

10.5.4

EE23BTECH11029 - Kanishk

Question:

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x .

Hint: $S_{x-1} = S_{49} - S_x$

Solution:

Parameter	Value	Description
$x(0)$	1	First house
d	1	Common difference
$x(n)$	$[n+1]u(n)$	$(n+1)th$ house

TABLE 0: Input Parameters

For an AP:

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \quad (1)$$

$$\Rightarrow X(z) = \frac{1}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2} \quad (2)$$

$$= \frac{1}{(1-z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$\Rightarrow Y(z) = X(z) \times U(z) \quad (5)$$

$$Y(z) = \frac{1}{(1-z^{-1})^3}, \quad |z| > 1 \quad (6)$$

Contour Integration to find inverse of Z-transform,

$$y(n) = \frac{1}{2\pi j} \oint_c Y(z) z^{n-1} dz \quad (7)$$

$$= \frac{1}{2\pi j} \oint_c \frac{1}{(1-z^{-1})^3} z^{n-1} dz \quad (8)$$

We can observe that Pole is repeated 3 times thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (9)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{z^{n+2}}{(z-1)^3} \right) \quad (10)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{n+2}) \quad (11)$$

$$= \frac{n+1}{2} (n+2) \quad (12)$$

$$\therefore y(n) = \frac{n+1}{2} (n+2) \quad (13)$$

Parameter	Value	Description
$y(48)$	$\frac{49}{2} (50)$	Sum of 49 terms
$y(x-2)$	$\left(\frac{x-1}{2}\right)x$	Sum of $x-1$ terms
$y(x-1)$	$\frac{x}{2} (x+1)$	Sum of x terms.

TABLE 0: Terms of $y(n)$

$$y(x-2) = y(48) - y(x-1) \quad (14)$$

$$\text{From Table 0 :} \quad (15)$$

$$\left(\frac{x-1}{2}\right)x = \frac{49}{2} (50) - \frac{x}{2} (x+1) \quad (16)$$

$$(x-1) = 49 \times 50 - x(x+1) \quad (17)$$

$$2x^2 = 49 \times 50 \quad (18)$$

$$x^2 = 49 \times 50 \quad (19)$$

$$x = 35 \quad (20)$$

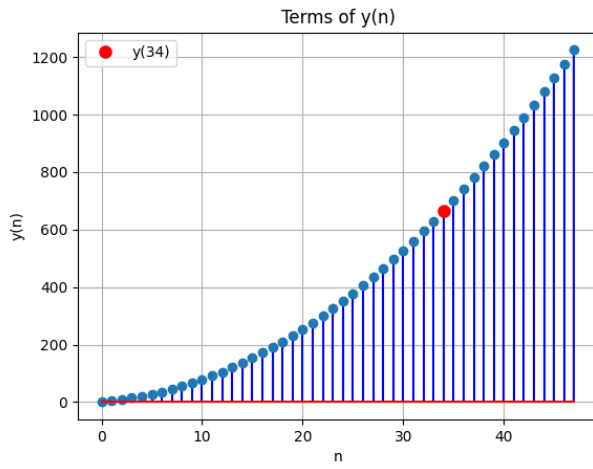


Fig. 0: Plot $y(n)$ vs n