

# Face Image Fitting\*

Kangcheng Hou<sup>†</sup>      Shuqi Wang<sup>‡</sup>      Tianhao Wei<sup>§</sup>

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## 1 Introduction

A large proportion of images people uploaded is facial images. Motivated by this, in this course project, we want to develop a software that ease the process of editing facial image. The core part of our course project is face fitting, i.e. given a image, find the corresponding 3D model of the image. We also build two applications based upon the face fitting module.

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\*This is the first time we try to write the course project report in English. Your understanding is much appreciated.

<sup>†</sup>kangchenghou@gmail.com

<sup>‡</sup>wangshuqi.cn@gmail.com

<sup>§</sup>phi.wth@gmail.com

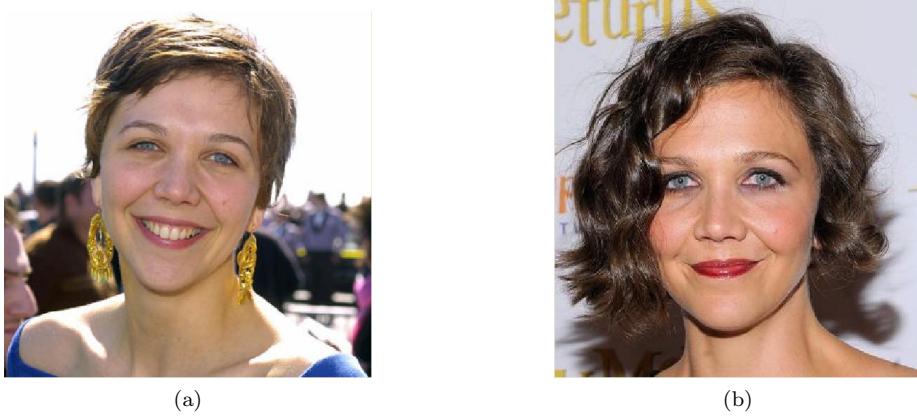


Figure 1: One application of face fitting is to transfer the smile from face (a) to face (b)

The first one is to substitute the mouth in one image to another. The second one is to provide accessible way to edit face.

### 1.1 Structure of the report

In this report, we will first introduce the core part of this project, i.e. to reconstruct the 3D face model from a single image. Then we will introduce pipelines of two applications and their underlying techniques. Lastly, we will discuss further possible improvements of this project.

## 2 Fitting a face model

Given a photo, we want to find a facial model fitted to the image. The overall procedure is to find the 2D feature points of the image, and use these 2D feature points as the proxy of the original image to fit a 3D face model.

### 2.1 Bilinear face model

First we introduce how to generate a 3D face model. A face model can be thought to be combined using two attributes, i.e. identity and expression. Bilinear face model [?] assumes any face model is generated from these two attributes.

$$\mathbf{f} = \mathcal{M} \times \mathbf{w}_{\text{id}}^\top \times \mathbf{w}_{\text{expr}}^\top$$

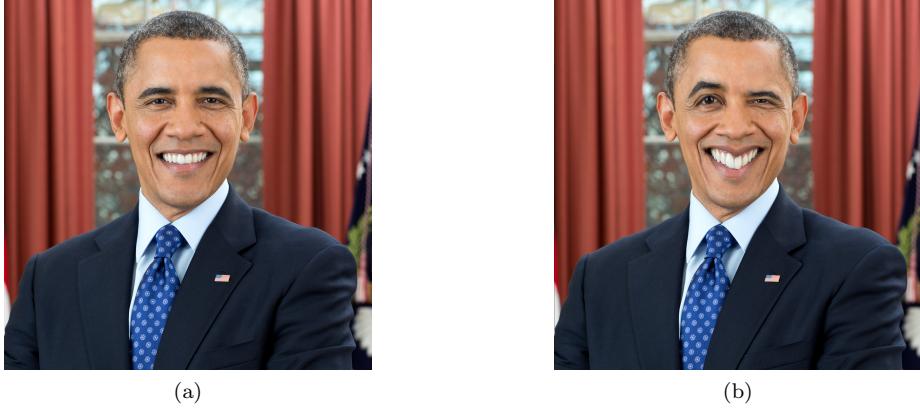


Figure 2: Another application of face fitting is to easily manipulate the face image without much artifact.

To implement this, we find the Basel Face Model [?]. What this face model database provides to us is slightly different from the original bilinear face model.

$$\begin{aligned} \mathbf{f} &= \bar{\mathbf{f}} + \mathcal{M}_{\text{id}} \times \mathbf{w}_{\text{id}}^\top + \mathcal{M}_{\text{expr}} \times \mathbf{w}_{\text{expr}}^\top \\ \mathbf{w}_{\text{id}} &\sim \mathcal{N}(0, \text{diag}(\sigma_{\text{id}}^{(1)}, \dots, \sigma_{\text{id}}^{(N_{\text{id}})})) \\ \mathbf{w}_{\text{expr}} &\sim \mathcal{N}(0, \text{diag}(\sigma_{\text{expr}}^{(1)}, \dots, \sigma_{\text{expr}}^{(N_{\text{expr}})})) \end{aligned}$$

It can be regarded as Taylor expansion approximation of the bilinear model. Though slightly different, the empirical result looks good so we didn't bother to spend time to find some database that use the original bilinear face model. So a face model is decided by two parameters, the identity vector  $\mathbf{w}_{\text{id}}$  and the expression vector  $\mathbf{w}_{\text{expr}}$ . To project the face model to the 2D plane, under the assumption of weak perspective camera model, with scale  $s$ , rotation  $\mathbf{R}$  and translation  $\mathbf{t}$ , we have

$$\hat{\mathbf{f}} = s(\mathbf{R}\mathbf{f} + \mathbf{t})$$

Now we know the generating process of a 3D face model:

$$\hat{\mathbf{f}} = s(\mathbf{R}(\bar{\mathbf{f}} + \mathcal{M}_{\text{id}} \times \mathbf{w}_{\text{id}}^\top + \mathcal{M}_{\text{expr}} \times \mathbf{w}_{\text{expr}}^\top) + \mathbf{t})$$

To fit 3D face model, we first use state-of-the-art commercial software provided by face++<sup>1</sup> to track the feature points  $\{l_1, \dots, l_{N_{\text{lm}}}\}$ . Then we solve the following optimization problem:

$$\min_{\mathbf{w}_{\text{id}}, \mathbf{w}_{\text{expr}}, s, \mathbf{R}, \mathbf{t}} \sum_{i=1}^{N_{\text{lm}}} \|l_i - h_i\|_2^2$$

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<sup>1</sup><https://www.faceplusplus.com.cn/>

where  $\{h_1, \dots, h_{N_{\text{lm}}}\}$  is the corresponding points on 3d model. This is a hard optimization problem which can't be solved by singly derive the derivative of the objective function and set the derivative to zero. However, it turns out we can solve this hard optimization problem by coordinate descent by alternating the optimizaion of  $s, \mathbf{R}, \mathbf{t}$  and  $\mathbf{w}_{\text{id}}, \mathbf{w}_{\text{expr}}$ .

$$\min_{\mathbf{w}_{\text{id}}, \mathbf{w}_{\text{expr}}, s, \mathbf{R}, \mathbf{t}} \sum_{i=1}^{N_{\text{lm}}} \|l_i - s(\mathbf{R}(\mathcal{M} \times [\mathbf{w}_{\text{id}}, \mathbf{w}_{\text{expr}}]^\top)_i + \mathbf{t})\|_2^2$$

It turns out we can calculate the derivative w.r.t  $s, \mathbf{R}, \mathbf{t}$  or  $\mathbf{w}_{\text{id}}, \mathbf{w}_{\text{expr}}$  and set them to 0. Thus we have the following algorithm.

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**Algorithm 1:** Fit face model to a single image

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**Input:** facial landmarks  $\{l_1, \dots, l_{N_{\text{lm}}}\}$  and PCA model  
**Output:** shape coefficients  $\mathbf{w}$  and camera parameters  $s, \mathbf{R}, \mathbf{t}$   
Set  $\mathbf{w} = \mathbf{0}$ ;  
**repeat**  
    Set  $\mathbf{f} = \bar{\mathbf{f}} + \mathcal{M} \times \mathbf{w}^\top$ ;  
    Find the camera parameters  $s, \mathbf{R}, \mathbf{t}$  using  $\mathbf{f}$  and  $\{l_1, \dots, l_{N_{\text{lm}}}\}$ ;  
    Project all vertices of  $\mathbf{f}$  onto the image plane:  $\hat{\mathbf{f}} = s(\mathbf{R}\mathbf{f} + \mathbf{t})$ ;  
    Find the convex hull of  $\hat{\mathbf{f}}$  as  $\text{hull}(\hat{\mathbf{f}})$ ;  
    For contour landmarks  $l_i$ , find the correspondence;  
    Solve  $\mathbf{w}$ ;  
**until**  $\mathbf{w}$  converges;

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Finding the optimal  $\mathbf{w}_{\text{id}}, \mathbf{w}_{\text{expr}}$  with  $s, \mathbf{R}, \mathbf{t}$  fixed is a simple regularized least square problem. Finding the optimal  $s, \mathbf{R}, \mathbf{t}$  with  $\mathbf{w}_{\text{id}}, \mathbf{w}_{\text{expr}}$  fixed is a classical problem in computational photography and can be solved by POSIT algorithm[?].

### 3 Joint fitting of the model

In some scenerios where we want to fit models for the same person that appeared in multiple images at the same time. We will need to fit the same identity weights  $\mathbf{w}^{\text{id}}$  and fit the expression weights  $\mathbf{w}^{\text{expr}}$  for different images. We adapt the previous algorithm as follows.

### 4 Introduction of the applications

Now we can fit a face model for a single image or fit multiple face models of the same person in multiple images, now we can do some interesting things. In the following sections, we will introduce three applications related to face model fitting and show the versatility of face fitting results.

## **5 Face Image Editing**

Do some introduction of face image editing.

### **5.1 Lapalacian Mesh Editing**

### **5.2 Face IK**

### **5.3 Results**