

10.5.4

EE23BTECH11029 - Kanishk

Question:

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x .

Hint: $S_{x-1} = S_{49} - S_x$

Solution:

Parameter	Value	Description
$x(0)$	1	First house
d	1	Common difference
$x(n)$	$(n+1)u(n)$	$(n+1)th$ house
$y(n)$	$\left(\frac{n+1}{2}\right)(n+2)u(n)$	Sum of $n+1$ number of houses.

TABLE 0: Input Parameters

For an AP:

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \quad (1)$$

$$\Rightarrow X(z) = \frac{1}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2} \quad (2)$$

$$= \frac{1}{(1-z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$\therefore y(n) = \frac{n+1}{2}(n+2) \quad (4)$$

$$y(x-2) = y(n-1) - y(x-1) \quad (5)$$

From Table 0:

$$\left(\frac{x-1}{2}\right)x = \frac{49}{2}(50) - \frac{x}{2}(x+1) \quad (6)$$

$$(x-1) + x(x+1) = n(n+1) \quad (7)$$

$$2x^2 = n(n+1) \quad (8)$$

$$x = \sqrt{\frac{n}{2}(n+1)} \quad (9)$$

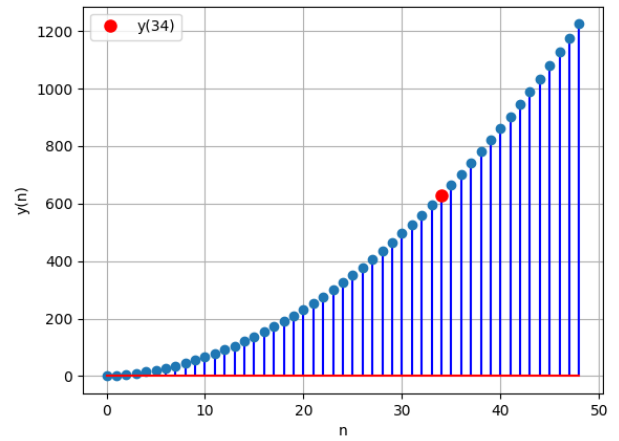


Fig. 0: Plot $y(n)$ vs n