Project Summary

This project models a simulation of a well-known card game, UNO. In this version of UNO, the player starts with 7 cards in their hand and must play one of these cards depending on the previously played card (i.e. the top card). This is determined through the colour and number of the top card. A card is playable if one of the following holds:

- *The colour of the card matches the colour of the top card;*
- *The number of the card matches the number of the top card; or*
- *They have a wildcard, in which case they can choose to change the current colour of the top card.*

Otherwise, the player must draw another card from a pile.

The objective of this game is to get rid of all of the player's cards in hand, which is something the code will determine.

Propositions

Below is a list of variables we will tentatively use for the model and their assigned meanings:

VARIABLE NAME	***REPRESENTATION***
A	*True if the player has cards in hand.*
B	*True if the player has playable cards.*
P	*True if the player has played a card.*
W	*True if the player can play a wildcard.*
D	*True if the player must draw a card from the pile.*
C	*True if the player has a matching colour card as the top card.*
T	*True if the player has a matching number card as the top card.*
E	*True if the top card is a "Draw 2" card.*
F	*True if the top card is a ``Draw 4" card.*
R	*True if the top card is a ``Reverse" card.*
S	*True if the top card is a ``Skip" card.*
U	*True if the player has met the winning condition (i.e. the player has no cards left).*

Next, we have a list of propositions we've come up with so far and their corresponding logic translation:

PROPOSITION	***LOGIC TRANSLATION***
Player has playable cards if they have a matching colour card, matching number card, or a wildcard.	$B \rightarrow C \vee T \vee W$
Player can play a wildcard regardless of whether they have matching colour or matching number cards.	$W \rightarrow (C \vee T) \vee \neg (C \vee T)$

If player is met with a ``Draw 4" or ``Draw 2" card, they cannot play and must draw a card from the pile instead.	$(F \lor E) \rightarrow (\neg P \land D)$
If the player has drawn a card and the card is playable, then the player can play said card.	$(D \land B) \rightarrow P$

Constraints

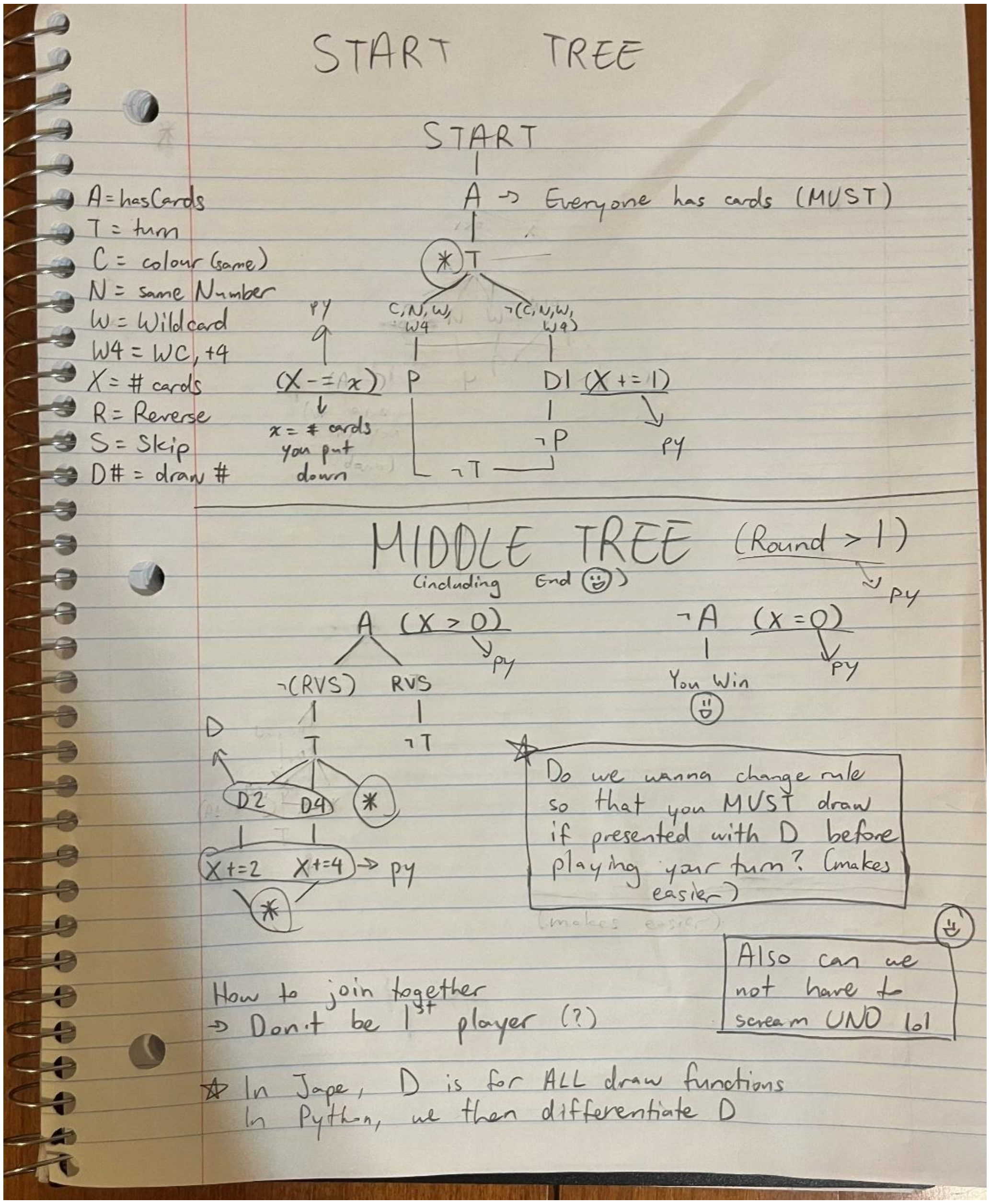
We have come up with the following constraints:

CONSTRAINT	***LOGIC TRANSLATION***
The player cannot simultaneously have cards in hand and not have cards in hand.	$A \lor \neg A$
If the player has cards in hand that include either a matching colour card, matching number card or wildcard AND the top card is neither a "Reverse" card nor a "Skip" card, then the player must play one of their playable cards.	$A \land (C \lor T \lor W) \land \neg (R \lor S) \rightarrow P$
If the player has cards in hand and the top card is a "Reverse" card or a "Skip" card, then the player cannot play a card.	$A \land (R \lor S) \rightarrow \neg P$
If the player has cards in hand but does not have any playable cards then they cannot play a card and they must draw instead.	$A \land \neg B \rightarrow D \land \neg P$
If the player has no cards in hand then they automatically win the game.	$\neg A \rightarrow U$

Model Exploration

Initially, when establishing an Uno game play we decided to consider multiple players, steps they take throughout the game, how every player draws or puts down a card. Through this, we then decided upon a list of propositions and constraints that mirror the original gameplay.

We came up with a tree model to connect all our variables together to figure out how our game will play out, from start to middle to finish. Through this, we figured out unnecessary variables and potential complications for our project. This tree drawing can be found below.



Later, after getting feedback from the proposal, we decided to shift our model's perspective to follow an individual player's gameplay for simplicity rather than an entire group, so that we can make specific propositions and constraints based on how the player reacts to the game.

As a "demonstration" or "something to go off of", we played a game of UNO and recorded the winning person's moves and the cards played immediately before her.

Once we really got the hang of what our main objective was, it was time to weed out the unnecessary variables, introduce new ones, and come up with new, better propositions and constraints – which are the ones we have now.

Jape Proof Ideas

We already had a few ideas and incorporated them into Jape; below are the screenshots and brief explanations of what they represent.

We also had to switch certain variables to accommodate them with Jape: W has been replaced with G and U has been replaced with H.

Proof 1: *Let the player have cards that include a wildcard, matching colour card, and matching number card. Then the player can play a wildcard regardless of their other cards.*

1:	G, C, T	premises
2:	$C \vee T$	$\vee$ intro 1.2
3:	G	assumption
4:	$(C \vee T) \vee \neg (C \vee T)$	$\vee$ intro 2
5:	$G \rightarrow (C \vee T) \vee \neg (C \vee T)$	$\rightarrow$ intro 3-4

Proof 2: *Let the player have playable cards, including a matching colour card, matching number card, and a wildcard. Then the player can play either the matching colour card or matching number card or matching number card.*

1:	B, C, T, G	premises
2:	$C \vee T$	$\vee$ intro 1.2
3:	$C \vee T \vee G$	$\vee$ intro 2
4:	B	assumption
5:	$C \vee T \vee G$	hyp 3
6:	$B \rightarrow C \vee T \vee G$	$\rightarrow$ intro 4-5

Proof 3: *Assume that the player has playable cards in hand, and the top card is neither be a "Reverse" card nor a "Skip" card. This means that the player can play one of their playable cards.*

1: A, B, R, S, P premises

2: $A \wedge B \wedge \neg(R \vee S)$ assumption

3: P hyp 1.5

4: $A \wedge B \wedge \neg(R \vee S) \rightarrow P$ → intro 2-3

Proof 4: *Let's assume that the top card is either a "Draw 4" or "Draw 2" card. Then the player cannot play a card and instead must draw a new card from the pile.*

1: $F, E, \neg P, D$ premises

2: $F \vee E$ ∨ intro 1.1

3: $F \vee E$ assumption

4: $\neg P \wedge D$ ∧ intro 1.3,1.4

5: $(F \vee E) \rightarrow (\neg P \wedge D)$ → intro 3-4

Proof 5: *This is a simple one; the player can either have cards in hand or not have cards in hand.*

1: A premise

2: $A \vee \neg A$ ∨ intro 1

Proof 6: *If the player does not have cards in hand, then they have won the game.*

1: A, H premises

2: $\neg A$ assumption

3: H hyp 1.2

4: $\neg A \rightarrow H$ → intro 2-3

Requested Feedback

1. *Are the propositions and constraints good enough and make sense? Do some of our propositions belong in the constraints and vice versa? Are there any other propositions/constraints that we might be missing?*
2. *Are the universal quantifiers appropriate for every proposition, or would some statements benefit from existential quantifiers? Do you have any ideas on how we can expand or make our quantifier statements better?*
3. *Are there any edge cases (such as when the player has only wildcards or draws consecutive "Draw 4" cards) that should be accounted for in the constraints or propositions? Should we include additional game mechanics, such as special card combinations or player-specific conditions, that could be represented by new variables?*

First-Order Extension

At first, we didn't have many ideas for a first-order extension so we decided to simply take our existing propositions and simply place universal quantifiers on all of our premises and conclusions and prove them in Jape. This resulted in the following proofs:

Extension 1 – *Corresponds to Proof 4*

1:	$\forall x.((F(x) \vee E(x)) \rightarrow (\neg P(x) \wedge D(x))),\ \exists x.(F(x) \vee E(x))$	premises
2:	actual i, $F(i) \vee E(i)$	assumptions
3:	$(F(i) \vee E(i)) \rightarrow (\neg P(i) \wedge D(i))$	$\forall$ elim 1.1,2.1
4:	$\neg P(i) \wedge D(i)$	$\rightarrow$ elim 3,2.2
5:	$\exists x.(\neg P(x) \wedge D(x))$	$\exists$ intro 4,2.1
6:	$\exists x.(\neg P(x) \wedge D(x))$	$\exists$ elim 1.2,2-5

Extension 2 – *Corresponds to Proof 6*

1:	$\forall x.(\neg A(x) \rightarrow H(x))$	premise
2:	$\forall x.(\neg A(x))$	assumption
3:	actual i	assumption
4:	$\neg A(i)$	$\forall$ elim 2,3
5:	$\neg A(i) \rightarrow H(i)$	$\forall$ elim 1,3
6:	$H(i)$	$\rightarrow$ elim 5,4
7:	$\forall x.(H(x))$	$\forall$ intro 3-6
8:	$\forall x.(\neg A(x)) \rightarrow \forall x.(H(x))$	$\rightarrow$ intro 2-7

Extension 3 – *Corresponds to Proof 3*

1:	$\forall x.A(x) \wedge \forall x.B(x) \wedge \forall x.(\neg R(x) \vee S(x))$	premise
2:	$\forall x.(\neg R(x) \vee S(x))$	$\wedge$ elim 1
3:	$\forall x.A(x) \wedge \forall x.B(x)$	$\wedge$ elim 1
4:	$\forall x.B(x)$	$\wedge$ elim 3
5:	$\forall x.A(x)$	$\wedge$ elim 3
6:	actual i	assumption
7:	$\neg R(i) \vee S(i)$	$\forall$ elim 2,6
8:	$B(i)$	$\forall$ elim 4,6
9:	$A(i)$	$\forall$ elim 5,6
10:	$A(i) \wedge B(i)$	$\wedge$ intro 9,8
11:	$A(i) \wedge B(i) \wedge (\neg R(i) \vee S(i))$	$\wedge$ intro 10,7
12:	$\forall x.(A(x) \wedge B(x) \wedge (\neg R(x) \vee S(x)))$	$\forall$ intro 6-11

Afterwards, we had some ideas:

- *Let x be a card. Then, for all x, there can only be one colour and one number assigned to each x.*
- *Let x,y be cards. Then, for all x, there exists a y that matches the colour or number of each x.*
- *Let x be a playable card. Then, for all x, it either has a matching colour or matching number or is a wildcard.*