Name: Kanit Mongkonrattanachai
UofT Student Number: 1007390538

MAT186 Modeling Project:

(Topic 8: Just Singing in the Rain)

Introduction:

Humans have relied heavily on weather forecasts to anticipate possible rainfall and plan ahead for the upcoming day. Although rainfall can be accurately forecasted by computer models, there always exists possibilities for rainfalls [1]. If such occasions occurred, how should the person move through the rain to get wet the least? The goal of this paper is to deduce how a person should move to be the least wet under different conditions of rain type and wind. We define the wetness of a person as the mass of rain the person contacts.

Assumption:

Several assumptions are made to simplify the model. First, we assume that a person will contact the rain in 2 directions: vertical and horizontal [2]. The amount of rain falling in the vertical direction depends on the time a person takes to reach the destination [2], whereas the amount of rain in the horizontal direction depends on the distance a person travels [2]. We assume every raindrop has equal volume, are equally spaced, and falls at constant velocity [3]. We assume a person can be represented as a cuboid and is moving at a constant velocity. [3].

Model:

1. Representation of a person:

Assume that a person is represented by a "cuboid". Let a = Length, b = width, and h = height. Hence; <u>Area of Head = ab</u> & <u>Area of Front = ah</u>. Let Person's speed = v_P and Rain's speed = v_R.

2. Angle:

= When a person moves, the rain is approaching them at a specific angle from the person's frame of reference [2]. The person is moving at velocity v_P; therefore, the rain is

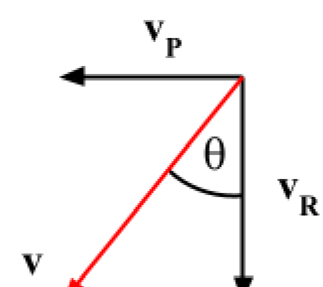


Figure 1: Resultant Vector Representation

traveling horizontally towards the person at equivalent speed, but opposite direction of v_P from the person's frame of reference. Therefore, we obtain Figure 1.

3. Constraints: Time

= The time for a person to reach a destination is given by: $t = d / v_P$. Thus, considering two cases: <u>Case #1: Person moves at Extremely Low Speed</u> - The velocity is presented as infinitely small. Thus we can use limit to compute the time a person stays contacts the rain when the velocity of the person approaches 0. We take $\lim_{v_P \to 0} \frac{D}{v_P} = \frac{D}{0} =+ \infty$. Thus, the slower a person moves, the more time the person contacts the rain. <u>Case #2: Person moves at High Speed</u> - The velocity can be presented as infinitely large. Thus we can use the notion of limit to compute the time a person stays contacts the rain when the velocity of the person approaches $+ \infty$. We take $\lim_{v_P \to \infty} \frac{D}{v_P} = \frac{D}{\infty} = 0$. Therefore, we see that the faster a person moves, the less time the person contacts the rain.

4. Conditions:

1. **Rain Type:**

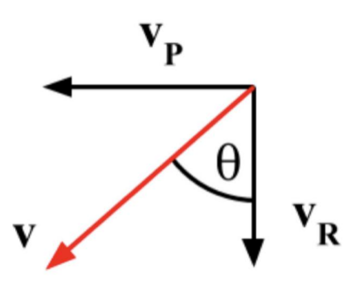


Figure 2: <u>Light Rain</u>

= The rain fall with
Low speed.

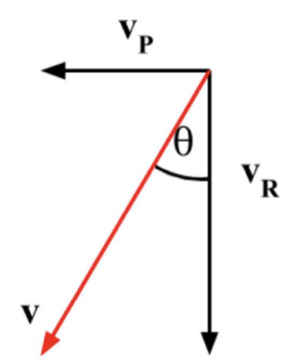


Figure 3: <u>Heavy Rain</u>

= The rain fall with
High Speed.

2. **Wind Conditions:**

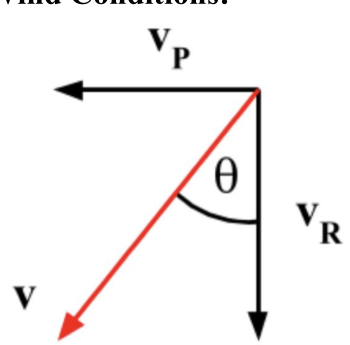


Figure 4: <u>No Wind</u>

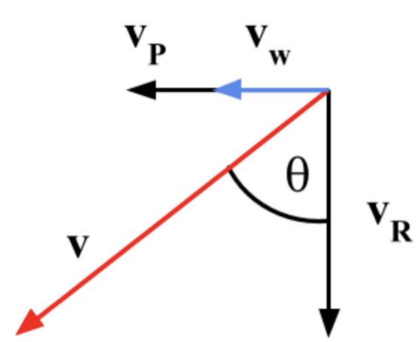


Figure 5: <u>Headwind</u>

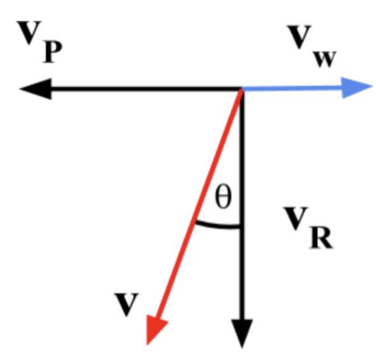


Figure 6: <u>Headwind</u>

Computation:

From the Resultant Velocity Vector v, we can now determine both the magnitude of v and the angle θ.

$|v| = \sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2}$ [2] $\quad\quad tan(\theta) = \frac{(v_P+v_W)}{v_R} \rightarrow \theta = tan^{-1}(\frac{v_P+v_W}{v_R})$ [2]

The area that rain contacts the head and the body of a person represented by a cuboid as:

Area of Head Contact by Rain: $\quad\quad$ Area of Body Contact by Rain:

$A_{Head} = abcos(\theta) = abcos(tan^{-1}(\frac{v_P+v_W}{v_R}))$ [2] $\quad\quad A_{Body} = ahsin(\theta) = ahsin(tan^{-1}(\frac{v_P+v_W}{v_R}))$ [2]

To find the volume of rain that the head/body contacts per unit time, we multiply the Area of Head & Body that contacts the rain to the Magnitude of the resultant velocity of rain.

$A_{Head}\times|v| = abcos(tan^{-1}(\frac{v_P+v_W}{v_R}))\times\sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2}$ [2] $[m^3/s]$ and

$A_{Body}\times|v| = ahsin(tan^{-1}(\frac{v_P+v_W}{v_R}))\times\sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2}$ [2] $[m^3/s]$

To derive the Mass of rain per unit time in kg/s, we multiply the rate of volume of rain the head/body contacts to the Density of rain. Thus, we have:

$A_{Head}\times|v| \times \rho = abcos(tan^{-1}(\frac{v_P+v_W}{v_R}))\times\sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2}\times(\rho)$ [2] [kg/s]

$A_{Body}\times|v| \times \rho = ahsin(tan^{-1}(\frac{v_P+v_W}{v_R}))\times\sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2}\times(\rho)$ [2] [kg/s]

The Mass of the rain a person contacts, can be obtained by multiplying the Mass of rain per unit time to the time a person contacts the rain. Since, time relates to the distance and speed a person moves by $t = \frac{d}{v_P}$.

Thereby, we have:

Mass of Rain contacting Person's Head [kg]:

$$M_{\text{Rain on Head}} = A_{\text{Head}} \times |v| \times \rho \times t = \text{abcos}(tan^{-1}(\frac{v_P + v_W}{v_R})) \times \sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2} \times (\rho) \times t \quad [2]$$

Mass of Rain contacting Person's Body [kg]:

$$M_{\text{Rain on Body}} = A_{\text{Body}} \times |v| \times \rho \times t = \text{ahsin}(tan^{-1}(\frac{v_P + v_W}{v_R})) \times \sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2} \times (\rho) \times t \quad [2]$$

Finally, sum of $M_{\text{Rain on Head}}$ & $M_{\text{Rain on Body}}$ can be represented by $M_{\text{Total}} = M_{\text{Rain on Head}} + M_{\text{Rain on Body}}$, which is equivalent to:

$$M_{\text{Total}} = [\text{abcos}(tan^{-1}(\frac{v_P + v_W}{v_R})) \times \sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2} \times (\rho) \times t] \quad [2]$$

$$+ [\text{ahsin}(tan^{-1}(\frac{v_P + v_W}{v_R})) \times \sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2} \times (\rho) \times t] \ [2] \quad [kg]$$

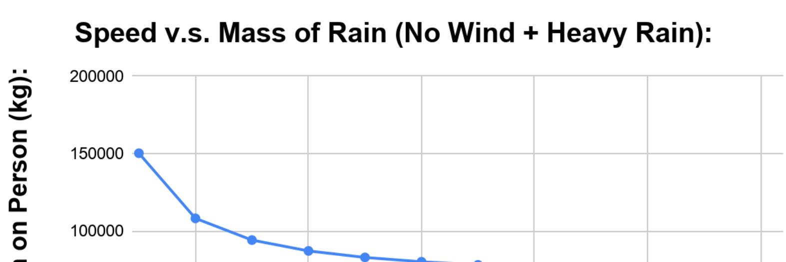
For practical purposes, we assumed values for several variables, shown in Table 1.

Table 1: Represents the Assumed Values for each Variables.

	Variable:	Assumed Value:	Unit:
	Person's Length (**a**):	0.411	m
	Person's width (**b**):	0.254	m
	Person's Height (**h**):	1.8	m
	Density (ϱ):	895	kg/m^3
	Angle (**θ**):	$\theta = tan^{-1}(\frac{v_P + v_W}{v_R})$	Degree
	Distance (**d**):	100	m
	Time (**t**):	$t = \frac{d}{v_R} = \frac{10}{v_R}$	m/s
	Person speed (v_P):	v_P	m/s
Rain Speed (v_R):	Heavy Rain speed (v_{Rl}):	$v_{Rl} = 9$	m/s
	Light Rain speed (v_{Rh}):	$v_{Rh} = 2.2$	m/s
Wind Speed (v_W):	No Wind (v_{W0}):	$v_{W0} = 0$	m/s
	Headwind (v_{Wh}):	$v_{Wh} = +8$	m/s
	Tailwind (v_{Wt}):	$v_{Wt} = -8$	m/s

Applying the derived equation with the assumed value, we can examine the Mass of water a Person's contacts at different moving speeds under the 6 circumstances.

Analysis:



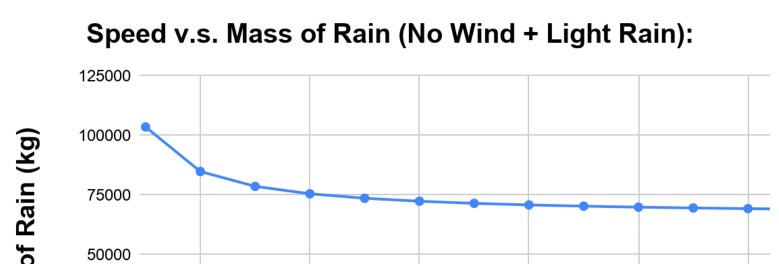


Figure 8: Mass of Rain under No Wind & Heavy Rain

Figure 9: Mass of Rain under No Wind + Light Rain

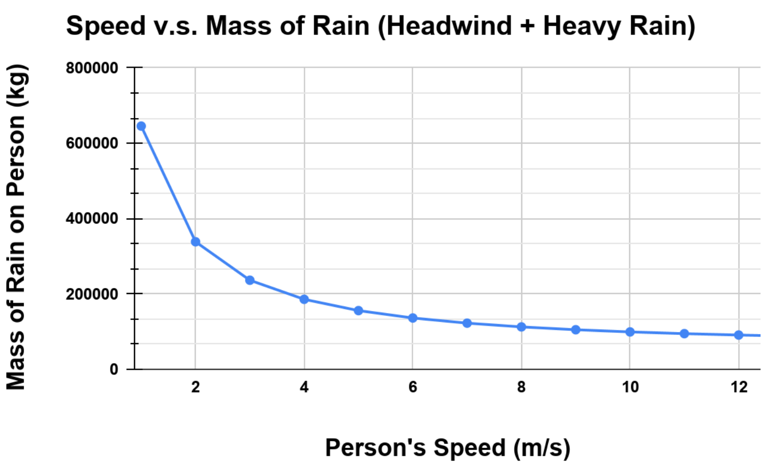


Figure 10: Mass of Rain under Headwind & Heavy Rain

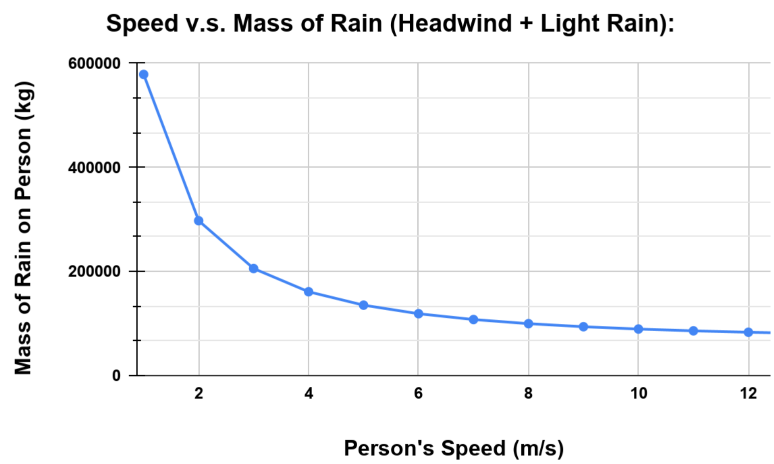


Figure 11: Mass of Rain under Headwind & Light Rain

In the case of no wind and headwind, Figure 8, 9, 10, 11 show that as the person moves at increasing speed, the total mass of rain a person contacts decreases. This suggests that $\lim_{v_p \to \infty} M_{Total} = 0$. Hence, under the closed domain of (0, 12.4], the least amount of rain a person can catch is at the right endpoint of 12.4 m/s or the fastest speed that a person can run. For no wind condition, the person will cover the same mass of rain in the front body regardless of the rate at which the person moves [5]. Moreover, as represented by Figure 4, the presence of a headwind increases the angle, causing the resultant vector becomes more horizontal. Since $A_{Head} = ab\cos(\theta)$ and $A_{Body} = ah\sin(\theta)$. As the angle increases towards 90 $\circ$, $\cos(\theta)$ decreases, while $\sin(\theta)$ increases. Therefore, the mass of rain contacting person's head decreases, while on person's front body increases. Since the A_{Body} scales more than the A_{Head} for a person, the resulting mass of rain that the person catches when headwind is present, is the greatest compared to other cases [5].

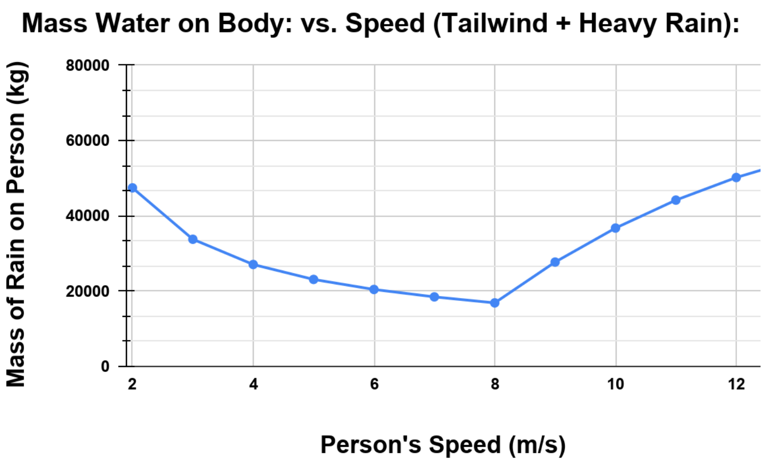


Figure 10: Mass of Rain under Headwind & Heavy Rain

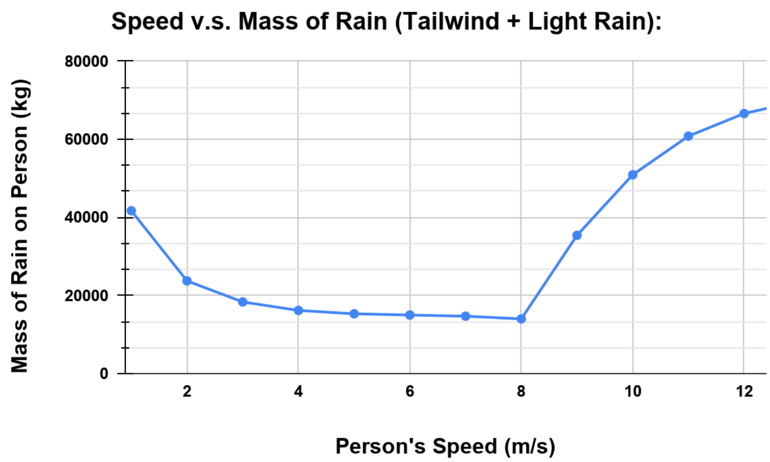


Figure 11: Mass of Rain under Headwind & Light Rain

From Figure 12 and 13, the local minimum of the graph can be found by taking the 1st derivative of the graph. Thus, we find that the local minimum exists when the person's speed is 8 m/s for both heavy and light rain, which is equivalent to the tailwind's speed. Therefore, a person contacts least mass of rain when moving at the same speed as tailwind [4]. From figure 6, the tailwind reduces the angle, causing the resultant vector to become more vertical. When the person runs at an equivalent speed to rain fall speed, the resultant vector in the horizontal direction becomes zero as $v_P = -v_W$ and $-v_W + v_W = 0$. Therefore, the angle between v_P & $-v_W$ becomes 0. Since, $M_{Body} = ahsin(\theta) \times \sqrt{(v_P)^2 + (v_R)^2 + (v_W)^2} \times (\rho) \times t$, when $\theta = 0 \rightarrow M_{Body} = 0$ kg. Thus, the person only contacts the rain on their head. Since A_{Head} scales less A_{Body}, this scenario results in the least mass of water.

Limitations:

This model does not take into account the influence of water drop temperature, air density, and atmospheric pressure on raindrop's Reynolds number and raindrop's surface tension, which determine the raindrop's shape and breakup velocity when it experiences aerodynamic forces [6]. The raindrop's size reduction decreases its terminal speed and results in varying velocity as it falls [6], which this model disregards. The irregular shapes of human heads and body were not considered in this model. Likewise, the model is only an accurate representation of an adult American male and is not an accurate representation for people of other nations or age ranges.

Reference List:

[1] "Explainer: how is rain forecast?," *Explainer: how is rain forecast? - Social Media Blog - Bureau of Meteorology*, 11-Feb-2020. [Online]. Available: http://media.bom.gov.au/social/blog/2332/explainer-how-is-rain-forecast/. [Accessed: 09-Oct-2020].

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