Essays on the Macroeconomics of the Family

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Fundamental Questions

1. What are the underlying forces driving the decline in fertility?
   - Ch. 1: roles of macro conditions due to technological change
2. What are the macroeconomic consequences of such policies?
   - Ch. 2: pro-natal cash transfers
   - Ch. 3: education subsidies (& policies insuring income risk)

Needs macro models w/ "family" perspectives (e.g., fertility choices)

Chapter 1

On the Trends of Technology, Family Formation, and Women's Time Allocation

(with Sagiri Kitao)

Background

- Big picture: what has driven the declining fertility over time?
- Concurrent changes in factors interconnected w/ fertility choice:
  - Marriage ↓
  - Educational attainment ↑ & parental investment ↑
  - (Married women's) Time allocation: Leisure ↑, Housework ↓, etc.
- What has driven these trends regarding/within the family?

  – Our hypothesis: **technological changes** (e.g., SBTC, GBTC)
  ⇒ affect relative prices (e.g., wage structure across gender-skill)
  ⇒ affect household decisions regarding/within the family

This Paper

- Constructs a unified framework of family formation and time allocation with (exogenous) technological changes in GE setup
- Calibration to replicate these facts:
  - Focuses on those in Japan over the past 50 years
  - Key time-variant parameters: technological changes
    - Skill-biased (SBTC)
    - Factor neutral (TFP growth)
    - Gender-biased (GBTC): e.g., Heathcote et al., (2010), Ngai and Petrongolo (2017)
- Counterfactual: What if a tech. change had not taken place?

Finding/Contribution

GBTC has played critical roles in shaping trends in family formation

- GBTC & SBTC explain 30% of fertility decline over 1970-2020
- GBTC explains 50% of marriage rate decline over 1970-2020

Relation to literature: e.g., Greenwood, Guner, Kocharkov, and Santos, 2016; Regalia, Ríos-Rull, and Short, 2019; Greenwood, Guner, and Marto, 2023, etc.

— The first to study/highlight the roles of GBTC in declining fertility/marriage

Family formation (fertility & marriage)

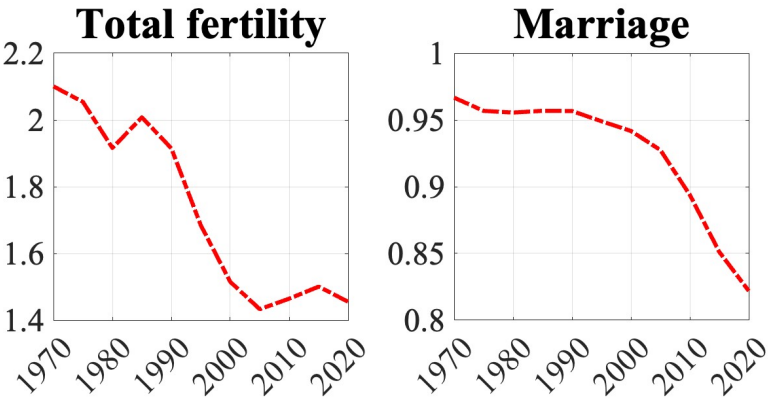


Figure: Trends in family formation. *Note*: "Marriage" indicates the fraction of people aged 50-54 who have ever been married.

Married women's time allocation

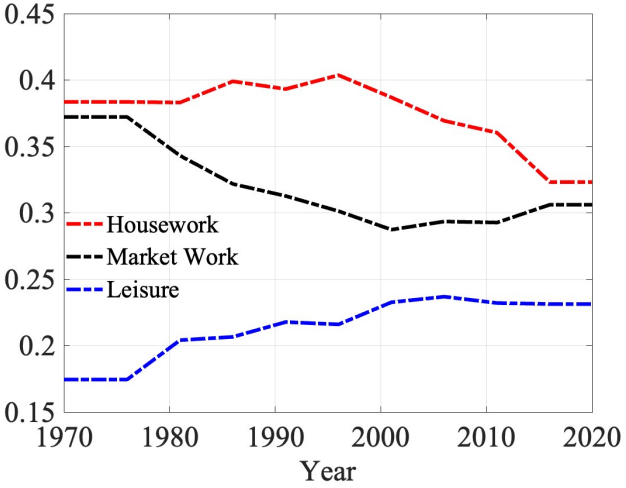


Figure: Trends in married women's time allocation (share).

Wage structure: gender wage gap

		1970	2020
Gender Gap	Non-college grad.	0.55	0.61
	College grad.	0.66	0.70
	Weighted Avg.	0.51	0.64

Table: Gender wage gap. *Gender gap* is defined as the ratio of women's wages to men's wages.

- 0.13 p.p. increase in female-male wage ratio (gender gap ↓),
- while women's labor supply (relative to men) ↑↑:
  – share of women in lab. market (in 1970-2020): 31.1% → 45.9%

▸ Skill premium

Facts: a quick review

Summary:

- Family formation: fertility & marriage ↓ (and education ↑)
- Time allocation: House- & market-work ↓, leisure ↑
- Wage structure: gender gap ↓

Underlying mechanisms? −Need a structural model

Model

A static GE model

- Households:
  - Economy consists of individuals with gender g: male (m) and female (f)
  - Males and females are randomly matched in the marriage market
  - Draws "joy shock" on marriage ($r \sim F(r)$) , decides to marry or not
    – compare values of marriage with those of being single, $M + r$ vs S
- Firm:
  - Produces final goods w/ four types of labor inputs
    – gender (male or female) and skill (low or high)

Choices (after marriage decision)

For both single and married households:

- Consumption (c)
- Housework (h) and durable goods (d) $\Rightarrow$ non-market goods (n)
- Leisure (l)

For married households:

- Fertility (k)
- Education/skill level (s): fraction of college graduates in children's gen.
  - Note: no heterogeneity across households (conditional on marital status)
  - ("Representative") Parent gen. chooses education level of children's gen.

Exogenous variables

For both single and married households:

- Price of household durable goods (π)

For married households:

- Monetary costs of children (b)
- Time costs of children (ζ_g)
- Unit costs of education (χ)
- Husband's time allocation (l_m, h_m)

Value of marriage

$$M = \max_{c,d,l_f,h_f,k,s} \left\{ u^M(c/\eta, n/\eta, l, k, s) \right\}$$

s.t.

$$c + \pi d + \chi s k + b k = \sum_g w_g (1 - \zeta_g k - l_g - h_g)$$

$$n = \left[\omega d^\sigma + (1-\omega)(h_m + h_f)^\sigma \right]^{1/\sigma}, \quad l = l_m + l_f$$

Value of being single

$$S_g = \max_{c,d,l,h} \left\{ u_g^S(c, n, l) \right\}$$

s.t.

$$c + \pi d = w_g(1 - l - h)$$

$$n = \left[\omega d^\sigma + (1 - \omega) h^\sigma\right]^{1/\sigma}$$

Marriage decision

- A woman who draws a joy shock $j \sim F(r)$ chooses to marry if the value of marriage surpasses that of remaining single:

$$\widehat{M}_f + r \geq S_f,$$

where $\widehat{M}_f = u^M(c/\eta, n/\eta, l_f, k, s)$ denotes the utility a woman receives when she is married.

Production

$$Y = Z\left[L^{\varphi} + AH^{\varphi}\right]^{1/\varphi},$$

where

$$L = \left[L_m^{\gamma} + BL_f^{\gamma}\right]^{1/\gamma} \quad \text{and} \quad H = \left[H_m^{\gamma} + A_f B H_f^{\gamma}\right]^{1/\gamma}$$

- Z : factor-neutral
- A : skill-biased (general)
- B : gender-biased
- A_f : skill-biased (female) c.f., Taniguchi and Yamada (2024)

Competitive equilibrium $\Rightarrow$ wage rates = MPL

Calibration

- Goal: replicate the trends over 1970-2020
  – fertility & marriage ↓, college enrollment ↑, time allocation
- 1st step: set parameters externally
  – Husband's time, childcare costs, price of durables, EoS, etc.
- 2nd step: pin down the rest internally
  - Preference (time-invariant): approx. the entire path of each variable
  - Technology (time-variant): replicate wage paths given labor input

Functional forms

- Married household:

$$u^M(c/\eta, n/\eta, l, k, s) = \alpha\frac{(c/\eta)^{1-\rho}-1}{1-\rho} + \beta\frac{(n/\eta)^{1-\nu}-1}{1-\nu} + \mu\frac{l^{1-\lambda}-1}{1-\lambda} + \phi\frac{k^{1-\kappa}-1}{1-\kappa} + \xi\frac{(s\cdot w_h + (1-s)\cdot w_l)^{1-\psi}-1}{1-\psi},$$

  where we set $\alpha = 0.1$ (normalization) and $\sigma = 2$ (standard).

  – Note: skill premium $\uparrow \Rightarrow \partial u^M/\partial s \uparrow$

- Joy shock $r \sim F(r)$: Gumbel distribution with parameters $\mathbf{a}, \mathbf{d}$.

Objective function

- Let $\boldsymbol{\theta} = \{\beta, \nu, \mu, \lambda, \phi, \kappa, \xi, \psi, \mathbf{a}, \mathbf{d}\}$
- Find a solution $\boldsymbol{\theta}^*$ for the following problem:

$$\min_{\boldsymbol{\theta}} \left\{ \sum_i \sum_t \left(\frac{x_{it}^M(\boldsymbol{\theta}) - x_{it}^D}{x_{it}^D} \right)^2 \right\},$$

where

- $x_{it}^M(\boldsymbol{\theta})$: model-implied moment for a variable i in year $t \in \{1970, ..., 2020\}$ given parameters $\boldsymbol{\theta}$
- x_{it}^D: data counterpart for the moment x_{it}^M

Parameter values

Target: fertility, marriage, & education

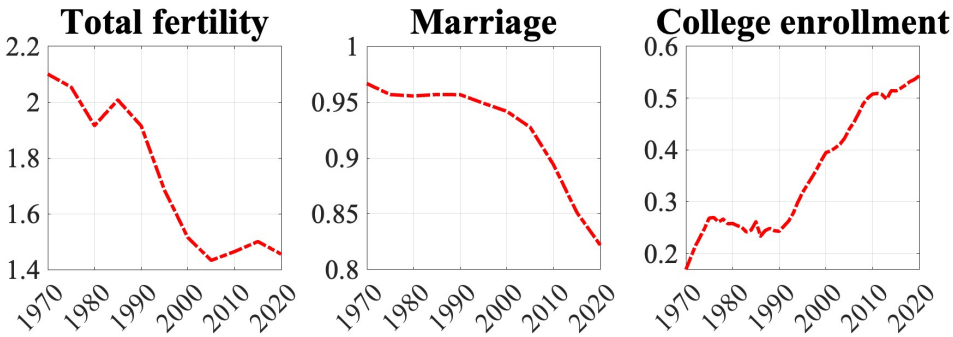


Figure: Family formation and education (data). *Note*: "Total fertility" stands for the total fertility rates. "Marriage" indicates the fraction of people aged 50-54 who have ever been married. "College enrollment" indicates college enrollment rates.

Baseline: fertility, marriage, & education

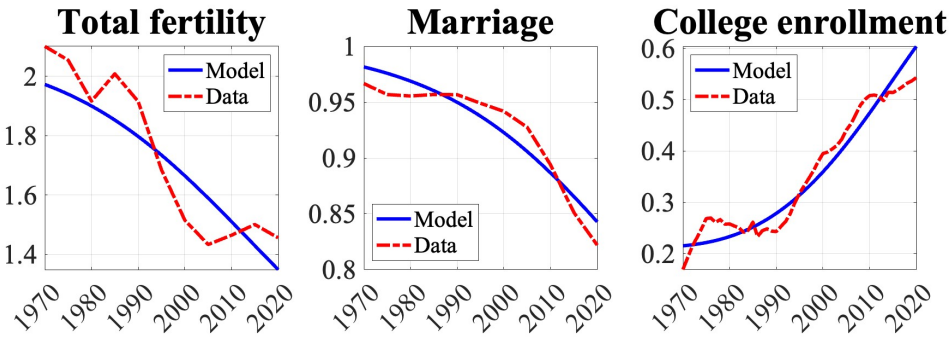


Figure: Family formation and education (model vs. data). *Note*: "Total fertility" stands for the total fertility rates. "Marriage" indicates the fraction of people aged 50-54 who have ever been married. "College enrollment" indicates college enrollment rates.

Target: Married women's time allocation

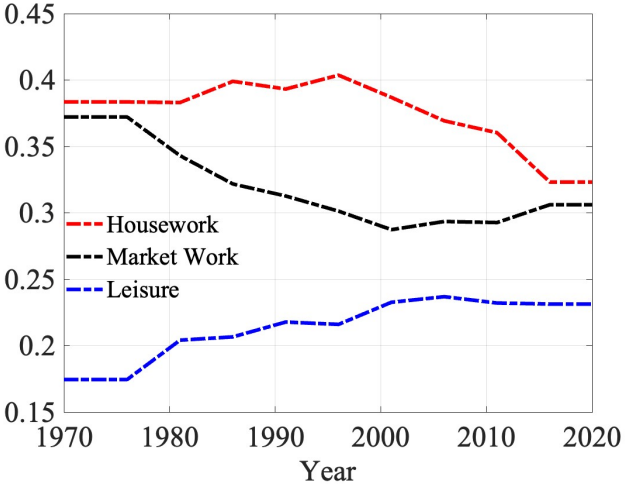


Figure: Trends in married women's time allocation (share).

Baseline: Married women's time allocation

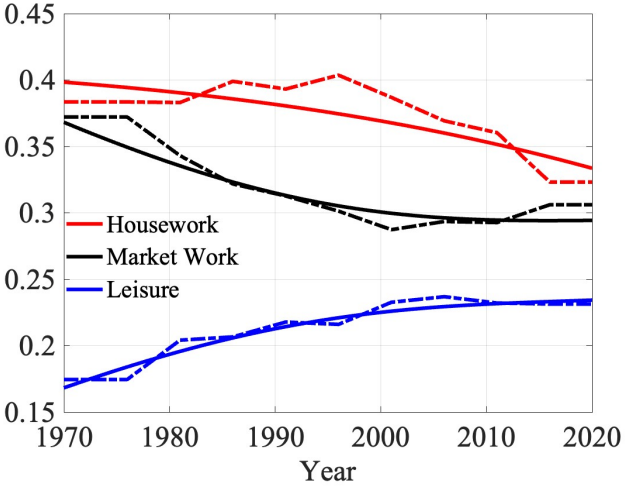


Figure: Trends in married women's time allocation (model vs. data).

Wage structure

		1970	2020
Gender Gap	Non-college grad.	0.55	0.61
	College grad.	0.66	0.70
	Weighted Avg.	0.51	0.64
Skill Premium	Men	1.36	1.35
	Women	1.65	1.55
	Weighted Avg.	1.46	1.42

Table: Gender wage gap and skill premium. *Gender gap* is defined as the ratio of women's wages to men's wages. *Skill premium* is defined as the ratio of college to non-college grad. wages.

Technology: annual growth (1970-2020)

- SBTC (general): 0.51%
- SBTC (female): 0.99%
- GBTC: 0.45%
- TFP growth: 0.33%

Fig

Counterfactual experiments

What if a technological change had not taken place?
– shut down a tech. change by fixing a parameter to 1970's value

	Baseline		Counterfactual outcomes (in 2020)			
	1970	2020	No-TFPG	No-SBTC general	No-SBTC female	No-GBTC
Family and Education						
Fertility (TFR)	1.972	1.349	1.342	1.403	1.429	1.400
College enrollment	0.215	0.604	0.543	0.334	0.402	0.542
Marriage	0.982	0.843	0.884	0.900	0.913	0.912
Time Allocation of Married Women						
Work Hours	0.368	0.294	0.374	0.381	0.283	0.254
Leisure	0.168	0.234	0.159	0.147	0.242	0.272
Housework	0.398	0.334	0.335	0.337	0.341	0.341

Table: Roles of Technology. Outcomes in 2020

Childcare costs Home production technology

Summary

- What has driven the trends regarding/within the family?
  – fertility/marriage ↓ & time allocation (work ↓, leisure ↓)
- Constructs a unified framework of family formation, time allocation, and endogenous wage structure
- Critical role of GBTC in declining trends in fertility/marriage
- Gender neutral tech. change ⇒ the trends in time allocation

Chapter 2

Macroeconomic Analysis of the Child Benefit: Fertility, Demographic Structure, and Welfare

Journal of the Japanese and International Economies, 2024, 73: 101325.

Child Benefit (CB)

- Cash transfers to household with children, unconditional on parents' labor market status, conducted in many countries
- Aimed at deterring demographic aging by increasing fertility
  – fertility $\uparrow \Rightarrow$ worker's pop. share $\uparrow \Rightarrow$ output & tax base $\uparrow$
- Side effects on maternal labor supply (e.g., Guner et al., 2020 REStud)
  – income effect $\Rightarrow$ labor supply $\downarrow \Rightarrow$ output & tax base $\downarrow$

Research Question: what are its macro & welfare consequences?
– Previous macro studies abstract the fertility channel (e.g., Guner et al.)

What I Do

- Embed fertility choices to a GE-OLG model
- Calibrate the model to Japan with the current CB policy
- Validation: the model implies the benefit elasticity of fertility consistent with empirical estimates
- Examines counterfactual CB policies (long-run & transition)

Model

- GE-OLG framework with heterogeneous households
  - GE needed to capture macro & fiscal implications in equilibrium
  - OLG needed to capture the change in demographic structure
  - Heterogeneity to examine broader welfare implications of CB
- Agents
  - Households
  - Firm
  - Government

Households

- Endowed with a certain (time-invariant) type defined by:
  - Gender (male or female)
  - Marital status (married or single),
  - Skills (Spouse's as well if married)
- Choices over the lifecycle:
  - Consumption, saving, and working hours (until retirement-age)
  - Fertility (only for married; one-shot decision at a given age)
- Fertility choices:
  - Having a child increases parents' utility captured by a function $v(\cdot)$
  - but takes monetary and time costs, both are exogenously given as a function of household's type
  - CB partially compensates the monetary costs (i.e., no market-childcare)
  - Affect demographic structure (age distribution μ_j) in the long run

Representative Firm

- Operates with the CRS technology $F(K, L)$
- Maximizes its profit by choosing labor (L) and capital (K) inputs in competitive factor markets,
- given the wage (w) and user costs of capital $(r + \delta)$:

$$\max_{(K,L)\geq 0} F(K, L) - w \cdot L - (\delta + r) \cdot K$$

Government

Expenditures:

1. Child benefit: payment function $CB(\boldsymbol{x}_{CB}; X)$ with household type $\boldsymbol{x}_{CB}$ and scale parameter X
2. Social security benefits
   2.1 PAYG pension (a fixed payment ss with replacement rate of ρ)
   2.2 Health insurance (with age-dependent co-payment rate ω_j, covering exogenous medical expenditures m_j incurred by households)
3. Others (G).

Funded by taxing three sources of income:

- Consumption (τ_c)
- Labor income (τ_l)
- Capital income (τ_a)

Government Budgets

- τ_l consists of a component used to finance social security (τ_{ss}), and a residual component (τ_{-ss}).
- Tax rates $(\tau_c, \tau_{ss}, \tau_{-ss}, \tau_a)$ satisfy the following budget:

$$\tau_{ss} wL = \underbrace{ss \sum_{j=J_R+1}^{J} \mu_j}_{\text{pension}} + \underbrace{\sum_{j=1}^{J} \omega_j m_j \mu_j}_{\text{health insurance}}$$

and

$$\tau_c C + \tau_{-ss} wL + \tau_a rK = CB + G$$

Definition 1 (Stationary equilibrium)

Given an initial asset endowment $\underline{a}$ *and some policy parameters* $(\rho, \{\omega_j\}_j, G, \tau_a)$*, a stationary equilibrium consists of (1) households' decision rules, (2) household distribution, (3) quantities, (4) factor prices, and (5) tax rates such that*

- *the decision rules solve their problems,*
- *prices are determined competitively,*
- *factor markets clear,*
- *government budgets are satisfied, and*
- *distributions are stationary.*

Calibration: Strategy

- Calibrate the model to 2020s Japanese economy
- Two-step calibration:
  - Step 1: A subset of parameters is chosen outside the model
  - Step 2: The rest is calibrated to match targeted moments
- Use several (aggregate) data source from Japan

Functional Forms

Flow utility for married couples:

$$u^C(c, l^w; b) = 2 \cdot \frac{(c/\Lambda(b))^{1-\sigma} - 1}{1-\sigma} - \varphi \frac{(l^w + \eta \cdot b)^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}}.$$

Utility of having b children:

$$v(b) = \kappa \cdot \frac{(1+b)^{1-\gamma} - 1}{1-\gamma}$$

- $\Lambda(b) = 1.5 + 0.3b$: the OECD modified equivalence scale
- Choose (φ, η, κ) endogenously
- Set $\gamma = 3.5$ arbitrary (subject to robustness check)

Endogenous Parameters

	Value	Targeted Moment	Data	Model
β	1.056	Capital/Output	2.8	2.6
η	0.339	Working/Childcare	0.721	0.720
φ	1.26	Average hours worked (men=1)	0.77	0.77
κ	367.8	Annual growth rate of birth	-1.15%	-1.16%
ρ	0.122	Pension expenditure to GDP	10%	10%

Table: Parameters calibrated inside the model.

Child Benefit Payment Function

$$CB(\boldsymbol{x}_{CB}; X) = \begin{cases} \mathbb{I}_{j_c \in [0,14]}[b \times ¥11,000] \times X & \text{if } \max\{y^h, y^w\} \leq \bar{I}, \\ \mathbb{I}_{j_c \in [0,14]}[b \times ¥5,000] \times X & \text{otherwise,} \end{cases}$$

where y^h (y^w) denotes husband's (wife's) labor earnings, j_c denotes children's ages, $X = 1$ in benchmark, and $\bar{I}$ is set to 9.6 M yen to mimic the actual scheme.

- More than 90% of eligible households receive $11,000$ yen (Employment Status Survey, ESS 2022).
- 11,000 yen $\simeq$ 5% of median labor earnings of husband who has a child under 18 (ESS 2022)

Non-targeted Moments

- The benefit elasticity of fertility
- Fertility differentials across education group

The Benefit Elasticity of Fertility

- %-change in fertility in response to a 1%-increase in cash benefit
- Empirical estimates from foreign countries $\simeq 0.1$ (e.g., Milligan, 2005)
- The model-implied elasticity is $0.026 \simeq$ Yamaguchi (2019 QE)
- What I Do:
  - Consider scale parameters $X \in \mathcal{X} \equiv \{2, 2.5, 3, 3.5, 4, 4.5, 5\}$, similar to those examined in empirical studies and those I examine in my policy experiments
  - Feed $X \in \mathcal{X}$ into the model, holding prices and tax rates constant
  - Compute the aggregate fertility and the implied elasticity with X

Fertility Differentials (across education)

	Education (wife)			
	$< HS$	HS	SC	CL
Data	1.00	0.88	0.84	0.84
Model	1.00	0.98	0.89	0.85

Table: Fertility differentials across education groups. *Note*: The fertility rates of less than high school graduates ($< HS$) are normalized to one. The data counterpart comes from the National Fertility Survey (2015). For the model part, I report the completed fertility of married females with each education background in the model. The column "CL" indicates the average fertility rate over college graduates (COL) and more-than-college graduates ($COL+$) so that the construction of education category in the model matches the data counterpart.

$-$Key: opportunity costs of having children $(\underbrace{\text{wage}}_{\sim\text{skill}} \times \underbrace{\eta}_{\text{fixed}} \times b)$

Policy Experiments: What I Do

- Examine expansion of CB by changing scale parameter X
  - Stationary (long-run) equilibrium
  - Transitional dynamics
- Welfare measure: consumption equivalence for new borns
- Government budget is balanced by τ_c
- Social security tax (τ_{ss}) is also adjusted
- Inspecting the mechanism
  - Endogenous vs. exogenous fertility
  - Decomposition: direct, price, taxation, and distribution effects.

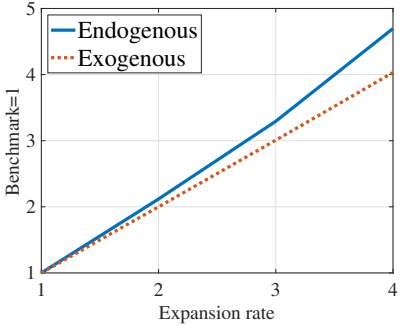
Macro Variables: Long-run

	$X = 0$	$X = 2$	$X = 3$	$X = 4$
TFR	1.39	1.45	1.48	1.53
Aggregate quantities ($\Delta\%$)				
Output	$-0.04(-0.10)$	$1.36(0.00)$	$2.33(0.25)$	$4.95(0.77)$
Capital	$-0.11(-0.50)$	$0.76(0.00)$	$1.97(0.70)$	$4.24(2.16)$
Labor	$0.00(0.11)$	$1.70(0.00)$	$2.54(0.00)$	$5.35(0.00)$
Women's working hours ($\Delta\%$)				
Married	$0.28(0.36)$	$-1.16(-0.74)$	$-1.86(-1.86)$	$-2.50(-1.88)$
Singles	$-0.09(-0.01)$	$0.04(-0.20)$	$0.24(-0.26)$	$1.28(-0.48)$

Table: Changes in aggregate variables in the long run with different per-child payments. *Note*: Values in the parenthesis in each cell are those obtained under exogenous fertility.

Consumption Tax

(a) Total expenditure on CB



(b) Consumption tax rate

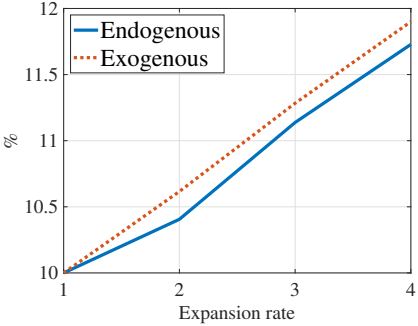
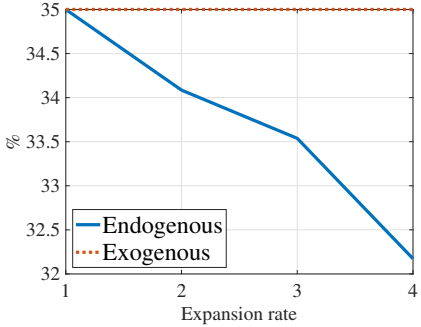


Figure: Payment expansion, government expenditures, and tax rates. *Note*: In each figure, "Endogenous" in the legend plots the results under endogenous fertility, while "Exogenous" plots the results under exogenous fertility where the number of children for each household is fixed as in the benchmark.

Labor Income Tax ($\tau_l = \tau_{ss} + \tau_{-ss}$)

(a) Labor income tax rate



(b) Social security spending

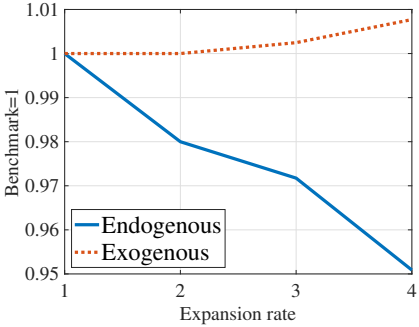


Figure: Payment expansion, government expenditures, and tax rates. *Note*: In each figure, "Endogenous" in the legend plots the results under endogenous fertility, while "Exogenous" plots the results under exogenous fertility where the number of children for each household is fixed as in the benchmark.

Welfare

	Expansion Rate (X)			
	0	2	3	4
Couples	−2.7(−2.7)	7.2(3.1)	11.4(5.5)	20.2(8.7)
High skilled	−2.3(−2.3)	6.6(2.5)	10.2(4.5)	18.8(7.2)
Low skilled	−3.7(−3.9)	8.2(4.5)	13.7(8.2)	23.5(12.7)
Singles	0.7(0.5)	1.3(−0.1)	1.1(−0.7)	1.9(−0.9)

Table: Changes in CEV (%) in the long run with different per-child payments. *Note*: The parenthesis in each cell reports the CEV of the agent with an expansion scenario under exogenous fertility. Here, a positive value of CEV means welfare improvement for the household, and vice versa. "High skilled" households are defined as those in which the husband and wife complete at least a college education, and "Low skilled" as those in which the husband and wife complete at most a high school education.

Decomposing Welfare Effects

	τ_l	τ_c	(w, r)	X	a_b	Overall
Couples	6.4	−2.8	−0.1	7.6	0.8	11.4
High skill	6.7	−2.8	−0.1	6.4	0.7	10.2
Low skill	5.9	−2.8	−0.1	10.2	0.9	13.7
Singles	2.0	−1.4	0.04	–	0.5	1.9

Table: Welfare decomposition under the expansion rate of three ($X = 3$). *Note*: Fours columns report the CEV of each household type, replacing labor income tax rate (τ_l), consumption tax rate (τ_c), factor prices (w, r), lump-sum transfers of accidental bequests (a_b), and the payment (X) in benchmark with that in the new equilibrium. Note that the decomposition method adopted here does not imply that each effect sums to the overall effect.

Transitional Dynamics: TFR and Output

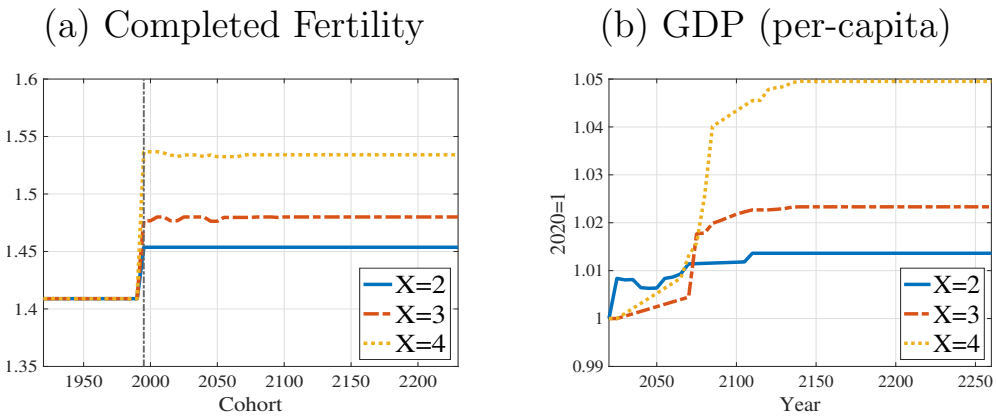
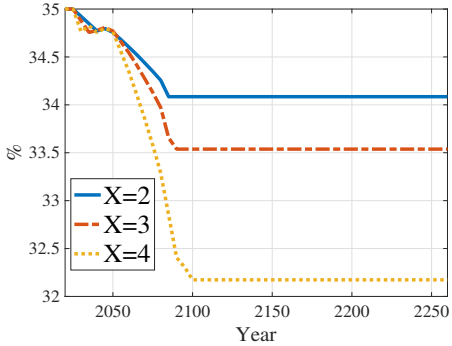


Figure: TFR and output during the transition.

Transitional Dynamics: Tax Rates

(a) Labor income tax rate (τ_l)



(b) Consumption tax rate (τ_c)

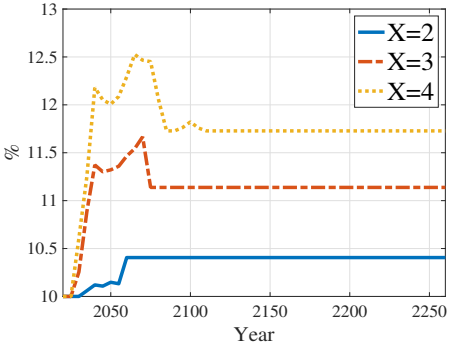


Figure: Tax rates during the transition.

Transitional Dynamics: Welfare

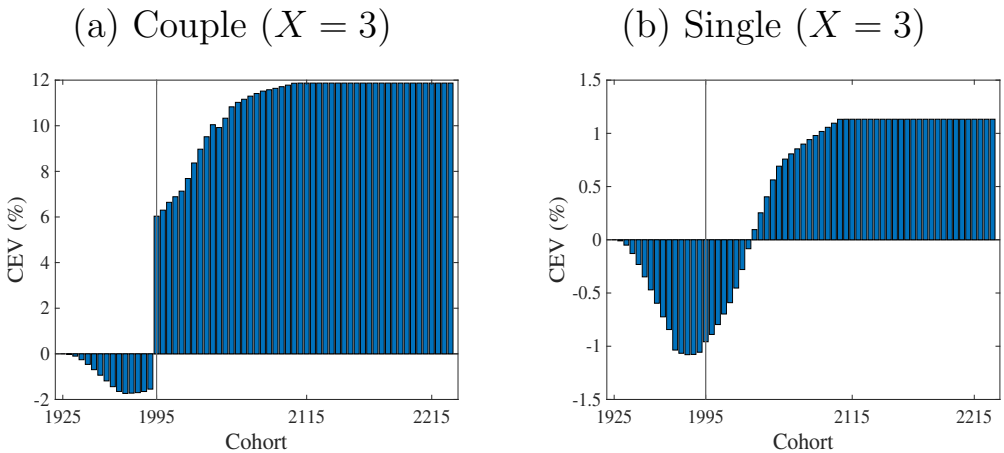


Figure: Welfare effects during the transition. *Note*: "1995" in the figure stands for the 1991-95 cohort, and "1925" for the 1921-25 cohort, whose age is 100-104 when the expansion occurs. The vertical line indicates the first cohort benefiting from the reform (i.e., the 1995 cohort).

Robustness

Qualitative results are robust to the following setup:

1. Human capital depreciation after birth
   – Introduce post-birth human capital depreciation (cf. Kleven et al., 2019)
2. Quantity–quality trade-off
   – Allow child quality to decline as quantity increases (cf. Zhou, 2022; Kim et al., 2024)
3. Benefit elasticity of fertility
   – Vary γ to generate different elasticities

Recap

- Expanding the CB leads to welfare gains even for non-recipients in the long run
- The gains accrue from lower tax burdens due to changes in demographic structure
- Given that the demographic shift takes a long time to take place, the accrual of welfare gains is gradual and spans $\simeq$ 100 years

Chapter 3

Education Subsidy as Insurance for Fertility Choices

Case of Japan as an Insightful Lesson

- A novel policy recently introduced: *college education subsidies*
  - College education: expensive due to little subsidies Background
  - Empirics: Costs of children $\downarrow \Rightarrow$ fertility $\uparrow$ (e.g., Malkova, 2018, etc.)
- Question: What are the macro consequences of college subsidies, considering endogenous fertility choices?

Key Findings and Contribution

- **Income-tested subsidies** serve as insurance for fertility choice
  - Income uncertainty $\Rightarrow$ Fertility $\downarrow$ (Modena et al, 2014; Guner et al., 2024, etc.).
  - Logic similar to $c \downarrow$ due to precautionary motives
- Fertility margins amplify macro effects of college subsidy
  - Income inequality $\downarrow$, output $\uparrow$, welfare $\uparrow$, etc.

What I Do

- Analytical model of fertility choices w/ uninsurable income risk
- Lifecycle Aiyagari w/ college education & endogenous fertility
- Model validation:
  - Cross-country estimates of col. subsidy elasticity of fertility
- Experiments: Intro./expansion of college subsidies
  - Long-run & transitional dynamics

An Analytical Model

$$\max_{n \geq 0} \{b(n) + \mathbb{E}_I V(c)\}$$
$$\text{s.t. } c + n \cdot a(I) = I, \; I \sim F(I),$$
$$\underbrace{a(I)}_{\text{(net) costs}} = \underline{a} - \underbrace{g(I)}_{\text{subsidy}} .$$

- Utility from # of children (n) and consumption (c)
- Income uncertainty: $I \sim F(I)$
- Timeline: (1) Choose n, (2) income I realized, (3) c pinned down

Question: What is effect of income-tested subsidy on fertility?

Assumptions and notation

- $g(I;\varepsilon) = (\mathbb{E}I - I)\cdot\varepsilon$ for some $\varepsilon \in \mathcal{E} \equiv \{\varepsilon \mid 1 - \varepsilon\cdot n(\varepsilon)\ \&\ \varepsilon \geq 0\}$.
  - $\mathbb{E}g(I;\varepsilon) = 0$, redistribution from "lucky" to "unlucky"
  - $\varepsilon = 0$ means no-subsidy: $g(I;0) = 0\ \forall I$
- $b(n) = n^{1-\sigma}/(1-\sigma)$, $V' > 0, V'' < 0, V''' > 0$.
- $n(\varepsilon), c(\varepsilon)$: optimal choices with policy parameter ε

Effects of income-tested subsidy

First-order condition:

$$n(\varepsilon) = \{\mathbb{E}[\underbrace{a(I) \cdot V'(c(\varepsilon))}_{\text{Marginal costs of } n}]\}^{-\frac{1}{\sigma}}$$

Implying that:

$$\frac{n(\varepsilon)}{n(0)} \equiv \frac{\text{Fertility w/ subsidy}}{\text{Fertility w/o subsidy}}$$

$$= \underbrace{\left\{\frac{\mathbb{E}V'(c(\varepsilon))}{\mathbb{E}V'(c(0))} - cov(g(I;\varepsilon), V'(c(\varepsilon)))\right\}^{-\frac{1}{\sigma}}}_{>1 \ (\therefore\ n(\varepsilon)>n(0))}$$

Intuition

$$\frac{n(\varepsilon)}{n(0)} = \underbrace{\left\{ \frac{\mathbb{E}V'(c(\varepsilon))}{\mathbb{E}V'(c(0))} - cov(g(I;\varepsilon), V'(c(\varepsilon))) \right\}^{-\frac{1}{\sigma}}}_{>1 \ (\therefore \ n(\varepsilon)>n(0))}$$

- Having more children is costly in utility $\because$ foregone consumption
- Even more costly upon bad income shock
  $\because$ very low consumption; fertility is an irreversible choice
- $g(I;\varepsilon)$ provides insurance: expected costs of children in utility $\downarrow$
  $-$ increases fertility due to an *insurance effect*

Next steps

1. Quantitative importance of the insurance effect:
   – relevant to policy design
2. Macro implications (esp. through fertility margins):

⇒ Need a quantitative macro model with:

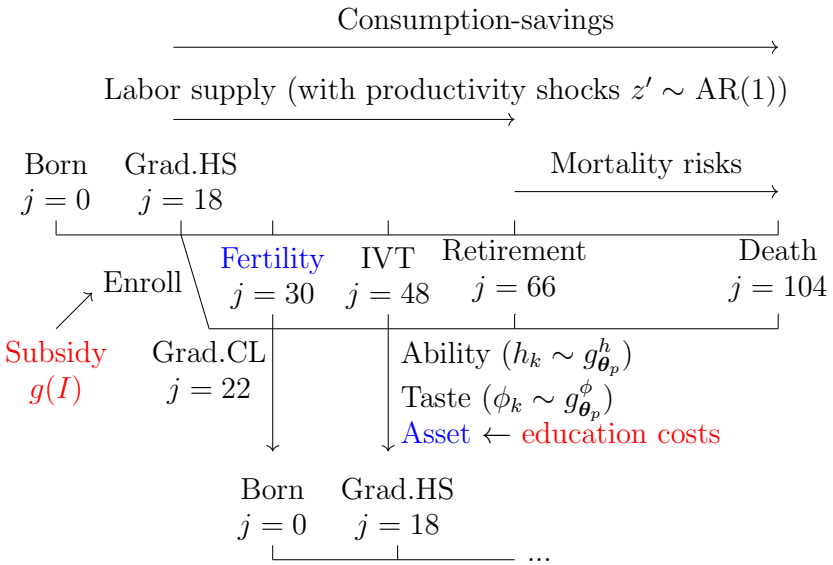
- realistic income process,
- savings as self-insurance,
- endogenous education investments, and more.

Quantitative Model

Basic framework: Lifecycle Aiyagari

- Firm: $Y = K^{\alpha} L^{1-\alpha}$ where $L = [\omega \cdot L_{HS}^{\rho} + (1-\omega) \cdot L_{CL}^{\rho}]^{1/\rho}$
- Government: Grants $g(I)$ for CL students (& loans)
- Households: $\underbrace{\text{Standard lifecycle + IG links + College}}_{\text{Standard (e.g., Krueger \& Ludwig, 2016; Abbott et al., 2019)}}$ + Fertility

Lifecycle


Consumption-savings
Labor supply (with productivity shocks $z' \sim$ AR(1))
Born
$j = 0$
Grad.HS
$j = 18$
Mortality risks
Enroll
Fertility
$j = 30$
IVT
$j = 48$
Retirement
$j = 66$
Death
$j = 104$
Subsidy
$g(I)$
Grad.CL
$j = 22$
Ability ($h_k \sim g^h_{\boldsymbol{\theta}_p}$)
Taste ($\phi_k \sim g^\phi_{\boldsymbol{\theta}_p}$)
Asset ← education costs
Born
$j = 0$
Grad.HS
$j = 18$
...

Calibration

- Goal: determine model parameters to match key moments
- Data: The Japanese Panel Survey of Consumers (JPSC) Details
- Procedure: Standard two-step calibration
  - 1st: Set or estimate 19 parameters externally
    - AR(1) income process (stochastic components) Detail
    - $g(I) = 0 \ \forall I$ (mimics before 2020)
  - 2nd: Pin down the rest (18 params) internally

Benchmark economy

Parameter	Value	Moment	Data	Model
Targeted				
λ_q	0.62	Average total educ. spend. (18 years, million yen)	6.60	6.60
λ_a	1.03	Average total transfer (4 years, million yen)	5.50	5.51
ψ_{CL}	20.8	College enrollment rate	0.38	0.38
ω	1.71	Intergenerational mobility of education		
		Pr(Child=HS | Parent=HS)	0.73	0.80
		Pr(Child=CL | Parent=HS)	0.27	0.20
		Pr(Child=HS | Parent=CL)	0.41	0.42
		Pr(Child=CL | Parent=CL)	0.59	0.58
σ_h	0.65	Variance of log(income) at age 28	0.27	0.24
α_{CL}	0.1	Log wage ratio (CL−HS) at age 28	0.34	0.38
β_{CL}	0.1	Var log wage for CL at age 28	0.14	0.24
ω_h	0.52	CL−HS wage ratio	1.36	1.48
$\bar{t}$	0.8	Students' income share of labor earnings	0.20	0.17
ι_s	0.055	Share of students using loans	0.44	0.34
ι	0.054	Household share with negative net worth	0.54	0.45

Benchmark economy (cont'd)

Parameter	Value	Moment	Data	Model
Targeted				
		Distribution of # of children		
		Share of childless	0.05	0.01
b_1	0.49	Share of one child	0.16	0.15
b_2	0.53	Share of two children	0.55	0.59
b_3	0.55	Share of three children	0.22	0.24
b_4	0.56	Share of four or more children	0.02	0.01
Non-targeted				
		Fertility differential (CL/HS)	0.91	0.80
		College subsidy elasticity of fertility	0.13	0.11
		Benefit elasticity of fertility	$\simeq 0.10$	0.13

Regression Back

Introducing income-tested grants

Consider a grant scheme (introduced in Japan in 2020):

$$g(I) = \begin{cases} g & \text{if } I \leq \bar{I} \\ 0 & \text{otherwise} \end{cases}$$

- $\bar{I}$: $\simeq$ the 15 PCTL
- g: $\simeq$ 2/3 of students' avg. expenses
- Gov. budget balance by labor income tax

Effects on fertility

	Total	Insurance	Other
Total fertility	+1.7%	+1.2%	+0.5%
College	+4.6%	+3.7%	+0.9%
High school	0.0%	0.0%	0.0%

Table: Direct effects of the income-tested subsidy on fertility

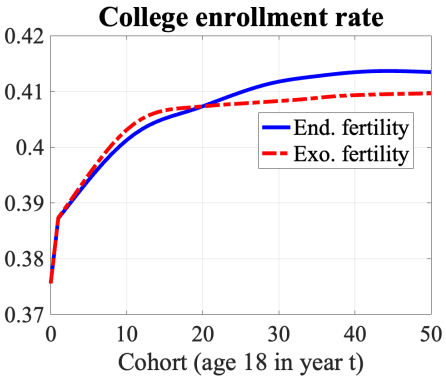
- Insurance effects have the most significant impact on fertility
  – Supporting evidence: Modena et al, 2014; Guner et al., 2024, etc.
- College graduate parents benefit from the subsidy ($\because$ IG links)
- Equilibrium effects play minor roles ▸ Equilibrium effects

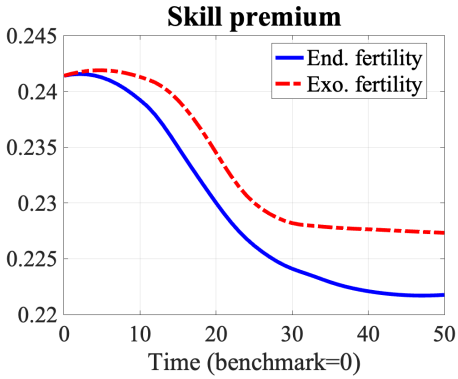
Roles of fertility margins: long-run effects

Table: Endogenous ("End.") vs. exogenous ("Exo.") fertility.

	Fertility setting		
	End.		Exo.
CL share (Δ p.p.)	+3.9	>	+2.9
Skill prem. (Δ %)	−5.9	<	−4.6
Wage SD (Δ %)	−1.3	<	−1.1
Output (Δ %)	+1.0	>	+0.4
Tax (Δ p.p.)	+0.04	<	+0.13
Welfare (%)	+5.1	>	+3.5
MVPF (WTP/Net-costs)	302.94	>	17.06

Transitional dynamics





Recap

- Income-tested college subsidies as insurance for fertility choice
- Fertility margins amplify macro effects of college subsidy

Conclusion

Takeaways and Future Research

Recap

- What has driven the declining fertility?
  – Ch.1: GBTC has been playing an important role

- What are the macro implications of pro-natal policies?
  – Ch.2: Pro-natal cash transfers benefit even non-recipients in the long run by deterring demographic aging
  – Ch.3: Income-tested education subsidies serve as insurance for fertility choices

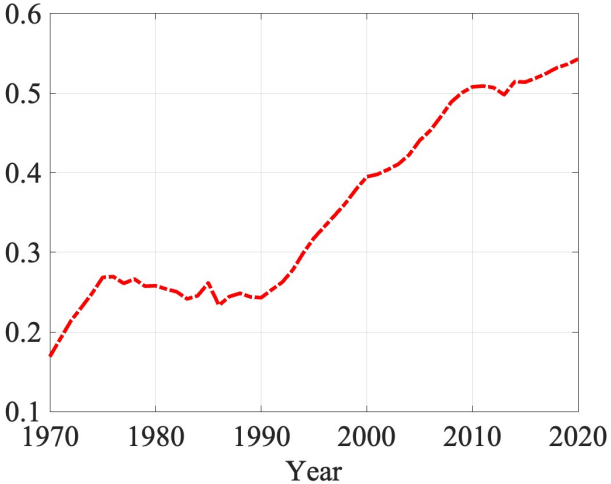
Appendix: Chapter 1

Wage structure

		1970	2020
Gender Gap	Non-college grad.	0.55	0.61
	College grad.	0.66	0.70
	Weighted Avg.	0.51	0.64
Skill Premium	Men	1.36	1.35
	Women	1.65	1.55
	Weighted Avg.	1.46	1.42

Table: Gender wage gap and skill premium. *Gender gap* is defined as the ratio of women's wages to men's wages. *Skill premium* is defined as the ratio of college to non-college grad. wages.

College enrollment rates in 1970-2020



Return

Worker distribution

Table: Distribution of Workers by Gender and Skill

	1970	2020
Women		
Low Skill	29.7%	28.5%
High Skill	1.4%	17.4%
Total	31.1%	45.9%
Men		
Low Skill	50.2%	29.0%
High Skill	18.7%	25.2%
Total	68.9%	54.1%
Total	100.0%	100.0%

Source: Population Census, Ministry of Internal Affairs and Communications.

Return

Time allocation (men & women)

Table: Time Allocation of Married Men and Women (% of disposable time)

	Men		Women	
	1976	2016	1976	2016
Market work	77.45	71.52	37.22	30.61
Housework	0.71	2.11	38.19	32.32
Childcare	0.49	2.72	7.14	13.93
(per child)	(0.22)	(1.68)	(3.24)	(8.61)
Leisure	21.34	23.65	17.46	23.14
Total	100.0%	100.0%	100.0%	100.0%

Source: Survey on Time Use and Leisure Activities. Ministry of Internal Affairs and Communications. Average time use of married men and women aged 25-59.

Return

Equilibrium

Competitive Equilibrium

Given a set of exogenous variables, a competitive equilibrium consists of choices by married households

$$(c, d, h_f, l_f, k, s)$$

and single households

$$(c, d, h)$$

and wage rates

$$\boldsymbol{w} = (w_{ml}, w_{mh}, w_{fl}, w_{fh})$$

that satisfy the following conditions:

- *Given* $\boldsymbol{w}$*, the allocation maximizes household utility.*
- *The firm maximizes its profit.*
- *Goods and labor markets clear.*

Return

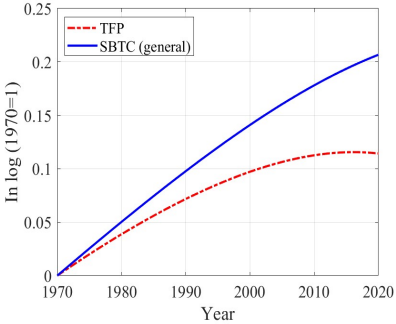
Parameter values

Table: Calibration Parameters

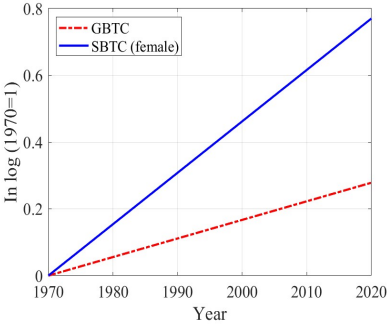
	Description	Value
ρ, α	Curvature and weight: consumption (married)	2.0, 1.0
ν, β	Curvature and weight: home goods (married)	2.007, 0.054
λ, μ	Curvature and weight: leisure (married)	0.113, 0.433
κ, ϕ	Curvature and weight: child (married)	2.085, 0.206
ψ, ξ	Curvature and weight: child quality (married)	0.969, 0.132
β_g	Weight: home goods (single)	0.001 (men)
		0.049 (women)
μ_g	Weight: leisure (single)	0.867 (men)
		2.040 (women)
d, a	Marriage joy shock distribution	0.740, -1.285

Return

Technology



(a) TFP and SBTC (general)



(b) GBTC and SBTC (female)

Figure: Technological Growth

Time allocation

	Baseline		Counterfactual (outcomes in 2020)			
	1970	2020	No-TFP	No-SBT (general)	No-SBT (female)	No-GBT
Work Hours	0.368	0.294	0.374	0.381	0.283	0.254
Leisure	0.168	0.234	0.159	0.147	0.242	0.272
Housework	0.398	0.334	0.335	0.337	0.341	0.341

Table: Roles of Technology. Outcomes in 2020

Return

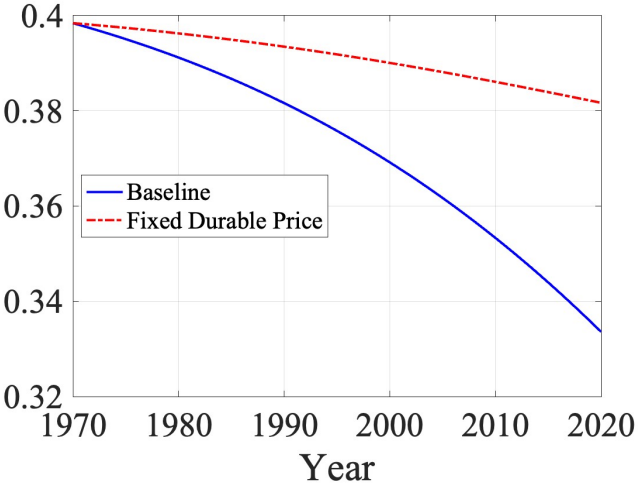
Roles of childcare costs

Table: Roles of Childcare Costs

	1970	2020	Basic Time	Basic Money	Edu Cost
Family and Education					
Fertility (TFR)	1.972	1.349	1.704	1.414	1.369
Schooling	0.215	0.604	0.533	0.593	0.664
Marriage	0.982	0.843	0.873	0.845	0.837
Time Allocation of Married Women					
Work Hours	0.368	0.294	0.323	0.283	0.286
Leisure	0.168	0.234	0.277	0.239	0.240
Housework	0.398	0.334	0.336	0.334	0.333
Childcare	0.065	0.138	0.063	0.144	0.141

Return

Home production



Return

Appendix: Chapter 3

Regression

$$\log(\text{Fertility}_{i,t}) = \alpha + \beta \text{Spending}_{i,t} + \boldsymbol{x}_{i,t}^{'}\boldsymbol{\gamma} + \zeta_t + \eta_i + \varepsilon_{i,t}$$

- $\text{Spending}_{i,t}$: government spending on tertiary education (per student / GDP per capita $\times 100$) in country i and year t.
- $\boldsymbol{x}_{i,t}^{'}$ includes GDP per capita, Gini, gov. expenditures on other educ. stages, and demographic variables.

Results

	(1)	(2)
$\text{Spending}_{i,t}$	$2.7e-04^{***}$	$1.3e-03^{*}$
	$(7.5e-05)$	$(7.6e-04)$
Sample	Full	OECD
Observations	456	201

Literature & Contribution

1. Macro analysis of college subsidy:
   Benabou, 2002; Krueger and Ludwig, 2016; Abbott et al., 2019; Matsuda and Mazur, 2022; Krueger et al., 2024, etc.
   ⇔ Endoginizing fertility and examines roles of fertility margins
2. Quantity-quality (macro) models:
   Doepke and de la Croix, 2004; Greenwood et al., 2016; Daruich and Kozlowski, 2020; Zhou, 2022; Kim et al., 2024, etc.
   ⇔ College choice, skill hetero., and the resulting GE effects.
3. Income risk and family structure:
   Vandenbroucke 2014; Santos and Weiss, 2016; Sommer, 2016; Coskun and Darugic, 2024, Bacher, Grübener, and Nord, 2024, etc.
   ⇔ Examines policies as insurance for fertility choices

What matters to enrollment choices?

1. IVTs made by parents (a_{IVT})
2. Psychic costs of education ($\phi \sim g^{\phi}_{h,e_p}$)
3. Innate ability ($h \sim g^{h}_{h_p}$)
4. Household income (I): determines eligibility for grants & loans

Budget constraints: college students

Expenditures:

- Tuition fees p_{CL} (exogenous)
- Consumption c (endogenous/choice)

Revenue:

- From parents, a_{IVT} (parent's choice)
- Labor earnings (endogenous/choice)
- Loans/Grants provided by the gov. (if eligible)

Education choices

$$V_{g0}(a_{IVT}, \phi, h, I) = \max_{e \in \{0,1\}} \Big\{ (1-e) \cdot \mathbb{E}_{z_0}[\underbrace{V^w(a_{IVT}, j = 18, z_0; e = 0, h)}_{\text{Value for high school graduates}}] + e \cdot [\underbrace{V_{g1}(a_{IVT}; h, I) - \phi}_{\text{Net value for college graduates}}] \Big\}, \tag{1}$$

where $e = 1$ represents enrolling in college.

Return

Fertility Choices

Problem at age 30:

$$V^f(a, z, e, h) = \max_{n \in \{0,1,..,N\}} \left\{ \underbrace{V^{wf}(a, J_F, z; e, h, n)}_{\text{Continuation value w/ } n \text{ kids}} \right\}$$

Derive utility from:

- n: The number of children ("quantity")
- a_{IVT}: IVT $\simeq$ education spending ("quality")

Working stage with children

For $j = J_F, ..., J_{IVT} - 1$,

$$V^{wf}(a,j,z;e,h,n)$$
$$= \max_{c,l,q,a'} \{u(c/\Lambda(n), l) + b(n) \cdot v(q)$$
$$+ \left\{ \begin{array}{ll} \beta \mathbb{E}_{z'}[V^{wf}(a', j+1, z'; e, h, n)] & \text{if } j < J_{IVT} - 1 \\ \beta \mathbb{E}_{z',\phi_k,h_k}[V^{IVT}(a', z'; \phi_k, h_k, e, h, n)] & \text{if } j = J_{IVT} - 1 \end{array} \right\}$$

s.t.

$$(1+\tau_c)(c + \underbrace{n \cdot q}_{\text{money costs}}) + a' = R \cdot a + n \cdot B + \psi$$
$$+ (1-\tau_w) w_e \eta_{j,z,e,h} (1 - l - \underbrace{\kappa \cdot n}_{\text{time costs}}),$$

$$a' \geq -\underline{A}.$$

Return

Inter-vivo transfers

Problem at the beginning of age 48 (w/ children aged 18):

$$V^{IVT}(a, z; \phi_k, h_k, e, h, n) = \max_{a_{IVT} \geq 0} \Big\{ \underbrace{V^w(a - n a_{IVT} R^{-1}, J_{IVT}, z; e, h)}_{\text{Own continuation value}} + \underbrace{b(n) \cdot \lambda_a}_{\text{Discounting}} \cdot \underbrace{V_{g0}(a_{IVT}, \phi_k, h_k, I)}_{\text{Children's ex. life. utility}} \Big\},$$

where $R = 1 + (1 - \tau_a)r$ and income $I = I(J_{IVT}, z, e, h)$.

1. Gains from investments (IVTs) depend on child's ability/taste
2. Having more children can be (ex-post) too costly at this stage

Data: JPSC

- Nationally represented sample of women & their family
- Since 1993 with sample of 1,500 women born in 1959-69
- Info.: income, education, fertility, education spending, etc.

Sample selection:

- Birth cohort: 1959-69
- Married

Return

Income Process

- Labor earnings: $w_e \times \eta_{j,z,e,h} \times$ Hours worked.
- Labor productivity $\eta_{j,z,e,h}$:

$$\log \eta_{j,z,e,h} = \underbrace{\log f^e(h) + \gamma_{j,e}}_{\text{deterministic}} + \underbrace{z}_{\text{stochastic}},$$

- where z follows an AR(1) process:

$$z' = \rho_e^z z + \zeta, \quad \zeta \sim N(0, \sigma_e^z).$$

Estimating income process

Use 2-year span (average) hourly wages (c.f., Daruich and Kozlowski, 2020).

	$<$ College	College
ρ_e^z	0.98	0.97
σ_e^z	0.02	0.03

Table: Estimated parameters for income process.

Return

Internally determined

Parameter	Value	Moment	Data	Model
μ	0.23	Work hours	0.33	0.30
$\bar{t}$	0.8	Income share of labor earnings	0.20	0.17
ι_s	0.055	Share of students using loans	0.44	0.34
ι	0.054	Household share with negative net worth	0.54	0.45
ω_h	0.52	CL−HS wage ratio	1.36	1.48
ψ	0.01	Var(log disposable income)/Var(log gross income)	0.60	0.68
λ_q	0.62	Average transfer / Average income at age 28	0.07	0.07
λ_a	1.03	Average transfer / Average income at age 28	0.27	0.27
ω	1.71	Intergenerational mobility of education	See Table **??**	
σ_h	0.65	Variance of log(income) at age 28	0.27	0.24
ψ_{CL}	20.8	College enrollment rate	0.377	0.376
α_{CL}	0.1	Log wage ratio (CL−HS) at age 28	0.34	0.38
β_{CL}	0.1	Var log wage for CL at age 28	0.14	0.24
b_1	0.49	Share of one child	0.16	0.15
b_2	0.53	Share of two children	0.55	0.61
b_3	0.55	Share of three children	0.22	0.24
b_4	0.56	Share of four or more children	0.02	0.00
Z	1.99	Low skill wage	1.0	1.0

Return

Effects on fertility rates

		Direct effect		Equilibrium effect		
	Overall	Average	Insurance	Price	Tax	Dist.
Total fertility	+1.7%	+0.5%	+1.2%	+0.6%	0.0	−0.4%
College	+4.5%	+0.9%	+3.7%	+1.6%	0.0	0.0
High school	0.0%	0.0%	0.0	0.0	0.0	0.0

Table: Decomposing the effects on fertility.

Return

Welfare across cohorts

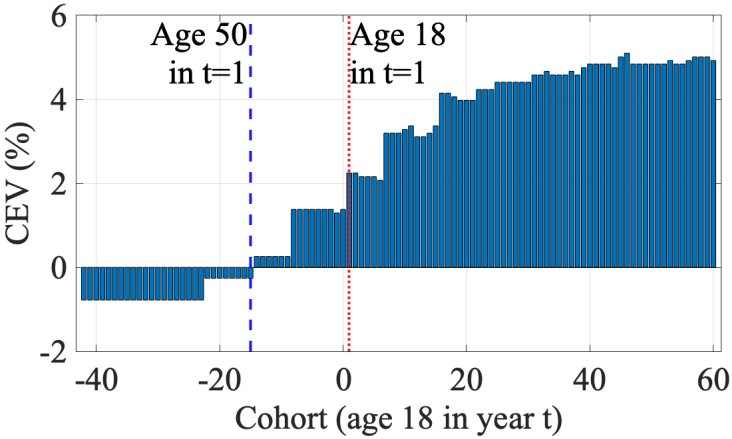


Figure: Welfare effects for each cohort.