# Robotic Manipulator Control Using PD-type Fuzzy Iterative Learning Control

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***Abstract*—In this paper, a single arm planar manipulator robot with a moving platform is controlled based on PD-type Fuzzy Iterative Learning Control (ILC). The manipulator robot is modeled based on the Euler Lagrange equation, and the Multi-Input-Multi-Output (MIMO) nonlinear model is obtained for simulation. The DC motor torque and horizontal force for moving platform are system inputs, and position of the moving platform and robot arm are system outputs. The linearized state-space linear model of the robot is obtained for analyzing stability and convergence of proposed controller. The results of comparing the proposed PD-type fuzzy ILC controller to P-type, PD-type, and P-type Fuzzy ILC illustrate fast and accurate reference tracking the performance of this proposed controller.**

***Index Terms*—Learning Control, Iterative Learning Control, PD-type ILC Control, Fuzzy Logic Control, Robotic Manipulator**

## I. INTRODUCTION

Iterative Learning Control (ILC) is using to improve the tracking performance of systems which have repeated dynamics. For example, a robot which is used to manipulate material without human supervision. The idea of ILC is similar to the human learning process for repetitive actions. Humans gain a new skill by repeating it over time. For instance, children improve their writing skill by learning through repeated training. Using ILC for a repeated dynamic system improves the tracking of the desired reference. ILC uses previous control inputs and errors to generate new control input for systems that repeat and this repeating action is called a cycle. This learning process improves the system's performance [6], [16]. Iteration learning based control has some unique advantages over the classic, non-linear, and model-based control method and can be used in conjunction with them. The four main ILC control features that make it useful, especially in the real-time, are [14]:

- the simple structure of ILC makes it reliable, computationally inexpensive, and easy to design,
- the ability to achieve perfect tracking both in steady state and during transients,
- the model-free design makes ILC design straight forward to design and implementable in real-time,
- the availability of a non-causal control signal for control compensation.

The term "Iterative Learning control (ILC)" was coined for the first time in 1984 and mathematically formulated in [2]. Since then, ILC control has been used for controlling various types of systems. P-type ILC control is used to control of dual fuel Homogeneous Charge Compression Ignition (HCCI) engine [13]. PD-type spatial iterative learning control (SILC) is used to control the pitch of wind turbine, and the convergence of the designed controller is derived based on tracking error in the form of Lebesgue-$\rho$ norm [8]. In [4], a PID-type ILC controller is designed to control a CNC machine tool, and the convergence of PID-type ILC is mathematically analyzed. Optimizing PID-type ILC is another method of using PID-type ILC which is used to control a aluminum extruder [12]. Passivity theory is used to design repetitive linear control for discrete-time dynamics and the control stability is analyzed [11]. The plant inversion method is used to design a switched ILC (SILC) controller for MIMO systems and improved performance of controller especially for uncertain system is shown [5]. Network-based ILC (NILC) is used to design a controller for SISO nonlinear system and the NILC controller shows improved performance compared to ILC [7]. Adaptive control is combined with iterative learning control to improve ILC performance [15]. Adaptive fuzzy ILC (AFILC) for non-parametrized nonlinear discrete-time systems with unknown dead zones is studied in [14]. The PD-type controller is designed for the class of linear systems with a relative degree of two by adding the second derivative of error to the common PD-type ILC control law [3]. N-parametric type ILC with optimal gains is introduced in [9]. The controller is designed by using an extended ILC (EILC) technique for SISO linear time-invariant (LTI) system to determine the optimal gain of the controller and the convergence of the controller is analyzed [9]. A Fuzzy Iterative Learning Control for Nonlinear Batch Processes which can guarantee the closed-loop performance by using fault-tolerant guaranteed cost controller is given in [17].

In this paper, a PD-type fuzzy ILC controller is used to control a two degree of freedom manipulator robot in simulation. A fuzzy logic mechanism is used to re-tune the PD-type ILC controller gains. First, the nonlinear and linear model of two degrees of freedom manipulator robot is calculated by the energy-based method (Euler-Lagrange



978-1-7281-0319-8/19/$31.00 ©2019 IEEE

method). Then, a PD-type fuzzy controller is designed and the convergence and stability of the controller is mathematically analyzed. The proposed ILC controller improves the performance resulting in fast and accurate reference tracking control which requires less parameter tuning compared to a classic ILC controller.

## II. Dynamic Modeling

### A. Nonlinear Model

A schematic single-link Planar Manipulator is shown in Fig. 1. The Euler-Lagrange method is used to derive the robot's dynamic equations of motion.

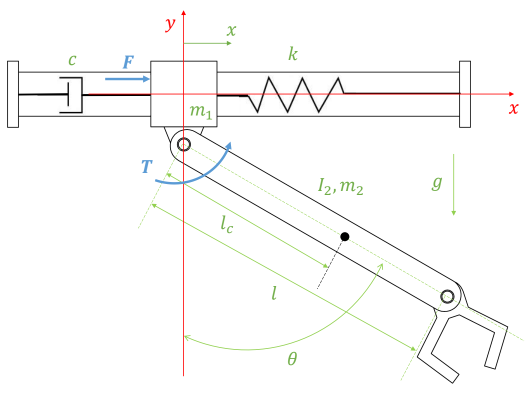


Fig. 1. kinematic diagram of the manipulated Robot

By using kinetic and potential energy of mass 1 and arm link, the Lagrangian is calculated as

$$\begin{aligned}\mathcal{L} &= \sum K - \sum U \\ &= \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{x}^2 + m_2\dot{x}\dot{\theta}l_c cos(\theta) \\ &+ \frac{1}{2}m_2 l_c^2\dot{\theta}^2 + \frac{1}{2}I_{2c}\dot{\theta}^2 - \frac{1}{2}kx^2 + m_2 l_c g\cos(\theta)\end{aligned} \tag{1}$$

A Rayleigh dissipation function $D = \frac{1}{2}c\dot{x}^2 + \frac{1}{2}c_\theta\dot{\theta}^2$ is used to add non-conservative forces to Lagranges equations. The equations of motion of robot are obtained by applying Lagranges equations

$$\frac{d}{dt}[\frac{\partial \mathcal{L}}{\partial \dot{q}_i}] - \frac{\partial \mathcal{L}}{\partial q} + \frac{\partial D}{\partial \dot{q}_i} = \tau_i \quad i = 1, 2 \tag{2}$$

where $q_1 = x$ and $q_2 = \theta$.

Substituting Eq. 1 into Eq. 2 the two DOF robot equations of motion are

$$\{\ddot{q}\} = [M^{-1}]\{C\} + [M^{-1}]\{U\} \tag{3}$$

where

$$\begin{aligned}\{\ddot{q}\} &= \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix}, [M]^{-1} = \begin{bmatrix} m_t & m\cos\theta \\ z_2\cos\theta & I_2 \end{bmatrix}^{-1}, \\ \{U\} &= \begin{bmatrix} F \\ T \end{bmatrix}, \{C\} = \begin{bmatrix} -c\dot{x} - kx + m\dot{\theta}^2\sin\theta \\ -c_\theta\dot{\theta} - z_1\sin\theta \end{bmatrix}\end{aligned} \tag{4}$$

and $m_t = m_1 + m_2$, $I_2 = m_2 l_c^2 + I_{c2}$, $z_1 = m_2 g l_c$ and $z_2 = m_2 l_c$.

### B. Linear Model

The linearized dynamic model is used for ILC stability analysis. The nonlinear model in Eq. 3 is linearized based on following assumptions:

- $\theta$ angle is assumed to be small so $\cos\theta \approx 1$ and $\sin\theta \approx \theta$
- high order states is neglected (such as $\dot{\theta}^2$)
- and the multiplication of states can be neglected

The linear system based on these assumptions is

$$\begin{aligned}&\dot{x} = Ax + Bu \\ &y = Cx \\ &x = [x \quad \dot{x} \quad \theta \quad \dot{\theta}]^T, y = [x \quad \theta]^T, u = [F \quad T]^T\end{aligned} \tag{5}$$

where $x$, $y$, and $u$ are the model states, outputs, and inputs respectively. A, B and C are the state space matrices of system.

A transfer function of system is used for the stability analysis. The transfer function is

$$G(s) = C[sI - A]^{-1}B \tag{6}$$

which $G(s)$ is the $2 \times 2$ transfer function matrix.

## III. Controller Design

### A. Iterative Learning Control

In ILC control the input can be calculated as

$$u_{j+1}(t) = Q(u_j(t)) + L(e_j(t)) \tag{7}$$

where L is a general learn operator or learn filter and Q is a control input filter or Q-Filter. The simplest ILC controller is P-type ILC which is function of previous cycle error and control input and the Q-filter is equal to identity matrix

$$u_{j+1}(t) = u_j(t) + Le_j(t) \tag{8}$$

A derivative term can be added to ILC control to achieve PD-type ILC controller. The PD-type control law is

$$u_{j+1}(t) = u_j(t) + Pe_j(t) + D\dot{e}_j(t) \tag{9}$$

where $P$ is the proportional and $D$ is derivative learning gain. [1].

### B. Proposed Controller

In this paper a PD-type ILC controller is combined with a fuzzy logic system to further improve reference tracking. The advantage of augmenting ILC with a fuzzy logic is that a large(small) learning gain can be applied for a large (small) error. This technique helps achieve fast learning and accurate tracking. The input and output of the fuzzy mechanism are normalized based on a maximum gain of $P$ and $D$ and the maximum two norm of error. Because the maximum two norm of each repetitive cycles occurs in the first cycle, $||e_1||_2$ is used to normalize the inputs as

$$(||e_{j-1}||_2)_n = \frac{||e_{j-1}||_2}{||e_1||_2} \tag{10}$$

$$(||e_j||_2 - ||e_{j-1}||_2)_n = \frac{||e_j||_2 - ||e_{j-1}||_2}{||e_1||_2} \tag{11}$$

where Eq. 10 and Eq. 11 are the inputs of $P$ and $D$ tuning respectively. The outputs of fuzzy mechanism are normalized $P$ and $D$ ($P_n$ and $D_n$). Then, $P$ and $D$ can be calculated by using the maximum value of $P$ and $D$ as

$$P = P_n P_{max},\ D = D_n D_{max} \tag{12}$$

So the $P$ and $D$ gains are tuned based on the second norm of error in each cycle using in the ILC controller. The PD-type fuzzy ILC controller structure is shown schematically in Fig. 2.

### C. Stability Analysis

For analyzing the stability of ILC, the term $Q - LG$ is considered as the criteria of stability and necessary but not sufficient conditions for ILC asymptotic stability is

$$||Q - LG||_\infty < 1 \tag{13}$$

or maximum singular value of $[Q - LG]$ is less than one. The $Q - LG$ term can be obtained based on tracking error [10].

Additionally, monotonic convergence of $e_{j+1} = Me_j$ with $M = [Q - G(s)L]$ occurs when using the Euclidean norm and when the maximum singular value $\bar{\sigma}(M) < 1$.

Now ILC control stability and convergence including stability criteria of PD-type controller can be calculated. The Q-filter is chosen to be $Q = I$, so $M$ is calculated as

$$\begin{aligned} e_{j+1}(s) &= r(s) - y_{j+1}(s) = r(s) - G(s)u_{j+1}(s) \\ &= r(s) - G(s)Qu_j(s) - G(s)Pe_j - sG(s)De_j \\ &= [I - G(s)(P + sD)]e_j \end{aligned} \tag{14}$$

So, for the PD-type controller $M = [I - G(s)(P + sD)]$. Thus, the conditions for ILC asymptotic stability and monotonic convergence are given with Eq. 15.

Additionally, to analyze the stability based on convergence, the euclidean norm of tracking error over the time in each iteration is used as

$$\begin{aligned} ||e_{x(j)}||_2 &= \sqrt{e^*_{x(j)} e_{x(j)}} \\ ||e_{\theta(j)}||_2 &= \sqrt{e^*_{\theta(j)} e_{\theta(j)}} \end{aligned} \tag{15}$$

Where $j$, $e_\theta$, and $e_x$ are number of each iteration, tracking error vector of $x$ and $\theta$ in each iteration respectively.

## IV. Results and Discussions

The PD-type Fuzzy ILC control is tested in simulation and compared to other types of ILC. Reference tracking of P-type ILC, PD-type ILC, P-type fuzzy ILC, and PD-type fuzzy ILC are shown in Fig. 3. The proposed controller performs has faster response and more accurate tracking than the other controllers. The fuzzy logic system applies a large learning gain to cycles which have the large error resulting in faster system convergence. As $||e_{x(j+1)}||_2 < ||e_{x(}j)||_2$ is satisfied in all types of controller, convergence of all controller are guaranteed.

## V. Conclusion

A PD-type Fuzzy ILC controller is developed for two degree of freedom manipulator robot. Using Euler-Lagrange the nonlinear equations of motion of manipulator robot are derived. The equations of motion are linearized and using the linear model, the stability and convergence is analyzed. In simulation the nonlinear model is used to evaluated the performance of PD-type Fuzzy ILC control compared with P-type and PD-type ILC controllers. The results show that the proposed controller has the fastest convergence and the most accurate reference tracking performance. In addition to the performance benefits of proposed controller, it has a reduced number tuning parameter in comparison to classic PD-type ILC controller. The proposed controller can be designed for different system by using maximum learning gains ($P_{max}$ and $D_{max}$) and maximum number of iteration ($j_{max}$). Perhaps adding a Q-filter or having a higher order L-filter possibly could further improve the performance but this is beyond the scope of this paper and is future work.

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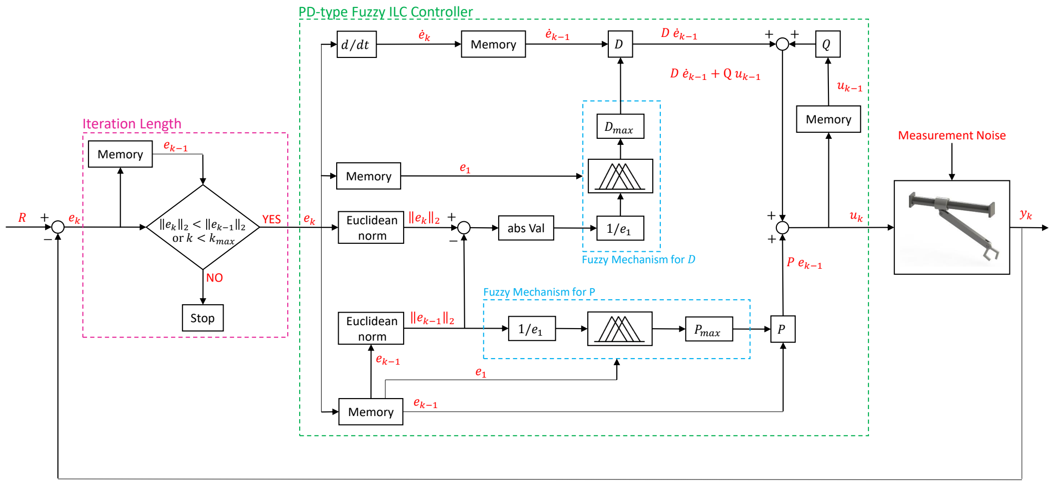


Fig. 2. Block diagram of Proposed Controller: PD-type ILC controller with a fuzzy PD gain

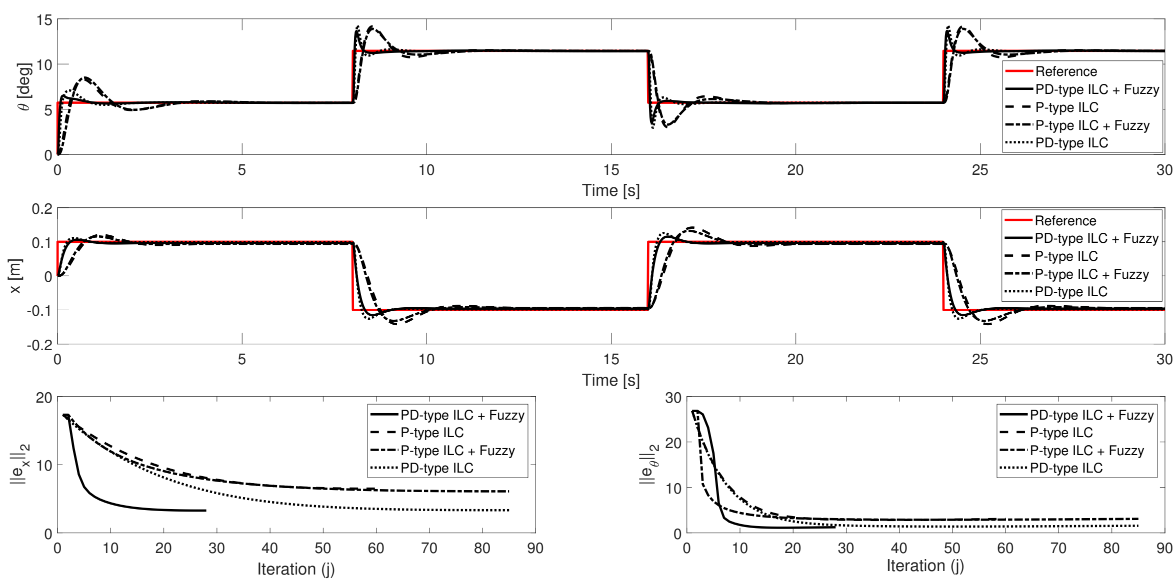


Fig. 3. Simulation: Input tracking to a reference input of P-type, PD-type, P-type with fuzzy gain, and PD-type with Fuzzy gain for reference input tracking simulation and euclidean norm for both $x$ and $\theta$ error. Control is Eq. 9 with PD gains updated by the cycle norm and given in Eq. 15

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