Visual Servoing based Model Reference Adaptive Control with Lyapunov Rule for a Ball on Plate Balancing System

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***Abstract* – The 2 Degree of Freedom (DoF) ball balancer system is a highly nonlinear system where the position of the ball is controlled by controlling the two servo motors simultaneously. In general, conventional Proportional Integral Derivative (PID) controller and Proportional Velocity (PV) is used to control the location of the ball on a plate in *X* and *Y* axes. But the major concerns are both classic PID and PV fails to track the ball position accurately due to non-linearity, process parameters variations and uncertainties. The above demerits are overcome by implementing Model Reference Adaptive Controller (MRAC) based PID controller. The closed loop reference model is chosen based on desired time domain specifications and by dominant pole placement technique. The gain values of the PID controller are tuned by MRAC to obtain optimal performance. MRAC based PID controller with MIT rule, modified MRAC with MIT approach, and modified MRAC with the Lyapunov rule are implemented in Simulink. The controller's performance on the benchmark Quanser 2 Dof ball balancer system is evaluated in real-time and comparative analysis is made. From the comparison, it is found modified MRAC based Lyapunov approach suppress the overshoot, provides better stability and tracks the system set-point accurately. On the other hand modified MRAC offers less Root Mean Square (RMS) value and faster tracking with less adaptation gain.**

Index Terms - Ball balancer system, MRAC, Lyapunov rule, Modified MRAC, MIT rule.

I. INTRODUCTION

A most commonly used benchmark system in control system is the ball and plate system. In the ball and plate system, the primary objective is to balance the movement of the ball on the plate within its boundary. The second objective is to identify the location of the ball. Using the camera based sensor and image processing algorithm the ball coordinates could be obtained. Based upon the ball's position the plate is adjusted by means of the controller. The ultimate aims are to track the ball trajectory and control the movement of the plate. The importance of ball and plate system is, using this system both classical and modern controller strategy could be evaluated. The challenge is to control the movement of the plate and tracking of the ball simultaneously are difficult. Further, the system stability should also be ensured. Some of the other difficulties are erroneous tracking of ball position by the image sensor and disturbances caused by the moving plate. Hence, there should be a mechanism to accommodate these disturbances or error in measurement and bring back the system to a stable state in a minimum time.

In [1] the location of the ball is identified using the visual servoing in the real-time. Machine vision technique is used to track the ball position. In order to identify the accurate location of the ball two sensors namely a touchscreen and a camera are used. The camera is placed at the top which senses the movement of the ball and its location [2]. There are some demerits in using these sensors. The foremost demerit is the ball may not have a continuous touching base with the plate when it is rolling. Hence, it causes an interruption in finding the location of the ball. The second demerit is the traditional cameras have low sampling rate i.e. 30Hz. By, using these cameras it would be difficult to identify the location of the ball in a real-time scenario.

An augmented image based visual servoing technique is implemented in [3] for controlling the 6 DoF system. The movement of the robot is controlled by Proportional-Derivative (PD) controller. The PD controller tracks the movement of the robot by means of receiving the feedback by visual servoing technique. Depending on the scenario, it controls the acceleration of the robot. Some of the shortcomings of this methods are the camera requires continuous calibration and computation time is high. Further, the visual servoing system suffers from local minima and singularities of the Jacobian matrix.

An MRAC along with MIT rule is implemented to control a chemical plant in [4]. The MRAC consists of an adjustment mechanism which tunes the controller parameters in the presence of disturbances and uncertainties. The disadvantages of this method are stability of the controller is not guaranteed and also controller performance is not evaluated in real-time. Moreover, for larger gain values the adaption mechanism produces more overshoot. To overcome the above short-comings an implicit MRAC is proposed in [5] to auto-tune the dominant-pole placement. Thus, instead of using MRAC controller along with the MIT rule, an IMRAC along with auto-tuning of dominant-pole placement may be adopted.

The modified MRAC method is implemented for controlling the inverted pendulum in [6]. In this paper, the MRAC is combined with PID controller which helps to control the oscillation of the pendulum. In [7] switching-driving Lyapunov method is used for balancing the ball and plate system. This method provides necessary stability condition for a non-linear system. A locally asymptotically stable control scheme is proposed for the system. The controller stability is guaranteed by Lyapunov function. A

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vision feedback along with xPC target together [8] controls and tracks the trajectory of the system. For the uneven dynamic systems, a modified Lyapunov function is proposed in [9] which helps to evaluate the stability of the system. Since the differential equations of the system are discontinuities. By, employing this function ease the process of computation.

In [10] two schemes are proposed to control the motion of the robot. The first scheme utilizes the scenes which is captured by the camera. Then by using the image processing algorithm, the trajectory for the robot movement is found. This command is given to the secondary loop which helps in controlling the position and movement of the robot. Even though by employing the above two techniques the accuracy of the movement of the robot is not satisfactory. Feature tracking is some of the other challenges which are not addressed in this paper.

A Microsoft Kinect camera [11] overcomes the demerits of the stereo camera. The Kinect camera gives depth information. Here, the first phase controller provides the information about the desired image. The second phase controller then correct the error between the desired image and current position. The drawbacks of two-phase controller is rotational alignment. Hence, the image which is captured by the cameras suffers from alignment problem. Therefore, some kind of image processing technique like affine transform along with Scale Invariant Fourier Transform (SIFT) has to be designed to get better results. Again the computation time is very high when SIFT is realized in hardware.

A webcam based tracking of ball's location is proposed in [12]. Fuzzy and sliding mode controller controls the location of the ball. Initially, a PID with a fuzzy controller tracks the ball location. But it is well-known, PID controller doesn’t perform for MIMO system. So, an intelligent sliding mode controller is developed. But the experimental result shows there is large steady state error and the stability of the system is not guaranteed. Hence, it is suggested to use more complex controllers like neural and an adaptive controller. Moreover, in a non-linear system there prevail some difficulties to design a controller which could accommodate all dynamics of a system.

In this paper, classical and adaptive controller's performances are evaluated in a real-time on the benchmark Quanser 2 Dof ball balancer system. Primarily, a mathematical model for the ball and plate system is formulated. Then the controllers are designed to control the movement of the plate and to track the ball position. PV and PID controllers are designed by means of pole placement technique based on the desired closed loop specifications. Using the Quanser real-time control software (QUARC), the controller interacts with the system and its performances are evaluated in real-time. The controllers are designed and implemented in Simulink. QUARC facilitate the Hardware-in-Loop (HiL) execution. Secondly, adaptive controllers are designed. MRAC controller on the basis of adaptation laws are designed. MRAC controller with three different approaches like MRAC with MIT rule, modified MRAC with MIT approach and modified MRAC with Lyapunov rule are designed and applied to the system. The performance of the controllers on the basis of RMS value is evaluated. Further, the system stability is verified by means of Lyapunov stability criterion. Finally, the conclusion is made by comparing the RMS values of each controllers. The paper is organized as follows. In section II, modeling technique for a ball balancer system is presented. MRAC design is delineated in section III, and the Lyapunov rule is also discussed in section IV. Experimental results and discussion are shown in sections V, and VI stated the conclusion.

II. SYSTEM MODEL

The classical electromechanical i.e. ball and plate system is carefully chosen for research to analyze its dynamic behavior. Various controller strategies are implemented to it. The significance of the system is it helps to realize the controller based on both classical and modern theory. The 2 DoF ball balancer uses two rotary servo base unit devices, a visual sensor, and a symmetrical plate.

Initially, the ball is positioned on the balanced plate and its coordinates are obtained using image processing technique. An assumption is made such that all axes have similar dynamics. It is therefore modeled as two de-coupled "ball and beam" systems where it is assumed an angle of the *X*-axis servo affects only the ball movement in the *X*-direction. The 2 DoF ball and plate system is illustrated in Fig. 1. The different parameters of the system are tabulated in Table I. The total forces acting on the ball along the beam are equals according to Newton's first law of motion. The movement of the ball along the *X*-axis relative to the plate angle is shown in (1). Using the equation (2), the movement of ball *X* to the angle of beam α could be found.

$$m_b\ddot{x}(t)=\sum F = F_{x,t} - F_{x,r} \qquad (1)$$

Where $F_{x,r}$ is the inertial force which is exhibited by the moving ball and $F_{x,t}$ is the rendering caused because of the gravitational force. Friction and viscous damping are neglected. The conventions of modeling are as follows:

- Applying a positive voltage causes the servo load gear to move in positive, counterclockwise direction. This moves beam upwards and causes the ball to roll in positive direction i.e., away from the servo towards the left. Thus $V_m > 0 \rightarrow \dot{\theta}_l > 0 \rightarrow \dot{x} > 0$
- Ball position is zero, x = 0, when located in the center of the beam
- Servo angle is zero, $\theta_l = 0$ when the beam is parallel to the ground, $\propto = 0$

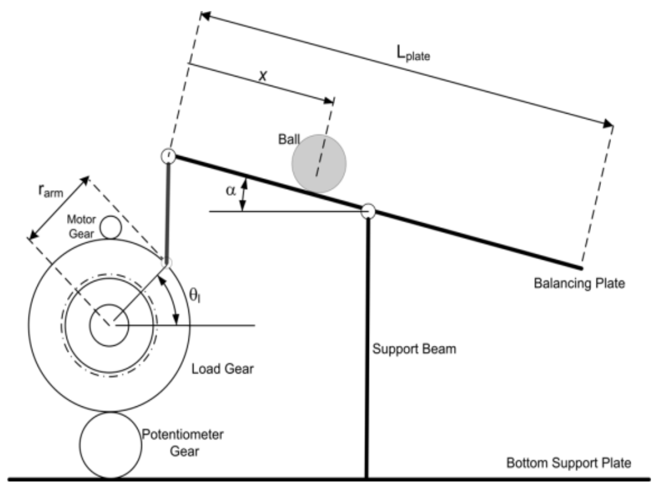

Fig. 1. Representation of 2 DoF ball on plate system

A general rule for a ball to be stationary at a particular location, two forces acting on an object must be same. The forces are a gravitational force and the force exhibited by the ball's momentum. Due to the influence of the gravity on the beam it produces a force $F_{x,t}$ in the *x*-direction and it is expressed in (2)

$$F_{x,t} = m_b g \sin \propto (t) \tag{2}$$

$$F_{x,r} = \frac{\tau_b}{r_b} \tag{3}$$

The turning of the ball creates a force in (3). The generated forces depend on two important factors they are ball's radius r_b and the torque τ_b. The relationship between r_b and τ_b is given in (4).

$$\tau_b = J_b \ddot{\gamma}_b(t) \tag{4}$$

Linear to angular displacement is obtained using sector formula $x(t) = \gamma_b(t) r_b$. Thus, by using this formula the force acting on the ball in the direction of *X*-axis is found in (5).

$$F_{x,r} = \frac{J_b \ddot{x}(t)}{r_b} \tag{5}$$

By substituting the rotational and translation forces in (1), a non-linear equation is obtained (6). The equation (6) is linearized and an acceleration is obtained in (7).

$$m_b \ddot{x}(t) = m_b g \sin \propto (t) - \frac{J_b \ddot{x}(t)}{r_b} \tag{6}$$

$$\ddot{x}(t) = \frac{m_b g \sin\propto(t) r_b^2}{m_b r_b^2 + J_b} \tag{7}$$

The equation of motion is nonlinear and therefore it is liberalized for the control design. To change the beam's height *h*, beam and servo angles are used (8).

$$\sin \propto (t) = \frac{2h}{L_{plate}} \tag{8}$$

By taking the sine of servo load shaft angle results in a

$$\sin\theta_l(t) = \frac{h}{r_{arm}} \tag{9}$$

Combining equations (8) and (9)

$$\sin \propto (t) = \frac{2 r_{arm} \sin\theta_l(t)}{L_{plate}} \tag{10}$$

TABLE I. PARAMETERS OF 2 DoF BALL ON PLATE SYSTEM

Symbol	Description	Value	Units
wXd	Plate dimensions	41.75 x 41.75	cm^2
L_t	Table length	27.5	*cm*
r_b	Radius of the ball	1.46	*cm*
m_b	Mass of the ball	0.003	*kg*
V_m	Motor nominal voltage	6	*V*
R_m	Motor armature resistance	2.6	Ω
L_m	Motor armature inductance	0.18	*mH*
K_t	Motor torque current constant	7.68 x 10^{-3}	*Nm/A*
K_{gi}	Internal gear box ratio	14	-
w_g	Maximum motor speed	628.3	*rad/S*

By substituting (10) in (7), the resulting acceleration $\ddot{x}(t)$ is given as

$$\ddot{x}(t) = \frac{2 r_{arm} \sin\theta_l(t) m_b g\, r_b^2}{L_{plate}(m_b r_b^2 + J_b)} \tag{11}$$

The sine function is approximated as $\sin\theta_l \approx \theta_l(t)$ and by applying this to the nonlinear equation of motion of the ball, the approximated version of (11) is

$$\ddot{x}(t) = \frac{2 r_{arm} m_b g\, r_b^2}{L_{plate}(m_b r_b^2 + J_b)} \theta_l(t) \tag{12}$$

In the equation (13), there are two important terms $P_s(s)$ and $P_{bb}(s)$. The term $P_s(s)$ which is the rotary servo unit transfer function. It is the output of load angle to the input voltage given to drive the servo motor. The term $P_{bb}(s)$ denotes the transfer function between the servo land gear angle and the ball's location. The model is derived and it is decoupled. This makes the *X*-axis independent from *Y*-axis i.e. the effect produced in one axis will not have any influence on another axis. So, the response is not affected.

In the Fig. 2, *X* (*s*) and *Y* (*s*) represents the measured ball position in the x and y-direction i.e. in the Cartesian coordinate system. $V_{m,x}(s)$ and $V_{m,y}(s)$ denotes voltages that are generated. Load angles are identified using these generated voltages. Similarly, the load angle for the shaft to rotate in the motor for the direction of x and y is denoted by $\theta_{l,x}(s)$ and $\theta_{l,y}(s)$ respectively. The equation (13) represents the 1DoF transfer function for a ball balancer system.

$$P(s) = P_s(s) P_{bb}(s) \tag{13}$$

Where the servo angle to ball position transfer function and voltage to servo angle transfer function is given in (14) & (15)

$$P_s(s) = \frac{X(s)}{\theta_l(s)} \tag{14}$$

$$P_s(s) = \frac{\theta_l(s)}{V_m(s)} \tag{15}$$

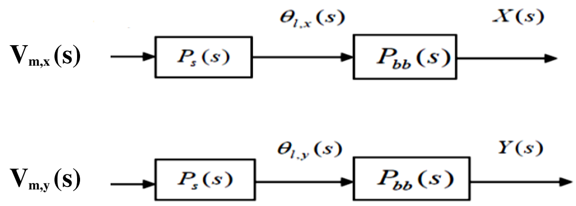

Fig. 2. Structure of a 2 DoF ball balancer system

The transfer function is derived in (16) based on the position of the gear in the servo load $\theta_l(t)$ to the amount of input voltage $V_m(t)$ applied to the servo motor.

$$P_s(s) = \frac{K}{s(s\tau + 1)} \tag{16}$$

Here, in the equation (16) steady-state gain *K* and time constant τ are calculated using the device parameters. The nominal model parameters are *K*=1.76*rad V/s*, τ =0.0248sec.

By taking the Laplace transform for (12) the transfer function of $P_{bb}(s)$ is found (17).

$$P_s(s) = \frac{X(s)}{\theta_l(s)} = \frac{K_{bb}}{s^2} \tag{17}$$

where $K_{bb} = \frac{2 r_{arm} m_b g\, r_b^2}{L_{plate}(m_b r_b^2 + J_b)}$

Since the systems are connected in a back to back form as shown in Fig. 2. The final transfer function for an entire system could be obtained, which is a servo voltage to ball displacement as expressed in (18).

$$P_s(s) = \frac{X(s)}{V_m(s)} = \frac{K_{bb} K}{s^3(s\tau + 1)} \tag{18}$$

III. MRAC DESIGN

In an MRAC, the parameters of the controller are tuned in such a way it tracks the reference of an input. The controller objective is to track the output of the reference model for the given reference input. It is achieved by tuning the controller parameters. Thus, the output of the ball balancer system should follow the output of the reference model. There is a mechanism which is provided to change the controller parameters. These help to track the reference model for the ball balancer system.

Many mathematical methods are framed to change the controller parameters in an efficient manner some of them are MIT rule, Lyapunov theory and theory of augmented error. In the Fig. 3 MRAC structure is shown, here $y_m(t)$ denotes the reference model output and $y(t)$ is actual plant output. The error is denoted by $e(t)$. PID controller is tuned based on the MRAC strategy. The gain of the controller is altered by means of proposing a novel technique called direct MRAIM (Model Reference Adaptive Internal Model) controller. The error is calculated using the (19)

$$e(t) = y(t) - y_m(t) \tag{19}$$

A. Design of modified MRAC based on MIT rule

In [13] the MIT rule was applied in designing an autopilot feature for the aircraft. In this research article, the same technique is applied to an MRAC controller, to develop a closed-loop system for 2DoF ball balancer system. $X_m(s)$ denotes the desired closed loop response. Then the error e is calculated using the difference between $X(s)$ and $X_m(s)$. In this rule, a cost function is defined as,

$$J(\theta) = {e^2}/{2} \tag{20}$$

Where θ is used to tune the controller gain values. By varying the θ the cost function J, will be decreased. The negative sign in the (21) indicates the value of θ trends to make the cost function J in the negative gradient area.

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} \tag{21}$$

From equation (20),

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta} \tag{22}$$

In the equation (22), the term $\frac{\partial e}{\partial \theta}$ denotes the derivative sensitivity of the ball balancer system. The term γ indicates the gain of the controller. The adaptive mechanism for the modified rule is given in (23).

$$\frac{d\theta}{dt} = -\frac{\gamma e \varphi}{\alpha + \varphi' \varphi} \tag{23}$$

Where $\varphi = \frac{\partial e}{\partial \theta}$ is a sensitivity derivative term. To calculate the φ (24) is used.

$$\varphi = \frac{\partial e}{\partial \theta} = \frac{K}{K_0} y_m \tag{24}$$

Whenever φ in equation (23) tends to zero. A zero division error occurs. Hence, the term ∝ introduced to avoid it. It is also to be noted the adaptive gain of MIT and modified MIT rule are different. The modified MIT rule provides more stable response and reference tracking.

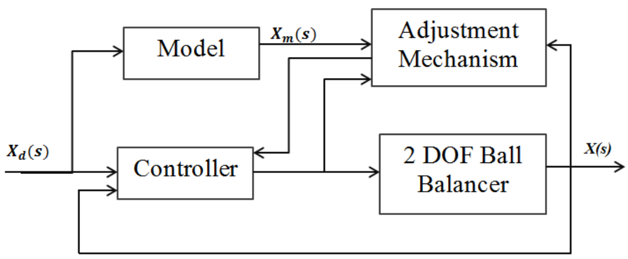

Fig. 3. Model Reference Adaptive Control

B. Condition for closed loop response

The time domain criteria are framed to balance the location of the servo gear load shaft are $e_{ss} = 0$, $t_p \leq$ 0.15 seconds and Po ≤ 5%. For tracking the ball position, the percentage overshoot must be ≤ 7.5% and the settling time must be 2.5 seconds which is 4% of its SS (Steady State).

IV. LYAPUNOV RULE

The Lyapunov theory is proposed to guarantee the stability of the system. Lyapunov investigated the nonlinear differential equation.

$$\frac{dx}{dt} = f(x) \qquad f(0) = 0 \tag{25}$$

Since $f(0) = 0$ and x(t)=0 which means there is a solution for it. Some assumptions are made to ensure the solution is unique for f(x). By Lipschitz the assumption for f(x) is made and it is given in (26).

$$\| f(x) - f(y) \| \leq L||x - y|| \qquad L > 0 \tag{26}$$

The equation in (25) is stable since there is a solution for x(t) = 0. The stability for x(t) = 0 is assured only if ϵ>0 and also δ(ε)>0 must exist.

$$||x(0)|| < \delta \tag{27}$$

$$||x(t)|| < \varepsilon \quad for\ 0 \leq t < \infty \tag{28}$$

The equation (27) is used whenever the solution is asymptotically stable. If x(t) ≠ 0 then it is unstable. The value of δ could be identified when all solutions in equation (27) obey a property $||x(t)|| < 0$ where t→∞ which is given in equation (28). A solution is globally asymptotically stable provided it is asymptotically stable for any of the initial values.

A. Lyapunov based modified MRAC design

For adjusting the parameters in adaptive systems Lyapunov's stability theory is used. The differential equation for error $e = y - y_m$ is derived subsequently for adjusting the parameters. Using this equation, the error could be brought down to zero by finding the appropriate value for adaptation technique and Lyapunov function. .

The adaption gains dv/dt is negative semi definite. This is founded by applying the Lyapunov theory. Thus to achieve the convergence, there should be uniform observability for the reference signal and system. Further, it is really essential to arrive at a determined excitation. First order MRAC based on stability theory. The desired response is given in (29)

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c \tag{29}$$

Where $a_m > 0$ and here the reference signal is bounded

$$\frac{dy}{dt} = -ay + bu \tag{30}$$

The controller is

$$u = \theta_1 u_c - \theta_2 y \tag{31}$$

Introducing the error

$$e = y - y_m \tag{32}$$

In order to minimize the value of the error, a differential term is introduced. This will ultimately reduce the value of the error. This is given in (33)

$$\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)u_c \tag{33}$$

$$\theta_1 = \frac{b_m}{b} \text{ and } \theta_2 = \frac{a_m - a}{b} \tag{34}$$

If the values of θ_1 and θ_2 are same as shown in (34) then the value of error will be zero. Thus, the value of θ_1 *and* θ_2 will arrive at the desired set-point. In the equation

(35), it is assumed $b\gamma > 0$ and a quadratic function is developed.

$$V(e,\theta_1,\theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2\right) \quad (35)$$

The quadratic function in (35) will tends to zero if the value of e = 0. The derivative dv/dt should be negative to ensure it satisfies the Lyapunov theory. The derivative equation is given in (36)

$$\frac{dV}{dt} = e\frac{de}{dt} + \frac{1}{\gamma}(b\theta_2 + a - a_m)\frac{d\theta_2}{dt} + \frac{1}{\gamma}(b\theta_1 - b_m)\frac{d\theta_1}{dt} \quad (36)$$

$$\frac{dV}{dt} = -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\left(\frac{d\theta_2}{dt} - \gamma y e\right) + \frac{1}{\gamma}(b\theta_1 - b_m)\left(\frac{d\theta_1}{dt} + \gamma u_c e\right) \quad (37)$$

If the parameter is updated as

$$\frac{d\theta_1}{dt} = -\gamma u_c e \quad (38)$$

$$\frac{d\theta_2}{dt} = \gamma y e \quad (39)$$

Therefore, $\frac{dV}{dt} = -a_m e^2$. By analyzing the derivative term *V* with time, it is found to be negative semi definite. Hence, V(t)≤V(0) and error, the parameters namely $\theta_1 and\ \theta_2$ are also bounded. Therefore, it could be concluded that $y=e+y_m$ should also be bounded.

B. PID controller design with Lyapunov rule for ball balance system

A stable closed-loop system cannot be guaranteed by MIT rule so the Lyapunov rule is used which is a more powerful theorem to design adaptive controllers. A function V which is either positive or negative. In case, if the V is positive then the system is stable. Because, the solution of the derivative term produces negative semi definite value. In case, if the V is negative and definite then the system is asymptotically stable. The function V is said to be a Lyapunov function. A quadratic Lyapunov function is developed for a linearized system. The parameter of the PI controller is adapted using Lyapunov rule. Controller output,

$$U = \theta_1 U_c(s) - \theta_2 Y(s) - \theta_2 sY(s) \quad (40)$$

The plant transfer function is

$$G(s) = \frac{a_1 s + a_2}{a_3 s^3 + a_4 s^2 + a_5 s + a_6} = \frac{Y(s)}{U(s)} \quad (41)$$

$$\dddot{y} = 1/a_3\,(a_1 \dot{u} + a_2\, u - (a_4\ddot{y} + a_5\dot{y} + a_6) \quad (42)$$

The model transfer function is

$$\frac{X_m(s)}{X_d(s)} = \frac{s+5}{s^3 + 3.78s^2 + 7.57s + 4.79} \quad (43)$$

$$\dddot{y} = \dot{u} + 5u - 3.78\ddot{y} - 7.57\dot{y} - 4.79y \quad (44)$$

Error $=Y(s) - Y_m(s)$. The Lyapunov function is defined as,

$$\frac{d\theta_1}{dt} = -\gamma u_c e \quad (45)$$

$$\frac{d\theta_2}{dt} = \gamma y e \quad (46)$$

$$\frac{d\theta_3}{dt} = \gamma \dot{y} e \quad (47)$$

V. RESULTS AND DISCUSSION

Real-time performance of designed classical and adaptive controllers for 2 Dof ball balancer system is obtained. The controllers are designed in Matlab with Simulink. The communication between the system and designed controllers are established by means of Hardware-in-Loop (HiL) technique. A real-time controller implementation and its behaviour analysis are made. The comparative analysis and stability of the system for various controllers like PV (Proportional Derivative Controller), PID, MRAC with MIT rule, Modified MRAC with MIT rule and Modified MRAC with the Lyapunov rule was analyzed and tabulated in Table II.

TABLE II. CONTROLLERS PERFORMANCE ANALYSIS

Performance Index	PV controller	2 DoF PID controller	MRAC based on MIT rule	Modified MRAC based on MIT rule	Modified MRAC based on Lyapunov rule
Controller & adaptation gains	Kp=3.45 Kd=2.11	Kp=5.385 Ki=3.692 Kd=2.971 α=0.35 β= 0.5	γ: -0.75	γ: -0.2 α: 2.24	θ_1: 0.75 θ_2: -0.75
RMS - *X* – axis position	0.0287	0.01453	0.0052	**0.000283**	0.0038
RMS - *Y* – axis position	0.0089	0.00243	0.01	**0.0028**	0.0061

PV controller gains are tuned to k_p=3.45, k_d=2.11 and its response is shown in Fig. 4. The set-point for real-time implementation is 2cm square wave. The significance of it is the ball motion will be clearly visible along the *X*-axis.

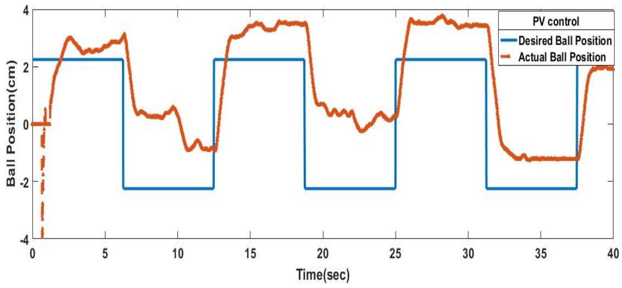

Fig. 4. Real-time response of PV controller for ball balancer system

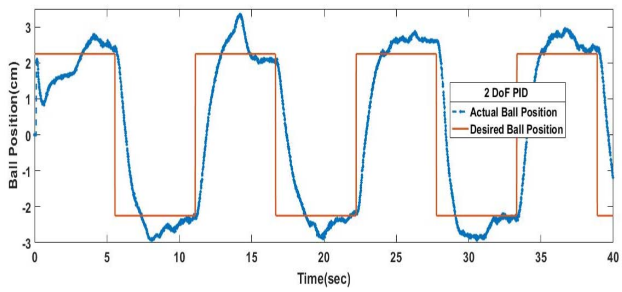

Fig. 5. Real-time response of PID controller for ball balancer system

From Fig. 4 it is observed controller couldn't track the ball position. The settling time and overshoots are higher. 2 DoF PID controller and its response is shown in Fig. 5. The controller able to improve the tracking of ball's position when compared to PV controller. RMS value of PID is less than PV which is an improved. But there exists some overshoots and ball's position is not precisely tracked.

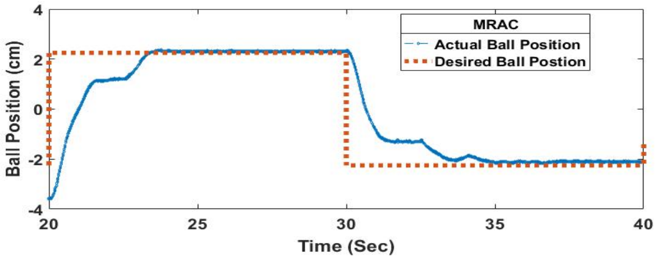

Fig. 6. Real-time response of MRAC controller based on MIT rule

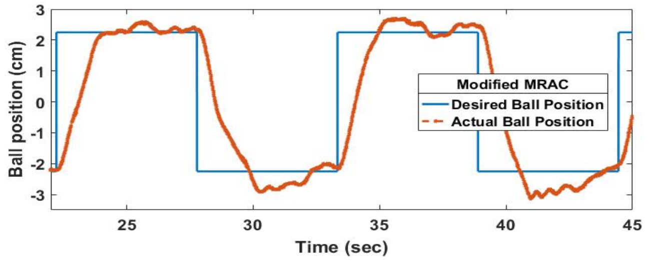

Fig. 7. Real-time response of modified MRAC controller based on MIT rule

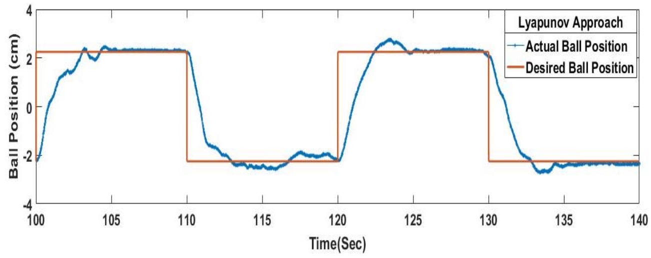

Fig. 8. Real-time response of modified MRAC controller based on Lyapunov

The response of MRAC using MIT rule having adaptation gain of value γ=0.75 is shown in Fig. 6. MRAC provides good tracking compared to classical controller and also overshoots are suppressed. But still exact tracking is not achieved. In the Fig. 7 modified MRAC with MIT rule response is shown. It is found the controller tracking is improved and also it provides minimum offset. But the stability of the system could be improved. Modified MRAC with the Lyapunov rule is implemented and its behaviour is shown in Fig. 8. Among all the controllers, Lyapunov provides best tracking and stable response. But on the basis of RMS value modified MRAC outperforms the Lyapunov.

VI. CONCLUSION

Classical and adaptive controller's performance on 2 Dof ball balancer system is evaluated in real-time. Initially, PV and PID controllers are designed based on desired closed loop specifications and evaluated in real-time to track the ball's position. PV controller lags in tracking and contains large overshoots. On the other hand PID controller provides good tracking response but the ball settling time is not observed at the given set-point. It indicates there exists more steady state error. Thus, it is clearly observed classical controller fails to track the ball's position in real-time and also RMS value is higher. Hence, MRAC controllers on the basis of adaptation laws are designed to improve the tracking performance. MRAC with MIT approach provides better tracking response compared to classical controllers. Further, the steady state error is minimized and there is no overshoot. But it is observed for higher adaptation gain the system becomes unstable and for lower adaptation gain the controller does not meet the desired specifications. Hence, reference input should be given in a very limited range. Modified MRAC with MIT approach provides minimum offset, low RMS value and less steady state error. But the stability of the system could be improved. Modified MRAC with Lyapunov approach gives stable response and control input is optimum for the same set-point tracking. But the adaptation gains are higher. The performance of all controllers on basis of RMS value is carried out. From the analysis, it is concluded modified MRAC has less RMS value, low adaptation gain and fast tracking in real-time among all designed controllers. But from the stability point of view Lyapunov approach is better and tracks the ball's position accurately.

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