Number System

Representation of Numbers in Radix r :

The number of unique digits used to form numbers which a number system is called radix of that system. For example; in the decimal number system 0,1,2,3,4,5,6,7,8,9 digits are used to form numbers, thus, its radix/base is 10.

There are two types of Number Systems :-

(i) Non-Positional Number System :- In Non-Positional number system 0 is missing absent. One example of this number system is Roman Number System.

I for 1, II 2, VI for 6, X for 10 L for 50 etc. there is no symbol for 0.

(ii) Positional Number System - Numbers are determined by a string of digit symbols. A number system of base or radix (r) uses distinct r digit symbols. It consist of two portions integer and fractional separated by a radix point.

$$(N)_r = \text{Integer portion} \cdot \text{fraction Portion} \quad (28.32)_{10}$$

$$= a_{n-1} a_{n-2} \cdots a_2 a_1 a_0 \cdot b_{-1} b_{-2} \cdots b_{-m}$$

$$\underset{\text{MSD}}{} \qquad\qquad\qquad\qquad\qquad \underset{\text{LSD}}{}$$

Position	$n-1$	$n-2$	....	2	1	0	-1	-2	$-m$
weight	r^{n-1}	r^{n-2}	...	r^2	r^1	1	r^{-1}	r^{-2}	r^{-m}

Quantity
or
Value

$$a_{n-1}r^{n-1} + a_{n-2}r^{n-2} + a_2 r^2 + a_1 r^1 + a_0 r^0 \cdot b_{-1}r^{-1} + b_{-2}r^{-2} + b_{-m}r^{-m}$$

$$2 \times 10^1 + 8 \times 10^0 + 3 \times 10^{-1} + 2 \times 10^{-2}$$

①

Number System	Base OR Radix	Symbols (Digits)	Examples
Binary	2	0,1	1011.11
Octal	8	0,1,2,3,4,5,6,7	345.27
Decimal	10	0,1,2,3,4,5,6,7,8,9,10	245.95
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F	2AB3.A53

Conversion between Number Bases:

Converting Decimal to binary, octal, and Hexadecimal

Decimal to Binary

· $(31)_{10}$ to Binary $(\ )_2$

$(31)_{10} = (11111)_2$

Fractional Conversion

Convert $(.125)_{10}$ to Binary $(\ )_2$

$.125$
$\underline{\times 2}$
0.250 $I_1 = 0$
$\underline{\times 2}$
0.500 $I_2 = 0$
$\underline{\times 2}$
1.000 $I_3 = 1$

$(.125) = (001)_2$

$/10$ $/16$

Example :-

Find the Binary equivalent of $(23.8125)_{10}$

2	23	
2	11	1
2	5	1
2	2	1
	1	0

$(23)_{10} = (10111)_2$

$$
\begin{array}{l}
.8125 \\
\times\ 2 \\
\hline
1.6250 \qquad I_1 = 1 \\
\times\ 2 \\
\hline
1.2500 \qquad I_2 = 1 \\
\times 2 \\
\hline
0.5000 \qquad I_3 = 0 \\
\times 2 \\
\hline
1.0000 \qquad I_4 = 1 \ \downarrow
\end{array}
$$

$(.8125)_{10} = (1101)_2$

Decimal to Hexadecimal

→ Convert $(5386)_{10}$ to Hexadecimal number.

16	5386	
16	336	10 - A
2	21	0
	1	5

$(5386)_{10} = (150A)_{16}$

→ Convert Decimal to Hexadecimal with fractional part

$(18.765625)_{10}$ to $(\qquad)_{16}$

16	18	
	1	2

$(18)_{16} = (12)_{16}$
$= 12$

$.765625 \times 16 = 12.250000$
$.25 \qquad \times 16 = 4.00$

$(.765625)_{10} = (C4)$

$(18.765625)_{10} = (12.C4)_{16}$

➤ Decimal to octal

$$(473)_{10} \text{ to } (\ ?\ )_8$$

$$
\begin{array}{c|c|c}
8 & 473 & \\
\hline
8 & 59 & 1 \uparrow \\
\hline
 & 7 & 3 \\
\end{array}
$$

$$(473)_{10} = (731)$$

• Decimal with Fraction to octal

$$(73.82)_{10} = (\qquad)_8$$

$$
\begin{array}{c|c|c}
8 & 73 & \\
\hline
8 & 9 & 1. \\
\hline
 & 1 & \\
\end{array}
$$

$$= (73)_{10} = (\cancel{101})_8 \quad (111)_8$$

$.82 \times 8 = \quad\quad 6.56$

$.56 \times 8 = \quad\quad 4.48$

$.48 \times 8 = \quad\quad 3.84$

$.84 \times 8 \quad\quad\quad .6.72 \downarrow$

$(.82) = (.6436)_8$
$= 10$

$$(73.82)_{10} = (111.6436)_8$$

➤ Decimal to octal

Conversion

Binary to Decimal $= (1101)_2$ to $(\quad)_{10}$

1 1 0 1

$1 \times 2^0 = 1$
$0 \times 2^1 = 0$
$1 \times 2^2 = 4$
$1 \times 2^3 = 8$

$\overline{13}$

$(1101)_2 = (13)_{10}$

· Fraction :

$(110.101)_2 = (\ ?\ )_{10}$

1 1 0 . 1 0 1

$1 \times 2^{-3} = .125$
$0 \times 2^{-2} = 0$
$1 \times 2^{-1} = .5$ $\Big\}$ .625

$0 \times 2^0 = 0$
$1 \times 2^1 = 2$
$1 \times 2^2 = 4$ $\Big\}$ 6

$(110.101)_2 = (6.625)_{10}$

$\begin{bmatrix} 2^{-1} = \tfrac{1}{2} = 0.5 \\ 2^{-2} = \tfrac{1}{4} = 0.25 \\ 2^{-3} = \tfrac{1}{8} = 0.125 \\ 2^{-4} = \tfrac{1}{16} = .0625 \end{bmatrix}$

$\boxed{0.0625}$

→ Octal to Decimal

$(5012)_8$ to $(\quad)_{10}$

5 0 1 2

$2 \times 8^0 = 2$
$1 \times 8^1 = 8$
$0 \times 8^2 = 0$
$5 \times 8^3 = 2560$

$(5012)_8 = (2570)_{10}$

OR

$5 \times 8^3 + 0 \times 8^3 + 1 \times 8^1 + 2 \times 8^0$
$= 2560 + 0 + 8 + 2$
$= 2570$

Convert Hexadecimal to Decimal

$(4CD)_{16} = (?)_{10}$

4C D

$13 \times 16^0 = 13$
$12 \times 16^1 = 192$ $= 1229$
$4 \times 16^2 = 1024$

$(4CD)_{16} = (1229)_{10}$

Convert Hexadecimal to Decimal

Binary Addition.

Rules for binary addition are as follows :-

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 0 \quad \text{with 1 carry.}$$

e.g.

	Decimal
carry 1 1 1 1 1	
1 1 0 1 1	27
(+) 1 0 1 0 1	21
1 1 0 0 0 0	48

Binary addition

$$(10001)_2 + (11101)_2$$

	Decimal
1 1	17
1 0 0 0 1	+ 29
(+) 1 1 1 0 1	46
1 0 1 1 1 0	

Binary Subtraction

Subtraction Rules :-

$$0 - 0 = 0$$
$$1 - 0 = 1$$
$$1 - 1 = 0$$
$$0 - 1 = 1 \quad \text{with 1 borrow}$$

e.g.

	Decimal
1 1 0 1 1 0 1	109
(-) 1 0 1 1 0 1 1	(-) 91
0 0 1 0 0 1 0	18

e.g.

	Decimal
1 1 0 1 1 0	54
(-) 1 0 1 1 1 1	(-) 47
0 0 0 1 1 1	7

r^s complement

Complement Number System

```
        ┌─────────────────────────────┴──────────────────────────┐
        ↓                                                          ↓
   $r^s$ Complement                                      $(r-1)'s$ Complement
                                                          (Deminished Radix/base)
   (Radix/base Complement)
                                                          $(B^K-1)-N$
        $B^K-N$
```
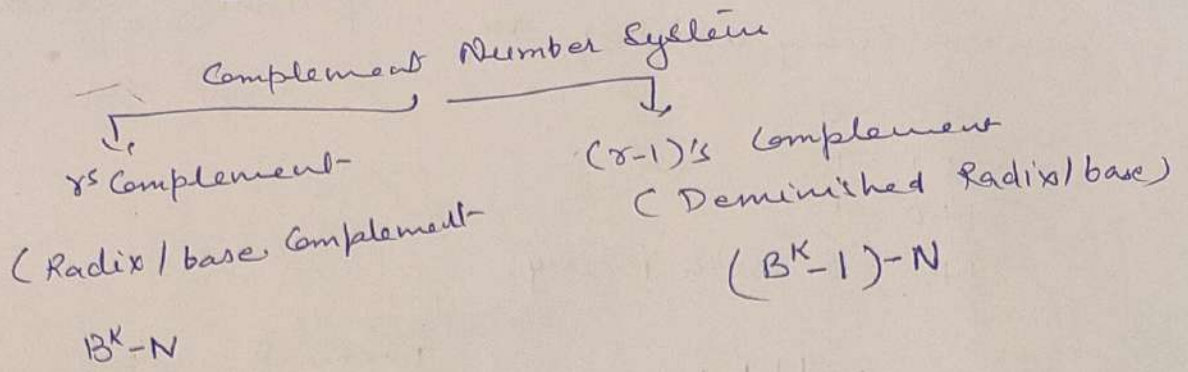
B = Base / Radix

K = Total digit of Number

N = Given Number

```
                     ┌→ 1ˢ complement    $(2^{K}-1)-N$
         Base 2  <
                     └→ 2's Complement   $(2^K-N)$
                                         $(8^K-1)-N$
                     ┌→ 7ˢ         "
         Base 8  <
                     └→ 8's        "     "   $(8^K-N)$
                     ┌→ 16'ˢ       "     "   $(16^K-1)-N$
         Base 16 <
                     └→ 15⁻'ˢ      "     "   $(16^K-N)$
                     ┌→ 10'ˢ       "     "   $(10^{K}-1)-N$
         Base 10 <
                     └→ 9'ˢ        "     "   $(10^K-N)$
```
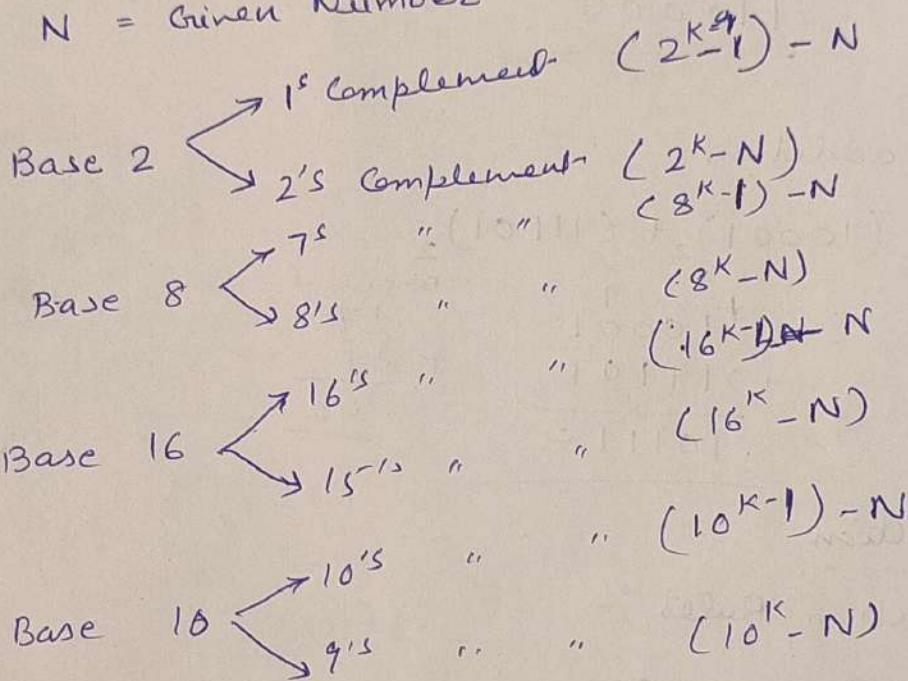
Ex. Find the $r's$ Complement of $(37)_{10}$

So: Base $=10$, K = 2, N = 37

$$B^K - N$$

$$10^2 - 37 = 100 - 37 = 63$$

S. $r-1^s = (B^K-1)-N$

$$(10^2-1) - 37$$

$$e\ 99 - 37$$

$$= \underline{62}$$

→ Second Method to find $r's$ and $r-1's$ complement.

Largest Number of base $10 = 9$
So subtract each digit from 9

$$\begin{array}{r} 99 \\ -37 \\ \hline 62 \end{array}$$

9's complement is 62 $(r-1)$ complement of base 10

10's complement $(r's) = $ 9's complement $+1$

so $= 62 + 1 = 63$

eg. Find the $r's$ complement of $(15)_8$

~~Method 1~~

$r's$ complement
$(8-1)$ $r's$ complement $(13\cancel{17}$ $\cancel{\text{or}}$ $)$ $-N$
$(64\cancel{91})_{10}$ $-(15)_8$
$4\cancel{8}$

Cor

Method 2.
~~Highest digit~~
Radix $-1 = 9$
$8 - 1 = 7$

$$\begin{array}{r} 77 \\ 15 \\ \hline 62 \\ + \quad 1 \\ \hline 63 \end{array}$$

$r's$ complement.

Method -1

Find the $r's$ complement of $(15)_8$

$B = 8$, $K = 2$, $N = 15$

$= 8^2 - 15 = (64)_{10} - (15)_8$

$= (15)_8 = 1 \times 8^1 + 5 \times 8^0$
$= 8 + 5$
$= 13$

Now Subtract

$64 - 13$
$= (51)_{10}$

Now Convert $(51)_{10}$ to $(?)_8$

$$\begin{array}{r|l} 8 & 51 \\ \hline & 6 \quad 3 \end{array}$$

$(63)_8$ is $r's$ $8's$ complement of $(15)_8$.

\# Find the Base : $(86)_{10} = (56)_x$

$\rightarrow \quad (86)_{10} = (5 \times x^1 + 6 \times x^0)_{10}$

$(86)_{10} = (5x + 6)_{10}$

$$86 = 5x + 6$$

$$86 - 6 = 5x$$

$$80 = 5x$$

$$x = \frac{80}{5} = 16$$

Hence $= (86)_{10} = (56)_{16}$