11.15

EE23BTECH11029 - Kanishk

Question:
A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km/hr. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 m/s.

Solution:

Parameter	Description	Value
V	Speed of sound in water	1450m/s
V_e	Speed of enemy submarine	100m/s
f	Frequency of SONAR wave	40kHz

The source(SONAR system) is at rest and the observer (enemy submarine) is moving toward it.
Enemy submarine(observer) is approaching the SONAR(source), So

$$Vrel = V + V_e \tag{1}$$

Wave equation of SONAR is given as:

$$y = A\sin(2\pi * ft - \frac{2\pi}{\lambda}x) \tag{2}$$
$$v = \frac{\omega}{k} \tag{3}$$
$$= \frac{2\pi * f}{2\pi/\lambda} \tag{4}$$
$$\lambda = \frac{V}{f} \tag{5}$$
$$\tag{6}$$

Frequency observed by the enemy submarine would be

$$f' = Vrel/\lambda \tag{7}$$
$$= (\frac{V + Ve}{V})f \tag{8}$$
$$= (\frac{1450 + 100}{1450})40 \tag{9}$$
$$= 42.76kHz \tag{10}$$

This frequency (f') is reflected by the enemy ship and is observed by the SONAR (which now acts as observer). Now the the enemy submarine has become source,
In time period T the wave reflected by source(enemy submarine) will travel distance VT and source will travel by distance V_eT

So the change in displacement by the wave and source would be the wavelength of the wave

$$\lambda_2 = T(V - V_e) \tag{11}$$
$$T = 1/f' \tag{12}$$
$$\lambda_2 = (\frac{V - V_e}{f'}) \tag{13}$$
$$f'' = V/\lambda_2 \tag{14}$$
$$f'' = (\frac{V}{V - V_e})f' \tag{15}$$
$$f'' = (\frac{1450}{1450 - 100})42.76 \tag{16}$$
$$= 45.93kHz \tag{17}$$

Parameter	Description	Value
λ	Wavelength of SONAR wave	3.625cm
y(x,t)	Equation of SONAR wave	$A\sin(2\pi * ft - \frac{2\pi}{\lambda}x)$
f'	Frequency observed by enemy submarine	42.76kHz
λ_2	Wavelength of reflected wave	3.157cm
$y_2(x,t)$	Equation of reflected wave	$A\sin(2\pi * f''t - \frac{2\pi}{\lambda_2}x)$
f''	Frequency of reflected wave	45.93kHz