Ball On The Plate Model Based on PID Tuning Methods*

Narong Aphiratsakun[1] and Nane Otaryan[2]

***Abstract*— Controller design is one of the important steps during the modelling of various systems, starting with constructing an electric kettle to an aircraft system. To achieve a good controller it is needed to define suitable values for the controller coefficients. This paper describes the research practices of the tuning technique by using MATLAB/Simulink compared with manually PID tuning based on the trial and error process, as well as, PID tuning methods from Ziegler-Nichols and Tyreus-Luyben (closed-loop proportional gain control or P-Control tests). The paper can be useful for readers who want to have a basic knowledge of research assignments based on PID tuning.**

I. INTRODUCTION

Finding the proper mathematical model description and good coefficients for the controller can save time on trying to define the controller coefficients, which ones are more accurate. This paper is dedicated analysing the efficiency of Ziegler-Nichols, Tyreus-Luyben PID tuning methods and through MATLAB/Simulink obtained coefficients on Ball On The Plate system, comparing the results with the values that were obtained through trial and error.

The report is based on the AU Ball On The Plate Balancing Robot, which is a non-linear system [1]. PID Tuning Methods require the transfer function and thus, the mathematical equation of the model. To find the transfer function is not easy because of the complexity and non-linearity of the system. In this case, the transfer function is based on linearized mathematical equations of the ball motion [2],[3],[10]. After obtaining the transfer function, PID Tuning experiments can be performed according to the methods mentioned above.

This report is divided into three main sections. Chapter II gives an introduction of the model and tuning methods. The mechanical and mathematical descriptions of the AU Ball On The Plate Balancing Robot are considered: knowledge about mechanical construction makes it possible to get the mathematical equations of ball motion, and the transfer function for the system is obtained after linearization. The tuning methods of Ziegler-Nichols and Tyreus-Luyben will be introduced. Chapter III describes the application of tuning techniques in MATLAB/Simulink, trial and error, as well as, tuning methods from Ziegler-Nichols and Tyreus-Luyben on the system. It also includes the comparison of experimental values. Finally, the experience, that was obtained through following research, is summarised in the conclusion.

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[1]Narong Aphiratsakun with Faculty of Mechatronics Engineering, Assumption University, Thailand nott.notty@gmail.com

[2]Nane Otaryan with Faculty of Computer Sciences, Clausthal University of Technology, Germany nane.otaryan@tu-clausthal.de

II. BALL ON THE PLATE MODEL AND TUNING METHODS

A. AU Ball On The Plate Balancing Robot

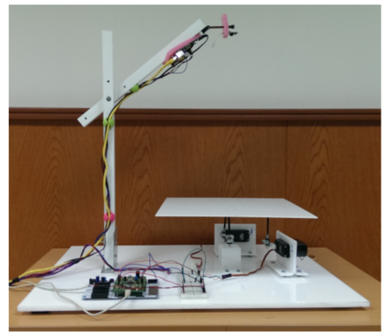

Fig. 1: AU Ball On The Plate Balancing Robot [1]

Figure 1 shows the mechanical construction of the AU Ball On The Plate Balancing Robot. The model contains two microcontrollers - Raspberry Pi 2 and STM32F0407 board, a camera on top of the plant, two DC motors attached to cables and pulleys and servo motors attached to arms, which are responsible for plate inclination. The camera on the top detects the position of the ball in real time and sends it to Raspberry Pi. The received coordinates' data is sent to the STM board. STM32F0407 calculates pulses that will be sent to servo motors, which are connected beneath the plate through arms and ball-sphere joints, and controlled with the PID algorithm. Therefore, the inclination angle depends not only on detected error, but also on the values of the PID controller [1].

B. Mathematical Model and Transfer Function

The angles of servo arms θ_x and θ_y are assumed to be the inputs, the ball position on x and y axis are assumed to be the output. Having the input and the output, the transfer function can be obtained. Thus, the mathematical model is needed and the linearization of the system is to be obtained due to the non-linear equations.

1) For the Ball On The Plate Model the following mathematical descriptions for the ball motion are considered with the parameters in Table I [2],[10]:

$$\left(m+\frac{I}{r^2}\right)\ddot{x}-m\left(x\dot{\alpha}^2+y\dot{\alpha}\dot{\beta}\right)+mgsin\alpha=0 \quad (1)$$

$$\left(m+\frac{I}{r^2}\right)\ddot{y}-m\left(y\dot{\beta}^2+x\dot{\alpha}\dot{\beta}\right)+mgsin\beta=0 \quad (2)$$

Since there was a slow rate of change, for the plate it is assumed [3]:

TABLE I: Parameters of the Ball On The Plate Mathematical Model

m:	mass of the Ball
r:	radius of the Ball
I:	mass moment of inertia
x,y:	position of axis
α, β:	inclination angles of the plate
$\dot{\alpha}, \dot{\beta}$:	angular velocity of the plate
g:	gravitation
l:	plate side length
d:	length between the joint and the centre of the gear

$$\dot{\alpha} \ll 0 \text{ , } \dot{\beta} \ll 0 \Rightarrow \dot{\alpha}, \dot{\beta} \simeq 0, \ \dot{\alpha}^2 \simeq 0, \ \dot{\beta}^2 \simeq 0.$$

2) The relationship between α and θ_x is [2]:

$$sin\alpha = \frac{2sin\theta_x d}{l} \tag{3}$$

3) The linearization of the motion at $\theta_x = 0$ and $\theta_y = 0$ [2], [10]:

$$\left(m + \frac{I}{r^2}\right)\ddot{x} + \frac{2mgd}{l}\theta_x = 0 \tag{4}$$

$$\left(m + \frac{I}{r^2}\right)\ddot{y} + \frac{2mgd}{l}\theta_y = 0 \tag{5}$$

The linearized equation (5) for the y-axis is equivalent to (4) because of the symmetry of the plate [2],[3].

4) With the Laplace transformation, the following transfer functions are obtained:

$$P(s_x) = \frac{x}{\theta_x} = -\frac{2mgdr^2}{l(mr^2 + I)}\frac{1}{s^2} \tag{6}$$

$$P(s_y) = \frac{y}{\theta_y} = -\frac{2mgdr^2}{l(mr^2 + I)}\frac{1}{s^2} \tag{7}$$

C. Tuning Methods: Ziegler-Nichols and Tyreus-Luyben

One of the popular manual closed-loop tuning techniques is the P-Control test from Ziegler-Nichols. The advantage of this method is its simple way of performing the tuning process. The procedure is the following:

- First, only P instead of PID controller is considered: at the very beginning all gains are set to zero.
- The experiment begins by increasing the proportional gain K_p until there are oscillations. K_p should be increased or decreased until the oscillations are symmetric. That value of K_p is the ultimate gain K_u.

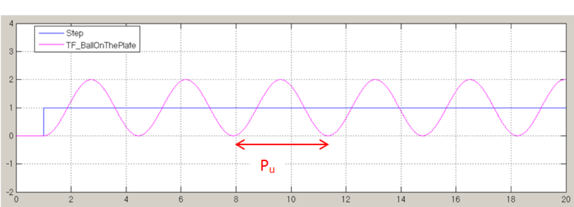

Fig. 2: Ultimate Period of Symmetric Oscillation

- The period of oscillation is the ultimate period P_u. Figure 2 shows how P_u can be measured.
- As soon as K_u and P_u are obtained, the controller parameters can be calculated according to Table II [4], [9].

TABLE II: Controller Parameters in the Ziegler-Nichols Closed-Loop Method

	K_p	T_i	T_d
P controller	$0.5K_u$	∞	0
PI controller	$0.45K_u$	$\frac{P_u}{1.2}$	0
PID controller	$0.6K_u$	$\frac{P_u}{2}$	$\frac{P_u}{8}$

Another closed-loop P-Control test tuning technique is Tyreus-Luyben method. K_u and P_u can be extracted from the description above. The PID parameters are calculated following Table III [5].

TABLE III: Controller Parameters in the Tyreus-Luyben Closed-Loop Method

	K_p	T_i	T_d
PI controller	$\frac{K_u}{3.2}$	$2.2P_u$	–
PID controller	$\frac{K_u}{2.2}$	$2.2P_u$	$\frac{P_u}{6.3}$

Both methods were applied on the real system considering transfer functions (6) and (7).

III. REAL SYSTEM VALUES FROM TRIAL AND ERROR COMPARED WITH SIMULATED AND TUNED CONTROLLER VALUES

A. MATLAB/Simulink: Closed-Loop of AU Ball On The Plate Balancing Robot

With the help of the transfer function, an experiment can be operated to tune the PID controller automatically. According to Figure 3, the closed-loop model was built in MATLAB/Simulink with sampling time set to 0.012.

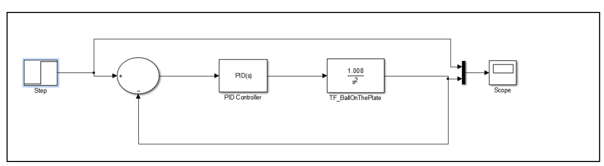

Fig. 3: Closed-Loop Simulink Model

Figure 4 shows the tuned response and the configurations, that were made after the auto tuning method was applied. As soon as the PID auto tuning experiment is completed through MATLAB/Simulink, the new controller values can be exported to MATLAB workspace and be extracted for the test on the real system. Figure 5 demonstrates the step response after operating the system with new parameters of the controller: the new step response exhibits strong oscillations. The controller parameters with $K_p = 2.07$ and $K_i = 0.0986$ are sufficient, but $K_d = 7.2$ is too high: the inclination of the plate is very steep, and the ball falls

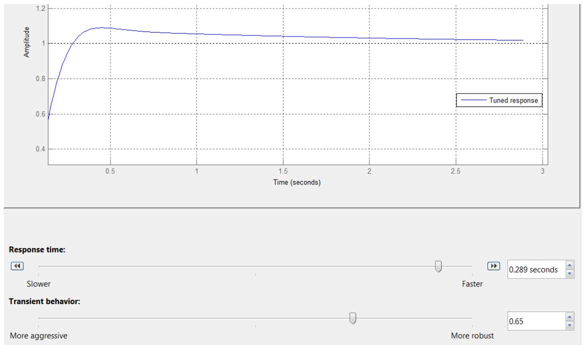

Fig. 4: Step Response After Tuning and Variation of the Response Time and Transient Behaviour

down. The first conjecture to this is: the tuning method is not adapted for the system.

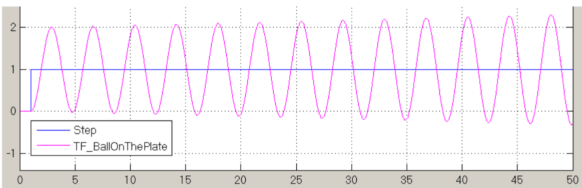

Fig. 5: Step Response with PID Parameters From Auto Tuning Through MATLAB/Simulink

B. Trial and Error Experiment

The trial and error experiment can be performed without any knowledge of the mathematical description. In the experiment, suitable PID coefficients were found through trial and error, which is based on Ziegler-Nichols method as well, but without considering the transfer function [1],[6],[7]. The integral and derivative gains K_i and K_d were also defined through trial and error. Increasing K_i was performed to eliminate the oscillation. K_d was increased to overcome overshooting and the fall of the ball. Finally, the following values were obtained for PID controller in the real experiment: $K_p = 2.0$, $K_i = 0.2$ $K_d = 0.2$ [1].

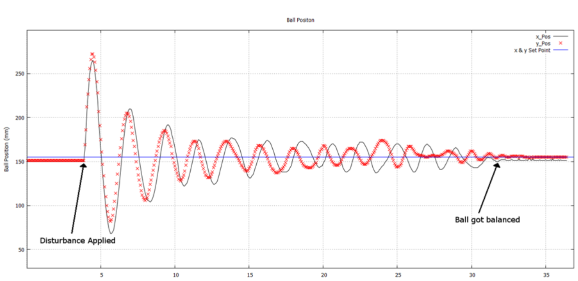

Fig. 6: Ball Position when $K_p = 2.0$, $K_i = 0.2$ And $K_d = 0.2$ [1]

According to Figure 6, the ball is balanced after 30 seconds: the monotonous curve is from servo motor 1 (along x-axis) and the red crosses - from servo motor 2 (along y -axis) [1].

C. PID Controller Coefficients Through Ziegler-Nichols and Tyreus-Luyben Methods

During the tuning experiment, the ultimate gain $K_u = 3.4$ and the ultimate period[1] $P_u = 3.5$ were measured[2]. Following the Ziegler-Nichols method, the following values were obtained and calculated:

$$K_{p_{zn}} = 0.6 * K_u \tag{8}$$

$$T_{i_{zn}} = \frac{P_u}{2} \tag{9}$$

$$T_{d_{zn}} = \frac{P_u}{8} \tag{10}$$

$\Rightarrow K_{p_{zn}} = 3.4 * 0.6 = 2.04$, $T_{i_{zn}} = \frac{3.5}{2} = 1.75$ and $T_{d_{zn}} = \frac{3.5}{8} = 0.44$

With $K_{p_{zn}}$, $T_{i_{zn}}$ and $T_{d_{zn}}$ the integral and derivative gains were calculated [8],[9]:

$$K_i = \frac{K_p}{T_i} \tag{11}$$

$$K_d = K_p T_d \tag{12}$$

$\Rightarrow K_{i_{zn}} = 1.16$ and $K_{d_{zn}} = 0.9$

By using the Tyreus-Luyben method with $K_u = 4.4$ and $P_u = 3.3$ the following results were calculated:

$$K_{p_{tl}} = \frac{K_u}{2.2} \tag{13}$$

$$T_{i_{tl}} = 2.2 * P_u \tag{14}$$

$$T_{d_{tl}} = \frac{P_u}{6.3} \tag{15}$$

$\Rightarrow K_{p_{tl}} = \frac{4.4}{2.2} = 2.0$, $T_{i_{tl}} = 2.2 * 3.3 = 7.3$ and $T_{d_{tl}} = \frac{3.3}{6.3} = 0.52$

According to (11) and (12), $K_{i_{tl}} = 0.27$ and $K_{d_{tl}} = 1.04$.

TABLE IV: Controller Parameter Values in Comparison

	K_p	K_i	K_d
Trial and error	2.0	0.2	0.2
Ziegler-Nichols	2.04	1.16	0.9
Tyreus-Luyben	2.0	0.27	1.04

In summary, the values from Tyreus-Luyben method are more similar to the controller values through trial and error. But still none of the P-Test methods delivers the expected results. Figure 7 and Figure 8 show the step responses with Ziegler-Nichols and Tyreus-Luyben parameters.

[1] For the definitions see chapter II, subchapter “Tuning Methods: Ziegler-Nichols and Tyreus-Luyben”.

[2] As the oscillations were always symmetric, the value for K_u is assumed: it is known through trial and error that $K_p = 2.0$ delivers adequate results.

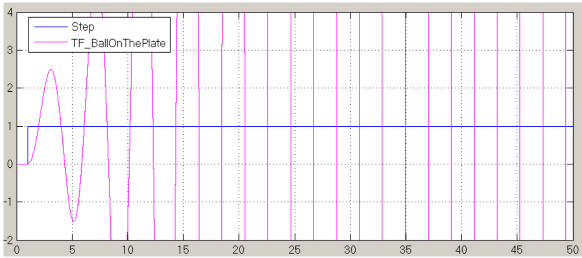

Fig. 7: Step Response with PID Parameters from Ziegler-Nichols Tuning Method

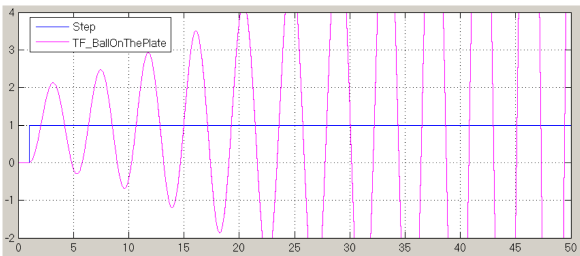

Fig. 8: Step Response with PID Parameters from Tyreus-Luyben Tuning Method

IV. CONCLUSION

During the integration of a controller in the system, there was one goal: to achieve the best balancing time. There are diverse ways of finding controller coefficient manually or automatically.

The auto tuning experiment of the PID controller using MATLAB/Simulink didn't deliver sufficient results for the model.

Some manual tuning methods, such as trial and error experiment or proportional gain control tests (Ziegler-Nichols or Tyreus-Luyben), were executed as well. The trial and error experiment had an advantage that it could be performed easily and without a mathematical model. This method delivered sufficient results for the system, and the ball could be balanced [1]. The disadvantages are the following aspects: it can require patience for a long period of time to find the values (continuous trial and error), and it does not guarantee a fast controller (because of the oscillations and long balancing time).

Proportional gain control tests can easily be performed without deep confrontation of the control system techniques as well. There is only one precondition: the mathematical model of the system is needed. The disadvantage is, that this method is not suitable for every system [8]. In the experiment, the oscillations were always symmetric and independent from the proportional gain value. If the transfer function has a higher order, it is suggested to use these tuning methods. The AU Ball On The Plate Robot has a second order transfer function. The research work gives knowledge that the proportional gain control tests above can not be used for PID Tuning of AU Ball On The Plate Balancing Robot. Further researches around the PID tuning problem can accelerate by using the information from the report.

There are several other auto tuning methods which require skill and experience. One of them is the System Identification Toolbox, supported from MATLAB. The knowledge of a mathematical model is not required: a big advantage for unknown systems or in the case that a model cannot be linearized. The System Identification Toolbox could be an extension of the research topic.

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