

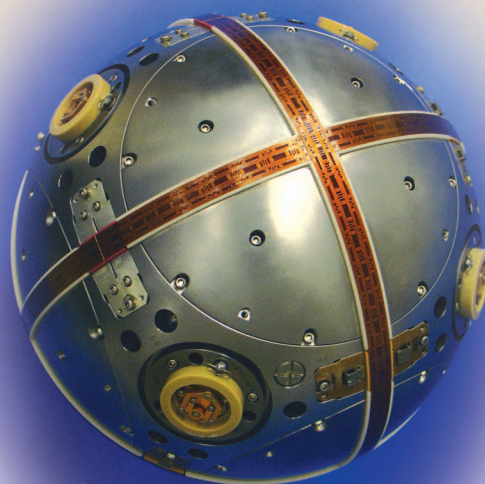
# Inertially Stabilized Platform Technology

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## CONCEPTS AND PRINCIPLES

Inertially stabilized platforms (ISPs) are used to stabilize and point a broad array of sensors, cameras, telescopes, and weapon systems. Since they began to be utilized about 100 years ago, ISPs have been used on every type of moving vehicle, from satellites to submarines, and are even used on some handheld and ground-mounted devices. Examples of the scientific, military, and commercial applications are shown in Figure 1. These applications include surveillance, target tracking, missile guidance, gun-turret control, communications, astronomical telescopes, and handheld cameras. For example, the entire Hubble Space Telescope is a gyro-stabilized ISP designed to point at distant stars and galaxies to within a few milliarcsec and hold the optical axis steady to a fraction of this angle to avoid blurring the magnified image.

Visible and infrared cameras are routinely pointed and held stable by ISPs on ground vehicles, ships, aircraft, and spacecraft for diverse missions such as scrutinizing military targets, mapping, and providing high-resolution imagery for environmental surveys. Other ISPs are mounted on vehicles to stabilize and point communication antennas and pencil-beam laser communication devices. Several of these applications, along with some of the many possible system configurations, are discussed in [1]–[18].



Inertially Stabilized Platforms

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ISP electromechanical configurations are as diverse as the applications for which they are designed. ISPs usually consist of an assembly of structure, bearings, and motors called a *gimbal* to which a gyroscope, or a set of gyroscopes, is mounted. The sensor or payload to be stabilized is mounted directly on the gimbal assembly in some configurations, while in others, mirrors or other optical elements are mounted to the gimbal, and the sensor is fixed to the vehicle. Typically, the gimbal must be designed to point and stabilize about two or more axes, and, therefore, most applications require at least two orthogonal gimbals. However, more than two gimbals are often required to provide additional degrees of freedom or to achieve better isolation from the host vehicle. Gimbal-sensor assemblies can range in weight from under a pound to several tons, and, although the weight of an ISP depends primarily on the size of the payload being controlled, the size and weight of the ISP also increase dramatically as additional gimbal axes are added.

Although requirements for ISPs vary widely depending on the application, they all have a common goal, which is to hold or control the *line of sight* (LOS) of one object relative to another object or inertial space. The LOS can be the aimpoint of a beam or weapon, the center of the *field of view* (FOV) of a telescope, or the direction a sensor is pointed. A typical scenario is shown in Figure 2 to illustrate the basic concepts. Much of the terminology surrounding ISP

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technology has evolved over the years. For example, a gimbal was originally thought of specifically as a concentric set of rings suspended on bearings, whereas, today, the term gimbal can refer to most any type of mechanism that allows the LOS of a stabilized object to be rotated and controlled. Likewise, *gyro* or *gyroscope*, terms once reserved to describe a particular spinning wheel device, are now commonly used to describe any instrument or device that measures rotational motion in inertial space.

In its most primitive form, an ISP attempts only to prevent the stabilized object from rotating in inertial space and therefore does not necessarily hold a fixed LOS to a target or object moving relative to it. However, with the exception of a few navigation or scientific applications, the motion that must be controlled is the motion between two objects, often moving relative to each other. Using Figure 2 as an example, the components of motion and apparent motion of the LOS between two moving objects can be described by

$$\begin{aligned}
 \underbrace{\omega_{\text{target/FOV}}}_{\substack{\text{Total motion of the} \\ \text{target in the sensor} \\ \text{FOV}}} &= \underbrace{V_{T\perp}/R - V_{i\perp}/R}_{\substack{\text{Parallactic motion} \\ \text{caused by} \\ \text{target and sensor motion} \\ \text{perpendicular to the LOS}}} \\
 &+ \underbrace{\omega_M}_{\substack{\text{Apparent motion} \\ \text{caused by} \\ \text{media distortion}}} \\
 &- \underbrace{\omega_i}_{\substack{\text{Inertial rotation} \\ \text{of the sensor}}} . \quad (1)
 \end{aligned}$$

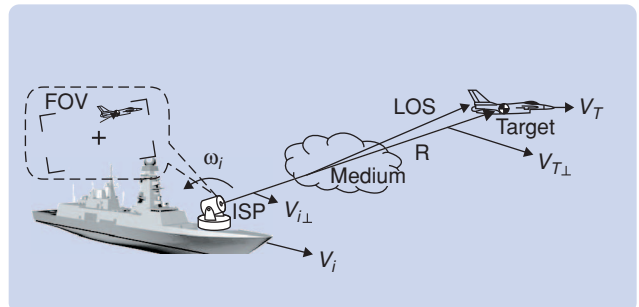
Parallactic motion of the aim point or of the image in the FOV, which is present whenever the target or sensor has a velocity component perpendicular to the LOS, requires that a pointing or tracking system be used in conjunction with the ISP. When the target motion is arbitrary or unknown, automatic radar or imaging trackers are necessary, but when the target has a known fixed location, some pointing systems use navigation sensors to remove the parallactic motion. Apparent motion caused by medium distortion, such as atmospheric scintillation, is far more difficult to deal with. Typically, these components are either accepted as inevitable in the motion error budget or, when not acceptable, they must be dealt with using special techniques such as adaptive optics [3].

### Requirements

In general, it is not sufficient to simply prevent the sensor from rotating in inertial space, but rather, the sensor must be controlled in a specific manner depending on what the

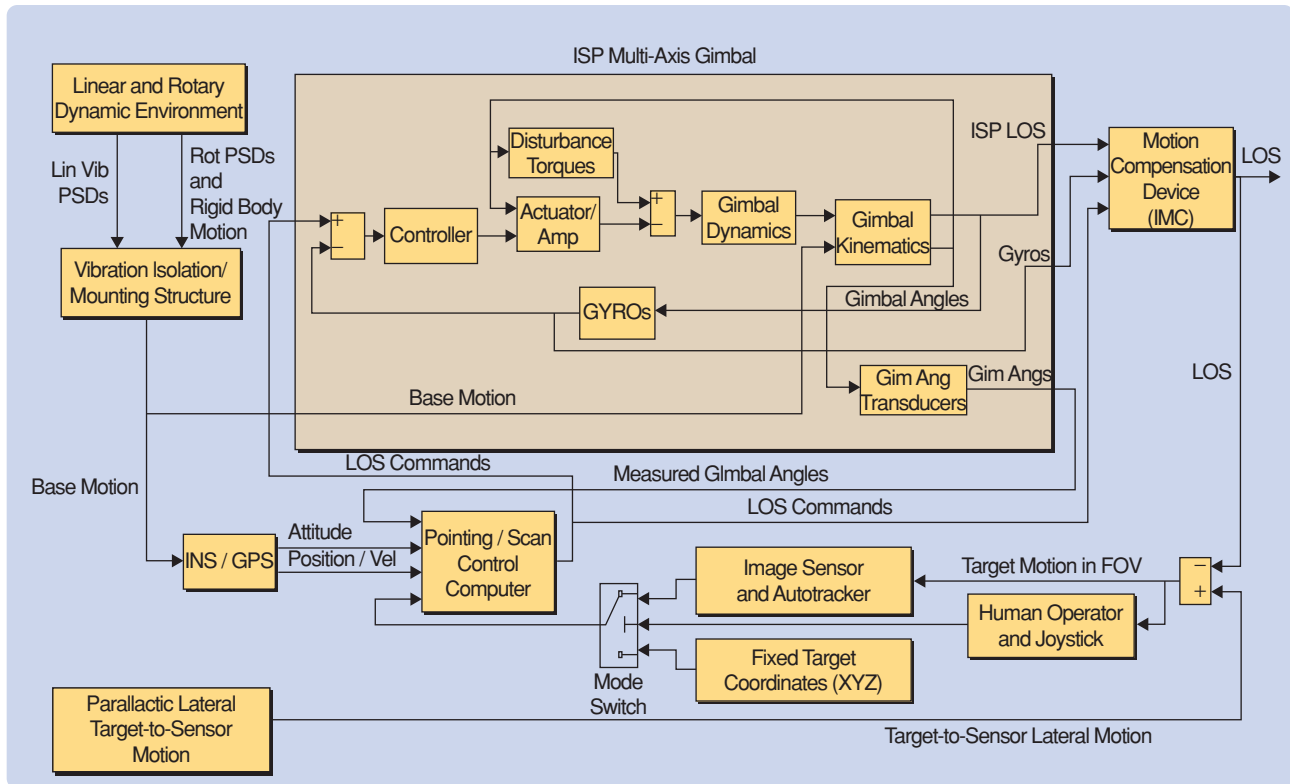


**FIGURE 1** Applications of inertially stabilized platforms. Inertially stabilized platforms (ISPs) are used in just about every type of moving vehicle as well as some handheld and ground-mounted applications. The Hubble Space Telescope shown in the upper left points at distant stars and galaxies with an accuracy of better than 10 milli-arcsec, which is equivalent to looking at a dime from 200 mi [1]. Military systems, such as the ones shown in the bottom right and left, commonly use multiple ISPs to control the targeting, or tracking, system as well as the weapon aimpoint. Commercial applications of ISPs, such as the handheld camera shown in the upper right, have been made possible in the last decade by the development of small, low-cost gyros. [Photos courtesy of (top left) NASA/Space Telescope Science Institute, (top right) Konica-Minolta, (bottom left) General Dynamics, and (bottom right) U.S. Air Force.]



**FIGURE 2** A typical inertially stabilized-platform (ISP) application. Many applications use ISPs to hold a line of sight (LOS) stationary. For beam- and weapon-pointing applications, the LOS is the aimpoint, whereas, for radars and electro-optical sensors, the LOS is defined by the field-of-view (FOV) of the sensor, where the motion of the target in the FOV is of interest. Although most applications require that only the two axes orthogonal to the LOS be controlled, some applications require control about all three axes.

sensor system is attempting to accomplish. For example, as illustrated in Figure 2, if the primary purpose of the system is to obtain a clear image of the target, it may suffice to hold the image steady within the sensor FOV without concern for the specific aimpoint. If, however, a beam, such as a laser rangefinder, is to be directed at the target, then holding the target precisely at the center of the FOV may be critical, while small amounts of high-frequency rotation or jitter might be inconsequential. For other systems, both the aimpoint and jitter must be controlled. Thus, the requirements for an ISP system depend on many factors



**FIGURE 3** A typical integrated inertially stabilized platform (ISP) block diagram. ISPs are commonly integrated into tracking or pointing systems as shown here. The ISP is designed to remove the effects of disturbances and provide a basis for controlling the line of sight (LOS) in response to inputs from the tracking and pointing systems. The ISP is thus often configured as a rate loop inside the tracking or pointing position loop used to point the LOS at a target or in a prescribed direction.

but are primarily determined by the characteristics of the device being stabilized. Various system requirements can be found in [19]–[25].

Requirements for an imaging system, for example, are driven by the resolution and integration time of the imaging sensor, whereas the requirements for a beam-pointing system or a weapon system may be driven more by pointing accuracy or the native accuracy of the weapon rather than by resolution considerations. For imaging systems, it is not uncommon to have stabilization requirements that are specified as a fraction of a resolution element over the sensor integration time period, which can easily translate to a fraction of a microradian for astronomical telescopes and high-resolution surveillance devices [1], [7], [24]. In contrast, devices such as handheld cameras and communication antennas may require stabilization performance only in the milliradians.

### Typical LOS Control Configuration

Although the primary purpose of an ISP is to control the rotation of a sensor or object in inertial space, an ISP is typically employed as the basic element of a complex pointing or tracking system. Specifically, the ISP is designed to control only the last component of motion on the right-hand

side of (1). When integrated into a control system with tracking or pointing sensors as in Figure 2, however, inertial rotation is controlled such that the total motion of the target image in the FOV can be controlled. The block diagram in Figure 3 represents one axis of a generic ISP control system configuration that includes a tracking mode, a geo-pointing mode, and a joystick mode to allow a human operator to point or slew the LOS. Typically, the ISP control system is configured as a high-bandwidth rate loop inside a lower bandwidth pointing or tracking position loop. Thus, the ISP might be viewed as a means for removing high-frequency disturbances and controlling the LOS, whereas the pointing and tracking loops have the task of removing the lower frequency parallactic motion and perhaps any bias or drift in the ISP rate loop.

In this article, we concentrate only on the ISP portion of systems such as the one depicted in Figure 3. First we consider an idealized single-axis system to establish the basic principles of operation. Next, we consider some of the more complex and less intuitive effects that must be dealt with, followed by a brief introduction to some alternative solutions to particular problems that are commonly encountered. Finally, we consider the technical skills that are common to most successful ISP design teams.

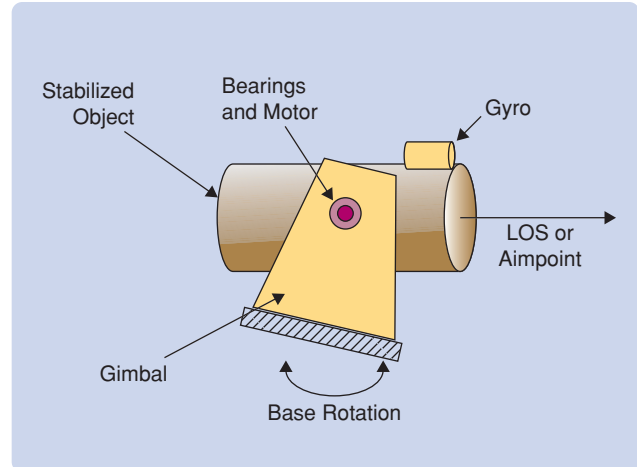
## SOME BASIC PRINCIPLES

Although many approaches are available for stabilizing the LOS of an object so that it doesn't rotate relative to inertial space, perhaps the most straightforward and most common approach is *mass stabilization*. This term applies to an ISP designed such that its LOS tends to remain stationary with respect to an inertial reference frame when the base on which it is mounted is rotated, whether or not a gyro or control system is utilized. The principle of mass stabilization is essentially an acknowledgment of Newton's first and second laws of motion. Newton's first law applied to rotational motion asserts that a body does not accelerate with respect to an inertial frame unless a torque is applied. Furthermore, Newton's second law establishes that if a net torque  $T$  is applied to a homogenous rigid mass having a moment of inertia  $J$ , then the body develops an angular acceleration  $\alpha$  according to

$$T = J\alpha. \quad (2)$$

Therefore, in principle, all that is required to prevent an object from rotating with respect to inertial space is to ensure that the applied torque is zero. However, despite careful electromechanical design, numerous sources of torque disturbances can act on a real mechanism causing excessive motion or jitter of the LOS. Also, a means for controlling the object so that it can be rotated in response to command inputs is usually required. Therefore, rate or displacement gyros are typically attached to the object to measure the inertial rotation about the axes that require stabilization and control. The gyro is used in a closed-loop servo system to counteract the disturbances and, at the same time, allow the object to be controlled from external command inputs.

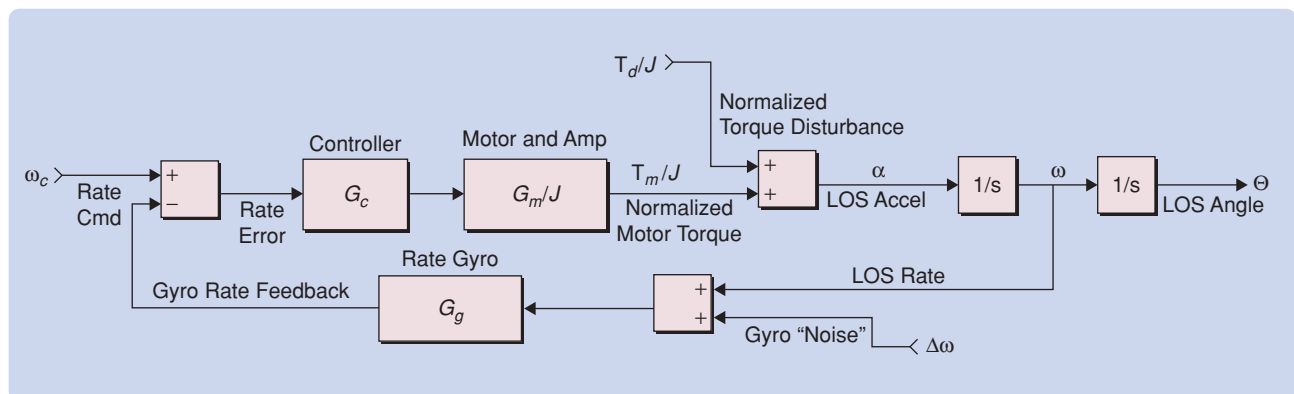
The single-axis stabilized gimbal shown in Figure 4 illustrates the basic concepts of a mass-stabilized system. Assume that the LOS of the sensor shown in Figure 4 is to be held inertially stationary in spite of any rotation of the base on which it is mounted. The gimbal on which the sen-



**FIGURE 4** A single-axis gimbal mechanism. The purpose of the gimbal is to isolate the stabilized objects from base rotation, and allow the line of sight (LOS) to be pointed.

sor is mounted is an axle or platform suspended on bearings or other type of suspension to allow it to freely rotate. The suspension is designed to minimize the friction and any other types of torque coupling to the rotating base. The entire rotating assembly is balanced about the suspension pivot axis to minimize imbalance torques due to linear vibration. To be successful, the mass properties and structural dynamics of all of the rotating components, including the gimbal structure and the payload, must be considered in the stabilization system design.

The block diagram in Figure 5 shows how a gyro feedback control system might be configured for the gimbal. While many types of gyros are used in ISP designs, gyros that sense angular rate are commonly used and the control system shown in Figure 5 is typically configured as a rate servo. That is, the control system attempts to null the difference between the rate command input and the angular rate of the gimbal; this scheme is used in the pointing and tracking system in Figure 3. When the rate command input is zero or absent, the system attempts to null the total



**FIGURE 5** Single-axis inertially stabilized platform (ISP) block diagram. The fundamental dynamics, major components, and common sources of error are shown in this block diagram of a mass-stabilized ISP with gyro feedback, such as the one shown in Figure 4. This system forms the rate loop for the multi-axis gimbal embedded in the overall LOS control system shown in Figure 3.



torque applied to the gimbal, which requires that the closed-loop control system generate a control torque at the motor that is equal and opposite to the net disturbance torque. The net torque  $T$  in Figure 5 is normalized by dividing by the inertia  $J$ , which effectively converts the system dynamics to a kinematic analysis. Disturbance and motor torques are often normalized to  $T/J$  in many aspects of ISP design to help make decisions before complete design details are available and also to draw conclusions applicable to a class of systems.

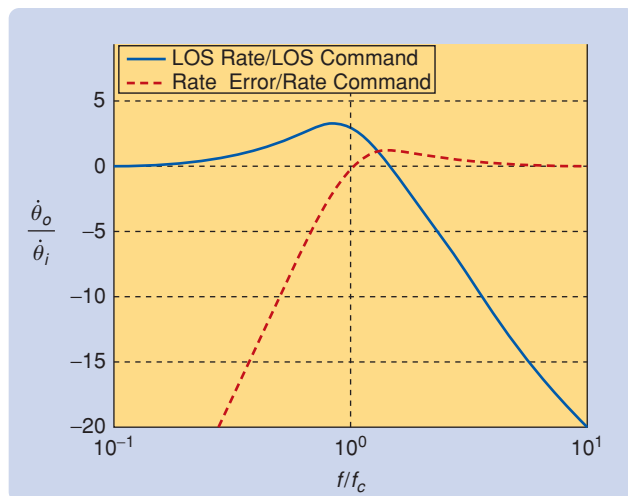
### The Role of Bandwidth

While the response characteristics and transfer functions for a realistic system can be complicated, understanding the properties of an idealized system can provide valuable insight into the basic principles of operation and sensitivities to disturbances and errors. The most indicative single parameter that predicts how well a control system can follow command inputs and how well it can compensate for disturbance torques is the closed-loop *bandwidth*. Although several definitions of bandwidth are available, one convenient definition is the frequency at which the closed-loop frequency response falls to 0.707 of its dc value. The ability to follow a command input constitutes the command tracking or following performance. The rate-command following error is reduced by a factor that is approximately proportional to the closed-loop bandwidth at low frequencies as shown in Figure 6, which shows the ratio of the rate-following error to the rate-command input versus frequency. The closed-loop bandwidth, which also deter-

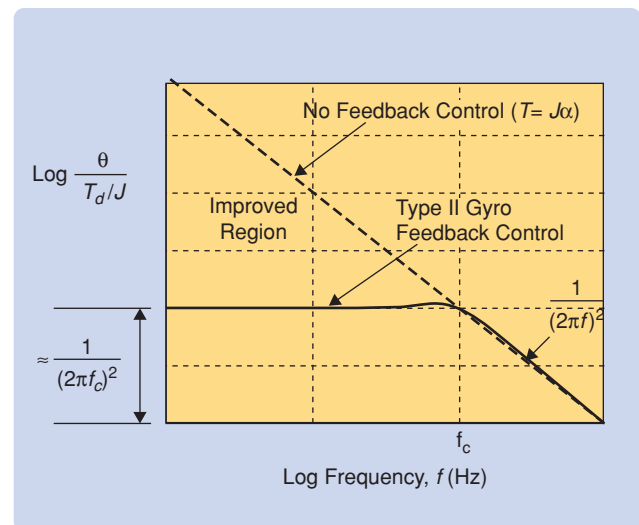
mines the disturbance rejection ratio  $\theta/(T/J)$ , can be used to estimate the LOS angular motion  $\theta$  due to the net disturbance torque. Figure 7 shows that, for a typical type-II control system design, the torque disturbance-rejection ratio is approximately proportional to the squared inverse of the closed-loop bandwidth for disturbances whose frequency content is below the closed-loop bandwidth. For higher frequency disturbances, the rejection characteristics of the system are governed by (2) as if no loop were present [7], [14], [26]–[28].

The loop bandwidth is usually determined by a combination of the specific control system design and the dynamic characteristics of the various components in the loop, such as the gyroscope and the actuator, as well as the torsional response of the structure discussed in “Structural Interactions.” The fundamental bandwidth limit is usually determined by structural resonances in large systems but is often determined more by the dynamics of the gyroscope or actuator in small systems. In all cases, obtaining optimal performance involves understanding the component dynamics as well as their noise and error characteristics.

The discussion and conclusions above, including figures 6 and 7, demonstrate the general characteristics and basic design goals of an ISP control system. However, it should be pointed out that the specific characteristics shown are for an idealized system using a simple proportional-plus-integral series compensator or control algorithm assuming no interactive structural resonances or other complications. While this type of control system is common even in many



**FIGURE 6** Command-following frequency response for a typical type II proportional-plus-integral control system. The ability of an inertially stabilized platform to faithfully follow a command is indicated by the control system bandwidth  $f_c$ . The ratio of the rate error to the rate command, shown by the dotted line, increases with the square of the frequency to approximately one at the loop bandwidth. Attenuation of the following error at frequencies below the loop bandwidth is thus enhanced by a factor approximately proportional to the loop bandwidth as the error curve moves to the right with increasing bandwidth.



**FIGURE 7** Torque disturbance frequency response. With a type-II feedback control loop, the gyro feedback provides disturbance rejection proportional to the inverse of the square of the loop bandwidth at frequencies below the loop bandwidth. With no loop feedback, the system responds to torque disturbances according to Newton’s second law. At frequencies above the loop bandwidth, the torque disturbance attenuation is the same as if no loop were present.

## Structural Interactions

Structural aspects of ISP design can be the most challenging part of the design effort and can easily dominate the performance. Structural design interacts with an ISP system as three separate and distinct effects. Since these effects impact different aspects of the design, they are sometimes either confused with one another or, worse, ignored altogether until problems arise. First we review the basic nature of structural dynamics and analysis techniques and then consider the effects mentioned above.

The structural dynamics of an ISP involve all of the components attached to the system including the structural characteristics of the payload as well as the gimbal and support structure. Because objects that require stabilization, such as optical sights, antennas, and weapons, tend to be geometrically and structurally complex, the structural dynamics of these objects combined with the gimbal systems are usually also complex, consisting of essentially an unlimited number of interactive *modes* of response. A structural mode can be thought of as a shape (see Figure S1) and a frequency at which a structural shape resonates. While the shape and frequency of a mode are primarily a function of the structural stiffness, damping and mass distribution, the amplitude of the response depends on the amplitude and spectrum of the vibration input or force causing the mode to respond.

### THE NATURE OF STRUCTURAL DYNAMICS

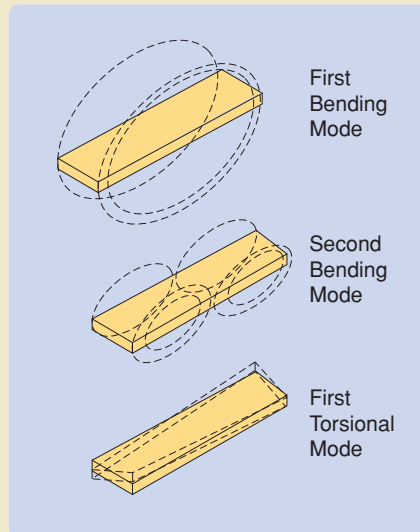
Figure S1 shows the first few bending and torsional modes in a beam structure. The first bending mode, or resonance, is excited by a force or motion input acting on the beam at the frequency of the first bending mode. While all modes respond to inputs at any frequency, the response is particularly pronounced when the forcing frequency coincides with a modal frequency. As the frequency increases, the second bending mode responds and so on indefinitely. When the excitation consists of a wideband spectrum, many modes are excited simultaneously. If a torque acts on the beam, the torsional modes respond in the same manner, as shown. If the beam is not symmetrical, as is the case with most real structures, a combination of bending and torsional modes can be excited with either a force or torque acting anywhere on the beam.

The frequency response defines the ratio of the amplitude and phase of the response at a particular point and direction on the structure as a function of frequency to the amplitude of the forcing function at another point and direction. A transfer function can be defined, for example, for the rotation at a point on the gimbal in response to a force or displacement input at another point. A typical structural frequency response is shown in Figure S2 [S1]–[S3].

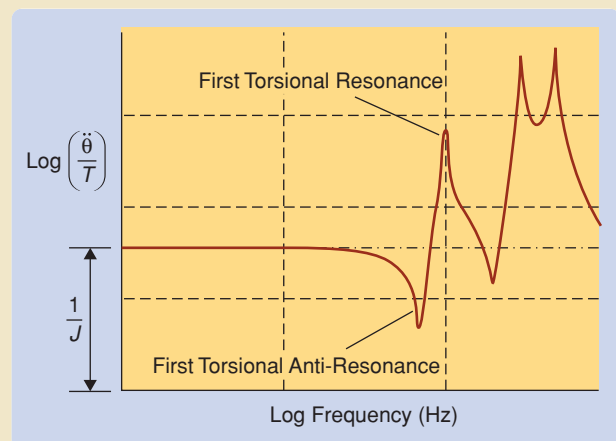
The frequency response shows all of the modes, including bending and torsional, that participate at that particular point and in the direction of interest. The peaks indicate the resonant frequencies for a point on the structure, whereas the valleys, commonly called antiresonances, show the frequencies at which little or no response occurs at that particular point on the structure. The amplitude of the frequency response at each modal frequency depends on the damping in the structure and how much that mode participates in the response to the excitation. Typically, metallic structural materials have low damping and resonant amplification of 15–25 is not uncommon. Thus, if an ISP system is vibrated at 1 g and the vibration input spectrum coincides with the frequency of a resonant mode, then response accelerations of 15–25 g can be expected.

Finite element analysis can be used to accurately estimate the various transfer functions and responses of a structural design. Typically, these analyses are iterated as necessary, along with appropriate modifications to the mechanical design, until a satisfactory design is achieved. If the structure, or some portion of it, already exists, the analysis can be augmented with experimental modal testing, which can provide guidance as to where modifications might be needed [S3]–[S6]. Usually, the above methods

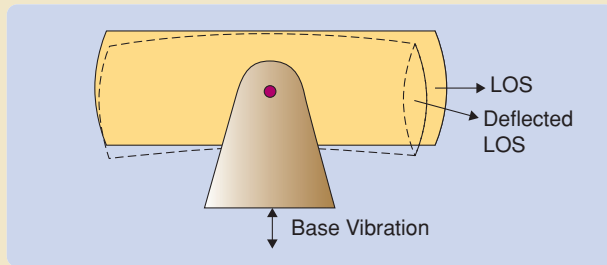
are followed by vibration testing to verify the performance. Whether analytical, experimental, or a combination of methods are used, however, a critical requirement is that realistic forcing functions are used to assess the structure. Ideally, this assessment must include



**FIGURE S1** Structural modes. A mode is defined by a shape and a frequency. All structural assemblies exhibit an unlimited number of resonant modes. This example shows the first two bending modes and the first torsional mode in a beam. The response amplitude of a mode depends on the amplitude, frequency, and location of the forces or torques applied to the structure.



**FIGURE S2** Typical structural transfer function. This structural transfer function shows the angular acceleration response to a torque excitation truncated to the first few resonances and antiresonances. Resonances in structures, which commonly have amplifications of 20 or more, are a major concern in the control system for ISPs.



**FIGURE S3** Structural effect #1: bending. When excited, the line of sight (LOS) deflects or jitters at the excitation frequency due to minute bending displacements within the stabilized payload. The excitation source can be either vibration in the mounting base or vibration sources within the gimbal or stabilized system. The amount of LOS deflection can depend on the sensitivity of the LOS motion to both tilt and decentering of optical elements within stabilized optical systems as well as on the bending of the overall structure. The gyro feedback loop is usually ineffective in dealing with this source of jitter.

the complete six-degree-of-freedom (6DOF) translational and rotational dynamic environment that the system is to operate in. The failure of many ISP designs to operate as intended can often be traced back to either an inadequate assessment of the dynamic 6DOF operating environment or a misunderstanding of how to interpret and apply the structural results.

We now discuss the categories of structural considerations that apply to most ISP system designs. The separate categories require that different transfer functions and responses be considered, each of which have different effects on the system.

#### EFFECT #1—BENDING

We first consider displacement of the line of sight (LOS) due to bending in the gimbal and payload as depicted by Figure S3. Displacement of the LOS can be caused by base-motion vibration, on-gimbal shaking forces and even torsional responses to the gimbal actuators. An analysis of this effect, which involves consideration specific transfer functions and realistic forcing functions, requires effort by the analyst to define the proper points in the system that are representative of the critical motion. Optical components, for example, can amplify or change the direction of the structural motion, thus requiring that coefficients be defined for each optical element. Displacement of the LOS due to structural bending is often not sensed by the gyros, while portions of the motion that can be sensed are usually at frequencies that are above the frequencies that can be effectively dealt with by the servo [S7], [S8].

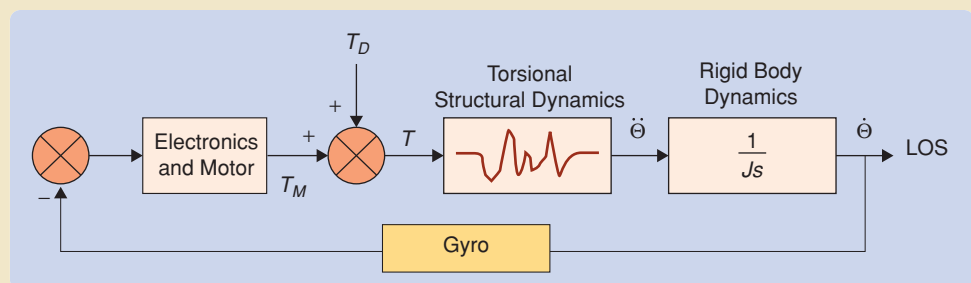
Usually, the first attempt to attenuate LOS motion due to bending involves trying to stiffen the structure. Given the constraints of the design, however, stiffening the structure alone often does

not suffice and alternative design approaches are used. In some precision optical ISP systems, devices such as electronic autocollimators and fast-steering mirrors are used to measure the displacement and compensate for the motion. In satellites that must perform as ISPs, the primary source of vibration can be the servo actuators themselves, in which case precision reaction wheels are used to prevent the actuators from reacting against the sensitive structure. When base-motion vibration is the primary culprit, the most common approach is to provide an active or passive vibration isolation system to attenuate vibration inputs. When this approach is used, the vibration isolation design must be carefully implemented to avoid inducing additional rotational motion. Also, interaction with other measurement systems and controls in the system must be addressed since motion that occurs between the gimbal and the base to attenuate the vibration complicates the measurements from the gimbal to the base [S9]–[S12].

#### EFFECT #2—TORSIONAL SERVO INTERACTION

Another structural effect that must be considered is the interaction of the structure with the stabilization control system. This effect, which is independent of the dynamic environment, limits the bandwidth of the control system and thus can critically affect the system performance. The relevant structural transfer function is the torsional response from the location of the torque motor or actuator to rotation at the location of the feedback gyro as shown in Figure S4. These resonances typically exhibit peaking by factors as much as 15–25, and therefore can limit the loop bandwidth to 1/10 or less of the first major resonant frequency.

The first approach to improving this situation is to stiffen the structure to modify the relevant structural transfer functions. However, adequate stiffening often cannot be achieved, and it is thus usually necessary to place one or more notch filters in the feedback control system to attenuate the loop gain in the vicinity of the resonance. However, notch filters add phase lag to the loop, and the resulting system bandwidth is typically limited to less than about 1/3 of the first torsional resonance in spite of the filtering. Also, modes in gimbal systems can shift as the gimbals rotate, thus leaving the filter ineffective. While several methods, such as input shaping and flexure control are available to control torsional modes in structures, these methods are difficult to apply to the complex high-frequency modes typically encountered in ISPs [S13]–[S15].



**FIGURE S4** Structural effect #2: torsional servo interactions. The torsional transfer function between the gimbal actuator and the gyro is often the primary factor that defines the limitation of the achievable control system bandwidth. This effect is not necessarily related to the bending described in effect #1, and thus its description usually requires a different set of transfer functions.

### EFFECT #3—MOUNTING COMPLIANCE INTERACTION

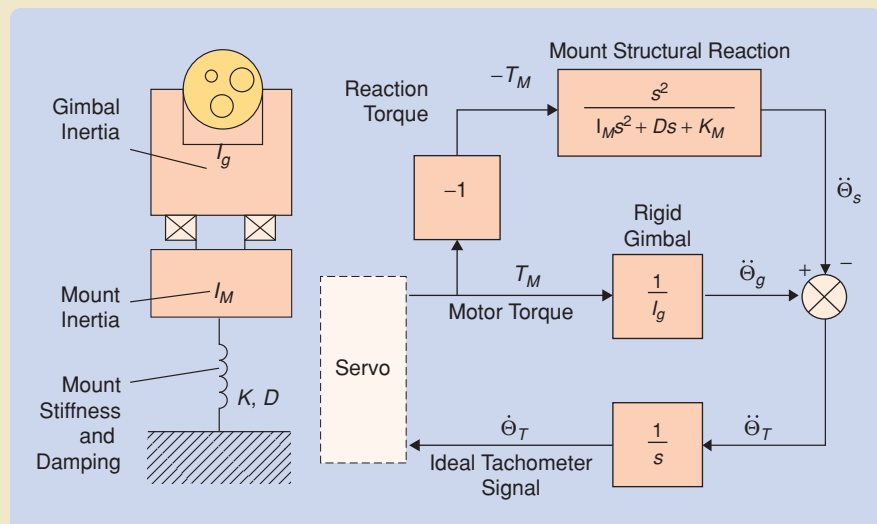
The last structural effect, which involves the interaction of the control system with the structure on which it is mounted, is similar to the previous effect except that it does not necessarily impact the design or performance of the inertial stabilization control system. This effect can, however, severely limit the bandwidth of any positioning or pointing system that relies on feedback from a sensor measuring relative motion between the gimbal and the base structure. Figure S5 shows that the gimbal actuators react against the structure they are mounted on which causes them to deflect and which is sensed by the gimbal transducers. This interaction can be pronounced if the system is mounted on vibration isolators or a flexible mounting structure.

Methods for improving the effects of this interaction include

stiffening the torsional response of the mounting structure, adding mass to the stationary gimbal structure, and employing notch filters in the pointing servo system. If isolators are the reason for the flexibility, the isolators can sometimes be positioned to increase the torsional resonant frequency of the system while maintaining the low lateral frequencies required for isolation. Another technique that may be applicable to precision ground-mounted systems is to use a gyro in combination or blended with a relative motion gimbal transducer to provide feedback [S16]–[S17].

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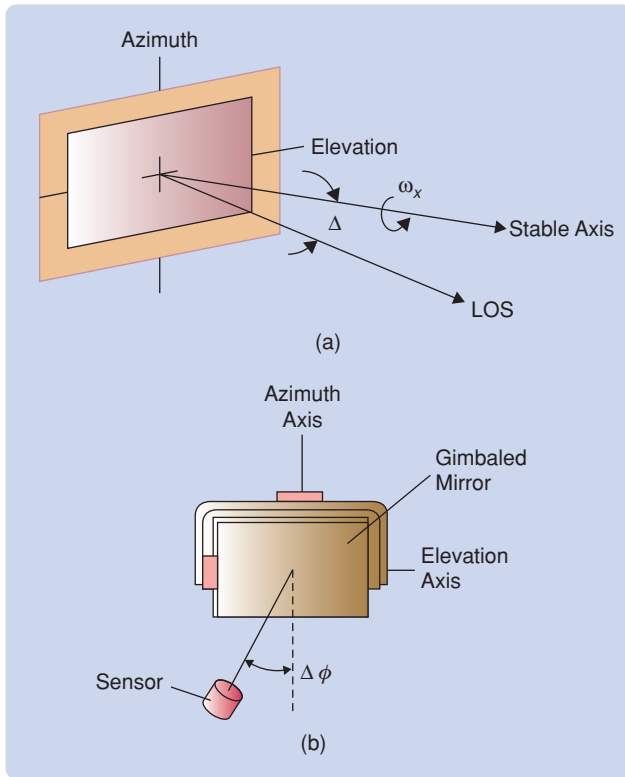
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**FIGURE S5** Structural effect #3: Servo interactions with the mounting structure. Pointing systems or other servo modes that use relative measurement transducers, such as resolvers, tachometers, or encoders, can interact with the structure on which the system is mounted, thus causing instability in the control system. This effect is caused by the reactions of the gimbal actuators, which act against the base structure and depend on the effective mass and stiffness of the mounting structure. Systems mounted on vibration isolators are particularly susceptible to this interaction. Gyro feedback loops are not inherently affected by this effect since they do not depend on the relative motion between the gimbal and the base.

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**FIGURE 8** Misalignment effects in gimballed systems. Misalignments between the line-of-sight (LOS) axes, the gyro axes, and the gimbal torque axes can cause unwanted LOS rotation since the axes being stabilized do not coincide with the LOS. Although the kinematics of misalignment effects are somewhat different in (a) mass-stabilized systems and (b) stabilized mirrors, the magnitude of the effect is similar. In either case, the LOS rotates in response to dynamic base rotations even though the gyro feedback loop is working properly and there are no torque disturbances. Therefore, increasing the loop bandwidth does not eliminate misalignment problems.

high performance ISPs, much better disturbance rejection and command following performance can be achieved with more complicated control algorithms using, for example, state variable feedback, adaptive techniques, and disturbance observer designs [29]–[33].

### Multiaxis Dynamics

Although a single-axis rigid system is used above to introduce the basic operation of mass-stabilized ISPs, single-axis ISPs are seldom used because most applications require the freedom to point the LOS in more than one plane and because isolation from base motion is generally required about more than a single axis. Therefore, most practical ISP systems have multiple gimballed axes. The kinematics of multiaxis gimbals can contribute several nonintuitive effects that are not foreseen by considering each axis individually (see “Multiaxis Gimbals”). Also, since realistic gimbal-sensor systems are not rigid symmetrical homogenous masses rotating about a single axis, (2) may not give an adequate description of the system dynamics.

For all three axes, the dynamics of a nonsymmetrical, nonhomogenous mass are given by Euler’s equations, which have the form

$$\begin{aligned} T_x &= \alpha_x I_x + \omega_y \omega_z (I_z - I_y) - (\omega_y^2 - \omega_z^2) I_{yz} \\ &\quad - (\omega_x \omega_y + \dot{\omega}_z) I_{xz} + (\omega_x \omega_z - \dot{\omega}_y) I_{xy}, \\ T_y &= \alpha_y I_y + \omega_x \omega_z (I_x - I_z) - (\omega_z^2 - \omega_x^2) I_{xz} \\ &\quad - (\omega_z \omega_y + \dot{\omega}_x) I_{xy} + (\omega_x \omega_y - \dot{\omega}_z) I_{yz}, \\ T_z &= \alpha_z I_z + \omega_x \omega_y (I_y - I_x) - (\omega_x^2 - \omega_y^2) I_{xy} \\ &\quad - (\omega_x \omega_z + \dot{\omega}_y) I_{yz} + (\omega_x \omega_z - \dot{\omega}_x) I_{xz}. \end{aligned}$$

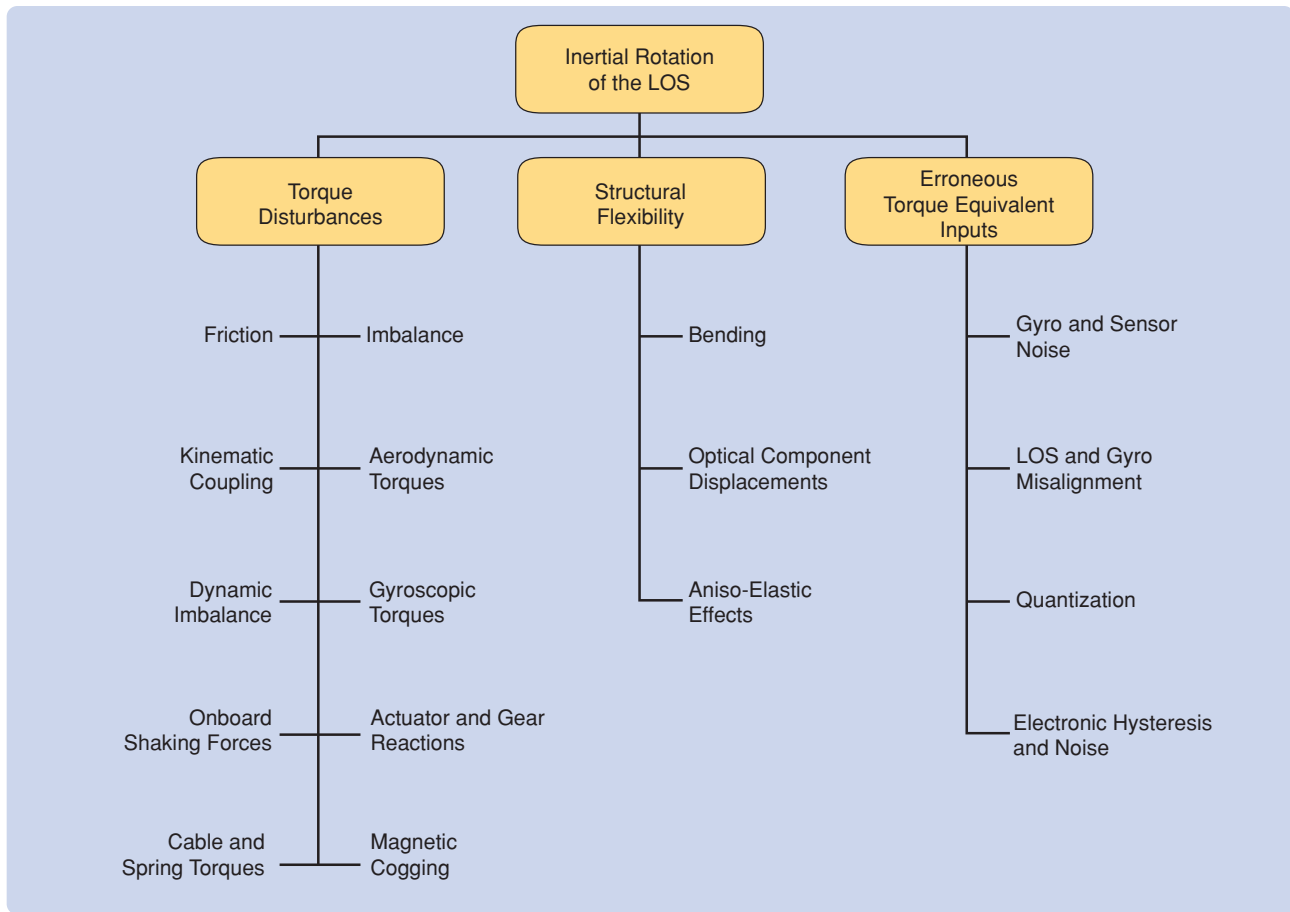
These equations can be written for each gimbal in a multi-axis system with  $N$  gimbals resulting in a set of  $3N$  nonlinear coupled equations that describe the entire multiaxis, multibody motion. When nonsymmetry and multiaxis rotations are present, the terms in Euler’s equations of motion reveal that disturbance torques due to gyroscopic effects and product-of-inertia coupling can occur in addition to friction and imbalance [34]–[37]. For a symmetrical homogenous mass, Euler’s equations reduce to (2) about each axis.

### Misalignments

Another source of unwanted LOS motion in ISPs is misalignments between the gyroscope-sensitive axis, the LOS axis, and the axis about which control torques are applied, as depicted in Figure 8. Ideally, two or more gyroscopes are mounted with their axes aligned perpendicular to the LOS on the gimbal that supports the object being stabilized. This alignment is not always easy to accomplish because the physical line that constitutes the LOS is often difficult to establish. The magnitude of the resulting LOS motion is the product of the residual misalignment and the angular motion of the axes orthogonal to the ideal gyro axis. The problem is further aggravated when more than one LOS, such as multiple sensors, are stabilized by the same gimbal system, in which case the alignment between sensors can be a source of unwanted motion as well as boresight error. A separate effect that can occur due to gyro misalignment is coupling between the actuator axes, which causes the LOS to deviate in one axis when another axis is commanded to slew.

### Summary of Effects

The above discussion introduces some of the primary sources of error in mass-stabilized ISPs and points out a few of the less intuitive effects. These sources can be classified into three categories as shown in Figure 9, which also lists many of the individual phenomena commonly encountered in each category. While the categories and error sources shown are primarily related to mass-stabilized ISP configurations, the same considerations, with appropriate modifications, also apply to most other types of ISPs.

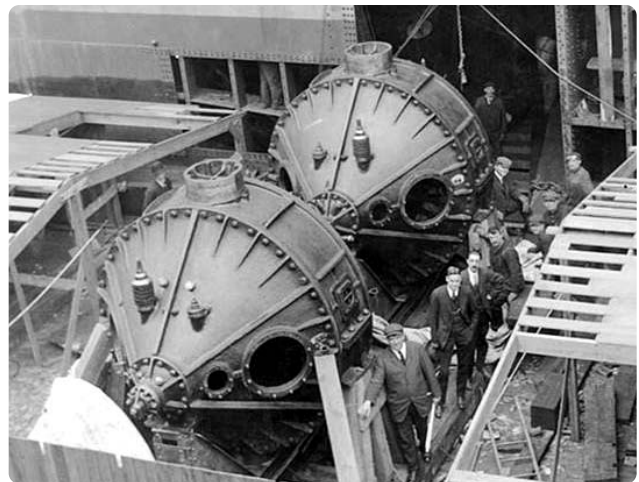


**FIGURE 9** Causes of inertial rotation of the line of sight (LOS). All causes of inertial rotation of the LOS of a stabilized platform can be traced to either a torque disturbance, flexibility in the system, or an erroneous input to the gimbal actuators.

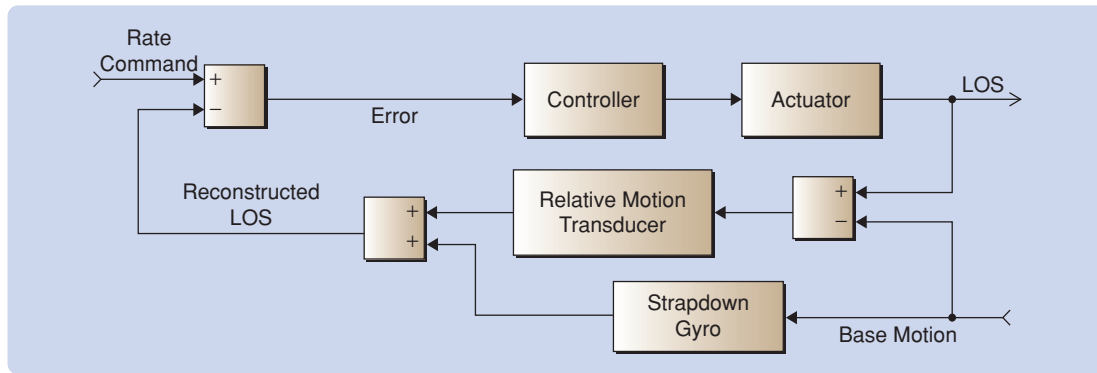
## ALTERNATIVE STABILIZATION TECHNIQUES

### **Momentum Wheel Stabilization**

The mass-stabilized ISP configuration discussed above utilizes instrument gyros, which are self-contained measurement systems that output an electrical signal proportional to the angular motion about the gyro's sensitive axis. However, long before instrument gyros became available, spinning wheels were attached directly to torpedoes, ships, and other vehicles to provide stabilization, in which case the entire vehicle or object it is attached to effectively becomes an ISP as shown in Figure 10. While the large spinning masses required in these applications have since been replaced by actuators controlled in closed loop using smaller, less expensive, instrument gyros, the direct stabilization approach is still used in some missile seekers as well as to stabilize and control satellites [38]–[41]. When used in this manner, the spinning mass devices are referred to as *momentum wheels* or *control-moment gyros* and, even though all of the principles of mass-stabilization apply, these systems are often referred to as momentum-wheel-stabilized systems to distinguish them from mass-stabilized systems, which utilize instrument gyros in closed loop.



**FIGURE 10** Momentum wheel stabilization. In this approach, a vehicle or object is held steady by the stabilizing nature of a spinning mass so that the vehicle or object to which it is attached effectively becomes an inertially stabilized platform. While most ship stabilizers such as the one shown, which was installed on the USS Henderson in 1917, have since been replaced with instrument gyros that control fin actuators, the momentum wheel approach is still used by some missile seekers as well as by many satellite attitude control systems. (Photo courtesy of U.S. Naval Historical Center.)



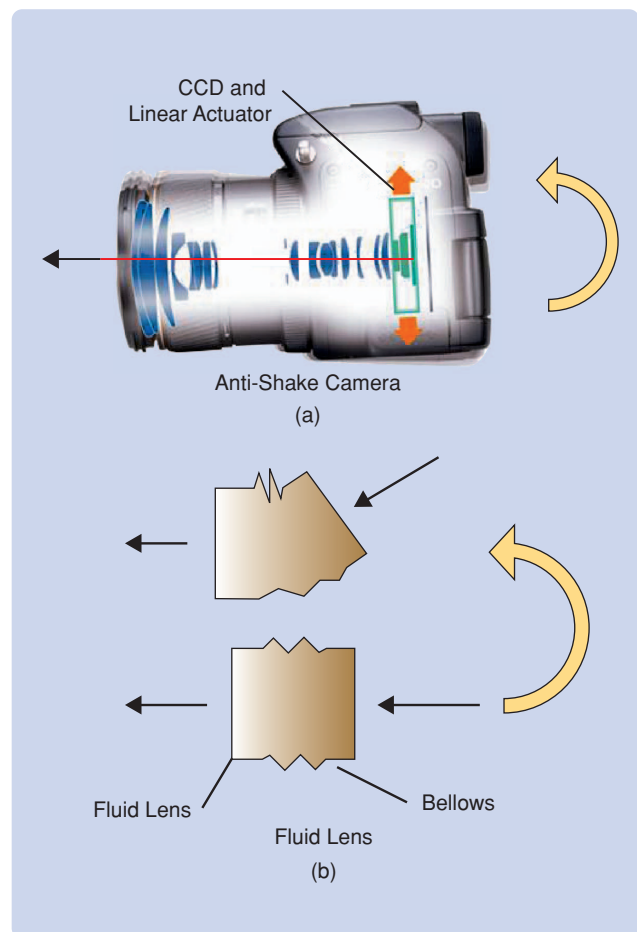
**FIGURE 11** The feedforward approach. When gyros cannot be mounted to provide a direct measurement of the line-of-sight (LOS) motion, a combination of sensors can often be used to reconstruct a signal that can then be used to control the LOS motion. This approach is called feedforward because the gyro output, which now measures the base motion, is fed forward to the loop similar to an input command and thus is not embedded in a feedback loop.

### Feedforward Techniques

An alternative to the common feedback configuration discussed above is the feedforward or *strapdown* configuration, which is effective in many applications. This approach is used when the required corrective motion does not coincide with the LOS being stabilized, and therefore there is no physical place to mount a feedback gyro, a situation commonly encountered in stabilized-mirrors or when an object, such as a small commercial camera, is to be stabilized and there is physically not enough room to mount the gyro on the gimbal. In this approach, an estimate of the LOS motion is constructed from base-mounted or strapdown gyros and a feedforward relative-motion transducer, such as a resolver or encoder, that measures the motion between the corrective element (such as a gimbal) and the base or vehicle to which it is attached [42], [43]. The measured relative motion, which may be either rotary or linear, relates the motion of the corrective element to an equivalent LOS rotation. With these considerations, the general form of the equation defining the feedforward control scheme is given by

$$\underbrace{\theta_{\text{los}}}_{\text{reconstructed LOS signal}} = \underbrace{\theta_{\text{base}}}_{\text{base-mounted strapdown gyro}} + \underbrace{\theta_{\text{gimbal/base}}}_{\text{relative motion feedforward transducer}} \quad (3)$$

The feedforward strapdown system is designed similarly to those with feedback gyros mounted on the gimbal except that the control loop is now closed on the estimated LOS signal as shown in Figure 11. Unlike the feedback approach, the strapdown gyro is not driven to null by the feedback loop and therefore must provide an accurate



**FIGURE 12** Two commercial applications. (a) The antishake camera removes the effect of camera rotation by using a linear actuator to move the CCD detector so that it stays centered on the image. In the fluid lens concept (b), the lens is deformed by an actuator to redirect the line of sight (LOS) based on the output of a gyro. The Dynalens, based on this concept, was successfully incorporated into movie cameras, binoculars, and other instruments. This invention won Juan De La Cierva an Oscar in 1969 for best technical innovation.

**Since they began to be utilized about 100 years ago, ISPs have been used on every type of moving vehicle, from satellites to submarines, and are even used on some handheld and ground-mounted devices.**

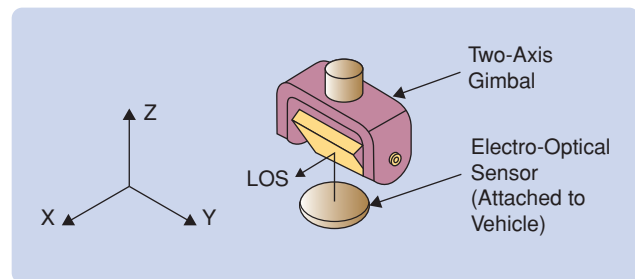
measurement over the entire range of motion of the base rather than just provide adequate sensitivity around the null point. This arrangement requires a gyro with a more accurate scale factor and higher dynamic range than the case in which the gyro is in the feedback loop. In addition, the error characteristics and dynamics of the relative-motion transducer, required to implement the approach, become a primary source of error in the LOS control.

An example of a feedforward system is shown in Figure 12(a), where the motion of the camera body is measured by a pair of gyros attached to the camera body, and a linear mechanism is used to move the light-sensitive CCD detector. The motion of the detector is controlled by a linear actuator and a linear transducer such that the image remains stationary on the detector in the presence of camera body rotation. Since the motion of the detector required to compensate for rotation of the camera is translational, rather than rotational, there is no suitable place to attach a feedback gyro, and therefore the feedforward estimate implied by (3) is constructed using gyros mounted on the camera body and linear transducers that measure the relative detector motion. These devices can achieve a reduction in apparent motion by a factor of three or better using small inexpensive solid-state gyros.

Another feedforward application is the fluid lens concept, which has been successfully used in stabilized binoculars and movie cameras. In this case, a fluid-filled lens is deformed by an actuator as shown in Figure 12(b) to achieve the desired deflection of the LOS. In both of these cases, (3) must be modified to reflect the translational-to-rotational kinematics that relate the linear motion of the corrective element to the LOS rotational motion. While these examples involve moderately low-performance commercial applications, the feedforward concept is routinely applied to high-performance applications such as mirror-stabilized systems, discussed in the following paragraph, as well.

### Mirror Stabilization

When the sensor and optics of the LOS being stabilized is large, a gimballed mirror is often placed in the optical path to stabilize and point the sensor LOS, rather than mass stabilizing the entire assembly, to reduce the size and weight of the system. However, the LOS kinematics of a mirror-stabilized system can be quite involved and, depending on the orientation of the mirror, the FOV required, and the



**FIGURE 13** A two-axis stabilized mirror. Stabilized mirrors are commonly used to reduce the moving mass in optical sights. While there are many configurations, this two-axis arrangement is popular in spite of the fact that it introduces a 2:1 coupling, which complicates stabilization of the line of sight (LOS) about the Y or inner axis, while the image the operator sees does not remain oriented upright as the gimbal is rotated about the Z or outer axis.

LOS being controlled, all of the axes of a mirror-stabilized system may or may not be mass stabilized [44]–[49].

A typical two-axis mirror-stabilized configuration, which is sometimes called a *heliostat* [13], is shown in Figure 13. Motion of the LOS about the Z axis in this case is one to one with the motion of the outer gimbal, and thus is mass stabilized. Motion about the Y or inner axis is far more complex, however, and requires special provisions to stabilize the LOS. Because of the reflective properties of mirrors, the LOS angle  $\theta_{LOS}$  about the Y axis is related to the mirror rotation  $\theta_M$  and base rotation  $\theta_B$  by

$$\theta_{LOS} = 2\theta_M - \theta_B. \quad (4)$$

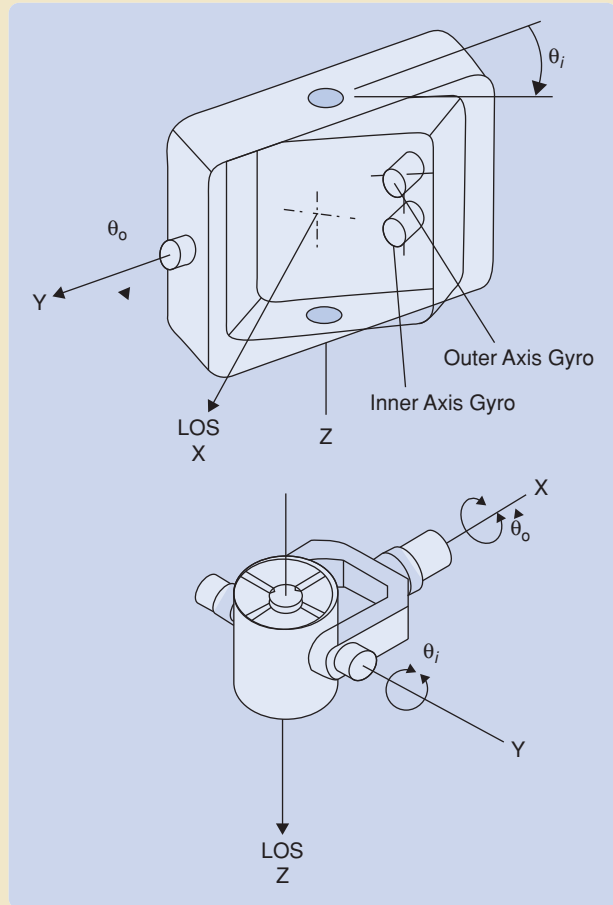
Equation (4) has several implications that greatly complicate the design. The mirror must move with respect to inertial space to hold the LOS stationary about the Y axis and thus is not mass stabilized. A feedback gyro cannot be attached directly to the mirror in this axis to stabilize the LOS since the motion of the mirror must satisfy (4), which involves a 2:1 factor as well as the base motion  $\theta_B$ . Therefore, rather than moving directly with the LOS, either some type of 2:1 drive mechanism must be employed to satisfy (4) or a feedforward scheme must be used to estimate the LOS motion to accomplish the stabilization task [49]. Despite these complications, two-axis mirror stabilization can achieve performance approaching that of mass stabilization in many applications.



## Multiaxis Gimbals

Single-axis gimbals are useful for introducing some of the fundamental issues of ISP design but, unfortunately, are not typically realistic. Most systems require at least two orthogonal stabilized axes to achieve isolation from base motion and to achieve the pointing field-of-regard dictated by the application. For most applications, it suffices to control only the motion about the two axes orthogonal to the LOS. This statement applies to most beam pointing and weapon systems as well as for many imaging systems. However, when orientation of the image is relevant or when rotation of the image about the LOS causes excessive motion at the periphery of the FOV, then a third axis of control must be implemented. In either case, two gyros mounted orthogonal to the LOS are required. These sensors are normally mounted on the innermost gimbal or on the gimbal whose LOS is to be stabilized as shown in Figure S6. When the gyros cannot be mounted on the innermost gimbal orthogonal to the LOS or when LOS control is required about all three axes, three orthogonal gyros are required to provide sufficient information to stabilize the LOS [S18], [S19].

Deciding how many gimbals are required is a tradeoff between the system requirements and the additional size, weight, cost, and structural flexibility inherent with additional gimbals. The decision is usually made to use as few gimbals as possible. However, kinematic phenomena, such as kinematic coupling and gimbal lock, can also influence the suitability and performance of a multi-axis gimbal, and therefore must be included in the configuration tradeoffs and any subsequent simulation or analysis of the gimbal system. We now review techniques for analyzing gimbal kinematics and then consider a two-axis gimbal example that provides insight into many issues common to all multi-axis gimbals. Also, two-axis gimbals are perhaps the most common among all of the various applications because two orthogonal axes are the minimum required to point in any direction in three-dimensional space.



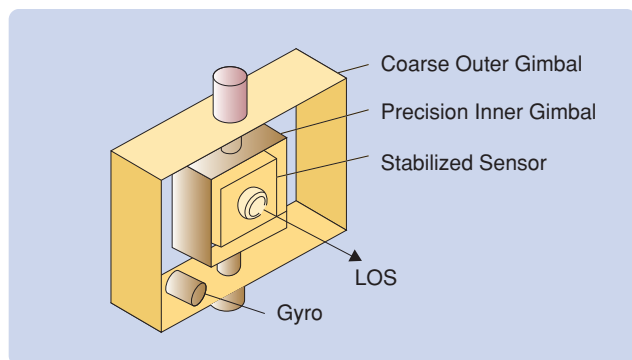
**FIGURE S6** Representative two-axis gimbal configurations. Two-axis gimbal designs are common since two is the minimum number of gimbals required to stabilize and point the line of sight (LOS) about two orthogonal axes, while additional gimbals add size, weight, and cost to the system.

## AUGMENTATION AND ENHANCEMENT TECHNIQUES

Various schemes are used to augment or improve the performance of the stabilization methods described above. These schemes, which include both electromechanical and software techniques, are particularly effective when the stabilized object is an electro-optical sensor, such as a camera, laser, or radar. A few of these approaches are reviewed below.

### Redundant Gimbals

The redundant gimbal configuration shown in Figure 14 is quite common and has several benefits. In this concept, the inner precision gimbal is mass stabilized, and the outer coarse gimbal is driven in closed loop to follow the inner gimbal, as shown in Figure 15. The gyros for this configuration are usually mounted on the fine inner gimbal but can be mounted on the outer gimbal using feedforward techniques with the appropriate number and orientation of the gyros, depending on the specific gimbal configuration. One benefit of the redundant gimbal approach, regardless of gyro placement, is that the outer gimbal



**FIGURE 14** The redundant gimbal concept. One or more redundant coarse outer gimbals are slaved to the inner gyro-stabilized gimbal. This configuration is often used in inertially stabilized platforms (ISPs) to provide protection to the inner gimbal against aerodynamic disturbances and contamination. However, properly configured, it can also reduce kinematic coupling torques on the inner gimbals. Although the gyros are normally mounted orthogonal to the line of sight (LOS) on the innermost gimbal, the concept has also been used in feedforward schemes, where the gyro is mounted on one of the outer gimbals as shown here.

Analyzing the kinematics of a multiaxis gimbal is facilitated using Euler transforms to transform a vector from one reference frame into a second that is rotated with respect to the first. An Euler transform is defined by  $V = Ev$ , where  $v$  is the vector in the first frame,  $V$  is the vector in the second frame, and  $E$  is an Euler matrix, which is a matrix of direction cosines that describe the rotation from the first frame to the second. The technique can be used with any right-handed orthogonal vector such as position or angular velocity, but it should be emphasized that angular displacements are not vectors and cannot be transformed using these techniques. Any sequence of orthogonal rotations can be analyzed by selecting the corresponding sequence of Euler matrices, taking care to multiply them in the proper order depending on the physical gimbal order [S20]–[S24]. The three Euler matrices about the X, Y, and Z axes, respectively, are

$$E_X(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\varphi & S\varphi \\ 0 & -S\varphi & C\varphi \end{bmatrix}, \quad E_Y(\theta) = \begin{bmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix},$$

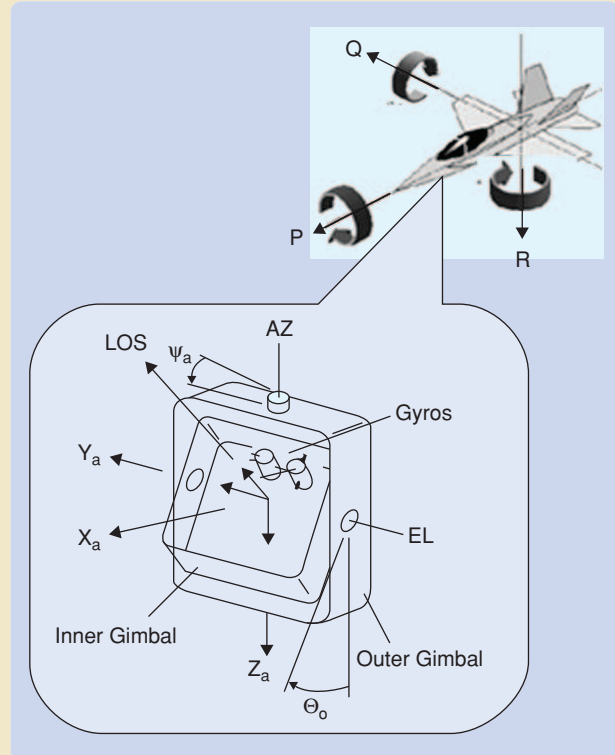
rotation about X                      rotation about Y

$$E_Z(\Psi) = \begin{bmatrix} C\Psi & S\Psi & 0 \\ -S\Psi & C\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

rotation about Z

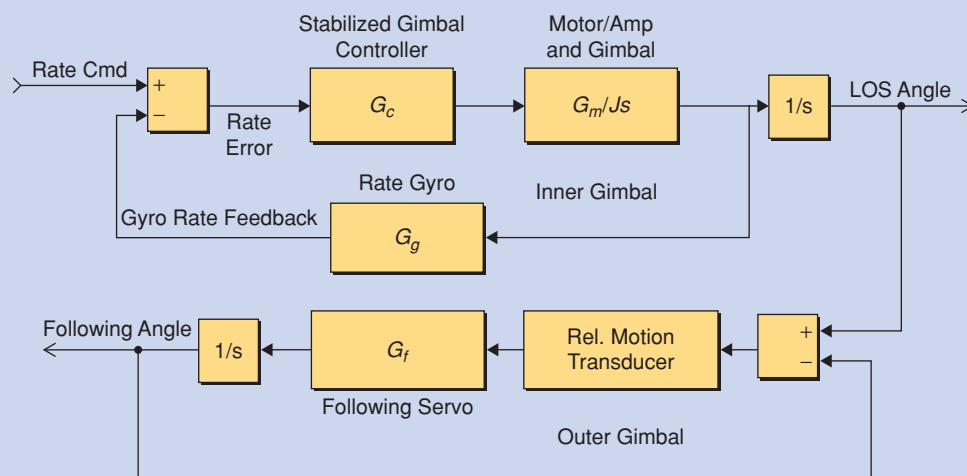
where  $S$  and  $C$  denote sine and cosine, respectively.

To illustrate the transformation process, consider a two-axis az-el gimbal mounted on an aircraft as shown in Figure S7 with the gimbal X axis nominally aligned with the aircraft X axis. We wish to determine the inertial rates that each gimbal must have to hold the LOS stable given the aircraft angular rates  $P$ ,  $Q$ , and  $R$  about the aircraft X, Y, and Z axes, respectively. In this

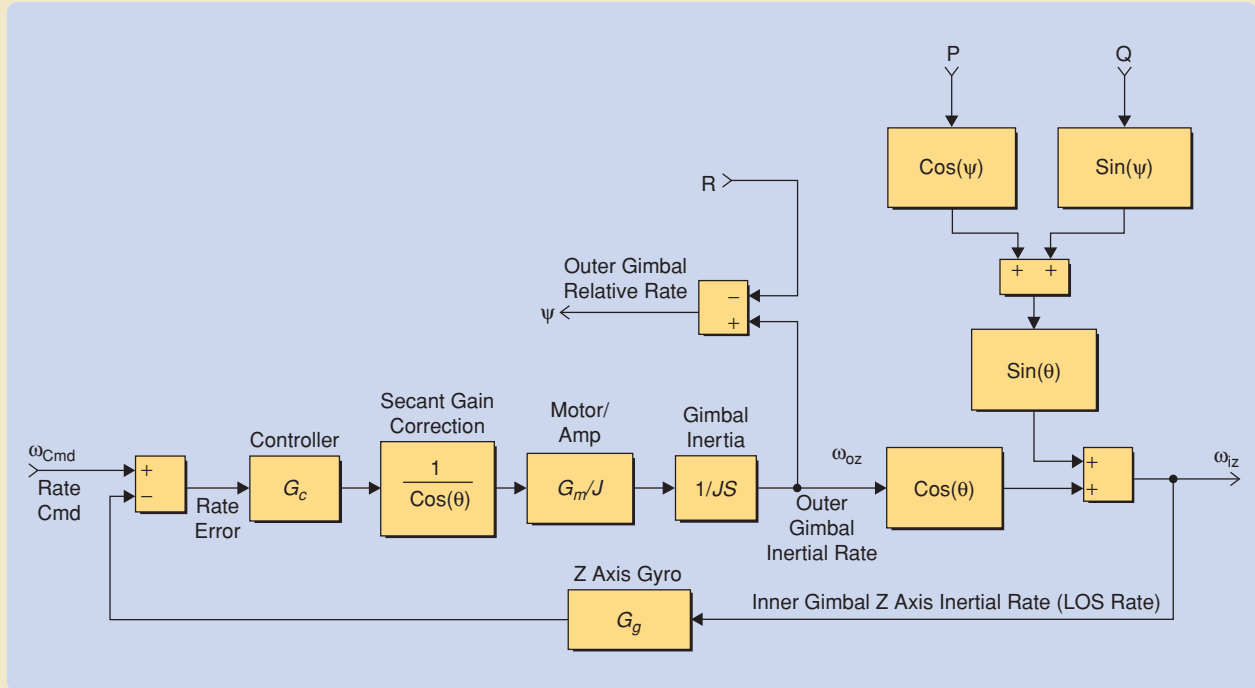


**FIGURE S7** A two-axis gimbal application. An az-el gimbal is attached to an aircraft with two gyros mounted on the inner (EL) axis orthogonal to the line of sight (LOS). The aircraft angular rates are  $P$ ,  $Q$ , and  $R$  about the X, Y, and Z axes of the aircraft, respectively.

case, the LOS is considered to be stable if the motion is removed from the two axes orthogonal to the LOS. Motion about the third axis X about the LOS cannot be removed with a two-axis gimbal.



**FIGURE 15** Redundant gimbal servo block diagram. The coarse outer gimbal servo is configured to follow the fine-stabilized inner gimbal. This configuration is commonly used when an outer windshroud is required to protect the inner stabilized gimbal from aerodynamic torques. A similar servo configuration is sometimes used when the inner gimbal or servo is configured as an image motion-compensation system.



**FIGURE S8** Outer (AZ- $\psi$ ) axis stabilization control system with kinematics for the gimbal shown in Figure S7. With the exception of the secant gain correction, the sine and cosine functions are natural kinematics, typical of all two-axis gimbals, which couple the body rates,  $P$ ,  $Q$ , and  $R$  into the outer gimbal dynamics. As shown, the gyro senses all of the kinematics, and attempts to hold the line of sight (LOS) steady in spite of the kinematic coupling torques. The secant gain is an electronic correction, which is sometimes used to compensate for the natural cosine function that occurs in the loop. However, the secant correction compensates only for the gain change in the loop, and does not compensate for the kinematic coupling torques seen by the outer gimbal.

Using the Euler matrix for rotation about the Z axis for the gimbal order shown, the outer gimbal rates are written in terms of the body rates  $P$ ,  $Q$ , and  $R$  as

$$\begin{bmatrix} \omega_{ox} \\ \omega_{oy} \\ \omega_{oz} \end{bmatrix} = \begin{bmatrix} C\psi & S\psi & 0 \\ -S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P \\ Q \\ R + \dot{\psi} \end{bmatrix} = \begin{bmatrix} PC\psi + QS\psi \\ -PS\psi + QC\psi \\ R + \dot{\psi} \end{bmatrix}, \quad (S1)$$

where the relative gimbal rate  $\dot{\psi}$  is added to the vehicle rate to account for gimbal rotation. Similarly, the inner gimbal rates are given by

$$\begin{bmatrix} \omega_{ix} \\ \omega_{iy} \\ \omega_{iz} \end{bmatrix} = \begin{bmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix} \begin{bmatrix} \omega_{ox} \\ \omega_{oy} + \dot{\theta} \\ \omega_{oz} \end{bmatrix} = \begin{bmatrix} C\theta(PC\psi + QS\psi) - S\theta(R + \dot{\psi}) \\ -PS\psi + QC\psi + \dot{\theta} \\ S\theta(PC\psi + QS\psi) + C\theta(R + \dot{\psi}) \end{bmatrix}. \quad (S2)$$

To stabilize the inner gimbal LOS about the Y and Z axes, the control system attempts to drive the inner gimbal inertial rates  $\omega_{iy}$  and  $\omega_{iz}$  to zero, which, from (S2), requires that the relative gimbal rates satisfy

$$\dot{\theta} = PS\psi - QC\psi \quad (S3)$$

can be configured to shield the inner gimbal from aerodynamic disturbances. Also, seals necessary to keep out rain or contaminants are incorporated in the outer gimbal, thus allowing the inner gimbal to be designed with only the friction from its suspension. This approach, however, has another benefit that may be more subtle, namely, that the redundant gimbal design can be used to reduce kinematic-coupling disturbances (see "Multiaxis Gimbals") in two-axis gimbals. Properly configured, the

inner gimbals are never displaced more than a few degrees relative to each other, and, therefore, the large kinematic-coupling torques are borne by the outer gimbal. Thus, the effect of the torques on the fine inner gimbal is attenuated.

### Image Motion Compensation

Figure 16 shows an entirely different electromechanical augmentation technique, which can achieve extremely

and

$$\dot{\Psi} = -T\theta(PC\Psi + QS\Psi) - R. \quad (S4)$$

By using these relative gimbal rates along with (S1) and (S2), it follows that the LOS is held stable if the inertial rates of the inner and outer gimbals are given by

$$\omega_{iy} = 0 \quad (S5)$$

and

$$\omega_{oz} = R + \dot{\Psi} = -T\theta(PC\Psi + QS\Psi). \quad (S6)$$

Equations (S5) and (S6) show that the inner gimbal remains mass stabilized and does not need to move in space to stabilize the LOS. However, the outer gimbal must move to stabilize the LOS and the amount of motion required increases with the tangent of the inner gimbal angle. This result, known as *kinematic coupling*, affects all two-axis gimbals and culminates in complete loss of control, or *gimbal lock*, as the inner gimbal angle  $\theta$  approaches  $90^\circ$ . At angles less than  $90^\circ$ , the gimbal can be controlled if enough motor torque is available, but kinematic coupling degrades the stabilization performance because any requirement to accelerate the gimbal in inertial space has the same effect on the apparent motion of LOS as a torque disturbance acting in the opposite direction. In other words, adequate torque must be provided by the outer gimbal motors to avoid complete loss of control, but the kinematic coupling still acts like any other torque disturbance and causes unwanted LOS motion.

Now consider what happens as  $\theta$  approaches  $90^\circ$  in Figure S7. The outer gimbal motor and rotating axis are now aligned with the LOS and thus are orthogonal to the gyro-sensitive axis. Therefore, the LOS motion is no longer sensed in this axis and no amount of motor torque can provide any correction, which constitutes gimbal lock.

A block diagram of the outer gimbal control system including the kinematics described above included is shown in Figure S8. The secant gain correction shown is added to cancel the cosine introduced by the kinematics. However, while the secant gain correction maintains a constant loop gain as the gimbal angle changes which keeps the control system gain from changing with gimbal angle, this technique cannot counteract the disturbance effects of kinematic coupling or prevent gimbal lock.

While feedforward approaches have been employed to counter kinematic coupling, the most common solution to both kinematic coupling and gimbal lock is to add additional gimbals. The system tradeoff, however, involves the additional size, weight, and structural flexibility attendant with each gimbal axis added. Surveys and discussions of different gimbal arrangements can be found in [S18] and [S26]–[S28].

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precise stabilization. In this approach, the residual stabilization error in a conventional mass-stabilized or mirror-stabilized system is further reduced by incorporating a secondary pointing mechanism such as a fast-steering mirror (FSM) at a point in the optical path where the ray bundle has a small diameter. This location in the optical path is critical because the FSM design relies on small size to

achieve high bandwidth and precision over a small angular range. Typically, stabilization is augmented by driving the FSM as shown in Figure 17, where the residual gimbal gyro error signal is used as a command input to the FSM feedforward loop [50]–[52]. In addition to improving stabilization, IMC mechanisms are also used to achieve the highspeed step-and-stare scan profiles discussed below.



**Requirements for ISPs vary widely, but they all have the common goal of holding or controlling the line of sight of one object relative to another object or inertial space.**

### Electronic and Software Augmentation Techniques

The above IMC concepts rely on electromechanical devices. However, over the last decade or so, signal processing speed has increased such that an inertial stabilization system can be augmented by software techniques, often in real time. Several system concepts are available, some of which make use of the image motion, while others attempt to remove its effects. While the algorithms used in these systems differ in detail, they all correlate information gathered at one instant to information gathered at another instant to either remove the effects of the motion between instants, stitch together a larger image, or, in some cases, provide a combination of the two functions.

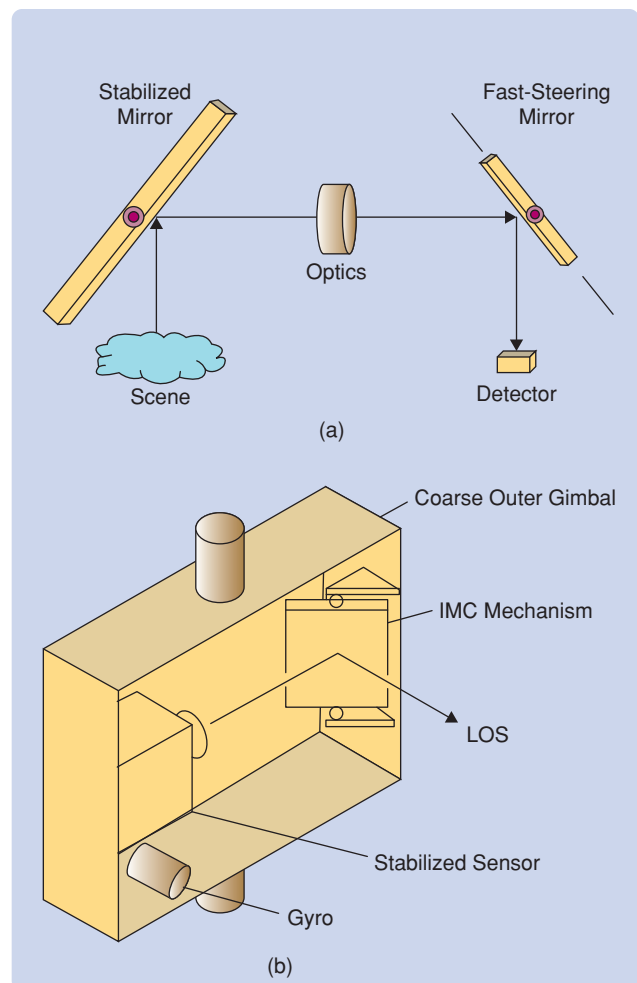
Deblurring algorithms attempt to remove the effects of image motion from one frame to the next. Some of these systems work in conjunction with the gimbal control system by using the gyros or other available sensors such as navigation system information to reconstruct the deblurred image. Alternative approaches that rely only on frame-to-frame image correlation are fundamentally bandwidth limited by the imaging system frame rate [53]–[56].

Examples of systems that stitch or blend information to achieve high resolution and a large FOV include step and stare or scanning electro-optical systems, which operate similarly to *synthetic aperture radar* (SAR) [57]. The scan direction and velocity can be controlled by means of the gimbal or determined solely by the vehicle motion as illustrated in Figure 18. When the scan profile is determined by the vehicle motion, sensors that measure the vehicle motion are used by the processing algorithm to provide software stabilization to the information collected, which is the approach is taken by many SAR systems. The step-and-stare concept, shown in Figure 18(c), uses the gimbal or an FSM to step or scan the LOS of an electro-optical sensor in a pattern over a target [58], [59]. The step-and-stare system pauses at each step to let the sensor integrate the image energy. The separate images are then stitched together to form a larger image called a *mosaic*. Thus the system basically

trades resolution for time since it takes more time to collect the necessary smaller high-resolution images than it would to take one large image with a low-resolution system. However, the effectively higher resolution also requires good stabilization while the sensor is integrating the energy at each step. The distance moved by the LOS between each step does not necessarily have to be highly accurate but must ensure overlap between frames so that the software can correlate the image and stitch a complete image together.

### ISP COMPONENTS AND DESIGN TRADEOFFS

Most ISP systems require gyros, some type of suspension device such as bearings, motors, or actuators, relative-motion transducers, and additional electromechanical components. The characteristics of available devices can be a key factor in the system configuration and subsequent



**FIGURE 16** Image motion compensation (IMC). IMC techniques are often used to improve performance or to step or scan the line of sight (LOS) with higher dynamic motion profiles than the primary stabilization system is capable of providing. Fast-steering mirrors are used in IMC configurations with both (a) mirror-stabilized and (b) mass-stabilized systems as shown. IMC is also used to compensate for structural flexibility in the gimbal and optical system.

performance [60], [61]. Although a thorough treatment would require a voluminous book, we provide an introductory overview of the salient features of some of the most critical of these devices as well as the system tradeoffs that must be made to make a proper selection.

### Gyros

The gyro is probably the most critical component in an ISP, and these devices are available in a wide range of size, cost, and performance options [4]. Probably the two most critical gyro performance parameters for ISPs are bandwidth and electrical noise. Since the gyro is usually in the feedback control loop, its bandwidth or dynamic response characteristics directly impact the achievable overall loop bandwidth. To avoid being the limiting factor in the loop bandwidth, the gyro bandwidth must be higher than that of the other dominant loop dynamics such as torsional resonances (see “Structural Effects”). For highly responsive systems, the gyro bandwidth is often several hundred hertz. The gyro noise over this bandwidth couples directly into the control system and therefore must be limited to a fraction of the overall LOS control specification. The gyro scale-factor accuracy and bias are often secondary considerations in ISPs but can be important for some applications.

Many types of gyros are used in ISPs. While the most demanding applications still use spinning mass rate-integrating displacement gyros because of their low noise and good dynamic characteristics, fiber optic rate gyros are increasingly used in ISPs because they also have good dynamic characteristics as well as a longer usable life expectancy than most spinning mass gyros. Ring laser gyros, which are displacement devices used extensively in inertial navigation applications because of their excellent drift and scale-factor characteristics, are seldom used in ISPs due to their large size and high quantization noise.

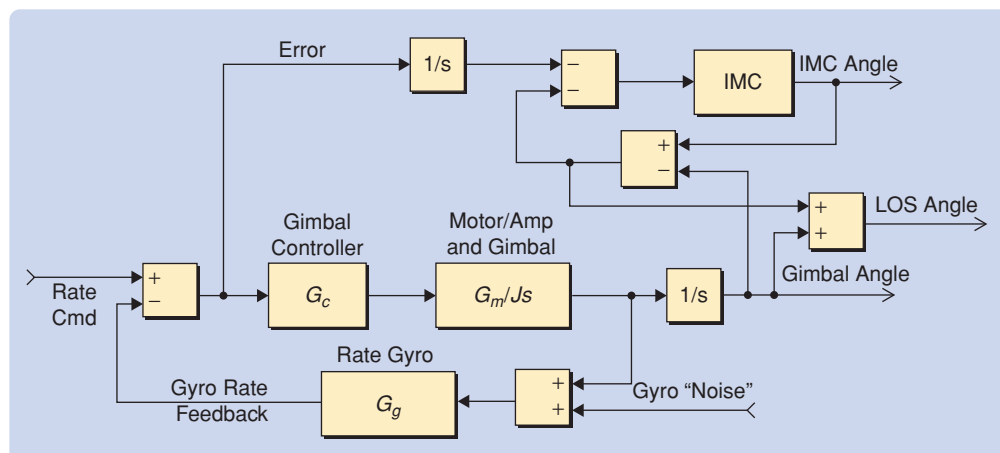
For moderate-to-low performance requirements, the small size and low cost of the quartz and silicon Coriolis rate gyros are deciding factors in many applications. For example, handheld camera applications would probably not have an image stabilization feature if it weren’t for the availability of small inexpensive gyros. Surveys of gyro technologies are given in [4]–[6].

### Bearings and Suspension

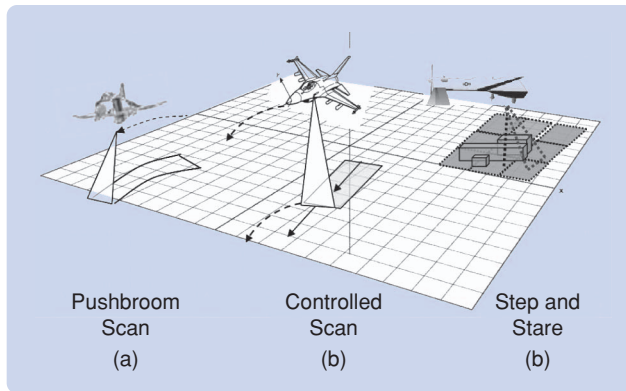
The critical parameters that affect the selection of the suspension technique are friction and structural stiffness. These requirements present a tradeoff because a stiff suspension often contributes more friction. Ordinary ball bearings are used in the majority of ISP applications, often preloaded to remove free motion and carefully implemented to avoid binding and increased friction over temperature. When the required gimbal motion is small, flexures or flex pivots can sometimes be used. Although these devices have very low friction, their spring effect must be accommodated by the control system. Additional suspension devices that can be considered when the application demands low friction and warrants additional size and complexity include magnetic suspension and gas-lubricated bearings, although adequate suspension stiffness can be difficult to achieve with these techniques [62], [63].

### Motors and Actuators

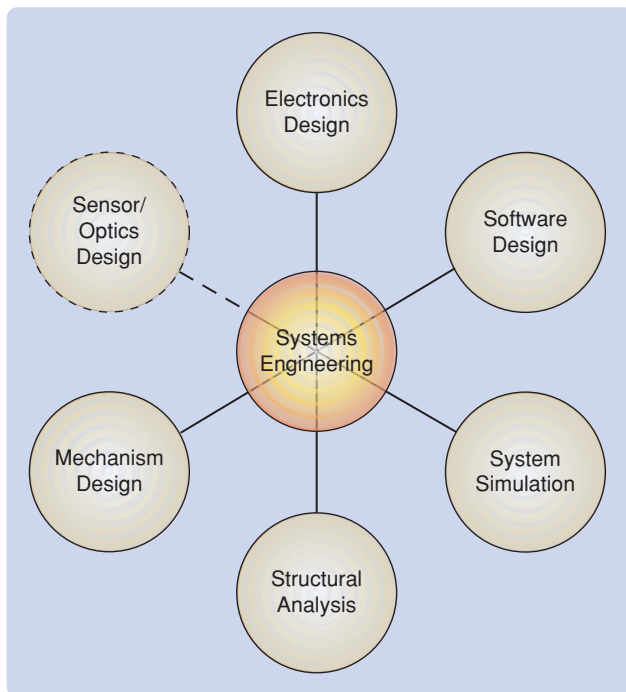
ISPs require actuators that provide adequate torque and velocity without excessive cogging or hysteresis. The actuator torque must be sufficient to suppress torque disturbances and to respond to command inputs to move the gimbal and LOS as required by the application. Another component of torque, which is sometimes overlooked, is the capacity to respond to high-frequency noise, inevitable in high-bandwidth systems, without saturating or overdriving the actuator.



**FIGURE 17** Image motion compensation (IMC) block diagram. The IMC servo controls the position of the IMC device relative to the primary gimbal to further reduce the error or jitter in the line of sight (LOS). The IMC system can also be given independent commands to step or scan the LOS or to compensate for other effects such as structural flexibility.



**FIGURE 18** Typical scan patterns. The line of sight (LOS) can be controlled relative to a moving vehicle or relative to a geographically fixed feature to collect energy or data for various types of sensors. Often, several small high-resolution images are combined or stitched together to form a larger image, thus creating a synthetic aperture effect. The scan pattern is determined by the vehicle motion with the (a) pushbroom scan but can be (b) controlled to provide some other pattern or (c) stepped in a predetermined sequence.



**FIGURE 19** The design team. The design and development of an inertially stabilized platform usually requires all of the engineering functions shown along with close coordination with other subsystem teams, such as the sensor-optics design team shown in the dashed circle.

Since the maximum required velocity of an ISP is usually low, rarely exceeding 100°/s, gearing can be considered in an attempt to reduce the size and weight of the actuator, particularly when the torque requirements are demanding. For systems having low angular displacement requirements, steel bands are sometimes used to achieve a transmission ratio while avoiding the

backlash and indexing anomalies characteristic of gear teeth. However, there are several other disadvantages of gearing in ISPs. Regardless of what mechanism is used to achieve the gear ratio, the reaction torques from a geared actuator constitute an equivalent torque disturbance that can degrade stabilization performance. Also, most gearing arrangements inevitably introduce additional friction and torsional resonances in the system, and therefore direct drive actuators are preferred in most cases except when practical configuration issues dictate otherwise.

Probably the most commonly used direct-drive actuators are permanent magnet dc torque motors wound with a high pole count to achieve high torque at low speeds. Voice-coil motors and similar limited motion permanent magnet dc devices are ideally suited for limited rotation applications because they are simple to control, have almost no cogging, and usually have characteristic response time constants of less than a millisecond.

### Relative-Motion Transducers

In addition to inertial sensors, ISPs usually require relative-motion transducers, sometimes referred to as *gimbal pickoff* sensors, to measure the displacement between gimbals and between the gimbal and its base. These measurements are required for pointing the gimbal relative to the base and also to determine where the LOS is pointed. Also, in feedforward configurations, relative motion transducers may be part of the fundamental stabilization as discussed above. Accuracy and resolution are the basic performance parameters that must be traded off against size and cost. Accuracy is required, for example, to control the aim point of an ISP when used in a pointing loop configuration, whereas resolution is significant when LOS stability or aimpoint jitter is impacted by the transducer.

With all relative measurement devices, accuracy is influenced by mounting eccentricity and other mechanical interface tolerances, whereas resolution is determined primarily by the device itself and electronic interfaces such as converters and analog signal noise. Accurately measuring over a small angular displacement is easier and there are more transducer options available than for measuring over an extended displacement. For applications where measurement over more than a few degrees is required, inductive devices such as multiwound resolvers and inductosyns are traditionally used in gimbals because they are rugged and, when properly implemented, can achieve arc-second accuracies since the multiple windings tend to average the effects of eccentricity. These devices also achieve good resolution limited primarily by analog-to-digital converters and electrical noise levels.

In the past decade, incremental encoders that can achieve sub- $\mu$ rad resolution by interpolation of the analog detector signals have been developed. These devices are

inherently smaller than inductive devices. However, achieving arc-second accuracies from an encoder requires special mounting or multiple read-heads to average the mounting eccentricity. Encoders also have a somewhat better dynamic response than inductive devices, whose effective bandwidth is generally limited to about a tenth of their excitation frequency. Calibration of the measurement system, once the device is mounted, is also an option that can be used to achieve a given accuracy requirement with either encoders or inductive devices but requires adequate repeatability and cannot be used to improve resolution [47].

### Rotating Electrical Interfaces

Another issue that must be addressed in ISP design is how to best traverse rotating interfaces with electrical power and signals without adding excessive friction and cable torques to the mechanism. For limited angle requirements, this function is often accomplished with flexible cables that are allowed enough freedom to flex at the interface. For extended angular travel, however, twist capsules and slip rings are often required. The twist capsule, which is a self-contained flex cable assembly specially designed to have low friction and spring torque, has an extended, but constrained, angular travel. A slip ring, on the other hand, is a set of conducting rings with brushes that allows unlimited rotation but adds size and cost to the design. When the circuits that traverse the gimbal interface consist entirely of signals, rather than power circuits, compact fiber-optic slip rings can be considered instead of brushes.

### CONCLUSIONS

We introduced some of the basic principles, techniques, and key design issues common to ISPs used in many diverse scientific, military, and commercial applications, and we touched on some of the less intuitive effects that must be dealt with. Many of these effects can influence the initial configuration of the system as well as design details later in the design process. The successful design of an ISP usually requires a multidisciplinary design team such as the one depicted in Figure 19. The design of an ISP must often be closely coordinated with that of other major subsystems such as the primary sensor and the optics.

The role of the systems engineer in the design process is perhaps the most critical because other members of the team may not be aware of the consequences of many of the effects discussed above. For example, the structural analyst might provide a dynamic analysis of the structure in the system but he or she would probably not be expected to fully understand the interaction of the structure with the control system. Thus, it is usually up to the systems engineer to understand the more subtle effects along with the system requirements and communicate to other team members how these requirements relate to specific hardware and software requirements.

A final comment regarding a critical aspect of the ISP design process is the need to understand the dynamic environment. Both the rigid body motion of the host vehicle as well as the frequency spectrum of the rotational and translational motion that the system must operate and the motion of intended targets can impact many decisions in the design process, including the system kinematics and dynamics, structural design, component selection, and even the basic system configuration. All too often, this information is either not utilized or difficult to acquire, primarily because its effects are not well understood. However, the aggressive acquisition of a complete understanding of the full spectrum of rotational and translational operating environment is perhaps one of the most effective risk-mitigation measures that an ISP design project can take.

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