



Bifurcations and chaos in passive dynamic walking: A review

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HIGHLIGHTS

- The chaos research in Passive Dynamic Walking (PDW) is covered in the reviewed literature.
- An account of chaos control techniques in PDW bipeds is presented. This area certainly necessitates further investigation.
- The need of new mathematical methods is emphasized so that PDW bipeds can be studied analytically.
- Potential research directions have been identified.

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ABSTRACT

Irrespective of achieving certain success in comprehending Passive Dynamic Walking (PDW) phenomena from a viewpoint of the chaotic dynamics and bifurcation scenarios, a lot of questions still need to be answered. This paper provides an overview of the previous literature on the chaotic behavior of passive dynamic biped robots. A review of a broad spectrum of chaotic phenomena found in PDW in the past is presented for better understanding of the chaos detection and controlling methods. This paper also indicates that the bulk of literature on PDW robots is focused on locomotion on slope, but there is a thriving trend towards bipedal walking in more challenging environments.

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1. Introduction

Passive Dynamic Walking (PDW) originated from the ability of the biped to act as an inverted-jointed pendulum during the single support phase. A passive biped robot walks by the dynamics only. Human-like walking and high efficiency are its two vital benefits. McGeer coined the term PDW and its principles in the early 1990s. He showed that an uncontrolled and unpowered biped robot could walk stably on shallow slopes if suitable initial conditions are applied [1–3].

Fig. 1 shows McGeer's passive bipedal machine—Dynamite. Its gait is generated automatically by gravity and inertia, without planning trajectory beforehand. This periodic gait is called the

"limit cycle" in terms of dynamical system theory. The organic evolution of limit cycle is a key characteristic of passive dynamic walking that imitates human walking—which is controlled by the neuro-muscular system, i.e., it is powered by muscles and controlled by the nervous system. Without nerves and muscles, this complex and controlled process can be modeled as an unpowered (passive) mechanical process. The better insight of human locomotion, development of cutting-edge prosthetic limbs and superior design of humanoid robots is the impetus for the investigation in PDW robots.

Passive bipeds are dynamical systems which are sensitive to initial conditions due to which they exhibit chaotic behavior. Several chaotic dynamics, e.g., bifurcation, intermittency and crises, etc., occur in consequence of variation in different parameters in the dynamical equations of many simple passive biped models. Many researchers from diverse fields like engineering, mathematics, biomechanics and computer science have been fascinated by these erratic dynamics. The chaos research in PDW bipeds has emerged as an interdisciplinary research area and it could be useful in diagnosing gait pathologies [4].

The chaos research in PDW has resulted in a tectonic shift in the way PDW phenomena is viewed now. This article describes the

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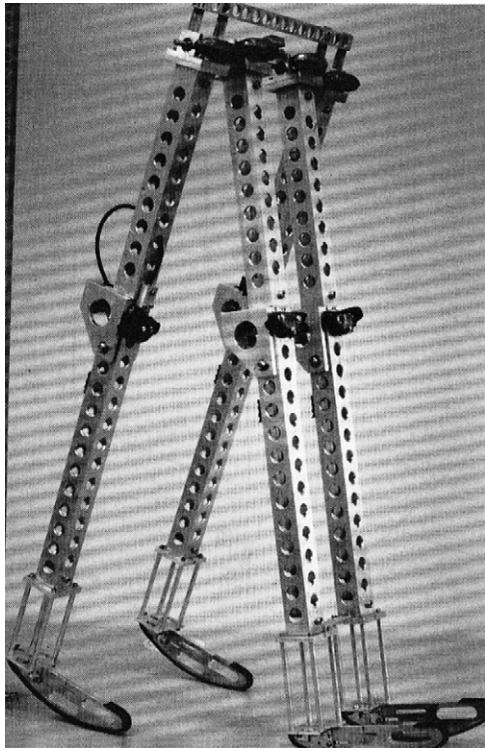


Fig. 1. McGeer's experimental passive biped [2].

current status and future of nonlinear dynamics study in passive walking. This paper is organized as follows: Section 2 introduces major terms of the chaos theory. In chronological order, the chaos exploration in passive bipeds is covered in Section 3. Similarly, Section 4 presents chaos control research in PDW sequentially and Section 5 discusses potential research directions in this research area. Section 6 gives concluding comments.

2. Chaotic dynamics: important terms

“Chaos” or “Deterministic Chaos” is aperiodic long-term behavior in a deterministic dynamical system that exhibits sensitive dependence on initial conditions. This sensitive dependence on initial conditions – that ensures uncertainty and unpredictability in the system behavior – is often called the “Butterfly Effect”. A tiny perturbation to the initial conditions significantly alters the long term system dynamics. The Butterfly effect is the trademark of chaos theory. The chaos theory is an interdisciplinary area of study including mathematics, physics and engineering. This field investigates the dynamical behavior of systems which are extremely susceptible to initial conditions. It is applied in various scientific fields e.g., biology, economics, finance, geology, meteorology, population dynamics, psychology, and robotics. Since the advent of digital computers, chaos theory has become more comprehensible and is growing quickly. A glossary of important terms of this theory is included for new researchers [5–7]:

Bifurcation or branching is a sudden qualitative change in the dynamics as a system parameter varies. The parameter value at which branching occurs is called “Bifurcation Point,” at which the system is unstable structurally. Technically, bifurcations are very vital as they provide paradigms of instabilities and transitions as some control parameter is modified. In both continuous and discrete systems bifurcations occur.

Bifurcation diagram is a chart which illustrates the steady-state behavior of a system over a range of parameter values. That is, it depicts the periodic orbits or fixed/equilibrium/critical points of a

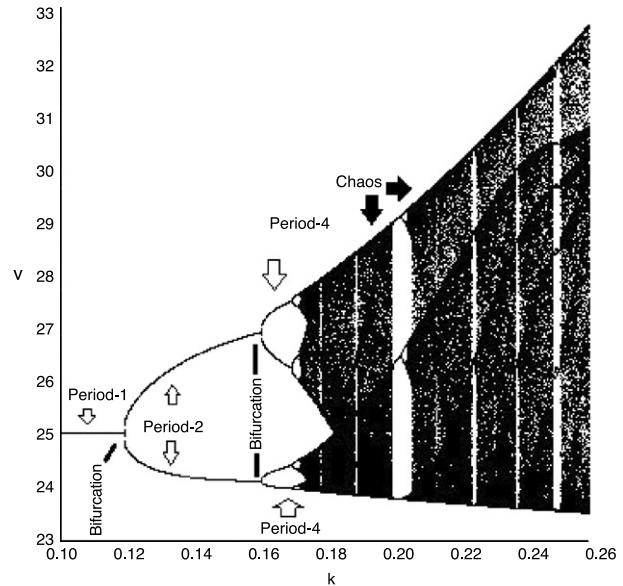


Fig. 2. Bifurcation diagram of a DC-DC buck converter [8].

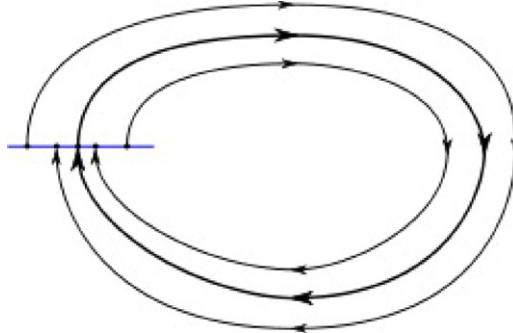


Fig. 3. Limit cycle.

system versus the bifurcation parameter as illustrated in Fig. 2. This graph summarizes the enormous information concisely so that system behavior as a function of a control (bifurcation) parameter can be observed. Fig. 2 shows the bifurcation diagram for a DC-DC buck converter [8].

Attractor is a set of points to which all adjacent trajectories converge as the number of iterations approaches to infinity i.e., points that get close enough to the attractor stay close even if slightly perturbed. It is a region in n -dimensional space. Since a trajectory can be periodic or chaotic, its instances are stable limit cycle, stable fixed point, and even an intricate strange attractor.

Limit cycle is an isolated periodic solution or orbit in the autonomous system as demonstrated in Fig. 3. Limit cycles are approached by nearby trajectories. They are categorized as “attractive or stable limit cycles” and “un-attractive or unstable limit cycles”.

Strange/chaotic attractor is an attractor of a dissipative system that has noninteger (fractal) dimension. It is also called “fractal attractor”. This shows sensitive dependence on initial conditions. Fig. 4 shows a strange attractor.

Basin of attraction also called “attracting basin”, is the set of all initial conditions that leads trajectories to a given attractor.

Lyapunov exponent quantifies the rate of divergence of infinitesimally nearby trajectories starting from close initial conditions. The largest one is called “Maximal Lyapunov Exponent (MLE)” and a positive MLE usually indicates a chaotic attractor.

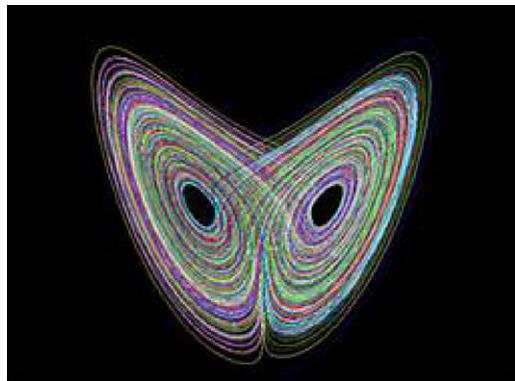


Fig. 4. Strange attractor.

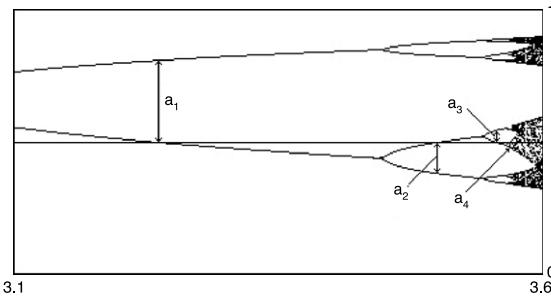


Fig. 5. Vertical and horizontal distances of wishbones in a bifurcation diagram.

Routes to chaos: a dynamical system may have several diverse kinds of chaotic behavior, i.e., transitions to chaos for different ranges of parameter values. The theory of dynamical systems is not developed enough to predict beforehand the routes to chaos with different control parameters for a given system. Familiar transitions to chaos are period-doubling, quasi-periodicity, intermittency and crisis.

Period-doubling bifurcation is a bifurcation, in a discrete system, in which the system changes to a new behavior with double the period of the original system, i.e., there are two points such that employing the dynamics to one of the points produces the other point. In continuous dynamical systems, a new limit cycle surfaces from the present limit cycle and the period of the new limit cycle is twice that of the old one. Many dynamical systems track this transition to chaos.

Feigenbaum constant δ given by (1) is a mathematical constant that describes ratios in a bifurcation diagram for a nonlinear discrete map. At every bifurcation point, each equilibrium point loses its stability and is substituted by two new attracting critical points in period-doubling bifurcation route to chaos as illustrated in Fig. 5.

Dynamical systems following this route bifurcate at similar pace. So δ is a universal constant of period-doubling.

$$\delta = \lim_{x \rightarrow \infty} \left[\frac{a_x - a_{x-1}}{a_{x+1} - a_x} \right] = 4.669 \dots \quad (1)$$

Intermittency: intermittent behavior consists of almost periodic motion interspersed with shorter bursts of chaos, i.e., system behavior switches back and forth between apparently periodic and aperiodic behavior. It is commonly observed in fluid flows that are turbulent or near the transition to turbulence.

Crisis is a bifurcation event in which a strange attractor and its basin of attraction disappear or suddenly expand in size. This occurs when the chaotic attractor collides with an unstable periodic orbit (UPO) as some control parameter of system is changed. Crises can generate intermittent behavior. Generally, crises result in discontinuous changes in the strange attractor and are classified according to different kinds of changes they generate.

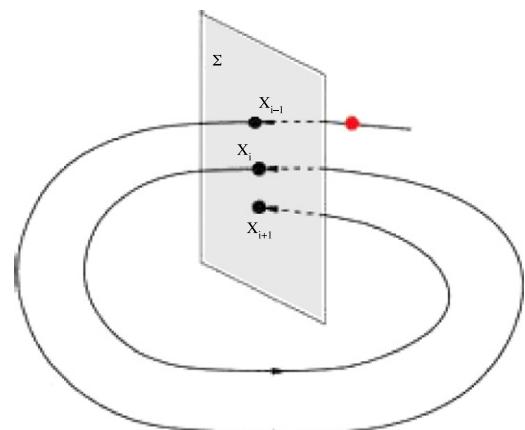


Fig. 6. Poincare map.

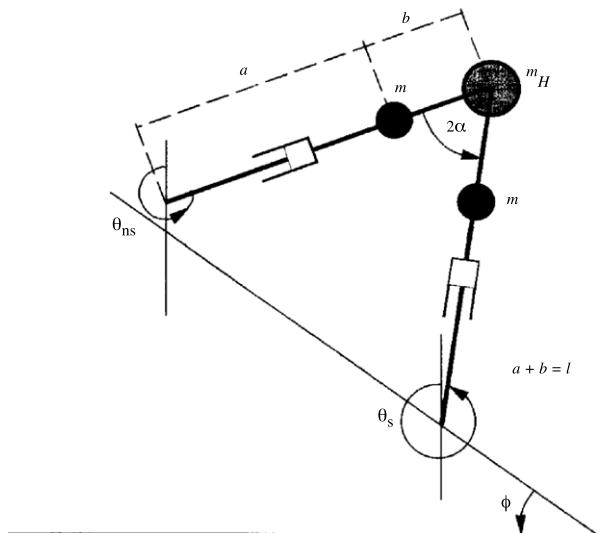


Fig. 7. The compass-gait biped [10].

- In exterior or boundary crisis, the strange attractor is abruptly destroyed.
- In interior crises the size of the chaotic attractor suddenly increases. The attractor bumps into a UPO or periodic solution that is inside the basin of attraction.

Poincare map substitutes the flow of an nth order continuous-time system with an $(n - 1)$ th order discrete-time system. So it can be taken as a discrete dynamical system with a state space that is one dimension lesser than the original continuous system. Fig. 6 shows a Poincare map for a 3D system.

3. Chaos and bifurcations in passive dynamic walking (PDW)

In the previous section, we have familiarized ourselves with a host of salient terms of chaos and bifurcation theory. This section puts forth a panoramic view of efforts for unearthing chaotic dynamics in passive bipeds. Goswami et al. were the first investigators who discovered bifurcations and chaos phenomenon in a simple nonlinear biped model called the “compass-gait biped”, as shown in Fig. 7 [9].

Goswami and coworkers showed that in response to continuous change in any one of the three parameters of the biped like ground slope, normalized mass and leg length—the compass-gait walker exhibited periodic and chaotic gaits proceeding from cascades of “period-doubling bifurcations” also termed as “flip bifurcations”

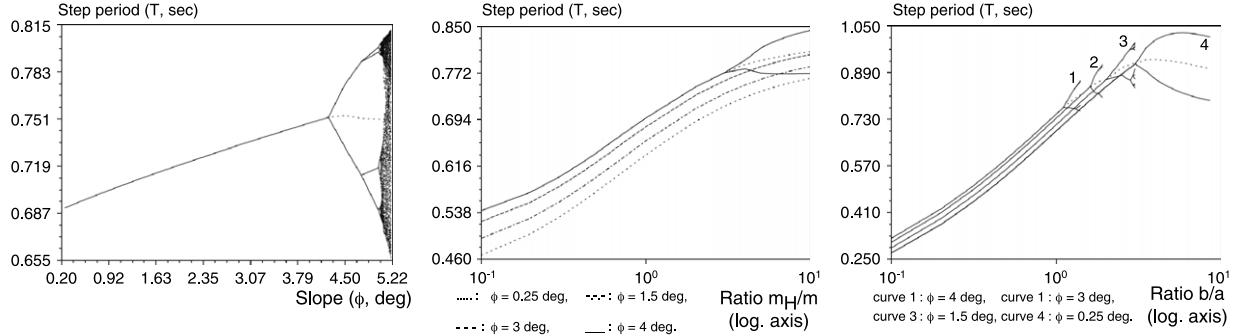


Fig. 8. Bifurcation diagrams for different bifurcation parameters [10].

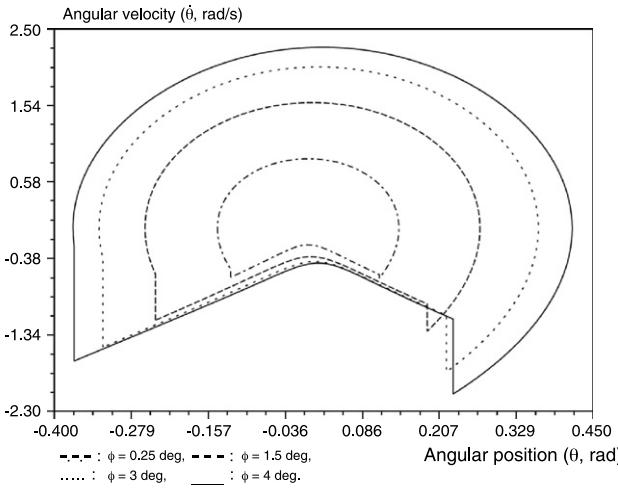


Fig. 9. Limit cycle for the compass-gait walker [10].

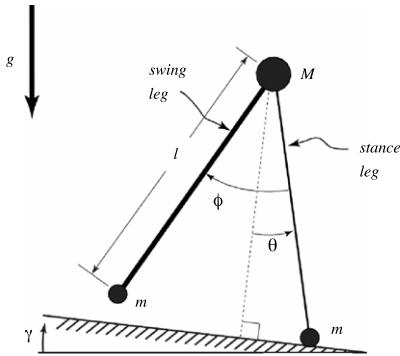


Fig. 10. Garcia's the simplest walking model [13].

as shown in Fig. 8. They reasoned that the complex nonlinear behavior of this outwardly plain biped mechanism is due to its governing hybrid algebro-differential equations [9–12]. They also introduced limit cycle representation of bipedal walking as depicted in Fig. 9. This representation is omnipresent now.

Garcia et al. [13] proposed irreducibly simple, point-foot straight-legged 2D walker referred to as the “simplest walking model” for the passive dynamic biped (see Fig. 10). They observed that as the ramp slope increased, the stable period-1 gait bifurcated into period-2 gait, then period-2 gait divided into period-4 and ultimately it became chaotic where no two steps were identical. They analyzed dynamics and efficiency of several PDW biped mechanisms like rimless wheel, straight leg walker, point-foot walker and kneeled walker (Fig. 11). They found period-doubling bifurcation and chaos in their kneeled walker [14–19]. The presence of numerous periodic and chaotic gaits of

a kneeled walker indicated the possibility of a large repertoire of passive dynamic motions (see Fig. 12).

Howell and Baillieul revisited McGeer's slope-driven biped and semi-passive biped driven by torso [20]. Using bifurcation diagrams (Fig. 13), they found the stable and periodic gaits of two bipeds evolved through a regime of period-doubling bifurcations, finally arriving at a seemingly chaotic gait. Besides period-doubling bifurcations, torso-driven biped also showed saddle-node bifurcations (Fig. 13(b)). They discovered an isolated stable period-three branch and following period-doubling gaits (period-six, period-twelve etc.) for slope-driven biped too.

Asano et al. found that for higher values of virtual slope, the compass-gait biped on horizontal floor experienced period-doubling bifurcations and demonstrated chaotic behavior just like original PDW mechanism [21] (see Fig. 14). Uchida and Furuta applied constant torque on the support leg of PDW biped for realizing steady bipedal walking on the horizontal ground [22]. Chaos and bifurcation phenomena were revealed by its analysis as shown in Fig. 15.

Osuka et al. developed a robot and conducted experiments [23–25]. They found chaos phenomenon that authenticated all previously done simulation work as illustrated in Fig. 16. Adolfsson [26–28] explored and figured out the dynamics of the 3D passive walker by using standard dynamical systems method like bifurcation diagrams; see Fig. 17.

Ikemata et al. investigated the compass-like biped model [29,30]. For short-period gait of the biped, the fixed-points were unstable and saddle-type (Fig. 18). In [31], they concluded that the walking motion became chaotic through a period doubling cascade, as the slope increased.

Bifurcation diagrams illustrate important behavioral changes under variations in system parameters. Piiroinen et al. illustrated the inherent dynamics of passive bipedal walkers [32–35]. Their numerical simulations yielded five types of bifurcations namely the saddle-node/fold, the pitchfork, the period-doubling/flip, the Hopf and grazing bifurcations (Fig. 19). Shannon explored passive bipeds and also reported these routes to chaos [36].

Borzova and Hurnuzlu conducted bifurcation analysis of a five-link biped by taking the upper body as a control parameter [37]. It was found that the variation of the bifurcation parameter produced chaotic trajectories through classical period-doubling cascade as shown in Fig. 20.

Likewise in [38–40], Wisse et al. verified the nonlinear behavior of the simplest walking model using simulations. The hip spring stiffness as the bifurcation parameter was varied and the biped yielded periodic motions as well as complex motions. They also developed 2D and 3D PDW robots with (and without) an upper body (see Fig. 21).

Kurz et al. investigated the chaotic dynamics of a bipedal PDW model [41]. They contended that this mathematical model might serve as a workable template for exploring biomechanical parameters responsible for nonlinear dynamics in human locomotion.

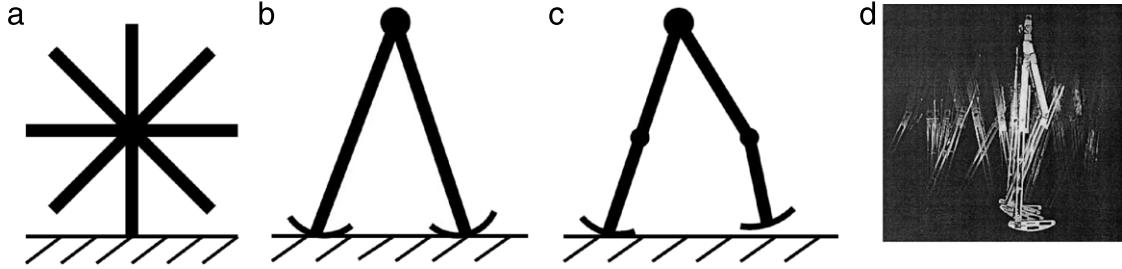


Fig. 11. (a) Rimless wheel, (b) straight leg walker (c) kneed walker and (d) its physical model [19].

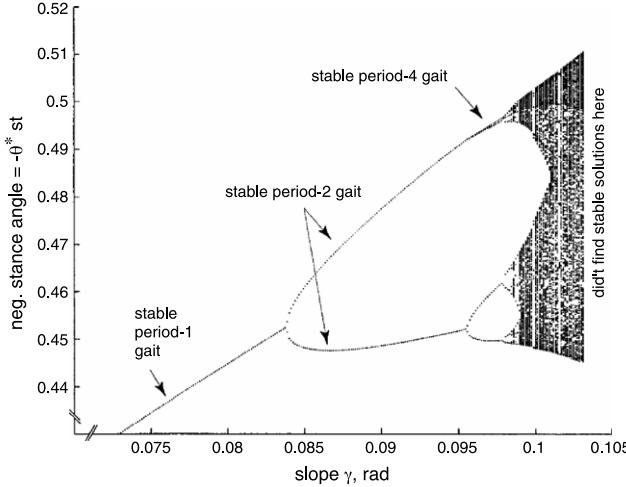


Fig. 12. Bifurcation diagram for Garcia's kneed walker [19].

Systematic numerical simulations verified that passive dynamic double-pendulum walking model could exhibit a cascade of bifurcations in the gait pattern that eventually became chaotic as the walking slope angles were increased (see Fig. 22).

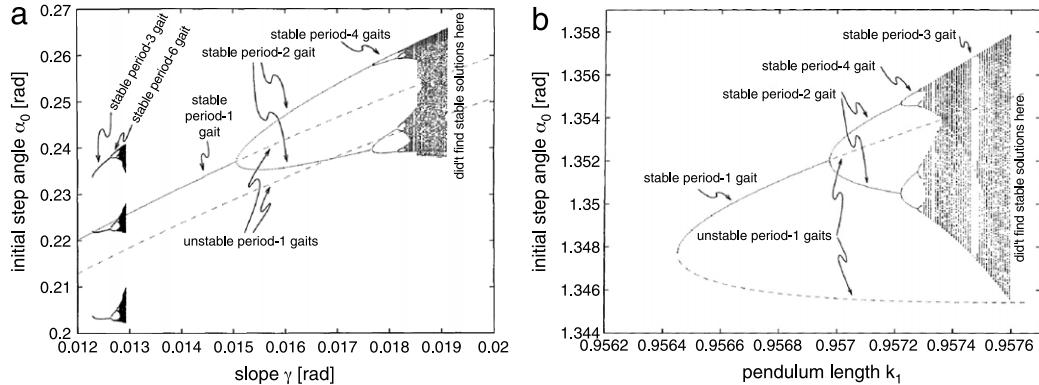


Fig. 13. Bifurcation diagrams for (a) the slope-driven and (b) torso-driven bipeds [20].

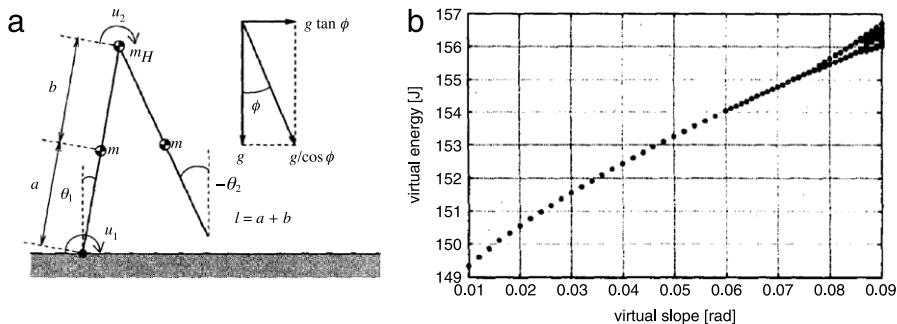


Fig. 14. (a) Virtual passive dynamic walker, (b) bifurcation diagram of virtual energy [21].

Narukawa et al. studied the compass model with a torso on the level ground [42]. Typical period-doubling bifurcations were resulted by numerical simulations. For swing leg control gain, a reverse period-doubling bifurcation from chaotic gaits to periodic gaits was also found (Fig. 23). Similarly, Asano and coworkers appraised Parametric Excitation Walking (PEW) on the level surface [43]. The generated PEW gaits exhibited period-doubling bifurcation route to chaotic motion analogous to original PDW gaits (Fig. 24).

As shown in Fig. 25, Berman explored the PDW model of Garcia et al. using simulations and experiments and computed bifurcation diagram [44]. In companion papers [45–48], Iribi and Osuka drew an analogy between compass biped and Phase Locked Loop (PLL) circuit. They ascertained from the analogous behavior that both dynamical systems are abundant in nonlinear dynamics, e.g., chaos, bifurcations, etc. (Fig. 26).

Aoi and Tsuchiya examined the dynamical behavior of a biped model driven by a rhythmic signal from a Central Pattern Generator (CPG). The 1-periodic motion became unstable, after which a stable 2-periodic motion emerged as the model parameter values were varied for the biped (Fig. 27). They discovered that this biped displayed period-doubling bifurcations and chaotic motion [49].

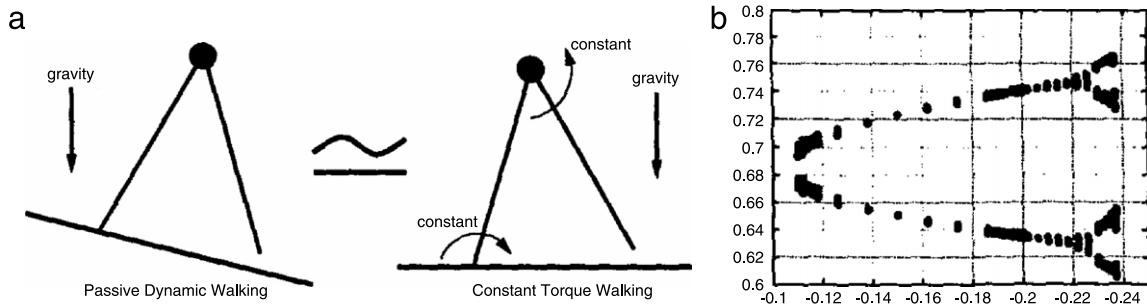


Fig. 15. (a) Constant torque walking model, (b) bifurcation diagram (step period versus torque) for the constant torque walker [22].

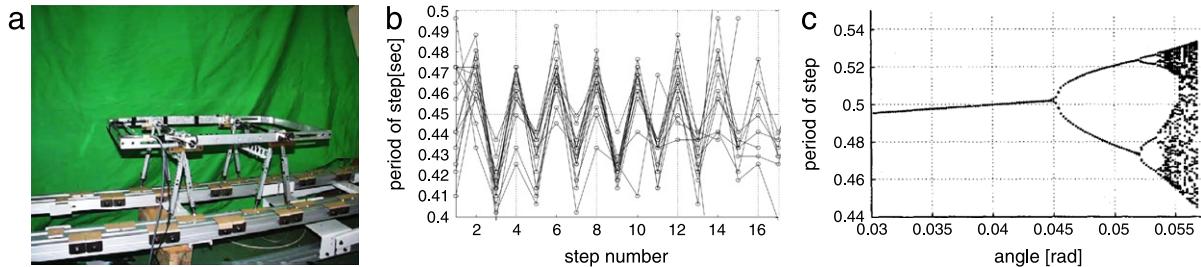


Fig. 16. (a) Physical robot, (b) time series plot, and (c) bifurcation diagram [24,25].

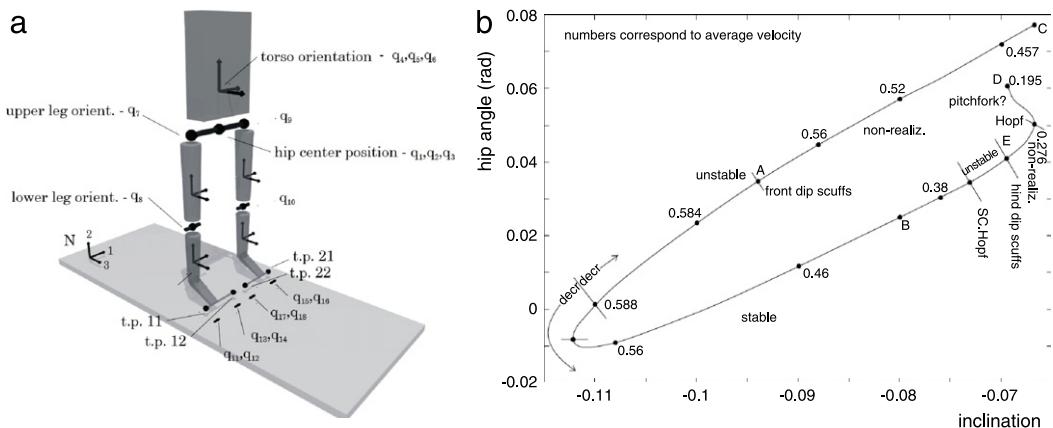


Fig. 17. (a) The geometry of 3D walker, (b) bifurcation diagram [28].

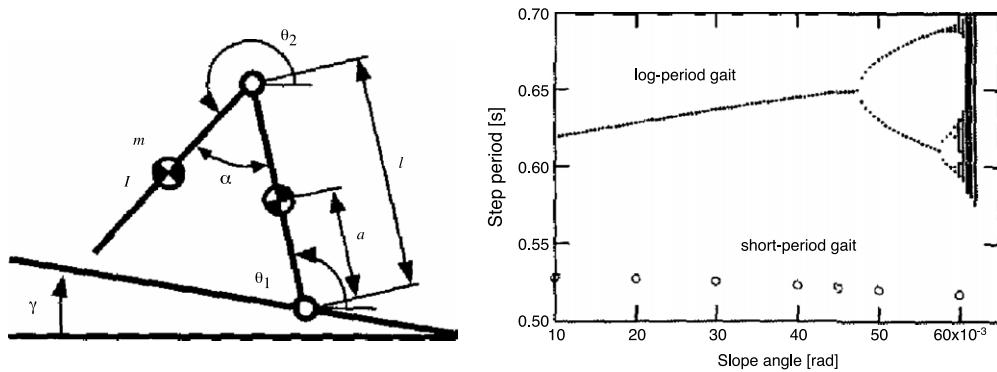


Fig. 18. Model of compass-like biped model and its bifurcation diagram [29,30].

Sugimoto and Osuka derived a linearized Poincaré map for a compass-gait biped which manifested bifurcation phenomenon [50] as depicted in Fig. 28. Tehrani et al. considered the simplest walking model for stepping motion [51]. Using bifurcation diagrams and Poincaré map, they proved that this biped has periodic as well as chaotic gaits (Fig. 29).

Max and Nicholas reported that the nonlinear composition of human motion is affected by the horizontal forces applied during the stance phase [52]. Their results revealed that the PDW model was a practical template for searching chaotic gait dynamics; see Fig. 30. As shown in Fig. 31, Kurz and workfellows applied innovative nonlinear techniques to determine the nonlinear gaits

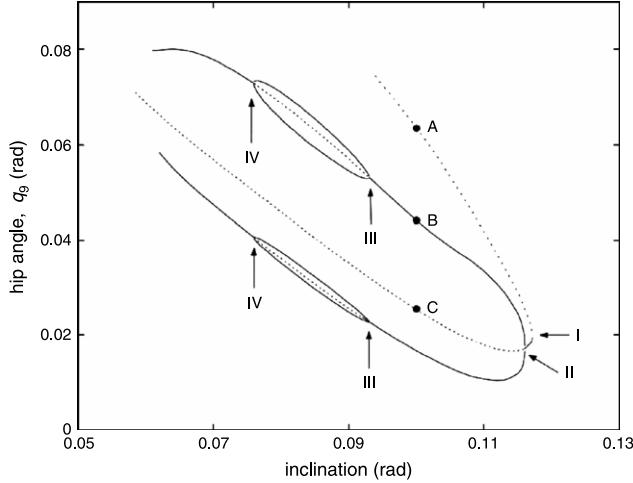


Fig. 19. Bifurcation diagram for a 2D walker [32–35].

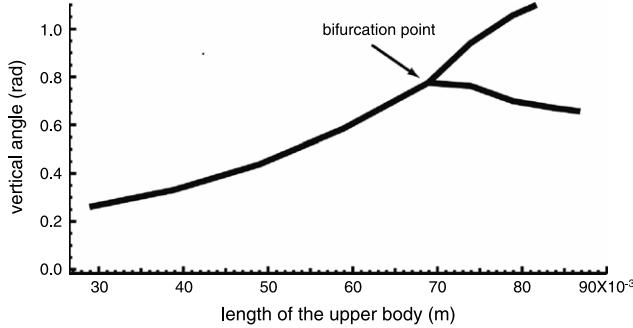


Fig. 20. Bifurcation diagram: vertical angle versus upper body length [37].

in the biped robots [53]. Norris et al. again observed that as the 2D compass biped model was sent down sharper slopes, the symmetric walking pattern bifurcated to a period-2, then a period-4 walking cycle, and so on until the periodic attractor turned chaotic [54] (see Fig. 32).

Liu et al. recorded period-doubling in the experiment that substantiated simulation work as illustrated in Fig. 33 [55].

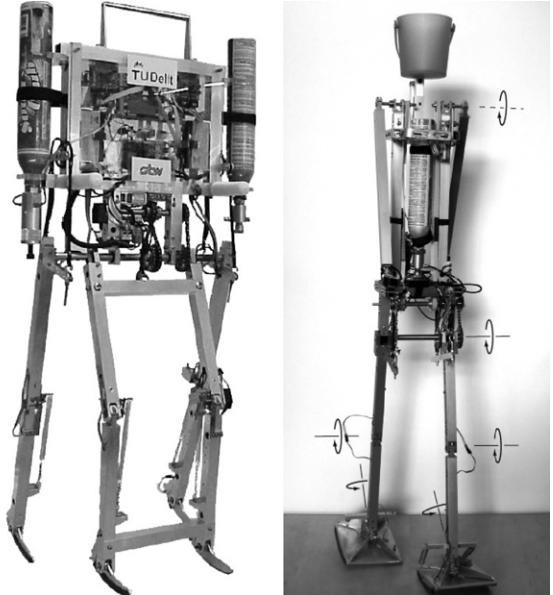
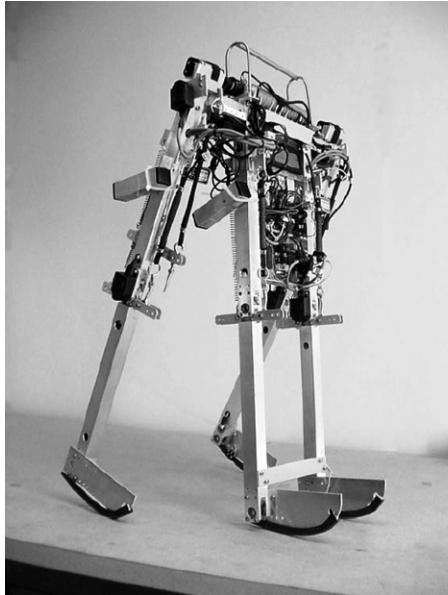


Fig. 21. 2D and 3D passive dynamic walking biped robots of Wisse et al. [39].

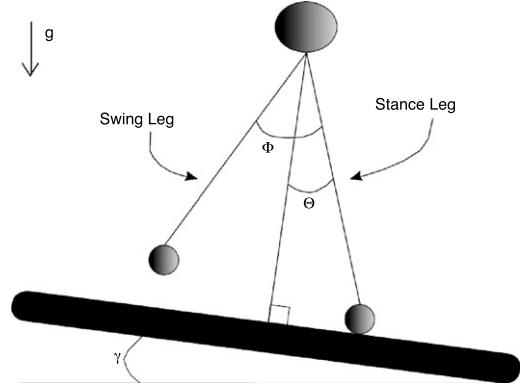


Fig. 22. The passive dynamic walking model of Kurz et al. [41].

Zhang et al. also investigated two passive walkers: compass-like biped and kneeled biped [56], as shown in Fig. 34. They confirmed bifurcation transition to chaos through bifurcation diagrams. Asano and Zhi-Wei mathematically explored an underactuated (passive) biped. They computed the gait efficiency and the results showed that after the first bifurcation, efficiency quickly diminished [57] (Fig. 35).

Zhenze et al. offered a systematic probe into a compass-gait biped model. Their computer simulations demonstrated that symmetric and steady gait of the biped evolved through an expanse of periodic gait; period-2 gaits eventually arrived at an apparently chaotic gait where two step-lengths are unequal, as any of the three gait parameters are altered [58] (Fig. 36).

In [59,60], Asano et al. investigated by making a simple rimless wheel asymmetric and noticed the variation of gait efficiency (Fig. 37). The simulations validated that the gait efficiency diminished monotonically due to the gait asymmetrization around the bifurcation point.

Owaki et al. developed a linearized analytical Poincaré map for passive dynamic walking [61]. They highlighted the period-doubling phenomenon in PDW as illustrated in Fig. 38. In related seminal papers [62–65], Jie et al. studied the nonlinear dynamics in the gait of the compass-gait biped model using bifurcation diagrams. Their studies revealed the intrinsic laws of the period-doubling bifurcations leading to chaos by considering the influence

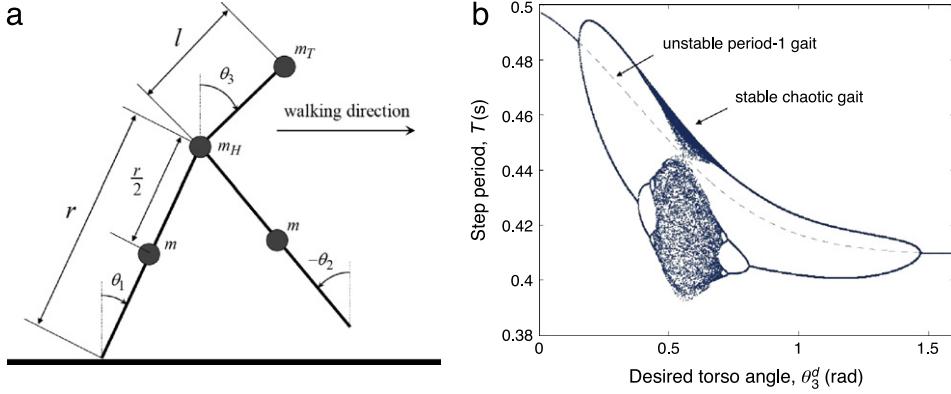


Fig. 23. (a) Torso-driven biped model on the level ground, (b) reverse period-doubling bifurcation diagram [42].

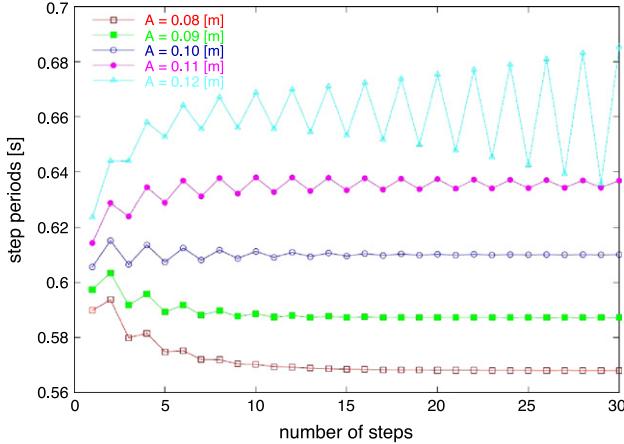


Fig. 24. Time-series plot for Parametric Excitation Walking (PEW) [43].

of parameter variation on the biped gait. They computed the Feigenbaum constant (see Table 1) and uncovered the chaos mirror of the period-doubling tree too. The biped gait endured different stages like periodic motion, bifurcation, chaos and merging of sub-bands, etc. (see Fig. 39).

Verdaasdonk et al. ascertained the typical bifurcation route to chaos from the bifurcation diagram of PDW without CPG-control [66,67]. Instead of period-doubling bifurcation, the pitch-fork bifurcation was found and the inception of chaos was determined using Feigenbaum's universal scaling law (Fig. 40).

Moon and Spong in affiliated papers [68–70] inspected gait asymmetry with a PDW compass-gait biped. They arranged bifurcation diagrams showing step periods versus two bifurcation parameters: the ground slope and the ratio of leg masses. They

Table 1
Computation of Feigenbaum constant [65].

Bifurcation case	Bifurcation value γ_n	Difference ratio $(\gamma_n - \gamma_{n-1}) / (\gamma_{n+1} - \gamma_n)$
1 → 2	0.1396	
2 → 4	0.1594	4.6817
4 → 8	0.1637	4.6457
8 → 16	0.1646	4.6547
⋮	⋮	⋮
Bifurcation → chaos	0.1647	4.669201661

grouped these bifurcation diagrams into six stages (Table 2) and confirmed that marginally stable limit cycles portrayed period-doubling, period-remerging and saddle-node bifurcations.

Gritli et al. examined walking patterns created by a compass-gait biped robot having “leg length discrepancy” or “leg length inequality” and reasoned that cyclic-fold bifurcation, shown in bifurcation diagrams (Fig. 41), was responsible for the collapse of the biped [71]. Similarly, Gregg et al. computationally probed two physically symmetric models: the planar passive compass-gait biped and a five-link 3-D biped to appraise the occurrence of functional gait asymmetry by making small changes in a mass distribution parameter for both legs [72–74]. They reasoned that functional asymmetry might be explained by period-doubling bifurcation also called “spontaneous symmetry breaking” (Fig. 42).

As shown in Fig. 43, Chyou et al. computed the bifurcation diagram for the “simplest ideal walker” and found bifurcation and chaos in gait patterns [75]. Kaygisiz et al. offered a panoramic view of the impact of the chaos theory and fractals on the field of robotics [76]. They covered the notion of chaos evolving through bifurcations under parametric variations in passive biped robots. Through the parametric study of the

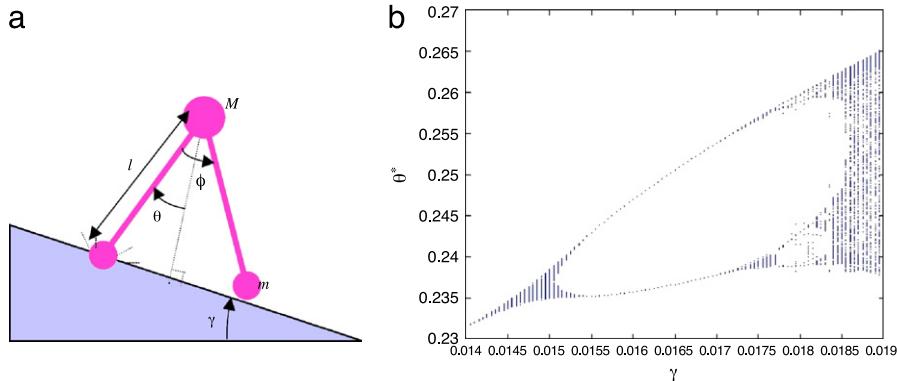


Fig. 25. (a) The PDW model from Garcia et al., (b) bifurcation diagram [44].

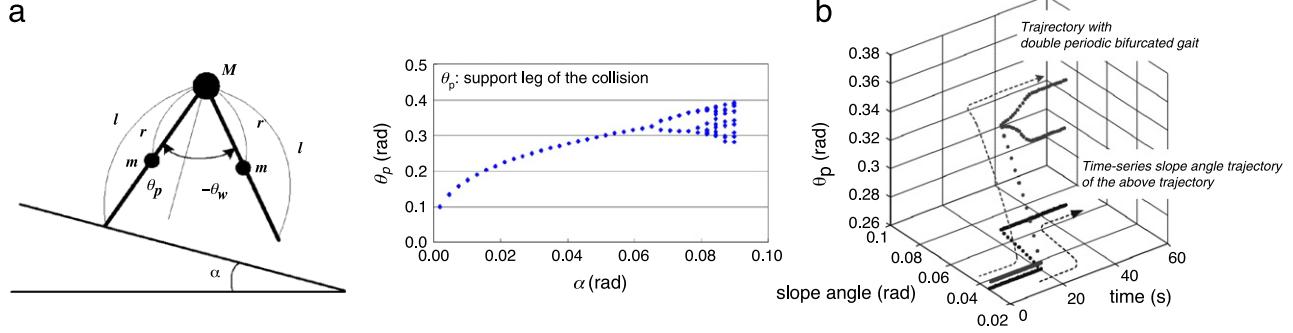


Fig. 26. (a) The compass biped model and (b) route to chaos [45–48].

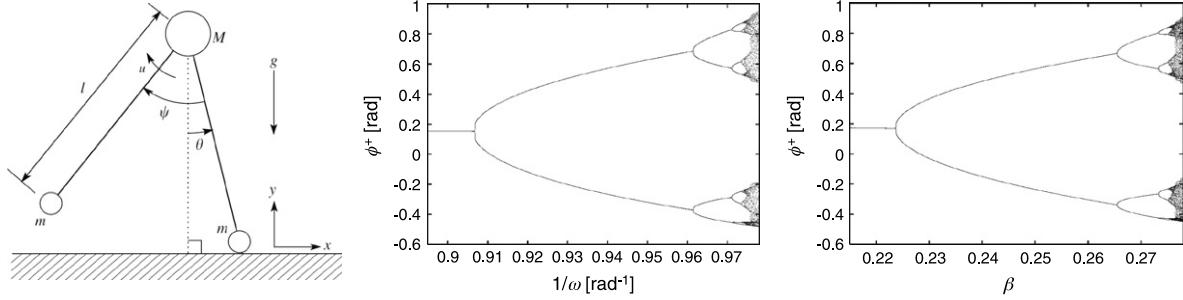


Fig. 27. The walking model of biped robot, period-doubling bifurcation and chaotic motion [49].

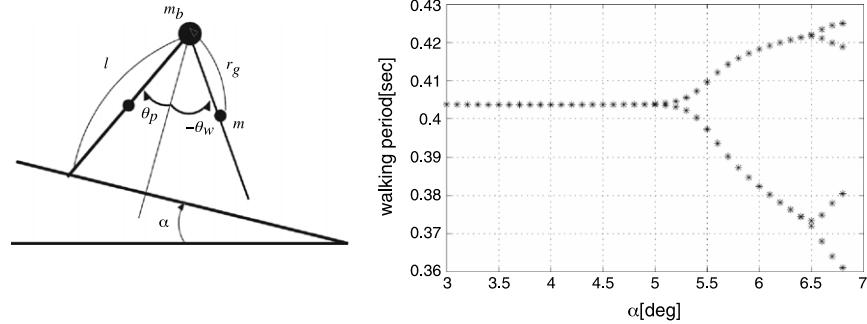


Fig. 28. A compass model of biped robot and its bifurcation diagram [50].

simplest biped model, Farshimi and Naraghi revealed different transitions to chaotic motion for different control parameters. Intermittency route to chaos was found as the slope angle increased [77]. For variations in parameters—ratio of body mass to hip mass and ratio of body length to leg length, quasi-periodic and period-doubling routes to chaos happened respectively (Fig. 44).

Q. Li and X.-S. Yang figured out bifurcation diagrams for Garcia's simplest walking model and located new stable periodic gaits with period-3 and period-4 [78]. They also proved the existence of chaos using the topological horseshoe theory (Fig. 45). Gritli et al. studied two biped models: the compass-gait biped (Fig. 46) and the semi-passive torso-driven biped (see Fig. 47). They found a pair of stable and unstable period-3 gaits generated through a cyclic-fold bifurcation and uncovered the period-3 route to chaos [79–82]. They reasoned that the same period-3 unstable periodic orbit (UPO) initiated the abrupt birth/annihilation of the bipedal chaos and hence triggered the fall of both robots. Besides bifurcation diagrams, fractal dimension and Lyapunov exponents were also used to quantify order and chaos in the passive walking. The authors discovered two novel routes to chaos: intermittency route and interior crisis route.

Mahmoodi et al. revisited Goswami's biped walker. Robot parameters such as the ground slope angle, length ratio and mass ra-

tio were taken as bifurcation parameters [83]. Bifurcation diagrams portrayed the evolution of gait descriptors like step period, average velocity, or inter-leg angle as a function of the bifurcation parameters. They suggested that developed bifurcation diagrams will help in developing effective controllers for active bipeds as well as understanding physics of human walking (Fig. 48).

Li and collaborators again re-examined Garcia's simplest walking model and found new bifurcation scenarios [84]; see Fig. 49. Previous research on modeling the biped foot mostly concentrated either on a point, a flat or a curved/circular foot. Over the last decade, the prosthetic research community has evolved roll-over shape model (Fig. 50). Mahmoodi et al. emphasized that insight gained from the bifurcation diagrams of the bipeds will help evolve the optimal design of prosthetic feet [85].

Goswami, Garcia and colleagues were the first researchers who discovered the chaotic behavior of PDW bipeds. Asano, Uchida and Furuta analyzed different bipeds to uncover nonlinear dynamics. This fact that when the slope increases, the gait period of biped increases by 2^n then chaotic gait appears and finally robot falls down, was proved by numerous computer simulations but Osuka et al. validated it experimentally. Thereafter, the biomechanics community along with engineers and mathematicians experimented with PDW biped models. Hurmuzlu, Spong, Moon, Nicholas, and Kurz integrated torso, arms and dampers in biped models which

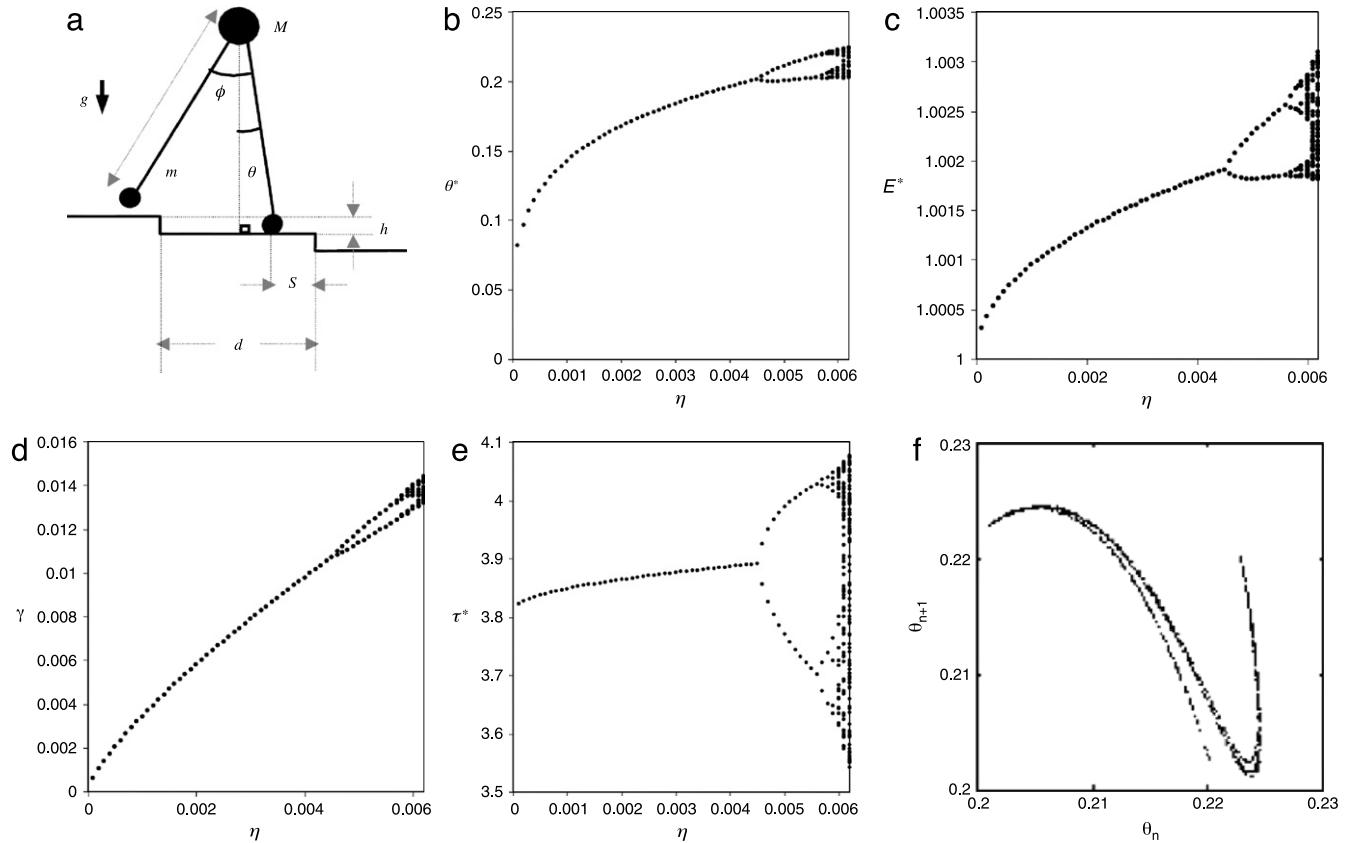


Fig. 29. (a) The model (b)–(e) bifurcation diagrams for different parameters, (f) Poincare map [51].

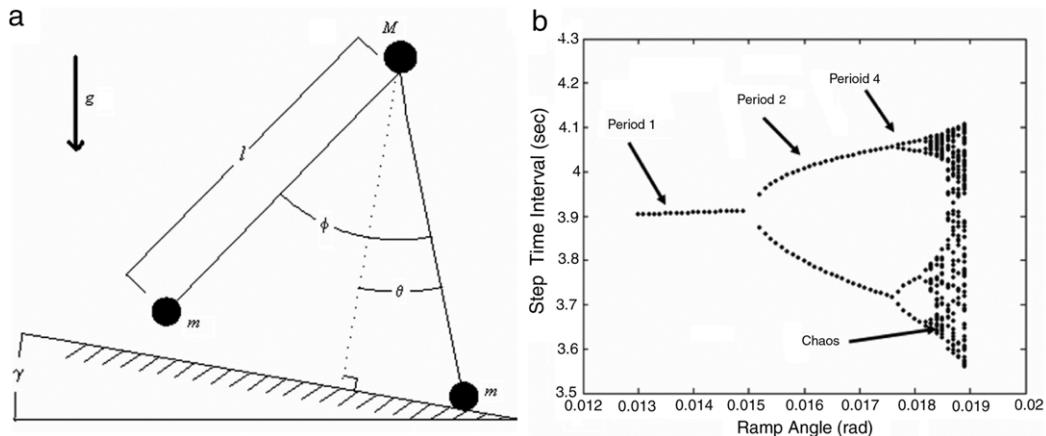


Fig. 30. (a) PDW model, (b) bifurcation diagram [52].

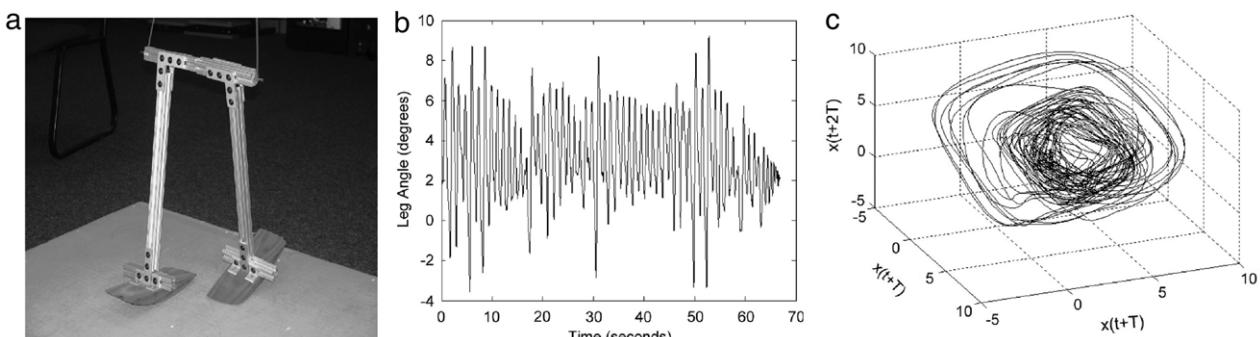


Fig. 31. (a) PDW walker, (b) time series plot, (c) locomotive attractor [53].

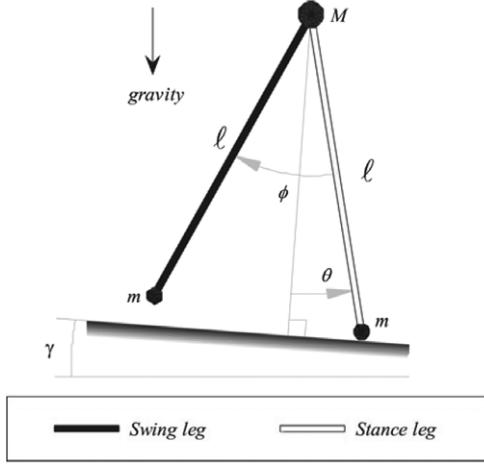


Fig. 32. Two-segment walker [54].

produced various kinds of chaotic behavior, e.g., bifurcation, intermittency, quasi-periodicity and crises. Several tools from mathematical chaos were employed to substantiate the occurrence of chaos in their locomotive patterns. Moon, Gregg and Gritli et al. investigated passive bipeds for physical handicap—leg-length discrepancy or leg-length inequality. Such physiological problems must be comprehended for working out their effective treatment.

4. Control of chaotic passive walking

“Control of chaos” is the stabilization of an unstable periodic orbit (UPO) by means of small system perturbations, i.e., directing chaotic trajectories to desired positions. For controlling chaos, numerous methods have been formulated, but most are enhancements of two major approaches: the Ott–Grebogi–Yorke (OGY) method and Delayed Feedback Control (DFC) method. In Ott–Grebogi–Yorke (OGY) method tiny, carefully selected, time-dependent parameter perturbations are applied to the system and one or more of the UPOs can be stabilized. In the DFC method, Pyragas proposed stabilization of UPO by applying feedback perturbations proportional to the deviation of the present state of the system from its state one period before, so that the control signal dies out when the chaotic system has been tamed.

We again refer to [60] as Asano used two approaches for the chaos control. Using the DFC scheme and a quasi-constraint on the impact posture, the stabilization to a 1-period gait of compass-gait biped was accomplished (Fig. 51).

Suzuki and Furuta proposed the chaos control approach based on the OGY-like method for expanding the walking range of the compass-gait biped (see Fig. 52). Simulations demonstrated that the biped could walk on more slanting slopes [86]. Osuka and Sugimoto proposed a new control method for stabilizing quasi-PDW of a biped model of the physical robot [87]. Computer simulations corroborated the efficacy of this control law. Likewise in [88–90], they put forward a novel control scheme of quasi-PDW that combined the DFC method and Weekly Guidance Control

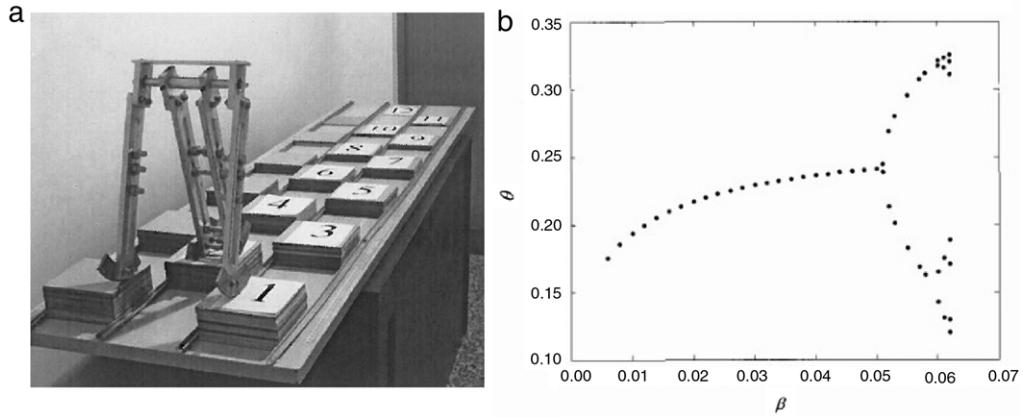


Fig. 33. (a) The experimental facility, (b) bifurcation route to chaos [55].

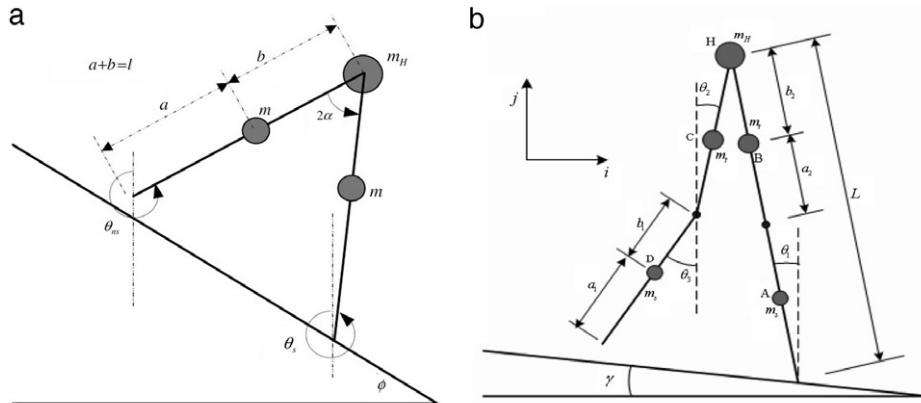


Fig. 34. Models of the (a) compass-like biped, (b) kneed passive biped robot [56].

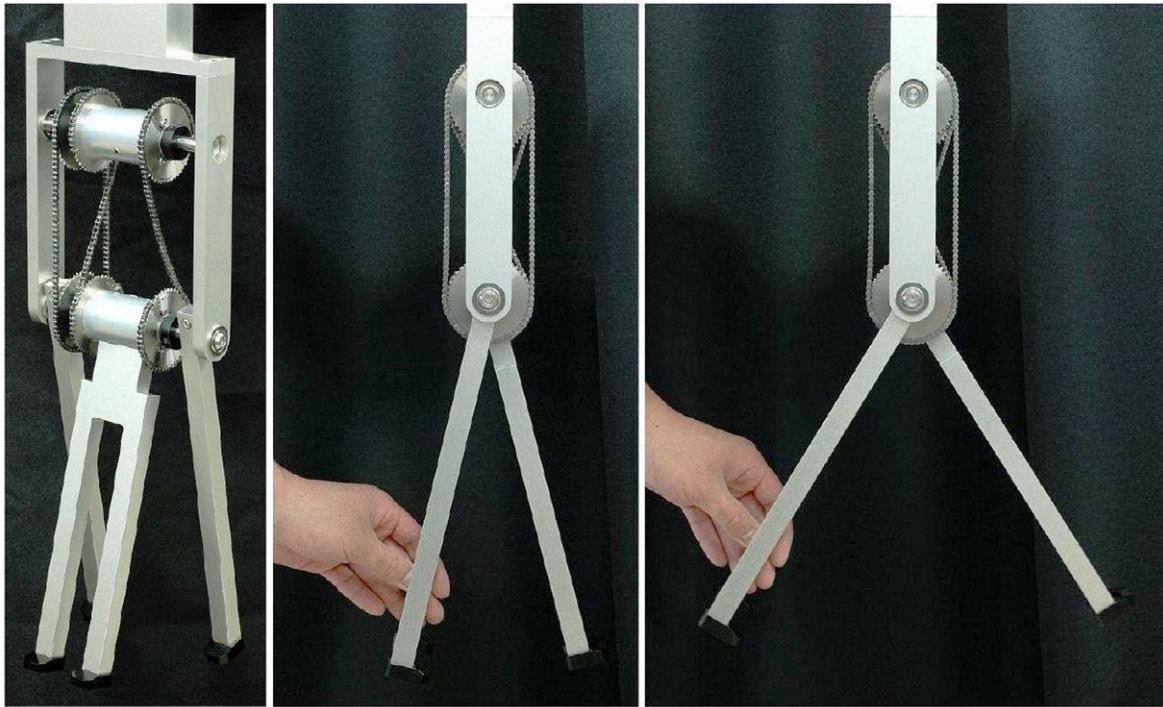


Fig. 35. The experimental biped robot [57].

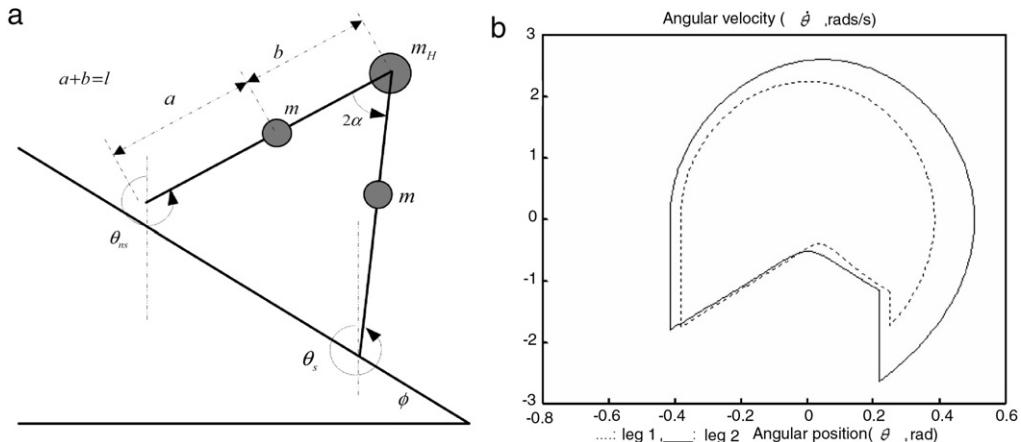


Fig. 36. (a) Model of compass-gait biped robot, (b) the limit cycle after period-doubling [58].

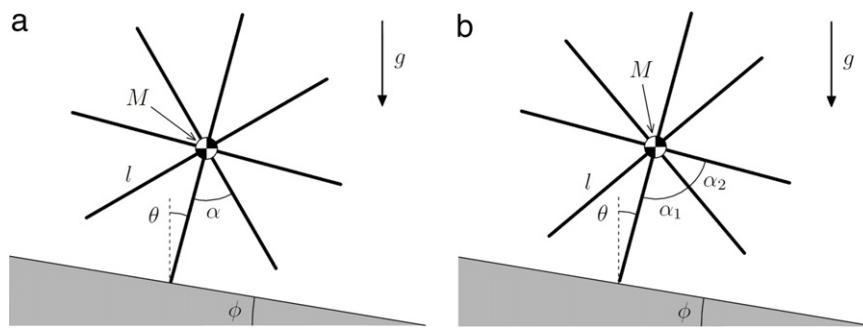


Fig. 37. Rimless wheel models: (a) symmetric model, (b) asymmetric model [59,60].

method. Experiments and simulations confirmed the usefulness of the designed technique (Fig. 53).

Suzuki et al. proposed the transition control from walking to running using a new chaos-control scheme based on the OGY

method [91]. This method generated fast nonchaotic walking on the sharp slope and running was realized (Fig. 54). In investigations [92,93], Kurz and Stergiou used Garcia's biped model with hip joint actuations of the swing leg to control

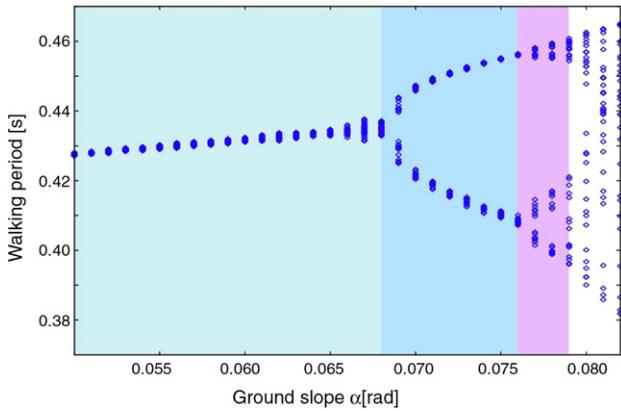


Fig. 38. Period-doubling bifurcation phenomenon in PDW [61].

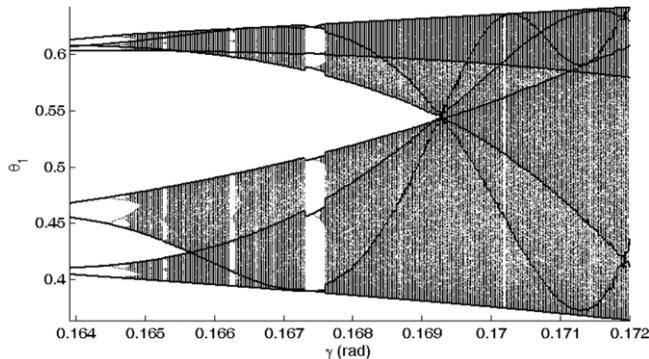


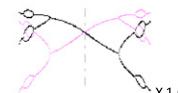
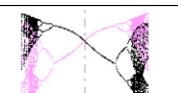
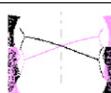
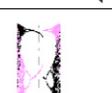
Fig. 39. Bifurcation diagram [63].

bifurcations and chaos in gait patterns. Like the OGY method, this actuator created perturbations to make the biped adjust to numerous walking patterns deep-rooted in the chaotic attractor (Fig. 55).

As shown in Fig. 56, they also used an artificial neural network (ANN) as the simulated nervous system that selected suitable hip joint actuations which moved the biped to a stable walking motion [92]. Kamath and Singh inspected the control of compass gait biped based on its impact dynamics using the Receding Horizon Control (RHC) approach [94]. Results showed that this control method stabilized the passive gait for those initial conditions that made the gait unstable (Fig. 57).

In [95,96], Harata et al. applied the DFC method to PEW to control bifurcations and chaos. Computer simulations validated the fact that walking efficiency with DFC was greater than that without DFC. Their devised method converted 2-period walking to 1-period walking, and consequently energy efficiency was

Table 2
Six stages of bifurcation diagrams [68–70].

Stage Slope	Bifurcation diagrams Period vs. Mass ratio r_m	Stage Slope	Bifurcation diagrams Period vs. Mass ratio r_m
A1 0.80°	 x 0.3	A2 3.50°	 x 1.0
B1 1.90°	 x 0.3	B2 3.60°	 x 1.0
C1 2.04°	 x 0.3	C2 3.74°	 x 1.0
D1 2.10°	 x 0.5	D2 3.80°	 x 1.0
E1 2.74°	 x 0.8	E2 4.70°	 x 1.0
F1 2.95°	 x 1.0	F2 4.96°	 x 1.0

boosted (Fig. 58). Harata et al. applied the DFC method to suppress bifurcations and chaos in compass-gait biped (Fig. 59).

Jamali et al. extended Kurz's algorithm for controlling chaos in Garcia's simplest biped robot using toe-off impulses as the control actuator through ANN [98]. As the control parameter the toe-off impulse was chosen because it was easily applicable and had a human locomotion look. This biologically inspired algorithm proved to be very useful and robust. Gritli et al. studied the under-actuated (semi-passive) torso-driven compass-gait biped walking down a ramp [99]. They demonstrated that the chaos could be controlled by the backward position of the torso; see Fig. 60. Gritli and colleagues put forward a new scheme of chaos control for a compass-gait biped, Fig. 61. Their simulation work verified the efficiency of this chaos control strategy [100].

During the last two decades, prominent researchers such as Osuka, Sugimoto, Furuta, Kurz, Harata and Gritli addressed the issue of controlling chaos in PDW bipeds. They proposed discrete solutions for solving this problem. From the scarcity of technical

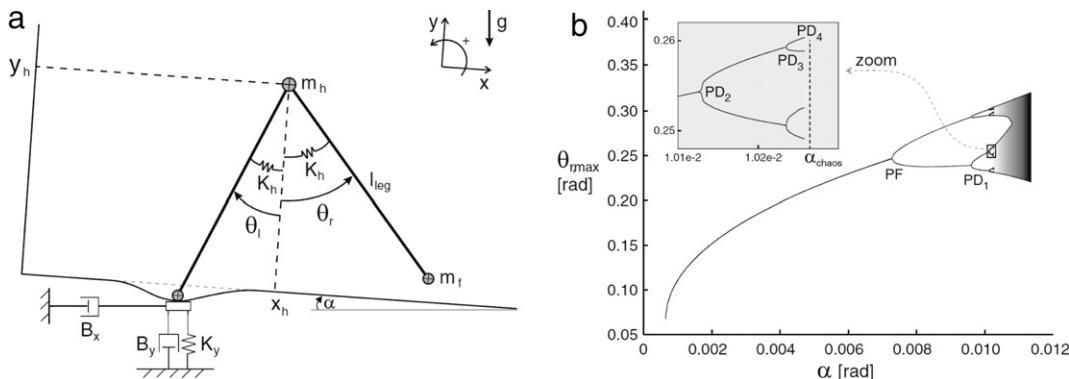


Fig. 40. (a) The passive dynamic walker, (b) period-doubling bifurcation route to chaos [66,67].

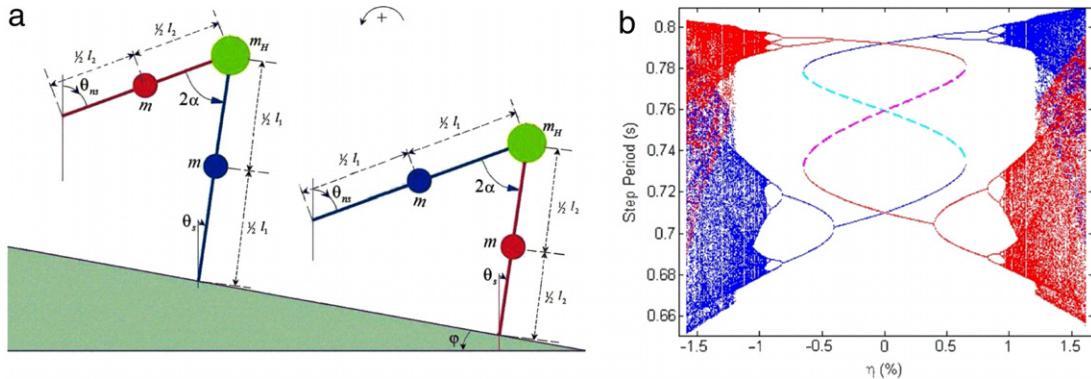


Fig. 41. (a) The compass-gait biped robots with leg length Discrepancy, (b) bifurcation diagram [71].

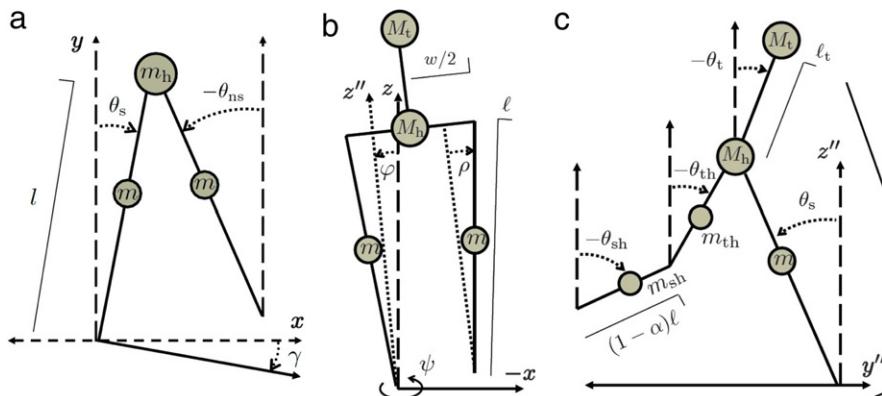


Fig. 42. (a) The planar passive compass-gait biped, (b) five-link 3-D biped frontal plane, (c) sagittal plane [72–74].

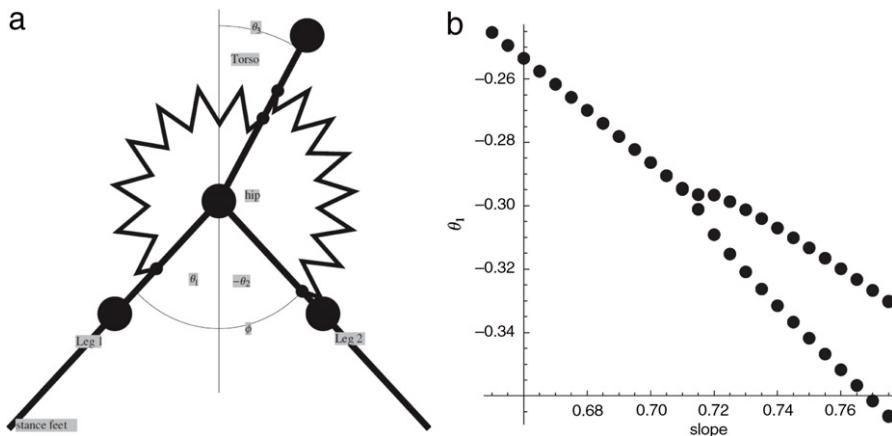


Fig. 43. (a) Ideal passive dynamic biped walker, with two straight legs and a torso, (b) bifurcation diagram [75].

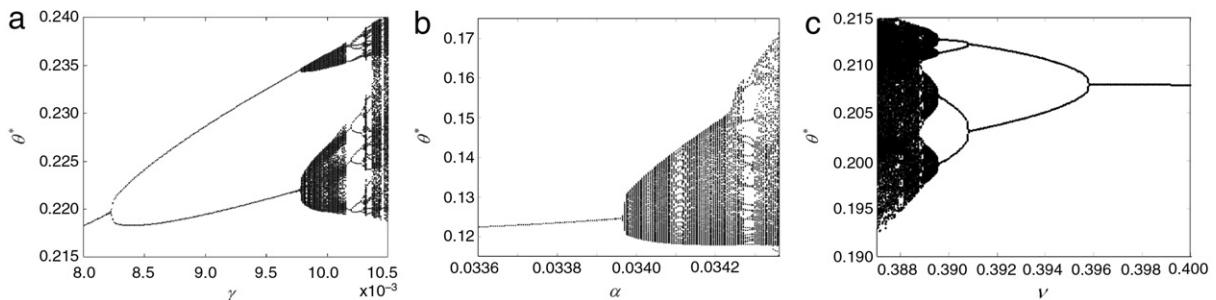


Fig. 44. Bifurcation diagrams for (a) slope angle, (b) ratio of upper body mass, (c) ratio of upper body length to leg length [77].

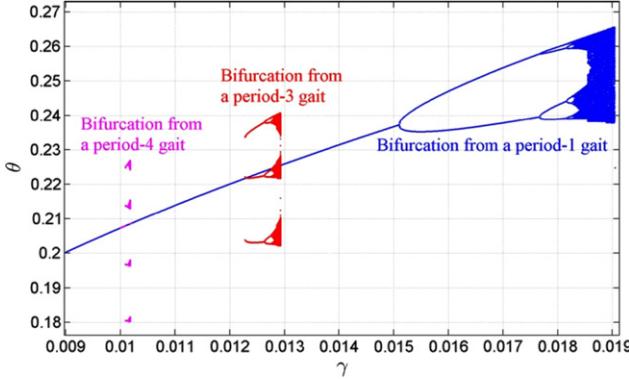


Fig. 45. Bifurcation diagram [78].

papers on chaos control in passive dynamic bipeds, it is evident that this area needs further investigation.

5. Discussion

Based on the presented literature review, we conclude this article with our thesis on the chaos study in passive dynamic walking as well as highlight potential research challenges in this field.

5.1. Research challenges for robotics engineers

The robotics community examined PDW for designing efficient humanoid robots. Legs are probably the most widely used biologically inspired locomotion system. Their two major dividends are efficiency and mobility. Since humanoid robots have been envisaged, two-legged robots have been extensively explored by many research groups [101]. **Table 3** catalogs passive bipeds covered in

Table 3
Major categories of PDW bipeds.

Passive walking on slope	[9–20,23–36,41,44–48,50–56,58–65,68–71,75,77–94,97–100]
Passive walking on level ground	[21,22,37–40,42,43,49,57,66,67,72–74,95,96]

this review article. Researchers from different research fields investigated PDW; they used 2D or 3D biped models or particular control strategies like CPG-control.

It is evident from **Table 3** that researchers have been studying dynamics of passive biped models walking downhill grade or on the flat ground and the engendered gait patterns revealed nonlinear dynamics. Future applications require these robots to walk and run in unfamiliar and unstructured environments. Such rough terrain will give rise to more complicated chaotic dynamics. Similarly, the research problem of adapting two-legged robots to environmental perturbations is of supreme importance. The question of dynamic stability in bipedal locomotion while contacting environment and manipulating objects remains open. The nonlinear dynamics of bipedal locomotion are also of paramount value from the control viewpoint, which is invariably a central subject in robotics. In the animal world, neural oscillators called CPGs provide basic rhythm for muscular activity in walking. Researchers are investigating the dynamics of these locomotive control systems that certainly affect the dynamical behavior of passive bipeds.

5.2. Research challenges for biomechanics researchers

The biomechanics researchers investigate this robotics theme for various reasons, for example – for comprehending human walking—for designing superior prosthetic limbs – and for rehabilitating gaits of patients. Gait pathologies are diseases that create difficulties in human walking. Walking is an ordeal for

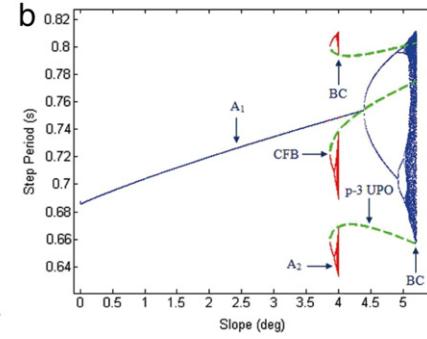
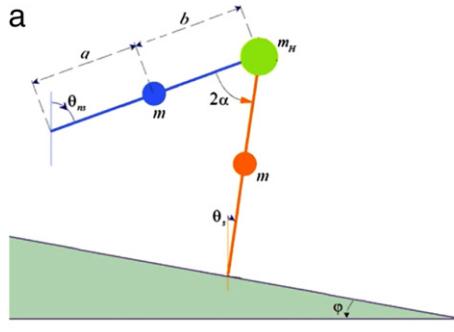


Fig. 46. (a) The compass-gait biped robot, (b) bifurcation diagram: variation of the step period as a function of the slope angle [80].

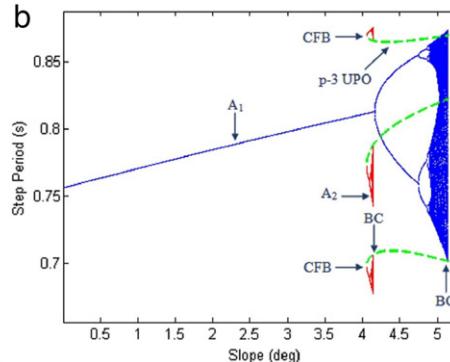
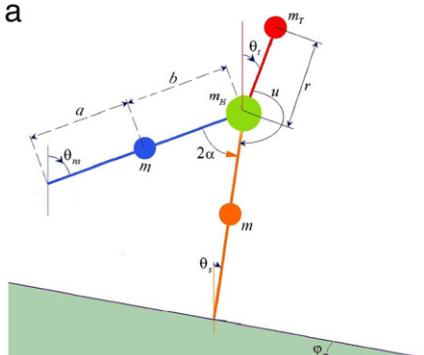


Fig. 47. (a) The torso-driven biped robot, (b) bifurcation diagram: step period as a function of slope angle [80,81].

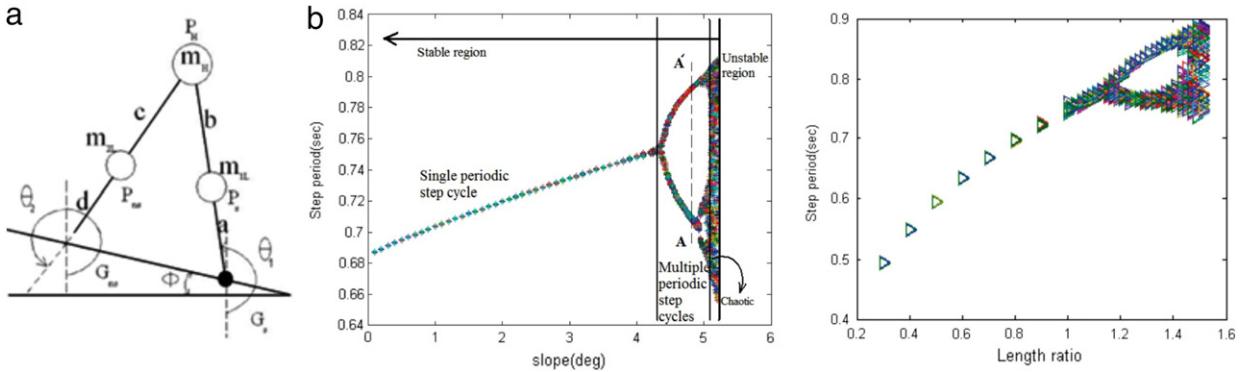


Fig. 48. (a) Two-linked robot, (b) bifurcation diagrams for different parameters [83].

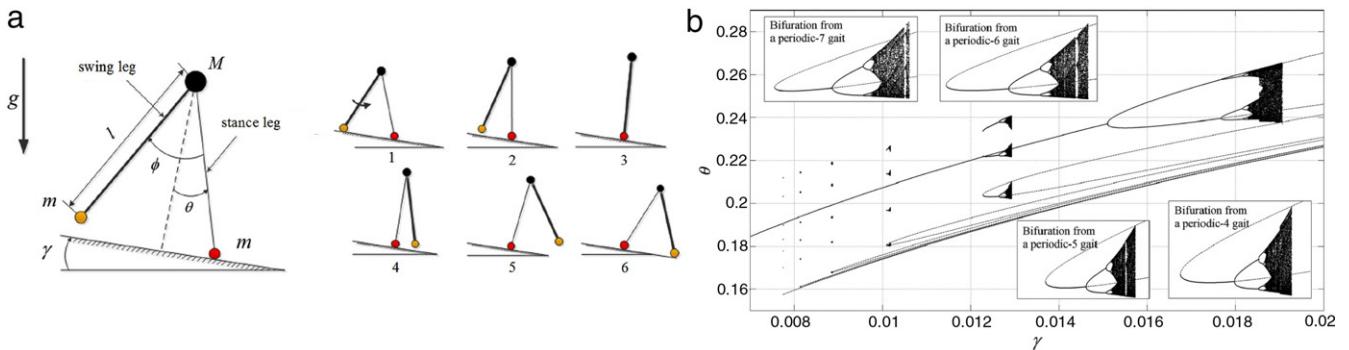


Fig. 49. (a) The walking model and a typical step, (b) the continuation and bifurcations for other periodic gaits [83].

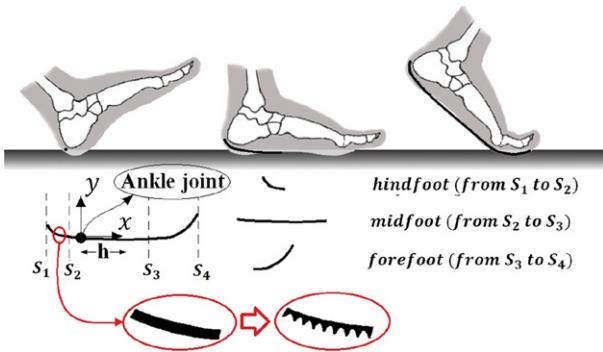


Fig. 50. Foot movement during human walking and the resulting roll-over shape [85].

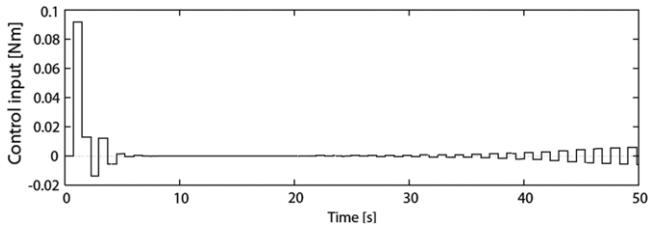


Fig. 51. Simulation of compass-gait biped with delayed feedback control (DFC) [60].

individuals with impaired gaits due to cerebral palsy, stroke, hemiplegia, spinal-cord injury and amputation. Gait rehabilitation addresses these issues [4]. Scientists are using the passive walking approach for designing experimental tests and associated results can help explain gait disorders. This research will develop diag-

nostic and testing strategies and bring about future expansion in rehabilitation engineering.

Human walking is accomplished through a complex orchestration of joint motions, muscle forces and neural motor commands. Two major theories have dominated walking research for the last six decades—the inverted pendulum analogy and the six determinants of gait. PDW advances the first theory and offers better understanding of human and animal walking [102]. Scientists have developed many mathematical models and physical prototypes for investigating PDW. Principal examples are McGeer's Dynamite, Garcia's simplest walking model and the compass-gait biped. The last two models have been exhaustively investigated in the reviewed papers. The passive biped robots produce periodic gaits but this does not guarantee that they imitate human walking fully. The impact of other important elements, e.g., nervous system, joints and muscles on walking, is gradually being discovered. Minimal biped models produce a gamut of chaotic dynamics. It is anticipated that simultaneous inclusion of features like knees, torso, feet with different shapes, and mass distributions in more complicated biped models will generate more complex dynamics.

5.3. Research challenges for chaos theorists

Since passive bipeds are dynamical systems which exhibit rich nonlinear dynamics, the chaos theorists—also called chaologists—explored them enthusiastically to find different routes to chaos and increase their basin of attraction by applying chaos controlling techniques. Following theory of relativity and quantum mechanics, chaos theory is regarded as the third revolution in physics. It encompasses virtually every field of human knowledge and endeavor and has been studied extensively for the past four decades. Chaos is universal because scientists have found similar nonlinear dynamics in dissimilar dynamical systems like double

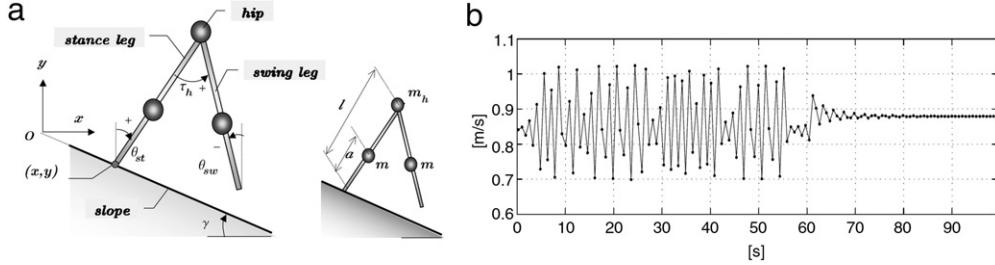


Fig. 52. (a) 2-link biped walking model, (b) stabilization of chaos in passive walking [86].

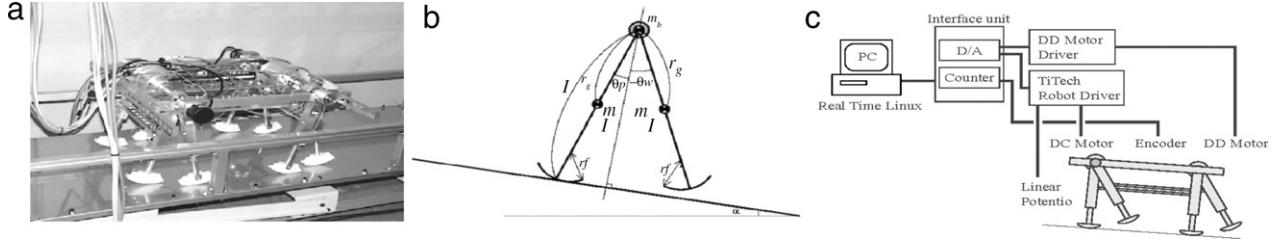


Fig. 53. (a) Experimental robot, (b) compass model, (c) experimental setup [90].

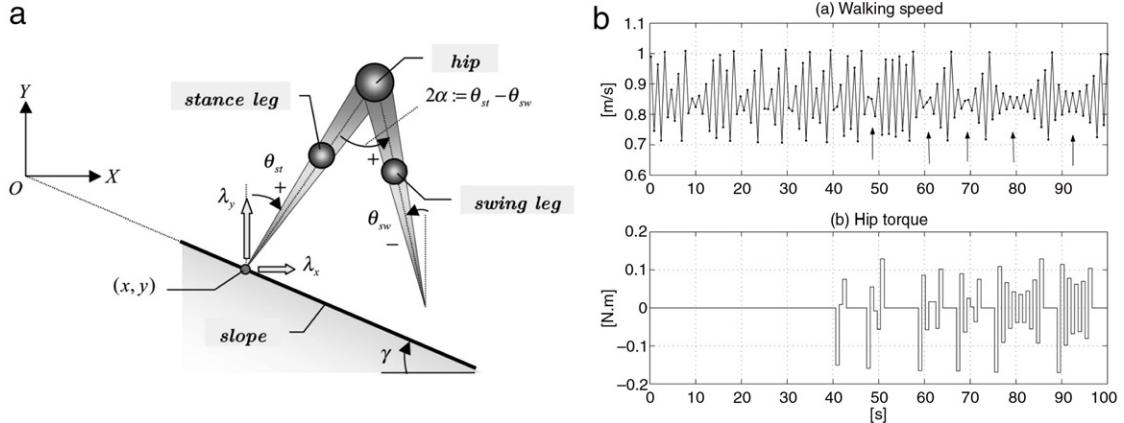


Fig. 54. (a) Two-link biped model, (b) stabilization of chaotic walking by the OGY method [91].

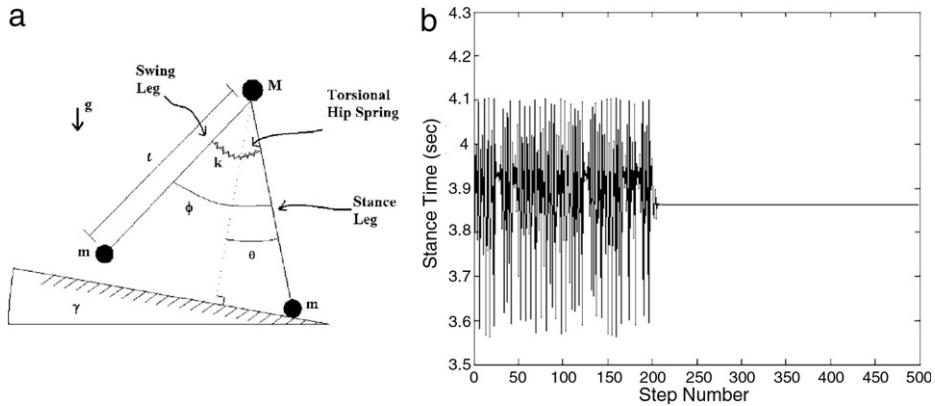


Fig. 55. (a) The PDW model; (b) application of hip actuation promoted the rapid transition of chaotic gait pattern to a period-1 gait [93].

pendulums and simple electronics circuits [5–7,103]. From the standpoint of chaos theory, it is important to investigate dynamics of passive bipeds. So far, researchers have found several routes to chaos in the gait patterns of very simple bipeds. The analyses of complex mathematical models, which are adequately close to the

behavior of the real biological systems, will provide deep insight about the origin of such chaotic dynamics.

Human walking is acknowledged to be a limit cycle behavior; it is conspicuous from the oscillatory motion of legs. Conventionally any deviation from this periodic behavior was regarded as noise

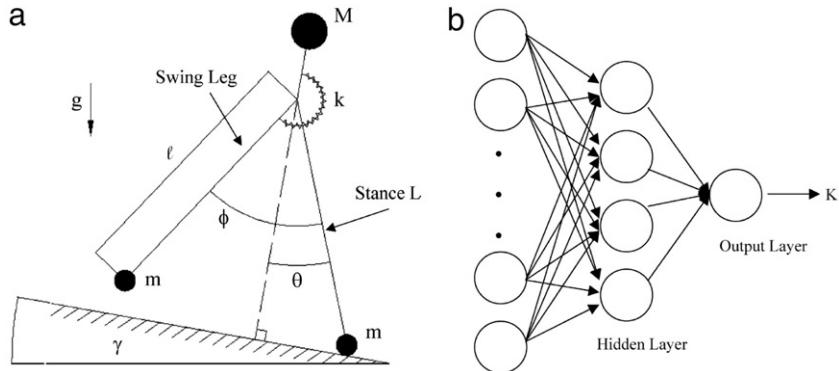


Fig. 56. (a) The PDW model, (b) schematic for the ANN for modulating hip joint actuations [92].

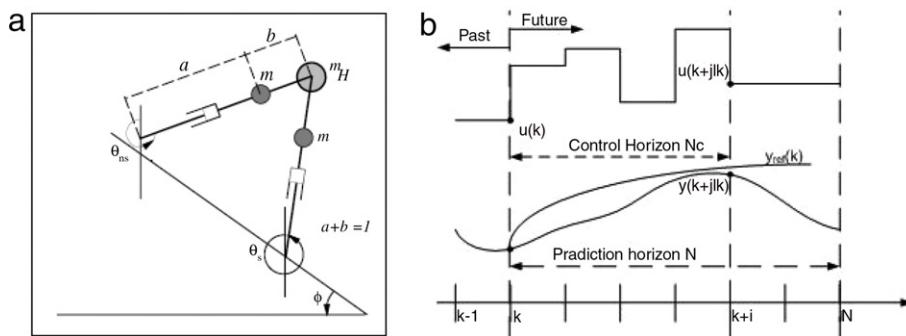


Fig. 57. (a) Model of a compass-like biped robot, (b) Receding Horizon Principle [94].

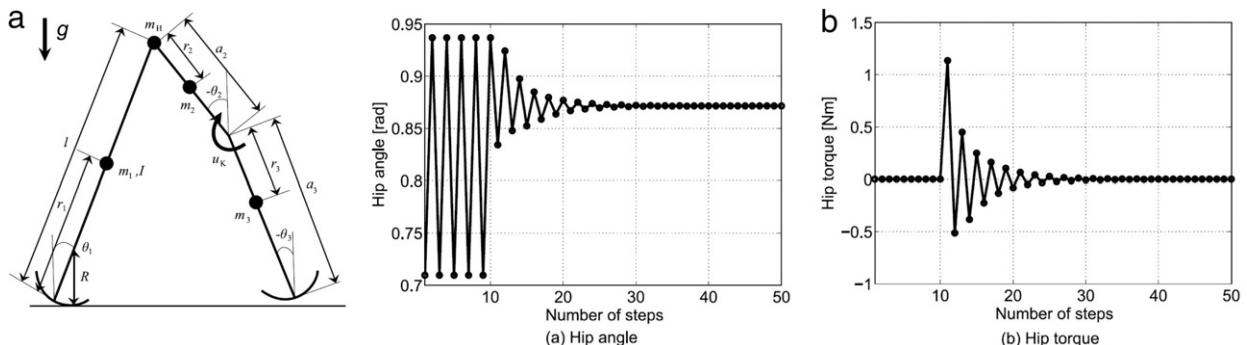


Fig. 58. (a) Model of planar kneeled biped robot with semicircular feet; (b) effect of DFC [95,96].

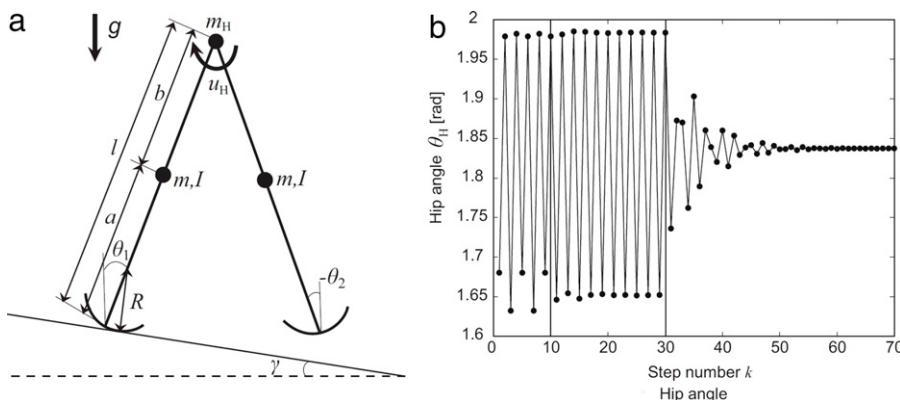


Fig. 59. (a) Model of compass-gait biped with semicircular feet; (b) effect of DFC [97].

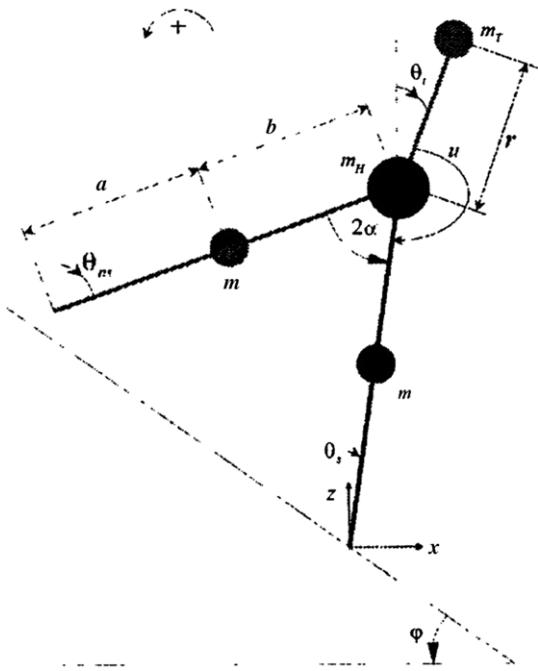


Fig. 60. A torso-driven compass-gait biped [99].

within the neuromuscular system. Recent studies of human locomotion have contradicted this concept. Assorted PDW computer models also indicate that these deviations are not instabilities but might have developed from the intrinsic dynamics of the locomotive mechanism. Chaotic dynamics are a central characteristic of human locomotion and have been proposed as a new prospect for controlling schemes of locomotion.

In the papers reviewed, various nonlinear analysis tools (e.g., bifurcation diagram, Poincare map and Feigenbaum number etc.) were used to locate a variety of chaotic dynamics, like period-doubling route to chaos, cyclic-fold bifurcation, intermittency and crises, revealed by the passive biped models. Asano and Gritli have the opinion that the gait models of PDW robots are characterized as impulsive hybrid systems defined by algebro-differential equations, so nonlinear phenomena in passive walkers are multifaceted and consequently their analyses have progressed with difficulty [60,81]. The founder of Zero-Moment Point (ZMP) method, Dr. Vukobratovic, pointed out the complexity in analysis of passive dynamic bipeds as follows [104]:

"The results achieved in the domain of human-like motion are still rather limited. The presence of unpowered joints highly complicates the stability investigation of such robotic mechanisms."

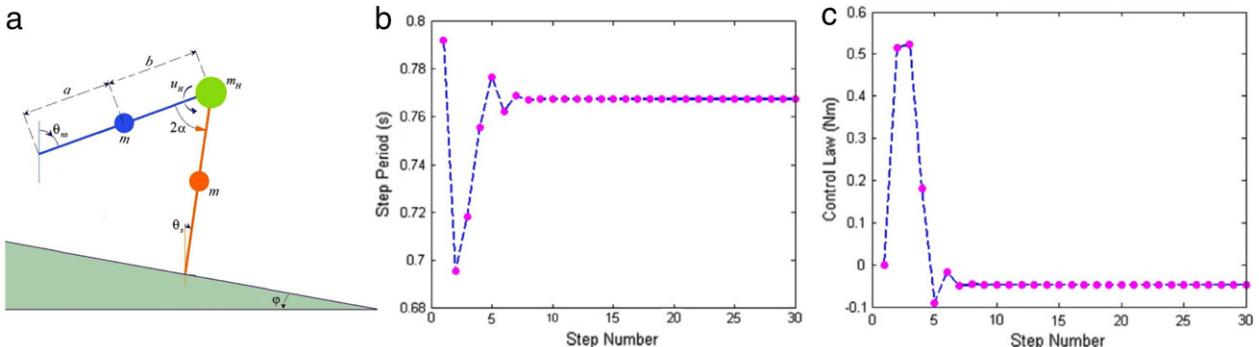


Fig. 61. (a) The compass-gait model down a slope, (b) step period and (c) control law as a function of step number [100].

In the past, it was accepted that the rationale for studying chaos was to pinpoint and avert it. But since the chaos control methods have started surfacing, it has been viewed in a complimentary manner. As revealed by reviewed studies, the bifurcation phenomena and multi-period walking patterns in PDW have been extensively analyzed. But still there is a dearth of literature on chaos taming techniques in this area. Famous chaos control methods—OGY and DFC—have been used to suppress bifurcations and chaos in passive bipedal walking. Physicists have proposed many other methods for taming chaos such as changing the system parameters, synchronization, application of a damper and occasional proportional feedback (OPF) [105]. All methods have their benefits and limitations. In future, scientists will certainly employ the existing distinct techniques to control chaotic dynamics in passive bipeds.

6. Conclusion

From this paper, it can be appreciated that research on the synergy of chaos theory and PDW has progressed through a number of stages. During the first stage, the chaotic dynamics in passive walking were uncovered. It was followed by formal recognition by researchers who employed innovative nonlinear analysis methods. This endeavor has entered the last phase, where researchers are applying chaos control techniques to resolve practical problems. Certainly there is an imperative need for improvised mathematical methods so that dynamical systems like passive bipeds can be studied analytically in place of computer simulations and numerical analysis. The worthwhile applications of multi-period or chaotic passive walking gait necessitate rigorous inquiry. For effective control of passive bipeds, further research is essential to harness the chaotic dynamics of these dynamical systems.

The chaologists will explore more complex passive biped models to uncover new routes to chaos. For comprehending human walking and for rehabilitating gaits disorders, this robotics theme will be investigated by the biomechanists. The suitable parameter configuration and proper feet shape are significant for the passive bipeds to achieve stability and disturbance rejection [62–65]. Hence the robotics community is striving to make use of the self-stabilization behavior seen in humans and animals to design robots that require less energy to maintain stability. The authors believe that the concepts of the chaos theory have enormous potential in passive dynamic walking and coming years will be a testimony to their confidence. The future of chaos research in PDW is likely to be exciting.

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