# Stabilization of Passive Walking Based on a Stability Mechanism of Fixed Point

*Yoshito Ikemata, Akihito Sano and Hideo Fujimoto*
Department of Mechanical Engineering, Nagoya Institute of Technology
Gokiso-cho, Showa-ku, Nagoya, Aichi, 466-8555 JAPAN

**Abstract:**
**Passive walker can walk down shallow slope without actuator and control. Though mechanical system of passive walker is simple, it is a sort of hybrid system that combines the continuous dynamics of leg-swing motion and the discrete event of leg-exchange. Passive walker can exhibit a stable limit cycle. In this paper, we demonstrate a global stabilization principle of fixed point from a simple passive walker with knees. And, the validity of stabilization principle is confirmed by walking experiment.**

## I. Introduction

Passive walker can walk down shallow slope with natural gait only by interaction between dynamics of passive walker and environment[1][2]. This motion is attractive because its gait is natural. Although mechanical system of passive walker is simple, passive walker exhibits a stable limit cycle and bifurcation phenomenon. Generation of limit cycle and its stability can be analyzed from fixed point.

Basic framework of local stability analysis of passive walking was formed by McGeer in 1990. Since then, Goswami et al. [3], Coleman et al. [4], and Garcia et al. [5] studied various passive walking in detail. These studies focus on the stability of fixed point of passive walking. However, the stabilization mechanism of fixed point is not demonstrated. In many studies, fixed points are searched by numerical method because it is thought that fixed point is formed by complex physical structure.

Although passive walker can generate an energy efficient gait [6]-[11], its stability is very low. Stabilizing control methods of passive walking are proposed [12][13]. These functions successfully only around fixed point because the control methods aim at high local stabilization around fixed point. To achieve high stability of fixed point, global stabilization principle is required.

In this paper, we focus on the stability mechanism of fixed point. It is thought that the essence of principle is common to any model. Therefore, we analyze the stability mechanism of fixed point from a simple passive walker with knees. At first, a global stabilization principle of fixed point is derived from Jacobian matrix of arbitrary state. Secondly, we demonstrate that the stabilization principle includes a generation principle of fixed point. Finally, the validity of the stabilization principle is confirmed by the walking experiment.

## II. Model of passive walker

### A. *Equation of leg-swing motion*

Figure 1 shows model of passive walker with knees. The model consists of stance and swing legs. Knee of the stance leg is locked straight. The motion is assumed to be constrained to saggital plane. For the purpose of simplicity and clarity of analysis as possible, assumptions are given as follows:

$$M \gg m, \quad M \gg m_1, \quad M \gg m_2 \tag{1}$$

Stance leg is assumed to be fixed on the ground as no slippage or take off. The equation of leg-swing motion can be written as

$$\boldsymbol{M}_K(\boldsymbol{\theta}_K)\ddot{\boldsymbol{\theta}}_{\boldsymbol{K}} + \boldsymbol{H}_K(\boldsymbol{\theta}_K, \dot{\boldsymbol{\theta}}_K) + \boldsymbol{G}_K(\boldsymbol{\theta}_K, \gamma) = \boldsymbol{0} \tag{2}$$

where

$$\boldsymbol{M}_K(\boldsymbol{\theta}_K) = \begin{bmatrix} l^2 & 0 & 0 \\ -(b_1 l + p l l_1)\cos(\theta - \phi_1) & b_1^2 + p l_1^2 & p b_2 l_1 \cos(\phi_1 - \phi_2) \\ -b_2 l \cos(\theta - \phi_2) & b_2 l_1 \cos(\phi_1 - \phi_2) & b_2^2 \end{bmatrix}$$

$$\boldsymbol{H}_K(\boldsymbol{\theta}_K, \dot{\boldsymbol{\theta}}_K) = \begin{bmatrix} 0 \\ (b_1 l + p l l_1)\sin(\theta - \phi_1)\dot{\theta}^2 + p b_2 l_1 \sin(\phi_1 - \phi_2)\dot{\phi}_2^2 \\ b_2 l \sin(\theta - \phi_2)\dot{\theta}^2 - b_2 l_1 \sin(\phi_1 - \phi_2)\dot{\phi}_1^2 \end{bmatrix}$$

$$\boldsymbol{G}_K(\boldsymbol{\theta}_K, \gamma) = \begin{bmatrix} -l\sin(\theta + \gamma) \\ (b_1 + p l_1)\sin(\phi_1 + \gamma) \\ b_2 \sin(\phi_2 + \gamma) \end{bmatrix} g$$

$\boldsymbol{\theta}_K (= [\theta, \phi_1, \phi_2]^T)$ is the vector of joint angles. $g$ is the acceleration of gravity. $p$ is $p = m_2/m_1$. Setting

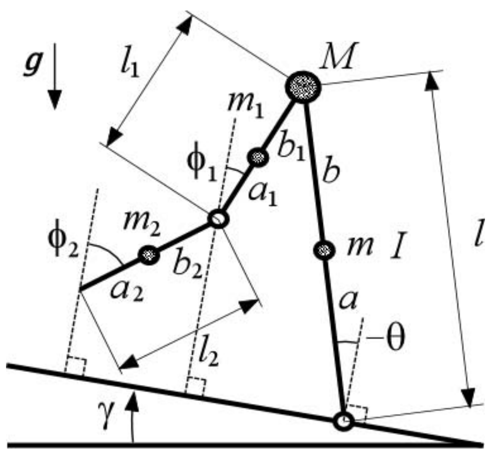



Fig. 1. Model of passive walker with knees

$a = \{m_1(l_2 + a_1) + m_2 a_2\}/(m_1 + m_2)$, $I = m_2(l_2 - b_2 - a)^2 + m_1(l_2 + a_1 - a)^2$, stance leg is equal to swing leg.

### B. Equation of leg-exchange

After knee-lock ($\phi_1 = \phi_2 = \phi$), the model can be regarded as compass-like biped model. It supplies a leg-exchange rule when the swing foot hits the ground. Collision occurs when the geometric condition

$$2\theta - \phi = 0 \tag{3}$$

is met. For an inelastic no-sliding collision with the ground, angular momentum is conserved through the collision for the whole walker about the swing foot contact point, and the former stance leg about the hip[14]. Relational expression is obtained from these conservations of angular momentum as

$$\boldsymbol{Q}^+(\alpha)\dot{\boldsymbol{\theta}}_C^+ = \boldsymbol{Q}^-(\alpha)\dot{\boldsymbol{\theta}}_C^- \tag{4}$$

where

$$\boldsymbol{Q}^+(\alpha) = \begin{bmatrix} l^2 & 0 \\ -bl\cos\alpha & b^2 + \dfrac{\bar{I}}{1+p} \end{bmatrix}$$

$$\boldsymbol{Q}^-(\alpha) = \begin{bmatrix} l^2\cos\alpha & 0 \\ -ab + \dfrac{\bar{I}}{1+p} & 0 \end{bmatrix}$$

The "+" superscript means "just after heel-strike," and the "−" superscript means "just before heel-strike". $\boldsymbol{\theta}_C (= [\theta, \phi]^T)$ is the vector of joint angles after knee-lock. $\alpha$ is inter-leg angle at heel-strike. $\bar{I}$ is $I/m_1$. We assume $0 < \alpha_k < \pi/2$ and $0 < \alpha_{k+1} < \pi/2$.

From Eq. (4), the vector of angular velocity after heel-strike can be given as

$$\dot{\boldsymbol{\theta}}_C^+ = (\boldsymbol{Q}^+(\alpha))^{-1}\boldsymbol{Q}^-(\alpha)\dot{\boldsymbol{\theta}}_C^- \tag{5}$$

## III. Global Stabilization Principle of Fixed Point

### A. Global stability of discrete-time system

In this section, we demonstrate that sufficient condition of global asymptotically stable of discrete-time system.

The discrete state is expressed as $\boldsymbol{x}_k$. Successive states is related as

$$\boldsymbol{x}_{k+1} = f(\boldsymbol{x}_k) \tag{6}$$

Fixed point is expressed as $\boldsymbol{x}_f$. Fixed point is related as $\boldsymbol{x}_f = f(\boldsymbol{x}_f)$ or $\boldsymbol{x}_k{=}\boldsymbol{x}_{k+1}{=}\boldsymbol{x}_f$. When the following equation holds,

$$|\boldsymbol{x}_{k+1} - \boldsymbol{x}_f| < |\boldsymbol{x}_k - \boldsymbol{x}_f| \tag{7}$$

$|\boldsymbol{x}_k - \boldsymbol{x}_f|$ decreases monotonically with $k$ steps. Hence, $\lim_{k\to\infty} |\boldsymbol{x}_k - \boldsymbol{x}_f| = 0$ holds. If Eq. (7) holds for arbitrary state $\boldsymbol{x}_k$, fixed point $\boldsymbol{x}_f$ is global asymptotically stable.

The state $\boldsymbol{x}_k$ can be written as

$$\boldsymbol{x}_k = \boldsymbol{x}_f + n\Delta\boldsymbol{x}_k \tag{8}$$

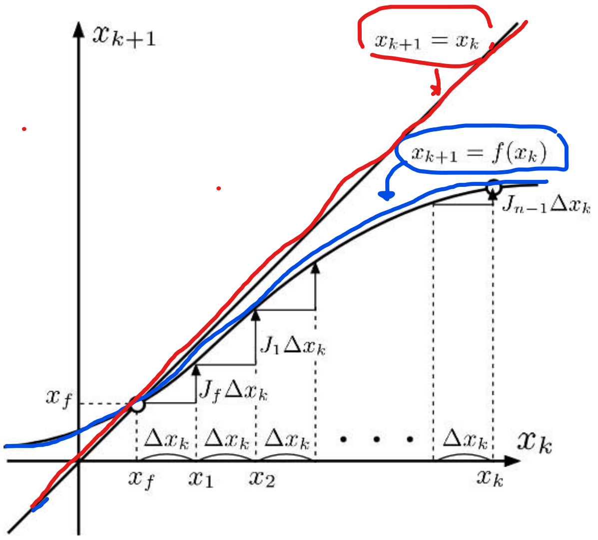



Fig. 2. Discrete state of $k+1$ steps (in case of one-dimension)

where $\Delta\boldsymbol{x}_k = (\boldsymbol{x}_k - \boldsymbol{x}_f)/n$ $(n \geq 1)$. When $\Delta\boldsymbol{x}_k$ is infinitesimal, $\boldsymbol{x}_{k+1}$ can be written as follows, and Fig. 2.

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_f + \boldsymbol{J}_f\Delta\boldsymbol{x}_k + \boldsymbol{J}_1\Delta\boldsymbol{x}_k \cdots + \boldsymbol{J}_{n-1}\Delta\boldsymbol{x}_k \tag{9}$$

where $\boldsymbol{J}_f$ is Jacobian matrix of fixed point. $\boldsymbol{J}_i$ is Jacobian matrix of $\boldsymbol{x}_f + i\Delta\boldsymbol{x}_k$.

If all absolute of eigenvalues of Jacobian matrix of arbitrary state, $|\Delta\boldsymbol{x}_k| > |\boldsymbol{J}_f\Delta\boldsymbol{x}_k|$ and $|\Delta\boldsymbol{x}_k| > |\boldsymbol{J}_i\Delta\boldsymbol{x}_k|$ holds. Consequently, Eq. (7) holds as follows:

$$\begin{aligned} |\boldsymbol{x}_{k+1} - \boldsymbol{x}_f| &= |\boldsymbol{J}_f\Delta\boldsymbol{x}_k + \boldsymbol{J}_1\Delta\boldsymbol{x}_k \cdots + \boldsymbol{J}_{n-1}\Delta\boldsymbol{x}_k| \\ &\leq |\boldsymbol{J}_f\Delta\boldsymbol{x}_k| + |\boldsymbol{J}_1\Delta\boldsymbol{x}_k| \cdots + |\boldsymbol{J}_{n-1}\Delta\boldsymbol{x}_k| \\ &< |\Delta\boldsymbol{x}_k| + |\Delta\boldsymbol{x}_k| \cdots + |\Delta\boldsymbol{x}_k| \\ &= n|\Delta\boldsymbol{x}_k| = n\left|\frac{\boldsymbol{x}_k - \boldsymbol{x}_f}{n}\right| \\ &= |\boldsymbol{x}_k - \boldsymbol{x}_f| \end{aligned} \tag{10}$$

Hence, fixed point is global asymptotically stable.

### B. Jacobian matrix

In this section, we derive the Jacobian matrix of arbitrary state of model as shown in Fig. 1.

In this study, the state just after heel–strike $\boldsymbol{x}_k^+$ is focused. State quantity of $\boldsymbol{x}_k^+$ is given as $\alpha_k$ and $\dot{\theta}_k^+$ because $\dot{\phi}_k^+$ is dependent variable of $\alpha$, $\dot{\theta}_k^+$ (see section IV-A). Successive states is related as $\boldsymbol{x}_{k+1}^+ = f(\boldsymbol{x}_k^+)$.

Linear discrete-time state equation of $\Delta\boldsymbol{x}_k^+$ can be derived as

$$\Delta\boldsymbol{x}_{k+1}^+ = \left.\frac{\partial f}{\partial \boldsymbol{x}^+}\right|_{\boldsymbol{x}^+=\boldsymbol{x}_k^+} \Delta\boldsymbol{x}_k^+ \equiv \boldsymbol{J}_{\boldsymbol{x}_k}\Delta\boldsymbol{x}_k^+ \tag{11}$$

where

$$\boldsymbol{J}_{\boldsymbol{x}_k} = \begin{bmatrix} \left.\dfrac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_k & \left.\dfrac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_k \\ \left.\dfrac{\partial\dot{\theta}_{k+1}^+}{\partial\alpha_k}\right|_k & \left.\dfrac{\partial\dot{\theta}_{k+1}^+}{\partial\dot{\theta}_k^+}\right|_k \end{bmatrix} \tag{12}$$

From energy conservation law and Eq. (4), discrete-time state equation of $\dot{\theta}_k^{+2}$ can be derived as

$$\dot{\theta}_{k+1}^{+2} = e_{k+1}^2 \left( \dot{\theta}_k^{+2} + \frac{2g}{l} \left\{ \cos\left(\frac{\alpha_k}{2} - \gamma\right) - \cos\left(\frac{\alpha_{k+1}}{2} + \gamma\right) \right\} \right) \quad (13)$$

where $e_{k+1} = \cos\alpha_{k+1}$. $(\partial\dot{\theta}_{k+1}^{+}/\partial\alpha_k)|_k$ and $(\partial\dot{\theta}_{k+1}^{+}/\partial\dot{\theta}_k^{+})|_k$ can be derived from Eq. (13) as

$$\left.\frac{\partial\dot{\theta}_{k+1}^{+}}{\partial\alpha_k}\right|_k = a_k \left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_k + b_k \quad (14)$$

$$\left.\frac{\partial\dot{\theta}_{k+1}^{+}}{\partial\dot{\theta}_k^{+}}\right|_k = a_k \left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^{+}}\right|_k + c_k \quad (15)$$

where $a_k$, $b_k$ and $c_k$ are given as

$$a_k = \frac{1}{d_k}\frac{\partial e_{k+1}}{\partial\alpha_{k+1}} \left\{ \dot{\theta}_k^{+2} + \frac{2g}{l}\left(\cos\frac{\alpha_k}{2} - \cos\frac{\alpha_{k+1}}{2}\right)\cos\gamma + \frac{2g}{l}\left(\sin\frac{\alpha_k}{2} + \sin\frac{\alpha_{k+1}}{2}\right)\sin\gamma \right\} + e_{k+1}\frac{g}{2ld_k}\left(\sin\frac{\alpha_{k+1}}{2}\cos\gamma + \cos\frac{\alpha_{k+1}}{2}\sin\gamma\right) \quad (16)$$

$$b_k = e_{k+1}\frac{g}{2ld_k}\left(-\sin\frac{\alpha_k}{2}\cos\gamma + \cos\frac{\alpha_k}{2}\sin\gamma\right) \quad (17)$$

$$c_k = e_{k+1}\frac{\dot{\theta}_k^{+}}{d_k} \quad (18)$$

$d_k^2$ is given as

$$d_k^2 = \dot{\theta}_k^{+2} + \frac{2g}{l}\left\{ \left(\cos\frac{\alpha_k}{2} - \cos\frac{\alpha_{k+1}}{2}\right)\cos\gamma + \left(\sin\frac{\alpha_k}{2} + \sin\frac{\alpha_{k+1}}{2}\right)\sin\gamma \right\} \quad (19)$$

From Eqs. (12), (14) and (15), Jacobian matrix $\boldsymbol{J}_{\boldsymbol{x}_k}$ of an arbitrary state $\boldsymbol{x}_k^{+}$ is written as

$$\boldsymbol{J}_{\boldsymbol{x}_k} = \begin{bmatrix} \left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_k & \left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^{+}}\right|_k \\ a_k\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_k + b_k & a_k\left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^{+}}\right|_k + c_k \end{bmatrix} \quad (20)$$

When $\alpha_{k+1} = \alpha_k$ and $\dot{\theta}_{k+1}^{+} = \dot{\theta}_k^{+}$, Eq. (20) is Jacobian matrix of fixed point.

### C. Global stabilization of fixed point

In this section, global stabilization principle of fixed point is derived from Jacobian matrix of arbitrary state.

$c_k$ in Jacobian matrix (20) can be written as

$$c_k = e_{k+1}\sqrt{\frac{\frac{1}{2}Ml^2\dot{\theta}_k^{+2}}{\frac{1}{2}Ml^2\dot{\theta}_k^{+2} + P}} \quad (21)$$

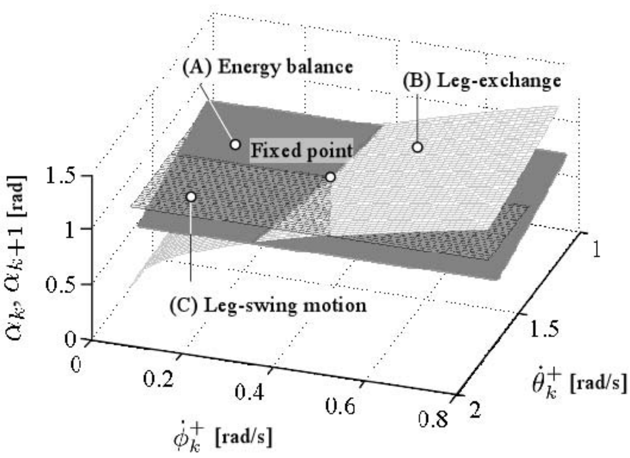



Fig. 3. Physical structure forming fixed point

where

$$P = Mgl\left\{\cos\left(\frac{\alpha_k}{2} - \gamma\right) - \cos\left(\frac{\alpha_{k+1}}{2} + \gamma\right)\right\}$$

$e_{k+1}$ is $0 < e_{k+1} < 1$. And, the term with root is positive value smaller than 1 because energy supplied by gravitational potential $P$ is positive. Hence, $|c_k| < 1$ holds always. If eigenvalue of Jacobian matrix (20) is $c_k$, fixed point is global asymptotically stable.

In this paper, we focus on the state as follows:

$$\frac{\partial\alpha_{k+1}}{\partial\alpha_k} = 0 \qquad \frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^{+}} = 0 \quad (22)$$

Eq. (22) denotes that inter-leg angle at heel-strike is always constant ($\alpha_k = \alpha_{k+1} = \alpha$). From Eqs. (20) and (22), Jacobian matrix (20) are $c_k$ and 0. Hence, fixed point is global asymptotically stable.

## IV. Fixed Point

### A. Energy balance

Walking system generates a cyclic trajectory. When the one cyclic trajectory is closed, the state just after heel-strike is fixed as one point. This point is called "fixed point".

The state just after heel-strike of $k$ steps is represent by inter-leg angle $\alpha_k$, angular velocities of stance and swing legs $\dot{\theta}_k^{+}$, $\dot{\phi}_k^{+}$. In fixed point, $\alpha_{k+1} = \alpha_k$, $\dot{\theta}_{k+1}^{+} = \dot{\theta}_k^{+}$ and $\dot{\phi}_{k+1}^{+} = \dot{\phi}_k^{+}$ hold.

In fixed point, discrete-time state equation (13) is written as

$$\dot{\theta}_k^{+} = \sqrt{\frac{4e_k^2 g}{l(1-e_k^2)}\sin\frac{\alpha_k}{2}\sin\gamma} \quad (23)$$

Surface (A) in Fig. 3 shows the relationship between $\alpha_k$, $\dot{\theta}_k^{+}$ and $\dot{\phi}_k^{+}$ of Eq. (23). Slope angle $\gamma$ is set to 0.073[rad]. The model parameters are $l$=0.7[m], $l_1$=$l_2$=0.35[m], $a$=$b$=0.35[m], $a_1$=$b_1$=$a_2$=$b_2$=0.175[m] and

$p$=0.4. Fixed point has a structure to be located on the surface (A). Equation (23) is rewritten as follows:

$$\frac{1}{2}Ml^2\dot{\theta}_k^{+2}\left(\frac{1}{e_k^2}-1\right)=2Mgl\sin\frac{\alpha_k}{2}\sin\gamma \tag{24}$$

Left and right parts of Eq. (24) are denoted to each the energy lost at heel-strike and the energy supplied by potential energy. The surface (A) represents energy balance.

### *B. Leg-exchange*

From leg-exchange equation (4), equation can be obtained as follows:

$$\dot{\phi}_k^+ = q(\alpha_k)\dot{\theta}_k^+ \tag{25}$$

where

$$q(\alpha_k)=\frac{-ab+\dfrac{\bar{I}}{1+p}+bl\cos^2\alpha_k}{\left(b^2+\dfrac{\bar{I}}{1+p}\right)\cos\alpha_k}$$

Surface (B) in Fig. 3 shows the relationship between $\alpha_k$, $\dot{\theta}_k^+$ and $\dot{\phi}_k^+$ of Eq. (25). The states just after heel-strike have a physical structure to be constrained on the surface (B) by leg-exchange.

### *C. Leg-swing motion*

From Eqs. (23) and (25), the angular velocity of swing leg $\dot{\phi}_k^+$ is derived as

$$\dot{\phi}_k^+ = q(\alpha_k)\sqrt{\frac{4e_k^2 g}{l(1-e_k^2)}\sin\frac{\alpha_k}{2}\sin\gamma} \tag{26}$$

From Eqs. (23) and (26), fixed point is represented by $\alpha_k$, $\dot{\theta}_k^+(\alpha_k)$ and $\dot{\phi}_k^+(\alpha_k)$. When $\alpha_{k+1}$=$\alpha_k$, a fixed point is generated. $\alpha_{k+1}$ is determined by leg-swing motion. In case that inter-leg angle at heel-strike is always constant ($\alpha_{k+1}$=$\alpha_k$), the relationship between $\alpha_k$, $\dot{\theta}_k^+$ and $\dot{\phi}_k^+$ is shown as surface (C) in Fig. 3. As shown in Fig. 3, one fixed point is formed by the physical structures of energy balance, leg-exchange and leg-swing motion.

In this section and section III-C, a global stable fixed point can be generated when inter-leg angle at heel-strike is always constant.

Fig. 4. Passive walker based on a global stabilization principle of fixed point

## V. EXPERIMENTS

### *A. Passive walker with knees*

Figure 4 shows passive walker with knees. Length and width of the walker are each 0.42[m] and 0.15[m]. Weight is 1.5[kg]. This passive walker has features as follows:

- Unified design by H-section leg with point of knee part

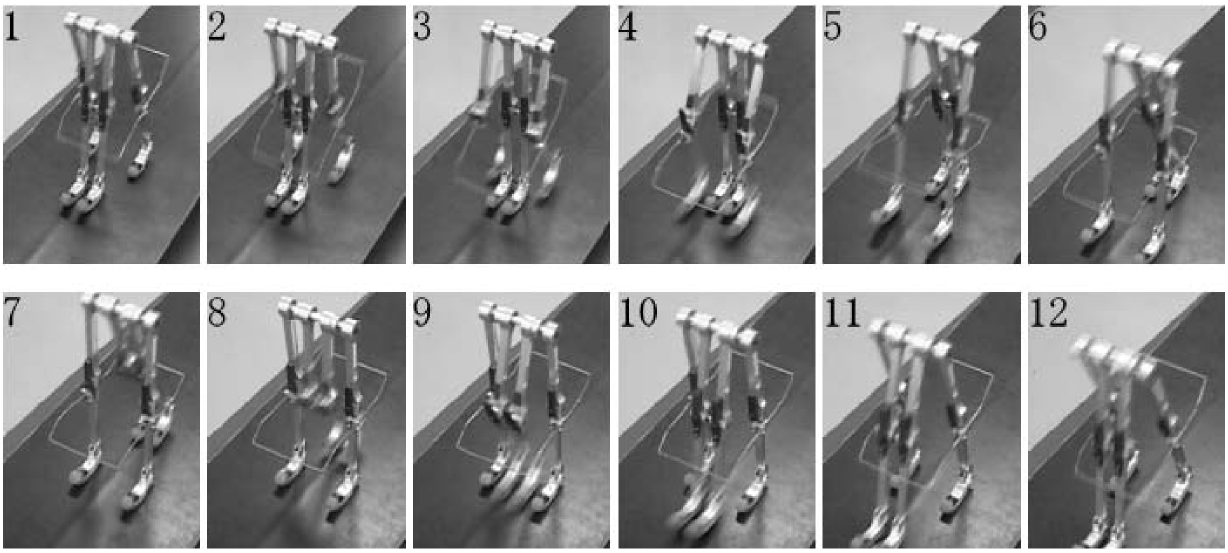
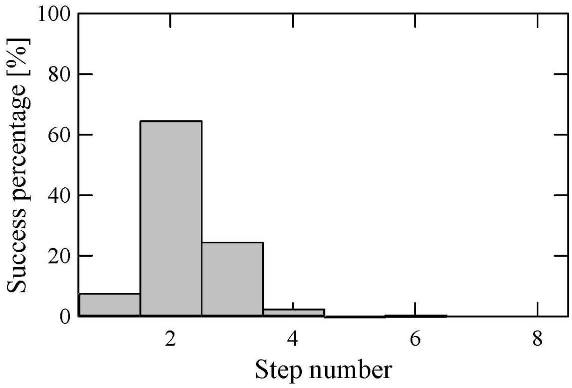
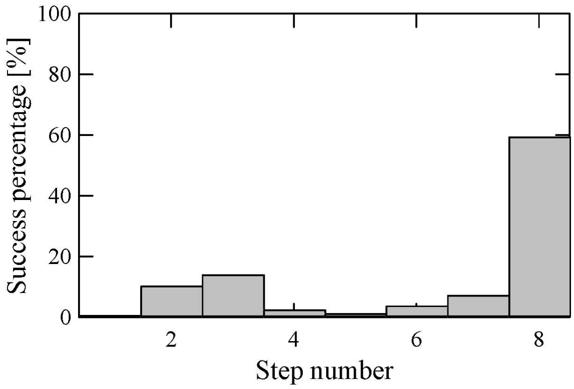



Fig. 5. A photographic playback




(a) Without stopper (normal)




(b) With stopper (stabilization)

Fig. 6. Experimental results

- Synchronizing legs and assuring twist stiffness by hollow shaft of hip
- Exchangeable foot attached by plane–plane
- High stiffness and accuracy by minimum parts

The walker has common four legs, which outside two legs and inner side two legs can swing in synchronization. The mechanical structure plays a role in keeping right-and-left balance. And, dynamics of outside legs is equal to the one of inner side legs.

Considering the stabilization principle of fixed point as mentioned in sections III-C, the passive walker has a stopper to maintain constant inter-leg angle at heel-strike. Weight of stopper (0.03[kg]) is enough small than total weight (1.5[kg]).

Figure 5 shows a photographic playback of passive walker with stopper. In case that inner side legs are stance legs, inter–leg angle at heel-strike is constant by stopper hitting front face of inner side legs. In case that outside legs are stance legs, inter-leg angle at heel-strike is constant by stopper hitting rear face of inner side legs.

### B. Verification experiment

Slope angle $\gamma$ is set to 5[deg]. Total length of slope is 1.8[m]. In this experimental condition, the passive walker can walk for maximum 8 steps. Proper initial condition for walking is found by the trial-and-error approach. Experimenter sets the proper initial condition as possible. But, the initial condition has a certain level of variations. The number of experiments is about 450 times.

Figures 6 (a) and (b) show each the experiment results of passive walker without and with stopper. Horizontal axis denotes the number of steps. Vertical axis denotes the success rate of number of steps. It is difficult for the passive walker without stopper to walk for more than 4 steps, as shown in Fig. 6 (a). While, the passive walker with stopper can achieve maximum 8 steps at success rate of about 60[%], as shown in Fig. 6 (b).

As explained before, the validity of stabilization principle of fixed point is demonstrated.

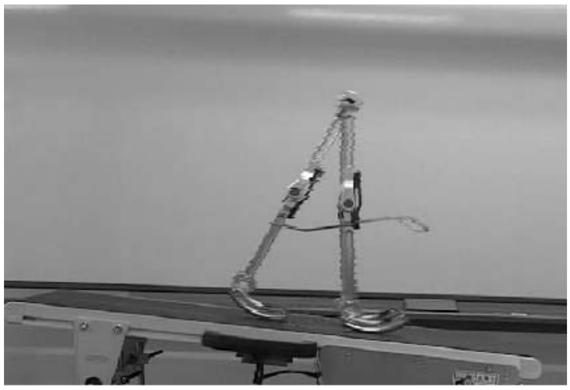


Fig. 7. Walking on treadmill

### C. Walking on treadmill

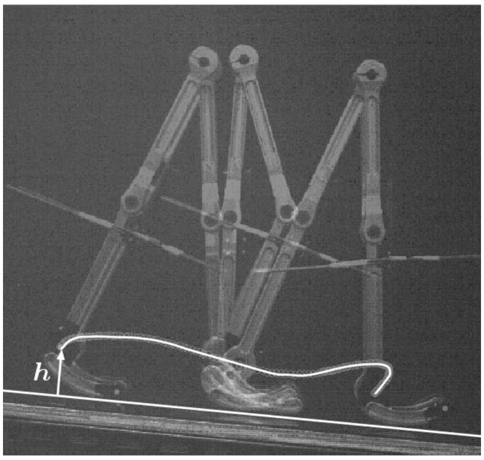
In experiment mentioned in section V-B, the maximum number of steps is limited. In this section, passive walker with stopper is experimented on treadmill, as shown in Fig. 7. Length and width of the treadmill are each 0.8[m] and 0.4[m]. Belt velocity is constant. In case of passive walker without stopper, it is very difficult to walk for even 2 or 3 steps. While, passive walker with stopper can walk for many steps. And, best record number of steps is 4010 steps (35 minutes) [1]!! This experiment result indicates that a stable fixed point is generated because the walker becomes steady-state on the constant-speed belt.

### D. Analysis of walking failure

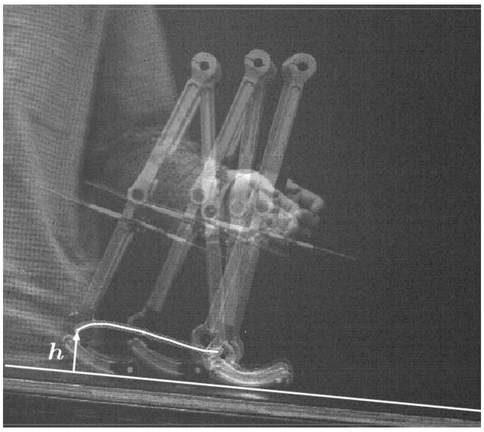
In this section, we analyze the walking failure in experiment as mentioned in section V-B.

Figures 8 (a) and (b) show the trajectories of ankle of swing leg in case of success and failure of walking. These trajectories are obtained from photographs taken by high-speed camera (frame rate : 500 [frame/s]). In success of walking, height of ankle $h$ is local minimal in the vicinity of crossing stance leg and swing leg. In failure of walking,

[1] Number of steps is count from the video. Actual number of steps is more than 4010 steps because the video is divided in two (first video: 3439 steps, second video: 571 steps).




(a) Success




(b) Failure

Fig. 8. Trajectory of ankle of swing leg

the foot of swing leg hits the ground around this vicinity. And the passive walker falls down. Most walking failure is this failure pattern.

Assuming that next state just after heel–strike $\boldsymbol{x}_{k+1}^{+}$ exists, stabilization principle of fixed point is derived. However, this assumption may not hold in experiment. To function effectively the stabilization principle, problem that swing leg doesn't swing through stance leg must be solved.

## VI. Conclusion

In this paper, we demonstrated a stabilization principle of fixed point of walking phenomenon. Also, we demonstrated that the stabilization principle includes a generation principle of fixed point.

Based on the stabilization principle, we made a simple improvement on passive walker with knees. High stability could be achieved by the improvement. Moreover, a great result is that the walker could walk for 4010 steps on treadmill.

In walking experiment, it is important that swing leg swings through stance leg without hitting the ground. we must understand the mechanism of leg-swing motion.

Video footage of the experiment can be seen on WWW (http://drei.mech.nitech.ac.jp/~fujimoto/sano/walk_jpn.html).

## References


[1] T. McGeer, "Passive Dynamic Walking," *The Int. J. of Robotics Research*, vol. 9, no. 2, pp.62–68, 1990.

[2] T. McGeer, "Passive Walking with Knees," *Proc. of the 1990 IEEE Int. Conf. on Robotics and Automation*, pp.1640–1645, 1990.

[3] A. Goswami, B. Thuilot, and B. Espiau, "A Study of the Passive Gait of a Compass-Like Biped Robot : Symmetry and Chaos," *The Int. J. of Robotics Research*, pp.1282–1301, 1998.

[4] M. J. Coleman, "A Stability Study of Three Dimensional Passive Dynamic Model of Human Gait," Ph. D. Thesis, Cornell University, 1998.

[5] M. Garcia, A. Chatterjee, A. Ruina, and M. Coleman, "The Simplest Walking Model : Stability, Complexity, and Scaling," *ASME J. of Biomechanical Engineering*, pp.281–288, 1998.

[6] R. Q. van der Linde, "Passive Bipedal Walking with Phasic Muscle Contraction," *Biological Cybernetics*, vol.81, no.3, pp.227–237, 1999.

[7] F. Asano, M. Yamakita, N. Kamamichi, and Zhi-Wei Luo, "A Novel Gait Generation for Biped Walking Robots Based on Mechanical Energy Constraint," *IEEE Trans. Robotics and Automation*, vol.20, no.3, pp.565–573, 2004.

[8] T. Takuma, S. Nakajima, K. Hosoda, and M. Asada, "Design of Self-Contained Biped Walker with Pneumatic Actuators," *SICE Annual Conference 2004*, WPI-2-4, 2004.

[9] S. H. Collins, A. Ruina, R. Tedrake, and M. Wisse, "Efficient Bipedal Robots Based on Passive Dynamic Walkers," *Science*, vol.307, pp.1082–1085, 2005.

[10] M. Wisse, A. L. Schwab, R. Q. van der Linde, and F. C. T. vd. Helm, "How to Keep From Falling Forward: Swing Leg Action for Passive Dynamic Walkers," *IEEE trans. on Robotics*, vol.21, no3, pp.393–401, 2005.

[11] M. W. Spong and F. Bullo, "Controlled Symmetries and Passive Walking," *IEEE trans on Automatic Control*, vol.50, no.7, pp.1025–1031, 2005.

[12] Y. Sugimoto and K. Osuka, "Walking Control of Quasi Passive Dynamic Walking Robot "Quartet III" based on Continuous Delayed Feedback Control," *Proc. of the IEEE Int. Conf. on Robotics and Biomimetics*, 2004.

[13] K. Hirata, H. Kokame, and K. Konishi, "On Stability of Simplified Passive Walker Model and Effect of Feedback Control," *Proc. of the 4th Asian Control Conference*, pp.1444–1449, 2002.

[14] Y. Hurmuzlu and T. Chang, "Rigid Body Collisions of a Special Class of Planar Kinematic Chains," *IEEE Trans. on Systems, Man and Cybernetics*, vol.22, no.5, pp.964–971, 1992.