~~$(D^2+3D+2)y = e^{2x}\sin x$~~

Unit 5

Seperation variable method -

① Let $U(x,t) = XT$ — depend on. here $X \to x$, $T \to t$

find $\frac{\delta U}{\delta x} = X'T$

$\frac{\delta U}{\delta T} = T'X$

Put the values in main question and seperate X & T variable. Then integrate them.

Laplace equation

$$\boxed{\frac{\delta^2 U}{\delta x^2} + \frac{\delta^2 U}{\delta y^2} = 0}$$

Let $U(x,y) = XY$ — $X \to x$, $Y \to y$

find $\frac{\delta U}{\delta x}$ ⇒ then $\frac{\delta^2 U}{\delta x^2} = X''Y$

$\frac{\delta U}{\delta y}$ ⇒ then $\frac{\delta^2 U}{\delta y^2} = Y''X$

by eq.

$X''Y + Y''X = 0$ — seperate them

$\frac{X''}{X} = -\frac{Y''}{Y}$ — (ii)

by eq. (iii) Let constant k

$$\frac{X''}{X} = -\frac{Y''}{Y} = k$$

Solve them Separately.

$$\frac{X''}{X} = k \quad \Big| \quad \frac{-Y''}{Y} = k$$

$$X'' - Xk = 0 \qquad Y'' + Yk = 0$$

double order partial differential equations

Now Solve them (let)

for $k > 0 \Rightarrow +\lambda^2$

$k = 0$

$k < 0 \Rightarrow -\lambda^2$

→ find X & Y put value in $U = XY$ Ans.

Q Wave equation $\frac{\partial^2 U}{\partial t^2} = c^2 \frac{\partial^2 U}{\partial x^2}$

let $U = XT$

find $\frac{\partial^2 U}{\partial x^2} = X''T$ $\qquad \frac{\partial^2 U}{\partial t^2} = XT''$

by eq. $XT'' = c^2 X''T$

saperate variable

$$c\frac{X''}{X} = \frac{T''}{T}$$

Let ~~xxx~~ Constant $= -k^2$

$$\frac{cX''}{X} = \frac{T''}{T} = -k^2$$

by eq $\frac{1}{c^2 T} T'' = k^2$ | $\frac{X''}{X} = -k^2$

$T'' + c^2 k^2 T = 0$ | $X'' + k^2 X = 0$

Now solve 2nd order Partial diff. equation.

(iii) Heat eq. $c^2 \frac{\partial^2 U}{\partial x^2} = \frac{\partial U}{\partial t}$

Let $U = XT$ $\quad \frac{\partial U}{\partial t} = XT'$

$\frac{\partial^2 U}{\partial x^2} = X''T$

· by eq.

$c^2 X'' T = XT'$ → ~~double ordy~~

$c^2 \frac{X''}{X} = \frac{T'}{T} = K$ → Const

↳ fi

$$\frac{c^2 X''}{X} = k$$

$$X'' - \frac{k}{c^2} X = 0$$

Second order

$$\frac{T'}{T} = k$$

$$T' - kT = 0$$

first order