Comparative Performance Analysis of Mamdani and Sugeno Fuzzy Controller using Application on 2DoF Ball Balancer System

2023 International Conference on Power, Instrumentation, Energy and Control (PIECON) | 979-8-3503-9976-9/23/$31.00 ©2023 IEEE | DOI: 10.1109/PIECON56912.2023.10085809

Nikita Nikita
Department of Electrical Engineering
Delhi Technological University
Delhi, India
nikita.dahiya2017@gmail.com

Bharat Bhushan
Department of Electrical Engineering
Delhi Technological University
Delhi, India
bharat@dce.ac.in

Abhishek Chaudhary
Department of Electrical Engineering
Delhi Technological University
Delhi, India
abhishek@dtu.ac.in

***Abstract*— This paper presents the design of Mamdani Fuzzy Inference System and Sugeno Fuzzy Inference System for controlling the ball position of two degree of freedom ball balancer system which is a nonlinear and unstable system with a complex transfer function. Intelligent control technique such as fuzzy helps in efficient control and position tracking of complex nonlinear systems. A comparative analysis has been performed to test the performance of the Mamdani FIS and Sugeno FIS on the ball balancer system for various characteristics such as ball position, ball plate angle and actuation voltage. The time domain specifications such as rise time, peak time, settling time, delay time and peak overshoot have been used as parameters to compare the two fuzzy systems. The simulations are performed using MATLAB/ Simulink environment.**

Keywords—ball balancer system, PID Controller, Fuzzy Systems, Sugeno Fuzzy, Mamdani Fuzzy

I. INTRODUCTION

A nonlinear system may be defined as a system on which the principle of superposition and homogeneity are not applicable. Almost all the systems found in real life are nonlinear system and the concept of linear system is only applied theoretically under certain assumptions. A few examples of nonlinear systems are inverted pendulum, robot arm manipulator, negative resistance oscillator, helicopter levitation system, etc. Nonlinear systems gain importance in the field of research due to their complexity and wide range of applications in real life. One such nonlinear system is a ball balancer system.

A ball balancer system consists of a ball which is kept on a 2-dimensional plate which can freely move about a center point. The ball is then needed to be stabilized with the help of various controllers. The 2 DoF Ball Balancer system gains importance in research due to its inherently unstable and non-linear behavior. This nature of a ball balancer system helps in designing of robots which can be used for various purposes such as automatic controlling and position tracking on rough terrains, unmanned aerial and land vehicles, satellite position correction, etc. Automatic ball balancers and magnetorheological dampers are used for designing of horizontal axis washing machine [1]. Ball balancer system has proved very useful application in the designing of humanoid robots wherein the object needs to be transported without grasping or letting it fall [2].

A. Literature Survey

In a ball balancer system, a camera is suspended vertically opposite to the plate and is utilized for tracking the ball position so that appropriate actions can be taken by the controller. For detecting the ball even when the light fluctuates frequently, blob analysis algorithm is used which helps in differentiating between the background and the foreground [3]. The main aim behind developing a control mechanism for ball balancer system is to train the system to keep an object in a balanced position on the unstable surface by adjusting the angle of inclination of the surface [4]. The objective here is to keep the ball near the set point and reduce the error between the set point and the current position [5]. The overhead camera provides feedback to the control system, and then accordingly control action is taken by the applied control strategy. The angle encoder collects the position of the motor as feedback [6]. Initial research on stabilizing the ball balancer includes using of a feedback compensation to stabilize the system [7]. With the help of mechatronic design principles, the system is realized using constraints such as performance, cost, functionality, extendibility, etc. With the help of the control scheme, the ball may be directed to follow any desired trajectory and not just be balanced at a single set point [8].

The modelling of a ball and plate system is done using the basis of First Principle's Method which is obtained from Newton-Euler method by balancing various forces along with the torques on motors, ball, plate as well as gears. Linearization is done for obtaining the state space model near the operating region, which is then utilized for discrete optimal control for trajectory tracking [9]. PID controllers based on Arduino microcontroller that use feedback signal for controlling the movement of the ball using linear potentiometer position sensor have been used [10]. In order to achieve point-to-point control with circular trajectory control, a nonlinear PID is used that also helps in solving the bigger programming problems that consume larger computation time [11].

Classical methods like the PID control were then used for position tracking and controlling in the 2DoF Ball Balancer systems. But soon intelligent control started to come in picture.

II. BALL BALANCER SYSTEM

The 2DBB system comprises of a plate and a ball, which are used for balancing purpose. The ball is kept on the surface of the plate where it is free to move in all the directions. The plate is made to move about a mid-point which makes to ball to move. A USB camera is placed overhead which acts as a vision system for measuring the position of the ball. Two servos, Rotary Servo Base Unit devices are placed just below the plate and joined to the sides of the plate using the 2DoF gimbals. The ball is balanced on the plate by adjusting its tilt angle. The tilt angle is adjusted by manipulating the servo load gears position.

The laboratory setup of 2 DoF ball balancer system comprises of a camera, data acquisition device, ball and plate, VOLTPAQ-X2 power amplifier, SRV02-ET unit and a result analysis system. The plate is attached to the gimble from its center and has 2 DoF for rotation while also being free to tilt either in x direction or y direction. The aim is to stabilize the ball on the surface of the 2D plate so that it does not fall down when the plate tilts. The purpose behind using a 2DBB system is balancing the ball at a point which set by user on surface of the 2D plate.

A. *Mathematical Modelling*

The modelling of the 2DBB is done with the use of first principles method [3]. Fig. 1 represents the block diagram of open loop structure of 2DBB system. The dynamics of input servo motor voltage along with the resultant load angle is found using transfer function of SRV02. $T_{bb}(s)$ represents the dynamics related to the servo load gear angle with respect to the ball position.

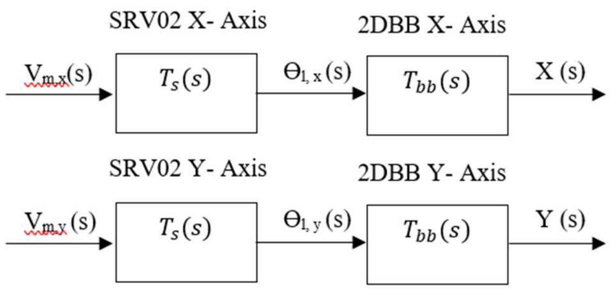

Fig. 1. 2DBB open loop block diagram

The 2DBB system is a decoupled system, which means that actuator in x-axis does not affect the response obtained through y-axis. Also, as the plate is symmetric and the SRV02 device has same hardware, the dynamics of both the axis are similar in nature, and consequently modelling of only 1 axis is done. Fig. 2 represents the open loop block diagram for 1DBB.

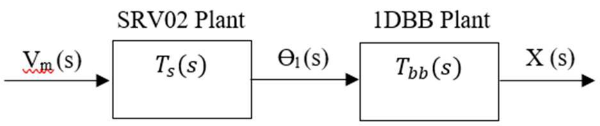

Fig. 2. 1DBB open loop block diagram

The diagram depicting all the forces on the ball balancer system is as in Fig. 3. The transfer function relating SRV02 and 1DBB is written as

$$T(s) = T_{bb}(s)T_s(s) \tag{1}$$

Here, $T_{bb}(s)$ represents 1DBB transfer function, which may further be written as

$$T_{bb}(s) = \frac{X(s)}{\theta_l(s)} \tag{2}$$

Eq (2) shows the relationship between displacement of ball w.r.t servo load angle.

While the transfer function of SRV02 is written as

$$T_s(s) = \frac{\theta_l(s)}{V_m(s)} \tag{3}$$

Let,

$$T_s(s) = \frac{k}{(\tau s+1)s} \tag{4}$$

The nonlinear time domain equation that describes the position of ball w.r.t the plate angle is

$$\frac{d^2}{dt^2}x(t) = f\big(\beta(t)\big) \tag{5}$$

Using Newton's second law on the ball, we get-

$$m_{ball}\left(\frac{d^2}{dt^2}x(t)\right) = \sum F \tag{6}$$

Here, m_{ball} represents the ball's mass.

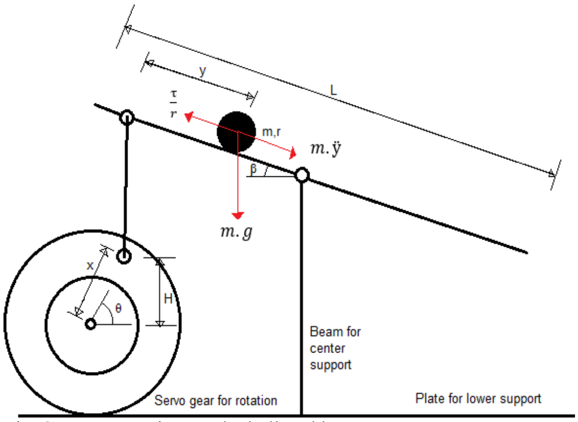

Fig. 3. Forces acting on the ball and beam system

The forces on the ball can be balanced as

$$m_{ball}\left(\frac{d^2}{dt^2}x(t)\right) = F_{x,t} - F_{x,r} \tag{7}$$

Here, $F_{x,t}$ represents the inertial force of ball while $F_{x,r}$ represents the force due to translation under the effect of gravity. The frictional force and viscous damping are not taken into consideration. In order to reach the state of equilibrium, equate the momentum of the ball with force acting as a result of gravity.

$$F_{x,t} = m_{ball}g\sin\big(\beta(t)\big) \tag{8}$$

Let, $\gamma_b(t)$ represents the ball's angle while r_b stands for the ball's radius. So, the angular displacement can be converted into linear displacement using the sector formula.

$$x(t) = \gamma_b(t)r_b \tag{9}$$

The rotational forces due to the ball's inertia can be written as

$$F_{x,r} = \frac{J_b\left(\frac{d^2}{dt^2}x(t)\right)}{r_b^2} \tag{10}$$

The acceleration of the ball in linear form is given as

$$\frac{d^2}{dt^2}x(t) = \frac{m_{ball}g\sin(\beta(t))r_b^2}{m_{ball}r_b^2+J_b} \tag{11}$$

The relation of the angle of the table with the servo angle can be given as

$$sin(\beta(t)) = \frac{2\,sin(\theta_l(t))r_{arm}}{L_{tbl}} \quad (12)$$

The linearized equation of motion for 1 DoF ball balancer system is

$$\frac{d^2}{dt^2}x(t) = \frac{2m_b g\theta_l(t)r_{arm}r_b^2}{L_{tbl}(m_{ball}r_b^2+J_b)} \quad (13)$$

Assuming zero initial conditions, the transfer function of one degree of freedom system is given by

$$T_{bb}(s) = \frac{K_{bb}}{s^2} \quad (14)$$

Where $k_{bb} = \frac{2m_{ball}g r_{arm}r_b^2}{L_{tbl}(m_{ball}r_b^2+J_b)}$

And combined 1DBB and SRV02 transfer function is obtained as

$$T(s) = \frac{K_{bb}K}{(\tau s+1)s^3} \quad (15)$$

While the relation of the displacement of ball with the servo voltage to x-axis is

$$x(s) = \frac{K_{bb}KV_m(s)}{(\tau s+1)s^3} \quad (16)$$

III. Controller Design

A. PID Controller

A cascaded control system is used for designing the ball balancer system. The closed loop block diagram for a ball balancer arrangement is as represented in Fig. 4. The inner loop comprises of SRV02 Controller and SRV02 Plant, while the outer loop comprises of the 1DBB Controller, a ball balancer and unity feedback. The error is fed to controller whose output is given to the inner loop. Combining this with the transfer function of the ball balancer an output is obtained. If it does not match the desired output, then using unity feedback, it is sent again to the controller, which then takes the appropriate action. While outer loop aims to adjust the angle of inclination of the plate. The compensator used in the outer loop calculates the angle of the servo load on basis of the measurement of position of the ball in order to obtain the required coordinates of the ball. The inner loop regulates any variations in angle and the voltage provided as input. The servo position control system is designed as the inner loop of the system. The required motor voltage is calculated using the servo compensator for load angle for tracking the load angle. Fig. 4. shows the cascaded control system which is utilized for controlling the ball position for the x-axis of the combined SRV02 and 2DBB plant. Here, $C_{bb}(s)$ represents the transfer function of 1DBB compensator, $C_s(s)$ represents the transfer function of SRV02 compensator, $T_s(s)$ represents the SRV02 plant transfer function and $T_{bb}(s)$ represents the transfer function for 1DBB plant. Fig. 5 represents the connections for PID compensator with derivative set point weighting.

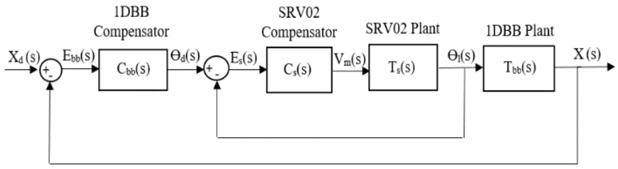

Fig. 4. Cascade control system for controlling ball position in x-axis of the combined SRV02 and 2DBB plant

Let,

$$\theta_l(t) = \theta_d(t) \quad (17)$$

The closed loop equation for Proportional-Integral-Derivative controller in time domain is given as

$$\theta_d(t) = k_p(x_d(t) - x(t)) + k_d\left(b_{sd}\left(\frac{d}{dt}x_d(t)\right) - \left(\frac{d}{dt}x(t)\right)\right) + k_i\int (x_d(t) - x(t))\,dt \quad (18)$$

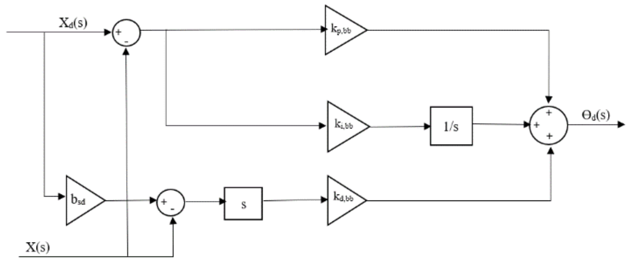

Fig. 5. PID Compensator with derivative set-point weighting

The steady state error is written as-

$$E(s) = X_d(s) - X(s) \quad (19)$$

B. Fuzzy Logic Controller

Fuzzy logic uses human-like understanding to reach judgements, especially when dealing with uncertain or imprecise data. It works with imprecise data rather than accurate data having clear borders. A rule-base, a fuzzifier, an inference mechanism, and a defuzzifier are the four main parts of a fuzzy controller. The defuzzification process transforms the formed conclusions back into numeric values, whereas the fuzzification process transforms the numerical data input into a form that the fuzzy system may use to draw conclusions. The rules developed to instruct the system how to control as well as map the input with the output is used to model the fuzzy controller.

In the designed fuzzy controller, there are two input variables and one output variable. For mamdani fuzzy model, the inputs are error and change in error and the output is control. The defuzzification method used is centroid. 49 if-then rules are designed for the inference mechanism. Triangular membership functions are used for both the input and the output variables. For the sugeno fuzzy model, the input variables are the error and velocity while the output variable is the angle. The two input variables have the triangular membership function while the output variable has constant type membership function. The defuzzification method used is wtaver. The controller design for the same is as shown by Fig. 6(a) and Fig. 6 (b) below.

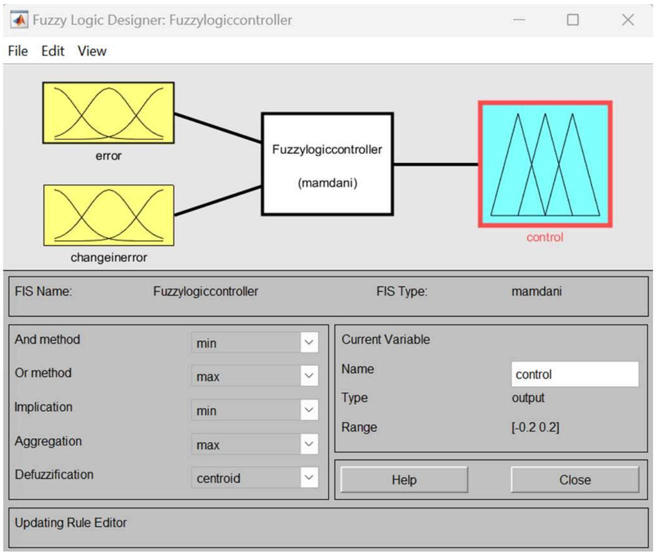

Fig. 6. (a) Fuzzy logic controller design using Mamdani fuzzy model

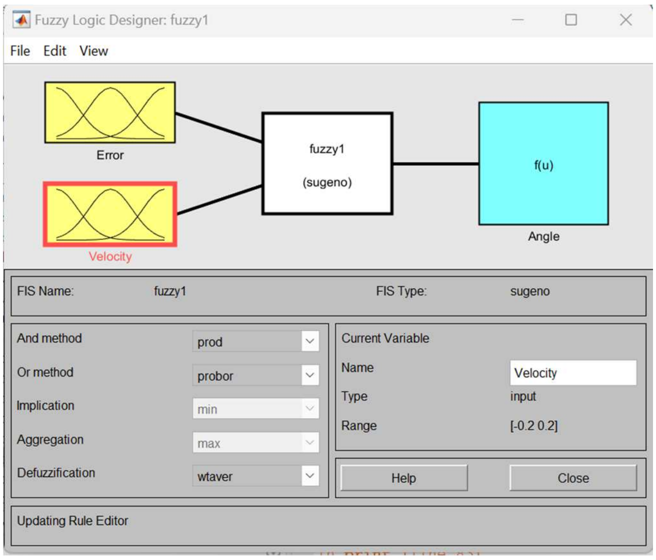

Fig. 6. (b) Fuzzy logic controller design using Sugeno fuzzy model

IV. Experimental Results

This section describes the results obtained after evaluating the working of the system using the simulation results. The performance of the fuzzy logic controller is evaluated and a comparison is done between the mamdani and sugeno FIS.

The following curve shows the characteristics for ball position for two degree of freedom ball balancer system. Fig. 7 shows the ball position characteristics, Fig. 8 shows the ball plate angle characteristics while the Fig. 9 shows the actuation voltage characteristics. The time domain analysis for the same is done as in Table I.

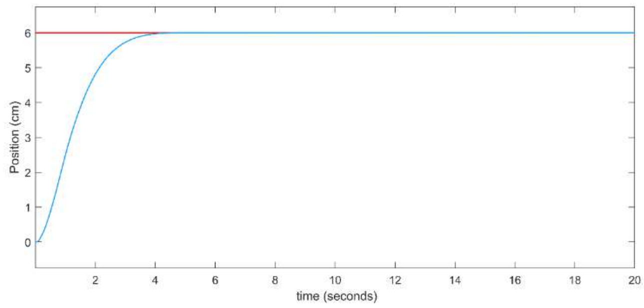

Fig. 7. (a) Ball position characteristics for 2DBB system using Mamdani FIS

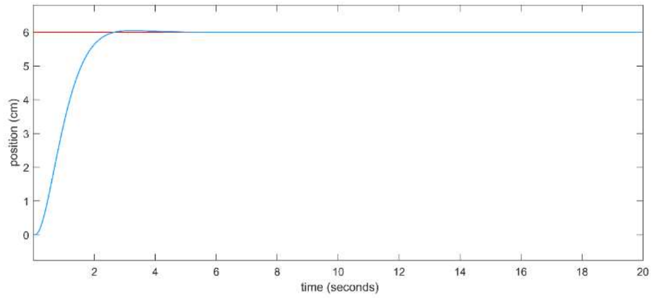

Fig. 7. (b) Ball position characteristics for 2DBB system using Sugeno FIS

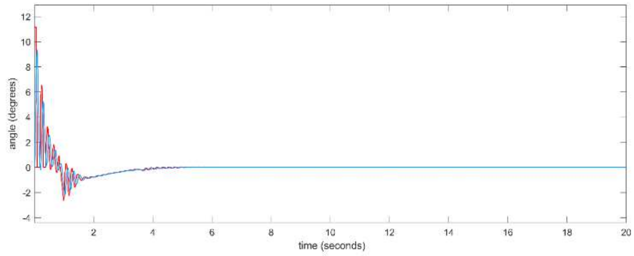

Fig. 8. (a) Ball plate angle characteristics for 2DBB system using Mamdani FIS

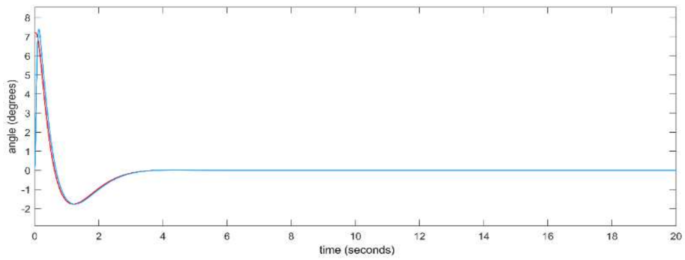

Fig. 8. (b) Ball plate angle characteristics for 2DBB system using Sugeno FIS

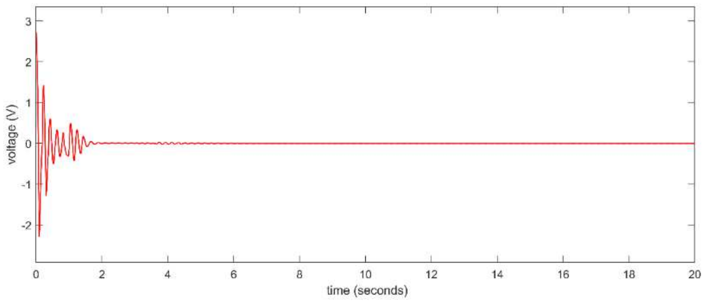

Fig. 9. (a) Actuation voltage characteristics for 2DBB system using Mamdani FIS

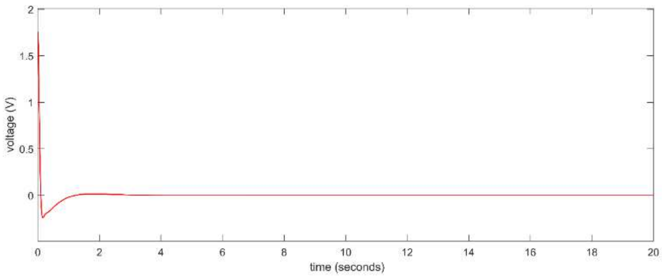

Fig. 9. (b) Actuation voltage characteristics for 2DBB system using Sugeno FIS

The time domain specifications are specified in Table I. The time domain analysis shows that the rise time, peak time, delay time and settling time is less while using the sugeno fuzzy model. This shows that the curve reaches the steady state and also its peak faster as compared to the mamdani fuzzy model and also has a more stable transient response. This means that the sugeno fuzzy model is superior than the mamdani fuzzy model.

TABLE I. COMPARATIVE TIME RESPONSE ANALYSIS

Time domain specification	Mamdani Fuzzy	Sugeno Fuzzy
Rise time	2.09945	1.44491
Peak time	4.6	3.242
Delay time	1.1654	0.9239
Settling time	2.93732	2.06316
Peak overshoot	0	0.04834

Furthermore, the surface generated using the two fuzzy models are shown as in Fig. 10. It is observed that while using the sugeno fuzzy model a smoother surface is obtained in comparison to the mamdani fuzzy model.

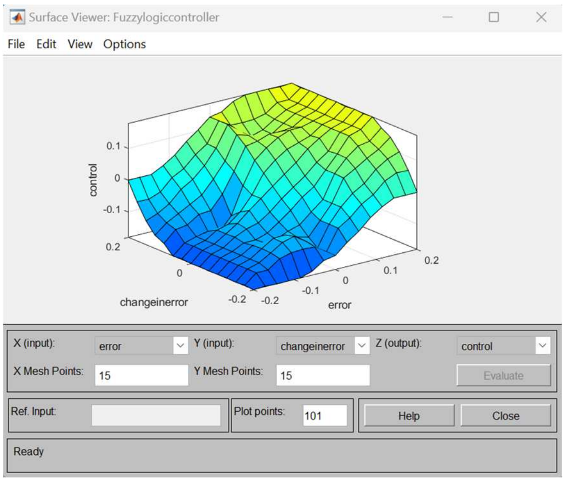

Fig. 10. (a) Surface generated using mamdani fuzzy model

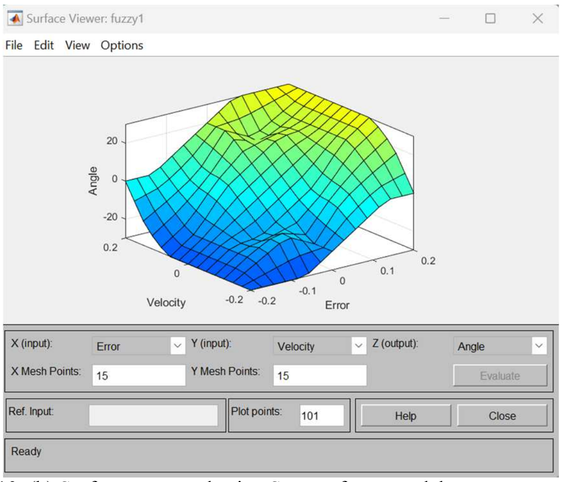

Fig. 10. (b) Surface generated using Sugeno fuzzy model

V. CONCLUSION

This paper aims to control the balancing of the 2 degree of freedom ball balancer system using a fuzzy tuned PID controller. A comparison has been done between the mamdani fuzzy model and sugeno fuzzy model to compare and evaluate the performance of the two fuzzy models. The ball position, ball plate angle and the actuation voltage characteristics have been studied to draw conclusions. Time domain analysis has been done to compare the two models on the basis of various parameters such as rise time, peak time, delay time, settling time and peak overshoot. It has been found that sugeno fuzzy model gives better steady state and transient response while mamdani fuzzy model gives no overshoot. The response generated using the sugeno fuzzy model is more stable and hence superior. For the actuation voltage and ball plate angle characteristics also, there are less oscillations using the sugeno fuzzy model, which increases the stability. The surface generated using the sugeno fuzzy controller is also smoother than the surface generated using the mamdani fuzzy model.

REFERENCES

[1] Chen, H., & Zhang, Q. “Design of horizontal axis washing machine with ball balancer and MR dampers”. International Journal of Precision Engineering and Manufacturing, 18, 1783-1793.

[2] Garcia-Haro, J. Miguel, S. Martinez, and C. Balaguer. "Balance Computation of Objects Transported on a Tray by a Humanoid Robot Based on 3D Dynamic Slopes." In 2018 IEEE-RAS 18th International Conference on Humanoid Robots (Humanoids), 2018, pp. 1-6.

[3] T. Muhammad, Y. Guo, Y. Wu, W. Yao and A. Zeeshan, "CCD Camera-Based Ball Balancer System with Fuzzy PD Control in Varying Light Conditions," 2019 IEEE 16th International Conference on Networking, Sensing and Control (ICNSC), 2019, pp. 305-310, doi: 10.1109/ICNSC.2019.8743305.

[4] Gao, Z., Wijesinghe, S., Pathinathanpillai, T., Dyer, E., & Singh, I. “Design and implementation a ball balancing system for control theory course”. International Journal of Mechatronics, Electrical and Computer Technology, 511, 2363-2374.

[5] B. Ahmad and I. Hussain, "Design and hardware implementation of ball & beam setup," 2017 Fifth

International Conference on Aerospace Science & Engineering (ICASE), 2017, pp. 1-6, doi: 10.1109/ICASE.2017.8374271.

[6] Zhao. B., Li. X., “On Multiple Model Adaptive Control Strategy Research of Ball and Plate System” Advanced Materials Research Vol. 898 (2014) pp 501-505. Trans Tech Publications, Switzerland. doi: 10.4028/www.scientific.net/AMR.898.501.

[7] C. N. Obiozor, "Feedback on a ball balancer," Proceedings IEEE Southeastcon'99. Technology on the Brink of 2000 (Cat. No.99CH36300), 1999, pp. 64-66, doi: 10.1109/SECON.1999.766092.

[8] Awtar, Shorya & Bernard, C & Boklund, N & Master, A & Ueda, D & Craig, Kevin. “Mechatronic design of a ball-on-plate balancing system. Mechatronics”. 12. 10.1016/S0957-4158(01)00062-9.

[9] F. Dušek, D. Honc and K. R. Sharma, "Modelling of ball and plate system based on first principle model and optimal control," 2017 21st International Conference on Process Control (PC), 2017, pp. 216-221, doi: 10.1109/PC.2017.7976216.

[10] Saad, Mustafa. “Design and Implementation of an Embedded Ball-beam Controller Using PID Algorithm”. Universal Journal of Control and Automation. 5. 63-70. 10.13189/ujca.2017.050402.

[11] Sun, Sheng & Li, Lin. “The Study of Ball and Plate System Based on Non-Linear PID. Applied Mechanics and Materials”. 187. 134-137. 10.4028/www.scientific.net/AMM.187.134.

[12] A. Surana and B. Bhushan, "Design and Comparison of PSO, SA and GA tuned PID Controller for Ball Balancer Arrangement," 2021 Fourth International Conference on Electrical, Computer and Communication Technologies (ICECCT), 2021, pp. 1-5, doi: 10.1109/ICECCT52121.2021.9616686.

[13] A. Surana and B. Bhushan, "Designing of PSO Tuned PID Controller for Ball Balancer Arrangement and Comparative Analysis with Classical PID and Fuzzy Logic Controller," 2021 IEEE International Conference on Electronics, Computing and Communication Technologies (CONECCT), 2021, pp. 1-5, doi: 10.1109/CONECCT52877.2021.9622355.

[14] N. Nikita and B. Bhushan, "Effect of Parameter Variation of Ball Balancer System Using PD and Fuzzy Control," 2022 IEEE Delhi Section Conference (DELCON), 2022, pp. 1-6, doi: 10.1109/DELCON54057.2022.9753641.