1. A large insurance agency services a number of customers who have purchased both a homeowner's policy and an automobile policy from the agency. For each type of policy, a deductible amount must be specified.

For an automobile policy, the choices are \$100 and \$250, whereas for a homeowner's policy, the choices are 0, \$100, and \$200.

Suppose an individual with both types of policy is selected at random from the agency's files. Let X = the deductible amount on the auto policy and Y = the deductible amount on the homeowner's policy.

Possible (X, Y) pairs are then (100, 0), (100, 100),
(100, 200), (250, 0), (250, 100), and (250, 200); the joint pmf specifies the probability associated with each one of these pairs, with any other pair having probability zero.

covariance $= \sum\sum (x-\mu_x)(y-\mu_y) f(x,y)$

$\mu_x = 0.5 \cdot 100 + 0.5 \cdot 250 = 175$

$\mu_y = 0 \cdot 0.25 + 100 \cdot 0.25 + 200 \cdot .5 = 125$

$cov = (0-125)(100-175) \cdot .2 + (100-175)(100-175) \cdot .1$

$+ \dots$ for all 6 cells

Suppose the joint pmf is given in the accompanying joint probability table:

joint Prob

$p(x, y)$		y = 0	100	200	
x	100	.20	.10	.20	0.5
	250	.05	.15	.30	0.5
		0.25	.25	.5	

a. What is the marginal distribution of X and Y?

$f(x) = \sum_y f(x,y) = \begin{cases} 0.5 & x = 100 \\ 0.5 & x = 250 \end{cases}$

$f(y) = \sum_x f(x,y) = \begin{cases} 0.25 & y = 0 \\ 0.25 & y = 100 \\ 0.5 & y = 200 \end{cases}$

b. What is the conditional distribution of X given that Y = 100?

$f(x \mid y = 100) = \begin{cases} 0.1/0.25 = 0.4 & x = 100 \\ 0.15/0.25 = 0.6 & x = 250 \end{cases}$

$f(x \mid y) = \dfrac{f(x,y)}{f(y)}$

c. Are X and Y independent?

No

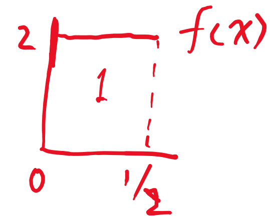
$f(x \mid y) = f(x)$ if ind.

But $f(x \mid y) \neq f(x)$

$f(x, y) = f(x) \cdot f(y)$ if ind But $0.1 \neq 0.25 \cdot 0.5$

2. A bank operates both a drive-up facility and a walk-up window. On a randomly selected day, let X = the proportion of time that the drive-up facility is in use (at least one customer is being served or waiting to be served) and Y = the proportion of time that the walk-up window is in use.

Then the set of possible values for (X, Y) is the rectangle D = {(x, y): 0 ⩽ x ⩽ 1, 0 ⩽ y ⩽ 1}.



no need

$f(x) \le 1$

$f(x,y) \le 1$

Suppose the joint pdf of (X, Y) is given by

$$f(x, y) = \begin{cases} \frac{6}{5}(x + y^2) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Verify that it is a legitimate pdf

$$\int_0^1 \int_0^1 \frac{6}{5}(x+y^2)\,dx\,dy = 1$$

b. What is the probability that neither facility is busy more than one-quarter (0.25) of the time in a day?

$P(X \le 0.25,\ Y \le 0.25)$

$$\int_0^{.25} \int_0^{.25} \frac{6}{5}(x+y^2)\,dx\,dy = 0.011$$

c. What is the conditional probability that Y ⩽0.5 given that X = .8?

$P(Y \le 0.5 \mid X = 0.8)$

$$= \int_0^{0.5} \frac{\frac{6}{5}(0.8+y^2)}{1.2 \cdot 0.8 + 0.4}\,dy$$

$= 0.39$

$f(y \mid X = 0.8)$

$$= \frac{f(y,x)}{f(x)} = \frac{f(x,y)}{\int_0^1 f(x,y)\,dy}$$

$$= \frac{f(x,y)}{1.2x + 0.4}$$

d. What is the marginal probability that Y ⩽ 0.5?

$f(y) = \int_0^1 f(x,y)\,dx$

$= 1.2y^2 + 0.6$

$P(y \le 0.5) = \int_0^{0.5} 1.2y^2 + 0.6\,dy = 0.35$

e. Are X and Y independent?

No! $P(y) \ne P(y \mid x)$

$$\underbrace{(1.2x+0.4)}_{f(x)}\underbrace{(1.2y^2+0.6)}_{f(y)} \ne f(x,y)$$