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Analysis and Experiment of Flat-Footed Passive Dynamic Walker with Ankle Inerter

Yuta Hanazawa, Hiroyuki Suda, and Masaki Yamakita

***Abstract*— To use biped walking as a locomotion method, many researchers have studied biped robots achieving high-performance walking. Many latest biped robots have arc-shaped feet to achieve more high-performance walking. However, the biped robots have many problems due to the arc-shaped feet. To overcome these problems, flat feet with elasticity and viscosity at ankles have been proposed, and it was shown that biped robots with the flat feet achieve high-performance walking. In this paper, for biped robots to achieve more high-performance walking, we propose a novel biped robot foot which has an inerter in addition to the spring and the damper at the ankle. By several simulations and experiments of passive dynamic walking, we show that the biped robot with our proposed flat feet realizes more excellent walking than conventional ones.**

I. INTRODUCTION

Biped walking has attracted attentions as a locomotion method which can be used in various environment, and many researchers study biped robots achieving high-performance walking that is energy efficient and high-speed. Many latest studies of biped walking robots [1], [2], [3], [4] consider an arc-shaped foot to achieve more high-performance walking. By various analyses of biped walking from biomechanics approaches and robotics approaches, effectiveness of the arc-shaped foot in biped walking has been shown experimentaly. Furthermore, Asano et al. [5] showed the effectiveness of the arc-shaped foot mathematically.

However, the biped robots have many problems due to the arc-shaped feet. The arc-shaped feet lack a friction torque between the foot and the ground to cancel yaw motion of biped robots. Moreover, since the arc-shaped feet do not have ankles, the arc-shaped feet restrict the robot behaviors in which ankles are actuated, such as push-off, jumping, and running. Thus, to overcome the problems of the arc-shaped feet, researchers studied biped robots using flat feet which have mechanical impedances of elasticity and viscosity at the ankles [6] [7]. By the analysis of passive dynamic walking using the flat feet, Wisse et al. [6] showed that the biped robot with the flat feet achieves high-performance walking since the ankle elasticity is nearly equivalent to the effect of the arc-foot. Narukawa et al. also showed that the flat foot cancels yaw motion of biped robots [7].

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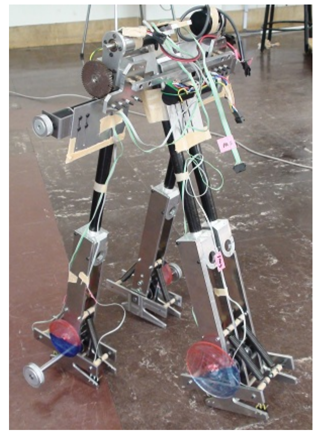


Fig. 1. Flat-footed passive dynamic walker with an ankle inerter

So, if we use the flat feet with the mechanical impedances at the ankles for biped robots, the biped robots can realize high-performance walking. In Sec IV-A of this paper, it will be shown that when we use the flat feet, we must use elasticity and viscosity property to realize biped walking. However, Sec. IV-A also shows that both high-efficiency and high-speed walking cannot be realized by the same mechanical impedance, since the ankle viscosity achieving energy-efficient walking is different from that achieving high-speed walking.

Therefore, if we can set an ankle mechanical impedance which resolves this trade-off by some elements modifyig the ankle mechanical impedance, we can realize biped robots achieving more excellent walking performance than conventional ones.

This trade-off problem of biped robots with the conventional flat feet resembles a trade-off problem of car suspension performances which are constructed by a spring and a damper. To resolve the trade-off problem of the suspension performance, and to realize high-performance suspension, Smith et al. [8] has proposed a passive mechanical suspension using a spring, a damper, and an inerter. The inerter which has a role as a mechanical impedance of inertia is proposed by Smith [9]. By attaching the inerter to the suspension, they also showed that the suspension with the inerter achieves more excellent performance than the conventional suspensions [8].

In this paper, to realize biped robots achieving more excellent walking performance, we propose a novel biped robot foot which is a flat foot with an elastic element, a viscous

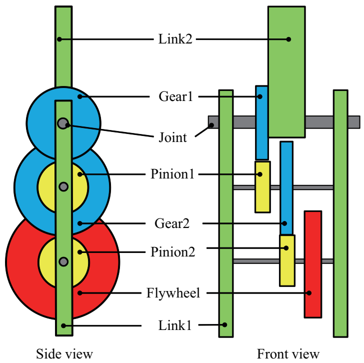


Fig. 2. Mechanism of the rotary inerter

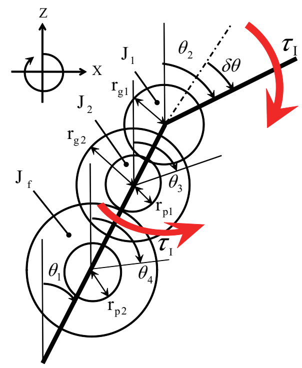


Fig. 3. Model of the rotary inerter

element, and the inerter at the ankle. Since various ankle mechanical impedances of biped robots can be obtained by the ankle inerter, biped robots with the proposed foot realize more high-performance walking.

We analyze the passive dynamic walking of the biped robot with the proposed flat feet by numerical simulations. Moreover, to confirm the validity of the simulation results, we constructed a passive dynamic walker with the proposed flat feet (Fig. 1) and carried out walking experiments.

II. MODELING AND MECHANISM OF ROTARY INERTER

A. Mechanism of the rotary inerter

The inerter of linear motion type was proposed by Smith et al. [9]. Since the linear inerter generates reaction force depending on relative acceleration between two links, it functions as a inertia of the mechanical impedance. Thus, in order to modify ankle mechanical impedance of the biped robots, we want to use the linear inerter. However, if we use the linear inerter at an ankle of biped robots, the ankle becomes a complicated mechanism and the complicated ankle increases the weight of the foot. Therefore, we propose a mechanism of a rotary inerter shown in Fig. 2. The mechanism of this rotary inerter is described as follows:

- The rotary inerter consists of two links, two gears, two pinions, and a flywheel,
- Two link can be rotated around a joint axis,
- The link 2 and the gear 1, the pinion 1 and the gear 2, the pinion 2 and the flywheel are fixed on the same axes, respectively.

B. Modeling of the rotary inerter

Fig. 3 shows a model of the rotary inerter. This rotary inerter generates reaction torque depending on relative angular acceleration of the link 1 (shank) and the link 2 (foot). We derive a dynamic equation of the rotary inerter with the following assumptions:

- Each element of the rotary inerter is a rigid body,
- The mass of gears, pinions and links is negligibly small.

The dynamic equation of the rotary inerter is given by

$$\tau_I = \beta(\delta\ddot{\theta}) = \beta(\ddot{\theta}_2 - \ddot{\theta}_1), \tag{1}$$

where $\beta = J_f \alpha_1^2 \alpha_2^2$, $\alpha_1 = r_{g1}/r_{p1}$, and $\alpha_2 = r_{g2}/r_{p2}$. Detail derivation of the equation is omitted due to space limitation.

III. MODELING OF BIPED ROBOT

Fig. 4 shows a biped robot model with proposed flat feet which have a spring, a damper, and an inerter at each ankle.

A. Dynamic equation

For simplicity of the modeling, we make the following assumptions:

- The biped robot consists of rigid links,
- The contact point of a foot never slips,
- The ankle viscosity by various elements (as an axis, gears by an inerter, and etc.) is modeled by a linear damper.

The dynamic equation of the biped model is given by

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q}) = -\boldsymbol{K}_m\boldsymbol{q}-\boldsymbol{D}_m\dot{\boldsymbol{q}}-\boldsymbol{\beta}_m\ddot{\boldsymbol{q}}+\boldsymbol{J}_c^{\mathrm{T}}\boldsymbol{\lambda}, \tag{2}$$

where $\boldsymbol{q} = [\theta_1\ \theta_2\ \theta_3\ \theta_4\ x_1\ z_1]^{\mathrm{T}}$ is a position vector (θ_i is a link angle, x_1 and z_1 is a position of the stance heel), $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{6\times 6}$ is a generalized inertia matrix, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \mathbb{R}^{6\times 6}$ is a coriolis and centrifugal force matrix, $\boldsymbol{G}(\boldsymbol{q}) \in \mathbb{R}^{6\times 1}$ is a gravity vector, $\boldsymbol{K}_m \in \mathbb{R}^{6\times 6}$ is a spring matrix, $\boldsymbol{D}_m \in \mathbb{R}^{6\times 6}$ is a damping matrix, $\boldsymbol{\beta}_m \in \mathbb{R}^{6\times 6}$ is an inerter matrix, $\boldsymbol{J}_c^{\mathrm{T}} \in \mathbb{R}^{N\times 6}$ is the constrain condition matrix, and $\boldsymbol{\lambda} \in \mathbb{R}^{N\times 1}$ is the constrain force vector where the N is number of constraints.

By rearranging (2), we get

$$\boldsymbol{M}_\beta(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q}) = -\boldsymbol{K}_m\boldsymbol{q}-\boldsymbol{D}_m\dot{\boldsymbol{q}}+\boldsymbol{J}_c^{\mathrm{T}}\boldsymbol{\lambda}. \tag{3}$$

where $\boldsymbol{M}_\beta(\boldsymbol{q}) := \boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{\beta}_m \in \mathbb{R}^{6\times 6}$ is the inertia matrix containing the inerter matrix. Hence, the inerter can modify the inertia matrix. Furthermore, even if the mass of the flywheel is small, a big reaction torque can be generated by increasing a gear ratio $\alpha_1\alpha_2$. Thus, we can only change the inertia matrix almost without changing the mass arrangement of the robot.

For simplicity, many studies of passive dynamic walking assume that a foot of the stance leg always contacts with the ground, and also many of them do not consider changing of contact conditions of the foot during support phase.

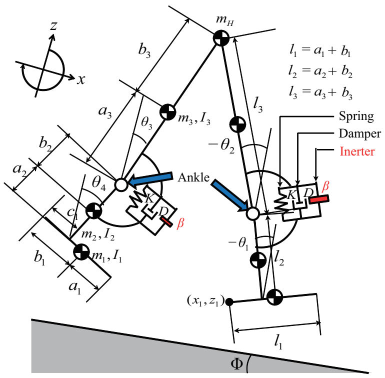


Fig. 4. Biped robot model with proposed flat foot

However, there is a possibility that real walking are not observed by the simplified modeling. Therefore, we consider several contact conditions without simplification, and the walking can take the following contact conditions:

- The stance heel contact,
- The stance heel and the toe contacts,
- The stance toe contact,
- The stance heel and the swing toe contacts,
- The stance heel, the toe, and the swing toe contacts,
- Float in the air.

By the impact of the foot and the constraint force in the walking simulation, the contact condition of the biped robot is changed. The constraint matrix $\boldsymbol{J}_c$ is determined from these contact conditions, and the constraint force λ is given by

$$\boldsymbol{\lambda} = \boldsymbol{X}(\boldsymbol{q})^{-1}(\boldsymbol{J}_c(\boldsymbol{q})\boldsymbol{M}_\beta(\boldsymbol{q})^{-1}(\boldsymbol{\Gamma}(\boldsymbol{q},\dot{\boldsymbol{q}}) - \dot{\boldsymbol{J}}_c\dot{\boldsymbol{q}})), \quad (4)$$

$$\boldsymbol{X}(\boldsymbol{q}) = \boldsymbol{J}_c(\boldsymbol{q})\boldsymbol{M}_\beta(\boldsymbol{q})^{-1}\boldsymbol{J}_c^{\mathrm{T}}(\boldsymbol{q}), \quad (5)$$

$$\boldsymbol{\Gamma}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{K}_m\boldsymbol{q} + \boldsymbol{D}_m\dot{\boldsymbol{q}}. \quad (6)$$

B. Impact equations

Considering the impact between the foot and the ground during walking, we derive impact equations. We make the following assumptions that simplify the derivation of the impact equations:

- There is no rebound and no slipping of the impact point,
- The impact takes place over an infinitesimally small period of time,
- The external forces during the impact can be represented by impulses.

Under these assumption, impulsive forces $\boldsymbol{\lambda}_I \in \mathbb{R}^{N\times 1}$ and velocity vector just after impact, $\dot{\boldsymbol{q}}^+ \in \mathbb{R}^{6\times 1}$, are obtained by

$$\boldsymbol{\lambda}_I = -\boldsymbol{X}_I(\boldsymbol{q})^{-1}\boldsymbol{J}_I\dot{\boldsymbol{q}}^-, \quad (7)$$

$$\boldsymbol{X}_I(\boldsymbol{q}) = \boldsymbol{J}_I(\boldsymbol{q})\boldsymbol{M}_\beta(\boldsymbol{q})^{-1}\boldsymbol{J}_I^{\mathrm{T}}(\boldsymbol{q}), \quad (8)$$

$$\dot{\boldsymbol{q}}^+ = (\boldsymbol{I} - \boldsymbol{M}_\beta(\boldsymbol{q})^{-1}\boldsymbol{J}_I^{\mathrm{T}}\boldsymbol{X}(q)^{-1}\boldsymbol{J}_I(\boldsymbol{q}))\dot{\boldsymbol{q}}^-. \quad (9)$$

where $\boldsymbol{J}_I \in \mathbb{R}^{N\times 6}$ is the impulsive constraint matrix derived from the geometric condition at impact, and $\dot{\boldsymbol{q}}^- \in \mathbb{R}^{6\times 1}$ is velocity vector just before impact(N is number of impulsive constraint).

TABLE I

MODEL PARAMETERS

Symbol	Unit	Value
l_1	m	0.16
l_2	m	0.04
l_3	m	0.70
$a_1 = b_1 = l_1/2$	m	0.06
$a_2 = b_2 = l_2/2$	m	0.02
$a_3 = b_3 = l_3/2$	m	0.35
c_1	m	0.04
m_1	kg	0.25
m_2	kg	0.25
m_3	kg	4.5
m_H	kg	10.0
I_1	kg· m^2	5.33×10^{-4}
I_2	kg· m^2	3.33×10^{-5}
I_3	kg· m^2	1.84×10^{-1}
K	N·m/rad	90.0

IV. ANALYSIS OF FLAT FOOT IN PASSIVE DYNAMIC WALKING

In the biped walking, efficient and high-speed walking is desired. Thus, we analyze energy efficiency and speed of passive dynamic walking with the flat feet by simulations. The energy efficiency of walking is evaluated by specific resistance (SR) [10], [1]. SR is defined by

$$SR := \frac{p}{Mgv} = \frac{MgX_g\sin\phi/T}{Mgv} = \sin\phi, \quad (10)$$

where p [J/s] is an average input power, M [m] is a total weight of biped robot, v [m/s] is a walking speed, X_g [m] is a length of step, T [s] is a steady step of period, and ϕ [rad] is a slope angle. Therefore, the energy efficiency of passive dynamic walking is determined by only a slope angle. For this reason, if the slope angle of passive dynamic walking is 0 rad (i.e. level grounds), the energy efficiency of the walking becomes maximum.

Table I shows model parameters. To determine these parameters, we made references to previous works [5], [11], and we set the ankle spring constant small so that passive dynamic walking can be realized.

In the simulations, we define that 1-period walking is success walking, and also we define that the following walks are failure:

Case 1 : 2-period walking,
Case 2 : Lack of energy required in walking,
Case 3 : Occurrence of slip between the stance heel and the floor.

The walking in the case 1 converes to 2-period walking after sufficient time passed, that in the case 2 is failed walking by lack of energy, and that in the case 3 is failed walking which does not satisfy the assumptions that a contact point is not slipping by slippage between a heel of the biped robot and the floor at the heel contact phase. We can judge the occurrence of the case 3 by a heel constraint force. To prevent slippage of the heel, the next condition should be satisfied,

$$\frac{|\lambda_x|}{\lambda_z} \leq \mu, \quad (11)$$

where λ_x is the heel constraint force in a direction of the slope, λ_z is the heel constraint force in the vertical direction, and μ is a friction coefficient. In a real environment, since walking in the situation where this coefficient exceeds 1 hardly exists, the coefficient is assumed to be 1 in the simulations. For this reason, we judge that the heel slips on the floor when the ratio of constraint force, $\frac{|\lambda_x|}{\lambda_z}$, exceeds 1.

A. Analysis of conventional flat foot

Many researchers have studied a characteristic of the conventional flat foot which has ankle elastic and viscous elements in biped walking from a view of passive dynamic walking. These results show that the ankle spring constant has similar effects of the radius of an arc-shaped foot in biped walking [6]. However, the characteristic of ankle viscosity in walking has not been most analyzed yet. Therefore, we firstly analyze the characteristic of ankle viscosity in passive dynamic walking.

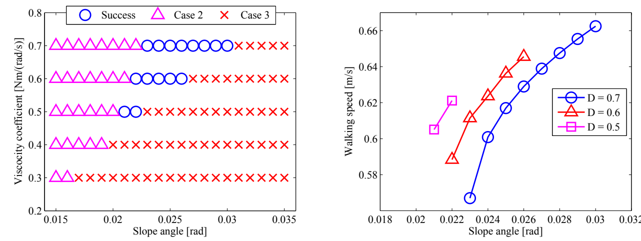
Fig. 5 shows the simulation results (Success, Case 1, Case 2 and Case 3) of passive walking using the conventional flat foot with respect to several slope angles, when the ankle viscosity coefficient is set to 5 values from 0.3 to 0.7 Nm/(rad/s) where the ankle inerter constant is set 0.0 Nm/(rad/s^2). Fig. 5(a) shows the walking result, and Fig. 5(b) shows the walking speed of success walking. From Fig. 5, we can see that the biped robot can walk on a gentle slope when the ankle viscosity is small. These results show that ankle viscosity impair energy efficiency of walking, and we can see that ankle viscosity increase energy loss of walking. Thus, we must set a smaller ankle viscosity to achieve energy efficient walking, as expected.

However, Fig. 5(a) shows that a biped robot with the small ankle viscosity can not walk on a steep slope since the case 3 arises, and Fig. 5(b) shows that the biped robot can not walk fast. So, if we want to achieve high-speed walking of biped robots, we must set the ankle viscosity to prevent a slippage between the foot and the floor on steep slopes.

Therefore, the ankle viscosity is an important element to determine energy-efficiency and speed of walking. Furthermore, these results show that we cannot realize the ankle mechanical impedances which achieve both energy efficient walking and high-speed walking by only using spring and damper.

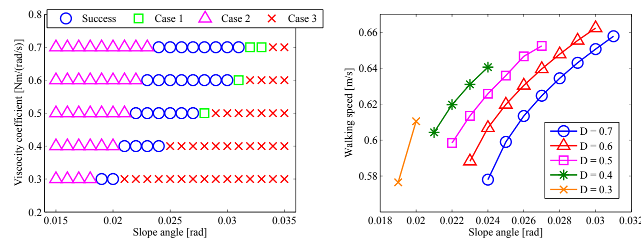
B. Analysis of proposed flat foot

In this section, we analyze the effects of the ankle inerter. Fig. 6 shows the results of the passive walking using the proposed flat foot with respect to several slope angles when an ankle viscosity coefficient is set to 4 values from 0.3 to 0.7 Nm/(rad/s), where the ankle inerter constant is set 0.005 Nm/(rad/s^2). Fig. 6(a) shows the walking result, and Fig. 6(b) shows the walking speed of the success walking. As same as the analysis of the conventional flat foot, these results show that a biped robot with a small ankle viscosity cannot walk on the steep slope. They also show that the biped robot with a big ankle viscosity can walk on a steep slope.



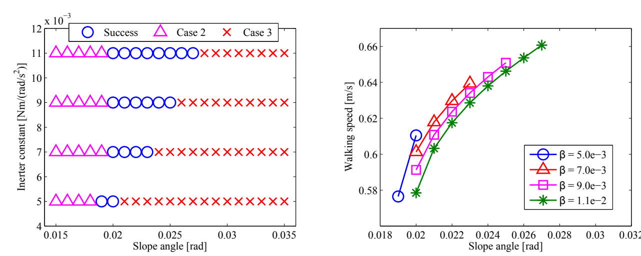
(a) Walking result (b) Walking speed

Fig. 5. Simulation results of passive dynamic walking by the biped robot with the conventional flat foot that is set ankle viscosity coefficient of 5 values, with respect to the slope angle ϕ



(a) Walking result (b) Walking speed

Fig. 6. Simulation results of passive dynamic walking by the biped robot with the proposed flat foot that is set ankle viscosity coefficient of 5 values, with respect to the slope angle ϕ



(a) Walking result (b) Walking speed

Fig. 7. Simulation results of the biped robot with proposed flat foot that is set ankle inerter constant of 4 values, with respect to the slope angle ϕ

Therefore, we can see that the ankle viscosity of the proposed flat foot determines energy efficiency and speed of biped walking, as for the conventional ones.

Moreover, we compare Fig. 5 to Fig. 6, to see the effect of the ankle inerter. In comparison of changing ankle viscosity coefficient from 0.5 to 0.7 Nm/(rad/s), the biped robot with the proposed flat foot needs a steep slope than that for the biped robot with the conventional flat foot to realize passive dynamic walking. Thus, this result show that the ankle inerter makes energy efficiency of walking worse.

In comparison of changing ankle viscosity coefficient from 0.3 to 0.4 Nm/(rad/s), the biped robot with the conventional flat feet cannot walk on a gentle slopes (from 0.019 to 0.020 rad) but the biped robot with the proposed flat feet can walk on the such slopes. As the simulation results, the biped robot with the proposed flat feet can make more energy efficient walking than the biped robot with the conventional flat feet, when the ankle viscosity of the biped robot is set very small.

In order to investigate the more detail effect of the ankle inerter, we analyzed the walking when the ankle viscosity coefficient is set to 0.3 Nm/(rad/s) with 4 distinct values of the inerter constant. Fig. 7(a) shows the walking result, and Fig. 7(b) shows the walking speed. These results show that the walking speed monotonically increase as the slope angle increases, whereas the energy-efficiency of walking worsens.

From these results, the ankle inertia has a similar effect of ankle viscosity which enable biped robots to walk fast on steep slopes by supressing ankle vibration. However, this effect is clearly different from the effect of ankle viscosity in terms of energetic consumption of walking. The ankle inertia increases the energy loss at impacts of foot, meanwhile, the ankle viscosity increase the energy loss during ankle rotations.

By taking adventage of the effects of the proposed flat feet as the different from that of conventional ones, we can realize biped robots achieving various walking than those of biped robot with conventional flat foot, which is shown in the next section.

V. Improving walking performance

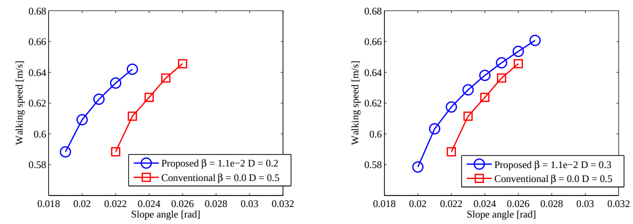
In this section, to confirm effectiveness of the proposed feet, by only changing ankle viscosity coefficient and inerter constant, we show that the biped robot with the proposed foot can achieve more high-performance walking than a conventional biped robots with the conventional flat feet.

Here, we assume that the biped robot with conventional flat feet (β = 0.0 Nm/(rad/s^2) D = 0.5 Nm/(rad/s)) shown in Fig. 5(b). As shown in the previous section (Fig. 5(b)), walking performance of this robot changes maximum speed and energy efficiency by changing the ankle viscosity. This biped robot cannot walk fast when the ankle viscosity is set small to improve energy efficiency, and it cannot walk energy efficiently when the ankle viscosity is set big to increase maximum speed. That is, there exists trade-off of the walking performances, and we cannot improve both energy efficiency and maximum speed simultaneously.

For this reason, to realize biped robots achieving high-performance walking which cannot be obtained by the conventional flat foot, we replace the conventional flat foot with a proposed flat foot which has the same ankle elasticity.

A. Improving energy efficiency

Initially, we show that the biped robot with the proposed flat foot achieves more energy-efficient walking than the biped robot with conventional flat foot without decrease of maximum walking speed. Fig. 8(a) shows that comparison of walking performances of the biped robot with the proposed flat foot (β = 0.011 Nm/(rad/s^2) D = 0.2 Nm/(rad/s)) and that of the biped robot with the conventional flat foot. From this



(a) Example of improving energy efficiency

(b) Example of improving maximum speed

Fig. 8. Comparison of the walking performances between the biped robot with the proposed foot and the biped robot with the conventional foot

figure, we can see that the biped robot with the proposed flat foot achieves energy-efficient walking than the biped robot with conventional flat foot almost without decrease of maximum walking speed.

B. Improving maximum speed

In addition, we show that the biped robot with the proposed flat foot achieves more high-speed walking than the biped robot with conventional flat foot without decrease of energy efficiency of walking. Fig. 8(b) shows that comparison of walking performances of the biped robot with the proposed flat foot (β = 0.011 Nm/(rad/s^2) D = 0.3 Nm/(rad/s)) and that of the biped robot with the conventional flat foot. From this figure, we can see that the biped robot with the proposed flat foot achieves high-speed walking than the biped robot with the conventional flat foot without decrease of energy efficiency of walking.

Since the proposed flat foot realizes more various mechanical impedances than conventional ones by changing the inerter parameter, mechanical impedances which achieves more high-performance walking than the conventional ones can be realized with the proposed flat foot.

VI. Walking Experiment

To investigate the validity of our simulation, we constructed a passive dynamic walker (Fig. 1) and carried out experiments. Since the parameters using simulations is too large to construct a prototype, we selected walker parameters as in Table II. Fig. 9 shows the snapshot of walking experiment. The slope angle is set about 0.028 rad which achieves 1-period walking, and footholds are placed about 0.4 m apart.

Then, we make comparison between simulation and experiment results. Fig. 10 shows phase plane trajectories which are a hip-joint relative angle and its angular velocity in the experiment and the simulation. This figure shows that the phase plane trajectory of the experiment converges the limit cycle of the simulation. Table III shows a comparison of the walking performaces. From these result, we can see that the experimental results have similar performances of the simulation results.

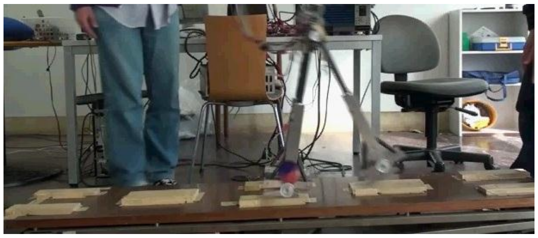


Fig. 9. Snapshot of walking experiment

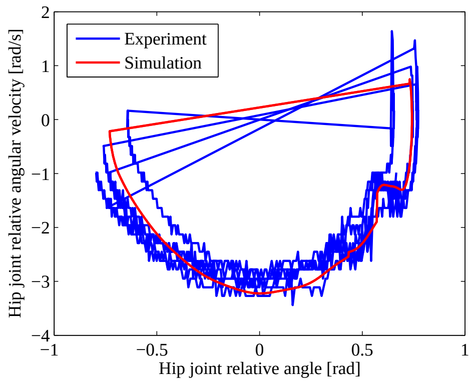


Fig. 10. Phase plane trajectories of the experiment and the simulation

TABLE II
PASSIVE WALKER PARAMETERS

Symbol	Unit	Value
l_1	m	0.12
l_2	m	0.04
l_3	m	0.55
$a_1 = b_1 = l_1/2$	m	0.06
$a_2 = b_2 = l_2/2$	m	0.02
$a_3 = b_3 = l_3/2$	m	0.275
c_1	m	0.04
m_1	kg	0.15
m_2	kg	0.15
m_3	kg	2.3
m_H	kg	1.8
I_1	kg$\cdot$ m^2	1.80×10^{-4}
I_2	kg$\cdot$ m^2	2.00×10^{-5}
I_3	kg$\cdot$ m^2	5.80×10^{-2}
K	N·m/rad	20.0
D	N·m/(rad/s)	0.05
β	N·m/(rad/s^2)	3.00×10^{-3}

TABLE III
WALKING PERFORMANCES OF THE SIMULATION AND THE EXPERIMENT

	Simulation	Experiment
Walk speed	0.542 m/s	0.532 m/s
Walk cycle	0.761 s	0.766 s

Therefore, we can confirm that our numerical simulations is valid, and we can infer that analysis by the simulations in this paper should be valid.

VII. CONCLUSION AND FUTURE WORK

In this paper, to realize biped robots achieving high-performance walking than conventional ones, we proposed the novel biped robot foot which is the flat foot with the elastic element, the viscosity element, and the inerter at the ankle, and also we analyzed passive walking of biped robot with the proposed flat feet to investigate effectiveness of the proposed ones.

Firstly, by the numerical walking simulations, we showed that the proposed flat foot can set more various ankle mechanical impedance by the ankle inerter than conventional one. Moreover, we showed this various ankle mechanical impedance realizes various high-performance walking which cannot be obtained by the biped robot with the conventional flat feet.

Furthermore, we constructed a passive dynamic walker with the proposed flat feet, and carried out walking experiments. By the experimental results, we confirmed that the biped walker can achieve the walking performance similar to our simulation, and we can infer that analysis by numerical simulations in this paper is valid.

More detailed analysis of the ankle inerter, and establishment of design method of the mechanical impedance of the ankle are left as future works. Moreover, we will apply the proposed mechanism for biped robots on level ground.

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