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New Benchmark Instances for the Capacitated Vehicle Routing Problem

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Abstract. The recent research on the CVRP is being slowed down by the lack of a good set of benchmark instances. The existing sets suffer from at least one of the following drawbacks: (i) became too easy for current algorithms; (ii) are too artificial; (iii) are too homogeneous, not covering the wide range of characteristics found in real applications. We propose a new set of instances ranging from 100 to 1000 customers, designed in order to provide a more comprehensive and balanced experimental setting. We report results with state-of-the-art exact and heuristic methods.

Keywords. Vehicle Routing Problem, Benchmark Instances, Experimental Analysis of Algorithms

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1 Introduction

The Vehicle Routing Problem (VRP) is among the most widely studied problems in the fields of operations research and combinatorial optimization. Its relevance stems from its direct application in the real world systems that distribute goods and provide services, vital to the modern economies. Reflecting the large variety of conditions present in those systems, the VRP literature is spread into dozens of variants. For example, there are variants that consider time windows, multiple depots, mixed vehicle fleet, split delivery, pickups and deliveries, precedences, complex loading constraints, etc.

In the vast landscape of VRP variants, the Capacitated VRP (CVRP) occupies a central position. It was defined by Dantzig and Ramser [12] as follows. The input consists of a set of n + 1 points, a depot and n customers; an $(n + 1) \times (n + 1)$ matrix $d = [d_{ij}]$ with the distances between every pair of points i and j; an n-dimensional demand vector $q = [q_i]$ giving the amount to be delivered to customer i; and a vehicle capacity Q. A solution is a set of routes, starting and ending at the depot, that visit every customer exactly once in such a way that the sum of the demands of the customers of each route does not exceed the vehicle capacity. The objective is to find a solution with minimum total route distance. Some authors also assume that the number of routes is fixed to an additional input number K (almost always defined as the minimum possible number of routes, K_{min}).

The CVRP plays a particular role on VRP algorithmic research, for both exact and on heuristics methods. Being the most basic variant, it is a natural testbed for trying new ideas. Its relative simplicity allows cleaner descriptions and implementations, without the additional conceptual burden necessary to handle more complex variants. Successful ideas for the CVRP are often later extended to more complex variants. For example, the classical CVRP heuristic by Clarke and Wright [9] was adapted for the VRP with Time Windows [29] and for many other variants, as surveyed in Rand [25].

The present work is motivated by the claim that the recent research on the CVRP is being hampered by the lack of a well-established set of benchmark instances able to push the limits of the state-of-the-art algorithms. Let us examine this claim:

• Since Augerat et al. [3], nearly all new exact methods for the CVRP have been tested over instances from six different classes. Classes A, B and P were proposed in [3], while classes E, F and M were proposed by Christofides and Eilon [7], by Fisher [13] and by Christofides et al. [8], respectively. Nearly all those instances are Euclidean. In the

literature of exact methods it became usual to follow the TSPLIB convention of rounding distances to the nearest integer [26] and also to fix the number of routes.

The ABEFMP instances provided a good benchmark until the mid 2000's decade. Although 90 out of its 95 instances have no more than 100 customers, several such instances were still very challenging for the branch-and-cut algorithms (like [1, 2, 3, 6, 20, 24, 34]) which prevailed at that time. Then, the branch-cut-and-price of Fukasawa et al. [14] solved all its instances with up to 100 customers, as well as instances M-n121-k7 and F-n135-k7 (the later was already solved since [3]). Only instances M-n151-k12, M-n200-k16 and M-n200-k17 remained unsolved. From that moment, the benchmark was not satisfactory anymore. While the majority of its instances became quite easy, the now more interesting range of 101 to 200 customers was very thinly populated. In particular, the column and cut generation algorithms of Baldacci et al. [4, 5] significantly reduced the CPU time required to solve many ABEFMP instances, but could not solve the three larger M instances. Later, the algorithms of Contardo and Martinelli [10] and Røpke [28] were capable of solving M-n151-k12. Very recently, the algorithm of Pecin et al. [22] solved the last two open instances in the benchmark.

• Since the 1980's, almost all articles proposing new heuristic and metaheuristic methods for the CVRP reported results on a subset of the instances by Christofides and Eilon [7] and Christofides et al. [8]. In this literature, it is usual to follow the convention of not rounding the Euclidean distances and not to fix the number of routes. This classical benchmark is also exhausted, since most recent heuristics find systematically the best known solutions on nearly all instances. In fact, Rochat and Taillard [27] already published in 1995 what we now know to be the optimal solutions, except on a single instance with 199 customers, where the reported solution was only 0.012% off-optimal.

The more recent set of instances by Golden et al. [17] is now the second most frequently used benchmark. Having larger instances, ranging from 240 to 483 customers, it still has a good discriminating power. In fact, the heuristics of Nagata and Bräysy [21] and Vidal et al. [32] are considered the best available for the CVRP basically due to their superior performance on Golden's instances. Nevertheless, we believe that those instances are not sufficient for a good benchmark. A first drawback is their artificiality. In all instances the customers are positioned in concentric geometric figures, either circles, squares or six-pointed stars. The demands also follow very symmetric patterns. As a result, the solution space is partitioned into groups of equivalent solutions obtainable by rotations

and flippings around several axis of symmetry. The second drawback is their relative homogeneity. For example, there are no instances with clusters of customers.

This work proposes a new set of 100 instances, ranging from 100 to 1000 customers, intended to be used by both exact and heuristic methods in the next years. They were designed in order to provide a more comprehensive and balanced experimental setting.

The remainder of the paper is organized as follows. Section 2 summarizes the current CVRP benchmark instances. Section 3 explains the procedure developed for generating new CVRP instances. Section 4 contains the results found by state-of-the-art heuristic and exact methods for the new set of instances. Section 5 briefly describes the features of the new CVRPLIB web site where all instances mentioned in this paper are available. Finally, Section 6 presents the concluding remarks of this work.

2 Current Benchmark Instances

In this section we provide detailed information about the existing benchmark instances. In particular, we tried to track back the generation process of all these instances.

Table 1 presents data regarding the E series. Although they are usually attributed to Christofides and Eilon [7], some instances actually come from Dantzig and Ramser [12] and from Gaskell [15], and some are modifications later suggested by Gillett and Miller [16]. As usual in the literature on exact methods, the naming of the instances reflects the number of points (including the depot) and the fixed number of routes. For example, E-n101-k8 is an instance with 100 customers and a requirement of 8 routes. Columns in Table 1 include the value of Q, the tightness (the ratio between the sum of all demands and KQ, the total capacity available in the fixed number of routes) and the optimal solution value.

Table 2 presents information about the M series and explains how each instance was generated. Instances M-n200-k16 and M-n200-k17 only differ by the required number of routes. In fact, M-n200-k17 is the only ABEFMP instance where the fixed number of routes does not match the minimum possible. This additional instance was created because M-n200-k16 has a tightness so close to 1 (0.995625) that finding good feasible solutions for it was very difficult. Surprisingly, it was recently discovered that the optimal solution of M-n200-k16 costs less than the optimal solution of M-n200-k17 [22].

Table 3 presents the three real-world instances that compose the F series. Tables 4 and 5 correspond to the A and B series, respectively. While in the A series the customers and the depot are randomly positioned; they are clustered in the B series. The instances

from series P were generated by taking some instances from the A, B and E series and changing their capacities. Consequently, the required number of routes also changes. For example, P-n101-k4 was obtained from E-n101-k8 by doubling the capacity.

Christofides et al. [8] defined the benchmark set shown in Table 7. Instances CMT1, CMT2, CMT3, CMT, CMT5, CMT11, and CMT12 correspond to instances E-n51-k5, E-n76-k10, E-n101-k8, M-n151-k12, M-n200-k16, M-n121-k7, and M-n100-k10, respectively. The only difference are the conventions: the euclidean distances are represented with full computer precision (without rounding), and the number of routes is not fixed. This set also contains instances for the duration constrained CVRP, obtained from the previous CVRP instances by adding maximum route duration (MD) and service time (ST) values. These values are also reported in Table 7. Column K_{BKS} gives the number of routes in the optimal/best known solution of each instance. Optimal solutions are marked with a *.

Table 8 corresponds to the benchmark proposed in Golden et al. [17]. There are 12 CVRP instances and 8 instances for Duration-constrained CVRP, the maximum durations (as there are no service times, this is equivalent to a bound on the maximum total distance traveled in a route) are given in column (MD). Table 8 also gives the geometric patterns used for positioning the customers in each instance.

Table 9 corresponds to a benchmark proposed in Rochat and Taillard [27]. Twelve instances from 75 to 150 customers were generated using a scheme where the depot is always in the center, the customers are clustered and demands are taken from an exponential distribution. An additional instance with 385 customers, obtained from real-world data, already appeared in [31]. Note that a best value of 2341.84 for instance tai150c was mentioned on some benchmark instance repositories and then relayed in some papers. Yet, the exact algorithm of [22] determined that a solution of 2358.66 is optimal and most recent heuristics found the same value. We thus assume that this previous solution was erroneous.

Although there are no pure CVRP instances in this set, for the sake of completeness, Table 10 presents a benchmark proposed in Li et al. [19], composed by 12 larger scale instances of the Duration-constrained CVRP. Typical CVRP heuristics can be easily adapted to handle duration/distance constraints, and some articles on the CVRP also reported results in the Li benchmark. However, most recent state-of-the-art methods for the CVRP [21, 32] did not. Therefore, in order to present solutions that correspond to the current point of algorithmic evolution, we performed runs with the algorithm of [32]. Table 10 has been updated to include the newly found best known solutions. The detailed computational results are reported in the Appendix. We also highlight an issue related

to the published solution of instance pr32, generated by a manual process in [19] with a distance value of 36919.24. This solution seems to have 10 routes from the figure in the paper, and as such cannot comply with the distance limit of 3600 units. For this reason, it was not included in the table.

3 Newly Proposed Instances

This section describes how the instances in the proposed CVRP benchmark were generated. As happens in almost all the existing instances, the distances are two-dimensional Euclidean. Depot and costumers have integer coordinates corresponding to points in a $[0, 1000] \times [0, 1000]$ grid. Each instance is characterized by the following attributes: number of customers, depot positioning, customer positioning, demand distribution, and average route size. The possible values of each attribute and their effect in the generation are described in the next subsection.

3.1 Instance Attributes

3.1.1 Depot Positioning

Three different positions for the depot are considered:

Central (C) – depot in the center of the grid, point (500,500).

Eccentric (E) – depot in the corner of the grid, point (0,0).

Random (R) – depot in a random point of the grid.

The TC, TE and TR instances [18], used as benchmark on rooted network design problems, present similar alternatives for root positioning.

3.1.2 Customer Positioning

Three alternatives for customer positioning are considered, following the R, C and RC instance classes of the Solomon set for the VRPTW [29].

Random (R) – All customers are positioned in random points of the grid.

Clustered (C) – At first, a number S of customers that will act as cluster seeds is picked from an uniform discrete distribution UD[3,8]. Next, the S seeds are randomly positioned in the grid. The seeds will then attract, with an exponential decay, the

Table 1: Instances of the set E

Instance	Q	Tightness	Opt	Original Source
$E-n13-k4^{1}$	6000	0.76	247	Dantzig & Ramser (1959)
$E-n22-k4^{1}$	6000	0.94	375	Gaskell (1967)
$E-n23-k3^{1}$	4500	0.75	569	Gaskell (1967)
$E-n30-k3^{1}$	4500	0.94	534	Gaskell (1967)
$E-n31-k7^2$	140	0.92	379	Clarke & Wright (1964)
$E-n33-k4^{1}$	8000	0.92	835	Gaskell (1967)
$E-n51-k5^{3}$	160	0.97	521	Christofides & Eilon (1969)
$E-n76-k7^{4}$	220	0.89	682	Gillett & Miller (1974)
$E-n76-k8^{4}$	180	0.95	735	Gillett & Miller (1974)
$E-n76-k10^3$	140	0.97	830	Christofides & Eilon (1969)
$E-n76-k14^4$	100	0.97	1021	Gillett & Miller (1974)
$E-n101-k8^3$	200	0.91	815	Christofides & Eilon (1969)
$E-n101-k14^5$	112	0.93	1067	Gillett & Miller (1974)

 $^{^{1}}$ No description about the generation

Table 2: Instances of the set M, original source: Christofides et al. (1979)

Instance	Q	${f Tightness}$	Opt
$M-n101-k10^1$	200	0.91	820
$M-n121-k7^{1}$	200	0.98	1034
$M-n151-k12^2$	200	0.93	1015
$M-n200-k16^3$	200	1.00	1274
$M-n200-k17^3$	200	0.94	1275

 $^{^{\}rm 1}$ Customers were grouped into clusters as an attempt to represent pratical cases

Table 3: Instances of the set F, original source: Fisher (1994)

Instance	Q	${f Tightness}$	Opt
$F-n45-k4^{1}$	2010	0.90	724
$F-n72-k4^2$	30000	0.96	237
$F-n135-k7^1$	2210	0.95	1162

 $^{^{\}rm 1}$ From a day of grocery deliveries from the Peterboro (Ontario terminal) of National Grocers Limited

 $^{^{2}}$ Example involving UK cities with customers located far from from the depot

 $^{^{3}}$ Locations generated at random from an uniform distribution

 $^{^4}$ Instance E-n76-k10 with modified capacity

 $^{^{5}}$ Instance E-n101-k8 with modified capacity

 $^{^2}$ Generated by adding customers from E-n51-k5 and E-n101-k8 and using the depot and capacity from E-n101-k8

³ Generated by adding customers from M-n151-k12 and the first 49 customers from E-n76-k10 and using the depot and capacity from M-n151-k12

 $^{^2}$ Data obtained from Exxon associated to the delivery of tires, batteries and accessories to gasoline service stations

Table 4: Instances of the set A, original source: Augerat $(1995)^1$

Instance	Q	Tightness	Opt
A-n32-k5	100	0.82	784
A-n33-k5	100	0.89	661
A-n33-k6	100	0.90	742
A-n34-k5	100	0.92	778
A-n36-k5	100	0.88	799
A-n37-k5	100	0.81	669
A-n37-k6	100	0.95	949
A-n38-k5	100	0.96	730
A-n39-k5	100	0.95	822
A-n39-k6	100	0.88	831
A-n44-k6	100	0.95	937
A-n45-k6	100	0.99	944
A-n45-k7	100	0.91	1146
A-n46-k7	100	0.86	914
A-n48-k7	100	0.89	1073
A-n53-k7	100	0.95	1010
A-n54-k7	100	0.96	1167
A-n55-k9	100	0.93	1073
A-n60-k9	100	0.92	1354
A-n61-k9	100	0.98	1034
A-n62-k8	100	0.92	1288
A-n63-k9	100	0.97	1616
A-n63-k10	100	0.93	1314
A-n64-k9	100	0.94	1401
A-n65-k9	100	0.97	1174
A-n69-k9	100	0.94	1159
A-n80-k10	100	0.94	1763

 $^{^1}$ Coordinates are random points in a $[0,100]\times[0,100]$ grid. Demands are picked from an uniform distribution U(1,30), however n/10 of those demands are multiplied by 3.

Table 5: Instances of the set B, original source: Augerat $(1995)^1$

Instance	Q	Tightness	Opt
B-n31-k5	100	0.82	672
B-n34-k5	100	0.91	788
B-n35-k5	100	0.87	955
B-n38-k6	100	0.85	805
B-n39-k5	100	0.88	549
B-n41-k6	100	0.95	829
B-n43-k6	100	0.87	742
B-n44-k7	100	0.92	909
B-n45-k5	100	0.97	751
B-n45-k6	100	0.99	678
B-n50-k7	100	0.87	741
B-n50-k8	100	0.92	1312
B-n51-k7	100	0.98	1032
B-n52-k7	100	0.87	747
B-n56-k7	100	0.88	707
B-n57-k7	100	1.00	1153
B-n57-k9	100	0.89	1598
B-n63-k10	100	0.92	1496
B-n64-k9	100	0.98	861
B-n66-k9	100	0.96	1316
B-n67-k10	100	0.91	1032
B-n68-k9	100	0.93	1272
B-n78-k10	100	0.94	1221

 $^{^1}$ Coordinates are points in a $[0,100]\times[0,100]$ grid, chosen in order to create NC clusters. In all instances, $K\leq NC-1.$

Demands are picked from an uniform distribution U(1,30), however n/10 of those demands are multiplied by 3.

Table 6: Instances of the set P, original source: Augerat $(1995)^1$

Instance	Q	Tightness	Opt
P-n16-k8	35	0.88	450
P-n19-k2	160	0.97	212
P-n20-k2	160	0.97	216
P-n21-k2	160	0.93	211
P-n22-k2	160	0.96	216
P-n22-k8	3000	0.94	603
P-n23-k8	40	0.98	529
P-n40-k5	140	0.88	458
P-n45-k5	150	0.92	510
P-n50-k7	150	0.91	554
P-n50-k8	120	0.99	631
P-n50-k10	100	0.95	696
P-n51-k10	80	0.97	741
P-n55-k7	170	0.88	568
P-n55-k8	160	0.81	588
P-n55-k10	115	0.91	694
P-n55-k15	70	0.99	989
P-n60-k10	120	0.95	744
P-n60-k15	80	0.95	968
P-n65-k10	130	0.94	792
P-n70-k10	135	0.97	827
P-n76-k4	350	0.97	593
P-n76-k5	280	0.97	627
P-n101-k4	400	0.91	681

 $^{^{1}}$ Modifications in the capacity of some instances from A, B and E series. Required number of routes are adjusted accordingly.

Table 7:	Instances	of	Christofides et al.	[8]

Instance	n	Q	MD	\mathbf{ST}	BKS	K_{BKS}
$CMT1^1$	50	160	∞	0	524.61*	5
$\mathrm{CMT}2^1$	75	140	∞	0	835.26*	10
$\mathrm{CMT3^1}$	100	200	∞	0	826.14*	8
$\mathrm{CMT4^1}$	150	200	∞	0	1028.42^*	12
$\mathrm{CMT5^1}$	199	200	∞	0	1291.29^*	16
$\mathrm{CMT}11^1$	120	200	∞	0	1042.11*	7
$\mathrm{CMT}12^1$	100	200	∞	0	819.56*	10
$\mathrm{CMT6^2}$	50	160	200	10	555.43	6
$\mathrm{CMT7^2}$	75	140	160	10	909.68	11
$\rm CMT8^2$	100	200	230	10	865.94	9
$\mathrm{CMT}9^2$	150	200	200	10	1162.55	14
$\mathrm{CMT}10^2$	199	200	200	10	1395.85	18
$CMT13^2$	120	200	720	50	1541.14	11
$CMT14^2$	100	200	1040	90	866.37	11

¹ The data of instances CMT1, CMT2, CMT3, CMT12, CMT11, CMT4 and CMT5 are the same of E-n51-k5, E-n76-k10, E-n101-k8, M-n101-k10, M-n121-k7, M-n151-k12 and M-n200-k16(k17), respectively. The only difference is the convention of not rounding the costs and not fixing the number of routes.

other n-S customers: the probability for a point p in the grid to receive a customer is proportional to

$$\sum_{s=1}^{S} \exp(-d(p,s)/40),$$

where d(p, s) is the distance between p and seed s. The divisor 40 in the above formula was chosen after a number of experiments. Smaller values lead to excessively dense and isolated clusters. On the other hand, larger divisors result in clusters that are too sparse and mingled. When two or more seeds happen to be close, their combined attraction is likely to form a single larger cluster around them. This means that the size of the clusters may differ significantly. When two seeds are a little more apart, their clusters will not coalesce but a "bridge" of customers between them may appear. A seed that is sufficiently apart from other seeds will form an isolated cluster. The overall clustering scheme was devised to mimic the densities found in some large urban agglomerations that have grown from more or less isolated original nucleus (the seeds).

² Instances CMT6, CMT7, CMT8, CMT14, CMT13, CMT9 and CMT10 were generated by adding maximum route duration and service time to CMT1, CMT2, CMT3, CMT12, CMT11, CMT4 and CMT5, respectively. Vehicles are assumed to travel at unitary speed.

Table 8: Instances of Golden et al. [17]

				L - J	
Instance	n	Q	MD	\mathbf{BKS}	K_{BKS}
$\mathrm{G}1^4$	240	550	650	5623.47	9
$\mathrm{G}2^4$	320	700	900	8404.61	10
$G3^4$	400	900	1200	11036.22	10
$G4^4$	480	1000	1600	13590.00	_
$\mathrm{G5^5}$	200	900	1800	6460.98	5
$ m G6^{5}$	280	900	1500	8400.33	_
$\mathrm{G7^5}$	360	900	1300	10102.70	8
$ m G8^{5}$	440	900	1200	11635.30	10
$G9^3$	255	1000	∞	579.71	14
$\mathrm{G}10^3$	323	1000	∞	735.66	_
$G11^{3}$	399	1000	∞	912.03	_
$G12^3$	483	1000	∞	1101.50	_
$G13^2$	252	1000	∞	857.19	26
$G14^2$	320	1000	∞	1080.55^*	30
$G15^2$	396	1000	∞	1337.87	_
$\mathrm{G}16^2$	480	1000	∞	1611.56	_
$\mathrm{G}17^1$	240	200	∞	707.76*	22
$\mathrm{G}18^1$	300	200	∞	995.13*	27
$\mathrm{G}19^1$	360	200	∞	1365.60*	33
$\mathrm{G}20^1$	420	200	∞	1817.89	_

 $^{^{1}}$ Concentric six-pointed pointed stars

Table 9: Instances of Rochat and Taillard [27]

Instance	n	Q	BKS	K_{BKS}
$tai75a^1$	75	1445	1618.36*	10
$tai75b^1$	75	1679	1344.64*	9
$tai75c^1$	75	1122	1291.01^*	9
${ m tai75d^1}$	75	1699	1365.42^*	9
$tai100a^1$	100	1409	2041.34*	11
$tai100b^1$	100	1842	1939.90*	11
$tai100c^{1}$	100	2043	1406.20*	11
$tai100d^1$	100	1297	1580.46*	11
$tai150a^1$	150	1544	3055.23*	15
$tai150b^1$	150	1918	2727.03*	14
$tai150c^1$	150	2021	2358.66*	15
${ m tai}150{ m d}^1$	150	1874	2645.40^*	14
$tai385^2$	385	65	24366.41	47

 $^{^{1}}$ Customers are non-uniformally spread in several clusters.

 $^{^{2}}$ Concentric squares, depot in the center

³ Concentric squares, depot in a corner

⁴ Concentric circles

⁵ Concentric rays

The number of clusters and their compactness are variable.

Demands are generated following a exponential distribution.

² The data for the customers were generated based on real information from the canton of Vaud in Switzerland. Already appeared in [31].

Table 10: Instances of Li et al. [19]¹

Instance	n	Q	MD	BKS	K_{BKS}
21	560	1200	1800	16212.74	_
22	600	900	1000	14499.04	15
23	640	1400	2200	18801.12	10
24	720	1500	2400	21389.33	_
25	760	900	900	16668.51	19
26	800	1700	2500	23971.74	_
27	840	900	900	17343.38	20
28	880	1800	2800	26565.92	_
29	960	2000	3000	29154.33	_
30	1040	2100	3200	31742.51	_
31	1120	2300	3500	34330.84	_
32	1200	2500	3600	37159.41	11

Random-Clustered (RC) – Half of the customers are clustered by the above described scheme, the remaining customers are randomly positioned.

It should be noted that superpositions are not allowed, all customers and the depot are located in distinct points on the grid.

3.1.3 Demand Distribution

Seven options of demand distributions have been selected in these instances.

Unitary (U) – All demands have value 1.

Small Values, Large Variance (1-10) – demands from UD[1,10].

Small Values, Small Variance (5-10) – demands from UD[5,10].

Large Values, Large Variance (1-100) – demands from UD[1,100].

Large Values, Small Variance (50-100) – demands from UD[50,100].

Depending on Quadrant (Q) – demands taken from UD[1,50] if customer is in an even quadrant (with respect to point (500,500)), and from UD[51,100] otherwise. This kind of demand distribution leads to solutions containing some routes that are significantly longer (in terms of number of served customers) than others.

Many Small Values, Few Large Values (SL) – Most demands (70% to 95% of the customers) are taken from UD[1,10], the remaining demands are taken from UD[50,100].

3.1.4 Average Route Size

The previous experience of the authors with CVRP algorithms indicated that the value n/K_{min} , the average route size (assuming solutions with the minimum possible number of routes) has a large impact on the performance of current exact methods. The impact of this attribute on heuristic methods is not so pronounced, but is still quite significant. We can not make a general statement that instances with shorter routes are easier than instances with longer routes or the opposite. What happens is that some methods are more suited for short routes and other methods for longer routes. Therefore, a comprehensive benchmark set should contain instances where this attribute vary over a wide range of values. Yet, generating an instance with exactly a given value of n/K_{min} would be difficult, since even computing K_{min} requires the solution of a bin-packing problem. The generator actually uses as attribute a value r representing the desired value of n/K_{min} . This causes the instance capacity to be defined as:

$$Q = \left\lceil \frac{r \sum_{i=1}^{n} q_i}{n} \right\rceil,$$

where the q vector represents the demands, already obtained according to the specified demand distribution. As $\sum_{i=1}^{n} q_i/Q$ is usually a good lower bound on K_{min} , instances have values of n/K_{min} that are sufficiently close to r.

3.2 Instance Generation

A benchmark set with instances corresponding to the cartesian product of so many parameter values (even restricting the "continuous" parameters n and r to a reasonably small number of values) would be huge. Thus, we generated a sample of 100 instances, presented in Tables 11 to 13. We believe that this number of instances is large enough to obtain the desired level of diversification, but still small enough to allow that future users of the benchmark can report detailed results for each instance. We now explain how the attribute values of those 100 instances were obtained.

• The instances are ordered by the number of customers n. For the first 50 instances, ranging from 100 to 330 customers, n is increased in linear steps. For the last 50 instances, n is increased in exponential steps from 335 to 1000. For the time being, instances with around 200 customers can already be hard for exact methods, and larger instances are sufficiently challenging to highlight significant quality differences between competing heuristics.

- The set of values of r were taken from a continuous triangular distribution T(3,6,25) (minimum 3, mode 6 and maximum 25). The 100 values of r were partitioned in quintiles, corresponding to very small routes, small routes, medium routes, long routes and very long routes. The k-th instance receives an r from the $((k-1) \mod 5) + 1$ quintile. In this way, it is guaranteed that every set containing 5t instances with consecutive values of n will have exactly t instances from each quintile. It can be observed in the end of Table 13 that the resulting set of n/K_{min} values have minimum 3.0, maximum 24.4, median 9.8 and average 11.1.
- A random permutation of the 3 possible values for the depot positioning attribute (C, E and R) provides the values for the first 3 instances. Another random permutation gives the values for the next 3 instances and so on. A similar scheme is used for obtaining the values for the customer positioning and demand distribution attributes. This guarantees that every subset of the instances with consecutive values of n will have a near-balanced number of instances having the same value of an attribute.

The name of an instance follows the ABEFMP standard, and has a format X-nA-kB, where A represents n + 1, the number of points in the instance including the depot, and B is the minimum possible number of routes K_{min} , calculated by solving a bin-packing problem. Since there are no two instances with the same number of points, in contexts where the K_{min} information is not considered much important, we propose the format XA as an alternative shorter name. For example, the shorter name for X-n284-k15 would be X284.

3.3 Two Decisions on Conventions

3.3.1 Rounding the Distances or Not?

Following the TSPLIB convention [26], the euclidean distances are rounded to the nearest integer in the literature on exact methods. On the other hand, the distances are seldom rounded in the literature on heuristics.

Advantages of Rounding – Most mathematical programming based algorithms (including standard MIP solvers) have a limited optimality precision. For example, CPLEX 12.5 default precision is only 10^{-4} (0.01%). It is possible to increase the precision up to a point by adjusting parameters (say, 10^{-6} or 10^{-7}). Going further requires special software, using more bits in the floating-point numbers or even exact rational arithmetic [11], that is not easily available or implementable. The practice of distance rounding is

convenient for avoiding those pitfalls and usually makes the optimal values found by exact methods based on standard mathematical programming software reliable. Remark that the practice of not rounding, but only reporting two decimal places does not solve the problem. For example, an algorithm with a precision of 10^{-6} may declare a solution having value 853.2351 (published as 853.24) as optimal. Later, someone finds a solution with value 853.2349 and publishes 853.23.

Disavantages of Rounding – A benchmark of rounded instances has less power for comparing competing algorithms. Especially for heuristics, the search space is formed by a relatively small number of plateaus, i.e., sets of solutions with the same value. Guiding the search on such plateaus is usually done by trial and error since there is no indication that the distance is –even slightly– reduced. There may also be in practice several distinct optimal solutions, leading to more frequent ties between competitors. This effect is quite significant on ABEFMP instances, where the optimal solution values have magnitudes around 10^3 . In addition, some people claim (but never publish, this is part of the community folklore) that rounding can artificially enhance the performance of exact methods. For example, if an upper bound of 1000 is known, a branch-and-bound node with lower bound 999 $+\epsilon$ can be fathomed. This effect is significant on ABEFMP instances, enough to make some algorithms to run at least twice as fast.

We took the decision that the newly proposed benchmark set will follow the TSPLIB convention of rounding distances. Nevertheless, the instances were devised in order to minimize the above mentioned disadvantages. The use of a $[0, 1000] \times [0, 1000]$ grid (instead of the $[0, 100] \times [0, 100]$ grids of most ABEFMP instances) makes the optimal solutions to have magnitudes between 10^4 - 10^5 . This is still quite safe for exact methods having a precision of 10^{-6} . However, the plateau effect and the artificial enhancement of exact methods are much reduced.

3.3.2 Fix the Number of Routes or Not?

The literature on exact methods usually follows the convention of fixing the number of routes to a value K. Except for instance M-n200-k17, K is always set to K_{min} . The standard explanation for that fixing is that K represents the number of vehicles available at the depot. A more sophisticated explanation is that fixing the number of routes to the minimum is an indirect way of minimizing the fixed costs for using a vehicle. We do not think that this is necessarily true: since the CVRP definition allows solutions containing routes that are much shorter than others, why not assigning the two shortest routes to

the same vehicle? In fact, the CVRP may be used to model real-world situations where a single vehicle (a truly homogeneous fleet!) will perform all routes in sequence. In other words, CVRP routes do not need to correspond to vehicles. Based on that reasoning, we decided that the newly proposed benchmark set will follow the convention of not fixing the number of routes. Therefore, the number K_{min} indicated in each instance should be taken only as a lower bound on the number of routes in a solution. A second reason for taking that decision is that the number of routes was not fixed in the original CVRP definition [12].

4 Experiments with State-of-the-Art Methods

In order to provide an initial set of results to the new benchmark, experiments have been conducted with recent state-of-the-art heuristic and exact methods. The experiments also provide an assessment of the level of difficulty of the proposed instances and even some hints on how the attributes used in their generation impact on that difficulty. Those results are reported jointly in Tables 11 to 13, which describe the characteristics of each instance, the average solution quality, best solutions quality and average CPU time of two state-of-the-art metaheuristics, the lower bound, root CPU time, number of search nodes and overall time of a recent state-of-the-art exact method, and finally the cost and number of vehicles of the best solution (BKS) ever found since the instances were created, including preliminary runs of those heuristics with alternative parameterizations. Table 13 also reports additional statistics on instances characteristics and results, such as the minimum, maximum, average and median results on the instance set for n, Q, r, n/K_{min} , as well as for the Gaps(%) and CPU time of ILS-SP, UHGS and BCP (root relaxation).

4.1 Heuristic solutions

We selected a pair of recent successful metaheuristics to illustrate the two main current types of approaches in the literature. We consider an efficient neighborhood-based method, the iterated local search based matheuristic algorithm (ILS-SP) of Subramanian et al. [30], and a recent population-based method, the unified hybrid genetic search (UHGS) of Vidal et al. [32, 33]. These two methods rely extensively on local search to improve new solutions generated either by shaking or recombinations of parents. They also include specific strategies to explore new choices of customer-to-route assignments: ILS is coupled with a integer programming solver over a set partitioning (SP) formulation, which seeks to create new solutions based on known routes from past local optimums, while UHGS implements

Ž \mathbf{BKS} Value 3065610856 28940 10916 47812 255692414558578 117595 4043725742 19230 27042 18880 55539 1359015700 43448 21220 16876 14138 20557 45607 16980 44225 19565 37274 38684 Table 11: New set of benchmark instances: characteristics and results of current state-of-the-art algorithms (Part I) 17.0 187.0 18.2 501.6819.0 399.1 15531024 7305 1303 303.51398T(min) 0.3 2695 253185 $_{\rm BCP}$ 26.8 2.0 264.9 13.2 133.0 22.8 0.5RT(min) 13234 28897 10840 13590 15634 43448 2114016876 14053 20476 45549 25511 2398058455 19484 30480 10809 117595 19070 26930 38473 16939 44225 25657 47721 40311 14971 3.19 3.32 5.927.97 8.62 12.08 10.2211.69 12.67 2.66 3.73 3.83 6.105.359.806.848.90 OHGS 55539 28940 10916 13590 15700 13448 21220 92891 14138 45607 47812 2556924145 16980 19565 30656 10856 25742 19230 27042 82804 38699 20557 44225117595 28948.5 21226.3 14141.3 47957.224147.2 44244.1 58626.417604.9 5700.2 20563.2 25591.116987.9 19571.5 10877.4 40499.0 25779.3 27067.3 14971.0 2747.0 3332.00934.913590.043448.00.978045607.030680.419288.4 82948.7 38796.40.0888126629.6 55542.11.72 0.20 2.07 1.60 0.840.490.760.5498.0 0.641.11 1.592.10 0.87 1.08 3.80 2.260.8517.83 2.066.6748140 ILS-SP 12747 28948 10916 13590 15726 43448 21340 16876 14138 20562 45607 25569 24145 17085 44225 5862619570 30667 10985 117595 25743 19266 27042 58697.221400.016876.014160.148249.8 17595.0[4971.0]12747.0 13337.6 15745.2 25571.5 24186.0 44234.3 19625.230765.411126.9 40533.5 25795.8 19336.727078.8 82874.2 37507.2 38840.0 18883.9 55673.8 28998.0 10947.413603.1 13452.120608.745616.1 17143.1 26869.014.6 12.3 13.0 n/K_{min} 6.7 25.014.0 22.6 15.5 17.8 13.0 5.63.4 6.8 3.8 13.5 19.4 Instance Characteristics 1190 133 138 643 106 1174 402 836 101 691 144 974 188 39161 181 631 1081 50 - 1001-10050 - 1001-10 1-10050 - 1005-1050 - 100 \mathbf{S} C(3)RC (8) C (6) C(3)RC (6) RC (8) RC (8) C (3) RC (5) RC (5) C(8)RC (3) C(5)C (4) RC (7) C(3)RC(5)C (8) RC (7) \otimes u166 180 185 189 194 199 161 171 X-n162-k11 X-n101-k25 X-n120-k6X-n129-k18 X-n153-k22 X-n176-k26 X-n186-k15 X-n209-k16 X-n214-k11 X-n223-k34 X-n233-k16 X-n237-k14 X-n256-k16 X-n261-k13 X-n110-k13 X-n115-k10 X-n125-k30 X-n134-k13 X-n139-k10 X-n143-k7 X-n148-k46 X-n157-k13 X-n167-k10 X-n172-k51 X-n181-k23 X-n190-k8 X-n195-k51 X-n200-k36 X-n204-k19 X-n219-k73 X-n228-k23 X-n242-k48 14 15 16 17 6 0 11 12 13

15 86 43 41 $\frac{29}{17}$ 130 140 N 94 \mathbf{BKS} 221909 95185 2174478355 2755642099259465150922814 147713 3826966243 19718 107798 55358 66633 69253 24201 33503 20226 47167 2585994044 29866 31103 139197 36395 24181 89535 66081 34231 Table 12: New set of benchmark instances: characteristics and results of current state-of-the-art algorithms (Part II) 115.034224.6 3.3 T(min) 150.1531.1Nds389 5 185 195 169 BCP221.9 25.770.4 82.3 49.4 247.7 247.9 133.9 804.9 16.5RT(min) 6.6 20.4 256.746.7318.12154627378 138706 25809147713 21245 3328629722 31027 5138122747 38167 19640 64930 362895492846911 2558793851 78334 41881 6568165971 15.1618.19 37.96 22.0228.26 49.52 53.1922.37 21.6733.7334.8540.20 28.65 34.71 41.4542.80 19.09 10.9324.4334.5564.9286.65 63.6119.91 15.31 22.4142099 139210 25946515092281466243 19718 55378 75517 2124533505 95244 217482585978355 29870 2756431103 147717 38269107798 36395 OHGS 20227 34231 94093 66081 6550195469.547259.039534.9 42208.8 22838.466270.2 65648.555553.127628.2 31159.626014.0 47750.2 38374.9 19743.8 24272.621280.633605.821850.994265.278387.8 29956.151721.7 66365.407924.122617.1 20286.425895.434292.136451.122.5813.13 60.36 22.19 38.22 19.13 15.7021.41 48.86 7.10 23.7614.15 9.5317.50 8.5614.68 25.21 20.8239.6359.9460.5950.406.9221812 4214666116 66453 2124520295 95315 34239 2759931105 139197 5170622902147713 38298 19792 6556336395 24209 ILS-SP 33624 25901 29877 26021 .07798 5576120448.542284.026150.352076.539461.0 47713.0 07838.0 22182.0 95450.621895.894297.3 31235.523003.2 66372.5 19954.9 36441.656204.921256.033769.434356.026101.178356.0 29991.327812.466715.1 24462.438457.416.3 4.0 Instance Characteristics 1106 61 68 1 - 10050 - 1005-105-101-10 50 - 1005-10C (8) RC (7) RC (5) RC (7) RC (6) RC (3) $\widehat{\mathfrak{S}}$ $\widehat{\infty}$ 4 5 6 $\widehat{\mathfrak{S}}$ 4 3 9 5 (3) $\widehat{\infty}$ ∞ \circ 358 307 326335 366 468 330 400 321 X-n317-k53 X-n327-k20 X-n331-k15 X-n336-k84 X-n344-k43 X-n351-k40 X-n359-k29 X-n376-k94 X-n384-k52 X-n401-k29 X-n411-k19 X-n420-k130 X-n449-k29 X-n266-k58 X-n280-k17 X-n284-k15 X-n308-k13 X-n313-k71 X-n322-k28 X-n367-k17 X-n275-k28 X-n289-k60 X-n294-k50 X-n303-k21 X-n393-k38 X-n429-k61 X-n439-k37 X-n459-k26 X-n469-k138 X-n298-k31 X-n513-k21 66 67 68 36 38 39 40

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Value NV \mathbf{BKS} 133105154594 106780 194266 329836 24201 90543 08813 146705 136366 89060 99715 5417285672 119194 59778 62366 63839 68425 82292 4352577700 114683 158611 7272773587Table 13: New set of benchmark instances: characteristics and results of current state-of-the-art algorithms (Part III) 212.1 41.5T(min) $_{\rm Nds}$ 21381 1429.8 302.31306.9 401.0 262.8 258.0257.2253.7 43.6 395.8RT(min) 264.3600.2 103.1 1643.2 680.6419.5 259.2332.8 433.2 984.5 937.5308.8 1643.2189.4 97.1 257.1 88839 71812 50575 62018 63228 43113 135748 114108 71728 53147 32858813249685328 86604 189950 108000 59323 106766 146211 67925 73124 98880 1183990.44%81694 76924 1576271932451.89%.75.29 150.48264.10244.53 382.99 374.28463.36321.89 531.50432.90553.96 84.24 60.60188.15 125.91 117.31 239.68156.71 253.17 264.28 313.88269.70 289.24 195.38 560.81549.03 560.81 98.7922.39158.81 288.43 63839 Best 95122190543 108813 59778 62366106829 146705 68425 82293 43525136366 77715 11468373587 194266 89118 99715 54172329836 133140 85672 24201 86822 42756 50780 158611 119194 72781 0.01% 0.13%95330.659960.0136587.677864.9 115147.9 73009.673731.0 194476.5 133512.9 85822.6 119502.10.19%86998.5 90838.0.09064.2 64192.0147222.7 82487.4 58899.3 89238.7 54439.8330198.3 42866.468654.1 43641.450915.1106899.199884.1 62524.163.9572.96 61.2473.85225.79111.56127.24 242.11 235.48432.64173.17 153.65410.17226.08202.50311.20 62.07 68.86 12.03 78.54 74.80 [40.42]47.24 210.08 148.91 109.31 687.22 17.6727.27 162.67 136832 115443 54713 24332 42952 190612 64462147045 68646 82888 77952 73447 7383089060 330639 13359285697 86710 51092 1090566022962783 106780 194804 95524 44021 159164 119994 1.42%0.25%0.10%ILS-SP 100177 134530.0115738.0195027.0100417.0 330948.0120253.095700.743131.3 190919.0 109384.060444.2 62905.6106782.0147676.0 68988.283042.2 44171.6 137045.078275.9 73722.9 74005.7 159425.089277.6 54958.585936.6 2.50%0.52%86874.1 51173.064606.15.9 10.8 16.3 20.4 20.0 14.8 n/K_{min} 9.113.5 10.0 14.518.6 16.0 16.520.221.04.610.8 24.24.8 Instance Characteristics 110 1816 5231007 396 166 764 866 131 1381 87 25 832 50 - 1001-100 5-101-10 50 - 1005-10 1 - 10050 - 10050 - 1001-1001-100RC (4) RC (7) Cust C(3)RC (4) RC (8) RC (6) C(3)C (8) RC (7) RC (7) RC (3) C (5) RC (4) RC (7) 9 <u>-</u> 3 4 260 598 626 654 684 700 732 748 765 782 836 699 800 X-n524-k153 X-n613-k62 X-n641-k35 X-n733-k159 X-n749-k98 X-n783-k48 X-n856-k95 X-n979-k58 X-n1001-k43 X-n536-k96 X-n548-k50 X-n561-k42 X-n573-k30 X-n586-k159 X-n599-k92 X-n627-k43 X-n655-k131 X-n670-k130 X-n685-k75 X-n701-k44 X-n716-k35 X-n766-k71 X-n801-k40 X-n837-k142 X-n876-k59 X-n895-k37 X-n916-k207 X-n957-k87 X-n513-k21 X-n819-k171 X-n936-k151 Median 82 81 85 # 83 84 90

a continuous diversification procedure by modifying the objective during parents and survivors selection to promote not only good but also diverse solutions. Both methods are known to achieve very high quality results on the previous sets of CVRP instances as well as on various other vehicle routing variants.

The two methods have been run with the same parameter setting specified in the original papers. To produce solutions that can stand the test of time, and counting on the fact that more computing power will be surely available in the coming years, we selected a slightly larger termination criterion for UHGS by allowing up to 50.000 consecutive iterations without improvement. These tests have been conducted on a Xeon CPU with 3.07 GHz and 16 GB of RAM, running under Oracle Linux Server 6.4. The average and best results on 50 runs are reported in the Table, as well as the average computational time per instance.

From these tests, it appears that both ILS-SP and UHGS produce solutions of consistent quality, with an average gap of 0.52% and 0.19%, respectively, with respect to the best known solutions (BKS) ever found during all experiments. Considering the subset of instances that are solved to optimality (see Section 4.2), ILS-SP and UHGS achieve average gaps of 0.18% and 0.09%, respectively. A direct comparison of the best solutions found by each method provides the following score: UHGS solutions are better 58 times, ILS-SP solutions are better 9 times and there are 33 ties.

UHGS produces solutions of generally higher quality than ILS for a comparable amount of CPU time, except for some instances containing few customers per route. For this type of problems, the set partitioning solver can produce new high-quality solutions from existing routes in a very efficient manner, while generating new structurally different solutions with other choices of assignments is more challenging for local searches and even for randomized crossovers. On the other hand, for problems with a large number of customers per route, the hybrid ILS exhibits a slower convergence and generally leads to solutions of lower quality.

Overall, these experiments show that some state-of-the-art methods may perform well on different classes of instances. Therefore, a promising research path would involve the extension of these methods and/or further hybridizations to cover all problems in the best possible way.

4.2 Exact solutions

The Branch-Cut-and-Price (BCP) in Pecin et al. [22] is a complex algorithm that combines and improves several ideas proposed by other authors [4, 5, 10, 28], as also detailed in [23].

The algorithm was run in a single core of an Intel i7-3960X 3.30GHz processor with 64 GB RAM. The value of the best solution found by the previously mentioned heuristics was inputted as an initial upper bound. Actually, in order to increase our confidence in the correctness of the implementation, we always add 1 to that upper bound. In this way, the BCP always has to find by itself a feasible solution with value at least as good as the upper bound.

The BCP could solve 40 out of the 100 new instances to optimality in reasonable times (maximum of 5 days, for X-n186-k15). For those instances we report the root node lower bound, the time to obtain that bound, the number of nodes in the search tree and the total time. For the remaining 60 instances we only report a lower bound and the corresponding time.

- The BCP can solve most instances with up to 275 customers. In the 38 instances in that range, it only failed when the routes are very long (X-n190-k8, X-n214-k11, X-n233-k16, and X-n261-k13). On the other hand, only 6 larger instances could be solved (X-n298-k31, X-n317-k53, X-376-k94, X-n420-k130, X-n524-k153, and X-nX655-k131). Those more favorable instances have short routes and small values of Q.
- The BCP attested the general good quality of the solutions that can be found by current heuristics. Consider the best solution provided either by ILS-SP or UHGS. In 96 out of the 100 instances, this solution is less than 1% above the computed lower bounds.

5 The CVRPLIB Web Site

The typical instance repository of today is a web page that allows downloading the instance files and includes additional textual information, like file format description, instance source, best known/optimal solution values, etc. The CVRLIB web page, where the new instances (and all the previous CVRP instances described in Section 2) are available (http://vrp.galgos.inf.puc-rio.br/index.php/en/), is more sophisticated:

• Its core is a full-fledged database containing, for each instance: (i) its actual data, like n, Q, customer coordinates and demands, (ii) its best known/optimal solution, represented as set of routes, (iii) a miscellanea of additional information, like original source, the values of the attributes used in its generation (for the new instances), and even comments about remarkable characteristics. The visible pieces of information

that appear in the web site are produced by queries. The objective of that design is having a larger degree of consistency and flexibility in the maintenance of the page.

- The best known solutions are automatically verified, their feasibility is checked
 and their costs are calculated from the original instance data. This eliminates the
 possibility that false best known solutions appear due to typos or misunderstandings
 about the conventions.
- Instances with euclidean distances (the vast majority) can be depicted graphically, along with their best known/optimal solutions.

6 Conclusions

We hope that the proposed set of instances will contribute to spark new interest on the "classic" CVRP, helping to test new algorithmic developments in the coming years. In particular, by proposing a common set of benchmark instances for both heuristic and exact methods, this work contributes to end a bizarre situation where minor convention details were sufficient for isolating both communities. For example, most of the CVRP instances in Table 7 could have their best known solutions proven to be optimal by the algorithms in [4, 5, 10, 14, 28]. But none of those authors effectively changed a few lines of code to adapt for the different conventions. This instance-induced isolation was harmful and prevented a more effective cross-fertilization between exact and heuristic methods. In fact, this paper has shown that it is already possible to perform direct comparisons of heuristic results with good lower bounds or even proven optimal solutions on fairly large-sized instances.

A major concern in the design of the new benchmark set was problem diversity. By having instances covering a wider set of characteristics, the benchmark will favor the development of more flexible methods (perhaps hybrids), that will eventually be applied to real-life situations with lesser risks of failure due to an uncommon instance type.

Finally, we point out that the proposed CVRP instances can be extended to other classical VRP variants as needed, by providing additional data fields like duration constraints, time windows, or heterogeneous fleet specifications.

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A Results on the instances of Li et al. [19]

Table 14 reports the results achieved with 10 runs of the hybrid genetic algorithm of [32], using the same parameter configuration as in the original paper and the same computing environment as other tests of this paper. The columns provide, in turn, the instance number and size, the average and best solutions quality, and CPU time per run.

Table 14: Instances of Li et al. [19] – results of [32].

Instance	n	Avg.	Best	T(min)
21	560	16212.83	16212.83	8.08
22	600	14545.76	14528.19	35.37
23	640	18826.22	18801.13	10.96
24	720	21389.43	21389.43	13.74
25	760	16753.56	16705.11	54.38
26	800	23977.73	23977.73	19.19
27	840	17411.96	17383.18	59.66
28	880	26566.04	26566.04	20.87
29	960	29154.34	29154.34	25.26
30	1040	31742.64	31742.64	30.11
31	1120	34330.94	34330.94	35.01
32	1200	37159.41	37159.41	51.07