing financial constraints on college enrollment decisions. Note that these direct effects do not explain the overall effects, so we need other forces accounting for the increase in college enrollment rate; that turns out to be the distributional effects.

The implied changes in the distribution in the long run, holding grant function, prices, and tax rate fixed as in the benchmark, lead to a 1.9 p.p. higher college enrollment rate. The most relevant factor is the change in the skill distribution of parents (i.e., the share of college graduates). The increase in the share of college graduates implies a higher share of those more likely to have children enrolling in college due to the intergenerational persistence of education. Then, the distributional effects amplify the short-run increase in college enrollment rate due to the direct effects in the long run.

Fertility: For the long-run increase in the TFR, vital forces are direct and GE effects. If the grant is introduced while other variables are fixed as in the benchmark, the TFR increases by 1.5% in the long run, which corresponds to the half of the overall effects. As in explaining the overall effects, that 1.5% increase due to the direct effects are largely driven by the fertility increase of college graduate parents; college graduate parents increase fertility by 4.5%, whereas high school graduate parents do by 1%.

To understand these direct effects on fertility, note that parents do not know whether they are beneficiaries of the grants when they make fertility choices, eighteen years before their children graduate high school and the eligibility is determined. They are beneficiaries if (1) their children enroll in college and (2) their earnings, at the period when their children choose whether to attend college, are sufficiently low. Those are subject to uncertainty regarding their productivity and children's characteristics (school tastes and human capital), which are not realized when they make fertility choices.

Now, let us look at changes in the average number of children across income distribution at the timing of IVT choices between the benchmark and new equilibrium. The average number of children for households in 1st income quintile, in which almost all children are eligible for grants if they enroll in college, remains the same. This means that the TFR increase is explained by fertility increases by ex-post ineligible households. Actually, households in 2nd and 3rd income quintiles increase fertility by 5.7%, and those in 4th and 5th income quintiles increase by 3.9%, on average. This may not be so surprising, given that I mentioned that college graduate parents increase fertility, and their income is relatively high.

Why can the subsidy introduction increase fertility, even that of ex-post ineligible households? The answer is due to an insurance effect of the income-tested subsidy. As I explained above, whether their potential children attend college is uncertain before giving birth. If their children have characteristics favoring college enrollment and thus they would like to make more IVT to support them, it significantly increases the financial costs of children. In other words, in addition to income risks throughout their working periods, they face a type of expenditure uncertainty until their children's education choices terminate.

Consider that those "shocks" are realized simultaneously; their children have characteristics willing to attend college, and they are also willing to make a sufficient amount of IVT, but they are poor due to the realization of negative income shocks. Without the income-tested grants, those unlucky households end up with less consumption and/or IVT, which may prevent their children from enrolling in college; these come at significant utility losses. Thus, in fertility decision-making, they may choose fewer children to avoid such a situation in the future, and the income-tested grants provide insurance against those risks and can improve fertility rates. Because college graduate parents are more likely to have children with characteristics favoring college enrollment, they benefit more, at least in the ex-ante sense, and their fertility behavior responds stronger.

Finally, the decomposition result shows that the GE effect also plays a role in accounting for the TFR increase. If the prices are set to the long-run equilibrium levels while other objects are fixed as in the benchmark, the college graduates' fertility increases by 2.5%, and the TFR then increases by 0.8%. The key is the decline of the wage rate for college graduate workers, w_{CL}. In the long-run equilibrium with the grants, w_{CL} decreases by 2.2%. First, the greater supply of college graduate workers depress w_{CL}. In addition, greater aggregate labor supply in efficiency units and lower capital accumulation discussed in Section 5.1 imply the lower marginal productivity of labor, further decreasing w_{CL}. The lower w_{CL} implies the lower opportunity costs of having children for college graduates, which makes some of them increase fertility.

	Bench.	Grant	Prices	Tax	Dist.	All
CL share	37.6	40.2	37.6	37.6	39.5	41.5
TFR	2.096	2.128	2.113	2.096	2.088	2.160
HS	2.282	2.304	2.283	2.283	2.280	2.290
CL	1.786	1.867	1.830	1.787	1.794	1.978

Table 7: Decomposition results. *Note*: Columns "Grant," "Prices," "Tax," and "Dist." report the results when only grant, prices, labor income tax rate, and distribution change, respectively. Columns "Bench." and "All" indicate the results in the benchmark and the long-run equilibrium with the grants. Rows "CL share" represent college enrollment rate, and rows "HS" and "CL" represent fertility rates for high school and college graduates.

5.2.2 Roles of Fertility Responses

I next solve the exogenous fertility version of the model, a standard setup used in the literature on financial aid for college students. More specifically, I follow the same procedure in the previous section 5.1, except that the policy functions for fertility are fixed as in the benchmark.

Table 8 reports the main results under exogenous fertility. First, the introduction of grants results in a 2.9 p.p. increase in the college enrollment rate under exogenous fertility, which is lower than a 3.9 p.p. increase under endogenous fertility. Under both exogenous and endogenous fertility, the grants make some children who otherwise cannot enroll in college. Under endogenous fertility, in addition to that effect, the college enrollment rate can further increase through fertility margins. As I discussed in Section 5.1, college graduate parents increase fertility than high-school graduate parents. Their children will likely attend college due to the intergenerational transmission of school tastes and human capital. And when those children become parents in the future, their children, if they have, are also likely to have similar characteristics to theirs, favoring college enrollment. Through this mechanism, the long-run share of college graduates increases, implying that the fertility margins amplify the effects on college enrollment.

From the second row onward, the changes in aggregate variables are reported in terms of percentage changes. Under exogenous fertility, per-capita labor in efficiency units increases by 0.2 % compared with the benchmark mainly because of the higher share of skilled labor. However, the extent of this increase is modest compared with a 1.3% increase under endogenous fertility because the college enrollment rate is lower and working-age population share remains constant.[32] However, because of the fixed demographic structure, the negative effects on capital realized under endogenous fertility disappear. Rather, the introduction would lead to a 0.6% increase in per-capita capital under exogenous fertility. The per-capita output would increase by 0.7 %, the same degree as under endogenous fertility.

	Bench.	Endogenous	Exogenous
CL share	37.6	41.5	40.5
Output	–	+0.7	+0.7
Capital	–	−1.8	+0.6
Labor	–	+1.3	+0.2
Tax	35.0	35.04	35.13

Table 8: Results under exogenous fertility. *Note*: Results for output, labor, and capital represent percentage changes compared with the benchmark.

5.3 Expansion

Finally, I consider the effects of raising the income threshold $\bar{I}$ so that students in households of broader income classes can be eligible. Recall that I set $\bar{I}$ so that the existing subsidy targets the students in households at the bottom 15 % of the income distribution. In this experiment, I increase $\bar{I}$ to correspond to the 40, 50, and 60 percentile of the income distribution, and solve the stationary equilibrium each case. For students in the household at the bottom 15 %, the payment is still given by g, which amounts to two-thirds of the average expenses of students. For students in households higher than the 15 percentile but less than x percentile of the income distribution, where x takes either 40, 50, or 60, the payment is given by $g/2$, which amounts to one-third of the average expenses of students. In other words, the payment tapers off in income. Letting $\bar{I}_{15\%}$ and $\bar{I}_{x\%}$ denote the income level of 15 and $x \in \{40, 50, 60\}$ percentiles of the income distribution, the grant function $g(h, I; x)$ with a threshold $\bar{I}_{x\%}$ can now be formulated as:

$$g(h, I; x) = \begin{cases} g & \text{if } I \leq \bar{I}_{15\%} \\ g/2 & \text{if } I \in (\bar{I}_{15\%}, \bar{I}_{x\%}] \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

[32]Note that fixing policy functions for fertility does not necessarily mean that the TFR is also the same as in the benchmark because the household distribution changes, which can change the TFR even though the policy functions are fixed. However, it turns out that the TFR does not change in this case of exogenous fertility.

Table 9 summarizes the results and there are several takeaways presented. First, the equilibrium college enrollment increases as the income threshold is higher. Setting the threshold at the 40, 50, and 60 percentiles of the income distribution, the enrollment rate would be 42.3%, 43.2%, and 43.8%, respectively. This is almost the case for per-capita output and labor income tax rates. With the income threshold at the bottom 60 % of the distribution, the per-capita output increases by 1.53%, which is more significant than 0.7% in the case of introduction. The expansion requires additional revenue, so the equilibrium tax rate also should increase by 0.3 p.p. for the case of the 60% threshold.

However, the expansion would not significantly increase the TFR. Recall that the introduction would lead to a TFR of 2.160 from the benchmark level of 2.096. With grant functions $g(h, I, 40)$, $g(h, I, 50)$, and $g(h, I, 60)$, the equilibrium TFR would be 2.158, 2.151, and 2.157, respectively; the TFR even decreases locally by expansions. Fertility rates for each skill help us understand this situation. First, college graduate parents continue to increase fertility. With grant functions $g(h, I, 40)$, $g(h, I, 50)$, and $g(h, I, 60)$, their equilibrium fertility rate would be 1.996, 1.998, and 2.021, all higher than the fertility rate with the existing subsidy, 1.978. This result is straightforward to understand given that the insurance and GE effects contribute to the fertility increase of college graduate parents when the grants are introduced, discussed in Section 5.2. On the contrary, high school graduate parents reduce fertility instead, making the TFR remain almost constant even though the income threshold is higher. Fig 8 depicts the changes in the TFR and fertility rates for high school and college graduates, where the equilibrium TFR with the existing program is normalized to one.

	Bench.	Threshold 15%	40%	50%	60%
CL share	37.6	41.5	42.3	43.2	43.8
TFR	2.096	2.160	2.158	2.151	2.157
HS	2.282	2.290	2.277	2.267	2.263
CL	1.786	1.978	1.996	1.998	2.021
CL/HS	0.78	0.86	0.88	0.88	0.89
Output	–	+0.70	+0.15	+1.07	+1.53
Tax	35.00	35.04	35.17	35.23	35.30

Table 9: Main results of higher income thresholds. *Note*: A row "CL share" represents the college enrollment rate, and rows "HS" and "CL" represent fertility rates for high school and college graduates. Output changes are expressed as percentage changes.

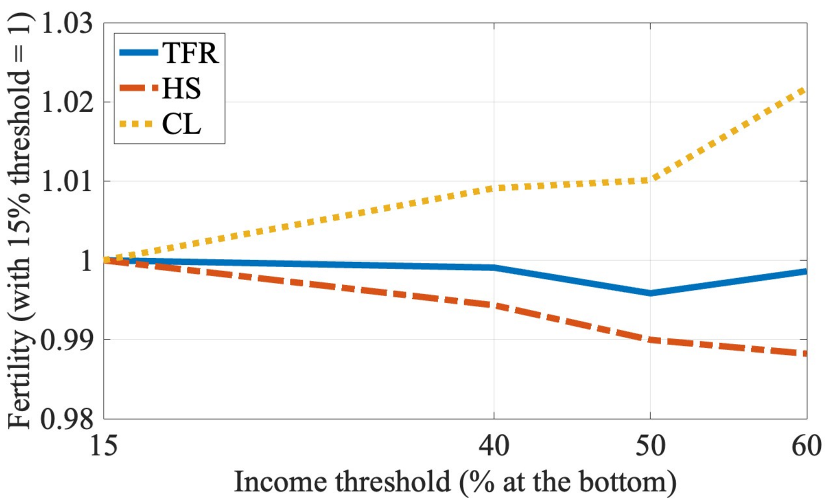

Fig 8: Changes in fertility rates with expansion. *Note*: The equilibrium TFR with the existing program (with the 15% threshold) is normalized to one. "HS" and "CL" represent fertility rates for high school and college graduates.

Why do high school graduate parents decrease fertility as the income threshold is higher? To understand this, I conduct the decomposition when the grant function is given by $g(h, I, 60)$, and the results are summarized in Table 10. Two effects are critical to explain the fertility decreases of high school graduates: the direct and GE effects.

In this case with the grant function $g(h, I, 60)$, the wage rate for high school graduates, w_{HS}, is 0.6% higher in the long run than in the benchmark, especially because of the significant supply of college graduates. This higher wage rate makes the opportunity costs of having children for high school graduates higher, which puts downward pressure on their fertility rates.

Next, the direct effects imply a lower fertility rate for high school graduates. This result is somewhat confusing given that we discuss the insurance effects of the subsidy on fertility, which is critical to account for the fertility increases of college graduates. Why do the direct effects lead to lower fertility for high school graduates? In the benchmark without grants, children of high school graduate parents are less likely to attend college. Hence, the probability that they face the "expenditure risk" arising from children's education choices is lower. If the grants are introduced and their target expands, education mobility increases in the sense that children of high school graduate parents are more likely to attend college than in the benchmark. Given that the grants are not generous enough to cover 100% of the costs to send their children to college, this higher probability of children going to college implies that those parents are more likely to have to make additional transfers upon children's college enrollment, increasing the expected costs of children and expenditure uncertainty. Thus, the expansion would lower fertility for high school graduates.

	Bench.	Grant	Prices	Tax	Dist.	All
HS	2.282	2.265	2.250	2.283	2.279	2.263
CL	1.786	1.948	1.863	1.829	1.797	2.021

Table 10: Decomposing the effects on fertility with $g(h, I, 60)$. *Note*: Rows "HS" and "CL" represent fertility rates for high school and college graduates.

A related work Zhou (2022) shows that public education subsidies are less cost-effective in increasing fertility than other pro-natal measures, including unconditional cash transfers such as baby bonuses. This result holds because the education subsidies increase parents with greater human capital, who tend to have fewer children. In other words, education subsidies put downward pressure on fertility via composition effects. Although the implication of the education policies for the fertility rate is similar to this study in terms of their limited effects, the mechanism is different. In his work, the composition effect is vital, while, in my study, the critical mechanism is that higher college enrollment and greater education mobility increase the expected costs of children for low-skilled households. The result builds on the model with discrete choice on college enrollment and the resulting uncertainty about the financial costs of children.

6 Concluding Remarks

This study constructs a GE-OLG model embedding choices on college enrollment, IVT, and fertility to investigate the macroeconomic implications of education subsidies for college students. The model is calibrated to the Japanese economy using panel data. I show that the existing means-tested grants introduced in 2020 would increase the fertility of high-skilled parents, including ex-post ineligible ones, by providing insurance against income and expenditure risks arising from having children. They would also increase the college enrollment rate, amplified by fertility margins and distributional effects in the long run. Setting a higher income threshold would increase high-skilled parents' fertility further. However, the low-skilled parents would decrease instead because, contrary to the high-skilled, the expected costs would significantly increase due to the higher probability of their children going to college, stemming from the greater education mobility thanks to the expansion.

This paper concludes by listing several points not considered in this preliminary work and to be examined further for future revision. First, while the current version focuses on the positive aspects of education subsidies, examining their welfare effects can also be relevant. In doing so, I would build on the work by Golosov et al. (2007) and adopt efficiency criteria that can be applied to models with endogenous fertility. Second, using the model to answer further questions actively discussed in Japan would also be a promising direction. For example, the Japanese government discusses the expansion of the income threshold for income-tested grants for college students conditional on the number of children a household has as a pro-natal measure. The model has a comparative advantage in simulating this potential reform, given that the model captures the discreteness of fertility choices, unlike previous studies that consider the number of children as a continuous variable.[33] Finally, solving the transition dynamics and investigating the time horizon of the policy effects will provide another picture of the effects of education subsidies with demographic change through policy effects on fertility.

[33] For example, see, De La Croix and Doepke (2003), De la Croix and Doepke (2004), Zhou (2022), Nakakuni (2023), etc.

References

Abbott, B. (2022). Incomplete markets and parental investments in children. *Review of Economic Dynamics*, 44:104–124.

Abbott, B., Gallipoli, G., Meghir, C., and Violante, G. L. (2019). Education policy and intergenerational transfers in equilibrium. *Journal of Political Economy*, 127(6):2569–2624.

Akyol, A. and Athreya, K. (2005). Risky higher education and subsidies. *Journal of Economic Dynamics and Control*, 29(6):979–1023.

Benabou, R. (2002). Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica*, 70(2):481–517.

Bick, A. (2016). The quantitative role of child care for female labor force participation and fertility. *Journal of the European Economic Association*, 14(3):639–668.

Cavalcanti, T., Kocharkov, G., and Santos, C. (2021). Family planning and development: Aggregate effects of contraceptive use. *The Economic Journal*, 131(634):624–657.

Cohen, A., Dehejia, R., and Romanov, D. (2013). Financial incentives and fertility. *The Review of Economics and Statistics*, 95(1):21.

Cordoba, J. C., Liu, X., and Ripoll, M. (2019). Accounting for the international quantity-quality trade-off. Working Paper.

Daruich, D. (2022). The macroeconomic consequences of early childhood development policies. Working paper.

Daruich, D. and Kozlowski, J. (2020). Explaining intergenerational mobility: The role of fertility and family transfers. *Review of Economic Dynamics*, 36:220–245.

De La Croix, D. and Doepke, M. (2003). Inequality and growth: why differential fertility matters. *American Economic Review*, 93(4):1091–1113.

De la Croix, D. and Doepke, M. (2004). Public versus private education when differential fertility matters. *Journal of Development Economics*, 73(2):607–629.

Erosa, A., Fuster, L., and Restuccia, D. (2010). A general equilibrium analysis of parental leave policies. *Review of Economic Dynamics*, 13(4):742–758.

European Commission (2018). First results of Poland's Family 500+ programme released. https://ec.europa.eu/social/main.jsp?langId=en&catId=1246&newsId=9104&furtherNews=yes.

Gauthier, A. H. (2016). Governmental support for families and obstacles to fertility in east asia and other industrialized regions. *Low fertility, institutions, and their policies: Variations across industrialized countries*, pages 283–303.

Golosov, M., Jones, L. E., and Tertilt, M. (2007). Efficiency with endogenous population growth. *Econometrica*, 75(4):1039–1071.

González, L. (2013). The effect of a universal child benefit on conceptions, abortions, and early maternal labor supply. *American Economic Journal: Economic Policy*, 5(3):160–88.

Guner, N., Kaygusuz, R., and Ventura, G. (2020). Child-related transfers, household labour supply, and welfare. *The Review of Economic Studies*, 87(5):2290–2321.

Hagiwara, R. (2021). Macroeconomic and welfare effects of the fiscal and social security reforms (in japanese). *Unpublished Manuscript*. https://hermes-ir.lib.hit-u.ac.jp/hermes/ir/re/71565/eco020202000503.pdf.

Jakobsen, K. M., Jørgensen, T., and Low, H. (2022). Fertility and family labor supply. *CESifo Working Paper*.

Kim, S., Tertilt, M., and Yum, M. (2023). Status externalities and low birth rates in korea. *Working paper*.

Kitao, S. (2015). Fiscal cost of demographic transition in japan. *Journal of Economic Dynamics and Control*, 54:37–58.

Kitao, S. and Nakakuni, K. (2023). On the trends of technology, family formation, and women's time allocations. *Working Paper*.

Kotera, T. and Seshadri, A. (2017). Educational policy and intergenerational mobility. *Review of economic dynamics*, 25:187–207.

Krueger, D. and Ludwig, A. (2016). On the optimal provision of social insurance: Progressive taxation versus education subsidies in general equilibrium. *Journal of Monetary Economics*, 77:72–98.

Kulikova, Y. (2015). Health policies and intergenerational mobility. Technical report, Mimeo.

Lawson, N. (2017). Liquidity constraints, fiscal externalities and optimal tuition subsidies. *American Economic Journal: Economic Policy*, 9(4):313–343.

Lee, S. Y. and Seshadri, A. (2019). On the intergenerational transmission of economic status. *Journal of Political Economy*, 127(2):855–921.

Malkova, O. (2018). Can maternity benefits have long-term effects on childbearing? evidence from soviet russia. *Review of Economics and Statistics*, 100(4):691–703.

Manuelli, R. E. and Seshadri, A. (2009). Explaining international fertility differences. *The Quarterly Journal of Economics*, 124(2):771–807.

Matsuda, K. (2022). Progressive taxation versus college subsidies with college dropout. *Journal of Money, Credit and Banking*.

Matsuda, K. and Mazur, K. (2022). College education and income contingent loans in equilibrium. *Journal of Monetary Economics*, 132:100–117.

Milligan, K. (2005). Subsidizing the Stork: New Evidence on Tax Incentives and Fertility. *The Review of Economics and Statistics*, 3(87):18.

Nakakuni, K. (2023). Macroeconomic analysis of the child benefit: Fertility, demographic structure, and welfare. *Working Paper*.

Santos, C. and Weiss, D. (2016). “why not settle down already?” a quantitative analysis of the delay in marriage. *International Economic Review*, 57(2):425–452.

Schoonbroodt, A. and Tertilt, M. (2014). Property rights and efficiency in olg models with endogenous fertility. *Journal of Economic Theory*, 150:551–582.

Sommer, K. (2016). Fertility choice in a life cycle model with idiosyncratic uninsurable earnings risk. *Journal of Monetary Economics*, 83:27–38.

The Guardian (2020). 'It's national preservation': Greece offers baby bonus to boost birthrate. https://www.theguardian.com/world/2020/feb/04/its-national-preservation-greece-offers-baby-bonus-to-boost-birthrate.

Yamaguchi, S. (2019). Effects of parental leave policies on female career and fertility choices. *Quantitative Economics*, 10(3):1195–1232.

Zheng, A. and Graham, J. (2022). Public education inequality and intergenerational mobility. *American Economic Journal: Macroeconomics*, 14(3):250–282.

Zhou, A. (2022). The macroeconomic consequences of family policies. *Available at SSRN*, 3931927.

Appendix A Equilibrium Definition

Let $\boldsymbol{x}_j^e$ be an age-specific state vector for agents with education level $e \in \{HS, CL\}$ and $\mu_j^e(\boldsymbol{x}_j^e)$ be the measure of agents with state vector $\boldsymbol{x}_j^e$. Let $I_l(h, I)$ and $I_g(h, I)$ be indicator functions for loans and grants, respectively, returning 1 if students with (h, I) are eligible and 0 otherwise.

Given exogenous parameters and policy rules $\{\tau_a, \tau_c, \iota, \iota_s, B, S, \psi, p, I_l(h, I), I_g(h, I)\}$, a *stationary recursive competitive equilibrium* consists of

- value functions $\{V_{g0}, V_{g1}, V_{g2}, V^w, V^f, V^{fw}, V^{IVT}, V^r\}$,
- policy functions for consumption, savings, leisure $\{c_j^e(\boldsymbol{x}_j^e), a_j^e(\boldsymbol{x}_j^e), l_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J}$, working hours $\{h_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J_R}$, fertility $\{n_{J_F}^e(\boldsymbol{x}_{J_F}^e)\}$, IVT $\{a_{J_{IVT}^e}(\boldsymbol{x}_{J_{IVT}}^e)\}$, and college enrollment $\{e_{J_E}(\boldsymbol{x}_{J_E})\}$,
- prices (r, w_{HS}, w_{CL}),
- labor income tax rate τ_w,
- aggregate quantities (K, L_{HS}, L_{CL}),
- measures for households $\{\mu_j^e(\boldsymbol{x}_j^e)\}_{j=J_E}^{J}$,

such that:

1. The decision rules of students, workers, and retired households solve their problems, and $\{V_{g0}, V_{g1}, V_{g2}, V^w, V^f, V^{fw}, V^{IVT}, V^r\}$ are the associated value functions.

2. The representative firm maximizes its profit and optimally chooses capital and labor inputs:

$$r + \delta = \alpha \cdot Z \cdot \left(\frac{K}{L}\right)^{\alpha-1}, \tag{8}$$

$$w_{HS} = \tilde{Z} \cdot \omega_{HS} \cdot L_{HS}^{\chi-1}, \tag{9}$$

$$w_{CL} = \tilde{Z} \cdot \omega_{CL} \cdot L_{CL}^{\chi-1}, \tag{10}$$

where

$$L = [\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}]^{1/\chi},$$

$$\tilde{Z} = (1-\alpha) \cdot Z \cdot \left(\frac{K}{L}\right)^{\alpha} \cdot [\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}]^{1/\chi-1}.$$

3. The labor market for each skill $e \in \{HS, CL\}$ clears:

$$L_e = \sum_{j=J_E}^{J_R} \int_{\boldsymbol{x}_j^e} \eta_j^e(\boldsymbol{x}_j^e) \cdot h_j^e(\boldsymbol{x}_j^e) \; d\mu_j^e(\boldsymbol{x}_j^e), \tag{11}$$

where $\eta_j^e(\boldsymbol{x}_j^e)$ represents the labor efficiency of agents with a state vector $\boldsymbol{x}_j^e$.

4. The capital market clears:

$$K = \sum_{j=J_E}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} a_j^e(\boldsymbol{x}_j^e) \; d\mu_j^e(\boldsymbol{x}_j^e). \tag{12}$$

5. The government budget is balanced:

$$\tau_c \cdot C + \tau_w \cdot (L_{HS} + L_{CL}) + \tau_a \cdot K + Q = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \leq 17} + S,$$

where

$$\begin{aligned}
C &= \sum_{j=J_E}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} c_j(\boldsymbol{x}_j^e) \; d\mu_j^e(\boldsymbol{x}_j^e), \\
Q &= \sum_{j=J_R+1}^{J} \frac{1 - \zeta_{j-1,j}}{\zeta_{j-1,j}} \sum_{e} \int_{\boldsymbol{x}_j^e} a_j^e(\boldsymbol{x}_j^e) \; d\mu_j(\boldsymbol{x}_j^e), \\
p \cdot \mu_{old} &= \sum_{j=J_R}^{J} \sum_{e} \int_{\boldsymbol{x}_j^e} p \; d\mu_j^e(\boldsymbol{x}_j^e), \\
K_s &= \int_{\boldsymbol{x}^s} \max\{0, -a^s(\boldsymbol{x}^e)\} \cdot I_l(h, I) \; d\boldsymbol{x}^s \\
G &= \int_{\boldsymbol{x}^s} g(h, I) \cdot I_g(h, I) \; d\boldsymbol{x}^s \\
B \cdot \mu_{j \leq 17} &= \sum_{j=J_F}^{J_{IVT}-1} \sum_{e} \int_{\boldsymbol{x}_j^e} B \cdot n \; d\mu_j^e(\boldsymbol{x}_j^e),
\end{aligned}$$

where $\boldsymbol{x}^s$, $\mu^s(\boldsymbol{x}^s)$, and $\{a^s(\boldsymbol{x}^s)\}$ represent a state vector for college students, measure of college students, and students' policy function for saving, respectively.

6. Distributions (measures) and households' behavior are consistent.

Appendix B Computational Algorithm: Stationary Equilibrium

For any variable or distribution x, let $\tilde{x}$ and $\hat{x}$ represent its guessed value and model-implied values, respectively. Also, let μ_j represent the distribution over state variables for age j. Dividing the computation process into three blocks makes it easier to understand. The first block is the outer loop, searching for equilibrium prices. The other two blocks are inner loops; one is the *optimization block*, solving household problems given prices, and another is the *distribution block*, searching for the stationary distributions given prices and policy functions obtained in the optimization block. The computational algorithm proceeds as follows:

1. Guess prices $\tilde{\boldsymbol{p}} = (\tilde{r}, \tilde{w}_{HS}, \tilde{w}_{CL})$.

2. *Optimization block*:

   - Guess the value function for agents at the beginning of age $j = 18$ ($\tilde{V}_{g0}$).
   - Given $\tilde{V}_{g0}$, solve backward from the period of IVT choice to that of the education choice, which gives the model-implied value function for V_{g0}, $\hat{V}_{g0}$.
   - Check if
   $$d(\tilde{V}_{g0}, \hat{V}_{g0}) < \varepsilon, \tag{13}$$
   where $d(\cdot)$ and $\varepsilon > 0$ represent an arbitrary metric function and error tolerance. If (13) is not satisfied, update $\tilde{V}_{g0}$ and follow the same procedure until convergence.
   - After obtaining the correct V_{g0}, solve backward from age $j = J$ to $j = J_{IVT}$, which gives all value functions and policy functions given a set of prices $\tilde{\boldsymbol{p}}$.

3. *Distribution block*:

   - Guess the distribution for age J_{IVT}. This $\tilde{\mu}_{J_{IVT}}$ and policy functions for IVT derive the implied distribution for agents aged 18, $\hat{\mu}_{18}$. Given $\hat{\mu}_{18}$ and policy functions, compute the implied distributions for age $j = 19, ..., J_{IVT}$, and obtain $\hat{\mu}_{J_{IVT}}$.
   - Check if
   $$d(\tilde{\mu}_{J_{IVT}}, \hat{\mu}_{J_{IVT}}) < \varepsilon. \tag{14}$$
   If (14) is not satisfied, update $\tilde{\mu}_{J_{IVT}}$ and follow the same procedure until convergence.

- After obtaing the correct distributions for age $j = 18, ..., J_{IVT}$, compute the distribution for age $j_{IVT} + 1$ onward, using those distributions and policy functions.

4. After computing value functions, policy functions, and distributions, compute the implied quantities, $\hat{K}$ and $\hat{L}$ based on (12) and (11), which gives the implied prices $\hat{\boldsymbol{p}}$ based on $\hat{K}$, $\hat{L}$, (8), (9), and (10).

5. Check if
$$d(\tilde{\boldsymbol{p}}, \hat{\boldsymbol{p}}) < \varepsilon. \tag{15}$$
If (15) is not satisfied, update $\tilde{\boldsymbol{p}}$, go back to the optimization block and follow the same procedure until convergence.