

MATHEMATICS

January 8, 2024

1 DISCRETE

1. Write the number of zeroes in the end of a number whose prime factorization is $2^2 \times 5^3 \times 3^2 \times 17$.
2. Use Euclid's division algorithm to find the HCF of 255 and 867.
3. Find the number of terms in the A.P. :

$$18, 15\frac{1}{2}, 13, \dots, -47.$$

4. Determine the A.P. whose third term is 16 and 7^{th} term exceeds the 5^{th} term by 12.
5. Find the value of x , when in the A.P. given below

$$2 + 6 + 10 + \dots + x = 1800.$$

6. Which term of the A.P. $-4, -1, 2, \dots$ is 101?
7. In an A.P., the first term is -4 , the last term is 29 and the sum of all its terms is 150. Find its common difference.
8. Prove that $2+3\sqrt{3}$ is an irrational number when it is given that $\sqrt{3}$ is an irrational number.

2 NUMBER SYSTEMS

9. Find a rational number between $\sqrt{2}$ and $\sqrt{7}$.
10. How many multiples of 4 lie between 10 and 205?

3 MATRICES

11. Find the solution of the pair of equations :

$$\frac{3}{x} + \frac{8}{y} = -1; \frac{1}{x} - \frac{2}{y} = 2, x, y \neq 0$$

12. Find the value(s) of k for which the pair of equations

$$\begin{aligned} kx + 2y &= 3 \\ 3x + 6y &= 10 \end{aligned}$$

has a unique solution.

4 ALGEBRA

13. Obtain all the zeroes of the polynomial $2x^4 - 5x^3 - 11x^2 + 20x + 12$ when 2 and -2 are two zeroes of the above polynomial.
14. Find the quadratic polynomial, sum and product of whose zeroes are -1 and -20 respectively. Also find the zeroes of the polynomial so obtained.
15. Sum of the areas of two squares is $157m^2$. If the sum of their perimeters is $68m$, find the sides of the two squares.
16. A plane left 30 minutes later than the scheduled time and in order to reach its destination $1500km$ away on time, it has to increase its speed by $250km/hr$ from its usual speed. Find the usual speed of the plane.
17. A motorboat whose speed is $18 km/hr$ in still water takes one hour more to go $24km$ upstream than to return downstream to the same spot. Find the speed of the stream.
18. Solve for x :

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}; x \neq 0, x \neq \frac{-2a-b}{2}, a, b \neq 0$$

19. The sum of the areas of two squares is $640m^2$. If the difference of their perimeters is $64m$, find the sides of the square.
20. For what values of k does the quadratic equation $4x^2 - 12x - k = 0$ have no real roots ?
21. Evaluate:

$$\frac{\tan 65^\circ}{\cot 25^\circ}$$

22. Express $(\sin 67^\circ + \cos 75^\circ)$ in terms of trigonometric ratios of the angle between 0° and 45° .

23. Prove that :

$$(\sin \theta + 1 + \cos \theta)(\sin \theta - 1 + \cos \theta) \cdot \sec \theta \csc \theta = 2$$

24. Prove that :

$$\sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = 2 \csc \theta$$

25. If $\sec \theta + \tan \theta = m$, show that $\frac{m^2 - 1}{m^2 + 1} = \sin \theta$.

26. Prove that :

$$2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$$

5 VECTORS

27. The point R divides the line segment AB , where $A(-4, 0)$ and $B(0, 6)$ such that $AR = \frac{3}{4}AB$. Find the coordinates of R .
28. In what ratio does the point $P(-4, y)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$? Hence find the value of y .
29. Find the distance between the points (a, b) and $(-a, -b)$.
30. Find the value of p for which the points $(-5, 1)$, $(1, p)$ and $(4, -2)$ are collinear.

6 CIRCLES

31. ABC is a right triangle in which $\angle B = 90^\circ$. If $AB = 8\text{cm}$ and $BC = 6\text{cm}$, find the diameter of the circle inscribed in the triangle.

7 GEOMETRY

32. In Figure ??, two concentric circles with centre O , have radii 21cm and 42cm . If $\angle AOB = 60^\circ$, find the area of the shaded region.

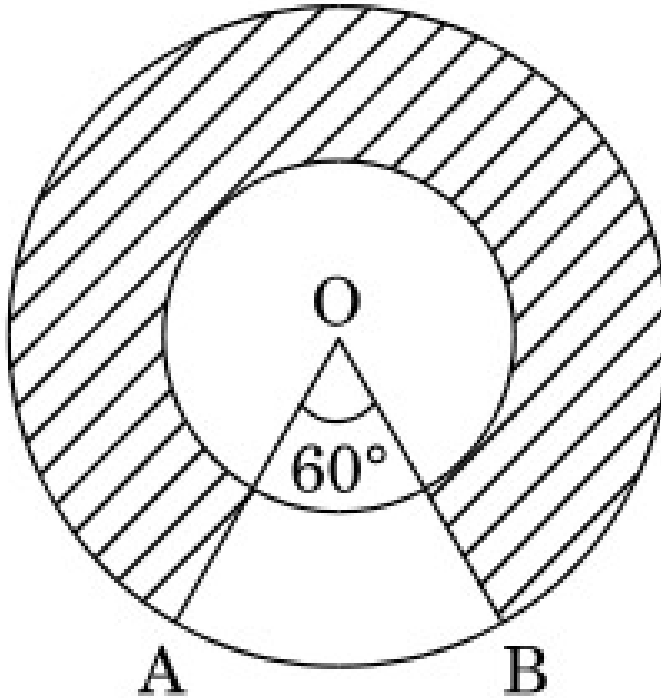


Figure 1: Circle AOB

33. A moving boat is observed from the top of a $150m$ high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/min .
34. There are two poles, one each on either bank of a river just opposite to each other. One pole is $60m$ high. From the top of this pole, the angle of depression of the top and foot of the other pole are 30° and 60° respectively. Find the width of the river and height of the other pole.
35. A cone of height $24cm$ and radius of base $6cm$ is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere and hence find the surface area of this sphere.
36. A farmer connects a pipe of internal diameter $20cm$ from a canal into a cylindrical tank in his field which is $10m$ in diameter and $2m$ deep. If water flows through the pipe at the rate of $3km/hr$, in how much time will the tank be filled ?
37. Find the dimensions of a rectangular park whose perimeter is $60m$ and area $200m^2$.

38. A container opened at the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container, at the rate of ₹50 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹10 per 100cm^2 . Take($\pi = 3.14$)

8 CONSTRUCTION

39. In $\triangle ABC$ Figure ??, $AD \perp BC$. Prove that
 $AC^2 = AB^2 + BC^2 - 2BC \times BD$

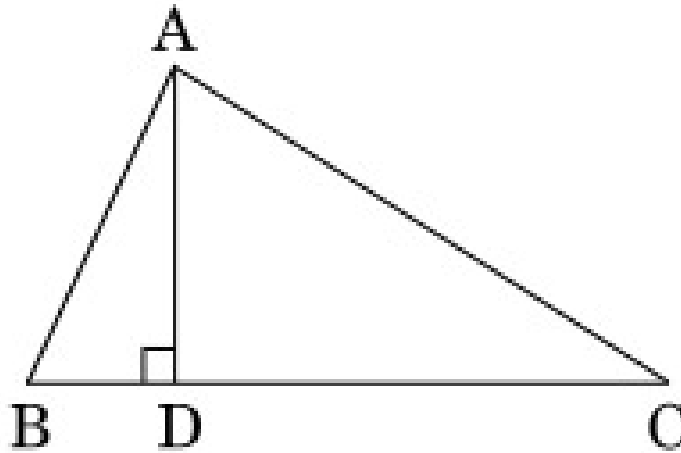


Figure 2

40. Draw a circle of radius 4 cm. From a point 6 cm away from its centre, construct a pair of tangents to the circle and measure their lengths.
41. Construct a triangle with sides 5cm, 6cm and 7cm and then another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the first triangle.
42. Let $\triangle ABC \sim \triangle DEF$ and their areas be respectively, 64cm^2 and 121cm^2 . If $EF = 15.4\text{cm}$, find BC .
43. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.
44. In Figure ??, BL and CM are medians of a $\triangle ABC$ right-angled at A . Prove that $4(BL^2 + CM^2) = 5BC^2$.

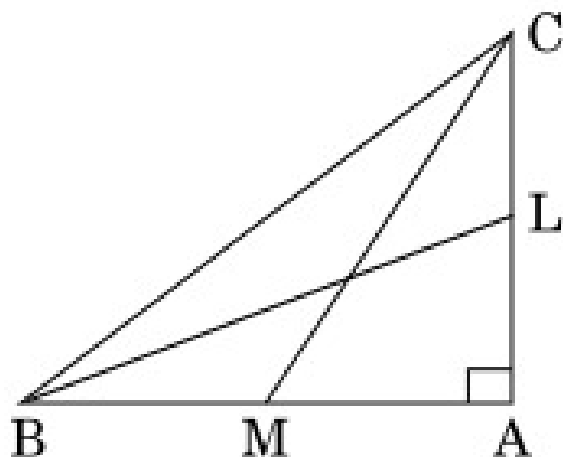


Figure 3: Triangle ABC

9 PROBABILITY

45. A bag contains 15 balls, out of which some are white and the others are black. If the probability of drawing a black ball at random from the bag is $\frac{2}{3}$, then find how many white balls are there in the bag.
46. A card is drawn at random from a pack of 52 playing cards. Find the probability of drawing a card which is neither a spade nor a king.
47. A die is thrown once. Find the probability of getting
 - (a) a prime number
 - (b) an odd number.
48. Three different coins are tossed simultaneously. Find the probability of getting exactly one head.