Neutrinos

Oscillations, Experimentation, Unification

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Introduction

- Within the Standard Model, neutrinos are massless and labeled by flavor, momentum, and helicity.
- Helicity is defined as $h = \mathbf{s} \cdot \hat{\mathbf{p}}$ and is a good quantum number for massless particles.
- Left-handed neutrinos appear in the Standard Model with $h = -\frac{1}{2}$.

CPT and Neutrinos

- CPT symmetry implies the existence of right-handed antineutrinos.
- These states are related to neutrinos but distinguished by helicity and lepton number.
- For massless neutrinos, helicity remains a valid quantum number in all reference frames.

Neutrinos with Mass

- When neutrinos have mass, helicity is no longer a good quantum number.
- Observers in different frames can disagree on the helicity of a given neutrino.
- Instead, spin direction in the rest frame is used: states labeled by $|\nu_i(p,\sigma)\rangle$ with $\sigma = \pm\frac{1}{2}$.

Dirac vs. Majorana Neutrinos

- If neutrinos carry a conserved charge (e.g., lepton number), they are Dirac particles:

$$|\nu_i\rangle \neq \mathrm{CPT}|\nu_i\rangle. \tag{1}$$

- If no such charge exists, neutrinos are Majorana particles:

$$|\nu_i\rangle = \mathrm{CPT}|\nu_i\rangle. \tag{2}$$

Field Representation

- Majorana neutrinos are described by Majorana spinor fields.
- If a conserved U(1) charge exists, neutrinos can be grouped into a Dirac spinor:

$$\psi_D = P_L \psi_1 + P_R \psi_2, \tag{3}$$

  with U(1) acting as $\psi_D \to e^{i\alpha} \psi_D$.
- This is analogous to how charged fermions in the Standard Model are treated.

Three Neutrinos Only

Suppose there are no additional neutrino states beyond the Standard Model. Then, CPT symmetry ensures neutrinos are their own antiparticles.

This implies:

- Lepton number is not an approximate symmetry.
- Neutrino masses break lepton conservation.
- Individual lepton numbers L_e, L_μ, and L_τ are separately violated.

Neutrino Mass Term

To describe neutrino masses phenomenologically, the Standard Model Lagrangian is modified:

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2}\left[m_{ij}(\bar{\nu}_i P_L \nu_j) + \text{c.c.}\right] \tag{4}$$

where m_{ij} is a complex symmetric 3×3 matrix.

Majorana Neutrinos

Since the Standard Model contains six neutrino and antineutrino states (counting spin labels), neutrinos can be taken as Majorana fields:

$$\nu = \begin{bmatrix} \chi \\ \chi^* \end{bmatrix} \tag{5}$$

The mass term then takes the form:

$$\mathcal{L}_{\nu,\text{mass}} = \frac{1}{2}\left[m^*_{ij}\chi_i^T\chi_j + \text{h.c.}\right] \tag{6}$$

This structure differs from quark and charged-lepton masses, where two distinct fields are coupled.

Majorana Mass Matrix

The Majorana mass matrix m_{ij} is symmetric, reducing the number of independent parameters compared to charged leptons.

Since the mass matrix transforms as:

$$m_{jk} \to m_{jk} e^{i(\omega_j + \omega_k)} \tag{7}$$

it is only invariant if $m_{jk} = 0$. Thus, the mass term breaks all lepton-number symmetries of the Standard Model.

Lepton Symmetries

The only conserved lepton number is a difference between two flavors, such as:

$$L_{12} = L_1 - L_2 \tag{8}$$

If the only nonzero mass matrix elements are m_{33} and $m_{12} = m_{21}$:

- ν_1 and ν_2 form a Dirac neutrino (with eigenvalue of L_{12}).
- ν_3 remains a Majorana neutrino if $m_{33} \neq 0$.

Neutrino Mixing

- Neutrino flavor states $|\nu_\alpha\rangle$ differ from mass states $|\nu_i\rangle$
- Described by the PMNS matrix U:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle \tag{9}$$

- PMNS matrix parameterization:

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \tag{10}$$

- CP-violating phase δ affects oscillations
- Experimental evidence from solar, atmospheric, reactor, and accelerator experiments

Neutrino Production and Evolution

- Neutrinos are produced in weak interactions with charged leptons:
  - $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$
  - $e^- X \to X' + \nu_e$
- Neutrino states evolve as:

$$i\frac{d}{dt}|\nu_i(t)\rangle = E_i|\nu_i(t)\rangle \tag{11}$$

- Mass eigenstates acquire phase factors due to propagation

Neutrino Oscillations

- Flavor transition probability:

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_i U_{\beta i} e^{-i \frac{m_i^2}{2E} L} U_{\alpha i}^* \right|^2 \tag{12}$$

- Key parameters:
  - Mass squared differences: $\Delta m_{ij}^2 = m_i^2 - m_j^2$
  - Mixing angles: $\theta_{12}, \theta_{13}, \theta_{23}$
  - CP-violating phase: δ
- Two-flavor approximation:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \tag{13}$$

Matter Effects on Neutrino Oscillations

Why is vacuum oscillation insufficient?

- Neutrinos in experiments often travel through Earth's bulk material.
- Matter can alter oscillation probabilities.

Two possible effects:

- Direct flavour change due to interactions (not allowed in SM).
- Forward scattering leading to an effective potential.

Effective Potential due to Matter

Two Contributions:

- Charged current interaction (via W boson exchange) affects only ν_e:
$$V_W = \pm\sqrt{2}G_F N_e$$
- Neutral current interaction (via Z boson exchange) affects all flavours equally:
$$V_Z = \mp\frac{\sqrt{2}}{2}G_F N_n$$

Schrödinger Equation for Neutrino Evolution

Lab-frame evolution equation:

$$i\frac{d}{dt}|\nu(t)\rangle = \mathcal{H}|\nu(t)\rangle$$

For two-neutrino mixing (ν_e, ν_μ):

$$|\nu(t)\rangle = \begin{bmatrix} f_e(t) \\ f_\mu(t) \end{bmatrix}$$

where $\mathcal{H}$ is a 2×2 Hamiltonian matrix.

Vacuum Hamiltonian

From mass eigenstates:

$$\mathcal{H}_{\text{Vac}} = \sum_i U^*_{\alpha i} U_{\beta i} E_i$$

Using the relativistic approximation,

$$\mathcal{H}_{\text{Vac}} = \frac{\Delta m^2}{4p} \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} + \left(p + \frac{m_1^2 + m_2^2}{4p} \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

.

Hamiltonian in Matter

Including matter potential:

$$\mathcal{H}_M = \mathcal{H}_{\text{Vac}} + V_W \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + (V_W + V_Z) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Substituting $V_W = 2\sqrt{2} G_F N_e E / \Delta m^2$:

$$\mathcal{H}_M = \frac{\Delta m^2}{4E} \begin{bmatrix} -\cos 2\theta + x & \sin 2\theta \\ \sin 2\theta & \cos 2\theta - x \end{bmatrix}$$

where $x = 2\sqrt{2} G_F N_e E / \Delta m^2$.

Modified m and θ

This can be interpreted as a change in $\theta \to \theta_M$ and $m \to m_M$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2} \tag{14}$$

$$\Delta m_M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - x)^2} \tag{15}$$

resulting in again

$$\mathcal{H}_M = \frac{\Delta m_M^2}{4E} \begin{bmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{bmatrix} \tag{16}$$

Finding the Oscillation Probability in Matter

From the eigenstates in matter:

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta_M + |\nu_2\rangle \sin\theta_M, \tag{17}$$

$$|\nu_\mu\rangle = -|\nu_1\rangle \sin\theta_M + |\nu_2\rangle \cos\theta_M. \tag{18}$$

Eigenvalues of $\mathcal{H}_M$:

$$\lambda_1 = +\frac{\Delta m_M^2}{4E}, \quad \lambda_2 = -\frac{\Delta m_M^2}{4E}. \tag{19}$$

Oscillation Probability in Matter

Solving for $|\nu(t)\rangle$ and computing $P_M(\nu_e \to \nu_\mu)$:

$$P_M(\nu_e \to \nu_\mu) = \sin^2 2\theta_M \sin^2 \left(\frac{\Delta m_M^2 L}{4E} \right) . \tag{20}$$

Replacing $\Delta m_M^2, \theta_M$ with $\Delta m^2, \theta$ recovers the vacuum case.

Discussions: Significance of Matter Effects

Matter effects depend on x:

$$|x| \approx \frac{E}{10.53\ \text{GeV}}. \tag{21}$$

Higher energy $\Rightarrow$ stronger matter effects.
Example: Atmospheric neutrinos traveling through Earth's mantle.

Numerical Estimate for x

- $\Delta m^2 = 2.4 \times 10^{-3}$ eV^2
- Fermi constant: 1.67×10^{-23} eV^{-2}
- Mantle density: 3000 kg/m^3
- $N_e = 6.89 \times 10^9$ eV^3

$$|x| = 9.50 \times 10^{-11} E \text{ eV}^{-1} \approx \frac{E}{10.53 \text{ GeV}}. \tag{22}$$

Resonance Effects in Matter

In special cases, matter effects can be dramatically large.

- If $\theta \approx 0$, then $\sin^2 2\theta_M \approx 1$.
- Resonant amplification: $\theta_M \gg \theta$.
- Known as the MSW effect (important for solar neutrinos).

Neutrino vs. Antineutrino Oscillations

Matter effects differ for neutrinos and antineutrinos:

- x changes sign for antineutrinos.
- Δm_M^2 and θ_M differ accordingly.
- Useful for distinguishing neutrinos from antineutrinos.

Experimental Observations

- Oscillations observed in various experiments:
  - Atmospheric neutrinos (Super-Kamiokande, IceCube)
  - Solar neutrinos (SNO, Borexino)
  - Reactor neutrinos (KamLAND, Daya Bay)
  - Accelerator neutrinos (MINOS, NOvA, T2K)
- Long-baseline experiments probe small mass differences
- Sensitivity increases with longer distances and lower energy

Neutrinoless Double Beta Decay

- Standard vs. neutrinoless beta decay
- Implications for Majorana neutrinos
- Experimental status

Possible Theoretical Frameworks: Overview

- The Standard Model (SM) cannot explain neutrino masses
- Key theoretical approaches:
  - Effective field theory: Dimension-5 operators (νSM)
  - Sterile neutrino models: Right-handed neutrinos
  - Seesaw mechanisms: Types I, II, and III
  - Left-right symmetric models: $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
  - Grand Unified Theories (GUTs): $SO(10)$, etc.
- Central question: Why are neutrino masses so small?

Gauge Invariance in Neutrino Models

- The Standard Model (SM) obeys $SU_L(2) \times U_Y(1)$ gauge symmetry.
- Neutrino masses require either:
  - Additional degrees of freedom.
  - Relaxing renormalizability.

Sterile Neutrinos

- Standard-model neutrinos may be supplemented by additional neutrinos.
- Introduce N Majorana neutrino fields s_x, with $x = 1, \ldots, N$.
- Mass terms are written without initial concern for gauge invariance.

Interaction Properties

- New neutrino fields interact primarily through their mixing with standard-model neutrinos.
- These new neutrinos, called **sterile neutrinos**, do not directly couple to standard-model matter.
- Consequently, sterile neutrinos interact much more weakly with matter.

Lagrangian for Sterile Neutrinos

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2} s_x \not\partial s_x - \frac{1}{2} \left(M_{xy} s_x P_L s_y + m_{ab} \nu_a P_L \nu_b + 2\mu_{ax} \nu_a P_L s_x + \text{c.c.} \right) \tag{23}$$

- The total left-handed neutrino mass matrix is given by:

$$\begin{bmatrix} m & \mu \\ \mu^T & M \end{bmatrix} \tag{24}$$

- The matrix is an arbitrary complex and symmetric $(3 + N) \times (3 + N)$ matrix.

Freedom in Neutrino Mass Choices

- The freedom to choose mass matrices m, μ, and M allows for many neutrino physics models.
- One important scenario: neutrino masses that preserve overall lepton number.
- Other constraints and possibilities regarding sterile neutrinos will be summarized in later sections.

Dirac Neutrinos

- Consider the case where lepton number is unbroken.
- Introduce $N = 3$ new neutrino states s_a.
- These transform under lepton symmetry as:

$$P_L\nu_a \to e^{i\omega} P_L\nu_a, \quad P_L s_a \to e^{-i\omega} P_L s_a. \tag{25}$$

Mass Structure of Dirac Neutrinos

- The mass term remains invariant for any μ, provided that:

$$m = M = 0. \tag{26}$$

- This symmetry allows us to group the six Majorana fields into three Dirac fields:

$$\psi_a = P_L \nu_a + P_R s_a. \tag{27}$$

- Under lepton number symmetry, $\psi_a \to e^{i\omega} \psi_a$.

Diagonalization of Neutrino Mass Matrix

- The diagonalization process follows the quark-mass matrix approach in the Standard Model.
- The neutrino spectrum consists of three massive states.
- These states are distinguished from their antiparticles by their lepton charge.
- Their masses are given by the positive square roots of the eigenvalues of $\mu\mu^{\dagger}$.

Yukawa Terms

- Another method is to introduce right-handed neutrinos N_m.
- They must be $SU_L(2)$ singlets with hypercharge $Y = 0$.
- Allows the gauge-invariant Yukawa term:

$$\mathcal{L}_N = -\frac{1}{2}\bar{N}_m \partial\!\!\!/ N_m - \frac{1}{2}M_m \bar{N}_m N_m - \left(k_{mn}\bar{L}_m P_R N_n \tilde{\phi} + h.c.\right) \tag{28}$$

Lepton Number Violation

- Assign $L = +1$ to $P_R N$.
- Yukawa interactions violate individual lepton numbers (L_e, L_μ, L_τ) if k_{mn} is non-diagonal.
- The Majorana mass term:

$$M_m \bar{N}_m N_m \tag{29}$$

breaks total lepton number $L = L_e + L_\mu + L_\tau$ by two units.

Neutrino Masses

- With Higgs VEV, model reduces to sterile-neutrino framework:

$$m_{ab} = 0, \quad \mu_{ab} = k_{ab}\frac{v}{\sqrt{2}}, \quad M_{ab} = M_a\delta_{ab} \tag{30}$$

- Dirac mass $\mu_{xa} \sim v$ (few hundred GeV or lower).
- Majorana mass M_m is naturally at weak scale or larger.

Helicity Suppression in Neutrino Scattering

- Neutrino production amplitudes are helicity suppressed, being proportional to powers of small neutrino mass.
- Consider a neutrino propagating from source to detector.
- Its evolution is governed by the propagator:

$$M = \frac{G_F^2}{2} J^{\mu}_{\text{prod}} J^{\nu}_{\text{det}} \left[(\ldots)\gamma_\mu (1+\gamma_5) \frac{-i \not{k} + m}{k^2+m^2} \gamma_\nu (1 \pm \gamma_5)(\ldots) \right] \tag{31}$$

- The sign $\pm$ depends on the detection interaction.

Detection of Neutrino Components

- Detection of the sterile component s_a requires $(1-\gamma_5)$.
- Detection of the active component ν_a requires $(1+\gamma_5)$.
- Only the $\not{k}+m$ term in the numerator contributes to s_a detection.
- Only the $\not{k}$ term contributes to ν_a detection.

Suppression of Sterile Component Detection

- The ratio of sterile to active detection amplitudes is $\sim k^0/m$.
- For MeV-scale neutrinos, this ratio is 10^{-6}, leading to a suppression factor of 10^{-12} in probability.
- Thus, sterile states have negligible impact on neutrino phenomenology.

Generic Sterile Neutrinos

- Diagonalization of neutrino and charged-lepton mass matrices leads to a complex neutrino mass pattern.
- Additional sterile neutrinos modify the PMNS matrix and neutral-current interactions.
- We discuss four main regimes distinguished by the relative sizes of mass matrix elements m_{ab}, μ_{ax}, and M_{xy}.

Effects of Mass Matrix Diagonalization

- Generates an extended PMNS matrix U_{ai} for charged-current interactions.
- Introduces new neutral-current mixing matrices H_{uv} and H'_{uv}.
- Additional sterile neutrinos impact oscillations but are constrained by experimental data.

Regime 1: $\mu \ll m, M$

- Sterile neutrinos do not significantly mix with active neutrinos.
- Standard-model neutrinos are described by the Majorana mass matrix m.
- Sterile neutrinos do not affect oscillation experiments but may impact cosmology.

Regime 2: Pseudo-Dirac Neutrinos ($\mu \gg m, M$)

- If $m = M = 0$, leads to three Dirac neutrinos when $N = 3$, with additional massless sterile states for $N > 3$.
- When m and M are small, neutrinos form almost-degenerate pairs with maximal mixing.
- Strong oscillations occur for distances $L \approx \frac{2E}{\Delta m^2}$.
- Constraints from solar neutrino data require $m, M \lesssim 10^{-9}$ eV.

Regime 3: Light Sterile Neutrinos ($m \sim \mu \sim M$)

- All mass matrices are of comparable size, leading to $(3 + N)$ Majorana neutrino states.
- Masses around 10^{-2} eV are required to match observed neutrino oscillations.
- Typically leads to large oscillation effects, but no experimental evidence supports this scenario.
- Some models avoid these constraints.

Regime 4: Seesaw Neutrinos ($m \ll \mu \ll M$)

- Results in N heavy eigenstates with masses from M and three light eigenstates from M^{-1}.
- Heavy states are almost purely sterile, with mixings of order μ/M.
- Light eigenstates remain mostly standard-model neutrinos.
- The seesaw mechanism explains small observed neutrino masses $O(\mu^2/M)$.
- Natural within the $SU_L(2) \times U_Y(1)$ framework.

Implications of Gauge Invariance

- Explains why sterile neutrinos do not couple directly to SM particles.
- Natural mass hierarchy:

$$m \ll \mu \ll M$$

suggests the seesaw mechanism.

Challenges

- Small neutrino masses require tiny Yukawa couplings:

$$\mathrm{Tr} k_{mn} < 4 \times 10^{-12} \tag{32}$$

- Majorana mass must satisfy:

$$M_m < 10^{-20}\mu \tag{33}$$

- Why should couplings be fine-tuned to such accuracy?

$B-L$ Symmetry and Anomalies

- Exact $B-L$ symmetry could set $M_m = 0$.
- However, it conflicts with:
  - Baryon asymmetry of the universe.
  - Standard charge assignments ensuring neutron neutrality.
- Without M_m, neutron charge neutrality requires fine-tuning.

$B-L$ Symmetry and Neutrino Masses

- $B-L$ is an accidental global symmetry in the SM
- If gauged: $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
  - Anomaly cancellation requires right-handed neutrinos
  - $B-L$ breaking at high scale connects to seesaw
- Left-Right symmetric models:

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \tag{34}$$

  - Explain parity violation as symmetry breaking
  - Right-handed neutrinos as part of $SU(2)_R$ doublets
  - $B-L$ breaking gives Majorana masses to N_R
- Connects neutrino physics to fundamental symmetries

νSM: Dimension-5 Interactions and Neutrino Masses

Introduce non-renormalizable interactions without new fields.

Effective Field Theory: Allows higher-dimension operators suppressed by a heavy mass scale Λ.

Non-Renormalizable Interactions: Physical Significance

- **SM as a Renormalizable Theory:** Contains all terms consistent with gauge symmetries up to dimension 4.
- **Historical Precedents:**
  - Fermi's four-fermion weak interactions: $G_F \sim M_W^{-2}$.
  - Chiral perturbation theory: Operators suppressed by $(4\pi F_\pi)^{-1}$.
- **Key Insight:** Predictivity is maintained at low energies by truncating to the lowest-dimension operators.

Effective Operators and Their Relevance

Dimension-p Interactions:

- Operators scale as $\mathcal{O}(M^{-p+4})$.
- Effects suppressed by inverse powers of Λ, dominant at low energies.

For neutrino physics: Lowest non-renormalizable operator is at dimension 5.

Unique Dimension-5 Operator in the SM

Gauge-invariant, Lorentz-scalar interaction:

$$\mathcal{L}_{\text{eff}} = -\tilde{k}_{mn}\tilde{\phi}^{\alpha}(\bar{L}^{\alpha}_{m} P_R L^{\beta}_{n})\tilde{\phi}^{\beta} + \text{h.c.} \tag{35}$$

- Involves Higgs doublet ϕ, lepton doublet L.
- **Lepton number violation:** $\Delta L = 2$.
- **Coupling structure:** $\tilde{k}_{mn} = c_{mn}/\Lambda$, where $c_{mn} \sim \mathcal{O}(1)$.

Implications for Neutrino Masses

In unitary gauge:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\tilde{k}_{mn}(\nu_m P_R \nu_n)(v+H)^2 + \text{h.c.} \tag{36}$$

Majorana Neutrino Mass in Higgs VEV:

$$m_{ab} = \tilde{k}^*_{ab} v^2 = c^*_{ab}\frac{v^2}{\Lambda} \tag{37}$$

- **Key Prediction:** $m_\nu \sim 50$ meV implies $\Lambda \sim 10^{14}$ GeV.
- **Natural Suppression:** Explains why neutrino masses are small without fine-tuning.

Possible Origins of the Dimension-5 Operator

- The Standard Model is incomplete at high energies.
- Low-energy consequences include higher-dimension operators.
- The lowest-order such operator is a unique dimension-5 term.
- This operator induces neutrino masses, which are observed.

Lepton Number Violation

- In νSM, total lepton number is violated, and neutrinos have Majorana masses.
- Experimentally, the nature of neutrino mass is still unknown.
- The observation of neutrinoless double beta decay would confirm lepton number violation.

Lepton Flavor Violation

- Observed only in neutrino oscillations so far.
- Possible searches in charged lepton decays:
  - $\mu \to e\gamma$
  - $Z \to \ell\ell'$
  - $h \to \ell\ell'$
- νSM predicts extremely suppressed rates, below experimental sensitivity.

PMNS Matrix Unitarity

- In νSM, the 3×3 PMNS matrix is unitary.
- If experiments establish deviations from unitarity, then additional leptons must exist.

The Λ Scale

- In the νSM, the scale of neutrino masses depends on the scale Λ in the coefficients of the dimension-five terms.
- For experiments with energies $E \ll \Lambda$, what is probed is the combination $z\nu/\Lambda$.
- The separation of the coefficient of a $d = 5$ term into a dimensionless coupling and a scale is meaningful when discussing a full high-energy theory that generates the effective term.

Neutrino Mass Scale

- The effective low-energy Lagrangian predicts that neutrino masses are much lighter than the weak scale:

$$m_{1,2,3} \approx \frac{v^2}{\Lambda} \ll v. \tag{38}$$

- Experimental findings that $m_\nu \ll m_Z$ support the idea that neutrino masses arise from $d = 5$ terms.
- Unlike other SM fermions, neutrinos are lighter by at least six orders of magnitude.

Implications for High-Energy Physics

- The SM cannot be valid above the Planck scale, $\Lambda \ll M_{\mathrm{Pl}}$.
- Expectation: $m_i \gtrsim \frac{v^2}{M_{\mathrm{Pl}}} \approx 10^{-5}$ eV.
- If $d = 5$ terms originate at the GUT scale $\Lambda_{\mathrm{GUT}} \approx 10^{16}$ GeV, and assuming $z_\nu \approx 1$:

$$m_\nu \approx 10^{-2}\ \mathrm{eV}. \tag{39}$$

Upper Bound on Λ

- Experimental lower bounds on neutrino masses provide an upper bound on Λ/z_ν.
- Assuming $z_\nu \lesssim 1$, this also gives an upper bound on Λ itself:

$$\Lambda \lesssim \frac{v^2}{m_\nu} \approx 10^{15}\ \text{GeV}. \tag{40}$$

- This upper bound is intriguingly close to the GUT scale.

Seesaw Mechanism - NSM

- Introduces heavy sterile neutrinos N_m.
- New terms in the Lagrangian:

$$\mathcal{L}_N = -\frac{1}{2}\bar{N}_m \partial\!\!\!/ N_m - \frac{1}{2} M_{mn} \bar{N}_m N_n - (k_{mn} \bar{L}_m P_R N_n \tilde{\phi} + h.c.) \quad (41)$$

- Large M leads to small neutrino masses via:

$$m_\nu \sim \frac{v^2}{M} \quad (42)$$

- Heavy N_m contribute only through virtual effects.

Integrating Out Heavy Neutrinos

- Virtual exchange of heavy neutrinos generates an effective interaction.
- Expansion in M^{-1} leads to:

$$\mathcal{L}_{\text{eff}} = -(kM^{-1}k^T)_{mn}\tilde{\phi}^{\alpha}\bar{L}_m P_R L_n \tilde{\phi}_{\beta} + h.c. \tag{43}$$

- Identifying coefficients:

$$\tilde{k}_{mn} = -(kM^{-1}k^T)_{mn} \tag{44}$$

Triplet Scalar Fields

- Modify the Higgs sector by introducing a scalar triplet Δ:

$$\Delta \sim (1, 3, -1) \tag{45}$$

- New terms in the Lagrangian:

$$\mathcal{L}_\Delta = -(D_\mu \Delta)^\dagger D^\mu \Delta - M_\Delta^2 \Delta^\dagger \Delta - y_{mn} \bar{L}_m (\Delta \tau^a \mathcal{C}) P_R L_n + h.c. \tag{46}$$

- Large M_Δ leads to small neutrino masses.

Integrating Out the Triplet Scalar

- Virtual exchange of Δ generates a similar effective interaction:

$$\mathcal{L}_{\text{eff}} = -\frac{2y_{mn}c}{M_\Delta^2}\tilde{\phi}^\alpha \bar{L}_m P_R L_n \tilde{\phi}_\beta + h.c. \tag{47}$$

- Another realization of the dimension-5 operator.

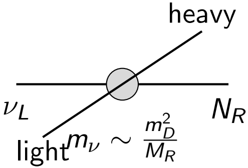
Type I Seesaw Mechanism

- Introduces right-handed neutrinos N_R
- Dirac and Majorana mass terms:

$$\mathcal{L}_N = -\bar{L} Y_\nu \tilde{\phi} N_R - \frac{1}{2} \bar{N}_R^c M_R N_R + \text{h.c.} \tag{48}$$

- Mass matrix in the basis (ν_L, N_R^c):

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \tag{49}$$



where $m_D = Y_\nu \frac{v}{\sqrt{2}}$

- For $M_R \gg m_D$, light neutrino masses:

$$m_\nu \approx -m_D M_R^{-1} m_D^T \tag{50}$$

Type II Seesaw Mechanism

- Introduces scalar triplet $\Delta \sim (1, 3, +2)$
- Yukawa coupling:

$$\mathcal{L}_\Delta = Y_\Delta \bar{L}^c i\tau_2 \Delta L + \text{h.c.} \tag{51}$$

- Triplet VEV $\langle \Delta^0 \rangle = v_\Delta$
- Neutrino mass:

$$m_\nu = 2 Y_\Delta v_\Delta \tag{52}$$

- $v_\Delta \sim \frac{\mu v^2}{M_\Delta^2}$ (naturally small)

Type III Seesaw Mechanism

- Introduces fermion triplets $\Sigma \sim (1, 3, 0)$
- Similar to Type I but with triplet instead of singlet
- Yukawa coupling:

$$\mathcal{L}_\Sigma = -\bar{L} Y_\Sigma \tilde{\phi} \Sigma - \frac{1}{2} \mathrm{Tr}(\bar{\Sigma}^c M_\Sigma \Sigma) + \text{h.c.} \tag{53}$$

- Light neutrino masses:

$$m_\nu \approx -m_D M_\Sigma^{-1} m_D^T \tag{54}$$

- Additional phenomenology due to charged components

Neutrinos in Grand Unified Theories

- GUTs unify SM gauge groups: $SU(3)_C \times SU(2)_L \times U(1)_Y \subset G$
- $SO(10)$ GUT:
  - One generation fits in 16-dimensional spinor
  - Includes right-handed neutrino automatically
  - Predicts seesaw mechanism naturally
- Connecting scales:

$$m_\nu \sim \frac{m_D^2}{M_R} \sim \frac{m_f^2}{M_{GUT}} \tag{55}$$

- Explains hierarchy: $m_\nu \ll m_f$ through ratio of weak to GUT scale
- Leptogenesis: Connection between neutrino masses and baryon asymmetry

Comparison of Neutrino Mass Models

Model	**New Fields**	**Mass Scale (GeV)**	**Predictions**
SS-I	N_R (singlets)	10^{10-15}	$m_\nu \sim \frac{m_D^2}{M_R}$
SS-II	Δ (triplet)	10^{12-14}	$m_\nu \sim \mu \frac{v^2}{M_\Delta^2}$
SS-III	Σ (triplets)	10^{10-15}	$m_\nu \sim \frac{m_D^2}{M_\Sigma}$

Challenges and Open Questions

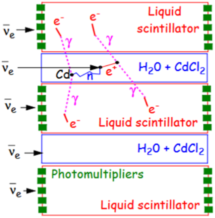
- Fine-tuning issues:
  - Why are Yukawa couplings so small? $Y_\nu < 10^{-12}$
  - Naturalness of neutrino mass scales
- Flavor puzzle:
  - Origin of specific mixing pattern (PMNS matrix)
  - Connection to quark mixing (CKM matrix)
- Testing high-scale models:
  - Neutrinoless double beta decay
  - Lepton flavor violation
  - Correlation with cosmological observables
- Majorana vs. Dirac nature: Fundamental question about neutrinos

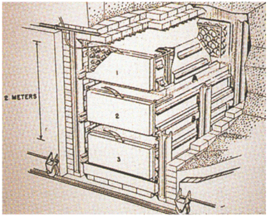
Conclusion: Theoretical Frameworks

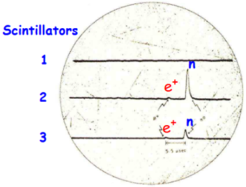
- Neutrino masses require physics beyond the Standard Model
- Multiple theoretical frameworks provide natural explanations:
  - Seesaw mechanisms explain small mass through scale hierarchy
  - Dimension-5 operators as effective description
  - Gauge extensions provide motivation for new particles
  - GUTs connect neutrino physics to fundamental unification
- Key experimental signatures:
  - Neutrinoless double beta decay
  - Lepton flavor violation
  - Connections to dark matter and cosmology
- Neutrino theory connects particle physics, cosmology, and symmetry principles

Discovery of ν_e - Cowan-Reines

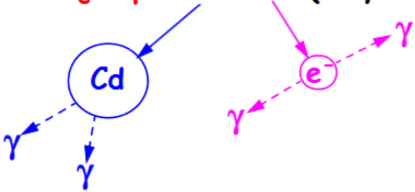
Signal signature: two small signals at the same time from e+ and large signal after few μs from the neutron.





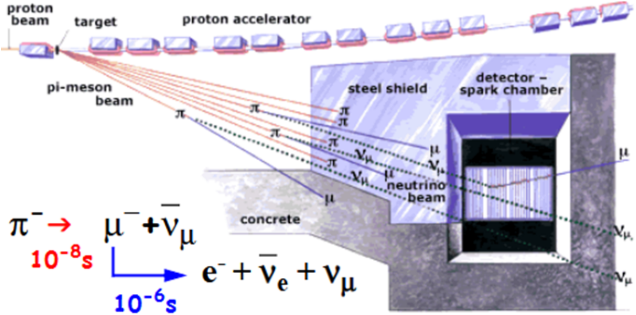


Detection process: $\bar{\nu}_e + p \rightarrow n + e^+$ **(very rare process)**



Discovery of ν_μ - AGS accelerator

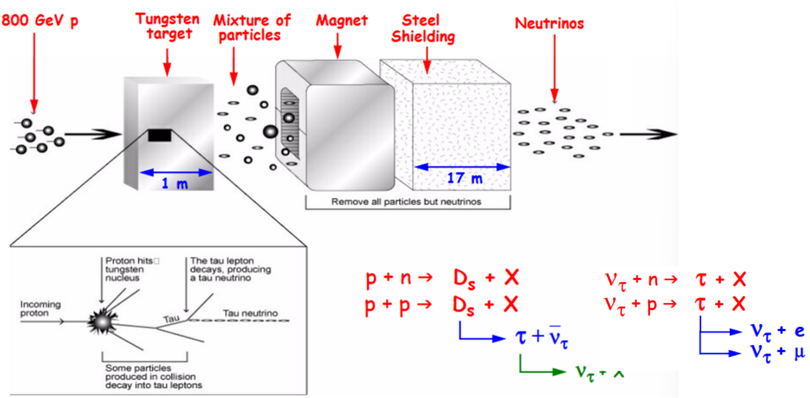
- The neutrinos interacted with the nucleons in the Aluminium and photos of the reaction products were recorded.
- 29 events recorded with muons and none with electrons.



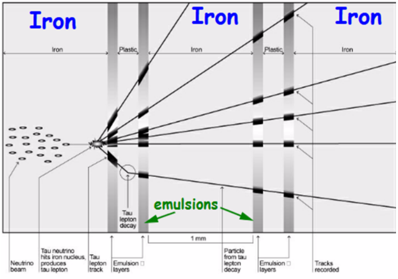
$$\pi^- \xrightarrow{10^{-8}s} \mu^- + \bar{\nu}_\mu$$
$$\mu^- \xrightarrow{10^{-6}s} e^- + \bar{\nu}_e + \nu_\mu$$

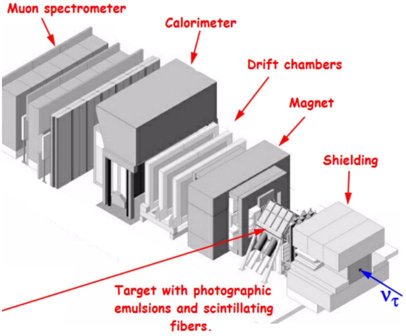
$$\pi^+ \xrightarrow{10^{-8}s} \mu^+ + \nu_\mu$$
$$\mu^+ \xrightarrow{10^{-6}s} e^+ + \nu_e + \bar{\nu}_\mu$$

Discovery of ν_τ - DONUT



Discovery of ν_τ - DONUT

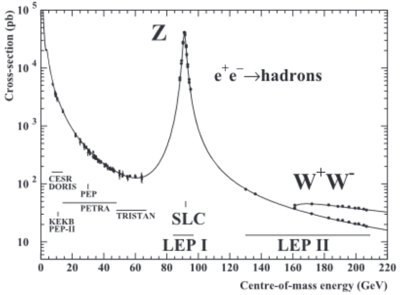


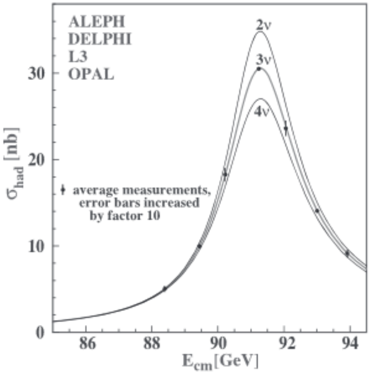


Constraints on Neutrino Number - Z Decay Width

$$\Gamma(inv) = \Gamma_Z - \Gamma(had) - \sum_i \Gamma(\ell_i \bar{\ell}_i) \quad (56)$$

$$\Gamma(inv) = (3 + \Delta N_\nu)\Gamma(\nu\bar{\nu}) \quad (57)$$





Constraints on Neutrino Number - Cosmology

- The energy density of relativistic particles is given by:

$$\rho(T) = \frac{g}{(2\pi)^3} \int dp \frac{4\pi p^3}{\exp(p/T) \mp 1} = \begin{cases} \frac{g\pi^2}{30} T^4 & \text{(Bosons)} \\ \frac{7}{8}\frac{g\pi^2}{30} T^4 & \text{(Fermions)} \end{cases} \tag{58}$$

- We can define the effective number of relativistic degrees of freedom in equilibrium:

$$g_\star^{\text{th}}(T) \equiv \sum_{i\in\text{bosons}} g_i + \frac{7}{8} \sum_{j\in\text{fermions}} g_j \tag{59}$$

- Decoupled species contribute with a different temperature:

$$g_\star^{\text{dec}}(T) \equiv \sum_{i\in\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{j\in\text{fermions}} g_j \left(\frac{T_j}{T}\right)^4 \tag{60}$$

Constraints on Neutrino Number - Cosmology

- The total energy density then takes a simple form:

$$g_\star(T) = g_\star^{\text{th}}(T) + g_\star^{\text{dec}}(T) \tag{61}$$

$$\rho(T) = g_\star(T)\frac{\pi^2}{30}T^4 \tag{62}$$

- We know the particle content of the Standard Model, so we know how $g_\star(T)$ evolves.
- Conservation of entropy gives us temperature evolution

$$T \propto g_{\star S}(T)^{-\frac{1}{3}}a^{-1} \tag{63}$$

- We can calculate the temperature of cosmic neutrinos relative to photons.
- After neutrino decoupling, prior to electron-positron annihilation, we have:

$$g_{\star S}^{\text{th}}(T_+) = 2 + \frac{7}{8}(2+2) = \frac{11}{2} \tag{64}$$

Constraints on Neutrino Number - Cosmology

- After electron-positron annihilation, we have only photons:

$$g_{\star S}^{\mathrm{th}}(T_{-}) = 2 \tag{65}$$

- Entropy conservation gives the temperature ratio after annihilation:

$$T_{\nu} = \left(\frac{g_{\star S}^{\mathrm{th}}(T_{-})}{g_{\star S}^{\mathrm{th}}(T_{+})}\right)^{\frac{1}{3}} T_{\gamma} = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_{\gamma} \tag{66}$$

Constraints on Neutrino Number - Cosmology

type		mass	spin	g
quarks	$t, \bar{t}$	173 GeV	$\frac{1}{2}$	$2 \cdot 2 \cdot 3 = 12$
	$b, \bar{b}$	4 GeV		
	$c, \bar{c}$	1 GeV		
	$s, \bar{s}$	100 MeV		
	$d, \bar{s}$	5 MeV		
	$u, \bar{u}$	2 MeV		
gluons	g_i	0	1	$8 \cdot 2 = 16$
leptons	$\tau^{\pm}$	1777 MeV	$\frac{1}{2}$	$2 \cdot 2 = 4$
	$\mu^{\pm}$	106 MeV		
	$e^{\pm}$	511 keV		
	$\nu_\tau, \bar{\nu}_\tau$	< 0.6 eV	$\frac{1}{2}$	$2 \cdot 1 = 2$
	$\nu_\mu, \bar{\nu}_\mu$	< 0.6 eV		
	$\nu_e, \bar{\nu}_e$	< 0.6 eV		
gauge bosons	W^+	80 GeV	1	3
	W^-	80 GeV		
	Z^0	91 GeV		
	γ	0		2
Higgs boson	H^0	125 GeV	0	1

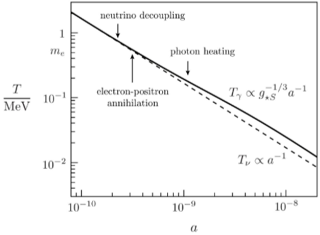
Constraints on Neutrino Number - Cosmology

- Total radiation energy density of the universe

$$\rho_r = \rho_\gamma \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} N_{\text{eff}}\right) \tag{67}$$

- For instantaneous decoupling, N_{eff} counts the number of neutrino species.
- A complete treatment of neutrino transport gives a slightly larger number in the Standard Model.

$$N_{\text{eff}}^{SM} = 3.046 \tag{68}$$



The Short Baseline Anomalies

Different data sets, sensitive to L/E values small enough that the known oscillation frequencies do not have "time" to operate, point to unexpected neutrino behavior. These include:

- $\nu_\mu \to \nu_e$ appearance — LSND, MiniBooNE;
- $\nu_e \to \nu_{\text{other}}$ disappearance — radioactive sources;
- $\bar{\nu}_e \to \bar{\nu}_{\text{other}}$ disappearance — reactor experiments.

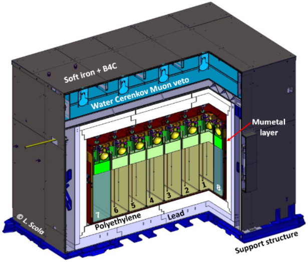
None are entirely convincing, either individually or combined. However, these anomalies are not statistical anomalies and may hint towards a 4th *sterile* neutrino. . . ?

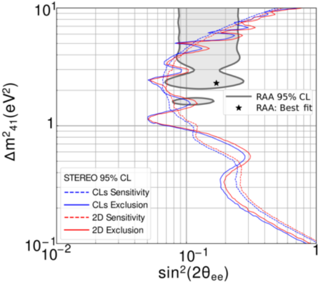
STEREO Experiment

Objective: To investigate the possibility of oscillations into sterile neutrinos

- Detector consists of organic liquid scintillator doped with Gd
- $\bar{\nu}_e + p \to n + e^+$
- e^+ produces scintillating light and neutron produces secondary signal through neutron capture
- 6 detectors measure the energy spectrum of incoming neutrinos to measure oscillation over 2m (expected oscillation wavelength for sterile neutrinos)

Most of the parameter space that could account for the RAA was excluded at a 90% confidence level.





Neutrino Oscillation Experiments: Solar

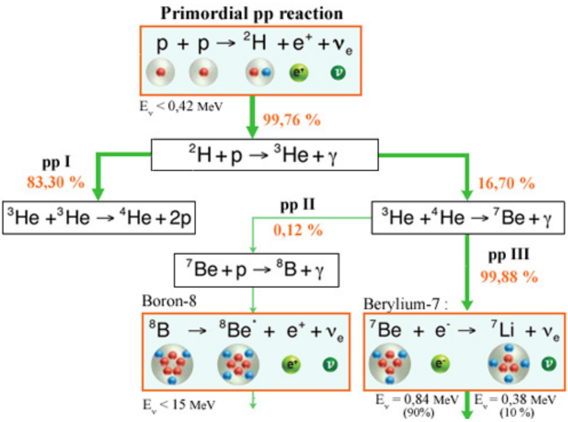


Figure: ν_e producing nuclear reactions

- pp neutrinos are abundant but low energy
- ^7Be and ^8B neutrinos are higher in energy but lower in number. Number sensitive to Solar environment (T&P)

Neutrino Oscillation Experiments: Solar

- **Homestake experiment:** $^{37}Cl + \nu_e \rightarrow e^- + {}^{37}Ar$ (814 keV treshold)
- Only $\sim$ 33% of flux predicted from Standard Solar Model (SSM) was measured
- **Kamiokande Experiments** - Water Cherenkov detectors
- $\nu_x e^- \rightarrow \nu_x e^-$ - sensitive to all neutrinos either through WCC (ν_e) and 1/7 probability of WNC (ν_x) though no unique signatures
- Thus $\sim$ 50% of SSM predicted flux measured
- Possible explanations?
  - Incomplete astrophysical understanding
  - Neutrino Oscillations

Neutrino Oscillation Experiments - Solar

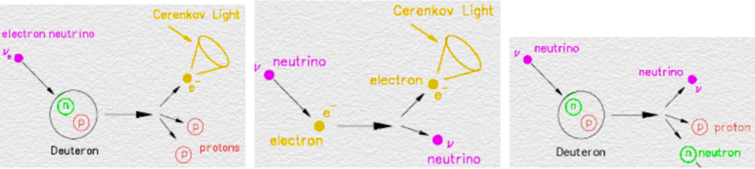
- To verify between the two possibilities, use **Gallium** based experiments (233 keV **threshold**):

$$\nu_e + {}^{71}Ga \rightarrow e^- + {}^{71}Ge$$

  which is sensitive to low energy pp neutrinos.
- If flux measured is
  - $\sim^8 B$ flux, neutrino oscillation
  - $\sim$ SSM predictions, unknown astrophysical effects
- GALLEX, GNO in Italy and SAGE in Russia all measured solar neutrino flux of $\gtrsim 50\%$
- Thus, astrophysical effects ruled out.

Neutrino Oscillation - Solar

- Sudbury Neutrino Observatory used D_2O to finally solve the solar neutrino problem.
- Deuterium is sensitive to all flavors of neutrinos, provides a unique signature for other two flavors.



$$\nu_e + d \rightarrow p + p + e^- \tag{69}$$

$$\nu_x + e^- \rightarrow \nu_x + e^- \tag{70}$$

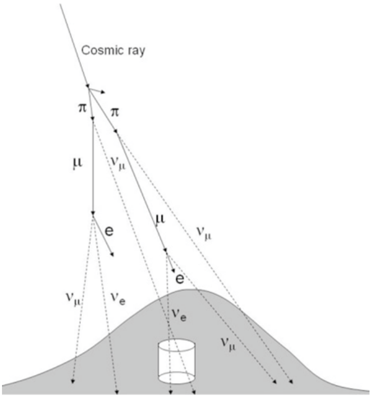
$$\nu_x + d \rightarrow p + n + \nu_x \tag{71}$$

Neutrino Oscillation - Atmospheric

- Primary cosmic rays (p^+) give rise to **pions** and muons in the upper atmosphere.

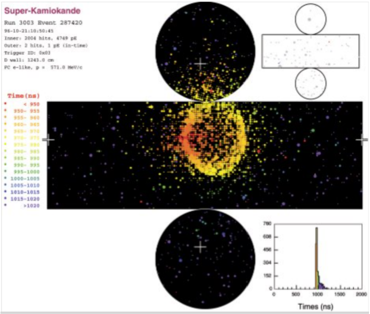
$$\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$$

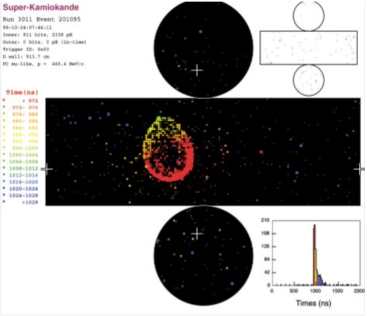
- Oscillations were unambiguously proved by measuring ν_μ/ν_e flux (expected value ~ 2, measured ≈ 1.2) and Zenith angle distribution asymmetry.



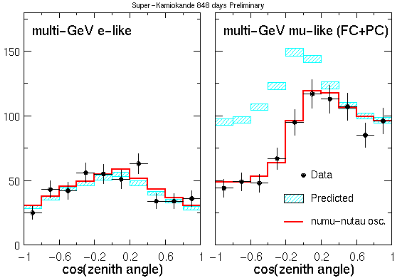
Neutrino Oscillation - Atmospheric

Super Kamiokande Experiment - Pattern recognition algorithm distinguishes between ν_e and ν_μ





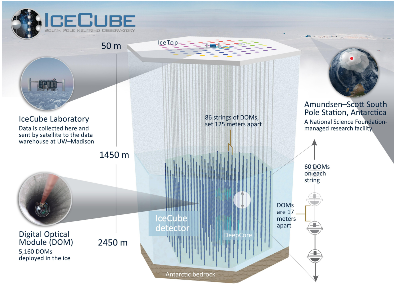
Neutrino Oscillation - Atmospheric



- Huge deficit in upward going multi-GeV ν_μ, around 6σ deviation from non-oscillating predictions.
- No such asymmetry observed for multi-GeV ν_e, even after considering $\nu_\mu \to \nu_e$ oscillations.
- Thus, ν_μ must be oscillating into a third flavor of neutrinos.
- Also, discrepancy between number of events from upward going muons did not match Monte-Carlo simulations.

IceCube Neutrino Observatory

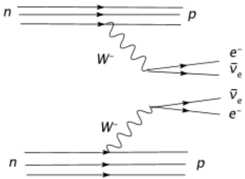
- Works on the basis of Cherenkov Radiation
- Glacial ice of volume 1 m^3 forms the passive detector volume
- DOM (Digital Optical Module) tubes are inserted into the ice and form a sensitive volume between the depths of 1450 and 2450 m

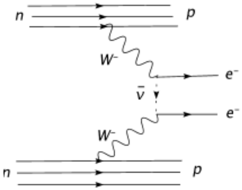


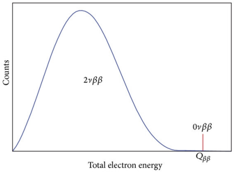
- IceTop: a surface detector array. a companion to the IceCube consisting of 162 ice-filled tanks, each with two optical sensors, used to detect cosmic ray air showers, and to serve as a veto and calibration detector for IceCube

The Neutrinos Anti-Particle

- Searching for Majorana neutrinos in double β-decay
  - For some radioactive isotopes, the single β-decay process is forbidden. In this case, it can be possible to see double β-decays.







Experiments like KamLAND-ZEN and NEMO-3 search for these decays. However, no conclusive signals of such decays have been found yet.

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