Low-Energy High-Precision Experiments

Standard Model

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Non-Newtonian Gravitational Interactions: Theory

- Newton's law of gravity (at large scale)

$$\mathrm{F} = \mathrm{G}\frac{\mathrm{m_1 m_2}}{\mathrm{r}^2}$$

$$\mathrm{V} = -\mathrm{G}\frac{\mathrm{m_1 m_2}}{\mathrm{r}}$$

where $\mathrm{G} = 6.6743 \cdot 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$

- Yukawa-like modified potential (at small scale)

$$\mathrm{V} = -\mathrm{G}\frac{\mathrm{m_1 m_2}}{\mathrm{r}}(1 + \alpha \cdot \mathrm{e}^{-\mathrm{r}/\lambda})$$

where, α = Strength Factor
λ = Yukawa distance

Non-Newtonian Gravitational Interactions: Theory

- To learn more about α and λ we need to probe the small distance gravitational interaction of particles.
- We need a particle which
  - Heavy
  - Long-lived
  - Neutral
- Neutron seems to be the perfect candidate.

Non-Newtonian Gravitational Interactions: Theory

Consider a neutron kept on top of a mirror with mass density ρ_m in the presence of a gravitational field (g).

Since the distance we are probing is much smaller than the size of mirror we can approximate it with an infinite plane.

Force on neutron due to earth is:

$$F_E = G\frac{m_E m_n}{R_E^2} = \frac{4}{3}\pi G \rho_E R_E m_n$$

Force on neutron due to the mirror:

$$F_m = 2\pi\rho_m \alpha\lambda G \cdot e^{-z/\lambda} m_n$$

Non-Newtonian Gravitational Interactions: Theory

Thus the gravitational acceleration (g) of the neutron is given by:

$$\mathrm{g} = \mathrm{g_E} + \mathrm{g_m}$$

$$\frac{\mathrm{g_m}}{\mathrm{g_E}} = \frac{3}{2}\alpha \cdot \frac{\lambda}{\mathrm{R_E}} \cdot \mathrm{e}^{-\mathrm{z}/\lambda}$$

$$\implies \frac{\mathrm{g}-\mathrm{g_E}}{\mathrm{g_E}} = \frac{3}{2}\alpha \cdot \frac{\lambda}{\mathrm{R_E}} \cdot \mathrm{e}^{-\mathrm{z}/\lambda}$$

Thus by measuring the value of g, we can put constraints on the value of α and λ

Non-Newtonian Gravitational Interactions: Theory

In classical physics, we measure by directly calculating the acceleration of a ball falling under gravity.

However, neutrons do not follow classical physics. We need quantum physics.

$$(-\frac{\hbar^2}{2m}\nabla^2 + V(z))\psi = E\psi$$

where,

$$V(z) = mgz \text{ for } z > 0$$
$$V(z) = \infty \text{ for } z < 0$$

Non-Newtonian Gravitational Interactions: Theory

The (un-normalized) solution to the Schrodinger equation is:

$$\psi_n(z) = \mathrm{Ai}(\frac{z}{z_0} - \frac{z_n}{z_0})$$

where,

$$z_n = z_0(\frac{3\pi}{2}(n - \frac{1}{4}))^{2/3}$$

$$z_0 = (\frac{\hbar^2}{2m^2 g})^{1/3}$$

$$E_n = mgz_n$$

Finding the value of z_n for various n will give us g.

Non-Newtonian Gravitational Interactions

Non-Newtonian Gravitational Interactions

- The Institut Laue Langevin (ILL), Grenoble, France

Non-Newtonian Gravitational Interactions: Experiment

- The ground state energy of the neutron is of the order of a few peV.
- Neutron mirrors make use of the strong interaction between nuclei and an ultracold neutron (UCN).
- We want the roughness of the mirror to be less than the de-Broglie wavelength of the neutron.
- For UCN the electromagnetic interaction with the mirror surface is of the order of 10^{-25}eV which is sub-gravitational and can be neglected.

Non-Newtonian Gravitational Interactions: Experiment

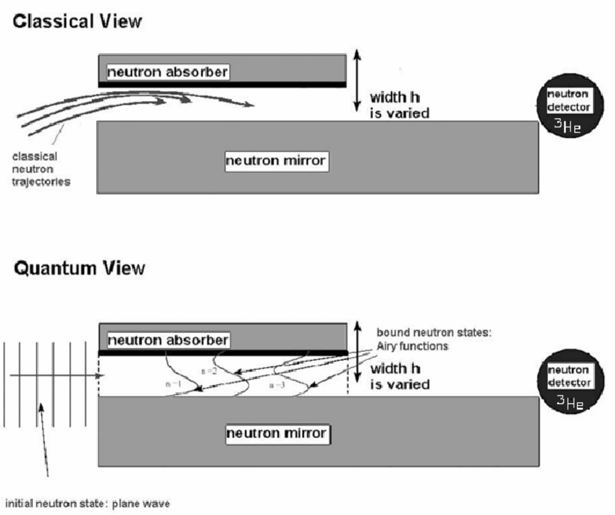
Table 1. from hot to ultracold: neutrons at the ILL

	fission neutrons	thermal neutrons	cold neutrons	ultracold neutrons	this experiment
Energy	2 MeV	25 meV	3 meV	100 neV	1.4 peV
Temperature	10^{10} K	300 K	40 K	1 mK	-
Velocity	10^7 m/s	2200 m/s	800 m/s	5 m/s	$v_\perp \sim 2$ cm/s

Non-Newtonian Gravitational Interactions: Experiment

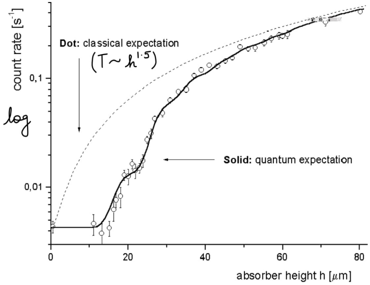
- We will be probing the scale where $3\mu\text{m} < \lambda < 10\mu\text{m}$
- The mirror has a roughness of about 2.2 ± 0.2 nm ($<< 50$ nm $<< 10\mu$m).
- We shall be using an absorber placed above the mirror to remove neutrons with high transverse energy.
- The absorber is made up of a rough glass ($\sigma = 0.75\mu$m) coated with Gd-Ti-Zr alloy.

Non-Newtonian Gravitational Interactions: Experiment



Non-Newtonian Gravitational Interactions: Experiment

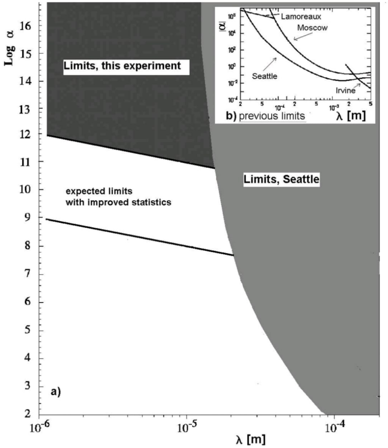
- A ^{3}He counter is used to measure the total neutron transmission (T).
- The absorber height is varied and T is measured as a function of h.



- No neutrons are observed below the height of 15μm

Non-Newtonian Gravitational Interactions: Experiment

- The results of the fit of potential to the measured data yield prediction for 90% confidence level exclusion bounds on α and λ
- The limit for α at $\lambda = 1\mu$m is 10^{11} and at $\lambda = 10\mu$m it is 10^{12}.



Non-Newtonian Gravitational Interactions: Future

- The neutron absorber used here has an absorption efficiency of 93% which can be improved in future to get stinger bounds on α and λ
- The limits can be further improved by an enhanced setup and improved statistics by new UCN sources as a Monte Carlo simulation shows.

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Pion Properties and Role in Strong Interaction

- Pion are the lightest Hadrons(~135-139 Mev/c^2) composed of u and d quark-antiquark pairs.
- They exist in three forms :π^+, π^0, π^-due to near mass -degeneracy of u and d quarks
- Act as effective low energy carriers of the strong nuclear force,originally proposed By Yukava.
- Their lightness is due to Spontaneous Chiral Symmetry Breaking
- Part of the pseudo-scalar meson octet along with kaons and η mesons

Pions in the Standard Model and Effective Theories

- Strong Interactions are governed by Quantum Chromodynamics (QCD),a Theory Of quarks and gluons
- At low energies,QCD becomes non perturbative -Mesons and Baryons are the observable degree of freedom
- Pions Play a Central role in Chiral Perturbation Theory ,an effective field theory built on QCD symmetries
- Light quark masses (u,d) are fundamental SM parameters but require indirect determinations
- Precision lattice QCD computations now allow accurate estimates of these masses, testing the SM at low energies

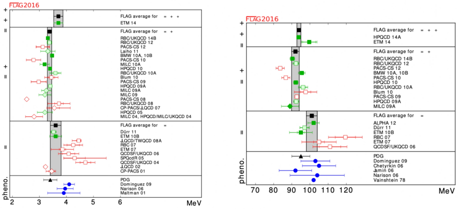


Figure 2: Some mass determinations from FLAG [20] of the average lightest quark masses and the s-quark mass. These are the mass parameters quoted at $\mu = 2$ GeV.

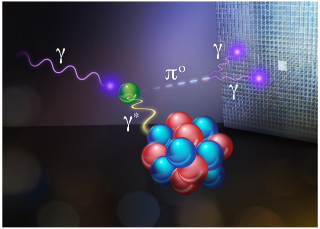
Chiral Perturbation Theory Meets Lattice QCD

- Chiral Perturbation Theory (CHPT) provides a low-energy effective framework for QCD
- To make precise predictions ,CHPT must be matched with lattice QCD calculations in a controlled manner
- Calculations of pion ,kaon, and eta masses and decay constants were completed upto two loop order nearly two decades ago
- However , some two loop (sunset) diagrams could only be evaluated numerically,limiting full analytic control

Mellin -Barnes Method for analytic Expansions

- Recent progress uses the mellin-Barnes technique to express problematic diagrams as double series expansion
- These expansions are functions of the mass ratios of the pseudoscalar mesons (π, η, K).
- This approach enables a Systematic and Controlled Comparision of Theoretical predictions with lattice data
- It improves analytic access to loop contributions and enhances precision in CHPT predictions .

The Primakoff Effect



The Neutral Pion Lifetime and Chiral Anamoly

- The π^0 lifetime is classic test of the chiral anomaly in the light-flavour sector of QCD
- It is governed almost entirely by :
    - The Charged pion decay Constant Fπ
    - The fine -structure constant α
    - The neutral pion mass M$_{\pi^0}$
- The decay width (from anomaly) is given By :

$$\Gamma(\pi^0 \to 2\gamma) = \frac{M_{\pi^0}^4 \alpha^2 F_\pi^2}{64\pi^3}$$

$$\Gamma(\pi^0 \to 2\gamma) = 7.760 \mathrm{eV} \mathrm{with} \tau = \frac{1}{\Gamma} = 8.38 \times 10^{-17} \mathrm{s}\,.$$

Isospin Violation and Corrections to π^0 Lifetime

- Isospin Violation especially $\pi^0 - \eta$ mixing ,leads to corrections in the predicted π^0 lifetime
- They are critical to achieving agreement between Theory and Experiment at high precision
- The correction account for difference in quark masses ($m_u \neq m_d$)and Electromagnetic effects .

Precision Measurements from the Primex Experiment

- The Primex Experiment at JLab measured the π^0 lifetime using the Primakoff process

  X-rays collide with nuclei, inducing π^0decay into two photons

- Result

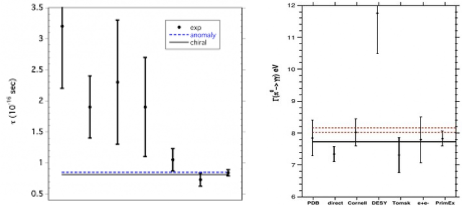
  Γ = 7.82 ±0.14(stat.) ±0.17(syst.) eV

In excellent agreement with corrected theoretical predictions

- 2018 Review of particle Properties lists the lifetime as:

  τ = (8.52±0.18)×10−17 s.

- Confirms the chiral anomaly prediction to high precision -remarkable success for QCD at low energies



The left panel shows early measurements of the neutral pion lifetime in comparison to the anomaly prediction and the leading chiral corrections due to pseudoscalar meson mixing . The right panel presents the preliminary PrimEx results, which were shared by Aron during his 2009 Chiral Dynamics talk.

Scattering Amplitudes and Partial Waves

- **Scattering Amplitude**: Measures the strength of interaction between particles. It is a complex function of energy, with real and imaginary parts.
- **Unitarity**: Ensures total probability conservation during the scattering process, meaning the sum of all possible scattering outcomes equals 1.
- **Partial Waves**: Scattering is decomposed into waves with different angular momenta (S-wave, P-wave, etc.).
- At **low energies**, the **S-wave** dominates.
- The **scattering length** is the main parameter for S-wave, which provides insight into the strength of interactions at low momentum transfer.

Pion -Pion Scattering and Predictions

- **Pions and Isospin**: Pions(π^+, π^0, π^-)**form an iso-triplet in isospin space, and scattering amplitudes can carry isospin 0, 1, or 2.**
- **Weinberg's Prediction (Leading Order)**: Pion-pion scattering length a_0^0 was predicted as 0.16, later revised to 0.20 ± 0.01 by Gasser and Leutwyler.
- This prediction stayed stable even at next-to-next-to-leading order.
- In the context of pion-pion scattering, a system of dispersion relations was established, revealing the presence of certain unknown functions of the momentum transfer, which limited the effectiveness of these relations.

- **Roy Equations :**

Roy's work provided a much more practical and precise framework for analyzing pion-pion scattering. By reducing the problem to just the two scattering lengths, he was able to avoid dealing with the unknown functions that had previously complicated the analysis. This was a significant advancement in the study of hadronic interactions, and it provided deeper insights into the nature of strong force interactions.

- The method used to determine pion scattering phase shifts became known as **Roy equation analysis**.

- This technique combined phase shift information from the rare **K_{l4} decay** with Roy's equations to enhance the understanding of pion scattering.

- Using data from **30,000 events** in the **Geneva-Saclay experiment**, the analysis determined the scattering length **a_{00}** to a precise value of **0.26 ± 0.05**.

NA48 Experiment and Scattering Length

- **NA48 Collaboration at CERN**:
  The experiment uses CERN's high-intensity proton beamline and a large detector.
  It provides two independent measurements of scattering lengths related to pion-pion scattering.
- **First Measurement**:
  Based on the **cusp effect** in the invariant mass distribution of pions from **kaon decay**.
  A **cusp** appears at energy $2m_\pi$ showing as an abrupt change in the derivative of the distribution.
- Analysis of **27 million events** ($K^\pm \to \pi^\pm\pi^0\pi^0$) resulted in a value of $|a_0^0\text{-}a_0^2|$ **= 0.264 ± 0.015**.
- **Second Measurement**:
  Based on a rare kaon decay involving lepton pairs and two pions.
  The analysis of **370,000 decays** provided a preliminary value of a_0^0 **= 0.256 ± 0.011**.
- **Other Notable Experiment**:
  The **E865 Experiment** at Brookhaven measured a_{00} as **0.216 ± 0.015**.

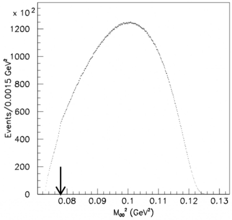
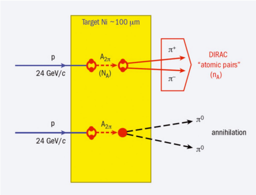


Figure 2. $M^2_{\pi^0\pi^0}$ mass distribution in $K^\pm \to \pi^\pm\pi^0\pi^0$. The arrow indicated the cusp position at $2m_{\pi\pi}$.

DIRAC Experiment and Pionioum Atom

DIRAC Experiment:

- Uses **pionium atoms** (charged pions bound electromagnetically) that scatter into neutral pions.
- The lifetime of pionium atoms provides an accurate measurement of the scattering length difference.
- **6,600 pionium atoms** were studied,**The lifetime of the ground state was measured to be approximately 2 femtoseconds (fs).**
- The result for the scattering length difference was **0.264**
- The pionium binding and lifetime were predicted over 50 years ago by **S. Deser, M. L. Goldberger, K. Baumann, and W. E. Thirring**.



Pion Beta Decay and Cabibbo-Kobayashi-Maskawa Unitarity

CKM Unitarity

- **CKM Matrix**: A 3x3 matrix describing quark mixing in the Standard Model (SM).
- **First row unitarity :**

$$\left|V_{ud}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}\right|^2 = 1$$

Neglecting $|V_{ub}|^2 \approx 2 \times 10^{-5}$,approximately the original Cabibbo two generational Relationship

$$\left|V_{ud}\right|^2 + \left|V_{us}\right|^2 = 1$$

- Dispersion Relation approach to super allowed beta decay loop effects gives radiative correction from 0.97420(21) to

$$|V_{ud}| = 0.97370(10)(10)$$

- A CMS calculation gives $V_{ud} = 0.97389(10)$
- From K_{l3} $(K \to \pi\ell\nu)$ and $K_{l2}(K \to \ell^{\pm}\nu)$ decays
    - $V_{us} = 0.2245(8)$

- This approaches led to roughly 3 and 2σ deviations from unitarity respectively.
- K_{l3} decays alone give a relatively small

$$V_{us} = 0.2234(8)$$

In contrast, the ratio R_A is considered a more dependable constraint, as common uncertainties in kaon and pion decays tend to cancel. This is particularly important for lattice gauge theory input calculations of $\frac{f_{K^+}}{f_{\pi^+}}$

- $$R_A = \frac{\Gamma(\pi \to \mu\nu(\gamma))}{\Gamma(K \to \mu\nu(\gamma))}$$
- From Experimental Constraint :

$$R_A = 1.3367(8)$$

- From this ,one finds. $$\frac{|V_{us}| f_{K^+}}{|V_{ud}| f_{\pi^+}} = 0.2760(4)$$
- Using the lattice value
  $\frac{f_{K^+}}{f_{\pi^+}} = 1.1932(19)$, one obtains

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23131(45)$$

- Agreement with Three Generation unitarity requires
  $|V_{ud}| = 0.97428(10) and |V_{us}| = 0.2253(4)$
- SM expectations are 2 or more σ different from some of the current values as shown here

Pion Beta Decay

- Branching ratio for $\pi^+ \to \pi^0 e^+ \nu(\gamma)$:

  $1.038(6) \times 10^{-8}$(updated upto +0.2% for normalization)

- Decay rate Calculations:

  $\Gamma(\pi^+ \to \pi^0 e^+ \nu(\gamma)) = 0.3988(23)\mathrm{s}^{-1}$

- **SM Theoretical Prediction:**

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu(\gamma)) = \frac{G_\mu^2 \,|\, V_{ud} \,|^2 m_{\pi^+}^5 \,|\, f_{\pi^+}(0) \,|^2}{64\pi^3} \left(1 + RC_\pi\right) I_\pi$$

$RC_\pi = 0.0334$ -The Radiative Correction

Electroweak correction: 0.0234

QED correction: 0.010(1)

- **Vud Value from Pion Beta Decay:** $\left| V_{ud} \right| = 0.9739(29)$

K_L(3e) Decay : Theory snd Importance

- The specific Ratio Rv:

$$R_v = \frac{\Gamma(K_L \to \pi^{\pm} e^{\mp} \nu(\gamma))}{\Gamma(\pi^{+} \to \pi^{0} e^{+} \nu(\gamma))}$$

- K_L(3e) decay is crucial for determining the Ckm matrix element V_{us}, a key parameter in particle physics

Decay Formula :

$$\Gamma(K_L \to \pi^{\mp} e^{\pm} \nu(\gamma)) = \frac{G_{\mu}^2 \, | \, V_{us} \, |^2 \, m_{K_L}^5 \, | \, f_K^{(0)^2} \, |}{192\pi^3} \left(1 + RC_K\right) I_K$$

Radiative corrections and Phase Factors

- Radiative correction :Short distance electroweak and QED radiative corrections mostly cancel in the ratio Rv.

  $$1 + R_{CK} \approx 1 + R_{CK} - R_{C\pi} = 1.000(2)$$

  The cancellation minimizes errors and simplifies extraction of |Vus|

- The pion energy dependence in the K_l rest frame is crucial for the calculation of phase space factor I_k

  Maximum Pion energy $E_m = 0.26838 GeV$

  Phase Space factor $\beta_m = \sqrt{1 - \left(\frac{m_{\pi^+}}{E_m}\right)^2}$

- The current experimental Value

$$R_{\text{exp}}(V) = \frac{\tau_\pi \times \mathrm{BR}(K_L \to \pi^\pm e^\mp \nu(\gamma))}{\tau_{K_L} \times \mathrm{BR}(\pi^+ \to \pi^0 e^+ \nu(\gamma))} = \frac{26.033(5)\mathrm{ns} \times 0.4056(9)}{51.16(21)\mathrm{ns} \times 1.038(6) \times 10^{-8}} = 1.9884(115)(93) \times 10^7$$

- The Theoretical value with SM prediction.

$$R_{\text{theory}}(V) = \frac{1}{3}\frac{m_{K^0}}{m_{\pi^+}}\frac{f_K^+(0)}{f_\pi^+(0)}\left(\frac{V_{us}}{V_{ud}}\right) \times \frac{I_K}{I_\pi} \times 1.000(2)$$

- Equating The Theory and experiment leads to

$$\frac{f_K^+(0)}{f_\pi^+(0)}\frac{V_{us}}{V_{ud}} = 0.22220(64)(58)$$

- Equating Rv experiment and theory followed by weighted averaging over all k_{l3}modes, allowing for correlated uncertainties as shown in table ,It leads to the average

$$\frac{f_K^+(0)V_{us}}{f_\pi^+(0)V_{ud}} = 0.22223(64)(40)$$

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Decay	$\frac{f_+^K(0)\lvert V_{\mathrm{us}}\rvert}{f_+^\pi(0)\lvert V_{\mathrm{ud}}\rvert}$	$\frac{f_+^K(0)}{f_+^\pi(0)}$
$K_L(e3)$	0.22220(64)(58)	0.9606(28)(19)(25)
$K_L(\mu 3)$	0.22250(64)(64)	0.9619(28)(19)(28)
$K_S(e3)$	0.22138(64)(134)	0.9571(28)(19)(52)
$K^\pm(e3)$	0.22220(64)(86)	0.9606(28)(19)(37)
$K^\pm(\mu 3)$	0.22200(64)(111)	0.9602(28)(19)(48)
Average	0.22223(64)(40)	0.9607(28)(19)(18)

TABLE I. K_{l3} results from five decay modes with approximate errors, weighted average (including some correlated theory uncertainties) for $f_+^K(0)\lvert V_{\mathrm{us}}\rvert/f_+^\pi(0)\lvert V_{\mathrm{ud}}\rvert$, [18, 20] based in part on the updated results in [19, 25]. Also shown are the individual $f_+^K(0)/f_+^\pi(0)$ values and their average for $\lvert V_{\mathrm{us}}\rvert/\lvert V_{\mathrm{ud}}\rvert = 0.23131(45)$ (see eq. (10)).

- For 2+1+1 quark flavours ,the discrepancy is about 2.2σ
- This discrepancy can also be illustrated by inserting :
  - $\frac{f_\pi^+(0)}{f_K^+(0)} = 0.970(2)$
- Leading to :

$$\frac{V_{ud}}{V_{us}} = 0.22910(91)$$

- If K_l(e3) was used ,the 2.2σ difference would have been 2.0σ
- Averaging over five decay modes demonstrates the consistency of experimental and theoretical inputs
- A potential explanation for the discrepancy is the need for an additional roughly -0.01 electromagnetic radiative corrections
- Such a correction would increase the control value of Vus to 0.2253 ,aligning with R_A and Unitarity

- Final Observations :

$$\frac{V_{ud}}{V_{us}} = 0.22910(102)$$

Suggested from Rv and lattice implies;

Vud=0.97474(22)

- This exceeds Three generational unitarity Vud value ,making it difficult to reconcile with the current super allowed nuclear beta decay discrepency…

References

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