10.5.4

EE23BTECH11029 - Kanishk

Question:
The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered is equal to the sum of the numbers of the houses following it. Find this value of x.
Hint:$S_{x-1} = S_{49} - S_x$
Solution:

Parameter	Value	Description
$x(0)$	1	First term
d	0	Common difference
$x(n)$	$[n+1]u(n)$	General term of series

TABLE 0: Input Parameters

For an AP:

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{1}$$
$$\implies X(z) = \frac{1}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2} \tag{2}$$
$$= \frac{1}{(1-z^{-1})^2}, \quad |z| > 1 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$
$$\implies Y(z) = X(z) \times (z) \tag{5}$$
$$Y(z) = \frac{1}{(1-z^{-1})^3}, \quad |z| > 1 \tag{6}$$

Contour Integration to find inverse of Z-transform,

$$y(n) = \frac{1}{2\pi j}\oint_c Y(z) z^{n-1}\,dz \tag{7}$$
$$= \frac{1}{2\pi j}\oint_c \frac{1}{(1-z^{-1})^3} z^{n-1}\,dz \tag{8}$$

We can observe that Pole is repeated 3 times thus m=3,

$$R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{9}$$
$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{z^{n+2}}{(z-1)^3}\right) \tag{10}$$
$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left(z^{n+2}\right) \tag{11}$$
$$= \frac{n+1}{2}(n+2) \tag{12}$$

$$\therefore y(n) = \frac{n+1}{2}(n+2) \tag{13}$$

Parameter	Value	Description
$y(48)$	$\frac{49}{2}(50)$	Sum of 49 terms
$y(x-2)$	$\frac{x-1}{2}(x)$	Sum of x-1 terms
$y(x-1)$	$\frac{x}{2}(x+1)$	Sum of x terms.

TABLE 0: Terms of y(n)

$$y(x-2) = y(48) - y(x-1) \tag{14}$$
$$FromTable\ 0: \tag{15}$$
$$\frac{x-1}{2}(x) = \frac{49}{2}(50) - \frac{x}{2}(x+1) \tag{16}$$
$$(x-1) = 49 \times 50 - x(x+1) \tag{17}$$
$$2x^2 = 49 \times 50 \tag{18}$$
$$x^2 = 49 \times 50 \tag{19}$$
$$x = 35 \tag{20}$$

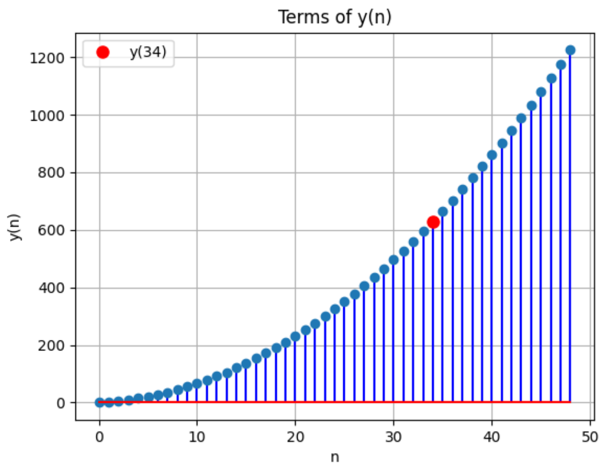

Fig. 0: Plot y(n) vs n