Assignment Problems
- Python for Civil Engineering -

Nelson Kandel

August 7, 2023

Please make individual *Jupyter Notebooks* for each problem sets.

Problem Set 1 - Computational Fundamentals

Q1.1. Determine the value of i^i as a real number, where $i = \sqrt{-1}$.

Q1.2. How many times must a sheet of paper (thickness, $t = 0.1mm$ but otherwise any size required) be folded to reach the Moon (distance from Earth, $d = 384400km$)?

Q1.3. The World Geodetic System is a set of international standards for describing the shape of the Earth. In the latest WGS-84 revision, the Earth's geoid is approximated to a reference ellipsoid that takes the form of an oblate spheroid with semi-major and semi-minor axes $a = 6378137.0m$ and $c = 6356752.314245m$ respectively.
Use the formula for the surface area of an oblate spheroid,

$$S_{obl} = 2\pi a^2 \left(1 + \frac{1-e^2}{e}\,\text{atanh}\,(e)\right), \text{ where } e^2 = 1 - \frac{c^2}{a^2}$$

to calculate the surface area of this reference ellipsoid and compare it with the surface area of the Earth assumed to be a sphere with radius $6371km$.

Q1.4. Write two functions which, given two lists of length 3 representing three dimensional vectors $\vec{a}$ and $\vec{a}$, calculate the dot product, $\vec{a} \cdot \vec{b}$ and the vector (cross) product, $\vec{a} \times \vec{b}$. Write two more functions to return the scalar triple product, $\vec{a} \cdot (\vec{b} \times \vec{b})$ and the vector triple product, $\vec{a} \times (\vec{b} \times \vec{b})$.

Q1.5. The normalized Gaussian function with mean μ and standard deviation σ is:

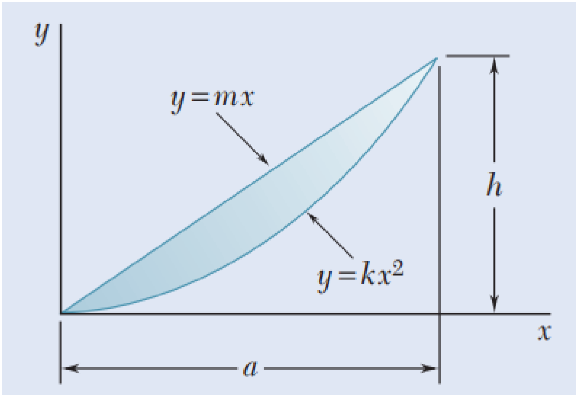
$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Write a program to calculate and plot the Gaussian functions with $\mu = 0$ and the three values $\sigma =$ 0.5, 1, 1.5. Use a grid of 1000 points in the interval $-10 \leq x \leq 10$. Verify (by direct summation) that the functions are normalized with area 1.

Q1.6. Perform quick plotting of the given equation using symbolic package *sympy*. Find the roots of function analytically if possible:

$$f(x) = x^2 + \sqrt{(x^3 - 23x + 1)}$$

Q1.7. Find the centroid of the following figure using method of integration:



Q1.8. Solve the following system of linear equation using sympy package.

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Q1.9. Simulate an experiment carried out n trials times in which, for each experiment, n coins are tossed and the total number of heads each time is recorded.
Plot the results of the simulation on a suitable histogram and compare with the expected binomial distribution of heads.

Q1.10. Using Runge-Kutta method of fourth order, solve the following differential equation and plot a graph of $y = f(x)$:

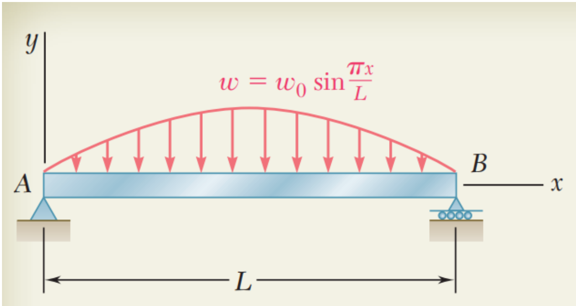
$$\frac{dy}{dx} = \frac{2xy + e^x}{x^2 + xe^x} \text{ given: } x_0 = 1, y_0 = 0$$

Problem Set 2 - Structural Analysis

Q2.1. For a simply supported beam loaded as shown below, compute horizontal and vertical reaction forces.
Q2.2. Shear force and Bending moment in a beam can be determined by consecutive integration of loading function with respect to length x.

$$\text{i.e, } V(x) = -\int w(x)\, dx + C_1$$
$$M(x) = -\int dx \int w(x)\, dx + C_1x + C_2$$

where, constants C_1 and C_2 can be determined by applying boundary conditions $[x = 0, M = 0]$ and $[x = L, M = 0]$.



Find the equation for Shear force $V(x)$ and Bending moment $M(x)$ for the beam shown in *Problem Q2.1*. Create a image for the plot of Loading, Shear and Bending Moment functions with appropriate textbook standard formatting.

Q2.3. Deflection by double integration is also referred to as deflection by the method of direct or constant integration. This method entails obtaining the deflection of a beam by integrating the differential equation of the elastic curve of a beam twice and using boundary conditions to determine the constants of integration. The first integration yields the slope, and the second integration gives the deflection.

$$\begin{aligned}
\text{i.e, } EI\frac{d^4y}{dx^4} &= -w(x) \\
EI\frac{d^3y}{dx^3} &= V(x) = -\int w(x)\,dx + C_1 \\
EI\frac{d^2y}{dx^2} &= M(x) = -\int dx \int w(x)\,dx + C_1x + C_2 \\
EI\frac{dy}{dx} &= EI\theta(x) = -\int dx \int dx \int w(x)\,dx + \frac{1}{2}C_1x^2 + C_2x + C_3 \\
EIy(x) &= -\int dx \int dx \int dx \int w(x)\,dx + \frac{1}{6}C_1x^3 + \frac{1}{2}C_2x^2 + C_3x + C_4
\end{aligned}$$

where, constants C_1 and C_2 are the same coefficients from previous problem. Constants C_3 and C_4 can be determined by applying boundary conditions $[x = 0, y = 0]$ and $[x = L, y = 0]$.
Determine equation of elastic curve $EIy(x)$ and maximum deflection. Also, modify the image produced in *Problem Q2.3* to include the elastic curve too.

Ans: $EIy(x) = -w_0 \frac{L^4}{\pi^4} \sin \frac{\pi x}{L}$
$y_{max} = \frac{w_0 L^4}{\pi^4 EI}$

Problem Set 3 - Data Analysis

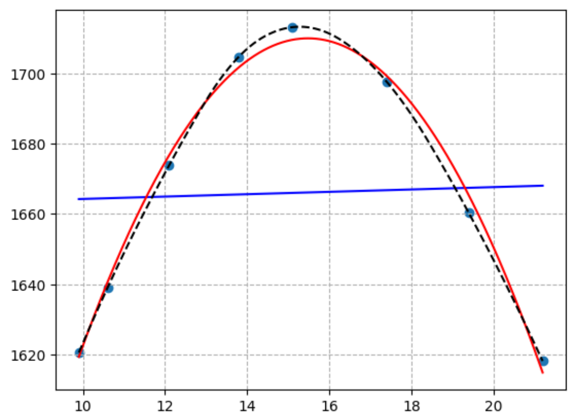
Compaction test of soil is carried out using Proctor's test to understand compaction characteristics of different soils with change in moisture content. Following table presents the observations from a standard proctor test. Perform data analysis as inquired below.

Volume of Proctor mold (cm3)	Mass of Wet soil in the mold	Moisture Content (%)	Moist Unit weight (kg/m3)	Dry unit weight
943.3	1.68	9.9	1780.98	1620.55
943.3	1.71	10.6	1812.78	1639.85
943.3	1.77	12.1	1876.39	1673.85
943.3	1.83	13.8	1940	1704.74
943.3	1.86	15.1	1971.8	1713.12
943.3	1.88	17.4	1993	1697.61
943.3	1.87	19.4	1982.4	1660.3
943.3	1.85	21.2	1961.2	1618.15

Q3.1. Make a Scatter Plot of moisture content vs dry unit weight.

Q3.2. Considering moisture content as cause (x variable) and dry unit weight as effect (y variable), make following curve fittings:
(i) one degree least square fit. (ii) two degree least square fit. (iii) quadratic spline curve.

Q3.3. Visulize three lines of fitting in graph along with scatter points. Then observe and determine whether least square fit of degree one or degree two is best suited for the data provided. Graph should be somewhat similar to this:



Q3.4. Considering Equation of Quadratic line of fit, Find it's derivative when moisture content is 15%.

Q3.5. Use Secant's method to determine two root of Quadratic line of fit.
(Different initial guess can be used to determine different lines of fit.)

Further Studies:

1. Basic Computational Mathematics:
Saha, A. (2015). Doing math with Python: Use programming to explore algebra, statistics, calculus, and more! No Starch Press.
2. Advanced Computational Mathematics:
Hill, C. (2020). Learning scientific programming with Python (Second edition). Cambridge University Press.
3. Core Programming Fundamentals:
Matthes, E. (2019). Python crash course: A hands-on. project-based introduction to programming (2nd edition). No Starch Press
4.*Mr. P Solver Python* Tutorial Playlist:
`https://youtube.com/playlist?list=PLkdGijFCNuVnGxo-1fSNcdHh5gZc17oRM`