# Hierarchical mixing matrices

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we assume that each region shares the proportion of infections that occur in the home region, $\delta_H$, and those that occur from outside the region, $\delta^A = 1 - \delta^H$, to produce the matrix to produce the single level mixing matrix

Similarly, $\delta^A$, the proportion of infections occurring outside an individuals home region can be divided into those infections which occur inside the home region at the next highest scale, $\delta^{H_2}$, and those that occur at outside this higher level home region $\delta^{A_2}$, to create the two level mixing matrix:

$$M = \begin{pmatrix} \delta^H & \delta^A \cdot L^2_{1,2} & \cdots & \delta^A \cdot L^2_{1,j} \\ \delta^A \cdot L^2_{2,1} & \delta^H & \cdots & \delta^A \cdot L^2_{2,j} \\ \vdots & \vdots & \ddots & \vdots \\ \delta^A \cdot L^2_{j,1} & \delta^A \cdot L^2_{j,2} & \cdots & \delta^H \end{pmatrix}$$

where

$$L^k_{i,j} = \begin{cases} \frac{\delta^H}{N_{j,k}} & \text{if } j \in R^k_i \\ \frac{\delta^A}{N_{j,k}} & \text{if } j \notin R^k_i \text{ \& } k = n \\ \delta^A L^{k+1}_{i,j} & otherwise \end{cases}$$

Where $R^k_i$, is the set of patches in the same $k$ level region as $i$, and $N_{j,k}$ is the number of patches in the same level $k$ region as $j$, and $n$ is the total number of levels.

The sum of each row is given by

$$\sum_{j=1}^{N} M_{i,j} = \delta^H + \delta^A\delta^H + \cdots + \delta^{H^{n-1}}\delta^A + \delta^{H^n}$$

Which factorises to

$$\sum_{j=1}^{N} M_{i,j} = \delta^H + \delta^A(\delta^H + \delta^A\delta^H + \cdots + \delta^{A^{n-2}}\delta^H + \delta^{A^{n-1}})$$

iteratively until

$$\sum_{j=1}^{N} M_{i,j} = \delta^H + \delta^A(\delta^H + \delta^A(\delta^H + \delta^A(\cdots(\delta^H + \delta^A)))$$

since $\delta^H + \delta^A = 1$, then

$$\sum_{j=1}^{N} M_{i,j} = \delta^H + \delta^A = 1$$