11.15

EE23BTECH11029 - Kanishk

Question:
A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km/hr. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 m/s.

Solution:
Operating frequency of the SONAR system, $f = 40kHz$
Speed of the enemy submarine, $Ve = 360km/h = 100m/s$
Speed of sound in water, $V = 1450m/s$
The source(SONAR system) is at rest and the observer (enemy submarine) is moving toward it.
Enemy submarine(observer) is approaching the SONAR(source), So

$$Vrel = V + V_e \tag{1}$$

Wave equation of SONAR is given as:

$$y = A\sin(2\pi * ft + -\frac{2\pi}{\lambda}x) \tag{2}$$

As we know,

$$v = \frac{\omega}{k} \tag{3}$$

$$v = \frac{2\pi * f}{2\pi/\lambda}$$

So,

$$\lambda = \frac{V}{f} \tag{4}$$

Frequency observed by the enemy submarine would be

$$f' = Vrel/\lambda \tag{5}$$

Now by putting equation 1 and equation 2 in equation 3 we got

$$f' = (\frac{V + Ve}{V})f \tag{6}$$

$$= (\frac{1450 + 100}{1450})40 = 42.76kHz$$

This frequency (f') is reflected by the enemy ship and is observed by the SONAR (which now acts as observer). Now the the enemy submarine has become source,
In time period T the wave reflected by source(enemy submarine) will travel distance VT and source will travel by distance V_eT
So the change in displacement by the wave and source would be the wavelength of the wave

$$\lambda_2 = T(V - V_e) \tag{7}$$

$$T = 1/f' \tag{8}$$

Putting equtaion 6 in equation 5 we get,

$$\lambda_2 = (\frac{V - V_e}{f'}) \tag{9}$$

As we know,

$$f'' = V/\lambda_2 \tag{10}$$

By putting equation 7 in equation 8 we got,

$$f'' = (\frac{V}{V - V_e})f' \tag{11}$$

$$f'' = (\frac{1450}{1450 - 100})42.76$$

$$= 45.93kHz$$

The equation of reflected wave is given as:

$$y = A\sin(2\pi * f''t + -\frac{2\pi}{\lambda}x) \tag{12}$$

where A is amplitude of wave generated.

Input Table:

Parameter	Value	Description
40kHz	45.93kHz	$f = 40kHz, f'' = 45.93kHz$
30kHz	34.44kHz	$f = 30kHz, f'' = 34.44kHz$
20kHz	22.96kHz	$f = 20kHz, f'' = 22.96kHz$
55kHz	63.14kHz	$f = 55kHz, f'' = 63.14kHz$
29kHz	33.29kHz	$f = 29kHz, f'' = 33.29kHz$
60kHz	68.88kHz	$f = 60kHz, f'' = 68.88kHz$
65kHz	74.62kHz	$f = 25kHz, f'' = 74.62kHz$