Course Code :22BM1101

R-2022

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING
(Autonomous)
Madhurawada, Visakhapatnam
Affiliated to Andhra University, Visakhapatnam.
B.Tech I-Semester Regular & Supplementary Examinations February 2024
Calculus and Linear Algebra
[Common to Chemical, Civil, ECE, EEE, CSE, IT, Mechanical Engineering &
Mechanical (Robotics)]

Date: 17-02-2024

Time: 3 Hours

Max. Marks: 70

1. Answer ONE Question from each UNIT
2. All parts of a Question must be answered in one place to get valued.
3. All questions carry equal marks.

UNIT-I

1. a) Examine the convergence of the series $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \ldots\infty$

    7 Marks

    b) Using Lagrange's mean value mean theorem find the value of $\sqrt[3]{245}$

    7 Marks

2. a) Test the convergence of the infinite series $\sum_{n=1}^{\infty} \frac{3.6.9\ldots 3n}{4.7.10\ldots(3n+1)} \frac{5^n}{(3n+2)}$

    7 Marks

    b) If $a < b$, prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$.

    Hence deduce $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$

    7 Marks

UNIT-II

3. a) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$

    7 Marks

    b) Expand $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-1)$ and hence compute $f(1.1, 0.9)$ approximately.

    7 Marks

4. a) If $x = r\cos\theta$, $y = r\sin\theta$, prove that $JJ' = 1$

    7 Marks

    b) Discuss the maximum and minimum of $f(x,y) = x^3 y^2 (1 - x - y)$

    7 Marks

UNIT-III

5. a) Evaluate $\iint\limits_{R}(x-y)\,dx\,dy$, where R is the region bounded by $y^2 = 4ax$ and $x^2 = 4ay$ 7 Marks

   b) Evaluate $\int\limits_{0}^{2}\int\limits_{0}^{\sqrt{2x-x^2}}\left(x^2+y^2\right)dy\,dx$ by changing into polar coordinates.   7 Marks

6. a) By changing the order of integration, evaluate $\int\limits_{0}^{3}\int\limits_{1}^{\sqrt{4-y}}(x+y)\,dx\,dy$ 7 Marks

   b) Evaluate $\int\limits_{0}^{\frac{\pi}{2}}\int\limits_{0}^{a\sin\theta}\int\limits_{0}^{\frac{a^2-r^2}{2}} r\,dz\,dr\,d\theta$ 7 Marks

UNIT-IV

7. a) Reduce the following matrix into echelon form and hence find its rank   7 Marks

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

   b) Find eigen values and eigen vectors of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ 7 Marks

8. a) Find the values of a and b for which the system of equations   7 Marks
   $x + 5y + 3z = 9$, $x + ay + z = 3$, $x + 2y + 2z = b$ will have   i) unique solution
   ii) infinite number of solutions   iii) no solution

   b) Reduce the following matrix A into normal form and hence find its rank   7 Marks
   where $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 1 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

UNIT-V

9. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence find the value   14 Marks

   of A^{-1} and A^4

10. Reduce the quadratic form $3x^2 + 2y^2 + 3z^2 - 2xy - 2yz$ into canonical form. Hence find rank,   14 Marks
    index, signature and nature of the quadratic form.