Proceedings of the 2006 IEEE International Conference on Robotics and Automation
Orlando, Florida - May 2006

A Physical Principle of Gait Generation and its Stabilization derived from Mechanism of Fixed Point

Yoshito Ikemata, Akihito Sano, and Hideo Fujimoto
Department of Mechanical Engineering
Nagoya Institute of Technology
Gokiso, Nagoya, Aichi 466-8555, JAPAN
{*ikemata, sano, fujimoto*}*@vier.mech.nitech.ac.jp*

***Abstract*— Passive walker with knees can walk down shallow slope in a natural gait and can exhibit a stable limit cycle. Though the passive walker is mechanically simple, it is a sort of hybrid system that combines the continuous dynamics of leg-swing motion and the discrete event of leg-exchange. In this paper, we focus on the mechanism of fixed point. We demonstrate a physical principle of gait generation and its stabilization derived from the mechanism of fixed point. Based on the principle, we made a simple improvement on passive walker with knees. The passive walker could have high stability, and could walk on treadmill for 4010 steps.**

***Index Terms*— Passive Walking, Fixed Point, Global Stability, Inter-Leg Angle**

I. Introduction

Passive walker can walk down shallow slope without actuator and control [1][2]. This motion is attractive because its gait is natural. Though mechanical system of passive walker is simple, it is a sort of hybrid system that combines the continuous dynamics of leg-swing motion and the discrete event of leg-exchange. Passive walker can exhibit a stable limit cycle. Gait generation and its stability of passive walking can be analyzed from the limit cycle.

Fixed point (cross-sectional point of limit cycle) of passive walking is determined by parameters of walker and slope angle. In many studies, fixed point is searched by numerical method because fixed point is formed by complex physical structures. It is difficult for passive walking to generate a flexible gait. It is desired to easily generate a fixed point with no consideration of the physical structures.

Local stability around fixed point of passive walking was first studied by McGeer [1]. Since then, Goswami et al. [3], Coleman et al. [4], and Garcia et al. [5] studied it in detail. However, stabilization mechanism of fixed point is not demonstrated in these studies. There is a trend in recent studied of passive walking toward adding actuators [6]-[11]. There are now few researchers investigating the stability mechanism.

Stabilizing control methods of passive walking are proposed [12][13]. It functions successfully only around fixed point because the control methods aim at high local stabilization around fixed point. On the other hand, It was report that there is no relevance between local stability and global stability [14]. To achieve high stability in passive walking, a global stabilization principle is required. Stability of fixed point is determined by eigenvalues of Jacobian matrix. To understand the stabilization principle of fixed point, it is necessary to understand a physical principle that governs the eigenvalues.

Fig. 1. Passive walker based on a a principle of gait generation and its stabilization

It is thought that essence of principle is common to any model. In this paper, we demonstrate a generation and stabilization principle of fixed point from a simple passive walker. And, the principle is confirmed by experiment of passive walker, as shown in Fig. 1.

II. Model of passive walker

A. Equations of swing motion

Figure 2 shows model of passive walker with knees. The model consists of stance and swing legs. Knee of the stance leg is locked straight. The motion is assumed to be constrained to saggital plane. For the sake of simplicity and clarity of analysis as possible, assumptions are given as follows:

$$M \gg m, \quad M \gg m_1, \quad M \gg m_2 \tag{1}$$

Stance leg is assumed to be fixed on the ground as no slippage or take off. The equations of swing motion can be

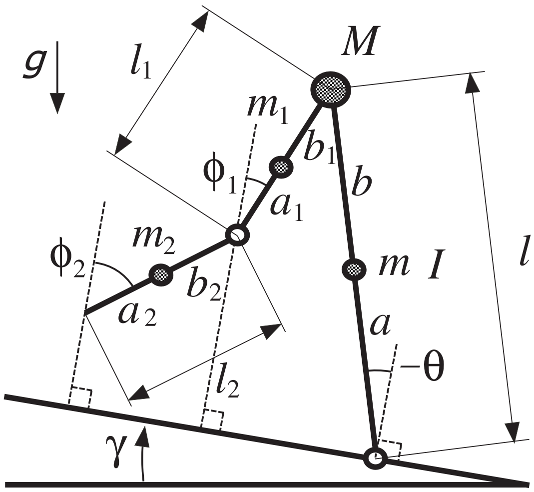


Fig. 2. Model of passive walker with knees

written as

$$\boldsymbol{M}_K(\boldsymbol{\theta}_K)\ddot{\boldsymbol{\theta}}_{\boldsymbol{K}} + \boldsymbol{H}_K(\boldsymbol{\theta}_K, \dot{\boldsymbol{\theta}}_K) + \boldsymbol{G}_K(\boldsymbol{\theta}_K, \gamma) = \boldsymbol{0} \tag{2}$$

where

$$\boldsymbol{M}_K(\boldsymbol{\theta}_K) = \begin{bmatrix} l^2 & 0 & 0 \\ -(b_1 l + p l l_1)\cos(\theta - \phi_1) & b_1^2 + p l_1^2 & p b_2 l_1 \cos(\phi_1 - \phi_2) \\ -b_2 l \cos(\theta - \phi_2) & b_2 l_1 \cos(\phi_1 - \phi_2) & b_2^2 \end{bmatrix}$$

$$\boldsymbol{H}_K(\boldsymbol{\theta}_K, \dot{\boldsymbol{\theta}}_K) = \begin{bmatrix} 0 \\ (b_1 l + p l l_1)\sin(\theta - \phi_1)\dot{\theta}^2 + p b_2 l_1 \sin(\phi_1 - \phi_2)\dot{\phi}_2^2 \\ b_2 l \sin(\theta - \phi_2)\dot{\theta}^2 - b_2 l_1 \cos(\phi_1 - \phi_2)\dot{\phi}_1^2 \end{bmatrix}$$

$$\boldsymbol{G}_K(\boldsymbol{\theta}_K, \gamma) = \begin{bmatrix} -l\sin(\theta + \gamma) \\ (b_1 + p l_1)\sin(\phi_1 + \gamma) \\ b_2 \sin(\phi_2 + \gamma) \end{bmatrix} g$$

$\boldsymbol{\theta}_K (= [\theta, \phi_1, \phi_2]^T)$ is the vector of joint angles. g is the acceleration of gravity. p is $p = m_2/m_1$. Setting $a = \{m_1(l_2 + a_1) + m_2 a_2\}/(m_1 + m_2)$, $I = m_2(l_2 - b_2 - a)^2 + m_1(l_2 + a_1 - a)^2$, stance leg is equal to swing leg.

B. Equations of knee-lock

Knee-lock occurs when the swing leg becomes straight ($\phi_1 = \phi_2 = \phi$). Assuming that the swing knee locks instantaneously, angular momentum is conserved through the knee-lock for the whole walker about the stance foot contact point, and the swing leg about the hip. Angular velocities of stance and swing legs just after knee-lock are obtained from these conservations of angular momentum as

$$\dot{\theta}^+ = \dot{\theta}^- \tag{3}$$

$$\dot{\phi}^+ = \frac{(b_1^2 + p l_1^2 + p l_1 b_2)\dot{\phi}_1^- + (p b_2^2 + p b_2 l_1)\dot{\phi}_2^-}{b_1^2 + p(l_1 + b_2)^2} \tag{4}$$

The "+" superscript means "just after knee-lock," and the "−" superscript means "just before knee-lock".

C. Equations of leg-exchange

After knee-lock, the model can be regarded as compass-like biped model. It supplies a leg-exchange rule when the swing foot hits the ground. Collision occurs when the geometric condition

$$2\theta - \phi = 0 \tag{5}$$

is met. For an inelastic no-sliding collision with the ground, angular momentum is conserved through the collision for the whole walker about the swing foot contact point, and the former stance leg about the hip[15]. Relational expressions are obtained from these conservations of angular momentum as

$$\boldsymbol{Q}^+(\alpha)\dot{\boldsymbol{\theta}}_C^+ = \boldsymbol{Q}^-(\alpha)\dot{\boldsymbol{\theta}}_C^- \tag{6}$$

where

$$\boldsymbol{Q}^+(\alpha) = \begin{bmatrix} l^2 & 0 \\ -bl\cos\alpha & b^2 + \dfrac{\bar{I}}{1+p} \end{bmatrix}$$

$$\boldsymbol{Q}^-(\alpha) = \begin{bmatrix} l^2\cos\alpha & 0 \\ -ab + \dfrac{\bar{I}}{1+p} & 0 \end{bmatrix}$$

The "+" superscript means "just after heel-strike," and the "−" superscript means "just before heel-strike". $\boldsymbol{\theta}_C (= [\theta, \phi]^T)$ is the vector of joint angles after heel-strike. α is inter-leg angle at heel-strike. $\bar{I}$ is I/m_1. We assume 0 $< \alpha_k < \pi/2$ and $0 < \alpha_{k+1} < \pi/2$.

From Eq. (6), the vector of angular velocity after heel-strike can be given as

$$\dot{\boldsymbol{\theta}}_C^+ = (\boldsymbol{Q}^+(\alpha))^{-1}\boldsymbol{Q}^-(\alpha)\dot{\boldsymbol{\theta}}_C^- \tag{7}$$

III. Generation of Fixed Point

A. Fixed point

Walking system generates a cyclic trajectory. When the one cyclic trajectory is closed, state just after heel-strike is fixed as one point. This point is called "fixed point". In this section, a generation principle of fixed point is demonstrated.

We focus on the discrete transition of the state just after heel-strike. The state just after heel-strike of k steps is represent by inter-leg angle α_k, angular velocities of stance and swing legs $\dot{\theta}_k^+$, $\dot{\phi}_k^+$. In fixed point ($\alpha_{k+1} = \alpha_k$, $\dot{\theta}_{k+1}^+ = \dot{\theta}_k^+$, $\dot{\phi}_{k+1}^+ = \dot{\phi}_k^+$), it keeps a balance between the energy lost and the energy supplied in one cycle. Therefore, physical structure of fixed point must meet energy balance equation as follows:

$$\frac{1}{2}Ml^2\dot{\theta}_k^{+2}\left(\frac{1}{e_k^2}-1\right)=2Mgl\sin\frac{\alpha_k}{2}\sin\gamma \tag{8}$$

where $e_k = \cos\alpha_k$. We call e_k ($0 < e_k < 1$) loss coefficient. Left and right parts of Eq. (8) are denoted to each the energy lost by heel-strike [1] and the energy supplied by potential energy. The angular velocity of stance leg just after heel-strike $\dot{\theta}_k^+$ is obtained from Eq. (8) as

$$\dot{\theta}_k^+=\sqrt{\frac{4e_k^2 g}{l(1-e_k^2)}\sin\frac{\alpha_k}{2}\sin\gamma} \tag{9}$$

From the matrix of leg-exchange (6), equation can be obtained as follows:

$$\dot{\phi}_k^+=q(\alpha_k)\dot{\theta}_k^+ \tag{10}$$

where

$$q(\alpha_k)=\frac{-ab+\dfrac{\bar{I}}{1+p}+bl\cos^2\alpha_k}{\left(b^2+\dfrac{\bar{I}}{1+p}\right)\cos\alpha_k}$$

The states just after heel-strike have a physical structure to be constrained by Eq. (10). From Eqs. (9) and (10), the angular velocity of swing leg $\dot{\phi}_k^+$ is derived as

$$\dot{\phi}_k^+=q(\alpha_k)\sqrt{\frac{4e_k^2 g}{l(1-e_k^2)}\sin\frac{\alpha_k}{2}\sin\gamma} \tag{11}$$

From Eqs. (9) and (11), fixed point is represented by α_k, $\dot{\theta}_k^+(\alpha_k)$, and $\dot{\phi}_k^+(\alpha_k)$. The fixed point is determined if α_{k+1}=α_k is determined. This means that a fixed point can be absolutely generated if inter-leg angle at heel-strike is always constant.

B. Energy efficiency

In this section, the relationship between inter-leg angle at heel-strike and energy efficiency of locomotion is demonstrated.

The specific cost of transport c_{et} (= (energy used)/(weight)(distance travel)) is a measure of energy efficiency of locomotion [9]. The specific cost of transport c_{et} of fixed point can be obtained as follows:

$$\begin{aligned} c_{et} &= \frac{2Mgl\sin\dfrac{\alpha_k}{2}\sin\gamma}{Mg\times 2l\sin\dfrac{\alpha_k}{2}} \\ &= \sin\gamma \end{aligned} \tag{12}$$

c_{et} doesn't include α_k. Therefore, the specific cost of transport is constant for any inter-leg angle at heel-strike. Passive walking in case of slope angle γ=3[deg] is c_{et}=0.05. While, human walking at speed 1.2[m/s] is c_{et}=0.3 from [16].

[1] Energy lost by knee-lock is negligible by assumptions of Eq. (1).

IV. Global stabilization of fixed point

A. Jacobian matrix of an arbitrary state

From Eq. (10), $\dot{\phi}_k^+$ is dependent variable of α, $\dot{\theta}_k^+$. As a result, an arbitrary state quantity just after heel-strike can be expressed as $\boldsymbol{x}_k^+$ (=$[\alpha_k, \dot{\theta}_k^+]^T$). Successive states is related as

$$\boldsymbol{x}_{k+1}^+=f(\boldsymbol{x}_k^+) \tag{13}$$

Fixed point is expressed as $\boldsymbol{x}_f^+$. Fixed point is related as

$$\boldsymbol{x}_f^+=f(\boldsymbol{x}_f^+) \tag{14}$$

For a small perturbation $\Delta\boldsymbol{x}_k^+$ around an arbitrary state $\boldsymbol{x}_k^+$, f is expressed in term of Taylor series expansion as

$$\boldsymbol{x}_{k+1}^+=f(\boldsymbol{x}_k^+)+\left.\frac{\partial f}{\partial\boldsymbol{x}^+}\right|_{\boldsymbol{x}^+=\boldsymbol{x}_k^+}\Delta\boldsymbol{x}_k^+ \tag{15}$$

From Eqs. (13) and (15), linear discrete-time state equation of $\Delta\boldsymbol{x}_k^+$ can be derived as

$$\Delta\boldsymbol{x}_{k+1}^+=\left.\frac{\partial f}{\partial\boldsymbol{x}^+}\right|_{\boldsymbol{x}^+=\boldsymbol{x}_k^+}\Delta\boldsymbol{x}_k^+\equiv\boldsymbol{J}_k\Delta\boldsymbol{x}_k^+ \tag{16}$$

where

$$\boldsymbol{J}_k=\begin{bmatrix}\left.\dfrac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_k & \left.\dfrac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_k \\ \left.\dfrac{\partial\dot{\theta}_{k+1}^+}{\partial\alpha_k}\right|_k & \left.\dfrac{\partial\dot{\theta}_{k+1}^+}{\partial\dot{\theta}_k^+}\right|_k\end{bmatrix} \tag{17}$$

From energy conservation law and Eq. (6), discrete-time state equation of $\dot{\theta}_k^{+2}$ can be derived as

$$\begin{aligned}\dot{\theta}_{k+1}^{+2} &= e_{k+1}^2\Bigg(\dot{\theta}_k^{+2}+\frac{2g}{l}\Bigg\{\cos\left(\frac{\alpha_k}{2}-\gamma\right) \\ &\quad -\cos\left(\frac{\alpha_{k+1}}{2}+\gamma\right)\Bigg\}\Bigg)\end{aligned} \tag{18}$$

where $e_{k+1}=\cos\alpha_{k+1}$. $(\partial\dot{\theta}_{k+1}^+/\partial\alpha_k)|_k$, $(\partial\dot{\theta}_{k+1}^+/\partial\dot{\theta}_k^+)|_k$ can be derived from Eq. (18) as

$$\left.\frac{\partial\dot{\theta}_{k+1}^+}{\partial\alpha_k}\right|_k=a_k\left.\frac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_k+b_k \tag{19}$$

$$\left.\frac{\partial\dot{\theta}_{k+1}^+}{\partial\dot{\theta}_k^+}\right|_k=a_k\left.\frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_k+c_k \tag{20}$$

where a_k, b_k, and c_k are given as

$$\begin{aligned}a_k=\frac{1}{d_k}\frac{\partial e_{k+1}}{\partial\alpha_{k+1}}&\Bigg\{\dot{\theta}_k^{+2}+\frac{2g}{l}\left(\cos\frac{\alpha_k}{2}-\cos\frac{\alpha_{k+1}}{2}\right)\cos\gamma \\ &+\frac{2g}{l}\left(\sin\frac{\alpha_k}{2}+\sin\frac{\alpha_{k+1}}{2}\right)\sin\gamma\Bigg\} \\ &+e_{k+1}\frac{g}{2ld_k}\left(\sin\frac{\alpha_{k+1}}{2}\cos\gamma+\cos\frac{\alpha_{k+1}}{2}\sin\gamma\right)\end{aligned} \tag{21}$$

$$b_k=e_{k+1}\frac{g}{2ld_k}\left(-\sin\frac{\alpha_k}{2}\cos\gamma+\cos\frac{\alpha_k}{2}\sin\gamma\right) \tag{22}$$

$$c_k=e_{k+1}\frac{\dot{\theta}_k^+}{d_k} \tag{23}$$

And, d_k^2 is given as

$$d_k^2 = \dot{\theta}_k^{+2} + \frac{2g}{l}\left\{\left(\cos\frac{\alpha_k}{2} - \cos\frac{\alpha_{k+1}}{2}\right)\cos\gamma + \left(\sin\frac{\alpha_k}{2} + \sin\frac{\alpha_{k+1}}{2}\right)\sin\gamma\right\} \quad (24)$$

From Eqs. (17), (19), and (20), Jacobian matrix $\boldsymbol{J}_k$ of an arbitrary state $\boldsymbol{x}_k^+$ is written as

$$\boldsymbol{J}_k = \begin{bmatrix} \left.\dfrac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_k & \left.\dfrac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_k \\ a_k\left.\dfrac{\partial\alpha_{k+1}}{\partial\alpha_k}\right|_k + b_k & a_k\left.\dfrac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+}\right|_k + c_k \end{bmatrix} \quad (25)$$

When $\alpha_{k+1} = \alpha_k$ and $\dot{\theta}_{k+1}^+ = \dot{\theta}_k^+$ hold, Eq. (25) becomes Jacobian matrix of fixed point.

B. Mathematical global stable condition

As a matter of convenience, fixed point is expressed as $\boldsymbol{x}_0$ in this section. State of k steps can be expressed as sum of fixed point and n infinitesimal change of state, as shown in Eq. (26)

$$\begin{aligned} \boldsymbol{x}_{n,k} &= \boldsymbol{x}_{n-1,k} + \Delta\boldsymbol{x}_{n-1,k} \\ &= \boldsymbol{x}_0 + \sum_{i=0}^{n-1}\Delta\boldsymbol{x}_{i,k} \end{aligned} \quad (26)$$

where $\boldsymbol{x}_{0,k} = \boldsymbol{x}_0$. Any initial state can be given by setting n and $\Delta\boldsymbol{x}_{i,0}$. Successive states can be written as

$$\begin{aligned} \boldsymbol{x}_{n,k+1} &= f(\boldsymbol{x}_{n,k}) \\ &= f(\boldsymbol{x}_{n-1,k} + \Delta\boldsymbol{x}_{n-1,k}) \end{aligned} \quad (27)$$

f is expressed by linearization of Eq. (27) as

$$\begin{aligned} \boldsymbol{x}_{n,k+1} &= f(\boldsymbol{x}_{n-1,k}) + \left.\frac{\partial f}{\partial\boldsymbol{x}}\right|_{\boldsymbol{x}=\boldsymbol{x}_{n-1,k}}\Delta\boldsymbol{x}_{n-1,k} \\ &\equiv f(\boldsymbol{x}_{n-1,k}) + \boldsymbol{J}_{n-1,k}\Delta\boldsymbol{x}_{n-1,k} \end{aligned} \quad (28)$$

where $\boldsymbol{J}_{n-1,k}$ is Jacobian matrix of the state $\boldsymbol{x}_{n-1,k}$. Repeating operations of Eqs. (27) and (28), equation can be obtained as

$$\boldsymbol{x}_{n,k+1} = f(\boldsymbol{x}_0) + \sum_{i=0}^{n-1}\boldsymbol{J}_{i,k}\Delta\boldsymbol{x}_{i,k} \quad (29)$$

Using $\boldsymbol{x}_0 = f(\boldsymbol{x}_0)$ and $\Delta\boldsymbol{x}_{i,k+1} = \boldsymbol{J}_{i,k}\Delta\boldsymbol{x}_{i,k}$, the state $\boldsymbol{x}_{n,k}$ can be derived from Eq. (29) as follows:

$$\boldsymbol{x}_{n,k} = \boldsymbol{x}_0 + \sum_{i=0}^{n-1}\left[\left(\prod_{j=0}^{k}\boldsymbol{J}_{i,j}\right)\Delta\boldsymbol{x}_{i,0}\right] \quad (30)$$

If all absolute eigenvalue of $\boldsymbol{J}_{i,j}$ is less then 1, $\lim_{k\to\infty}\boldsymbol{x}_{n,k} = \boldsymbol{x}_0$ holds.

As explained above, if all absolute eigenvalue of Jacobian matrix of an arbitrary states is less then 1, fixed point is global asymptotically stable.

C. Global stabilization

In section 3. A, we demonstrated that a fixed point can be absolutely generated if inter-leg angle at heel-strike is always constant. In this section, global stability of the gait is analyzed.

$(\partial\alpha_{k+1}/\partial\alpha_k)|_k$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_k$ in Eq. (25) denote each the rates of change of inter-leg angle at heel-strike α_{k+1} for small perturbations of inter-leg angle α_k and angular velocity of stance leg $\dot{\theta}_k^+$. When inter-leg angle at heel-strike is always constant ($\alpha_k = \alpha_{k+1} = \alpha$), $(\partial\alpha_{k+1}/\partial\alpha_k)|_k$ and $(\partial\alpha_{k+1}/\partial\dot{\theta}_k^+)|_k$ is given as

$$\frac{\partial\alpha_{k+1}}{\partial\alpha_k} = 0 \qquad \frac{\partial\alpha_{k+1}}{\partial\dot{\theta}_k^+} = 0 \quad (31)$$

Eigenvalues of Jacobian matrix $\boldsymbol{J}_k$ of $\boldsymbol{x}_k$ can be derived from Eqs. (25) and (31) as follows:

$$\begin{aligned} R_k &= c_k \\ &= e\sqrt{\frac{\frac{1}{2}Ml^2\dot{\theta}_k^{+2}}{\frac{1}{2}Ml^2\dot{\theta}_k^{+2} + 2Mgl\sin\frac{\alpha}{2}\sin\gamma}} \end{aligned} \quad (32)$$

And $R_k = 0$ is derived. $R_k = 0$ is given little thought because it doesn't get involved in the stability of fixed point.

In Eq. (32), $2Mgl\sin\frac{\alpha}{2}\sin\gamma$ denotes energy supplied by potential energy. The term with root is positive value smaller than 1 if energy supplied is positive. Loss coefficient e is $0 < e < 1$. Consequence, $|R_{rk}| < 1$ consists always. This indicates that the fixed point becomes global asymptotically stable if the walker steps ahead maintaining constant inter-leg angle at heel-strike,

From physical approach to mechanism of fixed point, we have reached a principle of gait generation and its stabilization. It's interesting to note that the principle is basically equivalent to the one of rimless spoked wheel [1]. Wissen et al. [10] achieve high stability by mimicking rimless spoked wheel. However, his study is not enough to investigate it physically.

V. Experiments

A. Verification experiment

Figure 1 shows the passive walker with knees. Considering the principle of gait generation and its stabilization as mentioned in sections 3. A and 4. C, the passive walker has a stopper to maintain constant inter-leg angle at heel-strike. To make experiment space, long scale of leg of the passive walker is half as many as the one of human. Total length and width of the walker are each 0.42[m] and 0.15[m]. Total weight is 1.5[kg]. Also, weight of stopper (0.03[kg]) is enough small than total weight.

Slope angle γ is set to 5[deg]. Total length of slope is 1.8[m]. Stopper is made to swing easily without stubbing swing leg's toe against ground. In this experimental condition, the passive walker can walk for maximum 8 steps. Proper initial condition for walking is found by the trial-and-error approach. Experimenter sets the proper initial

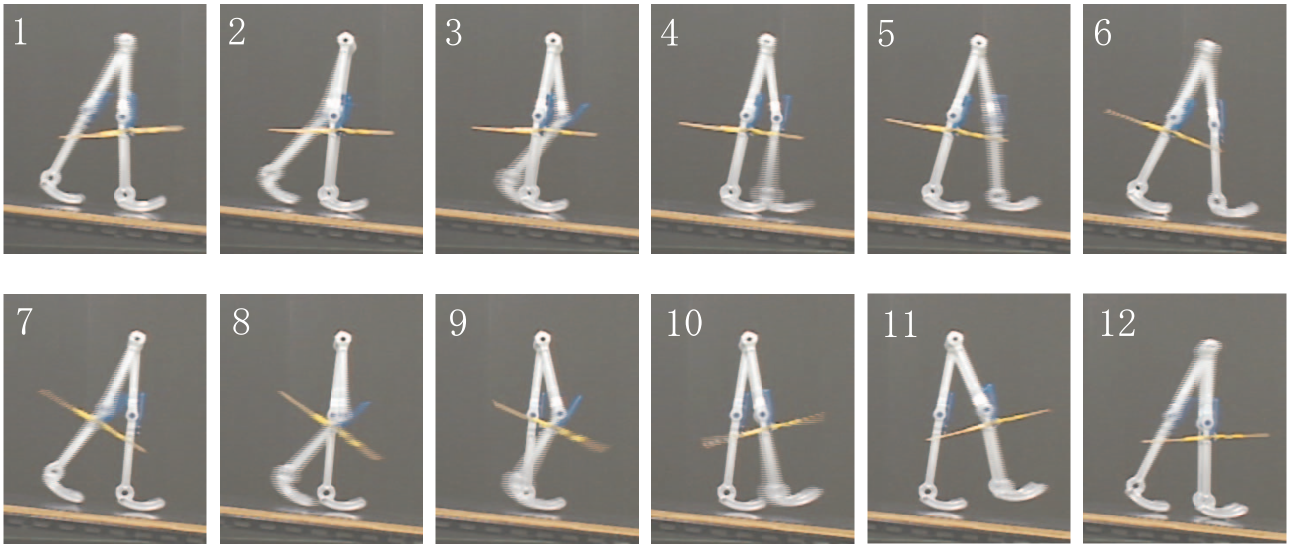
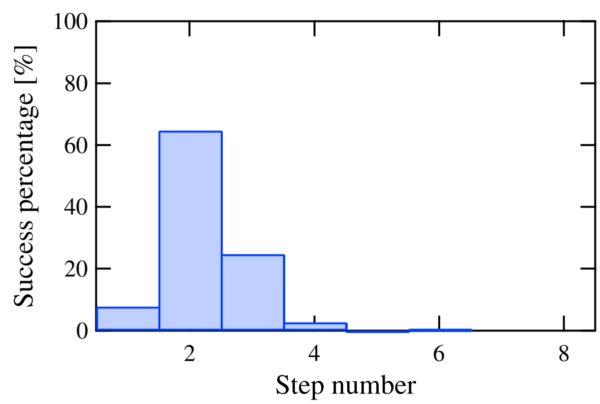


Fig. 3. A photographic playback

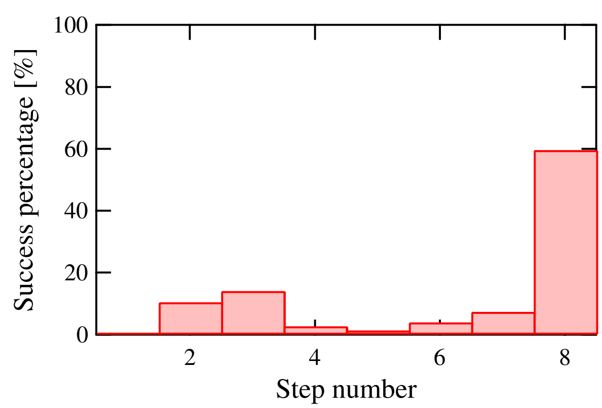
condition as possible. But, the initial condition has a certain level of variations. The number of experiments is about 450 times.

Figure 3 shows a photographic playback of passive walker with stopper. By the stopper, passive walker can step at approximately constant inter-leg angle at heel-strike, as shown in Figs. 3 (6) and (12).

Figures 4 (a) and (b) show each the experiment results of passive walker without and with stopper. Horizontal axis denotes the number of steps. Vertical axis denotes the percentage of number of steps. It is difficult for the passive walker without stopper to walk for more than 4 steps, as shown in Fig. 4 (a). While, the passive walker with stopper can achieve maximum 8 steps at success rate of about 60[%], as shown in Fig. 4 (b). Otherwise of the maximum steps, the walker stubs swing leg's toe against ground.



(a) Without stopper (normal)



(b) With stopper (stabilization)

Fig. 4. Experimental results

B. Walking on treadmill

In experiment mentioned in section 5.A, the maximum number of steps is limited. In this section, passive walker with stopper is experimented on treadmill, as shown in Fig. 5. Length and width of the treadmill are each 0.8[m] and 0.4[m]. Belt velocity is constant. In case of passive walker without stopper, it is very difficult to walk for even 2 or 3 steps. While, passive walker with stopper can walk for many steps. And, best record number of steps is 4010 steps (35 minutes) [2]!! this experiment result indicates that fixed point is generated because the walker becomes steady-state on the constant-speed belt.

As explained before, the validity of the principle is demonstrated.

[2]Number of steps is count from the video. Actual number of steps is more than 4010 steps because the video is divided in two (first video: 3439 steps, second video: 571 steps).

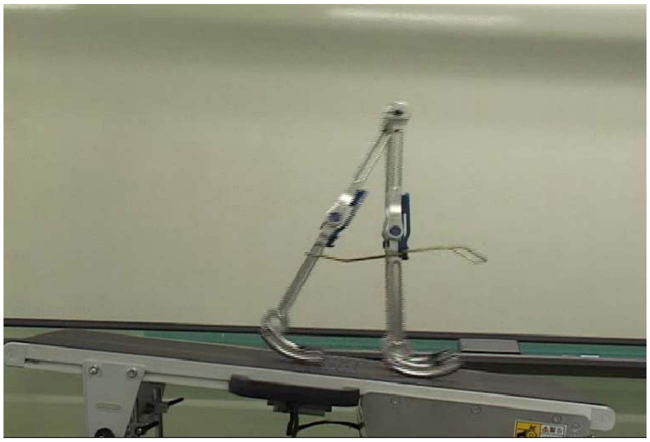


Fig. 5. Walking on treadmill

VI. Conclusions

In this paper, we focus on the generation and stability mechanisms of fixed point. The results of this study are summarized as follows:

1. We theoretically demonstrated that the fixed point can be generated when the inter-leg angle at heel-strike is constant. This generation principle is not required thought about physical structures of fixed point. And, it can give a flexibility gait by changing the value of constant inter-leg angle. In addition, energy efficiency of locomotion is not affected by the inter-leg angle.
2. We derived the mathematical global stable condition of fixed point from Jacobian matrix of an arbitrary state. We theoretically demonstrated that the fixed point of passive walking becomes global asymptotically stable if the walker steps ahead maintaining constant inter-leg angle at heel-strike.
3. Based on the principle, we made a simple improvement on passive walker with knees. High stability could be achieved by the improvement. Moreover, a great result is that the walker could walk for 4010 steps on treadmill. These results indicate the validity of the principle.

Our final goal is to establish design theory and control method of biped robot based on a principle of fixed point. Video footage of the experiment can be seen on WWW (http://drei.mech.nitech.ac.jp/~fujimoto/sano/walk_jpn.html).

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