**PROBABILITY RULES**

**I. THEORY**

**1. Operations on events**

**1. Joint events**

Let  and  be two events. In that case  are subsets of the sample space . Let , we have  as an event, and it is called the union of the two events  and , denoted as 

*Chú ý:* Note: Considering a favorable outcome  for event , which means . Since  should  or . In other words,  s a favorable outcome for either event  or event . This signifies that event  or event has taken place. As a result, event  can be articulated as the statement “Either happens or happens ” or “At least one of the events  takes place”.

**Example *1:*** In a sealed box, there are 10 blue balls and 8 red balls, all of which are identical in size and mass. Two balls are randomly drawn at the same time. Consider the events:

A : “The two balls drawn are blue”.

B : “The two balls drawn are red”.

Choose the correct statement among the following:

a) The union of events  and  is “The two balls drawn are both red or both blue”.

b) The union of events  and  is “The two balls drawn are of different colors”.

c) The union of events  and  is “The two balls drawn are of the same color”.

*Solution:*

Statement a) is correct ; statement b) is incorrect; statement c) is correct.

**2. Intersecting events**

Given two events  and . when  are subsets of the sample space . Let , then  is an event and is referred to as the intersection of the two events  and , symbolized as  or .

*Note: When considering a favorable outcome*   for the event , that means . Since  so  and. In other words,  is a favorable result for both events  and . This means both events  and  occur simultaneously. Thus, event  can be stated as the occurrence of “Both  and happening at the same time”.

**Example 2.** Given a box containing 52 identical cards, each card is numbered separately with the set {1, 2, 3, …, 52}; Two different cards have two different numbers written on them. A card is drawn at random from the box. Suppose event A is "The number on the drawn card is divisible by 3" and event B is "The number on the drawn card is divisible by 4". Determine the subsets of the sample space corresponding to events A,B,.

**Solution**

We have ; .

**3. Mutually exclusive events**

Let two events be A and B. Then A and B are subsets of the sample space.. If  then *A* and *B* are said to be mutually exclusive events.

*Note***:** Consider an outcome favorable  for event *A*, tức là . Vì  nên , This means that  is not a favorable outcome for event B. Therefore, events A and B are mutually exclusive if and only if when one event occurs, the other does not.

**Example 3.** Toss a fair and identical coin twice consecutively. Consider the events:

*A*: “The coin shows S on the first toss”;

*B*: “The coin shows N on the first toss”.

Are these two events mutually exclusive?

**Solution**

We find that: .

It follows . Therefore, A and B are mutually exclusive events.

**II. INDEPENDENT EVENTS**

Given two events A and B. Events A and B are said to be independent if the occurrence or non-occurrence of one event does not affect the probability of the occurrence of the other event.

*Note***:** If A and B are two independent events, then each of the following pairs of events are also independent: *A* and ;  and *B*;  and.

**Example 4.** A box contains 3 blue balls and 4 red balls; all the balls are of the same size and weight. Draw a ball at random twice consecutively, where each time a ball is drawn at random from the box, record the color of the ball drawn, and then return the ball to the box. Consider the events:

*A*: “A blue ball is drawn on the first try”;

*B*: “A red ball is drawn on the second try”.

a) Are events A and B independent? Why?

b) Are events A and B mutually exclusive? Why?

**a)** Firstly, event  occurs after event  so whether event  ccurs or not does not affect the probability of the occurrence of event .

Furthermore, we have: the probability of event  occurring given that event  has occurred is equal to ; the probability of event  occurring given that event  has not occurred, which is also equal to . Therefore, the occurrence or non-occurrence of event  does not affect the probability of the occurrence of event . Thus, events  and are independent.

**b)** We see that the outcome (blue; red) is favorable for both events  and . Therefore,  and  are not mutually exclusive events.

**III. Addition Rule of Probability**

**1. Addition Rule of Probability**

For two events  and . then **.**

If two events  and  are mutually exclusive, then , simplifies to . Therefore, we have the following consequence:

**Consequence:** If two events  and  are mutually exclusive, then .

**Example 5** Randomly select a positive two-digit intege. Consider the event : "The number written is divisible by 8 " and the event  : "The number written is divisible by 9 ". Calculate 

**Solution**

In the set of 90 two-digit numbers, there are 11 numbers divisible by 8, 10 numbers divisible by 9, and 1 number divisible by both 8 and 9. Therefore, we have:

.

So .

**Example 6** A box contains 12 cards of the same type, with each card bearing one of the numbers ; if two cards are different, they have different numbers. A card is drawn at random from the box. Consider the event  : " The number on the drawn card is divisible by 3" and the event  : " The number on the drawn card is divisible by 5". Calculate P ..

**Solution**

The sample space of the experiment has 12 elements, which means:: .

The number of favorable outcomes for the events are ,  respectively: . Suy ra



In the set of numbers ,there is no number that is divisible by both 3 and 5 . Therefore, ,  are two mutually exclusive events. Consequently: 

**2. multiplication rule of probability**

Given two events  and .

If events  and  are independent .

**Note**: If  then events  and  are not independent.

**Example 7** Friends Hanh and Ha play a shooting game independently. Each person shoots only once. The probabilities that Hanh and Ha hit the target on their respective shoots are 0.6 and 0.7. Calculate the probability of the event  : "Both Hanh and Ha hit the target"

**Solution**

Considering the event  : "Hanh hits the target", we have: .

Considering the event  : "Ha hits the target", we have: .

We see that A, B are two independent events and . Therefore:



**Example 8:** Friends Trung and Dung of class 11A participated in the singles badminton tournament organized by the school. The two friends were not in the same elimination bracket which only selected one person to the final round. The probabilities that Trung and Dung passed the elimination round to the final were 0.8 and 0.6 respectively. Calculate the probability of the following events:

a) : "Both friends made it to the final ".

b) : " At least one friend made it to the final" "

c) : " Only friend Trung made it to the final round ".

**Solution**

Considering the events  : "Trung made it to the final round" and  : "Dung made it to the final round".

From the assumption, we can deduce that E and G are two independent events and .

a) Since  ,therefore .

b) We see that , Therefore,



c) Considering the complementary event  of even . We see  and  are two independent events. Since  , therefore .

**IV. QUESTIONS**

**Bloom's Taxonomy cognitive levels:**

1. Remember (Remembering)

2. Understand (Understanding)

3. Apply (Applying)

4. Analyze (Analyzing)

5. Evaluate (Evaluating)

6. Create (Creating)

**Question 1(Remembering):** What is a sample space?

**Question 2(Remembering):** What is an event?

**Question 3(Understanding):** What are joint events?

**Question 4(Understanding):** What is an intersecting event?

**Question 5(Understanding):** When are two events called independent of each other?

**Question 6(Understanding):** What are mutually exclusive events?

**Question 7(Understanding):**  What is a complementary event?

**Question 8(Understanding):** State the classical formula for probability?

**Question 9(Understanding):** State the formula for adding probabilities of two events

**Question 10(Understanding):** State the formula for multiplying probabilities of two events

**Question 11(Understanding):** State the formula for calculating the probability of a complementary event.

**Question 12.** A shooter continuously shoots 4 bullets at the target. Let  be the events “ he shooter hits the target on the shot” with . Represent the following events through the events .

1. **(Analyzing**): "Hit the target on the fourth shot".
2. **(Evaluating):** "Hit the target at least once".
3. **(Evaluating):** "Hit the target exactly three times".

**Question 13(Applying):** Given  and  are two mutually exclusive events satisfying  và  Then the value of  is?

**Question 14(Applying):** Two athletes A and B throw balls into the basket independently of each other. The probabilities that athletes A and B score in the basket are  and  respectively. What is the probability of the event Both score in the basket is?

**Question 15(Analyzing):** Two shooters A and B each shoot one bullet at the target independently of each other. The probabilities that shooters A and B hit the target are  và respectively. Calculate the probability of the event Shooter A hits, shooter B misses.

**Question 16(Analyzing):** The probability of hitting a target for an athlete when shooting one bullet is The person shoots twice independently. What is the probability that one bullet hits the target and one misses?

**Question 17(Analyzing):** Three people shoot at the target (circle) independently. The probabilities that the first, second and third person hit the target are  and  respectively. Calculate the probability that exactly  people hit the target.

**Question 18(Analyzing):** An airplane has two engines I and II operating independently of each other. he probability that engine I runs well is  and the probability that engine II runs well is  What is the probability that at least one engine runs well?

**Question 19(Analyzing):** Two shooters each shoot one bullet at the target independently of each other. The probabilities that the two shooters hit the target are  and respectively. What is the probability that at least one shooter misses the target?

**Question 20(Analyzing):** The probability of hitting the bullseye of a person is  Calculate the probability that in three independent shots, the person hits the bullseye at least once.

**Question 21(Analyzing):** Two shooters Toan and Tinh both shoot at the target (circle) independently. The hitting probability of shooter Toan is Given that the probability of at least one person hitting the target is  What is the hitting probability of shooter Tinh?

**Question 22(Evaluating):** A person has a bunch of  that look exactly the same from outside and only  The person tries randomly each key (if it does not open will discard). What is the probability of opening the door in the third try?

**Question 23(Evaluating):** Box A contains  white balls,  red balls and  blue balls. Box B contains  white balls,  red balls and  blue balls. Taking one ball randomly from each box, calculate the probability that the two drawn balls have the same color.

**Question 24(Evaluating):** The probability of hitting the bullseye of a person is  Calculate the probability that in three independent shots, the person hits the bullseye exactly once.

**Question 25** **(Creating):** Given  and  are two independent events. Knowing the probabilities that events  and  occur are and  What is the probability that event  occurs?