25 $x_0 = 2$ $x_1 = 7$

$x^2 = 25$

$f(x) = x^2 - 25$

$x_2 = \frac{2+7}{2} = 4.5$

$f(x_2) = (4.5)^2 - 25$

$= -4.75 < 0$

$f(x_0) = -21 < 0$

$f(x_1) = 24 > 0$

$f(x_2) < 0$ and $f(x_1) > 0$

here root lies b/w x_2 and x_1

$x_3 = \frac{4.5+7}{2} = 5.75$

$f(5.75) = (5.75)^2 - 25 = 8.0625 > 0$

$f(5.75) > 0$ and $f(4.5) < 0$

root lies b/w 5.75 and 4.5

$x_4 = \frac{5.75+4.5}{2} = 5.125$

$f(5.125) = (5.125)^2 - 25 = 1.2656 > 0$

$f(5.125) > 0$ and $f(4.5) < 0$

root lies b/w 5.125 and 4.5

$f(5) = \frac{5.125+4.5}{2} = 4.8125$

$f(4.8125) = (4.8125)^2 - 25 = -1.8398 < 0$

$f(4.8125) < 0$ and $f(5.125) > 0$

root lies b/w 4.8125 and 5.125

$x_6 = \frac{4.8125+5.125}{2} = 4.9687$

$f(4.9687) = -0.31\ldots < 0$

$f(4.9687) < 0$ and $f(5.125) > 0$

root lies between

4.9687 and 5.125

$x_7 = \frac{4.9687+5.125}{2} = 5.0468$

$f(5.0468) = 0.47\ldots > 0$

$f(5.0468) > 0$ and $f(4.9687) < 0$

root lies between

$$x_8 = \frac{5.0468 + 4.9687}{2} = 5.00775$$

$f(5.00775) > 0$

$f(5.00775) > 0$ and $f(4.9687) < 0$

$$x_9 = \frac{5.00775 + 4.9687}{2} = 4.9887$$

$f(4.9882) = -0.1176 < 0$

$f(4.9882) < 0$ and $f(5.0075) > 0$

$$x_{10} = \frac{4.9882 + 5.00775}{2} = 4.9971$$

$f(4.9971) = -0.0209 < 0$

$f(4.9971) < 0$ and $f(5.00775) > 0$

$$x_{11} = \frac{4.9971 + 5.00775}{2} = 5.0024$$

$f(5.0024) > 0$ and $f(4.9971) < 0$

$$x_{12} = \frac{5.0024 + 4.9971}{2} = 5.0036$$

$f(5.00036) = 0.0036$

* (2) Find approximate root of eqn $\sin x = \frac{1}{x}$ $x_0 = 1$ and $1.5 = x_1$ carry out commutations upto 7 stages

$x \sin x = 1$

$f(x) = x \sin x - 1$

$f(1) = -0.158529 < 0$

$f(1.5) = 0.496242 > 0$

$x_2 = 1.25$

$x_3 = 1.125$

$x_4 = 1.0625$

$x_5 = 1.0937$

$x_6 = 1.0935$

$x_7 = 1.117175$

$x_8 = 1.1132$

(3) Find the real root of $x \log_{10} x = 1.2$ between 2 and 3

$f(x) = x\log_{10} x - 1.2$

$f(2) = 2\log_{10} 2 - 1.2 = \cancel{-0.5979 < 0}$ 2.5

$f(3) = 2.75$

$x_4 = 2.625$

$x_5 = 2.6875$

$x_6 = 2.71875$

$x_7 = 2.734$

$x_8 = 2.742$

$x_9 = 2.7303$

$x_{10} = 2.736$

$x_{11} = 2.739$

$x_{12} = 2.7405$

(4) $3x = e^x$ 0 & 1

(5) $3x = \cos x + 1$

(4) $f(x) = e^x - 3x$

$f(x_0) = 1 > 0$

$f(x_1) = -0.2187 < 0$

$x_2 = \frac{0+1}{2} = 0.5$

$f(x_2) = 0.1487 > 0$

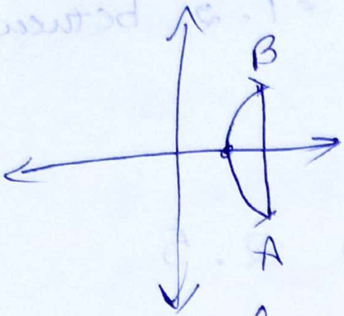
$x_3 = \frac{1+0.5}{2} = 0.75$

$f(x_3) = -0.1329$

$x_4 = \frac{0.5+0.75}{2} = 0.625$

$\cancel{f(x_4) =}$

Regula-false method



$$\text{Slope} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$\frac{y - f(x_0)}{x - x_0} = \frac{f(x) - f(x_0)}{x - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$\frac{x - x_0}{f(x) - f(x_0)} = \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x - x_0 = \frac{(x_1 - x_0) f(x) - f(x_0)}{f(x_1) - f(x_0)}$$

$$x - x_0 = \frac{x_1 f(x) - x_1 f(x_0) - x_0 f(x) + x_0 f(x_0)}{f(x_1) - f(x_0)}$$

$$x = x_0 + \frac{x_1 f(x) - x_1 f(x_0) - x_0 f(x) + x_0 f(x_0)}{f(x_1) + f(x_0)}$$

$$\boxed{x = x_0 - \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_1) - f(x_0)}}$$

$\boxed{f(x) = 0}$

① Find an appoximate root $x^4 - x - 10 = 0$ that lies between 1.8 and 2, carry out three approximations.

$f(x) = x^4 - x - 10$

$f(x_0) = f(1.8) = (1.8)^4 - 1.8 - 10 = -1.3024 < 0$

$f(x_1) = f(2) = (2)^4 - 2 - 10 = 4 > 0$

$$\boxed{\cancel{x_{i+1} = \frac{x_i f(x_i - 1) - x_{i-1} f(x_i)}{f(x_i) - f(x_i - 1)}}}$$

$$\boxed{x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}}$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{1.8(4) - 2(-1.3024)}{4 - (-1.3024)}$$

$= 1.84939$

$f(x_2) = -0.15132 < 0$

$f(x_1) = 4 > 0$

x_1, x_2

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{2(-0.1513) - 1.8493(4)}{-0.1513 - 4} = 1.8548$$

$f(x_3) = -0.019 < 0$

$x_1 \quad x_3$

$$x_4 = \frac{x_1 f(x_3) - x_3 f(x_1)}{f(x_3) - f(x_1)}$$

$$= \frac{4(-0.019) - 1.8548(4)}{(-0.019) - 4} = 1.8554$$

hence root is 1.8554

3.5 and 4

(2) $2x - \log_{10} x = 7$

$f(x) = 2x - \log_{10} x - 7$

$x_0 = 3.5 \quad x_1 = 4$

$f(x_0) = -0.5440 < 0$

$f(x_1) = 0.3979 > 0$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{3.5(0.3979) - 4(-0.5440)}{0.3979 - (-0.5440)} = 3.788 > 0$$

$f(x_2) = -0.002409 < 0$

$x_1 \quad x_2$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{4(-0.002409) - 3.788(0.3979)}{(-0.002409) - 0.3979}$$

$= 3.7888$

$f(3.7888) = -0.000901 < 0$

$x_1 \quad x_3$

$$x_4 = \frac{x_1 f(x_3) - x_3 f(x_1)}{f(x_3) - f(x_1)}$$

$$= \frac{4(-0.000901) - 3.7888(0.3979)}{(-0.000901) - 0.3979}$$

$= 3.7892$

(3) Find the real root of $xe^x = 3$ using Regula-falsi method.

$f(x) = xe^x - 3$

$x_0 = 0 \quad f(x_0) = 0 - 3 = -3 < 0$

$x_1 = 1 \quad f(1) = 1e^1 - 3 < 0 = -0.2817$

$x = 2 \quad f(2) = 2e^2 - 3 = 11.77 > 0$

$x_0 = 1 \quad x_1 = 2$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{1(11.77) - 2(-0.2817)}{11.77 - (-0.2817)} = 1.02337$$

$f(1.02337) = 1.02337\, e^{1.02337} - 3 = -0.1524 < 0$

The real roots lies between $x_2 = 1.02337$ and $x_1 = 2$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{2(-0.1524) - 1.02337(11.77)}{-0.1524 - 11.77} = 1.0358$$

$f(1.0358) = 1.0358\, e^{1.0358} - 3 = -0.0817 < 0$

The root lies between $x_3 = 1.0358$ and $x_1 = 2$

$$x_4 = \frac{x_1 f(x_3) - x_3 f(x_1)}{f(x_3) - f(x_1)}$$

$$= \frac{2(-0.0817) - 1.0358(11.7)}{-0.0817 - 11.77} = 1.0424$$

$f(1.0424) = -0.0435 < 0$

The root lies between $x_4 = 1.0424$ and $x_1 = 2$

$$x_5 = \frac{2(-0.0435) - 1.0424(11.7)}{-0.0435 - 11.7} = 1.0459$$

$f(1.04159) = 1.04159\, e^{1.04159} - 3$

(4) $\log x = \cos x$ by using Regular falsi method

$f(x) = \log x - \cos x$

$x_0 = 1 \qquad f(x_0) = -0.5403 < 0$

$x_1 = 2 \qquad f(x_1) = \cancel{0.7171 > 0} \; 1.1092 > 0$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{1(1.1092) - 2(-0.5403)}{1.1092 - (-0.5403)} = \cancel{1.4296}$$

$= 1.3275$

$f(1.3275) = 0.04239 > 0$

$$x_3 = \frac{x_0 f(x_2) - x_2 f(x_0)}{f(x_2) - f(x_0)}$$

$$= \frac{1\,\cancel{(1.3275)}\,(0.04239) - 1.3275\,(-0.5403)}{0.04239 - (-0.5403)}$$

$= 1.3037$

$f(1.3037) = 0.00127 > 0$

$x_0 \qquad x_3$

$$x_4 = \frac{x_0 f(x_3) - x_3 f(x_0)}{f(x_3) - f(x_0)}$$

$$= \frac{1(0.00127) - 1.3037(-0.5403)}{0.00127 - (-0.5403)} = 1.3030$$

Newton - Raphson — merits & Demerits

$x_1 = x_0 + h$

$f(x_1) = f(x_0+h)$

$= f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0) + \dots$

$f(x_0+h) = f(x_0) + hf'(x_0)$

$hf'(x_0) = f(x_0+h) - f(x_0)$

$h = \frac{f(x_1)}{f'(x_0)} - \frac{f(x_0)}{f'(x_0)}$

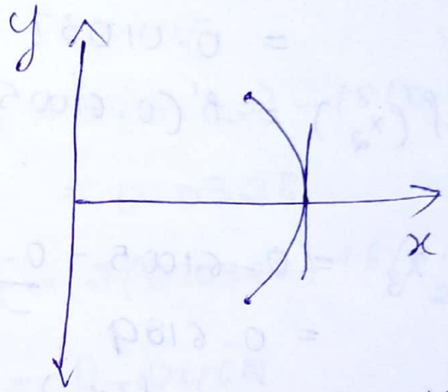
$h = \frac{-f(x_0)}{f'(x_0)}$

x_1, x_0

$x_1 = x_0 + h$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$



move the tangent till it touches the root.

1) using Newton raphson method find the root of $e^x - 3x$ that lies between 0 and 1.

$f(x) = e^x - 3x$

$f'(x) = e^x - 3$

$x_0 = 0$

$f(0) = e^0 - 3(0) = 1$

$f'(0) = e^0 - 3$

$= 1 - 3 = 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 0 - \frac{1}{-2} = \frac{1}{2} = 0.5$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$f(x_1) = f(0.5) = e^{0.5} - 3(0.5)$

$= 0.1487$

$f'(x_1) = f'(0.5) = e^{0.5} - 3$

$= 1.3512$

$x_2 = 0.5 - \frac{0.1487}{(-1.3612)}$

$= 0.61005$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$f(x_2) = f(0.61005)$

$= e^{0.61005} - 3(0.61005)$

$= 0.01037$

$f'(x_2) = f'(0.61005) = e^{0.61005} - 3$

$= -1.1594$

$x_3 = 0.61005 - \frac{0.01037}{-1.1594}$

$= 0.6189$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 0.6189 - \frac{0.000184}{-1.1431}$

$= 0.61906$

**

(2) Find the square root of a number

* Find the reciprocal of the number using Newton raphsson method

Sol $x = \sqrt{N}$

$x^2 = N$

$f(x) = x^2 - N$

$f'(x) = 2x$

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$f(x) = x^2 - N \quad f'(x) = 2x$

$f(x_i) = (x_i)^2 - N \quad f'(x_i) = 2x_i$

$$x_{i+1} = x_i - \frac{(x_i)^2 - N}{2x_i}$$

$$= \frac{2x_i^2 - x_i^2 + N}{2x_i}$$

$$\boxed{x_{i+1} = \frac{x_i^2 + N}{2x_i}}$$

$x = \frac{1}{N}$

$N = \frac{1}{x}$

$f(x) = \frac{1}{x} - N$

$f'(x) = -\frac{1}{x^2}$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = x_i - \frac{\left(\frac{1}{x_i} - N\right)}{-\frac{1}{x_i^2}}$$

$$\boxed{x_{i+1} = 2x_i - Nx_i^2}$$

$$x_{i+1} = \frac{x_i^2 + N}{2x_i}$$

Square root of 10

$x_0 = 3$

$$x_1 = \frac{x_0^2 + N}{2x_0}$$

$$x_1 = \frac{9+10}{6} = 3.1666..$$

$$x_2 = \frac{x_i^2 + N}{2x_i} = \frac{(3.1666)^2 + 10}{2 \times 3.1666}$$

$= 3.16222$

$$x_3 = \frac{(3.1622)^2 + 10}{2(3.1622)}$$

$= 3.16227$

$x_{i+1} = 2x_i - Nx_i^2$

$N = 18$

$x_0 = 0.1$

$x_1 = 2(0.1) - 18(0.1)^2$

$= 0.02$

$x_2 = 2(0.02) - 18(0.02)^2$

$= 0.0328$

$x_3 = 2(0.0328) - 18(0.0328)^2$

$= 0.0462$

$x_4 = 2(0.0462) - 18(0.0462)^2$

$= 0.0539$

$x_5 = 2(0.0539) - 18(0.0539)^2$

$= 0.055$

$x_6 = 2(0.055) - 18(0.055)^2$

$= 0.05555$

(3) Find the root of the equation

$x \sin x + \cos x = 0$

$f(x) = x \sin x + \cos x$

$f(0) = 1$

$f(1) = 1.3817$

$f(2) = 1.4024$

$f(3) = -0.15666$

$x_0 = 2$ ~~3~~

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$f(x_0) = 2\sin 2 + \cos 2$

$= 1.4024$

$f'(x_0) = x\cos x + \sin x - \sin x$

$= x\cos x$

$f'(x_0) = 2\cos 2$

$= -0.832$

$x_1 = 2 - \frac{1.402}{-(0.832)}$

$= 3.6851$

$x_2 = 3.6851 - \frac{f(3.6851)}{f'(3.6851)}$

$f(3.6851) = 3.6851\sin(3.6851) + \cos(3.6851)$

$= -2.7616$

$f'(3.6851) = 3.6851\cos(3.6851)$

$= -3.15407$

$x_2 = 3.6851 - \frac{(-2.7616)}{-3.15407}$

$= 2.8096$

$x_3 = 2.8096 - \frac{f(2.8096)}{f'(2.8096)}$

$= 2.798$

$x_4 = \cancel{2.80257}\ 2.7983$

$x_5 = 2.7984$

(4) Find the real root of

(i) $x\tan x + 1 = 0$

(ii) $x + \log_{10} x = 3.375$

(i) $x\tan x + 1 = 0$

$f(x_0) = 1\tan 1 + 1$

$= 2.5574$

$f(x_1) = 2\tan 2 + 1$

$= -3.3700$

$x_0 = 1$

~~$f(x_1) =$~~

$f'(x_0) = (1)\sec^2 x + \tan x$

$= \sec^2 x + \tan x$

$= \sec^2(1) + \tan(1)$

$= \frac{1}{\cos^2(1)} + \tan(1)$

$= 4.9829$

$x_1 = \cancel{2.5574} - $

$= 1 - \frac{2.5574}{4.9829}$

$= 0.4868$

(ii) $x+\log_{10} x = 3.375$

$f(x_0) = x + \log_{10} x - 3.375$

$= -2.375$

$f(2) =$

$x_0 = 2$

$f(x_0) = -1.073$

$f'(x_0) = 1 + \frac{1}{x}$

$= 1.5$

$x_1 = 2 - \left(\frac{-1.073}{1.5}\right)$

$= 2.715$

$f(x_1) = 2.715 + \log_{10} 2.715 - 3.375$

$= -0.226$

$x_2 = 2.8804$

$x_3 = 2.9064$

$x_4 = 2.9103$

$x_5 = 2.9108$

$x \tan x$

$\cancel{\tan(1)}$ $x \sec^2 x + \tan x$

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Iteration method

Root of an eq^n $f(x)=0$ is found by Iteration method considering $x=\phi(x)$ where $\phi(x)$ satisfies the cond $|\phi'(x)| \leq 1$

$\Rightarrow \phi(x)$ is covergent

then by iteration method or iterative process then

$$x_n = \phi(x_{n-1})$$

① By single point iteration method find the root of eq^n $x^3-2x-5=0$ which lies near $x=2$

$x^3-2x-5=0$

$x^3=2x+5$

$x=(2x+5)^{1/3}$

$\phi(x)=(2x+5)^{1/3}$

$\phi'(x)=\frac{1}{3}(2x+5)^{\frac{1}{3}-1}(2)$

$=\frac{1}{3}(2x+5)^{-2/3}(2)$

$=\frac{2}{3}\cdot\frac{1}{(2x+5)^{2/3}}$

$|\phi''(x_0)| \leq 1$

$x_1=\phi(x_0)$

$=(2(2)+5)^{1/3}$

$=2.0800$

$x_2=(2(2.0800)+5)^{1/3}$

$=2.0923$

$x_3=(2(2.0923)+5)^{1/3}$

$=2.0942$

$x_4=(2(2.0942)+5)^{1/3}$

$=2.0944$

$x_5=(2(2.0944)+5)^{1/3}$

$=2.0945$

$x_6=(2(2.0945)+5)^{1/3}$

$=2.0945$

② $3x=\cos x+1$

$x=\frac{1+\cos x}{3}$

$\phi(x)=\frac{1+\cos x}{3}$

let $x=\pi/4$

$\phi'(x)=\frac{-\sin x}{3}$

$x_0=\frac{\pi}{4}$

$x_1=\phi(x_0)$

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$= \frac{1 + \cos(\pi/4)}{3}$

$= 0.5691$

$x_2 = \phi(x_1)$

$= \frac{1 + \cos(0.5691)}{3}$

$= 0.6141$

$x_3 = \frac{1 + \cos(0.6141)}{3}$

$= 0.6057$

$x_4 = 0.6073$

$x_5 = 0.6070$

$x_6 = 0.6071$

(3) Find the real root of eq^n

$2x - \log x = 7$

$x_0 = 1$

$\cancel{f(x_0) = 2(1) - \log 1 - 7}$

$x = \frac{7 + \log x}{2}$

$\phi(x) = \frac{7 + \log x}{2}$

$f(x) = 2x - \log x - 7$

$x_0 = 1$

$f(1) = -5 < 0$

$f(2) = -3.13 < 0$

$f(3) = -0.402 < 0$

$f(4) = +0.397 > 0$

$f(5) = 1.39 > 0$

root lies b/w 3 and 4

$\phi(x) = \frac{7 + \log x}{2}$

$\phi'(x) = \frac{1}{2x}$

$x = 3$

$\phi'(3) = \frac{1}{6} < 1$

Hence $\phi(x)$ is convergent

$x_1 = \phi(x_0)$

$= \frac{7 + \log 3}{2} = 3.738$

$x_2 = \phi(x_1)$

$= \frac{7 + \log 3.738}{2}$

$= 3.786$

$x_3 = \phi(x_2)$

$= \frac{7 + \log 3.786}{2}$

$= 3.789$

$x_4 = \frac{7 + \log 3.789}{2}$

$= 3.789$

Interpolation:

The process of finding a function satisfying the given data is interpolation.

If the data is given as

x	y
x_0	y_1
x_1	y_2
x_2	y_3
x_3	y_4
;	;
x_n	y_n

~~f(x)~~ finding out $f(x)$ satisfying interplotion,

Predicting for the values above x_n is extraplotion.

while interploting or extraploting the fns have errors which can be found by the formula of errors in polynomials.

Interplotion can be by newton forward and backward interplotion formulae and gauss forward and backward, or legrange's.

Interplotion can be done by finite difference method. finite differences can be forward difference denoted by Δ. backward difference denoted by ∇. central diff denoted by δ.

Δ first difference, the diff are always $y_1 - y_0$, $y_2 - y_1$, $y_3 - y_2$, $y_4 - y_3$, $y_5 - y_4$ denoted by Δy_0. and no of value

x	$y = f(x)$			
x_0	y_0	Δy_0	$y_1 - y_0$	∇y_1
x_1	y_1	Δy_1	$y_2 - y_1$	∇y_2
x_2	y_2	Δy_2	$y_3 - y_2$	∇y_3
x_3	y_3	Δy_3	$y_4 - y_3$	∇y_4
x_4	y_4	Δy_4	$y_5 - y_4$	∇y_5
x_5	y_5			

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$

$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$

$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$

$\Delta^2 y_3 = \Delta y_4 - \Delta y_3$

Δ^3

$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$

$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$

$\Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2$

Δ^4

$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$

$\Delta^4 y_1 = \Delta^3 y_2 - \Delta^3 y_1$

Find the forward difference & backward dif table for the function $y = x^3$

x	$y = f(x)$	Diff	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	1	$\Delta y_0 = y_1 - y_0 = 7$	$\Delta^2 y_0 = 12$	$\Delta^3 y_0 = 6$	$\Delta^4 y_0 = 0$	$\Delta^5 y_0 = 0$
2	8	19	18	6	0	
3	27	37	24	6		
4	64	61	30			
5	125	91				
6	216					

$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$

x	$y = f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1	1	$\nabla y_1 = 7$	$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$	6	0	0
2	8	19	12	6	0	
3	27	37	18	6		
4	64	61	24			
5	125	91	30			
6	216					

Central difference

x	y	δ	δ^2
x_0	y_0	$\delta_{\frac{0+1}{2}} = \delta_{\frac{1}{2}} = y_1 - y_0$	$\frac{\delta^2_{\frac{1}{2}+\frac{3}{2}}}{2} = \delta_1^2$
x_1	y_1	$\delta_{3/2} = y_2 - y_1$	$= \delta_{3/2}y - \delta_{1/2}y$
x_2	y_2	$\delta_{5/2} = y_3 - y_2$	
x_3	|	|	
|	|	|	
x_n	y_n	$\delta_{\frac{n+n-1}{2}} = y_n - y_{n-1}$	

Operators

The average operator 'μ' is defined as $\mu y_r = \frac{1}{2}\left(y_{r+1/2} + y_{r-1/2}\right)$

$$r = \frac{r+\frac{1}{2} + r-\frac{1}{2}}{2}$$

Shift operator

Shift operator denoted by E is defined as

$$Ey_r = y_{r+1}$$

E^{-1}

$$E^{-1}y_r = y_{r-1}$$

$$\frac{1}{E}(y_r) = y_{r-1}$$

Relation b/w Δ, ∇ & E

$$\Delta y_0 = y_1 - y_0$$

$$= E y_0 - y_0$$

$$\Delta y_0 = (E-1)y_0$$

$$\Rightarrow \boxed{\Delta = E-1}$$

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$\nabla y_1 = y_1 - y_0$

$\nabla y_1 = y_1 - E^{-1} y_1$

$\nabla y_1 = (1 - E^{-1}) y_1$

$\nabla = 1 - E^{-1}$

$\boxed{\nabla = 1 - \frac{1}{E}}$

$\Rightarrow \Delta^3 y_0 = (E-1)^3 y_0$

$= (E^3 - 1 - 3E^2 + 3E) y_0$

$= E^3 y_0 - y_0 - 3E^2 y_0 + 3E y_0$

$= E^2 (E y_0) - y_0 - 3E(E y_0) + 3E y_0$

$= E^2 (y_1) - y_0 - 3E y_1 + 3y_1$

$= E(E y_1) - y_0 - 3y_2 + 3y_1$

$= y_3 - 3y_2 + 3y_1 - y_0$

$\rightarrow \delta = E^{1/2} - E^{-1/2}$

$\delta y_r = (E^{1/2} - E^{-1/2}) y_r$

$= \frac{y_{r+1/2} + r - 1/2}{2}$

$= y_{r+1/2} - y_{r-1/2}$

$= E^{1/2} y_r - E^{-1/2} y_r$

$\delta y_r = (E^{1/2} - E^{-1/2}) y_r$

$\delta_r = E^{1/2} - E^{-1/2}$

$\Rightarrow \Delta^4 y_0 = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$

$(E-1)^4 y_0$

☆☆

① x	0	1	2	3	4
y	1	3	9	–	81
	y_0	y_1	y_2	y_3	y_4

$$\Delta^4 y_0 = (E-1)^4 y_0$$

$$= (y_4 - 4y_3 + 6y_2 - 4y_1 + y_0)$$

$$= 81 - 4y_3 + 6(9) - 4(3) + 1$$

$$= 81 - 12 + 54 + 1 - 4y_3 = \Delta^4 y_0$$

let $\Delta^4 y_0 = 0$

$$4y_3 = 124$$

$$y_3 = \frac{124}{4}$$

$$= 31$$

① $E\nabla = \Delta$

$\nabla y_1 = y_1 - y_0$

$\therefore \nabla y_r = y_r - y_{r-1}$

$$E\nabla y_r = \Delta y_r$$

$$E(\nabla y_r) = E y_r - E y_{r-1}$$

$$= y_{r+1} - y_r$$

$$= \Delta y_r$$

$$E\nabla y_r = \Delta y_r$$

$$E\nabla = \Delta$$

② $\delta E^{1/2} = \Delta$ $\quad \delta E y_r = E y_{r+1}$

$E^{1/2} y_r = y_{r+1/2}$ $\quad \delta y_{1/2} = y_1 - y_0$

$\delta(E^{1/2} y_r) = \delta y_{r+1/2}$

$= y_{r+1} - y_r$

$= \Delta y_r$

$\delta(E^{1/2} y_r) = \Delta y_r$

$\delta E^{1/2} = \Delta$

③ $\mu^2 = 1 + \frac{\delta^2}{4}$

$\mu = \frac{1}{2}(E^{1/2} + E^{-1/2})$

$\delta = E^{1/2} - E^{-1/2}$

$\mu y_r = \frac{1}{2}(y_{r+\frac{1}{2}} + y_{r-1/2})$

$= \frac{1}{2}(E^{1/2} y_r + E^{-1/2} y_r)$

$= \frac{1}{2}(E^{1/2} + E^{-1/2}) y_r$

$\mu^2 = \left[\frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]\right]^2$

$= \frac{1}{4}\left[E^1 + E^{-1} + 2E^{1/2}E^{-1/2}\right]$

$= \frac{1}{4}\left[E + E^{-1} + 2\right]$

$= \frac{1}{4}\left[(E^{1/2} - E^{-1/2})^2 + 4\right]$

$= \frac{1}{4}\left[\delta^2 + 4\right]$

$= 1 + \delta^2/4$

(4) $(1+\Delta)(1-\nabla) = 1$

$\Delta = E-1$

$E = 1+\Delta$

$\nabla = 1-E^{-1}$

$E^{-1} = 1-\nabla$

$E(E^{-1}) = 1$

(5) $1+\mu^2\delta^2 = \left(1+\frac{1}{2}\delta^2\right)^2$

$1+\mu^2\delta^2$

$= 1+\left(\frac{1}{2}(E^{1/2}+E^{-1/2})\right)^2 (E^{1/2}-E^{-1/2})^2$

$= 1+\frac{1}{4}(E^{1/2})^4-(E^{-1/2})^4$

$= \cancel{\frac{1}{2}}+\frac{1}{4}(E^2-E^{-2})$

$= 1+\frac{1}{4}\left(E^{2/2}+E^{-2/2}+2E^{1/2}E^{-1/2}\right)$

$= 1+\frac{1}{4}\left[(E^{1/2}+E^{-1/2})(E^{1/2}-E^{-1/2})\right]^2$

$= 1+\frac{1}{4}\left[(E^{1/2})^2-(E^{-1/2})^2\right]^2$

$= 1+\frac{1}{4}\left[E^1-E^{-1}\right]^2$

$= 1+\frac{1}{4}\left[(E^{1/2}-E^{-1/2})^2+4\right](E^{1/2}-E^{-1/2})$

$= 1+\frac{1}{4}[\delta^2+4][\delta^2]$

$= 1+\left[\frac{\delta^2}{4}+1\right]\delta^2$

$= 1+\frac{\delta^4}{4}+\delta^2$

$= \left(1+\frac{\delta^2}{2}\right)^2$

(7) $E^{1/2} = \mu+\delta/2$

$\frac{1}{2}(E^{1/2}+E^{-1/2})+\frac{E^{1/2}-E^{-1/2}}{2}$

$= \frac{E^{1/2}+\cancel{E^{-1/2}}+E^{1/2}-\cancel{E^{-1/2}}}{2}$

$= E^{1/2}$

(6) $\nabla\Delta = (1-E^{-1})(E-1)$

$= E-1-1+E^{-1}$

$= E+E^{-1}-2$

$= (E^{1/2}-E^{-1/2})^2$

$= \delta^2$

⑦ $\Delta \cos x$

$= \cos(x+h) - \cos x$

$= -2\sin\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right) + \mu^2 \delta^2$

⑧ $\Delta \log x$

$= \log(x+h) - \log x$

$= \log\left(\frac{x+h}{x}\right)$

$= \log\left(1+\frac{h}{x}\right)$

⑨ $\Delta \tan^{-1} x = \tan^{-1}(x+h) - \tan^{-1} x$

① ~~Find the forward difference & backward difference table for the function $y = x^3$~~

x ~~$y = f(x)$~~   ~~Diff~~

→ $\Delta y_0 = y_1 - y_0$

→ $\nabla y_1 = y_1 - y_0$

$\delta y_{1/2} = y_1 - y_2$

$\mu = \frac{1}{2}\left(y_{r+1/2} + y_{r-1/2}\right)$

$E y_r = y_{r+1}$

$E^{-1} y_r = y_{r-1}$

Starting - forward

last - backward

Newton - Interpolation :

forward

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots +$$

$$\frac{p(p-1)(p-2)\cdots p-(n-1)}{(n)!}\Delta^n y_0$$

$$\boxed{p = \frac{x - x_0}{h}}$$

backward

$$y = f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \cdots + \frac{p(p+1)(p+2)\cdots(p+(n-1))}{(n)!}\Delta^n y_n$$

$$\boxed{p = \frac{x - x_n}{h}}$$

1) construct the forward diff table for the following table which gives melting point of alloy of lead & zinc. find the melting point of alloy containing 54% of lead using appropriate interpolation formula.

percentage of lead in alloy	50	60	70	80
Temperature $0^\circ C$	205	225	248	274

Sol

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
50	y_0 205	$\Delta y_0 = 20$	$\Delta^2 y_0 = 3$	$\Delta^3 y_0 = 0$
60	y_1 225	$\Delta y_1 = 23$	$\Delta^2 y_1 = 3$	
70	y_2 248	$\Delta y_2 = 26$		
80	y_3 274			

$$p = \frac{x - x_0}{h} = \frac{54-50}{10} = .4$$

$$y = f(54) = 205 + (.4)(20) + \frac{.4(.4-1)}{2!}3 + \frac{.4(.4-1)(.4-2)}{3!}(0)$$

$$= 212.64$$

(2) The population of a crowd in a decimal size is given below. Estimate the population of 1925.

year	1891	1901	1911	1921	1931
popularity	46	66	81	93	101

Sol

x	y	∇y	∇^2	∇^3	∇^4
1891	46	$\nabla y_1 = 20$	$\nabla^2 y_2 = -5$	$\nabla^3 y_3 = 2$	$\nabla^4 y_4 = -3$
1901	66	$\nabla y_2 = 15$	$\nabla^2 y_3 = -3$	$\nabla^3 y_4 = -1$	
1911	81	$\nabla y_3 = 12$	$\nabla^2 y_4 = -4$		
1921	93	$\nabla y_4 = 8$			
1931	101				

$$p = \frac{1925 - 1931}{10} = \frac{-6}{10}$$

$$y = f(1925) = 101 + \left(\frac{-6}{10}\right)8 + \frac{\left(\frac{-6}{10}\right)\left(\frac{-6}{10}+1\right)(-4)}{2!} + \frac{\left(\frac{-6}{10}\right)\left(\frac{-6}{10}+1\right)\left(\frac{-6}{10}+2\right)(-1)}{3!} + \frac{\left(\frac{-6}{10}\right)\left(\frac{-6}{10}+1\right)\left(\frac{-6}{10}+2\right)\left(\frac{-6}{10}+3\right)(-3)}{4!}$$

$$= 96.696$$

(3) The following table corresponds for values of x & y construct the dif table and express y as fun of x

x	0	1	2	3	4
y	3	6	11	18	27

Sol

x	y	Δ	Δ^2	Δ^3	Δ^4
0	3	$\Delta y_0 = 3$	$\Delta^2 y_0 = 2$	$\Delta^3 y_0 = 0$	$\Delta^4 y_0 = 0$
1	6	$\Delta y_1 = 5$	$\Delta^2 y_1 = 2$	$\Delta^3 y_1 = 0$	
2	11	$\Delta y_2 = 7$	$\Delta^2 y_2 = 2$		
3	18	$\Delta y_3 = 9$			
4	27				

$p = \frac{x - x_0}{h} = \frac{x-0}{1} = x$

$y = f(x) = 3 + x(3) + \frac{x(x-1)2}{2!} + \frac{x(x-1)(x-2)}{3!}(0) + \frac{x(x-1)(x-2)(x-3)}{4!}$

$y = 3 + 3x + x^2 - x$

$= x^2 + 2x + 3$

$y = f(5) = 25 + 10 + 3$

$= 38$

(4) for $x = 0, 1, 2, 3, 4$ $f(x) = 1, 14, 15, 5, 6$

find $f(3.5)$ using appropriate interpulation formula

x	$f(x)$	∇	∇^2	∇^3	∇^4
0	1	$\nabla y_1 = 13$	$\nabla^2 y_2 = -12$	$\nabla^3 y_3 = 1$	$\nabla^4 y_4 = 21$
1	14	$\nabla y_2 = 1$	$\nabla^2 y_3 = -11$	$\nabla^3 y_4 = 22$	
2	15	$\nabla y_3 = -10$	$\nabla^2 y_4 = 11$		
3	5	$\nabla y_4 = 1$			
4	6				

$p = \frac{x - x_n}{h} = \frac{3.5 - 4}{1} = -0.5$

$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \cdots + \frac{p(p+1)(p+2)\cdots + (n-1)}{(n-1)!}\nabla^n y_n$

$= 6 + (0.5)(1) + \frac{(-0.5)(-0.5+1)}{2!}(11) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(22)$

$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(21)$

Central difference

x_{-2}	y_{-2}	$\Delta y_{-2} = y_{-1} - y_{-2}$	
x_{-1}	y_{-1}	$\Delta y_{-1} = y_0 - y_{-1}$	
x_0	y_0	$\Delta y_0 = y_1 - y_0$	
x_1	y_1	$\Delta y_1 = y_2 - y_1$	
x_2	y_2		

forward
when lower values are high

backward
when top values are high

Gauss forward interpulation

formula

$$y_p = f(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_{-1} + \frac{P(P-1)(P+1)}{3!}\Delta^3 y_{-2} + \frac{P(P-1)(P+1)(P-2)}{4!}\Delta^4 y_{-3} + \dots$$

Gauss backward interpulation formula

$$y_p = f(x) = y_0 + P\Delta y_{-1} + \frac{P(P+1)}{2!}\Delta^2 y_{-1} + \frac{(P+1)P(P-1)}{3!}\Delta^3 y_{-2} + \frac{(P+1)P(P-1)(P+2)}{4!}\Delta^4 y_{-2}$$

(1) From the table interpulate value of y when $x = 1.91$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.7	5.4739					
		0.5757				
1.8	6.0496		0.0606			
		0.6363		0.0063		
1.9	y_0 6.6859		Δy_{-1} 0.0669 y_{-2}		0.0007 Δy_{-4}	
		Δy_0 0.7032		Δy_{-3} 0.0070		0.0001
2	7.3891		0.0739		0.0008	
		0.7771		0.0078		
2.1	8.1662		0.0817			
		0.8588				
2.2	9.0250					

$$p = \frac{x - x_0}{h} = \frac{1.91 - 1.9}{0.1} = \frac{0.01}{0.1} = 0.1$$

$$f(1.91) = 6.6859 + (0.1)(0.7032) + \frac{0.1(0.1-1)}{2!} 0.0669 + \frac{(0.1)(0.1-1)(0.1+1)}{3!} 0.0070 + \frac{0.1(0.1-1)(0.1+ \ldots}{4!}$$

$$= 6.7531$$

(2) To find $f(30)$ given that $f(21) = 18.4708$, $f(25) = 17.8144$, $f(29) = 17.1070$, $f(33) = 16.3432$, $f(37) = 15.5754$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2} 21	y_{-2} 18.4708				
		$\Delta y_{-2} = -0.6564$			
x_{-1} 25	y_{-1} 17.8144		$\Delta^2 y_{-2} = -0.051$		
		-0.7074		$\Delta^3 y_{-2} = -0.0054$	
x_0 29	y_0 17.1070		-0.0564		-0.002
		-0.7638		-0.0076	
x_1 33	y_1 16.3432		-0.064		
		-0.8278			
x_2 37	y_2 15.5154				

$$f(30) = 17.1070 + 0.25(-0.7638) + 0.25\frac{(0.25-1)}{2!}(-0.0\ldots) + \frac{(0.25)(0.25+1)(0.25-1)}{3!}(-0.007) + \frac{(0.25)(0.25+1)\ldots}{4!}(-0.0024)$$

$$= 16.921$$

$$p = \frac{x - x_0}{h} = \frac{30 - 29}{4} = 0.25$$

(3) find $y(8)$

x	0	5	10	15	20	25
y	7	11	14	18	24	32

0-1-2)

0.0001

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_{-2} 0	y_{-2} 7					
		$\Delta y_{-2} = 4$				
x_{-1} 5	y_{-1} 11		$\Delta y_{-2} = -1$			
		$\Delta y_{-1} = 3$		2		
x_0 10	y_0 14		1		-1	
		$\Delta y_0 = 4$		1		0
x_1 15	y_1 18		2		-1	
		$\Delta y_1 = 6$		0		
x_2 20	y_2 24		2			
		$\Delta y_2 = 8$				
x_3 25	y_3 32					

$$P = \frac{x - x_0}{h} = \frac{8 - 10}{5} = \frac{-2}{5}$$

$$y(8) = 14 + \left(\frac{-2}{5}\right)(3) + \frac{\left(\frac{-2}{5}\right)\left(\frac{-2}{5}+1\right)}{2!}(1) + \frac{\left(\frac{-2}{5}\right)\left(\frac{-2}{5}+1\right)\left(-\frac{2}{5}-1\right)}{3!}(2) + \frac{\left(\frac{-2}{5}\right)\left(\frac{-2}{5}+1\right)\left(-\frac{2}{5}-1\right)\left(-\frac{2}{5}+2\right)}{4!}(-1)$$

64)

(0.25-1)(0.25-2)

$= 12.7696$

Lagranges Interpolation formula for unevenly spaced points

If $x_0, x_1, x_2 - - x_n$ be unevenly spaced point let $y_0, y_1, \cdot - y_n$ be the corresponding functional elements then $f(x) =$

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$$f(x) = \frac{(x-x_1)(x-x_2) \cdots (x-x_n)}{(x_0-x_1)(x_0-x_2) \cdots (x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3) \cdots (x-x_n)}{(x_1-x_0)(x_1-x_2) \cdots (x_1-x_n)} f(x_1)$$

$$\frac{(x-x_0)(x-x_1) \cdots (x-x_n)}{(x_2-x_0)(x_2-x_1) \cdots (x_2-x_n)} f(x_2) + \cdots$$

$$+ \frac{(x-x_0)(x-x_1) \cdots (x-x_{n-1})}{(x_n-x_0)(x_n-x_1) \cdots (x_n-x_{n-1})} f(x_n)$$

① find $f(3)$

x	0	1	2	$\cancel{4}$ 4	5	6
$f(x)$	1	14	15	5	6	19

$$f(3) = \frac{(x-1)(x-2)(x-4)(x-5)(x-6)}{(0-1)(0-2)(0-4)(0-5)(0-6)} \cdot 1 + \frac{(x-0)(x-2)(x-4)(x-5)(x-6)}{(1-0)(1-2)(1-4)(1-5)(1-6)} \cdot 14$$

$$+ \frac{(x-0)(x-1)(x-4)(x-5)(x-6)}{(2-0)(2-1)(2-4)(2-5)(2-6)} \cdot 15 + \frac{(x-0)(x-1)(x-2)(x-5)(x-6)}{(4-0)(4-1)(4-2)(4-5)(4-6)} \cdot 5$$

$$+ \frac{(x-0)(x-1)(x-2)(x-4)(x-6)}{(5-0)(5-1)(5-2)(5-4)(5-6)} \cdot 6 + \frac{(x-0)(x-1)(x-2)(x-4)(x-5)}{(6-0)(6-1)(6-2)(6-4)(6-5)} \cdot 19$$

$$= 10.475$$

② Find the parabola passing through the points $(0,1)$ $(1,3)$ and $(3,55)$

$x_0 \quad x_1 \quad x_2$; $y_0 \quad y_1 \quad y_2$

Sol $f(x) = \frac{(x-1)(x-3)}{(0-1)(0-3)} \cdot 1 + \frac{(x-0)(x-3)}{(1-0)(1-3)} \cdot 3 +$

$$\frac{(x-0)(x-1)}{(3-0)(3-1)} \cdot 55$$

$$= \frac{x^2-4x+3}{3} - \frac{x^2-3x}{+2} \cdot 3 + \frac{x^2-x}{6} \cdot 55$$

$$= \frac{2(x^2-4x+3)+3(-x^2+3x)+x^2-x}{6}$$

$$= \frac{2x^2-8x+6(-3x^2+9x)3+(x^2-x)55}{6}$$

$$= 8x^2-6x+1$$

(3) 0, 1, 4, 5 $f(x)$

4   3   24   29

$$f(x) = \frac{(x-1)(x-4)(x-5)}{(0-1)(0-4)(0-5)} \cdot 4 + \frac{(x-0)(x-4)(x-5)}{(1-4)(1-24)(1-29)} \cdot 3 + \frac{(x-0(x-1)(x-5)}{(4-0)(4-1)(4-5)} \cdot 24 + \frac{(x-0)(x-1)(x-4)}{(5-0)(5-1)(5-4)} \cdot 29$$

$$= 2x^2-3x+4$$