1.1 $\boxed{\vec{x}_t = \begin{Bmatrix} x_t \\ v_t \end{Bmatrix}}$

$x_t \Rightarrow$ position

$\dot{x} = v_t \Rightarrow$ velocity

$\ddot{x} = a_t \Rightarrow$ acceleration

where $x_t = x_{t-1} + v_{t-1} + \frac{1}{2}a_t$

where $v_t = v_{t-1} + a_t$

$\vec{u}_t \Rightarrow$ control vector

1.2 $\vec{x}_t = A_t \vec{x}_{t-1} + B_t \vec{u}_t + \vec{\varepsilon}_t$

$A_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\quad B_t = 0 \quad$ $\vec{\varepsilon} = a_t \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$

$\sigma_2^2 = a^2 \sigma_x^2 = (\frac{1}{2})^2 \sigma_1' = \frac{1}{4}$

$Cov = \rho \sigma_1 \sigma_2 = 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$

$R = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{pmatrix}$

1.3 $t = 0$

$\vec{\mu}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Sigma_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$t = 1$

$\bar{\mu}_t = A_t \vec{\mu}_{t-1} + B_t \vec{u}_t = 0$ for all rest of $\vec{\mu}$

$\bar{\Sigma}_t = A_t \Sigma_{t-1} A^T + R$

$= 0 + R$

$= \begin{pmatrix} 1/4 & 1/2 \\ 1/2 & 1 \end{pmatrix}$

for rest of $\Sigma \rightarrow$

```
In [8]:  #t=1
         Cov1 = R
         Cov1
```

Out[8]: $\begin{bmatrix} 0.25 & 0.5 \\ 0.5 & 1 \end{bmatrix}$

```
In [9]:  #t=2
         Cov2 = A * Cov1 * A.T + R
         Cov2
```

Out[9]: $\begin{bmatrix} 2.5 & 2.0 \\ 2.0 & 2 \end{bmatrix}$

```
In [10]: #t=3
         Cov3 = A * Cov2 * A.T + R
         Cov3
```

Out[10]: $\begin{bmatrix} 8.75 & 4.5 \\ 4.5 & 3 \end{bmatrix}$

```
In [11]: #t=4
         Cov4 = A * Cov3 * A.T + R
         Cov4
```

Out[11]: $\begin{bmatrix} 21.0 & 8.0 \\ 8.0 & 4 \end{bmatrix}$

```
In [12]: #t=5
         Cov5 = A * Cov4 * A.T + R
         Cov5
```

Out[12]: $\begin{bmatrix} 41.25 & 12.5 \\ 12.5 & 5 \end{bmatrix}$

1.4 On Python

2.1 $\vec{x}_t = \begin{bmatrix} x_t \\ v_t \end{bmatrix}$ $\quad v_t = \dot{x}_t$

$\vec{z}_t = [x_t]$

$\vec{z}_t = C_t \vec{x}_t + \delta_t$

let's define $C_t = [1, 0]$

$\delta_t \sim N(0, \underbrace{8}_{Q})$

$\bar{\mu}_5 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\bar{\Sigma}_5 = \begin{bmatrix} 41.25 & 12.5 \\ 12.5 & 5 \end{bmatrix}$

2.2 $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$

$K_5 = \bar{\Sigma}_5 C_5^T (C_5 \bar{\Sigma}_5 C_5^T + Q)^{-1}$

$\overset{\text{Python}}{\approx} \begin{bmatrix} 0.838 \\ 0.254 \end{bmatrix}$

$\mu_5 = \bar{\mu}_5 + K_5 (z_5 - C\bar{\mu}_5)$

$= \begin{bmatrix} 8.380 \\ 2.540 \end{bmatrix}$

$\Sigma_5 = (I - K_5 C)\bar{\Sigma}_5$

$\approx \begin{bmatrix} 6.701, & 2.030 \\ 2.030, & 1.827 \end{bmatrix}$

2.3 On Python

3.1 $\vec{x}_t = A_t \vec{x}_{t-1} + B_t \vec{u}_t + \vec{\varepsilon}_t$

$\bar{x}_t = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ v_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{u_t} + \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} at$

where $5 = \cancel{x_{t-1}} + \cancel{v_{t-1}} + a + 0 \cdot b + \frac{1}{2} at$

$a = 4.5$

$at \to 1$

where $1 = \cancel{v_{t-1}} + c + 0 \cdot d + at$

$c = 0$

$\therefore B = \begin{bmatrix} 4.5 & 0 \\ 1 & 0 \end{bmatrix}$

Rest of things shown on Python