

## Schema Refinement

Problems due to Redundancy:-

1. Repeated Storage.
2. Insertion Problems
3. Updation Problems
4. Deletion Problems

Requirements:-

Professors of same Seniority should have same Salary.

Professor:-

SSN	Name	Seniority	Salary	functional dependency
1	A	5	50K	+10K X
2	B	5	50K	
3	C	5	50K	
4	D	10	1L	{ } delete X
5	E	10	1L	
6	F	10	1.2L	> Insert X

Prof

SSN    Name    Seniority FK

1	A	5
2	B	5
3	C	5
4	D	10
5	E	10

6 F 10 > Insert

Prof - Salary

Seniority PK Salary

5	5	50K
10	10	1L

10	10	1L

10	10	1L

10	10	1L

10	10	1L

10	10	1L

## Functional dependency:- (FD)

A Functional dependency  $X \rightarrow Y$  says that if any two tuples agree on the values on attribute  $X$ , they must also agree on the values of attribute  $Y$ .

Ex:-  $X \rightarrow Y$

$$\begin{array}{l} \xrightarrow{} X_1 \rightarrow Y_1 \\ \xrightarrow{} X_1 \rightarrow Y_1 \\ \xrightarrow{} X_1 \rightarrow Y_1 \\ \xrightarrow{} X_2 \rightarrow Y_4 \\ \xrightarrow{} X_2 \rightarrow Y_4 \\ \xrightarrow{} X_3 \rightarrow Y_7 \\ \xrightarrow{} X_4 \rightarrow Y_7 \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

If  $X \rightarrow Y$  is a functional dependency, then  $X$  may become Key for the relation.

Q. Consider the following relation instance,

R			find which of the following dependencies are satisfying on the above relation instances.
A	B	C	
a <sub>1</sub>	b <sub>1</sub>	g	
a <sub>1</sub>	b <sub>2</sub>	g	
a <sub>2</sub>	b <sub>1</sub>	g	

- |                         |  |   |
|-------------------------|--|---|
| a) $A \rightarrow B$ ✗  | Not<br>trivial<br>functional<br>dependencies | (f) $AC \rightarrow B$ ✗  |
| b) $A \rightarrow C$ ✓  |  | (g) $AC \rightarrow C$ ✗ ✓ → Trivial<br>"odd man-out" dependency. |
| c) $B \rightarrow C$ ✓  |  | $AC \rightarrow C$  |
| d) $C \rightarrow A$ ✗  |  | $C \subset AC$  |
| e) $AB \rightarrow C$ ✓ |  |   |

A functional dependency  $X \rightarrow Y$  is said to be trivial if and only if  $Y \subseteq X$ .

Q1P

①	A	B	C	$A \rightarrow BC$	X
	1	2	1	$AC \rightarrow B$	X
	1	2	2	$AB \rightarrow C$	X
	1	3	1	$BC \rightarrow A$	✓
	2	3	2		

②

X	Y	Z	a) $X$
1	4	2	b) ✓
1	5	3	c) X
1	6	3	d) X
3	2	2	

③

A	B	C	D	A	B	C	D	A	B	C	D			
a)	2	2	3	4	b)	3	2	3	4	✓	4	2	3	4
	2	2	3	4		2	3	3	4		4	2	5	6
	2	2	3	6		3	2	3	6		4	2	3	6

### Properties of Functional Dependency:

#### 1. Reflexivity:

If  $X \supseteq Y$  then  $X \rightarrow Y$  is a Functional Dependency.

Ex:- Producer, Hero  $\rightarrow$  Hero

#### 2. Augmentation:

If  $X \rightarrow Y$  then  $XZ \rightarrow YZ$  is a Functional Dependency.

Ex:- Producer  $\rightarrow$  Hero

Producer, Banner  $\rightarrow$  Hero, Banner

#### 3. Transitivity:

If  $X \rightarrow Y$  and  $Y \rightarrow Z$  then  $X \rightarrow Z$  is a Functional

Dependency.

Ex:- Producer  $\rightarrow$  Banner }  $\rightarrow$  Producer  $\rightarrow$  Director.  
 Banner  $\rightarrow$  Director }

4. Union: if  $x \rightarrow y$  and  $x \rightarrow z$  then,  
 $x \rightarrow yz$  is a functional dependency.

Ex:- Producer  $\rightarrow$  Banner }  
Producer  $\rightarrow$  Heroine }

Producer  $\rightarrow$  Banner, Heroine.

5. Decomposition: if  $x \rightarrow yz$  then  $x \rightarrow y$  and  
 $x \rightarrow z$  is a functional Dependency.

Ex:- Producer  $\rightarrow$  Banner, Heroine

Producer  $\rightarrow$  Banner

Producer  $\rightarrow$  Heroine

Closure Set of Functional Dependencies ( $F^+$ ) :-

It is a set of all functional dependencies,  
that can be determined, using the given set F  
of functional dependencies.

Ex:- R(ABC)  $\text{but } F: \{ A \rightarrow B, B \rightarrow C \}$

$$F^+ = \{ A \rightarrow B, B \rightarrow C \}$$

- $A \rightarrow C$  Transitivity
- $A \rightarrow BC$  Union
- $AC \rightarrow BC$  Augmentation
- $AC \rightarrow B, AC \rightarrow C$  decomposition
- $AC \rightarrow A, AC \rightarrow AC$  Reflexivity

Attribute closure :-  $[x^+]$

It is a set of all attributes that can be  
determined using the set (given set) X of

attributes and it is denoted by  $X^+$ .

Ex:-  $\rightarrow R(ABC)$   $F: \{ A \rightarrow B, B \rightarrow C \}$

$$A^+ = \{ A, B, C \} - S.Key, C.K, P.K$$

$$B^+ = \{ B, C \}$$

$$C^+ = \{ C \}$$

$$AB^+ = \{ A, B, C \} - S.Key$$

$$AC^+ = \{ A, C, B \} - S.Key$$

$$BC^+ = \{ B, C \}$$

$$ABC^+ = \{ A, B, C \} - S.Key$$

Q.  $R(ABCDE)$

$$f: \{ AB \rightarrow C, A \rightarrow D, B \rightarrow E, E \rightarrow B \}$$

$$AB^+ = \{ A, B, C, D, E \}$$

$$A^+ = \{ A, D \}$$

$$B^+ = \{ B, E \}$$

$$E^+ = \{ E, B \}$$

$$AE^+ = \{ A, E, D, B \}$$

Applications of Attribute closure :-

1. Additional functional dependencies.
2. Equivalence of functional dependencies.
3. Minimal set of functional dependencies.
4. Keys of a relation (Primary Key, Candidate Key, Foreign Key).

## I. Additional Functional dependencies:-

$$X \rightarrow Y$$

$X^+$  contains Y.

A Functional Dependency  $X \rightarrow Y$ , is possible, if and only if  $X^+ \subset X$  closure) contains Y.

Q. R (ABC) F: {A  $\rightarrow$  B, B  $\rightarrow$  C}

i) IS  $AB \rightarrow C$  possible?

$$AB^+ = A, B, C$$

↑

Yes.  $AB \rightarrow C$  is possible.

ii) IS  $BC \rightarrow A$  Possible?

$$BC^+ = B, C$$

↑

Not possible.

Q1P

(4) A  $\rightarrow$  B ED  $\rightarrow$  A E  $\rightarrow$  F

BC  $\rightarrow$  E EF  $\rightarrow$  G

$$AC^+ = A, C, B, E, F, G$$

(5) a) CF<sup>+</sup> = {C, F, G, E, A, D}.

b) BG<sup>+</sup> = B, G, A, C, D,

c) AF<sup>+</sup> = A, F, E, D

d) AB<sup>+</sup> = A, B, G, D, G

(6) Trivial = subset of CC

$$CD^+ = C, D, E, A, B, C \checkmark \quad BD^+ = B, D, X$$

$$BC^+ = B, C, D, \checkmark \quad AC^+ = A, C, B, D, E \checkmark$$

(8) A<sup>+</sup> = A X

C<sup>+</sup> = J, C, I X

$$AC^+ = A, C, J, I \checkmark \quad AGH^+ = A, G, H, X$$

## 2. Equivalence of Functional Dependency :-

The two sets of functional dependencies  $F$  and  $G$  are said to be equivalent if and only if

$$F^+ = G_1^+$$

[OR]

If every dependency of  $F$  is determined using  $G_1$  and every dependency of  $G_1$  is determined using  $F$  then  $F$  and  $G_1$  are said to be equivalent. i.e.,

Both  $F$  covers  $G_1$  and  $G_1$  covers  $F$  HOLDS.

satisfies

Q.  $F: \{ A \rightarrow B, AB \rightarrow C, D \rightarrow AC \}$

$$G: \{ A \rightarrow BC, D \rightarrow A \}$$

$F$  covers  $G$

$$\bullet A \rightarrow BC$$

$$\bullet A^+ = A, B, C$$

$$D \rightarrow A$$

$$\bullet D^+ = D, A, C$$

$G_1$  covers  $F$

$$A \rightarrow B$$

$$\bullet A^+ = A, B, C$$

$$AB \rightarrow C$$

$$\bullet AB^+ = A, B, C$$

$$D \rightarrow AC$$

$$\bullet D^+ = D, A, B, C$$

$$\therefore F = G_1$$

Q.  $F: \{ A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H \}$

$$G: \{ A \rightarrow CD, E \rightarrow AH \}$$

$F$  covers  $G$

$$A \rightarrow CD$$

$$A^+ = \{ C, A, D, \}$$

$$E \rightarrow AH$$

$$E^+ = E, A, D, C, H$$

$G$  covers  $F$

$$A \rightarrow C$$

$$A^+ \rightarrow A, C, D$$

$$AC \rightarrow D$$

$$AC^+ = A, C, D$$

$$E \rightarrow AD$$

$$E^+ \rightarrow A, H, C, D$$

$$E \rightarrow H$$

$$E^+ \rightarrow A, H, C, D$$

$$\therefore F = G$$

30/P

⑨

F covers G ✓

$$A \rightarrow BC$$

$$A^+ = A, B, C$$

$$D \rightarrow AB \times$$

$$D^+ = D, A, C, B, E$$

only F covers G ✓

G<sub>1</sub> covers F X

$$A \rightarrow B$$

$$A^+ = B, C$$

$$AB \rightarrow C$$

$$AB^+ = A, B, C$$

$$D \rightarrow AC$$

$$D^+ = D, A, B, C$$

$$D \rightarrow E$$

$$D^+ = D, A, B, C$$

⑩

$$C \rightarrow D$$

$$1 \rightarrow 2$$

$$c^+ = c,$$

$$2 \rightarrow 1$$

(b)  $E \rightarrow AH$   
 $E^+ \rightarrow E, H, AD$

$$C \rightarrow D$$

$$c^+ =$$

(c)  $D \rightarrow C$   
 $D^+ =$

$$C \rightarrow D$$

$$c^+ =$$

canonical set / irreducible set :- (OR)

Minimal set of Functional dependency :-

A minimal cover for the set of functional dependencies F is a set of functional dependencies G<sub>1</sub> such that every dependency of F is in the closure of G<sub>1</sub>.

Step-1:- if  $X \rightarrow Y$  is a functional dependency and if both X and Y are present in LHS/RHS of dependency, then remove 'Y'.

case ii)  $[ \begin{array}{l} X \rightarrow Y \\ XY \rightarrow Z \end{array} ]$  case ii

(i)  $Z \rightarrow XY$

$X \rightarrow Y$

$X \rightarrow Z$

(ii)

$X \rightarrow Y$

$Z \rightarrow X$

Step 2:-

Verify each functional dependency such that it can be determined from the remaining set of dependencies, if possible consider such dependency as redundant and can be eliminated.

$X \rightarrow Y$  ✓

$Y \rightarrow Z$  ✓

$X \rightarrow Z$  ✗

Ex:-

$A \rightarrow B$

$AB \rightarrow C$

$D \rightarrow AC$

$A \rightarrow B$

$AB \rightarrow C$

$D \rightarrow AC$

$A \rightarrow B$

$A \rightarrow C$

$D \rightarrow AC$

$A \rightarrow B$

$A \rightarrow C$

$D \rightarrow A$

$A \rightarrow BC$

$D \rightarrow A$

Ex:-

$A \rightarrow B$

$AB \rightarrow C$

$B \rightarrow C$

$A \rightarrow B$

$A \rightarrow C$

$B \rightarrow C$

$A \rightarrow B$

$B \rightarrow C$

30P

(11)

$A \rightarrow B$

$AB \rightarrow C$

$A \rightarrow BC$

$B \rightarrow C$

$A \rightarrow B$

$AB \rightarrow C$

$A \rightarrow BC$

$B \rightarrow C$

$A \rightarrow BC$

$B \rightarrow C$

$A \rightarrow B$

$B \rightarrow C$

(12)

$A \rightarrow B$

$CD \rightarrow A$

$CB \rightarrow D$

$CE \rightarrow D$

$AE \rightarrow F$

$AC \rightarrow D$

$A \rightarrow B$

$A \rightarrow D$

$CD \rightarrow A$

$CB \rightarrow D$

$CE \rightarrow D$

$AE \rightarrow F$

$A \rightarrow BD$

$CD \rightarrow A$

$CB \rightarrow D$

$CE \rightarrow D$

$AE \rightarrow F$

\* If at the right-hand side, only one time repetition, no need to check.  
 If repetition, there is a possibility for deduction.

$$CB \rightarrow D \quad CB^T = CB$$

$$CE \rightarrow D \quad CE^T = CE$$

$$AC \rightarrow D \quad AC^T = ACBD$$

(13)  $\boxed{BCD} \rightarrow A$

$$\begin{matrix} C \rightarrow E \\ E \rightarrow B \end{matrix} \quad \left. \begin{matrix} C \rightarrow E \\ E \rightarrow B \end{matrix} \right\} \quad C \rightarrow B$$

$\therefore B$  can be dropped from X.

(14)  $\boxed{BC} \rightarrow A \quad BC^T = B, C, D, E, F, A$

$$B \rightarrow D \checkmark$$

$$D \rightarrow EF \checkmark$$

$$C \rightarrow F \checkmark$$

$$EF \rightarrow A \checkmark \quad EF^T = E, F,$$

Ans:-  $BC \rightarrow A$  (S)  $X \rightarrow A$

(15) R {v, w, x, y, z}

(a)  $v \rightarrow w \checkmark \quad \textcircled{b} \quad v \rightarrow w$

$$v \rightarrow x \checkmark \quad \cancel{w \rightarrow x}$$

$$y \rightarrow v \checkmark \quad \cancel{y \rightarrow v}$$

$$y \rightarrow z \checkmark \quad \cancel{y \rightarrow z}$$

$$\begin{matrix} v \rightarrow w \\ v \cancel{w} \rightarrow x \\ v \rightarrow y \end{matrix}$$

$$y \rightarrow vx$$

$$y \rightarrow z$$

(16)  $A \rightarrow BC \checkmark \quad \cancel{\frac{A \rightarrow B}{A \rightarrow C}}$

$$ABE \rightarrow CDGH \quad ABET = A, B, E, F, \underline{C}, \underline{G}, D$$

$$C \rightarrow GD \checkmark$$

$$D \rightarrow G \checkmark$$

$$E \rightarrow F \checkmark$$

$A \rightarrow B$   
 $A \rightarrow C$   
 $AE \rightarrow CH$  ←  $AE \rightarrow C$   
 $AE \rightarrow H$   
 $C \rightarrow GD$   
 $D \rightarrow G$   
 $E \rightarrow F$

$A \rightarrow B$   
 $A \rightarrow C$   
 $AE \rightarrow C$  AE → C  
 $AE \rightarrow H$   
 $C \rightarrow G$   
 $C \rightarrow D$   
 $D \rightarrow G$   
 $E \rightarrow F$

$A \rightarrow B$   
 $A \rightarrow C$   
 $AE \rightarrow CH$   
 $C \rightarrow D$   
 $D \rightarrow G$   
 $E \rightarrow F$

18-10-19

### Finding the Keys of a Relation:-

If  $x + (x \text{ closure})$  contains all the attributes of a relation, then  $x$  is called Super-Key of the relation. If  $x$  is minimal set, then  $x$  is called candidate Key of the relation. We can

Select Primary Key among candidate Keys.

If  $x \rightarrow Y$ , then  $x$  may become Key.

Ex:- R (ABC)  $F : \{A \rightarrow B, B \rightarrow C\}$

$A^+ = A, B, C \rightarrow S.K$

$B^+ = B, C \times$

candidate Key : A

$\begin{cases} \phi = A \\ B = AB \\ C = AC \\ BC = ABC \end{cases}$

$(B, C) \rightarrow 2^2 = 4 S.K$

R (ABCD)  $AB \rightarrow C, B \rightarrow D$

$AB^+ = A, B, C, D \checkmark \rightarrow S.K$

~~$c^+ = C \times B^+ = B, D$~~

candidate Key = AB

$\begin{cases} \phi = AB \\ C = ABC \\ D = ABD \\ CD = ABCD \end{cases}$

$(C, D) \rightarrow 2^2 = 4 S.K$

③ R (ABCD) F:  $\{AB \rightarrow C, A \rightarrow B, B \rightarrow D\}$

$$AB \rightarrow C$$

$$AB^t = A, B, D, C \checkmark . SK$$

$$A \rightarrow B$$

$$A^t = A, \cancel{B}, C, D \checkmark SK$$

$$B \rightarrow D$$

$$B^t = BD \times "AB" \text{ cannot be a c.k.}$$

Candidate Key :- A

$$\begin{aligned} \emptyset &= A \\ B &= AB \\ C &= Ac \\ D &= AD \\ BC &= ABC \\ BD &= ABD \\ CD &= ACD \\ BCD &= ABCD \end{aligned} \quad \left. \begin{array}{l} \text{→ 8 Super Keys} \\ \text{→ } B, C, D \end{array} \right\} \rightarrow 2^3 = 8 S.K.$$

If the relation contains one candidate key,  
then the no. of Super keys are equals to  
2 power of remaining keys.

④ R: (ABCD) F:  $\{AB \rightarrow C, BC \rightarrow AD\}$

$$AB \rightarrow C$$

$$AB^t = A, B, C, D$$

$$BC \rightarrow AD$$

$$BC^t = B, C, A, D$$

Candidate Key = AB

$$\begin{aligned} AB &\leftarrow \begin{array}{l} \emptyset - AB \\ C - ABC \\ D - AB \cancel{D} \\ CD - ABCD \end{array} \end{aligned}$$

Candidate Key = BC

6 Super-Keys

$$\begin{aligned} BC &\leftarrow \begin{array}{l} \emptyset - BC \\ A - ABC \\ B - BCD \\ AD - ABCD \end{array} \end{aligned}$$

∴ "8" S.K 2 repeated.

(5) R(ABC) F: {AB → C, C → AB}

$$AB \rightarrow C$$

$$AB^+ = A, B, C$$

$$C^+ = C \cdot K$$

$$\begin{array}{c} \phi \rightarrow C \\ C \leftarrow A \rightarrow CA \\ C \leftarrow B \rightarrow CB \end{array}$$

$$AB \rightarrow CAB$$

$$C \rightarrow AB$$

$$C^+ = C, A, B$$

$$AB^+ = C \cdot K$$

$$AB \begin{cases} \phi \rightarrow AB \\ C \rightarrow ABC \end{cases}$$

∴ No. of Super Keys = 5.

$$\begin{array}{c} X \quad Y \\ AB \quad C \end{array} = XUY = ABUC = 2^2 + 2^2 - 2^0 = 5 \checkmark$$

$$(4) \quad \begin{array}{c} AB \quad BCD \end{array} = ABUCD = 2^2 + 2^2 - 2^1 = 6 \checkmark$$

(6) {A → B, B → C, C → A} R(ABC)

$$A \rightarrow B$$

$$A^+ = A, B, C$$

$$B \rightarrow C$$

$$B^+ \rightarrow B, C, A$$

$$C \rightarrow A$$

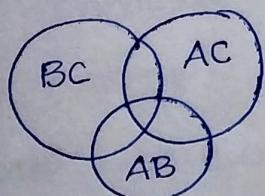
$$C^+ = C, A, B$$

A, B, C are candidate keys.

$$\begin{array}{c} A \rightarrow \phi \rightarrow A \\ A \rightarrow B \rightarrow AB \\ A \rightarrow C \rightarrow AC \\ A \rightarrow BC \rightarrow ABC \end{array}$$

$$\begin{array}{c} B \rightarrow \phi \rightarrow B \\ B \rightarrow A \rightarrow AB \\ B \rightarrow C \rightarrow BC \\ B \rightarrow AC \rightarrow ABC \end{array}$$

$$\begin{array}{c} C \rightarrow \phi \rightarrow C \\ C \rightarrow A \rightarrow AC \\ C \rightarrow B \rightarrow BC \\ C \rightarrow AC \rightarrow ABC \end{array}$$



$$\rightarrow (BC \cup AC \cup AB)$$

$$= 2^2 + 2^2 + 2^2 - 2^1 - 2^1 - 2^1 + 2^0$$

$$= 4 + 4 + 4 - 6 + 1 = 7. S.K$$

(7) R(ABCD) F: {A → B, B → C}

$$A \rightarrow B$$

$$A^+ = A, B, C \times$$

$$B \rightarrow C$$

$$B^+ = B, C \times$$

For a relation, there is atleast one candidate Key.

So, we need to find the independent Attributes [Those are not present at RHS]

In this problem, A & D not present at R.H.S  
So, they cannot be derived from other attributes. So, they can be derived from themselves.

$$\frac{AD^+}{C \cdot K} = A, D, B, C$$

⑧  $R: (ABCD) \quad F \{A \rightarrow B, C \rightarrow D\}$

$$A \rightarrow B \quad C \rightarrow D$$
$$A^+ = AB \quad C^+ = CD$$

$$AC^+ \rightarrow C \cdot K$$

⑨  $R: (ABCDE) \quad F \{AB \rightarrow C, BD \rightarrow E, ED \rightarrow B\}$

$$AB \rightarrow C \quad BD \rightarrow E \quad ED \rightarrow B$$

$$AB^+ = A, B, C, X \quad BD^+ = B, D, E, X \quad ED^+ = E, D, B, X$$

(A, D)  $\rightarrow$  Independent

$$AD^+ = A, D, X$$

along with Independent add other attributes.

$$ABD^+ = A, B, D, C, E \quad \checkmark$$

$$ACD^+ = A, C, D,$$

$$AED^+ = A, E, D, B, C \quad \checkmark$$

⑩ R (ABCDE)

$$F: \{ A \rightarrow B, CD \rightarrow E, CE \rightarrow D \}$$

$$A \rightarrow B$$

$$A^+ = A, B$$

$$CD \rightarrow E$$

$$CD^+ = C, D, E,$$

$$CE \rightarrow D$$

$$CE^+ = C, E, D$$

Independent terms: A, C

$$AC^+ = A, C, B,$$

$$ABC^+ = A, B, C,$$

$$ADC^+ = A, D, C, E, B \checkmark$$

$$AEC^+ = A, E, C, B, D \checkmark$$

∴ Candidate Keys = ACD, ACE.

⑪ R(ABC) F:  $\{ AB \rightarrow c, c \rightarrow A \}$

$$AB \rightarrow c$$

$$AB^+ = A, B, C$$

$$c \rightarrow A$$

$$c^+ = c, A$$

$$C \cdot K = AB$$

attributes of candidate Key = Prime attributes.

$AB \rightarrow$  attributes  $\rightarrow A, B, c$

See, whether they are at RHS. If present Replace  
with LHS

$c \rightarrow A$  So, Replace "A" with "c"

∴ Candidate Keys are AB, CB.

⑫ R (ABCDE) F:  $\{ AB \rightarrow c, B \rightarrow D, c \rightarrow AE \}$

$$D \rightarrow B, c \rightarrow A$$

$$AB^+ = A, B, C, D, E \checkmark$$

$$B^+ = B, D,$$

$$c^+ = C, A, E$$

Candidate Key = AB

Check prime attributes

$$D \rightarrow B$$

$$c \rightarrow A$$

$$AB$$

$$AD$$

$$CB$$

$$CD$$

Q. R(A,B,C,D,E) F: { AB → C, B → D, C → AE }

$$AB^+ = A, B, C, E, D$$

$$B^+ = B, D$$

$$C^+ = A, E, C$$

$$C \cdot K = AB \checkmark \quad C \rightarrow AE$$
$$= CB \checkmark$$

31/P

(17) R(A,B,C,D,E,F)

Candidate Keys = AB, AE

$$= 2^4 + 2^4 - 2^3 = 16 + 16 - 8 \\ = 24$$

(18) R(A,B,C,D,E)

Candidate Keys = AB, BC, CD

$$= 2^3 + 2^3 + 2^3 - 2^2 \\ - 2^2 - 2^1 + 2^1 \\ = 8 + 8 + 8 - 4 - 4 + 2 = 16$$

(19) R = (A,B,C,D,E,F)

$$C \rightarrow F \quad E \rightarrow A \quad EC \rightarrow D \quad A \rightarrow B$$

$$C^+ = C, F$$

$$E^+ = E, A$$

$$E^+ = E, C, A, B, D$$

∴ Candidate Key (Key) = EC.

Shortcuts - To see Given attributes are present in RHS

$$C \rightarrow F \checkmark$$

$$E \rightarrow A \checkmark$$

$$EC \rightarrow D \checkmark$$

$$AB \rightarrow B \checkmark$$

among these "C" and "E" are not present

∴ check the options. "Ec" is correct.

(20) In the given dependencies "ABC, CD, K" are not there.

So, ABCDK is answer.

(21) A, D are not there But AD present in (a) & (d)

So, check closure for both of them. So, ABD is correct.

(22) "A, H" both are present in (b) so, ACEFH is ans.

(23) To see "EH" common, so, option (c) is correct.

(24)  $F = \{CH \rightarrow G\}$  ABCDEF GH

$$A \rightarrow BC$$

$$CH^+ = C, H, G, X$$

$$B \rightarrow CFH$$

$$AT = A, B, C, F, H, E, G, X$$

$$E \rightarrow A$$

$$B^+ = B, C, F, H, E, G, X$$

$$F \rightarrow EG$$

$$E^+ = X$$

$$F^+ = X$$

$$AD^+ = A, D, B, C, F, H, E, G \checkmark$$

BD  
CD  
ED  
FD  
GD  
HD

Candidate Key : AD

$$E \rightarrow A \quad ED$$

$$F \rightarrow EG \quad FD$$

$$B \rightarrow CFH \quad BD$$

$$A \rightarrow BC \quad AD$$

Repeated.

∴ No. of candidate Keys = 4.

$$(25) AB^+ = A, B, C, D, E, F$$

$$F^+ = F, C, A, B, D, E$$

$$C^+ \times B^+ \times D^+ \times F^+$$

Candidate Key: F, AB

$$C \rightarrow A \quad CB$$

$$F \rightarrow C \quad FB \rightarrow \text{No use Because } F \text{ is}$$

already candidate key.

$\therefore$  No. of candidate Keys = 3.

32/p  
(26)

$$A^+ = A, C, E,$$

$$AB^+ = A, B, D, C, E \checkmark$$

$$B^+ = B, D,$$

$$C^+ = C, E, A, C$$

$$E^+ = E, A, C$$

$$D^+ = D, B,$$

$$AB$$

$$(E \rightarrow A)$$

$$EB$$

$$(D \rightarrow B)$$

$$ED$$

$$(C \rightarrow E)$$

$$AD$$

$$CD$$

$$CB$$

(28)

✓ CK  $\rightarrow$  all attributes

~~X~~ CK part of C.K

$\rightarrow$  all attributes

✓ S.K  $\rightarrow$  all attributes.

ABH is super key because

AH is candidate key.

$\therefore$  No. of candidate Keys = 2

D, AH.

$$ABH \leftarrow \begin{array}{l} (A \rightarrow BC) \\ (A \rightarrow B) \\ (A \rightarrow C) \end{array}$$

$$AAH$$

$$AH$$

$$CD^+ = C, D, E, X$$

$$E^+ = E, C, X$$

$$D^+ = D, A, E, H, B, C \checkmark$$

$$ABH^+ = A, B, H, D, C, E \checkmark$$

$$DH^+ = D, H, B, C$$

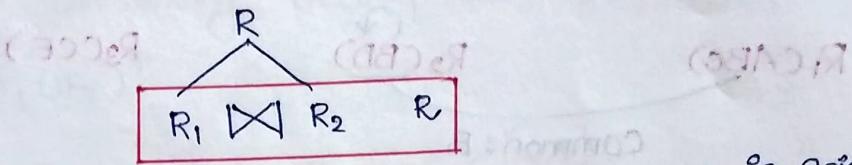
super keys because

D  $\rightarrow$  candidate key.

## \* Properties of decomposition \*

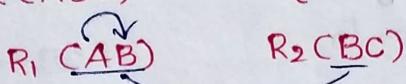
### 1. Lossless join decomposition:-

The decomposition of R into  $R_1$  and  $R_2$  is said to be lossless if and only if  $R_1 \text{ join } R_2 = R$ .



The decomposition of R into  $R_1$  and  $R_2$  is said to be lossless if and only if the attribute common to  $R_1$  and  $R_2$ , must be a key in either  $R_1$  or  $R_2$  or both.

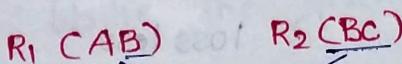
Ex:-  $R(ABC)$   $F: \{A \rightarrow B\}$



Common: B

Key: A      B is not a Key      it is lossy.

②  $R(ABC)$   $F: \{B \rightarrow C\}$

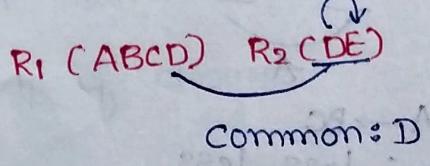


Common: B

Key: B  $\rightarrow$  it is lossless.

③  $R(ABCDE)$

$F: \{AB \rightarrow C, B \rightarrow D, D \rightarrow E\}$

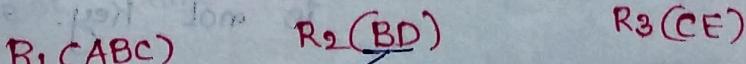


Common: D

D is Key in  $R_2$

$\therefore$  it is lossless.

④  $R(ABCDE)$   $F: \{AB \rightarrow C, B \rightarrow D, C \rightarrow E\}$



Common: B

B is Key in  $R_2$ .

$R_{12} (ABCD)$        $R_3 (CCE)$

common

'c' is Key in  $R_3$

$\therefore$  it is lossless.

(5)  $R (ABCDE)$        $F = \{ AB \rightarrow C, B \rightarrow D, E \rightarrow C \}$

$R_1 (ABC)$

$R_2 (BD)$

$R_3 (CCE)$

Common: B

$R_{12} (ABCD)$

$R_3 (CE)$

Common: C

"C" is not Key in  $R_3$

$\therefore$  it is lossy.

30P  
①

$R (ABCD)$

(1)  $B \rightarrow C, D \rightarrow A$

BC

AD

No - Common

attribute  $\therefore$  lossy

decomposition.

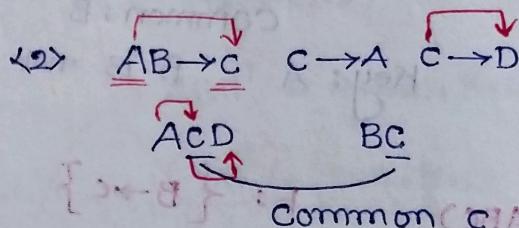
(2)  $A \rightarrow BC, C \rightarrow AD$

ABC, AD

A

$A \rightarrow BC$

$\therefore$  loss-less



(3)  $A \rightarrow B, B \rightarrow C, C \rightarrow D$

AB      AD      CD

A

(ABD)

(CD)

D

"not loss-less".

Because A is Key in

Both  $R_1$  and  $R_2$  But "D" is not Key. It is

lossy.

## 2. dependency Preserving decomposition:-

The decomposition of R with dependencies F into  $R_1$  and  $R_2$  (is said to be dependent) with dependencies  $F_1$  and  $F_2$  respectively is said to be dependency preserving decomposition if and only if

$$F^+ = (F_1 \cup F_2)^+$$

Q.  $R(ABC)$   $F: \{A \rightarrow B, B \rightarrow C, C \rightarrow A\}$

$R_1(AB)$   $R_2(BC)$

$R_1(AB)$

$A \rightarrow B$

$B \rightarrow A$

$B^+ = BCA$

$R_2(BC)$

$B \rightarrow C$

$C \rightarrow B$

$C^+ = CAB$

$C \rightarrow A$ ;  $C^+$  using  $(F_1 \cup F_2)$

$C^+ = CBA$

$\therefore$  decomposition is dependency Preserving.

Step 1:- Write the given dependencies Based on Relation.

Step 2:- if for any relation, the dependency is not given check whether indirect relation is present or not with the help of closures.

Step 3:- After finding all, check whether the remaining are obtained indirectly or not, if not came i.e., decomposition preservation is not satisfied. otherwise decomposition Preservation is possible.

(2)  $R(ABCD)$   $F = \{A \rightarrow B, B \rightarrow A, AC \rightarrow D\}$

$R_1(CAB)$

$F_1: A \rightarrow B$

$B \rightarrow A$

$A^+ = A, B$

$B^+ = B, A$

$R_2(CBD)$

$F_2: BC \rightarrow D$

$B^+ = B, A \times$

$C^+ = C \times$

$D^+ = D \times$

$\underline{BC^+ = A, B, C, D} \checkmark$

$\underline{CD^+ = C, D}$

$\underline{BD^+ = B, D, A}$

Now, find the dependency  $AC \rightarrow D$

$AC^+ = A, C, B, D$

$\therefore$  decomposition preservation possible.

391P

(2)  $R(ABCD)$

$A \rightarrow B \quad B \rightarrow C \quad C \rightarrow D \quad D \rightarrow B$

$R_1(CAB)$

$R_2(CBC)$

$R_3(CBD)$

$A \rightarrow B$

$B \rightarrow C$

"B common"

$\therefore$  lossless join

$R_{12}(ABC)$   $R_3(CBD)$

B common

$A \rightarrow B$

$B \rightarrow C$

$\therefore$  lossless join.

Better to do first check D.P.

$$\begin{array}{lll}
 R_1(CAB) & R_2(CBC) & R_3(BD) \\
 F_1: A \rightarrow B & F_2: B \rightarrow C \\ 
 & C \rightarrow B & F_3: B^+ = \underbrace{B, C, D} \checkmark \\
 B^+ = BCD & C^+ = CDB & D^+ = D, B, C \\
 & & B \rightarrow D \\
 & & D \rightarrow B \checkmark
 \end{array}$$

$$C \rightarrow D ; \quad C^+ \text{ using } (R_1 \cup R_2 \cup R_3)$$

$$C^+ = C, B, D$$

$\therefore$  dependency preserving ~~not~~ satisfied.  $\checkmark$

②  $AB \rightarrow C, C \rightarrow A, C \rightarrow D$

$$\begin{array}{ll}
 R_1(CACD) & R_2(CB \rightarrow C) \\
 F_1: C \rightarrow A & B^+ = B X \\
 C \rightarrow D & C^+ = A, C, D X
 \end{array}$$

$$(AB \rightarrow C): \quad AB^+ = (F_1 \cup F_2) \\ = A, B,$$

$\therefore$  Not decomposition Preservivence.

③  $A \rightarrow BC, C \rightarrow AD$

$$\begin{array}{ll}
 R_1(CABC) & R_2(AD) \\
 A \rightarrow BC & A^+ = \overbrace{A, B, C, D} \checkmark \quad A \rightarrow D \\
 C^+ = C, \underline{A}, \underline{D}, \underline{B} & D^+ = D,
 \end{array}$$

$$(C \rightarrow AD): \quad C^+ = (F_1 \cup F_2)$$

$$\Rightarrow C^+ \Rightarrow \begin{array}{l} C \rightarrow AB \\ A \rightarrow BC \\ A \rightarrow D \end{array}$$

$$C^+ = C, \underline{A}, \underline{B}, \underline{D}$$

$\therefore$  decomposition Preservivence.

$$\textcircled{4} \quad A \rightarrow B \quad B \rightarrow C \quad C \rightarrow D$$

$$R_1(AB) \quad R_2(AD) \quad R_3(CD)$$

$$A \rightarrow B \quad A \rightarrow D \quad C \rightarrow D$$

$$B^+ = B, C, D$$

$$A^+ = A, B, C, D$$

$$C^+ = C, D$$

$$D^+ = D$$

$$(B \rightarrow C) : \quad B^+ = (F_1 U F_2 U F_3) \\ = B,$$

$\therefore$  Not decomposition Preservence.

### Normal Forms

1NF :-

One - NF was designed to disallow multi-valued attributes, composite attributes and their combinations. i.e., 1NF allows only atomic values [The attribute of a tuple must be either one value or NULL value].

Unnormalized Relation

↓ Remove MVA, Composite Attributes  
1NF (Only Atomic Values)

Ex:- Professor : Table Name

SSN	Name	Professor Salary
1	Sai	{ 40K, 30K, 20K, 10K }

Unnormalized Relation

SSN	Name	Basic	TA	DA	HRA
1	Sai	40K	30K	20K	10K

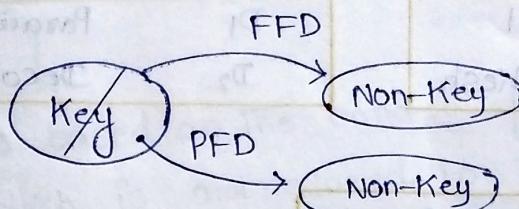
} INF

Note:-

Every relation in the Relational Model must be in 1NF.

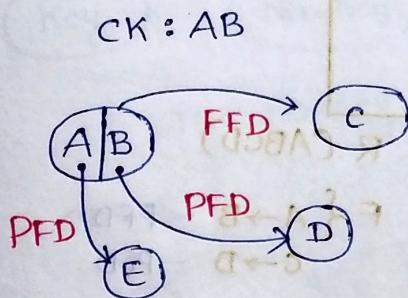
## 2-NF:-

It is based on the concept of full functional dependencies and not allows Partial functional dependencies.



PFD:- Part of Key  $\rightarrow$  Non-Key.

Ex:-



1NF  
Remove PFD  
2NF (only FFD).

definition:- A relation schema "R" is in 2NF, if no non-prime attribute of R is partial functional dependency on Primary Key of R.

Medication

Patient No	Drug No	No. of Units	Pname	Dname
P <sub>1</sub>	D <sub>1</sub>	10	Anil	Paracetomal
P <sub>1</sub>	D <sub>2</sub>	20	Anil	decold
P <sub>2</sub>	D <sub>2</sub>	15	Mukesh	decold

FFD:- Patient No, DrugNo  $\rightarrow$  No. of units

PFD:- Patient NO  $\rightarrow$  Pname

PFN:- drugNO  $\rightarrow$  Dname

CK:- (PatientNO, DrugNo)

Medication is in 1NF but not in 2NF.

Patient	
Patient No	Pname
P <sub>1</sub>	Anil
P <sub>2</sub>	Mukesh

Drug	
DrugNo	DrugName
D <sub>1</sub>	Paracetamol
D <sub>2</sub>	Decold

PatientNo	DrugNo	No.of units
P <sub>1</sub>	D <sub>1</sub>	10
P <sub>1</sub>	D <sub>2</sub>	20
P <sub>2</sub>	D <sub>2</sub>	15

Q. R (ABCDE)

F { AB  $\rightarrow$  C }

PFD A  $\rightarrow$  D

PFD B  $\rightarrow$  E }

CK: AB

R is in 1NF

(A, D) R<sub>1</sub>

(B, E) R<sub>2</sub>

(A, B, C) R<sub>3</sub>

Q. R (ABC)

F: { A  $\rightarrow$  B, B  $\rightarrow$  C }

A<sup>t</sup> = A, B, C

B<sup>t</sup> = B, C

Q. R (ABCD)

F { A  $\rightarrow$  B } PFD  
C  $\rightarrow$  D PFD

A<sup>t</sup> = A, B

C<sup>t</sup> = C, D

A<sup>t</sup>C<sup>t</sup> = A, B, C, D

CK = AC

R is in 1NF.

(A, B) R<sub>1</sub>

(C, D) R<sub>2</sub>

(AC) R<sub>3</sub>

"C.K = A"

$\therefore B \rightarrow C$  is't in PD

Because  $A \rightarrow B$   
 $B \rightarrow C$

indirectly  $A \rightarrow C$

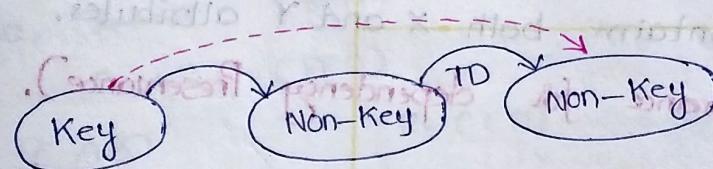
$\therefore$  No Partial dependency.

$\therefore R$  is in 2NF and also in 1NF.

A relation that contains a Key with only one attribute is always in 2NF.

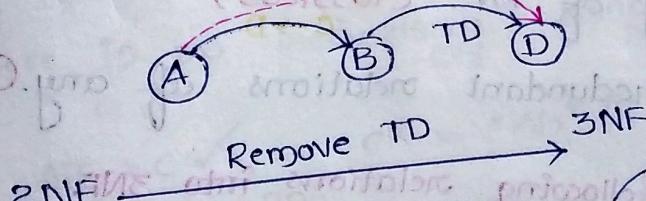
### 3NF

3NF is based on the concept of Transitive dependencies and which is not allowed in 3NF.



TD: NonKey → Non-Key

Ex- CK: A



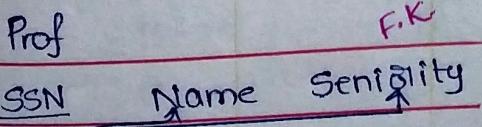
Ex- ① Professor (SSN, Name, Seniority, Salary)

FD: SSN → Name, Seniority

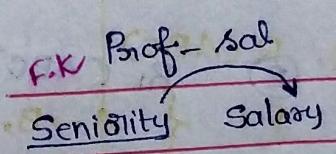
TD: Seniority → Salary

CK: SSN

Professor is in 2NF, but not in 3NF.



3NF

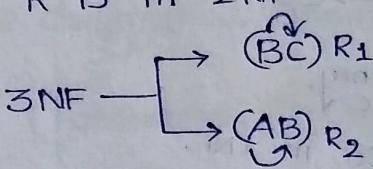


Q:-  $R(ABC)$   $F: \{A \rightarrow B, B \rightarrow C\}$

TD

CK = A

R is in 2NF



Algorithm: Lossless join and dependency preserving decomposition into 3NF.

Step 1:- find minimal set  $G_1$  for F.

Step 2:- For each functional dependency of the form  $X \rightarrow Y$  create a separate relation that contains both X and Y attributes.  
(assurance for dependency preservation).

Step 3:- If no decomposition relation contains key of R, then create a separate relation contains key of R. (lossless)

Step 4:- Remove redundant relations (if any). (3NF)

Q. Decompose the following relations into 3NF.

Sol:-  $R(ABCD)$   $F: \{AB \rightarrow C, A \rightarrow B, B \rightarrow D\}$

①  $G_1: \rightarrow$  find

$$\begin{array}{l} AB \rightarrow C \\ A \rightarrow B \\ B \rightarrow D \end{array} \xrightarrow{\quad} \begin{array}{l} A \rightarrow C \\ A \rightarrow B \\ B \rightarrow D \end{array} \xrightarrow{\quad} \begin{array}{l} A \rightarrow BC \\ B \rightarrow D \end{array} \xrightarrow{\quad} \begin{array}{l} A \rightarrow BC \\ B \rightarrow D \end{array}$$

TD

CK: A

R is in 2NF

②  $R_1(ABC)$   $R_2(BD)$

② R (ABCDE) F: { AB → C, B → D, D → E }

find minimal set

$$AB \rightarrow C$$

$$B \rightarrow D \text{ -PFD}$$

$$\underline{D \rightarrow E} \quad \text{TD}$$

$$CK = "AB"$$

"R" is in 1NF.

$$R_1 (ABC)$$

$$R_2 (BD)$$

$$R_3 (DE)$$

③ R (ABCD) F: { A → B, C → D }

$$A \rightarrow B$$

$$C \rightarrow D$$

$$A^+ = A, B$$

$$C^+ = C, D$$

$$AC^+ = A, C, B, D$$

$$CK = "AC"$$

$$\text{So, } A \rightarrow B \quad (\text{PFD})$$

$$C \rightarrow D \quad (\text{PFD})$$

$$R_1 (AB)$$

$$R_2 (CD)$$

$$R_3 (AC)$$

as "AC" is present in either of them? No.

$$\text{So, } R_1 (AB)$$

$$R_2 (CD)$$

$$R_3 (AC)$$

Hence it is in 3NF.  
There is no redundancy.

3NF

$$AB \rightarrow C \checkmark$$

$$CK = "AB"$$

$$A \rightarrow DE \text{ (CPD)}$$

$$B \rightarrow F \text{ (CPD)}$$

$$F \rightarrow GH$$

$$D \rightarrow IJ$$

$$R_1 (ABC) \quad R_2 (ADE) \quad R_3 (BF), R_4 (FGH), R_5 (DIJ)$$

④

$$AC \rightarrow F$$

$$A \rightarrow B \text{ (CPD)}$$

$$B \rightarrow E$$

$$C \rightarrow D \text{ (CPD)}$$

$$AC^+ = A, C, F, B, E, D$$

$$A^+ = A, B, E$$

$$B^+ = B, E,$$

$$C^+ = C, D$$

$CK = "AC"$

3NF  $R_1(ACF)$   $R_2(AB)$   $R_3(BE)$   $R_4(CD)$

(5)	$A \rightarrow FC$	$A \rightarrow F$ (PD)
	$C \rightarrow D$	$A \rightarrow C$ (PD)
	$B \rightarrow E$	$C \rightarrow D$
		$B \rightarrow E$ (PD)

$$AB^+ = A, F, C, E, D$$

$$\therefore CK = AB$$

$R_1(ACF)$   $R_2(CD)$   $R_3(BE)$   $R_4(AB)$

Q. What is the NF for the following Relation.

$R(ABC)$   $F: \{AB \rightarrow C, C \rightarrow A\}$

$$AB \rightarrow C \quad AB^+ = A, B, C$$

$$C \rightarrow A \quad \therefore CK = AB, CB$$

No partial dependency. So, it is in 2NF

No Transitive dependency. So, it is in 3NF.

definition:- A relation schema R is in 3NF,

if whenever a non-trivial functional dependency of the form  $X \rightarrow A$  holds, then either X is superkey and A is Prime Attribute.

If all the attributes in a relation must be prime attributes, then that relation is in 3NF.

BCNF (Boyce-Codd NF) :-

It is a modification of 3NF and removes the problem of prime transitivity.

definition:- A relation schema R is in BCNF if whenever a non-trivial functional dependency of the form  $X \rightarrow A$  holds, then X is a superkey of R.

Q. R(ABC) F:  $\{\overline{AB} \rightarrow C, \overline{C} \rightarrow AB\}$

CK: AB, C

R is in BCNF also in 3NF

② R(ABC) F:  $\{\overline{AB} \rightarrow C, C \rightarrow \overline{A}\}$

CK: AB, CB

R is in 3NF but not in BCNF

BCNF  $\begin{cases} \rightarrow R_1(CA) \\ \rightarrow R_2(BC) \end{cases}$

Lost: -  $(\overline{AB} \rightarrow C)$

### Note:-

1. The dependency preserving decomposition into BCNF may not be possible always.

2. Every relation in BCNF is also in 3NF but need not vice-versa.

3. A Relation with two attributes is always in BCNF.

### Proof:-

$F_1: \overline{A} \rightarrow B$

CK: A

B

$F_2: \overline{B} \rightarrow A$

CK: B

C

$F_3: \overline{A} \rightarrow B$   
 $\overline{B} \rightarrow A$

CK: A

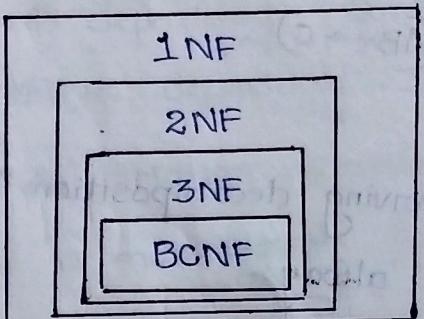
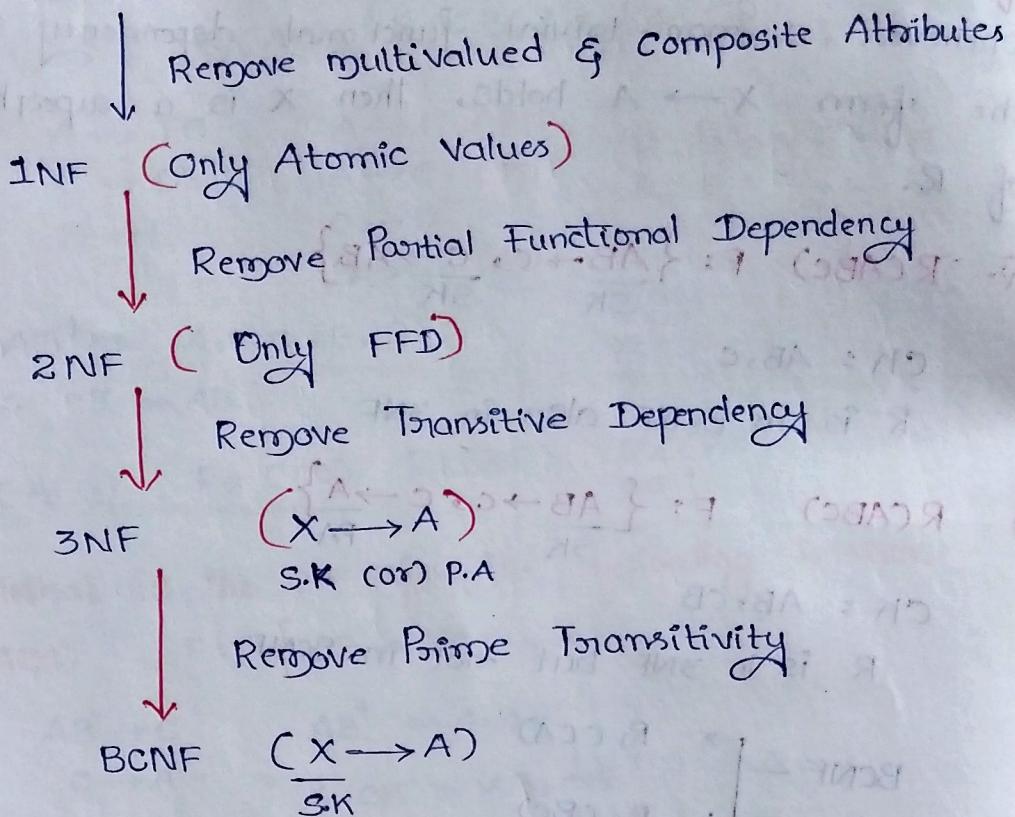
N

$\overline{SK}$

B

F

## Un-Normalized Relation



3alP

⑥ 1)  $C \rightarrow D, C \rightarrow A, B \rightarrow C$   
 $TD$  (Non-Key  $\rightarrow$  Non-Key)  
 $CK = B \rightarrow 2NF$

2)  $CK = BD$  1NF

3)  $CK = ABC$  3NF  
 $DBC$

4)  $A \rightarrow B, BC \rightarrow D, A \rightarrow C$   
 $TD$  2NF  
 $CK = A$

5)  $AB \rightarrow C, AB \rightarrow D, C \rightarrow A, D \rightarrow B$

$CK = AB, CB, AD, CD$

all are prime Attributes  $\rightarrow 3NF$

391P  
7 $R_1 (ABC)$  $R_2 (BD)$  $C.K = A, B, C, D \rightarrow (BCNF) \rightarrow BCNF$ 

The decomposition of BCNF turn into BCNF  
not into lower NF.

8

 $\frac{Y (PR)}{R \rightarrow P}$ 

BCNF

 $Z (QRS)$ 
 $\frac{QR \rightarrow S}{S \rightarrow Q}$ 
D.P  
Lossless

CK: QR, SR

3NF (All are prime Attributes).

9

 $c_i \rightarrow F, L \quad PFD$  $o_i \rightarrow D, C, Z \quad PFD$ CK:  $(c_i, o_i)$ 

1NF but not in 2NF

10

(VNSETYP)

 $VNSE \rightarrow T$  $VN \rightarrow Y$  $VNSE \rightarrow P$ 

21.10.19

## Relational Algebra:-

It is a procedural language, in which each query describes the step by step procedure for computing the desired result. i.e., the relational algebra expression represents Query Evaluation

Plan.

### 1. Selection Operator: ( $\sigma$ )

It is used to select "Rows".