KANISHKA ARORA
SC20B152
MSc ASTRONOMY & ASTROPHYSICS

Assignment 1

<u>Question – 1</u>

A plot with synthetic Planck curves showing the intensity of radiation emitted by stars of spectral type O, B, A, F, G, K, and M. The vertical axis should be the Planck function for black body in units of Joule s^{-1} m^{-2} sr^{-1} Hz^{-1}. The horizontal axis should be frequency of the electromagnetic radiation in units of Hz. You might want to plot the quantities along the axes in log units to accommodate the wide range of values.

Mark the location of 1 Rydberg in the plot by a dashed vertical line.

<u>Answer – 1</u>

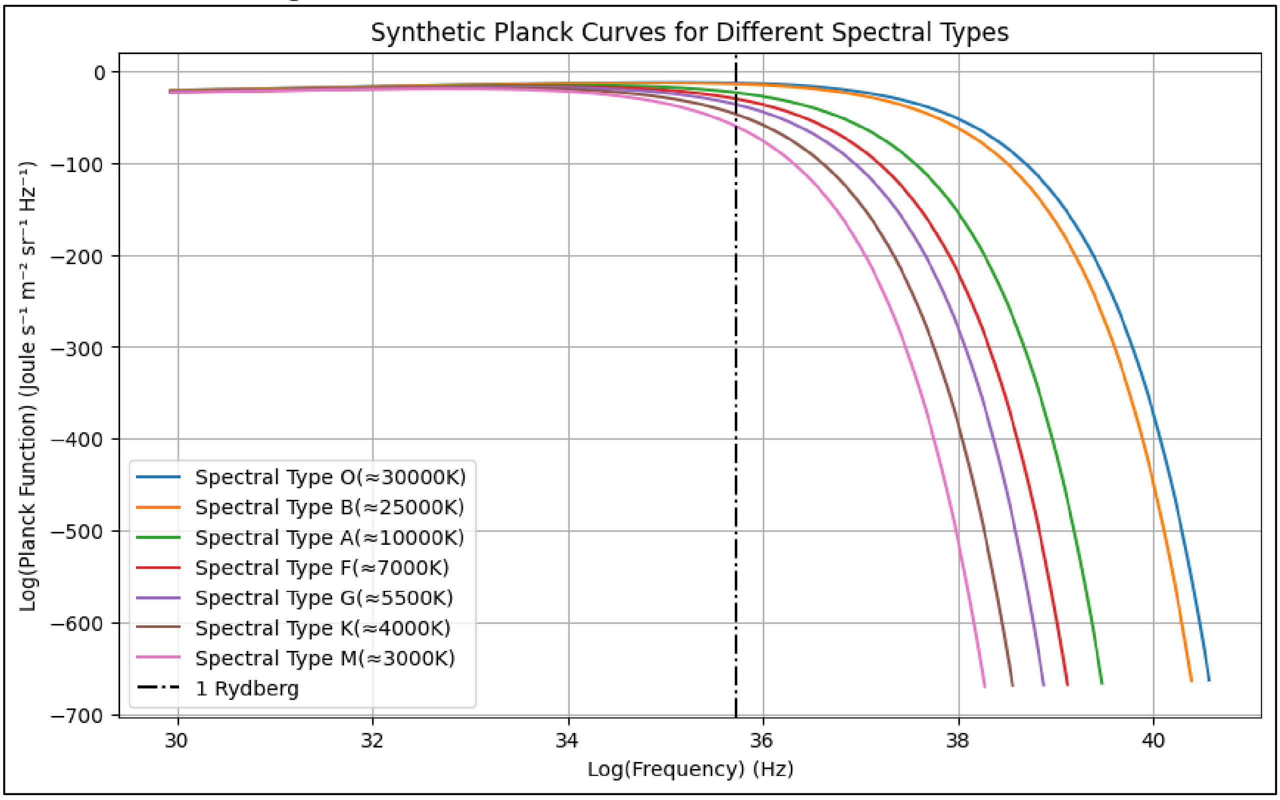
The Planck equation is represented by.

$$\boxed{B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}}$$

I have plotted this equation in python.

Below is the plot for the Planck curves for different spectral types (each at a given temperature).

Please note that the plotting is done in logarithmic scales to accommodate large values.

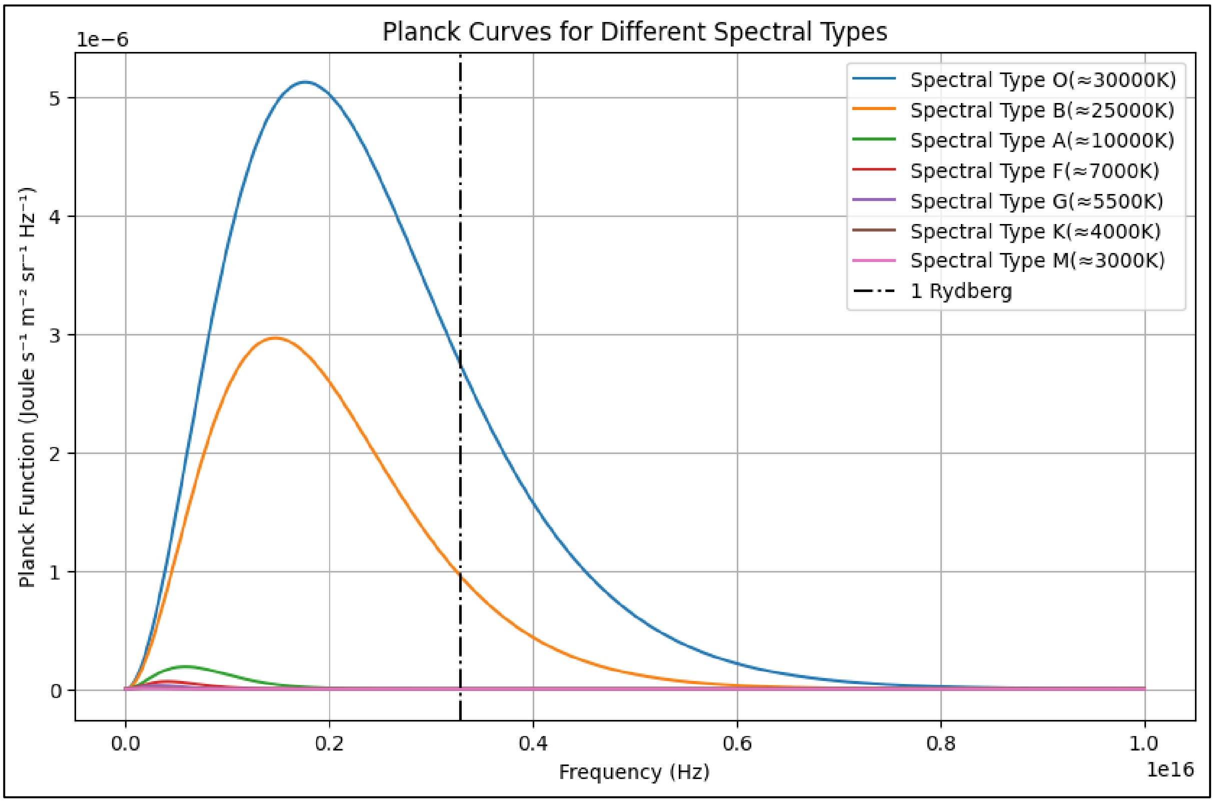


Below is the code attached to obtain the following plot.

```
import numpy as np
import matplotlib.pyplot as plt

# constants
h = 6.62607015e-34
c = 299792458.0
k = 1.380649e-23

# Function to calculate Planck's Law
def planck_function(frequency, temperature):
    return ((2 * h * frequency**3) / ((c**2) * (np.exp((h * frequency) / (k * temperature)) - 1)))

# Spectral types and their corresponding temperatures (in Kelvin)
spectral_types = ['O', 'B', 'A', 'F', 'G', 'K', 'M']
temperatures = [30000, 25000, 10000, 7000, 5500, 4000, 3000]

# Frequency range (in Hz)
frequency_range = np.logspace(13, 20, 10000)

# Plotting the synthetic Planck curves
plt.figure(figsize=(10, 6))

for i in range(len(spectral_types)):
    intensity = planck_function(frequency_range, temperatures[i])
    plt.plot(np.log(frequency_range), np.log(intensity), label=f'Spectral Type
{spectral_types[i]}(≈{temperatures[i]}K)')

# Marking the location of 1 Rydberg with a dashed vertical line
rydberg_frequency = 3.28984e15  # Hz
plt.axvline(x=np.log(rydberg_frequency), color='k', linestyle='dashdot', label='1 Rydberg')

# Adding labels and legend
plt.xlabel('Log(Frequency) (Hz)')
plt.ylabel('Log(Planck Function) (Joule s⁻¹ m⁻² sr⁻¹ Hz⁻¹)')
```

```python
plt.title('Synthetic Planck Curves for Different Spectral Types')
plt.legend()
plt.grid(True)
plt.show()
```

I have also plotted it in the linear scale for clarification on how the function originally looks.



So this gives us an overall perspective about the Planck function and how it varies with frequency at a particular temperature.

Question-2

A five-column table with the following values listed in the columns

Column 1: Spectral type

Column 2: Effective photosphere temperature assumed

Column 3: Radius of the star assumed (in units of solar radius)

Column 4: Hydrogen ionizing photon flux (in units of photons / sec)

Column 5: Size of the H II region (in units of parsec)

Answer-2

The below table gives you all the required information.

Spectral Type	T_{eff}(K)	Radius of star($R_\odot$)	Hydrogen Ionizing Flux(photons / sec)	Size of H II region(pc)
O	30000	15	4.88e+48	2.09
B	25000	10	5.93e+47	4
A	10000	2	6.05e+41	0.01
F	7000	1	1.17e+38	0.0006
G	5500	0.8	1.23e+35	6.143e-05
K	4000	0.5	7.25e+29	1.109-06
M	3000	0.08	2.65e+22	3.682e-09

The Hydrogen Ionizing Flux had to be calculated through integration methods in python. I will show the code for which does the same.

```
import numpy as np
import matplotlib.pyplot as plt
import math
```

```
# constants
h = 6.62607015e-34
c = 299792458.0
k = 1.380649e-23

# Initializing the lists
spectral_types = ['O', 'B', 'A', 'F', 'G', 'K', 'M']
temperatures = [30000, 25000, 10000, 7000, 5500, 4000, 3000]
radius = [15, 10, 2, 1, 0.8, 0.5, 0.08]
Flux = []

# Frequency range (in Hz)
frequency_range = np.linspace(3.29e15, 2.418e17, 100000)

# Defining the function for the Hydrogen Ionizing Flux in the units of Photon per second
def Ionizing_Flux(frequency, temperature, radius):
    return (((2 * frequency**2) * (4 * np.pi * (radius * (6.957e08))**2) * (np.pi)) / ((c**2) * (np.exp((h *
frequency) / (k * temperature)) - 1)))

# Calculating the Hydrogen Ionizing Flux for each spectral type by integrating using the trapezoid
function
for i in range(len(spectral_types)):
    Unit_Frequency = Ionizing_Flux(frequency_range, temperatures[i], radius[i])
    Flux.append(np.trapz(Unit_Frequency, frequency_range))

print(Flux)
```

The output would give us the values that I have shown in the table above. Below is the code for the Stromgen radius or the simply called the radius of the HII region. Please note that the following code is a continuation of the previous one.

```
# Now finding the Stromgen Radius
strongen_radius = []
```

```
# alpha_H is the recombination coefficient
alpha_H = 3 * pow(10, -13)

# Number density of electrons and protons is equal here
n = 120

# Finding the Stromgen Radius for each spectral type in pc
for i in range(len(spectral_types)):
    stromgen_radius.append((pow(((3 * Flux[i]) / (4 * np.pi * alpha_H *  pow(n,2))), 1/3)) * 3.2408 * 
pow(10, -19))

print(stromgen_radius)
```

This would print the values of the stromgen radius as mentioned in the table.

Please note the following values

$\alpha_H = 3 \times 10^{-13}$ cm^3 s^{-1} (recombination coefficient)

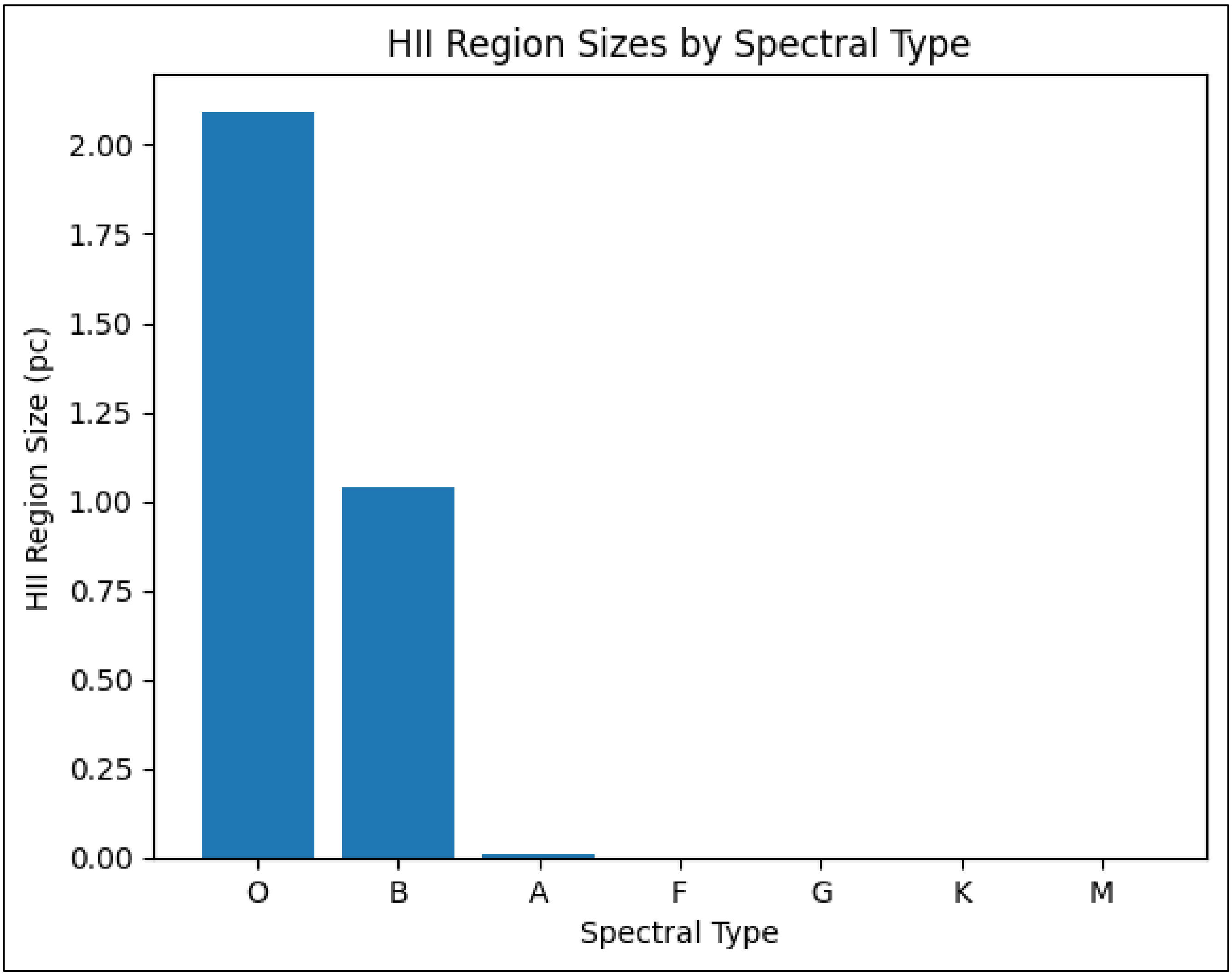
$n = 120$ cm^{-3} (number density)

Question-3

A histogram plot that shows the size of the H II region against stellar spectral type. The horizontal axis should be O, B, A, F, G, K, M. The vertical axis should show the size of the H II region in units of parsec.

Answer-3

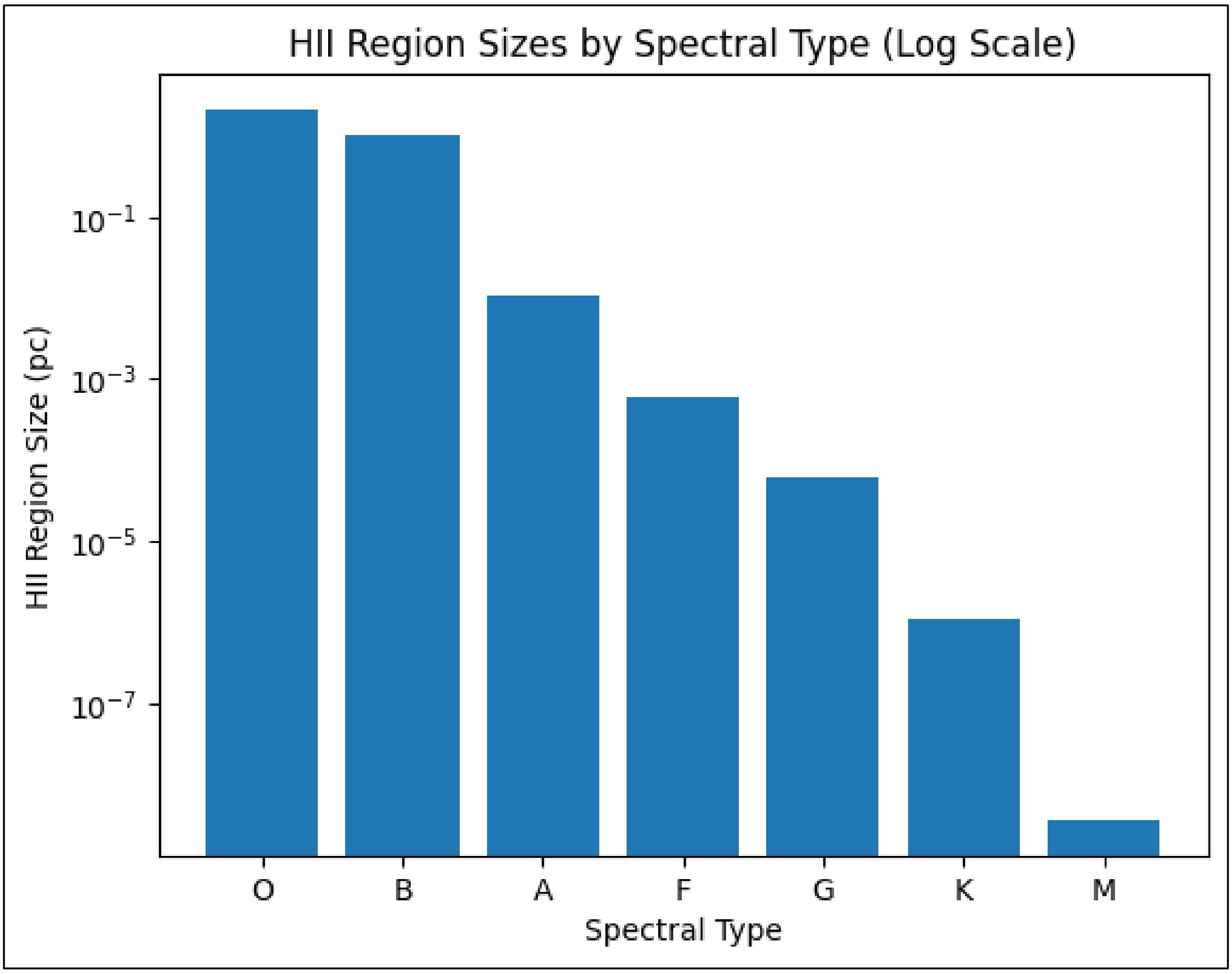
The below diagram gives a good idea.



I will now show the code that I had written to obtain such a plot in python. Please note that the below code is a continuation of the before, as only the plotting has to be done.

```
# plotting the graph for size of HII region vs spectral type
plt.bar(spectral_types, stromgen_radius)
plt.title('HII Region Sizes by Spectral Type')
plt.xlabel('Spectral Type')
plt.ylabel('HII Region Size (pc)')
plt.show()
```

I will also show a plot where the y-axis is in log scale for better understanding.


HII Region Sizes by Spectral Type (Log Scale)
HII Region Size (pc)
10^{-1}
10^{-3}
10^{-5}
10^{-7}
O
B
A
F
G
K
M
Spectral Type

Question-4

Based on your analysis write answers to the following questions

a) Stars of which spectral type can produce large regions of warm ionized gas?
b) What will be the rough temperature of the warm ionized plasma, and why?
c) What does the presence of bright WIM tell us about star formation in a galaxy?

Answer-4

a) From the data, stars O and B which are massive have a large capability of producing large regions of ionised gas around their photosphere. This is practically because they have a higher temperature and have the intensity required for the ionization of the medium around them.

   We receive the emission lines which help us conclude the ionization of hydrogen.

b) The Ultraviolet radiation of the star is what ionizes largely to produce this medium. The temperature is dependent on the energy of the ionized atoms which are distributed spatially round the star. The temperature will not be constant throughout. It will in fact vary and will have a higher temperature close to the photosphere of the star and will decrease as you go further out. Obviously the temperature of the WIM cannot be higher than the star itself.

   Typically the range of temperatures will be in the range of 6000K ~ 12000K

c) The ionization of the ISM to create bright WIM occurs from hot massive stars such as the spectral type O and B. This gives us some information as these stars would have been formed in regions where there is high activity and chance of star formation in the galaxy.

   Plus like I have mentioned, O and B are massive stars. High mass stars have a shorter lifetime compared to low mass stars as they used up their nuclear fuel faster. And observing the flux from the bright WIM tells us that there might indicate other such stars around in the region, as stars are formed in regions of high gas density.

Thanking Anand Sir for this assignment.