2008 IEEE International Conference on
Robotics and Automation
Pasadena, CA, USA, May 19-23, 2008

A Study of the Leg-swing Motion of Passive Walking

Yoshito Ikemata, Kiyoshi Yasuhara, Akihito Sano, and Hideo Fujimoto

***Abstract*— In our earlier work, we showed that high stability of passive walking can be achieved by the global stabilization principle of fixed point. The principle has been established, providing that the state just after heel-strike exists at the next step. However, this condition may not always hold. The passive walker with knees can execute the leg-swing motion with no control, only by gravity effect. Unfortunately, while the walker takes a step forward, the swing leg may strike its toe on the slope at unsuitable point. Therefore, understanding of the mechanism of leg-swing motion is very important for assuring the next step. In this paper, we focus on the leg-swing motion of passive walking, and demonstrate the mechanism of the flexion and extension of knee joint of swing leg.**

I. Introduction

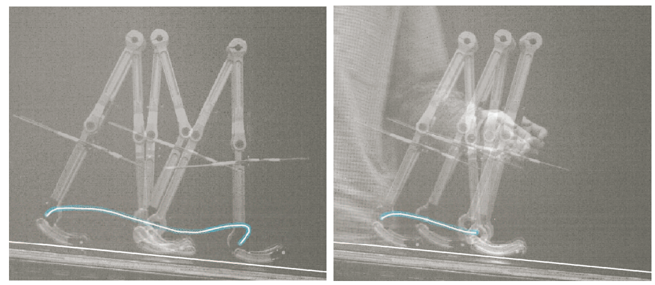
The bipedal walking is a central aspect of human behavior in human motion. Human motion is controlled by the nervous system and powered by muscle. While, passive walker can walk down without actuators, sensors, and controls[1][2]. This motion is very attractive because its gait is natural. Passive walking may give us an insight into understanding human locomotion and developing biped robots.

Though the mechanical system of passive walker is simple, it is a sort of hybrid system that combines the continuous dynamics of leg-swing motion and the discrete event of leg-exchange. Passive walking can exhibit a stable limit cycle. When the state keeps on the stable limit cycle, the walking system is stable. The generation and stability of limit cycle can be analyzed from fixed point (cross-section point of limit cycle).

McGeer [1][2] first demonstrated the local stability of fixed point from Jacobian matrix obtained by linearizing the discrete-time state equation (called “step to step equation”). Since then, Goswami et al. [3], Coleman et al. [4], and Garcia et al. [5] studied it in detail. They have demonstrated that the fixed point of passive walking is stable. However, they have not demonstrated the stability mechanism of fixed point in passive walking. In addition, they have not considered the physical mechanism behind the fixed point. They merely searched the fixed point by numerical methods.

In our earlier work, we have demonstrated the generation mechanism of a fixed point of passive walking, and have proposed a generation method of a fixed point based on its mechanism [6]. Moreover, we have derived the local stabilization control method from a stability mechanism of fixed point [6]. Though our local stabilization control method is very simple, it can realize the highest local stability of discrete-time system.

Y. Ikemata, K. Yasuhara, A. Sano, and H. Fujimoto are with Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya, Aichi, 466-8555, JAPAN ikemata@vier.mech.nitech.ac.jp



(a) Success (b) Failure

Fig. 1. Leg-swing motion of passive walking

In addition, we have derived a global stabilization principle from a stability mechanism of fixed point [7]. Based on the principle, high stability of passive walking can be achieved only by a simple improvement. A great result is that our passive walker could walk for 4,010 steps on a treadmill [7].

The global stabilization principle has been established, providing that the state just after heel-strike exists at the next step. However, this condition may not always hold. As shown in Fig. 1 (a), the passive walker with knees can execute the leg-swing motion with no control, only by gravity effect. Unfortunately, while the walker takes a step forward, the swing leg may strike its toe on the slope at unsuitable point as shown in Fig. 1 (b). Therefore, understanding of the mechanism of leg-swing motion is very important for assuring the next step.

Many researchers have studied the passive walking [1]-[5][8]. Moreover, many researchers have proposed robots based on passive walking [9]-[15]. However, they have not considered the mechanism of leg-swing motion of passive walking. Therefore, we focus on the flexion and extension of knee joint of swing leg. In this paper, first, an equation of angular acceleration of knee joint is derived from the simplified and linearized model of passive walking. Secondly, we demonstrate the mechanism of the flexion and extension of knee joint of swing leg. Finally, we demonstrate the influence of leg and foot on its mechanism.

II. Model of passive walking

A. Leg–swing motion

Figure 2 shows the model of kneed passive walker with feet. The model consists of stance and swing legs. The knee of the stance leg is locked straight. The motion is assumed to be constrained to the saggital plane. For the purpose of simplicity and clarity of analysis, as possible, we give a simplification of the model as follows:

$$M \gg m, \quad M \gg m_1, \quad M \gg m_2 \tag{1}$$

In addition, we assume that inertia moments of thigh and shank are very small.

1) Motion equation of 3 links (with Knees): Stance leg is assumed to be fixed on the ground with no slippage or take off. The equation of 3 links can be written as

$$\boldsymbol{M}_K(\boldsymbol{\theta}_K)\ddot{\boldsymbol{\theta}}_{\boldsymbol{K}} + \boldsymbol{H}_K(\boldsymbol{\theta}_K, \dot{\boldsymbol{\theta}}_K) + \boldsymbol{G}_K(\boldsymbol{\theta}_K, \gamma) = \boldsymbol{0} \tag{2}$$

where

$$\boldsymbol{M}_K(\boldsymbol{\theta}_K) = \begin{bmatrix} \rho^2 + d^2 + 2\rho d\cos(\theta - \zeta) & 0 & 0 \\ -(b_1 + l_1/k_m)\{\rho\cos\phi_1 + d\cos(\theta - \phi_1 - \zeta)\} & b_1^2 + l_1^2/k_m & b_2 l_1/k_m \cos(\phi_1 - \phi_2) \\ -b_2\{\rho\cos\phi_2 + d\cos(\theta - \phi_2 - \zeta)\} & b_2 l_1\cos(\phi_1 - \phi_2) & b_2^2 \end{bmatrix}$$

$$\boldsymbol{H}_K(\boldsymbol{\theta}_K, \dot{\boldsymbol{\theta}}_{\boldsymbol{K}}) = \begin{bmatrix} -\rho d\sin(\theta - \zeta)\dot{\theta}^2 \\ (b_1 + l_1/k_m)d\sin(\theta - \phi_1 - \zeta)\dot{\theta}_1^2 + b_2 l_1/k_m \sin(\phi_1 - \phi_2)\dot{\phi}_2^2 \\ b_2 d\sin(\theta - \phi_2 - \zeta)\dot{\theta}^2 - b_2 l_1\sin(\phi_1 - \phi_2)\dot{\phi}_1^2 \end{bmatrix}$$

$$\boldsymbol{G}_K(\boldsymbol{\theta}_K, \gamma) = \begin{bmatrix} -\rho\sin\gamma - d\sin(\theta + \gamma - \zeta) \\ (b_1 + l_1/k_m)\sin(\phi_1 + \gamma) \\ b_2\sin(\phi_2 + \gamma) \end{bmatrix} g$$

$\boldsymbol{\theta}_K (= [\theta, \phi_1, \phi_2]^T)$ is the vector of joint angles. g is the acceleration of gravity. k_m is $k_m = m_1/m_2$. Setting $m = m_1 + m_2$, $a = \{m_1(l_2 + a_1) + m_2 a_2\}/(m_1 + m_2)$, $I = m_2(l_2 - b_2 - a)^2 + m_1(l_2 + a_1 - a)^2$, stance leg is equal to swing leg. d and ζ are given as follows:

$$d = \sqrt{(\rho\sin\delta)^2 + (l - \rho\cos\delta)^2} \tag{3}$$

$$\zeta = \tan^{-1}\left(\frac{\rho\sin\delta}{l - \rho\cos\delta}\right) \tag{4}$$

2) Equation of knee-lock: Knee-lock occurs when the swing leg becomes straight ($\phi_1 = \phi_2 = \phi$). Assuming that the swing knee locks instantaneously, angular momentum is conserved through the knee-lock for the whole walker about the stance foot contact point, and the swing leg about the hip. Angular velocities of stance and swing legs just after knee-lock are obtained from these conservations of angular momentum as

$$\dot{\theta}^+ = \dot{\theta}^- \tag{5}$$

$$\dot{\phi}^+ = \frac{\{b_1^2 + (l_1^2 + b_2 l_1)/k_m\}\dot{\phi}_1^- + (b_2^2 + b_2 l_1)\dot{\phi}_2^-/k_m}{b_1^2 + (l_1 + b_2)^2/k_m} \tag{6}$$

The "+" superscript means "just after knee-lock," and the "–" superscript means "just before knee-lock".

3) Motion equation of 2 links (Compass-type): After knee-lock, the model can be regarded as compass-like biped model. The equation of 2 links can be written as

$$\boldsymbol{M}_C(\boldsymbol{\theta}_C)\ddot{\boldsymbol{\theta}}_C + \boldsymbol{H}_C(\boldsymbol{\theta}_C, \dot{\boldsymbol{\theta}}_C) + \boldsymbol{G}_C(\boldsymbol{\theta}_C, \gamma) = \boldsymbol{0} \tag{7}$$

where

$$\boldsymbol{M}_C(\boldsymbol{\theta}_C) = \begin{bmatrix} \rho^2 + d^2 + 2\rho d\cos(\theta - \zeta) & 0 \\ -b\{\rho\cos\phi + d\cos(\theta - \phi - \zeta)\} & b^2 + I/m \end{bmatrix}$$

$$\boldsymbol{H}_C(\boldsymbol{\theta}_C, \dot{\boldsymbol{\theta}}_C) = \begin{bmatrix} -\rho d\sin(\theta - \zeta)\dot{\theta}^2 \\ bd\sin(\theta - \phi - \zeta)\dot{\theta}^2 \end{bmatrix}$$

$$\boldsymbol{G}_C(\boldsymbol{\theta}_C, \gamma) = \begin{bmatrix} -\rho\sin\gamma - d\sin(\theta - \zeta - \gamma) \\ b\sin(\phi + \gamma) \end{bmatrix} g$$

$\boldsymbol{\theta}_C$ (=$[\theta, \phi]^T$) is the vector of joint angles after knee-lock.

4) Equation of inter-leg angle lock: We stabilize the fixed point by maintaining constant inter-leg angle at heel-strike [7]. So, the inter-leg angle is locked when it becomes the set inter-leg angle α. Angular velocity of stance leg just after inter-leg angle lock can be obtained as

$$\dot{\theta}^+ = \dot{\theta}^- \tag{8}$$

The "+" superscript means "just after inter-leg angle lock," and the "–" superscript means "just before inter-leg angle lock". After inter-leg angle lock, the model can be regarded as an inverted pendulum. The equation of 1 link can be written as

$$\left\{\rho^2 + d^2 + 2\rho d\cos(\theta - \zeta)\right\}\ddot{\theta} - \rho d\sin(\theta - \zeta)\dot{\theta}^2 - \{\rho\sin\gamma + d\sin(\theta + \gamma - \zeta)\}g = 0 \tag{9}$$

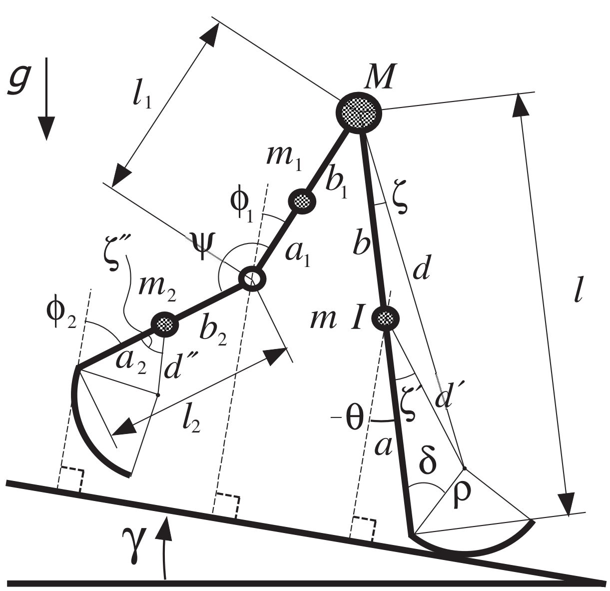
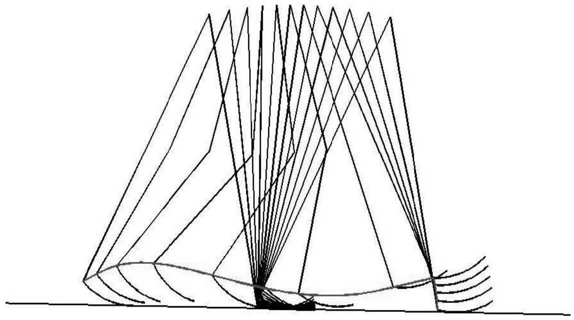
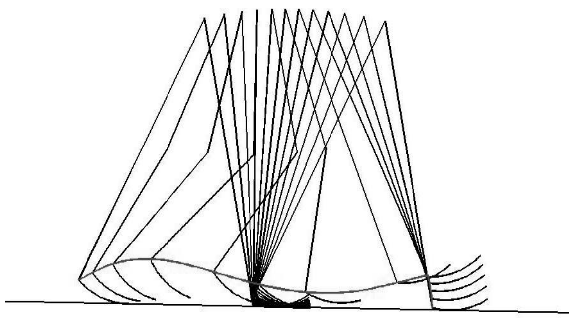


Fig. 2. Model of kneed passive walker with feet



(a) Nonlinear model



(b) Linear model

Fig. 3. Stick diagrams and ankle trajectories of swing leg

B. Equation of leg–exchange

For an inelastic no-sliding collision with the ground, angular momentum is conserved through the collision [16]. Relational expression can be obtained from these conservations of angular momentum as

$$\boldsymbol{Q}^{+}\dot{\boldsymbol{\theta}}_K^{+} = \boldsymbol{Q}^{-}\dot{\boldsymbol{\theta}}_C^{-} \tag{10}$$

where

$$\boldsymbol{Q}^{+} = \begin{bmatrix} \rho^2 + d^2 + 2\rho d\cos\alpha/2 & 0 & 0 \\ \{b_1 + (l_1 + b_2)/k_m\}\{\rho\cos(\alpha/2+\zeta) + d\cos(\alpha+\zeta)\} & -b_1^2 - l_1(l_1+b_2)/k_m & -b_2(l_1+b_2)/k_m \\ b_2\{\rho\cos(\alpha/2+\zeta) + d\cos(\alpha+\zeta)\} & -b_2 l_1 & -b_2^2 \end{bmatrix}$$

$$\boldsymbol{Q}^{-} = \begin{bmatrix} \rho^2 + d^2\cos\alpha + 2\rho d\cos\alpha/2 & 0 \\ -(1+1/k_m)b\,\{\rho\cos(\alpha/2+\zeta) + d'\cos\zeta'\} + I/m_1 & 0 \\ b_2\,\{\rho\cos(\alpha/2+\zeta) + d''\cos\zeta''\} & 0 \end{bmatrix}$$

The "+" superscript means "just after heel-strike," and the "–" superscript means "just before heel-strike". d' and ζ' are given as follows:

$$d' = \sqrt{(\rho\sin\delta)^2 + (a - \rho\cos\delta)^2} \tag{11}$$

$$\zeta' = \tan^{-1}\left(\frac{\rho\sin\delta}{a - \rho\cos\delta}\right) \tag{12}$$

d'' and ζ'' are given as follows:

$$d'' = \sqrt{(\rho\sin\delta)^2 + (a_2 - \rho\cos\delta)^2} \tag{13}$$

$$\zeta'' = \tan^{-1}\left(\frac{\rho\sin\delta}{a_2 - \rho\cos\delta}\right) \tag{14}$$

From Eq. (10), the vector of angular velocity after heel-strike can be given as

$$\dot{\boldsymbol{\theta}}_K^{+} = (\boldsymbol{Q}^{+})^{-1}\boldsymbol{Q}^{-}\dot{\boldsymbol{\theta}}_C^{-} \tag{15}$$

III. Knee Joint of swing leg

A. Linear model

The swing leg can swing through the stance leg without striking its toe on the slope, and then the knee joint of swing leg becomes straight. These motions are chiefly generated by the flexion and extension of knee joint of swing leg. Therefore, we focus on the knee joint motion of swing leg.

Linearized equation of leg-swing motion (1) can be obtained as follows:

$$\begin{bmatrix} (\rho+d)^2 & 0 & 0 \\ -(b_1 + l_1/k_m)(\rho+d) & b_1^2 + l_1^2/k_m & b_2 l_1/k_m \\ -b_2(\rho+d) & b_2 l_1 & b_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi}_1 \\ \ddot{\phi}_2 \end{bmatrix} + \begin{bmatrix} -\rho\gamma - d(\theta+\gamma-\zeta) \\ (b_1 + l_1/k_m)(\phi_1+\gamma) \\ b_2(\phi_2+\gamma) \end{bmatrix} g = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

Figures 3 (a) and (b) show the stick diagrams of nonlinear and linear models in fixed point respectively. The stick diagram of linear model has the same features as the one of nonlinear model. In addition, the ankle trajectory of swing leg has the same features as the one from the experiment result as shown in Fig. 1. Therefore, the simplified and linearized model is sufficient to analyze the knee joint motion of swing leg.

B. Angular acceleration of knee joint

From Eq. (16), we can obtain the angular acceleration of knee joint of swing leg as follows:

$$\begin{aligned} \ddot{\psi} &= \ddot{\phi}_1 - \ddot{\phi}_2 \\ &= p_1(\phi_1+\gamma)g + p_2(\phi_2+\gamma)g + p_3(\theta+\gamma)g + p_4 g \end{aligned} \tag{17}$$

where

$$p_1 = -\frac{b_1(l_1+b_2) + l_1(l_1+b_2)/k_m}{b_1^2 b_2}$$

$$p_2 = \frac{b_1^2 + l_1(l_1+b_2)/k_m}{b_1^2 b_2}$$

$$p_3 = \frac{d}{\rho+d}\frac{l_1 - b_1 + b_2}{b_1 b_2}$$

$$p_4 = -\frac{1}{\rho+d}\frac{l_1 - b_1 + b_2}{b_1 b_2}(d\zeta - \rho\gamma)$$

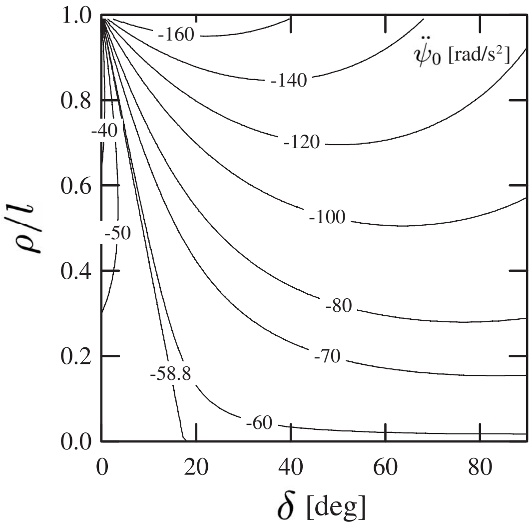


Fig. 4. Initial angular acceleration of knee joint of swing leg

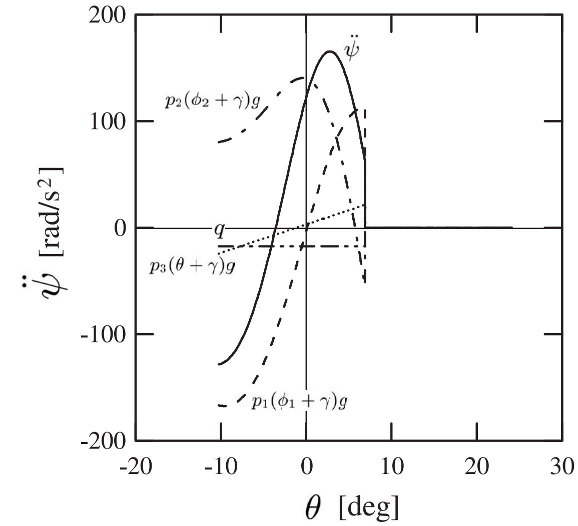


Fig. 5. Angular acceleration of knee joint of swing leg

In case of $\ddot{\psi} < 0$, the flexion moment acts on the knee joint of swing leg. In case of $\ddot{\psi} > 0$, the extension moment acts on the knee joint. The first term of the right-hand side in Eq. (17) depends on the swing leg's thigh posture (angle). The second term depends on the swing leg' shank posture. These equations are equivalent to the one of double pendulum.

The third term of the right-hand side in Eq. (17) depends on the stance leg posture (angle). By the simplification of Eq. (1), the motion of stance leg is equivalent to that of inverted pendulum. The fourth term depends on the circular arc foot. This term is a constant number if the model parameters are fixed.

IV. Mechanism of the flexion and extension of knee joint of swing leg

A. Initial phase

Initial state is set to the state just after heel-strike. We assume that the initial state can be given as follows:

$$\theta^+ = -\alpha/2 + \zeta, \ \ \phi_1^+ = \phi_2^+ = \alpha/2 + \zeta, \ \ \dot{\phi}_1^+ \leq \dot{\phi}_2^+ \tag{18}$$

From Eq. (18), initial angle and angular velocity of knee joint of swing leg can be obtained as follows:

$$\psi_0 = \pi + \phi_1^+ - \phi_2^+ = \pi \tag{19}$$

$$\dot{\psi}_0 = \dot{\phi}_1^+ - \dot{\phi}_2^+ \leq 0 \tag{20}$$

Initial angular acceleration of knee joint can be derived from Eqs. (17) and (18) as follows:

$$\begin{aligned}\ddot{\psi}_0 = & -\frac{(l_1-b_1)+b_2}{b_1 b_2(\rho+d)}\left\{\frac{\alpha}{2}(\rho+2d)+\rho(\zeta+\gamma)\right\}g \\ & -\frac{1}{\rho+d}\frac{(l_1-b_1)+b_2}{b_1 b_2}(d\zeta-\rho\gamma)g \end{aligned} \tag{21}$$

If $d\zeta - \rho\gamma > 0$, $\ddot{\psi}_0 < 0$ holds. In the case when the foot shape is point (δ=0, ρ=0, d=l, ζ=0), $\ddot{\psi}_0 < 0$ holds always. If $\ddot{\psi}_0 < 0$ and Eq. (20) holds, the knee of swing leg always inflects at initial phase.

Figure 4 shows the initial angular acceleration of knee joint as the parameters of circular arc foot are varied. The horizontal axis denotes the center angle of circular arc of foot δ. The vertical axis denotes the curvature radius of foot ρ/l (dimensionless). d, ζ, d', and ζ' are obtained from Eqs. (3), (4), (11), and (12) respectively. d'' and ζ'' are obtained from Eqs. (13) and (14) respectively. Slope angle and inter-leg angle are set to γ=0.02 and α=0.6[rad] respectively. In the case when the foot shape is point, $\ddot{\psi}_0$ can be obtained as $\ddot{\psi}_0$ =−58.8[rad/s^2]. If δ is bigger than 18[deg], the flexion of knee joint at initial phase is bigger than the one of point foot.

B. Leg-swing phase

One swing cycle is defined as the period from the state just after heel-strike to the state just before the next heel-strike. Figure 5 shows the angular acceleration of knee joint of swing leg $\ddot{\psi}$ in one swing cycle of fixed point. The horizontal axis denotes the angle of stance leg θ. The vertical axis denotes $\ddot{\psi}$. In addition, all terms of Eq. (17) are written in Fig. 5. Continuous line denotes $\ddot{\psi}$. Short dashed line and chain line denote the contributions of thigh and shank of swing leg respectively. Dot-line denotes the contribution of stance leg. Chain double-dashed line denotes the contribution of the circular arc foot.

As passive walker steps forward (θ changes from minus to plus), mechanical actions of stance leg and the thigh of swing leg change from flexion to extension. These are desirable mechanical action for the leg-swing motion. Mechanical action of shank of swing leg is almost extension. While, mechanical action of the circular arc foot is always flexion. All these mechanical actions generate the knee motion of swing leg in passive walking.

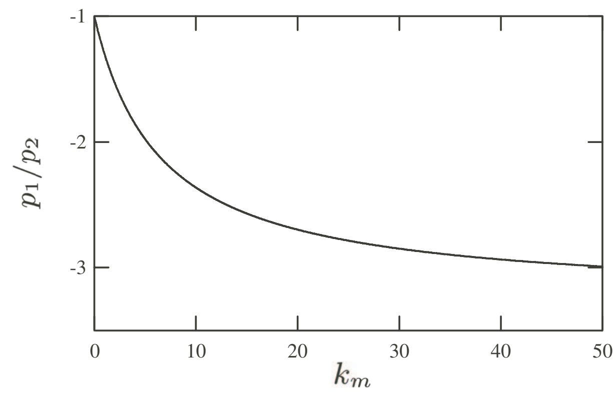


Fig. 6. Effect of mass ratio k_m on the coefficient ratio p_1/p_2

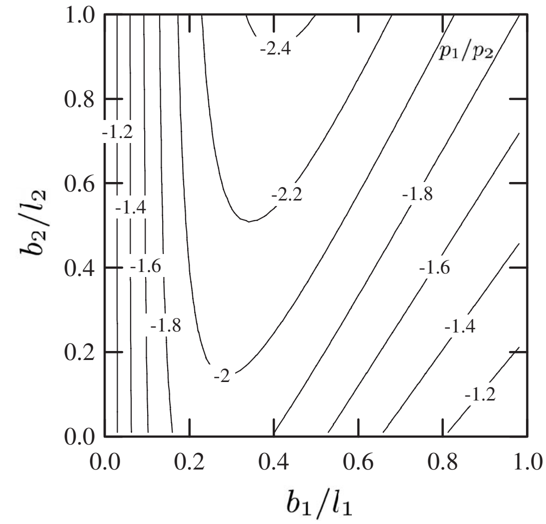


Fig. 7. Effect of center of gravity on the coefficient ratio p_1/p_2

V. Effects of leg

A. Mass ratio

The mass ratio of thigh and shank of swing leg $k_m(= m_1/m_2)$ influences p_1 and p_2 in Eq. (17). Figure 6 shows the relationship between k_m and the coefficient ratio p_1/p_2. Regardless of the model parameters, if k_m approaches 0, p_1/p_2 converges as follows:

$$\lim_{k_m \to 0} \frac{p_1}{p_2} = -1 \tag{22}$$

In addition, from the motion of equation (16), we can obtain the following equation.

$$\phi_1 = \phi_2 \tag{23}$$

Equation (23) denotes that the knee of swing leg is always straight. This means that the model always becomes compass-like passive walking.

Conversely, if k_m approaches infinity, p_1/p_2 converges as follows:

$$\lim_{k_m \to \infty} \frac{p_1}{p_2} = -\frac{l_1 + b_2}{b_1} < -1 \tag{24}$$

Extremal value of p_1/p_2 depends only on the length of thigh l_1, center of mass of thigh and shank b_1, b_2. In section IV-B, it is desirable that the mechanical action of thigh of swing leg is bigger than the one of shank. Therefore, it is desirable that the mass of thigh m_1 is bigger than the mass of shank m_2. It is more desirable that p_1/p_2 approaches the value of Eq. (24).

B. Center of mass

Effect of center of mass of thigh and shank is analyzed in the case of k_m fixed as $k_m = 7$. Figure 7 shows the relationship between the center of mass and the coefficient ratio p_1/p_2. The horizontal axis denotes the length ratio b_1/l_1. The vertical axis denotes the length ratio b_2/l_2. The contour line denotes p_1/p_2.

It is desirable that mechanical action of thigh of swing leg is bigger than the one of shank. Therefore, it is desirable that the center of mass of thigh and shank are set to the center position and the lower position respectively.

VI. Effects of foot

A. Mechanical action of stance leg

From Eq. (17), the terms determined by the posture of thigh and shank of swing leg (the first and second terms of the right-hand side) do not depend on the parameters of circular arc foot. While, the term determined by the posture of stance leg (the third term) depends on them. In the case when the foot shape is point, coefficient p_3 in Eq. (17) can be derived as follows:

$$p_{3p} = \frac{l_1 - b_1 + b_2}{b_1 b_2} \tag{25}$$

The "p" subscript means "point foot". The ratio of coefficient p_3 of point foot and circular arc foot can be obtained as follows:

$$\frac{p_3}{p_{3p}} = \frac{d}{\rho + d} \tag{26}$$

From Eq. (26), $0 < p_3/p_{3p} < 1$ holds. This indicates that the circular arc foot decreases the mechanical actions of stance leg.

B. Action of flexion

As mentioned in section IV-B, the circular arc foot exerts a flexion action on the knee joint of the swing leg. In this section, the relationship between the flexion actions and the parameters of circular arc foot is demonstrated. Figure 8 shows the coefficient q in Eq. (17) when the parameters of circular arc foot are varied. The horizontal axis denotes the center angle of circular arc of foot δ. The vertical axis denotes the curvature radius of foot ρ/l (dimensionless). The contour line denotes the coefficient q. If δ and ρ/l are increased, the flexion action of foot is enhanced. In addition, ρ/l has more influence than δ.

As shown in Fig. 3 (a), the swing leg swings through the stance leg while the knee of swing leg is inflecting. During

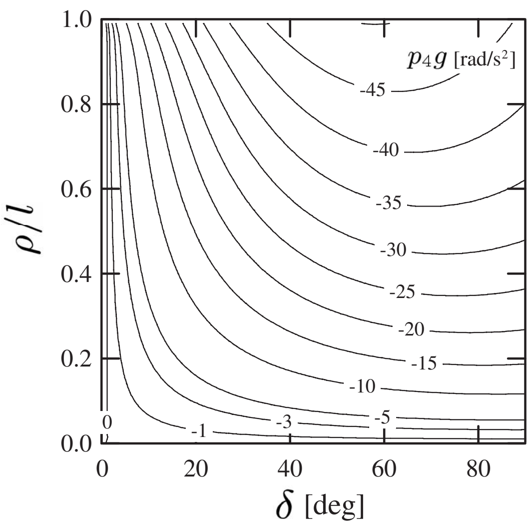


Fig. 8. Flexion effect by circular arc foot

this, passive walker has a high risk for striking its toe on the slope. In order to reduce the collision risk, the flexion action of knee of swing leg must be enhanced. Therefore, it is desired to take advantage of the flexion action of circular arc foot. At the same time, we must consider the disadvantage of the circular arc foot becoming an obstacle of swinging through the stance leg.

VII. Conclusion

In this paper, we derived the equation of angular acceleration of knee joint from the simplified and linearized model. From this equation, we demonstrated the mechanism of the flexion and extension of knee joint of swing leg, which is generated by the mechanical actions of thigh and shank of swing leg, stance leg, and circular arc foot.

From a viewpoint of the mechanism above, it is desired that the mass of thigh is bigger than the mass of shank. In addition, it is desirable that the center of mass of thigh and shank are set to the center position and the lower position respectively.

The circular arc foot exerts a flexion action on the knee joint of the swing leg. We demonstrated the relationship between the flexion action and the parameters of circular arc foot. We believe that the flexion action of circular arc foot is important for making an improvement on the leg-swing motion of passive walking.

In the future, we will develop a passive walking robot based on the mechanism of the flexion and extension of knee joint of swing leg. We aim at realizing a robust passive walking robot, which can walk stably on irregular terrain.

VIII. Acknowledgments

This work was partially supported by a Grant-in-Aid for Scientific Research, (B) No. 19360115, provided by the Japan Society for the Promotion of Science.

References

[1] T. McGeer, "Passive Dynamic Walking," *The Int. J. of Robotics Research*, vol. 9, no. 2, pp.62–68, 1990.

[2] T. McGeer, "Passive Walking with Knees," *Proc. of the 1990 IEEE Int. Conf. on Robotics and Automation*, pp.1640–1645, 1990.

[3] A. Goswami, B. Thuilot, and B. Espiau, "A Study of the Passive Gait of a Compass-Like Biped Robot : Symmetry and Chaos," *The Int. J. of Robotics Research*, pp.1282–1301, 1998.

[4] M. J. Coleman, "A Stability Study of Three Dimensional Passive Dynamic Model of Human Gait," Ph. D. Thesis, Cornell University, 1998.

[5] M. Garcia, A. Chatterjee, A. Ruina, and M. Coleman, "The Simplest Walking Model : Stability, Complexity, and Scaling," *ASME J. of Biomechanical Engineering*, pp.281–288, 1998.

[6] Y. Ikemata, A. Sano, and H. Fujimoto, "Generation and Local Stabilization of Fixed Point Based on a Stability Mechanism of Passive Walking," *Proc. of the 2007 IEEE Int. Conf. on Robotics and Automation*, pp.3218–3223, 2007.

[7] Y. Ikemata, A. Sano, and H. Fujimoto, "A Physical Principle of Gait Generation and its Stabilization derived from Mechanism of Fixed Point," *Proc. of the 2006 IEEE Int. Conf. on Robotics and Automation*, pp.836–841, 2006.

[8] K. Osuka and K. Kirihara: "Motion Analysis and Experiments of Passive Walking Robot QUARTET II," *Proc. of the 2000 IEEE Int. Conf. on Robotics and Automation*, pp.3052–3056, 2000.

[9] R. Q. van der Linde, "Passive Bipedal Walking with Phasic Muscle Contraction," *Biological Cybernetics*, vol.81, no.3, pp.227–237, 1999.

[10] Y. Sugimoto and K. Osuka, "Walking Control of Quasi Passive Dynamic Walking Robot "Quartet III" based on Continuous Delayed Feedback Control," *Proc. of the IEEE Int. Conf. on Robotics and Biomimetics*, 2004.

[11] F. Asano, M. Yamakita, N. Kamamichi, and Zhi–Wei Luo, "A Novel Gait Generation for Biped Walking Robots Based on Mechanical Energy Constraint," *IEEE Trans. Robotics and Automation*, vol.20, no.3, pp.565–573, 2004.

[12] T. Takuma, S. Nakajima, K. Hosoda, and M. Asada, "Design of Self-Contained Biped Walker with Pneumatic Actuators," *SICE Annual Conference 2004*, WPI-2-4, 2004.

[13] S. H. Collins, A. Ruina, R. Tedrake, and M. Wisse, "Efficient Bipedal Robots Based on Passive Dynamic Walkers," *Science*, vol.307, pp.1082–1085, 2005.

[14] M. Wisse, A. L. Schwab, R. Q. van der Linde, and F. C. T. vd. Helm, "How to Keep From Falling Forward: Swing Leg Action for Passive Dynamic Walkers," *IEEE Trans. on Robotics*, vol.21, no3, pp.393–401, 2005.

[15] M. W. Spong and F. Bullo, "Controlled Symmetries and Passive Walking," *IEEE Trans. on Automatic Control*, vol.50, no.7, pp.1025–1031, 2005.

[16] Y. Hurmuzlu and T. Chang, "Rigid Body Collisions of a Special Class of Planar Kinematic Chains," *IEEE Trans. on Systems, Man and Cybernetics*, vol.22, no.5, pp.964–971, 1992.