

# 10.5.4

EE23BTECH11029 - Kanishk

## Question:

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of  $x$  such that the sum of the numbers of the houses preceding the house numbered  $x$  is equal to the sum of the numbers of the houses following it. Find this value of  $x$ .

Hint:  $S_{x-1} = S_{49} - S_x$

## Solution:

Parameter	Value	Description
$x(0)$	1	First house
$d$	1	Common difference
$x(n)$	$(n+1)u(n)$	$(n+1)th$ house
$y(n)$	$\left(\frac{n+1}{2}\right)(n+2)u(n)$	Sum of $n+1$ number of houses.
$x_2(n)$	$(49-n)u(n)$	$(n+1)th$ house from last house
$y_2(n)$	$\left[49n - \left(\frac{n}{2}\right)(n+1)\right]u(n)$	Sum of $n+1$ houses from last house.

TABLE 0: Input Parameters

For an AP:

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (1)$$

$$\Rightarrow X(z) = \frac{1}{1 - z^{-1}} + \frac{z^{-1}}{(1 - z^{-1})^2} \quad (2)$$

$$= \frac{1}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$\therefore y(n) = \frac{(n+1)}{2}(n+2) \quad (4)$$

$$y(x-2) = y(n-1) - y(x-1) \quad (5)$$

From Table ??:

$$\left(\frac{x-1}{2}\right)x = \frac{n}{2}(n+1) - \frac{x}{2}(x+1) \quad (6)$$

$$(x-1) + x(x+1) = n(n+1) \quad (7)$$

$$2x^2 = n(n+1) \quad (8)$$

$$x = \sqrt{\frac{n}{2}(n+1)} \quad (9)$$

$$x = 35 \quad (10)$$

### Result Confirmation:

To prove:

$$y(33) = y_2(13) \quad (11)$$

**LHS:**

$$y(n) = x(n) * u(n) \quad (12)$$

$$\Rightarrow Y(z) = X(z) \times U(z) \quad (13)$$

$$Y(z) = \left(\frac{1}{(1-z^{-1})^2}\right) \left(\frac{1}{1-z^{-1}}\right) \quad (14)$$

$$= \frac{1}{(1-z^{-1})^3}, \quad |z| > 1 \quad (15)$$

$$(16)$$

Using Contour Integration to find inverse Z-transform,

$$y(33) = \frac{1}{2\pi j} \oint_C Y(z) z^{32} dz \quad (17)$$

$$= \frac{1}{2\pi j} \oint_C \frac{z^{32}}{(1-z^{-1})^3} dz \quad (18)$$

We can observe that the pole is repeated 3 times and thus  $m = 3$ ,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (19)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{35}) \quad (20)$$

$$= 595 \quad (21)$$

**RHS:**

From Table ??:

$$X_2(z) = \frac{49 - 50z^{-1}}{(1 - z^{-1})^2} \quad (22)$$

$$y_2(n) = x_2(n) * u(n) \quad (23)$$

$$\Rightarrow Y_2(z) = X_2(z) \times U(z) \quad (24)$$

$$y_2(z) = \frac{49 - 50z^{-1}}{(1 - z^{-1})^3} \quad (25)$$

$$(26)$$

Using Contour Integration to find inverse Z-transform,

$$y_2(13) = \frac{1}{2\pi j} \oint_C Y(z) z^{12} dz \quad (27)$$

$$= \frac{1}{2\pi j} \oint_C \frac{49 - 50z^{-1}}{(1 - z^{-1})^3} (z^{12}) dz \quad (28)$$

We can observe that the pole is repeated 3 times and thus  $m = 3$ ,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (29)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (49z^{15} - 50z^{14}) \quad (30)$$

$$= 49.15.14 - 50.14.13 \quad (31)$$

$$= 595 \quad (32)$$

$$LHS = RHS$$

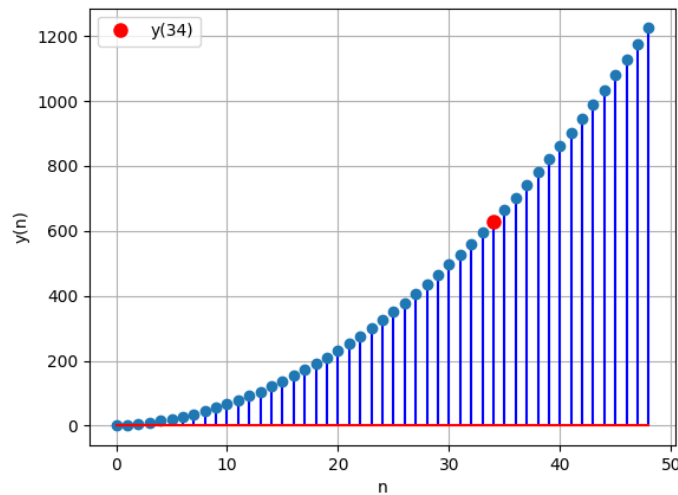


Fig. 0: Plot  $y(n)$  vs  $n$