10.5.4

EE23BTECH11029 - Kanishk

Question:

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered is equal to the sum of the numbers of the houses following it. Find this value of x.

Hint:$S_{x-1} = S_{49} - S_x$

Solution:

$$\left(\frac{x-1}{2}\right)x = \frac{n}{2}(n+1) - \frac{x}{2}(x+1) \quad (6)$$

$$(x-1) + x(x+1) = n(n+1) \quad (7)$$

$$2x^2 = n(n+1) \quad (8)$$

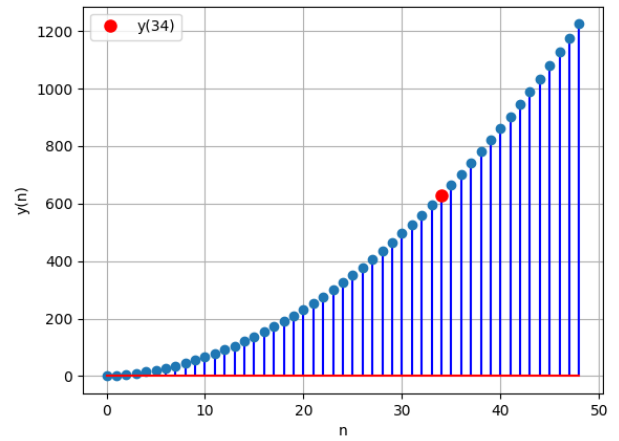$$x = \sqrt{\frac{n}{2}(n+1)} \quad (9)$$

Parameter	Value	Description
$x(0)$	1	First house
d	1	Common difference
$x(n)$	$(n+1)u(n)$	$(n+1)th$ house
$y(n)$	$\left(\frac{n+1}{2}\right)(n+2)u(n)$	Sum of $n+1$ number of houses.

TABLE 0: Input Parameters

Fig. 0: Plot y(n) vs n

For an AP:

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \quad (1)$$

$$\implies X(z) = \frac{1}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2} \quad (2)$$

$$= \frac{1}{(1-z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$\therefore y(n) = \frac{n+1}{2}(n+2) \quad (4)$$

$$y(x-2) = y(n-1) - y(x-1) \quad (5)$$

From Table 0: