Cognitive Radar for Target Tracking

A thesis submitted in partial fulfilment of the requirements for the degree of

Master of Technology in

Electronics and Communication Engineering

(Radar and Communication)

By

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Dedicated to my Father Shri Rajaram Rajpoot, Mother Mrs. Madhu Rajpoot,

and Sister Ms. Rashmi Singh Rajpoot for being my strength and my guide

Dr A. A. Bazilraj for supporting me throughout my project

Approval Sheet

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Sr. No.	Course Code	Course Name	Credits
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3	EE603	Wireless and Mobile Communication	4
4	EE604	Radar System Design	4
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7	EE607	Detection and Estimation Theory	4
8	EE609	Antenna Systems	4
9	EE610	Radar Signal Processing	4
10	EE611	Array Signal Processing	4
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This is to certify that the Dissertation report comprises original work (except where indicated) carried out by me and due acknowledgments have been made in the text for all other material used. The dissertation does not contain any classified information which will be detrimental to national security and has not been submitted in any form for another degree or diploma at any other institute/university.

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Kanika Singh Rajpoot

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DECLARATION FOR PLAGIARISM CHECK

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Abstract

For over six decades, the theory and design of radar systems have been dominated by probability theory and statistics, information theory, signal processing and control. However, the similar encoding-decoding property that exists between the visual brain and radar has been sadly overlooked in all radar systems. This thesis lays down the foundation of a new generation of radar systems, namely cognitive radar, that was described in a 2006 seminal paper by Haykin. Four essential elements of cognitive radar are Bayesian filtering in the receiver, dynamic programming in the transmitter, memory, and global feedback to facilitate computational intelligence.

All these elements excluding the memory compose a well-known property of mammalian cortex, the perception-action cycle. As such, the cognitive radar that has only this cycle is named as the basic cognitive radar (BCR). For tracking applications, this thesis presents the underlying theory of BCR, with the emphasis being placed on the cubature Kalman filter to approximate the Bayesian filter in the receiver, dynamic optimization for transmit-waveform selection in the transmitter, and global feedback embodying the transmitter, the radar environment, and the receiver all under one overall feedback loop.

Built on the knowledge learnt from the BCR, this thesis expands the basic perception action cycle to encompass three more properties of human cognition, that is, memory, attention, and intelligence. Specifically, the provision for memory includes the three essential elements, i. e., the perceptual memory, executive memory, and coordinating perception-action memory that couples the first two memories. Provision of the three memories add an advanced version of cognitive radar, namely the nested cognitive radar (NCR) considering the nesting of three memories in the perception-action cycle.

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List of Abbreviations

CKF: Cubature Kalman Filter

EKF: Extended Kalman Filter

UKF: Unscented Kalman Filter

KF: Kalman Filter

RMSE: Root Mean Square Error

BCR: Basic Cognitive Radar

NCR: Nested Cognitive Radar

AI: Artificial Intelligence

LFM: Linear Frequency Modulation

CTR: Cognitive Tracking Radar

CWS: Cognitive Waveform Selection

Chapter 1 Radar Systems

1.1 Introduction

Radar works on the principle of echolocation used by Bats and Dolphins. Echolocation is also known as bio-sonar which is used by many animal species to identify and track their prey. Bats generate the ultrasound frothe m larynx and emit sound ththe rough mouth. These waves travel in directed direction and after hitting an object or desired target, the waves revert back to its source with some time delay. The time delay signifies the range of the target whereas the frequency change accounts for the velocity change of the target. Radar is an acronym of RAdio Detection And Ranging.

A radar is an electromagnetic sensor that can be used for tracking, detection, and surveillance. Not only it is able to identify the range and velocity of the target but also the shape and size of the target. Civilian applications of radar are weather observations, air traffic control, space and planetary surveillance, vehicular navigation, industrial application, etc. Military applications are navigation in ships, surveillance, target detection, and tracking, etc. Still military use radar widely for various applications with technological advancements.

Radar range equation is:

$$R = \frac{c\Delta T}{2} \tag{1}$$

Where, c= speed of light

T= Time delay

As we know speed is the distance upon time so putting the values we get $c = \frac{2R}{\Delta T}$. 2R represents the to and fro motion of the radiated electromagnetic wave emitted by the radar. The waves travel to the target, hits the target, and return back so these two traveling is taken into account. Ambiguous range is defined as transmitting and receiving the first pulse before transmitting the second pulse. The unambiguous range is defined as:

$$R_{unam} = \frac{cT_{prt}}{2} \tag{2}$$

Where, T_{prt} = pulse repetition time

T_p = Pulse time

f_{prf} = pulse repetition frequency

T_{prt} can be defined in terms of frequency and due to this unambiguous range can defined again as:

$$T_{prt} = \frac{1}{f_{prf}} \tag{3}$$

$$R_{unam} = \frac{c}{2f_{prf}} \tag{4}$$

To transmit a waveform for the target which is at a larger distance we need to have a larger transmitted power. Larger the height of pulse is directly proportional to the larger power.

$$\text{Peak Power}[\mathrm{P_p}] = \text{Height of the pulse}$$

During T_p time transmitter is ON and during T_{prt} transmitter is OFF so

$$\text{Average Power} = \frac{\mathrm{P_p T_p}}{\mathrm{T_{prt}}} \tag{5}$$

The duty ratio and duty cycle of the radar is calculated by

$$\text{Duty ratio}(\%) = \frac{\mathrm{T_p}}{\mathrm{T_{prt}}} \times 100 \qquad (6)$$

$$\text{Duty cycle} = \text{Duty ratio} \times 100 \qquad (7)$$

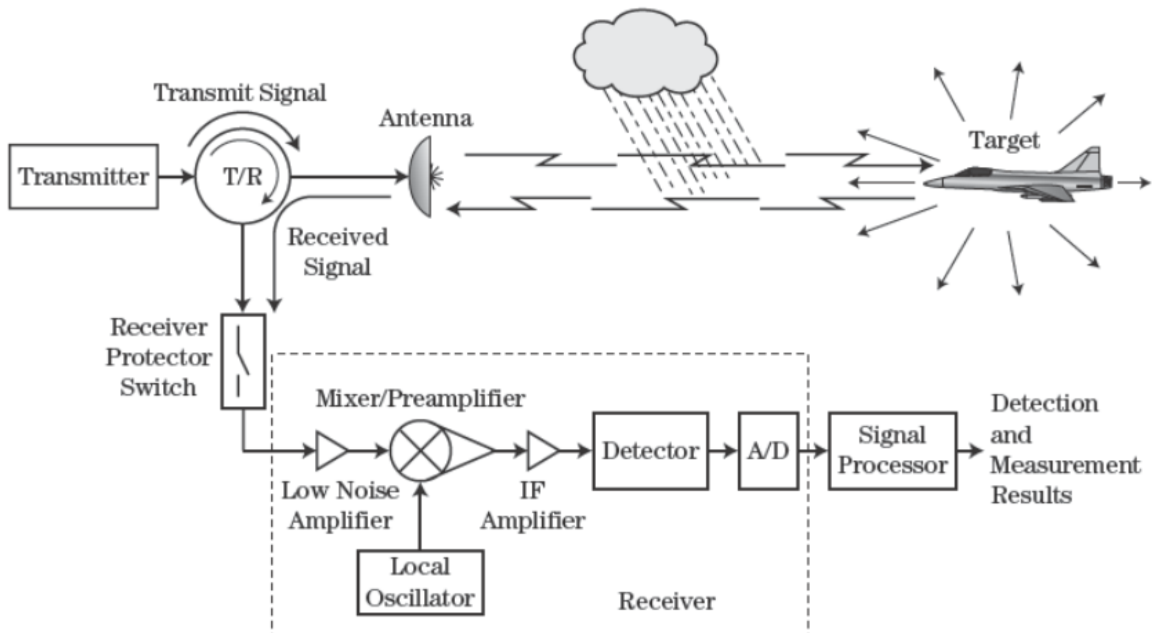


Figure1. Basic block diagram of radar [31]

1.2 Radar Resolution

There are two types of radar known as the radial and the angular resolution.

1.2.1 Radial Resolution

The resolution capability is described over a distance which means will radar be able to differentiate between the two objects if they are in the same line of sight. Such objects are considered apart by distance detection of each object although this method alone doesn't remove that issue. Radial Resolution depends on T_{prt}.

$$\text{Range Resolution} = \frac{cT_p}{2} \qquad (8)$$

1.2.2 Angular Resolution

When we have larger beamwidth of antenna, we will not be able to distinguish between the two targets. To differentiate the two objects instead of using a broader beamwidth, we can use pencil beams or we can using electronic beam scanning.

Angular distance between the two objects can be defined by calculating azimuth (Horizontal) and elevation (vertical) angles.

Here, the field of view of the antenna plays a key role in angular resolution. We prefer to use MIMO which stands for multiple input multiple output. MIMO radar has a number of antennas to emit and perceive waves from different angles, this helps to identify objects at different angles.

1.3 Monostatic vs Bistatic Radar

1.3.1 Monostatic Radar

Monostatic radar uses a single antenna for transmitting and receiving the pulse as shown in Fig. To separate the receiving and transmitting function we prefer to use a duplexer. It is more common than the bistatic radar and it is beneficial to detect the velocity of the target. Monostatic radar equation is defined as:

$$S_{min} = P_r = \frac{P_t G_t^2 \sigma \lambda^2}{(4\pi)^3 R^4} \text{ Watt} \tag{9}$$

Where, P_t = Transmitted power

G_t = Gain of transmitted antenna

σ = radar cross section

λ = wavelength

R = Range

P_r = Received Power

S_{min} = Minimum detectable signal

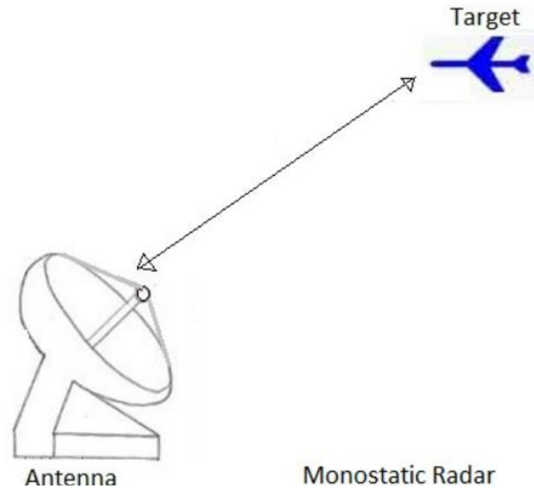


Figure 2. Block diagram depiction of a monostatic radar

1.3.2 Bistatic Radar

Bistatic radar has separate transmitter and receiver separated by a distance. Mostly the distance between the transmitter and the receiver is kept equivalent to the target distance. Bistatic radar is beneficial in cases where the reflected signal power is very low so in weather and long-range radar. In military it can be used to detect a stealth target as in such cases it is more useful than monostatic radar. Bistatic radar equation can be defined as:

$$P_r = \frac{P_t G_t G_B \sigma_B \lambda^2}{(4\pi)^3 dt^2 dr^2} \tag{10}$$

G_B = Gain of the receiver antenna

dt = distance of the target from the transmitter

dr = distance of the target from the receiver

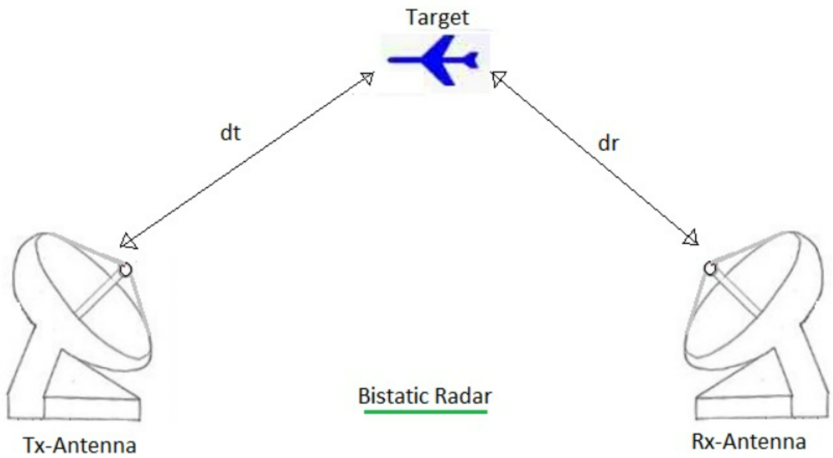


Figure 3. shows block diagram of a bistatic radar

1.4 Tracking Radar

In tracking radar detection of the target is already done and now this radar is tracking it. The beam will be centered at the target even though it is changing the path and speed continuously. Here, we can integrate infinite number of pulses which was not possible in search radar.

Tracking radar can track in doppler frequency shift, elevation angle, azimuth angle, range or any combinations of four. But track in angle makes it different from other radars. Tracking radar is of two types: Continuous Tracking radar (CTR) and Track While Scan (TWS).

The antenna beam is angled by servomechanism actuated by an error signal. There are various methods to generate this error signal like sequential lobing, conical scan, and simultaneous lobing or monopulse.

1.5 Literature Survey

The background of cognitive radar spans over radar signal processing, information theory, control theory and neuroscience. For over four decades, most of the research efforts devoted to this area share the same motivation: to increase the flexibility of the radar waveform and enhance the performance of radar detection and tracking. But few researchers have tried to address this issue in an *outside-of-the-box* manner. It is noteworthy to point out that these early attempts were derived directly from the fact that a radar is first a sensor. With the advance of digital computer and signal processing technology, the functions that a radar can perform are both diversified and specialized [19]. For example, two of the main functions are *detection* and *tracking.*

An Italian author Maria Sabrina Greco [1] gave us a brief idea about the basic working of cognitive radar and how is it possible in real world applications. Along with cognitive radar we get information about passive radar and cognitive radio [17].

The very first idea of incorporating cognition in radars was proposed by Simon Haykin [3]. In his initial works [8],[27] we notice the subtle technological advancements.

With the advent of digital waveform generators that can generate almost any form of radar signals, the waveform design should be considered as an integral part of the overall tracking system. A radar equipped with different waveforms have a different resolution and thus, leads to a different measurement error. Mathematically, we denote by θ the radar waveform and z as the measurement, the covariance matrix for the measurements noise can be represented as a function of θ, i.e., $R(\theta)$.

From the systems engineering viewpoint, the waveform design problem is to find the optimal tracking filter and waveform parameters that give us a minimum covariance of the target state in an on-line fashion.

Dai Hong De, Dai Shao Wu, Cong Yuan-Cai, Wu Guang-bin [10] in their paper have compared various types of Kalman Filtering that can be used at the receiver for better results. The comparison shows different filters with different pros and cons and how we can incorporate it in our application.

Following this line of thinking and thanks to the development of nonlinear filtering, e.g. the extended Kalman filter, the unscented Kalman filter (UKF), and particle filter (PF), in recent years, many other researchers have extended Kershaw and Evans's work to encompass the tracking of maneuvering targets [7] in both clutter-free and cluttered environments. Through the comparison between the Kalman Filters, we come to know that EKF is good for mild nonlinearities, and UKF is good for high nonlinearities but the performance of CKF is considered more accurate as it uses cubature points for filtering at the receiver as described by Ienkaran Arasaratnam[6].

When we talk about the word cognition Simon Haykin in his paper published in 2006 [27], first tried to incorporate this function in radar to increase accuracy, efficiency, speed, and resource-efficient. Cognition can be applied in any function with any parameter according to our application. An intelligent radar can be used in surveillance in which the cognitive part can be to maintain the beamwidth and length or number of rotations [28].

In the case of detection, it can be used to either detect through the material or by the number of propellers by taking into account their movement or it can detect simply by the motion of the target in various environments. In the case of tracking, we can focus on the direction of arrival or pulse width and its duration [26]. This is how we can say that cognition is a function that can be applied to any part of the radar function which can make it highly efficient and versatile in many new emerging fields.

From the works of Simon Haykin [13] we get to know that in Cognitive tracking radar, we use Cognitive Waveform-Selection (CWS) algorithm which selects the next transmitting waveform by analyzing the data given by the receiver to it. The waveform selection is done from the prescribed library which is provided at the transmitter [29]. This tracking algorithm can track a linear target, a ballistic target, or a maneuvering target. From the works of Yanbo Xue [2], the framework of basic cognitive radar is discussed. The tracking is done by reducing the root-mean-square error

1.6 Research Gap - Project Formulation

Therefore, what needs to be done in this field to address the following-

a) Need to incorporate memory in the transmitter, receiver, and feedback, link to make the radar adapt to different environments.
b) Adding attention and knowledge to the radar system is not easy, but with the development in AI fields, the idea does not look vague.
c) More advanced filtering methods need to be used for more accurate prediction and estimation of target state if we are tracking whether single or multiple targets at a time.
d) Advanced machine learning methods are needed to incorporate the neural network in feedback loop.
e) Cognition part can be applied in any feature of the radar in future.

Summary

- In this chapter we come across the basic idea of radar and its working.
- There are two types resolution while tracking or detecting a target that are radial resolution and angular resolution. It helps us to distinguish two targets close to each other.
- The major difference between monostatic and bistatic radar is the location between receiver and the transmitter. In monostatic they are co-located and in bistatic they are separated by some distance.
- For are future reference we discussed about the basic idea of tracking radar.
- In literature survey there is a brief explaination of the research papers which are inspired the project.

- Research gap helps us to discover the area which are needed to be improved in future for better performance.

Chapter 2 Basic Cognitive Radar

2.1 Introduction

The first introduction of Cognition in radar systems was done by Dr. Simon Haykin. Cognition is the most emerging field in radar technology. Cognitive Dynamic Systems include both Cognitive Radar and Cognitive Radio but, in this thesis, we are going to learn about Cognitive radar which is a monostatic radar whereas cognitive radio is a bistatic radar. Research on cognitive radio is more prominent as compared to cognitive radar.

Cognitive radar is a versatile field that needs to be explored more for both military and civilian purposes. The Perception-Action cycle for Basic Cognitive Radar (BCR) helps to develop a link between the transmitter and receiver to increase a better understanding of the nonlinear environment and prediction of the target's state. In Basic Cognitive Radar the perception-action cycle helps to differentiate it from traditional radar systems. The significance of this cycle is the repetition of it over several loops in order to reduce error and improve accuracy [30].

As soon as the radar is turned on, it becomes connected to its surrounding environment electromagnetically and the environment influences the radar returns effectively. The designed radar does not store the entire past data in fact it takes the past values as knowledge built over successive cycles. With the help of the state-space model of the environment, the requirement of past data is eliminated which is done by updating the state vector that accounts for the parameters according to our applications. This is ensured by assuming the environment as non-stationary which is true in the real world.

At the receiver, we identify the nonlinear environment using Cubature Kalman Filter (CKF) as this filter is the most effective filtering for high nonlinearities [4]. At the transmitter for waveform selection, we try to implement using Dynamic programming. The link between the transmitter and the receiver is developed using neural networks referring to the brain the link is developed using a complex system of neural network. The addition of three more elements i.e., memory, attention, and intelligence converts Basic Cognitive Radar (BCR) to Nested Cognitive Radar (NCR).

Nesting of the three memories allows the radar to work efficiently and take decisions according to the present environment. Memory is an important element present at the transmitter, receiver and the link joining them as it stores the information gained by the radar over successive cycles across time [20].

This information stored is used for future predictions and allows proper working of waveform transmission. The second element is attention which helps radar to be more attentive and utilize resources efficiently. The third element is intelligence which helps radar to work smartly and reduce the computational complexities of the system. This working of Cognitive radar resembles the working of the visual brain along with the echolocation of bats and dolphins [2],[21]. As this field is an emerging field still a lot of research in this area is needed as incorporating attention and intelligence particularly is not an easy task as it requires advanced smart algorithms.

2.2 Baseband Model of Radar Measurements

With optimal performance as the goal, the ideal way to build a cognitive radar is to look to the optimal Bayesian filter as the central functional block in the receiver for perception of the environment, and Bellman's dynamic programming as the central functional block in the transmitter for action to control the environment [22].

Naturally, there must be feedback from the receiver to the transmitter to make it possible for the receiver to send relevant information about the environment to the transmitter. In so doing, global feedback embodies the two parts of the radar system and the environment under a single overall loop operating in an *on-line* manner, and with it, the radar becomes computationally intelligent. Here again, if we are to examine the visual brain, we will find that, unlike perception and action, there is no single functional block that takes care of intelligence; rather, this important function is distributed through feedback across many parts of the brain [23].

To design the waveform generator, we have opted for Linear Frequency Modulation (LFM) combined with Gaussian pulse for Amplitude Modulation. The transmitted signal is defined as:

$$s_T\,(\mathrm{t}) = \sqrt{2}\mathrm{Re}\left\{\sqrt{E_T}\tilde{s}(t)\exp(j2\pi f_c t)\right\} \qquad (11)$$

Where E_T =Transmitted signal energy

$\tilde{s}$(t)=complex envelope of s_T(t)

The radar echo reflected from the target received at the Receiver input is correspondingly defined by

$$r(t) = s_R(t) + n(t) \qquad (12)$$

Where $s_R(t) = signal\ component\ of\ r(t)$

n(t) = AWGN

$s_R(t)$ is defined as:

$$s_R = \sqrt{2}Re\{\sqrt{E_R}\tilde{s}(t-\tau)\exp\big(j2\pi(f_c t + f_D t)\big)\} \qquad (13)$$

Where $E_R = Received\ signal\ energy$

$$\tau = \frac{2\rho}{c}\,; \qquad \rho = \text{ range of the target} \tag{14}$$

$$f_D = \frac{2 f_c \dot{\rho}}{c}\,; \tag{15}$$

Where $\quad \dot{\rho} = range\ rate\ of\ the\ target$

$\tilde{n} = complex\ envelope\ of\ the\ noise\ n(t)$

The transmitted radar signal is assumed to be narrowband, which means the complex envelopes $\tilde{s}(t)$and $\tilde{n}(t)$in the baseband model occupy a frequency band small in comparison to the carrier frequency f_c.

Recognizing that a matched filter is basically equivalent to a correlator, it follows that the bank of matched filters acts as a time-frequency correlator of the complex transmitted signal envelope with itself. In the absence of receiver noise, the squared magnitude of this correlation constitutes the ambiguity function. Every matched filter in the filter bank is therefore followed by a square-law envelope detector. The resulting real-valued two-dimensional output of each envelope detector, involving time delay and Doppler shift, defines an inter-pulse vector denoted by $\boldsymbol{z_k}$, where the subscript $\boldsymbol{k}$ denotes discrete time. This vector performs the role of *measurement vector* in the state-space model of the radar target.

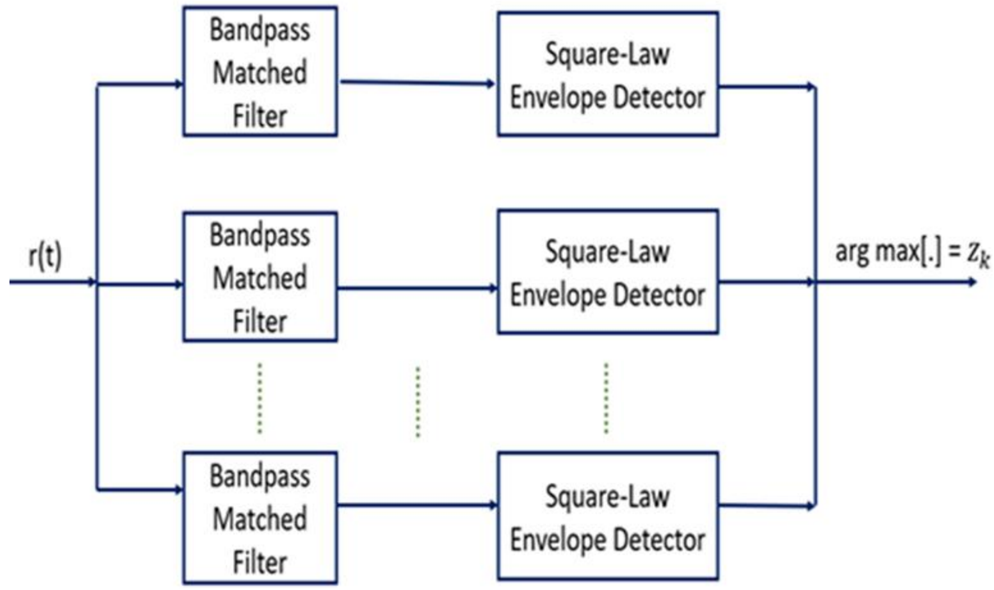


Fig.4 Bank of matched filters

The vector z_k performs the role of measurement vector in the state-space model of the radar target.

2.3 State-Space Model of the Target

There are two equations in the discrete state-space model of a radar target:

System equation:

System equation, which describes evolution of the target's state across time in accordance with the nonlinear equation:

$$x_k = f(x_{k-1}) + v_k \qquad (16)$$

Where $x_k =$ the state of the radar target at discrete time k

v_k= additive system noise accounting for environmental uncertainty about the target.

Measurement equation:

$$z_k = h(x_k) + w_k(\theta_{k-1}) \qquad (17)$$

Where $w_k(\theta_{k-1}) = \text{The measurement noise}$

$$\theta_{k-1} = \text{Waveform parameter}$$

The dependence of this noise w_k on the waveform-parameter vector θ_{k-1} that the transmitter influences the accuracy of the state estimation in the receiver

2.4 Basic Perception-Action cycle

In a cognitive radar, perception of the environment in the receiver leads to an action taken by the transmitter on the environment in accordance with the basic perception-action of the cycle [8]. It is through the continuation of this cycle across time that the system acquires its ability to adapt to changes in the environment by making successive internal changes of its own through lessons learned from continuing interactions with the environment.

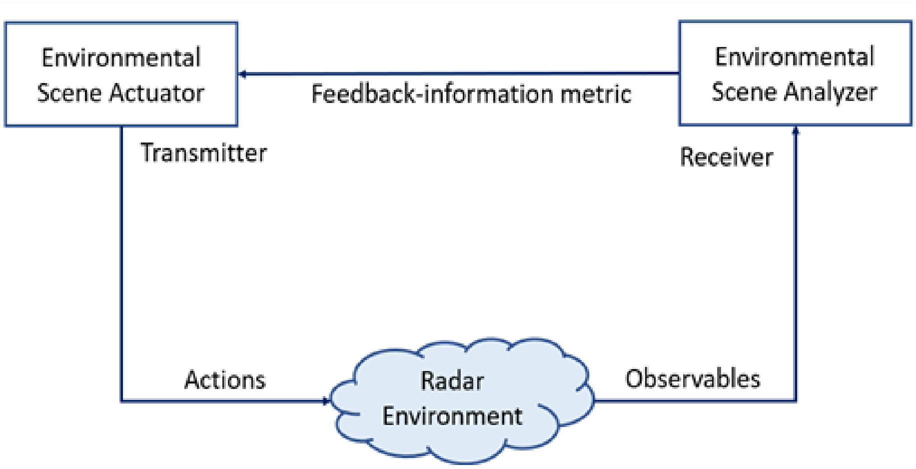


Fig.5 The perception-action cycle in its most basic form

2.5 Optimal Bayesian Filtering

The optimal Bayesian filter is the ideal tool for tracking the target 's state in the radar receiver. Optimal Bayesian filtering is a general probabilistic approach to estimate the posterior density of the state over time by using new measurements.

Kalman Filter works on two states:

1. Predicted state
2. Update state

(1) Predicted state

In Predict we just predict the new value called the **predicted value** based on the initial value and then predict the uncertainty/error/variance in our prediction according to the various process noises present in the system.

(2) Update state

In Update, we consider the actual measurement coming from the device and we call this the **measured value**. Here we calculate the difference between our predicted value and measured value and then decide which value to keep by calculating the Kalman Gain. We then calculate the new value and new uncertainty/error/variance based on our decision made by Kalman Gain. These calculated values will finally be the predictions done by our Kalman Filter in iteration 1.

Real world problems are basically nonlinear functions whereas Kalman Filter works for linear functions so in order to deal with these real-world issues new filter was developed known as Extended Kalman Filter (EKF). It uses Taylor series to linearize the nonlinear function.

Extended Kalman Filter predict the density function after passing through nonlinear function using mean. It is good for mild nonlinearities but it is not the right choice for high nonlinearities.

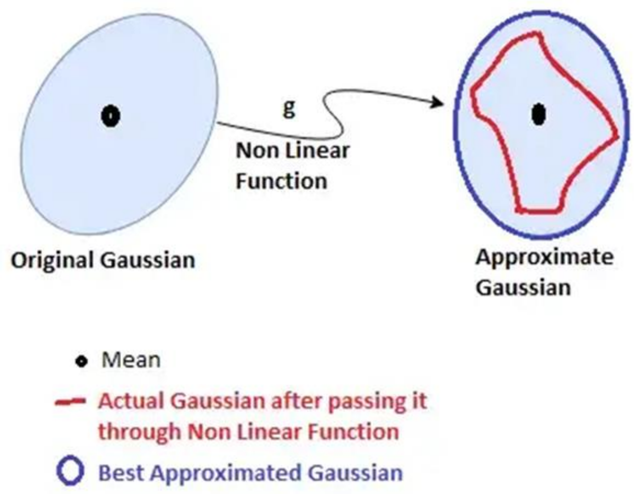


Fig.6 EKF Working

For high nonlinear functions, a new filter was formulated known as the Unscented Kalman Filter. It predicts the density function using sigma points and mean. Mean is given for weightage compared to sigma points. Sigma points are given as:

$$\begin{cases} \boldsymbol{\chi}_{k,k}^{0} = \hat{X}_{k,k} \\ \boldsymbol{\chi}_{k,k}^{i} = \hat{X}_{k,k} + \left(\sqrt{(n+\lambda)\,\boldsymbol{P}_{k,k}}\right)_i \\ \boldsymbol{\chi}_{k,k}^{i+n} = \hat{X}_{k,k} - \left(\sqrt{(n+\lambda)\,\boldsymbol{P}_{k,k}}\right)_i \end{cases}$$
$$i = 1, 2 \cdots n \tag{18}$$

UKF is good for high nonlinearities but for more accuracy and less computation we use modified version of KF i.e. Cubature Kalman Filter (CKF).

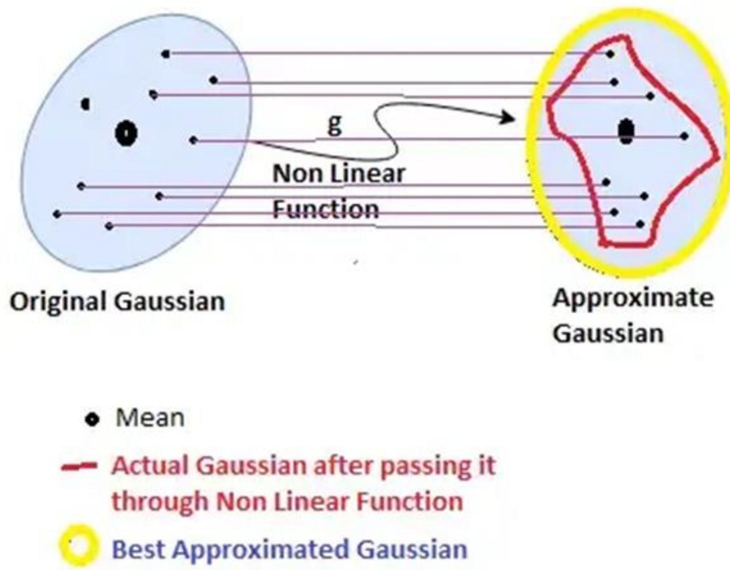


Fig.7 UKF Working

Unscented Kalman Filter

The newly developed Kalman Filter is known as Cubature Kalman Filter (CKF) which used cubature points to predict the density function after through the nonlinear function.

- In CKF the integrands are all of the form:

Nonlinear function * Gaussian density

- The cubature points can be achieved by a third-degree spherical-radial rule, where a total of 2n points are employed when the dimension of the state is n. the cubature points can be achieved as:

$$\xi_i = \sqrt{n}[1]_i \qquad (19)$$

$$\omega_i = \frac{1}{2n} \qquad i = 1, 2\ldots..,2n$$

Where $\xi_i = cubature\ points$

$\omega_i = weights\ of\ the\ cubature\ points$

$n = dimension\ of\ the\ state$

Factorize the know covariance matrix $P_{k,k}$

$$P_{k,k} = S_{k,k} S_{k,k}^{T} \tag{20}$$

The cubature point can be computed as

$$\chi_{k,k}^{i} = S_{k,k}\xi_i + \hat{X}_{k,k} \qquad \text{i=1, 2…..,2n} \tag{21}$$

CKF Algorithm

Prediction:

$$\boldsymbol{\chi}_{k+1,k}^{i} = f_k(\boldsymbol{\chi}_{k,k}^{i}, \boldsymbol{W}_k)$$

$$\hat{\boldsymbol{X}}_{k+1,k} = \frac{1}{2n}\sum_{i=0}^{2n}\boldsymbol{\chi}_{k+1,k}^{i}$$

$$\boldsymbol{P}_{k+1,k} = \frac{1}{2n}\sum_{i=0}^{2n}\boldsymbol{\chi}_{k+1,k}^{i}\boldsymbol{\chi}_{k+1,k}^{i\mathrm{T}} - \hat{\boldsymbol{X}}_{k+1,k}\hat{\boldsymbol{X}}_{k+1,k}^{\mathrm{T}} + \boldsymbol{Q}_k$$

Update:

$$P_{k+1,k} = S_{k+1,k} S_{k+1,k}^{\mathrm{T}}$$

$$\chi_{k+1,k}^{i} = S_{k+1,k}\xi_i + \hat{X}_{k+1,k} \quad i = 1, 2, \cdots, 2n$$

$$Z_{k+1,k}^{i} = h_{k+1}(\chi_{k+1,k}^{i}, V_{k+1})$$

$$\hat{Z}_{k+1,k} = \frac{1}{2n}\sum_{i=0}^{2n} Z_{k+1,k}^{i}$$

$$\hat{X}_{k+1,k+1} = \hat{X}_{k+1,k} + K_{k+1}[Z_{k+1} - \hat{Z}_{k+1,k}]$$

$$P_{k+1,k+1} = P_{k+1,k} - K_{k+1} P_{ZZ,k+1,k}^{-1} K_{k+1}^{\mathrm{T}}$$

Where

$$K_{k+1} = P_{XZ,k+1,k} P_{ZZ,k+1,k}^{-1}$$

$$P_{XZ,k+1,k} = \frac{1}{2n}\sum_{i=0}^{2n} \chi_{k+1,k}^{i} Z_{k+1,k}^{i\mathrm{T}} - \hat{X}_{k+1,k}\hat{Z}_{k+1,k}^{\mathrm{T}}$$

$$P_{ZZ,k+1,k} = \frac{1}{2n}\sum_{i=0}^{2n} Z_{k+1,k}^{i} Z_{k+1,k}^{i\mathrm{T}} - \hat{Z}_{k+1,k}\hat{Z}_{k+1,k}^{\mathrm{T}} + R_k$$

2.6 Dynamic Optimization for waveform selection

The measurement covariance $R(\theta_{k-1})$ as given in measurement equation depends on the transmitted waveform parameter vector $\theta = [\lambda, b]$ which applies to LFM with Gaussian amplitude modulation.

Where $\lambda = duration\ of\ the\ gaussian\ pulse$

$b = chirp\ rate\ of\ the\ LFM\ pulse$

- Dynamic optimization assumes the role of a controller in a nonlinear feedback system that tunes the transmit-waveform parameters so as to tame the behavior of the receiver in an effort to minimize the tracking errors in some statistical sense.
- It is assumed that z_0 is the initial condition for the perception-action cycle, and the cognition is wired to start from this initial observable. The transmitter then operates on the cost-to-go function (computed from the receiver output) to

produce a waveform parameter vector θ_0 on which radar waveform will be transmitted at next cycle $k = 1$, which correspondingly leads to a new observable z_1. This cycle runs recursively for k = 2, 3, ... Unfolding the perception-action cycle, we have

$$z_0 \rightarrow \theta_0 \rightarrow \ . \ . \ . \ . \ . \ . \ . \ . \ \rightarrow z_k \rightarrow \theta_k$$

- The total time, namely cycle time τ_{cycle}, needed for the directed information to flow across the cognitive radar system is composed of three parts, expressed as

$$\tau_{cycle} = \tau_{target} + \tau_{receiver} + \tau_{transmitter} \qquad (22)$$

Where

$\tau_{target} =$ time for the radar signal to reach to the target and bounce back

$\tau_{receiver} =$ time for the radar echo to be processed in the receiver

$\tau_{transmitter} =$ time for the transmitter to perform its action in the environment

A necessary property needs to be satisfied for a cognitive radar system to work:

$$\tau_{cycle} < \text{Pulse Repetion Time} \qquad (23)$$

Formulation of the dynamic optimization algorithm for action to control the waveform selection in the transmitter. The waveform selection is done at the transmitter and this plays a major role in incorporating cognition in the radar system. Waveform selection is a complex process as the idea of change in parameters leading into changing the waveform requires high level algorithm [9]. This is an essential part of the basic perception-action cycle.

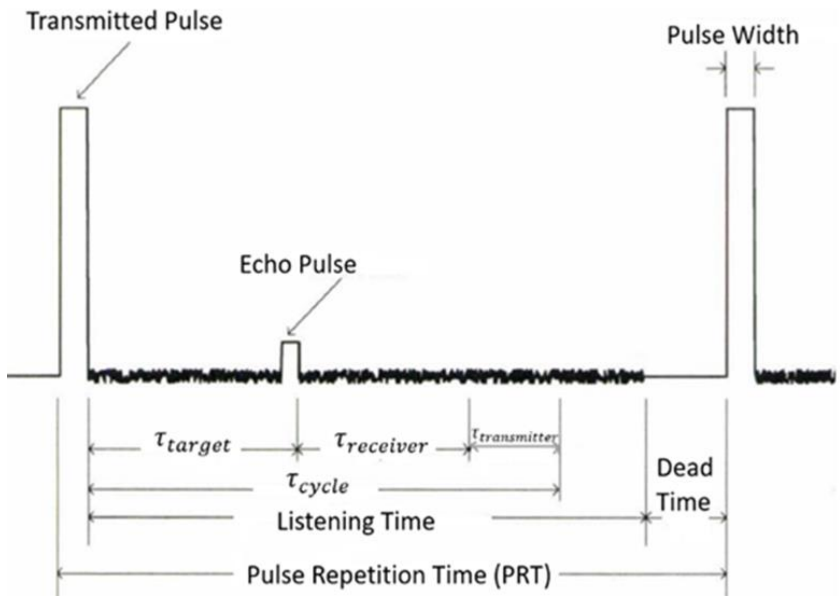


Fig.8 Diagram of radar pulse timing

Summary

- Basic cognitive radar works on the closed loop perception-action cycle.
- Perception of the environment where target is present, is done at the receiver where the state of the environment is unknown. This is achieved using Cubature Kalman Filter for Optimal Bayesian Filtering using cubature points.
- Waveform selection is done at the receiver. Change is two waveform parameters is observed according to which waveform is selected and transmitted as an action on the environment.
- Mean square error is obtained by calculating the difference between predicted and measured value and the error is tried to reduce at each cycle.

Chapter 3 Nested Cognitive Radar

3.1 Introduction

The nested cognitive radar distinguishes itself from the basic cognitive radar in the “nesting" of three memories within the perception-action cycle.

- *perceptual memory* in the receiver,
- *executive memory* in the transmitter, and
- *coordinating perception-action memory* that reciprocally couples the first two memories.

Three new features in NCR are *attention, memory, and intelligence.*

- *Memory,* stores the data for the receiver and from the transmitter for further learning. It is dynamic across time.
- *Attention,* which provides for the selective allocation of resources to the receiver and transmitter.
- *Intelligence,* which is enhanced significantly by virtue of the fact that we now have two global feedback loops embodying the environment and four visible local feedback loops within the three memories. It is static and timeless.

In Cognitive Radar we are taking our inspiration from visual brain although we are nowhere near mimicking brain cortex [11]. In human brain the impulse response is sent by the brain through the interconnection of several neurons to the desired organ.

Here, brain can be considered as the transmitter, the interconnection of the neurons can be considered as the feedback loop and the human body other than brain can be considered as the receiver.

Any environmental change is perceived by the human body, it observes and the signal is sent to the brain through back-propagation. Brain analyzes the information given by various organs and it takes actions according to the information given to it. This is the basic structure of cognitive radar but as we discuss visual brain, it has properties like memory, attention, and intelligence [12].

Brain can store past experiences as knowledge and utilize it effectively without the loss of information. For example, if a person is sent to a place where he has never been to, the environment will be new to him so his brain will store the information for future experiences [13]. Next time when the same person is sent to such an environment again, he will have knowledge about it and the brain will take actions accordingly.

Brain has the property of attention which conserve the resources available to it. This property ensures the right amount of resource extraction from all the data stored and utilize it perfectly according to the situation. Another property of brain is memory and because of this property cognitive radar is able to store information after every cycle. Memory is provided at transmitter, receiver and at the feedback too [15].

The basic block diagram of Nested Cognitive Radar is shown in Fig.9.

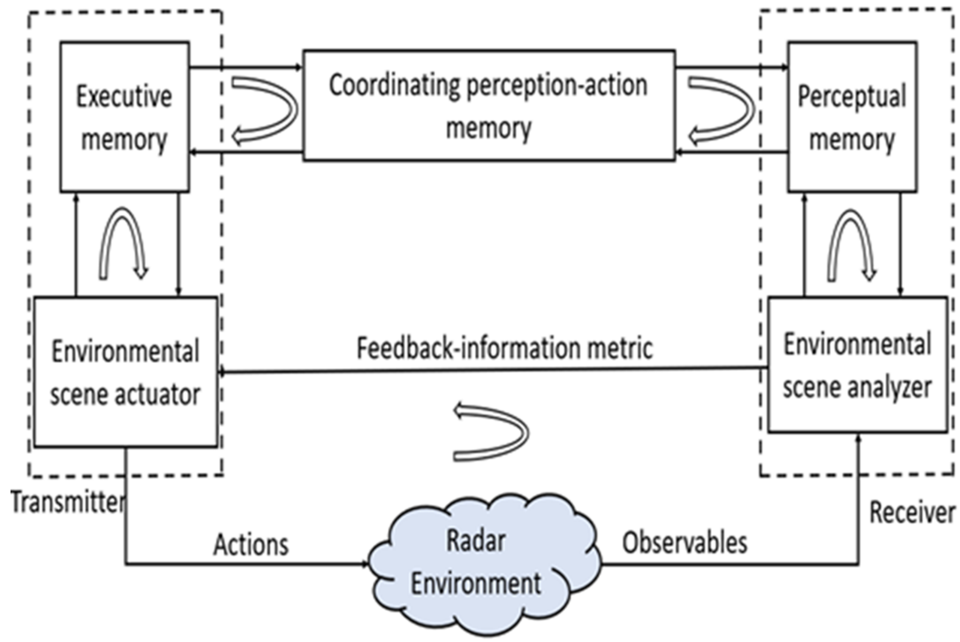


Fig.9 Nested perception-action cycle

3.2 Memory

Memory is the data stored at each cycle and thus changing the action taken by the transmitter. Contents of the memory is changed after every cycle and it improves its working with the help of new information given to it [14]. This way it differs from knowledge as memory continuously changes over time and learns from new information but knowledge is related to facts and it does not change over time so basically knowledge is timeless i.e. it is static whereas memory is dynamic in nature.

As can be observed from Fig.6, memory is divided into three parts, one is provided at the receiver another is provided at the transmitter and one links the two memories.

3.2.1 Perceptual memory

Perceptual memory is a part of the receiver that enables it to perceive new incoming stimuli and extract new information gathered in the form of new changes in the environment. Such changes are repeatedly stored and updated to learn more about the environment in which our target is present. To link this memory to the environment, memory is reciprocally coupled to the environmental scene analyzer [18].

Here, two types of learning occur in perception-action cycle i.e., top-bottom, and bottom-up. Top-bottom signifies the incoming stimuli information flow from environmental scene analyzer to perceptual memory [24]. This approach I it to give the new information to the perceptual memory for updating and in return the desired relevant information is given back to the environmental scene analyzer according to the new changes. The feedback link is the bottom-up approach.

3.2.2 Executive Memory

Executive memory is provided at the transmitter to store the past data regarding the information given to it by the receiver through feedback link [16]. Memory works in two ways, first is top-bottom in which the information of the receiver is given to the environmental scene actuator and then finally to the memory to update its data. In this manner the contents of the memory is updated with the new observations.

The second approach is bottom-top, in this the relevant information is extracted from the memory and given to the environmental scene actuator whose role is to take the action accordingly.

3.2.3 Coordinating Perception-Action Memory

Executive memory and Perceptual memory are co-dependent in cognitive radar. The reciprocal linking of the two memories is necessary to make radar cognitive. The linking between the two memories is done with the help of the third memory which is known as Coordinating Perception-Action memory. This memory plays a key role in the perception-action cycle as it connects the receiver and the transmitter together to perform a give task as a single unit [25].

3.3 Attention

Attention property is not related to a specific block of the cognitive radar in fact it is a property of whole system just like the visual brain in which we cannot define attention to a specific part.

Attention is the property which signifies the effective use of the respources available to the radar. At each element it's function is to restore and extract only the useful data in the particular cycle.

At the receiver this property makes sure that the incoming stimuli is sent to memory but obtaining useful data from the memory is attention property. At the transmitter the attention property resides in extracting useful data from the memory so that the desired waveform is transmitted.

This attention property ensures the environment perceiving property of the receiver and dynamic optimization of the transmitter.

3.4 Intelligence

It is not easy to define intelligence in a system element form and other three properties like perception, attention, and memory contribute to the intelligence property. Intelligence can be defined as the ability to learn from new situations for future references and taking actions according to the interrelationships of the previous data. The constant adaptivity of radar is termed as intelligence.

Intelligence can be related to the efficiency of information processing. The property of intelligence in radar is a different research area as it includes the feedback loop between receiver and the transmitter. The feedback loop is the key feature of the cognitive radar as because of the feedback there is a relation between the transmitted waveform and the environmental changes. If feedback was not present than the transmitted waveform would not be related to the target and efficiency will be low as the resource conservation is more when feedback is present.

The idea of cognitive radar itself suggest that the transmitted waveform should be changed at every cycle according to the environmental changes and this is performed with help of feedback so this itself is intelligence.

Summary

- Nested Cognitive radar is the advanced version of Basic Cognitive Radar with some modifications.
- The new featured added to it are memory, attention, and intelligence
- We can see memory is employed to transmitter, receiver, and the feedback to store the information for future reference.
- Attention is simply preserving and efficiently using the resources.
- Intelligence is inspired from the visual brain that means how well our radar can work on its own by updating the information.

Chapter 4 Design and implementation of Cognitive Radar

4.1 Kalman Filter

The Kalman filter is a mathematical technique that employs a sequence of measurements taken over time that contain noise and other imperfections to provide estimates of unknown variables that are more accurate than those based just on a single measurement. Rudolf Kalman created it in the 1960s and it is widely used in control systems, navigation, signal processing, and many other domains.

The Kalman filter is simply a recursive algorithm that guesses a system's state at each time step based on measurements collected up to that point. It works in two stages: prediction and updating. The filter predicts the state of the system at the next time step based on the present state and a dynamic model of the system during the prediction phase. The filter integrates fresh measurements and modifies the expected state estimate based on their accuracy throughout the update phase.

The Kalman filter is intended to function with systems that have continuous, linear dynamics and Gaussian noise, and it is predicated on the assumption that the system can be described using a set of equations that connect the current state to the prior state and the current measurement. It may, however, be expanded to deal with nonlinear systems by employing techniques such as the extended Kalman filter or the unscented Kalman filter.

Several applications employ the Kalman filter, including robotics, aeroplane navigation, satellite tracking, and financial forecasting. It is well-known for its speed, precision, and capacity to deal with noisy and unclear data.

We can calculate the present estimate as:

$$\mathrm{Est}_t = \mathrm{Est}_{t-1} + \mathrm{K}[\ \mathrm{MEA} - \ \mathrm{Est}_{t-1}] \qquad (24)$$

Where, $\mathrm{Est}_t = \mathrm{Present\ estimated\ state}$

$\mathrm{Est}_{\mathrm{t-1}}$= Previous estimated state

MEA = Measured Value

K = Kalman Gain

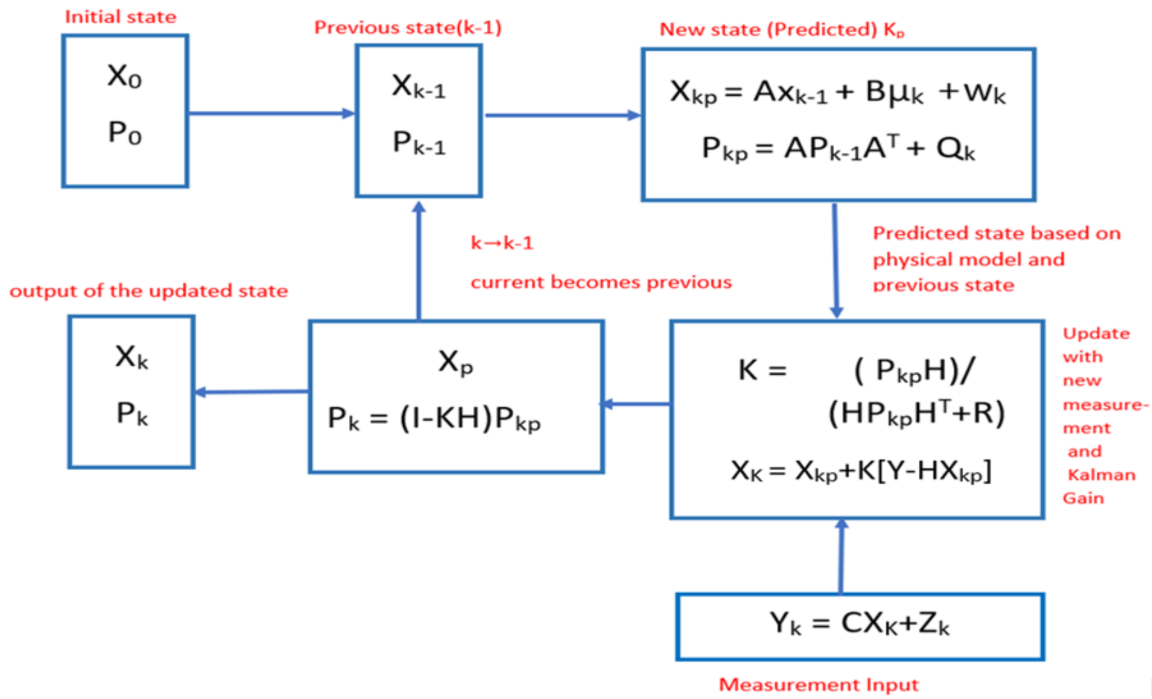
We calculate Kalman Gain as:

$$\mathrm{K} = \frac{\mathrm{E_{EST}}}{\mathrm{E_{EST}} + \mathrm{E_{MEA}}} \qquad (25)$$

Where, $\mathrm{E_{EST}} = \mathrm{Error\ in\ estimated\ value}$

$\mathrm{E_{MEA}} = \mathrm{Error\ in\ measured\ value}$

The value of the K ranges from 0 to 1. If we have a large error in measurement, K is nearer to 0, which means our predicted value is close to the actual value. If we have a large error in prediction, K is closer to 1, which means our measured value is closer to the actual value. The following Flowchart 1 shows a multidimensional model for the Kalman Filter.



Flowchart 1 of Kalman Filter

μ = control variable matrix

w = predicted state noise matrix

Q = Process covariance matrix

X = State Matrix

P = Process covariance matrix (represents error in the estimate/process)

K = Kalman Gain

R = Sensor noise covariance matrix (measurement error)

I = Identity matrix

Y = Measurement of the state

ΔT = Time for 1 cycle

Working procedure of Kalman Filter shown in flowchart can be explained step by step as:

1. State matrix contains position and velocity of the object we are tracking. It can be 1D, 2D, or 3D
   For 2D, Velocity -> x, y axis
   Position -> x, y axis
   For 3D, Velocity -> x, y, z axis
   Position -> x, y, z axis
2. In predict state we use μ. Control variable matrix contains information about what controls an object of our state of object. 'A' and 'B' matrices are simply a matrix that are used to convert the input to a state to the new state matrix (X_{kp}).
3. Once prediction is completed, we add actual measurement to it. 'C' matrix changes X_{KH} to Y_K (measurable vector).
4. Fourth step include the update state after considering the measured values. First we calculate Kalman gain. 'K' decides how much of the estimate we have to impart on measurement and predicted state respectively. K decides what fraction of measurement and predicted value it wants to use then combine that information to update the new state [X_K].
5. After updating state X_P, we update P_K too.
6. The outputs of updated state are X_K, P_K.
7. The current becomes input to the next cycle and becomes previous state for it.

4.2 Markov Chains

Markov chains are a form of stochastic process that simulates a series of events in which the probability of each occurrence is solely determined by the state of the system at the previous event. In other terms, a Markov chain is a process in which the system's future state is determined only by its current state and not by any previous states. Markov chains may be thought of as a set of states with transition probabilities between them. These probabilities may be expressed as a matrix called the transition matrix, with each entry representing the likelihood of transitioning from one state to another.

Markov chains are employed in physics, economics, finance, biology, and computer science, among other fields. Markov chains are commonly used to describe natural language processing and speech recognition systems. They are also utilized in image processing, signal processing, modelling, and optimization. The concept of a stationary distribution, which is a probability distribution that does not vary over time, is a critical characteristic of Markov chains. The chance of being in any given state in a stationary distribution is independent of the beginning state.

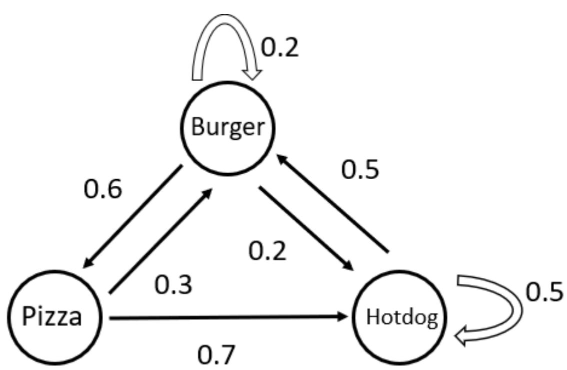
Property:

Future value depends only on present value.

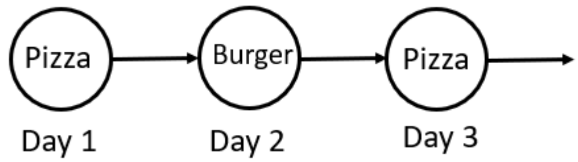
$$\mathrm{P}(\mathrm{X}_{\mathrm{n}+1} = \mathrm{x} \mid \mathrm{X}_{\mathrm{n}} = x_n) \tag{26}$$

Example:

Suppose the following condition is provided and we need to find the probability that a restaurant will serve hotdog on 4th day.



In order to find the probability of hotdog being served on 4th day depends only on the 3rd day serving.



$P(X_4 = \text{Hotdog} \mid X_3 = \text{Pizza}) = 0.7$

4.3 Monte Carlo Methods

Using interacting empirical measures, it is a class of mean-field particle methods for sampling and estimating the posterior distribution of a signal process given certain noisy and particle observations. Simulations using the Monte Carlo method change randomly. For example if we want to find out average height of all people on earth using monte carlo simulations.

It is impossible to determine the average height of billions of people. We can estimate global height and determine the height of smaller groups. But, we must keep in mind that I the group should be chosen randomly because we can't take into account the next five people who live close by because our neighbourhood may include tall or short people. Picking people at random from all across the world is a better approach. (ii) More accuracy results from sampling more persons, as anticipated value approaches actual value.

4.4 Cubature Kalman Filter at the receiver

The power of the Cubature Kalman Filter/Smoother (CKF/CKS) is demonstrated. Figure 10. depicts two common postures of a two-link robot arm, namely "elbow-up" and "elbow-down."

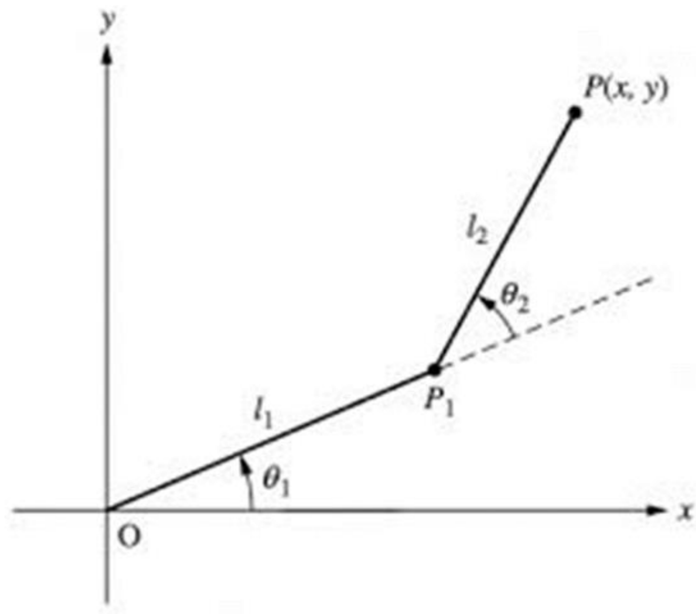


Fig.10 Two-link arm

Given the angles (α_1, α_2) the end effector position of the robot arm can be described in the Cartesian coordinate as follows:

$$y_1 = r_1 \cos(\alpha_1) - r_2 \cos(\alpha_1 + \alpha_2) \quad (27)$$
$$y_2 = r_2 \cos(\alpha_2) - r_2 \cos(\alpha_1 + \alpha_2) \quad (28)$$

where r1 = 0.8 and r2 = 0.2 are the lengths of the two links; $\alpha_1 \in [0.3, 1.2]$ and $\alpha_2 \in [\pi/2, 3\pi/2]$ are the joint angles confined to a specific region.

The forward kinematic is the mapping from to (α_1, α_2) $(y_1,\ y_2)$, whereas the inverse kinematic is the mapping from $(y_1,\ y_2)$ to (α_1, α_2). Because the inverse kinematic is not a one-to-one mapping, the solution is not unique.

For the inverse kinematic problem, let the state vector x be x = $[\alpha_1, \alpha_2]^T$ and the measurement vector y be y = $[y_1,\ y_2]^T$. The state-space model of the problem is written as State equation: $x_{k+1} = x_k + w_k$ (29)

Measurement equation:

$$\mathbf{y}_k = \begin{pmatrix} \cos(\alpha_{1,k}) & -\cos(\alpha_{1,k} + \alpha_{2,k}) \\ \sin(\alpha_{1,k}) & -\sin(\alpha_{1,k} + \alpha_{2,k}) \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} + \mathbf{v}_k \tag{30}$$

Here, we assume the state equation to follow a random-walk model perturbed by white Gaussian noise w ~ $\mathcal{N}$ (0, diag[0.01, 0.1]) The measurement equation is nonlinear, with additive measurement noise v ~ $\mathcal{N}$ (0, 0.005I), where I is the identity matrix in two dimensions. As shown in Fig. 11, α_1 is a gradually expanding process with a periodic random walk, whereas α_2 is a periodic, rapid, and linearly increasing/decreasing process.

Summary

- Kalman Filter has two states i.e., Prediction state and Update state. Prediction state predicts the future state of the target using only the present state. The new changes using measurements is incorporated in update state.
- Markov chains is the internal procedure of using only the present state for predicting the future state. This method restores and maintain the data without overconsuming the memory.
- Monte Carlo method signifies how a random process can give a desired output. Even in randomness there is a pattern to yield an output.

Chapter 5 Results

Following are the results of the output obtained using CKF at the receiver for two-link robotic arm. The third plot shows the output of cognitive radar using Kalman filter.

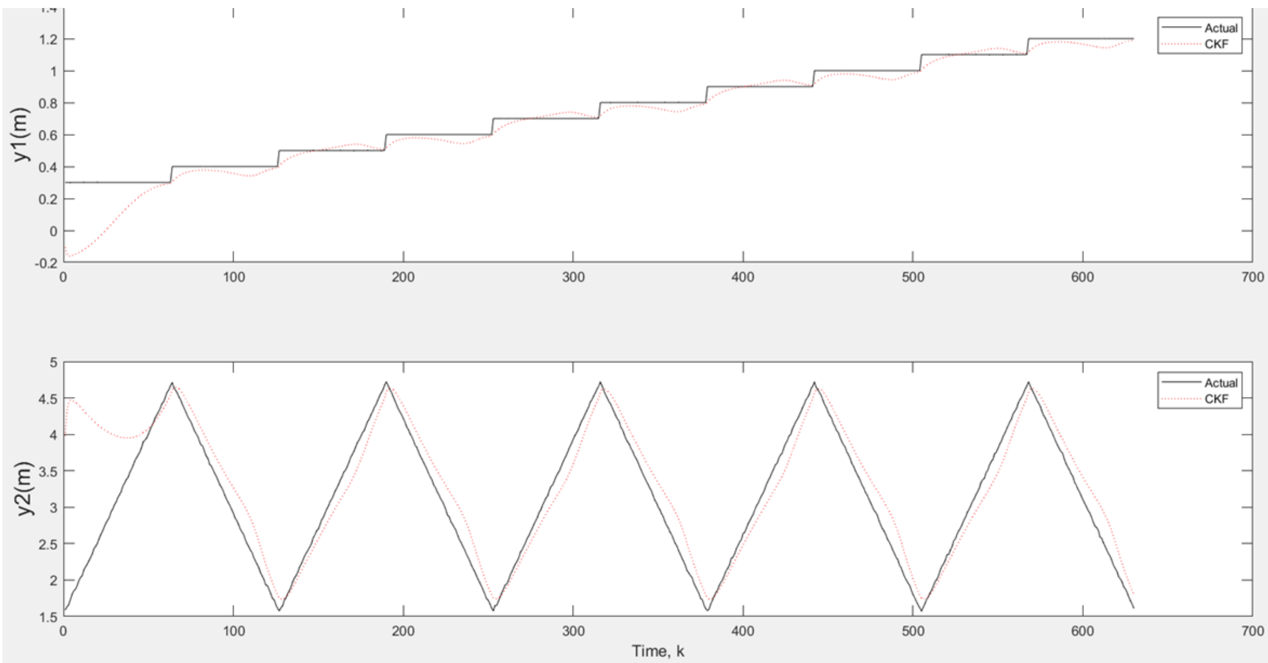


Fig.11 Tracking results (True trajectory- Solid line, Estimated Trajectory- Dotted line)

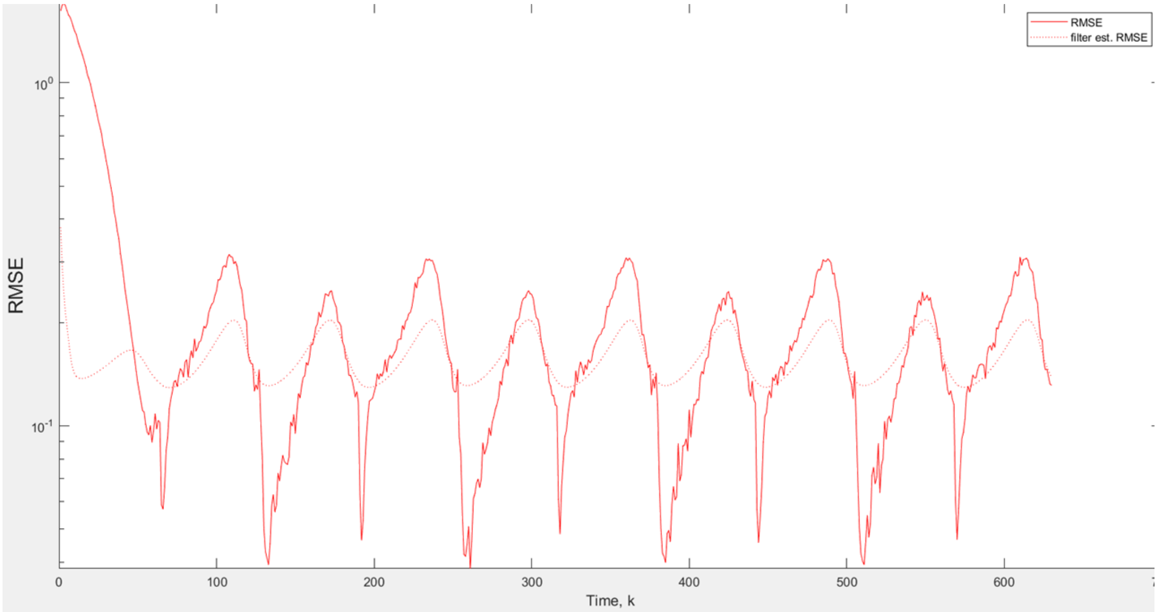


Fig. 12 Ensemble averaged (over 50 runs) root mean-squared error (RMSE) results (true rmse- continuous line, estimated rmse- dotted.

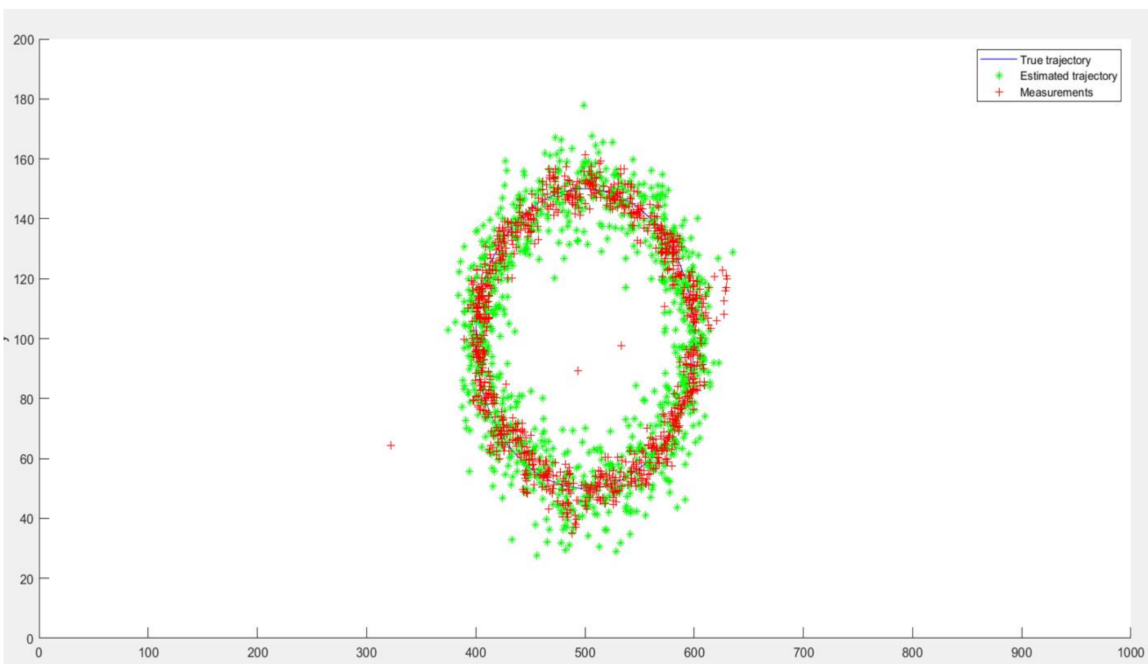


Fig. 13 Cognitive radar for tracking using Kalman Filter

Summary

- In the above results the output shows how accuracy of the system is improved over several cycles and the error is reduced.
- Kalman Filter is the basic filtering process used at the receiver in which we can see that how estimated value is brought near to the true value using measured value taken from sensors.
- Cubature Kalman Filter is the new modified form of Kalman Filter in which cubature points are used to predict the future vales. As of now this is the most accurate state prediction filter for highly non-linear environment which signifies the real-world problems.
- Tracking two-link arm can be used in future predicting an arm of human or bionic bird. Cognition can be applied in any form like predicting state or velocity or any parameter we need to identify.

Conclusion

- In my project it can be observed Kalman filter is used to predict the future state of a target.
- Kalman filter improves the error between the true value and the estimated state using measured values. Over the repetitive cycles this error is brought to minimum.
- Accuracy is improved over 1000 iterations.
- The tracking of a two-link robotic arm is done using Cubature Kalman Filter. This shows this filter is most suitable to be used for real-world problems as the real environment is highly non-linear.
- This filter can be used in the Cognitive radar for perceiving the environment.
- Smoother is used to bring the output more accurate.
- Cognition can be applied in any form in any part of the radar. Kalman filter is the most suitable filtering at the receiver for perception.
- Dynamic programming is used at the transmitter for waveform selection.
- Neural networks same as the visual brain are used in developing the feedback between, the receiver and the transmitter. Because of this feedback the cognitive radar is considered unique as the working of the transmitter and the receiver are co-dependent.

Future Scope

Cognitive radar is an emerging field for future research that constantly requires updating itself for better performance. Future research in this area can be applied to any part of the radar because cognition is an ability.

For increasing result accuracy, we can opt for different types of the filtering process at the receiver. Recently developed filters like Cubature Kalman Filter or Continuous-Discrete Cubature Kalman Filter (CD-CKF). These filters are more useful for highly non-linear environment which signifies real-world problems.

Future directions that need to be focused on include the use of lower bound to increase the performance of estimator, optimal use of chattering effect in Nested cognitive radar.

Future of cognitive radar is Nested Cognitive Radar as it includes new advanced machine learning algorithms and neural networks along with classical radar system. More research is needed in this field as incorporation of intelligence and attention is difficult.
Research done in past is in its initial stage and will take some time to fully include it in radar to make it an intelligent radar completely.

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