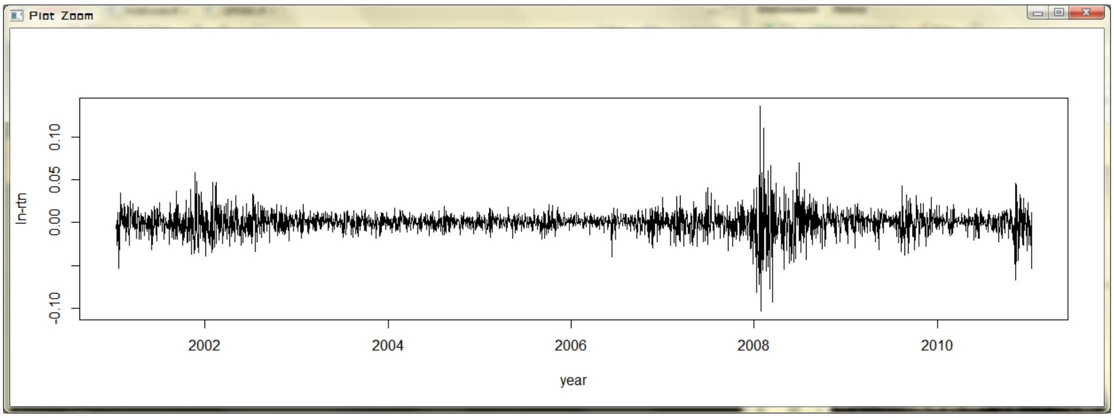
参考书《金融数据分析导论：基于 R 语言》
P187——1、3
（数据集可以在课程资源下载）

下面的习题要求：1）检验中应用 5%的显著性水平；2）对收益率序列应用 10 阶滞后自相关性

1.考虑道富环球顾问的 SPDR 标普 500ETF 日收益率，时间区间为 2001 年 9 月 4 日到 2011 年 9 月 30 日，共计 2535 个观测值，其交易代号（tick symbol）为 SPY。其简单收益率可以从 CRSP 获得，数据文件为 d-spy-0111.txt.把简单收益率变换为对数收益率。

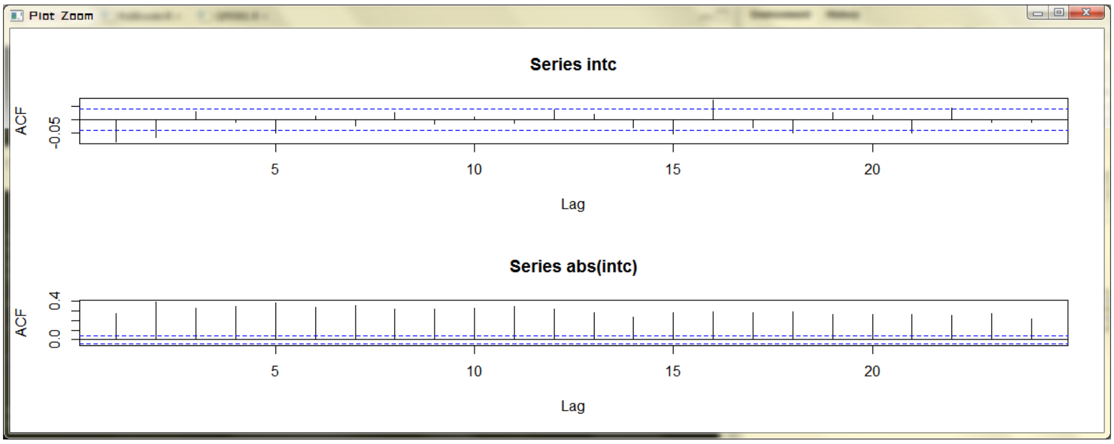
（a）期望的对数收益率为 0 吗？对数收益率中有没有明显的前后相关性？此对数收益率存在 ARCH 效应吗？

```
> ######波动率模型######
> da=read.table("E:/DATA/data mining/fts08/data&code/d-spy-0111.txt",header=T)
> head(da)
      date       rtn
1 20010904 -0.006395
2 20010905  0.002469
3 20010906 -0.025770
4 20010907 -0.018507
5 20010910  0.012233
6 20010917 -0.052249
> intc=log(da$rtn+1) #计算对数收益率
> rtn=ts(intc,frequency=254,start=c(2001,9,4),end=c(2011,9,30))   #转化为时间序列
> plot(rtn,type='l',xlab='year',ylab='ln-rtn') # 画时间序列图
> t.test(intc)  # 检验收益率的均值是否为0

 One Sample t-test

data:  intc
t = 0.2651, df = 2534, p-value = 0.7909
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.0004633792  0.0006082874
sample estimates:
  mean of x 
7.24541e-05 
```



p-value=0.7909，不能拒绝原假设，均值为零，2002 年、2008 年附近波动比较大以外，其他时间都比较平稳

```
> Box.test(intc,lag=12,type='Ljung')

 Box-Ljung test

data:  intc
X-squared = 44.0413, df = 12, p-value = 1.503e-05

> par(mfcol=c(2,1))
> acf(intc,lag=24) # ACF plots
> acf(abs(intc),lag=24)
> Box.test(abs(intc),lag=12,type='Ljung')

 Box-Ljung test

data:  abs(intc)
X-squared = 3414.97, df = 12, p-value < 2.2e-16
```



从对数收益率及其绝对值的 ACF 图，以及其对数收益率的 Ljung-Box Q 统计相关检验 X-squared=44.0413,p-value=1.503e-05，和其对数收益率的绝对值的 Ljung-Box Q 统计相关检验 X-squared=3414.97,p-value<2.2e-16，因此该对数收益率是前后不相关的，也是不独立的

```
> par(mfcol=c(1,1))
> ######ARCH效应检验######
> y=intc-mean(intc)    #计算at
> Box.test(y^2,lag=12,type='Ljung')

 Box-Ljung test

data:  y^2
X-squared = 2377.377, df = 12, p-value < 2.2e-16
```

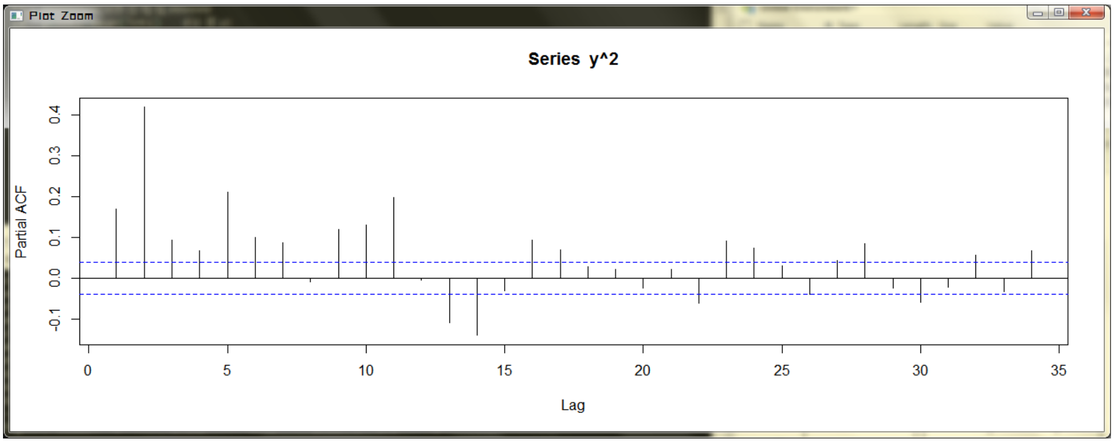
X-squared=2377.377，p-value<2.2e-16，可以拒绝原假设，也就是具有 ARCH 效应

```
> source("E:\\DATA\\data mining\\QF05\\archTest.txt")
> archTest(y,12)

Call:
lm(formula = atsq ~ x)

Residuals:
       Min         1Q     Median         3Q        Max
-0.0050324 -0.0000996 -0.0000440  0.0000230  0.0153584

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.027e-05  1.242e-05   2.438 0.014855 *
x1          -2.739e-02  1.996e-02  -1.372 0.170072
x2           3.093e-01  1.957e-02  15.803  < 2e-16 ***
x3          -4.084e-03  2.035e-02  -0.201 0.841000
x4          -2.374e-02  2.032e-02  -1.168 0.242895
x5           1.322e-01  2.030e-02   6.515 8.78e-11 ***
x6           7.077e-02  2.044e-02   3.462 0.000545 ***
x7           5.068e-02  2.037e-02   2.489 0.012892 *
x8          -4.995e-02  2.023e-02  -2.469 0.013608 *
x9           5.316e-02  2.024e-02   2.626 0.008693 **
x10          1.329e-01  2.027e-02   6.558 6.60e-11 ***
x11          1.965e-01  1.950e-02  10.076  < 2e-16 ***
x12         -5.229e-03  1.988e-02  -0.263 0.792510
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0005609 on 2510 degrees of freedom
Multiple R-squared:  0.3112, Adjusted R-squared:  0.3079
F-statistic:  94.5 on 12 and 2510 DF,  p-value: < 2.2e-16
```

p-value<2.2e-16，不能拒绝原假设，也就是具有很强的 ARCH 效应

（b）对该对数收益率序列建立 ARMA-GARCH 模型。进行模型检验，绘制标准化残差的 QQ 图，并给出拟合的模型【提示：尝试 GARCH（2，1）模型】

```
> ######GARCH模型######
> library(fGarch)
> pacf(y^2)#通过pacf定阶
```



```
> #题目提示GARCH (2, 1) 模型
> m4=garchFit(~1+garch(2,1),data=intc,trace=F)
> summary(m4)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(2, 1), data = intc, trace = F)

Mean and Variance Equation:
 data ~ 1 + garch(2, 1)
<environment: 0x1604fc54>
 [data = intc]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1      alpha2       beta1
5.7243e-04  2.3226e-06  1.9896e-03  1.1165e-01  8.7049e-01

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     5.724e-04   1.707e-04    3.353   0.0008 ***
omega  2.323e-06   4.938e-07    4.704 2.56e-06 ***
alpha1 1.990e-03   1.031e-02    0.193   0.8469
alpha2 1.117e-01   1.691e-02    6.604 4.01e-11 ***
beta1  8.705e-01   1.457e-02   59.730  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 7976.237    normalized:  3.146445

Description:
 Mon Jun 01 20:21:36 2015 by user: Administrator
```

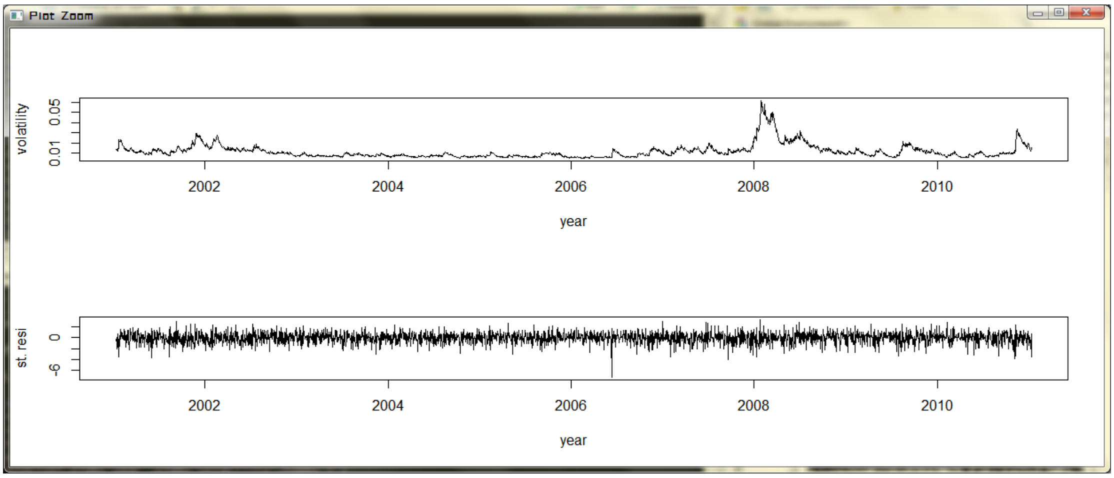
```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  341.8334  0
 Shapiro-Wilk Test  R    W      0.9847918 7.758723e-16
 Ljung-Box Test     R    Q(10)  16.68652  0.08159472
 Ljung-Box Test     R    Q(15)  23.20357  0.07991226
 Ljung-Box Test     R    Q(20)  27.01178  0.1349329
 Ljung-Box Test     R^2  Q(10)  7.011441  0.7243644
 Ljung-Box Test     R^2  Q(15)  8.756468  0.8899028
 Ljung-Box Test     R^2  Q(20)  9.758848  0.9723368
 LM Arch Test       R    TR^2   7.741883  0.8049621

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.288944 -6.277430 -6.288952 -6.284767
```

拟合模型：

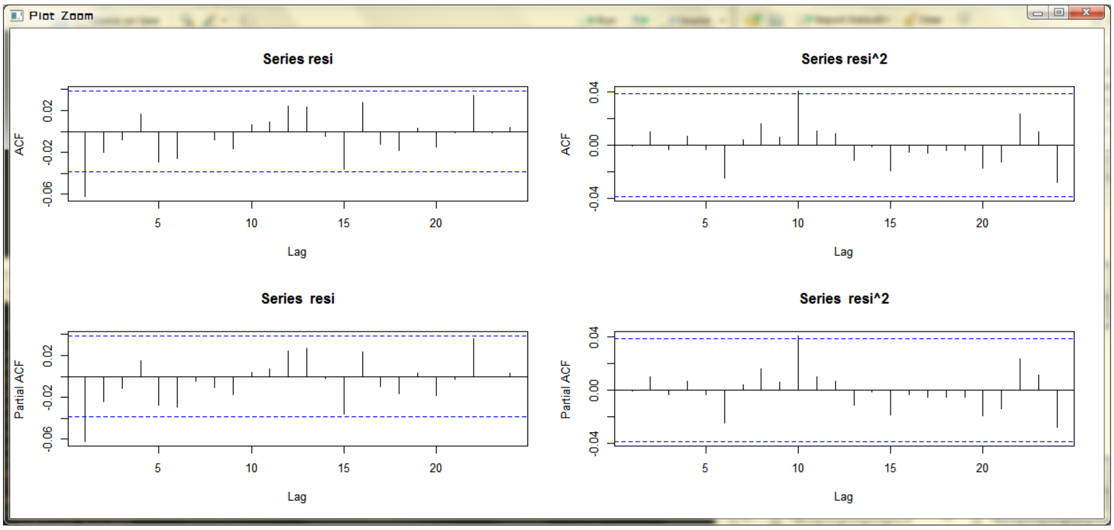
$$r_t = 5.7243\times10^{-4} + a_t, a_t = \sigma_t\varepsilon_t, \varepsilon_t \sim N(0,1)$$

$$\sigma_t^2 = 2.3226\times10^{-6} + 1.9896\times10^{-3}a_{t-1}^2 + 1.1165\times10^{-1}a_{t-2}^2 + 8.7049\times10^{-1}a_{t-3}^2$$

```
> v1=volatility(m4)  # Obtain volatility
> resi=residuals(m4,standardize=T) # Standardized residuals
> vol=ts(v1,frequency=254,start=c(2001,9,4),end=c(2011,9,30))
> res=ts(resi,frequency=254,start=c(2001,9,4),end=c(2011,9,30))
> par(mfcol=c(2,1))  # Show volatility and residuals
> plot(vol,xlab='year',ylab='volatility',type='l')
> plot(res,xlab='year',ylab='st. resi',type='l')
```



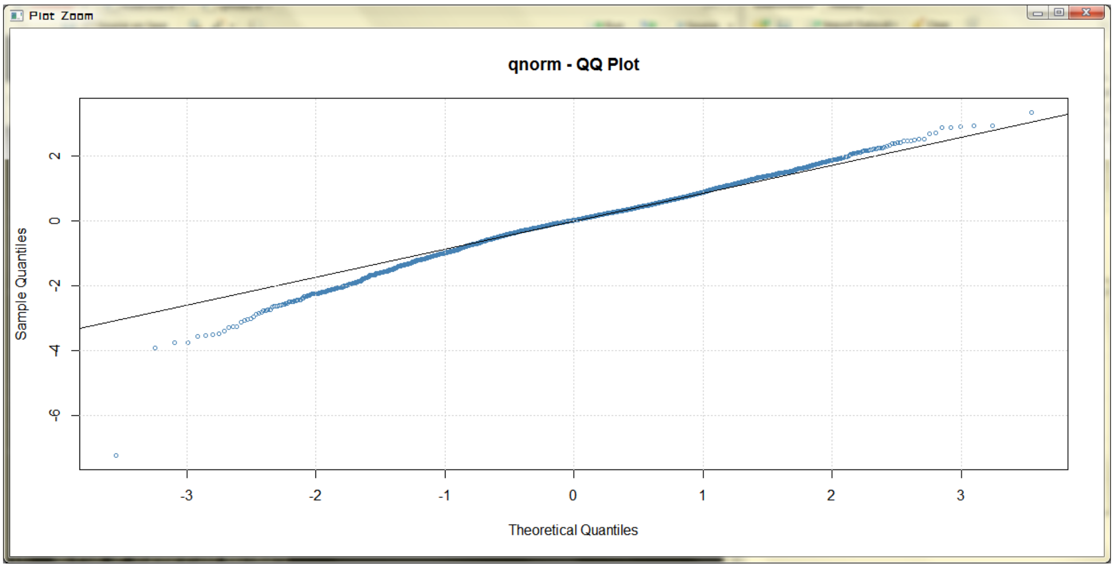

```
> par(mfcol=c(2,2)) # Obtain ACF & PACF
> acf(resi,lag=24)
> pacf(resi,lag=24)
> acf(resi^2,lag=24)
> pacf(resi^2,lag=24)
```




```
> Box.test(resi^2)

 Box-Pierce test

data:  resi^2
X-squared = 0.0014, df = 1, p-value = 0.9705
```

只有在 10 阶的时候有少许超出，可以满足拟合模型

```
> par(mfcol=c(1,1))
> plot(m4)#QQ图

Make a plot selection (or 0 to exit):

 1:    Time Series                               2:    Conditional SD
 3:    Series with 2 Conditional SD Superimposed  4:    ACF of Observations
 5:    ACF of Squared Observations               6:    Cross Correlation
 7:    Residuals                                 8:    Conditional SDs
 9:    Standardized Residuals                   10:    ACF of Standardized Residuals
11:    ACF of Squared Standardized Residuals    12:    Cross Correlation between r^2 and r
13:    QQ-Plot of Standardized Residuals

Selection: 13
```



（c）对该对数收益率序列建立带学生 t 新息的 ARMA-GARCH 模型，进行模型检验，并给出拟合的模型

```
> # Student-t innovations
> m5=garchFit(~1+garch(2,1),data=intc,trace=F,cond.dist="std")
> summary(m5)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(2, 1), data = intc, cond.dist = "std",
    trace = F)

Mean and Variance Equation:
 data ~ 1 + garch(2, 1)
<environment: 0x1621b4d4>
 [data = intc]

Conditional Distribution:
 std

Coefficient(s):
        mu       omega      alpha1      alpha2       beta1       shape
7.2454e-04  1.5793e-06  6.1117e-03  1.1435e-01  8.7418e-01  7.5067e+00

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     7.245e-04   1.613e-04    4.493 7.01e-06 ***
omega  1.579e-06   4.988e-07    3.166  0.00155 **
alpha1 6.112e-03   1.272e-02    0.480  0.63094
alpha2 1.143e-01   2.086e-02    5.482 4.21e-08 ***
beta1  8.742e-01   1.629e-02   53.656  < 2e-16 ***
shape  7.507e+00   1.137e+00    6.604 4.00e-11 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

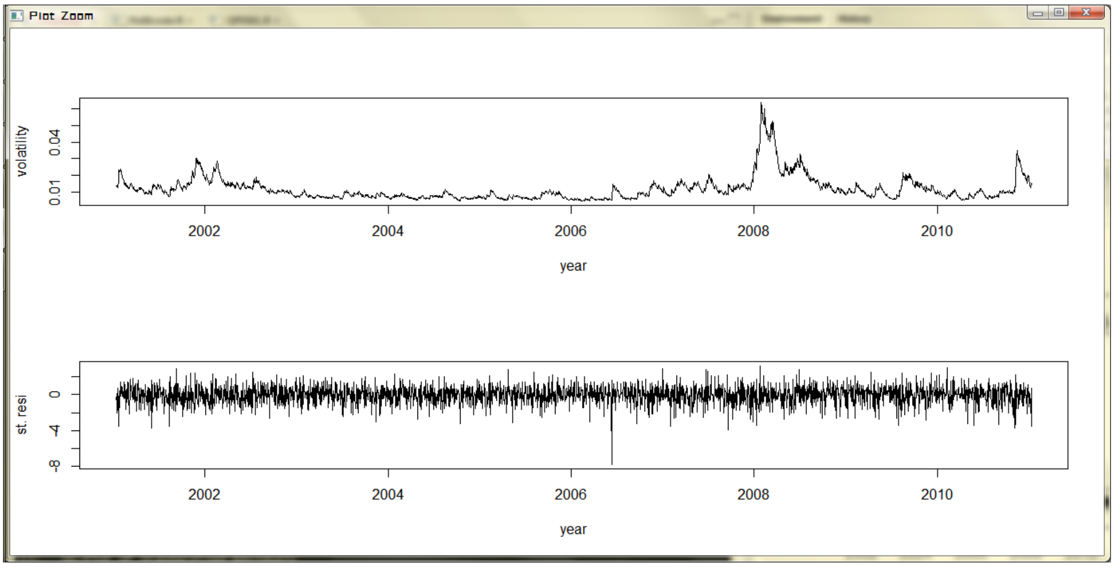
```
Log Likelihood:
 8015.731    normalized:  3.162024

Description:
 Mon Jun 01 20:34:25 2015 by user: Administrator


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  484.6893  0
 Shapiro-Wilk Test  R    W      0.9827891 0
 Ljung-Box Test     R    Q(10)  16.43528  0.08783153
 Ljung-Box Test     R    Q(15)  22.44416  0.09667575
 Ljung-Box Test     R    Q(20)  26.27182  0.157021
 Ljung-Box Test     R^2  Q(10)  5.461153  0.8583243
 Ljung-Box Test     R^2  Q(15)  7.906812  0.9274643
 Ljung-Box Test     R^2  Q(20)  9.649631  0.9740893
 LM Arch Test       R    TR^2   6.005839  0.9157874

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.319314 -6.305497 -6.319326 -6.314302
```

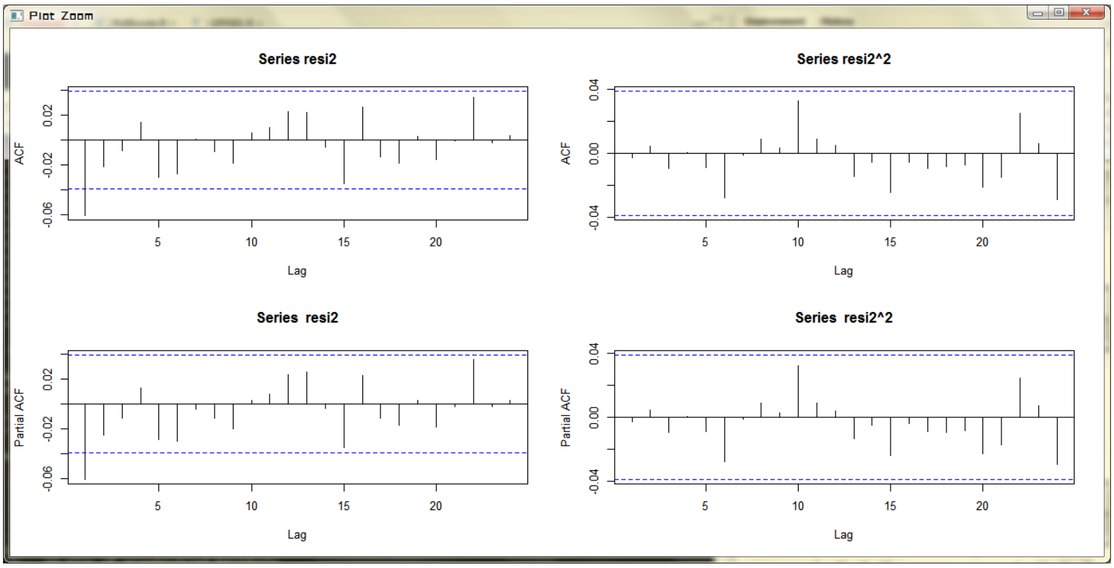
拟合带学生 t 新息模型

$$r_t = 7.2454\times10^{-4} + a_t, a_t = \sigma_t\varepsilon_t, \varepsilon_t \sim t_{7.5068}$$

$$\sigma_t^2 = 1.5793\times10^{-6} + 6.1117\times10^{-3}a_{t-1}^2 + 1.1435\times10^{-1}a_{t-2}^2 + 8.7418\times10^{-1}a_{t-3}^2$$

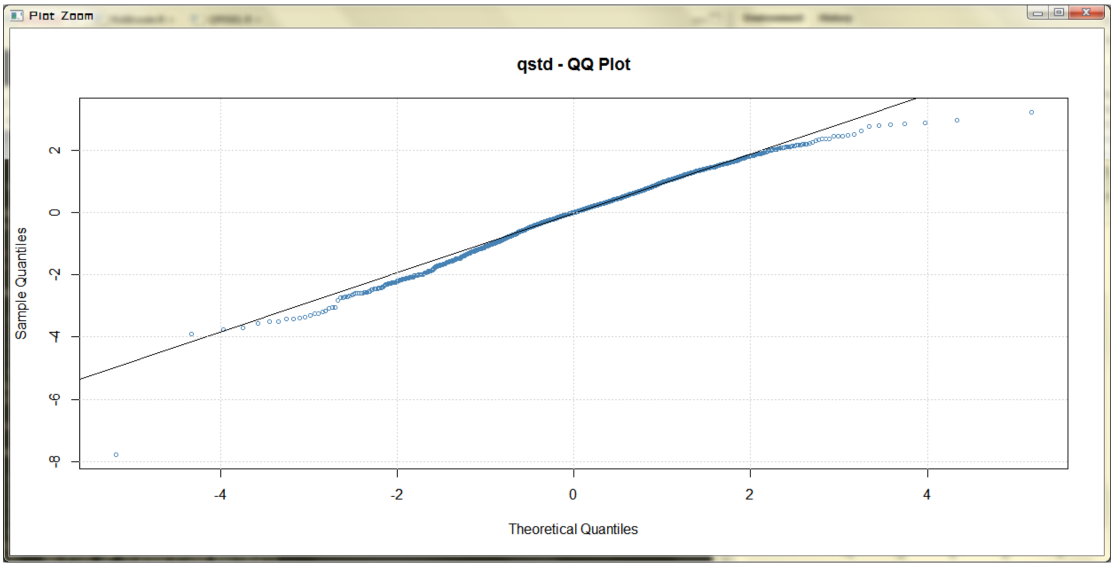
```
> v2=volatility(m5)
> resi2=residuals(m5,standardize=T) # Standardized residuals
> vol2=ts(v2,frequency=254,start=c(2001,9,4),end=c(2011,9,30))
> res2=ts(resi2,frequency=254,start=c(2001,9,4),end=c(2011,9,30))
> par(mfcol=c(2,1))  # Show volatility and residuals
> plot(vol2,xlab='year',ylab='volatility',type='l')
> plot(res2,xlab='year',ylab='st. resi',type='l')
> par(mfcol=c(2,2)) # Obtain ACF & PACF
> acf(resi2,lag=24)
> pacf(resi2,lag=24)
> acf(resi2^2,lag=24)
> pacf(resi2^2,lag=24)
> Box.test(resi2^2)

 Box-Pierce test

data:  resi2^2
X-squared = 0.0209, df = 1, p-value = 0.8851
```





没有超出，可以满足拟合模型

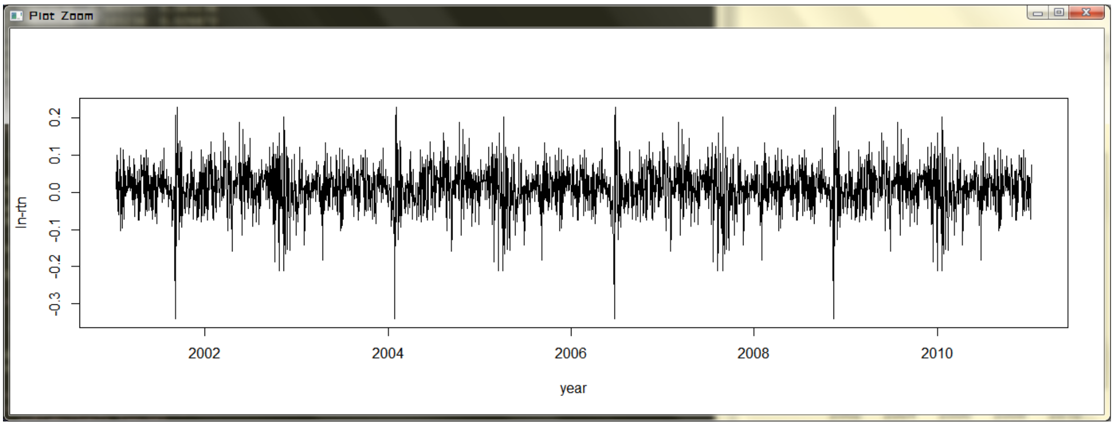
```
> par(mfcol=c(1,1))
> plot(m5)#QQ图

Make a plot selection (or 0 to exit):

 1:   Time Series                                2:   Conditional SD
 3:   Series with 2 Conditional SD Superimposed  4:   ACF of Observations
 5:   ACF of Squared Observations                6:   Cross Correlation
 7:   Residuals                                  8:   Conditional SDs
 9:   Standardized Residuals                    10:   ACF of Standardized Residuals
11:   ACF of Squared Standardized Residuals     12:   Cross Correlation between r^2 and r
13:   QQ-Plot of Standardized Residuals

Selection: 13
```



3.考虑从 1961 年 1 月到 2011 年 9 月可口可乐公司的月股票收益率，简单收益率可以从 CRSP 获取，这里由文件 m-ko-6111.txt 给出，转换简单收益率为对数收益率

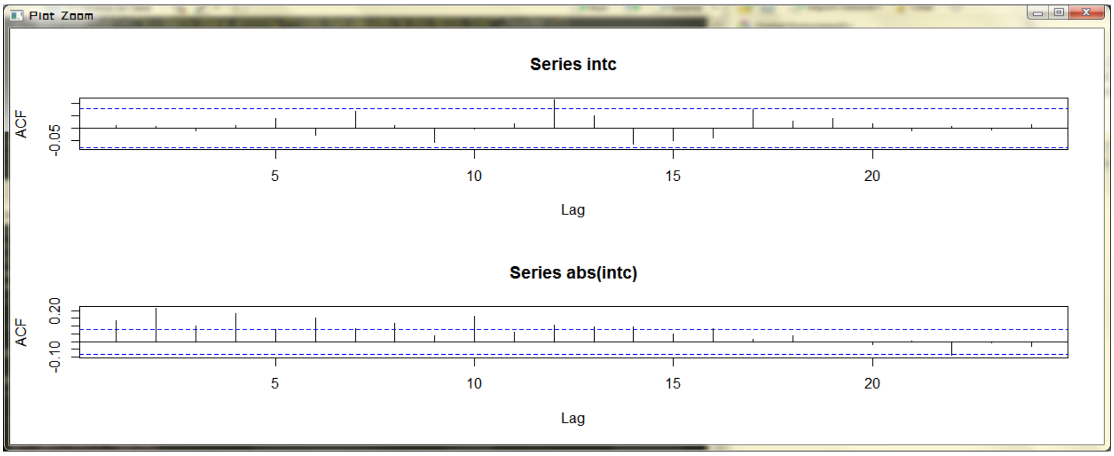
（a）期望的对数收益率为 0 吗？对数收益率中有没有明显的前后相关性？此对数收益率存在 ARCH 效应吗？

```
> da=read.table("E:/DATA/data mining/fts08/data&code/m-ko-6111.txt",header=T)
> head(da)
      date        ko        sp
1 19610131  0.009331  0.063156
2 19610228  0.103236  0.026870
3 19610330  0.012291  0.025536
4 19610428 -0.050000  0.003843
5 19610531  0.087719  0.019139
6 19610630 -0.058065 -0.028846
> intc=log(da$ko+1) #计算对数收益率
> rtn=ts(intc,frequency=254,start=c(2001,9,4),end=c(2011,9,30))   #转化为时间序列
> plot(rtn,type='l',xlab='year',ylab='ln-rtn') # 画时间序列图
> t.test(intc)  # 检验收益率的均值是否为0

 One Sample t-test

data:  intc
t = 4.2198, df = 608, p-value = 2.819e-05
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.005655242 0.015501584
sample estimates:
 mean of x
0.01057841
```



p-value=2.819e-05，拒绝原假设，均值不为零

```
> Box.test(intc,lag=12,type='Ljung')

 Box-Ljung test

data:  intc
X-squared = 14.8209, df = 12, p-value = 0.2514

> par(mfcol=c(2,1))
> acf(intc,lag=24) # ACF plots
> acf(abs(intc),lag=24)
> Box.test(abs(intc),lag=12,type='Ljung')

 Box-Ljung test

data:  abs(intc)
X-squared = 121.8302, df = 12, p-value < 2.2e-16
```



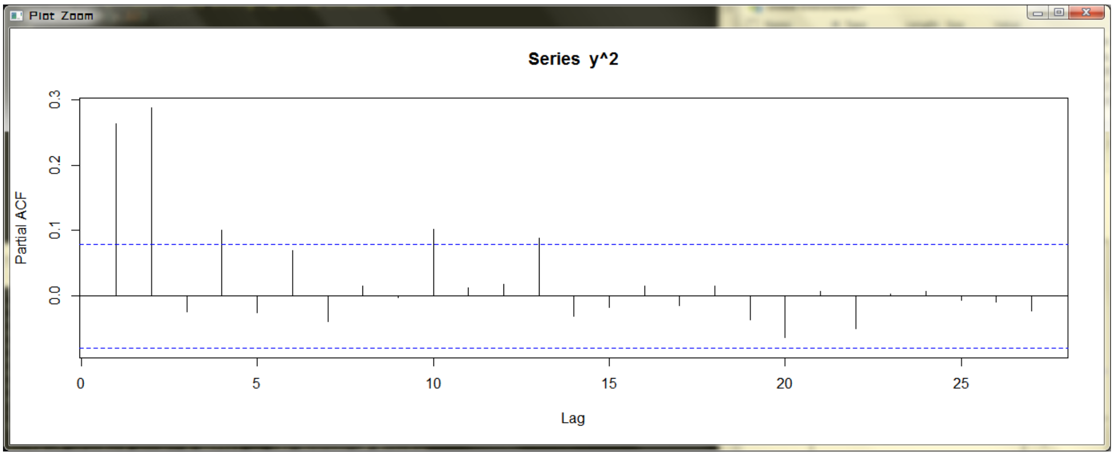
从对数收益率及其绝对值的 ACF 图，以及其对数收益率的 Ljung-Box Q 统计相关检验 X-squared=14.8209,p-value=0.2514，和其对数收益率的绝对值的 Ljung-Box Q 统计相关检验 X-squared=121.8302,p-value<2.2e-16，因此该对数收益率是前后不相关的，但是独立的

```
> par(mfcol=c(1,1))
> ######ARCH效应检验######
> y=intc-mean(intc)    #计算at
> Box.test(y^2,lag=12,type='Ljung')

 Box-Ljung test

data:  y^2
X-squared = 184.9282, df = 12, p-value < 2.2e-16

> source("E:\\DATA\\data mining\\QF05\\archTest.txt")
> archTest(y,12)

Call:
lm(formula = atsq ~ x)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.020771 -0.002807 -0.001545  0.000666  0.099244 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.0015671  0.0004379   3.578 0.000374 ***
x1           0.2056943  0.0413608   4.973 8.67e-07 ***
x2           0.2481802  0.0421593   5.887 6.65e-09 ***
x3          -0.0263466  0.0432095  -0.610 0.542271    
x4           0.0781499  0.0432051   1.809 0.070995 .  
x5          -0.0262333  0.0433213  -0.606 0.545048    
x6           0.0631800  0.0432992   1.459 0.145062    
x7          -0.0381006  0.0432942  -0.880 0.379200    
x8          -0.0101886  0.0433062  -0.235 0.814083    
x9          -0.0276711  0.0431917  -0.641 0.521997    
x10          0.0968461  0.0431845   2.243 0.025296 *  
x11          0.0086381  0.0421235   0.205 0.837592    
x12          0.0180950  0.0412754   0.438 0.661262    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.007753 on 584 degrees of freedom
Multiple R-squared:  0.1723,	Adjusted R-squared:  0.1553 
F-statistic: 10.13 on 12 and 584 DF,  p-value: < 2.2e-16
```

从上图 p-value 可看出，此对数收益率具有明显的 ARCH 效应

（b）对该对数收益率序列建立高斯 GARCH 模型，进行模型检验，并给出拟合的模型

```
> ######GARCH模型######
> library(fGarch)
> pacf(y^2)#通过pacf定阶
```



选择 1 阶

```
> m4=garchFit(~1+garch(1,1),data=intc,trace=F)
> summary(m4)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(1, 1), data = intc, trace = F)

Mean and Variance Equation:
 data ~ 1 + garch(1, 1)
<environment: 0x1632e380>
 [data = intc]

Conditional Distribution:
 norm

Coefficient(s):
       mu      omega     alpha1      beta1
0.0123677  0.0002592  0.0987809  0.8297573

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.237e-02   2.267e-03    5.455 4.90e-08 ***
omega  2.592e-04   8.641e-05    3.000   0.0027 **
alpha1 9.878e-02   2.261e-02    4.368 1.25e-05 ***
beta1  8.298e-01   3.393e-02   24.458  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 869.3329    normalized:  1.427476

Description:
 Mon Jun 01 21:04:01 2015 by user: Administrator
```

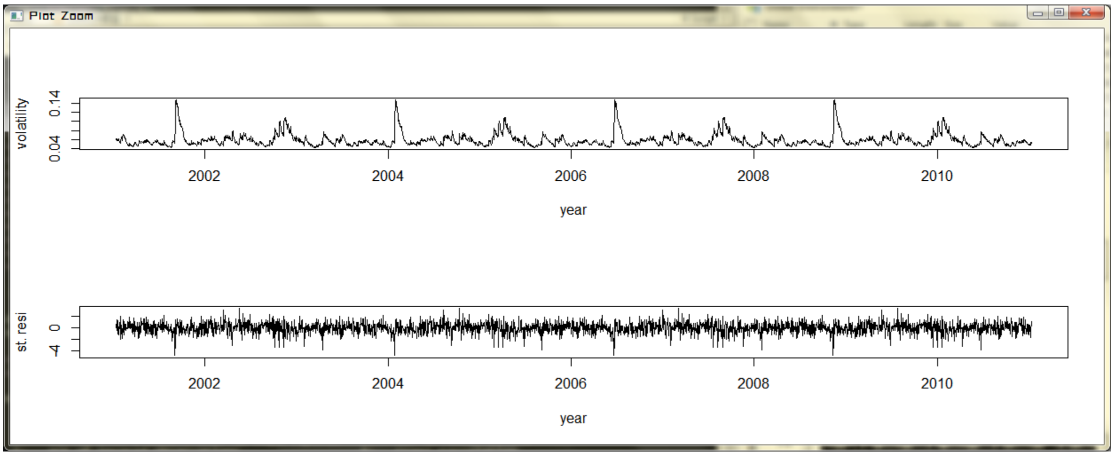
```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  83.09438  0
 Shapiro-Wilk Test  R    W      0.982898  1.470701e-06
 Ljung-Box Test     R    Q(10)  9.877629  0.4512942
 Ljung-Box Test     R    Q(15)  18.69547  0.2278667
 Ljung-Box Test     R    Q(20)  21.54345  0.3657889
 Ljung-Box Test     R^2  Q(10)  12.54335  0.2503354
 Ljung-Box Test     R^2  Q(15)  13.04873  0.5985339
 Ljung-Box Test     R^2  Q(20)  14.33025  0.8133643
 LM Arch Test       R    TR^2   10.79746  0.5463519

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.841816 -2.812838 -2.841901 -2.830543
```

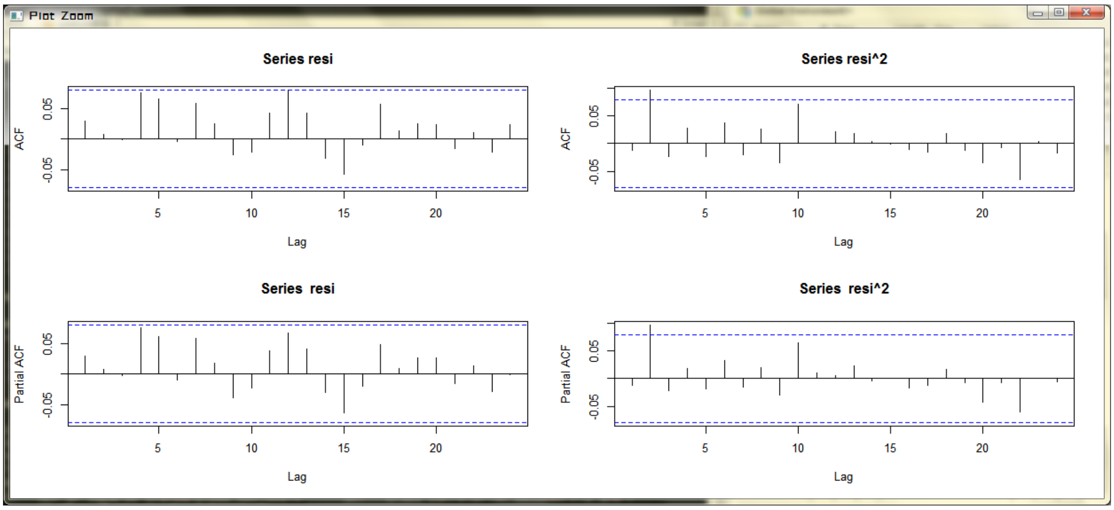
拟合模型

$$r_t = 1.23677\times 10^{-2} + a_t, a_t = \sigma_t \varepsilon_t, \varepsilon_t \sim N(0,1)$$

$$\sigma_t^2 = 2.592\times 10^{-4} + 9.87809\times 10^{-2} a_{t-1}^2 + 8.297573\times 10^{-1} a_{t-1}^2$$

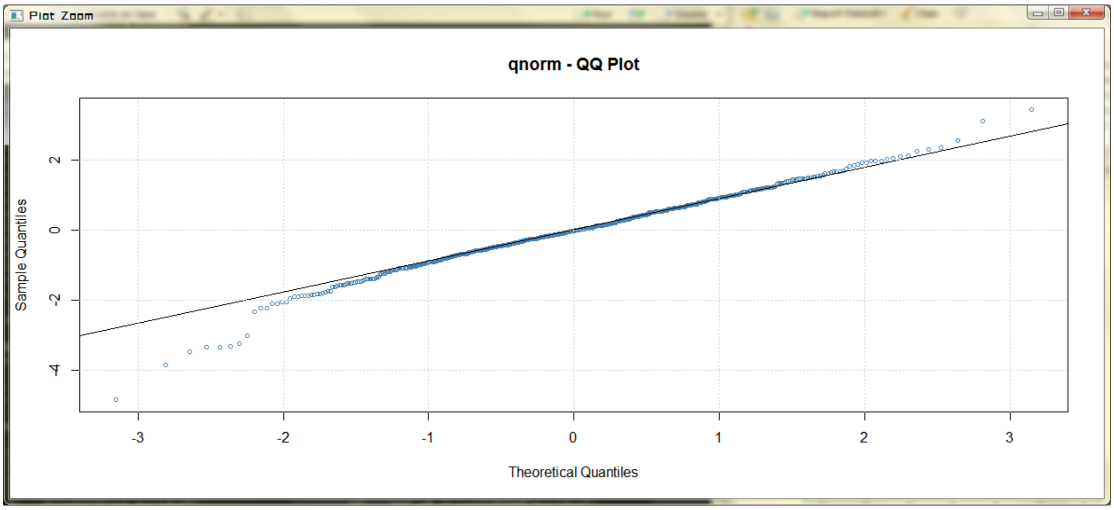
```
> v1=volatility(m4)  # Obtain volatility
> resi=residuals(m4,standardize=T) # Standardized residuals
> vol=ts(v1,frequency=254,start=c(2001,9,4),end=c(2011,9,30))
> res=ts(resi,frequency=254,start=c(2001,9,4),end=c(2011,9,30))
> par(mfcol=c(2,1))  # Show volatility and residuals
> plot(vol,xlab='year',ylab='volatility',type='l')
> plot(res,xlab='year',ylab='st. resi',type='l')
> par(mfcol=c(2,2)) # Obtain ACF & PACF
> acf(resi,lag=24)
> pacf(resi,lag=24)
> acf(resi^2,lag=24)
> pacf(resi^2,lag=24)
> Box.test(resi^2)

        Box-Pierce test

data:  resi^2
X-squared = 0.0989, df = 1, p-value = 0.7531
```





除 **1** 阶有少许超出，可以满足拟合模型

```
> par(mfcol=c(1,1))
> plot(m4)#QQ图

Make a plot selection (or 0 to exit):

 1:   Time Series                              2:   Conditional SD
 3:   Series with 2 Conditional SD Superimposed 4:   ACF of Observations
 5:   ACF of Squared Observations              6:   Cross Correlation
 7:   Residuals                                8:   Conditional SDs
 9:   Standardized Residuals                  10:   ACF of Standardized Residuals
11:   ACF of Squared Standardized Residuals   12:   Cross Correlation between r^2 and r
13:   QQ-Plot of Standardized Residuals

Selection: 13
```



（c）对该对数收益率序列建立带学生 t 新息的 GARCH 模型，进行模型检验，绘制标准化残差的 QQ 图并给出拟合的模型。同时，给出该序列波动率的超前 1 步到超前 5 步预测。

```
> # Student-t innovations
> m5=garchFit(~1+garch(1,1),data=intc,trace=F,cond.dist="std")
> summary(m5)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(1, 1), data = intc, cond.dist = "std",
    trace = F)

Mean and Variance Equation:
 data ~ 1 + garch(1, 1)
<environment: 0x162b0768>
 [data = intc]

Conditional Distribution:
 std

Coefficient(s):
        mu       omega      alpha1       beta1       shape
0.01270783  0.00022276  0.10421045  0.83594060  7.09676842

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     0.0127078   0.0021240    5.983 2.19e-09 ***
omega  0.0002228   0.0000909    2.451 0.014256 *
alpha1 0.1042104   0.0283506    3.676 0.000237 ***
beta1  0.8359406   0.0380904   21.946  < 2e-16 ***
shape  7.0967684   1.8562635    3.823 0.000132 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

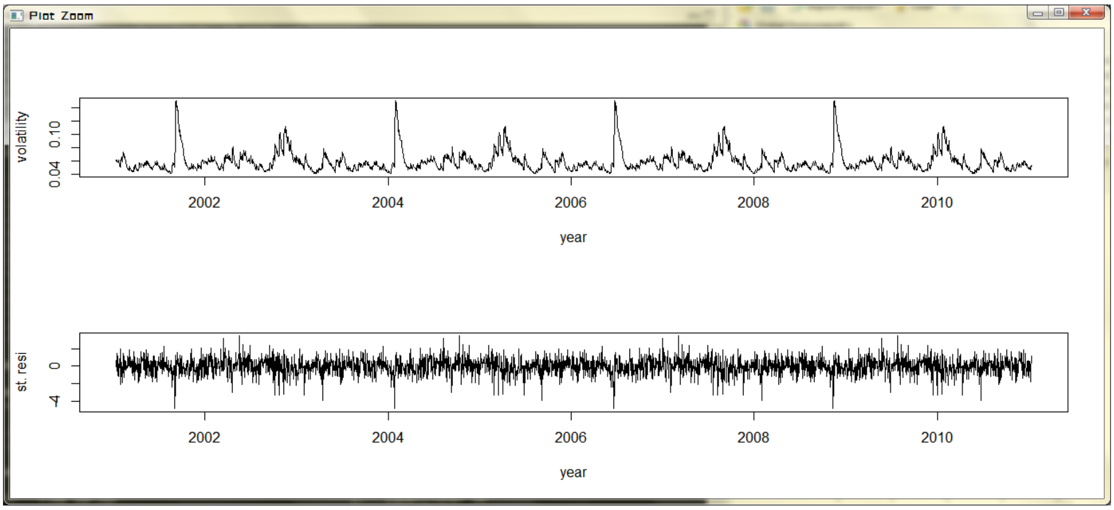
```
Log Likelihood:
 881.8586    normalized:  1.448044

Description:
 Mon Jun 01 21:11:08 2015 by user: Administrator


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  84.74462  0
 Shapiro-Wilk Test  R    W      0.982921  1.494274e-06
 Ljung-Box Test     R    Q(10)  10.28596  0.4157731
 Ljung-Box Test     R    Q(15)  18.9476   0.2161183
 Ljung-Box Test     R    Q(20)  21.6197   0.3614981
 Ljung-Box Test     R^2  Q(10)  11.67235  0.3075833
 Ljung-Box Test     R^2  Q(15)  11.94762  0.6829891
 Ljung-Box Test     R^2  Q(20)  13.25639  0.8661143
 LM Arch Test       R    TR^2   9.814017  0.6322729

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.879667 -2.843445 -2.879800 -2.865576
```

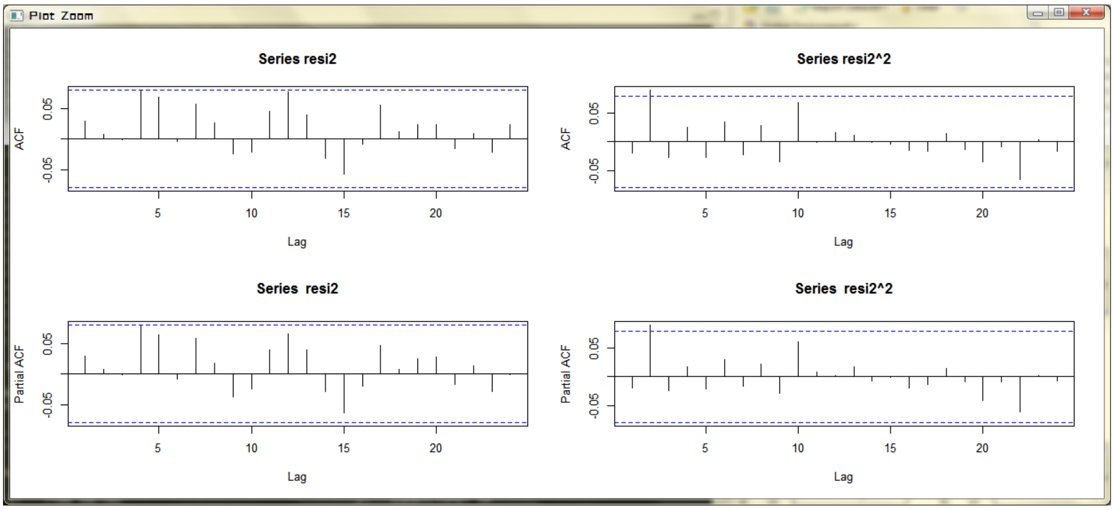
拟合带学生 t 新息模型

$$r_t = 0.01270783 + a_t, a_t = \sigma_t \varepsilon_t, \varepsilon_t \sim t_{7.09676839}$$

$$\sigma_t^2 = 0.00022276 + 0.10421045 a_{t-1}^2 + 0.83595060 a_{t-1}^2$$

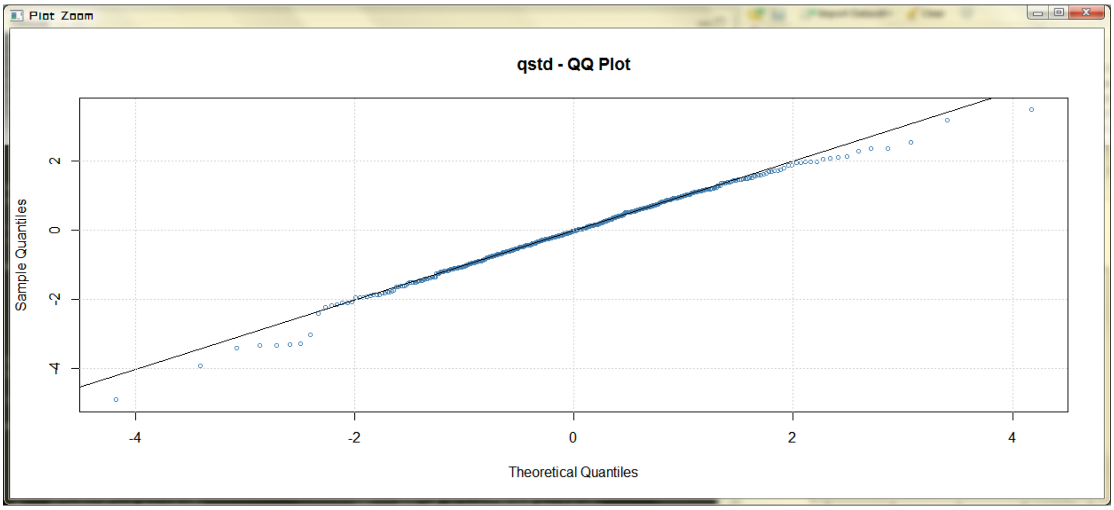
```
> v2=volatility(m5)
> resi2=residuals(m5,standardize=T) # Standardized residuals
> vol2=ts(v2,frequency=254,start=c(2001,9,4),end=c(2011,9,30))
> res2=ts(resi2,frequency=254,start=c(2001,9,4),end=c(2011,9,30))
> par(mfcol=c(2,1))  # Show volatility and residuals
> plot(vol2,xlab='year',ylab='volatility',type='l')
> plot(res2,xlab='year',ylab='st. resi',type='l')
> par(mfcol=c(2,2)) # Obtain ACF & PACF
> acf(resi2,lag=24)
> pacf(resi2,lag=24)
> acf(resi2^2,lag=24)
> pacf(resi2^2,lag=24)
> Box.test(resi2^2)

 Box-Pierce test

data:  resi2^2
X-squared = 0.2178, df = 1, p-value = 0.6408
```





除 2 阶有少许超出，可以满足拟合模型

```
> par(mfcol=c(1,1))
> plot(m5)#QQ图

Make a plot selection (or 0 to exit):

 1:   Time Series                                   2:   Conditional SD
 3:   Series with 2 Conditional SD Superimposed     4:   ACF of Observations
 5:   ACF of Squared Observations                   6:   Cross Correlation
 7:   Residuals                                     8:   Conditional SDs
 9:   Standardized Residuals                       10:   ACF of Standardized Residuals
11:   ACF of Squared Standardized Residuals        12:   Cross Correlation between r^2 and r
13:   QQ-Plot of Standardized Residuals

Selection: 13
```




```
> #预测
> predict(m5,5)
  meanForecast  meanError standardDeviation
1   0.01270783 0.04482957        0.04482957
2   0.01270783 0.04595843        0.04595843
3   0.01270783 0.04699500        0.04699500
4   0.01270783 0.04794910        0.04794910
5   0.01270783 0.04882910        0.04882910
```