1E3101

Roll No. : ....

1E3101

B.Tech. I sem(Main/Back) Exam 2024

1FY2-01 / Engineering Mathematics-I

Time : 3 Hours   Maximum Marks : 70

Instructions to Candidates :

Attempt all ten questions from Part-A, five questions out of seven questions from Part-B and three questions out of five questions from Part-C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used / calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in Form No. 205)

1. ....   2. ....

PART-A

(Answer should be given up to 25 words only)

All questions are compulsory

Q.1. What is the value of integral $\int\limits_{0}^{\infty} e^{-x^2} dx$?

Q.2. Write the formula of surface area of solid of revolution when the revolution is about x-axis.

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Q.3. What do you mean by convergence of a sequence?

Q.4. Find whether the following series is convergent or not?

$$\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\dots$$

Q.5. State Parseval's theorem.

Q.6. Find the value of a_0 for the function $f(x)=|x|$ in the interval $(-\pi,\pi)$.

Q.7. State the necessary and sufficient conditions for the minimum of a function $f(x,y)$.

Q.8. Find the gradient of $f(x,y,z)=x^2y^2+xy^2-z^2$ at $(3, 1, 1)$.

Q.9. Evaluate $\int\limits_{0}^{b}\int\limits_{0}^{x}xy\,dx\,dy$.

Q.10. State the Gauss Divergence theorem.

PART - B

(Analytical/Problem solving questions)

Attempt any five questions

Q.1. Use beta and gamma functions, to evaluate :

$$\int\limits_{0}^{\infty}\frac{x^2(1+x^4)}{(x+x)^{10}}dx .$$

Q.2. Expand $\sin x$ in the powers of $(x-\pi/2)$ using Taylor's series.

Q.3. Find Fourier series of x^2 in $(-\pi,\pi)$, and use Parseval's identity to prove :

$$\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots$$

Q.4. If $u = e^{xyz}$, then show that :

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$$

Q.5. Whether the fluid motion given by $V = (y+z)i + (z+x)j + (x+y)k$ is incompressible or not?

Q.6. Change the order of integration and hence evaluate :

$$\int_0^1 \int_{e^x}^{e} \frac{1}{\log y} \, dx \, dy .$$

Q.7. Evaluate $\displaystyle\int_1^2 \int_1^z \int_0^{yz} (xyz) \, dx \, dy \, dz.$

PART - C
(Descriptive/Analytical/Problem Solving/Design question)
Attempt any three questions

Q.1. Use beta and gamma functions, to evaluate :

(a) $\displaystyle\int_0^\infty \frac{x}{1+x^6} \, dx.$

(b) $\displaystyle\int_0^1 \sqrt{\left(\frac{1-x}{x}\right)} \, dx.$

Q.2. Find the Fourier series expansion of the following periodic function with period 2π.

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 0, & x = 0 \\ 1, & 0 < x < \pi \end{cases}$$

Hence, show that $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} - \ldots = \dfrac{\pi}{4}$.

Q.3. Use Lagrange's method to find the maximum and minimum distance of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$.

Q.4. If $u = f(r)$, where $r^2 = x^2 + y^2$, then prove that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$

Q.5. Verify Green's theorem for $\int_C \left[(xy + y^2)dx + x^2 dy \right]$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.