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Review

Static balancing with elastic systems of *DELTA* parallel robots

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ABSTRACT

This paper presents new design solutions and the corresponding mathematical models for static balancing of *DELTA* parallel robot. The static balancing of parallel robots is a more difficult task than the serial robots one, because the parallel robots consist of multi-loop spatial kinematic chains. Generally, these robots can be easily statically balanced only approximately.

To statically balance the *DELTA* robots, two constructive solutions are proposed: (*i*) a new constructive solution for full balancing and (*ii*) a simplified full balancing version, with an acceptable balancing accuracy.

The term full balancing is used here to denote that the masses of all links and springs are taken into account. To obtain full balance of the *DELTA* parallel robot, three elastic systems for each connecting kinematic chain are assembled. It is further noted that in this study only the balancing by using the *zero-free-length* elastic systems is considered.

The static balancing result according to the first solution is very good, but the structure is not simple, using many additional links. The second simplified version uses one single elastic system for each connecting kinematic chain.

Finally, a case study is presented in order to evaluate the performances of the simplified proposed version. It was found that the maximum driving moment decreases about 20 times relative to unbalanced version.

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1. Introduction

This paper proposed two constructive solutions for static balancing of the DELTA parallel robot, namely (*i*) a new constructive solution for full balancing, and (*ii*) a simplified full balancing version, with an acceptable balancing accuracy. The term full balancing is used here to denote that the masses of all links and springs are taken into account. To obtain full balance of the DELTA parallel robot, three elastic systems for each connecting kinematic chain are assembled. It is further noted that in this study, only balancing by using the zero-free-length elastic systems is considered.

The paper is structured as follows: Section 2 presents the structure of the *DELTA* parallel robot, while Sections 3 and 4 present the static balancing of a rocking element around a horizontal fixed axis and around a fixed point respectively. In Section 5 is shown the first constructive solution while Section 6 shows the second one. Subsequently, in Section 7 the kinematic analysis is formulated and Section 8 deals with the zero-free-length elastic systems. Finally, Section 9 presents an illustrative example and conclusions are presented in Section 10.

The problem of static balancing came up long time ago as a necessity to reduce the driving energy consumption to any mechanical system that does not work in horizontal plane. For example, nowadays, the static balancing became a usual task for mechanical structures of industrial robots [1].

A spatial parallel robot consists of two platforms, one fixed and the other mobile; in the case of non-redundant robots, platforms are linked in parallel by three to six connecting kinematic chains. The connecting kinematic chains can be planar or spatial and each one has one or two driving pairs [2]. Sometimes, the connecting chains are referred as legs [3].

The static balancing of serial robots was studied in many papers [4–8, 25], but static balancing of parallel robots is a relatively new direction of study. One of the researchers that started the studying of static balancing of parallel robots is Prof. Clement Gosselin from Laval University in Quebec, Canada. He wrote together with his coworkers many papers [3, 9–14] about static balancing of parallel robots of different structures, with ideal pairs, without friction. In paper [9], the parallel structure is made of planar kinematic chains with two DOF each, resembling the pantograph mechanisms. For each leg, two springs are used in order to statically balance the pantograph mechanism.

Static balancing of the spatial parallel robots of general shape, e.g. Gough-Stewart platform, can be done only partially, by using complex elastic systems [11]. The other parallel structures, e.g. *DELTA* robot [15] 3 with only 3 DOF all of translational type, can be statically balanced much simpler. In paper [16] the parallel connecting kinematic chains of *Penta-G* mechanism with 5 DOF, used like a haptic device, are statically balanced by the aid of helical springs.

Another author that studied the static balancing of parallel robots is Prof. Vigen Arakelian from INSA Rennes, France. In paper [17], a new parallel robot called *PAMINSA*, with a structure composed of connecting kinematic chains of planar pantograph shape, is presented. The static balancing of *PAMINSA* robot is done by using counterweights. In paper [18], a pantograph with 2 DOF connected between the fixed platform and the mobile platform is used again to statically balance the *DELTA* robots. In paper [19], another variant with a multi-loop pantograph linkage, joined between the fixed and the mobile platforms, in their mass centers, is presented. The same idea is used in paper [20], where the static balancing of a hexapod parallel robot is done by using a multi-loop pantograph and a counterweight. In paper [21], a design solution which is using a multi-loop pantograph statically balanced by a normal helical spring, is presented.

2. Structure of the *DELTA* parallel robot

The parallel robot *DELTA* [15] consists of a fixed platform (**1**) and a mobile platform (**6**); between them are connected equally radial and angular distanced, three spatial identically connecting kinematic chains (Fig. 1). Each connecting kinematic chain has five elements as follows: a link (**2**) joined in *A* to the fixed platform (**1**), a transversal link (**3**) joined to link (**2**) by revolute pair *B* with axis parallel to the axis of joint *A*, and two identical links (**4**) and (**4'**), which together with link (**5**) create a parallelogram.

Axes of *D* and *D′* revolute joints, between identical links (**4**) and (**4'**) respectively, and link (**5**), are parallel one with the other and also are parallel with axes of *C* and *C′* revolute pairs between identical links (**4**) and (**4'**) respectively, and the link (**3**). All these axes are perpendicular to the axes of *A* and *B* joints. The mobile platform (**6**) is connected to the links (**5**) by revolute pairs denoted by *E*, which axes are parallel with axes of joints *A* and joints *B*.

The *DELTA* parallel robot has 3 DOF, the driving pairs being the joints *A* between fixed platform (**1**) and links denoted by (**2**). All movements of mobile platform (**6**) are of translational type. All joints are considered ideal ones, without friction; the frictional forces cannot be taken into consideration in equilibrium equations because their senses depend on the senses of relative motions.

Static balancing of a *DELTA* parallel robot requires the static balancing of three types of elements:

i) rocking links around fixed horizontal axes, with movements in vertical planes;
ii) rocking links around two perpendicular and concurrent axes, one of the axis horizontal, with movements in space;
iii) links with translational movements along vertical directions.

Links (**2**) and (**3**) are of the first type, while links (**4**) and (**4**) are of the second type (Fig. 1). Mobile platform (**6**) and links (**5**) are of the third type. It is considered that the center of mass of link (**5**) is located upon the axis of joint *E*.

In order to design the static balancing systems it is considered that the axes of joints *A* are horizontal. If the fixed platform (**1**) is not horizontal then the following proposed static balancing systems will not work properly.

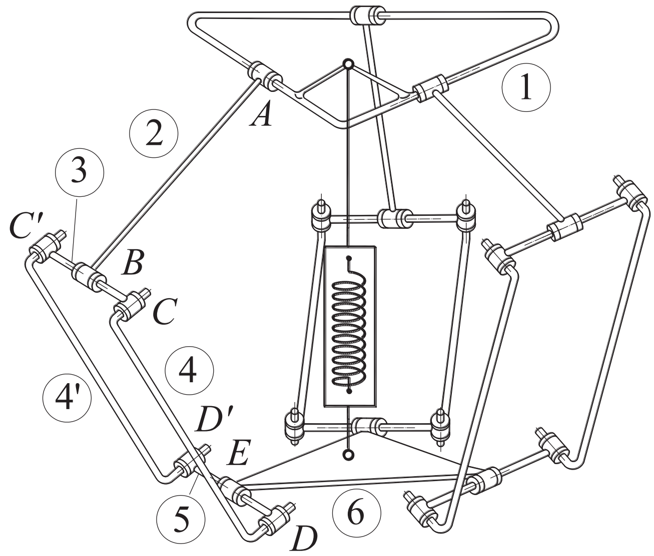

Fig. 1. The *DELTA* parallel robot.

To compute the characteristics of elastic systems used to statically balance the *DELTA* robot, the masses of links (**2**), (**4**) and (**4'**) are considered as been statically concentrated at two points on the axes of the corresponding joints [27–29]. These two points and the mass center must be collinear. Also, the masses of elastic systems are considered statically concentrated in the same way. The variation of the values of concentrated masses with respect to the elastic system length is neglected. The masses of links (**3**) and (**5**) are considered statically concentrated in their mass centers. The mass of the mobile platform (**6**) is considered statically concentrated on three radial and angular equally distanced points, each placed on the axes of the rotational pairs *E*.

Subsequently, a new design solution for full static balancing of the *DELTA* robot is proposed and also, a second simplified version, for approximate full static balancing, is presented in the following. By full static balancing all the weight forces of all links including springs are considered.

3. Static balancing of a rocking element around a horizontal fixed axis

The problem of balancing the weight forces of rocking elements around a horizontal fixed axis was studied by many researchers [4–8, 22, 23] who proposed several solutions, more or less complicated, with different levels of accuracy.

Let us consider link (**2**) which rotates around a horizontal fixed axis of joint *A* (Fig. 2). The simplest solution to statically balance this link is to assembly a *zero-free-length* elastic system joined at one end to the link in point *V* and at the other end joined in fixed point

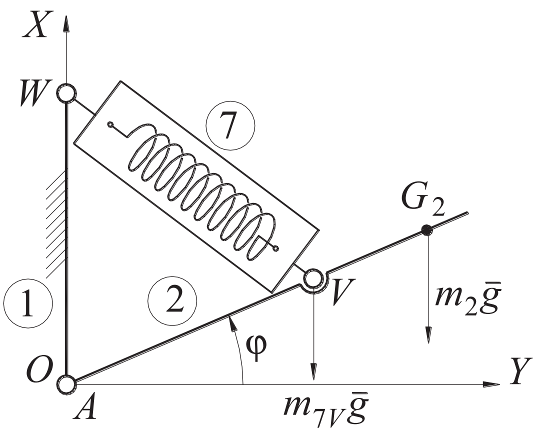

Fig. 2. Static balancing of a rocking element around a horizontal fixed axis.

W [6]. The mass center G_2 of link (**2**) is supposed to be in the plane of axes of joints A and V. In these conditions, the equilibrium equation of forces' moments which act on link (**2**) is:

$$F_{e7}\frac{X_W AV\cos\varphi}{VW} - (m_2 AG_2 + m_{7V} AV) g\cos\varphi = 0, \tag{1}$$

where:

- φ is the positioning angle of vector $\overline{AG_2}$, measured from OY axis of fixed system of coordinates;
- VW is the length of elastic system (**7**).

If the force developed by elastic system (**7**) has the expression:

$$F_{e7} = k_{e7}\, VW = k_7\sqrt{(X_W - AV\sin\varphi)^2 + AV^2\cos^2\varphi}, \tag{2}$$

then the spring rate is:

$$k_{e7} = \frac{(m_2 AG_2 + m_{7V} AV) g}{X_W\, AV}. \tag{3}$$

The static balancing is theoretically exact for any position of link, i.e. $\varphi \in [0,\ 2\pi]$.

4. Static balancing of an element which is moving around a fixed point

To statically balance the weight forces of links (**4**) and (**4'**) of *DELTA* robot (Fig. 1), the very well known balancing solutions [4, 6–8, 23, 24] cannot be used. Link (**4**) has a complex spherical movement, around axes of joint C, and together with link (**3**) around horizontal axes of revolute pair B (Figs. 1 and 3b). These two axes are concurrent and perpendicular to one another. A third rotation, around the axis which is passing through the center of mass G_4 of the link and the intersection point of the first two, does not influence the static balancing [25].

The problem of static balancing of a link which is rocking around two axes, one horizontal and another in vertical position, was studied by G. J. Walsh, D. A. Streit and B. J. Gilmore [26]. This approach is not useful for the present case due to the movement of the link around the vertical axis in the gravitational field does not modify its potential energy.

When the rotation axis of a link is inclined with an angle ψ with respect to the horizontal plane, then the vector of its weigh force is not in the rotation plane (Fig. 3b). Therefore, the elastic system for static balancing should compensate the effect of the projection of the weight force onto the plane of rotation of the link:

$$G_4 = m_4 g\cos\psi.$$

For this reason, it is necessary that the position of the revolute pair axis R (Fig. 3a) to be relocated, because the elastic system (**10**) for static balancing of the link (**4**), and the link analogous to frame, is moving according to the magnitude of the ψ angle. This is possible by adding an intermediary link (**8**) connected to the frame by joint F and to the slider element (**9**) by joint H (Fig. 3b).

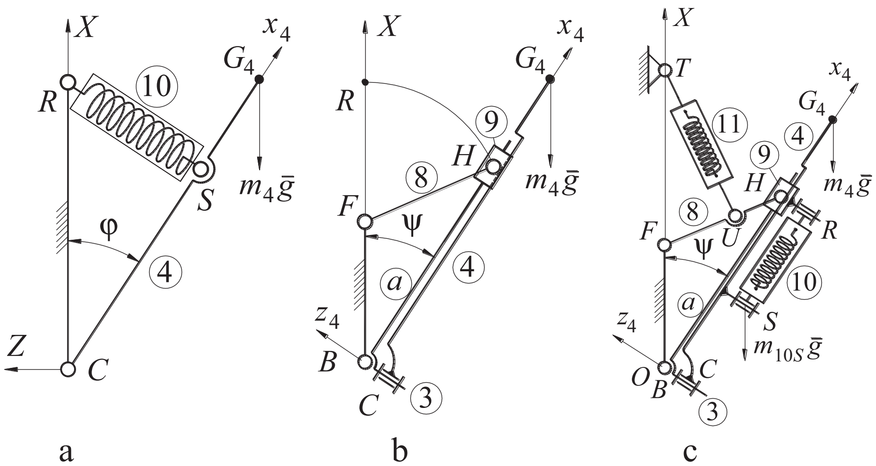

Fig. 3. Static balancing of an element which is rocking around a fixed point in space.

The slider (**9**) is moving upon the bar (***a***) fixed to link (**3**), which is rocking around the fixed axis of joint *B*, together with link (**4**). Axes of *B* and *F* joints are in the same vertical plane.

The *zero-free-length* elastic system (**10**) is joined to the slider (**9**) by revolute pair *R* and to the link (**4**) by joint *S* (Figs. 3c and 4). The axes of *H*, *B* and *F* joints are parallel. Same are the axes of *C*, *S* and *R* joints. The axes of *B* and *C* joints are perpendicular to one another.

From triangle *BFH* results (Fig. 3b):

$$BH^2 + BF^2 - 2BH\,BF\cos\psi - FH^2 = 0, \tag{4}$$

and taking into account that:

$$BH = BR\cos\psi, \tag{5}$$

one obtains:

$$BF = \frac{BR}{2} = FH. \tag{6}$$

To statically balance link (**4**), the elastic system (**10**) is joined (Figs. 3c and 4) between slider (**9**) and link (**4**). The stiffness constant of this elastic system has the expression:

$$k_{e10} = \frac{g(m_4 CG_4 + m_{10S} CS)}{CS(BF + FH)}. \tag{7}$$

In order to balance the weight forces of added elements: bar (***a***) fixed to link (**3**), link (**8**), slider (**9**) and elastic system (**10**), an elastic system (**11**) joined to link (**2**) by joint *T* and to the link (**8**) by joint *U* is added (Figs. 3c and 4). The axes of joints *B*, *F* and *T* are in the same vertical plane.

The stiffness constant of the elastic system (**11**) is:

$$k_{e11} = \frac{g\left(m_8 FG_8 + \left(m_9 + m_{3Q} + m_{10R}\right) FH + m_{11U} FU + m_4 CG_4 + m_{10S} CS\right)}{FU \quad FT}. \tag{8}$$

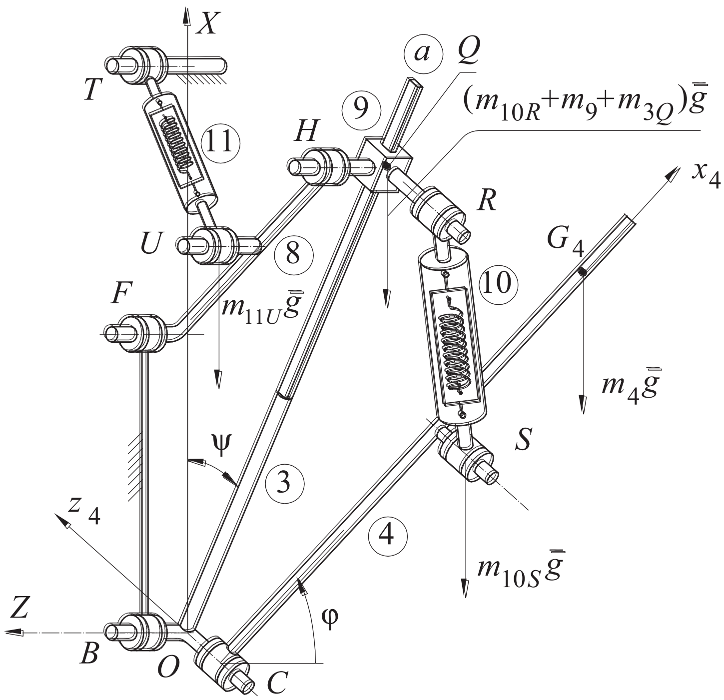

Fig. 4. Static balancing of an element which is rocking around a fixed point.

The portion m_{3Q} of the mass of link (**3**), supposed to be concentrated at the point Q of intersection between axes of joints R and H, is function of the variable distance BH. This disposition of the link mass is statically equivalent to the original link if the total mass is the same and if the position of the mass center is unchanged. To compute the stiffness constant k_{e11}, a mean value of this distance should be considered. For this reason, the balancing is approximately, but the error is small because the mass of link (**3**) is much smaller than the other elements' masses. This is an auxiliary element because it has the role to guide the slider (**9**) only. Anyway, the stroke of slider (**9**) along the bar (***a***) is very small compared to the length of bar (a). For example, for the angle $\psi = 0.523599$ the distance BH is reduced with only 13.39%.

5. Full static balancing of *DELTA* parallel robot

To fully statically balance of this type of parallel robot, three *zero-free-length* elastic systems (Fig. 5), denoted by (**7**), (**10**) and (**11**), are necessary for each connecting chain.

The following considerations are true if the axes of driving pairs are into the horizontal plane only.

5.1. Static balancing of links (4), (4'), (5) and (6)

To statically balance the weight forces of links (**4**),(**4'**), (**5**) and (**6**), two auxiliary elements must be added (Fig. 5):

- link (**2'**), with length equal with the length of link (**2**),
- link (**12**), which together with links (**2**), (**2'**) and fixed platform (**1**) is forming a parallelogram with two vertical sides ($B'F' = AT$); this link has a circular-translational motion.

Also, links (**8**) and (**9**), and a bar (***a***) fixed (welded) to the link (**3**), must be considered.

The kinematic chain of added links is provided with a joint R for the elastic system (**10**), articulated by joint S to link (**4**). Stiffness constant of this elastic system is:

$$k_{e10} = \frac{g\left(2m_4CG_4 + \left(m_5 + \frac{m_6}{3}\right)CD + m_{10S}CS\right)}{CS(BF + FH)}. \tag{9}$$

It is assumed that the weight force of mobile platform (**6**) is equally distributed on all three links denoted by (**5**).

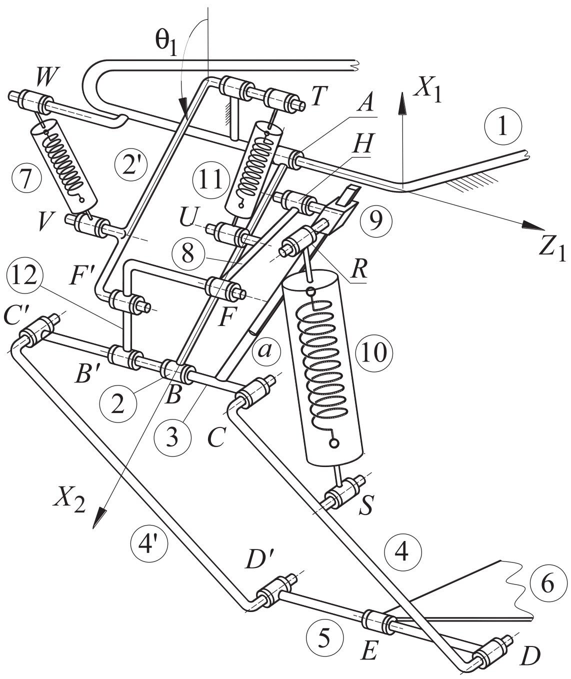

Fig. 5. Full static balancing of *DELTA* parallel robot.

5.2. Static balancing of links (3), (4), (4'), (5), (6), (8), (9), (10) and (12)

The elastic system (11), used to balance the weight forces of above mentioned elements, is joined to the mobile platform by revolute pair T and to the link (8) by revolute pair U (Fig. 6). Its stiffness constant is:

$$k_{e11} = \frac{g\left(m_8 FG_8 + m_{11} FU + (m_9 + m_{3Q} + m_{10R}) FH + 2m_4 CG_4 + m_{10S} CS + \left(m_4 + \frac{m_6}{3}\right) CD\right)}{FU\ \ FT}. \tag{10}$$

5.3. Full static balancing of the connecting kinematic chain consists of links: (2), (2'), (3), (4), (4'), (5), (6), (8), (9), (10), (11) and (12)

The balancing of the weights forces of the mentioned links is done by the elastic force of elastic system (7) (Fig. 5). According to the virtual work principle [27–29], the needed elastic force F_{e7} generated by the elastic system (7) is given by the solution of equation:

$$\begin{aligned} F_{e7}\frac{dVW}{d\theta_1} + g\Big(&\frac{dX_{1V}}{d\theta_1} m_{7V} + \frac{dX_{1G_2}}{d\theta_1}(m_2 + m_{2'}) + \frac{dX_{1G_3}}{d\theta_1} m_3 + 2\frac{dX_{1G_4}}{d\theta_1} m_4 + \\ &+ \frac{dX_{1G_5}}{d\theta_1}\left(m_5 + \frac{m_6}{3}\right) + \frac{dX_{1G_8}}{d\theta_1} m_8 + \frac{dX_{1G_9}}{d\theta_1} m_9 + \frac{dX_{1R}}{d\theta_1} m_{10R} + \frac{dX_{1S}}{d\theta_1} m_{10S} + \\ &+ \frac{dX_{1U}}{d\theta_1} m_{11U}\Big) = 0, \end{aligned} \tag{11}$$

whence the spring stiffness results:

$$k_{e7} = \frac{F_{e7}}{VW}, \tag{12}$$

where:

$$VW = \sqrt{(Y_{1V} - AW)^2 + X_{1V}^2}, \tag{13}$$

$$X_{1V} = AV \cos\theta_1,$$

$$Y_{1V} = AV \sin\theta_1.$$

The angle θ_1 is one of the generalized coordinates of parallel robot, i.e. is the variable of driving pair A (Fig. 5).

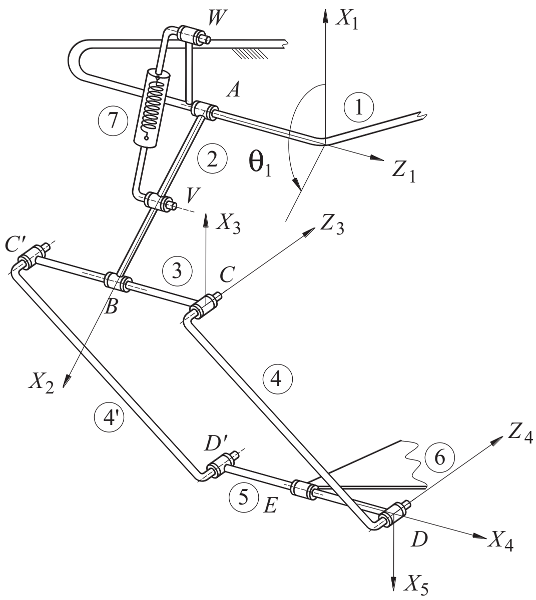

Fig. 6. Approximately static balancing of *DELTA* parallel robot.

6. Approximate static balancing of the elements of *DELTA* type parallel robot

The above solution for full static balancing of *DELTA* robot has a considerable complexity. In certain cases, a very simple balancing technique of approximate type could be accepted. This solution is much simpler and leads to very good results.

The static balancing of *DELTA* parallel robot can be done approximately, with small errors, if only the three *zero-free-length* elastic systems (**7**), joined between the fixed platform (**1**) and link (**2**), are assembled (Fig. 6). The static balancing is theoretically exact if the mechanism remains symmetric, i.e. the movement of the mobile platform (**6**) is along the vertical symmetry axis, so that the generalized coordinates of driving pairs are equal. The balancing is full because the weight forces of all elements of *DELTA* robot are taken into consideration. The mass of the mobile platform includes the mass of the end-effector and possibly half of the working object mass.

By using the virtual work principle [27–29] is obtained:

$$F_{e7}\frac{\mathrm{d}VW}{\mathrm{d}\theta_1}+g\left(\frac{\mathrm{d}X_{1G_2}}{\mathrm{d}\theta_1}m_2+\frac{\mathrm{d}X_{1G_3}}{\mathrm{d}\theta_1}m_3+2\frac{\mathrm{d}X_{1G_4}}{\mathrm{d}\theta_1}m_4+\frac{\mathrm{d}X_{1G_5}}{\mathrm{d}\theta_1}\left(m_5+\frac{m_6}{3}\right)+\frac{\mathrm{d}X_{1V}}{\mathrm{d}\theta_1}m_{7V}\right)=0. \tag{14}$$

Eq. (14) has three unknowns, namely: stiffness constant k_{e7}, and the distances *AV* and *AW*. If the distances *AV* and *AW* are imposed from a constructive point of view, then the stiffness constant k_{e7} of elastic system (**7**) results:

$$k_{e7}=-VW\frac{g\left(\frac{\mathrm{d}X_{1G_2}}{\mathrm{d}\theta_1}m_2+\frac{\mathrm{d}X_{1G_3}}{\mathrm{d}\theta_1}m_3+2\frac{\mathrm{d}X_{1G_4}}{\mathrm{d}\theta_1}m_4+\frac{\mathrm{d}X_{1G_5}}{\mathrm{d}\theta_1}\left(m_5+\frac{m_6}{3}\right)+\frac{\mathrm{d}X_{1V}}{\mathrm{d}\theta_1}m_{7V}\right)}{\frac{\mathrm{d}VW}{\mathrm{d}\theta_1}}. \tag{15}$$

The distance *VW* is computed with formula (13).

7. Kinematic analysis of the mechanism of *DELTA* parallel robot

The spatial mechanism of this kind of robot consists of 17 links connected by 21 revolute pairs (Fig. 7). The upper platform (**1**) is considered to be fixed. Three of the pairs are driving pairs, the ones denoted by *A* between the fixed platform and the links (**2**). Also, two independent loops can be identified in the structure of spatial mechanism; four mobile links are counted twice in adjacent independent loops.

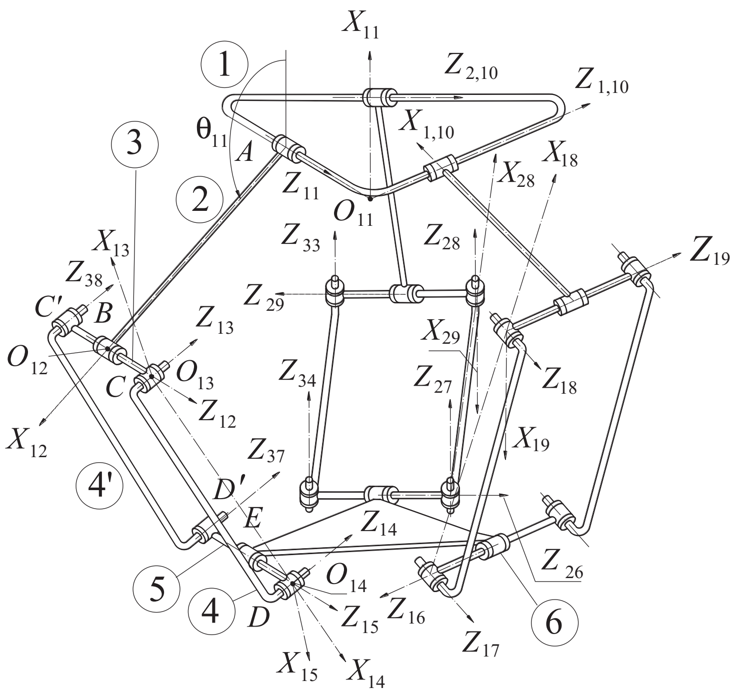

Fig. 7. The Denavit–Hartenberg coordinate axes systems allocation.

For kinematic analysis of this mechanism the Denavit–Hartenberg formalism is proposed, which requires a Cartesian coordinate system attached to each link [30]. Each axis is denoted by two subscripts [32]: the first one corresponds to the number of the loop and the second indicates the number of the kinematic pair from the current loop. The numbering starts from the fixed platform.

The kinematic dimensions of the links and the location of the axes of mobile Cartesian coordinate systems (Fig. 7) are in accordance with indications from [30].

In these conditions, the matrix of transformation of the coordinates of a point from $O_{ij}X_{ij}Y_{ij}Z_{ij}$ coordinate system to $O_{i,j+1}X_{i,j+1}Y_{i,j+1}Z_{i,j+1}$ coordinate system is:

$$\mathbf{A}_{ij} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ a_{ij}\cos\theta_{ij} & \cos\theta_{ij} & -\sin\theta_{ij}\cos\alpha_{ij} & \sin\theta_{ij}\sin\alpha_{ij} \\ a_{ij}\sin\theta_{ij} & \sin\theta_{ij} & \cos\theta_{ij}\cos\alpha_{ij} & -\cos\theta_{ij}\sin\alpha_{ij} \\ s_{ij} & 0 & \sin\alpha_{ij} & \cos\alpha_{ij} \end{Vmatrix}, \tag{16}$$

and the coordinates transformation equation in matrix form is:

$$\begin{Vmatrix} 1 \\ X_{ijP} \\ Y_{ijP} \\ Z_{ijP} \end{Vmatrix} = \mathbf{A}_{ij} \begin{Vmatrix} 1 \\ X_{i,j+1P} \\ Y_{i,j+1P} \\ Z_{i,j+1P} \end{Vmatrix}. \tag{17}$$

The Denavit–Hartenberg matrix equations [30–32], written for the two independent loops of spatial parallel mechanism (Fig. 7), are:

$$\begin{aligned} &\mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{16}\mathbf{A}_{17}\mathbf{A}_{18}\mathbf{A}_{19}\mathbf{A}_{1,10} = \mathbf{I}; \\ &\mathbf{A}_{1,10}^{-1}\mathbf{A}_{19}^{-1}\mathbf{A}_{23}\mathbf{A}_{24}\mathbf{A}_{16}^{-1}\mathbf{A}_{26}\mathbf{A}_{27}\mathbf{A}_{28}\mathbf{A}_{29}\mathbf{A}_{2,10} = \mathbf{I}. \end{aligned} \tag{18}$$

The matrix Eqs. (18) are equivalent with a system of 12 scalar nonlinear equations which is solved by a common Newton-Raphson numerical method, with respect to the following unknowns: θ_{12}, θ_{13}, θ_{14}, θ_{15}, θ_{16}, θ_{17}, θ_{18}, θ_{19}, θ_{26}, θ_{27}, θ_{28} and θ_{29}. The angles θ_{11}, $\theta_{1,10}$ and $\theta_{2,10}$ are parameters of driving pairs, i.e. are the generalized coordinates of *DELTA* robot mechanism.

By differentiating the matrix Eqs. (18) with respect to the angle θ_{11}, results a linear system of equations with following unknowns: $\frac{d\theta_{12}}{d\theta_{11}}, \frac{d\theta_{13}}{d\theta_{11}}, \frac{d\theta_{14}}{d\theta_{11}}, \frac{d\theta_{15}}{d\theta_{11}}, \frac{d\theta_{16}}{d\theta_{11}}, \frac{d\theta_{17}}{d\theta_{11}}, \frac{d\theta_{18}}{d\theta_{11}}, \frac{d\theta_{19}}{d\theta_{11}}, \frac{d\theta_{26}}{d\theta_{11}}, \frac{d\theta_{27}}{d\theta_{11}}, \frac{d\theta_{28}}{d\theta_{11}}$, and $\frac{d\theta_{29}}{d\theta_{11}}$. The angular velocities of driving pairs, $\frac{d\theta_{11}}{dt}$, $\frac{d\theta_{1,10}}{dt}$ and $\frac{d\theta_{2,10}}{dt}$, are known. If it is supposed that $\frac{d\theta_{11}}{dt} = 1$, then the values of the angular generalized velocities $\frac{d\theta_{1,10}}{d\theta_{11}}$ and $\frac{d\theta_{2,10}}{d\theta_{11}}$ are also known.

The coordinates of the mass center G_{1i} of element (i) from loop 1, with respect to $O_{11}X_{11}Y_{11}Z_{11}$ coordinate system attached to fixed platform, is computed with following relation:

$$\begin{Vmatrix} 1 \\ X_{11G_{1i}} \\ Y_{11G_{1i}} \\ Z_{11G_{1i}} \end{Vmatrix} = \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\ \ldots\ \mathbf{A}_{1,i-1} \begin{Vmatrix} 1 \\ X_{1iG_{1i}} \\ Y_{1iG_{1i}} \\ Z_{1iG_{1i}} \end{Vmatrix}, \quad i = \overline{2,\ 10}, \tag{19}$$

where $X_{1iG_{1i}}$, $Y_{1iG_{1i}}$, and $Z_{1iG_{1i}}$ represent the coordinates of this center of mass in mobile coordinate system $O_{1i}X_{1i}Y_{1i}Z_{1i}$ attached to element (i).

The derivatives of the mass center coordinates G_{1i} with respect to angle θ_{11} are:

$$\begin{Vmatrix} 0 \\ \dfrac{dX_{11G_{1i}}}{d\theta_{11}} \\ \dfrac{dY_{11G_{1i}}}{d\theta_{11}} \\ \dfrac{dZ_{11G_{1i}}}{d\theta_{11}} \end{Vmatrix} = \left(\frac{\partial \mathbf{A}_{11}}{\partial \theta_{11}} \mathbf{A}_{12}\mathbf{A}_{13}\ldots\mathbf{A}_{1,i-1} + \mathbf{A}_{11}\frac{\partial \mathbf{A}_{12}}{\partial \theta_{12}}\mathbf{A}_{13}\ldots\mathbf{A}_{1,i-1}\frac{d\theta_{12}}{d\theta_{11}} + \ \ldots\ + + \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\ldots\mathbf{A}_{1,i-2}\frac{\partial \mathbf{A}_{1,i-1}}{\partial \theta_{1,i-1}}\frac{d\theta_{1,i-1}}{d\theta_{11}} \right) \begin{Vmatrix} 1 \\ X_{1iG_{1i}} \\ Y_{1iG_{1i}} \\ Z_{1iG_{1i}} \end{Vmatrix}. \tag{20}$$

Similarly, the coordinates of the center of mass G_{2j} of element (j) from loop 2, with respect to coordinate system $O_{11}X_{11}Y_{11}Z_{11}$ attached to the fixed platform, are calculated with the following relation:

$$\begin{Vmatrix} 1 \\ X_{21G_{2j}} \\ Y_{21G_{2j}} \\ Z_{21G_{2j}} \end{Vmatrix} = \mathbf{A}_{1,10}^{-1}\mathbf{A}_{19}^{-1}\mathbf{A}_{23}\mathbf{A}_{24}\ \ldots\ \mathbf{A}_{2,j-1} \begin{Vmatrix} 1 \\ X_{2jG_{2j}} \\ Y_{2jG_{2j}} \\ Z_{2jG_{2j}} \end{Vmatrix}. \tag{21}$$

To calculate the characteristics of the elastic system (**7**) (Fig. 6), the masses of the connecting kinematic chain links and one third of the mass of mobile platform (**6**) are concentrated at the point O_{12} of element (**2**) (Fig. 7). If the axes $O_{11}Z_{11}$, $O_{1,10}Z_{1,10}$ and $O_{2,10}Z_{2,10}$ are horizontal, then the weight forces of the mechanism elements are parallel with axis $O_{11}X_{11}$.

The equivalence of the weight forces is done by equating the mechanical work of the equivalent weight force $\widetilde{m}g$ and the mechanical work of the weight forces:

$$\widetilde{m}\frac{dX_{11O_{12}}}{d\theta_{11}} = m_2\frac{dX_{11G_{12}}}{d\theta_{11}} + m_3\frac{dX_{11G_{13}}}{d\theta_{11}} + 2m_4\frac{dX_{11G_{14}}}{d\theta_{11}} + m_5\frac{dX_{11G_{15}}}{d\theta_{11}} + \frac{m_6}{3}\frac{dX_{11G_{16}}}{d\theta_{11}}. \tag{22}$$

The movements of links (**4**) and (**4'**) are identical because elements (**3**), (**4**), (**5**) and (**4'**) create a parallelogram.

The vertical projection of the velocity of the mobile platform gravity center is equal with the vertical projection of the velocity of the origin O_{14}.

The stiffness constant of the elastic system (**7**), assembled like in Fig. 6, is computed with next formula:

$$k_{e7} = \frac{(\widetilde{m}a_{12} + m_{7A}AW)g}{AV\,AW}, \tag{23}$$

where m_{7A} represents a portion of the mass of elastic system (7) concentrated at lower extremity *J*.

For symmetry reason, all the three connection kinematic chains of robot are identical. Also, the three elastic systems used for static balancing the robot *DELTA* are identical.

If the lengths of links (**2**) and (**4**) are equal ($a_{12} = a_{14}$) and the mobile platform (**6**) moves along the vertical direction, so that axes $O_{11}Z_{11}$ and $O_{15}Z_{15}$ of joints *A* and *E*, respectively, are in the same vertical plane, then the equivalent mass $\widetilde{m}$ concentrated at point O_{12} is constant and the static balancing is theoretically exact.

When the movement of mobile platform (**6**) is done so that the axes of joints *A* and *E* are not in the same vertical plane, then the static balancing of *DELTA* robot is approximately. The static un-equilibrium value is function of: the lengths a_{11} and a_{13} of links (**2**) and (**4**) respectively, the position of mobile platform (**6**) and the co-ordinates of origin O_{14} with respect to fixed Cartesian system $O_{11}X_{11}Y_{11}Z_{11}$ (Fig. 9).

8. *Zero-free-length* elastic systems

The *zero-free-length* elastic systems are elastic systems which are working at extension, with straight characteristics passing through the origin of the axes of system (F_e, l_e), where l_e is the length of elastic system and F_e is the elastic force developed by the elastic system [33].

These elastic systems have various design solutions [4, 5, 34]; a common one is using a compression spring (**1**) joined between links (**2**) and (**3**) which have relative translational movement (Fig. 8a). The joining points of elastic system are the revolute kinematic pairs *A* and *C*, with parallel axes [34].

The advantages of this elastic system are:

- a long stroke of relative movement $l_{e\ max}$ between links (**2**) and (**3**);
- if the friction forces from prismatic pairs *B* are neglected, then the proportionality between elastic force of system and the deformation l_e is theoretically exact.

The main disadvantage is the possible buckling of compression spring (Fig. 8a). If the buckling occurs, supplementary friction forces between spring (**1**) and case (**2**) appear, and also in prismatic pairs *B*, so that the elastic system characteristic is changed. This disadvantage can be overcome by elastic system from Fig. 8b, where the helical spring (**1**) is of the extension type.

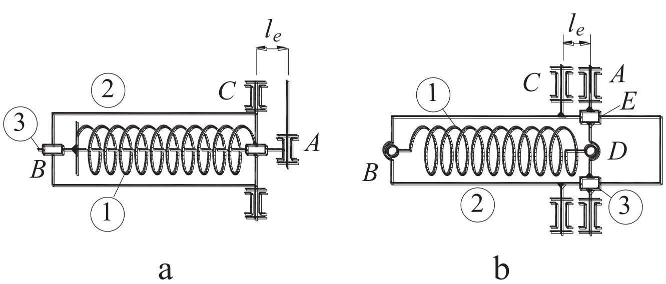

Fig. 8. *Zero-free-length* elastic systems.

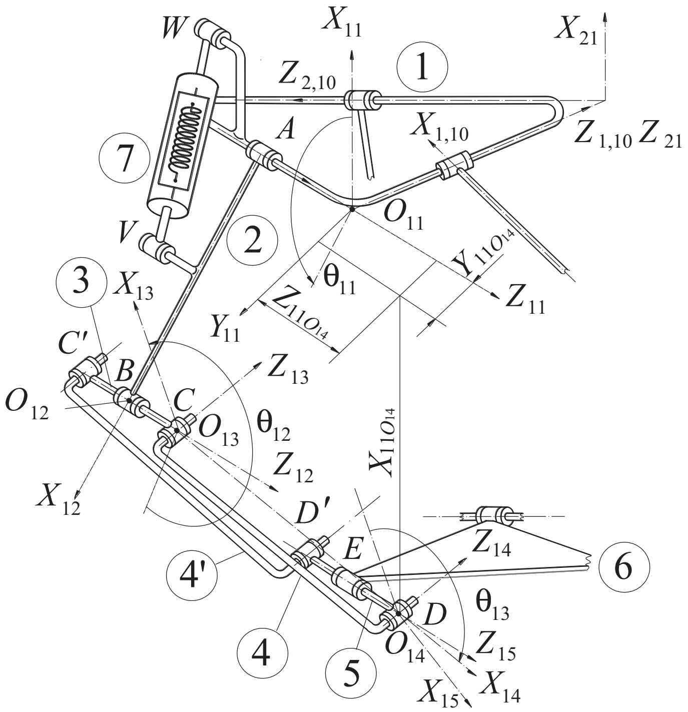

Fig. 9. Location of mobile platform with respect to the frame coordinate system.

9. Example

To illustrate the performances of the proposed static balancing method of parallel robot of *DELTA* type, an example was solved. The following dimensions of *DELTA* parallel robot (Fig. 7) were considered: $AB = a_{11} = 0.5$ m, $a_{12} = 0.0$ m, $CD = a_{13} = 0.5$ m, $\alpha_{11} = 0.0$, $\alpha_{12} = 0.5\pi$, $\alpha_{13} = 0.0$, $s_{11} = -0.2$ m, $s_{12} = 0.15$ m, and $s_{13} = 0.0$ m. The masses of links are: $m_2 = 1.225$ kg, $m_3 = m_5 = 0.4901$ kg, $m_4 = m_{4'} = 1.225$ kg, and $m_6 = 3.687$ kg.

For the first connecting kinematic chain, the range of the independent variable θ_{11} is $\theta_{11\ \min} = -4.354817$ rad to $\theta_{11\ \max} = -3.45915$ rad.

For each connecting kinematic chain (Fig. 6), a *zero-free-length* elastic system (Fig. 8) with stiffness constant $k_{s7} = 583.699$ Nm and mass $m_7 = 1.8$ kg, is used. The stiffness constant k_{s7} has been calculated with formula (11). The positions of the joints *V* and *W* (Fig. 9) are defined by the following dimensions: $AV = 0.25$ m and $AW = 0.3$ m.

When θ_{13} reach π value, the unbalancing moment is vanishing. For $\theta_{13} = \pi$, when the mobile platform is moving only to the vertical direction, the balancing is exact and the maximum of computed unbalancing moment is 0.006 Nm. This small value is due to inevitable errors of numerical computation.

The reduction of the driving moment magnitudes, as a result of the static balancing, is obvious and is shown in the diagrams from Figs. 10, 11 and 12. The maximum driving moment decreases from 40 Nm to 2 Nm approximately.

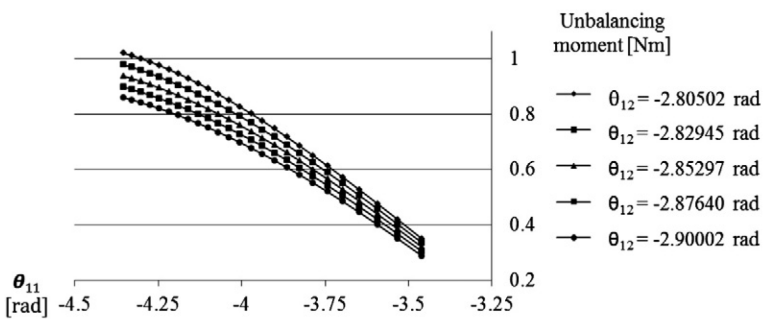

Fig. 10. Diagrams of variation of unbalancing moment from joint *A*, with respect to the angle θ_{11} of link (**2**), for $\theta_{13} = 3.5$ rad.

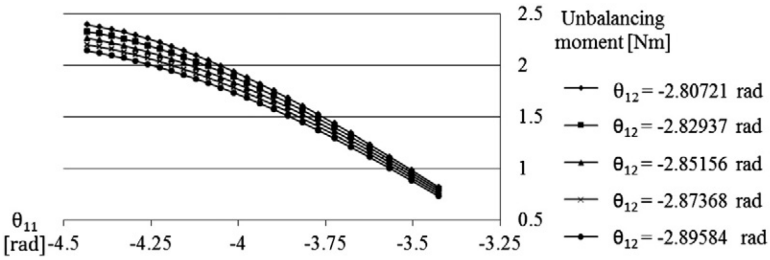

Fig. 11. Diagrams of variation of unbalancing moment from joint *A*, with respect to the angle θ_{11} of link (2), for $\theta_{13} = 2.6$ rad.

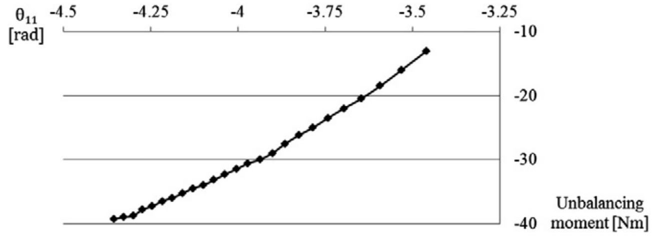

Fig. 12. Diagrams of variation of driving moment from joint *A*, when the *DELTA* robot is statically unbalanced, for $\theta_{13} = \pi$ rad.

10. Conclusions

In this paper, two new constructive solutions for static balancing by elastic systems of the *DELTA* parallel robots are presented: (*i*) a first solution for quasi-exact full balancing and (*ii*) a second solution for approximate full balancing, where the term full balancing refers to taking into account the masses of all links and springs, and were all elastic systems used for static balancing are of zero-free-length type.

For quasi-exact balancing, three elastic systems for each connecting kinematic chain of the *DELTA* parallel robot are used. The static balancing is very well realized, but the structure is not simple, using many additional links. The static balancing is quasi-exact because for one joint an average configuration was to be selected for computation of the spring stiffness.

For approximate balancing, only an elastic system for each connecting kinematic chain of the *DELTA* parallel robot is used. The static balancing is realized theoretically exactly if the mobile platform is moving onto vertical direction. When the movement of the mobile platform is random, the driving moment required for acting is very small, about 5% from the driving moment required for acting the unbalanced robot.

Appendix A

AB	distance between points or joints *A* and *B*,
AG_i	distance between point *A* and the center of mass G_i of link (*i*),
F_{ei}	elastic force of elastic system (*i*),
g	gravity acceleration,
k_{ei}	stiffness constant of elastic system (*i*),
k_{sj}	stiffness constant of helical spring (*j*),
m_i	mass of link (*i*), considered to be concentrated in its mass center G_i,
m_{jA}	a portion of the mass of element (*j*) supposed to be concentrated in its extremity *A*,
X_A	the coordinate onto *OX* direction of the point *A*,
Y_A	the coordinate onto *OY* direction of the point *A*,

References

[1] A. Fattah, S.K. Agrawal, Gravity-balancing of classes of industrial robots, Proceedings of the IEEE International Conference on Robotics and Automation, Orlando, Florida, May, 2006, 2006 (dc identifier citation 0-7803-9505-0/06: 2872-2877).
[2] J.-P. Merlet, Parallel Robots, 2nd edition Springer, Dordrecht, 2006, ISBN 1-4020-4132-2.
[3] C.M. Gosselin, J. Wang, Static balancing of spatial six-degree-of-freedom parallel mechanisms with revolute actuators, J. Robot. Syst. 17 (3) (2000) 159–170.
[4] E. Shin, D. Streit, Spring equilibrator theory for static balancing of planar pantograph linkages, Mech. Mach. Theory 26 (7) (1991) 645–657.
[5] T. Rahman, R. Ramanathan, R. Seliktar, W. Harwin, A simple technique to passively gravity balance articulated mechanisms, J. Mech. Des. 117 (4) (1995) 655–658.
[6] I. Simionescu, L. Ciupitu, The static balancing of the industrial robot arms: Part I: discrete balancing, Mech. Mach. Theory 35 (9) (2000) 1287–1298.

[7] I. Simionescu, L. Ciupitu, The static balancing of the industrial robot arms: Part II: continuous balancing, Mech. Mach. Theory 35 (9) (2000) 1299–1311.
[8] J.L. Herder, Energy-free Systems. Theory, conception, and design of statically balanced spring mechanisms(PhD Thesis) Technische Universiteit Delft, The Netherlands, 2001, ISBN 90-370-0192-0.
[9] I. Ebert-Uphoff, C.M. Gosselin, T. Laliberte, Static balancing of spatial parallel platform mechanisms — revisited, J. Mech. Des. 122 (March 2000) 43–51.
[10] J.-P. Merlet, C.M. Gosselin, Parallel Mechanisms and Robots, chapter 12 in Handbook of Robotics — part B, in: Siciliano, Khatib (Eds.), Springer, Heidelberg, 2008, pp. 269–285.
[11] M. Carricato, C.M. Gosselin, A statically balanced Gough/Stewart-type platform: conception, design, and simulation, J. Mech. Robot. 1 (August 2009) 031005-1–031005-16.
[12] C.M. Gosselin, Static balancing of spherical 3-dof parallel mechanisms and manipulators, Int. J. Robot. Res. 18 (8) (1999) 812–829.
[13] M. Jean, C.M. Gosselin, Static balancing of planar parallel manipulators, Proc. IEEE Int. Conf. on Robotics and Automation, April 22–28, 1996, Minneapolis, 1996, pp. 3732–3737. http://dx.doi.org/10.1109/ROBOT.1996.509149.
[14] J. Wang, C.M. Gosselin, Static balancing of spatial three-degree-of-freedom parallel mechanisms, Mech. Mach. Theory 34 (3) (1999) 437–452.
[15] R. Clavel, Conception d'un robot parallele rapide a 4 degres de liberte(PhD Thesis No. 925) Ecole Polytechnique Federale de Lausanne, 1991. (in French).
[16] T. van Dam, P. Lambert, J.L. Herder, Static balancing of translational parallel mechanisms, Proceedings of the ASME 2011 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference IDETC/CIE 2011 August 28–31, 2011, Washington, USA, DETC2011-47525, 2011.
[17] S. Briot, V. Arakelian, S. Guégan, PAMINSA: a new family of partially decoupled parallel manipulators, Mech. Mach. Theory 44 (2009) 425–444.
[18] C. Baradat, P. Maurine, V. Arakelian, Conception d'une systeme de compensation des charges sur les actionneurs du robot parallele DELTA, 17-eme Congres Francaise de Mecanique, 29 August–2 September, 2005, Troyes, France, paper No. 427, 6, 2005, p. 6 (in French).
[19] C. Baradat, V. Arakelian, S. Briot, S. Guégan, Design and prototyping of a new balancing mechanism for spatial parallel manipulators, J. Mech. Des. 130 (7) (2008) 072305-1–072305-13.
[20] A. Russo, R. Sinatra, F. Xi, Static balancing of parallel robots, Mech. Mach. Theory 40 (2) (2005) 191–202.
[21] A.A. Kozlov, ИСПОЛНИТЕЛЪНЫЙ ОРГАН МАНИПУЛЯТОРА (An executing manipulator organ), Patent SU 975386, 1982. (in Russian).
[22] A. Agrawal, S.K. Agrawal, Design of gravity balancing leg orthosis using non-zero free length springs, Mech. Mach. Theory 40 (2005) 693–709.
[23] F.L.S. Riele, J.L. Herder, Perfect static balance with normal springs, Proceedings ASME Design Engineering Technical Conference, September 9–12, 2001, Pittsburgh, Pennsylvania, DETC2001/DAC21096, 2001, p. 8.
[24] D.A. Streit, B.J. Gilmore, 'Perfect' spring equilibrators for rotatable bodies, J. Mech. Transm. Autom. Des. 111 (December 1989) 451–458.
[25] L. Ciupitu, S. Toyama, E. Purwanto, Robotic arm with 9 DOF driven by spherical ultrasonic motors, IFAC Volume with selected papers from Workshop "Intelligent Assembly and Disassembly IAD'2003", Elsevier, Oxford, ISBN: 0 08 044065 7, 2003, pp. 85–90.
[26] G.J. Walsh, D.A. Streit, B.J. Gilmore, Spatial spring equilibrator theory, Mech. Mach. Theory 26 (2) (1991) 155–170.
[27] P. Appell, Traité de mécanique rationnelle, Gauthier Villars, Paris, 1928. (in French).
[28] V.M. Starzhinskii, An Advanced Course of Theoretical Mechanics, Mir Publishers, Moscow, 1982.
[29] F.P. Beer, E.R. Johnston Jr., Vector Mechanics for Engineers: Statics, McGraw-Hill Book Company, New-York, 1988, ISBN 0-07-079946-6.
[30] J. Denavit, R.S. Hartenberg, A kinematic notation for lower-pair mechanisms based on matrices, Trans. ASME J. Appl. Mech. 23 (1955) 215–221.
[31] J.J. Uicker Jr., Displacement Analysis of Spatial Mechanisms by an Iterative Method Based on Matrices(M. S. Thesis) Northwestern University, Evanston, Illinois, 1963.
[32] J.J. Uicker Jr., J. Denavit, R.S. Hartenberg, An iterative method for the displacement analysis of spatial mechanisms, J. Appl. Mech. Trans. ASME Ser. E 86 (2) (1964) 309–314.
[33] L. Ciupitu, I. Simionescu, C.-C. Lee, Static balancing — an overview, Proceedings of the First IFToMM Asian Conference on Mechanism and Machine Science 2010, Taipei, Taiwan, 21–25 October 2010, 2010, pp. 250084-1–250084-8.
[34] G. Tsuda, H. Kada, T. Sekino, Y. Nagahama, Gravity Balancing Device for Rocking Arm, U. S. Patent 4,592,697, 1986.