Minimum ISE Fractional-order PID (FOPID) Controller for Ball and Beam Mechanism

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***Abstract* – In this paper, fractional-order PID controller (FOPID) for ball and beam is being considered. Ball and beam system is one of the control engineering prototypes used to illustrate balancing mechanism of dynamical systems. The challenge in this system is to make the ball to stay on a beam at a desired position while it is naturally unstable during open-loop. This paper demonstrates the results obtained for this application using fractional-order PID (FOPID). FOPID is an improvement of PID controller with more degree of freedom. This paper also presents the effect of different fractional-orders on the closed-loop response. For comparison, FOPID controller that was tuned using FOMCON toolbox integrated in MATLAB software was also presented. The results also demonstrated the comparison between FOPID and PID controller to see the improvement gains from FOPID controller. In overall, FOPID can reduce overshoot in the output response significantly and improved the response speed if the gains are optimally tuned. This research also presented the optimal FOPID gain that was obtained through minimum ISE of output response which is better compared to the one tuned using FOMCON.**

Index Terms—Fractional order PID (FOPID) controller, PID controller, ball and beam system, FOMCON.

I. INTRODUCTION

Ball and beam process is one of the most commonly used processes in control engineering field to study about unstable system. This system development is a safe way to study the dynamics and control of an unstable system that is dangerous to be tested on site. This mechanism can be related to the real problem of stabilizing an airplane during landing and a liquid transporting in a tanker [1]. The aim of this system is to control the position of a ball at a desired position in the presence of a disturbance. This is done by manipulating the angular position of the DC motor which is used to control the angle of the beam. The position of the ball on the beam can be measured using a linear position sensor. The control voltage is send to the DC motor and then the torque generated by the motor drives the beam at a desired angle. The dynamic structure of the ball and beam system consists of two segments which are ball and beam body and motor control system. Fig 1 shows the Googol ball and beam plant used as a case study in this research.

It should be noted that the system is open-loop unstable. However, it can be stabilized by applying a feedback controller. Many researchers were satisfied with PD control performance on ball and beam since it is a double-integrator system. PD controller can improved the system stability while it is easier to tune compared to PID[1]–[3]. However, PID will be better in terms of overshoot reduction and thus, produced better response [4]–[6].

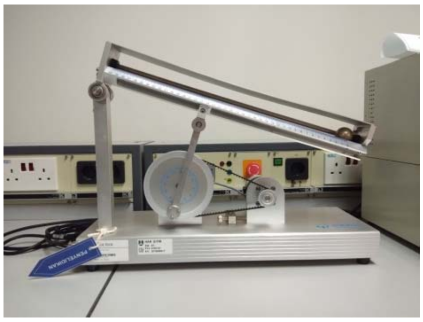

Fig 1: Ball and Beam Educational Tool GBB2004

PID controller had been applied successfully on ball and beam with different sets of performance achievements gained from different sets of tuning. Recently, studies had shown that PID performances can be greater by using fractional-order PID (FOPID) controller [7], [8]. FOPID controller is design to make a system more robust. The drawback is that it makes tuning procedures more complicated as there are more parameters to be tuned. FOPID tuning methods can be categorized into three main approaches which are analytical, rule-based, and numerical tuning methods.

In this research, FOPID will be applied to control a ball and beam system shown in Figure 1. The FOPID controller will be tuned using FOMCON Toolbox [9] in MATLAB where Oustaloup's algorithm [10] was applied to realize the fractional-order terms. Concurrently, this study simulated the behaviour of the controlled output as the fractional-order is varied and the evaluation in terms of Integral of Squared Error (ISE) will be reported. The outcomes of this study will identify the optimal fractional-order for the FOPID for the system under study. The optimized ISE FOPID was compared with FOMCON-FOPID

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and PID controllers tuned using Ziegler-Nichols and AMIGO (Approximate M-constrained integral gain Optimisation) methods.

II. BALL AND BEAM MODEL

In order to design a PID controller, the mathematical model of the ball and beam was derived. Ball and beam system can be separated into two parts which is the ball and beam itself and the motor to control the movement of the beam. A ball is placed on a beam where it can roll along the length of the beam. To stabilize the position of the ball, a servo motor will adjust the motor rotation angle, ϴ.

Based on the schematic diagram shown in Figure 2, a transfer function for ball and beam can be obtained using Langrange mechanic. The Lagrange mechanic used an energy based approach to derive an equation of motion for mechanical systems.

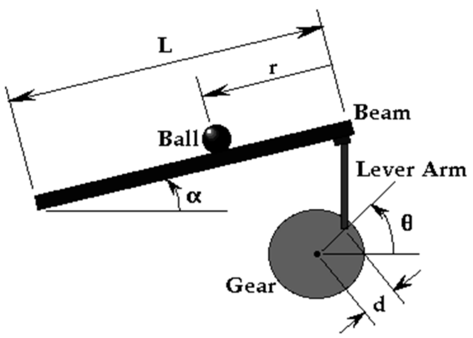

Fig 2: The schematic of servo motor and beam angle

The Lagrangian equation of motion for ball and beam is written as [11]:

$$\left(\frac{J}{R^2}+m\right)\ddot{x}+mg\sin\alpha-mx(\dot{\alpha})^2=0 \quad (1)$$

where J is the moment of inertia of the beam. All other parameters are defined in Table 1 with their values as described by Googol Technology.

Table 1: Parameters of Ball and Beam

Parameter's Name	Unit	Value
Mass of the ball (m)	kg	0.011
Radius of the ball (R)	m	0.015
Gravitational of acceleration (g)	m/s^2	-9.8
Length of the beam (L)	m	0.4
Radius of the gear (d)	m	0.1059

When the angle α is very small, (1) can be linearized which give the following linear approximation of the system:

$$\left(\frac{J}{R^2}+m\right)\ddot{x}=-mg\alpha \quad (2)$$

$$\alpha L=\theta d \quad (3)$$
$$\alpha=\frac{d}{L}\theta$$

Substituting (3) into (2);

$$\left(\frac{J}{R^2}+m\right)\ddot{x}=-mg\frac{d}{L}\theta \quad (4)$$

Taking Laplace transform of (4), and simplifying the equation into transfer function;

$$\left(\frac{J}{R^2}+m\right)X(s)s^2=-mg\frac{d}{L}\theta \quad (5)$$

$$\frac{X(s)}{\theta(s)}=\frac{-mgd}{Ls^2\left(\frac{J}{R^2}+m\right)} \quad (6)$$

Substituting all the values specified in Table 1, the transfer function of the ball and beam becomes:

$$\frac{X(s)}{\theta(s)}=\frac{-(0.011)(-9.8)(0.1059)}{0.4\left(\frac{2\times 0.011}{5}+0.011\right)s^2}$$

$$G(s)=\frac{1.853}{s^2} \quad (7)$$

III. FRACTIONAL-ORDER PID (FOPID)

Fractional-order calculus (FOC) is a generalization of integration and differentiation operation including the non-integer orders. The equation is described in (8) where α is a real number.

$$ {}_{\alpha}D_t^{\alpha}=\begin{cases}\frac{d^{\alpha}}{dt^{a}} & ,\alpha>0\\ 1 & ,\alpha=0\\ \int_a^t (d\tau)^{\alpha} & ,\alpha<0\end{cases} \quad (8)$$

Linear operator *D* was interpreted as a differentiator when α positive and an integrator when α is negative. Otherwise, D is a unity when α is zero. FOPID was introduced by I. Podlubny in 1999 [12]. FOPID equation is described in (9).

$$G(s)=Kp+\frac{Ki}{s^{\lambda}}+K_d s^{\mu} \quad (9)$$

FOPID generalized the integer PID in such a way that:

- If λ =1 and μ=1, a classical PID is obtained.
- If λ=1 and μ=0, a PI controller is obtained.
- If λ=0 and μ=1, a PD controller is obtained.
- If λ=0 and μ=0, a P controller is obtained.

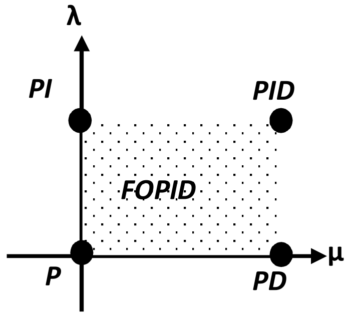

Fig. 3 Fractional-order PID control space

Hence, if λ and μ were set to arbitrary value between 0 and 1, the controller can be configured to behave within these four possibilities of FOPID control space shown in Figure 3. FOPID is more flexible but on the other hand can increase complexity in the tuning procedures.

Aleksei *et.al.* [9] had published a new MATLAB Toolbox known as FOMCON to facilitate modelling and control tuning of fractional-order system and control design. The toolbox is very convenient and offers optimization of FOPID parameters with certain objective functions to be minimized including an integral of squared error (ISE).

Many mathematical techniques had been used to approximately derive the fractional-order differ-integrator functions. These include Oustaloup's approximation algorithm which is based on recursive distribution of poles and zeros in frequency domain. This method was derived based on approximation of a function given in (10) where μ is the order of a function and μ is a positive real number.

$$H(s) = s^{\mu}\,, \qquad \mu \in \Re^{+} \tag{10}$$

The function in (10) can be approximated into a rational transfer function represented by (11).

$$s^{\lambda}_{[\omega_L,\omega_H]} = k\prod_{n=-N}^{N}\frac{1+ {s}/{\omega_{z,n}}}{1+ {s}/{\omega_{p,n}}} \tag{11}$$

where N, $\omega_{z,n}$, $\omega_{p,n}$ and k are the number of poles and zeros, the location of zero and pole and a constant gain respectively. From here, the magnitude and phase response of each function can be described by (12).

$$\left.\begin{aligned} 20\log\left|\hat{s}^{\lambda}\big|_{s=j\omega}\right| &= 20\lambda\log(\omega)dB \\ \angle\hat{s}^{\lambda}\big|_{s=j\omega} &= \frac{\pi\lambda}{2} \end{aligned}\right\} \quad \omega_l \leq \omega \leq \omega_h \tag{12}$$

Equation (12) shows that gain and phase is directly dependant on λ and can be adjusted between ±20 dB/dec and ±90º as the order varies. The frequency plots of the integral and derivative in fractional-order had been published by the authors and can be referred in [13].

IV. Results and Discussions

The first part of analysis is focusing on the effect of μ and λ towards the output performance. For this purpose, the set point has been set to 0.25m and the value of PID gain is fixed. For the first part, λ is maintained constant at 1.0 and μ is varied as presented in Table 2. Measurement of ISE is tabulated. From the table, it is apparent that ISE increases with μ until it achieved maximum value when μ=0.5 and decrease as μ increase.

Table 2: Effect of $\boldsymbol{\mu}$ (derivative) when $\boldsymbol{\lambda}$ = 1

$\boldsymbol{\mu}$	ISE
0.05	0.0109
0.25	0.0257
0.35	0.0173
0.50	0.1322
0.65	0.0074
0.80	0.0052
0.90	0.0041
1.00	0.0128

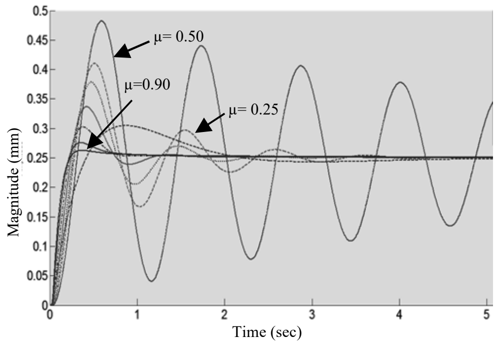

Fig. 4 Effect of μ on output response when λ=1.0

The effect of μ on the output response can be observed in Fig. 4. When μ is set to 0.5, highest overshoot occurred and the output was oscillated even after 6 sec. The overshoot will decay if μ was set to a value smaller or larger than 0.5. The optimal value of μ is found to be at 0.90 based on minimum ISE shown in Table 2.

Meanwhile, Table 3 summarizes the ISE measurements when $\boldsymbol{\mu}$ = 1 and $\boldsymbol{\lambda}$ is varied. From the table, it was observed that the effect is trivial and thus, insignificant to the output response as shown in Fig. 5. However, the trend shows that ISE is increased as λ increased. So, the optimal value of λ for this problem is 0.05 or 0.0(PD).

Table 3: Effect of $\boldsymbol{\lambda}$ (integral) when $\boldsymbol{\mu}$ = 1

λ	ISE
0.05	0.003271
0.25	0.003273
0.35	0.003274
0.50	0.003276
0.65	0.003278
0.80	0.003280
0.90	0.003282
1.00	0.003284

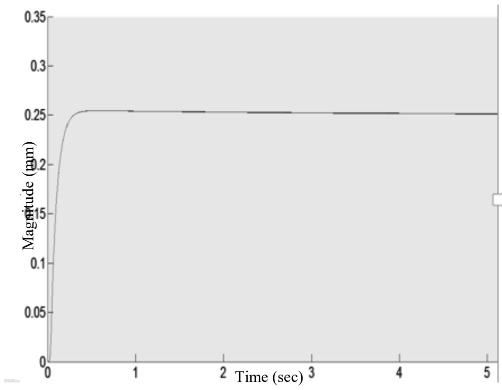

Fig. 5 Effect of λ on output response when μ =1.0

The next part of the analysis will show the comparison between PID, FOMCON-FOPID (μ=0.5, λ=0.5) and minimum ISE FOPID (μ=0.05, λ=0.9).

Table 5: Step Response of FOPID, PID and Optimum FOPID controller

Controller	OS (%)	Tr (sec)	Tp (sec)	ISE
FOMCON-FOPID	34.8	0.338	0.627	0.01094
PID	17.6	1.79	3.13	0.04563
Minimum ISE FOPID	4.8	0.32	0.482	0.00412

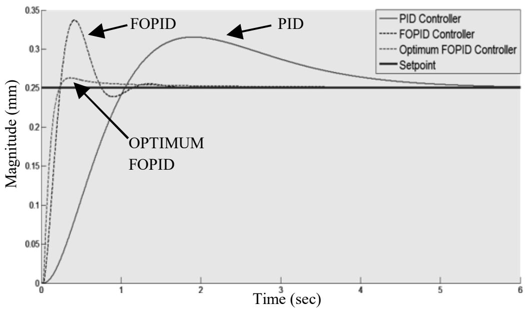

Fig 6: Comparison between PID, FOMCON-FOPID and minimum ISE FOPID.

By referring to Fig.6, it can be observed that PID response is slow and has high overshoot. When the order of integral and derivative were reduced to 0.5 by FOMCON-FOPID, the response is faster but the overshoot is higher than the PID. With the same value of gain, minimum ISE FOPID recommended the optimal order of integral is 0.05 and derivative is 0.9. This setting produced faster response and less overshoot in the output.

V. Conclusion

This study presented the results from analysis on the effect of fractional-order integral and derivative on the ball and beam system. The results show that the recommended setting for minimum ISE is approaching a PD while the presence of integral is only trivial. The proposed optimal fractional-order based on ISE produced better output response compared to the value proposed by FOMCON while the PID gains were kept constant. However, from the results it is obvious that FOPID can produced better response compared to PID.

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