MAKING GUIDE

Question 1: Know $\lim u_n = a; \lim v_n = b.$ Calculate $\lim\left(u_n + v_n\right)$ and $\lim\left(u_n.v_n\right)$.

- Step 1: $\lim\left(u_n + v_n\right) = a + b.$
- Step 2: $\lim\left(u_n.v_n\right) = a.b.$
- Complete two steps to get a perfect score (0.5 points)
- Doing 1 out of two steps correctly gets 0.25 points
- If you do both steps wrong, you will not get points

Question 2: Calculate $\lim u_n$, with $u_n = \dfrac{5n^2 + 3n - 7}{n^2}$.

- Step 1: We have $\lim u_n = \lim\left(\dfrac{5n^2}{n^2} + \dfrac{3n}{n^2} - \dfrac{7}{n^2}\right)$
- Step 2: $= \lim\left(5 + \dfrac{3}{n} - \dfrac{7}{n^2}\right)$
- Step 3: $= 5 + 0 - 0 = 5$
- Doing all three steps correctly gets a perfect score (0.75 points).
- Doing steps 2 and 3 correctly will get a perfect score (0.75 points).
- Doing step 3 incorrectly gets 0.5 points.
- Doing steps 2 and 3 incorrectly gets 0.25 points.
- If the steps are shortened or converted incorrectly, points will not be given.

Question 3: Calculate $\lim\dfrac{3.2^n - 5^n}{5.4^n + 6.5^n}$

- Step 1: We have $\lim\dfrac{3.2^n - 5^n}{5.4^n + 6.5^n} = \lim\dfrac{3.\dfrac{2^n}{5^n} - \dfrac{5^n}{5^n}}{5.\dfrac{4^n}{5^n} + 6.\dfrac{5^n}{5^n}}$
- Step 2: $= \lim\dfrac{3.\left(\dfrac{2}{5}\right)^n - 1}{5.\left(\dfrac{4}{5}\right)^n + 6}$
- Step 3: $= \dfrac{3.0 - 1}{5.0 + 6} = -\dfrac{1}{6}.$
- Doing all three steps correctly gets a perfect score (0.75 points).
- Doing steps 2 and 3 correctly will get a perfect score (0.75 points).
- Doing step 3 incorrectly gets 0.5 points.
- Doing steps 2 and 3 incorrectly gets 0.25 points.
- If the steps are shortened or converted incorrectly, points will not be given.

Question 4: Calculate $\lim\dfrac{\sqrt{n^4 + 2n + 2}}{n^2 + 1}$

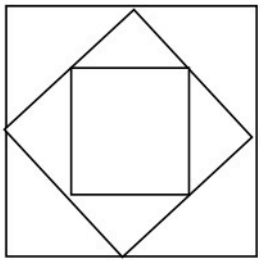
- Step 1: We have $\lim\dfrac{\sqrt{n^4 + 2n + 2}}{n^2 + 1} = \lim\dfrac{\sqrt{n^4\left(1 + \dfrac{2}{n^3} + \dfrac{2}{n^4}\right)}}{n^2\left(1 + \dfrac{1}{n^2}\right)}$

- Step 2: $= \lim \frac{n^2 \sqrt{1+\frac{2}{n^3}+\frac{2}{n^4}}}{n^2\left(1+\frac{1}{n^2}\right)}$
- Step 3: $= \lim \frac{\sqrt{1+\frac{2}{n^3}+\frac{2}{n^4}}}{1+\frac{1}{n^2}}$
- Step 4: $= \frac{\sqrt{1+0+0}}{1+0} = 1.$
- Do step 2, step 3 and step 4 correctly to get a perfect score (1 point)
- Doing step 4 incorrectly gets 0.75 points
- Doing steps 3 and 4 incorrectly gets 0.5 points
- Doing step 2, step 3 and step 4 incorrectly gets 0.25 points.
- If the steps are shortened or converted incorrectly, points will not be given.
- If step 2 is incorrectly shortened or written incorrectly, no points will be given
- If step 3 is incorrectly shortened or written incorrectly, no points will be given

Question 5: Calculate $\lim\left(\sqrt[3]{2n-3n^3}+n-1\right)$.

- Step 1: We have $\lim\left(\sqrt[3]{2n-3n^3}+n-1\right) = \lim\left(\sqrt[3]{n^3\left(\frac{2}{n^2}-3\right)}+n\left(1-\frac{1}{n}\right)\right)$
- Step 2: $= \lim\left(n\sqrt[3]{\frac{2}{n^2}-3}+n\left(1-\frac{1}{n}\right)\right)$
- Step 3: $= \lim n\left(\sqrt[3]{\frac{2}{n^2}-3}+1-\frac{1}{n}\right)$
- Step 4: $= \lim n.\lim\left(\sqrt[3]{\frac{2}{n^2}-3}+1-\frac{1}{n}\right)$
- Step 5: $= \lim n.\lim\left(1+\sqrt[3]{-3}\right) = -\infty.$
- Do 5 steps correctly to get a perfect score (1.25 points)
- Do step 2, step 3, step 4 and step 5 correctly to get a perfect score (1.25 points)
- If you do step 5 wrong, you get 1 point.
- Doing steps 4 and 5 incorrectly gets 0.75 points
- Doing step 3, step 4 and step 5 incorrectly gets 0.5 points.
- Doing step 2, step 3, step 4 and step 5 incorrectly gets 0.25 points.
- If the steps are shortened or converted incorrectly, points will not be given.

Question 6: Let square $ABCD$ have length 1. We inscribe in this square a second square, whose vertex is the midpoint of its sides. And so we continue to inscribe as shown below. Calculate the total perimeter of those squares.



- Step 1: We have $S_n = 4 + 4.\frac{1}{\sqrt{2}} + 4.\left(\frac{1}{\sqrt{2}}\right)^2 + ... + 4.\left(\frac{1}{\sqrt{2}}\right)^{n-1}$
- Step 2: $= 4.\left(1 + \frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}}\right)^2 + ... + \left(\frac{1}{\sqrt{2}}\right)^{n-1}\right) = 4.\frac{1-\left(\frac{1}{\sqrt{2}}\right)^n}{1-\frac{1}{\sqrt{2}}}$
- Step 3: $\lim S_n = \lim 4.\frac{1-\left(\frac{1}{\sqrt{2}}\right)^n}{1-\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2}-1} = 4\left(2+\sqrt{2}\right).$
- Doing all three steps correctly gets a perfect score (0.75 points).
- Doing step 1 incorrectly gets 0.25 points.
- Doing steps 1 and 2 incorrectly gets 0.5 points.
- Doing steps 3 or steps 2 and 3 correctly will not get points