Practical No. 1

Topic :- Limit & continuity

$$\lim_{x\to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$$

$$= \lim_{x\to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}}$$

$$= \lim_{x\to a} \frac{a+2x-3x}{3a+x-4x} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$$

$$= \lim_{x\to a} \frac{a-x}{3(a-x)} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$$

$$= \lim_{x\to a} \frac{1}{3} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$$

$$= \frac{1}{3} \lim_{x\to a} \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$$

$$= \frac{1}{3} \times \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}} = \frac{1}{3} \times \frac{\cancel{4}^{2}\sqrt{a}}{\cancel{2}\sqrt{3a}}$$

$$= \frac{2\sqrt{a}}{3\sqrt{3a}}$$

2] $$\lim_{y\to 0} \frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}$$

$$= \lim_{y\to 0} \frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}$$

$$= \lim_{y\to 0} \frac{\cancel{a}+y-\cancel{a}}{y\sqrt{a+y} \times \sqrt{a+y}+\sqrt{a}}$$

$$= \lim_{y\to 0} \frac{\cancel{y}}{\cancel{y}(\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}$$

$$= \lim_{y\to 0} \frac{1}{(\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}$$

$$= \frac{1}{\sqrt{a} \times (\sqrt{a}+\sqrt{a})}$$

$$= \frac{1}{\sqrt{a} \times (2\sqrt{a})}$$

$$= \frac{1}{2a}$$

3] $$\lim_{x\to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$$

Put $x - \pi/6 = h$

$x = h + \pi/6$

$x \to \pi/6$ hence $h \to 0$

$$= \lim_{h\to 0} \frac{\cos(h+\pi/6) - \sqrt{3}\sin(h+\pi/6)}{\pi - 6(h+\pi/6)}$$

$$= \lim_{h\to 0} \frac{\cos h \times \cos\pi/6 - \sin h \times \sin\pi/6 - \sqrt{3}\sin h \cdot \cos\pi/6 - \sqrt{3}\cdot\sin\pi/6 \cdot \cos h}{\cancel{\pi} - 6h - \cancel{\pi}}$$

$$= \lim_{h\to 0} \frac{\cancel{\cos h \times \frac{\sqrt{3}}{2}} - \sin h \times \frac{1}{2} - \sqrt{3}\sin h \times \frac{\sqrt{3}}{2} - \cancel{\cos h \times \frac{\sqrt{3}}{2}}}{-6h}$$

$$= \lim_{h\to 0} \frac{-\frac{\sin h}{2} - \sin h \times \frac{3}{2}}{-6h}$$

$$= \lim_{h\to 0} \frac{-\frac{\cancel{4}^{2}\sin h}{\cancel{2}}}{-6h}$$

$$= \lim_{h\to 0} \frac{\cancel{-}\cancel{2}\sin h}{\cancel{-}\cancel{6}_3 h} = \lim_{h\to 0} \frac{1}{3}\cdot\frac{\sin h}{h}$$

$$= \frac{1}{3}\lim_{h\to 0}\frac{\sin h}{h} = \frac{1}{3}$$

4] 11E.

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$$

By rationalizing numerator & Denominator

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}$$

$$= \lim_{x\to\infty} \frac{(x^2+5-x^2+3)}{(x^2+3-x^2-1)} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$= \lim_{x\to\infty} \frac{48}{\cancel{2}} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$= 4 \lim_{x\to\infty} \frac{\cancel{x}\left(\sqrt{1+\frac{3}{x^2}} + \sqrt{1+\frac{1}{x^2}}\right)}{x\left(\sqrt{1+\frac{5}{x^2}} + \sqrt{1-\frac{3}{x^2}}\right)}$$

$$= 4 \times \frac{\cancel{\sqrt{1}+\sqrt{1}}}{\cancel{\sqrt{1}+\sqrt{1}}}$$

$$= 4$$

5] Examin the continuty of the following function at given points.

i]
$$f(x) = \left.\begin{array}{ll} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}} & \text{for } 0 < x \le \frac{\pi}{2} \\ = \dfrac{\cos x}{\pi - 2x} & \text{for } \frac{\pi}{2} < x < \pi \end{array}\right\} \text{ at } x = \frac{\pi}{2}$$

$$f\left(\frac{\pi}{2}\right) = \frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2\left(\frac{\pi}{2}\right)}}$$

$$= \frac{\sin \pi}{\sqrt{1-\cos \pi}}$$

$$= 0$$

$$L.H.L = \lim_{x\to\frac{\pi}{2}} f(x)$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos x}{\pi - 2x}$$

Put $x - \frac{\pi}{2} = h$ $\quad x \to \frac{\pi}{2}$

$x = h + \frac{\pi}{2}$ $\quad h \to 0$

$$\therefore \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2}+h\right)}{\pi - 2\left(\frac{\pi}{2}+h\right)}$$

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$$= \lim_{h\to 0} \frac{-\sin h}{\pi-\pi+2h}$$

$$= \lim_{h\to 0} \frac{-\sin h}{2h}$$

$$= -\frac{1}{2} \lim_{h\to 0} \frac{\sin h}{h}$$

$$= -\frac{1}{2}$$

$$R.H.L = \lim_{x\to \frac{\pi}{2}^+} f(x)$$

$$= \lim_{x\to \pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$= \lim_{x\to \pi/2} \frac{\sqrt{2}\,\cancel{2\sin x}.\cos x}{\cancel{\sqrt{2}\sin x}}$$

$$= \sqrt{2} \lim_{x\to \pi/2} \cos \frac{\pi}{2}$$

$$= \sqrt{2}\cdot 0$$

$$= 0$$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ f is not continuous at $x = \pi/2$

ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \le x < 6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x=3$, $x=6$

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At $x=3$

$f(3) = x+3 = 3+3 = 6$

$$L.H.L = \lim_{x\to 3^-} f(x) = \lim_{x\to 3} x+3$$

$$= 3+3$$

$$= 6$$

$$R.H.L \lim_{x\to 3^+} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$$

$$= \lim_{x\to 3} (x+3)$$

$$= 3+3$$

$$= 6$$

$\therefore$ L.H.L = R.H.L = $f(3)$

$\therefore$ f is continuous at $x=3$

At $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{(x-3)(\cancel{x+3})}{(\cancel{x+3})} = 6-3 = 3$

$L.H.L = \lim_{x\to 6^-} \frac{x^2-9}{x+3}$

$= \lim_{x\to 6^-} \frac{(x-3)(\cancel{x+3})}{(\cancel{x+3})}$

$= \lim_{x\to 6} x-3$

$= 6-3$

$= 3$

$R.H.L = \lim_{x\to 6^+} f(x)$

$= \lim_{x\to 6} x+3$

$= 6+3$

$= 9$

$\therefore$ $L.H.L \neq R.H.L$

f is not continuous at $x=6$

Find value of k, so that the function $f(x)$ is cts at the indicated point.

$$f(x) = \left.\begin{array}{ll} \frac{1-\cos 4x}{x^2} & x<0 \\ = k & x=0 \end{array}\right\} \text{ at } x=0$$

$f(x)$ is continuous at $x=0$

$\therefore \lim_{x\to 0} f(x) = k$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$

$\frac{2}{4} \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2 = k$

$\frac{1}{2}\left(\lim_{x\to 0} \frac{\sin 2x}{2x}\right)^2 = k$

$\frac{1}{2} = k$

$\therefore$ $k = \frac{1}{2}$

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ii) $f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x} & x \neq 0 \\ = k & x = 0 \end{cases}$ at $x = 0$

f is continuous at $x = 0$

$\therefore \lim_{x \to 0} f(x) = k$

$\lim_{x \to 0} (\sec^2 x)^{\cot^2 x} = k$

$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}} = k$

$\therefore \lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

$e = k$

$\therefore k = e$

iii) $f(x) = \begin{cases} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} & x \neq \frac{\pi}{3} \\ = k & x = \frac{\pi}{3} \end{cases}$ at $x = \frac{\pi}{3}$

f is continuous at $x = \frac{\pi}{3}$

$\therefore \lim_{x \to \frac{\pi}{3}} f(x) = k$

$\lim_{x \to \frac{\pi}{3}} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} = k$

put $x - \frac{\pi}{3} = h$ $\quad x \to \frac{\pi}{3}$

$x = h + \frac{\pi}{3}$ $\quad h \to 0$.

$\lim_{h \to 0} \dfrac{\sqrt{3} - \tan(h + \frac{\pi}{3})}{\pi - 3(h + \frac{\pi}{3})} = k$

$\lim_{h \to 0} \dfrac{\sqrt{3} - \dfrac{\tan h + \tan \frac{\pi}{3}}{1 - \tan h \cdot \tan \frac{\pi}{3}}}{\cancel{\pi} - 3h - \cancel{\pi}} = k$

$\lim_{h \to 0} \dfrac{\sqrt{3} - \dfrac{\tan h + \sqrt{3}}{1 - \tan h \cdot \sqrt{3}}}{-3h} = k$

$\lim_{h \to 0} \dfrac{\dfrac{\sqrt{3}(1 - \tan h \cdot \sqrt{3}) - \tan h \mp \sqrt{3}}{1 - \tan h \cdot \sqrt{3}}}{-3h} = k$

$\lim_{h \to 0} \dfrac{\cancel{\sqrt{3}} - \tan h \cdot 3 - \tan h \mp \cancel{\sqrt{3}}}{\cancel{(1 - \tan h \cdot \sqrt{3})} \times -3h} = k$

$\lim_{h \to 0} \dfrac{2\sqrt{3} - \tan h \cdot 4}{-3h} \times \lim_{h \to 0} \dfrac{1}{1 - \tan h \cdot \sqrt{3}}$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{1-\sqrt{3}\tan h} = k$$

$$\frac{4}{3} \times \frac{1}{1}$$

$$k = \frac{4}{3}$$

7) Discuss the continuity of the following functions which of these functions have a removable discontinuity? Redefine the function so as to remove the discontinuity

i) $f(x) = \frac{1-\cos 3x}{x \tan x} \quad x \neq 0$
$\quad = 9 \quad x = 0$ } at $x = 0$

Sol:

$$f(x) = \frac{1-\cos 3x}{x \tan x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 3/2\, x}{x \tan x}$$

$$= \lim_{x\to 0} \frac{\frac{2\sin^2 3/2\, x}{x^2}}{\frac{x \cdot \tan x}{x^2}}$$

$$= \frac{2\lim_{x\to 0}\left(\frac{\sin \frac{3}{2}x}{x \times \frac{3}{2}}\right)^2 \times \frac{9}{4}}{\lim_{x\to 0} \frac{\tan x}{x}}$$

$$= 2 \times \frac{9}{4_2}$$

$$= \frac{9}{2}$$

$\lim_{x\to 0} f(x) = \frac{9}{2} \quad 9 = f(0)$

$\therefore$ f is not continuous at $x = 0$

Redefin function

$f(x) = \frac{1-\cos 3x}{x \tan x} \quad x \neq 0$

$\quad = \frac{9}{2} \quad x = 0.$

Now $\lim_{x\to 0} f(x) = f(0)$.

f has removable discontinuity at $x=0$

ii) $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$ $\quad x \neq 0$

$= \dfrac{\pi}{60}$ $\quad x = 0$ $\Big\}$ at $x=0$

Solu:

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\frac{x\pi}{180}}{x^2}$$

$$= 3\lim_{x\to 0}\frac{e^{3x}-1}{3x}\lim_{x\to 0}\frac{\sin\frac{x\pi}{180}}{x\times\frac{\pi}{180}}\times\frac{\pi}{180}$$

$$= 3\log e \times \frac{\pi}{\cancel{180}\,60}$$

$$= \frac{\pi}{60}$$

f is continuous at $x=0$

8. If $f(x) = \dfrac{e^{x^2}-\cos x}{x^2}$ for $x \neq 0$ is continuous at $x=0$ find $f(0)$

$f(x) = \dfrac{e^{x^2}-\cos x}{x^2}$ $\quad x=0$

is continuous at $x=0$

$\therefore$ Given

f is continuous at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$= \lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2} = f(0)$$

$$\lim_{x\to 0}\frac{e^{x^2}-\cos x+1-1}{x^2} = f(0)$$

$$\lim_{x\to 0}\frac{(e^{x^2}-1)+(1-\cos x)}{x^2} = f(0)$$

$$\lim_{x\to 0}\frac{(e^{x^2}-1)+(\cancel{1-\cos x}\ 2\sin^2\frac{x}{2})}{x^2} = f(0)$$

$$\lim_{x\to 0}\frac{e^{x^2}-1}{x^2} + 2\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{1}{4} = f(0)$$

$$1 + 2\times\frac{1}{\cancel{4}\,2} =$$

$$\frac{3}{2} = f(0)$$

$$\therefore f(0) = \frac{3}{2}$$

9] If $f(x) = \frac{\quad}{\cos^2 x}$ is cts at $x = \frac{\pi}{2}$ find $f(\frac{\pi}{2})$

Solu:- $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$

$f(x)$ is continuous at $x = \pi/2$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \lim_{x \to \pi/2} \frac{2 - 1 - \sin x}{(1-\sin^2 x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{\cancel{1-\sin x}}{(1+\sin x)(\cancel{1-\sin x})(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{(1+\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{(1+1)(\sqrt{2} + \sqrt{1+1})}$$

$$= \frac{1}{2 \times 2\sqrt{2}}$$

$f(\pi/2) = \frac{1}{4\sqrt{2}}$

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Practical = 2

Topic :- Derivative

Q.1 Show that the following function defined from IR to IR are differentiable.

i] Cot x

$f(x) = \cot x$.

$$\therefore Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\therefore = \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

Put $x - a = h$ $\quad x \to a$

$x = h + a$ $\quad h \to 0$

$$\therefore f(x) = \lim_{h \to 0} \frac{\cot(a+h) - \cot a}{a+h-a}$$

$$= \lim_{h \to 0} \frac{\frac{\cos(a+h)}{\sin(a+h)} - \frac{\cos a}{\sin a}}{h}$$

$$= \lim_{h \to 0} \frac{\cos(a+h)\sin(a) - \cos a \cdot \sin(a+h)}{\sin(a+h) \cdot \sin a \times h}$$

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$$= \lim_{h\to 0} \frac{\sin(a-h-a)}{\sin(a+h)\cdot \sin a \times h}$$

$$= \lim_{h\to 0} \frac{1}{\sin(a+h)\cdot \sin a} - \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{\sin a \cdot \sin a} \times -1$$

$$= -\frac{1}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a.$$

function is differentiable at IR to IR.

ii) Cosec x.

$f(x) = \operatorname{cosec} x.$

$$Df(a) \quad \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$f(x) = \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

Put $x - a = h$ $\quad x \to a$

$x = h + a$ $\quad h \to 0$

$$= \lim_{h\to 0} \frac{\operatorname{cosec}(h+a) - \operatorname{cosec} a}{h+a-a}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sin(h+a)} - \frac{1}{\sin(a)}}{h}$$

$$= \lim_{h\to 0} \frac{\sin(a) - \sin(h+a)}{\sin(h+a)\cdot \sin(a) \times h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(a+\frac{h}{2}\right)\cdot \sin\left(-\frac{h}{2}\right)}{\sin(h+a)\cdot \sin a \times h}$$

$$= 2\lim_{h\to 0} \frac{\cos\left(a+\frac{h}{2}\right)}{\sin(h+a)\cdot \sin a} \times -\frac{1}{2}\lim_{h\to 0} \frac{\sin\left(-\frac{h}{2}\right)}{-\frac{h}{2}}$$

$$= -\frac{\cos a}{\sin a \times \sin a}$$

$$= -\cot x \cdot \operatorname{cosec} x$$

function is differentiable at IR to IR

iii) Sec x.

$f(x) = \sec x$.

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

Put $x - a = h$ $\quad x \to a$

$x = h + a$ $\quad h \to 0$

$$= \lim_{h \to 0} \frac{\sec(h+a) - \sec a}{h + a - a}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\cos(h+a)} - \frac{1}{\cos a}}{h}$$

$$= \lim_{h \to 0} \frac{\cos a - \cos(h+a)}{\cos(h+a) \cdot \cos a \times h}$$

$$= \lim_{h \to 0} \frac{-2 \sin\left(a + \frac{h}{2}\right) \cdot \sin\left(-\frac{h}{2}\right)}{\cos(h+a) \cdot \cos a \times h}$$

$$= \cancel{2} \lim_{h \to 0} \frac{\sin\left(a + \frac{h}{2}\right)}{\cos(h+a) \cdot \cos a} \cdot \frac{-1}{\cancel{2}} \lim_{h \to 0} \frac{\sin\left(-\frac{h}{2}\right)}{-\frac{h}{2}}$$

$$= \frac{\sin a}{\cos a \times \cos a} \times 1$$

$$= \tan a \cdot \sec a.$$

function is differentiable at $\mathbb{R}$ to $\mathbb{R}$.

Q.2 If $f(x) = 4x + 1 \quad x \le 2$

$= x^2 + 5 \quad x > 2$ g at $x = 2$

Then find f is differentiable or not?

Solⁿ:

R.H.D $= Df(2^+)$

$$\lim_{x \to 2^+} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2^+} x + 2$$

$= 2+2$

$R.H.D = 4$

$L.H.D = Df(2^-)$

$$\lim_{x \to 2^-} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 4$$

$R.H.D = L.H.D$

hence function is differentiable.

Q.3] If $f(x) = 4x+7 \quad , x < 3$

$= x^2+3x+1 \quad x \geq 3$ at $x=3$

then find f is differentiable at or not?

Solu:-

$R.H.D = Df(3^+)$

$$= \lim_{x \to 3^+} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= \lim_{x \to 3} x+6$$

$$= 3+6$$

$R.H.D = 9$

L.H.D : $Df(3^-)$

$$= \lim_{x\to 3^-} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4\cancel{(x-3)}}{\cancel{x-3}}$$

$$= \lim_{x\to 3^-} 4$$

L.H.D = 4

R.H.D ≠ L.H.D at $x=3$

hence function is not differentiable at $x=3$

Q.4]. If $f(x) = 8x-5 \quad x<2$
$= 3x^2-4x+7 \quad x>2$ } at $x=2$

Then find function is differentiable or not?

R.H.D = $Df(2^+)$

$$= \lim_{x\to 2^+} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{\cancel{(x-2)}(3x+2)}{\cancel{x-2}}$$

$$= \lim_{x\to 2^+} 3x+2$$

$$= 3\times 2+2$$

R.H.D = 8

$L.H.D = Df(2^-)$

$= \lim_{x \to 2^-} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$

$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{x-2}}$

$L.H.D = 8$

$R.H.D = L.H.D$ at $x = 8$

Hence function is differentiable at $x = 8$.

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Practical => 3

Topic => Application of Derivative

Q1.

i) $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3x^2 > 5$

$x > \pm\sqrt{\frac{5}{3}}$

$x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

f is decreasing iff $f'(x) < 0$

$3x^2 - 5 < 0$

$x^2 < \frac{5}{3}$

$x = \pm\sqrt{\frac{5}{3}}$

$+ \quad - \quad +$

$-\sqrt{\frac{5}{3}} \qquad \sqrt{\frac{5}{3}}$

$x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

f is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$x > \frac{4}{2}$

$x > 2$

$\therefore x \in (2, \infty)$

$\therefore f$ is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$x < 2$

$\therefore x \in (-\infty, 2)$

$-\infty \quad + \quad 2 \quad - \quad \infty$

~~∴ f is de~~

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$6x^2 + 2x - 20 > 0$

$(6x - 10)(x + 2) > 0$

$\therefore x \in (-\infty, -2) \cup (\frac{10}{6}, \infty)$

f is decreasing iff $f(x) < 0$

$6x^2 + 2x - 20 < 0$

$(6x - 10)(x + 2) < 0$

$+ \quad -2 \quad -$

$\therefore x \in (-2, \frac{10}{6})$

iv) $f(x) = x^3 - 27x + 5$

$= 3x^2 - 27$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$3(x^2 - 9) > 0$

$(x - 3)(x + 3) > 0$

$x \in (-\infty, -3) \cup (3, \infty)$

f is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$(x - 3)(x + 3)$

$+ \quad 3 \quad - \quad 3 \quad +$

$\therefore x \in (-3, 3)$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

$= 6(x^2 - 3x - 4)$

f is increasing iff $f'(x) > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$(x + 1)(x - 4) > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$x \in (-\infty, -1) \cup (4, \infty)$

f is decreasing iff $f'(x) < 0$

$x^2 - 3x - 4 < 0$

$(x + 1)(x - 4) < 0$

$+ \quad 1 \quad - \quad 4 \quad +$

$x \in (1, 4)$

Q.2.

i) $y = 3x^2 - 2x^3$

let

$f(x) = y = 3x^2 - 3x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$f''(x)$ is concave upword iff. $f''(x) > 0$

$6(1-2x) > 0$

$1 - 2x > 0$

$x < \frac{1}{2}$

$x \in (-\infty, \frac{1}{2})$

$f''(x)$ is concave downwords iff

$f''(x) < 0$

$6(1-2x) < 0$

$x > \frac{1}{2}$

$\therefore x \in (\frac{1}{2}, \infty)$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

$f''(x)$ is concave upwords iff $f''(x) > 0$

$12x^2 - 36x + 24 > 0$

$x^2 - 3x + 2 > 0$

$(x-2)(x-1) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

$f''(x)$ is concave downword iff

$f''(x) < 0$

$x^2 - 3x + 2 < 0$

$(x-2)(x-1) < 0$

$x \in (1, 2)$

iii) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is concave downword iff $f''(x) < 0$

$6x < 0$

$x < 0$

$x \in (-\infty, 0)$

$f''(x)$ is concave upword iff $f''(x) > 0$

$6x > 0$

$x > 0$

$x \in (0, \infty)$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24x - 18x + 6x^2$

$f''(x) = -18 + 12x$

$f''(x)$ is concave upword iff $f''(x) > 0$

$-18x + 12x > 0$

$12x > 18$

$x > \frac{18}{12}$

$x > \frac{3}{2}$

$x \in (\frac{3}{2}, \infty)$

$f''(x)$ is concave downword iff $f''(x) < 0$

$-18 + 12x < 0$

$x < \frac{18}{12}$

$x < 3/2$

$x \in (-\infty, \frac{3}{2})$

v) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$f''(x)$ is concave upwords iff $f''(x) > 0$

$12x + 2 > 0$

$6x + 1 > 0$

$x > -\frac{1}{6}$

$x \in (\frac{1}{6}, \infty)$

$f''(x)$ is concave down word iff $f''(x) < 0$

$12x + 2 < 0$

$6x + 1 < 0$

$x < -\frac{1}{6}$

$x \in (-\infty, -\frac{1}{6})$

AV 20/12/19

Practical = 4

Topic: Application of Derivative & Newton's method

Q.1 find maxima & minima value of following fun

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider $f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x^4 - 32 = 0$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{16} = 2 + 6 = 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore$ $f(2) = 2^2 + \frac{16}{4} = 4 + 4 = 8.$

$f''(-2) = 2 + \frac{96}{(-2)^4} = 2 + \frac{96}{16} = 2 + 6 = 8 > 0$

f has minimum value at $x = 2$

function reaches minimum value at $x = 2$ and $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Now consider $f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$x^4 = x^2,$

$x = \pm 1$

$f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60 = 30 > 0$

f has minimum value at $x = 1$

$f(1) = 3 - 5 + 3 = 6 - 5 = 1.$

$f''(-1) = 30 + (-60) = -30 < 0$

f has maximum value at $x = -1$

$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

f has the maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

iii) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x.$

Now consider $f'(x) = 0$

$3x^2 - 6x = 0$

$3x^2 = 6x$

$x = 2$ or $x = 0$

$f''(x) = 6x - 6$

iii) $f''(0) = -6 < 0$

f has maximum value at $x = 0$

$f(0) = 0 - 3(0) + 1$

$= 1$

$f''(2) = 6 \times 2 - 6 = 12 - 6 = 6 > 0$

f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 12 + 1$

$= -3$

$\therefore f$ has maximum value 1 at $x = 0$ and

f has minimum value -3 at $x = 2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

Now consider $f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 12 \times 2 - 6 = 24 - 6 = 18 > 0$

f has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = -12 - 6$

$= -18 < 0$

f has maximum value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

f has maximum value at -18 at $x = 2$

f has minimum value 18 at $x = 2$

Q.2. Find the root of following equation by Newton's method.
(Take 4 iteration only) correct upto 4 decimal

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0 \leftarrow$ Given

$f'(x) = 3x^2 - 6x - 55$

By Newton's method

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 0 + \dfrac{9.5}{55} = 0.1727$

$$f(x_1) = (0.1727)^3 - 3(0.1727)^3 - 55(0.1727) + 9.5$$
$$= -0.0829$$

$$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$
$$= -55.9467$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 0.1727 - \frac{0.0829}{55.9467}$$
$$= 0.1712$$

$$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$
$$= 0.0011$$

$$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$$
$$= -55.9393$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 0.1712 + \frac{0.0011}{55.9393}$$
$$= 0.1712$$

The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$ $[2, 3]$

$f'(x) = 3x^2 - 4$

$$f(2) = 2^3 - 4(2) - 9$$
$$= -9$$
$$f(3) = 3^3 - 4(3) - 9$$
$$= 6$$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 3 - \frac{6}{23}$$
$$x_1 = 2.7392$$

$$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$$
$$= 0.596$$
$$f'(x_1) = 3(2.7392)^2 - 4$$
$$= 18.5096$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 2.7392 - \frac{0.596}{18.5096}$$
$$= 2.7071$$

$$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$$
$$= 0.0102$$

$$f'(x_2) = 3(2.7071)^2 - 4$$
$$= 17.9851$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7071 - \frac{0.0102}{17.9851}$$

$$= 2.7015$$

$$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$$

$$= -0.0901$$

$$f'(x_3) = 3(2.7015)^2 - 4$$

$$= 17.8943$$

$$x_4 = 2.7015 + \frac{0.0901}{17.8943}$$

$$= 2.7065$$

iii] $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$$f'(x) = 3x^2 - 3.6x - 10$$

$$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$$

$$= 6.2$$

$$f(2) = (2)^3 - (1.8)(2)^2 - 10(2) + 17$$

$$= -2.2$$

Let $x_0 = 2$ be initial approximation

By Newton's method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{2.2}{5.2}$$

$$= 1.577$$

$$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$$

$$= 0.6755$$

$$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$$

$$= -8.2164$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.577 + \frac{0.6755}{8.2164}$$

$$x_2 = 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$$

$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$

$$= -7.7143$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.6592 + \frac{0.0204}{7.7143}$$

$$= 1.6618$$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= -7.6977$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.6618 + \frac{0.0004}{7.6977}$$

$= 1.6618$

The root of equation is 1.6618

Practical = 5

Topic = Integration

Q.1. Solve the following integration

i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{x^2+2x+1-4}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

$I = \log\left|x+1+\sqrt{(x+1)^2-(2)^2}\right|+c$ $\quad \therefore I = \log\left|x+\sqrt{x^2+a^2}\right|+c$

$I = \log\left|(x+1)+\sqrt{(x+1)^2-(2)^2}\right|+c$

ii) $\int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}dx + \int dx$

$= \frac{4e^{3x}}{3} + x + c$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

OR

$= \int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$= 2\int x^2 dx - 3\int \sin x\, dx + 5\int x^{\frac{1}{2}} dx$

$= \frac{2x^3}{3} + \cos x + \frac{5 \times x^{3/2}}{3/2}$

$= \frac{2x^3}{3} + \cos x + \frac{10x^{3/2}}{3} + c$

iv) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$= \int \frac{x^3}{\sqrt{x}} dx + 3\int \frac{x}{\sqrt{x}} + 4\int \frac{1 dx}{\sqrt{x}}$

$= \int x^{5/2} dx + 3\int x^{\frac{1}{2}} dx + 4\int x^{-\frac{1}{2}} dx$

$= \frac{2}{7} x^{7/2} + 3 \times \frac{2}{3} x^{3/2} + 4 \times 2 x^{\frac{1}{2}}$

$= \frac{2}{7} x^{7/2} + 2x^{3/2} + 8x^{\frac{1}{2}}$

v) $\int t^7 \sin(2t^4)\, dt$

Let $t^4 = x$

$4t^3 dt = dx$

$dt = \frac{dx}{4t^3}$

$= \frac{1}{4} \int 4t^3 . t^4 \sin(2t^4)\, dt$

$= \frac{1}{4} \int x \cdot \sin(2x) dx$

$= \frac{1}{4} \left[x \int \sin 2x - \int \left[\int \sin 2x \cdot \frac{dx}{dx} \right] \right]$

$= \frac{1}{4} \left[-\frac{x\cos 2x}{2} + \frac{1}{2} \int \cos 2x \cdot 1 \right]$

$= \frac{1}{4} \left[-\frac{x\cos 2x}{2} + \frac{1}{4} \sin 2x \right]$

$= -\frac{1}{8} x \cos 2x + \frac{1}{16} \sin 2x + c$

Resubstituting $x = t^4$

$= -\frac{1}{8} t^4 \cos(2t^4) + \frac{1}{16} \sin(2t^4) + c$

vi) $\int \sqrt{x} (x^2 - 1)\, dx$

$= \int x^{5/2} dx - \int x^{\frac{1}{2}} dx.$

$= \frac{2}{7} x^{7/2} - \frac{2}{3} x^{3/2} + c$

x) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \, dx$

Let $x^3 - 3x^2 + 1 = t$

$3x^2 - 6x \, dx = dt$

$(x^2 - 2x) dx = \frac{dt}{3}$

$I = \int \frac{1}{t} \frac{dt}{3}$

$= \frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log t + c$

Resubstituting $t = x^3 - 3x^2 + 1$

$= \frac{1}{3} \log (x^3 - 3x^2 + 1) + c$

AK
03/01/2020

Practical - 6

Topic: Application of integration & Numerical integration

Q.1 Find the length of the following curve.

i) $x = t\sin t$, $y = 1 - \cos t$, $t \in [0, 2\pi]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$\therefore \frac{dx}{dt} = 1 - \cos t$

$x = t - \sin t$

$y = 1 - \cos t \quad \therefore \frac{dy}{dt} = 0 - (-\sin t) = \sin t$

$$L = \int_0^{2\pi} \sqrt{(\sin t)^2 + (1 - \cos t)^2} \, dt$$

$$= \int_0^{2\pi} \sqrt{\sin^2 t + 1 + \cos^2 t - 2\cos t} \, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t} \, dt$$

$$= \sqrt{2} \int_0^{2\pi} \sqrt{2\sin^2\left(\frac{t}{2}\right)} \, dt$$

$$= \cancel{\sqrt{2}} \cdot \sqrt{2} \int_0^{2\pi} \sin \frac{t}{2} =$$

$$= 2 \left[-\cos\left(\frac{t}{2}\right) \right]_0^{2\pi}$$

$= -4[-1-1]$

$= 8$

2] $y = \sqrt{4-x^2}$ $\quad x \in [-2, 2]$

$\cancel{\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}}$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{1(-2x)}{2\sqrt{4-x}}$$

$$= \frac{-x}{\sqrt{4-x}}$$

$$L = \int_{-2}^{2} \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1 + \frac{x^2}{4-x^2}}$$

$$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}$$

$$= 2\int_{-2}^{2} \sqrt{\frac{1}{2^2-x^2}}\, dx$$

$$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$$

$$= 2[\sin^{-1}(1) - \sin^{-1}(-1)]$$

$$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$= 2\pi$$

3] $y = x^{3/2}$ $\quad x \in [0, 4]$

$$L = \int_a^b \sqrt{\left(1 + \left(\frac{dy}{dx}\right)^2\right)}\, dx$$

$$\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$$

$$L = \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{27}\left[40^{3/2} - 8\right]$$

4] $x = 3\sin t$ $\quad y = 3\cos t$ $\quad t \in [0, 2\pi]$

$$L = \int_0^{5} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$\frac{dy}{dt} = -3\sin t$ $\quad \frac{dx}{dt} = 3\cos t$

$$L = \int_0^{2\pi} \sqrt{(-3\sin^2 t) + (3 \ldots}$$

$$= 3\int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}\; dt$$

$$= 3\int_0^{2\pi} \sqrt{1}\; dt$$

$$= 3[t]_0^{2\pi}$$

$$= 6\pi$$

5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ $\quad y \in [1,2]$

$$L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\; dy$$

$$\frac{dx}{dy} = \frac{3}{6}y^2 - \frac{1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{1}{2}y^2 - \frac{1}{2y^2}\right)^2}\; dy$$

$$= \int_1^2 \sqrt{1 + \left(\frac{y^4 - 1}{2y^2}\right)^2} dy$$

$$= \int_1^2 \sqrt{\frac{(y^4-1)^2 + 4 \times y^4 \times 1}{4y^4}}\; dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\; dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\; dy$$

$$= \frac{2}{1}\int \frac{y^2}{2}\; dy + \int_1^2 \frac{1}{2y^2}\; dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3}\right]_1^2 + \frac{1}{2}\left[\frac{1}{y}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{3}\right] + \frac{1}{2}\left[\frac{1}{2} - 1\right]$$

$$= \frac{7}{6} - \frac{1}{4}$$

$$= \frac{14-3}{12}$$

$$= \frac{11}{12}$$

Q.2

1) $\int_0^2 e^x\, dx$ with $n = 4$

$a = 0$, $b = 2$, $n = 4$

$h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

x	0	0.5	1	1.5	2
y	1	1.284	2.7182	9.4877	54.5981

By Simpson's Rule.

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182)\right]$$

2] $\int_0^4 x^2\,dx$ with $n=4$

$a=0$ $b=4$ $n=4$

$h=\frac{b-a}{n}=\frac{4-0}{4}=1$

x	0	1	2	3	4
y	0	1	4	9	16

By Simpson's Rule

$$\int_0^4 x^2dx=\frac{1}{3}\left[(0+16)+4(1+9)+2(4)\right]$$

$$=\frac{64}{3}$$

3] $\int_0^{\pi/3}\sqrt{\sin x}\,dx$ with $n=6$

$a=0$, $b=\frac{\pi}{3}$, $n=6$

$h=\frac{\frac{\pi}{3}-0}{6}=\frac{\pi}{18}$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$
y	0	0.4167	0.5848	0.7071	0.8017	0.8752

$$\int_0^{\pi/3}\sqrt{\sin x}=\frac{h}{3}\sqrt{(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)}$$

$$=\frac{\pi}{54}\left[(0+0.9306)+4(0.4167+0.7071+0.8752)+2(0.5848+\ldots\right.$$

$=0.6806$

Practical = 7

Topic = Differential Equation

Q.1. Solve the following differential equation

i) $x\frac{dy}{dx}+y=e^x$

$$\frac{dy}{dx}+\frac{1}{x}y=\frac{e^x}{x}$$

Comparing with $\frac{dy}{dx}+P(x)y=Q(x)$

$P(x)=\frac{1}{x}$; $Q(x)=\frac{e^x}{x}$

$I.F=e^{\int \frac{1}{x}dx}$

$=e^{\log x}$

$I.F=x$

$y(I.F)=\int Q(x)(I.F)\,dx$

$=\int \frac{e^x}{x}\cdot x\,dx$

$yx=e^x\,dx$

$xy=e^x+c$

ii) $e^x \frac{dy}{dx} + 2e^x y = 1$

$e^x\left(\frac{dy}{dx} + 2y\right) = 1$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = 2 \qquad Q(x) = \frac{1}{e^x}$

$I.F = e^{\int 2\,dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)(I.F)\,dx$

$y(e^{2x}) = \int \frac{1}{e^x} \cdot e^{2x}\,dx$

$= \int e^{-x} \cdot e^{2x}\,dx$

$= \int e^x\,dx$

$y e^{2x} = e^x + c$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} + 2y = \frac{\cos x}{x}$

$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = \frac{\cos x}{x^2}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = \frac{2}{x}, \quad Q(x) = \frac{\cos x}{x^2}$

$I.F = e^{\int \frac{2}{x}\,dx}$

$= e^{2\log x}$

$= e^{\log x^2}$

$I.F = x^2$

$y(I.F) = \int Q(x)(I.F)\,dx$

$= \int \frac{\cos x}{x^2}(x^2)\,dx$

$x^2 y = \sin x + c$

iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{\sin x}{x^3}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = 3x^{-1} \qquad Q(x) = \frac{\sin x}{x^3}$

$I.F = e^{\int \frac{3}{x}\,dx}$

$= e^{3\log x}$

$I.F = x^3$

$y(I.F) = \int Q(x)(I.F)\,dx$

$= \int \frac{\sin x}{x^3} \cdot x^3\,dx$

$x^3 y = -\cos x + c$

8] 28.

$$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

$$\frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$$

put $2x+3y=v$

$$2+3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{v-1}{3(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2}+2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$\frac{dv}{dx} = \frac{3v+3}{v+2}$$

$$\frac{(v+2)dv}{3(v+1)} = dx$$

$$\frac{1}{3}\int\frac{v+1+1}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\int\left(\frac{1}{v+1}+1\right)dv = \int dx$$

$$\frac{1}{3}\left(v+\log(v+1)\right) = x+c$$

put $v=2x+3y$

$\therefore\ 2x+3y+\log|2x+3y+1| = 3x+c$

$3y = x-\log|2x+3y+1|+c$

10/01/2020

Practical No: 8

Topic : Euler's method

1) $\frac{dy}{dx} = y + e^x - 2$

$\Rightarrow f(x,y) = y + e^x - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.87	3.57435
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8421	9.28305
4	2	9.2831		

$\therefore$ By Euler's formula

$y(2) = 9.2831$

2) $\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$ $\therefore$ $y_0 = 0$, $x_0 = 0$. $h = 0.2$

Using Euler's Iteration formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

By Euler's formula

$y(1) = 1.2942$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$, $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

4) $\frac{dy}{dx} = 3x^2+1$ $y_0 = 2$, $x_0 = 1$, $h > 0$

=> for $h = 0.5$

Using Euler's iteration formula
$y_{n+1} = y_n + h f(x_n - y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

By Euler's formula

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		

By Euler's formula

$y(2) = 8.9048$

19/01/2020

Practical no : 10

~~Q.1~~ Aim : Directional derivative gradien vector & mazima , minima Tangent & normal vectors.

Q.1. Find the directional derivative of the following function of given points & in the direction of given vector.

i) $f(x,y) = x + 2y - 3 \quad a = (1,-1), \quad u = 3i - j$

Sol^n : Here

$u = 3i - j$ is not a unit vector

$\bar{u} = 3i - j$

$|u| = \sqrt{10}$

$\therefore$ Unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3i - j)$

$= \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

Now

$f(a+hu) = f\left[\cancel{(1,-1)} + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right]$

$= 1 + 2(-1) - 3 + \left(\frac{3h}{\sqrt{10}}, \frac{-h}{\sqrt{10}}\right) + \left(1 + \frac{3h}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right)$

$$= -4 + \frac{3h}{\sqrt{10}} - 4 \quad 1 + \frac{3h}{\sqrt{10}} + 2\left(-1 - \frac{h}{\sqrt{10}}\right)$$

$$= 1 - 2 - 3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$$

$$= -4 + \frac{h}{\sqrt{10}}$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-4 + \frac{h}{\sqrt{10}} - (-4)}{h}$$

$$= \lim_{h \to 0} \frac{h}{\sqrt{10}} \cdot \frac{1}{h}$$

$$= \frac{1}{\sqrt{10}}$$

ii) $f(x,y) = y^2 - 4x + 1$ $\quad a = (3,4)$ $\quad u = i + 5j$

Here

$u = i + 5j$ is not a unit vector

$\bar{u} = i + 5j$

$|\bar{u}| = \sqrt{26}$

Unit vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{26}}(i + 5j)$.

$$= \frac{1}{\sqrt{26}}(1, 5)$$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

Now

$$f(a+hu) = f\left[(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right]$$

$$= f\left[3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right]$$

$$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \lim_{h \to 0} \frac{25h^2}{26} + \frac{36h}{\sqrt{26}}$$

$$= \lim_{h \to 0} \frac{\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right) \cancel{h}}{\cancel{h}}$$

$$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$$

$$= \frac{36}{\sqrt{26}}$$

iii) $f(x,y) = 2x + 3y$ $\quad a = (1,2) \quad u = 3i + 4j$

Here

$u = 3i + 4j$ is not a unit vector

$\bar{u} = 3i + 4j$

$|\bar{u}| = \sqrt{25} = 5$

Unit vector along $u = \frac{\bar{u}}{|\bar{u}|} = \frac{1}{5}(3i + 4j)$

$$= \frac{1}{5}(3, 4)$$

$$= \frac{1}{5}$$

Now

$$f(a+hu) = f\left((1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)\right)$$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4h}{5}\right)$$

$$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$$

$$= 8 + \frac{18h}{5}$$

$$\therefore D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{8 + \frac{18h}{5} - 8}{h}$$

$$= \frac{18}{5}$$

Q.2 Find gradient vector for the following function at given point.

i) $f(x,y) = x^y + y^x \qquad a = (1,1)$

$f_x = y(x^{y-1}) + y^x \log y$

$f_y = x(y^{x-1}) + x^y \log x$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y \,,\; xy^{x-1} + x^y \log x)$

$\nabla f(x,y)$ at $(1,1)$

$= (1 \times 1^{1-1} + 1^1 \log 1 \,,\; 1(1)^{1-1} + 1^1 \log 1)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1} x) \cdot y^2 \qquad a = (1,-1)$

$f_x = \cancel{y^2\left(\frac{1}{1+x^2}\right)} = \frac{y^2}{1+x^2}$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2} \,,\; 2y \tan^{-1} x\right)$

$\nabla f(x,y)$ at $(1,-1)$

$= \left(\frac{1^2}{1+1^2} \,,\; 2 \times (-1) \tan^{-1} 1\right)$

$= \left(\frac{1}{2} \,,\; -\frac{2\pi}{4}\right)$

$= \left(\frac{1}{2} \,,\; -\frac{\pi}{2}\right)$

iii) $f(x,y,z) = xyz - e^{x+y+z} \qquad a = (1,1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (\cancel{yz - e^{x+y+z}} \,,\; xz - e^{x+y+z} \,,\; xy - e^{x+y+z})$

$\nabla f(x,y,z)$ at $(1,-1,0)$

$= (-1(0) - e^{1-1+0} \,,\; 1(0) - e^{1-1+0} \,,\; 1(-1) - e^{1-1+0})$

$= (-1,-1,-2)$

Q.3 Find the equation of tangent & normal to each of the following curves at given points.

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f(x,y) = x^2 \cos y + e^{xy} - 2$

$f_x = 2x \cos y + y e^{xy}$

$f_y = -x^2 \sin y + x e^{xy}$

$(x_0, y_0) = (1,0)$

f_x at $(1,0) = 2(1)\cos 0 + 0$
$= 2$

f_y at $(1,0) = -(1)^2 \sin 0 + 1 e^{1(0)}$
$= 1$

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

Equation of tangent.

for equation of normal.

$bx + ay + d = 0$

$x + 2y + d = 0$

$(1) + 2(0) + d = 0$ at $(1,0)$

$1 + d = 0$

$d = -1$

$x + 2y - 1 = 0$ Equation of normal

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

f_x at $(2,-2) = 2(2) - 2 = 2$

f_y at $(2,-2) = 2(-2) + 3 = -1$

Equation of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + -1(y+2) = 0$

$2x-4-y-2=0$

$2x-y-6=0$

for equation of normal

$bx + ay + d = 0$

$-x + 2y + d = 0$

$-2 + 2(-2) + d = 0$

$d = 6$

$-x + 2y + 6 = 0$

equation of normal.

Q.4. Find the equation of tangent & normal line to each of the following surface.

1) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f(x,y,z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x - 0 + 0 + z - 0$

$= 2x + z$

$f_y = -2z + 3 + 0 - 0$

$= -2z + 3$

$f_z = 0 - 2y + 0 + x - 0$

$= -2y + x$

f_x at $(2,1,0) = 2(2)+0$

$= 4$

f_y at $(2,1,0) = -2(0)+3$

$= 3$

f_z at $(2,1,0) = -2(1)+2$

$= 0$

Equation of tangent

$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$

$4(x-2) + 3(y-1) + 0(z-0) = 0$

$4x + 3y - 11 = 0$

Equation of normal

$\frac{x-x_0}{f_x} \neq \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$

$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z-0}{0}$

2) $3xyz = x - y + z = -4$

$f(x,y,z) = 3xyz - x - y + z + 4$

$f_x = 3yz - 1 - 0 + 0 + 0 = 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 - 0$

$= 3xy + 1$

f_x at $(1,-1,2) = 3(-1)(2) - 1$

$= -7$

f_y at $(1,-1,2) = 3(1)(2) - 1$

$= 5$

f_z at $(1,-1,2) = 3(1)(-1) + 1$

$= -2$

Equation of tangent

$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$

$-7(x-1) + 5(y+1) + (-2)(z-2) = 0$

$-7x + 5y - 2z + 16 = 0$

Equation of Normal

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q.5 Find the local maxima & minima for the following function.

1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$ — (1)

$f_y = 2y - 3x + 0 - 4$

$= 2y - 3x - 4$ — (2)

$f_x = 0$

$6x - 3y + 6 = 0$

$2x - y + 2 = 0$

$2x - y = -2$ — (3)

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ — (4)

Multiplying (3) by 2 and subtracting with 4

$2y - 3x = 4$

$-\ \ -2y + 4x = -4$

$x = 0$

put value of x in equation (3)

$0 - y = -2$

$y = 2$

Critical points are $(0,2)$

Now

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12 - 9$

$= 3 > 0$

Here $r > 0$ and $rt - s^2 > 0$

$\therefore f$ has minimum at $(0,2)$

$f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy + 0$

$= 8x^3 + 6xy$

$f_y = 0 + 3x^2 - 2y$

$= 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 6y) = 0$

$4x^2 + 6y = 0 \quad -(1)$

$f_y = 0$

$3x^2 - 2y = 0 \quad -(2)$

multiplying equation (2) with (3) and adding with (1)

$9x^2 - 6y = 0$

$4x^2 + 6y = 0$

$13x^2 = 0$

$x = 0$

put x in equation (1)

$0 + 6y = 0$

$y = 0$

Critical point are $(0,0)$

\# $r = f_{xx} = 24x^2 + 8y$

$t = f_{yy} = -2$

$s = f_{xy} = 6x$

$$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$$
$$= -48x^2 - 12y - 36x^2$$
$$= -84x^2 - 12y$$

At $(0,0)$

$$r = 24(0)^2 + 6(0)$$
$$= 0$$

$$s = 6(0)$$
$$= 0$$

$$rt - s^2 = -84(0)^2 - 12(0)$$
$$= 0$$

$r = 0$ and $rt - s^2 = 0$

Nothing can be said.

3) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$	$f_y = 0$
$2x + 2 = 0$	$-2y + 8 = 0$
$x = \frac{-2}{2}$	$2y = 8$
$x = -1$	$y = 4$

Critical points are $(-1, 4)$

Now

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$$rt - s^2 = 2 \times -2 - 0$$
$$= -4$$

Here $r > 0$ and $rt - s^2 < 0$

$\therefore$ Nothing can be said.

07/02/2020