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Adaptive control using stochastic approach for unknown but bounded disturbances and its application in balancing control

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Abstract

This paper presents a simultaneous perturbation stochastic approximation (SPSA) approach for unknown but bounded disturbances in a typical closed loop system. These random disturbances are represented by a sequence of uncertainties along with the measured system output. Further, the proposed approach constructs a sequence of estimates and formulates an optimization problem which is minimized to achieve the adaptive control. In addition, the convergence conditions and additional generalizations are proposed to enhance the operation of SPSA algorithms for trail perturbation properties. To address the major challenges with unknown disturbances on the closed loop system, a balancing control problem with two degree of freedom (2DoF) ball balancer system and proportional integral derivative (PID) controller is considered. The proposed SPSA method minimizes the optimization problem to obtain the gain values of the PID controller. Simulation and experimental analysis are carried out, and the results substantiate that the PID gains optimized using SPSA provide better control when compared to conventional control approach in terms of tracking response under random uncertainties.

KEYWORDS

ball balancer system, proportional integral derivative (PID), simultaneous perturbation and stochastic approximation, two degree of freedom (2DoF), uncertainties

1 | INTRODUCTION

The effect of unknown disturbances during the control of non-holonomic balancer systems has been the subject of research due to their usefulness in various applications. In general, the robotic balancer systems are a special kind of automatic balancer systems that belong to a class of nonlinear, coupled, and underactuated systems. As a result, the modeling and control of these systems are dealt in a theoretical way by achieving steady-state operation through a feedback control loop. Recently, the development of various efficient control schemes has been proposed for balancer systems. Many of these design methods and techniques are specified towards handling unknown disturbances and system uncertainties. In a practical application,[1,2] proportional integral derivative (PID) handles the uncertainty in wide manner but limited to handle only constant parametric uncertainty and required an accurate system model for implementation purpose. In Huang and Xu,[3] the control of nonlinear system with periodic parameters is achieved using a discrete time adaptive control technique. The developed approach established a link between periodic adaptive and classic adaptive control problems and adapts lift technique to convert the periodic parameters into augmented constant parametric vectors. This achieved the direct application of discrete time adaptive control even for the conditions where the linear growth of the plant is not satisfying, parametric nonlinearity, and having unknown control directions. In Liu,[4] the dynamical and static uncertainties of a nonlinear system are considered for an adaptive stabilization problem. Besides, a condition on converging the estimated and true parameters is given to develop an adaptive feedback control law. The developed approach achieved an asymptotic synchronization of the system output for large parameter variations by maintaining the global stability of the system. In another control approach,[5] H∞/H2 controllers have been designed to deal with uncertainty. These controllers are robust in nature under the assumption that the uncertainties are norm-bounded and designed for the worst-case scenario. The H∞/H2controllers are responsible to provide guaranteed stability but within the bounds due to the modeling error[6] or filtering error.[7]

But the robustness of these controllers against unbounded disturbances, large uncertainties and nonlinearities remained as a drawback for the balancer systems.[8] Further, to find accurate bounds for the uncertainties to be allowed in system, a constraint handling-based controller has been designed.[9,10] This H∞ controller combined with model predictive controller (MPC) ensured the closed-loop performance under the explicit control law of MPC. The generated control law focuses on enhancing the utilization of given constraints with the help of optimization of receding horizon. This will improve the closed-loop disturbance attenuation level and optimizes towards the lower bound of the unconstrained H∞ control. The problem with this technique is that, here, the MPC optimizes the H∞ performance for predefined trajectory but ignores the previous states for achieving closed loop stability. This problem is explained by Chen and Scherer,[10] as past states to define H∞ performance is finite. This difficulty in linear system is handled by assigning multi-step sets associated with scaling parameters which guarantee the closed-loop H∞ performance using random system trajectories.[11] Further, in Basin et al.,[12] a direct extension is provided for the unknown state and disturbance initial conditions of a super-twisting control algorithm by designing a fixed-time convergent smooth second-order observer. The developed approach identified the drawback of zero initial value for the disturbance while calculating the upper estimate of control algorithm convergence time and overcomes it by admitting a disturbance with unknown but bounded initial value. In Zhang et al.,[13] the parametric uncertainties and additive disturbance in constrained linear systems were controlled by developing an adaptive interpolating controller. The approach updated the parametric uncertainty set at each time step to achieve robust stabilization for the uncertain system. Besides, the developed controller is recursively feasible and guarantees robust asymptotic stability and constraint satisfaction for the closed loop system. However, the practical implementation of these generalized methods needs repeated computations for multiple iterations and are mostly applicable for off-line tuning conditions or systems. This indicates the inability of the algorithms while performing online tuning, where the control activity of the feedback system is operational.

To further improve the performance of the closed loop system, achieve online tuning, and deal with random uncertainty, a simultaneous perturbation stochastic approximation (SPSA) algorithm which recursively generates estimates along random directions has been designed.[14–17] Conventionally, the stochastic approximation methods were used for statistical computations and later emerged as a separate field of control theory.[18] In the initial stage, these methods were proven for minimization of stationary functionals.[19] Later, the drawbacks of the gradient and newton methods while dealing with time-varying functionals due to the known bounds resulted in the development of stochastic approximation algorithms.[20] But the issue of constant step size in stochastic approximation limited their applications to time-varying systems, tracking, and position control problems.[21,22] Further, these drawbacks are overcome by developing a distributed asynchronous stochastic approximation algorithm.[23] Apart from the development, the constant step size stochastic approximation method has been used in the presence of arbitrary noises and stochastic disturbances with multi-agent systems under dynamic state changes.[24]

Moreover, the SPSA algorithm was adapted for achieving a data driven control with the neural network structure to solve the problems of nonlinear tracking in Dong et al.[25] The research in Tahir et al.[26] identified that SPSA can recursively solve the calibration problems in a system by posing them as stochastic optimization problems. The results depicted no parametric restrictions and fast convergence advantages of the SPSA algorithm. In Zhu et al.,[15] the second-order SPSA algorithm is adapted with a second-order stochastic gradient to solve the high-dimensional problems involving both gradient-based and gradient-free scenarios. The numerical studies identified the improvement in feasibility, convergence rate, efficiency, and numerical stability of the approach.

Considering the flexibility and advantages of SPSA, this paper develops an adaptive control strategy for solving the problem of balancing control in a closed loop system with unknown but bounded disturbances. The proposed approach is aimed at achieving a finite bound of residual between estimates and time-varying unknown parameters when observations are made under an unknown but bounded noise. This provides an intuitive tuning method for the PID controller and achieves balancing control for the closed loop system by updating the adaptive parameters in real time. Compared with the related work in the literature, the novel aspects of this paper are listed as follows:

- The proposed approach achieves self-balancing control in the presence of unknown but bounded disturbances by developing an adaptive control technique based on stochastic approximation algorithm.
- It overcomes the drawbacks of off-line training process of conventional optimization techniques.
- It also overcomes the drawbacks due to unknown but bounded disturbances on stochastic gradient estimation process.
- The effect of parametric uncertainties on the operation of the controller and system is overcome by estimating an optimal design parameter with the SPSA for tuning of PID controller.

Further, the remaining sections of the paper are arranged as follows: Section 2 discusses the problem statement by considering a closed loop system with sequence of uncertainties, and Section 3 defines the SPSA approach for minimizing the optimization problem. Further, the convergence conditions and additional generalizations necessary for enhancing the operation of SPSA algorithms for trail perturbation properties are discussed in the same section. Section 4 develops a balancing problem by considering a PID controlled two degree of freedom (2DoF) ball balancer system, and the stability analysis of the controller is discussed in Section 5. In Section 6, the developed controller is tested with the simulation and real-time experimental setup of ball balancer system, and the research is concluded in Section 7.

2 | PROBLEM STATEMENT

Initially, a typical closed loop system is described for formulating a task that can be characteristically solved by SPSA. The closed loop system consists of a plant and a controller. A sequence of uncertainties expressed as w_n and v_n is considered, where w_n corresponds to indeterministic behavior of internal system and v_n is an external noise combined with the measured output of the system. This optimization problem can be further divided in to online, off-line, and stochastic classes.[27] In the off-line class, the approach is classical, whereas for the online class, a new function is measured for every iteration. This resulted in the need for calculating the average of all the generated cost functions which makes the system complex. Further, the stochastic approach involves one function, and it is measurable with noise. Hence, in this research, the stochastic approach is considered, where the function $F(x,w)$ is optimized and measured with noise.

Consider $F(x,w):\mathbb{R}^q \times \mathbb{R}^p \to \mathbb{R}^1$ is differentiable by an argument function $(x_1, x_2, \ldots)$ at every instant $n = 1,2,3\ldots$. The value of optimization function with additive noise v_n is given as

$$y_n = F(x_n, w_n) + v_n, \tag{1}$$

where $w_n \in \mathbb{R}^p$ corresponds to a sequence of uncontrollable random values with equal but unknown distribution $P_w(\cdot)$.

The problem in hand is to construct a sequence of estimates $\{\hat{\theta}_n\}$ of an unknown vector θ using observations $y_1, y_2, y_3 \ldots y_n$ to minimize an average cost type function $f(x)$ given as

$$f(x) = \int_{\mathbb{R}^p} F(x,w) P_w(dw). \tag{2}$$

Usually, a simple model of observation is considered for the problem of minimization function $f(.)$ as given in 3 which is easily suitable for the proposed scheme.

$$y_n = f(x_n) + v_n \tag{3}$$

But to accommodate the sequence of uncertainties with the cost type function, the complicated model of minimizing is given as

$$y_n = w_n f(x_n) + v_n \tag{4}$$

for a general model with $F(x,w) = wf(x)$.[28]

Typically, for any condition in this complicated model, if the distribution $P_w(\cdot)$ is unknown, the problem lies outside the scope of a classical optimization theory. Further, this condition can be overcome by measuring the function $F(x_n, w_n)$ with an additive random zero mean disturbance $v_n \in \mathbb{R}$ which is independent and identically distributed. Hence, by adding an additional component v to the vector w which forms $\bar{w} = \begin{pmatrix} w \\ v \end{pmatrix}$, the measured function can be rewritten as

$$\bar{F}(x, \bar{w}) = F(x,w) + v. \tag{5}$$

This forms a new observation scheme with an additive disturbance which involves an unknown distribution

$P_{w,v}(.)$ instead of $P_w(.)$. Further, if the noise added with the measurement does not have any significant statistical properties, the complexity of the problem cannot be simplified.

Hence, from the above considerations, the problem formulation can be simplified as follows: using the inputs $x_1, x_2, x_3, \ldots, x_n$ and observations $y_1, y_2, y_3, \ldots, y_n$, an estimate $\hat{\theta}_n$ is constructed for an unknown vector θ which minimizes the time varying mean risk functional in (4).

3 | SPSA ALGORITHMS

The SPSA algorithm is used for efficient estimation of unknown vectors based on small measurements for application with signal identification and adaptive control. Generally, the SPSA works on selected coefficients[29] with two observations which generate estimations recursively in random directions at each iteration. The flow process of SPSA for optimal design parameter is shown in Figure 1.

Initially, the SPSA algorithm perturbs the current design parameter in random directions and measures the objective function of each observation. The measured observation is used to estimate the unknown vector which updates a new design parameter until the termination criterion is achieved. This provides an opportunity for achieving better convergence towards optimal solution for updating the design parameter. The motivation for using SPSA for adaptive control of a system with unknown but bounded disturbances is due to its easy to implement searching algorithm especially for the real-time control applications. Furthermore, the SPSA has advantages due to its iterative process which implies the idea of online learning with adaptability of new data and memory saving. The algorithm remains operational while accommodating the growing dimension of the estimated parameters and is resistant to arbitrary external noise at the point of input data. Besides, it exhibits less computation time due to small number of measurements and is capable of solving high-dimensional optimization

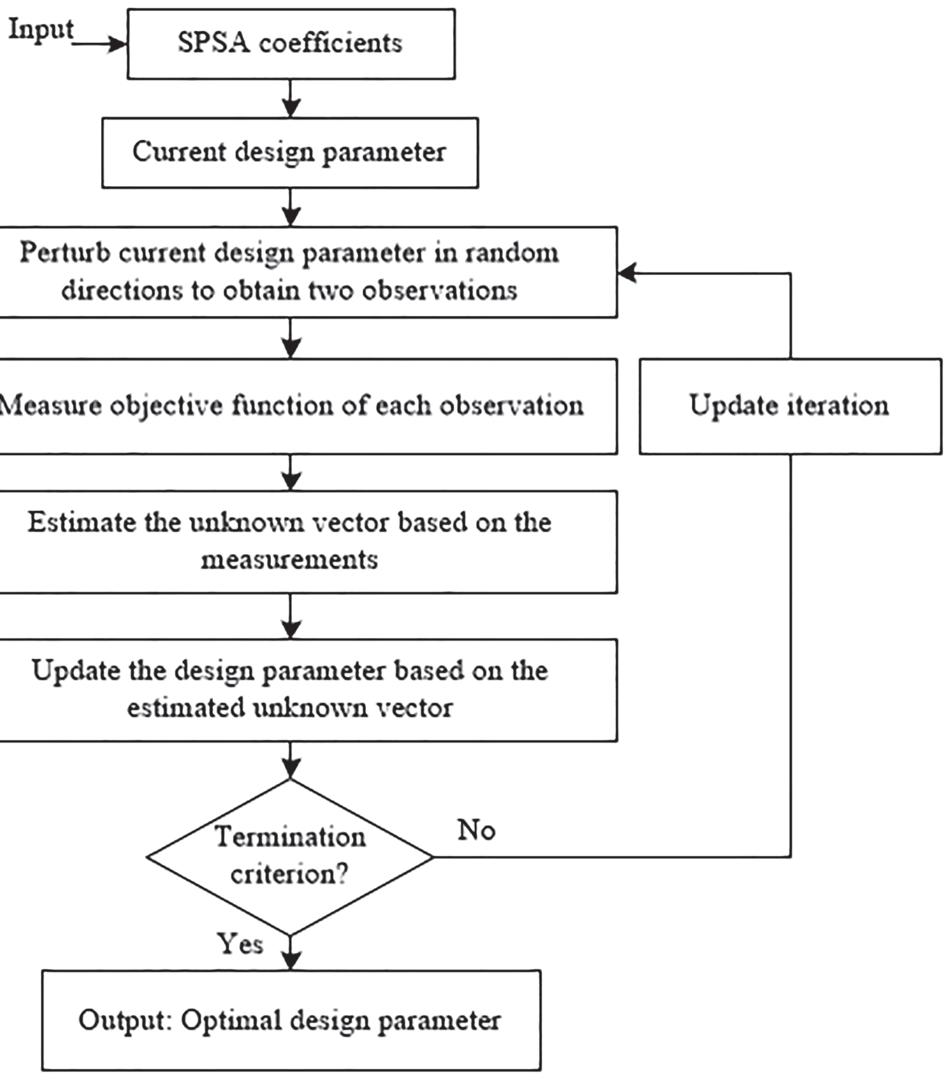

FIGURE 1 Flow process of parameter estimation with simultaneous perturbation and stochastic approximation

problems. The simultaneous perturbations for building the estimates of unknown vector θ are denoted as

$$\Delta_n \in \mathbb{R}^q. \tag{6}$$

Generally, the simultaneous perturbation vector is generated using the Monte Carlo approach which provides a two-dimensional random perturbation vector Δ_n. Besides, a zero mean probability distribution is used to generate the components of Δ_n independently. The common choice for all the components of Δ_n is the use of ± 1 Bernoulli distribution with $\frac{1}{2}$ probability for every ± 1 outcome.

In addition, two sets of sequence of positive numbers $\{\alpha_n\}$ and $\{\beta_n\}$, where $\{\alpha_n\}$, $\{\beta_n\} \to 0$ and a fixed initial vector $\hat{\theta}_0 \in \mathbb{R}^q$, are defined. Three different scenarios are considered for estimating $\{\hat{\theta}_n\}$ and constructing the sequence of points for measurements $\{x_n\}$ as follows:

Case 1. Estimating using one observation.

$$\begin{cases} x_n = \hat{\theta}_{n-1} + \beta_n \Delta_n, y_n = F(x_n, w_n) + v_n, \\ \hat{\theta}_n = \hat{\theta}_{n-1} - \dfrac{\alpha_n}{\beta_n} \mathcal{K}_n(\Delta_n) y_n. \end{cases} \tag{7}$$

Cases 2 and 3. Estimating using two observations on each iteration.

$$\begin{cases} x_{2n} = \hat{\theta}_{n-1} + \beta_n \Delta_n, x_{2n-1} = \hat{\theta}_{n-1} - \beta_n \Delta_n, \\ \hat{\theta}_n = \hat{\theta}_{n-1} - \dfrac{\alpha_n}{2\beta_n} \mathcal{K}_n(\Delta_n)(y_{2n} - y_{2n-1}), \end{cases} \tag{8}$$

$$\begin{cases} x_{2n} = \hat{\theta}_{n-1} + \beta_n \Delta_n, x_{2n-1} = \hat{\theta}_{n-1}, \\ \hat{\theta}_n = \hat{\theta}_{n-1} - \dfrac{\alpha_n}{\beta_n} \mathcal{K}_n(\Delta_n)(y_{2n} - y_{2n-1}). \end{cases} \tag{9}$$

From all the three cases, it can be observed that a kernel function $\mathcal{K}_n(\cdot) : \mathbb{R}^q \to \mathbb{R}^q$ is used to satisfy the simultaneous perturbation distribution $P_n(\cdot)$. This condition is denoted as

$$\int \mathcal{K}_n(x) P_n(dx) = 0, \int \mathcal{K}_n(x) x^T P_n(dx) = I, \tag{10}$$

where $I \to I_{q \times q}$ is a unit matrix.

The general form of $\mathcal{K}_n(\cdot)$ in the situation of uniform testing perturbation is formulated in a special case by Polyak and Tsybakov[30] corresponding to (7) and Spall[17] corresponding to (8). This makes an assumption about the centralization and independency of the observed noise. This special case defines the rule for distribution of trail perturbation with kernel $\mathcal{K}_n(\cdot)$ and finite inverse moments as follows:

$$\mathcal{K}_n(\Delta_n) = \begin{pmatrix} \dfrac{1}{\Delta_n^{(1)}} \\ \dfrac{1}{\Delta_n^{(2)}} \\ \vdots \\ \dfrac{1}{\Delta_n^{(q)}} \end{pmatrix}. \tag{11}$$

Considering the same kernel $\mathcal{K}_n(\cdot)$, a new scenario is developed as per (9) with constraints on trail simultaneous perturbation distribution.[31] In this paper, the sequence of estimates is constructed from (7) by formulating kernel $\mathcal{K}_n(\Delta_n) = \Delta_n$ with a projection which is given as

$$\begin{cases} x_n = \hat{\theta}_{n-1} + \beta_n \Delta_n, y_n = F(x_n, w_n) + v_n, \\ \hat{\theta}_n = \mathcal{P}_{\Theta_n}\left(\hat{\theta}_{n-1} - \dfrac{\alpha_n}{\beta_n} \mathcal{K}_n(\Delta_n) y_n\right), \end{cases} \tag{12}$$

where $\mathcal{P}_{\Theta_n}$ corresponds to projecting operators on bounded closed convex subsets $\Theta_n \subset \mathbb{R}^q$, which contain θ and start from $n \geq 1$. The bounded subsets Θ_n can extend up to infinity if the θ is not known, and for any case if Θ: $\theta \in \Theta$ is known, then the bounded set $\Theta_n = \Theta$. For some specific cases, the bounded subsets Θ_n can construct a decreasing sequence.

3.1 | Conditions for estimation

Further, the estimates are constructed using different scenarios as follows:

Let $E\{\cdot\}$ be an expectation; $\|\cdot\|$, $\|\cdot\|_\rho$, and $(\cdot, \cdot)$ be a norm in l_ρ space and scalar product in $\mathbb{R}^q$ for Euclidean norm with $\rho \in (1,2]$; and $\mathcal{F}_{n-1}$, derived from a set of random values $\left(\hat{\theta}_0, \hat{\theta}_1, \ldots, \hat{\theta}_{n-1}\right)$ for a σ-algebra of probabilistic events. The estimates are constructed using the predefined scenarios (8) and (9) as

$$\bar{w}_n = \begin{pmatrix} w_{2n} \\ w_{2n-1} \end{pmatrix}, \bar{v}_n = \kappa(v_{2n} - v_{2n-1}), \tag{13}$$

$$\kappa = \begin{cases} \dfrac{1}{2} & \text{for } (5) \\ 1 & \text{for } (6) \end{cases}, \tag{14}$$

$$F_w = \max_{x \in \mathbb{R}^q} E_{w'}\left\{E_{w''}\left\{\kappa^\rho |F(x, w') - F(x, w'')|^\rho\right\}\right\}. \tag{15}$$

Similarly, while constructing the estimates using (12),

$$\bar{v}_n = v_n, \bar{w}_n = w_n, F_w = E_w\{|F(\theta, w)|^\rho\}. \tag{16}$$

Further, the assumptions required to find the optimal vector θ are formulated[32] by considering a function as follows:

$$V(x) = \|x-\theta\|_\rho^\rho = \sum_{i=1}^{q} \left|x^{(i)} - \theta^{(i)}\right|^\rho. \tag{17}$$

Assumption 1. A unique minimum and (18) are considered for the function $f(x)$ along with a constant $\mu > 0$.[33]

$$(\nabla V(x), \nabla f(x)) \geq \mu V(x), \forall x \in \mathbb{R}^q \tag{18}$$

Assumption 2. For all the non-deterministic behaviors of the system, the gradients of the function $F(\cdot, w)$ must satisfy the condition in (19) with a constant $M > 0$.

$$\|\nabla_x F(x,w) - \nabla_x F(y,w)\|_\rho \leq M\|x-y\|_\rho, \forall x,y \in \mathbb{R}^q \tag{19}$$

Assumption 3. A local condition is considered from the Lebesgue integration,[34] where $\nabla_x F(x,\cdot): \forall x \exists$, neighborhood $U_x: \forall x' \in U_x \exists$ and function $\Phi_x(\cdot): \mathbb{R}^p \to \mathbb{R}$, $E_w\{\Phi_x(w)\} < \infty : |\nabla_x F(x', w)| \leq \Phi_x(w)$ for all of w.

Assumption 4. For kernel $\mathcal{K}_n(\cdot)$ and simultaneous perturbation distribution $P_n(\cdot)$, $n = 1,2,\ldots$ satisfies the conditions as

$$\bar{K} = F_w \sup_{n=1,2,\ldots} \int \|\mathcal{K}_n(x)\|_\rho^\rho P_n(dx) < \infty, \tag{20}$$

$$\tilde{K} = \sup_{n=1,2,\ldots} \int \|\mathcal{K}_n(x)\|_\rho \|x\|_\rho \|x\|_{\frac{\rho}{\rho-1}} P_n(dx) < \infty. \tag{21}$$

Assumption 5. For every value of $n > 1$,

$$\xi_n = \left\| E\{\mathcal{K}_n(\Delta_n)\bar{v}_n | \mathcal{F}_{n-1}\} \right\|_\rho^\rho \leq C_{\Delta v}\beta_n^2, E\left\{ \|\mathcal{K}_n(\Delta_n)\bar{v}_n\|_\rho^\rho \right\} \leq \sigma_n^\rho. \tag{22}$$

Similarly, for $\rho = 2$, Assumptions 1 and 2 can be formulated as

- Assumption 1′: The function $f(\cdot)$ is strictly convex.

$$\langle x-\theta, \nabla f(x) \rangle \geq \mu \|x-\theta\|^2, \forall x \in \mathbb{R}^q. \tag{23}$$

- Assumption 2′: For all the indeterministic behaviors of the system, the gradients of the function $F(\cdot, w): \forall x, \theta \in \mathbb{R}^q$ are satisfied by Lipschitz condition[35]:

$$\|\nabla_x F(x,w) - \nabla_x F(y,w)\| \leq M\|x-\theta\|. \tag{24}$$

3.2 | Convergence of estimates

Further, the convergence of sequence of estimates at a point θ is given in the following sense:

Note 1: For a linear regression model, the problem of estimating parameters with observations in (4) and when $\theta_n = \theta$ corresponds to the minimization of average risk functional.

$$f(x) = \frac{1}{2}(x-\theta)^T (x-\theta). \tag{25}$$

Note 2: Even though the scenarios in (8) and (9) are similar, the usage of (9) for real-time systems during arbitrary noise in observations is better. This is because of the repetitive moment of a vector $2n - 1$ in the system of scenario (8) restricts the independency of noise v_{2n} from trail perturbation Δ_n, whereas in (9), the vector of trail perturbation Δ_n and noise v_{2n} simultaneously enter the system allowing them to hope on their independency.

Note 3: For generalization of conditions for convergence of scenarios in (8), (9), and (12), sequence of positive numbers $\{\alpha_n\}$ and $\{\beta_n\}$ can be measured randomly, and σ-algebra $\mathcal{F}_n$ can be measured relatively. This is necessary for instances where quality of the estimation is being assessed. Depending upon the quality of the estimation, the speed of sequence convergence is either lowered to zero or expanded to a bigger value.

Note 4: For generalization of the properties of noise, i.e., the existence of $\rho \in (1,2]$-momemnts for w, indicates that the SPSA algorithms can be used for all sorts of adaptive control.

3.3 | Estimation evaluation

Since the system is designed for unknown but bounded disturbance case, its behavior under relatively high noise level needs to be evaluated. Besides, the concept of estimation evaluation tests whether the estimated parameter is correct for the corresponding dynamics of the system. The following steps outline the estimation evaluation process:

Step 1: When a new estimation $\hat{\theta}_{nnew}$ is obtained, discretize the estimated system, and set its initial value at a certain time point k_0.

Step 2: Analyze the estimated system over an input from $[k_0$ to $(k_0 + LTs)]$, where L is a positive integer indicating the length of evaluation and T_s is the sampling time. The resulted output is denoted as $y_e(k)$, the evaluation output.

Step 3: Compare the simulation output with measurements $x_n(k)$ along the same time sequence. This can be done by calculating their difference as

$$\Delta\left(\hat{\theta}_{nnew}\right) = \frac{1}{L-L'}\sqrt{\sum_{k=k_0+L'}^{k_0+LT_s}[y_e(k)-x_n(k)]^2}. \tag{26}$$

Here, it should be noted that the simulation starts from $k_0 + L'$ instead of k_0, where L' is the positive integer number. This eliminates the influence of the noisy initial value. Further, the value of L' is to be selected long enough such that the impact of initial value can be neglected.

Step 1: For the condition where the previously or old estimated value is less than the newly estimated value, i.e., $\alpha\Delta\left(\hat{\theta}_{nnew}\right) > \Delta\left(\hat{\theta}_{nold}\right)$, the $\hat{\theta}_{nnew}$ is considered to be invalid or at least worse than the $\hat{\theta}_{nold}$. Here, the new estimation is abandoned, and the old estimation is used. For an otherwise condition where $\hat{\theta}_{nold}$ is greater than $\hat{\theta}_{nnew}$, the new estimation is valid and considered as a better estimation for updating the system. In this condition, the α associated with the new estimation is a constant positive real number which controls the standard of selection. If the value of $\alpha \geq 1$, it indicates strict selection, and the only estimation leading to smaller $\Delta\left(\hat{\theta}_{nnew}\right)$ is accepted. Moreover, all the estimations are accepted if $\alpha = 0$. A perfect estimation should lead to $\Delta\left(\hat{\theta}_{nnew}\right) = 0$.

Further, to test the action of SPSA based adaptive control, a balancing problem is considered in closed loop with a PID control. The system description and implementation of developed approach are discussed in further sections.

4 | BALANCING PROBLEM OF 2DOF BALL BALANCER

A ball balancer system is a typical benchmark problem for achieving, balancing control, ball position tracking, and visual servo control. The typical representation of the ball balancer system is given in Figure 2.

This is a underactuated system with an access to four DoF controlled by two actuators. Hence, its operation is referred as 2DoF ball balancer. The other parts of the system consist of a digital camera which captures the images of X and Y coordinates for the ball movement on the plate. Further, a vision algorithm computes the reads the ball coordinates from the image and provides information to the controller[36] for adjusting the angle of plate through the X and Y servo motors to make the ball track the time varying reference. In general, the servo dynamics of the ball balancer coordinates are similar. Hence, the modeling of control through the direction X is presented in this research. The X-axis control of ball and plate system is presented in Figure 3.

The nonlinear equation of motion of the 2DoF ball balancer is obtained from Quanser 2DoF ball balancer model[37] as follows:

$$\ddot{x}_d(t) = \frac{2M_{ball}g\beta_l R_{arm}R_{ball}^2}{L_{plate}\left(M_{ball}R_{ball}^2 + J_b\right)}, \tag{27}$$

$$\ddot{\beta}_l(t) = \frac{\left(K_m u(t) - \dot{\beta}(t)\right)}{\tau}, \tag{28}$$

where x_d is the position of the ball, β_l is the load angle, M_{ball} represents the ball mass, J_b corresponds to the moment of inertia, α indicates the plate angle, R_{ball} represents the ball radius, R_{arm} measures the distance between the output gear shaft and the servo motor, g indicates the gravitational constant, L_{plate} represents the plate length, and K_m and τ correspond to the speed and time constant of the motor, respectively. A detailed modeling of 2DoF ball balancer system is discussed by the authors in their previous work.[38]

The transfer function for the control of ball position for an input β_l and output x_d is given as

$$P_b(s) = \frac{x_d(s)}{\beta_l(s)} = \frac{K_b}{s^2}, \tag{29}$$

where $K_b = \frac{2M_{ball}g\beta_l R_{arm}R_{ball}^2}{L_{plate}\left(M_{ball}R_{ball}^2 + J_b\right)}$.

Similarly, the plate angle control by the servo motor is mathematically formulated with a transfer function as

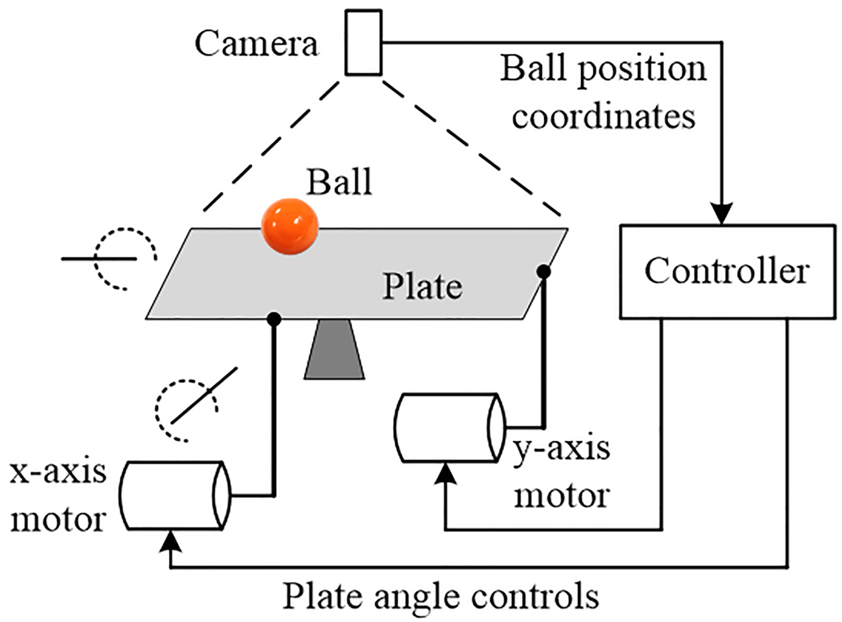

FIGURE 2 A typical representation of ball and plate system

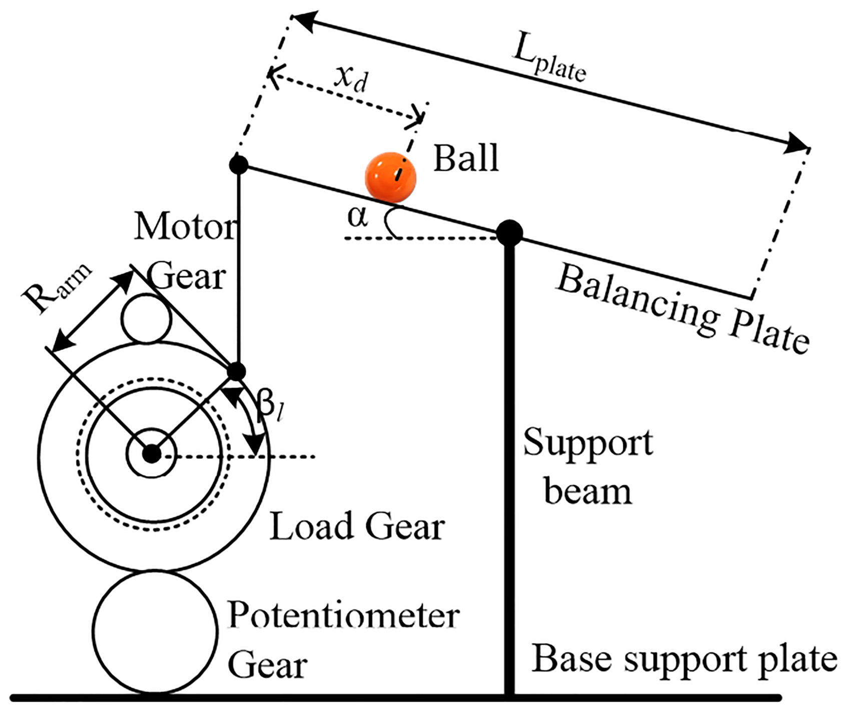

FIGURE 3 Schematic representation for X axis control of ball and plate system

$$P_s(s) = \frac{\beta_l(s)}{V_m(s)} = \frac{K_g}{s(\tau s + 1)}, \tag{30}$$

where K_g is the static gain.

The overall transfer function of the cascaded connection between servo motor and ball balancer module is given by

$$P(s) = P_s(s)P_b(s) = \frac{\beta_l(s)}{V_m(s)} = \frac{K_g}{s^3(\tau s + 1)}. \tag{31}$$

Thus, representing the system in state space variable is given by

$$\begin{bmatrix} \dot{x}_d(t) \\ \ddot{x}_d(t) \\ \dot{\beta}_l(t) \\ \ddot{\beta}_l(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & K_b & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} x_d(t) \\ \dot{x}_d(t) \\ \beta_l(t) \\ \dot{\beta}_l(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_m}{\tau} \end{bmatrix} u(t). \tag{32}$$

Since the closed loop control of ball and plate system is being realized by a PID control system, the basic block representation of the closed loop system is depicted in Figure 4.

The closed loop representation deals with a reference $r(t)$, controlled input $u(t)$, uncertainties $d(t)$, and

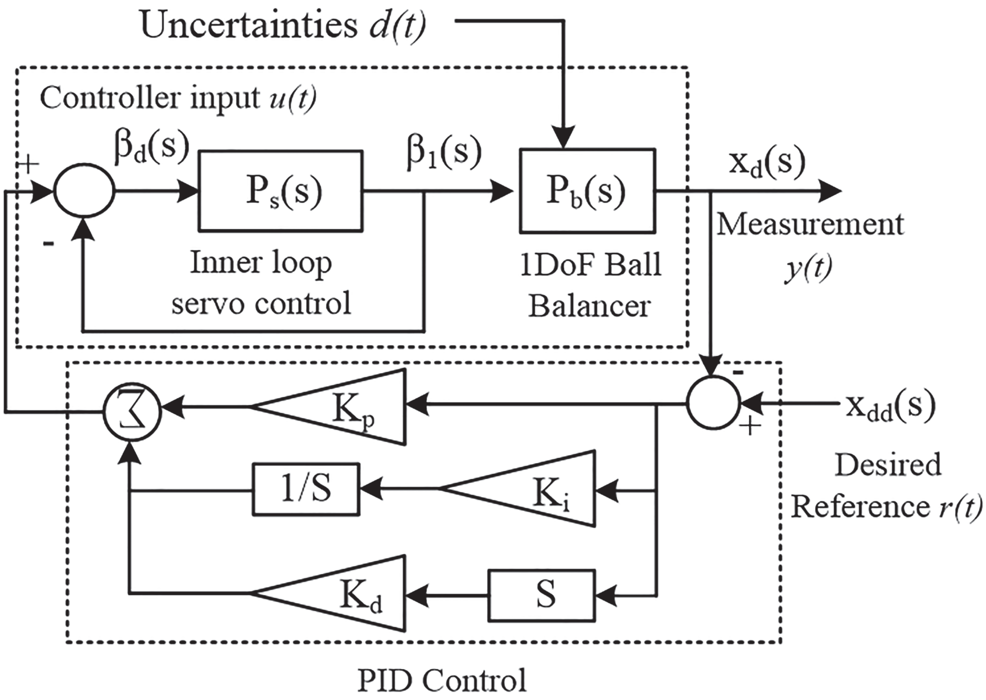

FIGURE 4 Closed loop control of ball balancer system

measurement $y(t)$. The controller $K(s)$ for a plant $G(s)$ is given by

$$K_{PID}(s) = \begin{bmatrix} h_{11}(s) & h_{12}(s) & \cdots & h_{1j}(s) \\ h_{21}(s) & h_{22}(s) & & h_{2j}(s) \\ \vdots & & \ddots & \vdots \\ h_{i1}(s) & h_{i2}(s) & \cdots & h_{ij}(s) \end{bmatrix}. \tag{33}$$

For a PID controller, the element $h_{ij}(s)$ is given by

$$h_{ij}(s) = P_{ij}\left(1 + \frac{1}{I_{ij}(s)} + \frac{D_{ij}(s)}{\frac{D_{ij}}{N_{ij}}(s)}\right). \tag{34}$$

Here, the terms P_{ij},I_{ij}, and D_{ij} correspond to the proportional gain, integral, and derivative time of the PID controller, and N_{ij} is the filter coefficient.

In order to assess the performance of the closed loop system depicted in Figure 4, the performance index is given by

$$\begin{aligned} \hat{e}_i &\coloneqq \int_{t_0}^{t_f} |r_i(t) - y_i(t)|^2 dt \\ \hat{u}_i &\coloneqq \int_{t_0}^{t_f} \left|u_i(t)^2\right| dt \end{aligned}, \tag{35}$$

where $r_i(t)$, $y_i(t)$, and $u_i(t)$ indicate the i_{th} elements of vectors $r(t)$, $y(t)$, and $u(t)$, respectively, $t_0 \in \{0\}$ defines a union of positive set of real numbers ($\mathbb{R}_+$) and $t_f \in \mathbb{R}_+$ which corresponds to the time interval or time period of performance evaluation. Hence, the optimization problem of the closed loop control is defined as

$$J(P,I,D,N) = \sum_{i=1}^{p} w_{1i}\hat{e}_i + \sum_{i=1}^{q} w_{2i}\hat{u}_i, \tag{36}$$

where $P \coloneqq [P_{11}, P_{12}, \ldots, P_{ij}]^{\mathrm{T}}$, $I \coloneqq [I_{11}, I_{12}, \ldots, I_{ij}]^{\mathrm{T}}$, $D \coloneqq [D_{11}, D_{12}, \ldots, D_{ij}]^{\mathrm{T}}$, and $N \coloneqq [N_{11}, N_{12}, \ldots, N_{ij}]^{\mathrm{T}}$. The weighting coefficients w_{1i} *and* $w_{2i} \in \mathbb{R}$, and p and q correspond to the dimensions $u(t)$ and $y(t)$, respectively. Theoretically, the performance index of the controller is defined by the sum of ball position error and the controlled input energy. Hence, the problem formulation for the system considered in Figure 3 is to find a controller $K_{PID}(s)$ by minimizing the optimization problem with respect to the values of P, I, D, and N by considering the controlled input and measurement data.

5 | STABILITY ANALYSIS

The stability analysis for the developed adaptive control loop is discussed here. Considering the linear matrix inequality-based stability analysis which adapts σ-modification for single input single-output system, the expansion for multi input multi-output systems is analyzed.

The arbitrary disturbance perturbation is not considered in this condition of closed loop stability analysis with adaptive control. Hence, the state space representation of the 2DoF ball balancer system is considered. Therefore, a simplified state space representation is given by

$$x_d(t) = A_p x(t) + B_p u(t). \tag{37}$$

Let $A_p{}'$ and $B_p{}'$ be represented as

$$A' = E_p^{-1} A_p, \tag{38}$$

$$B' = E_p^{-1} B_p, \tag{39}$$

where $E_p^{-1} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ [39] and the controller gain K_{PID} is divided as an integration gain which is given by

$$K_{PID_r} \in \mathbb{R}_{2\times 2}, \tag{40}$$

and state gain

$$K_{PID_x} \in \mathbb{R}_{2\times 4}, \tag{41}$$

where $K_{PID} = [K_{PID_r},\ K_{PID_x},]$

The actual plant model is given as

$$\begin{gathered} \dot{x}_{d_p} = A' x_{dp} + B'\left(u + W^T \phi(x)\right) \\ y_d = c_p x_{d_p} \end{gathered}, \tag{42}$$

where $W(t) = [W_1(t) \quad W_2(t)] \in \mathbb{R}_{4\times 2}$ is a matrix of uncertainties and $\phi(x) \in \mathbb{R}_{4\times 1}$ is a smooth basis function set. $W^T(t)\phi(x)$ is a matched uncertainty. Input for the actual plant model is defined as

$$\begin{gathered} u = u_{nom} - u_{ad} \\ u_{nom} = -K_{PID_x} x + K_{PID_r} \int (r(t) - y) dt, \\ u_{ad} = \hat{W}(t)^T \phi(x) \end{gathered} \tag{43}$$

where u_{nom} is the nominal input for reference model, u_{ad} is the adaptive signal, and $r(t)$ is step reference.

The u_{ad} acts as a function for canceling matched uncertainty by estimating the uncertainty matrix $W(t)$ with $\hat{W} = \left[\hat{W}_1 \quad \hat{W}_2\right] \in \mathbb{R}_{4\times 2}$. Further, a reference model corresponding to (42) which generates an ideal output given by

$$\dot{x}_{dm} = A_m x_m + B_m \int (r(t) - y) dt, \tag{44}$$

where $A_m = A' - B' K_{PID_x}$, and $B_m = B' K_{PID_r}$.

Consider $e = x_{dm} - x_d$ as the tracking error, and $\tilde{W}(t) = \hat{W}(t) - W(t) \left(\tilde{W}_1(t) = \hat{W}_1(t) - W_1(t), \tilde{W}_2(t) = \hat{W}_2(t) - W_2(t)\right)$ is the estimation error. Finally, deviation of actual plant from reference model is obtained as

$$\dot{e} = A_m e + B' \tilde{W}(t)^T \phi(x). \tag{45}$$

Let B_1 and B_2 equal to B' as $[B_1 \quad B_2]$, and from B_1, B_2, $W_1(t)$, and $W_2(t)$, (46) can be described as

$$\dot{e} = A_m e + B_1 \tilde{W}_1(t)^T \phi(x) + B_2 \tilde{W}_2(t)^T \phi(x), \tag{46}$$

where $W_1(t)$ and $W_2(t)$ can be updated using the adaptive control system with σ-modification.

$$\begin{gathered} \dot{\hat{W}}_1 = -\gamma \phi(x) e^T P B_1 - \sigma \hat{W}_1 \\ \dot{\hat{W}}_2 = -\gamma \phi(x) e^T P B_2 - \sigma \hat{W}_2 \end{gathered}, \tag{47}$$

where the adaptive gain is denoted as γ, $(\gamma > 0 \in \mathbb{R})$, and the σ-modification gain is given by σ. Further, the values of $P > 0$ satisfy the Lyapunov (48) for the values of $Q > 0$.

$$P A_m + A_m^T P + Q = 0. \tag{48}$$

Further, the values of γ and σ which satisfy the linear matric inequalities are used for assessing the stability of the system.

6 | APPLICATION

6.1 | Simulation

In this section, numerical simulations are developed with the SPSA-PID controller for a closed loop operation of the modeled 2DoF ball balancer system. To begin with, the movement of ball on the plate and the variation in plate balancing angle with reference to X and Y axis are measured as plant responses. As the plate angle is adjusted to balance the ball without falling off its surface, the position of the ball varies accordingly. The parameters used for development of the ball balancer setup are mentioned in Table 1, and the implementation of SPSA with the closed loop control of ball balancer system is shown in Figure 5.

Algorithm 1

Implementation of the developed SPSA-PID controller

Step 1: The error between the measured and desired position is obtained and provided as input to both PID controller and the SPSA algorithm.
Step 2: The SPSA algorithm is initialized with the selected coefficients. Here the counter index n is initially set to 0, unknown parameter vector $_\theta \in^{\mathbb{R}}$, and the positive coefficient $\alpha > 0$.
Step 3: The iterations are initiated: $n \rightarrow n+1$
Step 4: The simultaneous perturbation vector is generated using Monte Carlo and Bernoulli ±1 distribution.
Step 5: Estimate the observations as discussed in Section III.
Step 6: Evaluate the estimated observation by measuring the distance between old estimate and new estimate.
Step 7: If the new estimation is valid, proceed to step 8, or continue with the old estimate and parallelly start estimating the new observation in the next iteration.
Step 8: Update the design parameters to achieve the optimum PID values
Step 9: Terminate the process

TABLE 1 Parameter specification for 2DoF ball balancer

Nomenclature		
Symbol	**Quantity**	**Values**
$V_m(s)$	Motor input voltage	6 volts
M_{ball}	Mass of the ball	0.003 kg
α	Plate angle	0–0.5239 rad
g	Gravitational constant	9.810 m/s^2
R_{ball}	Ball radius	0.0196 m
J_b	Inertia of the ball	4.6217e−07 (kg/m^2)
L_{plate}	Length of the plate	0.2750 m
R_{arm}	Distance between SRV02 output gear shaft and coupled joint	0.0254 m

The steps for implementing the developed controller with the ball balancer system are discussed in Algorithm 1 as follows:

Based on the estimated design parameter, the optimal gains for PID are obtained, such that the controller output varies the plate angle efficiently to balance the ball even in the presence of disturbances. The gain value calculation for the PID controller is obtained as follows:

$$K_{PID}(s) = \begin{bmatrix} h_{11}(s) & h_{12}(s) \\ h_{21}(s) & h_{22}(s) \end{bmatrix}. \tag{49}$$

Further, to compensate the complexity and minimize the number of iterations for identifying the optimal values, it is considered that some of the gain values of PID has equal attributes as follows: $I_{12} = I_{11}$, $D_{12} = D_{11}$, $N_{12} = N_{11}$, $I_{21} = I_{22}$, $D_{21} = D_{22}$, and $N_{21} = N_{22}$.

The resultant parameters are calculated by minimizing the objective function J for values of $p = 2$, $q = 2$, $w_{11} = 10$, $w_{12} = 1000$, $w_{2i} = 1$ $(i = 1,2)$, $t_0 = 0$, and $t_f = 20$.

The response of the 2DoF ball balancer system has been assessed by implementing SPSA-PID method to control the ball position on plate. For analysis purpose, a comparative result between $H\infty$-proportional integral derivative (H-PID) and SPSA-PID has been shown in Figure 6.

From the Figure 6A, the ball position on plate settled at 2.4 s without any oscillations and follow the reference square trajectory of amplitude 5. On the other side, the H-PID takes 4.12 s to settle with large overshoot as compare to SPSA-PID and takes more time to follow the trajectory. It also has been analyzed from the Figure 6B, the plate angle variation goes up to 5.8^0 to -1^0 and resulted in less oscillation when controlled with SPSA-PID, while H-PID varies from 18.3^0 to -8.7^0. So the variation in angle movement is also less in SPSA-PID. Figure 6C shows the voltage waveform of both the controllers and voltage remains steady throughout the process and vary only for 0 to 0.8 s, which is signal initiation time, while for H-PID, voltage shows variation from 0 to 3.2 s. The amplitude of voltage is also small in case of SPSA-PID, and even small voltage is enough to track and able to balance the ball on plate. Further, the time response characteristics of the plate angle output for both the controllers are calculated to estimate the superiority of the proposed approach. Table 2 shows the peak time, settling time, peak overshoot, and steady-state error of SPSA-PID controller are 0.63 s, 1.4 s, 7.32e−06%, and 1.3e−06 cm, respectively, which is less in comparison with H-PID. As the peak time reduces, the reaching time of the ball to its specified amplitude is minimized. As per the output waveform, it has been noticed that SPSA-PID control has a suitable response with the lowest error and overshoot which carries the perfect ball balance on the plate.

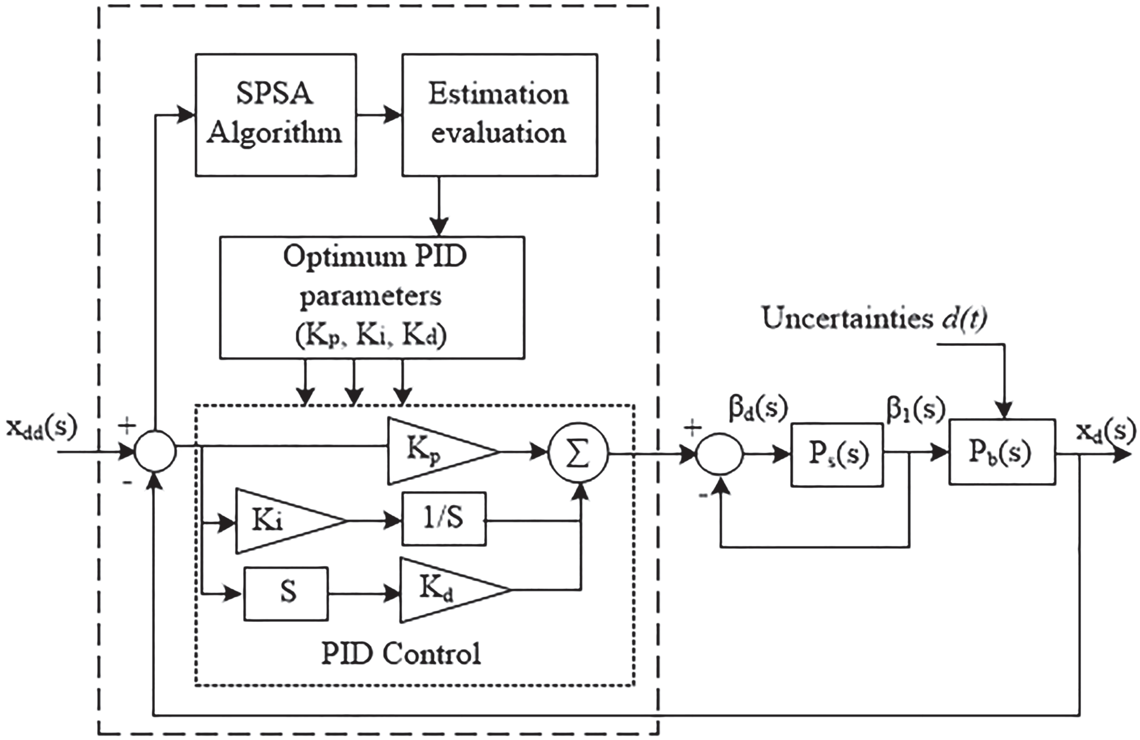

FIGURE 5 Overall structure of ball balancer system with SPSA algorithm and estimation evaluation for optimal PID controller

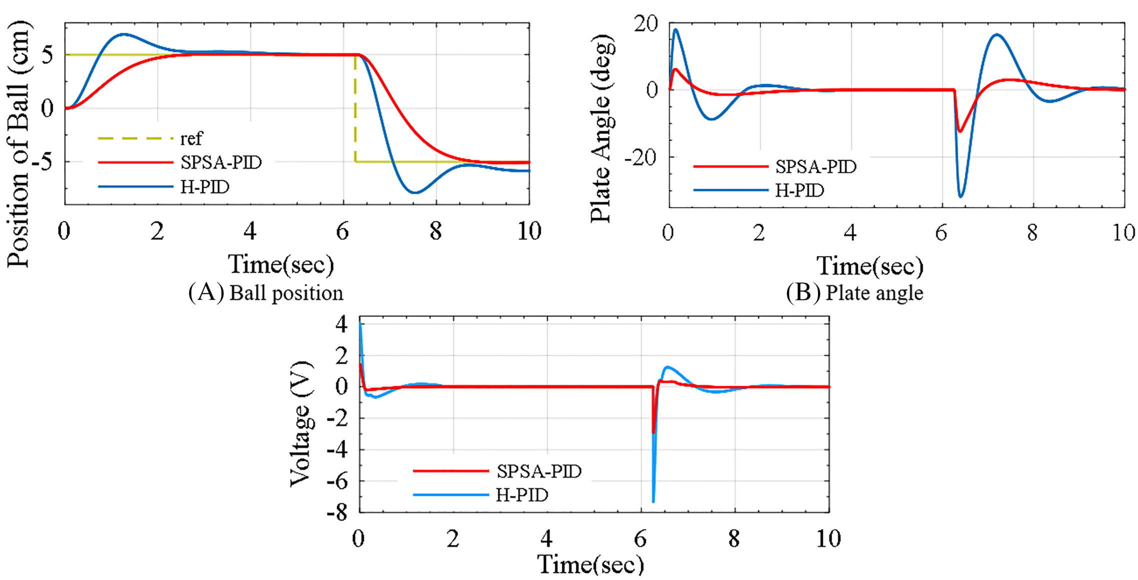

FIGURE 6 Response of simulated 2DoF ball balancer system for SPSA-PID and *H*-PID controller

TABLE 2 Time response characteristics of plant angle for SPSA-PID and H-PID controllers

Controllers Simulink	Peak time (t_p)	Settling time (t_s)	Peak overshoot (M_p)	Steady-state error (e_{ss})
H-PID	0.67 s	3.62 s	1.76%	0.0049 cm
SPSA-PID	0.63 s	1.4 s	7.32e−06%	1.3e−06 cm

6.2 | Controller fitness

The fitness of the optimization algorithm towards achieving the performance maximization is characterized by the tracking error. The most commonly used feedback controller fitness testing functions are integral of the square error (ISE),[40] integral of the absolute error (IAE),[41] and integral of the time weighted absolute error (ITAE).[42] In the research, the ISE is chosen to estimate the fitness function.

$$ISE = \int e^2(t)dt, \tag{50}$$

where e is the tracking error between reference and measured trajectory.

Furthermore, to highlight the improvement in the tracking control, the tracking error of both SPSA-PID and H-PID for positioning the ball within a square trajectory in numerical simulations are illustrated in Figure 7 and Table 3.

6.3 | Real time

The system comprises of a quadrangular metallic plate, fixed at the center through a gimble joint. The gimble joint has a two degree of rotational freedom to tilt in 2-dimensional direction, x and y axes. A Faulhaber DC micromotor series 2338 motor[43] along with a potentiometer and tachometer is used for balancing the system in both the directions. The main objective of the system is to balance the ball without falling off the quadrangular metallic plate. Initially, the ball movement concerning the operating voltage is observed, and the control signal related to the disposition of the plate angle is identified and sustained to the data acquisition (DAQ)–Q2USB.[37] In the second step, the signal measured is provided as an input to the controller, and its coordinates are captured through the camera and combined. Further, the output is forwarded to amplify the power using amplifier through the DAQ for processing the hardware. The experimental setup of 2DoF ball balancer as per the specifications developed by Quanser[37] is depicted by Figure 8.

Further, the control implementation with the experimental setup is achieved by performing the data exchange between the simulated models and the ball balancer setup. In this event, the assumptions made while developing the controller with the simulated models are encoded and validated based on the assumption management framework (AMF) associated with the function suite of data exchange. Here, the AMF flags the operating conditions of the system as valid and invalid in a machine checkable format for each assumption considered while developing the controller. The relevant set of conditions for which the assumptions are flagged as invalid is automatically avoided to improve the scalability and performance of the real-time operating system. Besides, the data exchange provides a whole suite of functions which use the features of supported data acquisition hardware from the C language. These functions configure the hardware and perform both synchronous and asynchronous I/O in various forms. The configuration functions provided by the Hardware-in-the-Loop Application Programming Interface (HIL API) gives the ability to open a hardware-in-the-loop card and configure it. The simplest I/O function of the HIL API allows single samples to be read or written immediately from the data

TABLE 3 Tracking error comparison for simulation of SPSA-PID and H-PID controllers for square trajectory

Controller	Ball position	Plate angle
SPSA-PID	7.2e+03	6.26e+03
H-PID	5.83e+04	3.31e+04

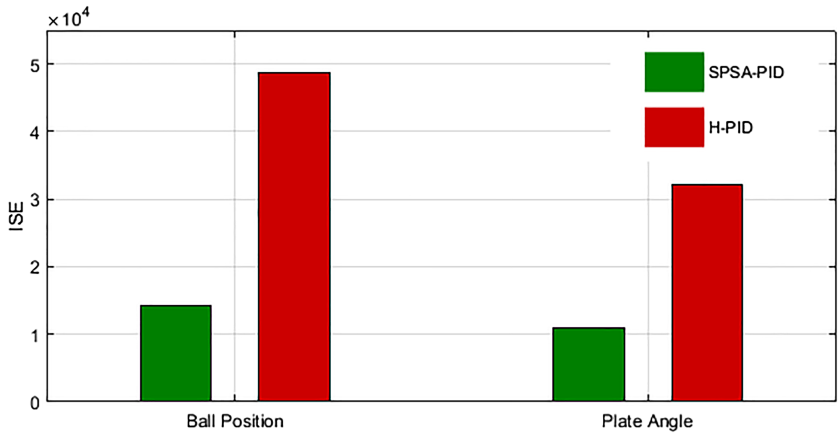

FIGURE 7 Tracking error comparison of SPSA-PID and H-PID controllers

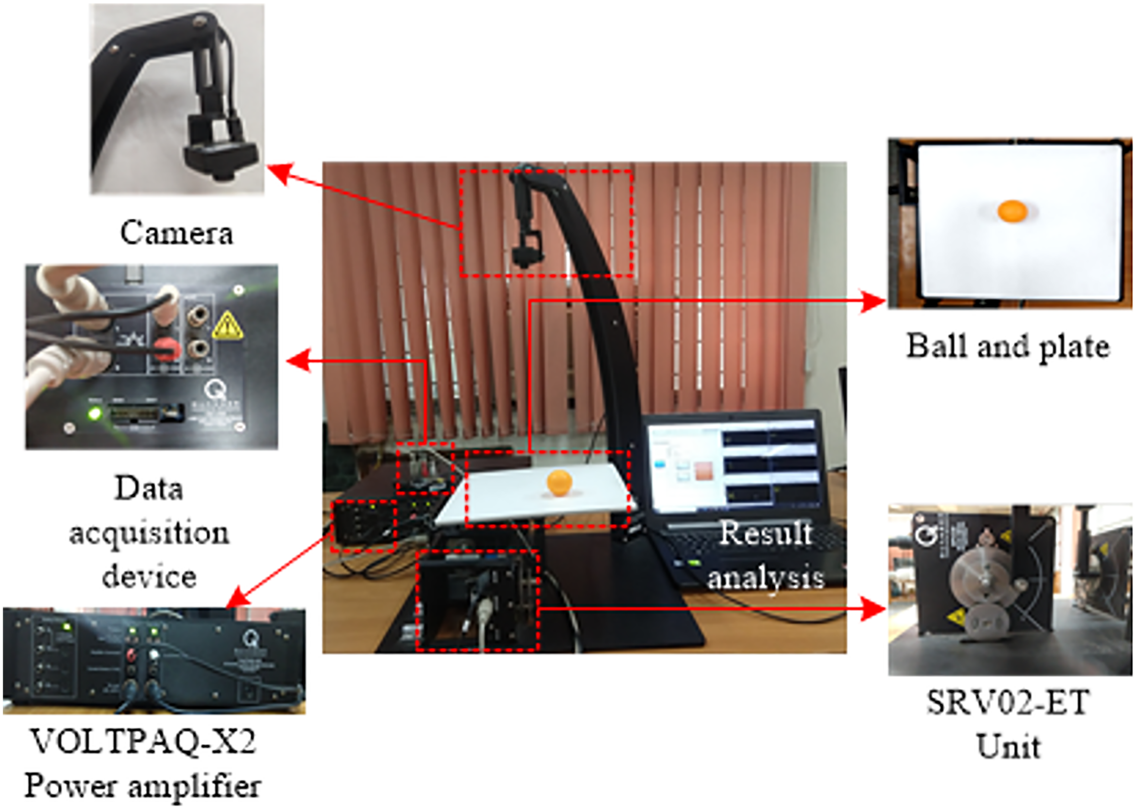

FIGURE 8 Laboratory setup of Quanser 2DoF ball balancer system

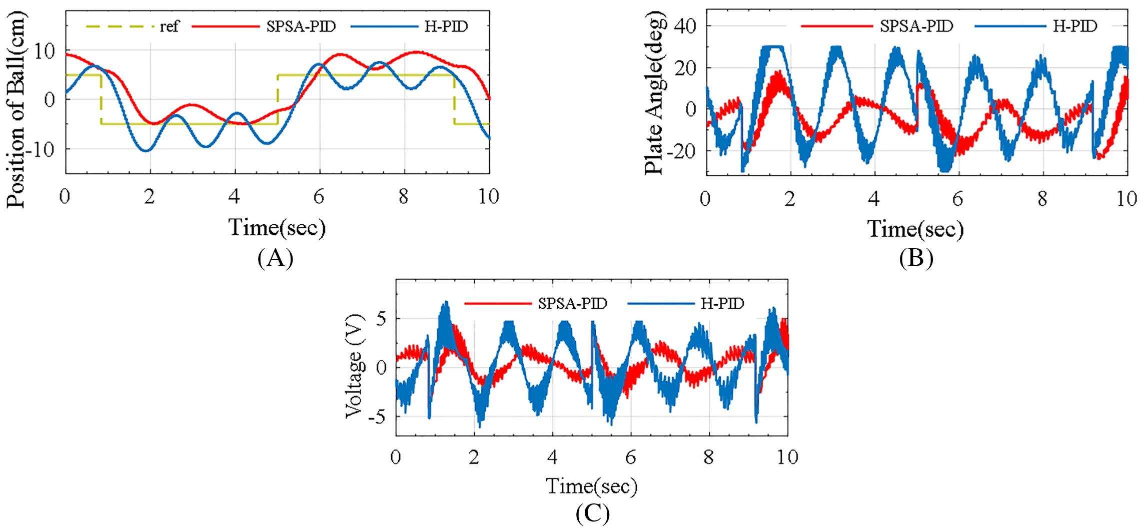

FIGURE 9 Response of Quanser 2DoF ball balancer system for SPSA-PID and $H\infty$-PID controller. (A) Ball position, (B) plate angle, and (C) voltage

TABLE 4 Time response characteristics of ball position in real-time system

Controllers	Peak time (t_p)	Settling time (t_s)	Peak overshoot (M_p)	Steady state error (e_{ss})
H-PID	2.4 s	2.76 s	22.9%	1.151 cm
SPSA-PID	1.62 s	2.09 s	12.4%	0.487 cm

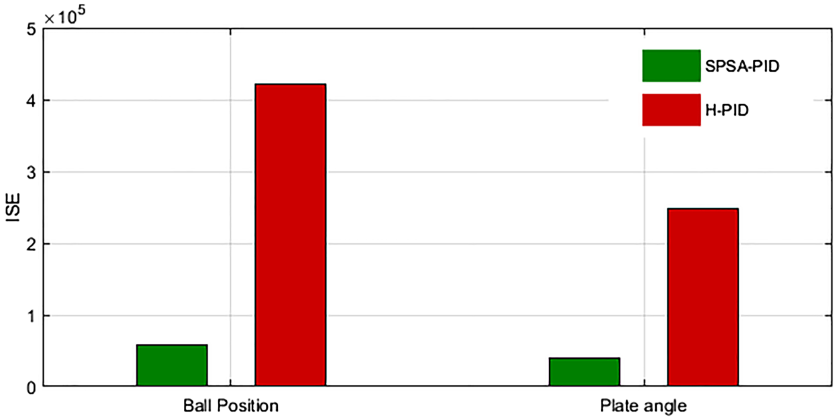

FIGURE 10 Tracking error comparison of SPSA-PID and H-PID controllers for ball position and plate angle

TABLE 5 Tracking error comparison of SPSA-PID and H-PID controllers for reference trajectory operation of real-time balancing control system

Controller	Ball position	Plate angle
SPSA-PID	6.87e+04	5.94e+04
H-PID	4.34e+05	2.47+05

acquisition card. This form of I/O is called “immediate I/O” and is supported by the Immediate I/O functions of the HIL API. This form of I/O is mainly useful for controlling the system with simulated outputs with various controllers.

As it was mentioned, the HIL API tasks are focused on the acquisition of ball position and DC motor positioning. The whole algorithm of ball positioning and control law may be easily implemented and changed in real-time in the running computer simulation, using graphical user interface (GUI). This user-friendly way of operation is ensured by a very simple, but effective, data exchange protocol of HIL C API function reference.

Further, the ball balancer system is controlled by applying SPSA-PID using MATLAB and for further interfacing with hardware, a Quarc HIL software has been used.

Figure 9A–C shows the ball position, plate angle, and motor voltage. The square input signal has been given to the setup with the frequency of 0.08 Hz and amplitude of 5. From the Figure 9A, the ball position follows the reference trajectory with fewer oscillations which make the system stable and ball moving speed will be slow down. While in *H*-PID, the number of oscillations is more, and it settles for the trajectory with more variation in ball movement. Subsequently, angle variation in Figure 9B is 18^0–20^0, made the movement of the ball stable and smooth for the SPSA-PID controller while *H*-PID responsible full plate angle variation of -30^0. As the freedom of movement of angle increase, the ball movement will also increase and causes for instability. Figure 9C shows the voltage also less and lies between 5.2 and −4.8 V which further responsible motor speed and control the plate angle smoothly. Further, the step response characteristics of the ball position are calculated to assess the superiority of the developed approach over the classical approach. As of Table 4, the response of improved SPSA-PID is better based on the peak time, settling time, and peak overshoot.

Furthermore, to highlight the improvement in the tracking control, the tracking errors of both SPSA-PID and H-PID for reference trajectory in real-time experiment are illustrated in Figure 10 and Table 5.

The results proved the effectiveness of adaptive controller using SPSA for position tracking and balancing control of balancer systems.

7 | CONCLUSION

A SPSA approach for unknown but bounded disturbances in a typical closed loop system is developed in the paper. The developed approach achieved adaptive control by constructing a sequence of estimates and formulating an optimization problem. To enhance the performance of the developed controller, additional generalizations and convergence conditions are mentioned. Stability analysis is carried out considering the linear matrix inequalities and Lyapunov stability. Further, the developed approach is assessed with a closed loop balancing control problem with two degree of freedom ball balancer system and PID controller. The developed method minimizes the optimization problem to achieve the gain values of the PID controller. The simulation and real time experiments have been conducted for balancing control using the developed adaptive controller. The results depicted improved tracking response of the SPSA-PID over conventional *H*-PID. The PID gains optimized using SPSA provide better control when compared to conventional control approach in terms of tracking response under random uncertainties.

AUTHOR CONTRIBUTIONS

Rupam Singh: Conceptualization, data curation, formal analysis, investigation, methodology, visualization. **Bharat Bhushan:** Funding acquisition, project administration, resources, software, supervision, validation.

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