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EFFECTS OF RAMP ANGLE AND MASS DISTRIBUTIONS ON PASSIVE DYNAMIC GAIT — AN EXPERIMENTAL STUDY

Q. WU and J. CHEN

Department of Mechanical and Manufacturing Engineering,
The University of Manitoba, Winnipeg, Manitoba,
Canada R3T 5V6

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A bipedal walking mechanism with knees is designed and built to study the passive dynamic gait. The effects of changing the ramp angle and the mass distributions of the thighs and the shanks on the gait patterns and walking robustness are studied. It is shown that the changes in the ramp angle and the mass distribution have significant effects on the step lengths and the robustness (the successful rate of launching and the step-count) of the passive gait. More specifically, as the ramp angle increases or the mass center of the entire walker is raised, the step length increases, which dictates the walking speed. However, our experiments show that the changes in the ramp angle and the mass distribution have slight effects on the step period. The optimal ramp angle and mass distribution of the passive walker are also identified, of which the passive walker has the highest successful rate of launching and the step-count. Our experimental results are compared with previous work based on simulations. This research can provide important information for validating/adjusting mathematical models of passive dynamic walking. The work also enables us to gain a better understanding of the mechanics of walking.

Keywords: Passive dynamic walking; ramp angle; mass distribution.

1. Introduction

Research on passive dynamic walking has attracted much attention in recent years for two reasons. One is to understand human locomotion, and another one is to develop energy-efficient bipedal walking robots. Passive dynamic walkers are a class of two-legged mechanisms, which consist of a chain of links connected by pin joints. For such mechanisms, walking is a natural dynamic mode. Once started on a shallow slope, these mechanisms settle into a steady gait remarkably similar to human walking without any active control. McGeer first demonstrated, through simulations and experiments,[1,2] the existence of such mechanisms. Following his work, researchers confirmed McGeer's finding, and further extended the research using two approaches. One is to build various physical mechanisms to demonstrate the passive dynamic walking. For example, Ruina's research group at Cornell University built a simple two-leg toy that is statically unstable in all standing positions, yet is stable in walking down a shallow slope.[3] They then built the first three-dimensional, two-legged and kneed passive-dynamic walking machine with specially curved feet, a compliant heel and mechanically constrained arms to achieve a harmonious and stable gait, which has been considered the most advanced fully passive dynamic walker to date.[4] With the success of the passive dynamic walkers, much research has been carried out to develop energy-efficient bipedal walking robots. For example, Wisse's research group at Delft University of Technology developed straight-legged walker with hip actuation based on McGeer's passive walker.[5] They later added an upper body to the passive dynamic walker by means of a bisecting hip mechanism.[6] Their most recent energy-efficient bipedal robot is actuated by series elastic actuation at the hip joint and the ankles with the knee joints being passive. Researchers from MIT developed a fully 3D walker with two legs and two arced feet. Its hip joint is fully passive and each ankle joint has two degrees of freedom (roll and pitch), which are activated by position controlled servo motors. Successful gait was obtained by applying reinforcement learning.[7,8] A detailed review of the actuated bipedal walkers based on the passive dynamic walking principle has been discussed in Refs. 9 and 10. The success of mimicking human walking using passive walkers suggests that natural dynamics may largely govern locomotion patterns. Thus, one needs to study the passive dynamics concept to fully understand human locomotion.[11]

The second approach of computer modeling and simulation has been widely used to study passive dynamic walking. For examples, using simple models, the nonlinear dynamics has been explored, such as a cascade of bifurcations in passive gait,[12–15] orbital stability and local stability of passive bipedal walking.[5,9,15,24] Recently, a new paradigm of limit cycle walking has been proposed, where only the orbital stability is required.[9] The computer modeling approach has also been used to study the effects of the system parameters on the gait patterns. To cite a few, McGeer[1,2] studied the effects of parameter variations on the step period and the step length of the straight-legged and kneed passive walkers. The varied parameters included the foot radius, hip mass, hip damping, leg inertia, height of the mass center and leg mismatch. Hass, *et al.*[16] investigated the optimal mass distribution for passive-based bipedal robots, and Kwan and Hubbard[17] have studied the optimal foot shape for a passive dynamic biped. Most of the research on passive walking using the modeling approach has been restricted to 2D models. Kuo[18] extended the planar motions to allow tilting side to side and found that passive walking cycles exist, but the rocking motion is unstable. Wisse *et al.*, researched a 3D passive dynamic biped with yaw and roll compensation.[19] These investigations provided important insights into the mechanics of bipedal walking. In addition, the computer simulation approach enables one to conduct simulations that are difficult even impossible for physical experiments. Computer modeling is a powerful method. However, the only study that intended to compare the simulations with the experimental results has shown poor agreements between the two.[1] Without careful validation, the simulation results from the computer models can be misleading.

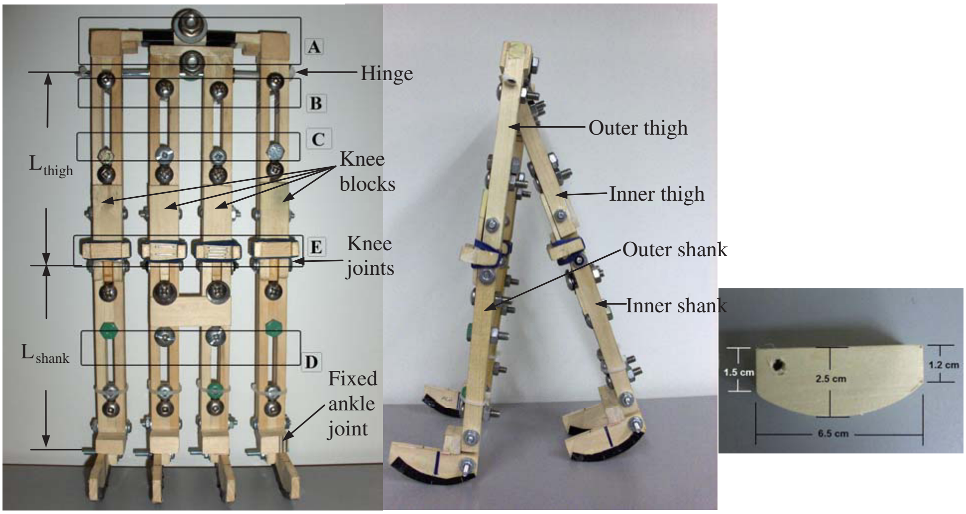
For example, it has been reported that as the ramp angle increases, the step period increases,[12] while in Refs. 1, 2, 16, and 20, simulations showed that the step period decreases significantly, and in Ref. 21, it was documented that the ramp angle has no effects on the step period. In all the above research, the computer modeling approach was used. Although there are some differences among the models, it is imperative to validate and justify the computer models by comparing simulations with the experimental results.

Currently, the physical models have been restricted to the demonstration of the existence of passive dynamic walking[4] and of various innovative ideas of energy-efficient bipedal walking robots as summarized in Refs. 5, 10, and 11. The experimental research on passive gait, especially the effects of dynamic parameters on passive gait, has been missing in the current literature. McGeer investigated the effects of ramp angles and mass offset on passive gait patterns.[1] In our previous work,[23] we designed and built a passive bipedal walker without knees, but with flat feet. Using such a walker, we studied the effects of the hip mass, ramp surface friction and size of the flat feet on the gait patterns. We believe that although quantitative comparisons among experimental results and simulations are limited due to uncertainties, it is important to investigate passive gait patterns experimentally for validating/adjusting computer models and for understanding the effects of the dynamic parameters on gait. As a continuation of our work, we present the design of a kneed passive dynamic walker similar to the one described by McGeer.[2] We further study, using the developed walking device, the effects of the ramp angle and the mass distributions of the thighs and the shanks on gait patterns. The gait patterns to be compared are: step lengths, step periods, walking speeds, and the robustness of walking. Robustness of walking, referred to as the step counts and successful rate of launches in this work, is a measure of walking stability and the basin of attraction. The overall objective is to reveal the effects of the ramp angle and mass distributions on passive gait, which can be used for model validations and justifications, and ultimately enable us to gain better understandings of the mechanics of passive dynamic walking.

The paper is organized as follows. The design of the passive walker is presented in Sec. 2. The measurement procedure, including the equipment set up, data acquisition and data analysis are detailed in Sec. 3. The results are presented in Sec. 4, followed by discussions in Sec. 5.

2. Design and Construction of Kneed Passive Dynamic Walker

The passive dynamic walker, designed in this work, consists of two parts: the wood structure of the walker and the added weights. Note that in this work, the weights refer to the bolts and the nuts as shown in Fig. 1. The walker consists of two pairs of four legs with knees and curved feet. The feet have 1 cm thickness, 10 cm diameter and other dimensions of the feet are shown in Fig. 1(b). The dynamic parameters of the walker are listed in Table 1.



(a) (b)

Fig. 1. The structure of the passive dynamic walker, (a) the skeleton and the weights, and (b) the foot.

Table 1. Dynamic parameters of the passive dynamic walker.

	Thighs				Shanks			
	Inner Thigh		Outer Thigh		Inner Shank		Outer Shank	
Mass of the link M (kg)	0.117		0.126		0.135		0.141	
Lengths of the thigh, L_{thigh}, and the shank, L_{shank} (m)	0.13		0.13		0.151		0.151	
Width of the walker (m)	0.13							
Location of mass center (m)* and its ratio to the link length	0.031	30%	0.031	24%	0.062	42%	0.064	41%
Moment of inertia about the mass center ($\times 10^{-4}$kg $\cdot$ m^2)	1.586		2.409		2.708		2.833	
Radius of gyration (m)	0.037		0.044		0.045		0.045	

*Note that the locations of the mass centers of the thigh and the shank are measured from the hip joint and the knee joint, respectively.

To improve the walking of the mechanism, weights (bolts and nuts) are added to the walker. The mass of the total added weights is 0.306 kg. The details of the weights added at different locations, as shown in Fig. 1, are: 0.0242 kg added at location A of outer legs, 0.02 kg added at location A of inner legs, 0.043 kg (0.011 kg$\times$ 4) added at location B, 0.08 kg added at location C (0.0093 kg $\times$ 2, 0.009 kg $\times$ 2 and 0.011 kg $\times$ 4), 0.043 kg added at the location right below the knees (0.011 g $\times$ 4), 0.038 kg added at location D (0.011 kg $\times$ 2 and 0.008 kg $\times$ 2) and 0.058 kg added at the location above the feet (0.011 kg × 4 and 0.004 kg × 4). The total mass of the entire walker is 0.519 kg.

During the building of the passive walker, several issues must be taken care of. First, as shown in the Fig. 1, the heights of the knee joints must be consistent, and it was achieved by constructing the four identical shanks with the holes of the knee joint drilled at the same height and making the thighs of the inner legs shorter. Secondly, a wooden block (0.053 m × 0.02 m × 0.003 m) was added to attach two inner thighs together to prevent the inconsistent swing of the two inner legs. Thirdly, the area of the curved feet and the material of the soles determine the coefficient of friction between the soles and the contact surface. It was found that preventing slipping is crucial for successful passive walking. Finally, the friction at the knee joints has significant effects on successful passive walking. The friction of the knees has to be reduced as low as possible, and to be kept consistent among all knees.

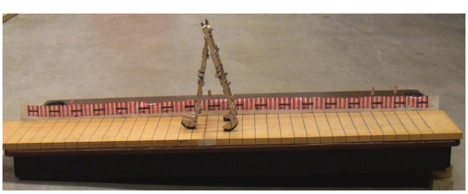
3. Measurement Protocol

The method for measuring passive gait parameters of the walker is discussed in this section. The protocol includes the equipment setup, data processing and data analysis.

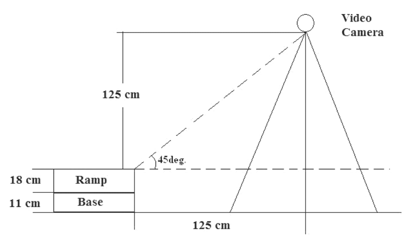
3.1. *Equipment setup*

One digital video camera recorder (Sony TRV340 (NTSC)) was mounted on an adjustable tripod and was connected to a computer to record the trials as the passive walker walking down the ramp. The camera has the frequency of 30 frames per second. The 45-degree observation angle of the camera was used for all measurements for the best visual results. A 1.8 m long by 0.25 m wide wooden block was used as the ramp. The ramp has marked division spacing 0.05 m each for calibration. The detailed setup is shown in Fig. 2.

In each trial, the walker was launched from the top of the ramp. The video camera started to record the walking motion, and after capturing the motion, a movie editing software (Adobe Premiere Pro) was used to digitize the motion. Once



(a)



(b)

Fig. 2. Experimental set-up (a) side view of the set-up and (b) ramp and ramp divisions.

all the desired data were extracted, they were entered into excel spreadsheets for analysis.

3.2. *Data process*

The recorded videos were loaded into Adobe Premiere Pro, the software was used for digitization, and the step period and the step length are determined as:

Step period: The step period is the duration of each step, which was calculated by subtracting the time instants between two adjacent foot-touch-downs.

Step length: The distance between the contact points of two feet is defined as the step length.

Walking speed: The walking speed is defined as the forward speed of the hip joint, which is the step length/the step period.

For each parameter variation, the experiment was repeated for ten trials. The measurements and digitization were conducted over five of the ten trials. The mean and standard deviation of all the above gait parameters were calculated over the five trials.

3.3. *Data analysis*

It is meritorious to discuss the measures of (in)efficiency of passive walking and of the merit for locomotion. The common dimensionless measure of inefficiency for gravity-powered locomotion is the "specific cost of locomotion" taking the distance as one step as[13]:

$$\eta = \frac{(\text{weight}) \times (\text{height drop over one step})}{(\text{walker weight}) \times (\text{step length})}. \tag{1}$$

The term, (weight) × (height drop over one step), is the amount of the energy input to the passive walker, which is converted from the gravitational energy. So based on path distance, the inefficiency measure is

$$\eta = \tan\gamma, \tag{2}$$

where γ is the ramp angle. For downhill locomotion on small ramp slope, the inefficiency can be approximated as

$$\eta = \gamma. \tag{3}$$

Perfect locomotion efficiency is achieved by passive walking with $\gamma = 0$.

Note that energetic efficiency does not always merit walking speed. A simply non-dimensional measure of the merit that rewards walking speed is the Froude number or the dimensionless velocity, $\frac{V}{\sqrt{gl}}$, where l is the length of the leg. In this work, the experimental results are presented in dimensional forms and non-dimensional forms for the purpose of comparisons with previous work. For the non-dimensional form, the step period is normalized with $\sqrt{l/g}$, and the walking speed is normalized with $\sqrt{gl}$. Unlike the previous work that the step length was characterized by the angle between the two legs when both pair of feet were in contact with the ramp, the dimensionless step length, in this work, is presented by the angle between the segments connecting the contact point of each individual foot and the hip joint to avoid the effects of the geometry of the foot design.

4. Results

Effects of parametric variations on passive gait patterns are presented in this section. The parameter variations include the changes in the ramp angles and in the mass distributions of the shanks and the thighs. The gait patterns include the step period, step length, walking speed and robustness of the gait. The robustness of the gait is referred to the step counts and the successful rate of launches.

4.1. *Effects of ramp angles*

The passive walker, as shown in Fig. 1, was used as the standard walker. The friction coefficient between the soles and the ramp is 0.09. The ramp angle, of which the walker can steadily walk for more than four steps, was ranged from 4.3° to 6.7°. The measured and calculated gait parameters are shown in Fig. 3 (the left column), and the dimensionless gait parameters are shown in the same figure (the right column).

From Fig. 3, one can observe that, as the ramp angle increases, the step length increases significantly from 0.14 to 0.20 m with the maximum standard deviation of 4.2×10^{-3} m. The step period, T, decreases slightly from 0.51 to 0.48 sec with the maximum standard deviation of 0.03 sec. Consequently, the walking speed increases with the ramp angle.

The step counts with various ramp angles are shown in Table 2. The mean step counts were calculated from ten trials. The successful rate of the launches is the ratio of the number of the launches with more than four steps to the total number of the launches. The results show that the successful rates, as the ramp angle increases from 4.3° to 6.1°, are above 90%.

The results also show that the most robust walking is at ramp angle of 5° which yields the successful rate of 100% and the highest step count of 8.4 steps. Thus, 5° is used as the standard ramp angle for the comparisons of the gait pattern affected by the mass distributions.

4.2. *Effects of mass distributions*

The alternations of the mass distributions vary the leg inertia and the height of the mass center of each thigh and shank, which changes the location of the mass center of the entire passive dynamic walker. As stated in McGeer's work,[1] the radius of gyration of the leg (inertia of the leg) and the location of the center-of-mass have similar effects. In this section, the gait patterns of the passive walker with various locations of mass centers will be discussed. The mass center is the one of the entire

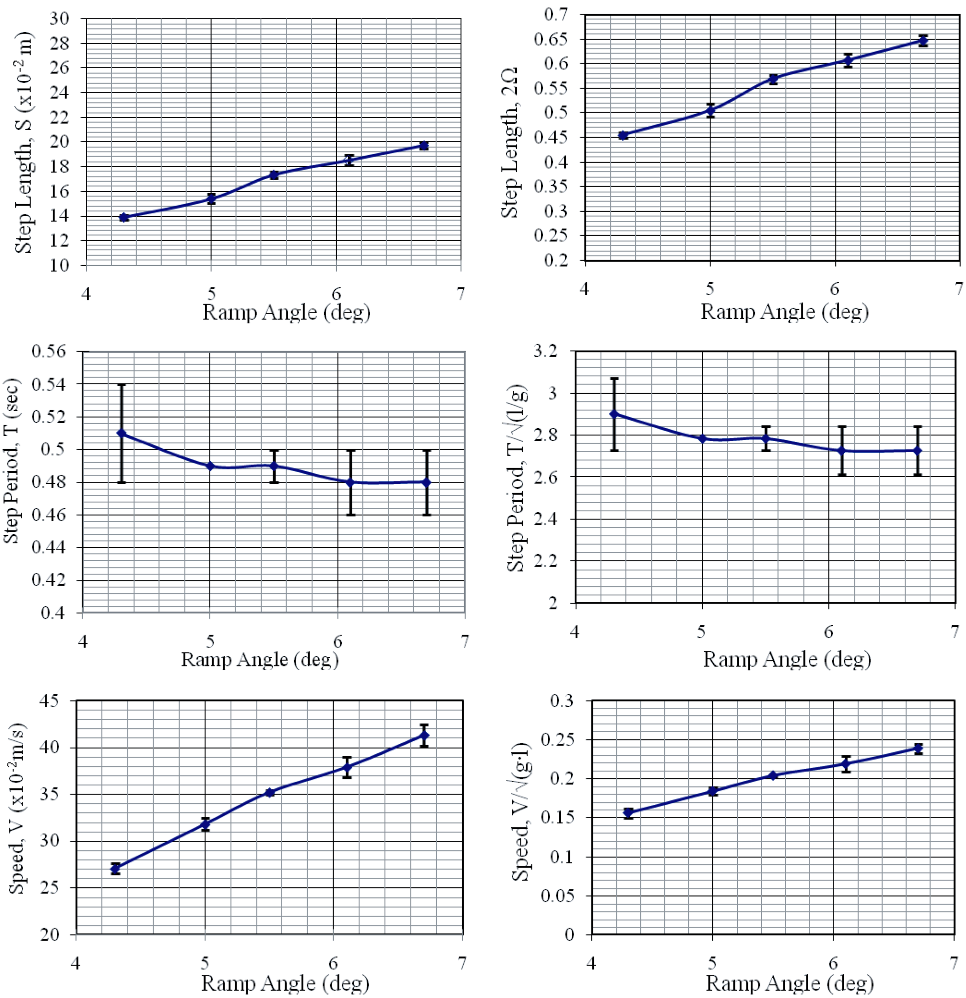


Fig. 3. Experimental results of the step length, step period and walking speed of the passive walker as the ramp angle changes. The left column shows the measured gait parameters, and the right column is the dimensionless gait parameters.

Table 2. Results of the step count with various ramp angles.

Ramp Angle (deg)	Step Count		
	Mean	Std.	Successful Rate (%)
4.3	4.7	0.9	90.0
5.0	8.4	1.7	100.0
5.5	5.7	1.5	90.0
6.1	4.9	1.3	90.0
6.7	2.5	1.2	20.0

Table 3. Locations of mass center with different mass distributions.

	Ramp Angle of 5.0 (deg)	Location of the Mass Center from the Hip ($\times 10^{-2}$m)	Ratio of the Mass Center to the Walker Height (%)
Raised mass center	Standard	12	43
	A(+43)	10.59	37.8
	B(+43)	10.90	38.9
	C(+43)	11.26	40.2
	D (−43)	11.27	40
	A (+86)	X	X
	B (+86)	10.07	36
	C (+86)	10.78	38.5
	E(+7)	11.58	41.4
	E(+14)	11.43	40.8
	E (+21)	11.28	40.3
Lowered mass center	A (−)	X	X
	B (−)	12.73	45.5
	C (−)	X	X
	D (+43)	12.22	43.6
	D (+86)	X	X

walker with the knees fully extended, which is referred to here as the static mass center. The static mass center of the standard passive walker, before the mass distributions are altered, is located close to the knee joint, 0.12 m measured from the hip. The locations of the static mass center were altered by adding/removing identical weights (bolts and nuts) at different locations or altering the mass of the weights at the same location. Note that changing the mass of the walker scales the forces, but does not change the gait.[1] Thus, the changes in the gait parameters are caused by the changes in the location of the mass center. The mass distributions were altered by adding/removing weights (nuts and bolts) at locations A, B, C, D, and E as shown in Fig. 1.

As shown in Table 3, fifteen experiments were carried out for different mass distributions. "A(−)" indicates that walking was tested using the standard walker by removing the existing weights with 0.0242 kg mass on the outer legs and of 0.0202 kg mass on the inner legs at location A. "A(+43)" indicates the addition of weights of 0.043 kg at the location A of the standard walker. "X" means the walker failed to walk for more than four steps. Table 3 also shows the locations of the mass centers with various mass distributions. There are two scenarios; either the static mass center is raised above the knee by adding the weights with various mass at A, B, C and E or removing the weights at D, or the static mass center is lowered below the knee by adding the weights at D or removing the weights from A, B and C. All experiments were tested on a ramp with a 5° angle.

Figure 4 shows the overall changes in the gait parameters due to the changes in the mass distributions. The measured and calculated gait parameters are shown as the left column of Fig. 4, and the dimensionless gait parameters are as the right column for later comparisons with the previous work. The vertical dash lines at

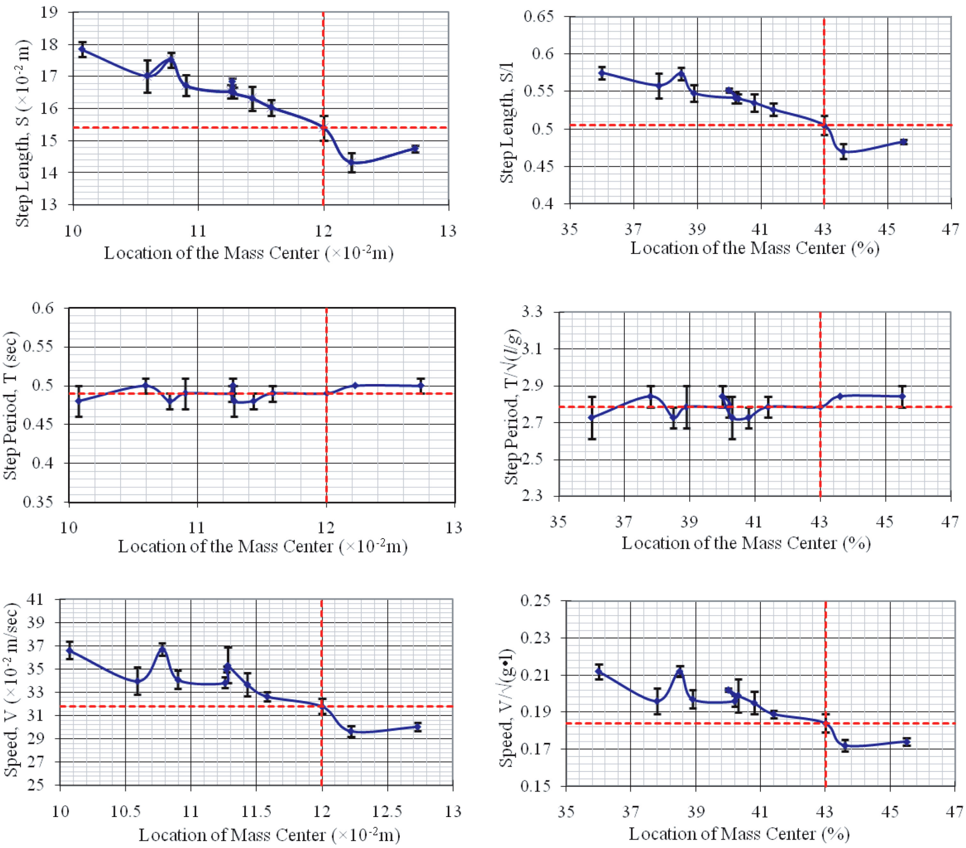


Fig. 4. Experimental results of the step lengths, step period and walking speed of the passive walker. as the mass distribution changes. The left column shows the measured gait parameters, and the right column is the dimensionless gait parameters.

0.12 m of the left column and 43% of the right column indicate the location of the static mass center of the standard walker without any mass alteration, and the horizontal dash line shows the dimensional and dimensionless gait parameters as references. Figure 4 shows that as the mass center is above the knee (the points left to the vertical dashed lines), the step length is higher than the one from the standard walker, and it increases as the mass center is close to the hip, while the mass center is below the knee (the points right to the vertical dash lines), the step length is lower than the one from the standard walker. However, the changes in the step period are insignificant, and no clear trend can be observed. Therefore, the walking speed is dominated by the step length, and follows the same trend as the step length.

To be more specific, as the static mass center is above the knee joint, Table 3 shows that walking is not sustainable as the weights with large mass (0.086 kg) are added at A, while as the weights with the mass of 0.043 kg are added at the locations A, B, C or removed from D, the walker is able to walk on the ramp with substantially increased step lengths, 0.165 m to 0.176 m versus the one from the standard walker, 0.154 m. However, the changes in the step period are insignificant (0.50 sec versus 0.49 sec). As the weights with the mass of 0.086 kg are added to the walker at the locations of B and C, the step lengths increases to 0.176 m and 0.175 m as compared to the standard one of 0.154 m. The step period is reduced to 0.48 sec with large deviations. The walking speed is dominated by the step length, and follows the same trend as the step length.

We also attempted to add weights close to the knee joints. It was found that the sustainable gait is very sensitive to the changes in mass at the knee joints. We were only successful to add the weights with low mass at the location E, which is slightly above the knee joints. It is shown in Fig. 4 that the step length is higher than the one from the standard walker of 0.154 m, and it increases from 0.16 m, to 0.163 m to 0.165 m as the weights with the mass of 0.007 kg, 0.014 kg and 0.021 kg is added to E individually. This is expected that adding the weights at E raises the mass center above the knee joints. Similar to other tests, the changes in the step period is insignificant, and the walking speed is dictated by the step length. Overall, the results show that raising the mass center increases the step length, but the changes in the step period is not significant.

As the mass center is below the knee, Table 3 shows that walking is not sustainable as the weights are removed from A and C, or the large mass (0.086 kg) are added at location D, while as the weights with the mass of 0.043 kg are removed from location B or added to location D, the walker is able to walk on the ramp with substantially reduced step lengths, 0.148 m and 0.143 m versus the one from the standard walker, 0.154 m. The step period is slightly above the one from the standard walker (0.50 sec versus 0.49 sec). Consequently, the walking speeds are lower than the one from the standard walker.

Comparing the walking robustness, as shown in Table 4, as the mass center is above the knee, although the successful rate of launches remain 100% as the weights with the mass of 0.043kg are added at the locations A, B and C, the step counts reduce noticeably, especially as the weights are added to the Location A, the step count is reduced by 36%. Removing the weights from the location D raises the mass center. However, in all the trials, the step counts are below 4 and the successful rate for having 3.3 steps is 60%, which indicates that the passive walking is sensitive to the changes in the mass distributions of the shank. Considering the results of adding the weights with large mass of 0.086 kg at the locations B and C, the successful rate of launches are both 100%. The step counts reduced to 5.6 (33%) and 6.9 (18%), respectively as compared with the standard walker of 8.4.

Comparing the walking robustness as the mass center is below the knee, it is noticed from Table 3 that although the launch successful rates are high above 90% for cases B(−43) and D(+43), the step counts, as the static mass center is below the knee, are significantly reduced by approximately 45%, as compared with the standard walker.

Table 4. Results of the step count with various mass distributions.

Location (Mass Added)	Step Count (5 degree)		
	Mean	Std.	Success (%)
Standard	8.4	1.7	100.0
A (−43)	X	X	X
B (−43)	4.9	1.4	90.0
C (−43)	X	X	X
D (−43)	3.3	0.9	60.0
A (+43)	5.4	1.2	100.0
B (+43)	6.9	1.2	100.0
C (+43)	6.2	1.2	100.0
D (+43)	4.7	1.1	100.0
A (+86)	X	X	X
B (+86)	5.6	1.0	100.0
C (+86)	6.9	1.0	100.0
D (+86)	X	X	X
E (7)	7.3	1.4	100.0
E (+14)	6.0	1.4	100.0
E (+21)	5.3	0.9	100.0

In summary, the results show that, as compared with the standard walker, raising the mass center increases the step length. The changes in the step period do not show a clear trend, and the walking speed is dictated by the step length. In terms of the step counts and successful rate of launches, raising the mass center does not affect the successful rate though the step counts reduced moderately as compared with the standard walker. However, the mass center should not be raised too high. On the other hand, lower the mass center, as compared with the standard walker, significantly reduces the step count and the successful rate for launching, which is not desirable for passive walking.

5. Discussions

In this section, we interpret the experimental results and compare, qualitatively, the gait parameters from previous work using the simulation approach to those from our experiments. We intend to identify the important components, which should be included in modeling and to improve future experimental studies.

5.1. *Effects of ramp angles*

Our results show that the ramp angle has significant effects on the step length and the walking speed, i.e., as the ramp angle increases, the step length increases significantly, and the walking speed is dominated by the step length. The finding of the increase in the ramp angle leading to the increase in the step length is consistent with previous research using the simulation approach.[1,2,12,23] Our observation on the increase in the ramp angle resulting in the slightly decrease in the step period is consistent with the simulation results from Refs. 1, 2, and 20. However, simulations from Ref. 12 shows the step period increases as the ramp angle increases, and Ref. 21 shows that the ramp angle has no effect on the step period as the ramp angle is low. Since the amount of changes in the step period is low and the standard deviation is relative high, more research is needed on experimental measurements to draw conclusions.

The ramp angle indicates the walking (in)efficiency.[1,13] Lower the ramp angle indicates higher walking efficiency in that lower mechanical work is required to execute passive walking. From the merit for walking point of view, our results show that lower ramp angles result in shorter step lengths, and, consequently, lower walking speeds, i.e., the lower merit of walking. Similar observations of the conflict between the efficiency and walking speed have also been made in Refs. 1, 2, 13, 16 and 20. Regarding the walking stability, we found that as the ramp angle was increased from 5°, although the successful rate of launching is consistently high, the step counts decreases significantly, indicating that the basin of attraction of the designed passive walker decreases as the ramp angle increases or the stability is weakened. This finding is consistent with the discussions in Ref. 16. However, we found the optimal ramp angle (5° in our case), at which the step count and the successful rate of launching are the highest. Reducing the ramp angle from 5° causes the decreases in the step count and successful rate of launching.

5.2. *Effects of mass distributions*

The mass distributions have significant effects on passive gait. As the mass center is raised, the step lengths increase, and step periods do not change significantly, and higher walking speeds are obtained. Changes in the mass distributions have no effects on walking efficiency, but have strong effects on the walking merit. As the mass center is lifted, the merit of walking is increased. This is consistent with the observations made in Refs. 1 and 16. Considering the robustness of the passive gait, there exists an optimal mass distribution. As the static mass center deviates from the optimal one, the robustness of walking, in terms of step counts and successful rate of launches, reduces significantly, especially when the mass center is below the knee joints. The decreases the step counts and the successful rate of launching indicate that the basin of attraction is reduced or the stability is weaker as compared with the standard walker. This observation is consistent with the results from Ref. 16 in that the maximum basin of attraction occurs at moderate speed, and conversely, for high walking speed, the gait tends to be unstable. In our experiments, we also observed that for low walking speed, the gait also tends to be unstable. These results indicate that as the input energy, i.e., the walking (in)efficiency is fixed, adjustment of the mass distributions, i.e., raising the mass center, can improve the merit and the stability of passive walking as compared with lowering the mass center. However, apart from the general and intuitive rule to put the mass near the hip, our results show that the center of mass should not be too close to the hip. These observations have been consistent with those in Ref. 16.

5.3. Comparison with McGeer's work

Since the computer model used in McGeer's work[1,2] is close to the one built in this research, and the effects of similar parameter variations on passive gait were investigated in Refs. 1 and 2, in this section, we compare our experimental results with those from Refs. 1 and 2. Note that in McGeer's work,[1,2] two computer models, one of a straight-legged passive walker and another one of a kneed passive walker were developed. Such models were used to simulate walking and the results on the step lengths, step periods and walking speeds versus the ramp angles were reported. In spite of the significant differences in the above two walkers, it was reported that the gaits form the kneed passive walker and the straight-legged walker have remarkably similar performance.[2]

Table 5 shows the key dynamic parameters of McGeer's simulation model and of our passive walker. Table 6 shows the dimensionless gait parameters from two walkers. The main differences between the dynamic parameters of McGeer's walker and our walker are: (1) the ratio of the mass of the thighs and the shanks, 1.6 of McGeer's walker versus 0.88 of our walker, and (2) the relative location of the mass center of the thighs measured from the hip, 43.5% versus 27.7%.

Although quantitative comparison is impossible, the trends of the changes of the gait parameters of the two walkers are comparable. For example, as the ramp angle increases, the step period decreases, the step length increases, which dominate the walking speed. Our observation that it is not desirable to have the mass center too high or too low to sustain passive gait, is consistent with the results in Ref. 1. These agreements indicate that the mathematical model[1,2] captured the main dynamics of the passive bipedal walker. However, the amounts of changes, especially the step period, are not comparable. For examples, with the range of ramp angles of 0.29° to 3.4°, McGeer reported that the dimensionless step period decreases from 3.5 to 2.5, the dimensionless step length (inter-leg angle) increases from 0.1 to 0.7, and the walking speed increases from 0.029 to 0.28. Our walker cannot walk on the ramp with the ramp angle below 3.4°. The ramp angle, of which our passive

Table 5. Dynamic parameters of the passive dynamic walker.

	McGeer's Model				Our Walker			
	Thighs		Shanks		Thighs		Shanks	
Mass of the link (kg)	0.1		0.062		0.243		0.275	
Length of the link (m)	0.46	46%	0.54	54%	0.13	46.3%	0.15	53.4%
Location of the mass center (m) and its ratio to the link length	0.2*	43.5%	0.24**	44%	0.036*	27.7%	0.065**	43.1%
Radius of gyration (cm)	0.135	29.3%	0.186	34%	0.041	31.5%	0.045	29.8%

Note: *Measured from the hip.
**Measured from the knee.

Table 6. Dimensionless gait parameters of McGeer's walker and our walker.

	Kneed Walker		Our Work	
Ramp angle (deg)	0.29°	3.4°	4.3°	6.7°
Step period/$\sqrt{l/g}$	3.5	2.5	2.900	2.727
Inter-leg angle (rad)	0.10	0.70	0.455	0.647
Walking speed/$\sqrt{gl}$	0.029	0.28	0.157	0.237

walker can walk, ranges from 4.3° to 6.7°. The step period decreases from 2.90 to 2.73, inter-leg angle increases from 0.455 to 0.647 rad. The experimental results alarm us in that although increasing the ramp angles enlarges the step length, our step lengths are significantly lower than the previous one.[1,2] Also, the simulation results from previous work[1,2] show the step period decrease significantly as the ramp angle increases, while our experimental results show insignificant changes in the step period, and our dimensionless step period is consistently higher than McGeer's simulated step period.

Although the simulation results[1,2] and our experimental measurements agree reasonably well in the trends of the gait kinematics, the range of ramp angles and the amount of the changes in such gait parameters, especially the step period, differs significantly. Improvements in measuring the step period are imperative for experimental research. However, in the experimental research, we noticed that friction, at the knee and hip joints as well as at the contact surface between the feet and the ground, plays an important role in sustainable gait. Our experiments indicates that it is crucial to include an adequate friction model to the mathematical model of passive bipedal walking, both rolling friction between the feet and the ground, and sliding friction at the knee joints and the hip joints. This finding is consistent with McGeer's work in that when the rolling friction was included in his model, the simulation results agreed with those from the experiments better.[1]

5.4. *Summary and recommendations*

In summary, a four-leg kneed passive dynamic walker, similar to McGeer's work, has been designed and built. Such a walker has been used to study the effects of the changes in the ramp angle and the mass distributions of the thighs and the shanks on gait patterns of passive walking. It was found that:

(1) Effects of the ramp angles on gait patterns: increases in the ramp angle increases the step lengths significantly, which dominates the walking speeds, The ramp angle is a measure of the walking (in)efficiency. Our results indicate that the increase in the ramp angle lowers efficiency, but more gravitational energy is available to execute the walking, which leads to a higher merit of walking.

(2) Effects of mass distributions on gait patterns: raising the mass center moderately increases the step lengths, which determining the walking speeds. Lowering the mass center reduces the step lengths and the walking robustness significantly.

Thus, lowering the mass center is not desirable for achieving passive walking. As the ramp angle is fixed, the walking efficiency is constant. The increases in the walking speed as compared with the standard walker indicate the increase in the merit of walking. Thus, to achieve the desired merit of walking, it is not necessary to have higher gravitational energy input, but changing the mass distributions can be an alternative way.

(3) Effects on gait stability: there exists an optimal ramp angle and optimal mass distributions, at which walking is most robust, i.e., the step count and the successful rate are the highest. The robustness of walking is a measure of walking stability. As the ramp angle and the mass distributions deviate from the optimal values, the step counts and the successful rate of launching decrease, which indicates the weaker walking stability and/or smaller basin of attraction.

The experimental results of the effects of ramp angles and the mass distributions in terms of step lengths, step periods, walking speed, and walking stability were compared, qualitatively, with the previous work based on simulations,[1,2,12,13,20] especially with McGeer's simulations results due to the similarity of the model.[1,2] The discrepancy between our results and those from previous work based on computer simulations has strong indications for future research. In our research, we noticed that friction plays an important role, while friction was not included in most of the computer models. Thus, it is recommended here to include friction in the mathematical model. Furthermore, friction can be a tool to sustain and to affect the patterns of passive walking. Its effects and role should be further explored.

On the other hand, as described in Sec. 3, the step period was measured by recording the time instants of two consequent foot-strikes. As the camera speed of 30 Hz, the accuracy in the measurements of the step periods with different ramp angle or mass distribution needs to be improved. Thus, to improve the future experiment, we plan to use force sensors to detect contact instants between the feet and the ground to improve the accuracy of the step period.

For the purpose of comparison, it is essential to document the results in a dimensionless form. Thus it is crucial to conduct dimensional analysis of bipedal gait adequately. In this work, the gait parameters were normalized following previous research,[1,2,13] which was based on the passive walking without knee joints. The mass distributions of the thighs and shanks are not uniform. The use of the length of the entire leg for normalizing the gait parameters is questionable. Thus, further research on dimensional analysis of gait is important.

Although the work is still at its early stage in terms of exploring theoretical insights into the mechanics of passive walking, we believe that such work provides (1) guidelines for developing mathematical models of passive walking relevant to the physical systems, such as including better friction models, and (2) the experimental results for validating the mathematical models. With active research on exploring nonlinear dynamics of passive walking using various mathematical models, experimental research on kinematics and dynamics of passive dynamic walking has been sparse. The research presented here is an initial step to fill such a gap.

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Qiong Wu received the B.Sc. degree in Applied Mechanics in 1986 from Beijing University, Beijing, China, the M.Sc. degree in Mechanical Engineering from The University of British Columbia, Vancouver, BC, Canada, in 1990, and the Ph.D. degree of Mechanical Engineering from The University of Manitoba, Winnipeg, Manitoba, Canada in 1996.

She is currently a Professor of mechanical engineering with the University of Manitoba. Her research interests include time series analysis, Lyapunov-related stability analysis of nonlinear/non-smooth dynamic systems, development of stable and energy-efficient bipedal walking robots, biomechanics and vehicle dynamics.

Dr. Wu is a Fellow of Canadian Society for Mechanical Engineering (CSME), a member of the American Society of Mechanical Engineers (ASME). She is a Registered Professional Engineer.

Jie Chen graduated from the University of Manitoba in 2008 with a B.Sc. degree in mechanical engineering. After that, he continued his graduate study in the University of Alberta, and graduated in 2010 with an M.Sc. degree in mechanical engineering. Currently, he continues on his graduate study in PhD level.

During the four and half years of undergraduate study at the University of Manitoba, he successfully completed all the requirements from the department of mechanical engineering. At the final year of undergraduate study, he did a thesis project with Dr. C. Wu about an experimental study of gait patterns of passive dynamic walking.

During the two years of graduate study at the University of Alberta, he did an industrial collaborated project about an experimental study of elastoplastic mechanical properties of coke drum materials.