UNIT-II

Numerical Integration

I. Trepezoidal Rule

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + - - - - + y_{n-1})\right]$$

II. Simphson 1/3 Rule

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + - - - - + y_{n-1}) + 2(y_2 + y_4 + - - - - + y_{n-2})\right]$$

III. Simphson 3 / 8 Rule

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 - - - - + y_{n-1}) + 2(y_3 + y_6 + - - - - + y_{n-3})\right]$$

1. **Find the value of $\int_0^1 \frac{1}{1+x^2}\,dx$, taking 5 sub internals & by using trapezoidal rule**

Ans.

$$f(x) = \frac{1}{1+x^2}, n = 5, a = 0, b = 1$$

$$\therefore h = \frac{b-a}{n} = \frac{1-0}{5} = 0.2$$

Construct a table of values of x_i & $yi = f(xi)$ as follows

Xi	0.0	0.2	0.4	0.6	0.8	1.0
Yi	1.00	0.961538	0.832069	0.735294	0.609755	0.50

Using Trapezoidal rule we get

$$I = \int_0^1 \frac{1}{1+x^2}\,dx = \frac{0.2}{2}\left[(1.0 + 0.50) + 2(0.961538 + 0.832069 + 0.735294 + 0.609759)\right]$$

$$= 0.783734$$

2. **Find the area bounded by the curve $f(x) = y$ and x-axis from $x = 7.47$ *to* $x = 7.52$**

xi	7.47	7.48	7.49	7.50	7.51	7.52
yi	1.93	1.95	1.98	2.01	2.03	2.06

Ans. Here h = 0.01

Area formed by the curve $y = f(x)$ and x – axis from $x = 7.47$ *to* $x = 7.52$ is

$$Area = \int_{7.47}^{7.52} f(x)\,dx$$

Applying Trapezoidal rule we get

$$Area = \int_{7.47}^{7.52} f(x)\,dx = \frac{h}{2}\left[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)\right]$$

$$= \frac{0.01}{2}\left[(1.93 + 2.06) + 2(1.95 + 1.98 + 2.01 + 2.03)\right]$$

$$= 0.0996$$

3. Evaluate $\int_0^1 x^3 dx$ with 5 sub intervals by Trapezoidal rule

Ans. Here $a=0, b=1, n=5 \;\&\; y=f(x)=x^3$

$\therefore h=\frac{b-a}{n}=\frac{1-0}{5}=0.2$ The values of x& y are tabulated below

x	0.2	0.4	0.6	0.8	1
y	0.008	0.064	0.216	0.512	1

By Trapezoidal rule $\int_0^1 x^3 dx=\frac{1}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$

$$=\frac{0.2}{2}\left[(0.008+1)+2(0.064+0.216+0.512)\right]$$

$=0.2592$

4. Evaluate $\int_0^\pi t\sin t dt$ using Trapezoidal rule

Ans. Divide the internal $(0,\pi)$ in to 6 parts each of width $h=\frac{\pi}{6}$

The values of $f(t)=t\sin t$ are given below

T	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
$f(t)=y$	0 y_0	0.2618 y_1	0.9069 y_2	1.5708 y_3	1.8138 y_4	1.309 y_5	0 y_6

By Trapezoidal rule

$$\int_0^\pi t\sin t dt=\frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$=\frac{\pi}{12}\left[(0+0)+2(0.2618+0.9069+1.5708+1.8138+1.309)\right]$$

$$=\frac{\pi}{12}(11.7246)$$

$$=3.0695$$

5. find the value of $\int_1^2 \frac{dx}{x}$ by simpson's 1/3 rule. Hence obtain approximate value of $\log_e^{\,2}$

Ans. Divide the interval (1,2) in to 8 parts each of width $h=0.125$

The value of x & y are tabulated below

x	1	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2
$y=\frac{1}{x}$	1 y_0	0.8888 y_1	0.8 y_2	0.7272 y_3	0.6666 y_4	0.6153 y_5	0.5714 y_6	0.5333 y_7	0.5 y_8

By simpson's 1/3 value

$$=\frac{h}{3}\left[(y_0+y_8)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6)\right]$$

$$=\frac{0.125}{3}\left[(1+0.5)+4(0.8888+0.7272+0.6153+0.5333)+2(0.8+0.6666+0.5714)\right]$$

$$=\frac{0.125}{3}[1.5+11.0584+4.076]$$

$$=\frac{0.125}{3}[16.6344]$$

$$=0.6931$$

By actual integration,

$$\int_1^2 \frac{dx}{x}=[\log x]_1^2 \;=\log 2-\log 1=\log 2$$

Hence $\log 2=0.6931$, correct to four decimal places

6. **A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using simpson's 1/3 rule, find the velocity of the rocket at t = 80 seconds**

t (sec)	0	10	20	30	40	50	60	70	80
$f\left(cm/\sec^2\right)$	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

Ans. We know that the rate of velocity is acceleration I.e., $f=\frac{\partial v}{\partial t}$

$\therefore$ velocity of the rocket at $t=80\sec$ is given by

$$v=\int_0^{80} f dt$$

$$=\frac{10}{3}\left[(30+50.67)+4(31.63+35.47+40.33+46.63)+2(33.34+37.75+43.25)\right]$$

$$=\frac{10}{3}[80.67+616.48+228.68]$$

$$=\frac{10}{3}(925.83)$$

$$=30.86\ m/\sec$$

7. **A river is soft wide. The depth 'd' in feet at a distance x ft from one bank is given by the table**

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find approximately the area of cross-section

Ans. Here $h=10, y_0=0, y_1=4, y_2=7, y_3=9, y_4=12, y_5=15, y_6=14, y_7=8\ \&\ y_8=3$

Area of cross section $=\int_0^{80} y dx$

Area = $\frac{h}{3}\left[\left(y_0+y_8\right)+4\left(y_1+y_3+y_5+y_7\right)+2\left(y_2+y_4+y_6\right)\right]$

$=\frac{10}{3}\left[(0+3)+4(4+9+15+8)+2(7+12+14)\right]$

$=\frac{10}{3}[3+144+66]$

$=710 sq.ft$

8. evaluate $\int_0^{\pi} \sin x\, dx$ by dividing the interval $(0,\pi)$ in to 8 sub intervals & using simpson's 1/3 rule

Ans. Given $a=0, b=\pi, n=8 \,\&\, f(x)=\sin x$

$\therefore h=\frac{b-a}{n}=\frac{\pi-0}{8}=\pi/8$

Tabulate the values of $\sin x$ as follows

xi	0	$\pi/8$	$\pi/4$	$3\pi/8$	$5\pi/8$	$6\pi/8$	$7\pi/8$	π
$\sin xi$	0	0.38	0.71	0.92	1.00	0.92	0.710	0

Simpson's 1/3 rule for n = 8 is

$$I=\int_a^b f(x)\,dx=\frac{h}{3}\left[\left(y_0+y_8\right)+4\left(y_1+y_3+y_5+y_7\right)+2\left(y_2+y_4+y_6\right)\right]$$

$$=\frac{\pi}{8.3}\left[(0+0)+4(0.38+0.92+0.92+0.38)+2(0.71+1.0+0.71)\right]$$

$=1.99$

9. evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ using simpson's 3/8 rule

Ans. Divide the interval into 6 sub intervals & tabulate the values of $f(xi)=\frac{1}{1+x^2}$ as follows

xi	0	1/6	2/6	3/6	4/6	5/6	6/6
$f(xi)$	1	0.9729	0.80	0.90	0.69231	0.59016	0.5

Here $h=1/6$

Using simpson's rule

$$I=\int_0^1 \frac{1}{1+x^2}\,dx=\frac{3h}{8}\left[\left(y_0+y_6\right)+3\left(y_1+y_2+y_4+y_5\right)+2y_3\right]$$

$$=\frac{3}{8.6}\left[(1.0+0.50)+3(0.9729+0.30+0.6931+0.59016)+2(0.80)\right]$$

$=0.785395$

10. find the area bounded by the curve $y=e^{-x^2/2}$, x axis between $x=0 \,\&\, x=3$ by using simpson's 3/8 rule

Ans. Divide the interval $(0,3)$ in to 6 sub intervals

$\therefore h = \frac{3-0}{6} = 0.5$

The values of $y_i = e^{-x^2/2}$ are tabulated as follows

xi	0.0	0.5	1.0	1.5	2.0	2.5	3.0
$y(xi)$	1.0	1.33	1.649	3.080	7.389	22.760	90.017

By simpson's 3/8 rule we get

$$I = \int_0^3 e^{-x^2/2} dx = \frac{3h}{8}\left[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3\right]$$

$$= \frac{3(0.5)}{8}\left[(1.00 + 90.017) + 3(1.133 + 1.649 + 7.389 + 22.760) + 2(3.080)\right]$$

$= 36.744$ square units

Numerical solutions of ordinary differential equations

1. The important methods of solving ordinary differential equations of first order numerically are as follows
   1) Picards method
   2) Euler's method
   3) Modified Euler's method of successive approximations
   4) Taylors series method
   5) Runge- kutta method

To describe various numerical methods for the solution of ordinary differential eqn's,we consider the general 1st order differential eqn

dy/dx=f(x,y)-------(1)

with the initial condition $y(x_0)=y_0$

The methods will yield the solution in one of the two forms:

i) A series for y in terms of powers of x,from which the value of y can be obtained by direct substitution.

ii) A set of tabulated values of y corresponding to different values of x

The methods of Taylor and picard belong to class(i)

The methods of Euler, Runge - kutta method, Adams, Milne etc, belong to class (ii)

Picards method of successive approaches

Consider the following diff eqn

dy/dx=f(x,y)-------(1)

initial condition is that

$y=y_0$ at $x=x_0$----(2)

the eqn is dy=f(x,y)dx

integrating the eqn between the limits x_0 and x_1 we get

$$\int_{x=x_0}^{x} dy = \int_{x_0}^{x} f(x,y)dx$$

i.e $\left[y\right]_{x=x_0}^{x} = \int_{x_0}^{x} f(x,y)dx$

$y(x)-y(x_0)= \int_{x_0}^{x} f(x,y)dx$

or $y(x)=y_0+\int_{x_0}^{x} f(x,y)dx$ ----(3)

we find that the R.H.S of (3)

contains the unknown y under the integral sign An eqn of this kind is called an integral eqn and it can be solved by a process of successive approximation

Picard's method gives a sequence of functions $y^1(x), y^{(2)}(x), y^{(3)}(x)$,….

Which form a sequence of approximation to y converges to y(x)

To get the 1st approximation $y^{(1)}(x)$, put $y = y_0$, in the integral of (3)

We get $y^1(x)= y_0+ \int_{x0}^{x} f(x,y_0)dx$ ------------(4)

Since $f(x,y_0)$ is a function of x it is a possible to integral it with respect to x

To get the 2nd approximation $y^{(2)}(x)$ for y, put $y = y^{(1)}(x)$ in the integral of (3) we get

$$y^{(2)}(x) = y_0 + \int_{x0}^{x} f(x,y^{(1)}(x))dx \rightarrow (5)$$

Similarly, a 3rd approximation of y^3 for y is $y^3 = y_0 + \int_{x_0}^{x} f\left[\left(x,y^{(2)}\right)(x)\right]dx \rightarrow (6)$

proceeding in this way we get the nth approximation $y^{(n)}(x)$ for y as

$$y_n(x) = y_0 + \int_{x_0}^{x} f\left[x, y^{(n-1)}(x)\right]dx \rightarrow (7) \text{ or } y_n = y_0 + \int_{x_0}^{x} f\left(x, y_{n-1}^{n-1} \right)dx, n = 1,2.....$$

TAYLOR'S SERIES METHOD

To find the numerical solution of the differential equation

$$\frac{dy}{dx} = f(x,y) \rightarrow (1)$$

With the initial condition $y(x_0) = y_0 \rightarrow (2)$

$y(x)$ can be expanded about the point x_0 in a Taylor's series in powers of $(x-x_0)$ as

$$y(x) = y(x_0) + \frac{(x - x_0)}{1} y'(x_0) + \frac{(x - x_0)^2}{2!} y''(x_0) + ... + \frac{(x - x_0)^n}{n!} y^n(x_0) \rightarrow (3)$$

In equ3, $y(x_0)$ is known from I.C equ2. The remaining coefficients $y'(x_0), y''(x_0), ...y^n(x_0)$ etc are obtained by successively differentiating equ1 and evaluating at x_0. Substituting these values in equ3, $y(x)$ at any point can be calculated from equ3. Provided $h = x - x_0$ is small.

When $x_0 = 0$, then Taylor's series equ3 can be written as

$$y(x) = y(0) + x.y'(0) + \frac{x^2}{2!} y''(0) + ... + \frac{x^n}{n!} y^n(0) + ... \rightarrow (4)$$

1. Using Taylor's expansion evaluate the integral of $y' - 2y = 3e^x, y(0) = 0$ **, at a)** $x = 0.2$

b) compare the numerical solution obtained with exact solution .

Sol: Given equation can be written as $2y + 3e^x = y', y(0) = 0$

Differentiating repeatedly w.r.t to 'x' and evaluating at $x = 0$

$$y'(x) = 2y + 3e^x, y'(0) = 2y(0) + 3e^0 = 2(0) + 3(1) = 3$$
$$y''(x) = 2y' + 3e^x, y''(0) = 2y'(0) + 3e^0 = 2(3) + 3 = 9$$
$$y'''(x) = 2.y''(x) + 3e^x, y'''(0) = 2y''(0) + 3e^0 = 2(9) + 3 = 21$$
$$y^{iv}(x) = 2.y'''(x) + 3e^x, y^{iv}(0) = 2(21) + 3e^0 = 45$$
$$y^{v}(x) = 2.y^{iv} + 3e^x, y^{v}(0) = 2(45) + 3e^0 = 90 + 3 = 93$$

In general, $y^{(n+1)}(x) = 2.y^{(n)}(x) + 3e^x$ or $y^{(n+1)}(0) = 2.y^{(n)}(0) + 3e^0$

The Taylor's series expansion of $y(x)$ about $x_0 = 0$ is

$$y(x) = y(0) + xy'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \frac{x^4}{4!} y''''(0) + \frac{x^5}{5!} y'''''(0) + ...$$

Substituting the values of $y(0), y'(0), y''(0), y'''(0), ...$

$$y(x) = 0 + 3x + \frac{9}{2}x^2 + \frac{21}{6}x^3 + \frac{45}{24}x^4 + \frac{93}{120}x^5 + ...$$

$$y(x) = 3x + \frac{9}{2}x^2 + \frac{7}{2}x^3 + \frac{15}{8}x^4 + \frac{31}{40}x^5 + ... \rightarrow \text{equ1}$$

Now put $x = 0.1$ in equ1

$$y(0.1) = 3(0.1) + \frac{9}{2}(0.1)^2 + \frac{7}{2}(0.1)^3 + \frac{15}{8}(0.1)^4 + \frac{31}{40}(0.1)^5 = 0.34869$$

Now put $x = 0.2$ in equ1

$$y(0.2) = 3(0.2) + \frac{9}{2}(0.2)^2 + \frac{7}{2}(0.2)^3 + \frac{15}{8}(0.2)^4 + \frac{31}{40}(0.2)^5 = 0.811244$$

$$y(0.3) = 3(0.3) + \frac{9}{2}(0.3)^2 + \frac{7}{2}(0.3)^3 + \frac{15}{8}(0.3)^4 + \frac{31}{40}(0.3)^5 = 1.41657075$$

Analytical Solution:

The exact solution of the equ $\frac{dy}{dx} = 2y + 3e^x$ with $y(0) = 0$ can be found as follows

$\frac{dy}{dx} - 2y = 3e^x$ Which is a linear in y.

Here $P = -2, Q = 3e^x$

I.F = $\int_e^{pdx} = \int_e^{-2dx} = e^{-24}$

General solution is $y.e^{-2x} = \int 3e^x.e^{-2x}dx + c = -3e^{-x} + c$

$\therefore y = -3e^x + ce^{2x}$ where $x = 0, y = 0 \quad 0 = -3 + c \Rightarrow c = 3$

The particular solution is $y = 3e^{2x} - 3e^x$ or $y(x) = 3e^{2x} - 3e^x$

Put $x = 0.1$ in the above particular solution,

$$y = 3.e^{0.2} - 3e^{0.1} = 0.34869$$

Similarly put $x = 0.2$

$$y = 3e^{0.4} - 3e^{0.2} = 0.811265$$

put $x = 0.3$

$$y = 3e^{0.6} - 3e^{0.3} = 1.416577$$

2. Using Taylor's series method, solve the equation $\frac{dy}{dx} = x^2 + y^2$ for $x = 0.4$ given that $y = 0$ when $x = 0$

Sol: Given that $\frac{dy}{dx} = x^2 + y^2$ and $y = 0$ when $x = 0$ i.e. $y(0) = 0$

Here $y_0 = 0$, $x_0 = 0$

Differentiating repeatedly w.r.t 'x' and evaluating at $x = 0$

$$y'(x) = x^2 + y^2, y'(0) = 0 + y^2(0) = 0 + 0 = 0$$
$$y''(x) = 2x + y'.2y, y''(0) = 2(0) + y'(0)2.y = 0$$
$$y'''(x) = 2 + 2yy'' + 2y'.y', y'''(0) = 2 + 2.y(0).y''(0) + 2.y'(0)^2 = 2$$
$$y''''(x) = 2.y.y''' + 2.y''.y' + 4.y''.y', y''''(0) = 0$$

The Taylor's series for f(x) about $x_0 = 0$ is

$$y(x) = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y''''(0) + ...$$

Substituting the values of $y(0), y'(0), y''(0), ...$

$$y(x) = 0 + x(0) + 0 + \frac{2x^3}{3!} + 0 + ... = \frac{x^3}{3} + \text{(Higher order terms are neglected)}$$

$$\therefore y(0.4) = \frac{(0.4)^3}{3} = \frac{0.064}{3} = 0.02133$$

3. Solve $y' = x - y^2, y(0) = 1$ **using Taylor's series method and compute y(0.1),y(0.2)**

Sol: Given that $y' = x - y^2, y(0) = 1$

Here $y_0 = 1$, $x_0 = 0$

Differentiating repeatedly w.r.t 'x' and evaluating at x=0

$$y'(x) = x - y^2, y'(0) = 0 - y(0)^2 = 0 - 1 = -1$$
$$y''(x) = 1 - 2y.y', y''(0) = 1 - 2.y(0)y'(0) = 1 - 2(-1) = 3$$
$$y'''(x) = 1 - 2yy' - 2(y')^2, y'''(0) = -2.y(0).y''(0) - 2.(y'(0))^2 = -6 - 2 = -8$$
$$y''''(x) = -2.y.y''' - 2.y''.y' - 4.y''.y', y''''(0) = -2.y(0).y'''(0) - 6.y''(0).y'(0) = 16 + 18 = 34$$

The Taylor's series for f(x) about $x_0 = 0$ is

$$y(x) = y(0) + \frac{x}{1!}y^1(0) + \frac{x^2}{2!}y^{11}(0) + \frac{x^3}{3!}y^{111}(0) + \ldots..$$

Substituting the value of y(0), $y^1(0)$, $y^{11}(0)$,…..

$$y(x) = 1 - x + \frac{3}{2}x^2 - \frac{8}{6}x^3 + \frac{34}{24}x^4 + \ldots..$$

$$y(x) = 1 - x + \frac{3}{2}x^2 - \frac{4}{3}x^3 + \frac{17}{12}x^4 + \ldots.. \quad \rightarrow(1)$$

now put x = 0.1 in (1)

$$y(0.1) = 1 - 0.1 + \frac{3}{2}(0.1)^2 + \frac{4}{3}(0.1)^3 + \frac{17}{12}(0.1)^4 + \ldots..$$

$$= 0.91380333 \simeq 0.91381$$

Similarly put x = 0.2 in (1)

$$y(0.2) = 1 - 0.2 + \frac{3}{2}(0.2)^2 - \frac{4}{3}(0.2)^3 + \frac{17}{12}(0.2)^4 + \ldots..$$

$$= 0.8516.$$

4. Solve $y^1 = x^2 - y$, y(0) = 1, using Taylor's series method and compute y(0.1), y(0.2), y(0.3) and y(0.4) (correct to 4 decimal places).

Sol. Given that $y^I = x^2 - y$ and y(0) = 1

Here $x_0 = 0$, $y_0 = 1$ or y =1 when x=0

Differentiating repeatedly w.r.t 'x' and evaluating at x = 0.

$Y^I(x) = x^2 - y$, $y^I(0) = 0 - 1 = -1$

$y^{II}(x) = 2x - y^I$, $y^{II}(0) = 2(0) - y^I(0) = 0 - (-1) = 1$

$y^{III}(x) = 2 - y^{II}$, $y^{III}(0) = 2 - y^{II}(0) = 2 - 1 = 1$,

$y^{IV}(x) = - y^{III}$, $y^{IV}(0) = -y^{III}(0) = -1$.

The Taylor's servies for f(x) about $x_0 = 0$ is

$$y(x) = y(0) + \frac{x}{1!} y^{I}(0) + \frac{x^2}{2!} y^{II}(0) + \frac{x^3}{3!} y^{III}(0) + \frac{x^4}{4!} y^{IV}(0) + \ldots\ldots$$

substituting the values of $y(0)$, $y^1(0)$, $y^{11}(0)$, $y^{111}(0)$,……

$$y(x) = 1 + x(-1) + \frac{x^2}{2}(1) + \frac{x^3}{6}(1) + \frac{x^4}{24}(-1) + \ldots\ldots$$

$$y(x) = 1 - x + \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} + \ldots\ldots \qquad \rightarrow (1)$$

Now put x = 0.1 in (1),

$$y(0.1) = 1 - 0.1 + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{6} - \frac{(0.1)^4}{24} + \ldots.$$

$$= 1 - 0.1 + 0.005 + 0.01666 - 0.0000416 - 0.905125 \sim 0.9051$$

(4 decimal places)

Now put x = 0.2 in eq (1),

$$y(0.2) = 1 - 0.2 + \frac{(0.2)^2}{2} + \frac{(0.2)^3}{6} - \frac{(0.2)^4}{64}$$

$$= 1 - 0.2 + 0.02 + 0.001333 - 0.000025$$

$$= 1.021333 - 0.200025$$

$$= 0.821308 \sim 0.8213 \text{ (4 decimals)}$$

Similarly y(0.3) = 0.7492 and y (0.4) = 0.6897 (4 decimal places).

5. Solve $\frac{dy}{dx}$ -1 = xy and y(0) = 1 using Taylor's series method and compute y(0.1).

Sol. Given that $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$

Here $\frac{dy}{dx} = 1 + xy$ and $y_0 = 1$, $x_0 = 0$.

Differentiating repeatedly w.r.t 'x' and evaluating at $x_0 = 0$

$y^{I}(x) = 1 + xy$, $\qquad y^{I}(0) = 1+0(1) = 1$.

$y^{II}(x) = x.y' + y$, $\qquad y^{II}(0) = 0+1=1$

$y^{III}(x) = x.y'' + y^{I} + y^{I}$, $\qquad y^{III}(0) = 0.(1) + 2\,g(1) = 2$

$y^{IV}(x) = xy^{III} + y^{II} + 2y^{II,}$ $\qquad y^{IV}(0) = 0+3(1) = 3.$

$y^{V}(x) = xy^{IV} + y^{III} + 2y^{III}$, $\qquad y^{V}(0) = 0 + 2 + 2(3) = 8$

The Taylor series for f(x) about $x_0 = 0$ is

$$y(x) = y(0) + x.y^{I}(0) + \frac{x^2}{2!} y^{II}(0) + \frac{x^3}{3!} y^{III}(0) + \frac{x^4}{4!} y^{IV}(0) + \frac{x^5}{5!} y^{V}(0) + \ldots..$$

Substituting the values of $y(0)$, $y^{I}(0)$, $y^{II}(0)$, ….

$$y(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}(2) + \frac{x^4}{24}(3) + \frac{x^5}{120}(8) + \ldots.$$

$$y(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} + \frac{x^5}{15} + \ldots. \qquad \rightarrow(1)$$

Now put x = 0.1 in equ (1),

$$y(0.1) = 1 + 0.1 + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} + \frac{(0.1)^4}{8} + \frac{(0.1)^5}{15} + \ldots..$$

$$= 1 + 0.1 + 0.005 + 0.000333 + 0.0000125 + 0.0000006$$

$$= 1.1053461$$

H.W

6. Given the differential equ $y^1 = x^2 + y^2$, $y(0) = 1$.Obtain $y(0.25)$, and $y(0.5)$ by Taylor's Series method.

Ans: 1.3333, 1.81667

7. Solve $y^1 = xy^2 + y$, $y(0) =1$ using Taylor's series method and compute $y(0.1)$ and $y(0.2)$.

Ans: 1.111, 1.248.

Note: We know that the Taylor's expansion of y(x) about the point x_0 in a power of $(x - x_0)$is.

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!}y^I(x_0) + \frac{(x-x_0)^2}{2!}y^{II}(x_0) + \frac{(x-x_0)^3}{3!}y^{III}(x_0) + \ldots \rightarrow(1)$$

Or

$$y(x) = y_0 + \frac{(x-x_0)}{1!}y_0^I + \frac{(x-x_0)^2}{2!}y_0^{II} + \frac{(x-x_0)^3}{3!}y_0^{III} + \ldots..$$

If we let $x - x_0 = h$. (i.e. $x = x_0 + h = x_1$) we can write the Taylor's series as

$$y(x) = y(x_1) = y_0 + \frac{h}{1!}y_0^I + \frac{h^2}{2!}y_0^{II} + \frac{h^3}{3!}y_0^{III} + \frac{h^4}{4!}y_0^{IV} + \ldots.$$

$$\text{i.e. } y_1 = y_0 + \frac{h}{1!}y_0^I + \frac{h^2}{2!}y_0^{II} + \frac{h^3}{3!}y_0^{III} + \frac{h^{IV}}{4!}y_0^{IV} + \ldots.. \qquad \rightarrow(2)$$

Similarly expanding y(x) in a Taylor's series about $x = x_1$. We will get.

$$y_2 = y1 + \frac{h}{1!}y_1^I + \frac{h^2}{2!}y_1^{II} + \frac{h^3}{3!}y_1^{III} + \frac{h^4}{4!}y_1^{IV} + \ldots\ldots. \qquad \rightarrow(3)$$

Similarly expanding y(x) in a Taylor's series about $x = x_2$ We will get.

$$y_3 = y_2 + \frac{h}{1!}y_2^I + \frac{h^2}{2!}y_2^{II} + \frac{h^3}{3!}y_2^{III} + \frac{h^4}{4!}y_2^{IV} + \ldots... \qquad \rightarrow(4)$$

In general, Taylor's expansion of y(x) at a point $x= x_n$ is

$$y_{n+1} = y_n + \frac{h}{1!} y_n^I + \frac{h^2}{2!} y_n^{II} + \frac{h^3}{3!} y_n^{III} + \frac{h^4}{4!} y_n^{IV} + \ldots.. \quad \rightarrow (5)$$

8. Solve $y^1 = x\text{-}y^2$, y(0) = 1 using Taylor's series method and evaluate y(0.1), y(0.2).

Sol: Given $y^1 = x - y^2$ $\rightarrow$(1)

and y(0) = 1 $\rightarrow$(2)

Here $x_0 = 0$, $y_0 = 1$.

Differentiating (1) w.r.t 'x', we get.

$y^{II} = 1 - 2yy^I \rightarrow (3)$

$y^{III} = -2(y.\, y^{II} + (y^I)^2)$ $\rightarrow$ (4)

$y^{IV} = -2[y.\, y^{III} + y.\, y^{II} + 2y^I.\, y^{II}]$ $\rightarrow$(5)

$= -2(3y^I.\, y^{II} + y.\, y^{III})$ ……

Put $x_0 = 0$, $y_0 = 1$ in (1),(3),(4) and (5),

We get

$y_0^I = 0\text{-}1 = -1,$

$y_0^{II} = 1\text{-}2(1)(-1) = 3,$

$y_0^{III} = -2[(-1)^2) + (1)(3)] = -8$

$y_0^{IV} = -2[3(-1)(3) + (1)(-8)] = -2(-9 -8) = 34.$

Take h=0.1

<u>Step1</u>: By Taylor's series, we have

$$y_1 = y_0 + \frac{h}{1!} y_0^I + \frac{h^2}{2!} y_0^{II} + \frac{h^3}{3!} y_0^{III} + \frac{h^4}{4!} y_0^{IV} + \ldots.. \quad \rightarrow \mathbf{(6)}$$

on substituting the values of y_0, y_0^I, y_0^{II}, etc in equ (6) we get

$$y(0.1) = y_1 = 1 + \frac{0.1}{1}(-1) + \frac{(0.1)^2}{2}(3) + \frac{(0.1)^3}{6}(-8) + \frac{(0.1)^4}{24}(34) + \ldots.$$

$= 1 - 0.1 + 0.015 - 0.00133 + 0.00014 + \ldots$

$= 0.91381$

<u>Step2</u>: Let us find y(0.2), we start with (x_1, y_1) as the starting value.

Here $x_1 = x_0 + h = 0+0.1 = 0.1$ and $y_1 = 0.91381$

Put these values of x_1 and y_1 in (1),(3),(4) and (5),we get

$y_1^I = x_1 - y_1^2 = 0.1 - (0.91381)^2 = 0.1 - 0.8350487 = -0.735$

$y_1^{II} = 1 - 2y_1 \cdot y_1^I = 1\text{-}\ 2(0.91381)(-0.735) = 1 + 1.3433 = 2.3433$

$y_1^{III} = -\ 2[(y_1^I)^2 + y_1 \cdot y_1^{II}] = -\ 2[(-0.735)^2 + (0.91381)(2.3433)] = -5.363112$

$y_1^{IV} = -\ 2[3.\ y_1^I\ y_1^{II} + y_1\ y_1^{III}] = -\ 2[3.(-0.735)(2.3433) + (0.91381)(-5.363112)]$

$= -2[(-5.16697) - 4.9] = 20.133953$

By Taylor's series expansion,

$$y_2 = y_1 + \frac{h}{1!}y_1^I + \frac{h^2}{2!}y_1^{II} + \frac{h^3}{3!}y_1^{III} + \frac{h^4}{4!}y_1^{IV} + \ldots.$$

$$\therefore y(0.2) = y_2 = 0.91381 + (0.1)(-0.735) + \frac{(0.1)^2}{2}(2.3433) +$$

$$\frac{(0.1)^3}{6}(-5.363112) + \frac{(0.1)^4}{24}(20.133953) + \ldots.$$

$$y(0.2) = 0.91381 - 0.0735 + 0.0117 - 0.00089 + 0.00008$$

$$= 0.8512$$

9. Tabulate y(0.1), y(0.2) and y(0.3) using Taylor's series method given that $y^1 = y^2 + x$ and $y(0) = 1$

Sol: Given $y^1 = y^2 + x$ $\rightarrow(1)$

and $y(0) = 1$ $\rightarrow(2)$

Here $x_0 = 0$, $y_0 = 1$.

Differentiating (1) w.r.t 'x', we get

$$y^{II} = 2y \cdot y^I + 1 \quad \rightarrow(3)$$

$$y^{III} = 2[y \cdot y^{II} + (y^I)^2] \quad \rightarrow(4)$$

$$y^{IV} = 2[y \cdot y^{III} + y^I y^{II} + 2\, y^I y^{II}]$$

$$= 2[y \cdot y^{III} + 3\, y^I y^{II}] \quad \rightarrow(5)$$

Put $x_0 = 0$, $y_0 = 1$ in (1), (3), (4) and (5), we get

$$y_0^I = (1)^2 + 0 = 1$$

$$y_0^{II} = 2(1)(1) + 1 = 3,$$

$$y_0^{III} = 2((1)(3) + (1)^2) = 8$$

$$y_0^{IV} = 2[(1)(8) + 3(1)(3)]$$

$$= 34$$

Take h = 0.1.

<u>**Step1**</u>: By Taylor's series expansion, we have

$$y(x_1) = y_1 = y_0 + \frac{h}{1!}y_0^I + \frac{h^2}{2!}y_0^{II} + \frac{h^3}{3!}y_0^{III} + \frac{h^4}{4!}y_0^{IV} + \ldots. \quad \rightarrow(6)$$

on substituting the values of y_o, y_0^I, y_0^{II} etc in (6),we get

$$y(0.1) = y_1 = 1 + (0.1)(1) + \frac{(0.1)^2}{2}(3) + \frac{(0.1)^3}{6}(8) + \frac{(0.1)^4}{24}(34) + \ldots.$$

$$= 1 + 0.1 + 0.015 + 0.001333 + 0.000416$$

$$y_1 = 1.116749$$

<u>**Step2**</u>: Let us find y(0.2),we start with (x_1,y_1) as the starting values

Here $x_1 = x_0 + h = 0 + 0.1 = 0.1$ and $y_1 = 1.116749$

Putting these values in (1),(3),(4) and (5), we get

$y_1^I = y_1^2 + x_1 = (1.116749)^2 + 0.1 = 1.3471283$

$y_1^{II} = 2y_1\, y_1^I + 1 = 2(10116749)\,(1.3471283) + 1 = 4.0088$

$y_1^{III} = 2(y_1\, y_1^{II} + (y_1^I)^2) = 2((1.116749)\,(4.0088) + (1.3471283)^2] = 12.5831$

$y_1^{IV} = 2y_1\, y_1^{III} + 6\, y_1^I\, y_1^{II} = 2(1.116749)\,(12.5831) + 6(1.3471283)\,(4.0088) = 60.50653$

By Taylor's expansion

$$y(x_2) = y_2 = y_1 + \frac{h}{1!}y_1^I + \frac{h^2}{2!}y_1^{II} + \frac{h^3}{3!}y_1^{III} + \frac{h^4}{4!}y_1^{IV} + \ldots.$$

$$\therefore y(0.2) = y_2 = 1.116749 + (0.1)\,(1.3471283) + \frac{(0.1)^2}{2}\,(4.0088) + \frac{(0.1)^3}{6}\,(12.5831) + \frac{(0.1)^4}{24}\,(60.50653)$$

$y_2 = 1.116749 + 0.13471283 + 0.020044 + 0.002097 + 0.000252$

$= 1.27385$

$y(0.2) = 1.27385$

<u>Step3</u>: Let us find y(0.3),we start with (x_2, y_2) as the starting value.

Here $x_2 = x_1 + h = 0.1 + 0.1 = 0.2$ and $y_2 = 1.27385$

Putting these values of x_2 and y_2 in eq (1), (3), (4) and (5), we get

$y_2^I = y_2^2 + x_2 = (1.27385)^2 + 0.2 = 1.82269$

$y_2^{II} = 2y_2\, y_2^1 + 1 = 2(1.27385)\,(1.82269) + 1 = 5.64366$

$y_2^{III} = 2[y_2\, y_2^{II} + (y_2^I)^2] = 2[(1.27385)\,(5.64366) + (1.82269)^2]$

$= 14.37835 + 6.64439 = 21.02274$

$y_2^{IV} = 2y_2 + y_2^{III} + 6\, y_2^I \cdot y_1^{II} = 2(1.27385)\,(21.00274) + 6(1.82269)\,(5.64366)$

$= 53.559635 + 61.719856 = 115.27949$

By Taylor's expansion,

$$y(x_3) = y_3 = y_2 + \frac{h}{1!}y_2^I + \frac{h^2}{2!}y_2^{II} + \frac{h^3}{3!}y_2^{III} + \frac{h^4}{4!}y_2^{IV} + \ldots..$$

$$y(0.3) = y_3 = 1.27385 + (0.1)\,(1.82269) + \frac{(0.1)^2}{2}(5.64366) + \frac{(0.1)^3}{6}(21.02274) + \frac{(0.1)^4}{24}(115.27949)$$

$= 1.27385 + 0.182269 + 0.02821 + 0.0035037 + 0.00048033$

$= 1.48831$

$y(0.3) = 1.48831$

10. Solve $y^1 = x^2 - y$, $y(0) = 1$ using Taylor's series method and evaluate

y(0.1),y(0.2),y(0.3) and y(0.4) (correct to 4 decimal places)

Sol: Given $y^1 = x^2 - y$ →(1)

and $y(0) = 1$ →(2)

Here $x_0 = 0$, $y_0 = 1$

Differentiating (1) w.r.t 'x', we get

$y^{II} = 2x - y^1$ →(3)

$y^{III} = 2 - y^{II}$ →(4)

$y^{IV} = -y^{III}$ →(5)

put $x_0 = 0$, $y_0 = 1$ in (1),(3),(4) and (5), we get

$y_0^I = x_0^2 - y_0 = 0-1 = -1,$

$y_0^{II} = 2x_0 - y_0^I = 2(0) - (-1) = 1$

$y_0^{III} = 2 - y_0^{II} = 2-1 = 1,$

$y_0^{IV} = - y_0^{III} = -1$ Take $h = 0.1$

Step1: by Taylor's series expansion

$$y(x_1) = y_1 = y_0 + \frac{h}{1!} y_0^I + \frac{h^2}{2!} y_0^{II} + \frac{h^3}{3!} y_0^{III} + \frac{h^4}{4!} y_0^{IV} + \ldots \rightarrow (6)$$

On substituting the values of y_0, y_0^I, y_0^{II} etc in (6), we get

$$y(0.1) = y_1 = 1 + (0.1)(-1) + \frac{(0.1)^2}{2}(1) + \frac{(0.1)^3}{6}(1) + \frac{(0.1)^4}{24}(-1) + \ldots$$

$$= 1 - 0.1 + 0.005 + 0.01666 - 0.0000416$$

$$= 0.905125 \simeq 0.9051 \text{ (4 decimal place).}$$

Step2: Let us find $y(0.2)$ we start with (x_1, y_1) as the starting values

Here $x = x_0 + h = 0 + 0.1 = 0.1$ and $y_1 = 0.905125$,

Putting these values of x_1 and y_1 in (1), (3), (4) and (5), we get

$y_1^1 = x_1^2 - y_1 = (0.1)^2 - 0.905125 = -0.895125$

$y_1^{II} = 2x_1 - y_1^1 = 2(0.1) - (-0.895125) = 1.095125,$

$y_1^{III} = 2 - y_1^{II} = 2 - 1.095125 = 0.90475,$

$y_1^{IV} = - y_1^{III} = -0.904875,$

By Taylor's series expansion,

$$y(x_2) = y_2 = y_1 + \frac{h}{1!} y_1^I + \frac{h^2}{2!} y_1^{II} + \frac{h^3}{3!} y_1^{III} + \frac{h^4}{4!} y_1^{IV} + \ldots$$

$$y(0.2) = y_2 = 0.905125 + (0.1)(-0.895125) + + \frac{(0.1)^2}{2}(1.09125) + \frac{(0.1)^3}{6}(1.095125) + \frac{(0.1)^4}{24}(-0.904875) + \ldots$$

$y(0.2) = y_2 = 0.905125 - 0.0895125 + 0.00547562 + 0.000150812 - 0.0000377$

$= 0.8212351 \simeq 0.8212$ (4 decimal places)

Step3: Let us find y(0.3), we start with (x_2,y_2) as the starting value

Here $x_2 = x_1 + h = 0.1+ 0.1 = 0.2$ and $y_2 = 0.8212351$

Putting these values of x_2 and y_2 in (1),(3),(4), and (5) we get

$y_2^1 = x_2^2 - y_2 = (0.2)^2 - 0.8212351 = 0.04 - 0.8212351 = - 0.7812351$

$y_2^{II} = 2x_2 - y_2^1 = 2(0.2) + (0.7812351) = 1.1812351,$

$y_2^{III} = 2 - y_2^{II} = 2 - 1.\ 1812351 = 0.818765,$

$y_2^{IV} = - y_2^{III} = -0.818765,$

By Taylor's series expansion,

$$y(x_3) = y_3 = y_2 + \frac{h}{1!} y_2^I + \frac{h^2}{2!} y_2^{II} + \frac{h^3}{3!} y_2^{III} + \frac{h^4}{4!} y_2^{IV} + \ldots$$

$$y(0.3) = y_3 = 0.\ 8212351 + (0.1)(-0.7812351) + \frac{(0.1)^2}{2}(1.1812351) + \frac{(0.1)^3}{6}(0.818765) + \frac{(0.1)^4}{24}(-0.818765) + \ldots$$

$y(0.3) = y_3 = 0.\ 8212351 - 0.07812351 + 0.005906 + 0.000136 - 0.0000034$

$= 0.749150 \simeq 0.7492$ (4 decimal places)

Step4: Let us find y(0.4), we start with (x_3,y_3) as the starting value

Here $x_3 = x_2 + h = 0.2+ 0.1 = 0.3$ and $y_3 = 0.749150$

Putting these values of x_3 and y_3 in (1),(3),(4), and (5) we get

$y_3^1 = x_3^2 - y_3 = (0.3)^2 - 0.749150 = -0.65915,$

$y_3^{II} = 2x_3 - y_3^1 = 2(0.3) + (0.65915) = 1.25915,$

$y_3^{III} = 2 - y_3^{II} = 2 - 1.\ 25915 = 0.74085,$

$y_3^{IV} = - y_3^{III} = -0.74085,$

By Taylor's series expansion,

$$y(x_4) = y_4 = y_3 + \frac{h}{1!} y_3^I + \frac{h^2}{2!} y_3^{II} + \frac{h^3}{3!} y_3^{III} + \frac{h^4}{4!} y_3^{IV} + \ldots$$

$$y(0.4) = y_4 = 0.\ 749150 + (0.1)(-0.65915) + \frac{(0.1)^2}{2}(1.25915) + \frac{(0.1)^3}{6}(0.74085) + \frac{(0.1)^4}{24}(-0.74085) + \ldots$$

$y(0.4) = y_4 = 0.\ 749150 - 0.065915 + 0.0062926 + 0.000123475 - 0.0000030$

$= 0.6896514 \simeq 0.6896$ (4 decimal places)

11. Solve $y^1 = x^2 - y$, $y(0) = 1$using T.S.M and evaluate $y(0.1)$,$y(0.2)$,$y(0.3)$ and $y(0.4)$ (correct to 4 decimal place) 0.9051, 0.8212, 07492, 0.6896

12. Given the differentiating equation $y^1 = x^1 + y^2$, $y(0) = 1$. Obtain $y(0.25)$ and $y(0.5)$ by T.S.M.

Ans: 1.3333, 1.81667

13. Solve $y^1 = xy^2 + y$, $y(0) = 1$ using Taylor's series method and evaluate $y(0.1)$ and $y(0.2)$

Ans: 1.111, 1.248.

EULER'S METHOD

It is the simplest one-step method and it is less accurate. Hence it has a limited application.

Consider the differential equation $\frac{dy}{dx} = f(x,y)$ $\rightarrow(1)$

With $y(x_0) = y_0 \rightarrow(2)$

Consider the first two terms of the Taylor's expansion of $y(x)$ at $x = x_0$

$$y(x) = y(x_0) + (x - x_0)\, y^1(x_0) \qquad \rightarrow(3)$$

from equation (1) $y^1(x_0) = f(x_0,y(x_0)) = f(x_0,y_0)$

Substituting in equation (3)

$\therefore y(x) = y(x_0) + (x - x_0)\, f(x_0,y_0)$

At $x = x_1$, $y(x_1) = y(x_0) + (x_1 - x_0)\, f(x_0,y_0)$

$\therefore y_1 = y_0 + h\, f(x_0,y_0)$ where $h = x_1 - x_0$

Similarly at $x = x_2$, $y_2 = y_1 + h\, f(x_1,y_1)$,

Proceeding as above, $y_{n+1} = y_n + h\, f(x_n,y_n)$

This is known as Euler's Method

1. **Using Euler's method solve for $x = 2$ from $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$, taking step size** (I) $h = 0.5$ and (II) $h=0.25$

Sol: here $f(x,y) = 3x^2 + 1$, $x_0 = 1$, $y_0 = 2$

Euler's algorithm is $y_{n+1} = y_n + h\, f(x_n,y_n)$, $n = 0,1,2,3,\ldots..$ $\rightarrow(1)$

(I) $h = 0.5$ $\therefore x_1 = x_0 + h = 1+0.5 = 1.5$

Taking $n = 0$ in (1) , we have $x_2 = x_1 + h = 1.5 + 0.5 = 2$

$y_1 = y_0 + h\, f(x_0,y_0)$

i.e. $y_1 = y(0.5) = 2 + (0.5)\, f(1,2) = 2 + (0.5)\,(3 + 1) = 2 + (0.5)(4)$

Here $x_1 = x_0 + h = 1 + 0.5 = 1.5$

$\therefore y(1.5) = 4 = y_1$

Taking n = 1 in (1),we have

$y_2 = y_1 + h\, f(x_1,y_1)$

i.e. $y(x_2) = y_2 = 4 + (0.5)\, f(1.5,4) = 4 + (0.5)[3(1.5)^2 + 1] = 7.875$

Here $x_2 = x_4 + h = 1.5 + 0.5 = 2$

$\therefore y(2) = 7.875$

(II) $h = 0.25$ $\therefore x_1 = 1.25,\ x_2 = 1.50,\ x_3 = 1.75,\ x_4 = 2$

Taking n = 0 in (1), we have

$y_1 = y_0 + h\, f(x_0,y_0)$

i.e. $y(x_1) = y_1 = 2 + (0.25)\, f(1,2) = 2 + (0.25)\,(3 + 1) = 3$

$y(x_2) = y_2 = y_1 + h\, f(x_1,y_1)$

i.e. $y(x_2) = y_2 = 3 + (0.25)\, f(1.25,3)$

$= 3 + (0.25)[3(1.25)^2 + 1]$

$= 4.42188$

Here $x_2 = x_1 + h = 1.25 + 0.25 = 1.5$

$\therefore y(1.5) = 5.42188$

Taking n = 2 in (1), we have

i.e. $y(x_3) = y_3 = h\, f(x_2,y_2)$

$= 5.42188 + (0.25)\, f(1.5,2)$

$= 5.42188 + (0.25)\, [3(1.5)^2 + 1]$

$= 6.35938$

Here $x_3 = x_2 + h = 1.5 + 0.25 = 1.75$

$\therefore y(1.75) =7.\ 35938$

Taking n = 4 in (1),we have

$y(x_4) = y_4 = y_3 + h\, f(x_3,y_3)$

i.e. $y(x_4) = y_4 = 7.35938 + (0.25)\, f(1.75,2)$

$= 7.35938 + (0.25)[3(1.75)^2 + 1]$

$= 8.90626$

Note that the difference in values of y(2) in both cases

(i.e. when h = 0.5 and when h = 0.25).The accuracy is improved significantly when h is reduced to 0.25 (Example significantly of the equ is $y = x^3 + x$ and with this $y(2) = y_2 = 10$

2. **Solve by Euler's method, $y^1 = x + y$, y(0) = 1 and find y(0.3) taking step size h = 0.1. compare the result obtained by this method with the result obtained by analytical solution**

Sol: $y_1 = 1.1 = y(0.1)$,

$y_2 = y(0.2) = 1.22$

$y_3 = y(0.3) = 1.362$

Particular solution is $y = 2e^x - (x + 1)$

Hence y(0.1) = 1.11034, y(0.2) = 1.3428, y(0.3) = 1.5997

We shall tabulate the result as follows

X	0	0.1	0.2	0.3
Euler y	1	1.1	1.22	1.362
Euler y	1	1.11034	1.3428	1.3997

The value of y deviate from the execute value as x increases. This indicate that the method is not accurate

3. **Solve by Euler's method $y^1 + y = 0$ given y(0) = 1 and find y(0.04) taking step size**

h = 0.01 Ans: 0.9606

4. **Using Euler's method, solve y at x = 0.1 from $y^1 = x + y + xy$, y()) = 1 taking step size h = 0.025.**

5. **Given that $\frac{dy}{dx}$ = xy ,y(0) = 1 determine y(0.1),using Euler's method.** h =0.1

Sol: The given differentiating equation is $\frac{dy}{dx} = xy$, y(0) = 1

a = 0

Here f(x,y) = xy , $x_0 = 0$ and $y_0 = 1$

Since h is not given much better accuracy is obtained by breaking up the interval (0,0.1) in to five steps.

i.e. $h = \frac{b-a}{5} = \frac{0.1}{5} = 0.02$

Euler's algorithm is $y_{n+1} = y_n + h\, f(x_n,y_n)$ $\rightarrow(1)$

$\therefore$ From (1) form = 0, we have

$$y_1 = y_0 + h\, f(x0,y_0)$$
$$= 1 + (0.02)\, f(0,1)$$
$$= 1 + (0.02)\,(0)$$
$$= 1$$

Next we have $x_1 = x_0 + h = 0 + 0.02 = 0.02$

$\therefore$ From (1), form = 1,we have

$$y_2 = y_1 + h\, f(x_1,y_1)$$
$$= 1 + (0.02)\, f(0.02,1)$$
$$= 1 + (0.02)\,(0.02)$$
$$= 1.0004$$

Next we have $x_2 = x_1 + h = 0.02 + 0.02 = 0.04$

$\therefore$ From (1), form = 2,we have

$$y_3 = y_2 + h\, f(x_2,y_2)$$
$$= 1.004 + (0.02)\,(0.04)\,(1.0004)$$
$$= 1.0012$$

Next we have $x_3 = x_2 + h = 0.04 + 0.02 = 0.06$

$\therefore$ From (1), form = 3,we have

$$y_4 = y_3 + h\, f(x_3,y_3)$$
$$= 1.0012 + (0.02)\,(0.06)\,(1.00012)$$
$$= 1.0024.$$

Next we have $x_4 = x_3 + h = 0.06 + 0.02 = 0.08$

$\therefore$ From (1), form = 4,we have

$$y_5 = y_4 + h\, f(x_4,y_4)$$
$$= 1.0024 + (0.02)\,(0.08)\,(1.00024)$$
$$= 1.0040.$$

Next we have $x_5 = x_4 + h = 0.08 + 0.02 = 0.1$

When $x = x_5$, $y \simeq y_5$

$$\therefore y = 1.0040 \text{ when } x = 0.1$$

6. **Solve by Euler's method $y^1 = \frac{2y}{x}$ given y(1) = 2 and find y(2).**

7. **Given that $\frac{dy}{dx} = 3x^2 + y$, y(0) = 4.Find y(0.25) and y(0.5) using Euler's method**

Sol: given $\frac{dy}{dx} = 3x^2 + y$ and y(1) = 2.

Here $f(x,y) = 3x^2 + y$, $x_0 = (1)$, $y_0 = 4$

Consider h = 0.25

Euler's algorithm is $y_{n+1} = y_n + h\, f(x_n,y_n)$ $\rightarrow$(1)

$\therefore$ From (1), for n = 0, we have

$$\begin{aligned} y_1 &= y_0 + h\, f(x_0,y_0) \\ &= 2 + (0.25)[0 + 4] \\ &= 2 + 1 \\ &= 3 \end{aligned}$$

Next we have $x_1 = x_0 + h = 0 + 0.25 = 0.25$

When $x = x_1$, $y_1 \simeq y$

$$\therefore y = 3 \text{ when } x = 0.25$$

$\therefore$ From (1), for n = 1, we have

$$\begin{aligned} y_2 &= y_1 + h\, f(x_1,y_1) \\ &= 3 + (0.25)[3.(0.25)^2 + 3] \\ &= 3.7968 \end{aligned}$$

Next we have $x_2 = x_1 + h = 0.25 + 0.25 = 0.5$

When $x = x_2$, $y \simeq y_2$

$$\therefore y = 3.7968 \text{ when } x = 0.5.$$

8. **Solve first order diff equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 and estimate y(0.1) using Euler's method (5 steps)** Ans: 1.0928
9. **Use Euler's method to find approximate value of solution of $\frac{dy}{dx}$ = y-x + 5 at x = 2-1 and 2-2with initial contention y(0.2) = 1**

Modified Euler's method

It is given by $y^{(i)}_{k+1} = y_k + h/2 f\left[\left(x_k, y_k\right) + f\left(x_{k+1},1\right)_{k+1}^{(i-1)}\right], i = 1,2.....,ki = 0,1.....$

Working rule :

i)Modified Euler's method

$$y^{(i)}_{k+1} = y_k + h/2 f\left[\left(x_k, y_k\right) + f\left(x_{k+1},1\right)_{k+1}^{(i-1)}\right], i = 1,2.....,ki = 0,1.....$$

ii) When $i = 1$ y^0_{k+1} can be calculated from Euler's method

iii) K=0, 1……… gives number of iteration. $i = 1,2...$

gives number of times, a particular iteration k is repeated

Suppose consider dy/dx=f(x, y) -------- (1) with $y(x_0) = y_0$----------- (2)

To find $y(x_1) = y_1$ at $x = x_1 = x_0 + h$

Now take k=0 in modified Euler's method

We get $y_1^{(1)} = y_0 + h/2\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(i-1)}\right)\right]$.......................... (3)

Taking i=1, 2, 3...k+1 in eqn (3), we get

$y_1^{(0)} = y_0 + h/2\left[f\left(x_0, y_0\right)\right]$ (By Euler's method)

$$y_1^{(1)} = y_0 + h/2\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(0)}\right)\right]$$

$$y_1^{(2)} = y_0 + h/2\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(1)}\right)\right]$$

$$y_1^{(k+1)} = y_0 + h/2\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(k)}\right)\right]$$

If two successive values of $y_1^{(k)}, y_1^{(k+1)}$ are sufficiently close to one another, we will take the common value as $y_2 = y\left(x_2\right) = y\left(x_1 + h\right)$

We use the above procedure again

1) using modified Euler's method find the approximate value of x when $x = 0.3$ given that $dy/dx = x + y \ and \ y\left(0\right) = 1$

sol: Given $dy/dx = x + y \ and \ y\left(0\right) = 1$

Here $f\left(x, y\right) = x + y, x_0 = 0, \ and \ y_0 = 1$

Take h = 0.1 which is sufficiently small

Here $x_0 = 0, x_1 = x_0 + h = 0.1, x_2 = x_1 + h = 0.2, x_3 = x_2 + h = 0.3$

The formula for modified Euler's method is given by

$$y_{k+1}^{(i)} = y_k + h/2\left[f\left(x_k + y_k\right) + f\left(x_{k+1}, y_{k+1}^{(i-1)}\right)\right] \rightarrow (1)$$

Step1: To find $y_1 = y(x_1) = y\ (0.1)$

Taking k = 0 in eqn(1)

$$y_{k+1}^{(i)} = y_0 + h/2\left[f\left(x_0 + y_0\right) + f\left(x_1, y_1^{(i-1)}\right)\right] \rightarrow (2)$$

when i = 1 in eqn (2)

$$y_1^{(i)} = y_0 + h/2\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(0)}\right)\right]$$

First apply Euler's method to calculate $y_1^{(0)} = y_1$

$$\therefore \ y_1^{(0)} = y_0 + h\,f\left(x_0, y_0\right)$$

= 1+(0.1)f(0.1)

= 1+(0.1)

= 1.10

$$now\left[x_0 = 0, y_0 = 1, x_1 = 0.1, y_1\left(0\right) = 1.10\right]$$

$$\therefore \ y_1^{(1)} = y_0 + 0.1/2\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(0)}\right)\right]$$

= 1+0.1/2[f(0,1) + f(0.1,1.10)

= 1+0.1/2[(0+1)+(0.1+1.10)]

= 1.11

When i=2 in eqn (2)

$$y_1^{(2)} = y_0 + h/2\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(1)}\right)\right]$$

= 1+0.1/2[f(0.1)+f(0.1,1.11)]

= 1 + 0.1/2[(0+1)+(0.1+1.11)]

= 1.1105

$$y_1^{(3)} = y_0 + h/2\left[f\left(x_0, y_0\right) + f\left(x_1, y_1^{(2)}\right)\right]$$

= 1+0.1/2[f(0,1)+f(0.1 , 1.1105)]

= 1+0.1/2[(0+1)+(0.1+1.1105)]

= 1.1105

Since $y_1^{(2)} = y_1^{(3)}$

$\therefore$ $y_1 = 1.1105$

Step:2 To find $y_2 = y(x_2) = y(0.2)$

Taking k = 1 in eqn (1) , we get

$$y_2^{(i)} = y_1 + h/2\left[f\left(x_1, y_1\right) + f\left(x_2, y_2^{(i-1)}\right)\right] \rightarrow (3)$$

i = 1,2,3,4,…..

For i = 1

$$y_2^{(1)} = y_1 + h/2\left[f\left(x_1, y_1\right) + f\left(x_2, y_2^{(0)}\right)\right]$$

$y_2^{(0)}$ is to be calculate from Euler's method

$$y_2^{(0)} = y_1 + h\ f\left(x_1, y_1\right)$$

= 1.1105 + (0.1) f(0.1 , 1.1105)

= 1.1105+(0.1)[0.1+1.1105]

= 1.2316

$$\therefore\ y_2^{(1)} = 1.1105 + 0.1/2\left[f\left(0.1, 1.1105\right) + f\left(0.2, 1.2316\right)\right]$$

= 1.1105 +0.1/2[0.1+1.1105+0.2+1.2316]

= 1.2426

$$y_2^{(2)} = y_1 + h/2\left[f\left(x_1, y_1\right) + f\left(x_2 y_2^{(1)}\right)\right]$$

= 1.1105 + 0.1/2[f(0.1 , 1.1105) , f(0.2 . 1.2426)]

= 1.1105 + 0.1/2[1.2105 + 1.4426]

= 1.1105 + 0.1(1.3266)

= 1.2432

$$y_2^{(3)} = y_1 + h/2\left[f\left(x_1, y_1\right) + f\left(x_2 y_2^{(2)}\right)\right]$$

= 1.1105+0.1/2[f(0.1,1.1105)+f(0.2 , 1.2432)]

= 1.1105+0.1/2[1.2105+1.4432)]

= 1.1105 + 0.1(1.3268)

= 1.2432

Since $y_2^{(3)} = y_2^{(3)}$

Hence y_2 = 1.2432

Step:3

To find $y_3 = y(x_3)$ = y y(0.3)

Taking k =2 in eqn (1) we get

$$y_3^{(1)} = y_2 + h/2\left[f\left(x_2, y_2\right) + f\left(x_3, y_3^{(i-1)}\right)\right] \rightarrow (4)$$

For i = 1 ,

$$y_3^{(1)} = y_2 + h/2\left[f\left(x_2, y_2\right) + f\left(x_3, y_3^{(0)}\right)\right]$$

$y_3^{(0)}$ is to be evaluated from Euler's method .

$$y_3^{(0)} = y_2 + h\, f\left(x_2, y_2\right)$$

= 1.2432 +(0.1) f(0.2 , 1.2432)

= 1.2432+(0.1)(1.4432)

= 1.3875

$\therefore\ y_3^{(1)}$ = 1.2432+0.1/2[f(0.2 , 1.2432)+f(0.3, 1.3875)]

= 1.2432 + 0.1/2[1.4432+1.6875]

$= 1.2432+0.1(1.5654)$

$= 1.3997$

$$y_3^{(2)} = y_2 + h/2\left[f\left(x_2, y_2\right) + f\left(x_3, y_3^{(1)}\right)\right]$$

$= 1.2432+0.1/2[1.4432+(0.3+1.3997)]$

$= 1.2432+ (0.1) (1.575)$

$= 1.4003$

$$y_3^{(3)} = y_2 + h/2\left[f\left(x_2, y_2\right) + f\left(x_3, y_3^{(2)}\right)\right]$$

$= 1.2432+0.1/2[f(0.2 , 1.2432)+f(0.3 , 1.4003)]$

$= 1.2432 + 0.1(1.5718)$

$= 1.4004$

$$y_3^{(4)} = y_2 + h/2\left[f\left(x_2, y_2\right) + f\left(x_3, y_3^{(3)}\right)\right]$$

$= 1.2432 + 0.1/2[1.4432+1.7004]$

$= 1.2432+(0.1)(1.5718)$

$= 1.4004$

Since $y_3^{(3)} = y_3^{(4)}$

Hence $y_3 = 1.4004$ $\therefore$ The value of y at x = 0.3 is 1.4004

2 . Find the solution of $\frac{dy}{dx}$ = x-y , y(0)=1 at x =0.1 , 0.2 ,0.3 , 0.4 and 0.5 . Using modified Euler's method

3. Find y(0.1) and y(0.2) using modified Euler's formula given that $dy/dx=x^2-y, y(0)=1$

[consider $h=0.1, y_1=0.90523, y_2=0.8214$]

4. Given $dy/dx = -xy^2, y\left(0\right) = 2$ compute y(0.2) in steps of 0.1

Using modified Euler's method

[$h=0.1$, $y_1=1.9804$, $y_2=1.9238$]

5. Given $y^1 = x+\sin y$, y(0)=1 compute y(0.2) and y(0.4) with h=0.2 using modified Euler's method [$y_1=1.2046$, $y_2=1.4644$]

Runge – Kutta Method Fourth order

$y_{i+1} = y_i + 1/6\ (K_1+2K_2+2K_3+K_4)$,

Where $K_1 = h\ (x_i, y_i)$

$K_2 = h\ (x_i+h/2,\ y_i+k_1/2)$

$K_3 = h\ (x_i+h/2,\ y_i+k_2/2)$

$K_4 = h\ (x_i+h,\ y_i+k_3)$

For i= 0,1,2-------

1. Using Runge-Kutta method of fourth order, find $y(2.5)$ from $\frac{dy}{dx} = \frac{x+y}{x}$, y(2)=2 , h = 0.25 .

Sol: Given $\frac{dy}{dx} = \frac{x+y}{x}$, y(2) = 2 .

Here $f(x, y) = \frac{x+y}{x}$, $x_0 = 0$, $y_0=2$ and h = 0.25

$\therefore\ x_1 = x_0+h = 2+0.25 = 2.25$, $x_2 = x_1+h = 2.25+0.25 = 2.5$

By R-K method of fourth order,

$$y_{i+1} = y_i + 1/2\left(k_1+k_2\right), k_1 - hf\left(x_i+h, y_i+k_1\right), i = 0,1.... \rightarrow (1)$$

step1:

$x_0=0, y_0=1, h=0.1$ To find y_1 i.e $y(x_1)=y(0.1)$
By 4th order R-K method, we have

$y_1 = y_0+1/6(k_1+2k_2+2k_3+k_4)$

Where $k_1=h\ f(x_0,y_0)=(0.1)f(0.1)= -0.1$

$k_2= h\ f\ (x_0+h/2,\ y_0+k1/2) = -0.095$

and $k_3= h\ f((x_0+h/2,y_0+k_2/2)=(0.1)f\ (0.1/2,1-0.095/2)$

$= (0.1)f(0.05,0.9525)$

$= -0.09525$

and $k_4= h\ f(x_0+h,y_0+k_3)$

$= (0.1)\ f(0.1,1-0.09525)=(0.1)f(0.1,0.90475)$

$=-0.090475$

Hence $y_1=1+1/6(-0.1)+2(-0.095)+2(0.09525)-0.090475)$

$=1+1/6(-0.570975)+1-0.951625 = 0.9048375$

Step2:

To find $y_2, i.e., y(x_2) = y(0.2), y_1 = 0.9048375, i.e., y(0.1) = 0.9048375$

Here $x_1 = 0.1$, $y_1=0.9048375$ and $h = 0.1$

Again by 4^{th} order R-K method, we have

$y_2 = y_1+1/6(k_1+2k_2+2k_3+k_4)$

Where $k_1=h\ f(x_1,y_1)=(0.1)f(0.1,0.9048375)=-0.09048375$

$k_2= hf\ (x_1+h/2,y_1+k_1/2)=(0.1)f(0.1+0.1/2,0.9048375 -0.09048375 /2)=-0.08595956$

and $k_3=hf(x_1+h/2,\ y_1+k_2/2)=(0.1)f(0.15,0.\ 8618577)= -0.08618577$

$k_4 =h\ f(x_1+h,y_1+k_3)=(0.1)f(0.2,0.86517)$

$= -0.08186517$

Hence $y_2 = 0.09048375+1/6(-0.09048375-2(0.08595956)-2(0.08618577)- 0.08186517$

$=0.9048375-0.0861065$

$=0.818731$

$y = 0.9048375$ when $x =0.1$ and $y =0.818731$

3. Apply the 4^{th} order R-K method to find an approximate value of y when x=1.2 in steps of 0.1,given that

$y^1 = x^2+y^2, y\ (1)=1.5$

sol. Given $y^1= x^2+y^2$,and $y(1)=1.5$

Here $f(x,y)= x^2+y^2$, $y_0 =1.5$ and $x_0=1,h=0.1$

So that $x_1=1.1$ and $x_2=1.2$

Step1:

To find y_1 i.e., $y(x_1)$

by 4^{th} order R-K method we have

$y_1=y_0+1/6\ (k_1+2k_2+2k_3+k_4)$

$k_1=hf(x_0,y_0)=(0.1)f(1,1.5)=(0.1)\ [1^2+(1.5)^2]=0.325$

$k_2= hf\ (x_0+h/2,y_0+k_1/2)=(0.1)f(1+0.05,1.5+0.325)=0.3866$

and k_3=hf((x_0+h/2,y_0+k_2/2)=(0.1)f(1.05,1.5+0. 3866/2)=(0.1)[$(1.05)^2$+$(1.6933)^2$]

=0.39698

k_4=hf(x_0+h,y_0+k_3)=(0.1)f(1.0,1.89698)

=0.48085

Hence

$$y_1 = 1.5 + \frac{1}{6}\left[0.325 + 2\left(0.3866\right) + 2\left(0.39698\right) + 0.48085\right]$$
$$= 1.8955$$

Step2:

To find y_2, i.e., $y\left(x_2\right) = y\left(1.2\right)$

Here x_1=0.1,y_1=1.8955 and h=0.1

by 4^{th} order R-K method we have

y_2 = y_1+1/6(k_1+2k_2+2k_3+k_4)

k_1=hf(x_1,y_1)=(0.1)f(0.1,1.8955)=(0.1) [1^2+$(1.8955)^2$]=0.48029

k_2= hf (x_1+h/2,y_1+k_1/2)=(0.1)f(1.1+0.1,1.8937+0.4796) =0.58834

and k_3=hf((x_1+h/2,y_1+k_2/2)=(0.1)f(1.5,1.8937+0.58743) =(0.1)[$(1.05)^2$+$(1.6933)^2$]

=0.611715

k_4=hf(x_1+h,y_1+k_3)=(0.1)f(1.2,1.8937+0.610728)

=0.77261

Hence y_2=1.8937+1/6(0.4796+2(0.58834)+2(0.611715)+0.7726) =2.5043

$\therefore$ y =2.5043 where $x = 0.2$

4. using R-K method, find y(0.2) for the eqn dy/dx=y-x,y(0)=1,take h=0.2

Ans:1.15607

5.Given that y^1=y-x,y(0)=2 find y(0.2) using R- K method take h=0.1

Ans: 2.4214

6. Apply the 4^{th} order R-K method to find $y\left(0.2\right)$ *and* $y\left(0.4\right)$ for one equation

$10\frac{dy}{dx} = x^2 + y^2, y\left(0\right) = 1$ **take h = 0.1** Ans. 1.0207, 1.038

7. using R-K method, estimate y(0.2) and y(0.4) for the eqn dy/dx=y^2-x^2/ y^2+x^2,y(0)=1,h=0.2

Ans:1.19598,1.3751

8. use R-K method, to approximate y when x=0.2 given that $y^1=x+y, y(0)=1$

Sol: Here $f(x,y)=x+y, y_0=1, x_0=0$

Since h is not given for better approximation of y

Take h=0.1

$\therefore x_1=0.1,\ x_2=0.2$

Step1

To find y_1 i.e $y(x_1)=y(0.1)$

By R-K method,we have

$y_1=y_0+1/6\ (k_1+2k_2+2k_3+k_4)$

Where $k_1=hf(x_0,y_0)=(0.1)f(0,1)=(0.1)\ (1)=0.1$

$k_2= hf\ (x_0+h/2, y_0+k_1/2)=(0.1)f(0.05,1.05)=0.11$

and $k_3=hf((x_0+h/2, y_0+k_2/2)=(0.1)f(0.05,1+0.\ 11/2)=(0.1)[(0.05)+(4.0.11/2)]$

$=0.1105$

$k_4=h\ f\ (x_0+h, y_0+k_3)=(0.1)f(0.1,1.1105)=(0.1)[0.1+1.1105]$

$=0.12105$

Hence $\therefore y_1 = y(0.1) = 1+\frac{1}{6}(0.1+0.22+0.240+0.12105)$

y = 1.11034

Step2:

To find y_2 i.e $y(x_2) = y(0.2)$

Here $x_1=0-1$, $y_1=1.11034$ and h=0.1

Again By R-K method,we have

$y_2=y_1+1/6(k_1+2k_2+2k_3+k_4)$

$k_1=h\ f(x_1,y_1)=(0.1)f(0.1,1.11034)=(0.1)\ [1.21034]=0.121034$

$k_2= h\ f\ (x_1+h/2,\ y_1+k_1/2)=(0.1)f(0.1+0.1/2,1.11034+0.121034/2)$

$=0.1320857$

and k_3=h f((x_1+h/2,y_1+k_2/2)=(0.1)f(0.15,1.11034+0.1320857/2)

=0.1326382

k_4=h f(x_1+h,y_1+k_3)=(0.1)f(0.2,1.11034+0.1326382)

(0.1)(0.2+1.2429783)=0.1442978

Hence y_2=1.11034+1/6(0.121034+0.2641714+0.2652764+0.1442978

=1.11034+0.1324631 =1.242803

$\therefore$ y =1.242803 when x=0.2

9.using Runge-kutta method of order 4,compute y(1.1) for the eqn $y^1=3x+y^2$,y(1)=1.2 h = 0.05

Ans:1.7278

10. using Runge-kutta method of order 4,compute y(2.5) for the eqn dy/dx = x+y/x, y(2)=2 [hint h = 0.25(2 steps)]

Ans:3.058

1. Use picard's method to approximate y when x=0.2

given that y=1 when x=0 and $\frac{dy}{dx} = x - y$

sol: Consider $\frac{dy}{dx} = f(x+y)$ where y=y_0 at x=x_0.

Here f(x,y)=x-y, x_0=0 and y_0=1.

By picard's method, picard's iteration formula is

$$\mathrm{y}^{(\mathrm{n})} = \mathrm{y}_0 + \int_{x_0}^{x} f(x, y^{(n-1)})dx$$

$$\therefore \mathrm{y}^{(\mathrm{n})} = 1 + \int_{0}^{x} f(x, y^{(n-1)})dx \rightarrow (1)$$

First approximation: put y=1 on R.H.S at (1).

$$\mathrm{y}^{(1)} = 1 + \int_0^x f(x,1)dx = 1 + \int_0^x f(x-1)dx = 1 + [\frac{x^2}{2} - x]_0^x = 1 - x + \frac{x^2}{2}$$

Second approximation:

$$\mathrm{y}^{(2)} = 1 + \int_0^x f(x, y^{(1)})dx = 1 + \int_0^x f(x, 1 + x + \frac{x^2}{2})dx$$

$$= 1 + \int_0^x [x - (1 + x + \frac{x^2}{2})]dx$$

$$= 1 + \int_0^x (2x - 1 - \frac{x^2}{2})]dx = 1 + x^2 - x - \frac{x^3}{6}$$

Third approximation:

$$y^{(3)} = 1 + \int_0^x f(x, y^{(2)})dx = 1 + \int_0^x f(x - y^{(2)})dx$$

$$= 1 + \int_0^x (x - 1 - x^2 + x + \frac{x^3}{6})dx$$

$$= 1 + x^2 - x - \frac{x^3}{3} + \frac{x^4}{24}$$

Fourth approximation:

$$y^{(4)} = 1 + \int_0^x f\left(x, y^{(3)}\right)dx = 1 + \int_0^x f\left(x - y^{(3)}\right)dx$$

$$= 1 + \int_0^x (x - 1 - x^2 + x + \frac{x^3}{3} - \frac{x^4}{24})dx$$

$$= 1 + \frac{x^2}{2} - x - \frac{x^3}{3} + \frac{x^2}{2} + \frac{x^4}{12} - \frac{x^5}{120}$$

$$= 1 - x + x^2 - \frac{x^3}{3} + \frac{x^4}{12} - \frac{x^5}{120}$$

Fifth approximation:

$$y^{(5)} = 1 + \int_0^x f(x, y^{(4)})dx = 1 + \int_0^x [x - y^{(4)}]dx$$

$$= 1 + \int_0^x (x - 1 + x - x^2 + \frac{x^3}{3} - \frac{x^4}{12} + \frac{x^5}{120})dx$$

$$= 1 + \int_0^x (2x - 1 - x^2 + \frac{x^3}{3} - \frac{x^4}{12} + \frac{x^5}{120})dx$$

$$= 1 - x + x^2 - \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} - \frac{x^6}{720}$$

when x=0.2, we have

$y_0=1$, $y^{(1)}=0.82$, $y^{(2)}=0.83867$, $y^{(3)} = 0.83740$, $y^{(4)} = 0.83746$ and $y^{(5)} = 0.83746$

y=0.83746 at x=0.2

2. Find an approximate value of y for x=0.1, x=0.2, if $\frac{dy}{dx} = x + y$ and y=1 at x=0 using picard's method.

Check your answer with the exact particular solution.

Sol:

Consider $\frac{dy}{dx} = f(x, y)$ where y=y_0 at x=x_0.

Here f(x,y)=x+y, x_0=0 and y_0=1.

By picard's method, a sequence of successive approximations are given by.

$$y^{(n)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)}(x))dx$$

(or)

$$y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})dx$$

For n=1,2,3,---- $\longrightarrow$ 1

$$y^{(n)} = 1 + \int_{0}^{x} f(x, y^{(n-1)})dx$$

For n=1,2,3,--- $\longrightarrow$ 2

when x=0.1

$$y^{(3)} = 1 + (0.1) + (0.1)^2 + \frac{(0.1)^3}{3} + \frac{(0.1)^4}{24}$$

$$= 1.1 + 0.01 + \frac{(0.001)}{3} + \frac{0.0001}{24}$$

=1.1+0.01+0.0003+0.0000041

= 1.1103041 ~ 1.1103

X=0.2

$$y^{(3)} = 1 + (0.2) + (0.2)^2 + \frac{(0.2)^3}{3} + \frac{(0.2)^4}{24}$$

=1.2+0.04+0.00266+0.0000666

=1.2427

Y=1.1103 at x=0.1

and y=1.2427 at x=0.2

Analytical solution:

The exact solution of $\frac{dy}{dx} = x + y$, y(0)=1 can be found as follows.

The equation can be written as $\frac{dy}{dx} - y = x$

This is a linear equation in y [i.e, $\frac{dy}{dx}+p.y=Q$]

then p=-1, Q=x. $I.F=e^{\int pdx}=e^{\int(-1)dx}=e^{-x}$

general solution is y X I.F= $\int QXI.Fdx+c$

y.e^{-x}= $\int x.e^{-x}dx+c$

y.e^{-x}=-e^{-x}(x+x)+c. or y=-(x+1)+ce^{+x}

when x=0, y=1 i.e, i=-(0+1)+c or c=2

Hence the particular solution of the equation is

Y=-(x+1)+2e^{x}=2e^{x}-x-1.

For x=0.1, y = e$^{0.1}$-0.1-1=2(1.1052)-0.1-1

=1.1104

For x=0.2, y=2e$^{0.2}$-0.2-1=2(1.2214)-0.2-1

=1.2428.

3. Find the value of y for x=0.4 by picard's method, given that $\frac{dy}{dx}=x^2+y^2$, y(0)=0.

Sol: Consider $\frac{dy}{dx}=f(x,y)$ and y=y_0 at x=x_0i.e $y(x_0)=y_0$

Here $f(x,y)=x^2+y^2$ and $x_0=0$, $y_0=0$.

By picard's method, the successive approximation are given by

$y^{(n)}(x)=y_0+\int_{x_0}^{x} f(x,y^{(n-1)})dx$, n=1,2,3,---

$y^{(n)}(x)=0+\int_{0}^{x} f(x,y^{(n-1)})dx$

$y^{(n)}(x)=\int_{0}^{x} f(x,y^{(n-1)})dx$, n=1,2,3-----$\longrightarrow$ 1

The first approximation:

$$y^{(1)}(x)=\int_{0}^{x} f(x,y^{(0)})dx=\int_{0}^{x} f(x,0)dx=\int_{0}^{x} x^2dx=\frac{x^3}{3}$$

The second approximation:

$$y^{(2)}(x)=\int_{0}^{x} f(x,y^{(1)})dx=\int_{0}^{x} f[x^2+(\frac{x^3}{3})^2]dx=\frac{x^3}{3}+\frac{x^7}{54}$$

Calculation of y$^{(3)}$ is tedious and hence approximate value is y$^{(2)}$.

For x=0.4, y$^{(1)}$=$\frac{(0.4)^3}{3}$=0.02133

$y^{(2)} = \frac{(0.4)^3}{3} + \frac{(0.4)^7}{54}$ =0.0213333+0.0000303.

=0.0213636 $\sim$ 0.0214(correct to 4 decimal places)

Y=0.0214 at x=0.4.

4. Given that $\frac{dy}{dx} = 1 + xy$ and y(0)=1, compute y(0.1) and y(0.2) using picard's method.

Sol:

$\frac{dy}{dx} = 1 + xy$ and y(0)=1

Consider $\frac{dy}{dx} = f(x, y)$ and $y(x_0)=y_0$.

Here f(x,y)=1+xy and $y_0=1$, $x_0=0$.

By picard's method, the successive approximations are given by

$$y^{(n)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})dx \text{ , n=1,2,3,---}$$

$$y^{(n)}(x) = 1 + \int_{0}^{x} f(x, y^{(n-1)})dx \text{ , n=1,2,3---}$$

The first approximation:

$$y^{(1)}(x) = 1 + \int_0^x f(x, y^{(0)})dx = 1 + \int_0^x f(x,1)dx = 1 + \int_0^x (1+x)dx = 1 + x + \frac{x^2}{2}$$

The second approximation:

$$y^{(2)}(x) = 1 + \int_0^x f\left(x, y^{(1)}\right)dx = 1 + \int_0^x \left[(1+x)\left(1 + x + \frac{x^2}{2}\right)\right]dx$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}$$

The third approximation:

$$y^{(3)}(x) = 1 + \int_0^x f(x, y^{(2)})dx = 1 + \int_0^x [1 + x(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8})]dx$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} + \frac{x^5}{15} + \frac{x^6}{48}$$

It is clear that the resulting expressions too big, as we proceed to higher approximations. Hence approximative value is $y^{(3)}$.

For x=0.1,

$$y^{(3)} = 1 + (0.1) + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} + \frac{(0.1)^4}{8} + \frac{(0.1)^5}{15} + \frac{(0.1)^6}{48}$$

=1+0.1+0.005+0.000333+0.0000125+0.000000666+0.00000002

$=1.105346 \cong 1.10535$

Y(0.1)=1.10534.

For x=0.2,

$$y^{(3)} = 1+(0.2)+\frac{(0.2)^2}{2}+\frac{(0.2)^3}{3}+\frac{(0.2)^4}{8}+\frac{(0.2)^5}{15}+\frac{(0.2)^6}{48}$$

=1.2+0.02+0.0026666+0.0002+0.00002133+0.000001333

$=1.222889 \sim 1.22289$

y(0.2)=1.22289.

5.Using picard's method, obtain the solution of $\frac{dy}{dx} = x - y^2$, y(0)=1 and compute y(0.1) correct to four decimal places.

Sol:

Consider $\frac{dy}{dx} = f(x,y)$ and $y(x_0)=y_0$.

Here $f(x,y)=x-y^2$, $y_0=1$ and $x_0=0$.

By picard's method, a sequence of successive approximation to y are given by

$$y^{(n)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})dx \text{ , n=1,2,3,---}$$

$$y^{(n)}(x) = 1 + \int_{0}^{x} f(x, y^{(n-1)})dx \text{ , n=1,2,3,---}$$

First approximation: we have

$$y^{(1)}(x) = 1+\int_0^x f(x,y^0)dx = 1+\int_0^x f(x,1)dx = 1+\int_0^x (x-1)dx = 1+\frac{x^2}{2}-x$$

Second approximation, we have

$$y^{(2)}(x) = 1+\int_0^x f(x,y^1)dx = 1+\int_0^x \left[x - \left(1+\frac{x^2}{2}-x\right)^2 \right] dx$$

$$= 1+\int_0^x \left[(x-(1+\frac{x^4}{4}+x^2+x^2-2x-x^3) \right] dx$$

$$= 1+\int_0^x (3x-1-\frac{x^4}{4}-2x^2+x^3)dx$$

$$= 1+\frac{3x^2}{2}-x-\frac{x^5}{20}-\frac{2x^3}{3}+\frac{x^4}{4}$$

It is clear that the resulting expressions too big as we proceed to higher approximations. Hence approximate value of y(x) is $y^{(2)}(x)$.

For x=0.1

$$y^{2)} = 1-0.1+\frac{3}{2}(0.1)^2-\frac{2}{3}(0.1)^3+\frac{1}{4}(0.1)^4-\frac{(0.1)^5}{20}$$

=1-0.1+0.015-0.0006666+0.000025-0.0000005

=1.015025-0.1006671

=0.9143579 $\sim$ 0.9143 (correct to four decimal places) y = 0.9143 at x=0.1.

6. Given the differential equation $\frac{dy}{dx} = x^2 + y^2$, y(0)=0. Obtain y(0.2) and y(1) by picard's method.

Sol: Consider $\frac{dy}{dx} = f(x, y)$ and $y(x_0)=y_0$.

Here $f(x,y)=x^2+y^2$, $y_0=0$ and $x_0=0$.

By picard's method, a sequence of successive approximation to y are given by

$$y^{(n)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})dx \text{ , n=1,2,3,---}$$

$$y^{(n)}(x) = \int_{0}^{x} f(x, y^{(n-1)})dx \text{ , n=1,2,3,--}$$

First approximation, we have

$$y^{(1)}(x) = \int_{0}^{x} f(x, y^0)dx = \int_{0}^{x} x^2 dx = \frac{x^3}{3}$$

Second approximation, we have

$$y^{(2)}(x) = \int_{0}^{x} f(x, y^{(1)})dx = \int_{0}^{x} (x^2 + \frac{x^6}{9})dx = \frac{x^3}{3} + \frac{x^7}{63}$$

Third approximation, we have

$$y^{(3)}(x) = \int_{0}^{x} f(x, y^{(2)})dx = \int_{0}^{x} \left[x^2 + (\frac{x^3}{3} + \frac{x^7}{63})^2 \right] dx$$

$$= \int_{0}^{x} (x^2 + \frac{x^6}{9} + \frac{x^{14}}{(63)^2} + \frac{2}{189} x^{10})dx$$

$$= \frac{x^3}{3} + \frac{x^7}{63} + \frac{x^{15}}{15.(63)^2} + \frac{2}{11.(189)} x^{11}$$

Calculation of $y^{4)}$ is tedious and hence approximative value for y is $y^{(3)}$

For $x = 0.2$

$$Y^{(3)} = \frac{(0.2)^3}{3} + \frac{(0.2)^7}{63} + \frac{(0.2)^{15}}{15.(63)^2} + \frac{2}{11.(189)} (0.2)^{11}$$

=0.0026666+0.000000203+

=0.0026668

For x=1, $y^{(3)} = \frac{1}{3} + \frac{1}{63} + \frac{1}{59535} + \frac{2}{11(189)}$

=0.3333333+0.0158730+0.000016796+0.000962

=0.350185.

7. Solve the differential equation $\frac{dy}{dx} = x^2 - y$, y(0)=1 by picard's method to get the value of y at x=1. Use terms through x^5,

Ans: $$y_4 = 1 - x + \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} + \frac{x^5}{60} - \frac{x^6}{360}$$

$$y_4(x=1) = 0.638888$$

8. Find the value of y for x=0.25, 0.5, 1 by picard's method, given that $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ and x_0=0, y_0 = 0.

Sol: Consider $\frac{dy}{dx} = f(x,y)$ and $y(x_0)=y_0$ or $y=y_0$ at $x=x_0$

Here $f(x,y) = \frac{x^2}{y^2+1}$ and x_0=0, y_0=0

By picard's method a sequence of approximations are given by

$$y^{(n)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})dx \text{ , n=1,2,3,---}$$

$$y^{(n)}(x) = 0 + \int_{0}^{x} f(x, y^{(n-1)})dx \text{ , n=1,2,3---} \qquad 1$$

First approximation: we have

$$y^{(1)}(x) = 0 + \int_0^x f(x, y^0)dx = 0 + \int_0^x f(x,1)dx = 0 + \int_0^x \frac{x^2}{0^2+1}dx = 0 + \frac{x^3}{3}$$

Second approximation, we have

$$y^{(2)}(x) = 0 + \int_0^x f(x, y^{(1)})dx = \int_0^x \frac{x^2}{(y^{(1)})^2+1}dx = \int_0^x \frac{x^2}{(\frac{x^3}{3})^2+1}dx$$

$$= \tan^{-1}(\frac{x^3}{3}) - 0 \text{ [by putting } \frac{x^3}{3}\text{=t]}$$

$$= \tan^{-1}(\frac{x^3}{3})$$

Third approximation, we have

$$y^{(3)}(x) = \int_0^x f(x, y^{(2)})dx = \int_0^x \frac{x^2}{[\tan^{-1}(\frac{x^3}{3})]^2+1}dx$$

The integration is difficult, this is the drawback of the method. Hence the approximation value of y is $y^{(2)}(x)$.

$$y^{(2)}(x)=\tan^{-1}(\frac{x^3}{3})=\frac{x^3}{3}-(\frac{x^3}{3})^3\frac{1}{3}+(\frac{x^3}{3})^5\frac{1}{5}---$$

$$=\frac{x^3}{3}-\frac{x^9}{81}+\frac{x^{15}}{1215}----\qquad [\tan^{-1}(2)=x-\frac{x^3}{3}+\frac{x^5}{5}---]$$

For x=0.25,

$$y^{(2)}(x)=\frac{(0.25)^3}{3}-\frac{(0.25)^9}{81}+\frac{(0.25)^{15}}{1215}=0.0052082$$

at x=0.5,

$$y^{(2)}(x)=\frac{(0.5)^3}{3}-\frac{(0.5)^9}{81}+\frac{(0.5)^{15}}{1215}=0.0416425$$

At x=1,

$$y^{(2)}(x)=\frac{1}{3}-\frac{1}{81}-\frac{1}{1215}=0.32180699$$

Y=0.0052082 at x=0.25

Y=0.0416425 at x=0.5

Y=0.32180699 at x=1

9. Given $\frac{dy}{dx}=xe^y$, y(0)=0, determine y(0.1), y(0.2) and y(1) using picard's method.

Sol: Consider $\frac{dy}{dx}=f(x,y)$ and $y(x_0)=y_0$

Here f(x,y)= xe^y, $x_0=0$ and $y_0=0$

By picard's method, a sequence of approximations are given by

$$y^{(n)}(x)=y_0+\int_{x_0}^{x}f(x,y^{(n-1)})dx\text{, n=1,2,3,---}$$

$$\therefore\ y^{(n)}(x)=\int_0^x f(x,y^{(n-1)})dx\text{, n=1, 2, 3,---- (1)}$$

First approximation, we have

$$y^{(1)}(x)=\int_0^x f(x,y^0)dx=\int_0^x x.e^0dx=\frac{x^2}{2}$$

Second approximation, we have

$$y^{(2)}(x)=\int_0^x f(x,y^{(1)})dx=\int_0^x x.e^{\frac{x^2}{2}}dx=e^{\frac{x^2}{2}}-1$$

Third approximation, we have

$$y^{(3)}(x) = \int_0^x f(x, y^{(2)})dx = \int_0^x x.e^{(x)\frac{x^2}{2}}dx$$

The integration is difficult, Hence the approximate value of y is $y^{(2)}(x)$.

$$y^{(2)}(x) = e^{\frac{x^2}{2}} - 1$$

for x=0.1, $y^{(2)}(x) = e^{\frac{(0.1)^2}{2}} - 1 = 0.005012$

for x=0.2, $y^{(2)}(x) = e^{\frac{(0.2)^2}{2}} - 1 = 0.02020$

for x=1, $y^{(2)}(x) = e^{\frac{1}{2}} - 1 = 0.648721$

Objectivetye Questions

1. If $\frac{dy}{dx}$ = x - y and y(0) = 1 using Picard method, the value of $y^I(1)$ = __________

   a) 1.0905 b) 0.9157 c) 0.905 d) None

2. If $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 and h = 0.02, using Euler's method the value of y_1 = __________

   a) 1.02 b) 1.04 c) 1.03 d) none

3. If $\frac{dy}{dx} = x^2+y^2$, y(0) = 0 using Taylor's series method, the value of y(0.4)=____________

   a) 0.2133 b) 0.02133 c) 0.002133 d) None

4. If $y^I = y - x^2$, y(0)=1 using Picard's method up to the second approximation, the value of y(x) =________

   a) $1 + x + \frac{x^2}{2} - \frac{x^4}{12} - \frac{x^3}{3}$ b) $1 + x + \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{12}$

   c) $1 + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{12}$ d) None

5. If $\frac{dy}{dx}$ = -y, y(0) = 1, h = 0.01 then by Euler's method, the value of y_1 = ___________

   a) 0.099 b) 0.0981 c) 0.99 d) None

6. If $\frac{dy}{dx} = x^2+y^2$, y(1) = 1.5, h = 0.1 then the value of k_1 in fourth order Runge – Kutta method is ___________

   a) 0.0325 b) 0.325 c) 0.235 d) None

7. The value of y at x= 0.1 using Runge – Kutta method of fourth order for the differential equation $\frac{dy}{dx}$ = x – 2y, y(0) = 1 taking h = 0.1 is __________

a) 0.825 b) 0.0825 c) 0.813 d) None

8. The value of y at x = 0.1 using modified Euler's method up to second approximation for $\frac{dy}{dx}$ = x – y, y(0) = 1 is _________

a) 0.909 b) 0.0909 c) 0.809 d) None

9. If $\frac{dy}{dx} = 1 + y^2$, $f(x_0,y_0) = 1$, h – 0.2, $K_1 = 0.2$, $K_2 = 0.202$, $K_3 = 0.20204$, $K_4 = 0.20216$, then the value of y_1 by fourth order Runge – Kutta method is ________

a) 0.0202 b) 0.202 c) 0.102 d) None

10. Using Runge – Kutta method, the approximate value of y(0.1) if $\frac{dy}{dx} = x + y^2$, y = 1 where x =0 and $f(x_0,y_0) = 1$ $K_1 = 0.1$, $K_2 = 0.115$, $K_3 = 0.116$, $K_4 = 0.134$ is

a) 1.116 b) 1.001 c) 1.211 d) None

Fill in the blanks

11. If $\frac{dy}{dx} = f(x,y), y(x_0) = y_0$ then the Taylor's series for solution of the differential equation is __________

12. Using Taylor's series, solution for $\frac{dy}{dx} = y^2 - x, y(0) = 1$ the value y(0.1) is __________

13. Using Taylor's series method from $\frac{dy}{dx} = x + y, y(1) = 0$ the value of y(1.1) is __________

14. The value of y(0.1) using Taylor's series method given that $\frac{dy}{dx} = 1 - y$, y(0) = 2 is _____

15. The Picard's method of solving the differential equation $\frac{dy}{dx} = f(x,y), y(x_0) = y_0$ using the integral equation _________

16. The second approximate solution of $\frac{dy}{dx} = 1 + xy$ using Picard's method is _______

17. Using Picard's method to third approximate of y when x = 0.2 given that y = 1 when $x_0 = 2, y_0 = 0$ x= 0, $\frac{dy}{dx} = x - y$ is ______

18. The solution of $\frac{dy}{dx} = 1 + xy$ with $x_0 = 2, y_0 = 0$ using Picard's method is_______

19. If $\frac{dy}{dx} = x^2 + y^2, y(0) = 0$ then by Picard method the value of $y^1(x)$ is __________

20. The value of y for x = 0.4 by Picard's method given that $\frac{dy}{dx} = x^2 + y^2, y(0) = 0$ is ______

21. Using the Picard's method the solution of $\frac{dy}{dx}$ = x + y^2,y(0) = 1 is ________

22. If $\frac{dy}{dx}$ = -xy^2, y(0) = 2,using Euler's method the first approximate value of y(0.1) is_____

23. Given $\frac{dy}{dx}$ = 4 + x^2 + y, y(0) = 1, using Euler's modified method the value of y(0.02) is_________

24. If $\frac{dy}{dx}$ = x + y^2,f(0) = 1 using Runge – Kutta method , the approximate value of y(0.1) is_________

25. If $\frac{dy}{dx}$ = f(x,y),y(x_0) = y_0,the formula for fourth order Runge – Kutta method is _______

26. Using Taylor's method the first approximate value of y(1.1) for the differential equation y^1 = x$y^{1/3}$,y(1) = 1 is ____________

27. The value of y(0.1) using Euler's method for the differential equation $\frac{dy}{dx}$ = x- y^2, y(0) = 1 is ________

28. Using Euler's modified method, find y(0.1) given $\frac{dy}{dx}$ = y – $\frac{2x}{y}$,y(0) = 1 is __________

29. In fourth order Runge – Kutta method for $\frac{dy}{dx}$ = x + y^2,y(0) = 1,h = 0.1 the value of K_2 is __________

30. IF $\frac{dy}{dx} = \frac{y-x}{y+x}$,y(0) = 1 then by Picard's method the value of $y^{(1)}$(x) is _________

31.Simpson's 3/8 rule for 9 ordinates is __________

32.if $I = \int_0^3 f(x)\,dx$, the value of I by trapezoidal rule for the data is________

x	0	1	2	3
$f(x)$	-2	4	6	12

33.The value of $\int_1^2 \frac{1}{x}dx$ using trapezoidal rule taking n = 4 is _________

34.The value of $\int_1^2 \frac{1}{x}dx$ using Simpson's 1/3 rule taking n = 4 is __________

35.The value of $\int_0^1 \frac{1}{1+x}dx$ using trapezoidal rule taking h = 0.5 is __________

36.The value of $\int_0^1 \sqrt{1-x^2}\,dx$ by using trapezoidal rule is _____________

37.The value of $\int_0^1 \frac{1}{1+x}dx$ using Simpson's 1/3 rule taking h = 0.25 is ________

38.The value of $\int_0^1 \sqrt{1-x^2}\,dx$ using Simpson's 1/ 3 rule is ___________

39.The value of $\int_0^2 \frac{1}{1+x^3}dx$ using Simpson's 1/3 rule with n = 4 is ___________

40.The value of $\int_{-2}^{2} \frac{x}{5+2x}dx$ using trapezoidal rule _____________

41.The value of $\int_0^{\pi} \frac{\sin x}{x}$ by using weedle's rule taking n = 6 is ______________

42.The value of $\int_0^5 \frac{dx}{4x+5}$ by usinf Simpson's 1/3 rule taking n = 10 is __________

43.The value of $\int_1^2 \left(x^3+1\right)dx$ using Simpson's 3/8 rule , dividing the range into three equal parts is ____________

44.The value of $\int_0^{\pi/2} \sqrt{\sin\theta}\; d\theta$ using Simpson's 1/3 rule considering 6 sub- intervals is _______

45. The value of $\int_0^{\pi/2} \sqrt{\sin\theta}\; d\theta$ using Simpson's 3/ 8 rule considering 6 sub- intervals is _________

46.The value of $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$ using Weddle's rule with n = 6 is _____________

47.The value of $\int_0^6 \frac{dx}{1+x}$ using Trapezoidal rule is ______________

48.The value of $\int_0^6 \frac{dx}{1+x}$ using Simpson's 3/8 rule is ___________

49.The value of $\int_0^6 \frac{dx}{1+x}$ using Weddle's rule is _________

50.Given $e^0 = 1, e^1 = 2.72, e^2 = 7.39, e^3 = 20.09, e^4 = 54.60$ find $\int_0^4 e^x dx$ using Simpson's 1/3 rule is ______________

51.The value of $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ dividing the range in six equal parts is ___________

52.The value of $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule is ____________

53.The value of $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's 1/3 rule is ____________

54.The value of $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's 3/8 rule is ____________

PROBLEMS

1. Tabulate y(0.1),y(0.2) & y(0.3) using Taylor's series method given that $y^1 = y^2 + x$ & y(0) = 1

(JNTU 2006)

2. Given that $\frac{dy}{dx}$ = 1 + xy, y(0) = 1 compute y(0.1) & y(0.2) using Picard's method

(JNTU 2006)

3. Solve $y^1 = y - x^2$, y(0) = 1 by Picard's method up to the fourth approximations. Hence find the value of y(0.1) & y(0.2)

(JNTU 2006)

4. Find the solution of $\frac{dy}{dx}$ = x –y,y(0) = 1 at x=0.1,0.2,0.3,0.4 & 0.5 using modified Euler's method

(JNTU 2006)

5. Find y(0.1) & y(0.2) using Euler's modified formula given that $\frac{dy}{dx} = x^2 - y$,y(0) = 1

(JNTU 2006)

6. Given y^1 = x + sin y, y(0) = 1 compute y(0.2),y(0.4) with h = 0.2 using Euler's modified method

(JNTU 2006)

7. Use Runge – Kutta method to evaluate y(0.1) & y(0.2) given that y^1 = x + y,y(0) = 1

(JNTU 2006)

8. Find y(0.1) & y(0.2) using Runge – Kutta 4th order formula given that $y^1 = x^2 - y$ & y(0) =1

(JNTU 2006)

9. If $\frac{dy}{dx} = 2ye^x$,y(0)=2 find y(0.4) using Adam's Predictor corrector formula by calculating y(0.1),y(0.2) and y(0.3) using Euler's modified formula.

(JNTU 2006)

10. Using 4th order Runge – Kutta method find y(0.1),y(0.2) and y(0.3) given $\frac{dy}{dx} = \frac{y^2 - 2x}{y^2 + 2x}$,y(0) = 1

11. Given $\frac{dy}{dx}$ = y – x, y(0) = 2.Using 4th order Runge – Kutta method. Find y(0.2),y(0.4) & y(0.6)

12. Given $\frac{dy}{dx} = \frac{y - x}{y + x}$,y(0) = 1.using 4th order Runge – Kutta method, find y(0.1) and y(0.2)

13. Given $\frac{dy}{dx} = \frac{1}{2}(1 + x)y^2$,y(0) = 1 using 2nd order Runge – Kutta method find y(0.3) taking h = 0.1

14. Given $\frac{dy}{dx}$ = x – 2y, y(0) = 1.Taking h = 0.1, determine y(0.1) and y(0.2) using 3rd order Runge – Kutta method.

15. Using Runge – Kutta of 4th order find y(0.1) and y(0.2),given $\frac{dy}{dx}$ = x + y,y(0) = 1.

16. Given $\frac{dy}{dx}$ = y^2 + 1,y(0) = 0,find y(0.2) using Taylor's series method.

17. Given $\frac{dy}{dx}$ = 3x + $\frac{y}{x}$ and y(0) = 1.Using Taylor's series method. Find y(0.1) and y(0.2).

18. Solve $\frac{dy}{dx}$ = x – y^2 by Taylor's series method for x = 0.2 to 0.6 with h = 0.2,given y(0) = 1

19. Using Picard's method, compute y(0.2) from $\frac{dy}{dx}$ = 1 – 2xy,y(0) = 0

20. Using Picard's method obtain the solution of $\frac{dy}{dx}$ = x + x^4y,y(0) = 3.Find the value of y for x = 0.1 and x = 0.2

21. Simpson's 3/8 rule for 9 ordinates is __________

22. if $I=\int_0^3 f(x)\,dx$, the value of I by trapezoidal rule for the data is________

x	0	1	2	3
$f(x)$	-2	4	6	12

23. The value of $\int_1^2 \frac{1}{x}dx$ using trapezoidal rule taking n = 4 is _________

24. The value of $\int_1^2 \frac{1}{x}dx$ using Simpson's 1/3 rule taking n = 4 is __________

25. The value of $\int_0^1 \frac{1}{1+x}dx$ using trapezoidal rule taking h = 0.5 is __________

26. The value of $\int_0^1 \sqrt{1-x^2}\,dx$ by using trapezoidal rule is _____________

27. The value of $\int_0^1 \frac{1}{1+x}dx$ using Simpson's 1/3 rule taking h = 0.25 is ________

28. The value of $\int_0^1 \sqrt{1-x^2}\,dx$ using Simpson's 1/ 3 rule is ___________

29. The value of $\int_0^2 \frac{1}{1+x^3}dx$ using Simpson's 1/3 rule with n = 4 is ___________

30. The value of $\int_{-2}^2 \frac{x}{5+2x}dx$ using trapezoidal rule _____________

31. The value of $\int_0^\pi \frac{\sin x}{x}$ by using weedle's rule taking n = 6 is ______________

32. The value of $\int_0^5 \frac{dx}{4x+5}$ by usinf Simpson's 1/3 rule taking n = 10 is __________

33. The value of $\int_1^2 \left(x^3+1\right)dx$ using Simpson's 3/8 rule , dividing the range into three equal parts is ______________

34. The value of $\int_0^{\pi/2} \sqrt{\sin\theta}\; d\theta$ using Simpson's 1/3 rule considering 6 sub- intervals is _______

35. The value of $\int_0^{\pi/2} \sqrt{\sin\theta}\; d\theta$ using Simpson's 3/ 8 rule considering 6 sub- intervals is __________

36. The value of $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$ using Weddle's rule with n = 6 is _____________

37. The value of $\int_0^6 \frac{dx}{1+x}$ using Trapezoidal rule is ______________

38. The value of $\int_0^6 \frac{dx}{1+x}$ using Simpson's 3/8 rule is ___________

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40. Given $e^0=1, e^1=2.72, e^2=7.39, e^3=20.09, e^4=54.60$ find $\int_0^4 e^x dx$ using Simpson's 1/3 rule is ______________

41. The value of $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ dividing the range in six equal parts is ___________

42. The value of $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule is ____________

43. The value of $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's 1/3 rule is ____________

44. The value of $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's 3/8 rule is ____________