Transfer learning → CNN Architectures

Conv → Pooling → Flattening → ANN → O/P

1.) Le-Net → 1998
2.) Alex Net → 2012
3.) VGG Net → 2014
4.) Inception Net → 2014
5.) ResNet → 2015

Pre-Trained Models

Statistics

1.) Descriptive → Mean
→ Median
→ Mode
→ Standard Deviation

2.) Inferential

Sample Population

Sample Population

(1 cr) $\longrightarrow$ (1 lakh)

50 lakh $\longrightarrow$ 90 k

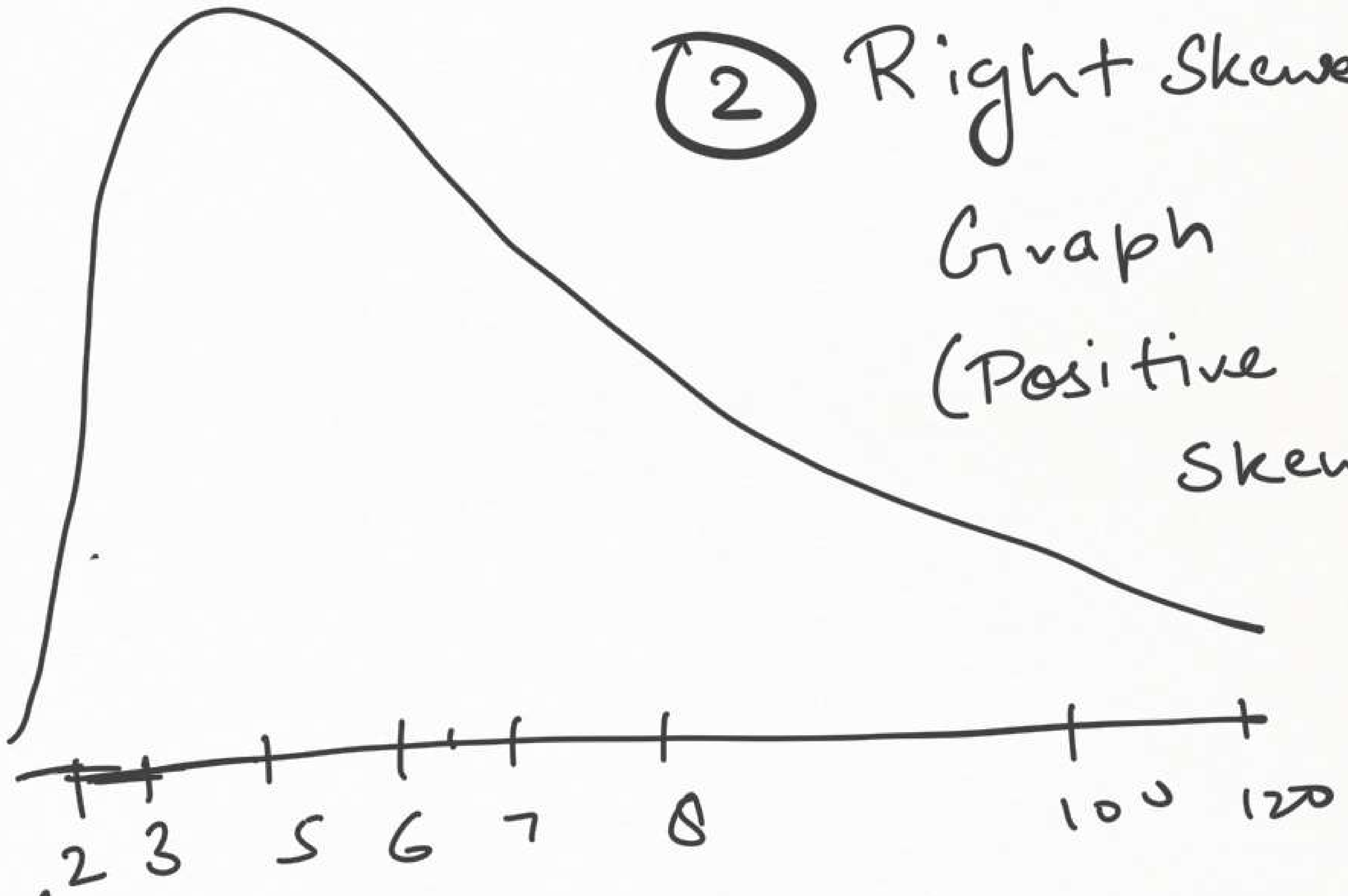
D → 3, 5, 7, 6, 2, 8, $\underbrace{100, 120}$ → outliers

$\mu = 31.3$

Median = 6.5

(1). Mean > Median

(2) Right Skewed Graph (Positive Skew)



(3) Median is ~~least~~ sensitive towards an outliers. & Mean is very much sensitive towards an outliers

$$D \rightarrow 100, 110, 120, 115, 105, 100, \underbrace{5, 3, 2}_{}$$

$\rightarrow$ outliers

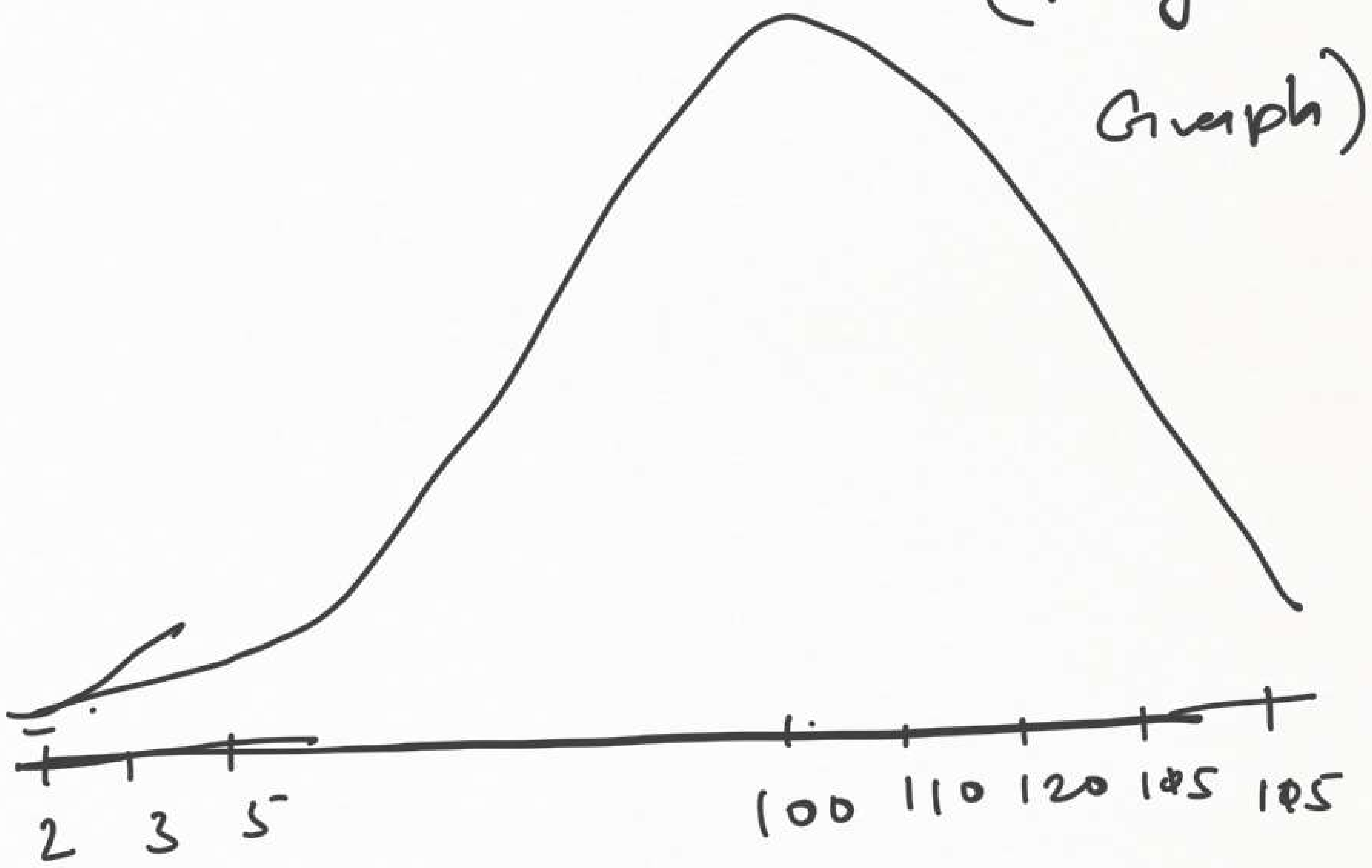
$\mu = 73.3$

Median = 100

(1) Median > Mean

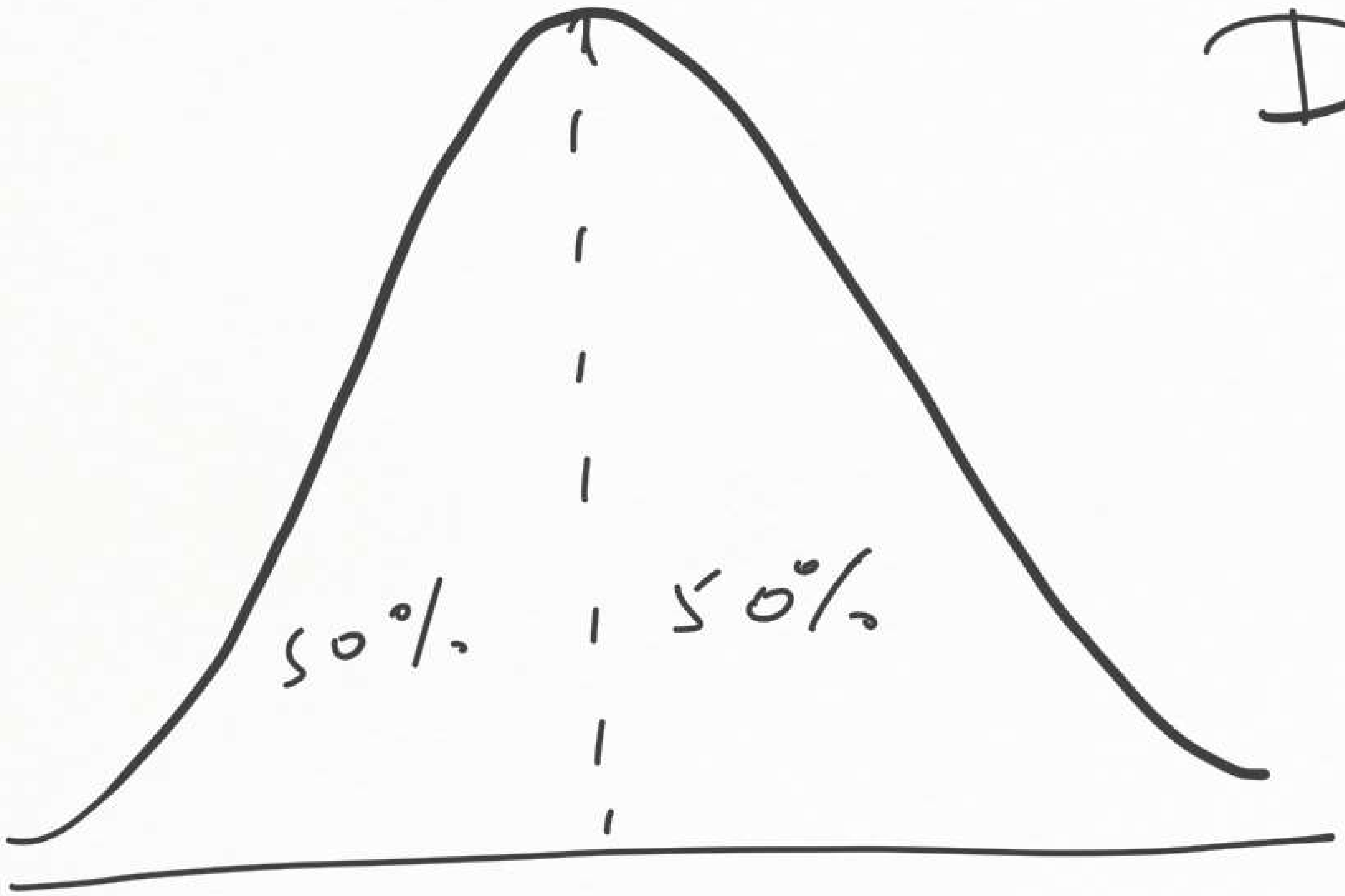
(2) Left Skewed (Negative Graph)

(3)



Mean ≈ Median ≈ Mode

Normal Distribution



Symmetrical Dist.

$D \rightarrow 2, 3, 5, 4, 7, 8, 10$

$\mu = 5.5$

Variance $= 7.1$

$\sigma = 2.66$

$$= \sum_{i=1}^{n} \left(\frac{x - \bar{x}}{n} \right)^2$$

$$= \frac{(2-5.5)^2}{7} + \frac{(3-5.5)^2}{7} + - \text{---} - = \frac{49.71}{7} = 7.$$

68 - 95 - 99 Rule (Empirical formula)

$$D = 2, 3, 5, 7, 4, 8, 10$$

$$\mu = 5.5$$

$$\sigma = 2.5$$

$$\mu \pm \sigma = 68\%$$

$$\mu \pm 2\sigma = 95\%$$

$$\mu \pm 3\sigma = 99.7\%$$

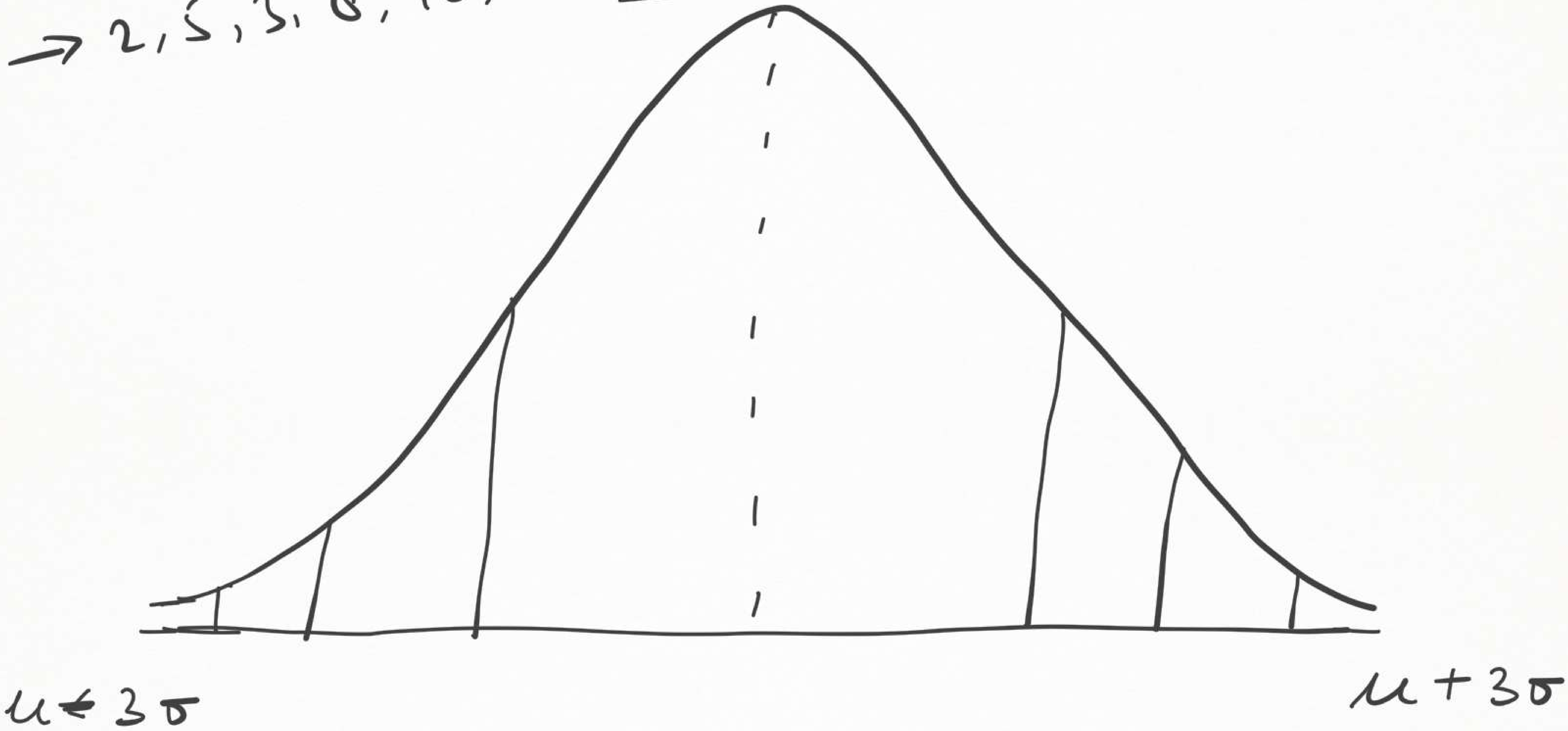
$$5.5 - 2.5 \le D \le 5.5 + 2.5$$

$$3 \le D \le 8$$

$$5.5 - 5 \le D \le 5.5 + 5$$

$$0.5 \le D \le 10.5$$

$D \rightarrow 2, 5, 3, 8, 10, 7, (\underline{\underline{80}})$



Probability Distribution

Random Variable

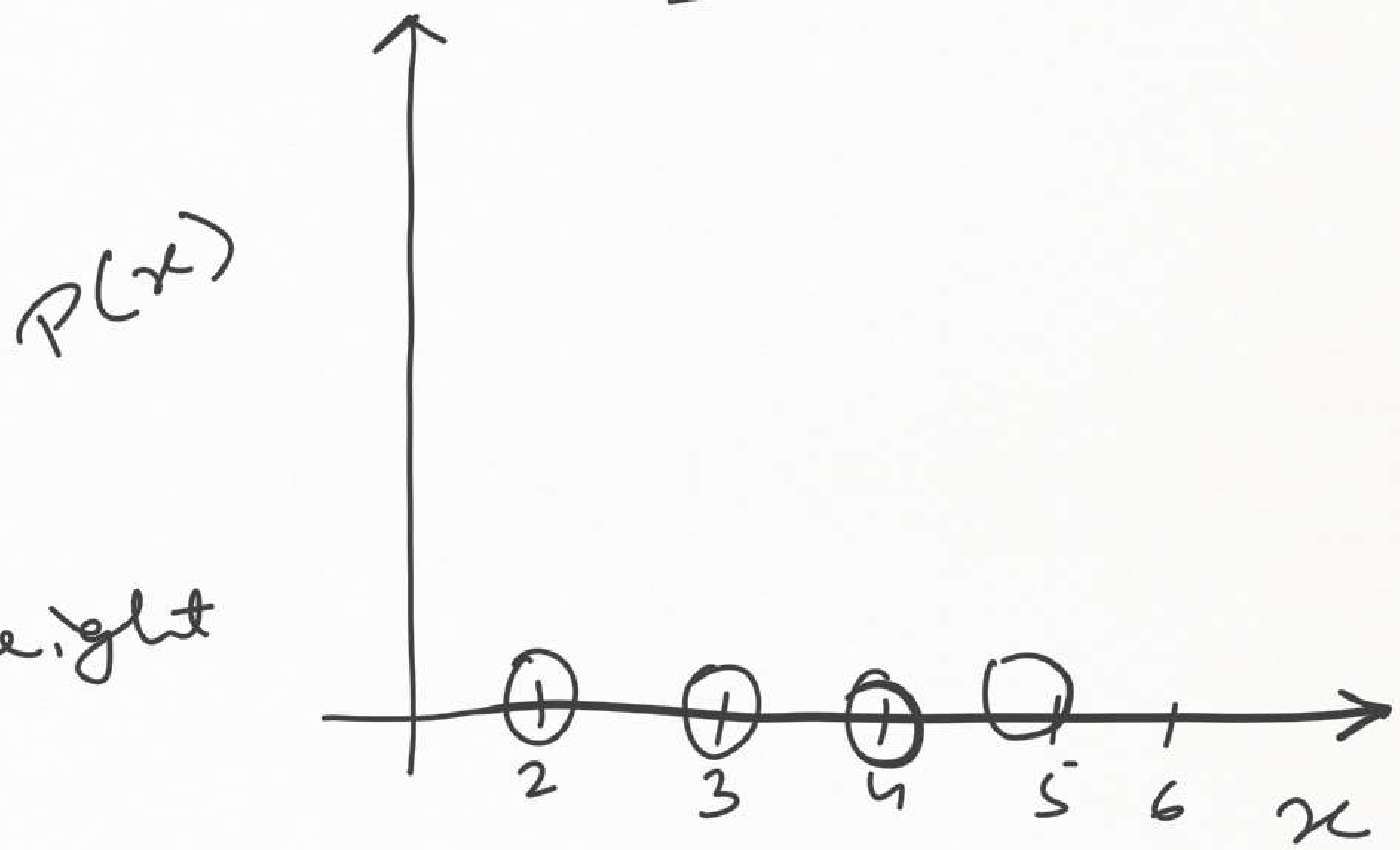
Discrete

ex:-

Continous

ex:- Weight, height

PDG



1.) Binomial Distribution

2.) Poisson Distribution

3.) Uniform Distribution

4.) Normal "

5. Standard N D

Binomial Distribution

$$\text{PDF of BD} \Rightarrow P(X=r) = {}^{N}C_{r}\, p^{r}\, q^{N-r}$$

$$= \frac{\lfloor N}{\lfloor N-r \;\lfloor r}\, p^{r} \cdot q^{N-r}$$

N = No. of trials

r = Total no. of desired success

p = pro. of getting success in 1 trial

q = " " " " failure

Q.) Tossed a coin for 7 times. What is the probability of getting 5 heads?

Sol:- $N = 7$

$r = 5$

$p = 1/2$

$q = 1 - p = 1/2$

$$P(x=5) = {}^{7}C_{5} \left(\tfrac{1}{2}\right)^{5} \left(\tfrac{1}{2}\right)^{2}$$

$$= \frac{7!}{5!\;2!} \left(\tfrac{1}{2}\right)^{5} \left(\tfrac{1}{2}\right)^{2}$$

$$= \frac{7 \cdot \cancel{6}^{\,3} \cdot \cancel{5!}}{\cancel{5!}\;\cancel{2}} \times \left(\frac{1}{2}\right)^{7} = 21 \times \frac{1}{} =$$

Q.)

$\boxed{100 \longrightarrow 65 \text{ die}}$

$6 \longrightarrow 4$ will recover?

Soln:

$N = 6$

$r = 4$

$p = 0.35$

$q = 0.65$

$P(X=4) = 0.09$

$N = 7$

$P(x > 3) = P(x=4) + P(x=5) + P(x=6) + P(x=7)$

CDF

Poisson Distribution

No. of desired

$$P(x) = \frac{\lambda^{x} e^{-\lambda}}{x!}$$

$e = 2.71$

$\lambda = \text{mean}$

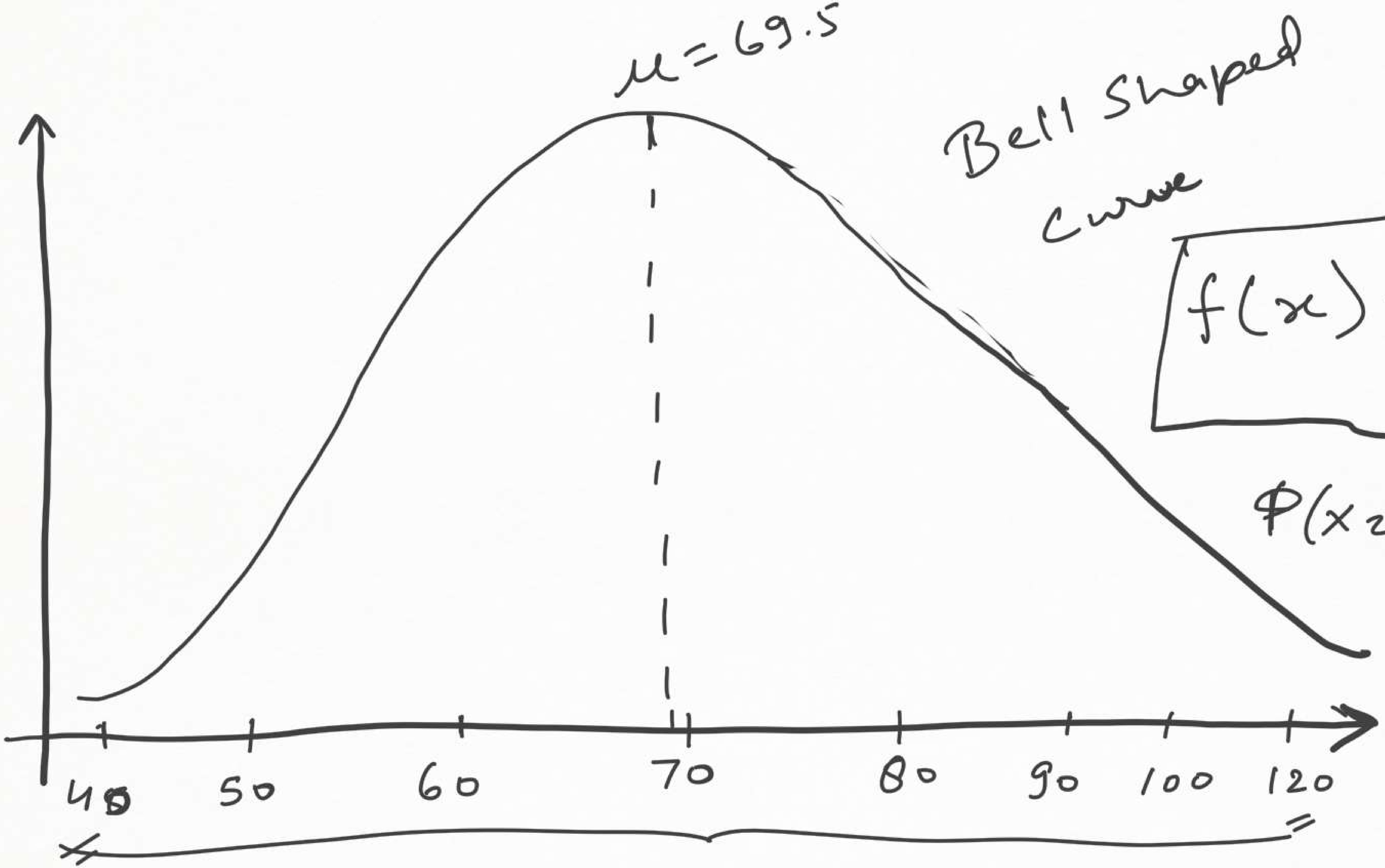
Q.)

3 insurance / week

$x = 0$

$\lambda = 3$

$$P(X=0) = \frac{3^0 \cdot e^{-3}}{0!} = 0.04$$



Bell Shaped Curve

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)}{2\sigma^2}}}$$

$$P(x \geq 85) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(85-\mu)}{2\sigma^2}}$$

0 to ∞

Standard Normal Distribution

$$\boxed{\begin{matrix}\mu = 0 \\ \sigma = 1\end{matrix}}$$

$D = 1, 3, 2, 5, 7, 6$, $\mu = 4$

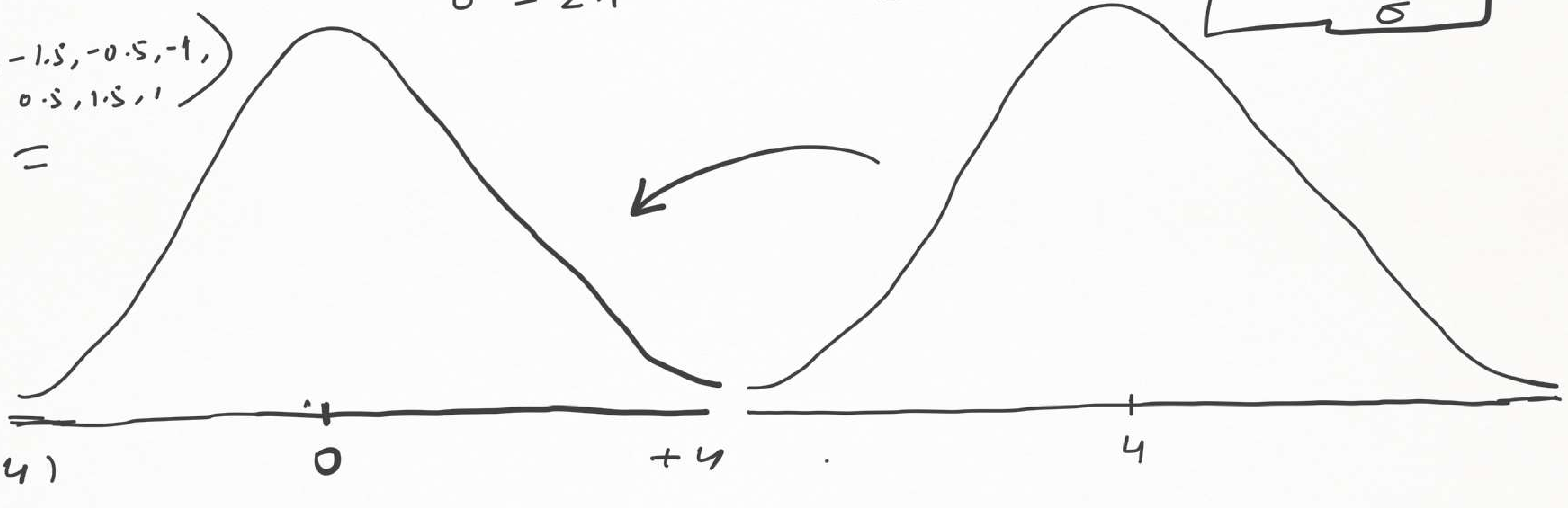
$\sigma = 2.1$

$Z = \frac{1-4}{2}, \frac{3-4}{2}$

$$\boxed{Z = \frac{x-\mu}{\sigma}}$$

$\left(D' = -1.5, -0.5, -1, 0.5, 1.5, 1\right)$

$=$



Q.)

$\mu = 50 \text{ mins}$

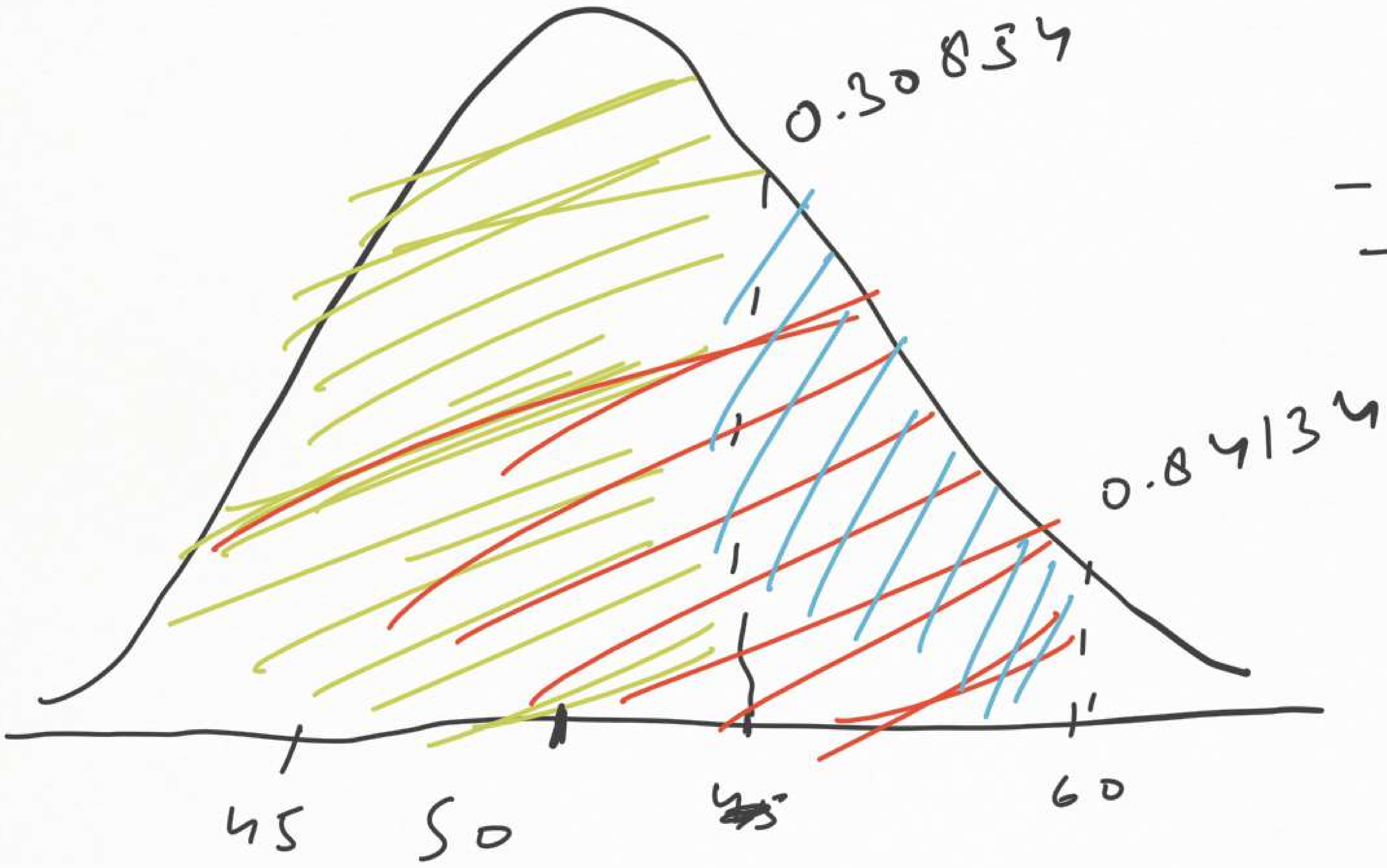
$\sigma = 10 \text{ mins}$

$$Z = \frac{45-50}{10} = -0.5$$

$$Z = \frac{60-50}{10} = 1$$

$P(45 \leq x \leq 60) = ?$

$P(-0.5 \leq x \leq 1) \Rightarrow 0.53280 \Rightarrow 53.28\%$



$$
\begin{array}{r}
0.84134 \\
-\,0.30854 \\
\hline
0.53280
\end{array}
$$

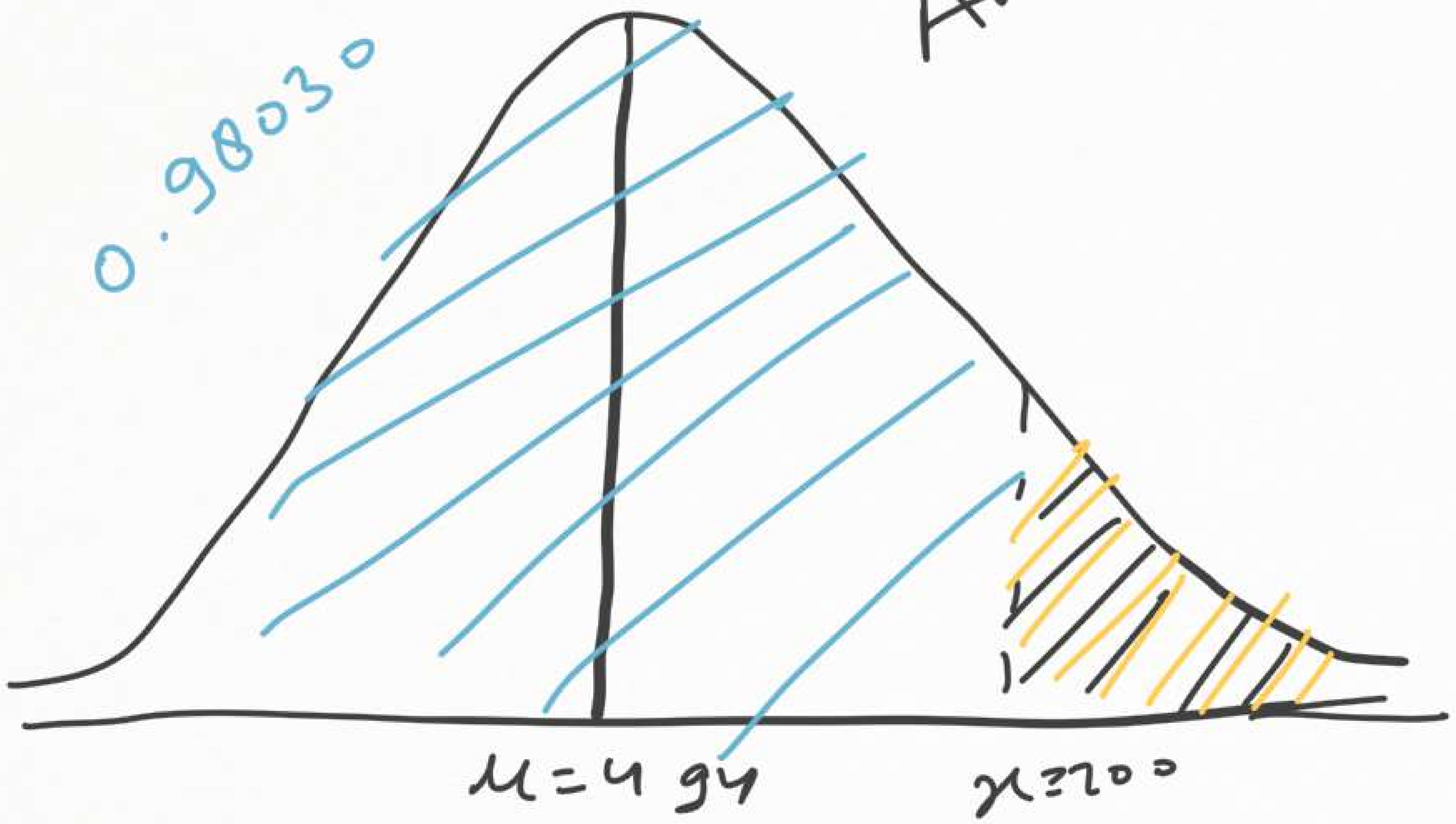
$\mu = 494$

$\sigma = 100$

$P(x > 700) = ?$

$z = \frac{700 - 494}{100} = 2.06$

$P(Z = 2.06) = 0.98030$



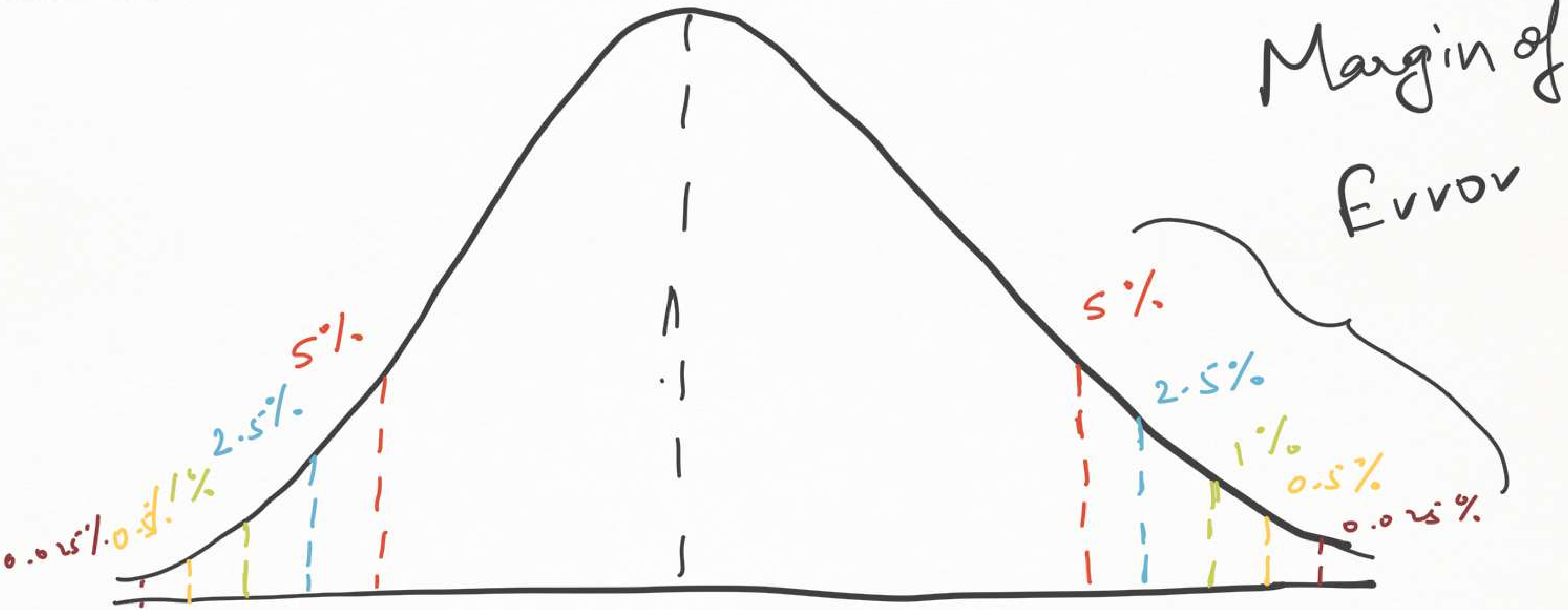
$P(x > 700) = 1 - 0.98030$

$= 0.0197$

1.97%

Estimation

Confidence Interval



$$\boxed{\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}}$$

Mean

Standerd deviation

Sample size

$\alpha \rightarrow$ level of significance

or

Margin of Error

$\alpha = 5\%$

$\alpha = 10\%$

$\alpha/2 = 5\%$

$= 0.05$

$\alpha/2 = 2.5\%$

$= 0.025$

$z_{0.05} = -1.64$

$z_{0.025} = -1.96$

Hypothesis Testing

Null hypothesis, H_0

Alternate hypothesis, H_A

H_0 99%

H_A $\neq$ 99%

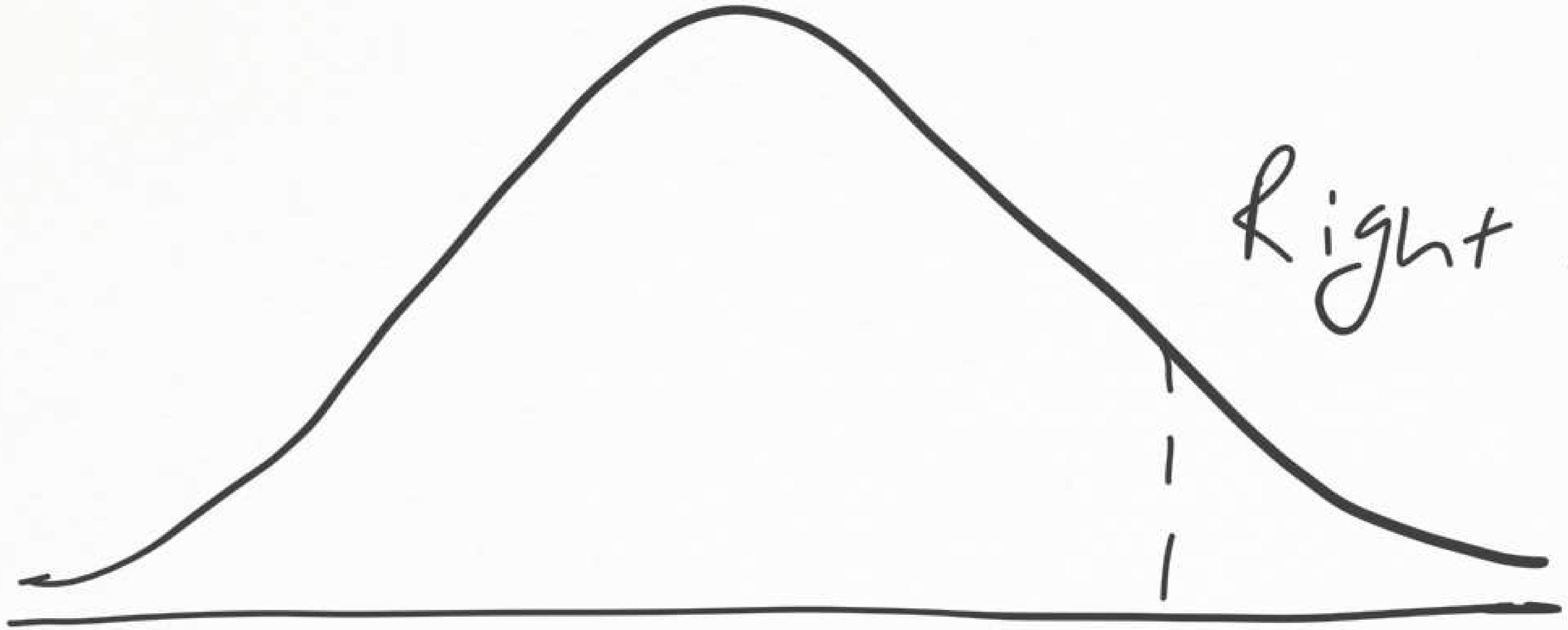
One Tail vs Two Tail Test

H_0 : Rent <=20000 (I. will buy)

H_A : Rent > 20k (I. will not buy)

One Tail Test

$\boxed{\leq \quad >}$

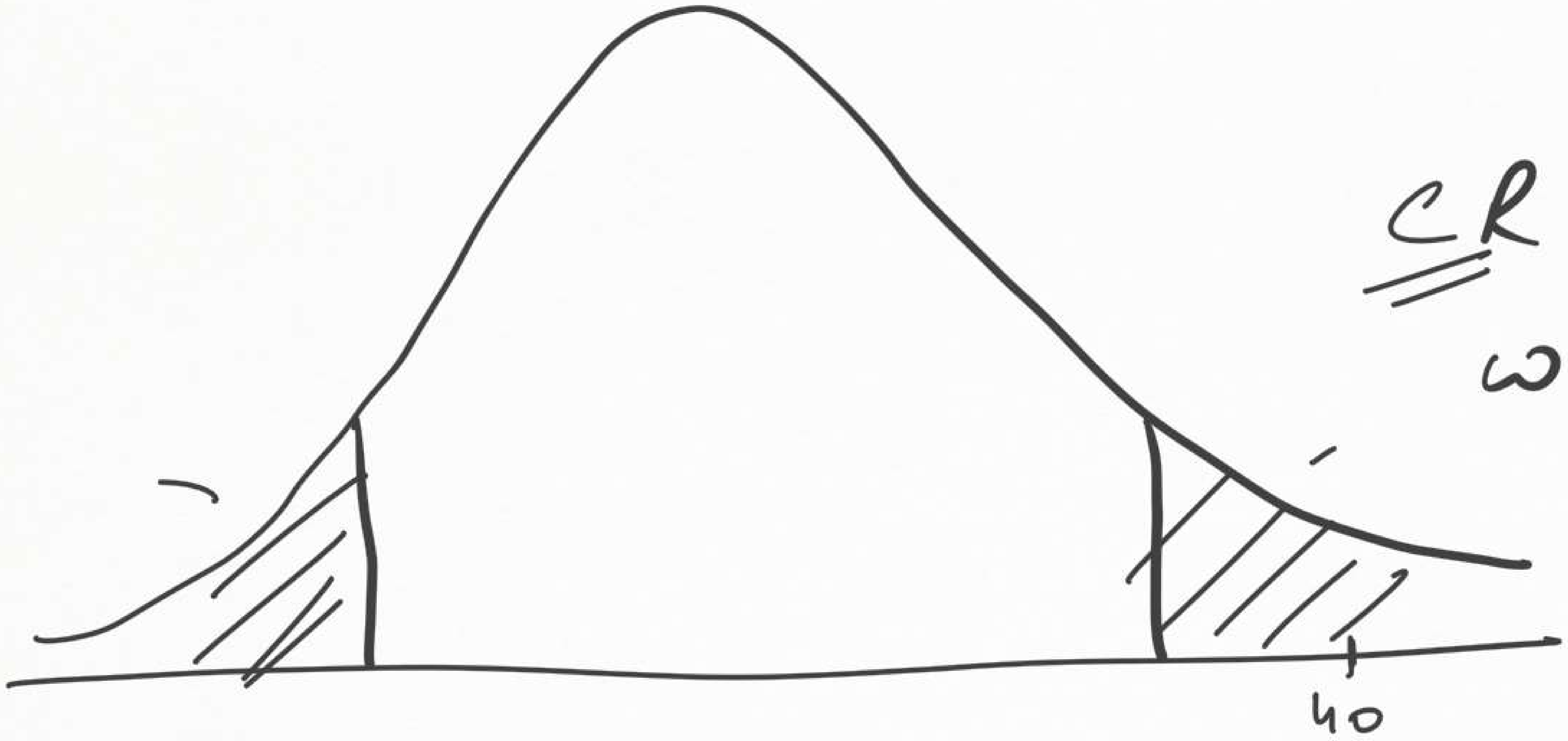


Right Hand Test

$H_0: D = 20\,mm$

$H_A: D \neq 20\,mm$

Two Tail Test



CR → It's a region where I can reject the H_0.

Type I and Type II error

Type 1 error ⇒ False Positive
2 error ⇒ False Negative

Decision	H_0 True	H_0 False
Reject H_0	Type I error	Correct Decision
Accept H_0	Correct Decision	Type II error

$\alpha = P$[rejecting H_0 when H_0 True]

$\beta = P$[accepting H_0 when H_0 is false]

1.) z - test
(sample size > 30)

2.) t - test
(Sample size < 30)

When we are cheking the significant difference b/w population mean & sample mean.

3.) Chi - Square Test ⇒ Population variance & Sample variance

4.) ANNOVA (Analysis of Variance)

Q) A manufacturer of printer cartridge clams that a certain cartridge manufactured by him has a mean printing capacity of at least 500 pages. A wholesale purchaser selects a sample of 100 printers and tests them. The mean printing capacity of the sample came out to be 490 pages with a standard deviation of 30 printing pages.

Should the purchaser reject the claim of the manufacturer at a significance level of 5%?

Soln:-

H_0: I will buy ($\mu \geq 500$)

$\alpha = 5\%$
$= 0.05$

$\boxed{H_A: \text{I will not buy } (\mu < 500)}$

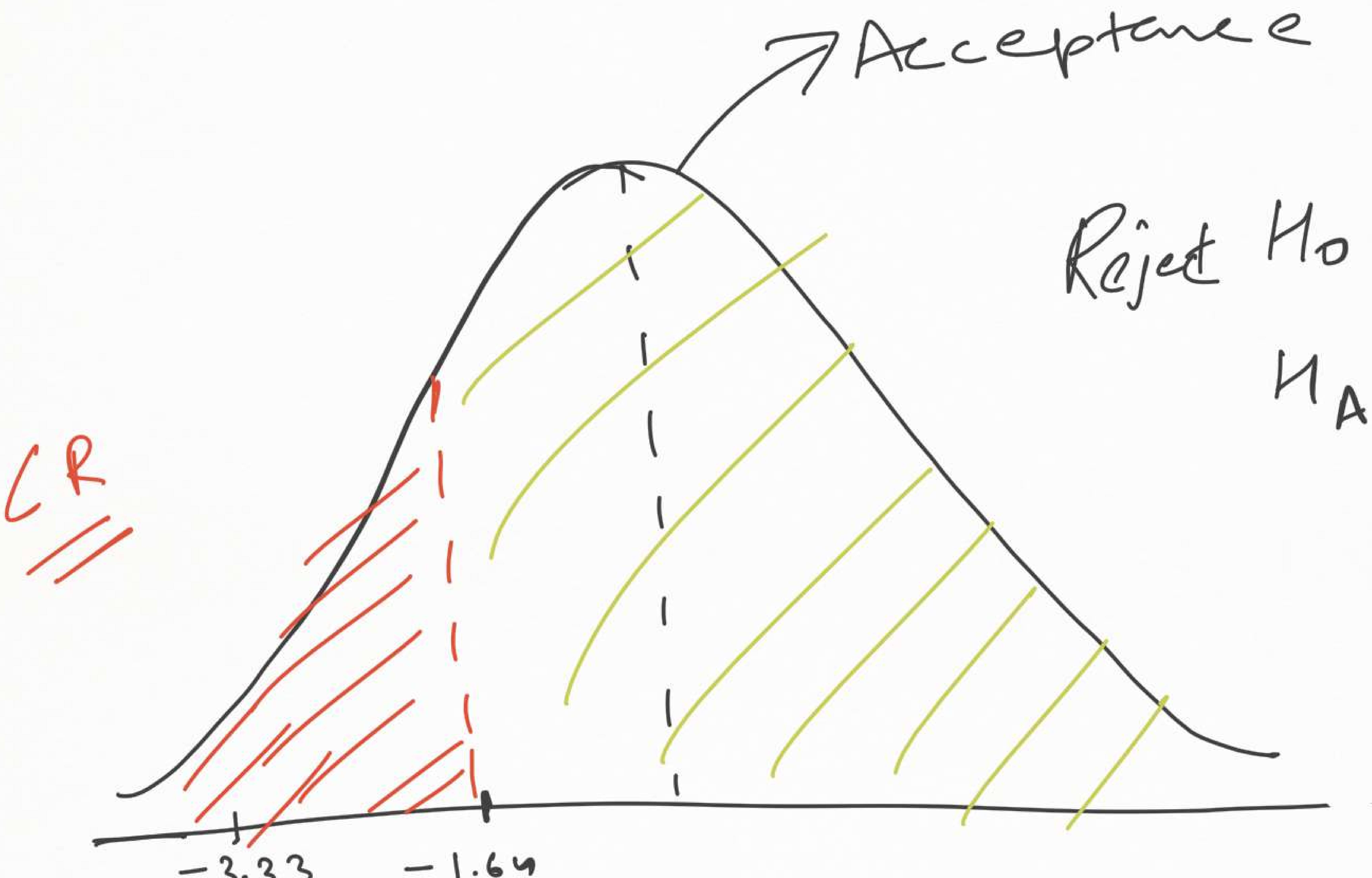
It's a one tail test. (left hand)

$$z\text{-test} = \frac{\text{Sample Mean} - \text{Popu. Mean}}{\text{Standard Error}}$$

$$SE = \frac{\text{Stand. Deviation}}{\sqrt{n}}$$

$$= \frac{490 - 500}{\frac{30}{\sqrt{100}}} = \frac{-10}{3} = -3.33$$

$$Z_{0.05} = -1.64$$


Acceptance
Reject H_0
H_A
CR
-3.33
-1.64

Q) A company used a specific brand of Tube lights in the past which has an average life of 1000 hours. A new brand has approached the company with new Tube lights with same power at a lower price. A sample of 120 light bulbs were taken for testing which yielded an average of 1100 hours with standard deviation of 90 hours. Should the company give the contract to this new company at a 1% significance level.

$H_0 = \mu \geq 1000$

$H_A = \mu < 1000$

$Z_{0.01} = 2.576$

Q) A tyre manufacturer claims that the average life of a particular category of its tyre is 18000km when used under normal driving conditions. A random sample of 16 tyres was tested. The mean and SD of life of the tyres in the sample were 20000 km and 6000 km respectively.

Assuming that the life of the tyres is normally distributed, test the claim of the manufacture at 1% level of significance.

$H_0 : \mu = 18000 \text{km}$

$H_A : \mu \neq 18000 \text{km}$

$\alpha = 1\%$

$$t\text{-test} = \frac{20000 - 18000}{\frac{6000}{\sqrt{16}}}$$

$$= \frac{\cancel{2000} \times 4}{\cancel{36000}} = 1.33$$

$df = n - 1$

$= 15$

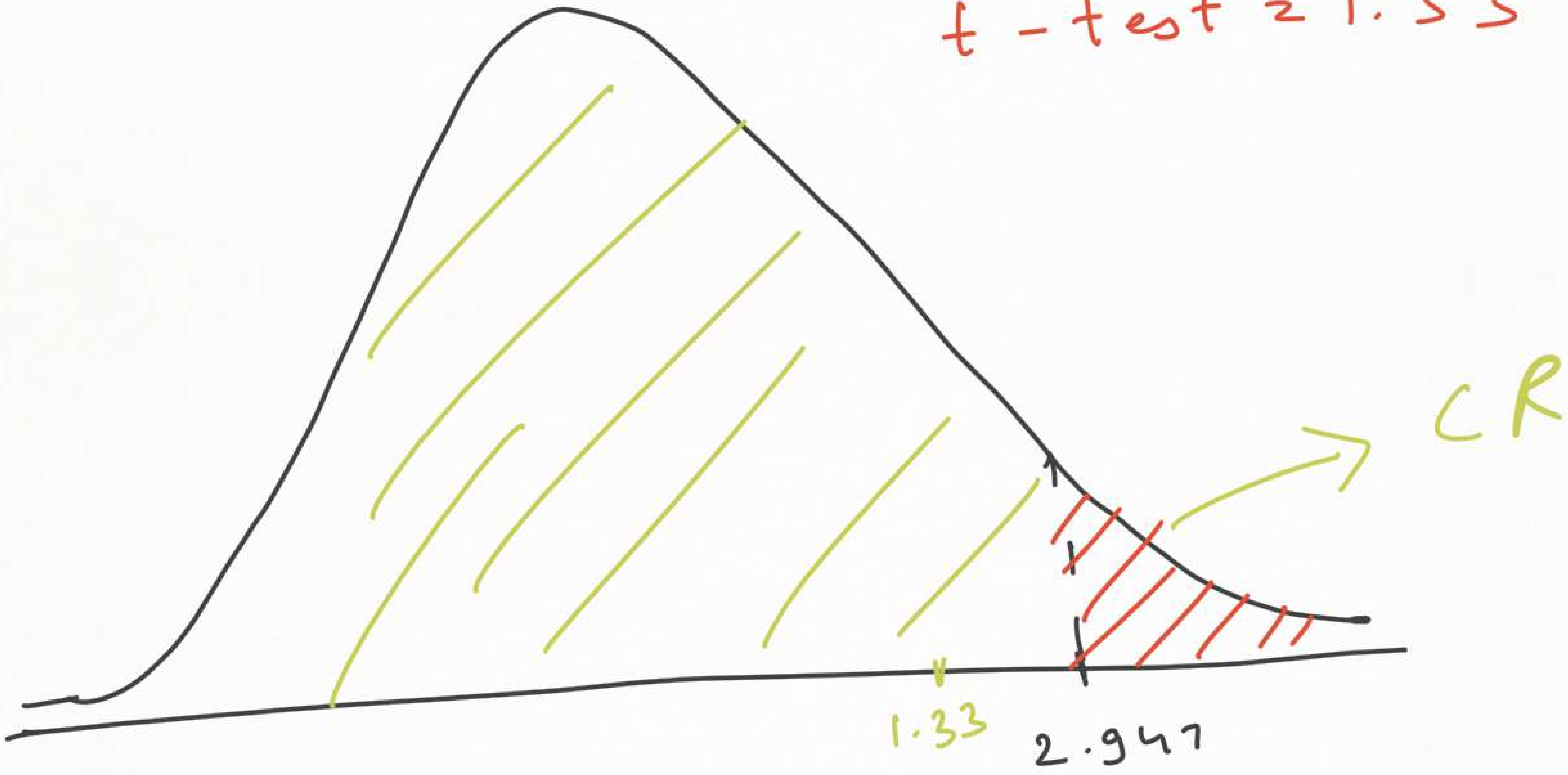
$\alpha = 0.01$

$t_{0.01} = 2.947$

t-test = 1.33

H_0 ✓

H_A ✗



$n-1$

$n_1 - 1 + n_2 - 1$

$$\boxed{df = n_1 + n_2 - 2}$$

Z-test & t-test

significant difference b/w means

chi-square

variance / fluctuation

$$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$$

$s^2 \rightarrow$ sample Variance

$\sigma^2 \rightarrow$ population variance

Q) The variance of a certain size of towel produced by a machine is 7.2 over a long period of time. A random sample of 20 towels gave a variance of 8. You need to check if the variability for towel has increased at 5% level of significance, assuming a normally distributed sample.

variance = $(s.d)^2$

H_0: Population Variance <= 7.2

H_A: " " > 7.2

$$\chi^2 = \frac{(n-1).s^2}{\sigma^2} = \frac{19 \cdot 8}{7.2} = 21.1$$

$\chi^2_{0.025} = 32.852$

$\chi^2 = 21.11$

H_0 ✓

H_A ×

Goodness of fit Test

H_0 : Population distribution of the variable is same as the proposed distribution

H_A : The distributions are different.

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

1.)

$E(1\ dog) = 0.60$

$E(2\ dog) = 0.28$

$E(3\ or\ more\ dogs) = 0.12$

$O(1\ dog) = 73$

$O(2\ dog) = 38$

$O(3\ or\ more) = 18$

H_0: Results are same.

H_A: Results are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

	1 Dog	2 Dog	3 or more
observed	73	38	18
Expected	$0.60 \times 129 = 77.4$	$0.28 \times 129 = 36.12$	$0.12 \times 129 = 15.48$
O - E	-4.4	1.88	2.52

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(-4.4)^2}{77.4} + \frac{(1.88)^2}{36.12} + \frac{(2.52)^2}{15.48}$$

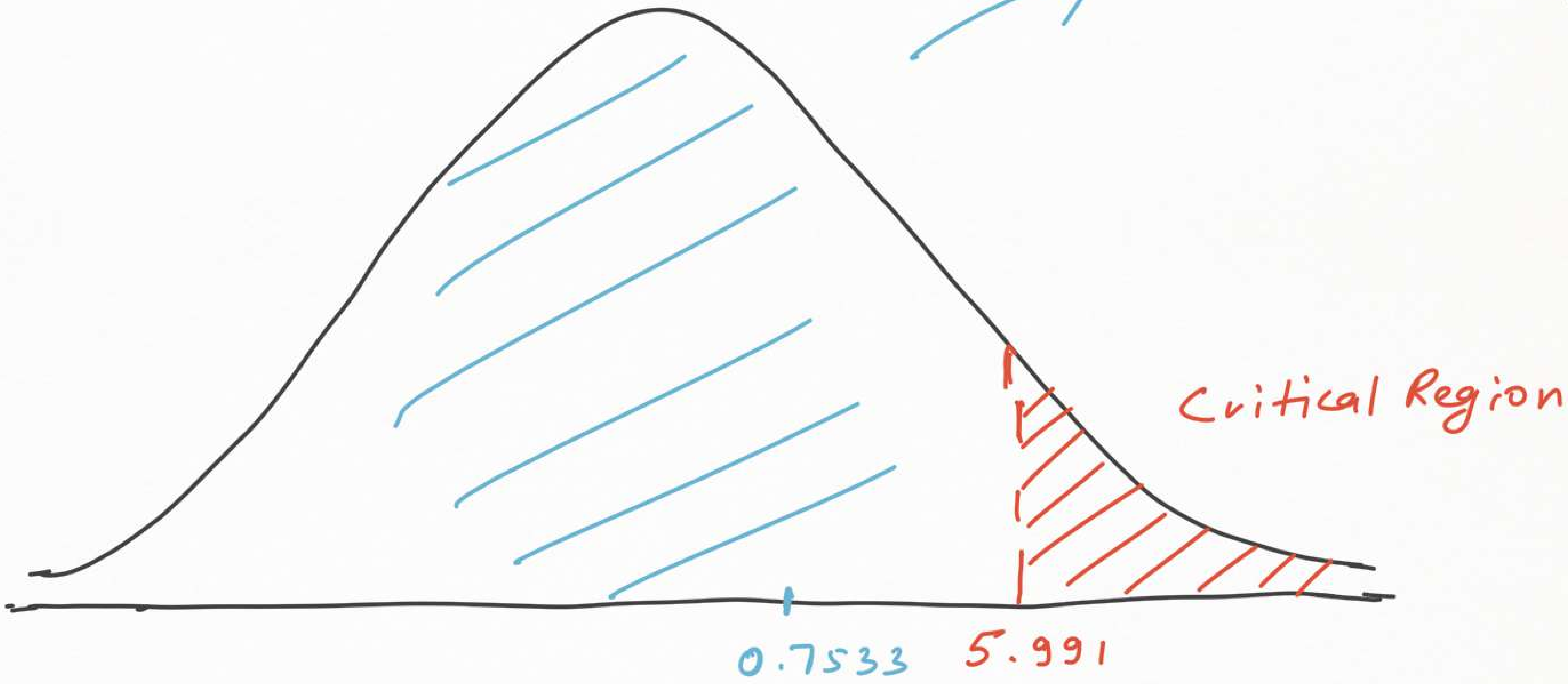
$$= 0.7533$$

$\chi^2 = 0.7533$

$df = n-1$
$= 2$

$\chi^2_{0.05} = 5.991$

We can accept H_0.



Analysis of Variance (ANOVA)

$$F\text{-test} = \frac{Mean_{ss\,between}}{Mean_{ss\,within}}$$

$SS_{between} \rightarrow$ Sum of square b/w the groups

$SS_{within} \rightarrow$ " " " within the groups

$H_0 \Rightarrow \mu_1 = \mu_2 = \mu_3 = \text{——} - . \mu_n$

$H_A \Rightarrow \mu_1 \neq \mu_2$

	School 1(S1)	School 2(S2)	School 3(S3)	School 4(S4)	(S1) - S1_mean)^2	(S2-S2_mean)^2	(S3-S3_mean)^2	(S4-S4_mean)^2
	8	6	6	5	1	0.111111556	0	1.361095556
	6	4	5	6	1	2.777775556	1	0.027775556
	7	6	5	6	0	0.111111556	1	0.027775556
	5	5	6	7	4	0.444443556	0	0.694455556
	9	6	7	6	4	0.111111556	1	0.027775556
		7	8	7		1.777779556	4	0.694455556
			5				1	
Total	35	34	42	37	10	5.333333333	8	2.833333334
Mean	7	5.666666667	6	6.166666667				
Grand mean	6.208333333							

$k = 4$

$N = 24$

$\mu_{g_1} = 7$

$\mu_{g_2} = 5.67$

$\mu_{g_3} = 6$

$\mu_{g_4} = 6.167$

$$GM = \frac{\mu_{g_1} + \mu_{g_2} + \mu_{g_3} + \mu_{g_4}}{4}$$

$$= \frac{7 + 5.67 + 6 + 6.167}{4}$$

$$SS_{Between} = \sum \underbrace{n_i}_{\text{Total no. of observations in each group}} (\underbrace{\bar{x}_i}_{\text{Mean of the group}} - \underbrace{GM}_{\text{Grand Mean}})^2$$

$$SS_{Between} = 5 * (7 - 6.21)^2 + 6 * (5.67 - 6.21)^2 + 7 * (6 - 6.21)^2 + 6 * (6.167 - 6.21)^2$$

$$= 5.18$$

$$MSS_{Between} = \frac{5.18}{k-1} = \frac{5.18}{3} = 1.78$$

$$SS_{within} = \sum (x_i - u)^2$$

$$= 10 + 5.33 + 8 + 2.83$$

$$= 26.16$$

$$\text{Mean}\ ss_{within} = \frac{26.16}{N-k} = \frac{26.16}{24-4} = \frac{26.16}{20}$$

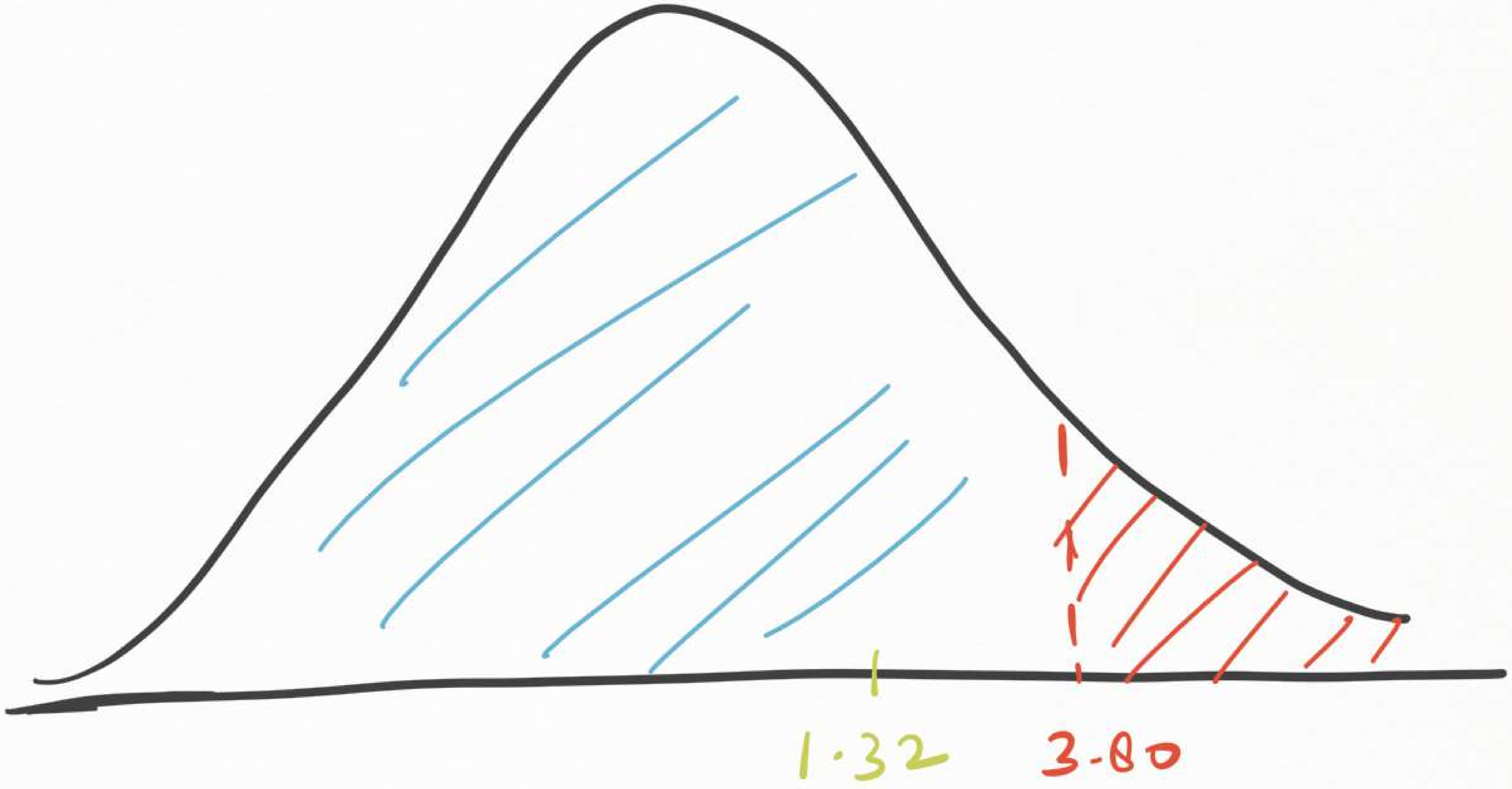
$$= 1.30$$

$$f\text{-test} = \frac{1.73}{1.30} = 1.32$$

$f\text{-test} = 1.32$

$f_{0.05} = 3.80$

H_0 ✓



1.) z-test → Checking the significant b/w Population Mean & Sample Mean. [Sample Size should be greater than 30]

2.) t-test → Sample Size is less than 30.

3.) χ^2-test → checking significant difference b/w Population Variance & Sample Variance.
Goodness of fit test on categorical data.

4.) ANOVA → Used when we need to compare more than 2 samples.

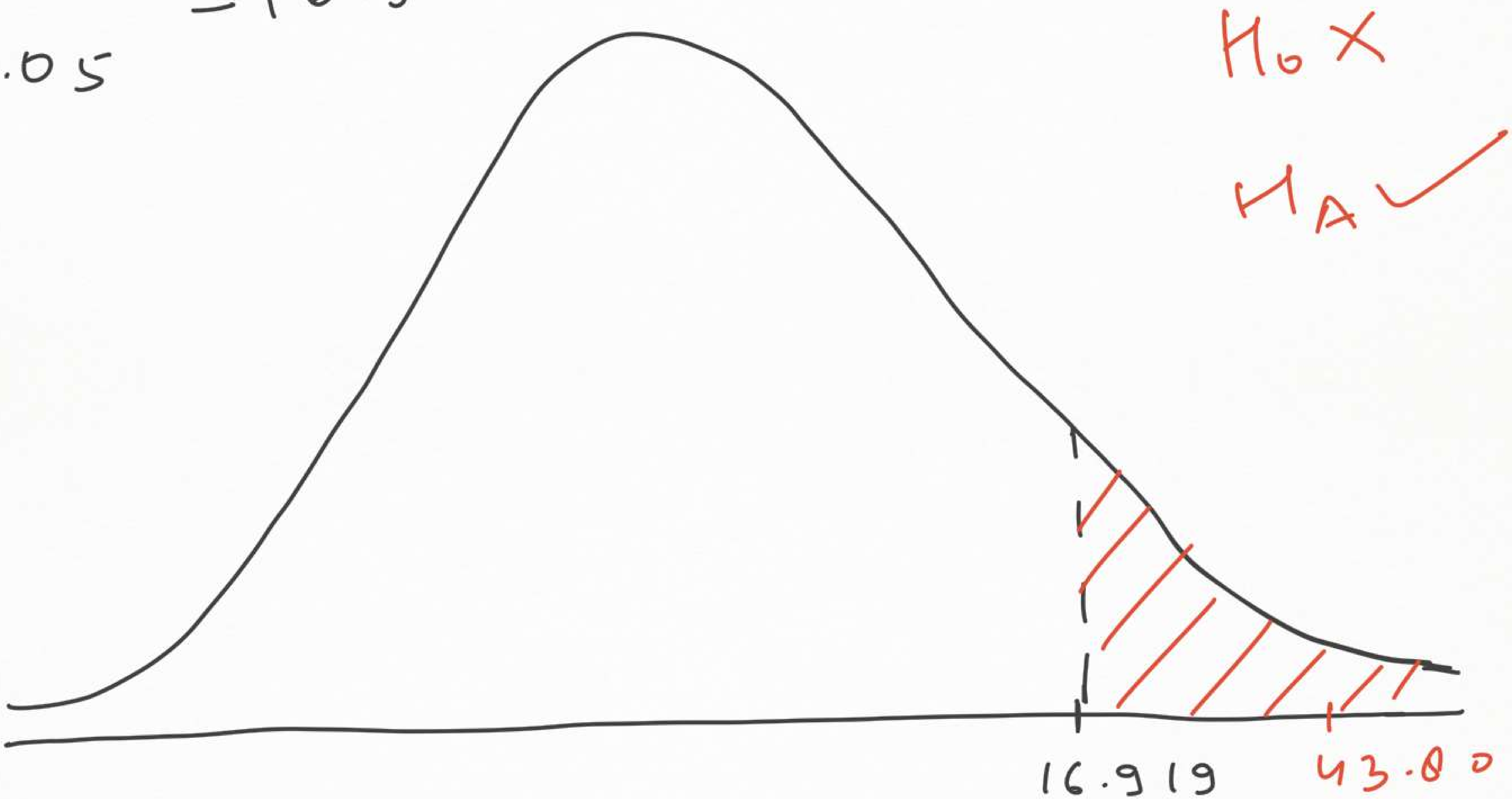
H_0 : Claim is correct. $(\sigma \leq 40)$

H_A : claim is not correct $(\sigma > 40)$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)\times 195}{40}$$

$$= \frac{9 \times 195}{40} = 43$$

$\chi_{0.05} = 16.919$



Sampling Distribution of Mean

X	P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

$$\boxed{\mu = \sum x \cdot P(x)}$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6}$$

$$+ 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$$

$$= \frac{21}{6} = 3.5$$

Sampling Distribution of Variance

$$\sigma^2 = \sum (x-\mu)^2 \, P(x)$$

$$= (1-3.5)^2 \times \frac{1}{6} + (2-3.5)^2 \times \frac{1}{6} + (3-3.5)^2 \times \frac{1}{6}$$

$$+ \ldots \ldots + (6-3.5)^2 \times \frac{1}{6}$$

$$= 2.92$$

X	$\bar{X}$
1, 1	1
1, 2	1.5
1, 3	2.0
1, 4	2.5
1, 5	3.0
1, 6	3.5
2, 1	1.5
2, 2	2.0

X	$\bar{X}$
2, 3	2.5
2, 4	3.0
2, 5	3.5
2, 6	4.0
3, 1	2.0
3, 2	2.5
3, 3	3.0
3, 4	3.5

X	$\bar{X}$
3, 5	4.0
3, 6	4.5
4, 1	2.5
4, 2	3.0
4, 3	3.5
4, 4	4.0
4, 5	4.5

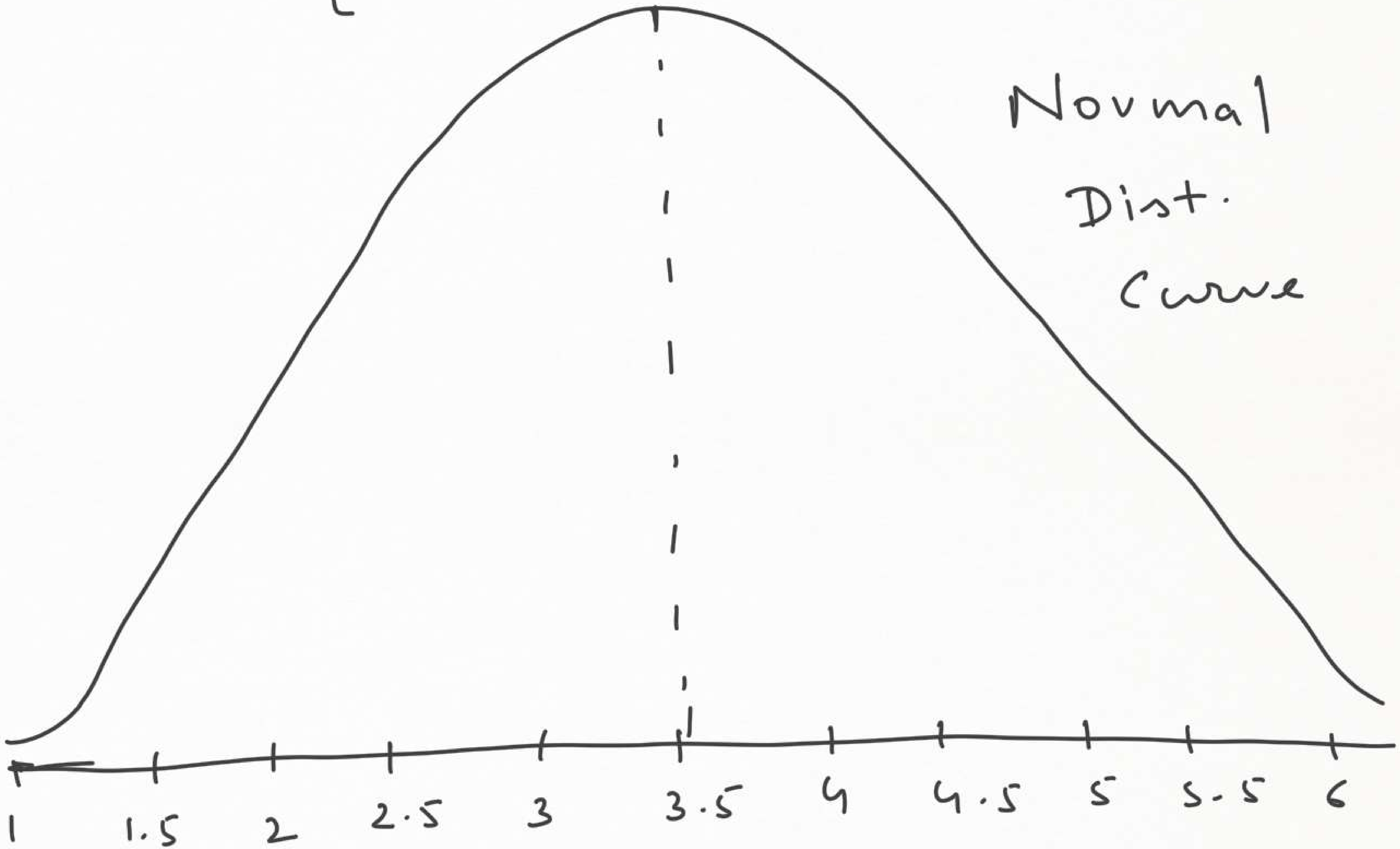
X	$\bar{X}$
4, 6	5
5, 1	3
5, 2	3.5
5, 3	4
5, 4	4.5
5, 5	5
5, 6	5.5
6, 1	3.5
6, 2	4
6, 3	4.5
6, 4	5

6, 5 → 5.5

6, 6 → 6

$\overline{X}$	$P(\overline{X})$
1	1/36
1.5	2/36
2.0	3/36
2.5	4/36
3.0	5/36
3.5	6/36
4.0	5/36
4.5	4/36
5.0	3/36
5.5	2/36
6.0	1/36

{Probability Dist. Graph}



$\Rightarrow$ Central Limit Theorem

C1 $\longrightarrow$ 10, 20, 15, 25, 30 $\Rightarrow$ 20

C2 $\longrightarrow$ 20, 5, 15, 25, 35 $\Rightarrow$ 20

C3 $\longrightarrow$ 10, 20, 30, 40, 10 $\Rightarrow$ 22

C4 $\longrightarrow$ 5, 15, 25, 35, 20 $\Rightarrow$ 20

C5 $\longrightarrow$ 5, 15, 50, 20, 30 $\Rightarrow$ 24

21.2K

$\downarrow$

Population Mean

Maximum Likelihood Estimation (MLE)

M

L

E

$$\boxed{L = \prod_{i=1}^{n} f(x_i, \theta)}$$

~~Properties~~

Efficient, consistent & invariant

example :- <u>Poisson Distribution</u>

$$f(x, \lambda) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$L = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$L = \frac{e^{-\lambda} \cdot \lambda^{x_1}}{x_1!} \times \frac{e^{-\lambda} \cdot \lambda^{x_2}}{x_2!} \times \cdots \frac{e^{-\lambda} \cdot \lambda^{x_n}}{x_n!}$$

$$\log L = -n\lambda + \sum x_i \log \lambda - \log \pi x_i!$$

$$\frac{\partial \log L}{\partial \lambda} = -n + \frac{\sum x_i}{\lambda} = 0$$

$$\frac{\sum x_i}{\lambda} = n$$

$$\lambda = \frac{\sum x_i}{n} = \overline{x}$$

$H_0 : \mu = 17\%$

$H_A : \mu > 17\%$

$p = 17\% = 0.17$

$\hat{p} = \dfrac{115}{550} = 0.20$

$z\text{-test} = \dfrac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}}$

$q = 1 - p$
$= 1 - 0.20$
$= 0.8$

$= \dfrac{0.20 - 0.17}{\sqrt{\dfrac{(0.20)(0.80)}{550}}}$

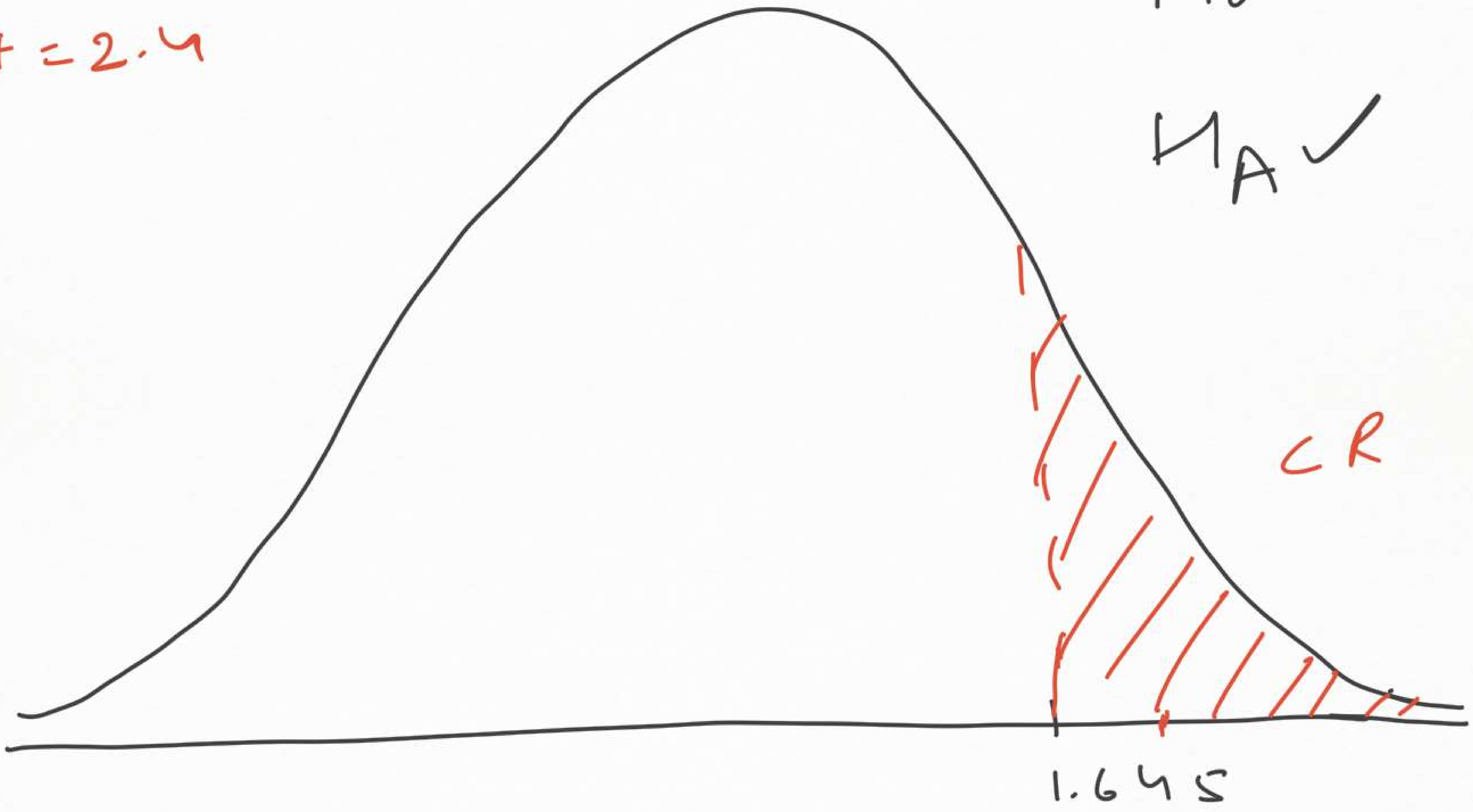
$= 2.4$

$Z_{0.05} = 1.645$

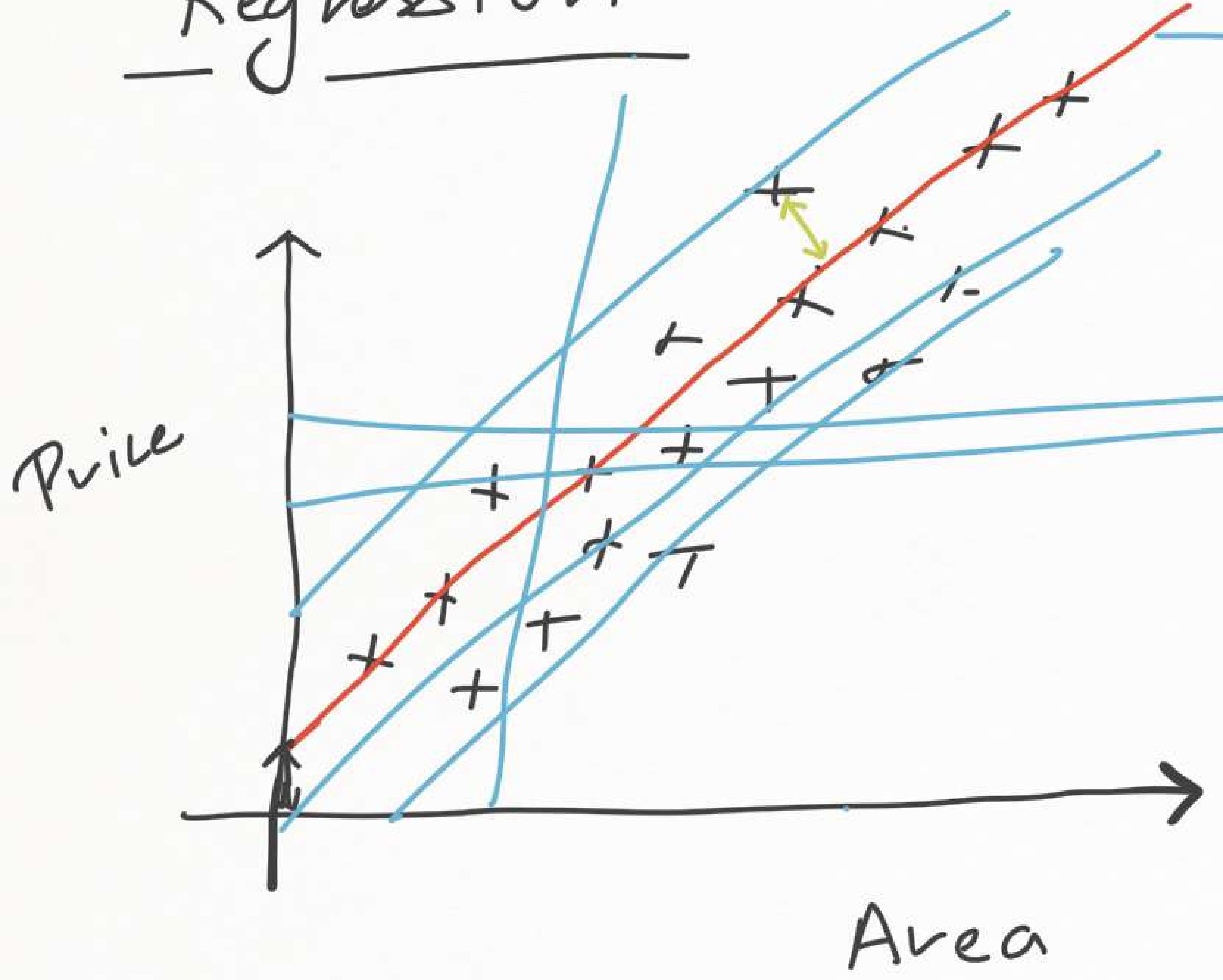
$z\text{-test} = 2.4$

H_0 ✗

H_A ✓



Linear Regression



Best fit line

$$\boxed{y = mx + c}$$

y-intercept

slope

weights

bias

$$\boxed{\hat{y} = \beta x + \beta_0}$$

$$= wx + b$$

$\hat{y} = \beta_1 x_1 + \beta_0$

$\beta = ?$ $\beta_0 = ?$ $\boxed{\begin{matrix} 20 \\ -56 \end{matrix}}$

x H	y W
5.8	60
5.9	62
5.7	58
5.5	57
5.4	62
5.6	?

$\hat{y} = 5.6\beta + \beta_0$

$60 = 5.8\beta + \beta_0$ — (i)

$62 = 5.9\beta + \beta_0$ — (ii)

$\beta_0 = 60 - 5.8\beta$

$62 = 5.9\beta + 60 - 5.8\beta$

$2 = 0.1\beta \Rightarrow$ $\beta = 20$

for 5.4

$$\hat{y} = 5.4 \times 20 - 56$$

$$= 108 - 56$$

$$= 52$$

$$y = 62$$

$$\text{Mean Squared Error (MSE)} = \frac{1}{2}(y - \hat{y})^2$$

$$= \frac{1}{2}(62 - 52)^2$$

$$= \frac{1}{2} \times 100 = 50$$

Optimization Eqn. for LR

$$\boxed{\underset{\beta,\ \beta_0}{\operatorname{argmin}} \ \frac{1}{n}\sum_{i=1}^{n}\left(y-\hat{y}\right)^2}$$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\boxed{M_x(t) = E\left(e^{xt}\right) = \int_{-\infty}^{\infty} e^{xt} f(x)\,dx}$$

$$M_x(t) = \int_{-\infty}^{\infty} e^{xt} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let, $\frac{x-\mu}{\sigma} = z$

$$\frac{\partial x}{\sigma} = \partial z$$

$$\partial x = \sigma \cdot \partial z$$

$$M_x(t) = \frac{1}{\sqrt{2\pi}\ \sigma} \int_{-\infty}^{\infty} e^{(\mu + \sigma z)t} \cdot e^{\frac{-z^2}{2}} \cdot \sigma\, dz$$

$$= \frac{\cancel{\sigma}}{\sqrt{2\pi}\ \cancel{\sigma}} \int_{-\infty}^{\infty} e^{\mu t} + e^{\sigma z t} \cdot e^{-z^2/2}\, dz$$

$$= \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2\left(z^2 - 2\sigma z t + (\sigma t)^2 - (\sigma t)^2\right)} dz$$

$$M_x(t) = \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2 (z - \sigma t)^2} e^{\sigma^2 t^2 / 2} \, dz$$

$$= \frac{2 . e^{\mu t + \sigma^2 t^2 / 2}}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-1/2 (z - \sigma t)^2} \, dz$$

Let, $\frac{1}{2}(z - \sigma t)^2 = \theta$

$$(z - \sigma t)\,dz = d\theta$$

$$dz = \frac{d\theta}{z - \sigma t}$$

$$= \frac{d\theta}{\sqrt{2\theta}}$$

$$M_{(x)}t = \frac{2\, e^{\mu t + \sigma^2 t^2/2}}{\sqrt{2\pi}} \int_0^{\infty} \frac{e^{-\theta}}{\sqrt{2\theta}}\, d\theta$$

$$= \frac{e^{\mu t + \sigma^2 t^2/2}}{\sqrt{\pi}} \int_0^{\infty} \theta^{-1/2} \cdot e^{-\theta}\, d\theta$$

$$\begin{cases} \int_0^\infty x^{n-1} \cdot e^{-x} \, dx = \Gamma n \\ \int_0^\infty \theta^{n-1} \cdot e^{-\theta} \, dx = \Gamma n \end{cases}$$

$$M_{(x)} t = \frac{e^{\mu t + \sigma^2 t^2/2}}{\sqrt{\pi}} \cdot \Gamma n/2$$

$$M_{(x)}t = \frac{e^{\mu t + \sigma^2 t^2/2}}{\cancel{\sqrt{\pi}}} \cdot \cancel{\sqrt{\pi}} \quad \left(\because \Gamma\tfrac{1}{2} = \sqrt{\pi}\right)$$

$$\boxed{M_{(x)}t = e^{\mu t + \sigma^2 t^2/2}}$$

Mean $= E(x)$

$$M_x(t) = e^{\mu t + \sigma^2 t^2/2}$$

$$E(x) = \frac{d}{dt}\left(M_x t\right)$$

$$= \frac{d}{dt}\left(e^{\mu t + \sigma^2 t^2/2}\right)_{t=0}$$

$$\frac{d}{dx} \cdot e^x$$

$$= e^x (e)$$

$$E(X) = \left[e^{\mu t + \sigma^2 t^2/2} \left(\mu + \sigma^2 t \right) \right]_{t=0}$$

$$= e^0 (\mu)$$

$$\boxed{E(X) = \mu}$$

$$\text{Variance} = E(X^2) - (E(X))^2 \qquad (u \cdot v)$$

$$E(X^2) = \frac{\partial^2}{\partial t^2}\left(M_{(X)}t\right)_{t=0}$$

$$= \frac{\partial}{\partial t}\left[e^{\mu t + \sigma^2 t^2/2}(\mu + \sigma^2 t)\right]_{t=0}$$

$$e^0 \quad 1. \, (\mu + 0)^2$$

$$E(x^2) = \Big[e^{\mu t + \sigma^2 t^2/2} \cdot (\mu + \sigma^2 t)^2 +$$

$$e^{\mu t + \sigma^2 t^2/2} \cdot \sigma^2 \Big]_{t=0}$$

$$= \mu^2 + \sigma^2$$

$$\text{Variance} = E(x^2) - (E(x))^2$$

$$= \cancel{\mu^2} + \sigma^2 - \cancel{\mu^2} = \sigma^2$$

Loc1	Loc2	Loc3	Loc4	SS1	SS2	SS3	SS4
5.7	6.2	5.4	3.7				
6.3	5.3	5.0	3.2				
6.1	5.7	5.6	3.9				
6.0	6.0	5.6	4.0				
5.8	5.2	4.9	3.5				
6.2	5.5	5.2	3.6				
6.01	5.65	5.35	3.65				

GM = 5.165