MATHEMATICS

January 4, 2024

1. For what values of k does the quadratic equation $4x^2 - 12x - k = 0$ have no real roots ?

2. Find the distance between the points (a, b) and $(-a, -b)$.

3. Find a rational number between $\sqrt{2}$ and $\sqrt{7}$.

4. Write the number of zeroes in the end of a number whose prime factorization is $2^2 \times 5^3 \times 3^2 \times 17$.

5. Let $\triangle ABC \sim \triangle DEF$ and their areas be respectively, $64cm^2$ and $121cm^2$. If $EF = 15.4cm$,find BC.

6. Evaluate:

$$\frac{\tan 65^\circ}{\cot 25^\circ}$$

7. Express $(\sin 67^\circ + \cos 75^\circ)$ in terms of trigonometric ratios of the angle between 0° and 45°.

8. Find the number of terms in the A.P. :

$$18, 15\frac{1}{2}, 13, ..., -47.$$

9. A bag contains 15 balls, out of which some are white and the others are black. If the probability of drawing a black ball at random from the bag is $\frac{2}{3}$,then find how many white balls are there in the bag.

10. A card is drawn at random from a pack of 52 playing cards. Find the probability of drawing a card which is neither a spade nor a king.

11. Find the solution of the pair of equations :

$$\frac{3}{x}+\frac{8}{y}=-1; \frac{1}{x}-\frac{2}{y}=2, x, y \neq 0$$

12. Find the value(s) of k for which the pair of equations $\begin{cases} kx+2y=3 \\ 3x+6y=10 \end{cases}$ has a unique solution.

13. How many multiples of 4 lie between 10 and 205 ?

14. Determine the A.P. whose third term is 16 and 7^{th} term exceeds the 5^{th} term by 12.

15. Use Euclid's division algorithm to find the HCF of 255 and 867.

16. The point R divides the line segment AB, where $A(-4, 0)$ and $B(0, 6)$ such that $AR = \frac{3}{4}AB$. Find the coordinates of R.

17. Prove that :

$$(\sin\theta + 1 + \cos\theta)(\sin\theta - 1 + \cos\theta) \,.\, \sec\theta \csc\theta = 2$$

18. Prove that :

$$\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\csc\theta$$

19. In what ratio does the point $P(-4, y)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$? Hence find the value of y.

20. Find the value of p for which the points $(-5, 1)$, $(1, p)$ and $(4, -2)$ are collinear.

21. ABC is a right triangle in which $\angle B = 90°$. If $AB = 8cm$ and $BC = 6cm$,find the diameter of the circle inscribed in the triangle.

22. In Figure 1, BL and CM are medians of a $\triangle ABC$ right-angled at A.Prove that $4(BL^2 + CM^2) = 5BC^2$.

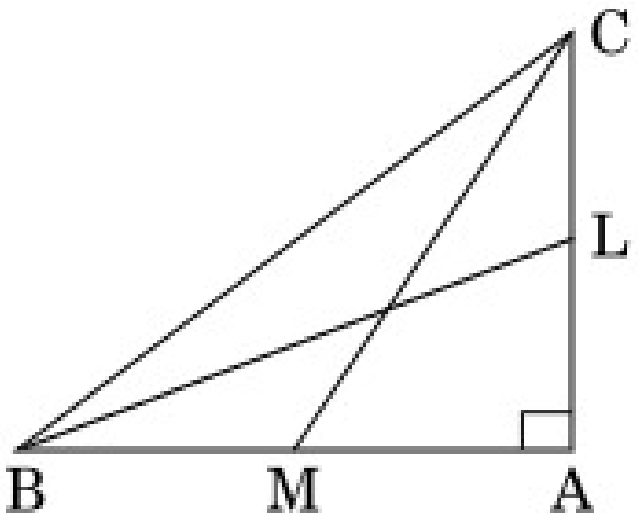

Figure 1: Triangle ABC

23. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

24. In Figure 2, two concentric circles with centre O, have radii $21cm$ and $42cm$. If $\angle AOB = 60°$, find the area of the shaded region.

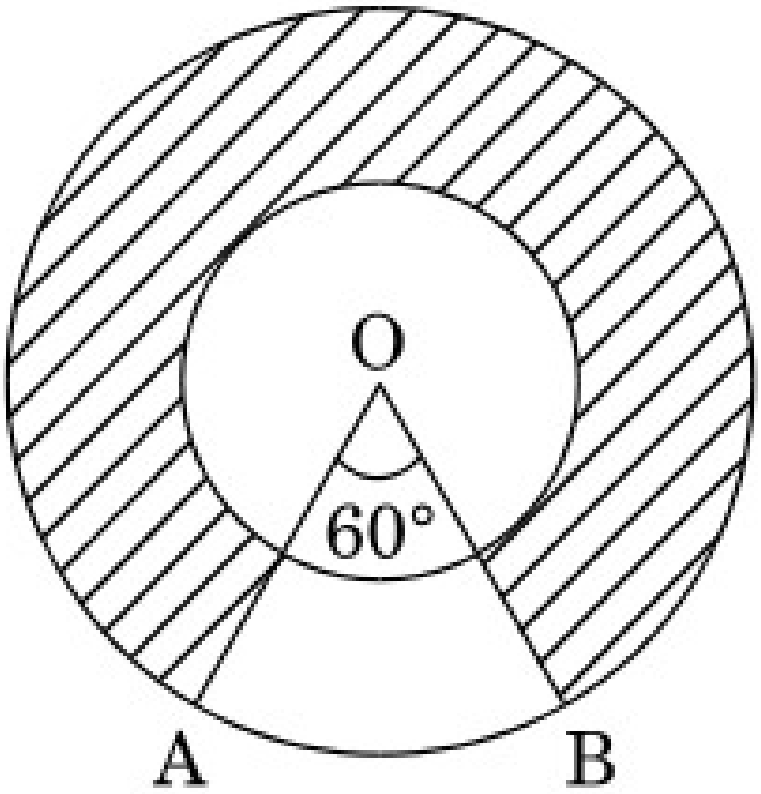

Figure 2: Circle AOB

25. A cone of height $24cm$ and radius of base $6cm$ is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere and hence find the surface area of this sphere.

26. A farmer connects a pipe of internal diameter $20cm$ from a canal into a cylindrical tank in his field which is $10m$ in diameter and $2m$ deep. If water flows through the pipe at the rate of $3km/hr$, in how much time will the tank be filled ?

27. Prove that $2 + 3\sqrt{3}$is an irrational number when it is given that $\sqrt{3}$ is an irrational number.

28. Sum of the areas of two squares is $157m^2$.If the sum of their perimeters is $68m$, find the sides of the two squares.

29. Find the quadratic polynomial, sum and product of whose zeroes are -1 and -20 respectively. Also find the zeroes of the polynomial so obtained.

30. A plane left 30 minutes later than the scheduled time and in order to reach its destination $1500km$ away on time, it has to increase its speed by $250km/hr$ from its usual speed. Find the usual speed of the plane.

31. Find the dimensions of a rectangular park whose perimeter is $60m$ and area $200m^2$.

32. Find the value of x, when in the A.P. given below

$$2 + 6 + 10 + ... + x = 1800.$$

33. If $\sec\theta + \tan\theta = m$, show that $\frac{m^2-1}{m^2+1} = \sin\theta$.

34. In $\triangle ABC$ Figure 3, $AD \perp BC$. Prove that
$AC^2 = AB^2 + BC^2 - 2BC \times BD$

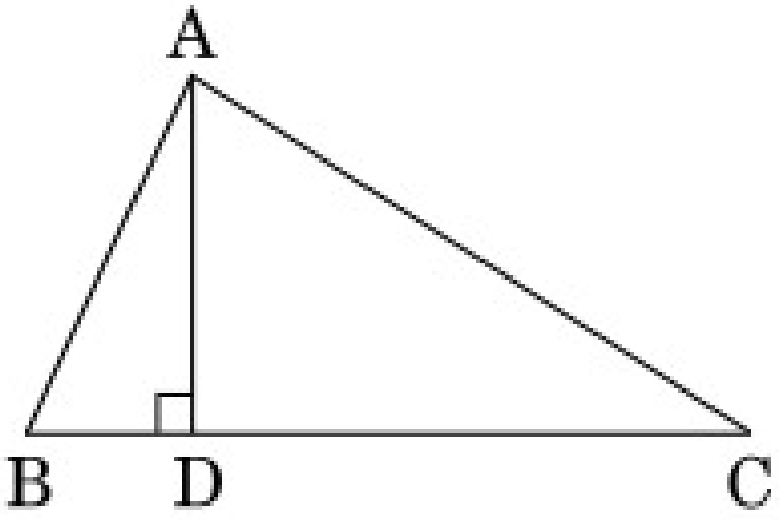

Figure 3

35. A moving boat is observed from the top of a 150m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/min.

36. There are two poles, one each on either bank of a river just opposite to each other. One pole is 60m high. From the top of this pole, the angle of depression of the top and foot of the other pole are 30° and 60°respectively. Find the width of the river and height of the other pole.

37. Construct a triangle with sides 5cm, 6cm and 7cm and then another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the first triangle.

38. A container opened at the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container, at the rate of ₹50 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹10 per 100cm^2. ($Take\pi = 3.14$)

39. Which term of the A.P.$-4, -1, 2, ...is101$?

40. Three different coins are tossed simultaneously. Find the probability of getting exactly one head.

41. A die is thrown once. Find the probability of getting

    (a) a prime number

    (b) an odd number.

42. Obtain all the zeroes of the polynomial $2x^4 - 5x^3 - 11x^2 + 20x + 12$ when2and -2 are two zeroes of the above polynomial.

43. A motorboat whose speed is 18 km/hr in still water takes one hour more to go 24km upstream than to return downstream to the same spot. Find the speed of the stream.

44. In an A.P., the first term is -4, the last term is 29 and the sum of all its terms is 150. Find its common difference.

45. Draw a circle of radius 4 cm. From a point 6 cm away from its centre,construct a pair of tangents to the circle and measure their lengths.

46. Prove that :

$$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$$

47. Solve for x :

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}; x \neq 0, x \neq \frac{-2a-b}{2}, a, b \neq 0$$

48. The sum of the areas of two squares is $640m^2$.If the difference of their perimeters is $64m$, find the sides of the square.