Mathematics and Engineering   Modules: None

In this introductory assignment, we ask you to examine the use of mathematics in the world around you. Pick an Engineering or other real-world topic that interests you.

1. In a few sentences, Explain what your topic is, its background, and why it interests you.

= A topic that interests me is a “Noise Canceling Headphone”. A noise canceling headphone, first engineered by Amar Bose [1], is a special type of headphone that uses active noise control to reduce unwanted ambient sounds [2]. Unlike passive noise control, which relies on the choices of materials to physically block external noise and reduce leakages of internal sound from audio, active noise control reduces unwanted noise by producing anti sound waves that cancels out the incoming ambient sound waves. The reason this topic intrigues me stemmed from my past experience with an ordinary headphone. Due to its poor ability to isolate external noise, I always struggle to hear the audio of my headphones clearly when I wear it in a loud environment. This led me to carelessly raise the volume of my headphones, which ended up damaging my eardrum, causing a temporary sharp pain and hearing loss to my ears. However, ever since I received an active noise canceling headphone, I never experienced any ear pain ever again. Therefore, I wanted to explore what makes a noise headphone so effective in reducing the ambient noise to such a large extent.

2. Pick one aspect of mathematics involved in this topic. Describe this aspect and how it aids our understanding of the topic.

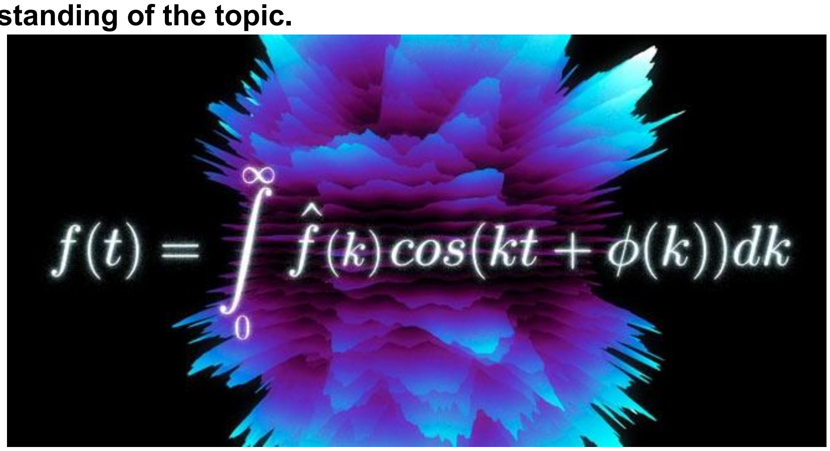

Figure 1 [3]: A Fourier transform expression that produces a wave that is 180 degrees out of phase to resulting incoming complex waves by breaking down the interferences of multiple external sound waves into its simple constituent waves.

= An aspect of mathematics relating to the mechanism of active noise control is Trigonometric Function and Fourier Transform of a function. These two mathematical concepts are combined together to form an equation in Figure 1 which represents the resulting wave that is 180 degrees out of phase to the resulting incoming complex waves. To interpret the expression, we must first understand what a sound wave is. A sound is a “ longitudinal pressure wave” which is created by vibration of an object [3]. When an object vibrates, it causes the particles in the surrounding medium to be in vibrational motion; therefore, energy is being transported through the medium [3]. Since sound is a pressure wave, we can represent a sound as a wave by trigonometric expression of cosine (cos(x)), as represented in Figure 1. Not every sound wave is identical. The uniqueness of each wave is determined by its property of frequency and amplitude. Because the oscillations of sound vary depending on how fast the particle in the medium vibrates, it is equivalent to the "frequency" of the sound wave which determines the sound pitch [4]. In other words, frequency is the measure of the number of wave cycles in a given unit of time. Frequency is determined by the constant "k" in Figure 1, which represents how much the trigonometric function is horizontally stretched/compressed. Thus, different frequencies sound waves will have different horizontal stretched/ compressed factors. Moreover, the loudness of the sound is determined by its amplitude [4], which measures how many particles are displaced by a vibration from its equilibrium position [5]. This is represented by $\widehat{f}(k)$in the expression of Figure 1, which is the amplitude of the wave at specific frequency "k" [4].

Another important property of a wave is its "Interference" property. Sound waves "interfere" with one another when two or more sound waves meet. There are two common types of interference: Destructive and Constructive interference. Destructive Interference occurs when the crest of 1st wave meets the trough of the 2nd wave and both waves are "out of phase" by 180 degrees(or pi radian) [6]. This results in a wave of 0 amplitude or zero loudness as the displacement cancels one another. Constructive interference is the opposite. In noise cancelling headphones, the goal is to get rid of the background noise. Therefore, headphones must generate an anti sound wave that is identical to the incoming sound wave in frequency and amplitude, but must be out of phase by 180 degrees or pi radian to destructively interfere with the incoming wave to a wave with zero amplitude or loudness. This correspond to the $\phi(k)$expression in Figure 1, which represents the degree of phase shift of the wave to be generated out by the headphone to diminish the amplitude of the incoming external sound wave [4].

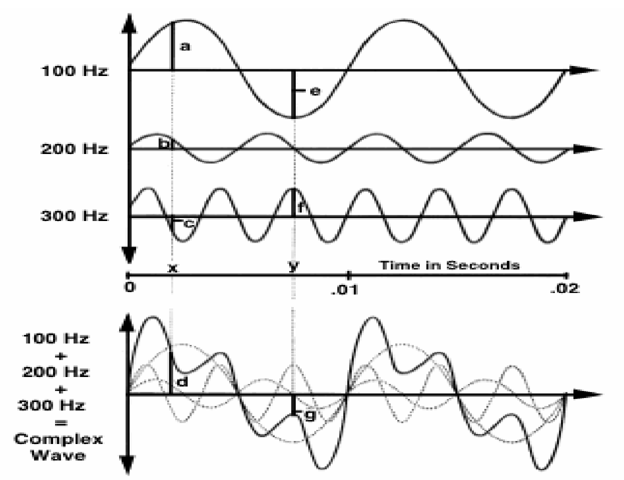

Figure 2 [6]: Idea of Fourier Transform. Breaks down the complex sound wave into simpler waves from the sources that produce them.

Since there are hundreds of sound waves with unique frequency and amplitude constantly generated from different sources on the background, these wave piles on top of one another and are constantly constructively and destructively interfering with one another. Hence, they create a complex resulting wave at every instant as multiple sound waves approach the headphone. This is when Fourier transform comes into play. Since the background noise is an interference of sound waves from various sources, the fourier transform simplifies the complex resulting wave by separating the sound into its simple constituent waves. As represented in **Figure 2**, the fourier transform breaks down the function of time of the resulting complex wave into its constituent frequencies of all the incoming waves that make it up [7]. This is equivalent to the integral expression $\int_0^\infty dk$, which indicates the summation of the formula inside the integral when evaluated for every frequency “k” between 0 and infinity [4]. Since the formula inside is a trigonometric function that produces an anti sound wave for each individual external sound waves that are generated from different sources, the integral will allow the anti sound waves generated for each incoming sound wave to interfere by addition of displacements to produce a single resulting antiwave. This single resulting anti wave will, hence, destructively interfere with the resulting incoming external sound wave. The whole expression $f(t)$, therefore, represents the air pressure(sound) at any given time t [4].

3. Is there anything else you want to know about this topic?

= I would like to know more about how the mathematical aspect of trigonometric functions and fourier transform is applied to the noise cancelling headphones. I wanted to explore deeper into the general physical components of every noise cancellation headphone, especially which components(s) the mathematical aspect has been integrated into. For instance, what physical parts receive the external signal and where and why is it placed on the specific location on the headphone. Is there an integrated circuit that can compute the Fourier transform expression, and how it transfers the information to another component that generates anti waves to destructively interfere with the external sound wave. Since sound can enter from any direction and angle around the ear pads of the headphone, does it mean that there must be more than one wave generator components that are placed around the ear pads? Being able to find answers to these questions will help me see the connections between the mathematical aspect and physical aspects of noise cancelling headphones, as well as the reasons behind why each physical component has been used and placed in a specific way on the headphone.

Reference List:

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Writing Task 1: Mathematics and Engineering (None) MAT186 Calculus I, due Sept. 27