10.5.4

EE23BTECH11029 - Kanishk

Question:
The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered is equal to the sum of the numbers of the houses following it. Find this value of x.
Hint:$S_{x-1} = S_{49} - S_x$
Solution:

Parameter	Value	Description
$x(0)$	1	First house
d	1	Common difference
$x(n)$	$(n+1)u(n)$	$(n+1)th$ house
$X(z)$	$\frac{1}{(1-z^{-1})^2}$	Z-transform of $x(n)$
$y(n)$	$\frac{n+1}{2}(n+2)u(n)$	Sum of $n+1$ number of houses.
$Y(z)$	$\frac{1}{(1-z^{-1})^3}$	Z-transform of $y(n)$

TABLE 0: Input Parameters

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{1}$$

$$\implies X(z) = \frac{1}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2} \tag{2}$$

$$= \frac{1}{(1-z^{-1})^2}, \quad |z| > 1 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$\implies Y(z) = X(z) \times U(z) \tag{5}$$

$$Y(z) = \frac{1}{(1-z^{-1})^3}, \quad |z| > 1 \tag{6}$$

Contour Integration to find inverse of Z-transform,

$$y(n) = \frac{1}{2\pi j}\oint_c Y(z) z^{n-1}\, dz \tag{7}$$

$$= \frac{1}{2\pi j}\oint_c \frac{1}{(1-z^{-1})^3} z^{n-1}\, dz \tag{8}$$

We can observe that Pole is repeated 3 times thus $m = 3$,

$$R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{9}$$

$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{z^{n+2}}{(z-1)^3}\right) \tag{10}$$

$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left(z^{n+2}\right) \tag{11}$$

$$= \frac{n+1}{2}(n+2) \tag{12}$$

$$\therefore y(n) = \frac{n+1}{2}(n+2) \tag{13}$$

$$y(x-2) = y(48) - y(x-1) \tag{14}$$

From Table 0:

$$\left(\frac{x-1}{2}\right)x = \frac{49}{2}(50) - \frac{x}{2}(x+1) \tag{15}$$

$$(x-1) = 49 \times 50 - x(x+1) \tag{16}$$

$$2x^2 = 49 \times 50 \tag{17}$$

$$x^2 = 49 \times 50 \tag{18}$$

$$x = 35 \tag{19}$$

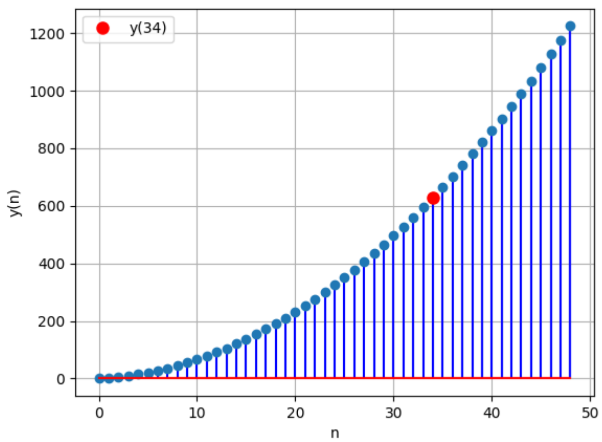

Fig. 0: Plot y(n) vs n