MAT188 Application Paper:

Introduction:

Circuits serve as a crucial element in any electronic systems today. The functionality of a circuit lies in its capability to transfer electricity [1]. This flow of electricity is current, which allows for the operation of circuits. The three fundamental components of a circuit are current, voltage, and resistance. Ohm's Law relates these 3 components of any circuit by the relationship: V = IR. Therefore, changing the voltage or resistance of each part of the circuit will, as a result, change the current. For any circuit, the ability to control the amount of current in the circuit is vital since excess current can cause a short circuit, posing safety concerns. This can result in a rapid release of energy and potentially to an "arc blast" explosion [2]. Each electronic device has a specific rated current requirement which is the most appropriate current range that can flow to operate without any danger. Therefore, manipulations on the voltage and resistance through the presence of, for instance, a battery or resistor, will allow the current generated to be within the rated current range. Thus, being able to analyze the current voltage of any circuit serves as a foundation of both simple and complex circuits.

How exactly do we measure the amount of current voltage of a more complex circuit system, consisting of many parallel and series circuits, and with different types of electrical providers such as battery, cell, or power supply? This can be done through Kirchhoff's Current Law and Kirchhoff's Voltage Law. This method helps simplify our analysis of the circuit for the current of the circuit. Linear Algebra comes in on this part. When we apply Kirchoff's law to construct several systems of equations of current and voltage, we can convert the systems of equations into an Augmented matrix form, where we can use Gaussian elimination to reduce the matrix into its Reduced row echelon form. This will allow us to compute for the resulting value of each current in each circuit of the whole circuit system.

Applying Linear Algebra:

Let's consider a circuit in Figure 1. It is a parallel circuit, which has a constant voltage source. By Kirchoff's Current Law, the sum of currents entering or leaving a node is equal to zero [3]. We assume that the current leaving the node is assumed to be positive [5]. To analyze the circuit, a reference node is chosen, where the voltage is equal to zero. This helps simplify the analysis of not having to specify the pairs of nodes for voltage measurement. Suppose that a reference node is chosen as indicated in Figure 1. We then assign variables to represent voltage at every other node [4]. Since a constant voltage source exists on the Left Circuit, it forces the voltage at V_1 to be exactly 10 volts higher than the reference node [4]. Therefore: $V_1 = 10$

Now, by applying Kirchhoff's Current Law to node V_2. Since node V_2 has 3 different paths for current leaving [4]. We can apply Ohm's Law of I = V/R to

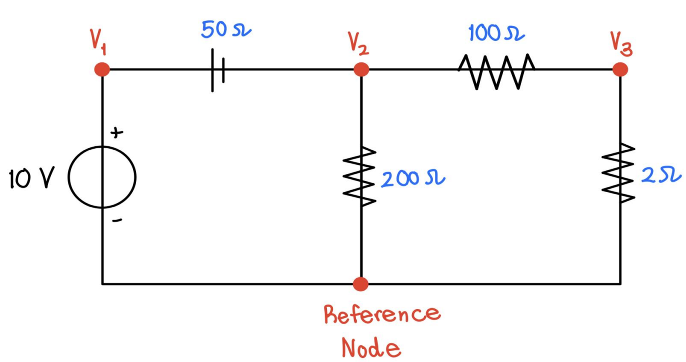

Figure 1: Sample Circuit

determine the current that leaves node V_2. 1.) For current I_1 leaving the 50 ohm resistor, the Voltage across resistor is $V_2 - V_1$. Therefore: $I_1 = \frac{v}{R} = \frac{(V_2 - V_1)}{50}$ .2.) For current I_2 leaving the 100 ohm resistor, the Voltage across resistor is $V_2 - V_3$ [4]. Therefore: $I_2 = \frac{v}{R} = \frac{(V_2 - V_3)}{100}$ 3.) For the current I_3 leaving the 200 ohm resistor, the Voltage across the resistor is $V_2 - 0$ [4]. Therefore: $I_3 = \frac{v}{R} = \frac{(V_2 - 0)}{200}$. Thus; by Kirchhoff's Current Law, we find that the Total Current at Node $V_2$is: $I_1 + I_2 + I_3 = 0 \rightarrow \frac{(V_2 - V_1)}{50} + \frac{(V_2 - V_3)}{100} + \frac{(V_2 - 0)}{200} = 0$

$4(V_2 - V_1) + 2(V_2 - V_3) + 1(V_2 - 0) = 0 \rightarrow -4V_1 + 7V_2 - 2V_3 = 0$

Lastly, by applying Kirchhoff's Current Law to node V_3. Since node V_3has 2 different paths for current leaving. We can apply Ohm's Law of $I = \frac{v}{R}$to determine the current that leaves node V_3 [4]. 1.) For current I_2 leaving the 100 ohm resistor, the Voltage across resistor is $V_3 - V_2$. Therefore: $I_2 = \frac{v}{R} = \frac{(V_3 - V_2)}{100}$. 2.) For current I_4 leaving the 2 ohm resistor, the Voltage across resistor is $V_3 - 0$. Therefore: $I_2 = \frac{v}{R} = \frac{(V_3 - 0)}{2}$. Therefore, the sum of current entering and leaving node $V_3$is: $\frac{(V_3 - V_2)}{100} + \frac{(V_3 - 0)}{2} = 0 \rightarrow (V_3 - V_2) + 50(V_3 - 0) = 0 \rightarrow$

$-V_2 + 51V_3 = 0$

From all the 3 Nodes: V_1, V_2, V_3, we obtain 3 Equations of Voltage:

$$\begin{aligned} V_1 + 0V_2 + 0V_3 &= 10 \\ -4V_1 + 7V_2 - 2V_3 &= 0 \\ 0V_1 - V_2 + 51V_3 &= 0 \end{aligned}$$

We can express the Equations in a Matrix form of A**x** = **b**, where A = Coefficient matrix ; **x** = Variables vector ; **b** = Constant Matrix. We can express the matrix as an "Augmented Matrix" Form of [A | **b**], where can perform Gaussian Elimination through using elementary row operations to reduce the augmented matrix into a Reduced Row Echelon Form (RREF).

$$\begin{bmatrix} 1 & 0 & 0 \\ -4 & 7 & -2 \\ 0 & -1 & 51 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ -4 & 7 & -2 & 0 \\ 0 & -1 & 51 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ -4 & 7 & -2 & 0 \\ 0 & -1 & 51 & 0 \end{array}\right] \end{array} \quad R_2': R_2 + 4R_1$$

$$\begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & -\frac{2}{7} & \frac{40}{7} \\ 0 & 0 & \frac{355}{7} & \frac{40}{7} \end{array}\right] \end{array} \quad R_3': \frac{7}{355} \times R_3$$

$$\begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & -\frac{2}{7} & \frac{40}{7} \\ 0 & 0 & 1 & \frac{8}{71} \end{array}\right] \end{array} \quad R_2'': R_2 + \frac{2}{7} R_3$$

$V_1 \quad V_2 \quad V_3$

Hence, we obtain the solution to this Matrix that:

$V_1 = 10\,V$

$V_2 = 408/71\,V$

$V_3 = 8/71\,V$

Knowing the voltage, we can also determ dd
The current by:

$I_1 = \frac{v}{R} = \frac{(408/71-10)}{50}$ = **-0.08507 Ampere**

$I_2 = \frac{v}{R} = \frac{(408/71-8/71)}{100}$ = **0.05634 Ampere**

$I_3 = \frac{v}{R} = \frac{(408/71-0)}{200}$ = **0.02873 Ampere**

Reference Lists:

[1] C. Flinn, “The Basic Anatomy of an Electrical Circuit,” *Basic Electricity*. [Online]. Available: https://pressbooks.bccampus.ca/basicelectricity/chapter/wires/. [Accessed: 10-Dec-2020].

[2] W. P. Mendenhall, "DC Arc Hazard Mitigation Design at a Nuclear Research Facility," in *IEEE Transactions on Industry Applications*, vol. 51, no. 1, pp. 69-72, Jan.-Feb. 2015, doi: 10.1109/TIA.2014.2351251.

[3] S. T. Karris, *Circuit Analysis I with MATLAB Applications*. Fremont: Orchard Publications, 2014.

[4] L. L. Scharf, *A First Course in Electrical and Computer Engineering*. OpenStax-CNX, 2009.

[5] J. G. Gottling, “Node and mesh analysis by inspection,” *Education, IEEE Transactions on*, vol. 38, no. 4, pp. 312–316, Nov. 1995.