PIGEON: Flying Beyond Strong Neuron Activation Coverage

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Abstract

Neuron Boundary Coverage (NBC) has been widely adopted to evaluate the test adequacy of deep neural networks. However, NBC suffers from key limitations, such as its sensitivity to activation outliers and lack of consideration for the underlying distribution of neuron activations. In this work, we propose a probabilistic refinement of NBC by incorporating confidence-thresholded bounds derived from neuron-wise activation statistics. Our coverage variants provide more robust and statistically meaningful assessments of neuron behavior. Empirical results across 147 model-dataset pairs demonstrate that our method correlates more strongly with established robustness metrics and captures nuanced differences in model responses that traditional coverage metrics often miss.

1 Introduction

Deep Neural Networks (DNNs) have demonstrated impressive performance across a wide range of tasks, yet their deployment in safety-critical applications demands rigorous testing and validation. Due to the non-linearity and high-dimensional nature of DNNs, traditional software testing methodologies often fail to capture vulnerabilities in these models, necessitating the development of specialized testing approaches.

Inspired by classical software engineering where code coverage serves as a proxy for test adequacy, various neuron coverage metrics have been proposed to analyze the internal behavior of DNNs. These metrics aim to quantify how thoroughly neuron activations are exercised by a given input set, offering a structured perspective on model exploration and potential failure modes.

Among existing metrics, Neuron Coverage (NC), k-Multisection Neuron Coverage (KMNC), and Neuron Boundary Coverage (NBC) have gained prominence. However, prior studies have shown that these metrics often lack a strong and consistent correlation with robustness—particularly against adversarial examples. This raises concerns about their reliability as indicators of model safety.

In this work, we revisit Strong Neuron Activation Coverage (SNAC), a variant of NBC that captures how often neurons exceed their historical maximum activation. While SNAC highlights over-activation phenomena commonly induced by attacks such as FGSM and c&w, its original formulation is limited by its reliance on a single maximum threshold, which can be skewed by outliers or noise.

To address these limitations, we propose confidence-thresholded variants of SNAC that incorporate neuron-wise activation distributions. By grounding the threshold in statistical confidence intervals, our method better reflects the rarity of activations and improves the correlation with robustness metrics. Through comprehensive experiments on adversarially retrained models, we demonstrate that our refined coverage metrics provide more meaningful insights into a model's robustness.

2 Related Work

2.1 Neuron Coverage

Metric Several neuron coverage metrics have been introduced to evaluate the testing adequacy of deep neural networks (DNNs). Neuron Coverage (NC) [6] measures the proportion of activated neurons beyond a threshold, while k-Multisection Neuron Coverage (KMNC) [5] refines this by dividing activation ranges into intervals to assess diversity. Neuron Boundary Coverage (NBC) [5] and Strong Neuron Activation Coverage (SNAC) [5] focus on whether activations exceed training-time boundaries.

Robustness Recent studies have raised critical concerns about the effectiveness of neuron coverage metrics in evaluating the robustness of deep neural networks. Harel-Canada et al. [4] and Yang et al. [9] show that higher neuron coverage does not guarantee better fault detection and often results in unnatural inputs, with gradient-based methods outperforming coverage metrics. Dong et al. [2] further find little correlation between neuron coverage, including NBC, and model robustness, highlighting its sensitivity to input data rather than robustness. The work of Dong et al. [2] critically analyzes existing coverage metrics (NC, KMNC, NBC, etc) and their weak correlation with adversarial robustness. Their experiments, especially the test configurations and coverage-evaluation methodology, have strongly influenced our study. We adopt their evaluation protocol and build upon their critiques to propose a refined SNAC-based metric that accounts for activation distributions and addresses the statistical limitations of traditional maximum-based thresholds.

2.2 Robustness Metric

Robustness evaluation methods such as Lipschitz [8] and CLEVER [7] aim to quantify a model's sensitivity to input perturbations. Lipschitz analysis derives theoretical upper bounds on output variations, while CLEVER estimates the minimal perturbation required for misclassification by leveraging local Lipschitz constants.

Formally, given a classifier f and input x, the global Lipschitz constant L satisfies:

$$\|f(x) - f(x')\| \leq L \cdot \|x - x'\|$$

for all admissible inputs x, x'. A smaller Lipschitz constant indicates that the model's output is less sensitive to input perturbations, implying better robustness.

The CLEVER score further estimates the minimal adversarial distortion required to change the model's prediction for a specific input x. Let $f_c(x)$ denote the logit corresponding to the true class c, and $f_k(x)$ denote the logits for all other classes $k \neq c$. The CLEVER score is defined as:

$$\text{CLEVER}(x) = \frac{f_c(x) - \max_{k \neq c} f_k(x)}{L_{\text{local}}}$$

where L_{local} is the local Lipschitz constant approximated around x by:

$$L_{\text{local}} \approx \max_{\|\delta\| \leq R} \frac{\|f(x+\delta) - f(x)\|}{\|\delta\|},$$

with δ sampled within a ball of radius R. In contrast to the global Lipschitz constant, a higher CLEVER score indicates better robustness, as it reflects the amount of perturbation required to induce misclassification for a given input.

2.3 Adversarial Attack

Fast Gradient Sign Method (FGSM) [3] generates adversarial examples by adding perturbations along the gradient direction. The Carlini & Wagner (C&W) attack [1] uses optimization to craft stronger, less perceptible perturbations. These attacks reveal the vulnerability of DNNs to small input changes.

3 Preliminaries

3.1 Neuron Boundary Coverage (NBC)

Neuron Boundary Coverage (NBC) quantifies the extent to which corner-case regions of neuron activations are exercised by a given test input set T. These corner cases are defined based on the boundary values observed during training. Specifically:

- The upper corner region is $[\text{high}_n, \infty)$.
- The lower corner region is $(-\infty, \text{low}_n]$.

Let UCN be the set of neurons with activations beyond their upper bound, and LCN the set below their lower bound, when evaluated on T. Then NBC is formally defined as:

$$NBC = \frac{|UCN| + |LCN|}{2 \times |N|} \tag{1}$$

where $|N|$ is the total number of neurons in the network. This metric measures how many neurons were activated in either extreme, providing an indication of the network's exposure to boundary behavior.

3.2 Strong Neuron Activation Coverage (SNAC)

Strong Neuron Activation Coverage (SNAC) focuses exclusively on the upper-bound region of neuron activations. It is defined as the ratio of neurons whose activations exceed their maximum observed value during training:

$$SNAC = \frac{|UCN|}{|N|} \tag{2}$$

This metric emphasizes how many neurons are triggered into unusually high activation states, which may indicate sensitivity to input perturbations, particularly under ReLU activation. SNAC has been studied as a proxy for robustness to over-activation attacks.

3.3 Why Measure Neuron Over-Activation?

Over-activation of neurons can be a symptom of abnormal input patterns, such as those induced by adversarial examples or distribution shifts. By capturing how often a network responds with extreme neuron activations, NBC and SNAC offer insight into whether the model is operating outside of its trained regime.

While Dong et al. (2019) argued that the correlation between structural coverage and robustness can be limited, their work does not invalidate the usefulness of coverage metrics altogether. Rather, it highlights the need for more carefully designed metrics that reflect meaningful behavioral deviations under attack.

3.4 Adversarial Attacks Considered

To evaluate the effectiveness of our proposed metric, we consider two representative adversarial attack methods:

Fast Gradient Sign Method (FGSM) FGSM, proposed by Goodfellow et al., is a one-step gradient-based attack that perturbs the input x in the direction of the gradient of the loss with respect to the input:

$$x' = x + \epsilon \cdot \text{sign}(\nabla_x J(x, y)) \tag{3}$$

where ϵ controls the perturbation magnitude, J is the loss function, and y is the true label. FGSM is simple and fast, and tends to cause over-activation in neurons due to its direct gradient-based perturbation.

Carlini & Wagner (C&W) Attack The C&W attack formulates the adversarial example generation as an optimization problem to find minimal perturbation adversarial samples:

$$\arg\min_{x'} \|x' - x\|_p + \lambda \cdot f(x', t) \tag{4}$$

where $f(x', t)$ is a loss encouraging misclassification to target label t, λ balances perturbation and misclassification, and p denotes the norm used (L_0, L_2, or L_∞). C&W is effective in crafting subtle yet strong attacks, often bypassing simple defenses.

4 Distribution-Aware Neuron Coverage

To overcome the limitations of existing neuron coverage metrics such as NBC and SNAC, which rely solely on fixed activation boundaries, we introduce **Pigeon**, Distribution-Aware Neuron Coverage. This metric characterizes each neuron's activation behavior based on its empirical distribution, enabling a more statistically grounded notion of coverage.

4.1 Pre-Activation Distribution

Pigeon uses the *pre-activation* values of neurons—i.e., the inputs to nonlinear functions such as ReLU—rather than post-activation outputs. This is because ReLU eliminates negative values, restricting the observable distribution to only non-negative activations. As a result, distributional information is lost. Pre-activation values preserve the full dynamic range of a neuron's response, enabling more accurate modeling of activation behavior using statistical methods. We assume that the distribution of pre-activation values for each neuron approximately follows a Gaussian distribution. This is a reasonable approximation in networks using batch normalization or standardized initializations (e.g., LeNet).

4.2 Confidence Interval Estimation

Given a set of profiling inputs, we compute the empirical mean μ_i and standard deviation σ_i of the pre-activation values z_i for each neuron i:

$$\mu_i = \mathbb{E}[z_i], \quad \sigma_i = \sqrt{\mathbb{E}[(z_i - \mu_i)^2]}.$$

Since ReLU restricts the output to non-negative values, we focus only on the upper tail of the distribution and define the following one-sided confidence intervals:

- **95% upper bound:** $\mu_i + 1.645\sigma_i$
- **99% upper bound:** $\mu_i + 2.326\sigma_i$
- **99.99% upper bound:** $\mu_i + 3.891\sigma_i$

These thresholds define the boundary for statistically rare (high) activations for each neuron.

4.3 Definition

Given a test dataset, we say that a neuron i is *covered* under Pigeon if any test input produces a pre-activation value z_i that exceeds the corresponding upper confidence bound. Formally:

$$\text{Pigeon}_i(x) = \begin{cases} 1 & \text{if } z_i(x) > \mu_i + k\sigma_i, \\ 0 & \text{otherwise} \end{cases}$$

where $k = 2$ for 95% confidence, or $k = 3.3$ for 99.9% confidence.

The overall Pigeon is computed as the proportion of neurons that are covered by at least one test input:

$$\text{Pigeon} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}\left[\exists x \in \mathcal{D}_{\text{test}} : \text{Pigeon}_i(x) = 1\right],$$

where N is the total number of neurons and $\mathbb{I}[\cdot]$ is the indicator function.

This approach allows us to measure how well test inputs cover unusual or atypical neuron behaviors, thereby improving the sensitivity of coverage metrics in detecting corner-case activations compared to NBC.

4.4 Interpretation

The Pigeon metric reflects how frequently test inputs elicit statistically rare pre-activation values across neurons. A high or low Pigeon score can indicate different underlying characteristics of the model, training data, or test data distribution.

High Pigeon score:

- The training data used for the model may have a narrow or well-normalized distribution, leading to tightly bounded pre-activation statistics.
- The model may exhibit reduced robustness, with many neurons prone to over-activation when exposed to unfamiliar or out-of-distribution inputs.
- The test dataset may differ significantly from the training distribution, potentially containing inputs that stimulate atypical neuron behavior.

Low Pigeon score:

- The model may have been trained on a dataset with broad or diverse input distributions, covering a wide range of neuron activations during training.
- The model may be more robust, exhibiting stable and bounded neuron responses to test inputs.
- The test dataset may be well-normalized or closely aligned with the training distribution.

5 Experiments

Our overall experiment pipeline is illustrated in Figure 1.

5.1 Adversarial Sample Generation via SNAC and FGSM-Based Neuron Boundary Perturbation

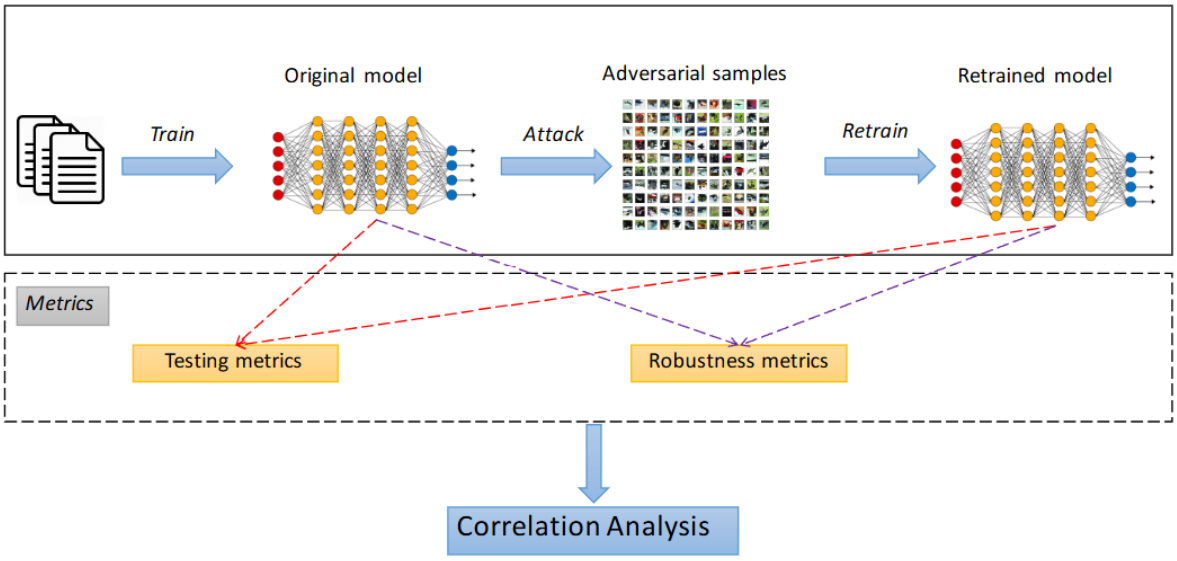

Figure 1: Overview of the experiment pipeline.

5.1.1 Objective

In this experiment, we investigate the behavior of internal neuron activations in a trained neural network and systematically generate adversarial inputs that exceed previously observed neuron activation upper boundaries. This follows the framework of SNAC, which evaluates how extensively individual neurons are activated during training and guides adversarial generation into unexplored activation regions.

5.1.2 Motivation

We hypothesize that once an input achieves maximal activation for a given neuron, slight perturbations using FGSM can often increase its activation beyond the previous maximum. This enables us to expose the model to novel activation states and serves both as a white-box testing method and a robustness evaluation technique.

Moreover, since SNAC is computed by counting the number of neurons whose activations exceed their previously observed maximum values, the SNAC score can be artificially inflated by generating additional boundary-crossing adversarial samples. In other words, by controlling the proportion of adversarial inputs that cross neuron boundaries, the overall SNAC score can be arbitrarily adjusted, thereby revealing its inherent vulnerability as a coverage metric.

5.1.3 Methodology

The experiment consists of the following steps:

1. **Model Setup and Hook Registration:** We utilize a pretrained LeNet model trained on the MNIST dataset. Forward hooks are registered to extract activation values from internal layers (`conv1`, `conv2`, `fc1`, `fc2`, `fc3`) during inference.

2. **Maximum Neuron Activation Collection:** The entire training set is processed through the model. For each neuron in the target layers, we record the maximum activation value observed across all samples, which serves as its SNAC boundary.

3. **Adversarial Sample Generation:** For each neuron, we identify the input sample that originally produced its maximum activation. Starting from these samples, we iteratively apply a gradient-based attack to further increase the neuron activation.

   Specifically, we optimize the following objective for neuron i at layer l:

   $$\mathcal{L} = -a_i^{(l)}(x)$$

   where $a_i^{(l)}(x)$ denotes the activation value of neuron i at layer l given input x. This encourages the neuron activation to increase as much as possible.

   The attack updates follow an FGSM-like rule:

   $$x_{t+1} = \text{clip}\left(x_t + \alpha \cdot \text{sign}\left(\nabla_x \mathcal{L}\right)\right)$$

   where α is the step size and the perturbation is projected within the ϵ-ball around the original input. The perturbed inputs are clipped to remain within the normalized data range of MNIST.

4. **Evaluation of Boundary Crossing:** After generating the adversarial examples, we verify whether the neuron activations successfully exceed their original SNAC boundaries. The boundary-crossing success rate is then computed as our evaluation metric.

5.2 Adversarial Experiment

5.2.1 Dataset Construction

We constructed a total of 21 datasets based on the MNIST dataset, utilizing FGSM and c&w adversarial attacks on the LeNet model. The datasets are summarized as follows:

5.2.2 Model Retraining

We trained seven models based on different dataset configurations as summarized in Table 2. Model **a** corresponds to the original model trained solely on the MNIST dataset without any adversarial examples. The remaining models (**b** to **g**) were obtained by retraining model **a** on adversarially-augmented datasets with different configurations and fine-tuning epochs.

5.2.3 Neuron Activation Statistics and SNAC Evaluation Results

For each of the seven trained models described in Section 2, we extracted neuron-wise activation statistics using their corresponding training datasets. We computed the following statistics for all neurons across the five layers (`conv1`, `conv2`, `fc1`, `fc2`, `fc3`):

- The maximum activation value.
- The mean activation value.
- The standard deviation of activations.
- Confidence-based boundaries computed as $\mu + z \cdot \sigma$, with $z \in \{1.645, 2.326, 3.891\}$ corresponding to 95%, 99%, and 99.99% one-sided confidence levels, respectively.

ID	Dataset Description
(1)	Original MNIST Train Set
(2)	Original MNIST Test Set
(3)	Full MNIST Set = (1) + (2)
(4)	FGSM Successful Adversarial on Train Set
(5)	FGSM Successful Adversarial on Test Set
(6)	FGSM Adversarial Full Set = (4) + (5)
(7)	c&w Successful Adversarial on Train Set
(8)	c&w Successful Adversarial on Test Set
(9)	c&w Adversarial Full Set = (7) + (8)
(10)	(1) + (4)
(11)	(2) + (5)
(12)	(3) + (6)
(13)	(1) + (7)
(14)	(2) + (8)
(15)	(3) + (9)
(16)	(4) + (7)
(17)	(5) + (8)
(18)	(6) + (9)
(19)	(1) + (4) + (7)
(20)	(2) + (5) + (8)
(21)	(3) + (6) + (9)

Table 1: Summary of constructed datasets.

Model	Training Dataset	Fine-tuning Epochs
a	Dataset (1)	-
b	Dataset (10)	5
c	Dataset (10)	10
d	Dataset (13)	5
e	Dataset (13)	10
f	Dataset (19)	5
g	Dataset (19)	10

Table 2: Model training configurations.

To improve computational efficiency and reduce memory usage during activation extraction, we employed a streaming cache mechanism where only the cumulative sum and sum of squares of activations were stored during forward passes. After processing the entire dataset, the mean and variance of each neuron were computed directly from these cached statistics without retaining the full activation history. This approach significantly reduces storage requirements while allowing exact calculation of both mean and standard deviation.

Using these statistics, we evaluated four neuron coverage variants: Using these statistics, we evaluated four variants of neuron coverage:

- **SNAC (baseline):** The number of neurons whose activations exceed their respective maximum activation observed during training (i.e., maximum-based boundary used in prior work).
- **SNAC_95 (ours):** The number of neurons exceeding the 95% confidence boundary based on $\mu + 1.645\sigma$.
- **SNAC_99 (ours):** The number of neurons exceeding the 99% confidence boundary based on $\mu + 2.326\sigma$.
- **SNAC_9999 (ours):** The number of neurons exceeding the 99.99% confidence boundary based on $\mu + 3.891\sigma$.

5.2.4 Comprehensive Coverage and Robustness Evaluation

To evaluate the effectiveness of our proposed SNAC-based metrics, we performed extensive experiments across multiple models and datasets. Specifically, we evaluated all possible combinations of 7 trained models and 21 constructed datasets, resulting in a total of 147 model-dataset pairs.

For each pair, we computed:

- The original **SNAC** metric (using maximum activation boundaries).
- Our proposed confidence interval-based metrics: **SNAC_95**, **SNAC_99**, and **SNAC_9999**.
- Four widely used robustness metrics: **Lipschitz constant**, and **CLEVER scores** under L_1, L_2, and L_∞ norms, denoted as **CL1**, **CL2**, and **CLi**, respectively.

To compute these metrics, we extracted activations from all layers using forward hooks, and applied our SNAC evaluation based on the cached neuron statistics (mean, standard deviation, confidence intervals, and maximums). The Lipschitz constant was estimated by multiplying the spectral norms of all weight matrices layer-wise. For CLEVER score computations, we used the implementation provided by the `art` library, averaging the scores across 10 randomly selected samples for each dataset.

After collecting all metric results across the 147 model-dataset pairs, we analyzed the pairwise correlations between the SNAC metrics and the robustness metrics. The results are summarized as correlation heatmaps, as shown in Figure 2.

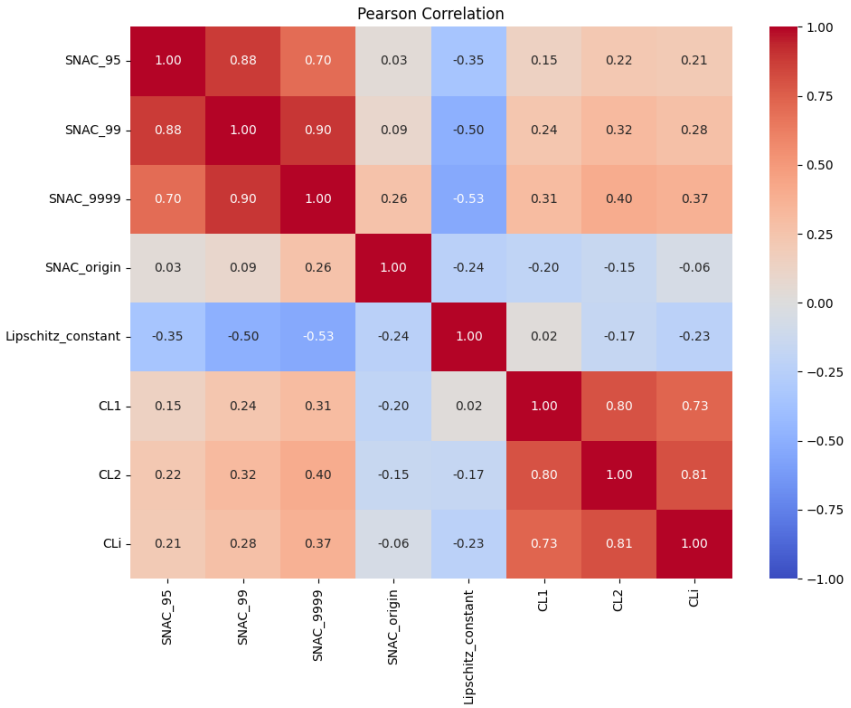

Figure 2: Correlation heatmap between neuron coverage metrics and robustness metrics across all model-dataset combinations. Other correlations are at the Appendix. Note that SNAC_{origin} refers to the original SNAC definition, while SNAC_{95}, SNAC_{99}, and SNAC_{9999} correspond to variants of the Pigeon method using one-sided confidence bounds at the 95%, 99%, and 99.99% levels, respectively.

6 Results and Analysis

6.0.1 Adversarial Sample Generation via SNAC and FGSM-Based Neuron Boundary Perturbation

Through this experiment, we achieved a boundary-crossing success rate of about 60%. This result demonstrates that even a simple FGSM-based algorithm can easily exploit the vulnerabilities of the existing SNAC metric, revealing its lack of robustness. Therefore, in this study, we propose a new metric that improves upon SNAC to address these limitations.

6.0.2 Adversarial Experiment Results

We analyzed the correlations between the SNAC-based neuron coverage metrics and various robustness metrics across all 147 model-dataset combinations. The correlation results are visualized as a heatmap in Figure **??**.

Overall, we observe several noteworthy patterns:

- The original SNAC metric (`SNAC_origin`) shows weak correlation with all robustness metrics, with correlation coefficients ranging from -0.24 to -0.06. This suggests that the maximum-based SNAC boundary may not reliably reflect model robustness.
- In contrast, the proposed confidence interval-based metrics (`SNAC_95`, `SNAC_99`, `SNAC_9999`) exhibit stronger and more consistent correlations with robustness metrics. For instance, `SNAC_9999` achieves up to 0.40 correlation with `CL2` and 0.37 with `CLi`.
- The Lipschitz constant shows negative correlations with all SNAC variants, especially with `SNAC_9999` (-0.53). This is expected since larger Lipschitz constants often indicate less robustness.
- Notably, our confidence-based SNAC metrics exhibit significantly higher correlation with both Lipschitz constants and CLEVER scores compared to the original SNAC, indicating better alignment with robustness characteristics.

These results demonstrate that replacing the maximum activation boundary with statistically-derived confidence thresholds provides a more informative and stable indicator of neural network robustness.

7 Conclusion

In this study, we revisited neuron coverage as a tool for robustness evaluation in neural networks and proposed confidence-threshold-based extensions to the traditional SNAC metric. By analyzing seven retrained LeNet models under various adversarial training conditions and applying our coverage metrics to 147 model-dataset pairs, we demonstrated that higher-threshold SNAC variants (SNAC_95, SNAC_99, SNAC_9999) generally yield stronger correlations with established robustness metrics.

Our findings suggest that replacing the traditional max-based SNAC with our confidence-aware variant, **PIGEON**, can provide more meaningful insights into model robustness. Furthermore, the proposed coverage metrics may serve as principled baselines for defining per-neuron upper activation bounds, which could inform future work in activation-level constraint design or formal verification frameworks.

Overall, confidence-thresholded neuron coverage offers a lightweight and generalizable diagnostic tool for robustness analysis. We hope this work encourages further research on adaptive coverage strategies and principled metric design for safety-critical machine learning systems.

8 Limitations and Future Work

8.1 Deviation from Gaussian Assumption in Neuron Activations

Our coverage metric implicitly assumes that neuron activations approximately follow a Gaussian distribution, which simplifies statistical treatment and coverage computation. However, recent theoretical findings suggest that this assumption becomes increasingly inaccurate as network depth increases. The cumulant expansion framework provides a principled way to quantify and understand such deviations from Gaussianity. Although cumulant-based correction methods could be introduced to mitigate this issue, we have not yet incorporated them into our current metric.

Gaussianity and Cumulants
A random variable X is Gaussian if and only if all cumulants of order $n \geq 3$ vanish:

$$\kappa_n(X) = 0 \quad \text{for all } n \geq 3.$$

Thus, Gaussianity implies that the distribution is fully characterized by its first two cumulants (mean κ_1 and variance κ_2).

Cumulant Expansion Formula
For a transformed random variable $Y = g(X)$ where X has a known distribution, the expectation of $g(X)$ can be approximated via a cumulant expansion:

$$\mathbb{E}[g(X)] = \sum_{n=0}^{\infty} \frac{\kappa_n(X)}{n!} g^{(n)}(\mu),$$

where $g^{(n)}(\mu)$ denotes the n-th derivative of g evaluated at the mean μ of X. This expression shows how higher-order cumulants of X affect the transformed expectation $\mathbb{E}[g(X)]$. In deep networks, $g(\cdot)$ represents nonlinear activation functions (e.g., ReLU), which induce and propagate higher-order cumulants across layers.

Width-Induced Gaussianity
In wide layers, pre-activations $z = \sum_{i=1}^{N} w_i a_i + b$ are formed by summing a large number of weakly correlated activation values. By the Central Limit Theorem and the additivity of cumulants, we have:

$$\kappa_n(z) \sim \mathcal{O}(N^{1-n/2}) \quad \text{for } n \geq 3.$$

Hence, as $N \to \infty$, we obtain $\kappa_n(z) \to 0$ for all $n \geq 3$, and z becomes increasingly Gaussian.

Depth-Induced Non-Gaussianity
In contrast, each nonlinear layer transformation of the form $a^{(l)} = f(W^{(l)}a^{(l-1)} + b^{(l)})$ introduces distortion via $f(\cdot)$, generating new non-zero cumulants. These accumulate with depth, and since the composition of nonlinear functions is not cumulant-preserving, we observe a compounding effect:

$$\kappa_n^{(L)} = \mathcal{F}_n\left(\kappa_3^{(L-1)}, \kappa_4^{(L-1)}, \ldots\right), \quad n \geq 3.$$

Even if $\kappa_n^{(1)} \approx 0$, deeper layers amplify higher-order cumulants significantly, leading to substantial deviation from Gaussianity.

Future Work
As future work, we aim to develop an extended coverage metric that explicitly accounts for the deviation from Gaussianity induced by both network depth and width. By modeling the contribution of higher-order cumulants to activation distributions, such a metric could more accurately reflect the statistical behavior of deep neural networks. This direction would allow us to quantify and correct for the structural sources of distributional error in a principled manner.

8.2 Discussion on Coverage Metric Sensitivity

Structural Limitations of Coverage Metrics
While the proposed SNAC_95, SNAC_99, and SNAC_9999 metrics provide a more refined perspective on model robustness than the traditional maximum-based SNAC, they remain structurally sensitive to the size of the test dataset. Since coverage is defined by whether a neuron's activation exceeds a threshold at least once across all test samples, the probability of such exceedance increases with the number of inputs. This causes coverage scores to rise not necessarily due to changes in model behavior, but as a statistical artifact of larger data volume. As a result, comparing coverage values across datasets of different sizes becomes inherently unfair.

Fixed Threshold Semantics
The use of fixed statistical thresholds (e.g., $z = 1.645$, 2.326, 3.891) assumes that these values have consistent semantic meaning across different models and datasets. However, variations in data distribution and internal activation dynamics mean that the same z-value can correspond to different practical levels of rarity. This limits the generalizability of fixed-threshold-based coverage metrics, especially when comparing across architectures or data domains.

Concluding Note
Although higher thresholds (e.g., SNAC_9999) showed stronger correlation with robustness metrics in our experiments, their semantic meaning depends heavily on data scale and rarity assumptions. We encourage future experiments to treat these thresholds as tunable hyperparameters, and to consider dataset-specific calibration strategies where appropriate.

A Appendix

A.1 Other Correlation Heatmaps Between Neuron Coverage Metrics and Robustness Metric

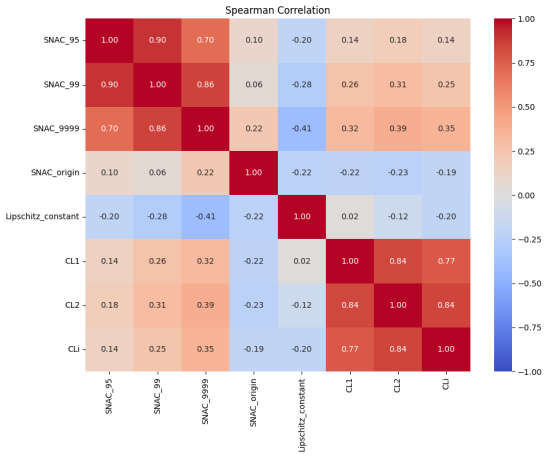

Figure 3: (a) Spearman Correlation Heatmap

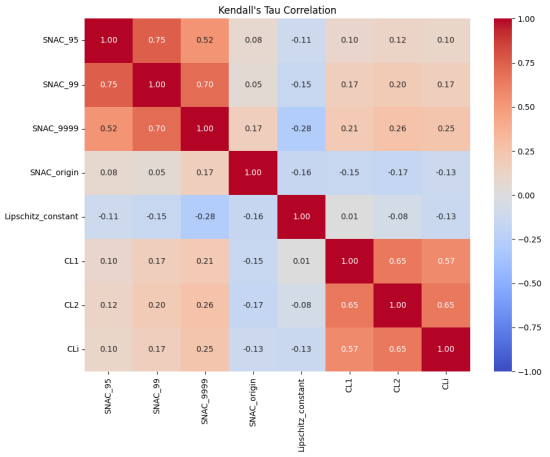

Figure 4: (b) Kendall Correlation Heatmap

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