DERIVATIVE HEDGING

PART-A>

1>a>To hedge the expected sale of 500,000 bushels of wheat, the farmer would need to enter into 100 wheat futures contracts. This is because each contract represents the delivery of 5,000 bushels of wheat, and 500,000 bushels / 5,000 bushels per contract = 100 contracts.
Since the farmer wants to sell 500,000 bushels of wheat at the end of a year, he would enter into 100 future contracts of the short hedge with expiry date at the end of the year when he plans to sell the wheat.

b>Since the farmer is short hedging, he is effectively making sure that he would be able to sell his wheat at the pre-determined price at the end of the year, protecting himself from the volatility of prices. **So, he is hedging against the rise and fall of prices, by locking down the price he would get for his wheat.**
By using futures contracts, the farmer can reduce the risk of losing money due to changes in the price of wheat, allowing him to focus on farming operations.

2> To find the final realized payoff per barrel of oil, let us move step by step and analyze the payoff for each of the futures contracts the company has entered.

In April 2021, the company has shorted 100 October 2021 futures contracts which they have closed in September 2021.
October 2021 futures price in April 2021 = 119.80 per barrel
October 2021 futures price in September 2021 = 119.20 per barrel

Since the price decreased when they closed the contract and the company had shorted the future contracts, they have made a profit of **119.80-119.20 per barrel=0.6 per barrel**

In September 2021, the company had shorted 100 March 2022 futures contracts which they have closed in February 2022.
March 2022 futures price in September 2021 = 118.80 per barrel
March 2022 futures price in February 2022 = 118.00 per barrel

Since the price decreased when they closed the contract and the company had shorted the future contracts, they have made a profit of **118.80-118.00 per barrel=0.8 per barrel**

In February 2022, the company had shorted 100 July 2022 futures contracts which they have closed in July 2022.
July 2022 futures price in February 2022 = 117.60 per barrel
The spot price in July 2022= 117.10 per barrel

Since the price decreased when they closed the contract and the company had shorted the future contracts, they have made a profit of **117.60-117.10 per barrel=0.5 per barrel**

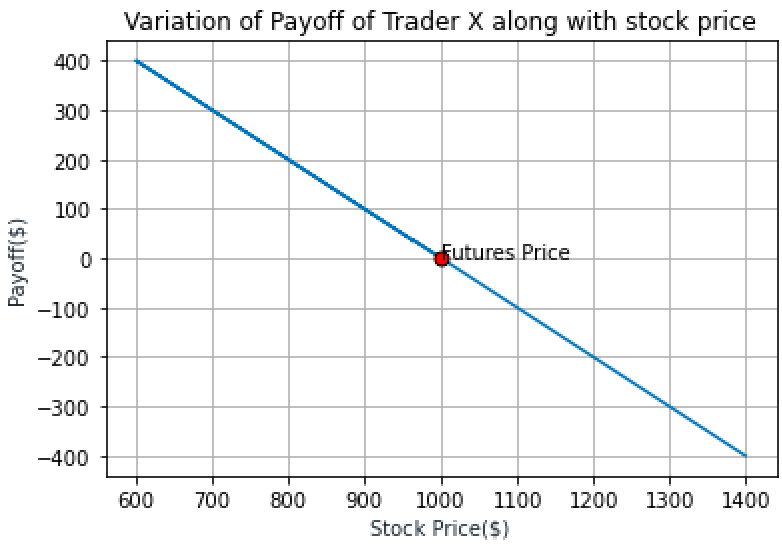
So, the total payoff they have made=0.6+0.8+0.5=1.9 per barrel

3>

Trader X-
Futures price(pre-determined price at which the trader has agreed to sell his asset at the expiry date)=1000$
Expiry date- one year from when the contract has been made

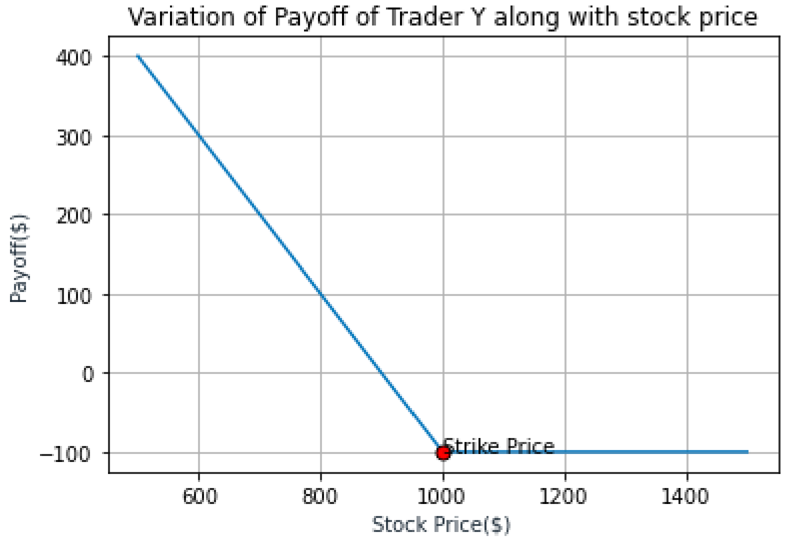
The trader will make a profit if the spot price at the end of the year(suppose x$) is less than the futures price as he would be able to sell his asset at a price higher than the current market price, and his profit would be equal to (1000-x)$. On the other hand, he would incur a loss if the spot price at the end of the year would be greater than the futures price as he would have to sell his asset at a price less than its current market price, and his loss would be equal to (x-1000)$.

So, we can see that his payoff would essentially be (1000-x)$.



Trader Y-
Put option to sell his asset
Strike Price- 1000$
Premium- 100$
Expiry date- one year from when the option has been purchased
Let the spot price at the end of the year be x$
Trader Y will exercise his put option only when the spot price at the end of the year is less than the strike price as then he could sell his asset at a price higher than the current market price. In this case, his profit would be equal to (1000-x)$- premium paid by him= (1000-x)-100= (900-x)$.

Now if the spot price at the end of the year is greater than the strike price then the put option would not be exercised as then he would incur a loss as he would have to sell his asset at a price less than its current market price. So, in this case, his payoff would be -(premium paid by him), i.e, -100$.



In the yellow region of the graph, where spot price is less than 1100$, future outperforms options. In the pink region of the graph,where spot price is more than 1100$, option outperforms future.

(NOTE- code for the graphs have been attached in file KannanRustagi_Derivatives_PartA_Q3_MAIN.ipynb)

PART-B>

a>Since the trader pays a premium for both instruments, he/she has a long position in both options.

Both the call and put options have the same expiry date and are exercised for the same underlying asset and the same strike price. *Let's assume the strike price to be K $.*

Assume that the premium paid for the call option is P1 $ and the premium paid for the put option is P2 $.

Let the spot price at expiration be St $.

Case-1> St<k, i.e, the spot price at expiry is less than the strike price. **In this case, the put option will be exercised** as the trader will make a profit of (k-St)$ by being able to sell at a price higher than the current market price.
The call option would not be exercised in this case as the trader would only incur a loss if he buys the asset at a price higher than the current market price.

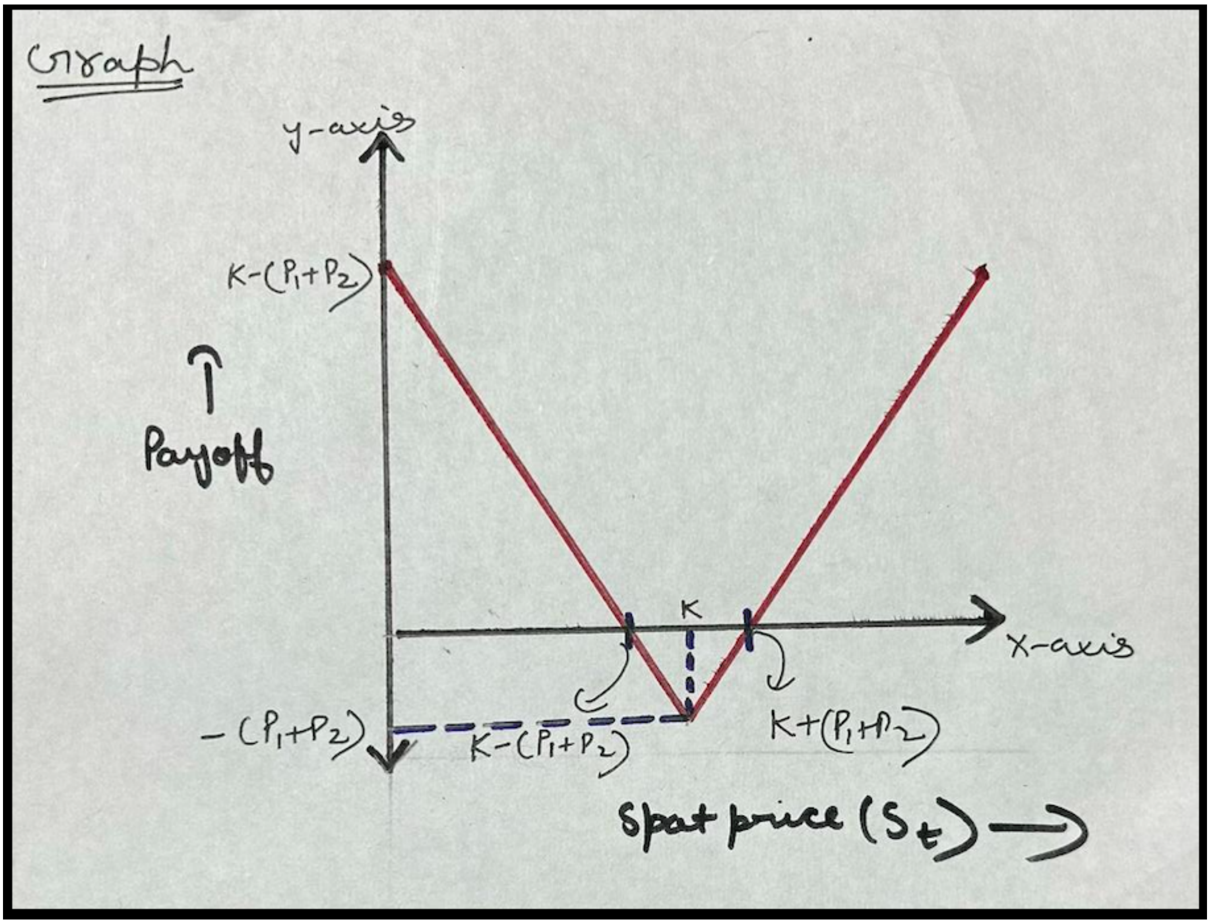
Payoff for case-1= (k-St)-(P1+P2)= ((k-P1-P2) - St) $

Case-2> St=k, i.e, the spot price at expiry is equal to the strike price. **In this case, neither the call option nor the put option will be exercised** as since the current market price is equal to the strike price, so neither buying the asset at strike price (call option) nor selling the asset at strike price (put option) would yield a profit. So, my payoff, in this case, would just be -(premiums paid for both options).

Payoff for case-2= -(P1+P2) $

Case-3> St>k, i.e, the spot price at expiry is greater than the strike price. **In this case, the call option would be exercised** as the trader will make a profit of (St-k)$ by being able to buy the asset at a price lower than the current market price. **The put option would not be exercised** in this case as the trader will only incur a loss by selling the asset at a price lower than the current market price.

Payoff for case-3= (St-k)-(P1+P2)= (St - (k+P1+P2)) $



This strategy is essentially what we call a **straddle options strategy** in which as visible from the graph, the trader will incur a profit in case there is a large fluctuation in the price of the asset in either of the directions, i.e, either there is a large decrease in the price of the asset from the strike price or there is a large increase in the price of the asset from the strike price.

Maximum loss incurred is (P1+P2)$ when the spot price at expiry is equal to the strike price of the call and put option.
Break-even points for the strategy is when the spot price at expiry is equal to either k-(P1+P2)$ or k+(P1+P2)$ as is visible from the graph.

b> The trader is trading in the following three instruments-
1.going long on an ITM(in the money) call option->

Suppose that Strike Price= k1 and premium paid= p1 for the above ITM call option

2.going long on an OTM(out of the money) call option->
Suppose that Strike Price= k2 and premium paid= p2 for the above OTM call option

3.going short on 2 ATM(at the money) call options->
Suppose that Strike Price= k and premium received= p3 for each of the above ATM call option

I am assuming that all the options are for the same underlying asset.

Hence, the trader pays a premium of p1+p2 and receives a premium of 2p3.

In the money(ITM) call option means that the strike price of the option is lower than the current prevailing market price at the time that the option is purchased.
Out of the money(OTM) call option means that the strike price of the option is higher than the current prevailing market price at the time that the option is purchased.
At the money(ATM) call option means that the strike price of the option is same as the current prevailing market price at the time that the option is purchased.

It has been assumed that all the options have same expiration date, and the distance between each strike price of the constituent leg is the same which means that the difference between the current prevailing market price, i.e, the strike price of ATM call option, and the strike price of the ITM call option is same as difference between the strike price of OTM call option and the current prevailing market price, i.e, the strike price of ATM call option. *Let this difference be x $.*

So, essentially,
k-k1=x => k1=k-x;
k2-k=x => k2=k+x;

Let the spot price of the asset at the expiry date be St.

Case-1> St<=k1 => St<=k-x, i.e, the spot price at the expiry date is less than the strike price of the ITM call option.

In this case, the ITM call option would not be exercised as the trader would not profit if he buys the asset at a price higher than or equal to the current market price. Similarly, as St<=k1, St<k2 as St<k+x, the OTM call option would not be exercised as the trader would incur a loss if he buys the asset at a price higher than the current market price. As, St<k-x => St<k, hence the ATM call options would not be exercised by the person to whom the trader has sold the ATM call options, as that person would incur a loss if he buys the asset at a price higher than the current market price. **So, in case-1, essentially none of the options would be exercised.**

Payoff for case-1= 2p3-(p1+p2) $

Case-2> k1< St<= k,i.e, the spot price lies between the strike price of the ITM call option and ATM call option.

In this case, the ITM call option would be exercised as the trader would incur a profit of (St-k1)$ if he buys the asset at a price lower than the current market price. As, St<=k, hence **the ATM call option would not be exercised** by the person to whom the trader has sold the ATM call options, as that person would not profit if he buys the asset at a price higher than or equal to the current market price. As St<=k => St<k2, the **OTM call option would not be exercised** as the trader would incur a loss if he buys the asset at a price higher than the current market price.

Payoff for case-2= (St-k1) + 2p3-(p1+p2) $= (St- (k-x)) + 2p3-(p1+p2) $

Case-3> k<St<=k2, i.e, the spot price lies between the strike price of the ATM call option and OTM call option.

In this case, the ITM call option would be exercised as the trader would incur a profit of (St-k1)$ if he buys the asset at a price lower than the current market price. **The ATM call option would be exercised** by the person to whom the trader has sold the ATM call option as that person would incur a profit of 2*(St-k)$ if he buys the asset at a price lower than the current market price. So, in turn, the trader would incur a loss of 2*(St-k). As St<=k2, **the OTM call option would not be exercised** as the trader would not incur a profit if he buys the asset at a price higher than or equal to the current market price.

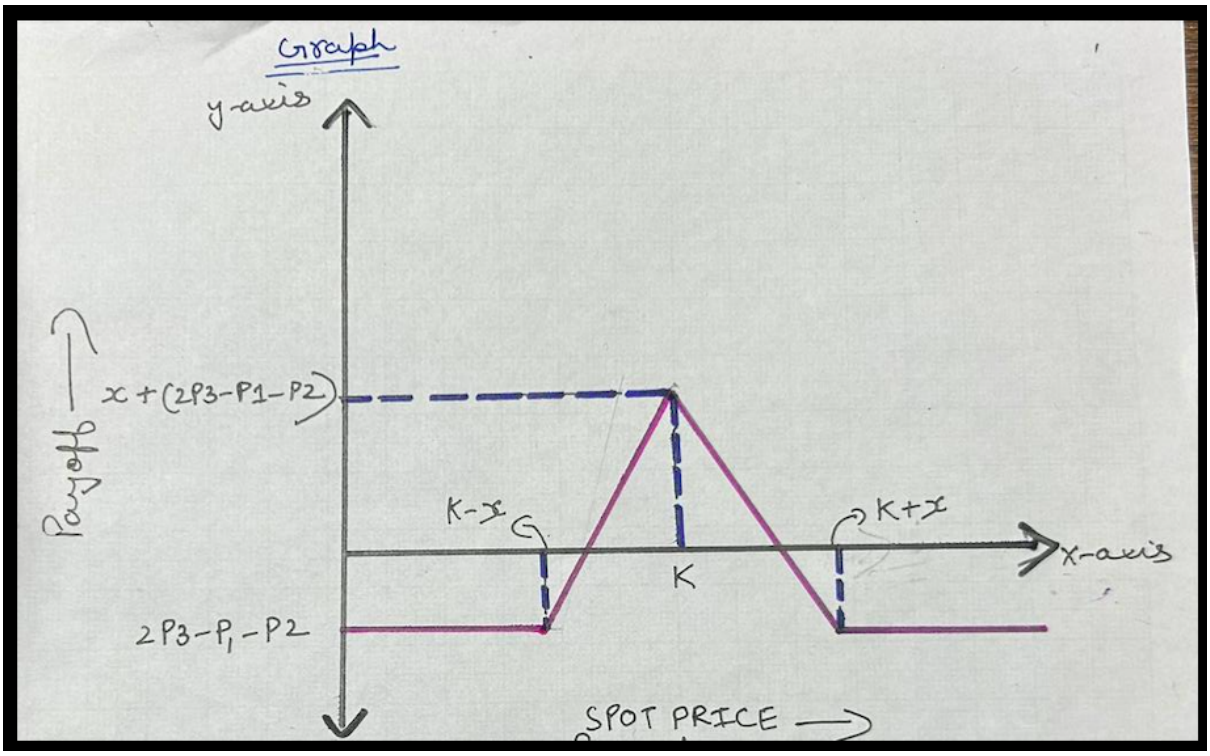
Payoff for case-3= (St-k1) - 2*(St-k) + 2p3-(p1+p2) $
= k+x-St + 2p3-(p1+p2) $

Case-4> St>k2 => St>k+x, i.e, the spot price at expiry date is higher than the strike price of the OTM call option.

In this case, the ITM call option would be exercised as the trader would incur a profit of (St-k1)$ if he buys the asset at a price lower than the current market price. The ATM call option would be exercised by the person to whom the trader has sold the ATM call option as that person would incur a profit of 2*(St-k)$ if he buys the asset at a price lower than the current market price. So, in turn, the trader would incur a loss of 2*(St-k). As St>k2, so the OTM call option would be exercised as the trader would incur a profit of (St-k2)$ if he buys the asset at a price lower than the current market price. **So, essentially, in this case, all the options would be exercised.**

Payoff for case-4= (St-k1) - 2*(St-k) +(St-k2) 2p3-(p1+p2) $
= 2p3-(p1+p2) $

If 2p3-(p1+p2) <0, the graph for variation of payoff with spot price is as follows-



The maximum payoff that can be incurred by the trader= x + (2p3-p1-p2)$
The maximum loss that can be incurred by the trader= (2p3-p1-p2)$
The breakeven point for the strategy will be when the spot price at expiry is equal to (k-x)-2p3-(p1+p2) $ or k+x+2p3-(p1+p2) $

If 2p3-(p1+p2)>=0, the graph for payoff will shift upwards accordingly and then the payoff incurred by the trader will always be positive.

c> The trader is trading in the following two instruments-
1.going long on an ITM(in the money) put option->
Suppose that Strike Price= k1 and premium paid= p1 for the above ITM put option

2.going short on an OTM(out of the money)put option->
Suppose that Strike Price= k2 and premium received= p2 for the above OTM put option.

It has been given that the expiration date and the underlying asset are the same for the above options.

For an ITM put option, the strike price is higher than the current market price at the time at which the option has been purchased. For an OTM put option, the strike price is lower than the current market price at the time at which the option has been purchased.

Let the current spot price at the time at which the options have been purchased be s.

So, according to the above definition, I can assume, k1=s+x and k2=s-y.

Let the spot price of the asset at the expiry date be St.

Case-1> St<k2 => St<s-x,i.e, the spot price at expiry is less than the strike price of the OTM put option.

In this case, **the OTM put option will be exercised** by the person to which the trader has sold the OTM put option as that person will make a profit of (k2-St)$ by being able to sell the asset at a price higher than the current market price. In turn, the trader will incur a loss of (k2-St)$. **The ITM put option will also be exercised by the trader** as St<k2 => St<s-x =>St<s+x => St<k1. So, the trader will incur a profit of (k1-St)$ for selling the asset at a price higher than the current market price. **Hence, essentially, in this case, both options would be exercised.**

Payoff for case-1= (k1-St) - (k2-St) + p2-p1= x+y+p2-p1

Case-2> k2<=St<k1 => s-x<=St<s+x, i.e, the spot price is higher than or equal to the strike price of the OTM put option and lower than the strike of the ITM put option.

In this case, **the OTM put option would not be exercised** by the person to which the trader has sold the OTM put option as that person would not profit from selling the asset at a price lower than the current market price. **The ITM put option will be exercised** by the trader as St<k1. So, the trader will incur a profit of (k1-St)$ for selling the asset at a price higher than the current market price.

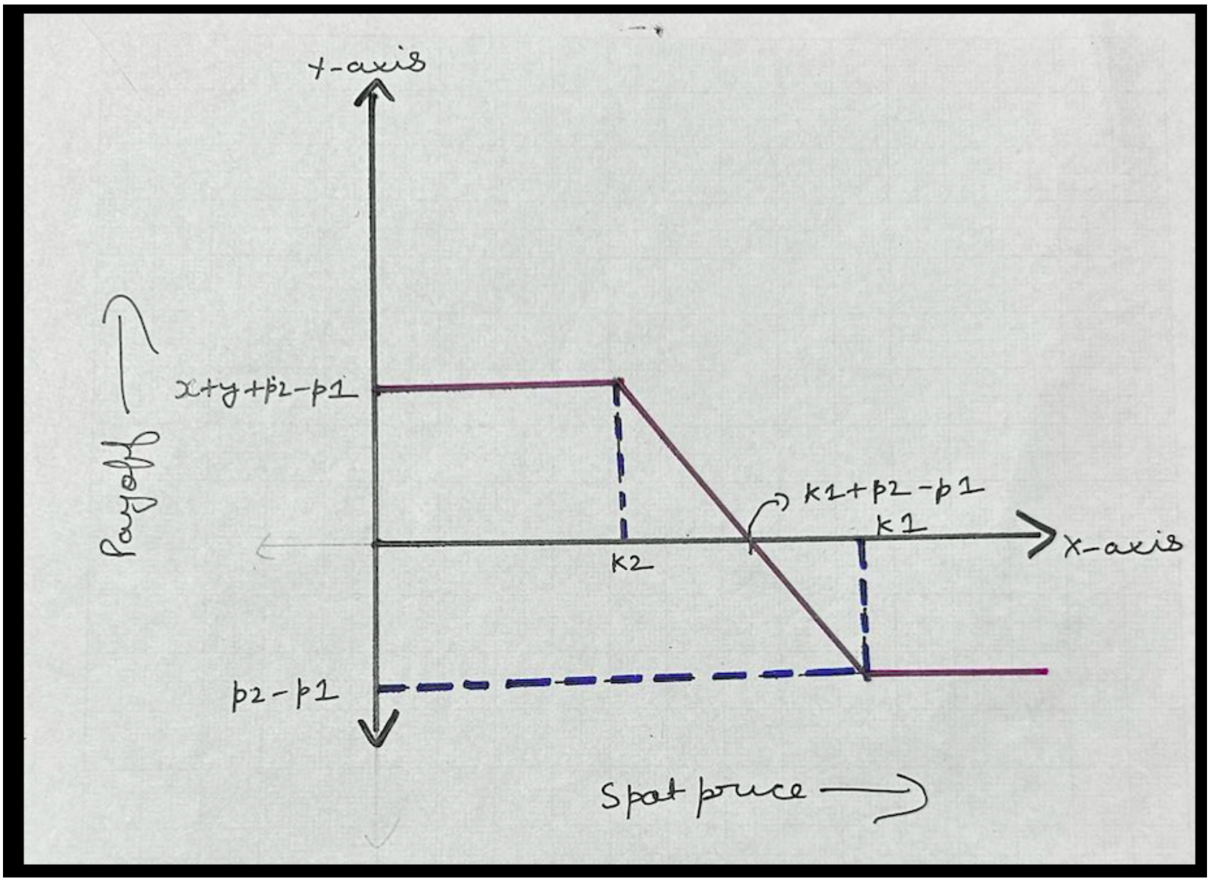
Payoff for case-2= (k1-St)+ p2-p1= s+x-St + p2-p1

Case-3> St>=k1, i.e, the spot price is higher than or equal to the strike price of the ITM put option

In this case, **the OTM put option would not be exercised** by the person to which the trader has sold the OTM put option as that person would not profit from selling the asset at a price lower than the current market price since St>k2. **The ITM option would not be exercised** by the trader either as he/she would not profit from selling the asset at a price lower than the current market price. **Hence, essentially, in this case, none of the options would be exercised**.

Payoff for case-3= p2-p1

If p2-p1<0, the graph for variation of payoff with spot price is as follows-



For the above strategy as is visible from the above graph,
The maximum profit that can be incurred by the trader= x+y+p2-p1
The maximum loss that can be incurred by the trader= p2-p1
Break-even point for the strategy is achieved when the spot price at expiry is equal to k1+p2-p1.

If p2-p1>=0, the graph for payoff will shift upwards accordingly and then the payoff incurred by the trader will always be positive.

MATHEMATICS

1.1 ans) The ant starts from $(0,0)$ and it is given that at each step it can move either upwards, i.e, $+y$ direction or eastwards ($+x$ direction). So, it will stay only in the first quadrant. Hence, after infinite steps have been taken, it will still be in the first quadrant. So, (x,y) lies in 1st quadrant.

1.2 ans) By definition,

d_m = manhattan distance b/w (x,y) & $(0,0)$

where $(x,y) = \lim_{i \to \infty}(x_i, y_i)$

$$d_m = |y-0| + |x-0|$$
$$= |y| + |x| = |x| + |y|$$

Since, we have shown in the above question that (x,y) lies in the 1st quadrant.

So, $\boxed{d_m = |x| + |y| = x+y.}$

At every step, no matter whether we choose to move in $+x$ direc^n or $+y$ direc^n, we can say that $x+y$ will increase at i^{th} step by $\frac{1}{2^{i-1}}$

$$\text{So,} \quad d_m = \sum_{i=1}^{\infty} \frac{1}{2^{i-1}}$$
$$= \left(1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} \cdots\right)$$
$$= \frac{1}{1-\frac{1}{2}} = 2$$

[sum of an infinite Geometric progression with ratio, $r<1$ & start term, $a = \frac{a}{1-r}$]

So, we have obtained $d_m = 2$ as a constant value.

Hence, $\boxed{E(d_m) = 2}$

1.3 ans) By definition,

d_e = euclidean distance b/w (x, y) & $(0,0)$

$= \sqrt{(y-0)^2+(x-0)^2} = \sqrt{x^2+y^2}$

$\Rightarrow d_e^2 = x^2+y^2$

We need to find,

$$\boxed{E(d_e^2) = E(x^2+y^2) = E(x^2)+E(y^2)} \quad -(1)$$

(using the property of linearity of expectation)

Since at every step we have equal probability of either choosing to take a step in $+x$ or $+y$ direcn.

Hence, we can say that, $\boxed{E(x^2) = E(y^2)}$ $-(2)$

(By symmetry).

$$E(x^2) = \sum_{i=1}^{\infty} \left(\begin{array}{c}\text{Probability we choose to move in} \\ +x \text{ direc}^n \text{ at step } i\end{array}\right) \times (\text{steplength of } i^{th} \text{ step})^2$$

$$= \sum_{i=1}^{\infty} \frac{1}{2} \times \left(\frac{1}{2^{i-1}}\right)^2$$

$$= \frac{1}{2} \sum_{i=1}^{\infty} \frac{1}{4^{i-1}}$$

$$= \frac{1}{2} \times \left[1+\frac{1}{4}+\frac{1}{4^2}+ \cdots\right]$$

$$= \frac{1}{2} \times \frac{1}{1-\frac{1}{4}} = \frac{1}{2} \times \frac{1}{\frac{3}{4}} = \frac{1}{2} \times \frac{4}{3}$$

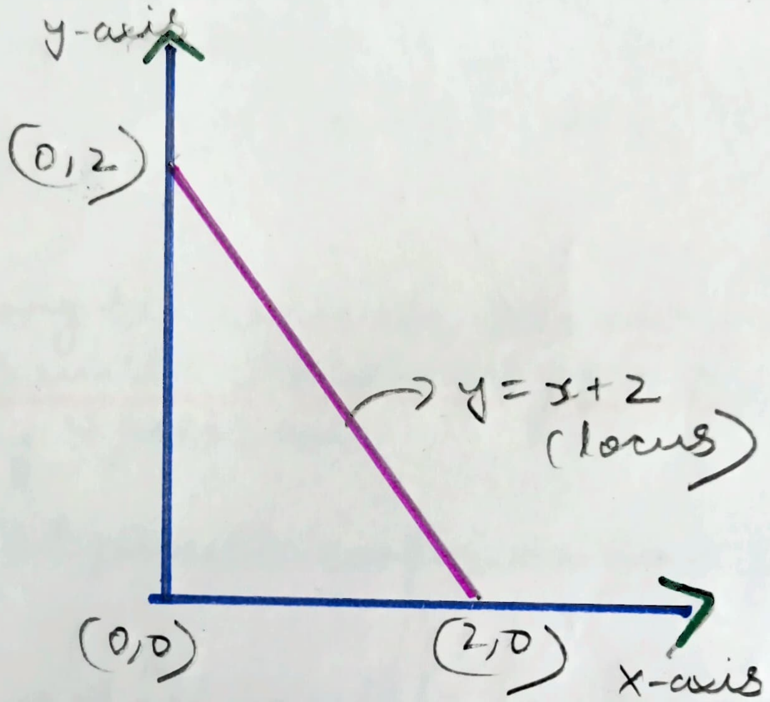
$$= \frac{2}{3}$$

$E(d_e^2) = E(x^2)+E(y^2) = 2E(x^2)$ [using (1) & (2)]

$$\boxed{E(d_e^2) = 2 \times \frac{2}{3} = \frac{4}{3}}$$

1.4 ans) In the answer to the question 1.2, we had shown that $d_m = x+y = 2$.

Hence, $x+y=2$ is the equation of the locus representing all the possible values of (x, y). where, $x \geq 0$ & $y \geq 0$ since the ant cannot move left or down.



2.1 ans) We have a total of 8 possible positions available to place 4 red and 4 black cards. So, in order to construct a possible configuration of this deck, I can choose 4 places out of the available 8 positions to place the 4 red cards. The no. of ways to do this is equal to the number of ways to choose 4 positions out of available 8 positions which is equal to $8C_4 = \frac{8!}{4!(8-4)!} = \frac{8!}{4!\,4!} = \frac{\cancel{8}\times 7\times \overset{2}{\cancel{6}}\times 5}{\cancel{4}\times\cancel{3}\times\cancel{2}}$

$= \boxed{70}$

After placing the red cards, the remaining 4 black cards will automatically occupy the remaining 4 positions

Hence, total possible configurations for this deck = 70.

2.2 ans) Let C = set of all possible configurations.

for a $c \in C$ where c is a possible configuration

let us define $P(c)$ = probability of occurrence of the given configuration c out of all possible configurations.

Since, in this case all possible configurations are equally likely.

So, $P(c) = \frac{1}{\text{Total configurations}} = \frac{1}{70}$.

We have to calculate the expected payoff (E) for this game,

$\boxed{E = \sum_{c\in C} P(c) \cdot \text{Payoff}(c)}$ where payoff(c) is the payoff for each possible configuration.

$$\text{So, } E = \frac{1}{70} \sum_{C \in C} \text{Payoff}(C).$$

⟹ code in file KannanRustagi_Mathematics_Q2_Subpart 2_MAIN.cpp

$\boxed{\text{answer} = 1.32857}$

2.3 ans> We can say that a payoff is a possible value of payoff when playing the game with 52 cards if there exists atleast one configuration for which that is the payoff.

Maximum payoff is equal to the number of red cards present in total with us as getting a red card gives us 1\$.

My maximum possible payoff of 26\$ will be achieved when we have the configuration -

$$\overset{1\,2\,3\,4}{\underbrace{RRRR\,-\,-\,\overset{26}{R}}_{26\text{ times}}}\;\underbrace{\overset{27}{B}BBB\,-\,-\,\overset{52}{B}}_{26\text{ times}}.$$

Here, we will stop at the 26th card to get the maximum possible payoff of 26\$.

Now in the above configuration if we replace the 26th card (R) with 27th card (B), then we'll get a payoff of 25\$.

Now if in the configuration using which we have obtained 25\$, we exchange the 25th card (R) with 28th card (B), then we'll get a payoff of 24\$.

We can keep repeating the above process in a similar manner to eventually get all possible payoffs ranging from 0 to 26\$.

The final configuration we'll get from the above process is $\underbrace{BBBBB\,-\,-\,B}_{26\text{ times}}\;\underbrace{RRRR\,-\,.\,R}_{26\text{ times}}$

Moreover, it is impossible to get a payoff less than 0 as if we have drawn more black cards than red cards, then we just keep drawing till the end to get 0 payoff.

Hence, possible payoffs which I can get are $\{0, 1, 2 \cdots 26\}$

2.4 ans) We have shown in the answer to the above question that it is impossible to get a payoff < 0. So, we can say that,

n_0 = number of deck configurations for which payoff is equal to 0.

If for every position, k, the no. of black cards encountered till ~~that~~ position, i.e, B_k is greater than or equal to the no. of red cards encountered till that position.

So, essentially $R_k \leq B_k$ for all k s.t $1 \leq k \leq 52$.

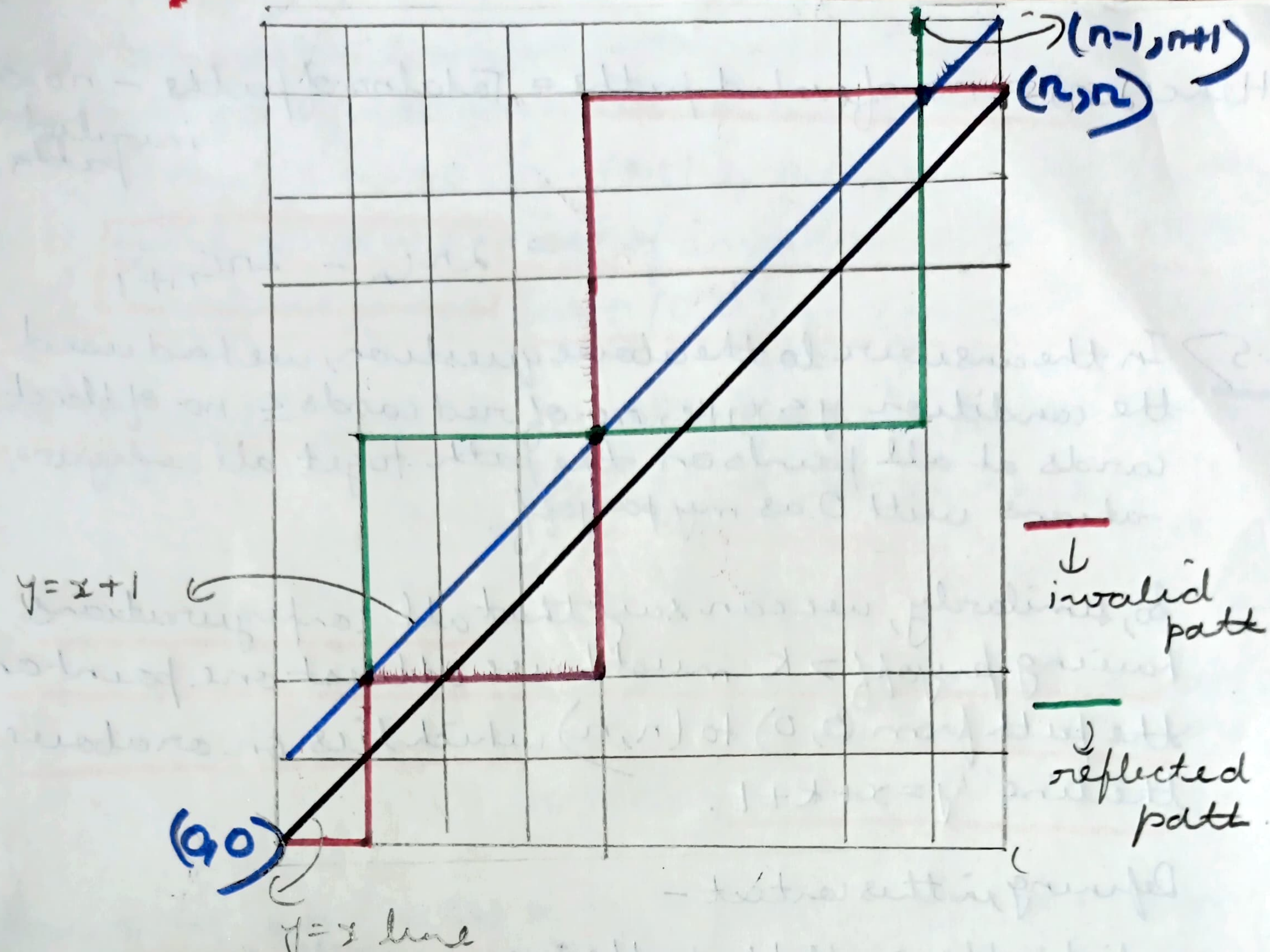
Let X-coordinate represent the number of black cards & Y-coordinate represent the number of red cards we have encountered till now. Essentially, a path from $(0,0)$ to ~~(...)~~ (n,n) represents a possible deck configuration of $2n$ cards with n black cards and n red cards.

Defining,

Valid paths $\Rightarrow$ All the paths passing only through the points on or below the line $y=x$ represent my configurations with 0 as my payoff.

Invalid paths $\rightarrow$ All paths with $y > x$ at any point on the path (i.e, no. of red cards > no. of black cards) is an invalid path.

Hence, we can say that if any path crosses or touches the line $y = x+1$, the path becomes an invalid path.



If we flip an invalid path from the point, it first touches / crosses the $y=x+1$ line up to the point (n, n). We get a path which starts from $(0,0)$ and ends at $(n-1, n+1)$ which is essentially the reflection of the point (n,n) about the line $y=x+1$.

Hence, for every invalid path we can find a monotonic path (in which we only move right or up) from $(0,0)$ to $(n-1, n+1)$. Hence, we get a bijection b/w the set of invalid paths and the set of monotonic paths from $(0,0)$ to $(n-1, n+1)$.

So, the no. of invalid paths = no. of monotonic paths from $(0,0)$ to $(n-1, n+1)$

$$= {}^{2n}C_{n-1} = \boxed{{}^{2n}C_{n+1}}$$

(same as choosing $n-1$ right steps out of a total of $2n$ steps)

Hence, n_0 = no. of valid paths = Total no. of paths − no. of invalid paths

$$= \boxed{{}^{2n}C_n - {}^{2n}C_{n+1}}$$

2.5) In the answer to the above question, we had used the condition $y \leq x$, i.e., no. of red cards $\leq$ no. of black cards at all points on the path to get all configurations with 0 as my payoff.

So, similarly, we can say that all configurations having payoff $> K$ must have atleast one point on the path from (0, 0) to (n, n) which lies on or above the line $y = x + K + 1$.

Defining, in this context –

Valid paths → all the paths passing only through points below the line $y = x + K + 1$. They essentially represent my configurations having payoff as less than equal to K.

Invalid paths → All paths that touch or cross the line $y = x + K + 1$.

If we have an invalid path, we can flip that path about the line $y = x + K + 1$ from the point it first touches/crosses that line up to the point (n, n).

After flipping, we will essentially get a path from (0, 0) to the point $(n - (K+1), n + (K+1))$

this point is the reflection of the point (n, n) in the line $y = x + K + 1$.

Hence, for every invalid path, we can find a monotonic path (a path in which we move only right or up) from $(0,0)$ to $(n-(k+1), n+(k+1))$. Hence, we get a bijection b/w the set of invalid paths and the set of monotonic paths from $(0,0)$ to $(n-(k+1), n+(k+1))$.

So, no. of invalid paths = no. of monotonic paths from $(0,0)$ to $(n-(k+1), n+(k+1))$.

$$= \boxed{{}^{2n}C_{n+k+1}}$$

(same as choosing $n+k+1$ up steps out of a total of $2n$ steps where $n-(k+1)$ are right steps)

Hence, no. of valid paths $= n_k$ = Total no. of paths from $(0,0)$ to (n,n) − no. of invalid paths

$$= \boxed{{}^{2n}C_n - {}^{2n}C_{n+k+1}}$$

2.6) Let k be a possible payoff where $0 \leq k \leq 26$.

We have n_k = no. of deck configurations having payoff less than or equal to k.

Total no. of possible deck configurations $= {}^{2n}C_n$

Since all the configurations are equally likely,

$P(x \leq k)$ where x is my payoff

$$= \frac{n_k}{{}^{2n}C_n} = \frac{{}^{2n}C_n - {}^{2n}C_{n+k+1}}{{}^{2n}C_n}$$

$$= 1 - \frac{\frac{2n!}{(n+k+1)!\,(n-k-1)!}}{\frac{2n!}{(n)!\,(n)!}}$$

$$= 1 - \frac{n!\,n!}{(n+k+1)!\,(n-k-1)!}$$

$$F_X(k) = \begin{cases} 1 - \frac{n!\,n!}{(n+k+1)!\,(n-k-1)!} & \forall\ k = 0, 1, \ldots n-1 \\ 1 & k >= n \\ 0 & \text{otherwise} \end{cases}$$

where $n = 26$

cummulative distⁿ funcᶜ for all the possible payoffs.

2-7 ans) Let n'_K be the no. of deck configurations such that maximum possible payoff for a configuration is greater than or equal to k.

n'_K = Total no. of configurations − configurations where payoff <= k−1

$$\boxed{n'_K = {}^{2n}C_n - n_{K-1}}$$

b_K = Probability that maximum possible payoff >= k

$$b_K = \frac{n'_K}{{}^{2n}C_n} = \frac{{}^{2n}C_n - n_{K-1}}{{}^{2n}C_n} = \frac{{}^{2n}C_n - ({}^{2n}C_n - {}^{2n}C_{n+k})}{{}^{2n}C_n} = \boxed{\frac{{}^{2n}C_{n+k}}{{}^{2n}C_n}}$$

similarly,

$$b_{K-1} = \frac{n'_{K-1}}{{}^{2n}C_n} = \frac{{}^{2n}C_n - n_{K-2}}{{}^{2n}C_n} = \frac{{}^{2n}C_{n+K-1}}{{}^{2n}C_n}$$

$$\boxed{\frac{b_k}{b_{k-1}} = \frac{{}^{2n}C_{n+k}}{{}^{2n}C_{n+k-1}}} = \frac{\dfrac{(2n)!}{(n+k)!\,(n-k)!}}{\dfrac{(2n)!}{(n+k-1)!\,(n-k+1)!}}$$

$$= \frac{(n-k+1)!}{(n-k)!} \times \frac{(n+k-1)!}{(n+k)!}$$

$$\boxed{\frac{b_k}{b_{k-1}} = \frac{n-k+1}{n+k}}$$

2-4 ans $\Rightarrow$ From the previous question, we know ~~that~~,

$$\frac{b_k}{b_{k-1}} = \frac{n-k+1}{n+k}$$

We know $b_0 = 1$,

So, we can calculate $\boxed{b_1 = \frac{n-1+1}{n+1} \times b_0 = \frac{n}{n+1} \times b_0 = \frac{n}{n+1}}$

Now, we know b_1

Similarly, we have $b_2 = \frac{n-2+1}{n+2} \times b_1$

Similarly, we can calculate b_k as

$\boxed{b_k = \frac{n-k+1}{n+k} \times b_{k-1}}$ when we know the value of b_{k-1} already.

Now, we have all values of b_i $\forall\ 0 \le i \le 26$.

In this question, we need to find $P(X=k)$.

$P(X=k)$ = No. of configurations having payoff $> k$ − No. of configurations having payoff $> k+1$

$$\boxed{P(X=k) = b_k - b_{k+1}}$$

The above expression can be used to calculate $P(X=k)$ for all $0 \le k \le 25$

For, $x = 26$,
since the payoff cannot be > 26.
so, $P(X=26) = P(X>=26) = b_{26}$

=> code in file KannanRustagi_Mathematics_Q2_Subparts_8_9_10_MAIN.cpp (for 2.8 & 2.9 & 2.10)

2.9 ans) From the previous question, I have with me the probabilities with which I get a configuration with a particular value of payoff ranging from 0 to 26. So, I can find the payoff with the highest probability of occurrence amongst the all possible configurations.

$\boxed{\text{answer} = 3}$

2.10 ans) Expected payoff is calculated using the formula below −

For $0 \le k \le 26$ where k is a possible payoff.

$$\text{Expected Payoff} = \sum_{k=0}^{26} P(k) \cdot k$$

where $P(k)$ is the probability of getting a configuration with payoff k.

$\boxed{E(\text{payoff}) = 4.04}$