MAT186 Writing Task 4:

Filling the decanter Modules: E1

Watch the video here: https://youtu.be/HMnuu_HyFIw. The rate of water flowing in is constant. How long will it take to fill the decanter?

1. Explain how you obtained your estimate.
(a) Volume of the decanter:

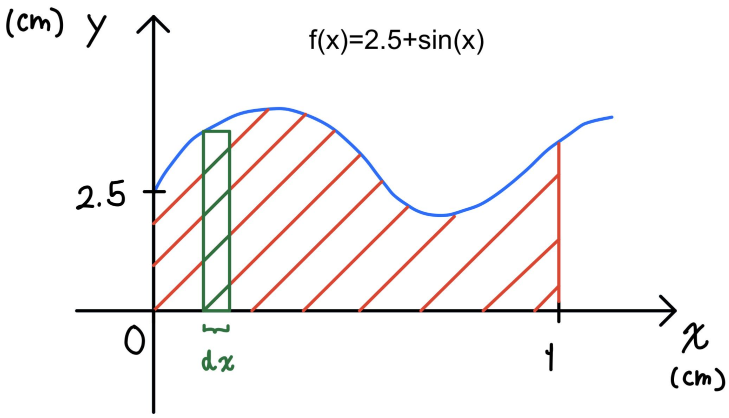

Figure 1: Horizontal Orientation Representation of Decanter.

= We can vertically cut the decanter in half and rotate it from the vertical by 90 degrees. This way, we can visualize the decanter in a horizontal orientation as illustrated in Figure 1. The curve of the decanter resembles the sine function. From the video, we can determine the radius of the decanter equal to approximately 2.5 cm and the height equal to 10 cm. Hence, we can determine the function representing the curve of the decanter to be: $f(x) = 2.5 + sin(x) \quad for\ domain\ [0, 9]$.By revolving the decanter in Figure 1 about the x-axis, its volume can be obtained by slicing the decanter into thin sheets of cylinder. If we take the cross section between $[x_{i-1}, x_i]$, then the volume of the cylinder can be approximated by multiplying the cross sectional area of the circle with the thickness of the sheet by the equation: $(\pi r^2) \times \Delta x$. By adding up the volumes of these cylindrical slices and taking the limit as $n \to \infty$, we can therefore obtain the volume of the decanter as:

$V = \lim_{n \to \infty} \sum_{i=1}^{n} \pi r^2 \Delta x$, which is equivalent to $V = \int_0^{10} \pi r^2 dx$ by the limit definition of definite integral.

Since the radius(r) depends on x by the function $f(x) = 2.5 + sin(x)$; we can further obtain the expression: $V = \int_0^{10} \pi(2.5 + sin(x))^2 dx$. Integrating the expression, we get:

$$V = \pi \int_0^{10} 6.25 + 5sin(x) + sin^2(x)dx \quad \to \quad V = \pi[6.25x - 5cos(x) + (-\frac{sin(2x)-2x}{4})] \Big|_0^{10}$$

$$\to V = \pi[6.25x - 5cos(x) + \frac{1}{2}x - \frac{1}{2}sin(x)cos(x)] \Big|_0^{10}$$

V = $71.467\pi - (-5\pi) = 240.23\ cm^3$

Therefore, the volume of the decanter according to the scale provided in the video is approximately $240.23\ cm^3$.

(b) Rate of water flowing in, and how to put the two together to get an estimate:

= By determining the height of water filled at a specific time t, we can determine the constant rate of the water flowing into the decanter. From the video, at time t=30s, the height of the water filling the decanter is at approximately 1.5 cm. Therefore, we can determine the volume of water that had filled the decanter at t=30 seconds. From part 1(a), we determined that the volume of the decanter can be expressed as: $\int_n^m \pi(2.5 + sin(x))^2 dx$. The decanter is initially empty at t=0s, and by t=30s, the decanter is filled up to the height 1.5 cm. Therefore, the lower and upper bound of the integral is [0,1.5] cm. Hence, the volume of the water that filled the decanter can be determined by: $\int_0^{1.5} \pi(2.5 + sin(x))^2 dx$. Integrating the expression, we get:

$$V = \pi \int_0^{1.5} 6.25 + 5sin(x) + sin^2(x)dx \quad \rightarrow \quad V = \pi[6.25x - 5cos(x) + (-\frac{sin(2x)-2x}{4})] |_0^{1.5}$$

$$\rightarrow V = \pi[6.25x - 5cos(x) + \frac{1}{2}x - \frac{1}{2}sin(x)cos(x)] |_0^{1.5}$$

V = $9.7360\pi - (-5\pi) = 46.295\ cm^3$ when time t=30 seconds.

Therefore, the constant rate of water flowing in can be determined by dividing the volume of water that filled the decanter by the time it takes to fill the respective amount as shown by:

$$Rate\ water\ flowing\ in\ = \frac{46.295\ cm^3}{30\ s} = 1.5432\ cm^3/s.$$

With the known rate of water flowing into the decanter, we can determine the time it takes to fill the entire decanter. Since the rate of water flowing into the decanter is $1.5432\ cm^3/s$ and the total volume of the decanter is $240.23\ cm^3$ from part 1(a), by proportionality, we have:

$$\frac{Volume\ of\ water(cm^3)}{Time\ (s)} = \frac{1.5432}{1} = \frac{240.23}{x}$$

Thus, solving for the total time “x”, we get:

$1.5432x = 240.23$

$x = 155.67s \approx 2\ minutes\ and\ 36\ seconds.$

2. (Optional) Put an upper bound on the error (of your “time to fill” estimate), and explain how you got this number.

Since the measurement of height of water is made based on the given scale of the video, it may be off by 0.1 cm. In such cases, the lower bound of the height of water would be 1.4 cm instead of 1.5 cm. Hence, the volume of water that filled the decanter at t=30s would be

$V = \pi \int_{0}^{1.4} 6.25 + 5sin(x) + sin^2(x)dx = 42.463\ cm^3$ when t=30s, giving the rate of water flowing in as: $1.4154\ cm^3/s$. Hence, by proportionality, $\frac{Volume\ of\ water(cm^3)}{Time\ (s)} = \frac{1.4154}{1} = \frac{240.23}{x} \rightarrow x = 169.72s$

Error in fill time $\leq |169.72 - 155.67| \leq 14.05\ s$

	Clarity and conciseness of written exposition; consideration of audience	Good movement between English & Mathematics	Mathematical Thinking
Below expectations (0)	Writing is confusing, or shows signs of carelessness through high level of error; does not demonstrate application of engineering communication; only understandable to the writer.	Math is presented without context; it is difficult to reconstruct the meaning of equations.	No demonstration of any significant level of mathematical analysis at an appropriate level. No serious attempt to apply mathematics.
Shows potential (1)	Writing requires some effort to understand, or has some errors rarely affecting understanding; some evidence of communication principles but may be awkward or simplistic; offers sufficient information and demonstrates mostly logical development of ideas; generally directed to the instructor or TA.	While the writing is clear, the math is kept completely separate from it; requires some effort to understand how the math and writing are connected.	A serious attempt is made to apply appropriate math and explain mathematical reasoning, regardless of completeness or correctness.
Meets expectations (2)	Writing shows appropriate selection and balance of text and image (if necessary); demonstrates clarity at the paragraph, sentence, and word choice levels, with minimal error; writing applies principles in ways that enhance the reading experience; generally directed towards a fellow student who understands the math but not this particular instance.	The interaction of the text with the math is adequate; any transitions between math and text are appropriately placed and make sense; any equations and variables are introduced and explained in an organized manner; any solutions are interpreted in a real-world context where appropriate; no guesswork is needed by the reader.	The response shows a good ability to analyze a situation using math; relevant topics are applied with proper notation; any solutions are critically analyzed for sensibility; any obviously incorrect solutions are remarked upon and followed up with a sketch of an alternative idea or plan.
Above expectations; bonus (3)	The writing demonstrates professional polish through word choice, sentence structure choices, clarity, concision, proofreading, etc. This solution is presentable as a textbook example or exposition.	The math is merged very well with the writing; any equations and variables are embedded, clear, and concise; any transitions between text and mathematics aid understanding; any solutions are contextualized and explained in a professional manner.	The analysis is excellent, creating a large amount of insight into the prompt and demonstrates mastery of the topic at or above the course level.