Physical Modeling and Control of Self-Balancing Platform on a Cart

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Abstract **–This paper demonstrates a self-balancing platform with 2-degrees of freedom on a cart which is generally treated as inverted pendulum for simplicity. The system can be used in various transportation devices, delivery and stabilization systems and is particularly suitable for working in outdoor where the surface of ground is not flat or structured. The platform can freely rotate with the help of a ball and socket joint at its center. Lateral and longitudinal movements are controlled by two servomotors for each axis. As the cart moves on slope or on a rough terrain, the instantaneous tilt of the platform is measured by a gyroscope assembly which is compared with the desired orientation of the platform and can take corrective measure for platform up to 5 degrees in both axes. In this paper,physical modeling method is used for rapid simulation of system and mathematical relationship between platform tilt with respect to servomotor's rotation angle is developed. Proportional-integral-derivative (PID) controller is used for desired smooth operation and jerk attenuated balance of the platform. Comparative study ofperformance of the system forthe physical model developed and the one that is implemented is made.**

Keywords – inverted pendulum; physical modeling; PID; mobile robot.

I. INTRODUCTION

Transportation and delivery systems used in service sectors like pharmaceutical and health care industries are now being automated. Research on control methods for complex and non-linear mechanisms are hot topic [1].In this paper, the inverted pendulum, which is a classic nonlinear control experiment, is analysed using MATLAB and Simulink's SimMechanics toolbox. Despite the advances in wheeled robotics, certain amount of vibrations and shocks are unavoidable. Hence, ensuring the safety of fragile goods that are being transported becomes more important as any delay or damage to goods can end up being very costly [2]. This paper aims to provide the design and implementation of a small scale working model of a self-balancing platform on a cart[3] that can be utilized to transport fragile materials across rough terrain and if developed on a larger scale can minimize these significant losses in maintaining balance. A multifunctional self-balancing mobile platform that was designed and implemented based on inverted pendulum model is studied [4]. A simple, convenient, stable and reliable, environment friendly platform controlled through intelligent automation is presented. The design of a two-wheeled balancing robot based on the concept of inverted pendulum [5] and implementation of PID control in developing a stable self-balancing platform on a mobile robot is analyzed [6], [7].

A parallel manipulator uses several serial chains to support a single platform and is designed in such a way that each chain is usually short, simple and is rigid enough to restrict unwanted movement.

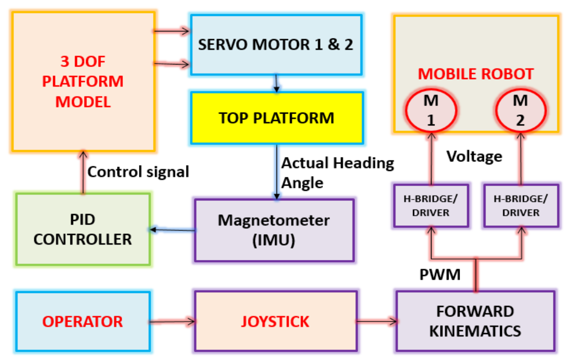

Fig.1. Block Diagram for the Working of Self-Balancing Platform

The self-balancing platform is also a type of parallel manipulator and hence its understanding and control becomes important. The lateral and longitudinal movement of the self-balancing platform is controlled by two links in this case. Unlike a traditional pendulum which has the center of mass lying below the pivot point, an inverted pendulum has its center of mass above its pivot point. The challenge involved in an inverted pendulum is to achieve stability. Fig.1 shows the block diagram which represents the working of Self-Balancing Platform. When an operator gives input command to the robot by using a joystick, signals are passed into the motor driver circuit making the robot to move in a rough terrain. The movement of the robot in such an environment makes the platform to tilt and thus unstable. The angle to which it is tilted is measured by using IMU sensor and then it is given to PID controller [8].The controller gives input to the servomotors which controls the platform to stay in the steady

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state desired angle.Major challenge in implementation is in finding the optimum offset of lateral motion of platform and in developing mathematical relationship for platform tilt with respect to individual actuation. All the parts are modeled with certain level of assumption to meet the practical scenario.

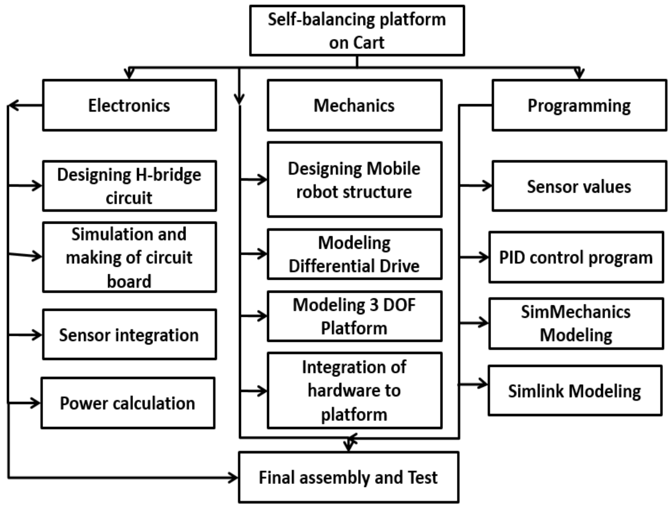

Fig.2. Block Diagram of Components Required for the System

Fig.2 describes in details different areas and sub-areas required for design, development and implementation of the self-balancing platform on a cart.

II. MECHANICAL DESIGN

A square shaped platform with each side measuring 30 cm is used. For constructing the platform, plywood or acrylic sheet is preferred to metal, due to its light weight and ease to drill holes for placing the support links and attaching the acrylic clamps which holds the servo motors. At the bottom, the center of the platform has a provision to connect to a ball and socket joint which allows free rotation of the platform. The ball and socket joint consists of a rotating portion and a fixed column structure connecting the platform with the bottom of the box like wooden structure meant for housing the electronic components and batteries. Fig.3 shows the isometric view of the mobile platform designed in CAD. When the cart starts moving in a rough terrain, the platform which is connected to it by means of ball and socket jointexperiences vibration and tilts. The friction made by bearing and joints are considered negligible so that the plate movement alone is considered.

The two links to the platform are connected to two revolute joints and their movement is controlled by connecting the links with the shaft of servomotors.If the platform is tilted by an undesired angle, then the links are tilted with certain angle. That angular tilt is converted into corresponding motor rotation angle by kinematics and is processed with the controller.The controller signals the driver circuit to rotate the servomotor by the same angle in opposite direction which compensates the tilt caused by robot movement.

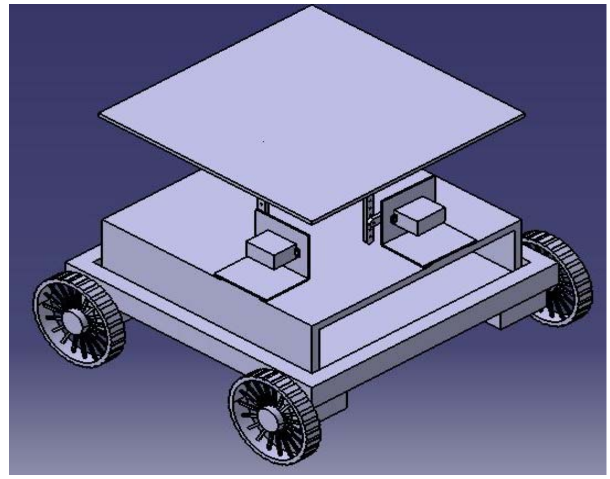

Fig.3. Isometric View of CAD Model of Mobile Platform

III. KINEMATIC MODEL AND ANALYSIS

The platform tilt axes are considered in x-z and y-z directions. The change of platform's orientation because of movement of the cart is measured by using gyroscope and is given as feedback signal to the controller. The feedback signal is compared with the desired angle and the error signal is given as input to the controller. The tilt angle is adjusted accordingly in order to maintain the platform in steady state.

The tilt of the platform measured in two axes is represented as roll (ϕ) and pitch (ψ). The linear velocity of the robot which makes the platform to tilt, is found in terms of roll and pitch. For the kinematic analysis [9] of the platform the following parameters are considered for formulating mathematical relationship between platform tilt and servomotor rotation angle. The length of the links (L_1 and L_2) joining the platform to motor links(S_{r1} and S_{r2}) connected with servomotor shaft are of dimension 10 cm and 2.8 cm respectively. Servomotor is positioned with an offset length (L_3) of 10 cm from center of cart in both the axes [10]. Pictorial representation of platform with base of cart is shown in Fig.4.

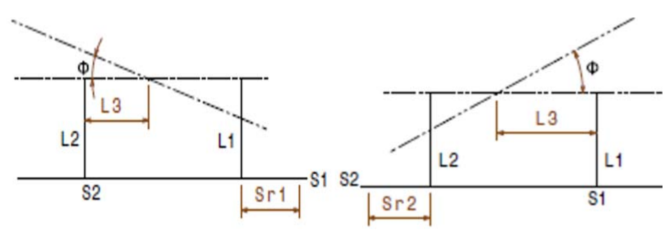

Fig.4. Front and Side-View of the Platform on Cart

When an actuation is given by servomotors S_1 and S_2, the motor links S_{r1} and S_{r2}rotates by angles α_1 and α_2 respectively. The change in postion of link S_{r1} or S_{r2} will give difference of distance 'd_2'. Fig.5 represents the sketch for understanding the measurement of 'd_2' from change in servomotor actuation.

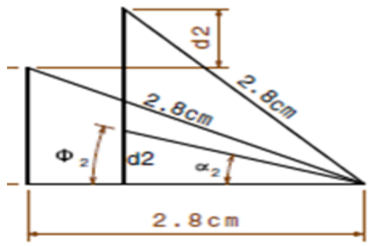

Fig.5. Representation of 'd_2' with respect to Motor Actuation

The maximum offset distances the platform would move laterally when servomotor actuation is given can be calculated by the relation formulated below. Distance between XZ and YZ is b_2 and O_2 respectively. Distance between Y and W is 2.8 cm and angle made between YW and XW is α_2.The offset with respect to actuation angle and the relationship between known actuation angle and platform tilt angle can be found by assuming parameters as shown in the Fig.6.

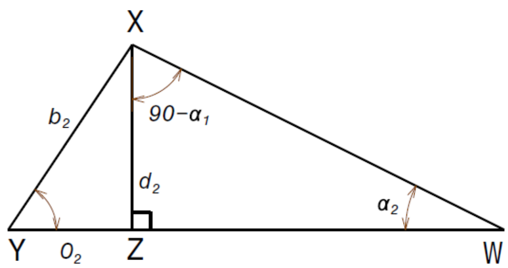

Fig.6. Pictorial Representation of Parameters for Motor Link and Offset Distance.

Assumed distance covered by the section arc of angle α_2 is b_2 and is given by the (1).

$$b_2 = (2*\pi*\alpha_2*S_{r1})/360 \tag{1}$$

Applying Pythagoras theorem to triangle XYZ, (2) and (3) gives the relationship between d_2, b_2 and o_2, and between α_2 and d_2, respectively.

$$d_2^2 = (b_2^2 - o_2^2)^{1/2} \tag{2}$$

$$\sin(90\text{-}\alpha_2) = d_2 / b_2 \tag{3}$$

For a desired fixed offset say,o_2 =0.5cm. The angle of rotation to which the servomotor needs to be restricted can be found by (4).

$$d_2^2 + (0.5e\text{-}2)^2 = ((2*\pi*\alpha_2*S_{r1})/360)^2 \tag{4}$$

Platform tilt angle is measured in the form of roll (ϕ) and pitch (ψ) can be given equations (5), (6) and (7).

$$\sin\phi = d_2/ L_3 \tag{5}$$

$$\text{Roll, } \phi = \sin^{-1}(d_2/ L_3) \tag{6}$$

$$\text{Pitch, } \psi = \sin^{-1}(d_1/ L_3) \tag{7}$$

Relationship between d_2 and α_2 is given in (8),

$$\sin\alpha_2 = d_2/ S_{r2} \tag{8}$$

From (5),

$$d_2 = L_3 * \sin\phi \tag{9}$$

Upon substituting (9) in (2), Roll is

$$\phi = \sin^{-1}\{((2*\pi*S_{r1}/L_3)*\alpha_2/360)^2 - (0.5e\text{-}2/ L_3)^2\}^{1/2} \tag{10}$$

For finding pitch, similar methods are used, Pitch is

$$\psi = \sin^{-1}\{((2*\pi*S_{r2}/ L_3)*\alpha_1/360)^2 - (0.5e\text{-}2/ L_3)^2\}^{1/2} \tag{11}$$

The roll and pitch angle measured from the above equations are given as input to the controller. By equating (5) and (8), the relationship between the input servomotor angle and platform tilt could be found out.

$$\sin\phi = (S_{r2} * \sin\alpha_2)/L_3 \tag{12}$$

From (4) it can be found out that, the maximum of 18.5 degrees (α_2) can be given as servomotor angle for maximum offset 'o_2'value of 0.5cm.

IV. Mathematical Model of DCMotor

Mathematical model of DC motor represents the behavior of motor by using mathematical equations. The DC motor is controlled by varying armature current or field current. Equation (13) gives the relationship between coil inductance, resistance, armature current and armature voltage for DC motor. The platform balance is done by negative feedback in which the angular displacement of motor and the disturbance is given as feedback input and is compared with the desired roll angle. The error signal is given as input to PID controller. The controller's output signal is again compared with the actual angle of rotation and control action takes place till steady state is reached. The equations given below gives the model of the DC motor used.

$$L\, di/dt + iR = E - E_b \tag{13}$$

The relationship between armature current and torque produced can be given by (14).

$$iK_t = T \tag{14}$$

Equations(15) and (16) gives the mechanical relationship between angular velocity of shaft and back EMF produced in coil.

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$$d^2\theta/dt^2 + B\, d\theta/dt = T \quad (15)$$

$$E_b = K_b * \omega \quad (16)$$

From (17) the angular velocity ω(s) taken as output for the corresponding input voltage E(s) is regulated for any angular change occurring in the platform.

$$K_t/[(sL+R)(sJ+B)+ K_t K_b] = \omega(s)/E(s) \quad (17)$$

The outer loop computes the angle to be maintained by the platform for stable position and then the inner loop gives the compensation as a motor control,for the angular tilt that occurred in outer loop. This transfer function is used in SimMechanicsfor motor model. The Table I given below shows the values for the parameter taken in DC motor model for the system.

TABLE I. MOTOR SPECIFICATIONS

Sl.No	Parameters	Values
1.	Operating voltage (V)	12V
2.	Operating current (I)	2.28A
3.	Motor Torque (T_{motor})	0.0187 N-m
4.	Armature inductance (L)	0.58mH
5.	Armature Resistance (R)	1.17Ω
6.	Speed (ω)	9960
7.	Viscous damping factor (K)	1.34e-6
8.	coulomb friction torque (B)	0.0025N-m
9.	Rotor inertia (J)	1.62e-6
10	Mass (M)	164.78Kg
11.	Motor torque constant (K_t)	0.011Nm/A
12.	Back EMF constant (K_b)	0.011V/rad/S

Fig. 7 shows the closed loop Simulink aided model of DC motor with position feedback for servomotor characteristics. Tuning of PID is done using auto tune feature present in Simulink software. The range of P-I-D values are realistic and implementable.

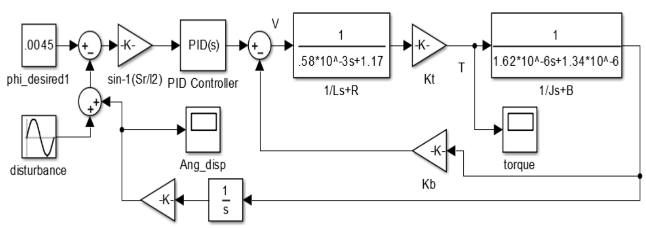

Fig.7. Mathematical Model of DC Motor

The relationship between armature angular velocity and useful motor velocity, inside the motor casing is found by using (18) and found that the gear ratio is 99.8 for chosen motor as shown in (19).

$$\omega_{motor} * T_{motor} = \omega_{shaft} * T_{shaft} \quad (18)$$

$$\text{Gear ratio} = \omega_{shaft}/\omega_{motor} = 99.8 \quad (19)$$

V. PHYSICAL MODELING WITH AID OF SIMMECHANICS

The identification of parameters used for modeling is tedious and based on some assumptions. Equation of motion for mechanism are manually derived and compared with physical model developed in SimMechanics [11]. Physical modeling using standard Newtonian dynamics of forces and torques for modeling environment. Mechanical systems are represented in graphical or Simulink block based to save time and effort to model. The self-balancing platform over a cart is modeled by using block set which consists of libraries for cart platform, balanced platform, links, sensors, joints and other blocks representing motors and soon. The below Fig.8 shows the visualization window for physical model in two different side views, top view and isometric view of self-balancing platform modeled using the aid of SimMechanics with actual dimensions.

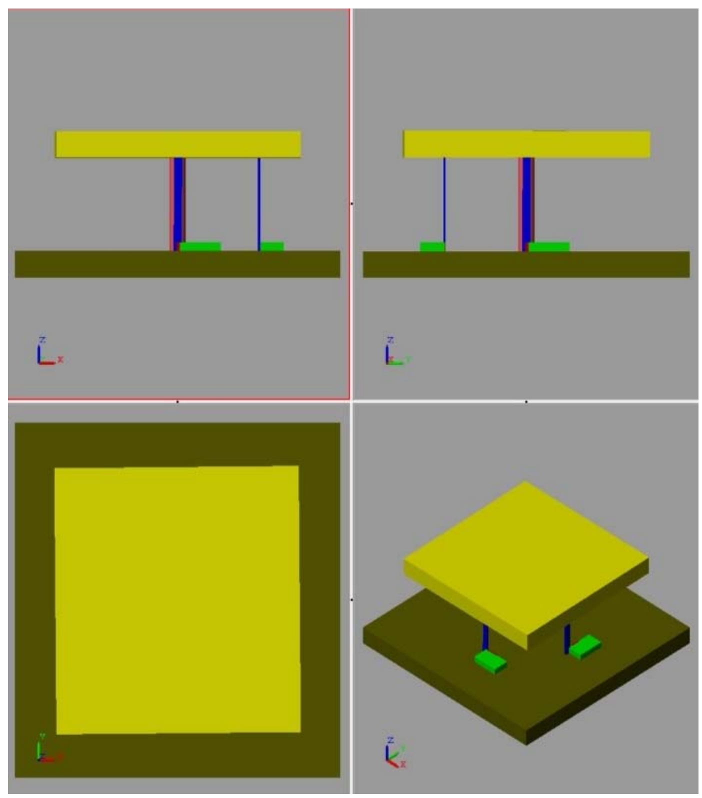

Fig.8. Visualization Window of Physical Model of Self-BalancingPlatform using SimMechanics.

From the spherical joint pitch and roll angle is measured using sensors from the top platform and is given to the error detector where it is compared with the desired values where the

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platform is in steady state and then the error value is given to PID controller as shown in figure. The PID controller output is given as a torque profile for revolute joints connected to the motor through the links are adjusted to make the platform in a steady state from the motor. In this model the disturbance of platform is given by supplying an impulse input to the platform then the respective components are estimated from calculation and the control of the platform is done by PID controller.

For proper realization of complete system, physical model with motor subsystem and transformations of tilt angle given by (12) are used. Angular position sensing is done at spherical joint kept at center of platform. The 2 axis (pitch, roll) sensor value is used as feedback for better control of the complete system. There are seven rigidly connected components, nine joints, seven tree joints, seven tree degree of freedom and minimum of one mechanism degrees of freedom.Fig.9 shows the Sim Mechanics model of platform control. After proper development of complete model, the system can be simulated and SimMechanics has feature of visualization window which animates the platform in motion for given simulation time.

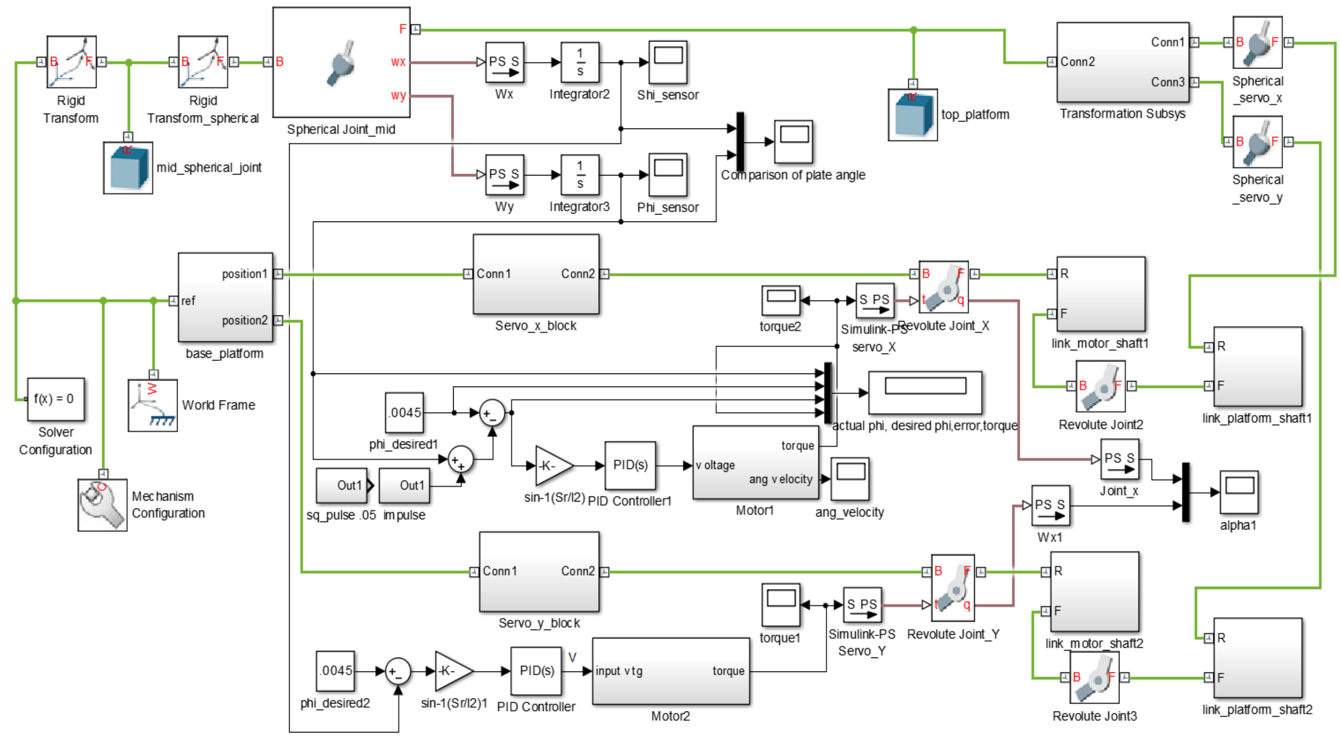

Fig.9. SimMechanics Model of Platform Control

VI. EXPERIMENT AND RESULT

The experiments are carried out in real system by giving the input and P-I-D values obtained by results from iterating physical model of the platform. The disturbance to platform is given impulse or square wave to simulate real condition encountered. PID values obtained after auto tune in SimMechanics are, K_p = 14.506, K_i = 163.151, K_d = 0.320 and N (filter coefficient) = 7366.88.

Fig.10 represents the roll and pitch angle, error and the motor torque. The observation made here is that the variation in roll or pitch angle are caused by external vibrations occurred in the robot, which makes the platform to tilt slightly and oscillate. To keep the platform at desired level, equal and opposite oscillations are produced by servo motors in pitch and roll axes of platform.

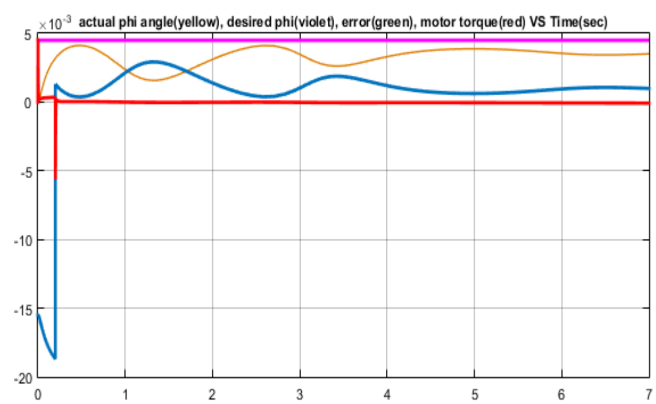

Fig.10.Effects of Roll (Yellow), Pitch (voilet), Error(Green), upon Motor Torque(red)Applied for a Certain Time.

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From previous figure it is clear that for disturbance of defined level can be damped with gradual and jerk free motion. The roll motion of platform behaves opposing the change from the disturbance and torque response is sharpe to correct the error created.When comparing the tilt of platform and actual joint angle (α) made by motor shaft for pitch and roll, and both servomotor respectively, it could be observed that both are in synch with each other to correct the change in gradual fashion.

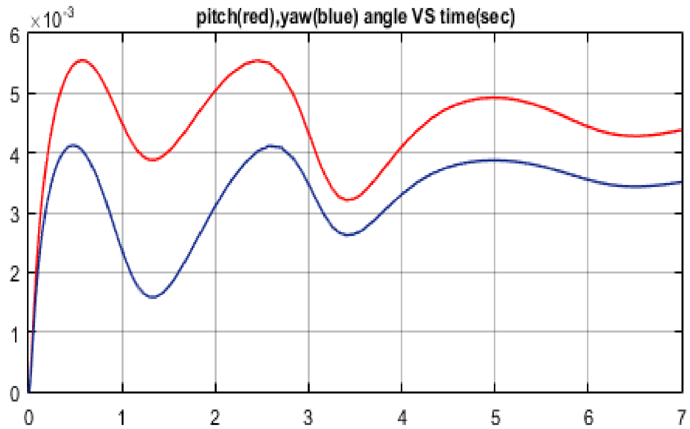

Fig.11.Real-time Responding Curves of Pitch(Red), Yaw(Green) vs Time.

The above Fig.11 shows the simultaneous action of both link mechanisms to keep the platform corrected for given disturbance without any abnormal spike during motion of robot. This is important for better handling of workpiece or medicine in real industrial or laboratory applications. It is found that the system can negotiate with maximum tilt disturbance of 0.385 radians with ease.

VII. CONCLUSION

In this paper, the complete design and implementation of a self-balancing platform on a moving cart was discussed to efficiently balance the object on top of it.It was observed that when moving across a small slope or a grassy terrain or a rough road, the platform could balance small objects on top of it with correction time of 0.7 second. From numerous iteration of experiment on platform, it is understood that the system can withstand disturbance of 0.385 radians at regular interval. The mechanical structure of the platform can further be stabilized by using three support links to the platform from the base. The method of modeling based on SimMechanics is more time efficient as it does not require to compute forward dynamics or differential equations. The physical modeling of platform is done using MATLABSimMechanics and modeling of DC motor is done using Simulink. The controller used here is PID controller and observedoptimum parameters discussed.

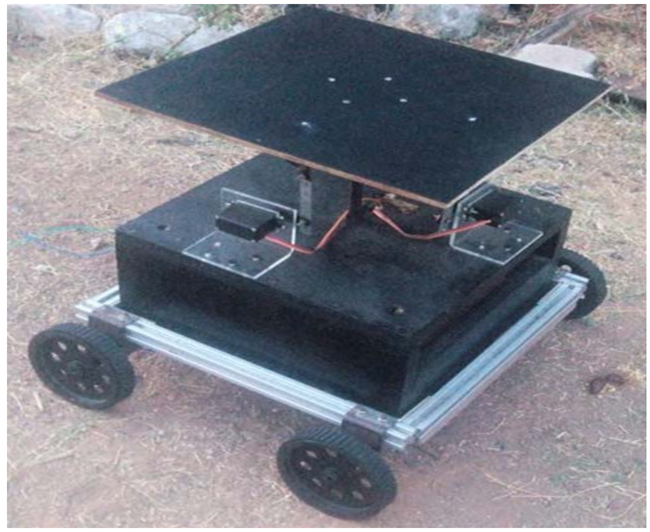
Fig.12. Actual Implementation of Cart and Platform

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