(Unit 1)

Matrices

$A = [a_{ij}]_{m \times n}$ $i \to i^{th}$ row $m \Rightarrow$ number of row
$j \to j^{th}$ Column $n \to$ Column,

Types $\to$
(i) Row matrix $\Rightarrow [a\ b\ c]$
(ii) Column matrix $\Rightarrow \begin{bmatrix} a \\ b \end{bmatrix}$
(iii) Null Matrix (zero) $\Rightarrow [0]$
(iv) Square $\Rightarrow m = n$
(v) Diagonal $\Rightarrow$ non main diagonal element $= 0$
$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$
(vi) Scaler matrix $\Rightarrow$ diagonal element constant
(vii) Unit matrix $\Rightarrow I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(Identity)
(viii) Transpose $\Rightarrow m \Leftrightarrow n \Rightarrow A^T$
(ix) Symmetic $\Rightarrow A = A^T$
(x) Skew symmetric $\Rightarrow A = -A^T$
(xi) Trace of matrix $\Rightarrow$ sum of diagonal element $= tr(A)$
(xii) Triangular $\to$ Upper $\Rightarrow \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$
$\to$ Lower $\Rightarrow \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$
(xiii) Equal matrix
$\Rightarrow$ order same
corresponding element are equal.
(xiv) Orthogonal $\Rightarrow AA^T = A^TA = I$
(xv) Idempotent $\Rightarrow A^2 = A$
(xvi) Involuntary $\Rightarrow A^2 = I$

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Alegbra — (i) Addition } (Same order)
(ii) Substraction
(iii) Multiplication
(a) Scaler :- const × matrix ⇒ kA

(b) two matrix ⇒ $A \cdot B$ where $A = [a_{ij}]_{m \times n}$

$C = AB$ $B = [b_{jk}]_{n \times p}$

where $C = [c]_{m \times p}$

Determinant

order (2) $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$

order 3 → $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \Rightarrow a_{11}(a_{22}a_{33} - a_{23}a_{32})$

$- (a_{12}(a_{21}a_{33} - a_{31}a_{23})$

$+ (a_{13}(a_{21}a_{32} - a_{31}a_{22})$

Minors : $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

for M_{11} (of a_{11}) = $\begin{bmatrix} a_{22} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$

$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$

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Cofactor $a_{11} \Rightarrow A_{11} = (-1)^{i+j} M_{ij}$

* Adjoint

$adj(A) = [A_{ij}]^{T}_{n \times n}$

here $A_{ij} \Rightarrow$ Cofactor matrix

Property $(adj\, A) A = A (adj\, A) = |A| I_n$

Singular matrix — $|A| = 0$

Non singular matrix — $|A| \neq 0$

Inverse of a Matrix

$A^{-1} = \frac{1}{|A|} adj(A)$

$|A| \neq 0$ means non-singu.

Property

$(AB)^{-1} = B^{-1}A^{-1}$ $\quad (A^{T})^{-1} = (A^{-1})^{T}$

Rank of a Matrix $\rho(A)$ non-negative integer. (0 to ∞)

* at least one square sub-matrix of r order.
  determinant $\neq 0$

Rank of Null matrix $= 0$

Matrix $(m \times n) = \rho(A) \leq \min(m, n)$

Echelon form (i) a row which has all its elements zero below a row of non zero element

(ii) No. of zero befor first non zero element in row. < no. of zeros in next row.

Note $\rho(A)$ in echelon = No of non zero rows

for echelon form ⇒ by elementry transformation make lower row = zero

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Normal form : I_r ⇒ $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, $[I_r \; 0]$ $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$

$\rho(A) = r$ — Normal form

after getting echelon form make identity matrix on constant by row & Column

Nullity of a Matrix for $AX = 0$

rank nullity theorem:

$r_A + N_A = n$ → n of Column

rank ↙ ↓ nullity

here N_A = dimension of space (Nullity)

* System of linear Simultaneous eqn

$AX = B$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Coeffi matrix, Vector (Solution), Column

$\rho(A:B)$ → Rank of augmented matrix

$AX = B$

- $B = 0$ Homogenous
  - $D \neq 0$ Trivial solution $\rho(A) = \rho(A:B) = n$ (Unique)
  - $D = 0$ Non trivial solution $\rho(A) = \rho(A:B) < n$ (∞ soln)
- $B \neq 0$ Inconsistent $\rho(A) \neq \rho(A:B)$

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Solution methods

(i) $AX = B$

$\Rightarrow X = A^{-1}B$

Inventible method

(ii) Gauss elimination

A → echelon form by row transformation.

Homogeneous system $AX = 0$

$|A| \neq 0 \Rightarrow \rho(A) = n \Rightarrow$ Trival solution.

all zero. $x_1 = x_2 = x_3 = 0$

$|A| = 0 \Rightarrow \rho(A) = r < n \Rightarrow$ non trival.

at least one $x \neq 0$

Linear Dependent (LD)

$\rho(A) \neq$ no of variable

Linear Independent (LI)

$\rho(A) =$ no. of variable

- Eigen values $A = [a_{ij}]_{n \times n}$

Characteristic matrix of A $= [A - \lambda I]$

$\lambda = |A - \lambda I| =$ Characteristic polynomial of A

$|A - \lambda I| = 0 \Rightarrow$ Characteristic equation

roots of equation = eigen values (λ')

- Sum of eigen value = tr(A)
- product = $|A|$
- A & A^T same eigen value
- set of eigen value = Spectrum of A

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Eigen vactor (Charactor vactor)

Put value of λ in equation
$(A - \lambda I)X = 0$
then find X
here X is eigen vactor of A

Diagonalization of matrix X_1 & X_2 $\Rightarrow$ Orthogonal
for
$X_1^T X_2 = X_2^T X_1 = 0$

$\therefore$ $A = \lambda I$
then B $\Rightarrow$ $B^{-1}AB$ is diagonal matrix
& diagonal element are
eigen value of A.

Caley - Hamilton Theorem
every square matrix satisfy its own
charastristic equation
$|A - \lambda I| = (-1)^n \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \dots a_n = 0$
Put $\lambda = A$

$(-1)^n A^n + a_1 A^{n-1} + a_2 A^{n-2} + \dots a_2$

$\downarrow$ multiply by A^{-1} then devide by a_2