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## Design of Dynamic Legged Robots

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## **Abstract**

Animals exhibit remarkable locomotion capabilities across land, sea, and air in every corner of the world. On land, legged morphologies have evolved to manifest magnificent mobility over a wide range of surfaces. From the ability to use footholds for navigating a challenging mountain pass, to the capacity for running on a sandy beach, the adaptability afforded through legs motivates their prominence as the biologically preferred method of ground transportation. Inspired by these achievements in nature, robotics engineers have strived for decades to achieve similar dynamic locomotion capabilities in legged machines. Learning from animals' compliant structures and ways of utilizing them, engineers developed numerous novel mechanisms that allow for more dynamic, more efficient legged systems. These newly emerging robotic systems possess distinguishing mechanical characteristics in contrast to manufacturing robots in factories and pave the way for a new era of mobile robots to serve our society. Realizing the full capabilities of these new legged robots is a multi-factorial research problem, requiring coordinated advances in design, control, perception, state estimation, navigation and other areas. This review article concentrates particularly on the mechanical design of legged robots, with the aim to inform both future advances in novel mechanisms as well as the coupled problems described above. Essential technological components considered in mechanical design are discussed through historical review. Emerging design paradigms are then presented, followed by perspectives on their future applications.

# 1

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## Introduction

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Developing legged machines that walk and run like humans and animals has long been a grand challenge in robotics. Mobility is one of the most critical, yet immature, technological components for future mobile robotics applications. Many engineers aim to develop robots capable of navigating in human environments, and legs are considered the biologically-preferred mode of ground locomotion. Current modes of ground transportation are primarily dominated by wheeled systems or variations such as tracks. Wheeled systems offer great simplicity and robustness in relatively well-structured environments and have impact in a variety of applications, whereas man-made legged machines have started demonstrating basic capabilities only recently. Although legged systems are designed to navigate rough terrains that wheeled vehicles cannot access, the performance of the legged robots to date has yet to unlock these benefits.

In order to envision critical applications for legged systems, it is important to understand the characteristics and unique advantages provided by legs at a broad scope. The next section discusses many benefits of legged systems and the special characteristics that distinguish them from more conventional means of transportation. Following this high-level motivation, Section 1.2 details a history of legged locomotion with focus on trends in design. In light

of this historical background, Section 1.3 details important underlying challenges remaining in robot design. These reflections will serve to motivate the remaining chapters of the review.

## 1.1 Legs vs. Wheels

A legged architecture for locomotion machines has attractive promise for high versatility operation, providing mobility in challenging environments. However, the complexity of legs dwarfs that of wheels due to an articulated morphology that requires additional degrees of freedom (DoFs). Are there appropriate roles for legged machines when mankind has invented (and dramatically benefited) from wheeled vehicles<sup>1</sup> throughout its history? For transportation in air, we have taken inspiration from birds and sought to embody their operation without explicitly copying the complexity of wings. With this in mind, it should not be expected that legs are universally optimal for transportation on land. However, while airplanes drastically outperform animals in nearly every aspect of flight, there are still animals on land with ground transportation capabilities that well exceed our wheeled solutions.

### 1.1.1 A Case for Legs

Comparing legged and wheeled systems is hardly black and white – the utility of these two modes of transportation depends heavily on the application. However legs offer main advantages in applications that require the use of intermittent contacts and an ability to shift the center of mass relative to the contact locations. These advantages chiefly manifest in situations that require both an ability to transverse and manipulate geometrically complex environments.

In modern ground transportation, artificial modification of the terrain is essential. Conventional wheeled vehicles maintain continuous contact with the terrain, and their design assumes good conditions for the roads to accomplish this. The chassis of vehicles is connected to the wheels via a passive suspension mechanism, allowing toleration of variations in roadway materials (gravel, dirt paths, asphalt, etc.) as well as roadway geometry. Through

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<sup>1</sup>Here, wheeled vehicles represent all vehicles that use wheels or tracks as a main means of transportation

this approach, wheeled systems can travel faster than most legged animals (ignoring scale differences) when the ground is fairly flat. Novel suspension designs, such as in the wheeled SHRIMP robot (Lamon et al., 2004), increase the ability to attenuate disturbances from contact irregularities but still maintain continuous contact with the terrain.

A practical middle ground between legs and wheels is the use of Whegs (Schroer et al., 2004), which combine the simplicity of wheels with discrete contact interactions provided by feet. Whegs designs were largely inspired by the RHex family of robots (Saranli, 2001), and can be described as discrete wheels. For instance, the rimless wheel represents the simplest embodiment of the Whegs concept. This morphology allows Whegs to change contacts from step to step and traverse varied terrain that is unable to be negotiated by wheels of the same radius. While Whegs can be seen as a middle ground between legs and wheels in terms of mechanical design, their maximum performance envelope represents a compromise between legs and wheels as well. Without articulation in the limbs, Whegs inevitably lack critical versatility for contact reconfigurability.

Legged machines provide improved mobility over wheeled vehicles chiefly through an ability to reconfigure and exploit discrete interactions in a large workspace. This ability to make and break contacts is important where the roughness of the ground varies, or continuous contact paths are unavailable. Whether for locomotion over bouldered grounds, stiff slopes, or even sheer cliffs, the ability to radically modify support structure from step to step can be critically necessary to negotiate the most extreme terrains. A large workspace amplifies these abilities, providing valuable additional options.

The ability to reconfigure contact geometries in legged machines further eliminates the need for a wide support polygon that stabilizes most wheeled systems solely based on their fixed geometry. Since the geometry and properties of contacts influence the ability to provide friction-limited forces, reconfigurable contacts allow for the generation of propulsive forces in a wider range of directions. This advantage allows legged systems to manage dynamic stability while subject to more narrow footprint requirements. Even in challenging passages found in disaster environments or a packed urban warehouse, legged systems can maintain balance despite their high center of mass and using only small footprints through the versatility of legs.

Articulation of the limbs also offers an ability to dynamically reconfigure the center of mass for high-power manipulation. In disaster response situations, for instance, being able to maintain balance with a high center of mass can be greatly advantageous. Simply opening a spring-loaded door requires high force generation at around 1.2 m above the ground where the door knobs are located. If the robot's center of mass is low, this task can be extremely difficult to achieve by solely relying on static stability. Using our dynamics, humans can generate much higher forces than in a static body posture. Throwing, kicking, and batting motions of humans well represent our ability to shift the center of mass of the body to generate momentum and thus generate greater power output. Although much less powerful, the mundane daily task of opening a spring-loaded door may require mastering the basics of such dynamic movements.

Until we mature the technologies for legged robots, it may be meaningless to argue which mode of transportation can be most useful for a given application. What is clear is that we need to advance legged locomotion technologies in order to develop mobile robots capable of operating in a wider range of environments. Across automation in agriculture and construction, assistance in the home, exploration of distant planets, search and rescue, or disaster response, mastering legged locomotion is a critical and logical step towards many future applications of mobile robots.

### **1.1.2 Steps towards future applications: A need for design-centered thinking**

Advancing these legged technologies will require addressing great complexity in design. A car needs two active degrees of freedom, propulsion and steering, which requires two actuators. In contrast, a legged system requires at least three degrees of freedom per leg to properly select and manage contact interactions in 3D. This complexity in structure drives up cost from many components. While this curse of complexity manifests in the mechanical design, a similar challenge accompanies the design of control algorithms, sensing systems, and other coupled components of these systems. To realize the full capabilities of legged machines, integrative challenges must be mastered across these intersecting domains. Ultimately, lagging capabilities in any of these domains may limit legged systems from achieving their full potentials.

It is a main hypothesis, however, that the treatment of mechanical design within locomotion robots is a limiting factor of their performance in current hardware. While better control algorithms will make current robots more capable, improvements in our design methodologies will yet simplify control and allow new levels of proficiency as mobile legged machines emerge from the laboratories and are let loose in real work. The past decades have provided a renaissance in the design of legged robots, and lend great credibility to this vision. The next section provides a review of this previous work. It is intended to provide a window into both how far the field has progressed as well as the challenges that remain to achieve biologically proven levels of legged performance.

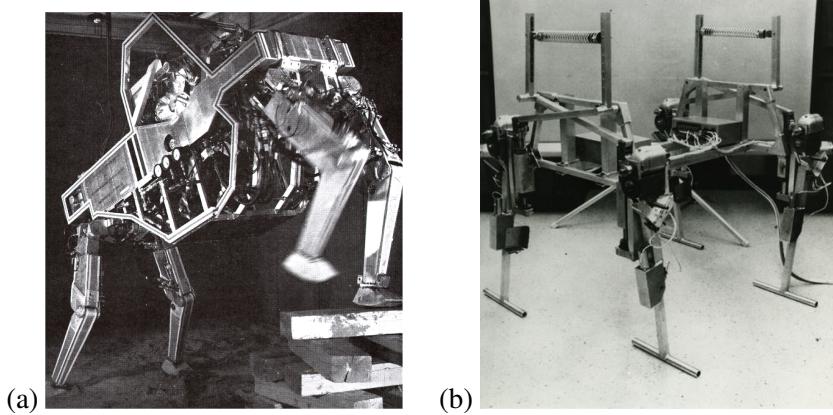
## 1.2 A Brief History of Legged Robots

The design of machines with legged mobility has been a pursuit of engineers for over a century. Dating back to as early as the mid 1800's, efforts first concentrated on the use of clever linkage-based designs to mechanically produce fixed leg motions. The celebrated Russian mathematician Chebychev is credited with the earliest of these designs (Lucas, 1894), with similar ideas appearing in US patents by the late 19th century (Rygg, 1893) and making their way into machines constructed more recently (Morrison, 1968).

While many of these systems were capable of rudimentary locomotion on prepared surfaces, their fixed gait patterns prevented truly adaptive locomotion and limited the classes of terrain they could traverse. Starting in the early 1960s, however, a shift began to occur. Rather than focusing on linkage-based designs with fixed limb trajectories, researchers started to pursue methods for active control, and slowly, adaptive legged machines began to emerge.

### 1.2.1 The Beginnings of Adaptive Legged Machines

In 1962, the General Electric Corporation and R.S. Mosher began work on a quadruped that was unlike any of its predecessors. The GE Walking truck (Mosher and Liston, 1968) as shown in Figure 1.1 was a hydraulically powered, 12 degree of freedom quadruped weighing 1400 kg. Without complex linkages to coordinate the motion of its limbs, the Walking Truck was designed to be controlled by a skilled human operator.

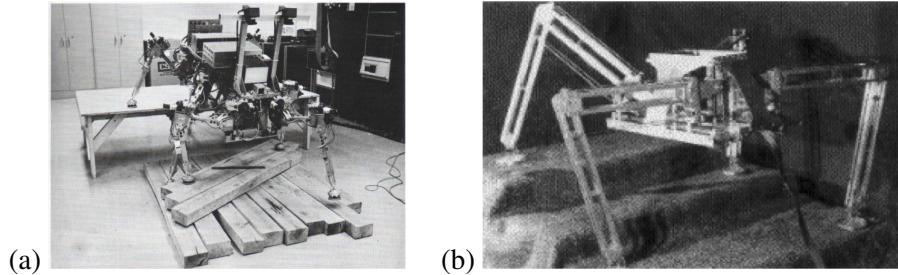


**Figure 1.1:** (a) The GE Walking Truck and (b) Phony Pony, two of the first legged robots.

The teleoperation interface for this landmark system was truly ahead of its time. All 12 degrees of freedom were commanded by a human driver using a series of handles and pedals for their hands and feet. The system also provided the operator with force feedback which enabled response to obstacles or other terrain disturbances. After roughly 20 hours of operator training, the system was capable to climb railroad ties and walk along at 5 mph (Raibert, 1986).

Rather than rely on a skilled human operator, R. McGhee of the University of Southern California realized that an automated system could instead be used to coordinate the rhythmic motions of locomotion. Born out of his collaborative theoretical work with R. Tomovic (Tomovic and McGhee, 1966), McGhee created the first legged machine to apply finite-state automata to robot walking (McGhee, 1968; McGhee and Frank, 1968). His robot, the Phony Pony (Figure 1.1), weighed 50 kg and consisted of 8 DoFs driven by electric drill motors. Using digital logic based on flip-flops, the system could perform a quadruped crawl and a diagonal walking trot.

It wasn't soon after until computer control of legged machines became a possibility. In 1977, following a move to the Ohio State University (OSU), McGhee built the OSU hexapod (Figure 1.2), the first computer-controlled walking robot (McGhee, 1985). His machine had 18 electrically actuated DoFs that were coordinated by the computer, which mainly used its processing power to solve kinematic equations and ensure static stability of the ma-

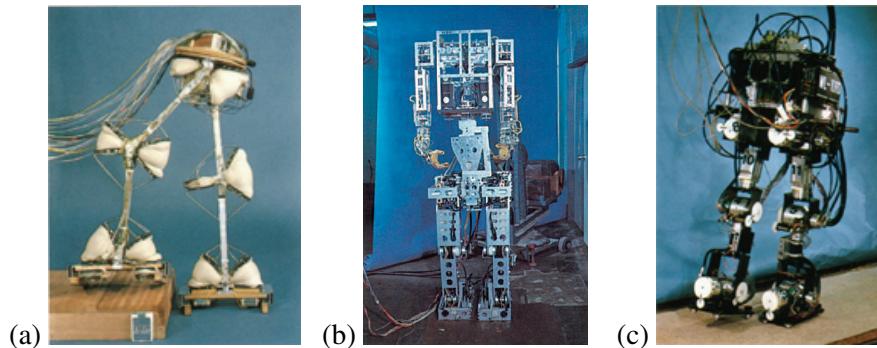


**Figure 1.2:** (a) The OSU Hexapod (b) Hirose's PV-II quadruped featuring a PANTOMECH leg.

chine. The hexapod was able to perform a variety of basic gaits and showed the ability to turn, walk sideways, and negotiate locomotion over piles of lumber. The nascentcy of computer control and available balance control theories dictated a great deal of the design in these systems. Gaits were designed to be statically stable, that is, their center of mass (CoM) was design to remain over their base of support at all times. To simplify application of this strategy, these early computer controlled machines were designed with a wide support base, not unlike their wheeled counterparts.

Despite the power of this early computer control approach, significant computational resources were required for basic kinematic computations. Drawing inspiration from the early days of legged machines, Shigeo Hirose, at the Tokyo Institute of Technology, showed how clever mechanical design could be revived to reduce the computational needs of adaptive machines. Roughly, he could embed portions of the kinematic computations into the design of his mechanisms. Hirose developed a three-DoF pantograph leg mechanism the PANTOMECH (Figure 1.2), where each actuator produced approximately linear motion of the foot in the primary Cartesian directions. Freeing up the control from kinematic computations, Hirose's quadrupeds (Hirose, 1984) could focus control on higher-level goals, enabling his machines to climb up and down stairs and handle obstacles. Hirose's machines are a representative early example of how strategic changes in mechanical design can alleviate the burdens on control towards unlocking new levels of performance.

During this period of active research on quadrupedal and multi-legged machines, great strides were being made in the bipedal realm with research



**Figure 1.3:** (a) WAP-3 (b) WABOT-1 (c) WL-10RD. Courtesy of the Humanoid Robotics Institute, Waseda University, Tokyo.

on quasi-static walking. Professor Ichiro Kato, a pioneer of robotics in Japan, began creating his famous bipedal machines in 1967 (Lim and Takanishi, 2007). The pneumatically actuated 2D biped WAP-1 and 3D biped WAP-3 (Figure 1.3) were capable of statically stable bipedal locomotion, with WAP-3 representing the first time this was accomplished in 3D.

Only 6 years after the start of his work, in 1973, Kato created the first full-scale anthropomorphic robot WABOT-1 (Kato et al., 1973). This hydraulically actuated machine (Figure 1.3) was capable of static walking and was equipped with artificial ears and eyes to detect distances to objects. In 1980, Kato developed the 10 DoF biped WL-9DR which could execute quasi-dynamic walking (during weight transfer between feet), taking an important step away from the current practice of focus on static stability for the first time to date.

Collectively, these advances across the globe provided the cornerstone for adaptive walking robots. It wasn't long, however, until new designs for dynamic legged machines disrupted much of this previous thinking.

### 1.2.2 Dynamic Balance and Raibert's Machines

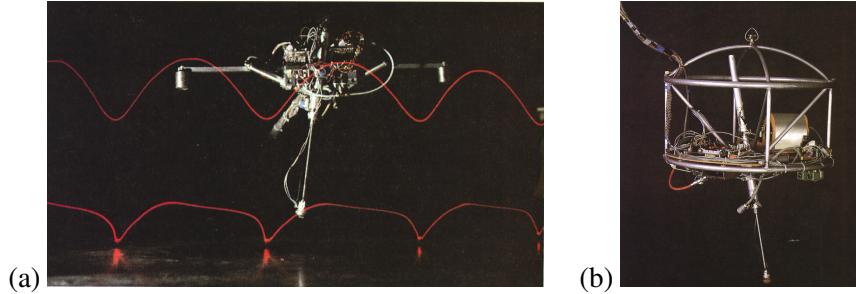
Prior to the 1980s, adaptive legged machines relied largely on their static stability to maintain balance. Whether through large feet or multiple limbs in contact, these early machines took great effort to place their center of mass (CoM) over their base of support. Although this strategy did not rigorously

guarantee that the systems would not tip over during motion, when confined to their slow conservative movements, balance was effectively ensured.

Ultimately, however, this strategy limited early machines. Indeed, humans and animals often purposefully "tip" over their support to reach far away footholds or to let gravity do work to propel the system forward. Even the most moderate of human walking gaits are marked by periods of static instability where we roll over our foot naturally, knowingly placing ourselves in a state where we require the next step to prevent a fall. This notion of a type of balance that requires continuous motion, coined dynamic balance by Marc Raibert, was a new idea that fueled a series of groundbreaking machines in his lab during the early 1980s. Machines that remain on the ground do possess the opportunity to study this type of balance. Raibert, however, set forth to design machines that would enable him to study dynamic balance in the extreme setting – the case where legged machines also experience flight. Creating machines which could fly through the air and regain stable contacts however, required as much redesign mechanically as it did in control.

In 1981, Marc Raibert founded the leg lab at Carnegie Mellon University and began work on a new class of hopping machines. Although the first hopping robot was actually built in Japan by Matsuoka (1980), Matsuoka's 2D machine simplified control, operating in low effective gravity by laying on shallow inclined table. Raibert's machines thus were the first to regulate balance during ballistic flight. The first machine he constructed at CMU was a single-legged 2D hopper (Raibert and H. B. Brown, 1984), shown in Figure 1.4, weighing 8.5 kg with height around 50 cm. The machine featured point feet and springy prismatic limbs, providing natural dynamics with a paradigm shift away from the higher-impedance designs of previous quasi-static locomotors. Raibert's machines again illustrate the degree to which paradigm changes in control and design have been historically intertwined.

As he explains in his book (Raibert, 1986), this machine was designed in order to focus on the general mechanisms of legged balance, without the need to focus on combinatorial issues of leg sequencing that had consumed much academic work in the preceding decades. It was Raibert's idea that general mechanisms to control a single leg should be immediately applicable to the control of so-called one-foot gaits such as human running, where a single foot was in stance at any given time. He further reasoned that these principles



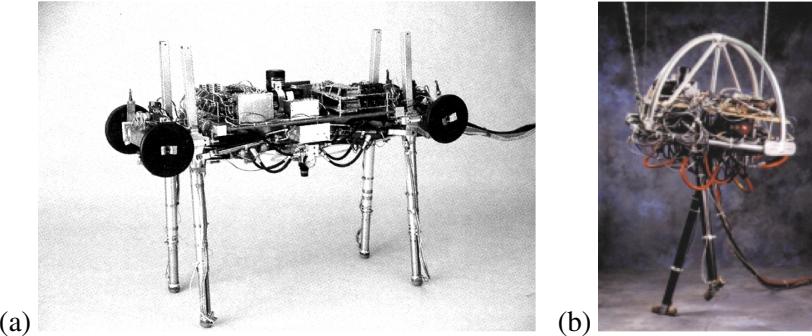
**Figure 1.4:** (a) Raibert’s 2D hopper and (b) 3D hopper.

of balance could be used in gaits with multiple feet in stance at a time, such as a trot, provided that these legs work together to simulate the operation of a single leg. These ideas ultimately came to fruition through a series of impressive machines.

Raibert’s original single-leg 2D hopper was effectively an actuated spring-loaded inverted pendulum (SLIP) model. The design consisted of an actuated pneumatic leg spring that could store and release energy passively during stance through compression and decompression of an air chamber. Another pneumatic actuator controlled the angle of this virtual leg. Two counter weighting masses were attached at a distance on the body of this machine, increasing its angular inertia, and placing the net CoM roughly along the axis of the leg spring. A floor-attached boom was used to approximate planar motion by constraining the machine to move in a sphere. This breakthrough machine was able to maintain dynamic balance by decomposing its control law into three roughly decoupled parts: hopping control, forward speed control, and body attitude control. With this three-part approach, the machine could travel up to 2.6 mph and was able to jump over small obstacles.

Following the success of this platform, Raibert built a 3D version of the machine (Figure 1.4) and employed a 3D generalization of his three-part control decomposition (Raibert et al., 1984). This one legged machine again utilized a pneumatic leg actuator for its stroke. The hopper was robust to push disturbances and was able to move fully unconstrained in 3D at speeds of up to 4.5 mph.

Around the same time as Raibert’s hoppers, Kato’s lab back in Japan was also making great strides on dynamic locomotion. Using the zero-moment



**Figure 1.5:** (a) Raibert’s quadruped and (b) 3D biped.

point (ZMP) criterion of Vukobratovic (Vukobratovic and Juricic, 1969; Vukobratović and Stepanenko, 1972), Kato and Takanishi realized fully-dynamic ZMP walking for the first time in the world on the WL-10RD in 1985 (Takanishi et al., 1985). This 3D biped (Figure 1.3) had 12 DoFs driven by hydraulic actuators, was 1.43m tall, and weighted 84.5 kg.

In 1986, Raibert moved the leg lab to MIT and began work on his multi-legged machines. Raibert and Hodgins demonstrated the applicability his previous design and control mechanisms on a 2D planar biped that was capable of top speeds of 9.5 mph (Hodgins et al., 1986). The planar biped had two telescoping legs driven by hydraulic actuators with passive series air springs. With the basic mechanisms of balance addressed through previous work, Hodgins concentrated on methods to modify the gait to hop over uneven terrain (Hodgins and Raibert, 1991) and to perform an open-loop flip (Hodgins and Raibert, 1990).

Following the success of the planar biped, Raibert constructed a 3D quadruped. The quadruped leg design mirrored that of its bipedal predecessor with hydraulically actuated prismatic legs positioned by a set of lower strength hydraulic cylinders. Even prior to construction, in Raibert’s mind, quadruped trotting was already a solved problem. Raibert reasoned that trotting is like having a biped at each instant, and two-legged hopping can be treated as single virtual leg, so you should be able to run on four legs as if they are one. By coordinating multiple legs of the quadruped to act as a single virtual leg, the three-part decomposition was in fact able to stabilize Raibert’s quadruped (Raibert et al., 1986).

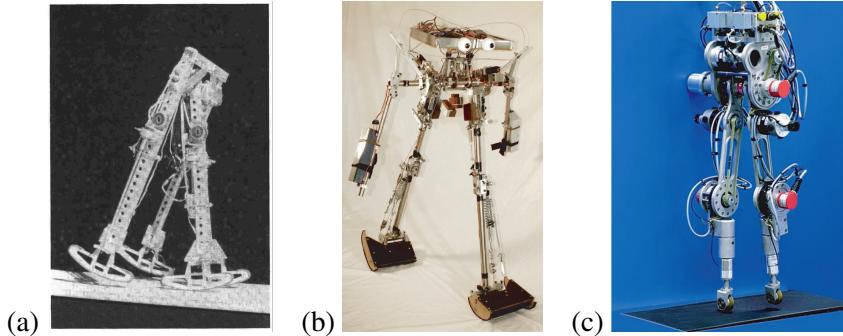
In his last years at MIT, Raibert worked with Robert Playter and created one of his most impressive dynamic machines to date. The 3D biped, shown in Figure 1.5, was able to run outside on grass, pull its operator along in a wheeled cart, and even execute a running somersault on a treadmill (Playter and Raibert, 1992). Like many of Raibert’s designs, control was facilitated by placing the hip joints nearly coincident with one another and the CoM, and by designing a high inertia torso in comparison to the legs. These strategic design decisions reduced influences of the leg motions on the body and prevented impulses along the leg from creating unwanted moments on the torso. This design, in part, enabled Raibert’s biped to execute a range of dynamic behaviors that still, in many ways, remain the gold standard to which other dynamic bipeds are compared.

### 1.2.3 Iterating Towards the State of the Art - Dynamic Legged Machines in the Wake of Raibert

#### Passive Dynamic Walking

Just as Raibert’s machines were demonstrating the capability to actively control dynamic balance, a provocative new idea was introduced by Tad McGeer from Simon Fraser University. Similar to how clever kinematic mechanisms were sought to simplify static locomotion in the early days of the field, McGeer carefully designed a completely passive planar walking machine (Figure 1.6) that led to a naturally stable dynamic gait down a gentle slope (McGeer, 1990). While relying simply on the energetic interplay between gravity and inertia, this passive machine seemed arguably the most lifelike when compared with any robot to date.

Passive dynamic walkers, by nature, are not able to walk on level terrains where inevitable dissipations prevent continuous steady state locomotion. More recently, a number of minimally actuated walkers have been constructed to glean the energetic benefits of passive dynamic designs while retaining the capability to locomote on flat or moderately inclined surfaces (Collins et al., 2005). Collins and Ruina (2005) designed the Cornell biped (Figure 1.6) which has an energetic efficiency on par with human walking. This system has five internal degrees of freedom and is powered by electric motors and springs that are primarily responsible for ankle push off to restore energy in each stride.



**Figure 1.6:** Bipeds on the spectrum of passive design. (a) Tad McGeer’s passive 2D walker (b) A 3D, minimally actuated walker designed by Collins and Ruina and (c) RABBIT, an underactuated point-foot biped.

The MIT learning biped developed by Tedrake et al. (2005) utilized passive dynamic principles to achieve a nominally passive baseline gait. This design simplified an online stochastic gradient descent algorithm enabling it to automatically discover actuated walking control policies from a blank slate in a matter of minutes. Martin Wisse developed a set of robots (Wisse et al., 2007; Hobbelen et al., 2008) inspired by McGeer’s designs with once-per-step active actuation that improved gait robustness in comparison to a purely passive approach.

More generally, a number of underactuated (Spong, 1998) walking machines have been constructed where the number of actuators are less than the number of degrees of freedom. Without the ability to manipulate the entirety of system dynamics at each instance, these systems must rely on mechanical couplings amongst the many degrees of freedom, much like in passive walkers. As one prominent approach, systems utilizing the framework of Hybrid Zero Dynamics (Westervelt et al., 2003) have demonstrated efficient, stable walking gaits in planar walkers controlled by DC motors. By optimizing gaits that are as close to passive as possible (as measured through actuated torque or work-based metrics), these methods are able to utilize natural dynamics to attain stability in spite of under actuation. Westervelt et al. (2004) showed the applicability of the framework on the position controlled 5-link planar biped RABBIT (Figure 1.6). Martin et al. (2014) have recently shown that the incorporation of curved feet, as employed in the original passive dynamic

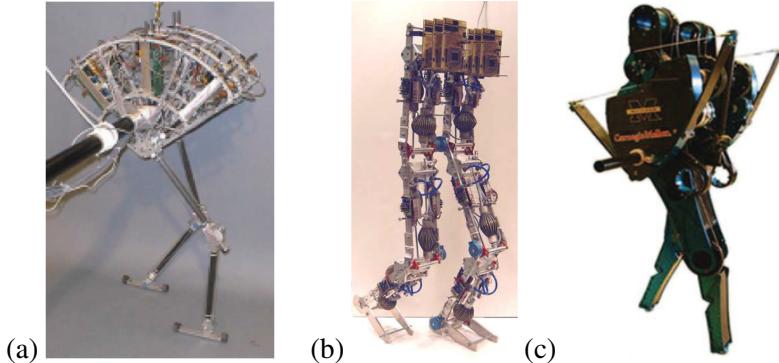
walkers, can further push these methods towards efficient gaits in experimental machines. In practice, however, the impedance of actuators employed in physical robots may limit the existence of any favorable natural dynamics for HZD-based control laws to leverage.

### Compliantly Actuated Bipeds

In 1992, Marc Raibert left the leg lab to found Boston Dynamics, pursuing the advancement of his legged machines to higher and higher levels of technological readiness. Following Raibert's departure, Gill Pratt inherited the MIT leg lab and began new lines of research to move away from inefficient hydraulic actuation technologies. Gill Pratt formalized a new actuation paradigm called series elastic actuation (Pratt and Williamson, 1995), in an attempt to provide a low-impedance high-power-density electric drive. Because of the low impedance of these actuators, they did not override the natural dynamics of the underlying mechanisms that they control, enabling those natural dynamics to potentially be exploited (Pratt, 2000b). Although Raibert's machines were able to accomplish this goal through the existence of series air springs, their reliance on messy, inefficient hydraulics in series was seen as an unnecessary downside.

Pratt's SEAs (Pratt and Williamson, 1995) were capable of 300 lbs. of linear force and had a force-control bandwidth of 20 Hz. SEA designs were incorporated into his robots Spring Turkey and Spring Flamingo (Figure 1.7), enabling them to execute continuous planar walking while attached to a boom (Pratt et al., 2001). Taking advantage of the force control capabilities of the SEAs, Pratt was able to implement virtual impedance behavior through a closed-loop control approach named virtual model control.

Series elastic actuation principles continue to find their way into more recent machines. The University of Michigan's MABEL biped (Figure 1.7), designed by Jonathan Hurst (Park et al., 2011), incorporated large nonlinear leaf springs into its design. This design gave rise to natural SLIP-like dynamics that could be leveraged through HZD control (Poulakakis and Grizzle, 2009). These concepts were able to be applied to generate walking at 3.4 mph (Sreenath et al., 2011), robust walking over uneven terrain through reactive gait modification (Park et al., 2013), and running at 6.8 mph (at the time a speed record for a kneeled bipedal machine) (Sreenath et al., 2013). In



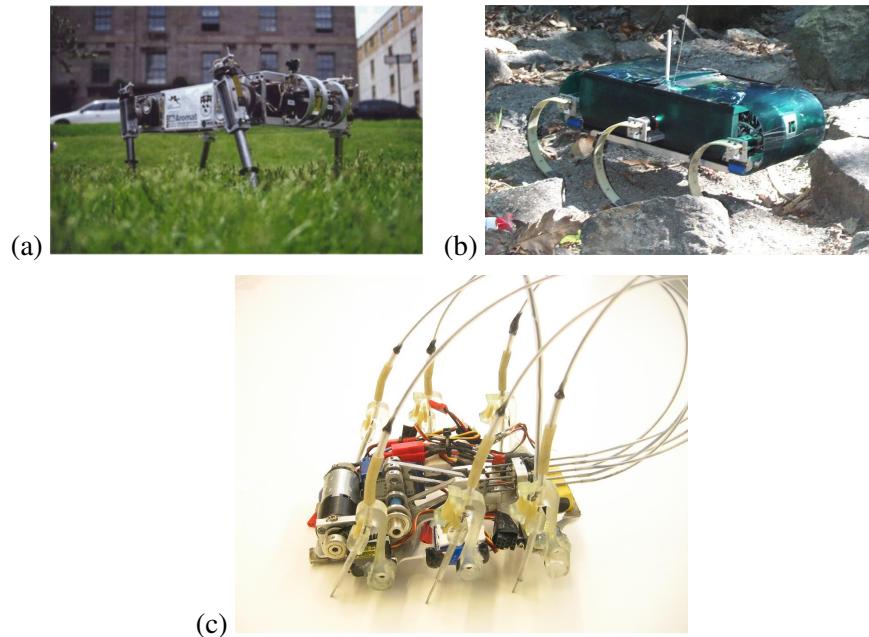
**Figure 1.7:** Compliant bipeds (a) Spring Flamingo with linear SEAs and (b) Lucy with variable compliance pneumatic artificial muscles and (c) MABEL actuated using a nonlinear leaf spring to provide compliance along the virtual leg.

comparison to DC motors alone, SEAs have also shown to enhance performance in hopping bipeds (Knox and Schmiedeler, 2009; Curran et al., 2009; Liu et al., 2011) and have been important in the design of modern passively compliant quadrupeds (Hutter et al., 2010, 2011) and humanoids (Tsagarakis et al., 2013).

Recently, other methods of achieving compliance in bipedal machines have been proposed (Vanderborght et al., 2008a; Verrelst et al., 2005) using pleated pneumatic artificial muscles. These actuators, used in the Lucy biped (Figure 1.7), can actively change their compliance, providing opportunity to tune the passive dynamics of the system and reduce energetic costs (Vanderborght et al., 2008b). Lucy was capable of walking and executing a jump, although running was never demonstrated.

### Quadrupeds and Multilegged Machines

Ideas to reduce actuation and rely partially on compliant passive dynamics for running permeated quadruped designs in this time as well. Scout II, designed by Papadopoulos and Buehler (2000), was potentially the most influential of the machines that followed. Scout II (Figure 1.8) weighed just under 21 kg and was 0.55 m long. The machine included very minimal actuation, with only a single hip actuator per leg along with passive leg springs. On-board power made Scout II the first self-contained quadruped capable of running,



**Figure 1.8:** Minimally actuated multi-legged machines (a) Scout II (b) RHex and (c) iSprawl.

with a top speed of 1.3 m/s (Poulakakis et al., 2005, 2006). This system also claimed the notable mark of also being the first to demonstrate galloping in a quadruped robot (Smith and Poulakakis, 2004), although its minimal actuation provided little authority over heading.

Other multilegged machines have been created using minimal actuation as inspired by principles in nature. The hexapod RHex, originally designed by Saranli et al. (2001), consisted of a single rigid body with six compliant legs, each driven by a single actuator. The design (Figure 1.8) was motivated by clock-driven, mechanically self-stabilizing, compliant sprawled-posture mechanics proposed by (Full et al., 1998) as inspired by observations in the cockroach *Blaberus discoidalis*. With its recirculating compliant legs, RHex was capable to travel at one body length per second over height variations exceeding its body clearance (Saranli et al., 2001). Despite its apparent lack of similarities to Raibert’s original hopping machines, the design of RHex similarly has been shown to anchor SLIP dynamics (Altendorfer et al., 2001).

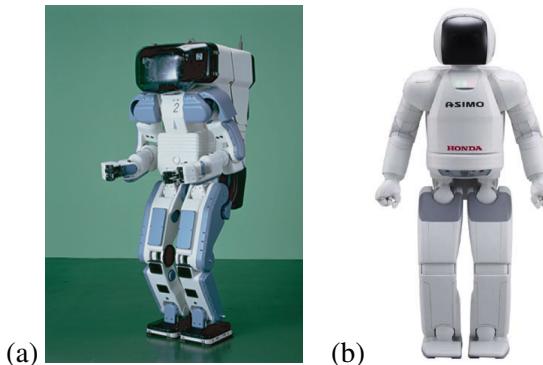
Inspiration from experiments with the cockroach informed the design of the Sprawl series of robots around this same time as well. Using shape deposition manufacturing (SDM), these robots out of the lab of Mark Cutkosky were able to embed actuators and sensors into structures with locally-varying compliance and damping (Cham et al., 2002; Kim et al., 2006). Sprawlita was capable of running at 3.5 body lengths per second using off board pneumatic pumps, while iSprawl (Figure 1.8) was capable of running at 15 body lengths per second (2.3 m/s) with a completely autonomous design. More recent minimal designs using the Smart Composite Microstructures (SCM) process have produced the X2-VelociRoACH (Haldane and Fearing, 2015) which is capable of running at 4.9 m/s (approximately 45 body lengths per second, which represents a current record).

### **Humanoids in the ZMP domain**

On the opposite end of the spectrum, the development of high DoF anthropomorphic humanoid systems has attracted a vast amount of research in the past 20 years. Bipedal systems without an upper body offer the opportunity to focus on the balance, but do not require difficult orchestration to coordinate the upper body with the legs. Despite this challenge, legged machines will ultimately need to be just as adept at manipulation as locomotion in order to interact meaningfully with the world. The upright posture of humanoids facilitates this interaction with a world designed to accommodate human forms, further driving the field forward.

With these among other consumer market motivations, HONDA launched a secret program to build a humanoid biped in 1986. Nearly a decade later, in 1998, Honda unveiled their humanoid robot P2 (Hirai et al., 1998). Using harmonic drives with a high-torque capacity and specially cast high-rigidity mechanical structures, P2 (Figure 1.9) was the first humanoid with on-board power and computing capable of stable walking. P2, much like its predecessors from Kato's lab, relied on the use of the ZMP for its walking control.

In the years following P2, Honda has continued to refine and improve its humanoid systems. Honda's ASIMO (Figure 1.9) is potentially the most well known humanoid to date (Sakagami et al., 2002) with current versions capable of walking, hopping, running amongst an impressive array of non locomotion-based intelligences (Takenaka et al., 2009d,a,b,c).



**Figure 1.9:** Evolution to Honda’s ASIMO (a) Original P2 (1998) to (b) Modern ASIMO (2009).

DC motors with harmonic drives have been a workhorse for many other humanoid designs. The HRP-2 (Kaneko et al., 2002b), SONY’s QRIO (Nagasaka et al., 2004), and HUBO (Park et al., 2005) have all converged to a similar actuation paradigm. All of these designs are particularly amenable to position control and employ force sensors on the feet which are used to measure the ZMP. All of these systems have shown the capacity to execute a running gait (Kajita et al., 2007; Nagasaka et al., 2004; Cho et al., 2009; Takenaka et al., 2009d), where running is defined as a bipedal gait having one foot in contact at a time with a flight phase between footfalls. While these systems technically execute a run, their high impedance actuators cause large sensitivity to the impact velocity of the foot, and their flat-footed gaits resemble conservative ZMP-based walking more closely than graceful compliant running gaits observed in nature (Blickhan, 1989).

#### 1.2.4 Today’s Machines

Today, laboratories and research centers around the world have access to dynamic legged machines actuated by high-power DC motors and hydraulics. These robots continue to push the boundaries of speed and performance through advances in materials, design, and control.

## Quadrupeds

Since creating Boston Dynamics (BDI), Raibert and colleagues have continued to innovate with their quadrupedal and bipedal machines. Using an on-board 15 HP internal combustion engine to power hydraulic pumps, Big Dog (Raibert et al., 2008) was created to be a rough-terrain robot capable of walking, running, climbing, and carrying heavy loads. The machine was about 3 feet long, 2.5 feet tall, and weighed 240 pounds. The machine could trot at 4 mph, walk across rubble, snow, and mud with slopes up to 35 degrees. Designed under DARPA funding, the machine was also able to carry a 340 pound load and had the capacity to autonomously follow a designated human leader.

BDI has introduced other groundbreaking descendants of Big Dog. The BDI cheetah, unveiled in 2012, was capable of running up to 28.3 mph with off-board power while constrained in 2D by a boom. The BDI cheetah had an articulated back that flexed back and forth on each step. In 2013, BDI released a self-contained version of the BDI cheetah, named WildCat, that was able to run untethered in 3D at speeds of up to 16 mph. WildCat could execute high-speed turns, although detailed specifications on their performance or the design have not been released.

Following acquisition by Google, BDI unveiled their smallest, most nimble quadruped, Spot, in early 2015. Very little is known about Spot other than a four sentence caption provided by BDI on their video release "Spot is a four-legged robot designed for indoor and outdoor operation. It is electrically powered and hydraulically actuated. Spot has a sensor head that helps it navigate and negotiate rough terrain. Spot weighs about 160 lbs.". This new robot is capable of walking up stairs and slopes, and has balance reflexes to respond to disturbances such as lateral kicks.

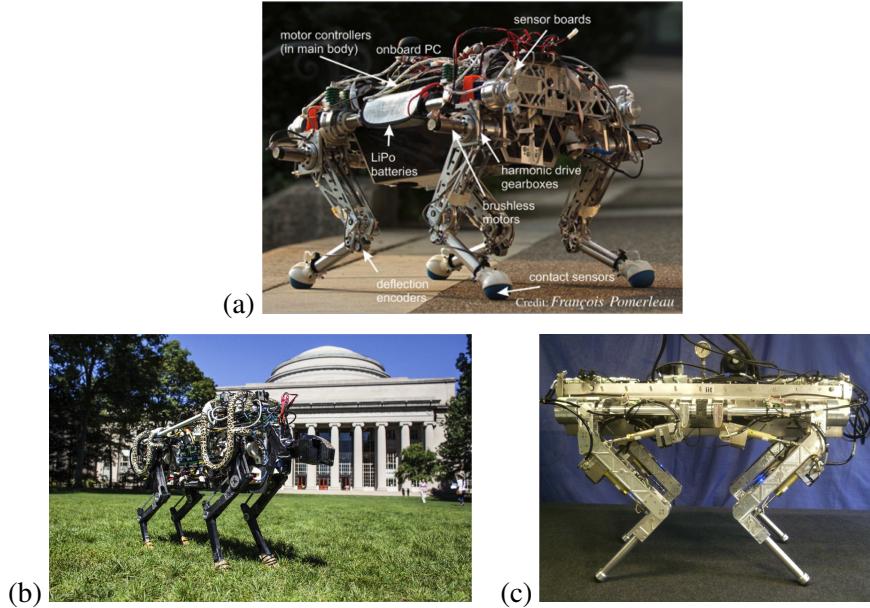
A variety of other dynamic quadrupeds have recently been developed in the academic realm, many outperforming BDIs machines in certain areas. Roland Siegwart's group at ETH recently designed the StarlETH quadruped (Hutter et al., 2014). StarlETH (Figure 1.10) is actuated by SEAs at its joints and is about 0.5 m long with a total weigh of 25 kg. The robot is capable to trot at speeds up to 0.7 m/s, with a cost of transport (COT) of 1.7. This dimensionless cost of transport measures how much energy it takes to move one kilogram one meter ( $COT = \frac{E}{mgd}$ ). StarlETH drastically outper-

forms BigDog, which has an estimated COT of 15. StarLETH was capable of a broad range of gaits (Gehring et al., 2013) and could trot over piles of lumber (Gehring et al., 2014). Marco Hutter, one of the original designers of StarLETH, recently introduced a more modular version of StarLETH, ANYmal (Hutter et al., 2016), which is able to climb steep stairs ( $50^\circ$  inclination) and trot dynamically at 0.8 m/s.

The Cheetah-Cub robot at EPFL utilized a variety of parallel and series compliant mechanisms to provide self-stable locomotion over a range of speeds (up to 1.4 m/s) in a small quadruped (Sprowitz et al., 2013). Their design includes a spring-loaded pantograph mechanism inspired by the spring-loaded inverted pendulum template observed in biology (Full and Koditschek, 1999). The emergent self-stability provided by the leg mechanisms enabled central pattern generators (CPGs) to be used to generate kinematic targets for leg trajectories, without higher-level reflex mechanisms as used in previous CPG studies (Kimura et al., 2007). Despite the use of compliance in the design of Cheetah-Cub, the use of high-geared RC servos led to a minimum cost of transport of 6.9 in experiments, over 15 times that of simulation predictions. The use of compliant actuation strategies with lower-impedance servo drives represents an interesting area of future potential for designs in the spirit of Cheetah-Cub. Although smaller than many of other quadrupeds described in this section, this 1.1 kg robot is comparatively inexpensive and safe to handle, making it suitable to test prototype leg designs and bioinspired control strategies.

The MIT Cheetah robots have further pushed the boundaries of energetic efficiency with their unique high-force proprioceptive actuators (Seok et al., 2015). By incorporating large gap radius brushless DC motors into the leg design (Seok et al., 2012), the MIT Cheetah robots are able to obtain high torque density actuation without the traditional need for a large, lossy, staged gearbox. The MIT Cheetah 1 was constrained to operate in 2D and was capable to run up to 6 m/s with a COT of 0.5, on par with the energetic efficiency of actual cheetahs. Due to the unique actuator design, the Cheetah is able to emulate passive springs and dampers without the need to incorporate them physically into the design.

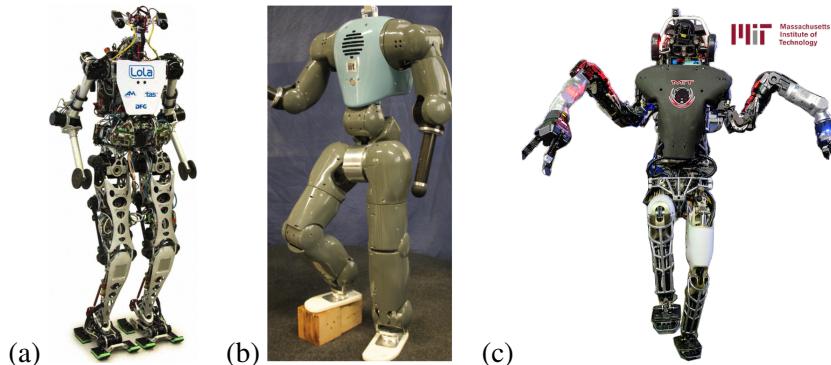
In a subsequent redesign, the MIT Cheetah 2 (Figure 1.10) has achieved a COT of 0.47 while running unconstrained in 3D with on-board power. Chee-



**Figure 1.10:** Modern Quadrupeds (a) StarlETH (b) MIT Cheetah 2 and (c) HyQ.

tah 2 is capable to run at speeds up to 6 m/s, and has shown the ability to autonomously jump over obstacles (Park et al., 2015a,b). The feat of landing an autonomous running jump had not previously been demonstrated in a experimental quadruped machine. While due in part to its control system, the ability to land the jump successfully is partially enabled by the backdrivability of the leg design which prevents otherwise prohibitively large impulses from causing structural damage.

Perhaps most similar in design to Big Dog and its predecessors, the hydraulic quadruped HyQ (Figure 1.10) was recently built in the advanced robotics department at IIT by Claudio Semini (Semini et al., 2011). HyQ is equipped with a combination of 12 torque-controlled hydraulic and electric actuators, is 1 m tall, and weighs around 90kg. While through a completely separate design paradigm from the MIT Cheetah robots, HyQ is also able to emulate passive springs and dampers with careful closed-loop force control on its hydraulic DoFs (Semini et al., 2015). Through the incorporation of vision, this platform has shown the capacity to navigate a variety of challenging terrains (Bazeille et al., 2014; Winkler et al., 2014).



**Figure 1.11:** Modern Humanoids (a) LOLA, actuated by DC motors, (b) COMAN, actuated by DC Motor SEAs, and (c) ATLAS, actuated by high-power hydraulics.

### Humanoids and Bipedes

Advances in DC motor, SEA, and hydraulic servo valve designs have also pushed the envelope in the performance capabilities of bipedal and humanoid machines. Taking insights from the design of JOHNNIE (Gienger et al., 2001), researchers at the Technical University of Munich designed the 25 DoF humanoid LOLA (Lohmeier et al., 2006) to study fast human-like walking. LOLA (Figure 1.11) is 180 cm tall and weighs approximately 55 kg. The robot is driven by modular brushless motor modules and features lightweight 7 DoF legs to enable dynamic performance sufficient for fast walking (Lohmeier et al., 2009). LOLA has been able to walk at speeds of up to 3.34 km/h. Modular design strategies have also been employed in the construction of TORO (Englsberger et al., 2014), DLRs new torque-controlled humanoid, which uses similar integrated DC motor and torque sense hardware to the DLR lightweight robot (Hirzinger et al., 2002).

Series elastic actuation has begun to be incorporated into many modern humanoid designs. Both the Valkyrie (Radford et al., 2015), built at NASA Johnson space center, and COMAN (Tsagarakis et al., 2013), built at IIT, include joint SEAs that enable naturally compliant operation and joint-torque control. Valkyrie stands 1.87m tall and weighs 129 kg, while COMAN (Figure 1.11) stands 0.95m tall and weighs 31.2kg. The incorporation of SEAs into humanoid designs is still a very new trend, with the capabilities of these machines yet to reach their peak performance.

A number of other humanoids were designed to compete in the recent DAPRA robotics challenge (DRC), many within the traditional DC-servo motor design framework. As a notable exception, the ATLAS robot (Figure 1.11) used by multiple teams at the DRC was powered by on-board hydraulics for 28 actuated joints. ATLAS, designed by Boston Dynamics, is 1.88m tall and weighs 150kg (Boston Dynamics) with on board batteries and an integrated vision sensor suite.

With the focus on humanoid robotics in recent years, there have been comparatively less developments in bipedal machines. ATRIAS, however, is a recent 3D biped, built at Oregon State by Jonathan Hurst and colleagues, (Grimes and Hurst, 2012; Hereid et al., 2014) that has been designed to operate like a physical spring-loaded inverted pendulum model. ATRIAS strives towards this aim by employing large series springs in similar spirit to Hurst's previous designs. The robot has shown the ability to walk at speeds up to 1.2 m/s using a single set of optimization-inspired heuristics for control (Reza-zadeh et al., 2015) and has shown empirical robustness to a wide variety of terrain disturbances in laboratory settings.

### **1.3 Challenges of Current Machines**

Across the legged robots that dominate today's state of the art, designs have slowly converged towards supporting an ability to regulate force-based interactions with the environment. Whether through series elastic actuators, hydraulic actuators, or transparent DC electric motors, many of the most successful legged robots today manage balance through torque control at their joints. As future robots transition into less structured environments, this ability to be cognizant of interactions with the world will remain a priority in designs. Chapter 2 discusses some of the main challenges to actuator design in legged robots and discusses a recently developed technology called proprioceptive actuators in order to meet the needs of today's legged machines.

The rapid progress in locomotion technologies in recent years makes it clear that legged robots may soon roam beyond the lab. For legged robots to reach their full impact, they will need to extend their operational lifetime both in terms of reliability and energetic economy. Both of these aspects are incredibly complex due to the underlying interplay between so many con-

tributing factors. Reliability will likely come with maturity of the field and technological components. Energetics however, are a concern that require careful consideration in design. Energetics seems like something that should be able to be modeled. However, the factors that influence it rank among those that are most difficult to capture with accuracy. Chapter 3 further discusses philosophical perspectives on designing for energetic efficiency.

Grown out of both footstep placement in the spring-mass machines of Raibert to ZMP control in the humanoid walkers in 90's, current machines are able to take full advantage of both stepping and ground force shaping in their methods to maintain balance. These diverse modes of operation place unique demands on the more than simply the actuation systems. The design of legs themselves must be capable to handle high force and high-bandwidth loading patterns while minimizing weight to enable rapid replacement in flight. While this tradeoff has held true throughout the history of legged locomotion, the diverse functional requirements induced from more flexible control have only added to the challenge. Chapter 4 discusses trends in leg design and offers a case study using principles from observations in biology to design a leg for the MIT Cheetah.

Following these three Chapters on more detailed considerations in design, Chapter 5 concludes with a summary of future directions and applications.

# 2

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## Actuator Design

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Legged locomotion introduces unique challenges to the process of actuator design. As legs make and break contact, actuators alternate between periods of high speed leg swing in flight, with high force delivery in stance, punctuated by shock loads at impacts in transition. This wide range in operating conditions places unique demands on the both the performance specifications and passive mechanical characteristics of the actuator. Actuator design faces objectives to maximize torque, bandwidth, and power while minimizing sources of loss from friction, inertia, and mass. Metrics to guide design for these individual requirements can be well defined. Yet, it is not clear how to design an actuator to satisfy many conflicting requirements given the numerous couplings among them.

Biological muscles might be considered an ideal actuator – capable of compliant yet high-power operation in a compact form factor. However, the intrinsic characteristics of muscles may not have necessarily evolved as optimal actuator solutions for the functional requirements of individual animals. Rather, the characteristics of muscles may be the outcome of underlying muscle mechanics, dictated by evolutionary baggage of the biological building blocks, actin and myosin, available at hand. For instance, the mechanics behind the force-displacement relationship are well-known, and are

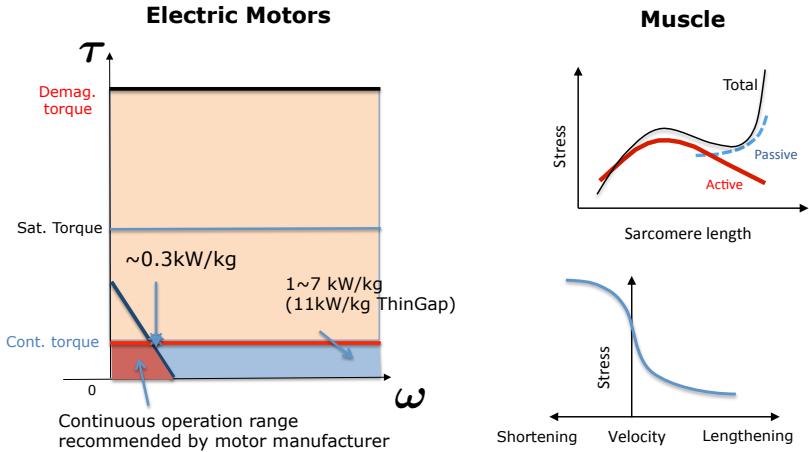
grounded in static properties of actin-myosin fiber overlapping. When muscles are stretched, most muscle cells have very small overlapping distance between two fibers. Therefore, the available force is small. When the muscle length gets near the shortest length, most muscle cells reach the end of stroke and cannot generate force anymore, and thus the available force drops. Hill's muscle model (Hill, 1950) shows the muscle's specific force-displacement relationship and force-velocity relationship. According to the Hill's model, muscles have complex coupling between the states (position and velocity) and the available force. See the Figure 2.1. For example, a bicep can generate the highest force when the elbow angle is 120 degrees. Further, its isometric force is much higher than the force capability when actively contracting.

Force velocity relationships are also dictated by fundamental properties of the interaction dynamics between actin and myosin fibers. The faster the muscle contracts, the lower the available force. The force-velocity relationship is known to be caused by the reduction in the total number of attached cross-bridges. The force is generated by the interaction in attached cross-bridges. The attachment process takes a fixed-amount of time and as the speed of muscle contraction increases, the force decreases due to the lower number of cross-bridges attached at any given time. In negative velocity, the available force reaches the maximum value. The characteristics of the muscles are inherently coupled with the details of the structure and the force generation mechanisms. Thus it could be conjectured that the incremental nature of evolution may not provide sufficient pressure to re-engineer these fundamental mechanisms without upsetting the delicate splendor of their integrated performance. Yet, as a result, there is little reason to believe that muscular properties represent a performance bound for actuator design in the machines we build as engineers.

While force generation in muscles is coupled with their length and contraction speed, electric motors can generate torques rather independent from both position and velocity. Typical torque-speed curves for electric motors at a constant voltage source do not apply once the voltage of the power source is sufficiently high and controlled on demand. The available torque is not a function of angular velocity of the motor once the voltage is high enough<sup>1</sup>.

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<sup>1</sup>In many cases, the operation voltage is recommended by a motor manufacturer to reduce risk of overheating the motor. As long as the current is controlled based on the thermal model



**Figure 2.1:** (left) Torque and velocity relationship in electromagnetic motor. With high voltage input, the torque is independent from the angular velocity, (right) Hill's model represents force-displacement relationship and force-velocity relationship of mammalian muscle. Muscles exhibit force-displacement dependency in both passive force production, as well as the maximum active force production.

The torque is only limited by magnetic saturation of iron for temporary usage and limited by maximum temperature for continuous operation.

Developing advanced actuators for legged robots capable of navigating a variety of environments requires a new paradigm not only in control algorithms but also in mechanical design. In contrast to the conventional robotics applications such as manufacturing, legged locomotion involves severe physical interactions with the environments. For example, in a typical human walking, the ground reaction force reaches typically over one-bodyweight within 150 milliseconds after the collision. The maximum normal ground reaction force on each leg is about three times the bodyweight in a human running at 4.5 m/s (Bobbert et al., 1992). It is 2.6 times the bodyweight in a dog galloping at 9 m/s (Walter and Carrier, 2007) where the ground phase is approximately 70 msec, or only 20% of the gait period.

Contrary to the highly dynamic physical interactions in locomotion, manufacturing manipulators rarely deal with impacts against the objects or environments. Most manufacturing manipulators employ high gear reduction

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of the motors, the supply voltage can be much higher than the recommended value.

ratios to maximize position accuracy and rigidity. This induces a very high reflected mass at the end effector and limits the force control capability in the situations that involve impacts or dynamic interactions. This architecture is not appropriate for the fast physical interactions where impacts have to be minimized. To address these differences, mobile legged robots need new actuator design architectures.

Apparent impedance can be modulated by impedance control (Hogan, 1985). However, in case where the controller relies on force-torque sensors at the end effector (Hirzinger et al., 2002; Kugi et al., 2008), the performance is then limited to relatively slow-speed dynamics and such systems are still vulnerable to impacts. The end effectors of robots and their joints often employ a force sensor to provide force feedback, but suffer from instability caused by non-collocated sensing (Eppinger and Seering, 1989) upon high frequency disturbances. Even in an ideal situation where the force sensing is perfect and signal delay is zero, the actuators have a limited bandwidth to control the interaction force caused by impacts which are typically much faster than the actuator. Therefore, in the event of a collision, the end effector will experience collisional forces due to the total reflected inertia and the controller cannot have any effects. The bottom line is that conventional electromagnetic actuators combined with gearboxes inevitably increase the total mechanical impedance, and this is majorly caused by the high inertias as amplified through the gear ratios. A high gear ratio reduction also subsequently increases frictional losses and reduces the overall mechanical robustness of the system because the collisional inertia is high. In order for a legged robot to dynamically interact with ground and perform robustly, it is critical to minimize the mechanical impedance for operating in unstructured environments.

To address this issue, there are two distinctive efforts that have been made in the field - series elastic actuators and proprioceptive force control actuators. Series elastic actuators minimize mechanical impedance by placing a compliant element in series with a high-impedance actuator, while proprioceptive actuators strive to directly minimize the impedance of the actuator through strategic modifications to motor geometry. The following two sections will discuss these paradigms in further detail.

## 2.1 Series Elastic Actuators

To handle high-impact collisions, researchers developed approaches that bypass the high inertia of high-gear-ratio actuators by adding springs in series with the gearbox output. Inspired by muscle-tendon units found in mammals, series elastic actuators (SEAs) (Pratt and Williamson, 1995) were chiefly introduced to minimize apparent mechanical impedance upon impact. Working with Pratt, Robinson studied the implications of series elasticity in closed-loop force control (Robinson, 2000). Along with analytic models, he conducted experiments on a hydraulic piston with a servo valve and an electric motor with geared linear transmission. His experiments showed a dropoff for force control bandwidth in the high-force operation ranges that would be required for locomotion.

After Pratt's introduction, there have been various approaches to achieve variable stiffness actuators. Tonietti (Tonietti et al., 2005) demonstrated a variable stiffness SEA using a transmission belt that was tensioned by linear springs and connect the joint shaft to an antagonistic pair of actuator pulleys. Schiavi presented a design that used antagonist actuators connected to a 4-bar spring mechanism that provided a nonlinear stiffness characteristic. It used a special case of the Grashof neutral linkage to achieve the desired stiffness nonlinearity by adjusting the mechanism geometry and using a linear spring (Schiavi et al., 2008). Hurst developed a 2 DOF leg where SEA stiffness was adjusted by pre-tensioning fiberglass bending springs using cables and spiral pulleys (Hurst et al., 2010). Petit introduced the design of a bidirectional antagonistic joint that explored the additive power of two actuators. The actuators together controlled joint force and stiffness in a differential faction (Petit et al., 2010). Wolf presented the design of the Floating Spring Joint (FSJ), a compact SEA that included a motor, Harmonic Drive, sensors, elastic element, and joint bearing (Wolf et al., 2011). A smaller motor was used to adjust the joint's nonlinear stiffness by rotary pretension with a pair of cams. Kong designed a SEA that used a worm gear transmission and a rotary spring. The contact angle of worm gear was evaluated for friction identification and transmission efficiency (Kong et al., 2012).

Other authors have explored the non-backdrivability of the transmission to isolate the motor from disturbances. Kim developed a Hybrid Variable Stiffness Actuator (HVSA) that used two motors per joint in order to regu-

late the stiffness by changing the moment arm of the mechanism (Kim et al., 2011). The design was called hybrid because it controls position and regulates the stiffness around that position. The joint was used in manipulators for assembly tasks that required precise fitting. Thorson designed HypoSEA, a SEA that used a hypocycloid transmission to load a linear spring in a nonlinear way to explore characteristic nonlinearities of legged locomotion (Thorson and Caldwell, 2011). This approach was intended to achieve very low reflected inertia and reflected stiffness around zero-torque. Groothuis presented a transmission design that used a hypocycloid and a double supported Bernoulli leaf spring beam to allow variable stiffness by changing the distance between supports. The output shaft was completely decoupled from actuator during zero stiffness and mechanically locked during infinite stiffness (Groothuis et al., 2014). Design of a linear SEA was applied for THOR, a humanoid robot, using a Maxon EC motor, a ball screw transmission, and a bending titanium beam as spring (Knabe et al., 2014). The beam stiffness could be varied by adjusting the clamping support. The actuator had a carbon fiber housing tube and PTFE bushing for the sliding contact. A load cell in series measured the applied force. A quadruped, StarlETH, demonstrated a successful execution of controlled leg impedance compliance using SEAs (Hutter et al., 2014)

There are a wide variations in VSA designs in terms of how to provide mechanical stiffness, how to change the stiffness, and the range of stiffness provided. On top of the available stiffness the system can generate, the weight of the system and the allowable force are very important particularly for legged robots. Due to these many dimensions of variability across actuator architectures for VSAs, comparative evaluation of VSAs is generally difficult. In spite of the variability, Vanderborght et al. (2013) recently proposed a helpful classification based on the principles through which the variable stiffness and damping are achieved by active impedance control, inherent compliance, inherent damping, and inertial actuators. SEAs are a great candidate solution for legged robots, yet we need to further understand the effects of design parameters on essential performance metrics for design optimization. In addition, the design of SEAs should be further simplified to minimize the mechanical complexity which increases the mass and decreases robustness.

## 2.2 Proprioceptive Force Control Actuators

Rather than explicitly including mechanical compliance, proprioceptive force control actuators provide a different outlook to minimize mechanical impedance and enable dynamic physical interaction through transparent force control. In general, to accurately control forces at the feet (or end-effector) a robot requires accurate joint-torque control, a detailed dynamic model, and accurate joint state (position and velocity) sensing. In principle, access to this information can be used to control the forces or even impedance rendered at an endpoint (Hogan, 1985). However, in practice, this approach is limited to relatively gentle (low-bandwidth) interaction with environments. Even in slow walking, let alone running, the leg experiences high-bandwidth collisions with ground at every step. As mentioned above, even with perfect measurements, and a perfect dynamic model, the bandwidth of available actuators is generally too low to catch up with such harsh impacts. We first note which properties of the robot determine the highest forces experienced during these impacts as motivation for the proprioceptive force control paradigm.

### 2.2.1 Impact Dynamics

Given a robot in generalized coordinates,  $q \in \mathbb{R}^n$ , its Lagrangian dynamics generally follow

$$M(q)\ddot{q} + V(q, \dot{q}) + G(q) = S^T \tau_{motor} + J(q)^T F_{ext} \quad (2.1)$$

where  $M(q) \in \mathbb{R}^{n \times n}$  is the mass matrix,  $V(q, \dot{q}) \in \mathbb{R}^n$  is a vector of velocity-dependent coriolis and centripetal terms, and  $G(q) \in \mathbb{R}^n$  captures the effects from conservative forces such as gravity or included springs. As other forcing terms to these dynamics  $\tau_{motor} \in \mathbb{R}^{n_m}$  is the vector of motor torques,  $S \in \mathbb{R}^{n_m \times n}$  is a selector Jacobian for the actuated coordinates,  $F_{ext} \in \mathbb{R}^{n_c}$  is the vector of external contact forces, and  $J(q) \in \mathbb{R}^{n_c \times n}$  is the Jacobian for external contacts.

When a system makes contact with the environment, it experiences a impulse that is determined by the inertial properties of the mechanism. For instance, when coming into contact with generalized velocities  $\dot{q}$ , the contact locations impact the environment with velocity  $v = J\dot{q}$ . This impact velocity

produces a impact impulse  $\hat{F}_{ext}$  at the contacts according to

$$\hat{F}_{ext} = \Lambda v \quad (2.2)$$

$$= (JH^{-1}J^T)^{-1} v \quad (2.3)$$

which is related to a impulse in the generalized coordinates through

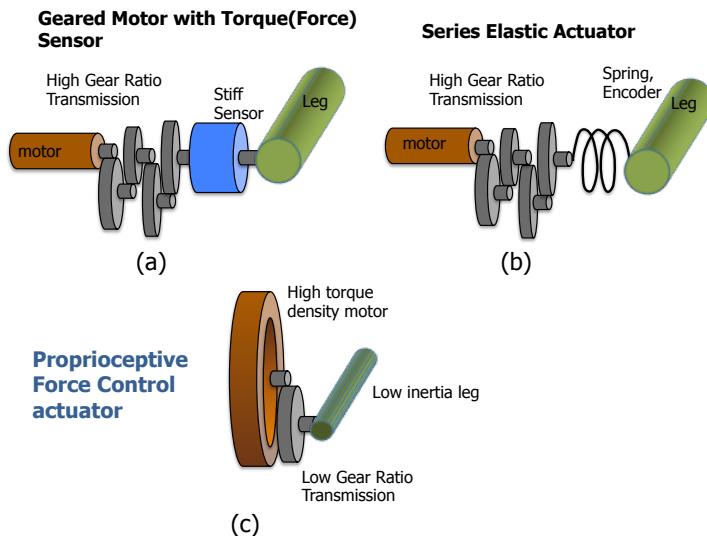
$$\hat{\tau} = J^T \hat{F}_{ext} \quad (2.4)$$

$\Lambda = (JH^{-1}J^T)^{-1}$  is the apparent inertia, more technically the operational-space inertia, felt at the contacts. Put another way, this is the total inertia of the robot as reflected to the foot point. As a result, an ability to handle impacts is inherently linked to reflected inertias.

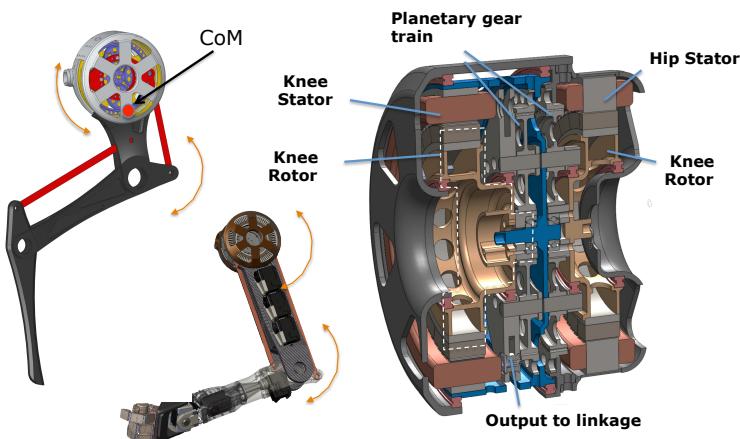
### 2.2.2 Proprioceptive actuators

Proprioceptive actuation (Wensing et al., 2016) is an approach to mitigate high impact forces by introducing a completely different architecture of the actuators. The overall architecture is shown in Figure 2.2. By minimizing total mechanical inertia and the transmission ratio, the system becomes 'light' upon impact, effectively minimizing  $\Lambda$ . Rather than using a spring to decouple high inertias from the output, directly minimizing the reflected inertia at the output enables the system to avoid high forces through its transmission chain. To compensate the traditionally low torque output of low gear ratio transmission, electric motors should be redesigned to meet torque density requirements (Seok et al., 2012). Such high-torque motors employ a large radius stator and rotor, as shown applied to the MIT Cheetah leg and MIT HERMES humanoid arm in Figure 2.3. This shift in outlook also has favorable energetic consequences as discussed in the following chapter.

The use of proprioceptive actuation has benefits in high-force high-bandwidth regimes in comparison to series elastic approaches. To purposefully create a large change in actuator force, proprioceptive actuators need only push additional electrons through their stator. In contrast, series elastic actuators have to physically move a mechanical spring to a different deflection, and may be limited in slew rate by the speed of the motor. As a result, force bandwidth in proprioceptive actuators conceptually is less variable to force magnitude.



**Figure 2.2:** (a) Conventional actuators with high gear ratio and stiff force sensors (b) Series elastic actuator (c) Proprioceptive actuator.



**Figure 2.3:** Proprioceptive actuators developed for the MIT Cheetah and HERMES humanoid platform.

Instead of measuring the force/torque of the end effector, force output in proprioceptive actuators can be controlled by motor torques only ( $= -J^T F_{ext}$ ). By effectively ignoring any of the other dynamic terms in (2.1), this approach results in inaccurate force control especially in high acceleration situations such as collision. Yet, designing for maximum inertial transparency indirectly minimizes these ignored inertial forces, improving proprioceptive force tracking errors. Further, proprioceptive actuators have stability benefits, as motor current sensors are collocated with the actuators. This is in contrast to noncollocated force control approaches wherein force sensors are placed away from the force sources. Noncollocated force control can classically be found in end-effector force control approaches, and can lead to instability due to unmodeled dynamics of structural compliance between the force source and force sensor (Eppinger and Seering, 1989). In proprioceptive actuation, collocated sensing allows for stable execution of force control and low inertia allows for limited and manageable impact forces. A pioneering haptic display device ‘PHANTOM’ was designed following such approach (Massie and Salisbury, 1994). Although the PHANTOM was not designed to handle collisions, the overall design concept became the source of inspiration for the development of the high-force proprioceptive actuators in the MIT Cheetah (Seok et al., 2012, 2015; Park et al., 2015a), and more recently in the direct drive Minitaur and Jerboa at UPenn (Kenneally et al., 2016).

### 2.3 Conclusion

Achieving actuators with performance that exceeds biological muscle in every aspect remains a grand challenge in engineering. However, as we have argued, these biological tissues should not be viewed as a hard and fast performance bound. Both series elastic and proprioceptive actuators have great promise as actuator solutions for legged machines, offering low-impedance solutions with an ability to make and break contact. These solutions further offer the ability to control force-based interactions that are of such importance to balance in legged robots. The closed-loop bandwidth of actuators in these paradigms already outmatch biological muscle, with proprioceptive actuators providing further benefits in high-force regimes. More recently, these same paradigms have displayed favorable energetic performance, nearing the

efficiency of animals in nature. The next chapter discusses these energetic considerations in further detail, touching on the systems-wide considerations that extend well beyond the choices of an actuator paradigm.

# 3

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## Energetic Considerations

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### 3.1 Measuring Energetic Economy for Legged Locomotion

Energy efficiency in locomotion is a critical aspect of legged robot design. Mobile robots will eventually operate in the field using on-board power sources and the efficiency of the locomotion will determine the operation time per charge. Efficiency in locomotion is defined significantly differently from a conventional notion based on input/output power ratios. Rather, efficiency is best represented by a non-dimensional metric, named the cost of transport (COT). Since this differs from traditional notions of efficiency, the COT might more accurately be described as a metric for the energetic economy of locomotion. The COT is commonly defined according to

$$\text{COT} = \frac{\text{power}}{\text{weight} \cdot \text{velocity}} = \frac{P}{mgv}$$

This metric appears to be a power ratio, but the denominator does not yield power for locomotion on flat ground. Since the velocity vector is perpendicular to the weight vector, their dot product is zero. Traveling in a horizontal surface does not produce any mechanical work since the kinetic and potential energy of the system stay constant. Thus, for an conventional definition of

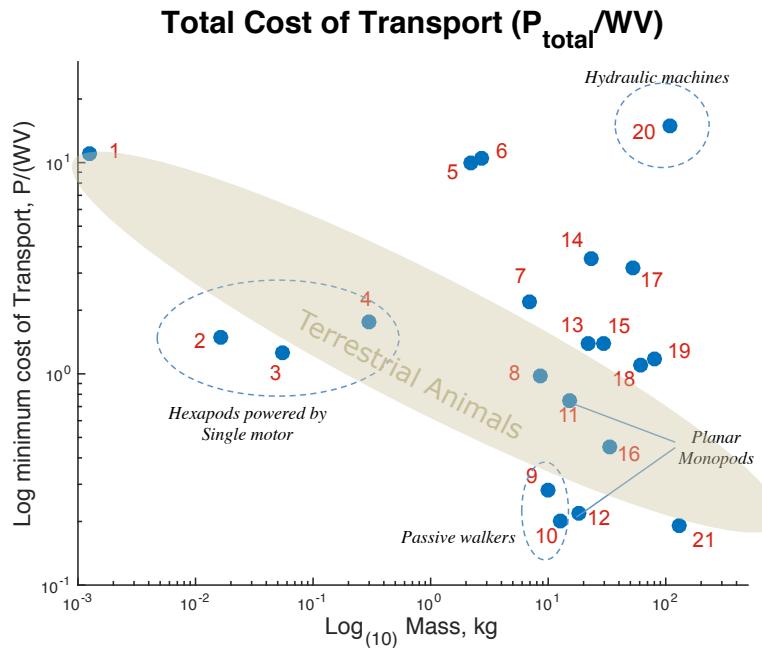
efficiency, this would mean that all locomotion systems have an efficiency of zero. Locomotion in flat ground is an energy dissipative process, where the most “efficient” machines are those that lose the least amount of energy per unit distance.

Recently two related approaches have been chiefly employed to improve the efficiency of legged locomotion: engineering passive dynamics and incorporating series elastic actuators. Passive dynamic walking, in particular, has received a great deal of attention. An exceptionally successful example of engineering passive dynamics is the Cornell Ranger ( $COT=0.19$ ), designed based on McGeer’s fully passive walker (Bhounsule et al., 2012).

Employing series elasticity also potentially benefits locomotion efficiency by utilizing the high energy recovery of mechanical springs in the work cycle of locomotion. iSprawl (Kim et al., 2006) shows a  $COT$  of 1.7 rivaling animal efficiency by utilizing series elastic actuation. To integrate series elastic actuation into the design of a general purpose robot is non-trivial as there needs to be ways of altering the mechanical stiffness and damping. This is usually achieved through the use of additional smaller actuators (Wolf and Hirzinger, 2008), which significantly increases complexity, weight, and overall energy consumption. Another promising approach is employing parallel springs (G. Folkertsma and Stramigioli, 2012) for saving torque-generating cost and optimizing swing leg retraction (Haberland et al., 2011) for minimizing impact losses. Hurst uses series elastic actuator in the bipedal robot ATRIAS (Rezazadeh et al., 2015) to recycle mechanical work and mitigate impact forces.

Figure 3.1 shows the  $COT$  for a wide range of robots across scales. The figure also highlights the range of  $COT$  where terrestrial animals traditionally reside. Across animal data,  $COT$  generally diminishes with overall mass. This trend is due in part to the different loading patterns placed on muscles with changing strength to weight ratios across scale (Biewener, 2005). The designs of passive dynamic walkers as well as a few small hexapod robots have managed to exceed the energetic performance observed in the biological realm. However, these machines have done so through sacrificing much versatility. The ARL Monopods (11 and 12 in the Figure) were able to achieve their energetic performance in part due to their use of SEAs. The MIT Cheetah (16 in the figure) has energetic performance on par with a biological cheetah.

tah, and has maintained a great deal of versatility in its design. Developing machines which drastically exceed the energetic performance observed in animals while maintaining great versatility remains a grand challenge within legged robot design and control. Achieving this aim will require great understanding and management of the many energy loss modes in locomotion.



1 HAMR-VP (Baisch et al., 2014)	12 ARL Monopod II (Ahmadi et al., 2006)
2 X2-VelociROACH (Haldane et al., 2013)	13 Scout II (Buehler, 2002)
3 DASH (Birkmeyer et al., 2009)	14 StarlETH (Hutter et al., 2013)
4 iSprawl (Kim et al., 2006)	15 Fastrunner (Cotton et al., 2012)
5 Cheetah Cub (Spröwitz, 2013)	16 MIT Cheetah (Seok et al., 2013)
6 MIT Learning Biped (Collins et al., 2005)	17 ATRIAS 2.1 (Renjewski et al., 2013)
7 Rhex hexapod (Salanli 2001)	18 Asimo (Collins et al., 2005)
8 RHex-biped (Neville et al., 2003)	19 KOLT (Estremera et al., 2008)
9 Cornell Ranger (Bhounsule, 2012)	20 Big Dog (Raibert et al., 2008)
10 Cornell Biped (Collins et al., 2005)	21 ETH Cargo (Günther et al., 2015)
11 ARL Monopod I (Buehler, 2002)	

**Figure 3.1:** The cost of transport ( $P/mgv$ ) of legged robots compared with animals' equivalent data.

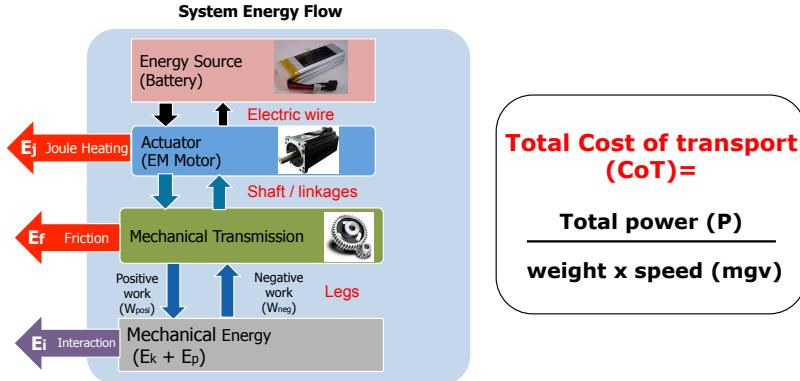
### **3.2 Contributions to the Cost of Transport**

Figure 3.2 shows common energy flows for a locomotive system, with focus on its main energy loss modes. There are three modes of energy dissipation. First is heat loss of the actuator. In an electromagnetic force/torque actuator, this is Joule heating loss, which is  $I^2R$ . The current  $I$  is linearly proportional to the torque in most cases. The second mode is transmission losses. This could be most underestimated in robotics<sup>1</sup>. All frictional and viscous losses (joints, shafts, gears and etc.) belong to this mode. Recently, Wang proved that the efficiency of the spur gears and harmonic drive depends on whether the motor is performing positive or negative mechanical work (Wang and Kim, 2015). In typical large gear ratio design, the difference is quite significant. In negative mechanical work regimes, the gearbox was found 5-10% less efficient than in positive work regimes (5% in a 1:247 Dynamixel, 10% in a 1:50 Harmonic drive). While this difference is never considered in modeling of robots, it is important to identify every energy dissipation path for energetic optimization. The third mode of dissipation is interaction losses. This loss occurs at the interface between the system and the environment. For example, air drag loss, collisional loss, and sliding loss belong to this mode. This mode of dissipation can be transformed to various forms such as sound, vibration, and abrasion.

The relative contributions and overall footprint of these three loss modes is tightly related to the design of legged robots. The motor constant, i.e. the ratio between the torque and the Joule heating power, will be critical for minimizing the first mode of energy dissipation. Of course, leg kinematics play a critical role as well since the torque requirement will be directly related to the ground reaction force through a standard Jacobian transpose mapping. There is, further, a clear trade off in selecting gear ratios. Higher gear ratios that require less motor torque will reduce the Joule heating. However, high reflected inertias increase the contact inertia  $\Lambda = (JM^{-1}J^T)^{-1}$ . Higher contact inertia will cause high interaction energy loss, but the effect is not fully amenable to rigorous analysis. Higher impact forces caused by the high inertia will cause

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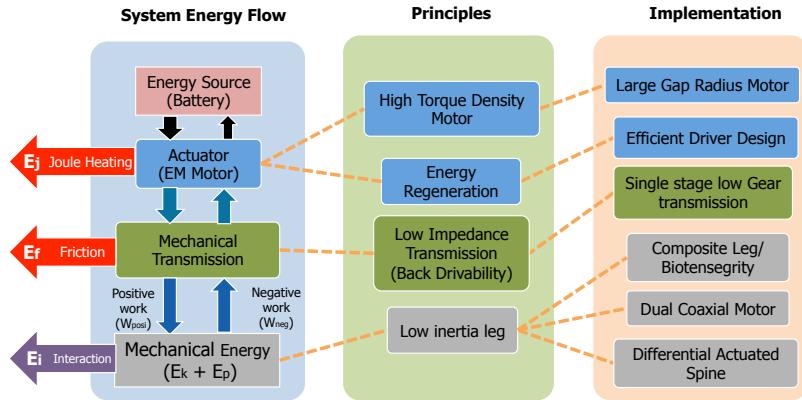
<sup>1</sup>Many researchers take the efficiency data from the gear manufacturer, which usually is very high, although the efficiency is measured in a particular condition. The actual efficiency of transmission in general varies significantly depending on the load, speed and lubrication type



**Figure 3.2:** Energy flow diagram of the robot showing how the energy flow between the source and mechanical interaction with environment.

high forces throughout the transmission chain and may excite unmodelled high-frequency load-dependent dynamics which are difficult to model. More importantly, higher reflected inertia will prevent the robot from dynamically interacting with ground. In other words, if the reflected inertia is too high, excessive actuator impedance might prevent dynamic behaviors (e.g. running speed) that might otherwise be possible in systems with lower reflected inertia. An ability to accurately model these three loss modes a-priori to the construction of a system remains beyond current modeling tools.

Considering that it is extremely difficult to track all the parameters for designing energy efficient robots, it is essential to embody principles in design process. Figure 3.3 shows the design principles implemented in the design of the MIT Cheetah. Design principles towards achieving high motor torque density, energy regeneration, low impedance transmissions, and low inertia legs are directly tied to minimize system energy losses through joule heating, friction, and contact interactions. High torque density motors are constructed through the use of a high air-gap radius between the stator and rotor, providing valuable mechanical advantage to the electromechanical interaction between these torque producing components. Efficient motor drivers provide the opportunity for negative mechanical work in these motors to result in usable regenerative power. A low-inertia limb is provided by a composite leg with biotensegrity as described in the following chapter, and dual coaxial mo-



**Figure 3.3:** (a) Energy flow diagram of the robot showing how the energy flow between the source and mechanical energy. Joule heating loss occurs at the motor, friction loss occurs in the mechanical transmission and interaction loss reduces the total mechanical energy. (b) Design principles for improve efficiency at the sources of energy loss. (c) Strategies for implementing the design principles for efficiency used on the MIT Cheetah Robot.

tors located at the hip to reduce distal leg mass. Actuator inertia can be further reduced by exploiting synergies and symmetries in the motions of many degrees of freedom. For instance, a differentially actuated spine driven by a synergy between the back hips allowed for longer leg stroke during galloping without the need for additional leg length or actuators.

### 3.3 Conclusion

This chapter has briefly introduced metrics of energetic economy in legged machines and provided systems-level insights into practices which can be sought to minimize loss modes. Improving efficiency in locomotion is purely a task of decreasing losses. These loss modes from actuators, transmission, and interactions are strongly dependent on target operational specifications and high-level design decisions centered on actuator topologies and paradigms. Due to this specificity, it is difficult to provide general yet accurate recommendations regarding optimization of system energetics. As we have highlighted, many of these loss modes are difficult to model with high-fidelity, despite their relative importance in the energetics of physically constructed machines. As a community, it is thus important that energetic perfor-

mance is characterized and documented for experimental robots in the lab, such that we might continue to iterate towards improved component level technologies and integrative design strategies.

# 4

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## Bio-inspired Leg Design

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Many animals have evolved to become fast runners since running speed is a critical aspect of survival for both predator and prey. It is natural to seek design principles that nature's best runners have adopted for survival. A critical trade-off, between the capability to withstand high ground reaction forces and an ability to rapidly reposition the limbs, is effectively managed in many high-strength low-inertia limbs in nature. This chapter introduces a bio-inspired leg design approach for robotics, with focus on a particular principle of biotensegrity. Sections 4.1-4.2 present an overview of the leg design problem and previous approaches, while Sections 4.3-4.7 describe a case study in leg design for the MIT Cheetah robot. A short conclusion is provided in Section 4.8.

### 4.1 Design Challenges in Leg structure

Legged locomotion involves higher ground reaction forces than in static weight support. In locomotion, each leg may spend only a fraction of the overall gait period on the ground. This ratio is referred to as the duty ratio of locomotion and is inversely proportional to the ground force requirements. This is due to the fact that, in steady state locomotion, the total vertical im-

pulse during one period of cyclic locomotion has to be the same as the total gravitational momentum loss to satisfy momentum conservation:

$$\int_0^T F_z dt = mgT$$

where  $T$  represents the gait period of locomotion.

The simple vertical momentum equation shows that a smaller duty factor inevitably entails higher ground reaction forces in the vertical direction. In general, higher speed running requires a higher stride frequency and lower duty factor. This trend is well shown in running data (Maes et al., 2008) for dogs. For example, the typical maximum normal ground reaction force on each leg is around three times the bodyweight in human running at 4.5 m/s (Bobbert et al., 1992) and 2.6 times the bodyweight in dog galloping at 9 m/s (Walter and Carrier, 2007). Such high ground reaction forces will cause high stresses throughout the leg structure and cause failures by stress concentration. In order to control the ground reaction forces in 3D, the leg should have three degrees of freedom, which naturally increases complexity of the leg. Compared with car design, which is already very mature and robust, this complexity and higher ground reaction force requirement poses a challenge in designing legged robots. Such discrete ground contact is not common in other modes of transportation using wheels and tracks, where the vehicle continuously stays in contact with ground.

Simply increasing the thickness of the leg may not solve the high stress problem since the maximum loading is dependent on the leg inertia, and high leg inertias limit rapid leg-swing motions to cycle the legs at high speeds. The leg actuation has to change direction twice (protraction and retraction) while in the flight phase to create a cyclic trajectory and requires high accelerations of the leg. Such a high stride frequency can be realized by either a high actuator capacity and/or decreasing the leg inertia.

The current actuation technology, as introduced in the Chapter 2, precludes increase in actuator capacity without considerably increasing the overall weight and effective inertia of the actuator. Therefore, increasing actuator capacity will increase the mass of the robots and increase torque requirement. Hence, the second option of decreasing the total inertia of the leg structure is essential. In order to decrease the leg inertia, the overall mass of the struc-

ture must be reduced and the distribution of the mass should be controlled. However while doing so, the strength of the structure should be maintained to handle high peak stress caused by discrete impacts.

Even in a slow speed walking, high-speed leg repositioning is required to stay balanced. Quickly positioning upcoming foot steps is a critical approach in rejecting disturbances (Pratt et al., 2006). Therefore, there is a serious trade off between agility and structural integrity in designing leg structures for a legged robot. This trade-off applies more severely in the distal part of the leg due to its larger contribution to the rotational inertia felt by the hip. Impact considerations similarly highlight the importance of this distal leg mass. The distal part of leg loses its kinetic energy upon impact, which suggests to minimize the distal mass of the leg. However, the design of the distal part of the leg, namely the foot, needs to satisfy other important requirements such as mechanical robustness to repeated impacts.

## 4.2 Existing robot leg design approaches

There are several leg design approaches for dynamic legged robots. Most bio-inspired morphologies aim to decrease the inertia of the distal leg and introduce compliance (Pratt, 2000a). Some common techniques found in the literature include locating the actuators closer to the body through pantograph mechanisms, cable drives, or other mechanisms. Using an under-actuated leg with a passive compliant joint, as in series-elastic actuation methods, in part attempts to replicate the muscle/tendon connection topology and reduce limb impedance.

Cable drives are commonly used to minimize the weight of a structure by remotely transmitting power from the base. A 6-DoF cable-driven manipulator (Perreault and Gosselin, 2008) was developed using a reconfigurable pin-jointed structure without buckling and bending. The use of cables allowed for the location of the actuator to be flexible and helped to minimize the weight and inertia of the robot. "Spring Turkey" (Pratt, 2002) was designed with cable drives to decrease the weight and consequently the inertia of the legs. This robot had an actuated knee and hip with actuators on the body actuating the joints through a cable drive system. An alternative to cable drive systems is to use rigid linkages, for instance as in the pantograph mechanisms of Hirose

(Hirose, 1984) or more recent parallelogram linkages which have appeared in the MIT Cheetah 2 (Wensing et al., 2016) or Cheetah-Cub (Sprowitz et al., 2013).

A prismatic leg design (Raibert et al., 1989; Pratt, 2000a) combined with hydraulic actuators enabled dynamic behaviors of early legged robots in MIT Leg lab. Their air piston designs provided necessary compliance for the prismatic joint. The planar one-leg hopper from MIT leg lab (Raibert and H. B. Brown, 1984) also included a pneumatically actuated hip. Prosthetic legs (Sup et al., 2008; Shen and Goldfarb, 2007) often employ adjustable compliance using pneumatic systems. More recently, hydraulically powered robots such as Big dog (Raibert et al., 2008) and HyQ (Semini et al., 2011) have commonly used hydraulic actuators with closed-loop force control to modulate the leg impedance.

The Rhex robot uses several variations of a simple compliant one DoF c-shape leg design. Using such simple and robust monolithic legs, Rhex has achieved a range of behaviors such as walking, running, pronking, leaping and flipping (Saranli, 2001), (Galloway et al., 2010). Stickybot (Kim et al., 2008), a lightweight climbing robot, employing world-first directional adhesives, used serial compliance made by SDM (Shape Deposition Manufacturing). The deformation of compliance made of soft polymers was measured for force balancing among legs to increase the robustness of climbing. RiSE platforms (Spenko et al., 2008) including RiSEV2, RiSEV3 (Haynes et al., 2009), DynoClimber (Clark et al., 2006) used a four-bar linkage or a slider-crank mechanism that allowed high torque during stance and high speed during leg recirculation with almost constant motor velocity.

Another common design technique to deal with the trade-off between strength and weight is using high strength-to-weight ratio materials such as carbon fibre (Pratt and Pratt, 1998), steel, Aluminium Alloys (Nichol et al., 2004), magnesium alloy (Kaneko et al., 2002a) and Titanium (Lambrecht et al., 2005). Using tubular elements are also common, especially in high-speed robots (Raibert et al., 2008, 1983; Playter and Raibert, 1992; Pratt and Pratt, 1998).

The common thesis in all the above mentioned techniques is the requirement for reduction of mass of the distal end to decrease the overall inertia of the robotic leg structures. The techniques use clever design strategies to

achieve this goal, with most designs that described being special to the specific robotic applications.

Seemingly missing for the discussion thus far has been considerations of limb morphology on leg design. Perhaps the simplest question would be to examine whether a forwards pointed knee as in humans, or a backwards pointed knee as in birds, should generally be preferable for a legged robot design. Computational experiments in bipeds suggest that, from an energetic standpoint, a backwards pointed knee is preferable on average for bipeds (Haberland and Kim, 2015). Yet, preference is dependent on dynamic parameters (masses, inertia, center of mass locations) of the limbs, and thus, a general rule of thumb does not exist that is universally applicable across bipeds. This question would be further complicated in multi-legged machines where individual leg pairs might exhibit independent configuration preferences. This picture of how morphology affects energetics lacks crispness already. Any conclusions would be further complicated by considering conflicting goals of minimizing limb loading, maximizing workspace, controllability, and other considerations.

As a result of the complexity of answering even such seemingly simple questions relating to morphology, we will not try to address optimal morphology. Instead we will focus the remainder to this section on detailing a high-level principle known as biotensegrity for a fixed limb morphology. From birds to humans, our musculoskeletal systems display a common use to biotensegrity strategies to increase the strength of limbs. We present the embodiment of this principle through a case study in our own work related to leg design for the MIT Cheetah robot.

### **4.3 Tendon Bone Co-Location Design (Biotensegrity)**

Musculoskeletal systems seem to achieve structural robustness without using special or bulky materials. Although the strength of biological materials is significantly lower than engineering materials, animals such as gazelles can run and jump using thin legs and a narrow foot structure. The yield strength of bones and tendons are around 100MPa whereas that of aluminum is 400MPa, Kevlar 3GPa, Steel 1GPa. It is intriguing to investigate how land animals deal with relatively brittle bones and such low tensile strength

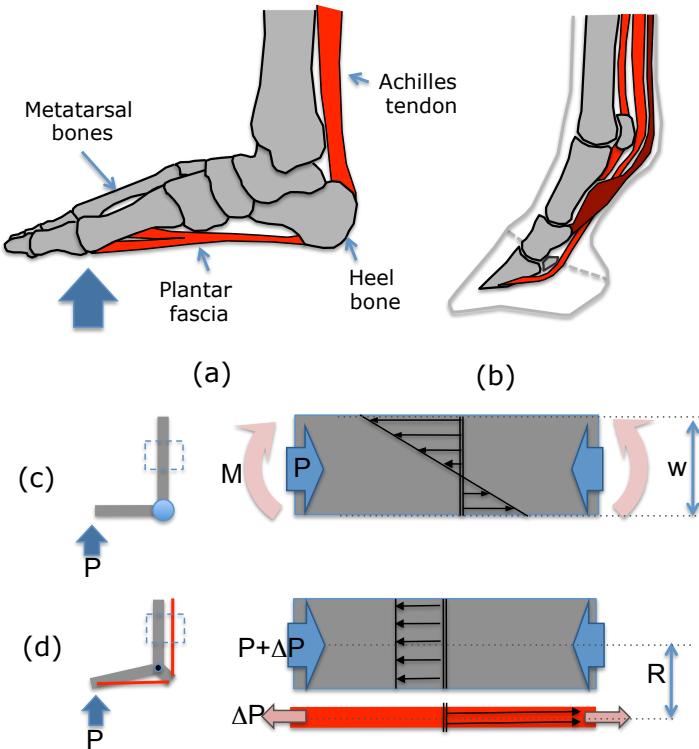


**Figure 4.1:** A picture of horse's front leg. The skin is removed and the rest is plastinated

while still maintaining dynamic performance capabilities. One interesting design principle derived from biomechanical observations is tendon bone co-location (Ananthanarayanan et al., 2012), also known as *biotensegrity*. Figure 4.1 shows this tendon/bone co-location in a horse's front leg. Work towards bio-inspired tensegrity develops feasible engineering solutions to design strong and light legs. A part of this work was implemented in the MIT Cheetah, capable of running at 6m/s. This section summarizes the biotensegrity approach as a potential way to handle the demanding strength-to-weight requirements in leg design for dynamic robots.

The hypothesis is that the synergistic co-location of bones and tendons reduces the bending moment at the bone structure. There has been an argument that the muscles, tendons and ligaments carry tension while the bones are under compressive loads during ground loading (Rudman et al., 2006). Bones have better strength under compression than tension (Carter and Hayes, 1976) and tendons and muscles have high strength in tension. Utilizing this characteristics of the materials, this distribution of the loads can be more effective and eventually yield a design with a higher strength-to-weight ratio.

Consider the anatomical arrangement of the human leg (Figure 4.2 (a)). When there is a load at the metatarso-phalangeal joints, which is ball of the foot, during running, the ankle would experience a high moment (roughly 309 Nm for a 70 kg human), assuming the length between the ball of the foot and ankle is around 0.15 m and the maximum ground reaction force is three times of weight. If the hypothesis applies, the ground reaction forces are distributed through the bones and tendon muscle units as axial forces and as a result avoid large bending stress. The Plantar Fascia, the Achilles tendon and the Gastrocnemius muscle carry tension and the bones including Metatar-



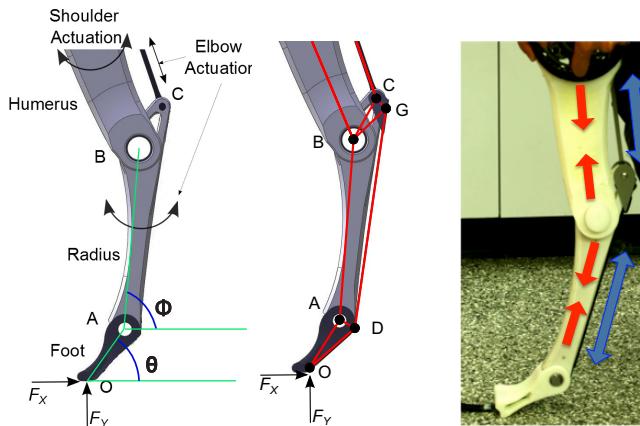
**Figure 4.2:** A synergistic arrangement of bones and tendons/ligaments shown in human (a), and horse foot(b). (c), (d) show the conceptual stress profile on the bone without and with the tendon respectively. (Ananthanarayanan et al., 2012)

sus bones and tibia are loaded mainly in compression. The same principle can be applied in the distal limb structures of many other animals such as digitigrades (e.g. cats and dogs) and ungulates. The figure 4.1 shows plasinated horse leg. The distal part of the leg bones are covered with tendons and ligaments, which can reduce bending moments in the bone structure. The hypotheses drawn from the animals are listed as follows.

- The tendons help reduce the bending stress at the bone by taking tension. Thus the stresses on the bone are more uniformly distributed along the cross-section.

- By employing biotensegrity, we can reduce the overall weight of the leg without loosing strength.

The principle is conceptualized in Figure 4.2. The stress distribution in the bone structures without and with a tendon are shown respectively. With the combination of a tendon, we can achieve a more uniform stress distribution along the cross-section of the bone structure. We call this concept 'Biotensegrity'. The tensegrity structure (S. Pellegrino, 1986) is composed of struts and strings, where no two rigid struts are in contact with each other. The tensegrity structures are known to be very robust because of the lack of bending moment in the rigid components. All mechanical stress are distributed uniformly in the cross-section of strings and the struts. The biotensegrity structure holds the core advantage of tensegrity structure.



**Figure 4.3:** Leg Design Concepts (a) Parameters indicated without tendon. (b) Biotensegrity design. The red lines represent an equivalent pin-jointed structure. (c) The leg prototype of the MIT Robotic Cheetah. The parts undertaking tensions are made of high strength material for minimizing bending on the bone. The blue arrow represents hypothesized tension in the tendon and red arrow represents compression in the bones.

Eliminating bending moments is a critical approach in cranes and large constructions. Pin-jointed structures with trusses are one such solution to reduce structure weight (S. Pellegrino, 1986). Like tensegrity, all elements are axially loaded without any bending moments. Such design paradigm improves the strength-to-weight ratio significantly and is widely used.

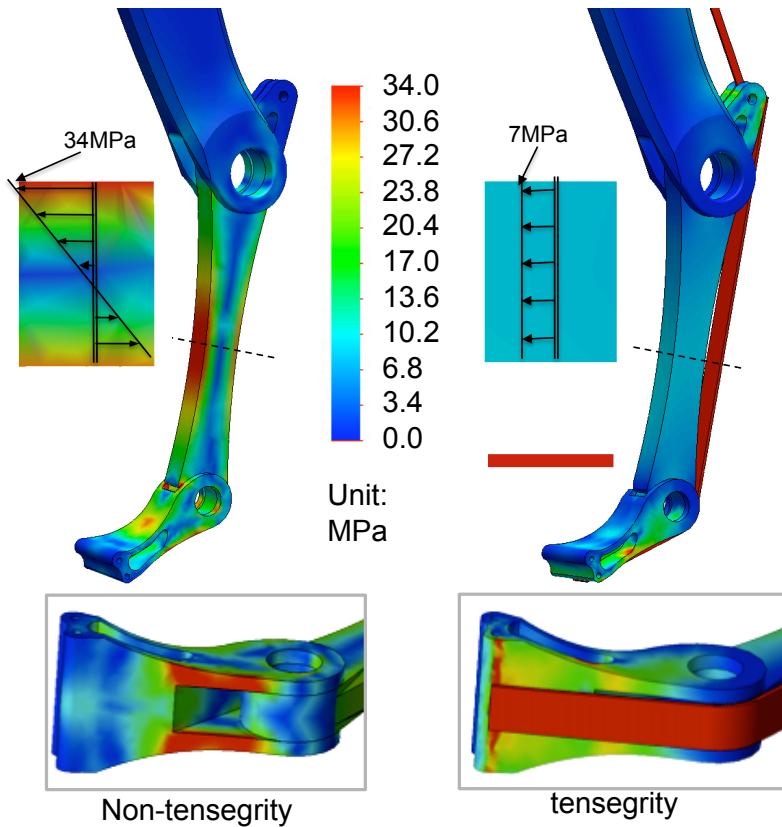
#### **4.4 Finite Element Analysis**

To evaluate the hypothesis in a robotic system, a lightweight leg was designed based on the biotensegrity concept. A static finite element analysis was performed using Solidworks 2011 for a specific loading condition to evaluate the stresses on the leg. It is assumed that the vertical ground reaction forces on the leg ( $F_y$ ) were at their maximum at this configuration. According to the dog running data from (Walter and Carrier, 2007), the maximum ground reaction force is assumed to act along the line connecting the foot to the shoulder. For this condition, the comparison of the finite element analysis for the two design concepts is shown in Figure 4.4. Two different models represent the locked ankle design (left) of leg and a design using biotensegrity method (right).

The von Mises stresses at the tendon is around 65 MPa and 195 MPa at the triceps linkage. These components can be made of a thin materials because they take tensions most of the time. The tensile strength of the tricep, made of hardened stainless steel, is around 750 MPa and the tendon is made out of Kevlar® with 3.5 GPa tensile strength. The maximum stress at the bottom of the foot in the non-tendon case is 45 MPa due to bending. Thanks to the tendon that redistributes the stress, the stresses are much lower and in pure compression at the same location for the biotensegrity design case. The experimental results of stresses for the same loading configuration on the actual leg design is followed in the following subsection.

#### **4.5 Static Loading Experiments**

In order to verify the FEA results, the experiments on the leg prototype illustrated in Figure 4.3 were performed using a Zwick-Roell Z005 Tensile testing machine. Figure 4.5 illustrates our experimental setup. Four strain gauges were attached on the sides of the smallest section in the radius. Table 4.1 shows the maximum von-mises stresses obtained from the experiments. The results clearly show considerable bending of the leg due to applied ground reaction force in the conventional design with no tendon. In contrast, in the biotensegrity design, the stresses on all sides of the leg were roughly uniform. The experimentally measured stresses on the biotensegrity method were found to be 22% of the stresses in the same configuration for a fixed ankle design. (Note that the Finite Element Analysis predicted about 21%).



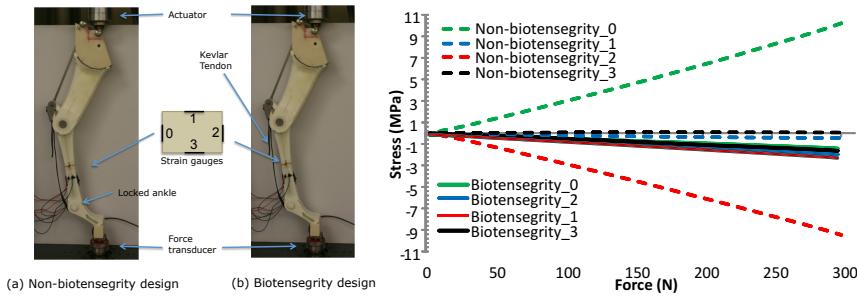
**Figure 4.4:** FEA analysis of locked-joint(L) and tensegrity leg(R) under 1000 N vertical load.  
(Ananthanarayanan et al., 2012)

The FEA and the experiments results strongly support the hypotheses introduced previously. However, it requires completely different experiments on the bodies of animals *in vivo* to verify the benefit of biotensegrity in animals. This is a representative case where engineers take only 'inspirations' from biology. Sometimes, it is much easier and faster to test a concept in engineering domains although the idea originated from animal study.

The next section will discuss several methods we can use to quickly prototype lightweight legs which is critical for shortening the design iteration cycle time necessary to evaluate candidate designs for running quadrupeds.

Applied Force	300N		1000N	
	FEA	Experiment	FEA	Experiment
Non-Biotensegrity	10.2 MPa	9.77 MPa	34 MPa	N/A
Biotensegrity	2.1 MPa	1.78 MPa	7 MPa	7.2 MPa

**Table 4.1:** Comparison of Maximum Von-mises Stresses for the two different design methods leg



**Figure 4.5:** Experimental setup for measuring stresses on the Radius for the two design concepts, Stresses on the Radius for the two design concepts. (Ananthanarayanan et al., 2012)

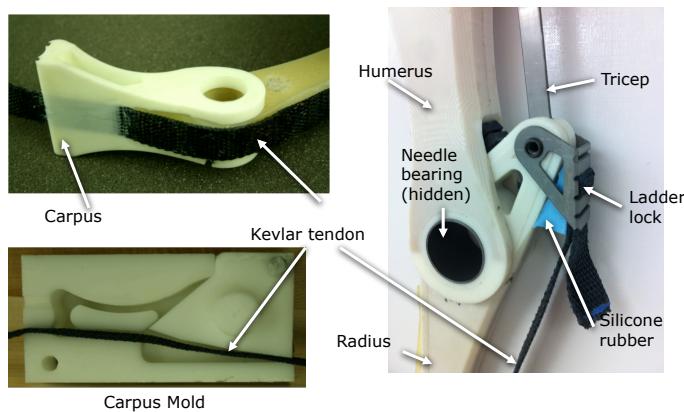
## 4.6 Fabrication of the light leg

We have shown that the tendon bone co-location between the foot and radius provided a light but stronger foot and radius. In this section, we will explain a bio-inspired fabrication method for the bone structures to enable a light-weight bone structure. The fabrication approach is inspired from biological structures such as bones which are made of a soft, light and low stiffness interior (Cancellous bone) and high stiffness shell (Cortical bone) (Rho et al., 1998). This allows bones to be light but strong. This design, while light, can withstand high bending moments due to its high area moment of inertia.

In this section, we will describe a potential prototyping method for fabricating light and strong legs. We will also describe a method we have developed for attaching the tendon to the bone structures.

## 4.7 Tendon connection

One of the other challenges in fabrication of leg based on biotensegrity designs is proper attachment of the tendon to the bone structure. Using adhesives to glue the tendon to the outside is vulnerable to fatigue failure as the peeling forces on the edge of the attachment point become stress concentrations. To address these issues we have taken two approaches: embedding and ladder-locking. Our mechanism of embedding eliminates a separate adhesive, instead routing the ribbon through the center of the foot to become encapsulated during molding. In doing so, stress is better distributed across substantial interlocking surface features of webbing. In order to prevent the unnecessary saturation of webbing by the resin, we coat wax on part of webbing that is not embedded inside of the foot. The upper left panel of Figure 4.6 shows the encapsulated tendon in the foot accomplished using the in-mold embedding technique that we have described. After exploring various high tensile-strength flexible materials for the tendon such as nylon (high tensile strength but low stiffness), we chose Kevlar® webbing as the tensile element of the design due to its minimal elongation (Cheng et al., 2005). To provide appropriate compliance, silicone rubber was inserted in between the radius and the webbing as easily tunable compliance shown in Figure 4.6, whereas the tendon is difficult to exchange or actively controlled.



**Figure 4.6:** In-mold fabrication of metacarpus with attached tendon

In addition to providing stress relaxation on the leg structure, the biotensegrity design concurrently introduces compliance to the leg which has been shown to be essential for running. However the design of the leg can be further optimized to reduce the total stresses on the leg while minimizing the overall weight of the leg.

Another connecting mechanism applied in the prototype is a ladder-lock, commonly used in rope climbing harnesses for ease of adjustment and modularity. The ladder-lock is used as a tendon extension connection and is made of steel using waterjet cutting and subsequent bending. Figure 4.6 shows how a metal ladder-lock connects the triceps linkage to the Kevlar® webbing. This design allows strong tricep tension transmission to the Extensor tendon without applying undesired torques to the bone structure. The self-locking ladder-lock design allows for convenient adjustment of tendon length.

#### **4.8 Conclusion**

This chapter has introduced design strategies for high-performance leg designs to manage the critical tradeoff of strength and weight. The design of our musculoskeletal systems has suggested a principle of biotensegrity to reducing bending moments through the strategic collocation of tension-only tendon elements with compression-only bones. While the strength-to-weight tradeoff is of critical importance in legged system design, other considerations such as kinematic workspace, and the controllability provided through available actuators manifest as potentially conflicting requirements that must be weighed in the design of any limb. As echoed in our conclusion to the chapter on actuation, there is reason to believe that we should yet exceed the performance specifications of our own limbs. Yet, after decades of work, our design tools remain more an art than a science. This observation is likely to remain true as we continue to uncover shreds of inspiration from the biological realm, with necessarily careful restraint. To exceed the performance of biological designs, the tools of engineering must be used to evaluate and emulate these guiding biological insights, while cognizant of the relative merits of our own materials and their limitations. The case study provided in this chapter may serve as an exemplar of such a process.

# 5

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## Future Directions and Applications

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Recent advancements in legged robot technology promise great potential in future mobility. Enhanced legged versatility will open up numerous mobile robot applications. Robotic service will be provided beyond information gathering and exchange. Robots will be able to pick up capabilities that allow operation in 3D (difficult, dirty, dangerous) work. Mastering legged locomotion and balance will enable stable control of systems with a high center of mass, allowing for high-force generation against environments in a large workspace. These collective advances will greatly expand the capabilities of mobile robots beyond those in wheeled designs.

There are several advantages of using robots in difficult, dirty, and dangerous environments. First, mobile robots will be more adaptable and resilient. Although we may design manual work to be as ergonomic as possible, there are still many tasks that are laborious to humans and prone to cause injuries. Robots are not only tireless but also easily reconfigurable. Electric motors, when properly designed and controlled can rapidly transition between regions of their workspace, seamlessly switching between high torque and high speed regimes without fatigue. Second, it is much easier to protect robots from hazardous elements such as chemicals, heat, fire, and so on. A robot design can reorganize critical components in a way that distal

limbs contain only crude and robust materials such as steel, while a well-protected center body frame contains all delicate components such as sensors, electronics, motors, and computers. For example, in a building fire, a specially designed robot can potentially perform physical work in a high temperature, high toxicity area where a human would not be able to perform at all. Third, it is less dangerous in general, if a robot fails to perform mission. In high-risk operations, if a robot damages itself, we can replace components and fix structures, removing operational decisions from the moral dilemma of risking human life.

There are several critical design challenges to realize this vision of legged robots serving for our society. First, robots need to have mechanical robustness comparable to animals. We often consider robotic systems to be stronger than biological systems. Potentially, robots can be significantly more robust against physical, chemical, and thermal damages. Despite access to comparatively higher strength engineering materials, the robots in current design paradigms still suffer from damage upon external impacts or any wear in components. We only discussed actuation and leg structure in this article, but there are many other components to be considered towards improving robustness. Wires, connectors, computers, bearings, sensors, motors, linkages all should be designed to survive repeated and multi-physical stains (thermal, radiational, chemical, mechanical) while addressing dust and water proofing where necessary.

While focusing on the functional advantages of legged systems, minimizing complexity is a critical step to achieve breakthroughs in ground locomotion beyond wheels and tracks. Trying to copy animals' morphologies could lead to unnecessarily complex designs. The main motivation behind legged robotics research is to emulate animals' versatile, agile and robust locomotion capabilities. Although such remarkable capabilities may be attributed to the exceptional complexity, realizing such complex features in mechanical domain could be nearly impossible. Aviation engineers were inspired by flying birds yet, did not copy the details of the birds. They focused on the major functions and principles of flying not on the detail design of wings. Extracting the essentials from the animals is the challenge to bio-inspiration for modern robotics engineers.

Although it is still debatable whether we need to copy the architecture of nature's legged systems, we need to advance mobility technologies in order to develop mobile robots capable of operating in environments outside of factories. While wheeled robots realize mobility with minimum mechanical complexity, their mobility is often limited to simple structured environments. Most promising applications for mobile robots such as elderly care, search and rescue, and disaster response require navigating unknown environments, or at least, environments with a large structural variations that current wheeled vehicles cannot manage to traverse. Mastering legged locomotion is a critical and logical step toward future applications of mobile robots.

These critical steps forward will assuredly occur through coordinated advances beyond the topics covered in this review. The design of next generation legged systems will be brought to life through breakthroughs in control architectures, perception algorithms, sensor technologies, human-machine interfaces, and yet other areas. As these groups come together, integrative challenges will illuminate the need for new theories which bridge these traditionally isolated problems. The paradigm shifts in design described here have already opened up new frontiers for high-bandwidth force-centered control methodologies and for physical human-machine interfaces that manage a legged embodiment. Through these collective advances, the next 10 years will be an bright time for legged locomotion research. With the historical context and outlook provided in this review, we hope that the both the design community and those in the areas above will have a stronger sense of the capabilities of emerging technologies. Ultimately, we hope these insights will pave the way for legged machines to yet exceed the performance of their biological counterparts while improving the quality of life for all.

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