

3E1136	Roll No. _____	Total No. of Pages: 4
<p style="font-weight: bold; font-size: 1.2em;">3E1136</p> <p style="font-weight: bold;">B. Tech. III - Sem. (Back) Exam., February - 2023</p> <p style="font-weight: bold;">Computer Science & Engineering</p> <p style="font-weight: bold;">3CS2 – 01 Advanced Engineering Mathematics</p> <p style="font-weight: bold;">Common For CS, IT</p>		

Time: 3 Hours

Maximum Marks: 120

Min. Passing Marks: 42

Instructions to Candidates:

Attempt all ten questions from Part A, five questions out of seven questions from Part B and four questions out of five from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL2. NIL**PART – A****(Answer should be given up to 25 words only)****[10×2=20]****All questions are compulsory**

Q.1 Let $X = \{-1, 1\}$ be random variable with probability density function $f(x = 1) = 1/2$ and $f(x = -1) = 1/2$, find the moment generating function.

Q.2 State the condition for which binomial distribution becomes symmetric.

Q.3 Write Chebyshev's Inequality.

Q.4 What will be the value of coefficient of correlation when two regression lines coincide?

Q.5 If X has the variance 9 and Y has the variance 5, then write the value of $\text{var}(2X + Y - 5)$.

- Q.6 Feasible region's optimal solution for a linear objective function always includes which points?
- Q.7 In transportation models designed in linear programming, write the name of "points of demand".
- Q.8 Which solutions are included in the convex set of equations in a linear programming equations?
- Q.9 Let X_1 and X_2 are two independent variables and $Y = a_1X_1 + a_2X_2$, then write the variance of Y .
- Q.10 What is the name of specific combination of decision variables to specify non – negativity and structural constraints in optimization problem?

PART – B

(Analytical/Problem solving questions)

[5×8=40]

Attempt any five questions

- Q.1 A firm manufacturing two type of electrical items. A and B can make a profit of ₹ 20 per unit of A and ₹ 30 per unit of B. Each unit of A requires 3 motors and 2 transformers and each unit of B requires 2 motors and 4 transformers. The total supply of these per month is restricted to 210 motors and 300 transformers. Type B is an expert model requiring a voltage stabilizer, which has supply restricted to 65 units per month. Formulate above as a linear programming problems for maximum benefit.
- Q.2 Let $f(x, y) = \begin{cases} 1 & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$ be the joint density function of X and Y . Find the density function of $Z = XY$.
- Q.3 Classify optimization problems based on existence of constraints and nature of design variables.

- Q.4 The regression lines of y on x and x on y are $y = ax + b$ and $x = cy + d$, respectively. Show that -

$$\frac{\sigma_y}{\sigma_x} = \sqrt{\frac{a}{c}}; \bar{x} = \frac{bc+d}{1-ac} \quad \bar{y} = \frac{ad+b}{1-ac}$$

- Q.5 Find maximum of the function $f(X) = 2x_1 + x_2 + 10$ subject to $g(X) = x_1 + 2x_2^2 = 3$ using the Lagrange multiplier method. Also find the effect of changing the right hand side of the constraint on the optimum value of f.
- Q.6 Find the dual of the following LPP:

$$\min z = x_1 + x_2 + x_3 \text{ such that } x_1 - 3x_2 + 4x_3 \leq 5;$$

$$2x_1 - 2x_2 \leq -3;$$

$$2x_2 - x_3 \geq 5; \quad x_1, x_2, \geq 0, x_3 \text{ is unrestricted.}$$

- Q.7 If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001. Determine the probability that out of 2000 individuals (a) exactly 3 (b) more than 2 individuals will suffer from bad reaction.

PART - C

(Descriptive/Analytical/Problem Solving/Design Questions) [4×15=60]

Attempt any four questions

- Q.1 Find first four moments and moment generating function of exponential distribution.

- Q.2 Consider the problem -

$$\text{minimize } f(x_1, x_2) = (x_1 - 1)^2 + x_2^2$$

$$\text{subject to } g_1(x_1, x_2) = x_1^3 - 2x_2 \leq 0;$$

$$g_2(x_1, x_2) = x_1^3 + 2x_2 \leq 0;$$

Determine whether the constraint qualification and the Kuhn - Tucker conditions are satisfied at the optimum point.

- Q.3 Four different jobs can be done on four different machines and take down time costs are prohibitively high for change overs. The matrix below gives the cost in rupees of producing job i on machine j :

Jobs	Machine			
	M_1	M_2	M_3	M_4
J_1	5	7	11	6
J_2	8	5	9	6
J_3	4	7	10	7
J_4	10	4	8	3

How the jobs should be assigned to the various machines so that the total cost is minimized?

- Q.4 Solve the following LPP:

$$\begin{aligned} \min z = x_1 - 3x_2 + 2x_3 \text{ such that } & 3x_1 - x_2 + 3x_3 \leq 7; \\ & -2x_1 + 4x_2 \leq 12; \\ & -4x_1 + 3x_2 + 8x_3 \leq 10; \quad x_1, x_2, x_3 \geq 0. \end{aligned}$$

- Q.5 Let X and Y be continuous random variables having joint density function

$$f(x, y) = \begin{cases} c(x^2 + y^2) & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine (a) constant c , (b) $P(X < 1/2, Y > 1/2)$, $P(1/4 < X < 3/4)$, $P(Y < 1/2)$ (c) marginal density functions of X and Y , (d) whether X and Y are independent (e) conditional distributions of X and Y .
