Financial Costs of Children, Education Subsidies, and Parental Choices in Equilibrium

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2. Model
3. Calibration
4. Numerical Analysis
5. Literature
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Low Fertility and Pro-natal Policies

- Low fertility and demographic aging has driven pro-natal policies.
  - A typical measure: cash benefits.
  - Empirical studies report their significant impacts on fertility.
    e.g., Milligan (2005, REStat), Cohen et al. (2013, REStat), González (2013, AEJEP) .
- In Japan, education subsidies for college students has garnered attention.

A significant financial cost gap

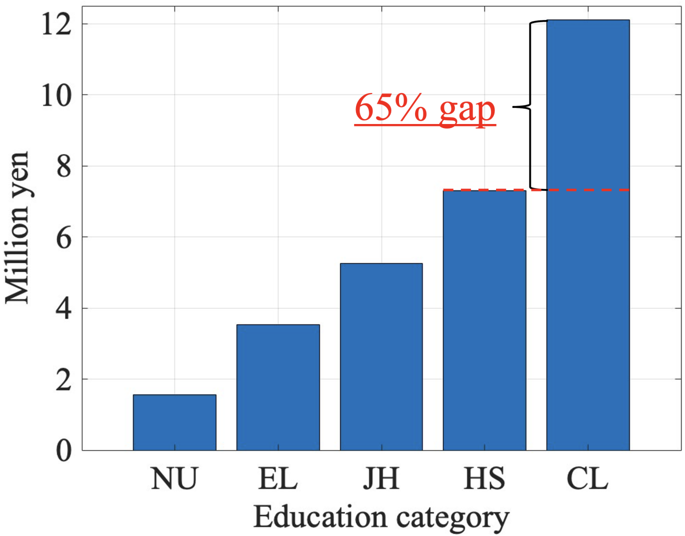


Figure: Cumulative education-expenditures. Sources: the SCLE (2021) and the SLS (2018) by the MEXT.

Japan's fewer subsidization in tertiary education

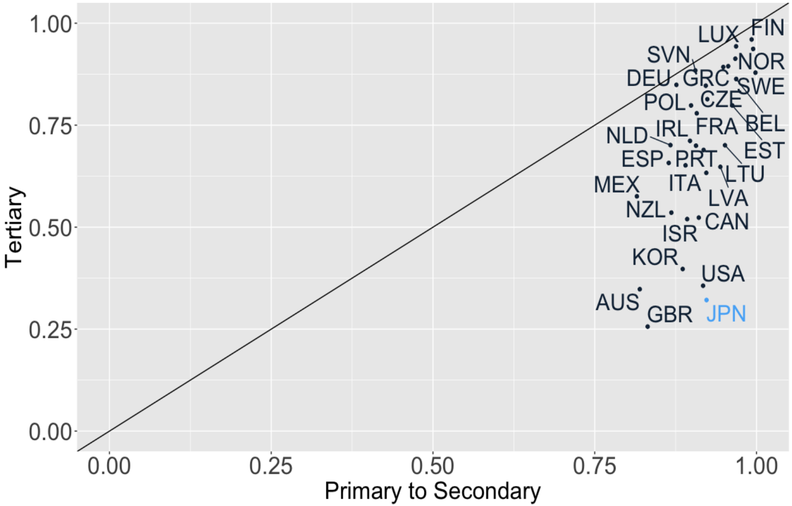


Figure: Subsidization rate for each education category (OECD, 2018).

Questions

- The subsidy is expected to increase (1) fertility and (2) workers' productivity.
  $\rightarrow$ "Two birds with one stone" against demographic aging.

- Does it work as expected? What are the macroeconomic consequences?

Questions

- The subsidy is expected to increase (1) fertility and (2) workers' productivity.
  $\rightarrow$ "Two birds with one stone" against demographic aging.
- Does it work as expected? What are the macroeconomic consequences?

– The answer is unclear:

- Education (Macro) literature abstracts fertility choices.

  e.g., Krueger and Ludwig (2016, JME), Abbott et al. (2019, JPE), Matsuda and Mazur (2022, JME).
- Macro models with fertility choices abstract college enrollment and IVT.

  e.g., De la Croix and Doepke (2003, AER), Sommer (2016, JME), Zhou (2022), Kim et al. (2023).

This Paper

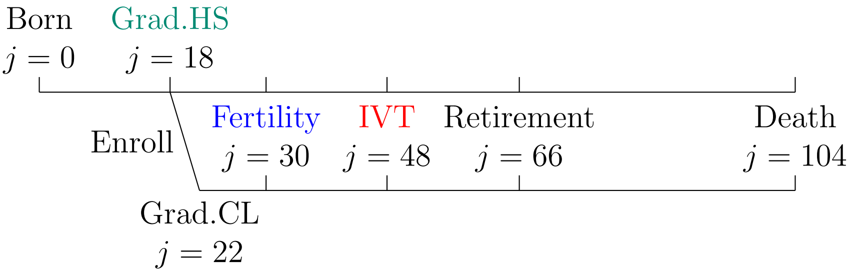
- Constructs an incomplete market GE-OLG model incorporating:
  - College enrollment choices,
  - Inter-vivo transfers (IVT),
  - Fertility choices.
- Calibrates the model to the Japanese economy using panel data.
- Validates if the model implies reasonable fertility behavior.
  - The benefit elasticity of fertility (comparison with empirical estimates).
  - Fertility differential across education groups.
- Examines the macroeconomic effects of education subsidies for college students.

Model

Model with fertility choices otherwise standard in (Macro) education literature.
Krueger and Ludwig (2016, JME), Abbott et al. (2019, JPE), Matsuda and Mazur (2022, JME), etc.

- Incomplete market GE-OLG framework.
- Standard lifecycle $\underbrace{+\text{ Education} + \text{IVT}}_{\text{Literature standard}}$ + Fertility
- Production: allows for the imperfect substitutability b/w skilled and unskilled.

Timeline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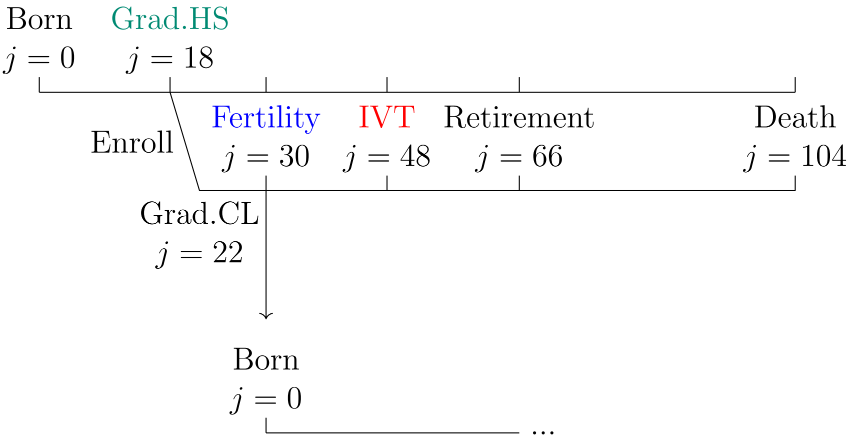
$j = 0$
Grad.HS
$j = 18$
Enroll
Fertility
$j = 30$
IVT
$j = 48$
Retirement
$j = 66$
Death
$j = 104$
Grad.CL
$j = 22$

Timeline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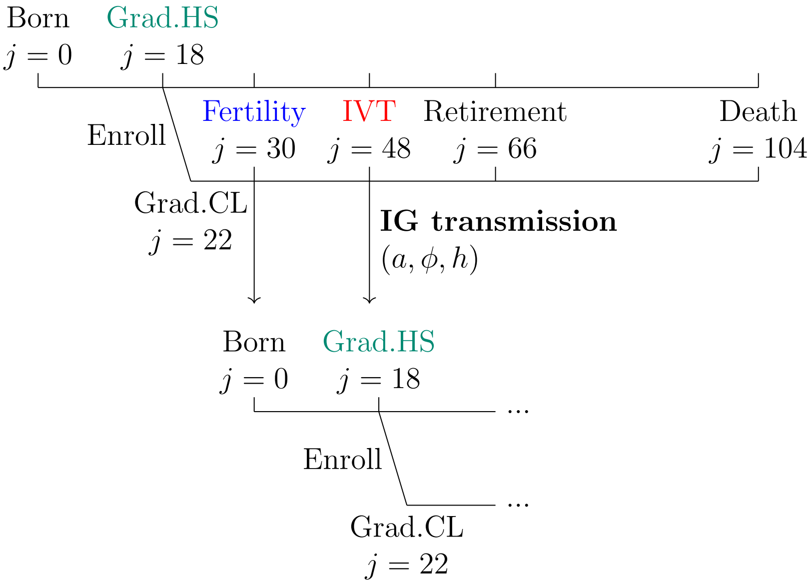
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IG transmission
(a, ϕ, h)
Born
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...
Enroll
...
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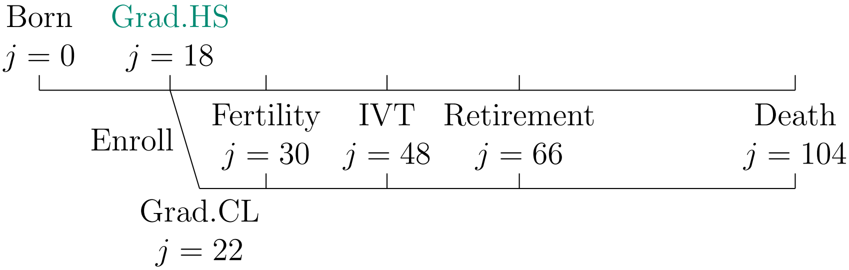
$j = 22$

Education choices



Three factors (IG linkages) influence their college enrollment decisions:

1. IVTs made by their parents (a_{CL}).
2. Psychic costs of education ($\phi \sim g^{\phi}_{h,e_p}$).
3. Their human capital ($h \sim g^{h}_{h_p}$).

Budget constraints: college students

Expenditures:

- Tuition fees p_{CL} (exogenous)
- Living expenses c (endogenous/choice)

Revenue:

- Transfers from their parents a_{CL} (parent's choice).
- Labor earnings by themselves (endogenous/choice).
- Loans provided by the government (depending on h and I).
- No grants in the benchmark (i.e., an economy before 2020).

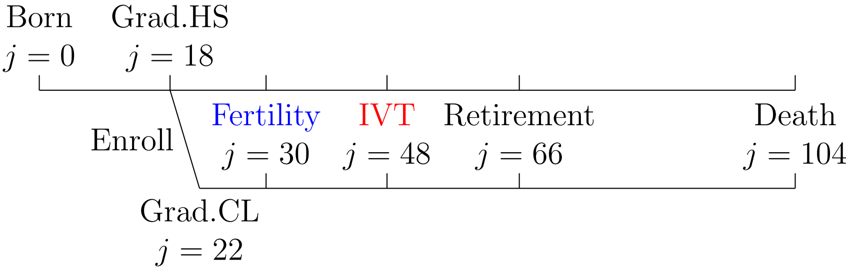
Education choices

$$V_{g0}(a_{IVT},\phi,h,I) = \max_{e\in\{0,1\}}\Big\{(1-e)\cdot\mathbb{E}_{z_0}[\underbrace{V^w(a_{IVT},j=18,z_0;e=0,h)}_{\text{Value for high school graduates}}] \\ +e\cdot[\underbrace{V_{g1}(a_{IVT};h,I)-\phi}_{\text{Net value for college graduates}}]\Big\}, \tag{1}$$

where $e = 1$ represents enrolling in college.

▸ Value functions

Fertility and IVT choices



Fertility choices are subject to multiple sources of uncertainty:

- Children's characteristics ($\phi_k \sim g^{\phi_k}_{h_k,\phi}$ and $h_k \sim g^{h_k}_h$) realized at $j = 48$.
- Future productivity ($\{z_j\}_{j>30}$) are also uncertain.
- Optimal IVT hinges on them, and so does the optimal number of children.

Inter-vivo transfers

$$
\begin{aligned}
V^{IVT}(a, z; \phi_k, h_k, e, h, n) = \max_{c,l,a',a_{IVT}} \Big\{ & V^w(a - \tilde{a}_{IVT}, j = J_{IVT}, z; e, h) \\
& + b(n) \cdot \lambda_a \cdot V_{g0}(a_{IVT}, \phi_k, h_k, I) \Big\} \\
\text{s.t.} \quad (1 + \tau_c)c + a' + n \cdot a_{IVT} &= Y_{IVT}, \\
a' &\geq -\underline{A},
\end{aligned}
$$

where $\tilde{a}_{CL} = \frac{n \cdot a_{CL}}{1+(1-\tau_a)r}$, $I = I(J_{IVT}, z, e, h)$, and Y_{IVT} is the disposable income.

Budget constraint details

Fertility choices and working stage with children I

The value function at age $j = J_f$:

$$V^f(a, z, e, h) = \max_{n \in \{0,1,..,N\}} \left\{ V^{wf}(a, j = J_F, z; e, h, n) \right\}$$

where, for $j = J_F, ..., J_{IVT} - 1$,

$$\begin{aligned} V^{fw}(a, j, z; e, h, n) = & \max_{c,l,q,a'} \{ u(c/\Lambda(n), l) + b(n) \cdot v(q) \\ & + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^{wf}(a', j+1, z'; e, h, n)] & \text{if } j < J_{IVT} - 1 \\ \beta \mathbb{E}_{z', \phi_k, h_k}[V^{IVT}(a', z'; \phi_k, h_k, e, h, n)] & \text{if } j = J_{IVT} - 1 \end{array} \right\} \\ \text{s.t.} & \\ & (1 + \tau_c)(c + nq) + a' = Y_{fw}, \\ & a' \geq -\underline{A}, \end{aligned}$$

Fertility choices and working stage with children II

where

$$
\begin{aligned}
Y_{wf} \equiv\ & (1-\tau_w) w_e \eta_{j,z,e,h} (1 - l - \kappa \cdot n) \\
& + n \cdot B + \psi + \begin{cases} (1+(1-\tau_a)r)a & \text{if} \quad a \geq 0, \\ (1+r^-)a & \text{otherwise.} \end{cases}
\end{aligned}
$$

Data

The Japanese Panel Survey of Consumers (JPSC)

- Panel survey of women and their household members.
- Starts in 1993 with 1,500 women aged 24-34.

Sample selection:

- Birth cohort: 1959-69
- Married (1993-2020)

Internally determined

Parameter	Value	Moment	Data	Model
μ	0.23	Work hours	0.33	0.30
$\bar{t}$	0.8	Income share of labor earnings	0.20	0.17
ι_s	0.055	Share of students using loans	0.44	0.34
ι	0.054	Household share with negative net worth	0.54	0.45
ω_h	0.52	CL−HS wage ratio	1.36	1.48
ψ	0.01	Var(log disposable income)/Var(log gross income)	0.60	0.68
λ_q	0.62	Average transfer / Average income at age 28	0.07	0.07
λ_a	1.03	Average transfer / Average income at age 28	0.27	0.27
ω	1.71	Intergenerational mobility of education	See Table 1	
σ_h	0.65	Variance of log(income) at age 28	0.27	0.24
ψ_{CL}	20.8	College enrollment rate	0.377	0.376
α_{CL}	0.1	Log wage ratio (CL−HS) at age 28	0.34	0.38
β_{CL}	0.1	Var log wage for CL at age 28	0.14	0.24
b_1	0.49	Share of one child	0.16	0.15
b_2	0.53	Share of two children	0.55	0.61
b_3	0.55	Share of three children	0.22	0.24
b_4	0.56	Share of four or more children	0.02	0.00
Z	1.99	Low skill wage	1.0	1.0

Education Mobility

Parents/Children	HS	CL
HS	0.725 (0.798)	0.275 (0.202)
CL	0.412 (0.423)	0.588 (0.577)

Table: Intergenerational transition matrix of education.

Validation and Non-targeted Moments

- The benefit elasticity of fertility. ▸ Detail
- Fertility differential across education. ▸ Table
- Composition of students' revenue.
  (IVT:Labor:Loan $\simeq$ 0.6:0.2:0.2)

Road Map

- Examining the effects of introducing grants.
- Expanding the income threshold for eligibility.
- Inspecting the mechanism:
  - Behavioral and distributional effects
  - Roles of endogenous fertility

Introducing Subsidies: What I Do

Introduce a grant $g(h, I)$ into students' BC and solve the equilibrium.

$$g(h, I) = \begin{cases} g & \text{if } I \leq \bar{I} \\ 0 & \text{otherwise} \end{cases}$$

- Threshold $\bar{I} \simeq$ the 15 percentile of income dist.
- Amount $g \simeq 2/3$ of the students' average expenses in the benchmark.
- Budget balance by adjusting the labor income tax rate.

Results: Education and Fertility

Education:

- College enrollment rate: 4 p.p. ↑
- Educational mobility (HS→CL): 2.5 p.p. ↑
- Skill premium (w_{CL}/w_{HS}): 0.02 points ↓

Fertility:

- TFR: 3% ↑
- Largely driven by fertility increases among college graduates.

Introducing Subsidies: Aggregate Quantity

- Efficiency labor (per-capita): 1.3% $\uparrow$
  - Higher CL share $\Rightarrow$ Larger share of skilled workers.
  - Higher TFR $\Rightarrow$ Larger working-age population share.
- Capital (per-capita): 1.8% $\downarrow$
  - Reduce saving incentives/Crowd out IVTs.
  - Higher TFR $\Rightarrow$ Larger share of younger generations, who hold fewer assets.
- Output (per-capita): 0.7% $\uparrow$

Inspecting the mechanism

Changes in the TFR and college enrollment can be driven by changes in:

1. Grant function $g(h, I)$
   "**Direct effects**"
2. Prices (in particular, w_{CL}/w_{HS})
   "**GE (Price) effects**"
3. Tax rate τ_l
   "**Taxation effects**"
4. Distributions (e.g., college graduates share)
   "**Distributional effects**"

Interpretations:

- Short-run (1) vs. Long-run (2-4)
- Behavioral (1-3) vs. Composition (4)

Decomposition: Fertility

	Bench.	Direct	Prices	Tax	Dist.	All
TFR	2.096					2.160
HS	2.282					2.290
CL	1.786					1.978

Table: Decomposing the effects on fertility.

Decomposition: Fertility

	Bench.	Direct	Prices	Tax	Dist.	All
TFR	2.096	2.128	2.113	2.096	2.088	2.160
HS	2.282	2.304	2.283	2.283	2.280	2.290
CL	1.786	1.867	1.830	1.787	1.794	1.978

Table: Decomposing the effects on fertility.

- Note: HHs do not know whether eligible when making fertility choices.
  - Source of uncertainty: income shocks (z) and children's states (ϕ_k and h_k).
- HHs who are (ex-post) not eligible increase fertility.
  - Direct: Provide insurance against income and/or "expenditure" risks.
- Prices: CL share $\uparrow \Rightarrow w_{CL} \downarrow \Rightarrow$ Opportunity costs $\downarrow \Rightarrow$ Fertility $\uparrow$

Education

Expansion: What I Do

Raise the threshold for the grant eligibility to cover students in middle income HHs.

- **Threshold** $\simeq$ the x percentile of income dist.

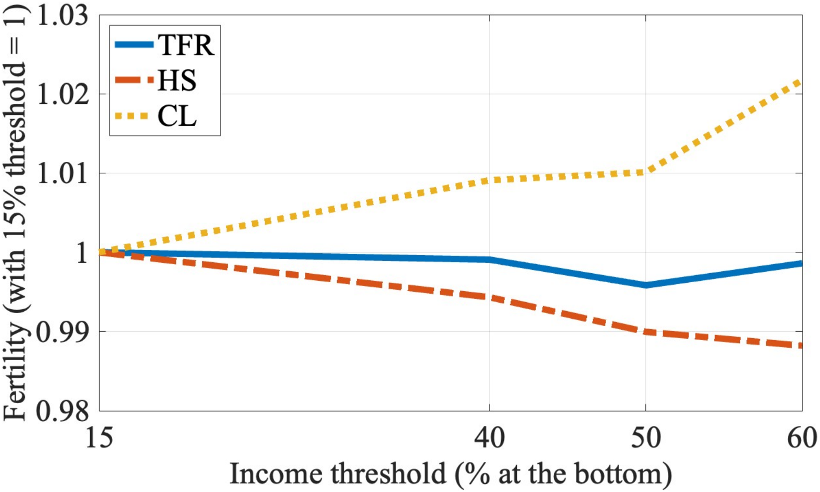
Expansion: Results

	Bench.	Threshold 15%	40%	50%	60%
CL share	37.6	41.5	42.3	43.2	43.8
TFR	2.096	2.160	2.158	2.151	2.157
Output (Δ%)	–	+0.70	+0.15	+1.07	+1.53
Tax	35.00	35.04	35.17	35.23	35.30

Table: Main results of higher income thresholds.

- CL share increases with the expansion.
- TFR would stagnate.

Expansion: Fertility



Details

Fertility Decomposition: Case of Threshold = 60%

	Bench.	Direct	Prices	Tax	Dist.	All
HS	2.282	2.265	2.250	2.283	2.279	2.263
CL	1.786	1.948	1.863	1.829	1.797	2.021

Table: Decomposing the effects on fertility when income threshold = 60%.

- Higher education mobility $\Rightarrow$ Higher expected costs of children $\Rightarrow$ Fertility $\downarrow$
- CL share $\uparrow \Rightarrow w_{HS} \uparrow \Rightarrow$ Opportunity costs $\uparrow \Rightarrow$ Fertility $\downarrow$

Roles of endogenous fertility

- Solve the equilibrium with each program under exogenous fertility.
- Policy functions for fertility are fixed as in the benchmark.

Exogenous fertility

Considering fertility responses implies:

- Higher college enrollment rates
- Greater output
- Lower tax rate (fiscal externalities)

Moments/Threshold	15%	40%	50%	60%
CL share	41.0 (41.5)	41.5 (42.3)	42.3 (43.2)	42.8 (43.8)
Output (Δ%)	0.65 (0.70)	0.55 (0.15)	0.85 (1.07)	1.23 (1.53)
Tax	35.14 (35.04)	35.36 (35.17)	35.37 (35.23)	35.40 (35.30)

Table: Values in parentheses in each cell represent the result under endogenous fertility.

Literature

(1) Fertility choices in incomplete market models:

Schoonbroodt and Tertilt (2014, JET), Santos and Weiss (2016, IER), Sommer (2016, JME), etc.

- This paper: Income volatility + Uncertainty about children's characteristics.
- The education subsidies would provide insurance against those multiple sources of uncertainty and increase fertility.

(2) Education subsidies for college students in macro models:

Benabou (2002, Ecta), Krueger and Ludwig (2016, JME), Abbott et al. (2019, JPE), Matsuda and Mazur (2022, JME).

- This paper: + Fertility choices
- Fertility margins amplify the effects on other macro variables.

Literature

(3) Macro effects of pro-natal policies:

Erosa et al. (2010, RED), Hagiwara (2021), Zhou (2022), Kim et al. (2023), Nakakuni (2023), etc.

- This paper: + Education choice (its discreteness matters to fertility choices),
- and investigates effects of education subsidies for college students on fertility.

(4) Macro models with fertility choices:

De la Croix and Doepke (2003, AER), De la Croix and Doepke (2004, JPubE), Daruich and Kozlowski (2020, RED), etc.

- This paper: + Full lifecycle + GE + uninsurable shocks + IG linkages,
- all crucial to study the effects of education subsidies for college students.

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Conclusion

1. The existing grants $\Rightarrow$ CL share 4 p.p. $\uparrow$, TFR 3% $\uparrow$, output 0.7% $\uparrow$.
   - Fertility of skilled parents $\uparrow$ due to **insurance effects**.
2. Marginal effects of its expansion on the TFR are limited:
   - **Expected costs of children** $\uparrow$ due to a greater mobility of education. $\Rightarrow$ downward pressure on the low skill parents' fertility.
3. **Fertility responses** $\Rightarrow$ higher CL share, greater output, and lower tax rate.

Costs of Children and Fertility Choices in Japan

1. Couples are most likely to give up having their ideal number of children due to its financial costs.
2. There is a significant financial cost gap between having children enrolling in college and those not.
3. A substantial fraction of parents desire a college education for their children.
4. Japan is one of the least in subsidizing tertiary education.

▸ Fig1 ▸ Fig2 ▸ Fig3 ▸ Fig4 ▸ Data

Data

The National Fertility Survey:

- Conducted almost every five years by the IPSS.
- Survey questions regarding marriage, childbirth, and child-raising.
- married couples and singles.

Sample selection:

- Year: 2015 (latest among available)
- Married couples.
- Age (wife): $25 - 39$.

Return

Ideal and planned number of children

A fraction of people give up having their ideal number of children.

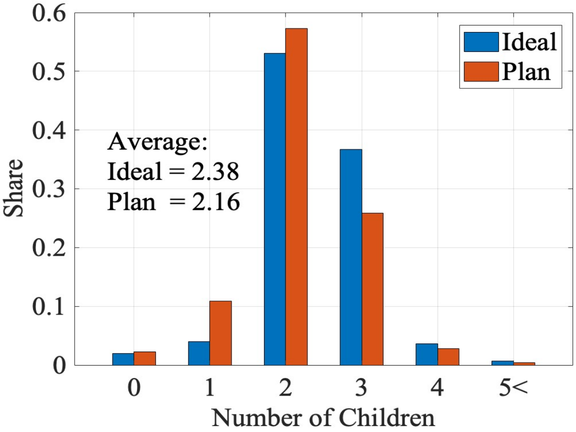

Figure: Distribution of ideal and planned number of children.

Reasons for the gap b/w ideal and plan

"Financial" ⇒ "**Child-raising and education are too expensive.**"

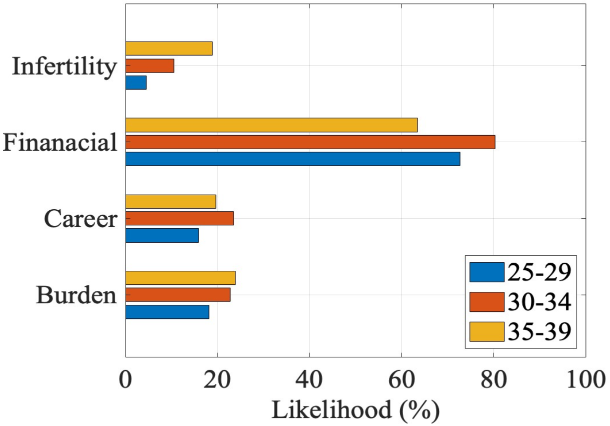

Figure: Reasons for the gap between the ideal and planned number of children.

▸ Return

Timeseries

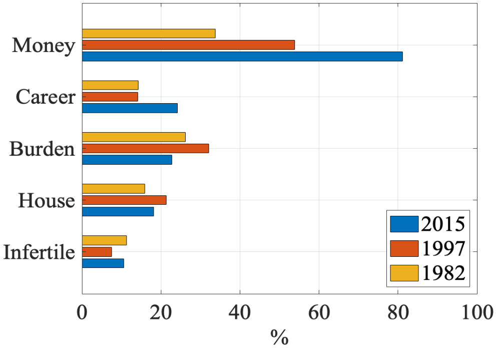

Figure: Reasons why the planned number of children is lower than the ideal one. The sample consists of wives aged 30-34, whose planned number of children is lower than the ideal number. They can choose more than one reason.

Source: The National Fertility Survey (2015).

Desired education for children

More than 75% of parents desire a college education for their children.

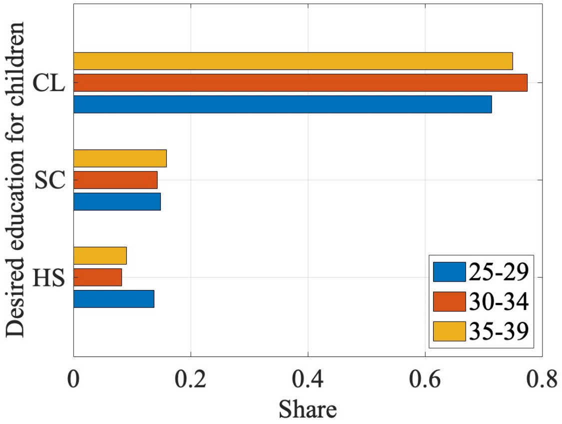

Figure: Wives' intention for children's education attainment.

Return

Preliminaries

- Technology
- Demographics
- Preferences
- Labor income
- Financial markets
- Government

Production Technology

A representative firm operates with the Cobb-Douglus production function:

$$Y = ZK^{\alpha}L^{1-\alpha},$$

where

$$L = \left[\omega_{HS} \cdot (L_{HS})^{\chi} + \omega_{CL} \cdot (L_{CL})^{\chi}\right]^{1/\chi}.$$

- L_e: total efficiency labor with skill $e \in \{HS, CL\}$.

Demographics

- The size of new cohort grows at rate g_n.
  - g_n is determined enodgenously.
- Mortality risks after retirement.
  - $\zeta_{j,j+1}$: Survival probability at age $j+1$ conditional on surviving until age j.
- g_n and $\{\zeta_{j,j+1}\}_{j \geq J_R}$ pin down the age distribution μ_j.

Preferences

- Households draw utility from consumption c and leisure l according to $u(c, l)$.
- If they have children under 18 (before completing HS), they further draw utility from the "quantity and quality" of children according to $b(n) \cdot v(q)$:
  - n: the number of children.
  - $b(n)$: increasing in $n \in \{0, 1, ..., N\}$ and concave.
    i.e., marginal utility gains from having additional child diminishes as n increases.
  - q: spending on children's quality.
    - Caveat: the spending does not affect children's human capital.

Preference: IVT

Utility from the IVT for households with n children is given as follows:

$$\underbrace{b(n) \cdot \lambda_a}_{\text{Discounting}} \cdot \underbrace{V_{g0}(a_{CL}, \phi_k, h_k, I)}_{\text{Value function for children}} .$$

Labor income

- Gross labor income is determined by:
  1. equilibrium wage rates w_e varying with skill (education levels) e,
  2. productivity $\eta_{j,z,e,h}$,
  3. hours worked.

- The productivity $\eta_{j,z,e,h}$ depends on:
  1. age j,
  2. skill e,
  3. human capital h,
  4. a stochastic component $z \sim \pi(z' \mid z)$.

Financial Markets

- Incomplete market: households can trade only claims for risk-free bonds.
- Households face debt limits that vary over the life-cycle:
  - (1) eligible college students and (2) households aged $j < (J_R - 1)$ can borrow.
    - $r^s = r + \iota_s$ for (1).
    - $r^- = r + \iota$ for (2).
  - Retired households cannot.

Government: revenue

- Consumption tax: τ_c
- Capital income tax: τ_a
- Labor income tax: τ_w
- Accidental bequests: Q

Government: expenditures

- Public pension: p per household
- Cash benefits for households with children under 17: B per child
- Lump-sum transfers ψ to generate the progressivity for τ_w.
- Education subsidies/loans for college students.
- The other expenditures: S

Financial aid for college students

Grants $g(h, I)$:

- Eligibility and payments can depend on:
  1. student' human capital (a proxy of "ability"),
  2. household income (I).
- No grants in the benchmark.

Loans:

- Eligible students can access to the subsidized loans with interest rate $r^s \leq r^-$.
- Eligibility is determined by (h, I).
- Government incurs the costs implied by the wedge b/w r^- and r^s.

Government Budget Constraint

$$\tau_c \cdot C + \tau_w \cdot (L_{HS} + L_{CL}) + \tau_a \cdot K + Q = p \cdot \mu_{old} + (\iota - \iota_s) \cdot K_s + G + \psi + B \cdot \mu_{j \leq 17} + S, \quad (2)$$

- C: total consumption,
- Q: total accidental bequests,
- μ_{old}: population mass of retired households,
- $\mu_{j \leq 17}$: population mass of children under age 17,
- K_s: total amount of borrowing by college students,
- μ_{es}: mass of students eligible for the grants.

Budget constraint for IVT choices

$$(1+\tau_c)c + a' + na_{IVT} = Y_{IVT},$$
$$a' \geq -\underline{A},$$

where

$$Y_{IVT} \equiv (1-\tau_w)w_e\eta_{j,z,e,h}(1-l) + \psi + \begin{cases} (1+(1-\tau_a)r)a & \text{if} \quad a \geq 0, \\ (1+r^-)a & \text{otherwise.} \end{cases}$$

Return

Value functions for college students

$$V_{g1}(a_{IVT}; h, I) = \max_{c,l,a'}\{u(c,l) + \beta V_{g2}(a'; h, I)\},$$
$$V_{g2}(a; h, I) = \max_{c,l,a'}\{u(c,l) + \beta \mathbb{E}_{z_0}[V^w(a^s(a'), j = 22, z_0; e = 1, h)]\}.$$

Budget constraints for college students

$$a^s(a') = a' \times \frac{r^s}{1-(1+r^s)^{-10}} \times \frac{1-(1+r^-)^{-10}}{r^-}.$$

Eligible to loans:

$$\begin{aligned} &(1+\tau_c)c + p_{CL} + a' && (3)\\ &-(1-\tau_w)w_{HS}(1-\bar{t}-l) - \psi - g(h,I) = \begin{cases} (1+(1-\tau_a)r)a & \text{if } a \geq 0, \\ (1+r^s)a & \text{otherwise.} \end{cases} && (4)\\ &a' \geq -\underline{A}_s. \end{aligned}$$

The rest:

$$\begin{aligned} (1+\tau_c)c + p_{CL} + a' &= (1+(1-\tau_a)r)a + (1-\tau_w)w_{HS}(1-\bar{t}-l) + \psi + g(h,I), \\ a' &\geq 0. \end{aligned}$$

Working stage without children

$$
\begin{aligned}
V^w(a,j,z;e,h) = \max_{c,l,a'} \{ & u(c,l) \\
& + \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^f(a',z',e,h)] & \text{if } \ j = J_F - 1 \\ \beta[V^r(a',j+1)] & \text{if } \ j = J_R - 1 \\ \beta \mathbb{E}_{z'}[V^w(a',j+1,z';e,h)] & \text{otherwise} \end{array} \right\} \\
\text{s.t.} & \\
& (1+\tau_c)c + a' = (1-\tau_w) w_e \eta_{j,z,e,h}(1-l) + \psi + (1+(1-\tau_a)r)a, \\
& z' \sim \pi(z',z), \\
& a' \geq \left\{ \begin{array}{ll} 0 & \text{if } \ j = J_R - 1, \\ -\underline{A} & \text{otherwise.} \end{array} \right.
\end{aligned}
\tag{5}
$$

Retirement stage

$$
\begin{aligned}
V^r(a, j; e) &= \max_{c,a'} u(c, 1) + \beta \xi_{j,j+1} V^r(a', j+1; e) \\
&\text{s.t.} \\
(1+\tau_c)c + a' &= p + (1 + (1-\tau_a)r)a + \psi, \\
a' &\geq 0 \ (a' = 0 \text{ when } j = J).
\end{aligned}
$$

▸ Return

Preferences

Instantaneous utility for students and adults:

$$u(c, l) = \frac{(c^{\mu} l^{1-\mu})^{1-\gamma}}{1-\gamma}$$

Instantaneous utility from quantity and quality of children:

$$v(q) = \lambda_q \frac{q^{1-\gamma_q}}{1-\gamma_q}$$

The discount function takes a non-parametric form (i.e., $b(n) = b_n$ and $b(0) = 0$).

Financial markets

Targets for

- ι: share of negative net worth,
- ι_s: share of students borrowing

Borrowing limits are set outside the model:

- $\underline{A}_s$: 2.88 million yen.
- $\underline{A}$: 20 million yen.

School taste

- Psychic costs ϕ are given as $\phi = \psi_{CL} \cdot \exp(-h) \cdot \tilde{\phi}$.
  - ψ_{CL} governs the college enrollment rate at the initial steady state.
  - $\tilde{\phi}$ is distributed on $[0, 1]$.
- As in Daruich and Kozlowski (2020), the CDF for $\tilde{\phi}$ is given as

$$G^{\tilde{\phi}}_{e^p} = \begin{cases} \tilde{\phi}^{\omega} & \text{if } e^p = 0 \\ 1 - (1 - \tilde{\phi})^{\omega} & \text{if } e^p = 1 \end{cases}$$

- Target for ψ_{CL}: the college enrollment rate (37.7%).
- Target for ω: intergenerational transition matrix of education.

Intergenerational transmission of human capital

- The initial draw of human capital:

$$\log(h) = \rho_h \log(h_p) + \varepsilon_h,$$
$$\varepsilon_h \sim N(0, \sigma_h).$$

- $\rho_h = 0.19$ following Daruich and Kozlowski (2020).
- Target for σ_h: Variance of log(income) at age 28-29.

Income Process

- Labor productivity $\eta_{j,z,e,h}$:

$$\begin{aligned} \log \eta_{j,z,e,h} &= \log f^e(h) + \gamma_{j,e} + z \\ z' &= \rho_z z + \zeta, \quad \zeta \sim N(0, \sigma_z). \end{aligned}$$

- $\gamma_{j,e}$: estimate the second-order polynomial of hourly wages on age.

Education returns

- Consider the following human-capital production function of the non-linear form:

$$f^e(h) = h + e \cdot (\alpha_{CL} h^{\beta_{CL}})$$

- Target for α_{CL}: Log(wage) ratio (CL$-$ $\leq$CL) at age 28-29 (0.34).
- Target for β_{CL}: Variance of log(wage) for college grad. workers (0.14).

Production

- $\chi = 0.39$ following Matsuda and Mazur (2022).
- $\omega_h = 0.52$: to replicate the wage ratio between CL and the rest.
- $Z = 1.99$: s.t. low skill wage $= 1$.
- $\alpha = 0.33$.
- $\delta = 0.07$ (annual).

Externally determined

Parameter	Value	Description
$\underline{A}_s$	2.88 million yen	Borrowing limit for students
$\underline{A}$	20 million yen	Borrowing limit
p_{CL}	1.05 million yen/year	Tuition fees
κ	0.044	Time costs
$\xi_{j,j+1}$	$-$	survival prob.
τ_c	0.10	Consumption tax
τ_a	0.35	Capital income tax
τ_w	0.35	Labor income tax
p	¥$160,000$/month	Pension benefits
b	¥$10,000$/month	Cash transfers
α	0.33	Capital share
δ	0.07	Depreciation rate
χ	0.39	Elasticity of substitution
ρ_z	0.95	Persistence
σ_z	0.02	Transitory
ν	1	Education sorting by ability
γ	0.5	Curvature
β	0.98	Discount factor
ρ_h	0.19	Transmission of h

Wage age-profile

	College graduates	The rest
Age	0.048	0.041
$\text{Age}^2 \times 10,000$	-5.364	-4.551

Table: Wage age-profile

Fertility differential across education

More (less) educated parents have fewer (more) children, robust to cohorts/data.

	Data
HS	2.12
CL	1.92

Table: Fertility differential across education in the benchmark.

Fertility differential across education

More (less) educated parents have fewer (more) children, robust to cohorts/data.

	Data	Model
HS	2.12	2.28
CL	1.92	1.79

Table: Fertility differential across education in the benchmark.

Validation: Fertility elasticity

- The most relevant moment: the subsidy elasticity of fertility (non-targeted).
  - Few empirical evidence.
- Second best: validation based on the benefit elasticity of fertility.
  - 1% of cash benefits for children increases fertility rates by $0.1 - 0.2\%$. e.g.) Milligan (2005), Cohen et al. (2013), etc.
- The benchmark model implies the elasticity of $0.13 \in [0.1, 0.2]$.

Procedure

Validation: Fertility elasticity

- Set the per-child payment $B = B_0 \cdot X$ where $X \in \{1.1, 1.2, ..., 3.0\}$.
- Solve the household decisions with particular X and compute the TFR.
- Compute the implied elasticity for each X, denoted by ξ_X.
- After having $\{\xi_X\}_X$, compute the average elasticity $\bar{\xi}_X$.

Go Back

Decomposition: Education

	Bench.	Direct	Prices	Tax	Dist.	All
CL share	37.6	40.2	37.6	37.6	39.5	41.5

Table: Decomposing the effects on education.

- Direct $\Rightarrow$ The subsidy relaxes the financial constraint (short/long run).
- Dist. $\Rightarrow$ The effects are amplified in the long run via IG linkages.
  CL share among parents $\uparrow$ $\Rightarrow$ share of children favoring college $\uparrow$ $\Rightarrow$ CL share $\uparrow$
- Fertility margins also play roles:
  College graduates have more children, who are likely to be college graduates.

Return

Expansion: Fertility

		Threshold			
	Bench.	15%	40%	50%	60%
TFR	2.096	2.160	2.158	2.151	2.157
HS	2.282	2.290	2.277	2.267	2.263
CL	1.786	1.978	1.996	1.998	2.021

Table: Fertility by different educational background.

- College graduates' fertility rates continue to increase.
- High school graduates' fertility rates rather decrease.

Return