

Hierarchical mixing matrices

we assume that each region shares the proportion of infections that occur in the home region, δ_H , and those that occur from outside the region, $\delta^A = 1 - \delta^H$, to produce the matrix to produce the single level mixing matrix

Similarly, δ^A , the proportion of infections occurring outside an individuals home region can be divided into those infections which occur inside the home region at the next highest scale, δ^{H_2} , and those that occur at outside this higher level home region δ^{A_2} , to create the two level mixing matrix:

$$M = \begin{pmatrix} \text{nbsp;} & \text{nbsp;} & \text{nbsp;} & \text{nbsp;} \delta^H & \delta^A \cdot L_{1,2}^2 & \dots & \delta^A \cdot L_{1,n}^2 \\ \text{nbsp;} & \text{nbsp;} & \text{nbsp;} & \text{nbsp;} \delta^A \cdot L_{2,1}^2 & \delta^H & \dots & \delta^A \cdot L_{2,n}^2 \\ \text{nbsp;} & \text{nbsp;} & \text{nbsp;} & \text{nbsp;} \vdots & \vdots & \ddots & \vdots \\ \text{nbsp;} & \text{nbsp;} & \text{nbsp;} & \text{nbsp;} \delta^A \cdot L_{j,1}^2 & \delta^A \cdot L_{j,2}^2 & \dots & \delta^A \cdot L_{j,n}^2 \end{pmatrix}$$

where

$$L_{i,j}^k = \begin{cases} \frac{\delta^H}{N_{j,k}} & \text{if } j \in R_i^k \\ \frac{\delta^A}{N_{j,k}} & \text{if } j \notin R_i^k \text{ \& } k = n \\ \delta^A L_{i,j}^{k+1} & \text{otherwise} \end{cases}$$

Where R_i^k , is the set of patches in the same k level region as i , and $N_{j,k}$ is the number of patches in the same level k region as j , and n is the total number of levels.

The sum of each row is given by

$$\sum_{j=1}^N M_{i,j} = \delta^H + \delta^A \delta^H + \dots + \delta^{H^{n-1}} \delta^A + \delta^{H^n}$$

Which factorises to

$$\sum_{j=1}^N M_{i,j} = \delta^H + \delta^A (\delta^H + \delta^A \delta^H + \dots + \delta^{A^{n-2}} \delta^H + \delta^{A^{n-1}})$$

iteratively until

$$\sum_{j=1}^N M_{i,j} = \delta^H + \delta^A (\delta^H + \delta^A (\delta^H + \delta^A (\dots (\delta^H + \delta^A))))$$

since $\delta^H + \delta^A = 1$, then

$$\sum_{j=1}^N M_{i,j} = \delta^H + \delta^A = 1$$